S.F. Amirov, M.S.Yoqubov, N.G’. Jabborov, X.A. Sattarov, N.E. Balgaev.

ELEKTROTEXNIKANING NAZARIY ASOSLARIDAN MASALALAR TO‘PLAMI

O’zbekiston Respublikasi Oliy va o’rta maxsus ta’lim vazirligi tomonidan
5310200 – Elektr energetikasi (tarmoqlar va yo‘nalishlar bo‘yicha);
5310700 – Elektr texnikasi, elektr mexanikasi va elektr texnologiyasi (tarmoqlar bo‘yicha);
5310800 – Elektronika va asbobsozlik (tarmoqlar bo‘yicha);
5311000 – Texnologik jarayonlar va ishlab chiqarishni avtomatlashtirish (tarmoqlar bo‘yicha);
5311100 – Radioelektron qurilmalar va tizimlar (tarmoqlar bo‘yicha);
5430200 – Qishloq xo‘jaligini elektrlashtirish va avtomatlashtirish
ta’lim yo’nalishlari bakalavriyat talabalari uchun o’quv qo’llanma sifatida tavsiya etilgan.

,,Adabiyot uchqunlari“
Toshkent – 2015

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O'quv qo'llanmada "Elektrotexnikaning nazariy asoslari" fanining barcha qismlariga tegishli bo'lgan tayanch tushunchalar, atamalar, qisqacha nazariy ma'lumotlar, qoidalar, misol va masalalar hamda shaxsiy hisob-grafik ishlarga oid mustaqil vazifalar keltirilgan. Kitob energetika, elektrotexnika, radiotexnika, avtomatika, elektr aloqa va qishloq xo'jaligini elektrlashtirish va avtomatlashtirish sohalariga oid ta'lim yo'nalishlari talabalari uchun mo'jallangan. Undan ishlab chiqarish sohalarining mutaxassislari, muhandislar, magistrantlar va ilmiy-texnik hodimlar ham foydalanishlari mumkin.

Mas'ul muharrir: texnika fanlari doktori, professor **Amirov S.F.**

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O'zbekiston Respublikasi oliy va o'rta maxsus ta'lim vazirligi 5310200, 5310700, 5310800, 5311000, 5311100, 5430200 ta'lim yo'nalishlari talabalari uchun o'quv qo'llanma sifatida tavsiya etgan (Grif № 484 -049).

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Kirish so'zi

Elektotexnika, elektroenergetika va elektr ta'minoti bakalavriat yo'nalishida ta'lim olayotgan talabalarga amaliyot va mustaqil mashg'ulotlarga mo'ljallangan o'quv qo'llanma "Elektrotexnikaning nazariy asoslari" (ENA) fanining Oliy va o'rta maxsus ta'lim vazirligi tomonidan 2014- yili tasdiqlangan namunaviy dastur talablari mavzulariga, mazmun-mohiyatiga va belgilangan hajmiga mos ravishda tuzilgan.

Ma'lumki, har qanday fanlarning o'zlashtirilganlik mezoni - bu ularga oid misol va masalalarni yecha olishdir. Masala yechish nazariy bilimlarni yanada chuqurroq tushunishga hamda mustahkamlashga imkon beradi, chuqur ta'lim olish va fikrlash qobiliyatini oshirishga xizmat qiladi.

Shu nuqtayi nazardan qo'llanmada ENA fanining barcha qismlariga oid to'la yechilgan misol va masalalar, mustaqil bajarish uchun vazifalar keltirilgan. Har bir bob boshida qisqacha nazariy ma'lumotlar va qoidalar bayon etilgan.

Bakalavr-elektrikda bilim va ko'nikma elektr zanjirlarni samarali yechish usullarini tanlab olish, hisoblash tenglamalarini tuzish va hosil bo'lgan yechimlarning to'g'riligini o'zlashtirish, o'zgarmas tok zanjirlarini hisoblash jarayonida shakllanadi. Ana shu maqsadda o'zgarmas tok zanjirlariga tegishli bobda masala yechishning asosiy usullariga alohida e'tibor berilgan.

O'quv qo'llanmaga kiritilgan misol, masalalar va mustaqil vazifalar Toshkent temir yo'l muhandislari instituti, Toshkent irrigatsiya va melioratsiya institutining tegishli yo'nalishlarida ta'lim olayotgan talabalar bilan ENA fani amaliy va mustaqil mashg'ulotlarida foydalanildi va ijobiy baholandi.

Mualliflar o'quv qo'llanmaning dastlabki nusxasini o'qib chiqib, uning sifatini yaxshilash bo'yicha o'z fikr-mulohazalarini bildirgan Toshkent temir yo'l muhandislari instituti "Elektr aloqa va radio" kafedrasi professori, texnika fanlari doktori A.A. Holiqovga, Toshkent Davlat texnika universiteti "Elektrotexnika va kasb ta'limi(energetika)" kafedrasi mudiri texnika fanlari nomzodi, dotsent Sh.E. Begmatovga va "Nazariy elektrotexnika va elektron texnologiyalar" kafedrasi mudiri texnika fanlari nomzodi, dotsent Q.G'. Abidovga samimiy minnatdorchilik bildiradilar. Bundan tashqari, ushbu kitobni kompyuterda sahifalashda amaliy yordam bergan katta ilmiy xodim- izlanuvchi Sh.A. Sharopov va magistrant N.R. Yuldashevlarga minnatdorchilik bildiramiz.

O'quv qo'llanma tasdiqlangan fan dasturi asosida yozilganligi bois mavzularga tegishli misol va masalalar soni va yechimlari chegaralangan. ENA fanidan davlat tilida yozilgan dastlabki qo'llanmalardan biri bo'lganligi sababli kamchiliklardan xoli emas. Soha mutaxassislari, talabalar, umuman o'quvchilar fikr-mulohazalari asosida qo'llanmani keyingi nashrlari mukammalroq bo'ladi degan umiddamiz.

Mualliflar

MUNDARIJA

1-BOB. O‘ZGARMAS TOK CHIZIQLI ELEKTR ZANJIRLARINI HISOBLASH

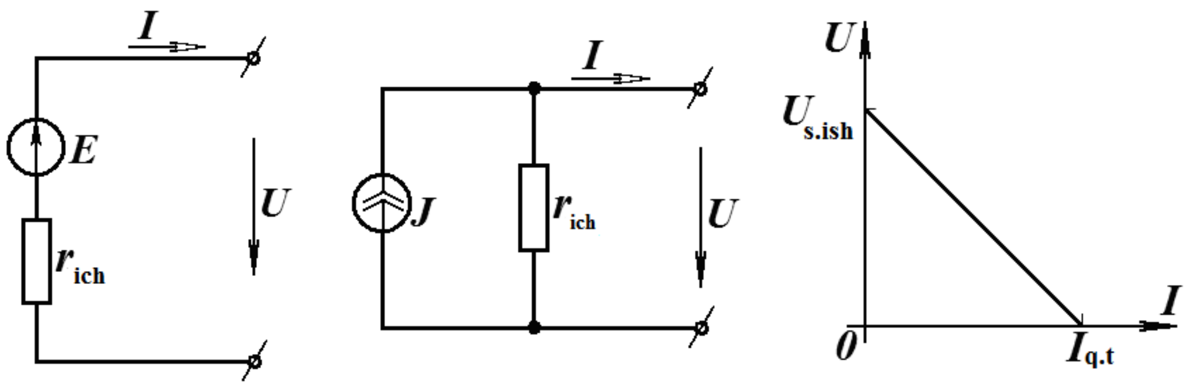
1.1. Asosiy ma’lumotlar

1. Elektr zanjirlar asosan *elektr energiya manbalaridan* va *iste’molchi (yuklama) lardan* tashkil topadi. Elektr energiya iste’molchisi (rezistor) zanjir qarshiligi bilan xarakterlanib, o‘zgarmas tokda R harfi bilan belgilanadi.

2. Elektr energiya manbayi tashqi xarakteristika $U = f(I)$ bilan ifodalanib, ikkita ekvivalent sxema yordamida tasvirlanishi mumkin: EYK manbayi va unga ketma-ket ulangan uning ichki qarshiligi (1.1- rasm, *a*) hamda tok manbayi va unga parallel ulangan uning ichki qarshiligi (1.1- rasm, *b*).

1.1- rasm, *d* sxemadagi EYK kattaligi son jihatdan manbaning salt ishlashidagi (yuklama ajratilgandagi) $U_{\text{s.ish}}$ kuchlanishiga teng.

1.1- rasm, *d* sxemadagi tok kattaligi son jihatdan manbaning qisqa tutashgandagi $I_{\text{q.t.}}$ tokiga tengdir.



a) *b*) *d)*

1.1- rasm.

3. EYK manbayi sxemasidan tok manbayi sxemasiga va aksincha ekvivalent o‘zgartirishlar quyidagi formulalar yordamida bajariladi:

$$I_r = \frac{E}{r_{\text{ich}}}; \qquad E = r_{\text{ich}} I \, .$$

4. Elektr zanjirlarni hisoblashda ideallashgan EYK va tok manbalaridan foydalaniladi. Ideal EYK manbayining ichki qarshiligi $r_{\text{ich}} = 0$, ideal tok manbayida esa ichki qarshilik $r_{\text{ich}} = \infty$. EYK manbayining qismalaridagi kuchlanish manbadan o‘tayotgan tokka bog‘liq emas. Tok manbayidan o‘tadigan tok uning qismalaridagi kuchlanishga bog‘liq emas.

5. Om qonuni yozilishining uchta shakli mavjud:

1)
$$I = \frac{\sum E}{\sum R},$$

bunda, $\sum E$ – EYK manbalarining algebraik yig‘indisi: agar EYK yo‘nalishi tanlab olingan tok yo‘nalishiga mos bo‘lsa, u holda bu EYK algebraik yig‘indiga “+” ishorasi bilan kiradi, aksincha bo‘lganda esa “–” ishorasi bilan olinadi; $\sum R$ – zanjir qarshiliklari(shu jumladan, EYK manbalarining ichki qarshiliklari)ning arifmetik yig‘indisi.

2) shoxobchasida EYK si bo‘lmagan elektr zanjirlar uchun tok:

$$I = \frac{\varphi_a - \varphi_b}{R_{ab}} = \frac{U_{ab}}{R_{ab}},$$

bunda, φ_a va φ_b – mos ravishda *a* va *b* tugunlar potensiallari.

U_{ab} – *a* va *b* tugunlar orasidagi kuchlanishlar yoki potensiallar ayirmasi;

R_{ab} – *a* va *b* tugunlar orasidagi shoxobchadagi qarshilik.

3) EYK manbalari bo‘lgan shoxobcha uchun tok:

$$I = \frac{\varphi_a - \varphi_c + \sum E}{\sum R_{ac}} = \frac{U_{ac} + \sum E}{\sum R_{ac}},$$

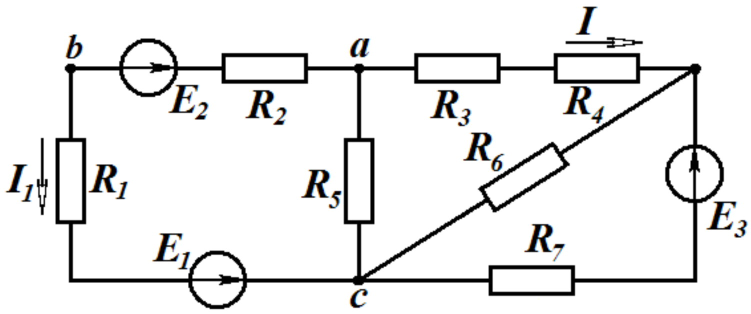
bunda, $U_{ac} = \varphi_a - \varphi_c$ – shoxobcha ulangan tugunlar kuchlanishi yoki potensiallar ayirmasi;

$\sum E$ – shoxobchadagi EYK larning algebraik yig‘indisi;

R_{ac} – shoxobchadagi qarshiliklarning arifmetik yig‘indisi.

Masalan, 1.2- rasmdagi *abs* shoxobcha uchun $\sum E = E_1 - E_2$; $\sum R_{ac} = R_1 + R_2$.

Elektr zanjirlarni hisoblash usullari Kirxgof qonunlariga asoslangan.



1.2- rasm.

6. Kirxgofning 1— qonuni. Kirxgofning 1— qonuni zanjirning tugunlariga tegishli boʻlib, unga koʻra zanjirning istalgan tugunida toklarning algebraik yigʻindisi nolga teng boʻladi, ya'ni:

$$\sum_{k=1}^{m} I_k = 0$$

yoki elektr zanjirning istalgan tuguniga kiruvchi toklarning arifmetik yigʻindisi shu tugundan chiquvchi toklarning arifmetik yigʻindisiga tengdir, ya'ni:

$$\sum_{i=1}^{m} I_i = \sum_{j=1}^{q} I_j .$$

Tugunga kirayotgan toklar bir xil ishora bilan, masalan ,,+" ishorasi bilan, chiqayotganlari esa boshqa ishora bilan, masalan ,,—" ishorasi bilan olinadi.

7. Kirxgofning ***2— qonuni.*** *Kirxgofning 2— qonuni zanjirning berk konturlariga tegishli boʻlib, unga koʻra elektr zanjirining istalgan berk konturida kuchlanishlar tushuvining algebraik yigʻindisi shu konturdagi EYKlarning algebraik yigʻindisiga teng*, ya'ni:

$$\sum_{k=1}^{n} R_k I_k = \sum_{i=1}^{m} E_i .$$

Agar konturni aylanib chiqish yo'nalishi bilan tok yoki EYK yo'nalishi bir xil bo'lsa, u holda yig'indiga tegishli tashkil etuvchilar **"musbat"** ishora bilan, aks holda esa **"manfiy"** ishora bilan kiradi.

Kirxgofning 2− qonunini boshqa ko'rinishda yozish ham mumkin: zanjirning ixtiyoriy konturida kuchlanishlarning algebraik yig'indisi nolga teng:

$$\sum_{k=1}^{n} U_k = 0.$$

8. Passiv zanjirlarda ekvivalent qarshiliklarni aniqlash:

a) parallel ulangan *n* ta qarshilikning ekvivalent qarshiligi

$$\frac{1}{R_{\text{ek}}} = \sum_{k=1}^{n} \frac{1}{R_k}.$$

Xususiy holda ikkita qarshilik parallel ulanganda

$$R_{\text{ek}} = \frac{R_1 R_2}{R_1 + R_2},$$

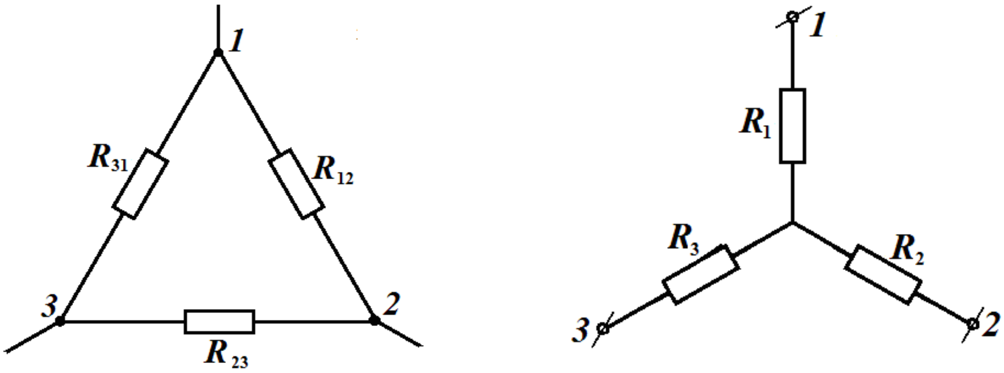
uchta qarshilik parallel ulanganda

$$R_{\text{ek}} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3};$$

b) qarshilik uchburchagini (1.3- rasm, *a*) ekvivalent yulduzchaga (1.3- rasm, *b*) va aksincha o'zgartirishlar mos ravishda quyidagi formulalar yordamida bajariladi:

$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}; \quad R_2 = \frac{R_{23} R_{12}}{R_{12} + R_{23} + R_{31}}; \quad R_3 = \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}};$$

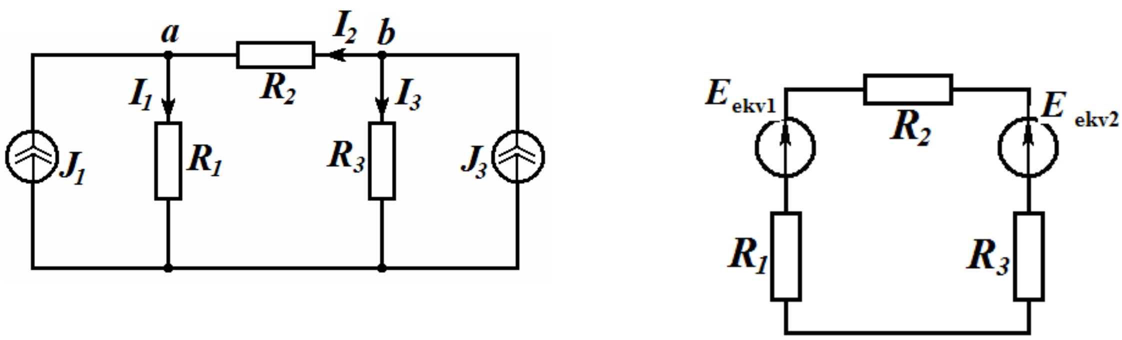
$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}; \quad R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1}; \quad R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2}.$$



1.3- rasm.

1.2. Chiziqli elektr zanjirlarda manbalarni va zanjirlarni ekvivalent o‘zgartirish

1.1-masala. 1.4- rasm, *a* da keltirilgan sxemada $J_1 = 3\text{A}$, $J_3 = 4\text{A}$, $R_1 = 10\ \Omega$ $R_2 = 20\ \Omega$; $R_3 = 30\ \Omega$ bo‘lganda, I_1, I_2 va I_3 toklar aniqlansin.



a) *b)*

1.4- rasm.

Yechish. Ichki qarshiligi R_1 bo‘lgan real J_1 tok manbayini xuddi shu R_1 qarshilik ketma-ket ulangan ekvivalent EYK manbayi bilan almashtiramiz:

$$E_{\text{ekv1}} = R_1 J_1 = 10 \cdot 3 = 30\ \text{V}$$

Shunga o‘xshash

$$E_{\text{ekv3}} = R_3 J_3 = 30 \cdot 4 = 120\ \text{V}.$$

Natijada 1.4-rasm, *b* da keltirilgan ekvivalent zanjirni hosil qilamiz va undagi tokni Om qonuni asosida quyidagicha aniqlaymiz:

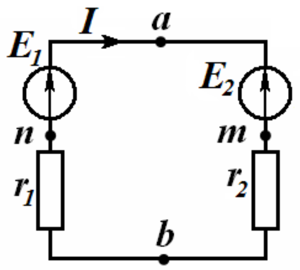
$$I_2 = \frac{\sum E}{\sum R} = \frac{E_3 - E_1}{R_1 + R_2 + R_3} = \frac{120 - 30}{10 + 20 + 30} = 1{,}5 \text{ A.}$$

I_1 va I_2 toklarni a va b tugunlar uchun Kirxgofning 1- qonuni asosida tuzilgan tenglamalardan quyidagicha aniqlaymiz:

$$I_1 = J_1 + I_2 = 3 + 1{,}5 = 4{,}5\,\text{A},$$

$$I_3 = J_3 - I_2 = 4 - 1{,}5 = 2{,}5\,\text{A}.$$

1.2- masala. Shoxobchalanmagan zanjirda (1.5- rasm) EYK $E_1 = 120$ V, uning ichki qarshiligi $r_1 = 12\ \Omega$, $E_2 = 40$V, $r_2 = 8\ \Omega$. a va b nuqtalar orasidagi kuchlanish aniqlansin.



1.5- rasm.

Yechish. Tok yo‘nalishini soat mili bo‘yicha belgilab, Om qonuni bo‘yicha tokni hisoblaymiz:

$$I = \frac{E_1 - E_2}{r_1 + r_2} = \frac{120 - 40}{12 + 8} = 4\,\text{A}..$$

Ko‘rsatilgan a va b nuqtalar orasidagi kuchlanishni zanjirning amb qismi uchun Om qonunidan foydalanib topamiz:

$$I = \frac{U_{ab} - E_2}{r_2}.$$

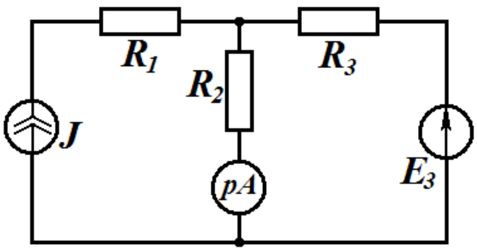
Unda

$$U_{ab} = E_2 + r_2 I = 40 + 4 \cdot 8 = 72 \quad \text{V}.$$

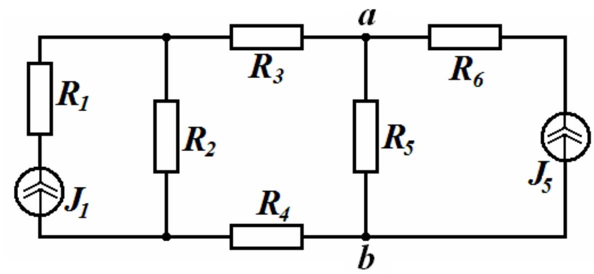
1.3- masala. 1.6- rasmdagi ampermetr ko‘rsatishini aniqlang.

$J = 2$ A; $E_3 = 4$ V; $R_1 = R_2 = R_3 = 1\ \Omega$.

1.4- masala. Tok manbayi qismalaridagi kuchlanish $U_{ab} = 32$ V bo‘lishi uchun sxemadagi (1.7- rasm) R_5 qarshilik qancha bo‘lishi kerak? $J_1 = 3$ A; $J_5 = 6$ A; $R_1 = 13\ \Omega$; $R_2 = 4\ \Omega$; $R_3 = R_4 = R_6 = 2\ \Omega$.

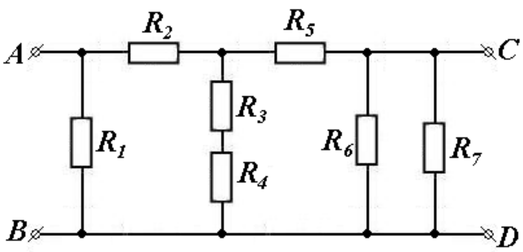


1.6- rasm.

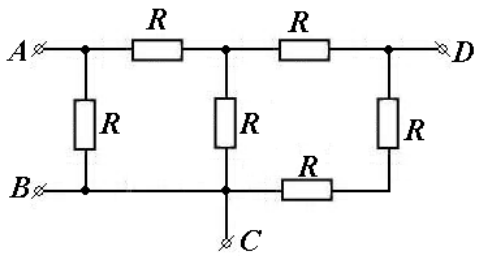


1.7- rasm.

1.5- masala. Elektr zanjirda (1.8- rasm) $R_1 = R_2 = 15\ \Omega$, $R_3 = R_6 = 20\ \Omega$, $R_4 = R_5 = 17{,}5\ \Omega$, $R_7 = 12\ \Omega$. Zanjirning *AB* va *CD* qismalari orasidagi qarshiliklar aniqlansin.



1.8- rasm.

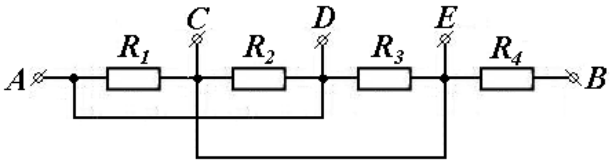


1.9- rasm.

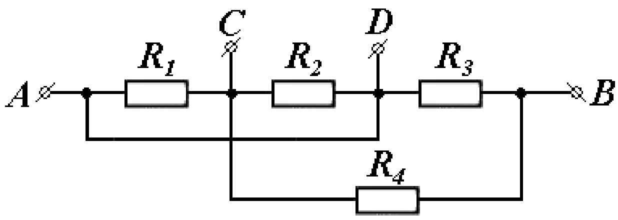
1.6- masala. Elektr zanjirining (1.9- rasm) *AB* va *CD* qismalari orasidagi umumiy qarshiligi topilsin.

1.7- masala. Elektr zanjirning (1.10- rasm) *AB*, *DE* va *AE* qismalari orasidagi umumiy qarshiliklari aniqlansin: $R_1=R_2=R_3=R_4=R$.

1.8- masala. Elektr zanjirining (1.11- rasm) *AB* qismalaridagi qarshilik aniqlansin. $R_1 = R_2 = 4\ \Omega$, $R_3 = 6\ \Omega$, $R_4 = 10\ \Omega$. Agar a) *C* va *D*; b) *A* va *C*; d) *C* va *B*; e) *D* va *B* qismalar qisqa tutashtirilsa, u holda zanjirning ekvivalent qarshiliklari qanday o‘zgaradi?



1.10- rasm.



1.11- rasm.

1.3. EYK manbayi bo‘lgan zanjirning bir qismi uchun Om qonuni

1.9- masala. EYKi $E = 2\text{V}$ bo‘lgan Akkumulatorga qarshiligi $R_1 = 1{,}5\Omega$ rezistor ulanganda $I_1 = 0{,}8\text{A}$ tok, $R_1 = 3{,}25\Omega$ li rezistor ulanganda esa $I_2 = 0{,}4\text{A}$ tok hosil bo‘lgan. Akkumulator qismalaridagi kuchlanish va ichki qarshiligini aniqlang.

1.10- masala. Ishqorli Akkumulator $R_1 = 4\Omega$ li tashqi qarshilikka ulanganda $I_1 = 0{,}2\text{A}$ tokni, $R_2 = 7\Omega$ li tashqi qarshilikka ulanganda esa $I_2 = 0{,}14\text{A}$ tokni beradi. Akkumulatorning qisqa tutashish tokini toping.

1.11- masala. Manbaning qisqa tutashish toki $I_{q.t}$= 48 A. Manbaga R=19,5 Ω qarshilik ulanganda zanjirdagi tok I=1,2 A bo‘lsa, manbaning EYKi va ichki qarshiligi aniqlansin.

1.12- masala. Manbaning salt ishlashidagi kuchlanishi $U_{s.i}$=250 V. Agar manbaning yuklangan holatdagi kuchlanishi U=242 V va ichki qarshiligi r_{ich}=2,5Ω bo‘lsa, zanjirdagi tok, yuklama qarshiligi va manbaning zanjirga berayotgan quvvati aniqlansin hamda quvvatlar balansi tuzilsin.

1.13- masala. EYKi E=300 V bo‘lgan o‘zgarmas tok manbayiga qarshiligi R=2,5 kΩ bo‘lgan yuklama reostat orqali ulangan. Yuklamaning tokini 10 dan 100 mA gacha o‘zgartirish uchun reostatning maksimal va minimal qarshiligi hamda maksimal va minimal toklarda yuklama va reostat quvvatlari aniqlansin.

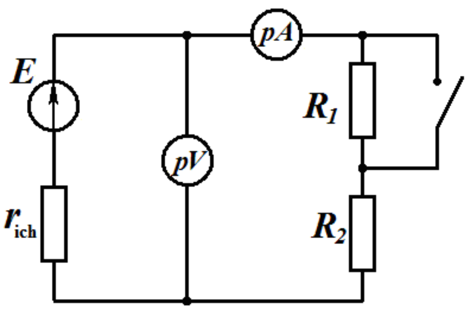
1.14- masala. Elektr yuklamaning qarshiligi R=3 Ω. Unda kattaligi P=3W quvvat ajralishi uchun yuklamaga EYKi 1,5 V va ichki qarshiligi r_{ich}=0,9 Ω bo‘lgan manba elementlaridan kamida nechtasi parallel ulanishi kerak?

1.15- masala. Agar elektr mashinaning EYKi $115\ \text{V}$, toki $25\ \text{A}$, ichki qarshiligi $R_0 = 0{,}2\ \Omega$ bo‘lsa, generator rejimida ishlayotgan elektr mashinaning qismalaridagi kuchlanishni aniqlansin.

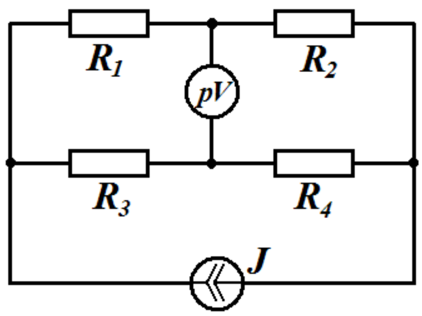
1.16- masala. Avvalgi masaladagi elektr mashina motor rejimida 25 A tok iste’mol qilib ishlayotgan bo‘lsa, uning qismalaridagi kuchlanish aniqlansin.

1.17-masala. Kalit ajralgan holda zanjirga ulangan ampermetr 5 A ni, voltmetr esa 48 V ni ko‘rsatgan (1.12-rasm). Kalit ulangan holda ularning ko‘rsatishi mos ravishda 10 A va 46 V ga teng bo‘ldi. Energiya manbayining EYKi va ichki qarshiligi aniqlansin.

1.18- masala. Agar zanjirda (1.13- rasm) J=3 A, R_1=4 Ω, R_2=1 Ω, R_3=2 Ω, R_4=8 Ω bo‘lsa, voltmetrning ko‘rsatshi aniqlansin.

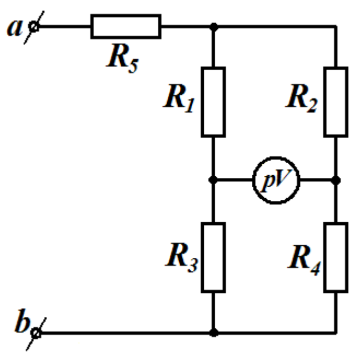


1.12- rasm.



1.13- rasm.

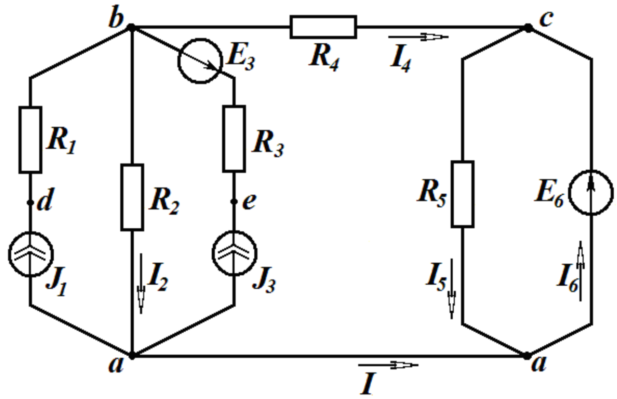
1.19- masala. Zanjir (1.14- rasm) qismalaridagi kuchlanish $U_{ab} = 8\,\text{V}$. Agar $R_1 = R_2 = R_5 = 8\,\Omega$, $R_3 = 12\,\Omega$, $R_4 = 7\,\Omega$ bo‘lsa, voltmetrning ko‘rsatishi aniqlansin.



1.14- rasm.

1.4. Elektr energiya va elektr quvvat. Quvvatlar balansi

1.20- masala. 1.15-rasmdagi zanjir uchun quvvatlar balansi tekshirilsin. Zanjirda $J_1 = J_3 = 1$ A; $E_3 = E_6 = 1$ V; $R_1 = R_2 = R_3 = R_4 = R_5 = 1\ \Omega$.



1.15- rasm.

Yechish. Quvvatlar balansini tekshirish uchun avval shoxobchalardagi noma'lum toklarni Kirxgof qonunlari asosida aniqlaymiz. Buning uchun to‘rtta tenglama tuzamiz: ikkita tenglamani Kirxgofning 1- qonuni va ikkita tenglamani 2- qonuni bo‘yicha tok manbayi bo‘lmagan konturlar uchun tuzamiz va quyidagi tenglamalar sistemasini hosil qilamiz:

a tugun uchun $-J_1 + I_2 - J_3 + I_5 - I_6 = 0,$

b tugun uchun $J_1 - I_2 + J_3 - I_4 = 0,$

bacb – kontur uchun $R_2 I_2 - R_5 I_5 - R_4 I_4 = 0,$

aca – kontur uchun $R_5 I_5 = E_6.$

Keltirilgan tenglamalarni birgalikda yechamiz:

$I_2 = 1{,}5$ A, $I_4 = 0{,}5$ A, $I_5 = 1$ A, $I_6 = 0{,}5$ A.

J_1 tok manbayi qismalaridagi kuchlanishni aniqlaymiz:

$$\varphi_d = \varphi_a + R_2 I_2 + R_1 J_1,$$

bundan

$$U_{da} = \varphi_d - \varphi_a = R_2 I_2 + R_1 J_1 = 2{,}5 \text{ V}.$$

Xuddi shunday tarzda U_{ea} ni aniqlaymiz:

$$U_{ea} = \varphi_e - \varphi_a = R_2 I_2 + E_3 + R_3 J_3 = 3{,}5 \text{ V}.$$

Endi quvvatlar balansi tenglamasini tuzamiz.

$$E_6 I_6 - E_3 J_3 + U_{da} J_1 + U_{ea} J_3 =$$
$$= J_1^2 R_1 + I_2^2 R_2 + J_3^2 R_3 + I_4^2 R_4 + I_5^2 R_5,$$

bu tenglamadan ushbu ayniyatni hosil qilamiz: 5,5=5,5.

Shuni qayd etish lozimki, E_3 manba quvvati manfiy, ya'ni ushbu manba zanjirdan energiya iste'mol qiladi. Tok manbayining quvvatini hisoblashga e'tibor bering.

1.21- masala. Quvvati P_m = 2,5 kW bo'lgan manbaga qarshiligi $R_l = 2{,}64\,\Omega$ bo'lgan liniya orqali yuklama ulangan. Agar yuklama toki I=12 A bo'lsa, yuklama quvvati, liniyada kuchlanish yo'qolishi va liniyaning FIK aniqlansin.

Yechish. Liniyadagi kuchlanish yo'qolishini topamiz:

$$\Delta U = R_l I = 2{,}64 \cdot 12 = 31{,}7 \text{V}.$$

Liniyadagi quvvat isrofi:

$$\Delta P_l = \Delta U \cdot I = 31{,}7 \cdot 12 = 380 \text{W}.$$

Yuklamada ajralgan quvvat:

$$P_{yuk} = P_m - \Delta P_l = 2500 - 380 = 2120 \text{W}.$$

Liniyaning FIK:

$$\eta = \frac{P_{\text{yuk}}}{P_{\text{m}}} \cdot 100\% = \frac{2120}{2500} \cdot 100\% = 85\%.$$

1.22- masala. O'zgarmas tok generatoriga quvvati $P_1 = 5{,}4$ kW ga ega yuklama ulanganda uning qismalaridagi kuchlanish $U_1 = 225$ V ga, $P_2 = 3{,}68$ kW li yuklama ulanganda esa kuchlanish $U_2 = 230$ V ga teng bo'ldi. Generatorning EYKi va ichki qarshiligi aniqlansin.

Yechish. Ikkala hol uchun yuklamalardan o'tuvchi toklarni topamiz.

$$I_1 = \frac{P_1}{U_1} = \frac{5{,}4 \cdot 10^3}{225} = 24 \text{A},$$

$$I_2 = \frac{P_2}{U_2} = \frac{3,68 \cdot 10^3}{230} = 16\,\text{A}.$$

Kirxgofning ikkinchi qonuni bo‘yicha tenglamalar tuzamiz:

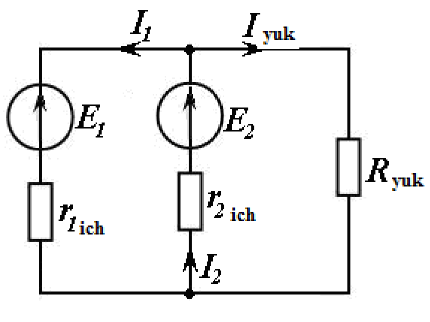
$$\begin{cases} E = I_1R + I_1r = 225 + 24r, \\ E = I_2R + I_2r = 230 + 16r. \end{cases}$$

Tenglamalar sistemasini yechib, E va r ni aniqlaymiz: $E = 240\,\text{V}, \quad r = 0,625\ \Omega$;

1.5. Kirxgof qonunlari. Potensial diagramma

1.23- masala. EYKlari va ichki qarshiliklari mos ravishda E_1=11,5 V, r_1=2,5Ω, va E_2=16,5 V, r_2=6 Ω bo‘lgan ikkita o‘zgarmas tok manbalari qarshiligi R_{yuk}=30 Ω li yuklamaga parallel ulangan (1.15- rasm). Manbalar va yuklamadan o‘tuvchi toklar qiymati va yo‘nalishini, har bir manba rejimini va ular qismalaridagi kuchlanish pasayishini aniqlang.

Yechish. Toklar yo‘nalishini shartli ravishda ixtiyoriy tanlaymiz. I_1, I_2 va I_{yuk} toklarni aniqlash uchun Kirxgofning birinchi va ikkinchi qonunlariga ko‘ra 3 ta tenglama sistemasini tuzamiz:



1.16- rasm.

$$\begin{cases} I_2 = I_1 + I_{yuk}, \\ E_2 - E_1 = I_1r_1 + I_2r_2, \\ E_2 = I_{yuk}R_{yuk} + I_2r_2. \end{cases}$$

Tenglamalarga EYK va qarshiliklarning qiymatlarini qo‘yib va sistemani yechib, noma’lum toklarni topamiz:

$I_1 = 0{,}3\text{A}, I_2 = 0{,}71\text{A}, I_{\text{yuk}} = 0{,}41\text{A}$.

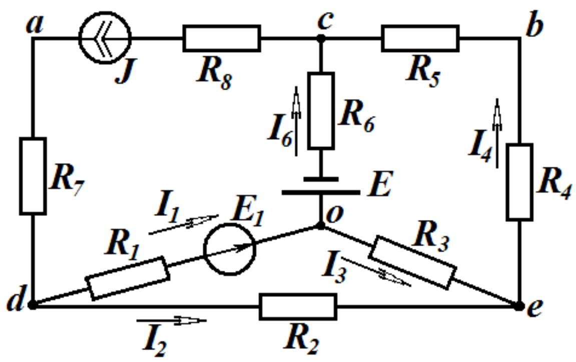
E_1 manba yuklama rejimida, E_2 esa – generator rejimida ishlaydi. Shunga ko‘ra quvvat balansi tenglamasini tuzamiz:

$E_2 I_2 - E_1 I_1 = I_1^2 r_1 + I_2^2 r_2 + I_{\text{yuk}}^2 R_{\text{yuk}}$,

$16{,}5 \cdot 0{,}71 - 11{,}5 \cdot 0{,}3 = 0{,}3^2 \cdot 2{,}5 + 0{,}71^2 \cdot 6 + 0{,}41^2 \cdot 30$,

$8{,}265\,\text{W} \approx 8{,}29\,\text{W}$.

1.24- masala. 1.17- rasmda keltirilgan zajirdagi R_3 qarshilikni va galvanik elementning EYKi E aniqlansin (manbaning ichki qarshiligi hisobga olinmasin).



1.17- rasm.

Zanjir parametrlari:

$J = 3$ A; $E_1 = 12\,\text{V}$; $R_1 = R_5 = 2\ \Omega$; $R_2 = 6\ \Omega$;

$R_6 = 20\ \Omega$; $R_4 = R_7 = R_8 = 3\ \Omega$; $I_2 = I_3 = 1$ A.

Yechish. Zanjirning d tuguni uchun qabul qilingan musbat tok yo‘nalishlarini e’tiborga olib, Kirxgofning 1- qonuni bo‘yicha tenglama tuzamiz:

$$J = I_1 + I_2 .$$

Bundan noma’lum I_1 tokni aniqlaymiz:

$$I_1 = J - I_2 = 3 - 1 = 2\ \text{A}.$$

doed kontur uchun Kirxgofning 2- qonunidan

$$R_1 I_1 + R_3 I_3 - R_2 I_2 = E_1 ,$$

$$R_3 = \frac{E_1 - R_1 I_1 + R_2 I_2}{I_3} = \frac{12 - 2\cdot 2 + 6\cdot 1}{1} = 14\ \Omega.$$

e tugun uchun tuzilgan Kirxgofning 1- qonunidan

$$I_4 = I_2 + I_3 = 2\ \text{A};$$

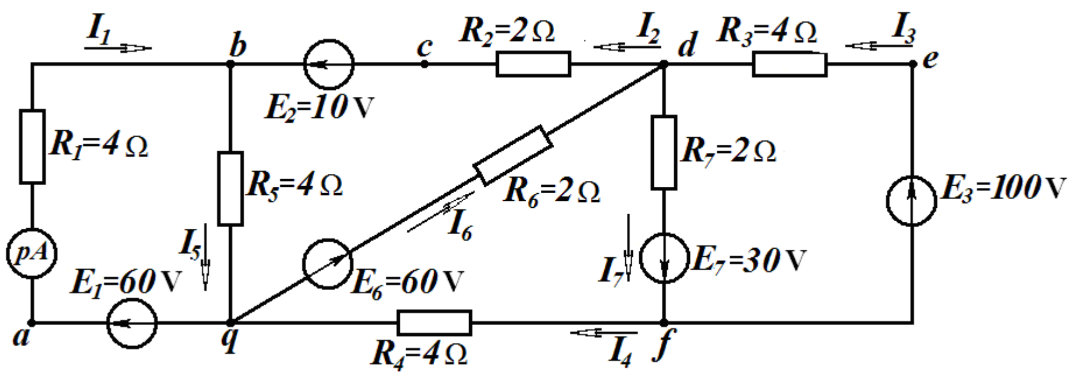
o- tugundan esa $\quad I_6 = I_1 - I_3 = 1\ \text{A}.$

EYK E ni aniqlash uchun Kirxgofning 2- qonuni asosida *oebco* kontur uchun tuzilgan tenglamadan foydalanamiz:

$$R_3 I_3 + (R_4 + R_5) I_4 - R_6 I_6 = E,$$

$$E = 14\cdot 1 + 5\cdot 2 - 20 = 4\,V.$$

1.25- masala. 1.18- rasmda keltirilgan zanjirdagi ampermetr 5 A tok kuchini ko‘rsatmoqda. Om va Kirxgof qonunlaridan foydalanib, boshqa shoxobchalardagi toklar aniqlansin. Zanjirning tashqi konturi uchun potensial diagramma qurilsin.



1.18- rasm.

Yechish. Zanjirning barcha parametrlari hamda bir tok $(I_1 = 5\ \text{A})$ berilgani sababli Kirxgofning birinchi va ikkinchi qonuniga ko‘ra bog‘lanmagan tugun va konturlar uchun tuzilgan tenglamalarni yechish shart emas. *abqa* – kontur uchun Kirxgofning ikkinchi qonuni asosida tuzilgan $R_1 I_1 + R_5 I_5 = E_1$ tenglamadan

$$I_5 = \frac{E_1 - R_1 I_1}{R_5} = \frac{60 - 4\cdot 5}{4} = 10\ \text{A}.$$

b tugun uchun Kirxgofning birinchi qonuni asosida tuzilgan tenglamadan

$$I_1 + I_2 - I_5 = 0$$

$$I_2 = I_5 - I_1 = 10 - 5 = 5 \text{ A}.$$

Keyingi bosqichlarda tugun va konturlarni shunday tanlab olamizki, har bir navbatdagi tenglamaga bitta noma’lum tok kirsin:

bdqb kontur uchun $R_5 I_5 + R_6 I_6 + R_2 I_2 = E_2 + E_6$; bundan $I_6 = 10 \text{ A}$;

q tugun uchun : $I_4 + I_5 - I_1 - I_6 = 0;$ bundan

$$I_4 = I_1 + I_6 - I_5 = 5 + 10 - 10 = 5 \text{ A}.$$

qdfq kontur uchun: $R_6 I_6 + R_7 I_7 + R_4 I_4 = E_6 + E_7$, bundan $I_7 = 25$ A,

f tugun uchun: $I_7 - I_4 - I_3 = 0$,

bundan $I_3 = I_7 - I_4 = 25 - 5 = 20 \text{ A}$.

Tashqi konturga tegishli potensial diagramma qurish uchun potensiali nol deb qabul qilinadigan biror nuqtaga nisbatan konturga tegishli tugun va oraliq nuqtalarning potensiallarini hisoblab chiqish kerak. $\varphi_a = 0$ deb qabul qilamiz. U holda:

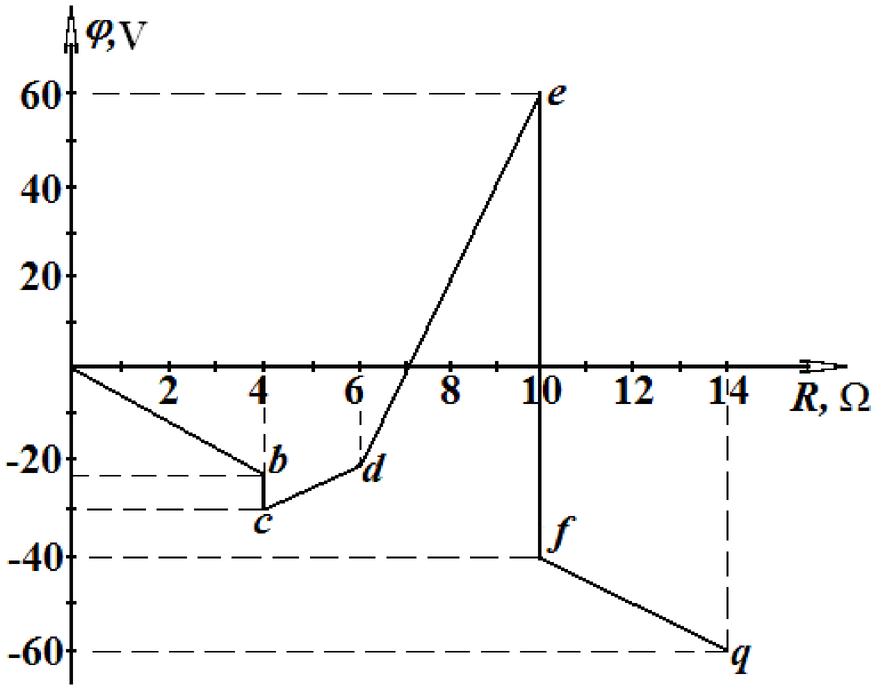
$$\varphi_b = \varphi_a - R_1 I_1 = 0 - 4 \cdot 5 = -20 \text{V},$$
$$\varphi_c = \varphi_b - E_2 = -20 - 10 = -30 \text{V},$$
$$\varphi_d = \varphi_c + R_2 I_2 = -30 + 2 \cdot 5 = -20 \text{V},$$
$$\varphi_e = \varphi_d + R_3 I_3 = -20 + 4 \cdot 20 = 60 \text{V},$$
$$\varphi_f = \varphi_e - E_3 = 60 - 100 = -40 \text{V},$$
$$\varphi_q = \varphi_f - R_4 I_4 = -40 - 4 \cdot 5 = -60 \text{V},$$
$$\varphi_a = \varphi_q + E_1 = -60 + 60 = 0.$$

Potensial diagramma 1.19- rasmda tasvirlangan. Diagramma kontur bo‘ylab potensialning taqsimlanishini ko‘rsatadi.

Potensial diagrammada:

a) vertikal chiziqlar EYK larga mos keladi;

b) diagramma masshtabida grafikning og‘ish burchaklari mos toklarga teng ($\text{tg}\alpha_1 = I_2$, $\text{tg}\alpha_2 = I_3$, $I_3 > I_2$), demak, potensial grafikning og‘ishi elektr zanjirining tuguniga mos keladi.

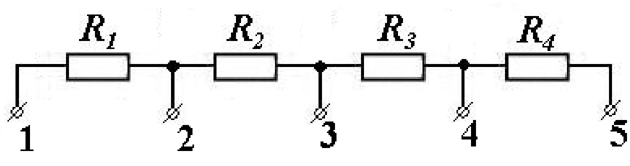


1.19- rasm.

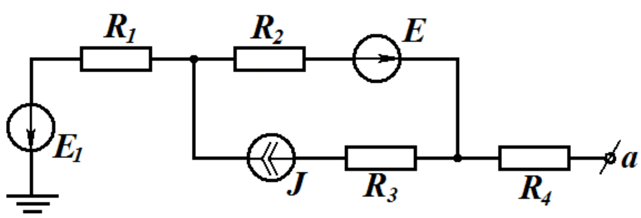
1.6. Chiziqli elektr zanjir sxemalarini ekvivalent o‘zgartirish

1.26-masala. Kuchlanish bo‘lgichining (1.20- rasm) kirishiga 100 V kuchlanish berilgan. Uning *1-2, 1-3, 1-4* qismalari orasidagi kuchlanishlar aniqlansin. Berilgan: $R_1 = 0{,}1$ kΩ, $R_2 = 0{,}9$ kΩ, $R_3 = 9$ kΩ, $R_4 = 90$ kΩ.

1.27- masala. 1.21- rasmda keltirilgan zanjirdagi “*a*” nuqtaning potensiali aniqlansin. Berilgan: $E_1 = 4$ V, $E_2 = 6$ V, $J = 3$ A, $R_1 = R_2 = R_3 = R_4 = 2$ Ω.



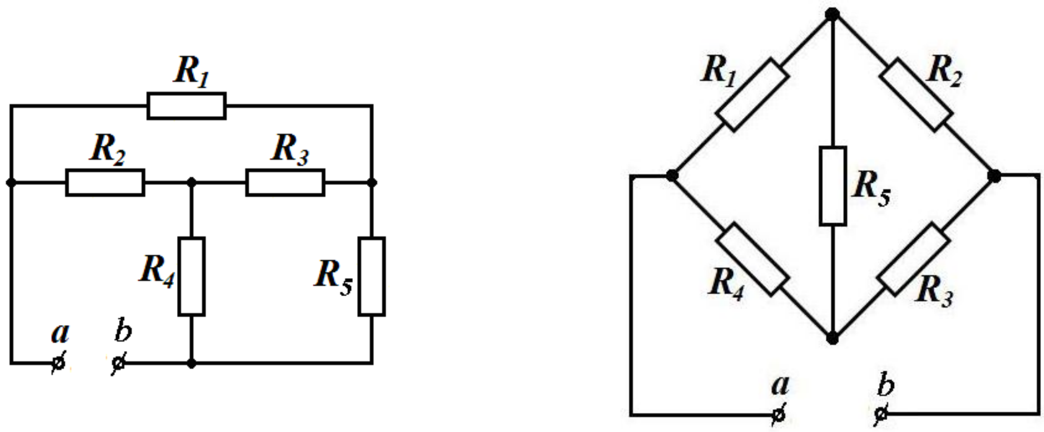
1.20- rasm.



1.21- rasm.

1.7. Qarshiliklar uchburchagini qarshiliklar yulduziga va aksincha ekvivalent oʻzgartirish

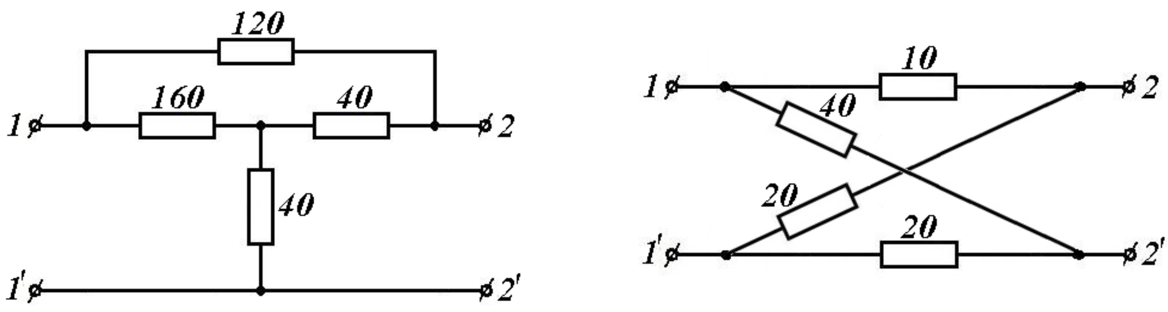
1.28-masala. 1.22-rasmda keltirilgan zanjirlarda $R_1 = 100\ \Omega$, $R_2 = 200\ \Omega$, $R_3 = 300\ \Omega$, $R_4 = 400\ \Omega$, $R_5 = 500\ \Omega$ boʻlsa, a va b qismalarga nisbatan qarshiliklar aniqlansin.



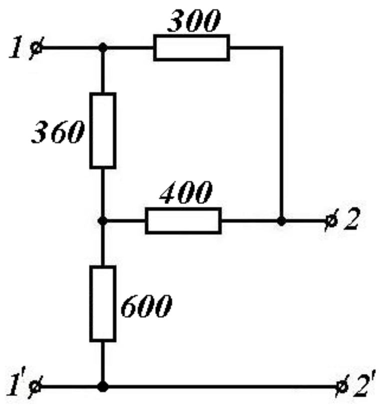
a)   *b*)

1.22- rasm.

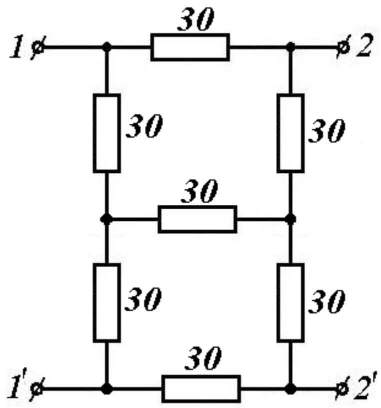
1.29-masala. 1.23-rasmda keltrilgan zanjirlarni *1* - *1*' qismalarga nisbatan ekvivalent qarshiliklar salt ishlash rejimida (*2* va *2*' qismalar ajratilgan holda) va qisqa tutashgan rejimda (*2* va *2*' qismalar birlashtirilgan holda) aniqlansin. Rezistorlar qarshiliklari (Ω hisobida) rasmda keltirilgan.



a)   *b)*



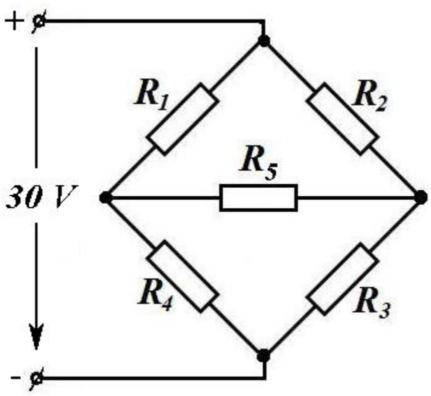
d)



e)

1.23- rasm.

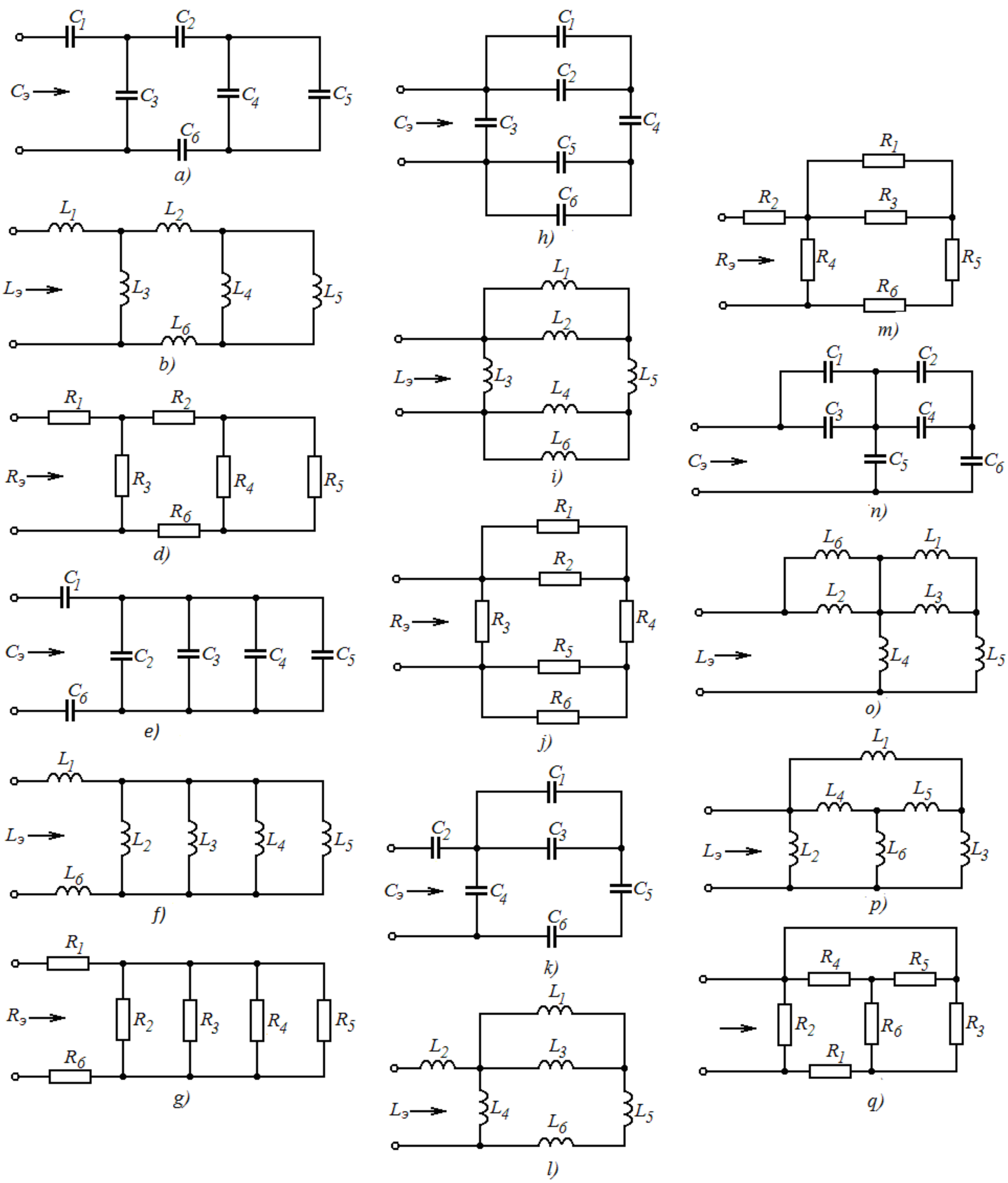
1.30- masala. 1.24- rasmda keltirilgan zanjir shoxobchalaridagi toklar aniqlansin. $R_1 = 180\,\Omega$, $R_2 = 60\,\Omega$, $R_3 = 120\,\Omega$, $R_4 = 80\,\Omega$, $R_5 = 120\,\Omega$.



1.24- rasm.

1.1-vazifa. Elementlar ulanishlarining ekvivalent parametrlarini hisoblash

Berilgan elektr zanjirlarning ekvivalent parametrlarini hisoblang (1.25-rasm). Zanjir elementlarining parametrlari 1.1-jadvalda keltirilgan.



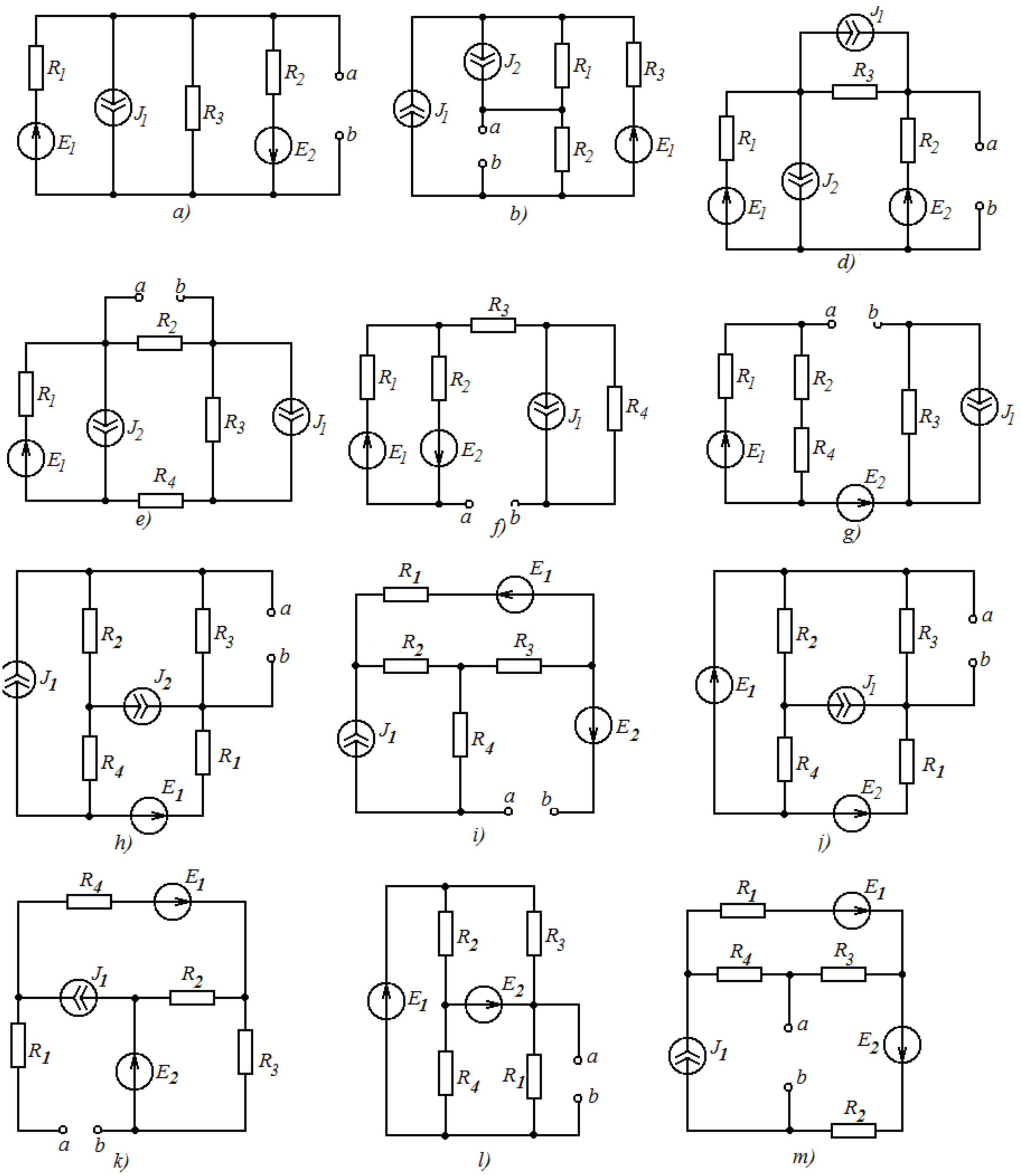
1.25- rasm.

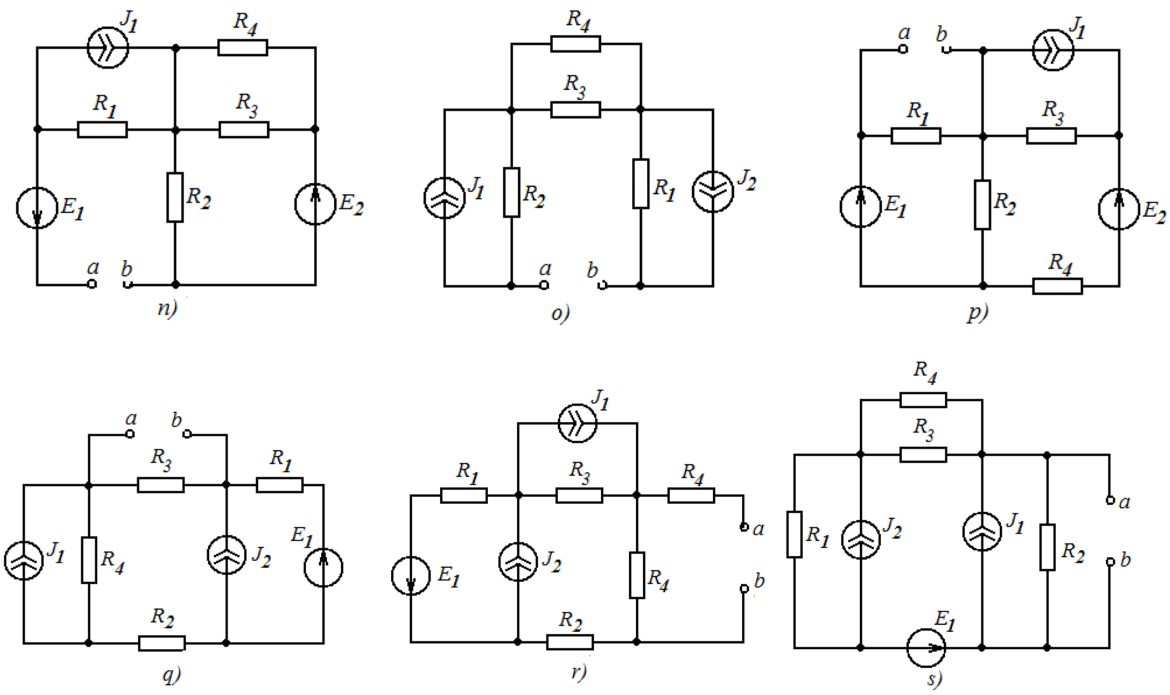
1.1- *jadval*

Variant	Rasm	Shoxobchalar elementlari parametrlarining qiymatlari Ω, H, mkF.					
		1	2	3	4	5	6
1	a	1	2	2	7	3	2
2	b	1	1	4	4	4	7
3	e	30	10	10	20	10	20
4	f	1	2	2	4	4	3
5	h	6	6	12	9	6	6
6	i	4	4	12	8	4	4
7	k	2	2	12	8	8	8
8	l	4	4	10	6	9	1
9	n	2	4	2	4	4	8
10	o	2	2	2	2	1	1
11	v	8	8	6	4	4	2
12	e	5	1	4	4	2	4
13	i	2	6	9	6	2	6
14	m	4	6	12	6	3	6
15	p	2	2	2	4	4	4
16	r	2	2	2	3	3	3
17	a	6	1	3	2	4	6
18	b	2	8	12	4	21	1
19	e	10	3	5	2	5	30
20	f	4	12	12	12	8	6
21	h	4	8	8	12	4	8
22	z	3	6	3	2	3	6
23	k	6	3	6	6	5	6
24	l	6	3	2	6	1	3
25	n	2	4	2	4	4	8
26	e	10	2	8	6	4	20
27	i	10	10	10	15	30	60
28	n	5	6	5	6	10	10
29	p	5	5	5	6	6	6
30	o	3	6	4	4	3	6

1.8. EYK va tok manbayiga ega bo‘lgan sxemalarni ekvivalent o‘zgartirish

1.2-vazifa. EYK va tok manbayiga ega bo‘lgan sxemalarni ekvivalent o‘zgartirish. 1.26- rasmda tasvirlangan sxemalarining *a* va *b* qismalarga nisbatan ekvivalent kuchlanishlar manbalarining r_{ekv} va $U_{ab\,\text{ekv}}$ parametrlari hisoblansin. Sxema elementlarining parametrlari 1.2 - jadvalda keltirilgan.





1.26- rasm.

1.2- jadval

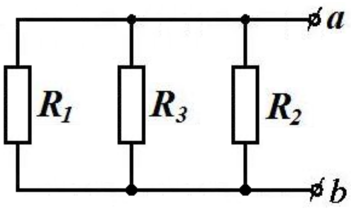
Variant	Rasm	Sxema elementlari parametrlarining qiymatlari, Ω,A,V							
		Qarshiliklar				Toklar		Kuchlanishlar	
		R_1	R_2	R_3	R_4	J_1	J_2	E_1	E_2
1	a	1	2	4	-	2	0	14	7
2	b	4	6	4	-	5	2	14	0
3	d	5	8	2	-	1	3	20	30
4	e	2	3	5	5	1	2	30	0
5	f	4	4	3	4	4	0	18	36
6	g	4	4	5	1	2	0	26	13
7	h	3	2	4	5	1	2	24	0
8	i	4	5	2	2	2	0	15	10
9	j	3	2	4	6	1	0	16	14
10	k	3	6	5	-	2	0	26	13
11	l	4	4	6	8	0	0	22	11
12	m	3	6	8	4	1	0	30	15
13	n	10	5	10	10	4	0	25	50
14	o	3	2	6	2	4	3	0	0
15	p	2	6	2	2	2	0	11	22
16	q	4	3	8	8	1	2	7	0
17	r	5	2	8	10	2	1	10	0

18	t	4	5	6	3	2	1	11	0
19	i	4	4	5	1	2	0	26	13
20	j	3	2	4	5	1	2	24	0
21	l	4	5	2	2	2	0	15	10
22	n	3	2	4	6	1	0	16	14
23	o	10	5	10	10	4	0	25	50
24	t	3	2	6	2	4	3	0	0
25	r	2	6	2	2	2	0	11	22
26	m	4	3	8	8	1	2	7	0
27	g	5	2	8	10	2	1	10	0
28	b	4	5	6	-	2	1	11	0
29	a	3	6	5	-	2	0	26	13
30	d	4	4	6	-	0	0	22	11

1.31- masala. 1.26- rasm, *a* da tasvirlangan sxemaning *a* va *b* qismalariga nisbatan $U_{\text{ab ekv}}$ ekvivalent kuchlanishi (1- variant parametrlari bo‘yicha):

$$U_{\text{ab ekv}} = \frac{E_1\dfrac{1}{R_1} - E_2\dfrac{1}{R_2} - J_1}{\dfrac{1}{R_1}+\dfrac{1}{R_2}+\dfrac{1}{R_3}} = \frac{14\cdot\dfrac{1}{1} - 7\cdot\dfrac{1}{2} - 2}{\dfrac{1}{1}+\dfrac{1}{2}+\dfrac{1}{4}} = \frac{8{,}5}{1{,}75} = 4{,}857\ \text{V}.$$

a va *b* qismalarga nisbatan ekvivalent qarshilikni aniqlash uchun zanjirdagi EYK manbalarini fikran qisqa tutashtiramiz va tok manbayini ichki qarshiligi cheksiz bo‘lganligi sababli zanjir quyidagi ko‘rinishga keladi: (1.27- rasm).



1.27- rasm.

Demak,

$$r_{\text{ekv}} = \frac{R_1R_2R_3}{R_1R_2+R_1R_3+R_2R_3} = \frac{1\cdot2\cdot4}{1\cdot2+1\cdot4+2\cdot4} = \frac{8}{14} = 0{,}571\,\Omega.$$

1.9. O‘zgarmas tok chiziqli elektr zanjirlarini hisoblash usullari

1.9.1. Proporsional kattaliklar usuli.

Proporsional kattaliklar usulini bitta manbadan ta’minlanayotgan va qarshiliklari o‘zaro aralash ulangan zanjirni hisoblashda qo‘llash maqsadga muvofiq.

Hisoblash ketma-ketligi quyidagicha:

1) manbadan eng uzoqda joylashgan shoxobchadagi tokka ixtiyoriy, masalan, *1* A qiymat beriladi;

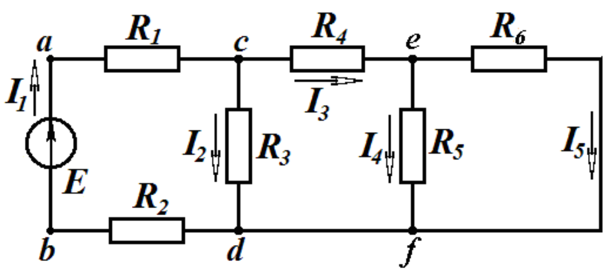
2) tanlangan shoxobchadagi tok qiymatidan foydalanib qolgan barcha shoxobchalardagi tok va kuchlanishlarni hisoblab topiladi;

3) qayta hisoblash koeffitsiyentini topamiz, ya’ni berilgan EYK ni uning o‘rniga hisoblab topilgan *U* kuchlanishga bo‘linadi;

4) barcha shoxobchalardagi hisoblab topilgan toklar va kuchlanishlarni qayta hisoblash koeffitsiyentiga ko‘paytirib tok va kuchlanishlarning haqiqiy qiymatlarini topiladi.

1.32- masala. 1.28- rasmda keltirilgan zanjir shoxobchalaridagi toklar proporsional kattaliklar usulida topilsin. $R_1 = R_2 = 3\ \Omega$; $R_3 = 26\ \Omega$; $R_4 = 6\ \Omega$; $R_5 = 4\ \Omega$; $R_6 = 8\ \Omega$; $E = 200$ V.

Yechish. $I_5' = 1\,\text{A}$ deb qabul qilamiz. Shunga ko‘ra boshqa shoxobchalardagi toklar va kuchlanishlarni hisoblaymiz:



1.28- rasm.

$$U_{ed} = R_6 I_5' = 8\cdot 1 = 8\,\text{V},\ \ I_4' = \frac{U_{ed}}{R_5} = \frac{8}{4} = 2\,\text{A},\ \ I_3' = I_5' + I_4' = 1 + 2 = 3\,\text{A},$$

$$U_{cd} = U_{ed} + R_4 I_3' = 8 + 3\cdot 6 = 26\,\text{V},\ \ I_2' = \frac{U_{cd}}{R_3} = \frac{26}{26} = 1\,\text{A},$$

$$I_1' = I_2' + I_3' = 1 + 3 = 4\ \text{A},\ \ U_{ab} = U_{cd} + I_1'(R_1 + R_2) = 26 + 4(3+3) = 50\ \text{V}.$$

Qayta hisoblash koeffitsiyenti:

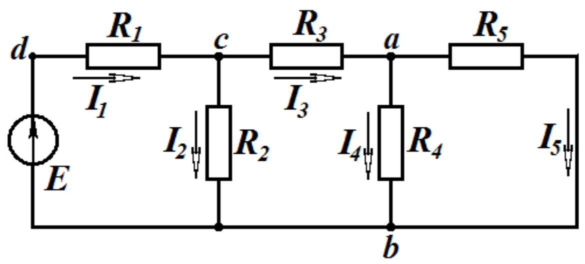
$$k = \frac{E}{U_{ab}} = \frac{200}{50} = 4.$$

Demak, toklarning haqiqiy qiymatlari:

$$I_1 = kI_1' = 4\cdot 4 = 16\ \text{A},\ \ I_2 = kI_2' = 4\cdot 1 = 4\ \text{A},\ \ I_3 = kI_3' = 4\cdot 3 = 12\ \text{A},\ \ I_4 = kI_4' = 4\cdot 2 = 8\ \text{A},$$

$$I_5 = kI_5' = 4\cdot 1 = 4\ \text{A}.$$

1.33- masala. 1.29- rasmda tasvirlangan zanjirdagi toklarni proporsional kattaliklar usulida aniqlang. Berilgan: $E = 48\ \text{V},\ \ R_1 = R_2 = R_3 = R_4 = R_5 = 2\,\Omega$.



1.29- rasm.

Yechish. Tok $I_5' = 1\ A$ deb qabul qilamiz. Qarshilik R_5 bo‘lgan shoxobchadan EYK manbali shoxobcha yo‘nalishiga ketma-ket siljib quyidagilarni topamiz:

$$U_{ab} = R_5 I_5' = 2\cdot 1 = 2\ \text{V},\ \ I_4' = \frac{U_{ab}}{R_4} = \frac{2}{2} = 1\ A,\ \ I_3' = I_4' + I_5' = 2\ A,$$

$$U_{cb} = R_3 I_3' + U_{ab} = 2\cdot 2 + 2 = 6\ \text{V},\ \ I_2' = \frac{U_{cb}}{R_2} = \frac{6}{2} = 3\ A,$$

$$I_1' = I_2' + I_3' = 5\ \text{A},\ U_{db} = R_1 I_1' + U_{cb} = 2\cdot 5 + 6 = 16\ \text{V}.$$

Shoxobchalardagi barcha toklarni qayta hisoblash koeffitsiyenti

$k = \dfrac{E}{U_{ab}} = \dfrac{48}{16} = 3$ ga ko‘paytirib, quyidagi haqiqiy toklarni aniqlaymiz:

$I_1 = 15\ \text{A},\ I_2 = 9\ \text{A},\ I_3 = 6\ \text{A},\ I_4 = 3\ A,\ I_5 = 3\ \text{A}.$

1.34-masala. Avvalgi masaladagi zanjir EYK manbayi o‘rniga tok manbayi

$J = 2\,\text{A}$ ulangan holat uchun barcha shoxobchalardagi toklar proporsional kattaliklar usulida aniqlansin.

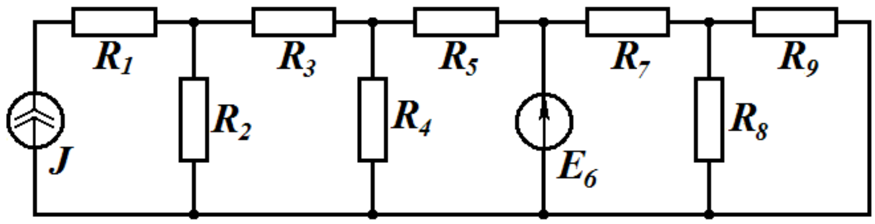
Yechish. Berilgan zanjir uchun $I_5' = 1\,A$ deb qabul qilib, avvalgi masalalardagi kabi ketma-ket ravishda shoxobchalardagi toklarni aniqlaymiz. Bunda $I_1' = 5\,A$ ga teng bo‘ldi. Bu holda qayta hisoblash koeffitsiyenti:

$$k = \frac{J}{I_1'} = \frac{2}{5} = 0{,}4.$$

Barcha toklarni shu koeffitsiyentga ko‘paytirib, ularning haqiqiy qiymatlarini aniqlaymiz:

$$I_2 = 1{,}2\,\text{A},\ I_3 = 0{,}8\,\text{A},\ I_4 = 0{,}4\,\text{A},\ I_5 = 0{,}4\,\text{A}.$$

1.35- masala. Proporsional kattaliklar usulidan foydalanib, 1.30- rasmdagi zanjir shoxobchalaridagi toklarni aniqlang. Berilgan: $J = 10\,\text{A}$, $E = 30\,\text{V}$, $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R_7 = R_8 = R_9 = 2\,\Omega$.

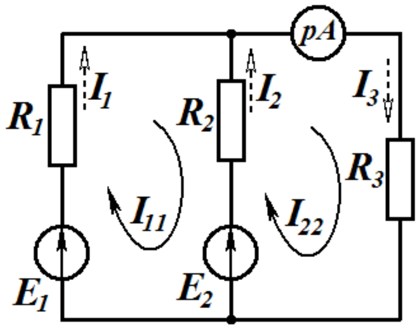


1.30- rasm.

1.9.2. Kontur toklar usuli.

Bu usulda o‘zaro bog‘liq bo‘lmagan (mustaqil) konturdan yagona kontur toki o‘tadi deb faraz qilinadi va shoxobchalar toki shu kontur toklari orqali aniqlanadi. Kontur toklar usuli Kirxgofning 2-qonuniga asoslanadi. Tenglamalar shu kontur toklariga nisbatan tuziladi. Tenglamalar sistemasi yechilib, noma’lum kontur toklar, ular orqali esa shoxobchalardagi haqiqiy toklar aniqlanadi. Shunday qilib, kontur toklar usuli asosida tuzilgan tenglamalarda noma’lumlar soni o‘zaro bog‘liq bo‘lmagan konturlar soniga teng va zanjir bu usulda hisoblanganda Kirxgof tenglamalari usuliga nisbatan tenglamalar soni va ularni yechishga sarf bo‘ladigan vaqt kam bo‘ladi. Kontur toklar usuli ko‘pincha murakkab zanjirdagi mustaqil konturlar soni tugunlar sonidan kam bo‘lganda qo‘llaniladi.

1.36-masala. Kontur toklar usulida zanjirdagi (1.31-rasm) toklar aniqlansin. Bunda E_1=52 V, E_2=48 V, R_1=4 Ω, R_2=8 Ω, R_3=10 Ω.



1.31- rasm.

Yechish. Zanjirda ikkita bog‘liq bo‘lmagan(mustaqil) kontur mavjud. Kontur toklarni aniqlash uchun kanonik tenglamalar sistemasi quyidagi ko‘rinishga ega:

$$\left.\begin{aligned} R_{11}I_{11} + R_{12}I_{22} = E_{11}, \\ R_{21}I_{11} + R_{22}I_{22} = E_{22}, \end{aligned}\right\}$$

bunda $R_{11} = R_1 + R_2 = 4 + 8 = 12\ \Omega$ – birinchi konturning xususiy qarshiligi; $R_{22} = R_2 + R_3 = 8 + 10 = 18\ \Omega$ – ikkinchi konturning xususiy qarshiligi; $R_{12} = R_{21} = -R_2 = -8\ \Omega$ – 1- va 2- konturlarning o‘zaro qarshiligi; $E_{11} = E_1 - E_2 = 52 - 48 = 4\ \text{V}$ – 1-konturdagi EYKlarning algebraik yig‘indisi; $E_{22} = E_2 = 48\ \text{V}$ – 2- konturdagi EYKlarning algebraik yig‘indisi.

Yuqoridagilarni sistemaga qo‘yamiz:

$$\left.\begin{aligned} 12I_{11} - 8I_{12} = 4, \\ -8I_{21} + 18I_{22} = 48. \end{aligned}\right\}$$

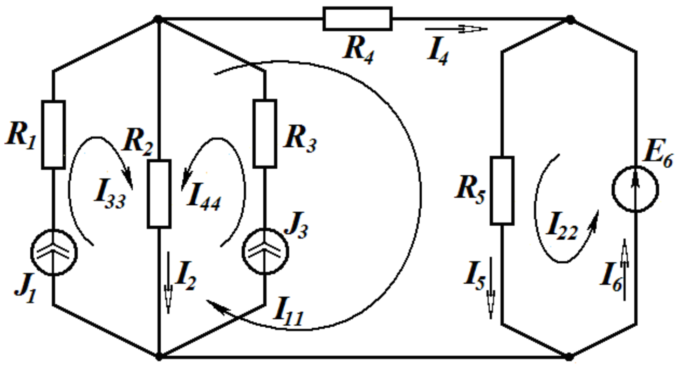
Sistemani yechib kontur toklarni aniqlaymiz:

$$I_{11} = 3\ \text{A}, \quad I_{22} = 4\ \text{A}.$$

Shoxobchalardagi haqiqiy toklar:

$$I_1 = I_{11} = 3\ \text{A},\ I_3 = I_{22} = 4\ \text{A},\ I_2 = I_{22} - I_{11} = 4 - 3 = 1\ \text{A}.$$

1.37- masala. 1.32- rasmda keltirilgan zanjirdagi toklar aniqlansin, bunda $J_1 = J_3 = 1\ \text{A},\ E_6 = 1\ \text{V},\ R_1 = R_2 = R_3 = R_4 = R_5 = 1\ \Omega.$



1.32- rasm.

Yechish. Zanjirda to‘rtta bog‘liq bo‘lmagan kontur mavjud. Ikkita shoxobchada toklari ma’lum bo‘lgan tok manbalari bor. Agar har bir shoxobchadan o‘tadigan kontur toklar tenglamasini tuzsak, ushbu kontur toklar «avtomatik» ravishda ma’lum bo‘lib qoladi:

$$I_{33} = J_1 = 1 \text{ A}, \; I_{44} = J_3 = 1 \text{ A}.$$

Boshqa ikkita bog‘liq bo‘lmagan konturlar toklarini aniqlash uchun Kirxgofning 2- qonuniga ko‘ra ikkita tenglama tuzamiz:

$$R_{11}I_{11} + R_{12}I_{22} + R_{13}I_{33} + R_{14}I_{44} = E_{11},$$
$$R_{21}I_{11} + R_{22}I_{22} + R_{23}I_{33} + R_{24}I_{44} = E_{22}.$$

E’tirof etish lozimki, yuqoridagi kanonik sistemadagi tenglamalar soni noma’lum kontur toklar soniga teng.

Koeffitsiyentlarni aniqlaymiz:

$$R_{11} = R_2 + R_4 + R_5 = 3 \; \Omega;$$
$$R_{22} = R_5 = 1 \; \Omega;$$
$$R_{12} = R_{21} = R_5 = 1 \; \Omega;$$
$$R_{13} = -R_2 = -1 \; \Omega;$$
$$R_{14} = -R_2 = -1 \; \Omega;$$

$$R_{23} = 0;$$
$$R_{24} = 0;$$
$$E_{11} = 0;$$
$$E_{22} = E_6 = 1 \text{ V}.$$

Koeffitsiyentlarni tenglamalarga qo‘yib, ozod hadlarni tenglamaning o‘ng tomoniga o‘tkazamiz:

$$\left.\begin{aligned} 3I_{11} + I_{22} &= 2, \\ I_{11} + I_{22} &= 1. \end{aligned}\right\}$$

Bundan $I_{11}=0{,}5$ A, $I_{22}=0{,}5$ A.

Shoxobchalardagi haqiqiy toklarni aniqlaymiz:

$$I_2 = I_{33} + I_{44} - I_{11} = 1{,}5 \text{ A},$$
$$I_4 = I_{11} = 0{,}5 \text{ A},$$
$$I_5 = I_{11} + I_{22} = 1 \text{ A},$$
$$I_6 = I_{22} = 0{,}5 \text{ A}.$$

1.9.3. Tugun potensiallar usuli.

Zanjirni tugun potensiallar usuli yordamida hisoblash tartibi quyidagicha:

1) shoxobchalardagi toklarga ixtiyoriy yo‘nalish beriladi;

2) istalgan bitta tugunning potensiali nolga teng deb qabul qilinadi;

3) har bir shoxobchaning noma’lum toki Om qonuni bo‘yicha ifodalanadi;

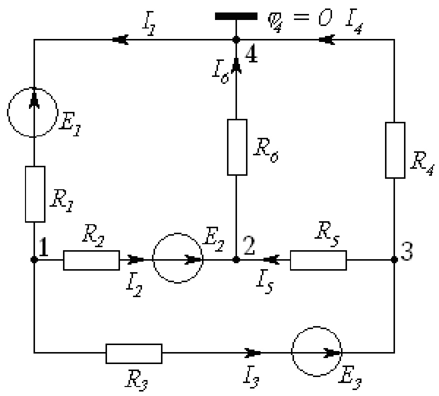
4) zanjirning qolgan tugunlari potensiallari uchun Kirxgofning 1- qonuni asosida va 3- banddan foydalanib, noma’lum potensiallarga nisbatan tenglamalar sistemasi tuziladi;

5) tenglamalar sistemasini yechib, tugunlar potensiallari topiladi;

6) 3- bandda yozilgan Om qonuni yordamida shoxobchalardagi noma’lum toklar topiladi.

1.38- masala. 1.33- rasmda keltirilgan zanjir shoxobchalaridagi toklar tugun potensiallar usuli yordamida topilsin.

E_1 = 6 V, E_2 = 12 V, E_3 = 18 V, $R_1 = R_2 = R_3$ = 2 Ω, $R_4 = R_5 = R_6$=6 Ω.



1.33- rasm.

Yechish. Shoxobchalardagi toklar yo‘nalishini ixtiyoriy tanlab olamiz va 4-tugun potensialini nolga tenglaymiz. Keyin boshqa tugunlar potensiallari uchun quyidagi tenglamalar sistemasini tuzamiz:

$$\varphi_1\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)-\varphi_2\frac{1}{R_2}-\varphi_3\frac{1}{R_3}=-E_1\frac{1}{R_1}-E_2\frac{1}{R_2}-E_3\frac{1}{R_3},$$

$$-\varphi_1\left(\frac{1}{R_2}\right)+\varphi_2\left(\frac{1}{R_2}+\frac{1}{R_5}+\frac{1}{R_6}\right)-\varphi_3\frac{1}{R_5}=E_2\frac{1}{R_2},$$

$$-\varphi_1\frac{1}{R_3}-\varphi_2\frac{1}{R_5}+\varphi_3\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)=E_3\frac{1}{R_3},$$

Hosil bo‘lgan sistemada chap tomonidagi tenglamalarning asosiy diagonalidagi koeffitsiyentlarning ishoralari musbat, qolgan koeffitsiyentlar esa manfiy olinadi. Sistemaning o‘ng tomonidagi koeffitsiyentlarning ishoralari qaralayotgan tugunga nisbatan EYKlarning yo‘nalishlariga bog‘liq bo‘ladi: agar EYK yoki tok manbayi yo‘nalishi tugunga yo‘nalgan bo‘lsa, u holda EYK yoki tok koeffitsiyenti musbat, aks holda esa manfiy ihora bilan olinadi.

Sistemani yechib tugunlar potensiallarini topamiz:

$$\varphi_1=-9\text{ V};\quad \varphi_2=3\text{ V};\quad \varphi_3=6\text{ V}.$$

Shoxobchalardagi toklarni Om qonuni asosida aniqlaymiz:

$$I_1=(\varphi_4-\varphi_1-E_1)\frac{1}{R_1}=(-\varphi_1-E_1)\frac{1}{R_1}=(9-6)\cdot\frac{1}{2}=1{,}5\text{ A},$$

$$I_2=(\varphi_1-\varphi_2+E_2)\frac{1}{R_2}=(-9-3+12)\cdot\frac{1}{2}=0,$$

$$I_3=(\varphi_1-\varphi_3+E_3)\frac{1}{R_3}=(-9-6+18)\cdot\frac{1}{2}=1{,}5\text{ A},$$

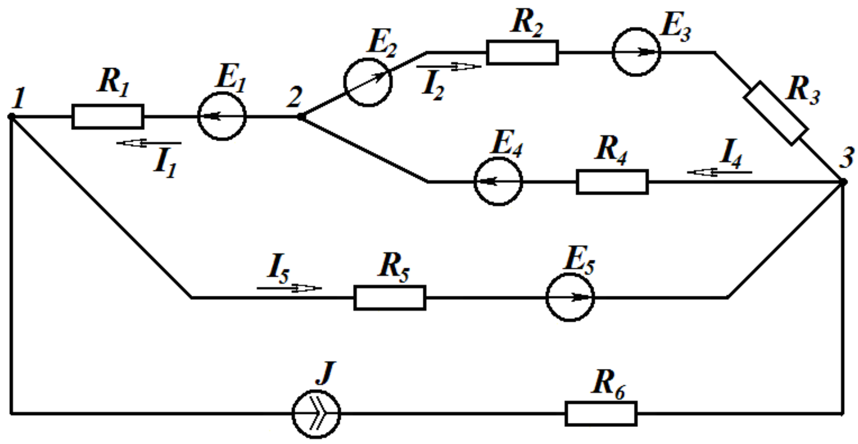
$$I_4=(\varphi_3-\varphi_4)\frac{1}{R_4}=(6-0)\frac{1}{6}=1\text{ A},$$

$$I_5=(\varphi_3-\varphi_2)\frac{1}{R_5}=(6-3)\frac{1}{6}=0{,}5\ \text{A},$$

$$I_6=(\varphi_2-\varphi_4)\frac{1}{R_6}=(3-0)\frac{1}{6}=0{,}5\ \text{A}.$$

1.39- masala. Berilgan zanjir uchun (1.34- rasm) shoxobchalardagi toklarni tugun potensiallari usulida aniqlansin.

Berilgan: $E_1=12\,\text{V},\ E_2=E_3=3\ \text{V},\ E_4=30\,\text{V},\ E_5=2\ \text{V},\ J=2\ \text{A},$
$R_1=R_4=R_5=R_6=2\ \Omega,\ R_2=R_3=1\ \Omega.$



1.34- rasm.

Yechish. Zanjirda uchta tugun mavjud. Bitta tugun potensialini, masalan 3- tugun potensialini $\varphi_3=0$ deb qabul qilib, kanonik tenglamalar sistemasini tuzamiz:

$$\left.\begin{aligned}G_{11}\varphi_1-G_{12}\varphi_2&=I_{11}\\-G_{21}\varphi_1+G_{22}\varphi_2&=I_{22}\end{aligned}\right\},$$

bu tenglamalarda $G_{11}=\frac{1}{R_1}+\frac{1}{R_5}=0{,}5+0{,}5=1\ \text{S}$ – 1- tugunda jamlangan barcha shoxobchalar o‘tkazuvchanliklarining yig‘indisi;

$G_{22}=\frac{1}{R_1}+\frac{1}{R_2+R_3}+\frac{1}{R_4}=1{,}5\ \text{S}$ – 2- tugunda jamlangan shoxobchalar o‘tkazuvchanligining yig‘indisi;

$G_{12}=G_{21}=\frac{1}{R_1}=0{,}5\ \text{S}$ – 1- va 2- tugunlar orasidagi o‘zaro o‘tkazuvchanlik;

$I_{11} = \frac{E_1}{R_1} - \frac{E_5}{R_5} - J = \frac{12}{2} - \frac{2}{2} - 2 = 3$ A – 1- tugun uchun hisoblangan tugun toki;

$I_{22} = -\frac{E_1}{R_1} - \frac{E_2 + E_3}{R_2 + R_3} + \frac{E_4}{R_4} = -\frac{12}{2} - \frac{6}{2} + \frac{30}{2} = 6$ A – 2 - tugun uchun hisoblangan tugun toki.

Yuqoridagi son qiymatlarini qo‘yib, quyidagi tenglamalar sistemasiga kelamiz:

$$\left.\begin{aligned} \varphi_1 - 0{,}5\varphi_2 &= 3, \\ -0{,}5\varphi_1 + 1{,}5\varphi_2 &= 6. \end{aligned}\right\}$$

Bundan $\varphi_1 = 6$ V; $\varphi_2 = 6$ V.

Shoxobchalardagi noma’lum toklarni Om qonuni bo‘yicha aniqlaymiz:

$$I_1 = \frac{\varphi_2 - \varphi_1 + E_1}{R_1} = \frac{6 - 6 + 12}{2} = 6 \text{ A},$$

$$I_2 = \frac{\varphi_2 - \varphi_3 + E_2 + E_3}{R_2 + R_3} = \frac{6 - 0 + 3 + 3}{1 + 1} = 6 \text{ A},$$

$$I_4 = \frac{\varphi_3 - \varphi_2 + E_4}{R_4} = \frac{0 - 6 + 30}{2} = 12 \text{ A},$$

$$I_5 = \frac{\varphi_1 - \varphi_3 + E_5}{R_5} = \frac{6 - 0 + 2}{2} = 4 \text{ A}..$$

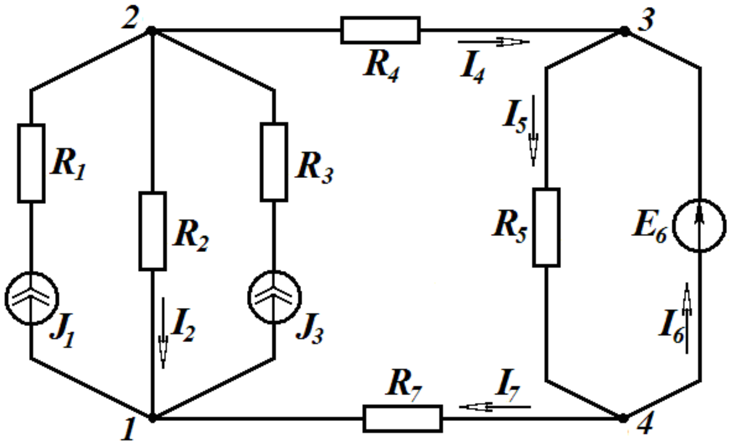
Masalaning to‘g‘ri yechilganligini bilish maqsadida ko‘rilayotgan zanjir uchun quvvatlar balansini tuzamiz:

$$E_1 I_1 + (E_2 + E_3) I_2 + E_4 I_4 + E_5 I_5 - U_{13} J = I_1^2 R_1 + I_2^2 (R_2 + R_3) + I_4^2 R_4 + I_5^2 R_5$$

yoki $12 \cdot 6 + (3 + 3) \cdot 6 + 30 \cdot 12 + 2 \cdot 4 - 6 \cdot 2 = 6^2 \cdot 2 + 6^2 \cdot (1 + 1) + 12^2 \cdot 2 + 4^2 \cdot 2$

ya’ni $464 = 464.$

1.40- masala. Zanjir shoxobchalaridagi (1.35- rasm) toklarni tugun potensiallar usulida aniqlansin. Bunda $J_1 = J_3 = 1$ A, $E_6 = 1$ V, $R_1 = R_2 = R_3 = R_5 = 1$ Ω, $R_4 = R_7 = 0{,}5$ Ω.



1.35- rasm.

Yechish. Zanjirdagi to‘rtinchi tugun potensialini $\varphi_4 = 0$, deb qabul qilamiz. Ichki qarshilik $r_6 = 0$ bo‘lganligi sababli $\varphi_3 = \varphi_4 + E_6 = 1$ V bo‘ladi. Shunday qilib, qolgan ikkita (birinchi va ikkinchi) tugunning potensiallarini aniqlash lozim. Tugun potensiallari tenglamalari sistemasini tuzamiz:

$$\left.\begin{aligned} G_{11}\varphi_1 - G_{12}\varphi_2 - G_{13}\varphi_3 &= I_{11}, \\ -G_{21}\varphi_1 + G_{22}\varphi_2 - G_{23}\varphi_3 &= I_{22}. \end{aligned}\right\}$$

Uchinchi tenglamani tuzishga ehtiyoj yo‘q, chunki uning potensiali ma’lum. Tuzilgan tenglamalarning koeffitsiyentlarini aniqlaymiz:

$$G_{11} = \frac{1}{R_2} + \frac{1}{R_7} = 3 \text{ S},$$

$$G_{22} = \frac{1}{R_2} + \frac{1}{R_4} = 3 \text{ S},$$

$$G_{12} = G_{21} = \frac{1}{R_2} = 1 \text{ S}, \quad G_{13} = 0,$$

$$G_{23} = \frac{1}{R_4} = \frac{1}{0{,}5} = 2 \text{ S},$$

$$I_{11} = -J_1 - J_3 = -2 \text{ A}, \; I_{22} = J_1 + J_3 = 2 \text{ A}.$$

Bu koeffitsiyentlarni tenglamalarga qo‘yib, ma’lum hadlarni o‘ng tomonga o‘tkazamiz:

$$\left.\begin{aligned} 3\varphi_1 - \varphi_2 &= -2 \\ -\varphi_1 + 3\varphi_2 &= 4 \end{aligned}\right\}$$

bundan $\varphi_1 = -0{,}25$ V, $\varphi_2 = 1{,}25$ V.

Shoxobchalardagi toklarni Om qonuni bo‘yicha aniqlaymiz:

$$I_2 = \frac{\varphi_2 - \varphi_1}{R_2} = \frac{1{,}25 - (-0{,}25)}{1} = 1{,}5 \text{ A},$$

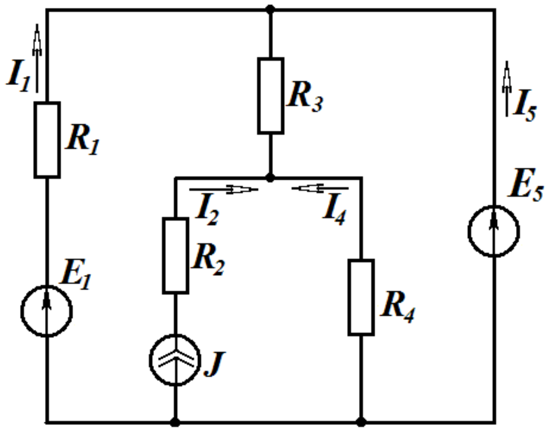
$$I_4 = \frac{\varphi_2 - \varphi_3}{R_4} = \frac{1{,}25 - 1}{0{,}5} = 0{,}5 \text{ A},$$

$$I_7 = \frac{\varphi_4 - \varphi_1}{R_7} = \frac{0 - (-0{,}25)}{0{,}5} = 0{,}5 \text{ A},$$

$$I_5 = \frac{\varphi_3 - \varphi_4}{R_5} = \frac{1 - 0}{1} = 1 \text{ A},$$

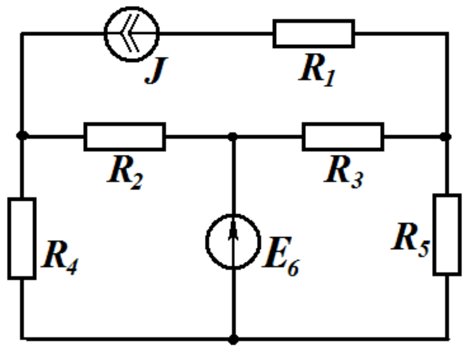
I_6 tokni 3- yoki 4- tugunlar uchun Kirxgofning qonunidan aniqlaymiz: $I_6 = I_5 - I_4 = 1 - 0{,}5 = 0{,}5$ A.

1.41- masala. Zanjir shoxobchalaridagi (1.36- rasm) toklarni tugun potensiallari usulida aniqlansin. $E_1 = E_5 = 4$ V, $J = 2$ A, $R_1 = R_2 = R_3 = R_4 = 1$ Ω.



1.36- rasm.

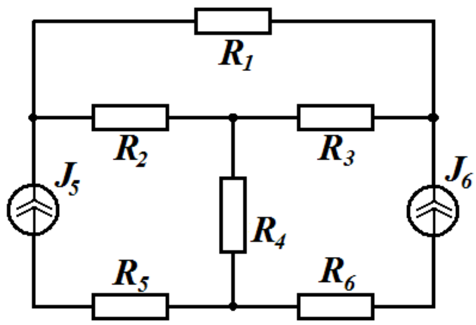
1.42- masala. Zanjir shoxobchalaridagi (1.37- rasm) toklarni tugun potensiallari usulida aniqlansin, bunda $E_6 = 48$ V, $J = 4$ A, $R_1 = 10\,\Omega$, $R_2 = 3$ Ω, $R_3 = 4$ Ω, $R_5 = 12$ Ω.



1.37- rasm

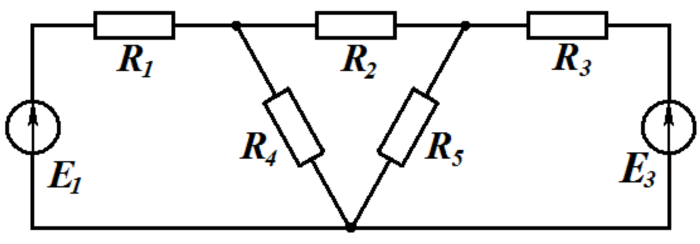
1.43- masala. Zanjir (1.38- rasm) shoxobchalaridagi toklarni tugun potensiallar usulida aniqlansin, bunda

$J_5 = 1$ A, $J_6 = 3$ A, $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 3\ \Omega$.



1.38- rasm.

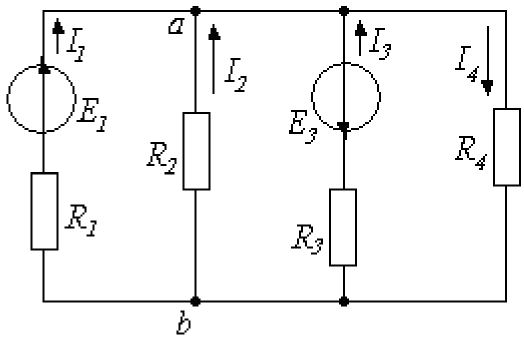
1.44- masala. Zanjir (1.39- rasm) shoxobchalaridagi toklarni tugun potensiallar usulida aniqlansin, bunda $E_1 = 8$ V, $E_3 = 12$ V, $R_1 = R_4 = 1\ \Omega$, $R_2 = R_5 = 2\ \Omega$, $R_3 = 3\ \Omega$.



1.39- rasm.

1.9.4. Ikki tugun usuli.

Ba’zi hollarda zanjirda faqat ikkita tugun bo‘lib, konturlar soni ko‘p bo‘ladi (1.40- rasm). Bunday sxemalarni hisoblash uchun eng qulay bo‘lgan usul - tugun potensiallar usulining xususiy holi – ikki tugun usulidan foydalaniladi. Bunda tenglamalar soni T–1=2 –1 = 1 tani tashkil etadi.



1.40– rasm.

"b" nuqtani tayanch tugun deb olib, uni shartli ravishda yerga ulaymiz. Bunda faqat "a" nuqtaning potensialini aniqlasak, masalani yechish uchun yetarli bo‘ladi. Buning uchun parallel ulangan shoxobchalarni bitta ekvivalent manba bilan almashtiramiz. Berilgan sxema uchun a va b tugunlar orasidagi kuchlanish:

$$U_{ab} = \frac{\Sigma E_k G_k + \Sigma I_k}{\Sigma G_k}.$$

Kuchlanishni hisoblaganimizdan keyin har bir shoxobchadagi tokni Om qonuniga binoan topamiz:

$$I_n = G_n\left(E_n - U_{ab}\right),$$

bunda n — parallel shoxobchalar soni.

1.45- masala. 1.40- rasmda berilgan elektr zanjiridagi toklarni ikki tugun usuli yordamida aniqlansin va quvvatlar balansini tekshirilsin. $E_1 = 110$ V, $E_3 = 48$ V, R_1 = 2 Ω, R_2 = 4 Ω, R_3 = 2 Ω, R_4 = 10 Ω.

Yechish.

$$U_{ab} = \frac{G_1E_1 - G_3E_3}{G_1 + G_2 + G_3 + G_4} = \frac{0{,}5\cdot 110 - 0{,}5\cdot 48}{0{,}5 + 0{,}25 + 0{,}5 + 0{,}1} = \frac{31}{1{,}35} = 22{,}96\ \text{V},$$

$$I_1 = \frac{E_1 - U_{ab}}{R_1} = \frac{110 - 22{,}96}{2} = \frac{87{,}04}{2} = 43{,}52\ \text{A},$$

$$I_2 = \frac{-U_{ab}}{R_2} = \frac{-22{,}96}{4} = -5{,}74\ \text{A},$$

$$I_3 = \frac{-E_3 - U_{ab}}{R_3} = \frac{-48 - 22{,}96}{2} = \frac{-70{,}96}{2} = -35{,}48\ \text{A},$$

$$I_4 = \frac{E_4 + U_{ab}}{R_4} = \frac{0 + 22{,}96}{10} = 2{,}3\ \text{A}.$$

Zanjirdagi E_1 manba a tugunga yo‘nalgan, shuning uchun yuqoridagi tenglamada E_1G_1 ko‘paytma musbat ishora bilan, E_3 manba esa b tugundan yo‘nalgan, shuning uchun E_3G_3 ko‘paytma manfiy ishora bilan olingan.

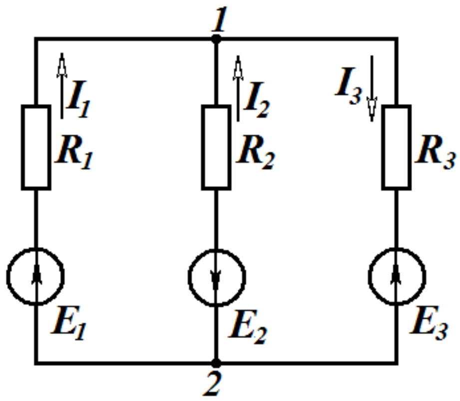
Quvvatlar balansini tekshirish uchun iste’molchi va manba quvvatlarini hisoblaymiz:

$$P_{\text{ist}} = R_1I_1^2 + R_2I_2^2 + R_3I_3^2 + R_4I_4^2 = 2\cdot(43{,}52)^2 + 4\cdot(-5{,}74)^2 +$$
$$+ 2\cdot(-35{,}48)^2 + 10\cdot(-2{,}29)^2 = 6489{,}87 \approx 6490\ \text{W},$$

$$P_{\text{manba}} = E_1I_1 - E_3I_3 = 110\cdot 43{,}52 - 48(-35{,}48) = 6490{,}2\ \text{W}.$$

Demak, $P_{\text{manba}} \approx P_{\text{ist}}$.

1.46- masala. Zanjirdagi toklar (1.41- rasm) ikki tugun usulida aniqlansin. $E_1 = 6$ V, $E_2 = 4$ V, $E_3 = 8$ V, $R_1 = 1\ \Omega$, $R_2 = R_3 = 2\ \Omega$.



1.41- rasm

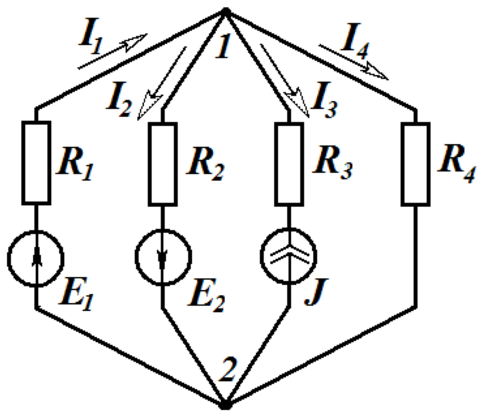
Yechish. Tugunlar orasidagi kuchlanishni topamiz:

$$U_{12} = \frac{\sum EG}{\sum G} = \frac{\frac{E_1}{R_1} + \frac{E_2}{R_2} - \frac{E_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{\frac{6}{1} + \frac{4}{2} - \frac{8}{2}}{\frac{1}{1} + \frac{1}{2} + \frac{1}{2}} = 2\ \text{V}.$$

Shoxobchadagi toklarni Om qonuni bo‘yicha aniqlaymiz:

$$I_1 = \frac{E_1 - U_{12}}{R_1} = \frac{6-2}{1} = 4 \text{ A},$$
$$I_2 = \frac{E_2 - U_{12}}{R_2} = \frac{4-2}{2} = 1 \text{ A},$$
$$I_3 = \frac{E_3 + U_{12}}{2} = \frac{8+2}{2} = 5 \text{ A}.$$

1.47- masala. Zanjirdagi (1.42- rasm) toklar ikki tugun usulida aniqlansin. $E_1 = E_2 = 4\,\text{V},\ J = 6\,\text{A},\ R_1 = R_4 = 2\,\Omega,\ R_2 = R_3 = 1\,\Omega.$



1.42- rasm.

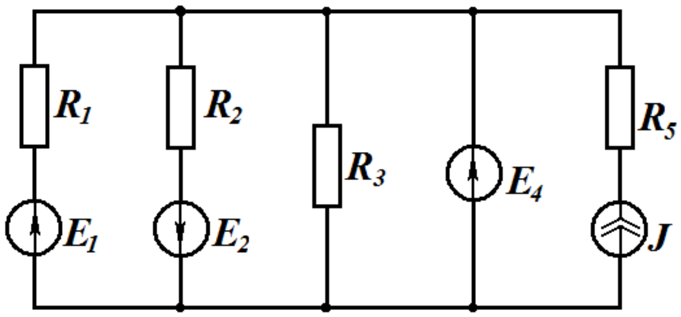
Yechish. Tugunlar orasidagi kuchlanishni topamiz:

$$U_{12} = \frac{\sum EG + \sum J}{\sum G} = \frac{\frac{E_1}{R_1} - \frac{E_2}{R_2} + J}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}} = 2 \text{ V}.$$

Shoxobchalardagi toklar

$$I_1 = \frac{E_1 - U_{12}}{R_1} = \frac{4-2}{2} = 1 \text{ A},\ I_2 = \frac{E_2 + U_{12}}{R_2} = \frac{4+2}{1} = 6 \text{ A},\ I_4 = \frac{U_{12}}{R_4} = \frac{2}{2} = 1 \text{ A}.$$

1.48- masala. Zanjirdagi (1.43- rasm) toklar ikki tugun usulida aniqlansin. $E_1 = E_2 = E_4 = 4\,\text{V},\ J = 1\,\text{A},\ R_1 = R_2 = R_3 = R_5 = 1\,\Omega$



1.43- rasm

1.49- masala. 1.37- masalani ikki tugun usulida yeching.

1.50- masala. 1.57- masalani ikki tugun usulida yeching.

1.51- masala. 1.59- masalani ikki tugun usulida yeching.

1.52- masala. Ichki qarshiklari mos ravishda $R_{01} = 0{,}6\,\Omega$, $R_{02} = 0{,}4\,\Omega$ va EYKlari $E_1 = E_2 = 120\,\text{V}$ bo‘lgan ikkita generator umumiy yuklamasi $100\,\Omega$ bo‘lgan liniyaga parallel ulangan. Har bir generator toki aniqlansin.

1.9.5. Ustma-ustlash usuli.

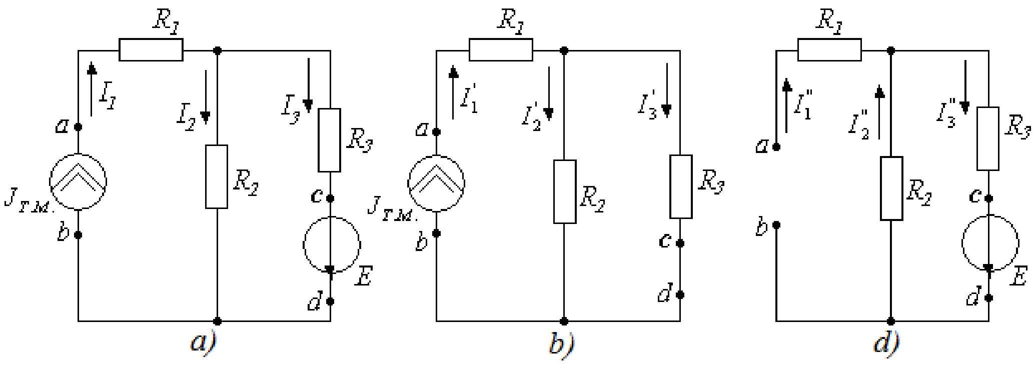
Murakkab elektr zanjirni ustma-ustlash usulida hisoblash quyidagi ketma-ketlikda bajariladi:

a) har bir EYK yoki tok manbayi ta’siridan shoxobchalarda hosil bo‘lgan xususiy toklar aniqlanadi, bunda fikran sxemada yagona EYK yoki tok manbayi qoldirilib, boshqa EYKlar va tok manbalari olib tashlanadi va ularning ichki qarshiliklari zanjirga manbalar o‘rniga ulangan deb qabul qilinadi;

b) shoxobchalardagi haqiqiy toklar esa alohida hisoblangan xususiy toklarning algebraik yig‘indisiga teng bo‘ladi.

1.53- masala. 1.44- rasmdagi sxema uchun ustma-ustlash usuli yordamida shoxobchalardagi toklarni aniqlansin va quvvatlar balansini tekshirilsin.

Berilgan: $R_1 = 2\ \Omega$, $R_2 = 4\ \Omega$, $R_3 = 6\ \Omega$, $J_{TM} = 5$ A, $E = 24$ V.



1.44- rasm.

Yechish. a) shoxobchalardagi toklar yo‘nalishini ixtiyoriy belgilaymiz, EYK manbayini sxemadan chiqarib tashlab, *c* va *d* nuqtalarni birlashtiramiz. EYK manbayining ichki qarshiligi $r_{\text{ich}} = 0$ (1.44-rasm, *b*). Shu sxema uchun shoxobchalarda tok manbayi ta’siridan hosil bo‘lgan xususiy toklarni aniqlaymiz:

$$I_1^{'} = J_{\text{TM}} = 5\text{ A},\ I_2^{'} = I_1^{'}\frac{R_2R_3}{(R_2+R_3)R_2} = 3\text{ A},\ I_3^{'} = I_1^{'}\frac{R_2R_3}{(R_2+R_3)R_3} = 2\text{ A}.$$

Endi EYK manbayi ta’siridan zanjir shoxobchalaridagi xususiy toklarni aniqlaymiz. Bunda tok manbayi sxemadan olib tashlangan hamda *a* va *b* nuqtalar orasi uzilgan, chunki tok manbayining ichki o‘tkazuvchanligi $G_{\text{ich}} = 0$ ga teng (1.44- rasm, *d*). Demak, 1.44- rasm, *d* uchun:

$$I_1^{''} = 0;\quad I_2^{''} = I_3^{''} = \frac{E}{R_2+R_3} = \frac{24}{4+6} = 2{,}4\ \text{A}.$$

b) ustma-ustlash prinsipidan foydalanib har bir shoxobchadagi haqiqiy toklarni aniqlaymiz:

$$I_1 = I_1^{'} + I_1^{''} = 5+0 = 5\text{ A},\ \ I_2 = I_2^{'} - I_2^{''} = 3-2{,}4 = 0{,}6\text{ A},$$

$$I_3 = I_3^{'} + I_3^{''} = 2+2{,}4 = 4{,}4\text{ A}.$$

a nuqtaning potensiali:

$$\varphi_a = \varphi_b + R_2I_2 + R_1I_1,\ U_{ab} = \varphi_a - \varphi_b = R_2I_2 + R_1I_1 = 4\cdot 0{,}6 + 2\cdot 5 = 12{,}4\text{ V}.$$

Tok manbayi quvvati:

$$P_{ab} = U_{ab}J_{TM} = 12{,}4\cdot 5 = 62{,}0\ \text{W}.$$

EYK manbayi quvvati esa

$$EI_3 = 20\cdot 4{,}4 = 88\text{ W}.$$

Quvvatlar balansi:

$$P_{\text{ist}} = P_{\text{manba}} = R_1 I_1^2 + R_2 I_2^2 + R_3 I_3^2 = U_{ab} J_{\text{TM}} + E I_3 .$$

yoki

$$P_{\text{ist}} = 2 \cdot 25 + 4 \cdot 0{,}36 + 6 \cdot 19{,}36 = 50 + 1{,}44 + 116{,}16 = 167{,}6\ \text{W},$$

$$P_{\text{manba}} = U_{ab} J_{\text{TM}} + E I_3 = 12{,}4 \cdot 5 + 24 \cdot 4{,}4 = 62 + 105{,}6 = 167{,}6\ \text{W}.$$

1.54- masala. 1.45- rasmda keltirilgan zanjir shoxobchalaridagi toklar ustma-ustlash usulida aniqlansin. E = 5 V, J = 1 A, $R_1 = R_2$ = 2 Ω, $R_3 = R_4$ = 3 Ω.

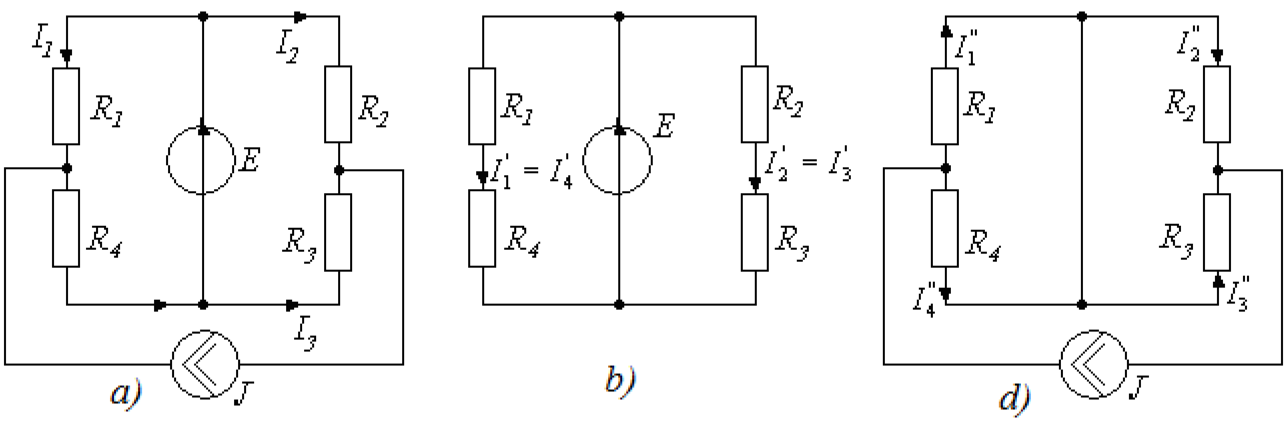
Yechish. Berilgan zanjirdagi toklarni aniqlash uchun ikkita alohida zanjir toklarini hisoblash kerak (1.45-*b* va *d* rasmlar). 1.45- rasmdagi toklar:

$$I_1' = I_4' = E/(R_1 + R_4) = 5/(2+3) = 1\ \text{A},$$
$$I_2' = I_3' = E/(R_2 + R_3) = 5/(3+2) = 1\ \text{A}.$$

EYK manbayi fikran olib tashlangani tufayli R_1 va R_4, R_2 va R_3 qarshiliklar parallel ulangan, shu sababli J tok manbayidan shoxobchalarda hosil bo‘lgan qismiy toklar R_1 va R_4 yoki R_2 va R_3 lar orqali hisoblanadi:

$$I_1'' = J\frac{R_4}{R_1 + R_4} = 1 \cdot \frac{3}{3+2} = 0{,}6\ \text{A}, \quad I_4'' = J\frac{R_1}{R_1 + R_4} = 1 \cdot \frac{2}{5} = 0{,}4\ \text{A},$$

$$I_2'' = J\frac{R_3}{R_2 + R_3} = 1 \cdot \frac{2}{2+3} = 0{,}4\ \text{A}, \quad I_3'' = J\frac{R_2}{R_2 + R_3} = 1 \cdot \frac{3}{2+3} = 0{,}6\ \text{A}.$$



1.45- rasm.

Berilgan sxema shoxobchalaridagi toklar alohida ajratilgan sxemalarning mos shoxobchalaridagi toklarning algebraik yig‘indisiga teng:

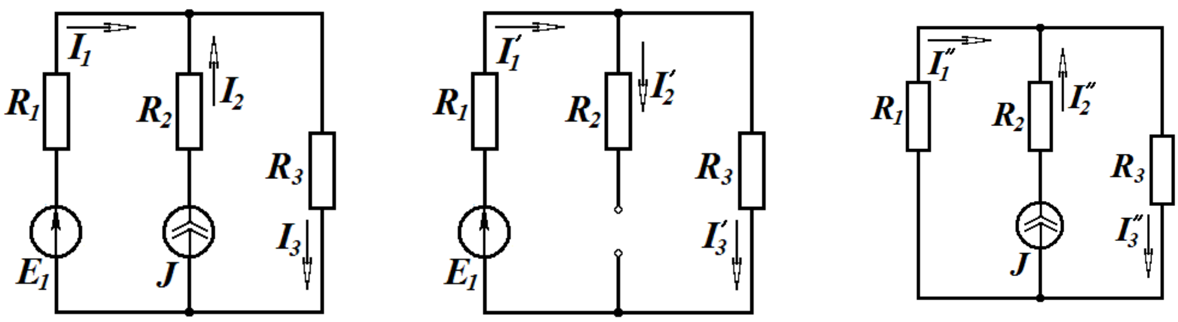
$$I_1 = I_1^{'} - I_1^{''} = 1 - 0{,}6 = 0{,}4 \text{ A}; \;\; I_4 = I_4^{'} + I_4^{''} = 1 + 0{,}4 = 1{,}4 \text{ A};$$

$$I_2 = I_2^{'} + I_2^{''} = 1 + 0{,}4 = 1{,}4 \text{ A}; \quad I_3 = -I_3^{'} + I_3^{''} = -1 + 0{,}6 = -0{,}4 \text{ A}.$$

1.55- masala. 1.46- rasm, *a* da keltirilgan zanjir shoxobchalaridagi toklar ustma-ustlash usuli bilan aniqlansin. Berilgan: $E_1 = 16$ V, $J = 2$ A, $R_1 = R_2 = R_3 = 2\,\Omega$.

Yechish. Zanjirdagi haqiqiy toklarni (1.46- rasm, *b* va 1.46- rasm, *d*) har bir energiya manbayi hosil qiladigan toklarning algebraik yig‘indisi sifatida aniqlaymiz.

1.46- rasm, *b* dagi zanjirga faqat EYK E_1 ta’sir etadi. Tok manbasining ichki qarshiligi cheksiz katta bo‘lgani uchun J tok manbasi fikran olib tashlangan.



a) *b)* *d)*

1.46- rasm.

$$I_1' = I_3' = \frac{E_1}{R_1 + R_3} = \frac{16}{4} = 4 \text{ A}.$$

1.46- rasm, *d* dagi zanjirga faqat J tok manbasi ta’sir etadi, ichki qarshiligi 0 ga teng bo‘lgan EYK manbasini fikran olib tashlaymiz va qolgan zanjirdagi toklarni aniqlaymiz:

$$I_1'' = I_2'' \cdot \frac{R_3}{R_1 + R_3} = 2 \cdot \frac{2}{2+2} = 1 \text{ A}. \qquad I_3'' = I_2'' - I_1'' = 1 \text{ A}.$$

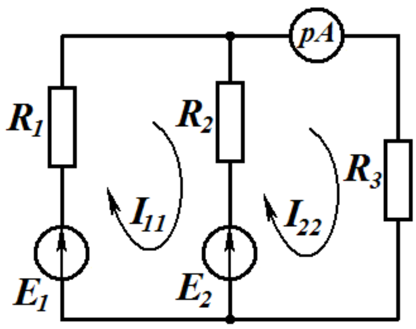
Zanjirdagi haqiqiy toklarni ustma-ustlangan toklarning algebraik yig‘indi sifatida aniqlaymiz:

$I_1 = I_1' - I_1'' = 3$ A; $I_2 = -I_2' + I_2'' = 2$ A; $I_3 = I_3' + I_3'' = 5$ A.

Qarshilik R_3 da ajralayotgan quvvatni hisoblaymiz:

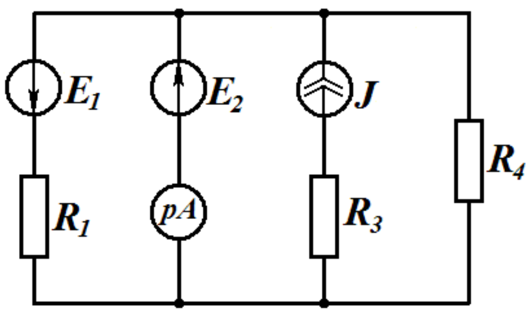
$P_3 = I_3^2 R_3 = 5^2 \cdot 2 = 50$ W.

1.56- masala. Zanjirdagi (1.47- rasm) ampermetr ko‘rsatishini ustma-ustlash usulida topilsin. $E_1 = 52$ V, $E_2 = 48$ V, $R_1 = 4$ Ω, $R_2 = 8$ Ω, $R_3 = 10$ Ω.

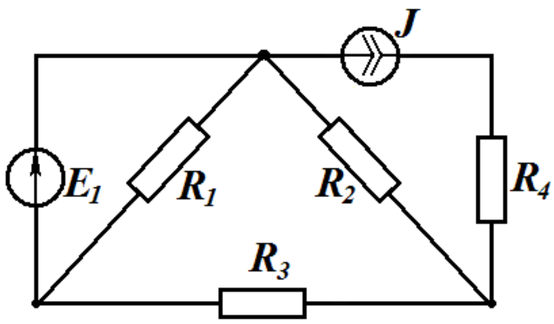


1.47- rasm.

1.57- masala. Ustma-ustlash usuli yordamida zanjirdagi (1.48- rasm) ampermetr ko‘rsatishini topilsin. Bunda $E_1 = E_2 = 4$ V, $J = 2$ A, $R_1 = R_3 = R_4 = 1$ Ω, $R_2 = 2$ Ω.



1.48- rasm.

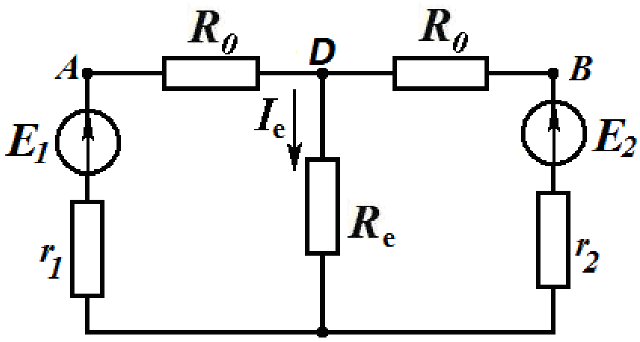


1.49- rasm.

1.58- masala. Ustma-ustlash usulida zanjirdagi (1.49- rasm) I_3 tok aniqlansin. $E_1 = 60$ V, $J = 6$ A, $R_1 = R_2 = R_3 = 3$ Ω, $R_4 = 6$ Ω.

1.59- masala. 1.38- masalani ustma-ustlash usulida yeching.

1.60- masala. Bir xil quvvatga ega *A* va *D* nimstansiyalar uzunligi 20 km bo‘lgan elektrlashtirilgan temir yo‘l uchastkasining oxirlarida joylashgan (1.50- rasm). Yuklama bo‘lmaganda tortuvchi nimstansiyalarning shinalaridagi nominal kuchlanish $U = E_1 = E_2 = 3300$ V. Nimstansiyalarning kirish qarshiliklari $r_1 = r_2 = 0{,}5\,\Omega$. Kontakt tarmog‘i va relslar barcha uchastkasining har bir kilometriga keltirilgan qarshiligi bir xil va $R_0 = 0{,}2$ Ω/km ni tashkil etadi. Agar elektrovozning nominal toki 410 A bo‘lsa unga tushayotgan kuchlanish va nimstansiyalarning toklari aniqlansin. *A* nimstansiya va elektrovoz turgan *D* nimstansiya oralig‘i 5 km ni tashkil etadi.



1.50- rasm.

1.9.6. Ekvivalent generator usuli.

Bu usul yordamida tokni aniqlash quyidagi ketma-ketlikda bajariladi:

a) elektr zanjir sxemasidan toki aniqlanishi kerak bo‘lgan shoxobchasini fikran uzib, uning uzilgan *ab* qismalaridagi kuchlanish $U_{abs.\text{ish}}$ aniqlanadi;

b) ab qismalariga nisbatan ichki EYK manbalari qisqa tutashtirilgan holda zanjirning kirish qarshiligi R_{kir} hisoblanadi;

d) shoxobchadan o‘tayotgan tok quyidagi formula yordamida topiladi:

$$I = \frac{U_{ab.\text{sish.}}}{R_{ab} + R_{\text{kir}}}.$$

Agar *ab* shoxobchadagi qarshilikni nolga tenglasak (R_{ab} = 0), u holda sxemada qisqa tutashish rejimi hosil bo‘ladi va shoxobchadan o‘tuvchi tok qisqa tutashish tokiga teng bo‘ladi ($I_{\text{q.t}}$). Yuqoridagi formulada R_{ab} bo‘lganida

$$I_{q.t} = \frac{U_{abs.\text{ish}}}{R_{\text{kir}}} \text{ yoki } R_{\text{kir}} = \frac{U_{ab.\text{s.ish}}}{I_{\text{q.t}}}.$$

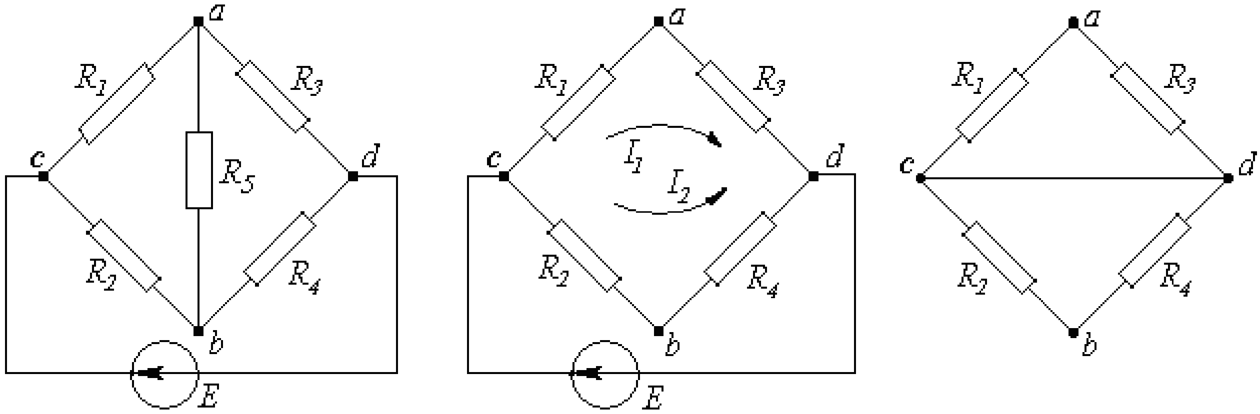
topiladi.

Oxirgi formuladan ko‘rinib turibdiki, $U_{ab.\text{s.ish}}$ va R_{kir} larni aniqlash uchun oddiy tajriba usulidan foydalanish mumkin.

Buning uchun *ab* qismalaridagi salt ish rejimidagi kuchlanish $U_{ab.\text{s.ish}}$ o‘lchanadi va zanjir qisqa tutashish rejimiga o‘tkazilib, shoxobchadagi qisqa tutashish toki $I_{\text{q.t}}$ aniqlanadi, keyin esa R_{kir} hisoblanadi.

1.61- masala. 1.51- rasm, a da keltirilgan ko‘prik sxemaning *ab* diagonalidagi tokni ekvivalent generator usuli yordamida aniqlansin Berilgan:
$R_1 = R_4 = 1\ \Omega,\ \ R_2 = 4\ \Omega,\ R_3 = 2\ \Omega,\ \ R_5 = 2\ \Omega,\ E = 10\,\text{V}.$

Yechish. *ab* shoxobchani uzamiz (1.51- rasm, b) va salt ish rejimi uchun U_{ab} kuchlanishni aniqlaymiz:



a) *b)* *d)*

1.51- rasm.

$$\varphi_a = \varphi_b + R_2 I_2 - R_1 I_1 = \varphi_b + \frac{R_2 E}{R_2 + R_4} - \frac{R_1 E}{R_1 + R_3},$$

$$U_{ab\text{s.ish}} = \varphi_a - \varphi_b = \left(\frac{R_2}{R_2 + R_4} - \frac{R_1}{R_1 + R_3}\right)E = \left(\frac{4}{4+1} - \frac{1}{1+2}\right)10 \approx 4{,}67\ \text{V}.$$

ab qismalariga nisbatan EYK manbayi qisqa tutashtirilgan holdagi kirish qarshiligini aniqlaymiz (1.51-rasm, *d*). Bu holda sxemadagi *c* va *d* nuqtalar o‘zaro qisqa tutashgan bo‘ladi. Shuni hisobga olsak:

$$R_{\text{kir}} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4} = \frac{1\cdot 2}{1+2} + \frac{4\cdot 1}{4+1} \approx 1{,}47\,\Omega,$$

ab shoxobchadagi tok esa

$$I=\frac{U_{ab.\text{s.ish}}}{R_5+R_{\text{kir}}}=\frac{4{,}67}{2+1{,}47}=\frac{4{,}67}{3{,}47}=1{,}34\text{ A.}$$

1.62- masala. Berilgan zanjirdagi (1.52- rasm, *a*) I_5 tok ekvivalent generator usulida aniqlansin. Bunda

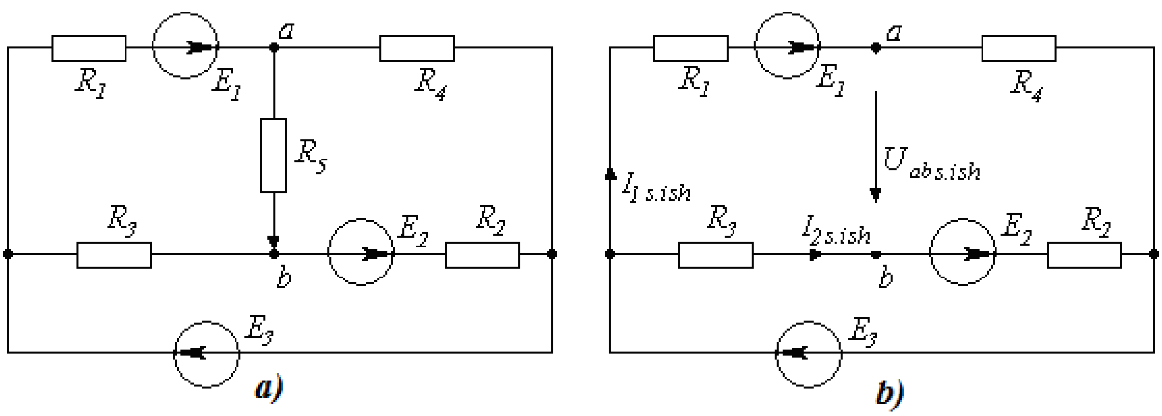
$E_1=50\text{ V};\ E_2=80\text{ V};\ E_3=100\text{ V};\ R_4=20\,\Omega;\ R_1=10\,\Omega;$
$R_2=20\,\Omega;\ R_3=40\,\Omega;\ R_5=20\,\Omega.$

Yechish. Tokni aniqlash kerak bo‘lgan shoxobchani uzamiz va uzilish nuqtalari *ab* orasidagi $U_{ab.\text{s.ish}}$ kuchlanishni aniqlaymiz. Hosil bo‘lgan sxema 1.52- rasm, *b* da keltirilgan. Kirxgofning ikkinchi qonunidan:

$$E_3+E_1=(R_4+R_1)I_{1\text{s.ish.}},\ E_3+E_2=(R_3+R_2)I_{2\text{s.ish.}}$$

1.52- rasm, b dagi zanjir shoxobchalaridagi toklarni aniqlaymiz:

$$I_{1\text{s.ish.}}=\frac{(E_3+E_1)}{(R_4+R_1)}=\frac{100+50}{20+10}=5\text{ A},$$
$$I_{2\text{s.ish.}}=\frac{(E_3+E_2)}{(R_3+R_2)}=\frac{(100+80)}{(40+20)}=3\text{ A.}$$

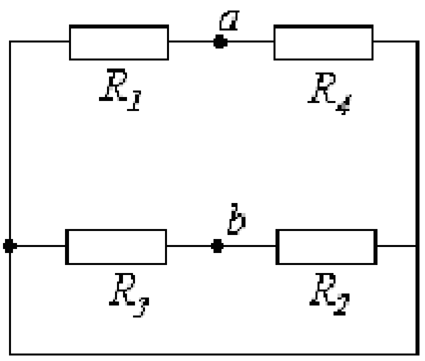


1.52- rasm.

2. Salt ish kuchlanishi:

$$U_{ab.\text{s.ish.}}=\varphi_a-\varphi_b=E_1-R_1I_{1\text{s.ish.}}+R_3I_{2\text{s.ish.}}=50-10\cdot5+40\cdot3=120\text{ V.}$$

Uzilgan nuqtalarga nisbatan butun zanjirning kirish qarshiligini aniqlaymiz. Bunda uchala EYK manbalarini fikran qisqa tutashtiramiz (1.53- rasm).



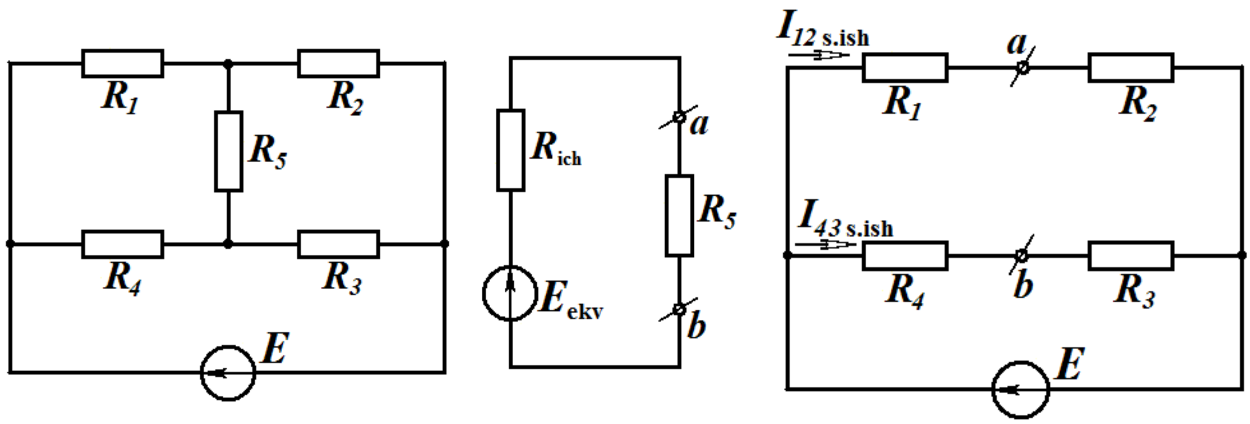
1.53- rasm.

$$R_{kir} = \frac{R_1 R_4}{R_1 + R_4} + \frac{R_2 R_3}{R_2 + R_3} = \frac{10 \cdot 20}{10 + 20} + \frac{20 \cdot 40}{20 + 40} = 20\,\Omega.$$

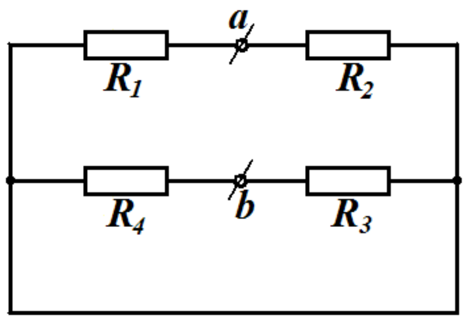
3. Izlanayotgan tokni hisoblaymiz:

$$I_5 = \frac{U_{ab.s.ish.}}{R_{kir} + R_5} = \frac{120}{20 + 20} = 3\text{ A}.$$

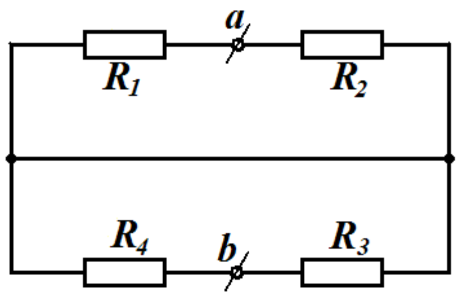
1.63- masala. Agar zanjir parametrlari (1.54- rasm, *a*) $E = 48\text{ V}$, $R_1 = R_2 = R_3 = 4\,\Omega$, $R_4 = 12\,\Omega$, $R_5 = 1\,\Omega$ bo‘lsa I_5 tokni ekvivalent generator usulida aniqlasin. Rezistor R_5 o‘zgarganda undagi P_5 quvvatning o‘zgarish qonuniyati qanday bo‘ladi?



a) *b*) *d*)



e)



f)

1.54- rasm.

Yechish. Qarshiligi R_5 ga teng bo‘lgan shoxobchani uzamiz. Zanjirning qolgan qismini ekvivalent generator ko‘rinishida (1.54- rasm, *b*) tasvirlaymiz. Ekvivalent generator EYKini aniqlash uchun 1.54- rasm, *d* zanjirdagi *a* va *b* qismalar orasidagi kuchlanishni aniqlaymiz, bunda $E_{\text{ekv}} = U_{ab\text{s.ish.}}$

Avval toklarni aniqlaymiz (tokdagi indekslar qarshiliklar nomerlariga mos):

$$I_{12\text{s.ish.}} = \frac{E}{R_1 + R_2} = \frac{48}{4+4} = 6\text{ A},\quad I_{43\text{s.ish.}} = \frac{E}{R_4 + R_3} = \frac{48}{12+4} = 3\text{ A}.$$

$U_{ab\text{s.ish.}}$ kuchlanishni aniqlaymiz:

$$U_{ab\text{s.ish.}} = \varphi_a - \varphi_b;$$

bundan $U_{ab\text{s.ish.}} = R_4 I_{43\text{s.ish.}} - R_1 I_{12\text{s.i.sh}} = 12\cdot 3 - 4\cdot 6 = 12\text{ V}$.

Ekvivalent generatorning $R_{\text{ich.}}$ ichki qarshiligi zanjirning (1.54- rasm, *e*) *a* va *b* qismalar orasidagi kirish qarshiligiga teng. Zanjirdan ajratib tashlangan EYK manbayi o‘rnida qisqa tutashtirilgan qism qolgan.

a va *b* qismalarga nisbatan kirish qarshiligi (1.54- rasm, *f*):

$$R_{\text{kir}ab} = \frac{R_1 R_2}{R_1 + R_2} + \frac{R_4 R_3}{R_4 + R_3} = \frac{4\cdot 4}{4+4} + \frac{12\cdot 4}{12+4} = 5\,\Omega.$$

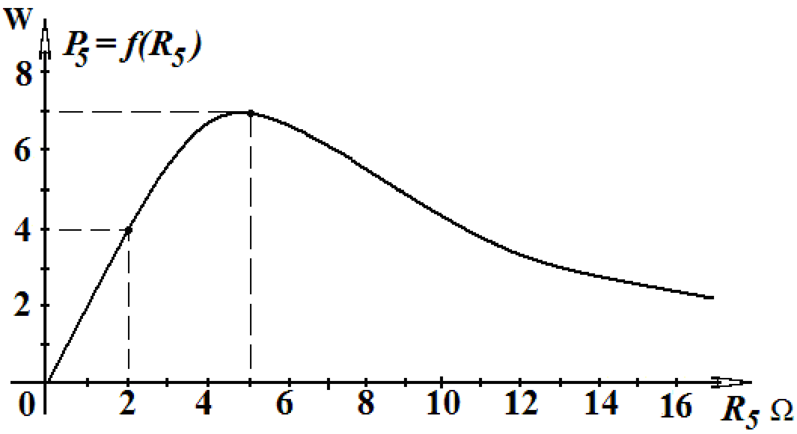
Izlanayotgan tok:

$$I_5 = \frac{U_{ab\text{s.ish.}}}{R_{\text{kir}ab} + R_5} = \frac{12}{5+1} = 2\text{ A}.$$

Qarshilik R_5 o'zgarganda undan ajraladigan quvvatni topish uchun $P_5 = I_5^2 R_5$ formuladan foydalanamiz. R_5 qarshilik qiymatlarini o'zgartirib, ularga mos bo'lgan tok va quvvatlarni ushbu jadvalga kiritamiz:

R_5	0	1	3	5	7	19	∞
I_5, A	2,4	2,0	1,5	1,2	1,0	0,5	0
P_5, W	0	4	6,75	7,2	7	4,75	0

$P_5 = f(R_5)$ grafigi 1.55- rasmda tasvirlangan.



1.55- rasm

1.64- masala. Zanjirdagi (1.56- rasm, *a*) I_4 tokni ekvivalent generator usulida aniqlang. $J_1 = J_3 = 1$ A, $E_6 = 1$ V, $R_1 = R_2 = R_3 = R_4 = R_5 = 1\,\Omega$.

Yechish. Qarshiligi R_4 bo'lgan shoxobchani ajratamiz (1.56- rasm, *b*) va $U_{abs.\text{ish.}}$ kuchlanishni aniqlaymiz:

$U_{abs.\text{ish.}} = \varphi_a - \varphi_b;$, bundan

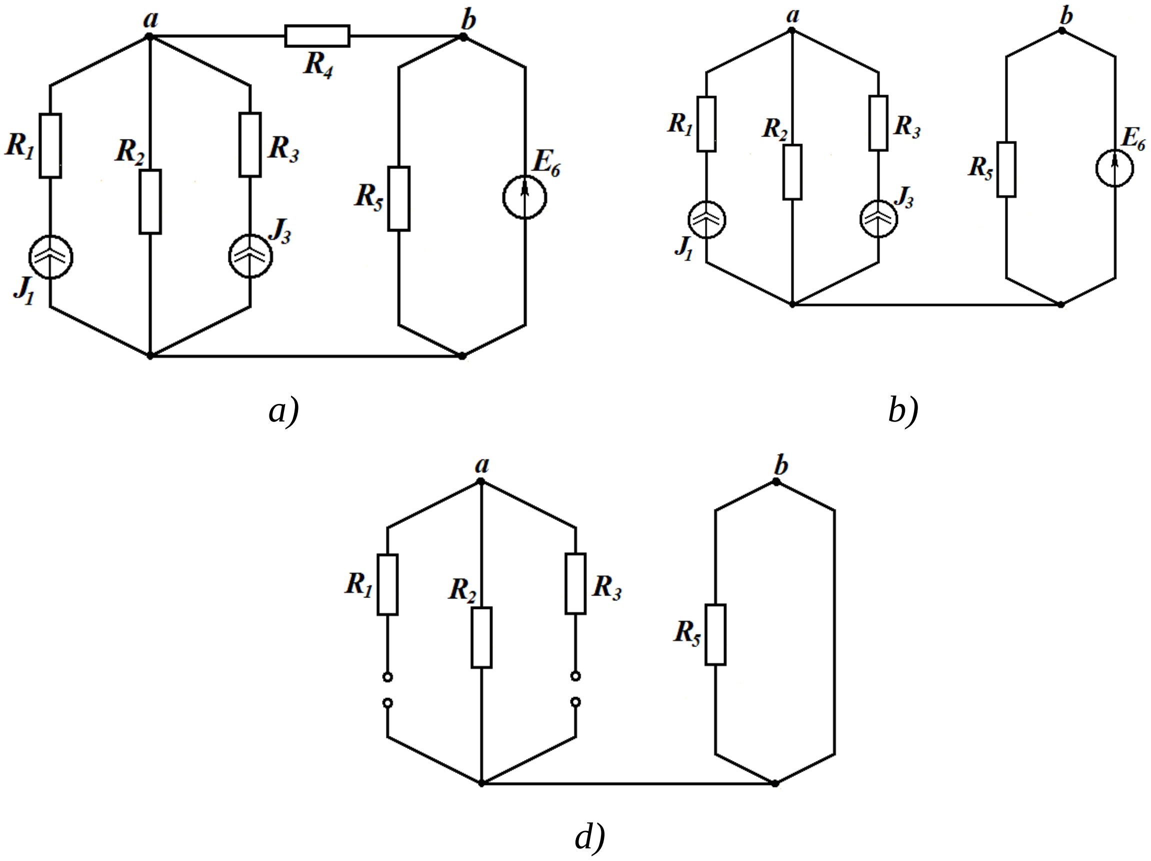
$U_{abs.\text{ish.}} = -E_6 + R_2(J_1 + J_3) = -1 + 1(1+1) = 1\,\text{V}.$

Kirish qarshiligini 1.56- rasm, *d* asosida topamiz.

$R_{\text{kir.}ab} = R_2 = 1\,\Omega.$

Shunday qilib,

$$I_4 = \frac{U_{abs.\text{ish.}}}{R_{\text{kir}\,ab} + R_4} = \frac{1}{1+1} = 0{,}5\text{ A}.$$



1.56- rasm.

1.65- masala. 1.50- masaladagi zanjir uchun tarkibida E_4 EYK manbayi bo‘lgan shoxobchadagi tokni ekvivalent generator usulida aniqlang.

1.66- masala. 1.32- rasmda tasvirlangan zanjirdagi I_5 tokni ekvivalent generator usulida aniqlang.

1.67- masala. 1.46- rasm, *a* da tasvirlangan zanjirdagi I_3 tokni ekvivalent generator usulida aniqlang.

1.68- masala. 1.6- rasmda tasvirlangan zanjirdagi ampermetr ko‘rsatishini ekvivalent generator usulida aniqlang.

1.69- masala. 1.15- rasmda keltirilgan zanjirdagi I_2 tokni ekvivalent generator usulidan foydalanib aniqlang.

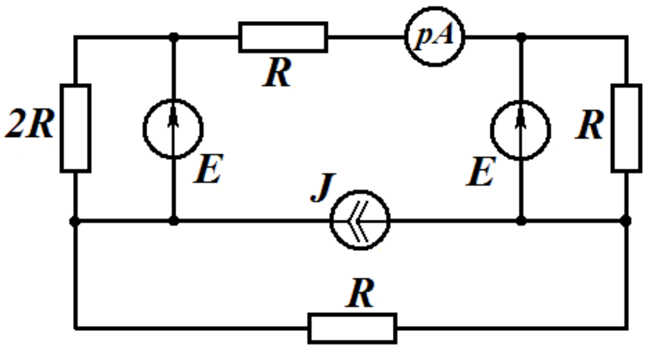
1.70- masala. 1.46- rasmda keltirilgan zanjirning ampermetr ko‘rsatishini ekvivalent generator usulidan foydalanib aniqlang.

1.71- masala. 1.34- rasmda keltirilgan zanjirning I_1 tokini ekvivalent generator usulidan foydalanib aniqlang.

1.72- masala. 1.36- rasmda keltirilgan zanjirning R_3 orqali o‘tadigan tokini ekvivalent generator usulidan foydalanib aniqlang.

1.73- masala. 1.39- rasmda keltirilgan zanjirning R_2 orqali o‘tadigan tokini ekvivalent generator usulidan foydalanib aniqlang.

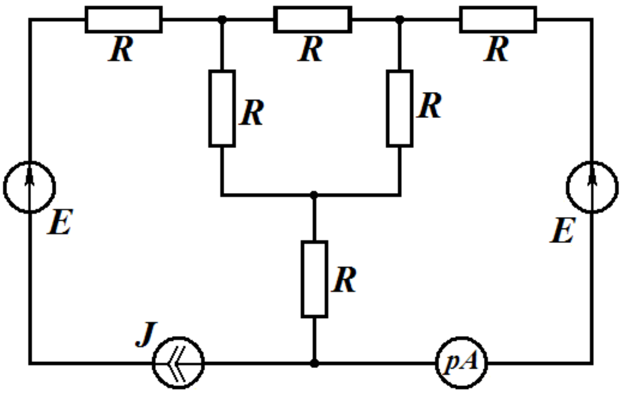
1.74- masala. 1.57- rasmda tasvirlangan zanjirga ulangan ampermetr ko‘rsatishini ekvivalent generator usulidan foydalanib aniqlang. Bunda $E = ?$, $R = ?$, $J = 4$ A.



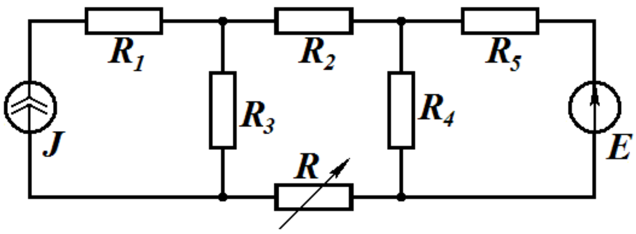
1.57- rasm.

1.75- masala. 1.58- rasmda tasvirlangan zanjirga ulangan ampermetr ko‘rsatishini ekvivalent generator usulidan foydalanib aniqlang. Bunda $E = 4$ V, $J = 3$ A, $R = 3\,\Omega$.

1.76- masala. Zanjirdagi (1.59- rasm) rostlanadigan R qarshilikning qanday qiymatida unda ajraladigan quvvat maksimum qiymatga ega bo‘ladi? Berilgan: $E = 48$ V, $J = 4\,A$, $R_1 = R_2 = R_3 = 2\,\Omega$, $R_4 = 12\,\Omega$, $R_5 = 4\,\Omega$.

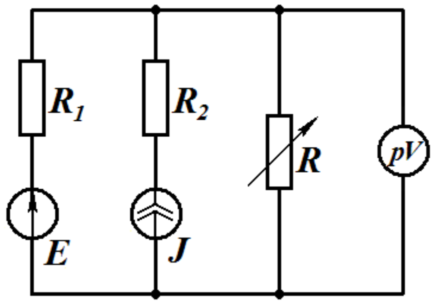


1.58- rasm.



1.59- rasm

1.77-masala. Zanjirdagi (1.60-rasm) R qarshilik $10\ \Omega$ dan $5\,\Omega$ gacha o‘zgarganda voltmetr ko‘rsatishi $20\,\mathrm{V}$ dan $15\ \mathrm{V}$ ga o‘zgargan. Agar $R = 25\,\Omega$ bo‘lsa, R qarshilikdagi tok aniqlansin.

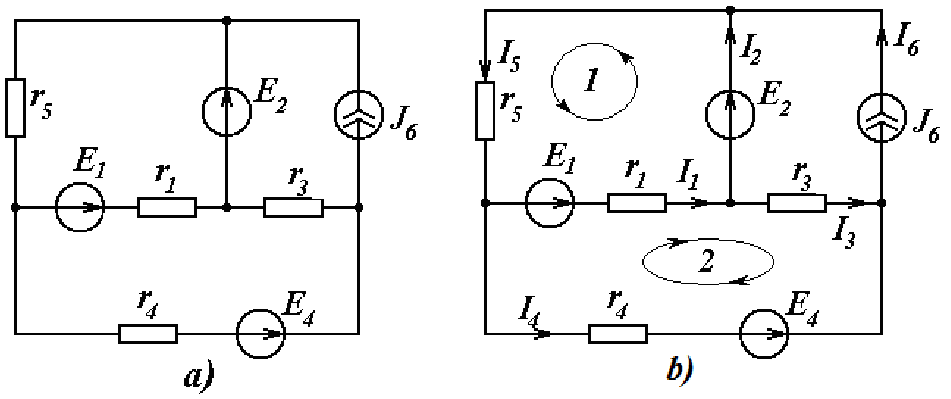


1.60- rasm.

1.3.vazifa. Kirxgof qonunlari bo‘yicha o‘zgarmas tok zanjirlarini hisoblash

1.78- masala. Kirxgof qonunlaridan foydalanib 1.61- rasm, *a* da tasvirlangan sxema shoxobchalaridagi toklarni aniqlang. Sxema parametrlari quyidagicha: E_1=40 V; E_2=20 V; E_4=10 V; J_6=3 A; r_1=5 **Ω**; r_3=5 **Ω**; r_4=20 **Ω**;

r_5=10 Ω .



1.61- rasm. 1.80- masalaga boshlang‘ich (*a*) va hisobiy (*b*) sxemalar.

Yechish. Zanjir oltita shoxobchadan iborat (n_m=6). 1, 2, 4-shoxobchalarda EYK manbalari, 6- shoxobchada esa tok manbayi mavjud. Zanjir uchta bog‘lanmagan tugunga ega. Shoxobchalardagi toklar yo‘nalishini tanlab olamiz (1.61- rasm, *b*) va 1,2,3- tugunlar uchun Kirxgofning birinchi qonuni bo‘yicha tenglamalar sistemasini tuzamiz:

$$\left.\begin{aligned} -I_2+I_5&=J_6,\\ I_1+I_4-I_5&=0,\\ -I_3-I_4+J_6&=0. \end{aligned}\right\} \tag{1}$$

Sxemada uchta bog‘lanmagan kontur mavjud, demak Kirxgofning ikkinchi qonuni bo‘yicha uchta tenglama tuzish mumkin. Lekin 6-shoxobchadagi tok tok manbayi tokiga teng bo‘lganligi sababli, 1- va 2- konturlar uchun tenglama tuzish kifoya. Kontur toklar yo‘nalishlarini ixtiyoriy tanlab olib (1.61- rasm, *b*), Kirxgof tenglamalarini tuzamiz:

$$\left.\begin{aligned} r_1I_1+r_5I_5&=E_1+E_2,\\ r_1I_1+r_3I_3-r_4I_4&=E_1-E_4. \end{aligned}\right\} \tag{2}$$

Kirxgof qonunlariga ko‘ra tuzilgan (1) va (2) tenglamalar sistemalari matritsa shaklida quyidagicha ifodalanadi:

$$\begin{vmatrix} 0 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & -1 & 0 \\ r_2 & 0 & 0 & 0 & r_5 \\ r_1 & 0 & r_3 & -r_4 & 0 \end{vmatrix} \times \begin{vmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{vmatrix} = \begin{vmatrix} J_6 \\ 0 \\ -J_6 \\ E_1+E_2 \\ E_1-E_4 \end{vmatrix} . \qquad (3)$$

Bu tenglamaga zanjir parametrlarini qo‘yib quyidagi matritsa ko‘rinishidagi tenglamaga kelamiz:

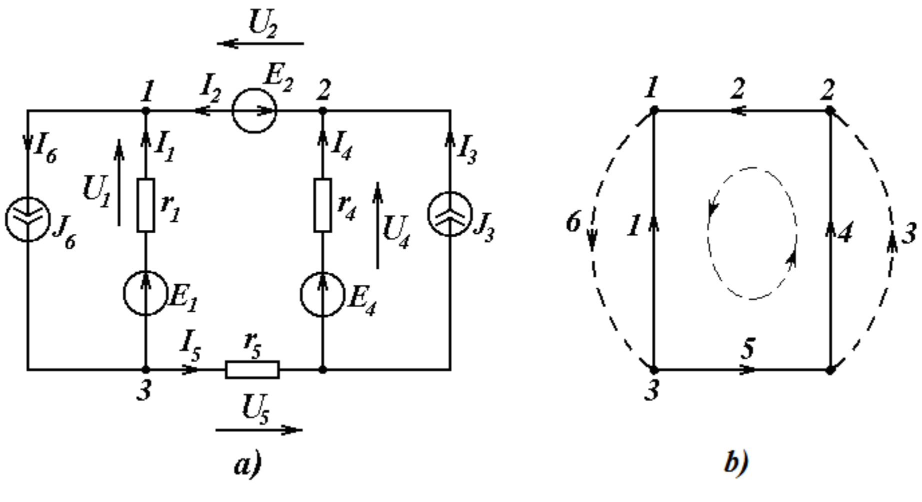
$$\begin{vmatrix} 0 & -1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & -1 & 0 \\ 5 & 0 & 0 & 0 & 10 \\ 5 & 0 & 5 & -20 & 0 \end{vmatrix} \times \begin{vmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{vmatrix} = \begin{vmatrix} 3 \\ 0 \\ -3 \\ 60 \\ 30 \end{vmatrix} . \qquad (3a)$$

(3a) tenglamani yechib toklar qiymatlarini hosil qilamiz:

I_1=3,88 A; I_2=1,06 A; I_3=2,83 A; I_4=0,176 A; I_5=4,058 A.

O‘zgarmas tok murakkab zanjirlarini hisoblash

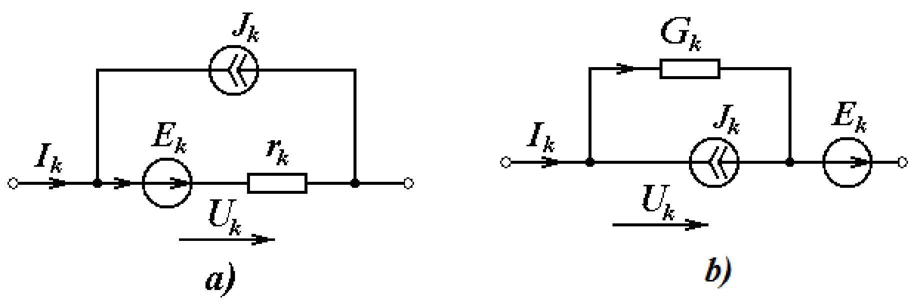
1.79- masala. 1.62- rasm, *a* da keltirilgan zanjir uchun topologik graf tuzilsin. Matritsa shaklida Kirxgof tenglamalari yozilsin va shoxobchalardagi toklar hisoblansin. Zanjir elementlarining parametrlari quyidagicha: E_1=1 V; E_2=5 V; E_4=9 V; J_3=3 A; J_6=6 A; R_1=1 Ω; R_4=2 Ω; R_5=3 Ω.



1.62- rasm.

Yechish. Masalani yechishdan avval quyidagilarni bayon qilamiz. Zanjirning topologik grafini qurganda *umumlashgan shoxobcha* tushunchasidan foydalaniladi. Bunday shoxobchaga kuchlanish manbayi E_k, tok manbasi J_k, qarshlik r_k yoki o'tkazuvchanlik G_k kiradi. 1.63- rasm, *a* da tasvirlangan shoxobcha parallel ulangan ideal tok manbayi J_k va ichki qarshiligi r_k bo'lgan real kuchlanish manbayi E_k dan iborat. Bunda ushbu shoxobcha uchun quyidagi tenglama o'rinli:

$$U_k + E_k = (I_k + J_k)R_k.$$



1.63- rasm. Ketma-ket-parallel (*a*) va parallel-ketma-ket (*b*) umumlashgan shoxobchalar.

Shunga o'xshash, 1.62- rasm, *a* va *b* dagi shoxobcha parallel ulangan ideal kuchlanish manbayidan va ichki o'tkazuvchanligi G_k bo'lgan real tok manbayi J_k dan iborat. Bunday shoxobcha uchun quyidagi tenglama o'rinli:

$$U_k + E_k = \frac{I_k + J_k}{G_k}.$$

Agar $G_k^{-1} = R_k$ bo'lsa, bunday shoxobchalar ekvivalent bo'ladi.

Masalada ko'rilayotgan zanjir oltita shoxobchadan iborat. 1-,2-,4- shoxobchalarga E_1, E_2, E_4 EYK manbalari, 3-,6- shoxobchalarga J_3, J_6 tok manbalari ulangan. Topologik grafni tuzishda shuni e'tiborga olish joizki, 1-,6- va 3-,4- shoxobchalar ikkita umumlashgan sxemalarga mosdir (1.63- rasm, *a*), 2- va 5- shoxobchalar esa umumlashgan to'liq bo'lmagan shoxobchalardir. Shunga ko'ra asosiy graf shoxobchalaridagi toklar yo'nalishi asosiy sxema shoxobchalaridagi toklar yo'nalishiga mosdir.

Ma'lumki, bu grafdan bitta shoxobchasini olib tashlansa, grafda konturlar paydo bo'lmaydi. 4- shoxobcha olib tashlangandan keyingi qismi *graf daraxti*, olib tashlangan 4- shoxobcha esa *bog'lanish shoxobchasi* deyiladi.

Bu sxemani hisoblash uchun to'rtta tenglama tuzish kerak bo'ladi. Bunda yagona bog'lanmagan kontur uchun Kirxgofning ikkinchi qonuni bo'yicha faqat bitta tenglama tuzish mumkin, boshqa uchta tenglamalarni tugunlar uchun, masalan 1-,2-,3- tugunlar uchun Kirxgofning birinchi qonuni bo'yicha tuziladi. Graf asosida tuzilgan tenglamalar quyidagi kurinishga ega:

$$\left.\begin{array}{c} -I_1 - I_2 + J_6 = 0, \\ I_2 - I_3 - I_4 = 0, \\ I_1 + I_5 - J_6 = 0, \\ -U_1 + U_2 + U_4 + U_5 = 0. \end{array}\right\}$$

Tuzilgan tenglamalar zanjir grafi tuzilishini (strukturasini) aks ettirganligi uchun *topologik* tenglamalar deyiladi. Lekin ular zanjir toklarini hisoblash uchun yaramaydi, chunki ular turli o'zgaruvchilar – shoxobchalarning toki va kuchlanishlari uchun yozilgan. Bu tenglamalarni yechish uchun kuchlanishlarni shoxobchalar toki va qarshiliklari orqali ifodalanishi kerak. Bunday tenglamalarni umumlashgan shoxobchalarning tenglamalaridan foydalanib ifodalash mumkin. 1.62- rasm, *a* dagi shoxobchalar uchun quyidagi tenglamalarni yozish mumkin:

$$\left.\begin{array}{c} U_1 = r_1 I_1 - E_1, \\ U_2 = E_2, \\ U_4 = r_4 I_4 - E_4, \\ U_5 = r_5 I_5, \\ I_6 = J_6, \\ I_3 = J_3. \end{array}\right\}$$

Hosil bo'lgan tenglamalarni topologik tenglamalarga qo'yib, zanjir toklari uchun to'ldirilgan tenglamalar sistemasini yozishimiz mumkin.

$$\left\{\begin{array}{c} -I_1 - I_2 = -J_6, \\ I_2 - I_4 = I_3, \\ I_1 + I_5 = J_6, \\ -r_1 I_1 + r_4 I_4 + r_5 I_5 = -E_1 - E_2 + E_4. \end{array}\right.$$

Bu sistemaning matritsa shaklidagi tenglamasi quyidagi ko‘rinishga ega:

$$\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 \\ -r_1 & 0 & r_4 & r_5 \end{vmatrix} \times \begin{vmatrix} I_1 \\ I_2 \\ I_4 \\ I_5 \end{vmatrix} = \begin{vmatrix} -J_6 \\ I_3 \\ J_6 \\ -E_1 - E_2 + E_4 \end{vmatrix}$$

Elementlar parametrlarini bu tenglamalarga qo‘yib zanjirning matritsali tenglamasini hosil qilamiz:

$$\begin{vmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 1 \\ -1 & 0 & 2 & 3 \end{vmatrix} \times \begin{vmatrix} I_1 \\ I_2 \\ I_4 \\ I_5 \end{vmatrix} = \begin{vmatrix} -6 \\ 3 \\ 6 \\ 3 \end{vmatrix}$$

Tenglamalar sistemasini yechib zanjir toklarni aniqlaymiz:

$$I_1 = 3{,}5\,\mathrm{A}; \quad I_2 = 2{,}5\,\mathrm{A}; \quad I_4 = -0{,}5\,\mathrm{A}; \quad I_5 = 2{,}5\,\mathrm{A}.$$

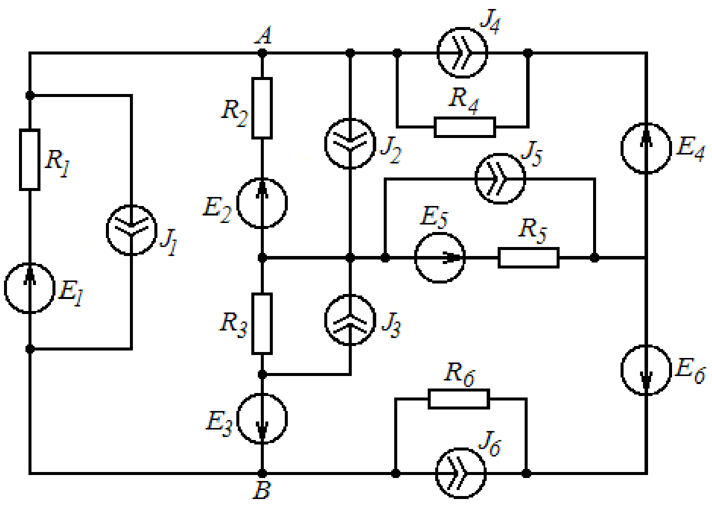
I_4 tokdagi “–” ishora tokning haqiqiy yo‘nalishi sxemada qabul qilinganiga qarama-qarshi ekanligini bildiradi.

1.3- vazifa. O‘zgarmas tok murakkab zanjirni hisoblash

1.64- rasmdagi zanjir uchun quyidagilar bajarilsin:

1. 1.3- jadvalda berilgan parametrlardan foydalanib elektr zanjirning hisobiy sxemasini tuzing;
2. Zanjirning barcha shoxobchalaridagi toklarni topish uchun Kirxgof tenglamalar sistemasini tuzing;
3. Zanjirning barcha shoxobchalaridagi toklarni kontur toklar usulida hisoblang;
4. Zanjirda uchburchak ulangan uchta qarshilikni tanlang va ularni ekvivalent yulduz ulangan sxema bilan almashtiring;
5. Hosil bo‘lgan zanjirni tugun potensiallar usulida hisoblang;
6. R_6 qarshilikdan o‘tadigan tokni ekvivalent generator usulida hisoblang;
7. Zanjirning *A* va *B* nuqtalari orasidagi kuchlanishni aniqlang;

8. Berilgan zanjir uchun quvvatlar balansini tuzing.



1.64- rasm. 1.3- vazifaga umumlashgan sxema.

1.3 -*jadval*

Variant	Sxema elementlari parametrlarining qiymatlari, Ω, V, A.																	
	Qarshiliklar						Kuchlanishlar						Toklar					
	R_1	R_2	R_3	R_4	R_5	R_6	E_1	E_2	E_3	E_4	E_5	E_6	J_1	J_2	J_3	J_4	J_5	J_6
1	2	1	4	5	4	3	0	0	0	10	0	15	0	0	0	0	6	0
2	4	3	1	4	5	3	10	0	16	0	0	0	0	4	0	0	0	0
3	6	5	2	1	4	5	15	20	0	0	0	0	0	0	0	8	0	0
4	1	2	7	4	3	5	0	0	0	15	0	0	0	0	0	0	5	3
5	5	4	3	4	7	8	0	0	0	0	0	22	5	0	8	0	0	0
6	7	2	5	8	3	4	18	0	0	0	0	0	0	8	0	2	0	0
7	2	3	6	7	4	5	0	0	0	0	10	20	0	0	0	3	0	0
8	4	7	4	3	6	3	20	0	12	0	0	0	0	0	0	0	0	7
9	6	1	2	3	2	5	16	0	0	21	0	0	0	12	0	0	0	0
10	9	4	7	2	3	6	0	0	0	0	10	0	0	0	0	8	0	3
11	3	2	1	4	5	4	0	0	20	0	0	0	6	0	0	0	0	4
12	7	4	5	4	3	2	18	0	0	0	0	0	0	5	0	3	0	0
13	6	1	2	5	4	3	0	0	0	15	10	0	0	0	0	0	0	6
14	4	7	4	3	8	5	20	0	12	0	0	0	0	0	0	0	0	4
15	2	3	6	5	2	3	0	14	0	20	0	0	8	0	0	0	0	0
16	3	6	7	4	1	4	0	0	0	20	0	0	0	0	0	6	0	5
17	7	2	5	6	3	2	0	0	15	0	0	0	3	0	0	0	0	9
18	5	4	3	2	7	5	0	20	0	0	0	0	4	0	0	9	0	0
19	8	3	4	7	4	5	0	0	0	0	20	10	0	0	0	3	0	0

20	4	5	8	3	6	7	0	0	17	0	0	15	5	0	0	0	0	0
21	6	1	4	5	8	3	0	14	0	10	0	0	3	0	0	0	0	0
22	5	8	3	6	3	4	0	0	0	15	0	0	0	0	0	0	0	5
23	3	4	7	8	1	6	14	0	0	0	0	0	0	0	3	0	0	4
24	7	6	5	4	3	8	0	0	0	24	0	0	2	3	0	0	0	0
25	6	3	4	7	4	5	0	0	0	0	0	20	0	0	0	3	5	0
26	2	7	2	1	8	3	0	0	18	0	0	0	7	0	0	0	0	8
27	4	5	8	3	2	9	0	20	0	0	0	0	5	0	0	7	0	0
28	3	6	7	2	5	2	0	0	0	16	0	18	0	0	0	0	4	0
29	9	4	3	6	3	4	25	0	18	0	0	0	0	0	0	0	0	5
30	7	2	9	4	3	6	0	15	0	23	0	0	3	0	0	0	0	0

1.10. Elektr zanjirlarni matritsa usulida yechish

Elektr zanjirlarni matritsa shaklida analitik ifodalash

Amaliy masalalarni yechishda elektr sxema shoxobchalarining ulanish xarakterini ikki usul bilan ifodalash mumkin: tugunlarda va bog‘lanmagan konturlarda.

Shuni aytish lozimki, shoxobchalarni bunday ulanishi o‘zaro bir-biriga bog‘liqdir. Ma’lum sxema shoxobchalarining ulanishini ifodalash uchun bog‘lanish, ya’ni *insidensiyalarning matritsasi* qo‘llaniladi. Barcha elektr kattaliklarni hisoblashda musbat yo‘nalishlarni belgilash katta ahamiyatga ega. Musbat yo‘nalishlar nafaqat shoxobchalardagi toklarni, balki sxemaning barcha EYK va kuchlanishlar yo‘nalishlarini ham belgilaydi. Musbat yo‘nalishlar shartli bo‘lib ixtiyoriy qabul qilinadi.

Shoxobchalar yo‘nalishlari belgilangan har qanday sxema *yo‘naltirilgan graf* deyiladi.

Har bir shoxobchaning yo‘nalishi u boshlang‘ich uchidan (tugundan) oxirgi uchigacha hisoblanadi.

Bog‘lanishlarning (insidensiyalarning) birinchi matritsasi.

Sxema holatining har qanday *i*- tuguni uchun Kirxgofning matritsa shaklidagi birinchi tenglamasi.

$$M\underline{I} = \underline{Y},$$

bunda $M-$ matritsa sxema tuguniga ulangan shoxobchalarning ulanishini aks ettiradi. *M* matritsa zanjir holatining birinchi tenglamasini tuzish uchun kerak bo‘ladi. Bu matritsa *bog‘lanishlarning birinchi matritsasi* deyiladi.

Shoxobcha yo‘nalishlari belgilangan bo‘lsa, *M* matritsani tuzish mumkin. Bu matritsaning har qanday *i*- qatori sxemaning bog‘lanmagan xuddi shu *i*- tartib raqamlari tuguniga to‘g‘ri keladi.

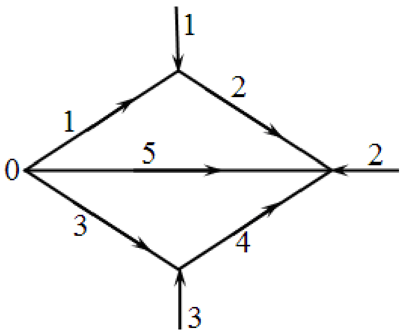
Shu matritsaning har qanday *j*- ustuni xuddi shu *j*- tartib raqamlari shoxobchaga to‘g‘ri keladi. Matritsaning *i*- qatori va *j*- ustuni kesishgan joyiga +1 yoziladi, agar *j*-shoxobcha *i*- tugun bilan boshlang‘ich uchi bilan ulangan, ya’ni yo‘nalishi $i-$ tugundan boshlangan bo‘lsa, *i*- qator va *j*- ustun kesishgan joyda “–1” qo‘yiladi, agar *j*- shoxobcha *i*- tugunga qarab yo‘nalgan, ya’ni oxirgi uchi bilan ulangan bo‘lsa. Nihoyat, *i*- qator va *j*- ustun kesishgan joyda 0 qo‘yiladi, agar *j*- shoxobcha *i*- tugun bilan bevosita bog‘lanmagan bo‘lsa.

M matritsa elektr sxemani to‘laligicha ifodalaydi va masalani analitik yechishga imkon beradi. Matritsaning har bir qatorida bittadan kam bo‘lmagan musbat yoki manfiy bir bo‘lish lozim. Aks holda bu tugun tizimning boshqa qismi bilan ulanmagan bo‘ladi. Qatordagi musbat birlar soni shu tugunga nechta shoxobcha ulanganini ko‘rsatadi. Manfiy birlar soni shu tugunga oxirgi uchlari bilan ulangan shoxobchalar soniga teng bo‘ladi.

M matritsaning har bir ustunida faqat bitta “+1” va “–1” bo‘lishi mumkin. Agar ustunda faqat bitta bir bo‘lsa (musbat yoki manfiy), bu shoxobchaning boshqa uchi bazis tugun bilan ulanganini bildiradi.

M matritsa to‘g‘ri burchakli bo‘ladi. Uning qatorlari soni sxemadagi bog‘lanmagan tugunlar soniga teng bo‘ladi, ustunlari soni esa shoxobchalar soniga teng bo‘ladi.

1.65-rasmda beshta shoxobcha va uchta bog‘lanmagan tugunga ega sxema tasvirlangan. Demak, sxemaning M matritsasi uchta qator va beshta ustunga ega bo‘lishi kerak. Agar shoxobchalarning yo‘nalishi 1.65-rasmda ko‘rsatilganidek bo‘lsa, bu matritsa quyidagi ko‘rinishga ega bo‘ladi:



$$M = \begin{array}{c} \\ \text{tugun} \end{array} \begin{array}{c} \text{Sh o x o b ch a} \\ \begin{Vmatrix} -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & -1 \\ 0 & 0 & -1 & 1 & 0 \end{Vmatrix} \end{array}$$

1.65- rasm.

Birinchi tugunga mos keladigan 1- qatorning 1- ustunida manfiy bir joylashgan, 2-ustunda esa — musbat bir, chunki birinchi tugun shoxobcha bilan birinchi shoxobcha o‘zining oxirgi uchi bilan ulangan, ikkinchi shoxobcha esa boshlang‘ich uchi bilan ulangan. Boshqa ustunlarda esa nollar joylashgan. 2-qatorda esa manfiy birlar 2-, 4-, 5- ustunlarda joylashgan, chunki mos shoxobchalar 2- tugunga oxirgi uchlari bilan ulangan va hokazo.

M matritsa tugun kattaliklari orasidagi munosabatlarni aniqlash, ya’ni bu matritsa sxemaning har bir bog‘lanmagan tugun va uning bazis tuguni orasidagi U_Δ kuchlanishlar tushishi bo‘yicha sxema shoxobchalaridagi kuchlanish pasayishlariningU_β matritsasini aniqlash uchun qo‘llaniladi. Bu munosabat quyidagicha ifodalanadi:

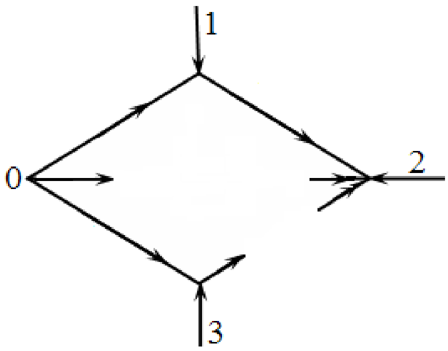
$$\underline{U}_\beta = M_t U_\Delta ,$$

bunda M_t – transponirlangan matritsa.

M_t matritsani U_Δ matritsaga o‘ng tomondan ko‘paytirilsa, sxemaning har bir shoxobchasi uchun kuchlanish ayirmasi hosil bo‘ladi, ya’ni shoxobchaning boshlang‘ich va oxirgi uchlari orasidagi kuchlanishlar farqi chiqadi.

Ochiq zanjirga kvadratli *M* matritsa to‘g‘ri keladi, chunki ochiq zanjirning har bir shoxobchasi bazis tugunni yangi tugun bilan ulaydi.

Misol uchun, 1.66- rasmdagi uchta bog‘lanmagan tugunli va uchta shoxobchali ochiq zanjirning birinchi bog‘lanish (insidensiya) matritsasi quyidagi ko‘rinishga ega bo‘ladi:



1.66- rasm.

$$M_p = \begin{array}{cc} & \text{sh o x o b ch a} \\ \text{tugun} & \begin{Vmatrix} -1 & 1 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{Vmatrix} \end{array}$$

Bu matritsa berk zanjir matritsasi kabi tuzildi.

Ochiq zanjirlarning bog‘lanish matritsasi M_p berilma toklar shoxobchalarda hosil qiladigan toklarni aniqlanishda qo‘llaniladi:

$$\underline{I} = M_p^{-1}\underline{J},$$

Xuddi shunga o‘xshash:

$$\underline{I} = C_0\overline{J},$$

chunki

$$M_p^{-1} = C_0,$$

bunda C_0 – taqsimlanish koeffitsiyentlari matritsasi.

C_0 matritsaning har bir *i*- ustuni xuddi shu raqamli bog‘lanmagan tugunga mos keladi, har bir *j*- qatoriga esa *j*- shoxobcha to‘g‘ri keladi. C_p matritsaning har bir ustuni sxemaning bog‘lanmagan tugunidan balans tuguniga qadar yo‘lni aniqlaydi. Grafning belgilangan yo‘liga kiradigan shoxobcha birlar bilan belgilanadi, ishorasi esa – musbat yoki manfiy – mos ravishda shoxobchaning yo‘nalishi graf yo‘li bilan mos kelsa, unga qarama-qarshi bo‘lganini ko‘rsatadi.

C_p matritsaning har bir qatori shu shoxobcha grafning qanday yo‘llariga (qanday tugunlariga) kirishini ko‘rsatadi. Matritsaning *i*-qatori va *j*-ustuni kesishgan joydagi “0” grafning *j*-tugunidan sxemaning balans tugunigacha bo‘lgan yo‘l tarkibiga *i*-shoxobcha kirmasligini ko‘rsatadi.

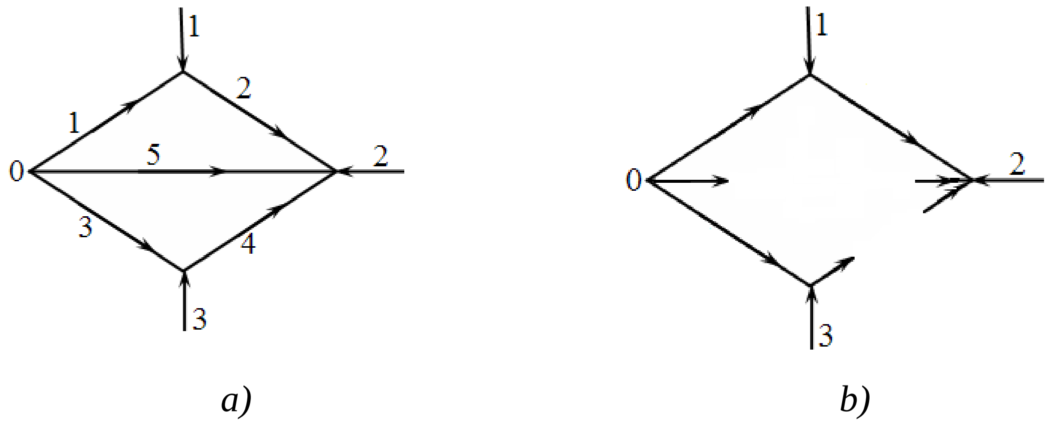
Shunday qilib, C_p matritsani bevosita ajratilgan sxema bo‘yicha tuzish mumkin.

1.80- masala. 1.67-rasmdagi sxema uchun vatarning 5- va 4- shoxobchalaridagi toklar aniqlansin: 1, 2, 3, 4, 5 – shu nomerdagi shoxobchalar qarshiligi, [Ω]; 1, 2, 3–shu nomerdagi tugunlarga kirayotgan berilma toklar qiymati, [A].

$$I_\beta = \begin{Vmatrix} 0{,}5211 \\ -1{,}0704 \end{Vmatrix}$$

Yechish. Bu holda sxemani daraxt ko‘rinishida tasvirlab uni soddalashtirish mumkin. Sxemaning boshqa barcha shoxobchalaridagi toklarni bevosita topamiz. Buning uchun quyidagi formuladan foydalaniladi:

$$\overline{I}_\alpha = C_o\left(\underline{J} - \underline{M}_\beta \underline{I}_\beta\right).$$



1.67- rasm.

Bu formulada $\overline{I}_\alpha$ matritsani $\underline{M}_\beta$ matritsasiga chap tomondan ko‘paytirish, vatarlardagi toklarni berilma toklarning mos yig‘indisi bilan almashtirilganligini aniqlatadi:

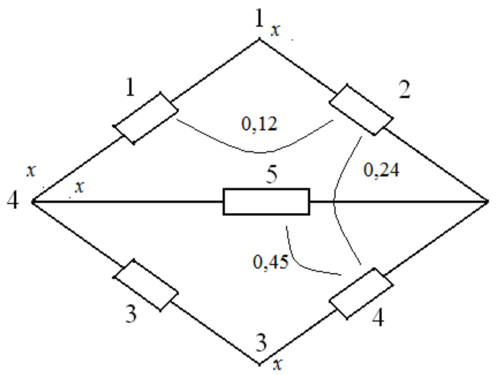
$$M_\beta I_\beta = \begin{Vmatrix} 0 & 0 \\ -1 & -1 \\ 1 & 0 \end{Vmatrix} \cdot \begin{Vmatrix} 0{,}5211 \\ -1{,}0704 \end{Vmatrix} = \begin{Vmatrix} 0 \\ 0{,}5493 \\ 0{,}5211 \end{Vmatrix}$$

Yuqoridagi formulada bu toklar sxemadagi berilma toklar bilan algebraik ravishda qo‘shiladi:

$$\underline{Y} - M_\beta \underline{I}_\beta = \begin{Vmatrix} 1 \\ 2 \\ 3 \end{Vmatrix} - \begin{Vmatrix} 0 \\ 0{,}5493 \\ 0{,}5211 \end{Vmatrix} = \begin{Vmatrix} 1 \\ 1{,}4507 \\ 2{,}4789 \end{Vmatrix}$$

1.81- masala. Almashlash sxemasi 1.68- rasmda tasvirlangan tarmoq uchun qarshiliklar matritsasi yozilsin («x» – ishorasi bilan shoxobchadagi tok yo‘nalishi belgilangan). O‘z qarshiligidan tashqari, sxema 1- va 2-, 2- va 4-, 4- va 5- shoxobchalar orasida o‘zaro qarshiliklarga ega. Matritsa yozilishi oson bo‘lishi uchun qarshiliklarning qiymatlari shoxobchalar raqami bilan bog‘langan: $Z_{12} = Z_{21} = 0{,}12\,; Z_{42} = Z_{24} = 0{,}24\,; Z_{45} = Z_{54} = 0{,}45$.

Yechish. Umumiy holda ikki o‘lchamli matritsa to‘g‘ri burchakli bo‘ladi. To‘g‘ri burchakli matritsada qatorlar soni ustunlar soniga teng bo‘lmasligi, ya’ni ko‘p yoki kam bo‘lishi mumkin bo‘lgan chiziqli matritsaning xususiy holidir. Misol uchun, ustunli matritsada ustunlar soni birga teng, kvadrat matritsada qatorlar soni ustunlar soniga teng.



$$\underline{Z}_b = \begin{Vmatrix} 1 & 0{,}12 & 0 & 0 & 0 \\ 0{,}12 & 2 & 0 & 0{,}24 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0{,}24 & 0 & 4 & 0{,}45 \\ 0 & 0 & 0 & 0{,}45 & 5 \end{Vmatrix}$$

1.68- rasm.

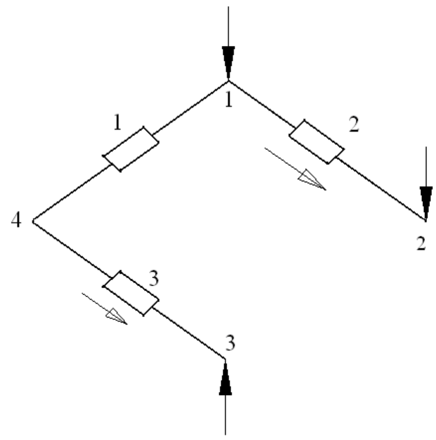
1.82- masala. 1.69- rasmda tasvirlangan sxema uchun balans tuguni 4 bilan belgilangan, berilma toklarning matritsasi va taqsimlanish koeffitsiyentlarining matritsasi

$$\underline{C} = \begin{Vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{Vmatrix}$$

Shu sxemada balans tugunini 1- tugunga o‘zgartirilganda $\underline{J}$ va $\underline{C}$ matritsalarni aniqlash kerak.

Yechish. Yechilishini aniqlashda kerak bo‘lgan matritsalarni yozish uchun 1- va 4- tugunlarni o‘zaro o‘zgartiramiz. Unda berilma toklarning matritsasi

$$\underline{J} = \begin{Vmatrix} -6 \\ 2 \\ 3 \end{Vmatrix}$$



1.69- rasm.

va taqsimlanish koeffitsiyentlarining matritsasi

$$\underline{C} = \begin{Vmatrix} 1 & 0 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{Vmatrix}, \qquad \underline{J} = \begin{Vmatrix} 1 \\ 2 \\ 3 \end{Vmatrix}$$

bo‘ladi

1.83- masala. Berilgan elektr sxema (1.70- rasm) toklarini holat tenglamalari usulida aniqlang. Bunda

$E_1 = 12$ V; $E_2 = 24$ V; $R_1 = 5{,}8\ \Omega$; $R_2 = 12{,}4\ \Omega$; $R_3 = 16{,}6\ \Omega$; $R_4 = 12{,}4\Omega$;

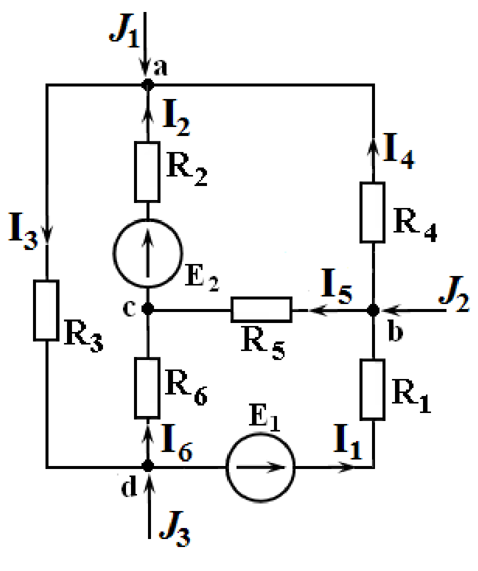
$R_5 = 5{,}8\Omega$; $R_6 = 6{,}2\ \Omega$; $J_1 = 4$ A; $J_2 = 3$ A; $J_3 = 4$ A .

Yechish.

1. Berilma toklarning matritsasini tuzamiz: $J = \begin{Vmatrix} 4 \\ 3 \\ 4 \end{Vmatrix}$;

2. Shoxobchalar qarshiliklarining matritsasini tuzamiz:

$$Z = \begin{Vmatrix} 5{,}8 & 0 & 0 & 0 & 0 & 0 \\ 0 & 12{,}4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16{,}6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5{,}8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6{,}2 \end{Vmatrix} ;$$

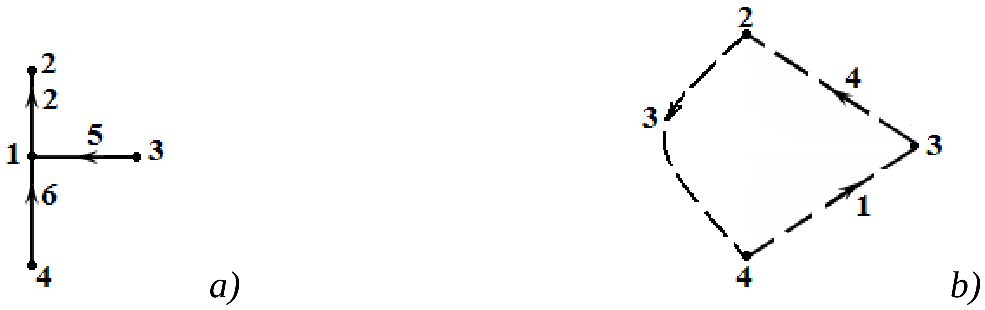


1.70- rasm.

3. EYK lar matritsasini tuzamiz:

$$E_{ij} = \begin{Vmatrix} 12 \\ 24 \\ 0 \\ 0 \\ 0 \\ 0 \end{Vmatrix} ;$$

Sxema grafini daraxtga va vatarga ajratamiz (1.71- rasm, *a* va *b*). Ma’lumki, bunda daraxt tarkibida barcha tugunlar bo‘lishi kerak.



1.71- rasm. Elektr sxemaning daraxti *a* va vatari *b*.

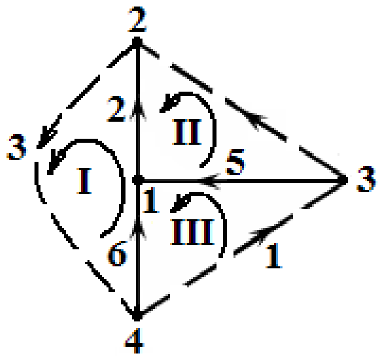
4. Daraxt $Z_{\alpha\alpha}$ va vatarlar $Z_{\beta\beta}$ shoxobchalarining qarshiliklari matritsalarini tuzamiz:

$$Z_{\alpha\alpha} = \begin{Vmatrix} 12{,}4 & 0 & 0 \\ 0 & 5{,}8 & 0 \\ 0 & 0 & 6{,}2 \end{Vmatrix} ; \qquad Z_{\beta\beta} = \begin{Vmatrix} 5{,}8 & 0 & 0 \\ 0 & 1{,}6 & 0 \\ 0 & 0 & 4 \end{Vmatrix} ;$$

5. Vatar tugunlarining bog‘lanish (insidensiyalarning) birinchi matritsasini tuzamiz:

$$\begin{array}{rc} & \text{shoxobchalar №} \\ M_{\beta} = \begin{array}{r} \text{tu-} \\ \text{gun -} \\ \text{lar №} \end{array} & \begin{Vmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{Vmatrix} ; \end{array}$$

6. Sxema daraxti uchun konturlar (insidensiyalar) matritsasining bloki tarkibiga kiruvchi shoxobchalarni hisobga olgan holda (1.72- rasm) matritsalar tuzamiz:



1.72- rasm.

shoxobchalar№

$$N_{\alpha}=\begin{array}{rc} & \begin{array}{ccc} 2 & 5 & 6 \end{array} \\ \begin{array}{r}\text{kon-}I\\ \text{tur-}II\\ \text{lar }III\end{array} & \left\|\begin{array}{ccc} 1 & 0 & 1\\ 0 & 1 & -1\\ -1 & -1 & 0\end{array}\right\| \end{array};$$

7. Sxema daraxti uchun taqsimlanish koeffitsiyentlarining matritsasi:

tugunlar№

$$C_{o\alpha}=\begin{array}{rc} & \begin{array}{ccc} 2 & 3 & 4 \end{array} \\ \begin{array}{r}\text{sho-}\ \ 2\\ \text{xobcha -}5\\ \text{lar}\ \ \ \ 6\end{array} & \left\|\begin{array}{ccc} -1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{array}\right\| \end{array};$$

8. To‘la sxema uchun konturlar insidensiyalarning ikkinchi matritsasi:

sh o x o b ch a l a r №

$$N=\begin{array}{rc} & \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{array}{r}\text{kon-}\ I\\ \text{tur-}\ II\\ \text{lar}\ \ III\end{array} & \left\|\begin{array}{cccccc} 0 & 1 & 1 & 0 & 0 & 1\\ 1 & 0 & 0 & 0 & 1 & -1\\ 0 & -1 & 0 & 1 & -1 & 0\end{array}\right\| \end{array};$$

9. Vatardagi toklar matritsasini quyidagi formuladan aniqlaymiz:

$$\mathrm{I}_{\beta} = (Z_{\beta\beta} + N_{\alpha} Z_{\alpha\alpha} N_{\alpha\ t})^{-1} \cdot (NE - N_{\alpha} Z_{\alpha\alpha} C_{o\ \alpha} J) =$$

$$= \left(\overset{Z_{\beta\beta}}{\begin{Vmatrix} 5{,}8 & 0 & 0 \\ 0 & 1{,}6 & 0 \\ 0 & 0 & 0 \end{Vmatrix}} + \overset{N_{\alpha}}{\begin{Vmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ -1 & -1 & 0 \end{Vmatrix}} \cdot \overset{Z_{\alpha\alpha}}{\begin{Vmatrix} 12{,}4 & 0 & 0 \\ 0 & 5{,}8 & 0 \\ 0 & 0 & 6{,}2 \end{Vmatrix}} \cdot \overset{N_{\alpha\ t}}{\begin{Vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{Vmatrix}} \right)^{-1} \cdot$$

$$\cdot \left(\overset{N}{\begin{Vmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ -1 & -1 & 0 \end{Vmatrix}} \cdot \overset{E}{\begin{Vmatrix} 12 \\ 24 \\ 0 \\ 0 \\ 0 \\ 0 \end{Vmatrix}} - \overset{N_{\alpha}}{\begin{Vmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ -1 & -1 & 0 \end{Vmatrix}} \cdot \overset{Z_{\alpha\alpha}}{\begin{Vmatrix} 12{,}4 & 0 & 0 \\ 0 & 5{,}8 & 0 \\ 0 & 0 & 6{,}2 \end{Vmatrix}} \cdot \overset{C_0}{\begin{Vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix}} \cdot \overset{Y}{\begin{Vmatrix} 4 \\ 3 \\ 4 \end{Vmatrix}} \right) = \begin{Vmatrix} -5{,}3 \\ -1 \\ -6{,}7 \end{Vmatrix}$$

Matritsalar ustida amallar bajarish quyida ko‘rsatilgan:

$$\text{a)} \begin{Vmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ -1 & -1 & 0 \end{Vmatrix} \cdot \begin{Vmatrix} 12{,}4 & 0 & 0 \\ 0 & 5{,}8 & 0 \\ 0 & 0 & 6{,}2 \end{Vmatrix} = \begin{Vmatrix} 12{,}4 & 0 & 6{,}2 \\ 0 & 5{,}8 & -6{,}2 \\ -12{,}4 & -5{,}8 & 0 \end{Vmatrix};$$

$$\text{b)} \begin{Vmatrix} 12{,}4 & 0 & 6{,}2 \\ 0 & 5{,}8 & -6{,}2 \\ -12{,}4 & -5{,}8 & 0 \end{Vmatrix} \cdot \begin{Vmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 1 & -1 & 0 \end{Vmatrix} = \begin{Vmatrix} 18{,}6 & -6{,}2 & -12{,}4 \\ -6{,}2 & 12 & -5{,}8 \\ -12{,}4 & -5{,}8 & 18{,}2 \end{Vmatrix};$$

$$\text{d)} \begin{Vmatrix} 5{,}8 & 0 & 0 \\ 0 & 16{,}6 & 0 \\ 0 & 0 & 4 \end{Vmatrix} + \begin{Vmatrix} 18{,}6 & -6{,}2 & -12{,}4 \\ -6{,}2 & 12 & -5{,}8 \\ -12{,}4 & -5{,}8 & 18{,}2 \end{Vmatrix} = \begin{Vmatrix} 24{,}4 & -6{,}2 & -12{,}4 \\ -6{,}2 & 28{,}6 & -5{,}8 \\ -12{,}4 & -5{,}8 & 22{,}2 \end{Vmatrix};$$

Teskari matritsani hisoblashda quyidagi formuladan foydalanamiz:

$$A^{-1}=\frac{1}{D}\cdot\begin{Vmatrix} \begin{vmatrix} A_{22} & A_{23} \\ A_{32} & A_{33} \end{vmatrix} & -\begin{vmatrix} A_{12} & A_{13} \\ A_{32} & A_{33} \end{vmatrix} & \begin{vmatrix} A_{12} & A_{13} \\ A_{22} & A_{23} \end{vmatrix} \\ -\begin{vmatrix} A_{21} & A_{23} \\ A_{31} & A_{33} \end{vmatrix} & \begin{vmatrix} A_{11} & A_{13} \\ A_{31} & A_{33} \end{vmatrix} & -\begin{vmatrix} A_{11} & A_{13} \\ A_{21} & A_{23} \end{vmatrix} \\ \begin{vmatrix} A_{21} & A_{22} \\ A_{31} & A_{32} \end{vmatrix} & -\begin{vmatrix} A_{11} & A_{12} \\ A_{31} & A_{32} \end{vmatrix} & \begin{vmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{vmatrix} \end{Vmatrix}$$

Demak,

$$\begin{Vmatrix} 24{,}4 & -6{,}2 & -12{,}4 \\ -6{,}2 & 28{,}6 & -5{,}8 \\ -12{,}4 & -5{,}8 & 22{,}2 \end{Vmatrix} = \frac{1}{8528{,}5}\begin{Vmatrix} -601 & 210 & 391 \\ 210 & 388 & 218 \\ 391 & 218 & 659 \end{Vmatrix}$$

Keyingi amallar:

$$\text{e)}\ \begin{Vmatrix} 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 1 & -1 & 0 \end{Vmatrix}\cdot\begin{Vmatrix} 12 \\ 24 \\ 0 \\ 0 \\ 0 \\ 0 \end{Vmatrix} = \begin{Vmatrix} 24 \\ 12 \\ -24 \end{Vmatrix};\quad \text{d)}$$

$$\begin{Vmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ -1 & -1 & 0 \end{Vmatrix}\cdot\begin{Vmatrix} 12{,}4 & 0 & 6{,}2 \\ 0 & 5{,}8 & -6{,}2 \\ -12{,}4 & -5{,}8 & 0 \end{Vmatrix};$$

$$\text{f)}\ \begin{Vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix}\cdot\begin{Vmatrix} 12{,}4 & 0 & 6{,}2 \\ 0 & 5{,}8 & -6{,}2 \\ -12{,}4 & -5{,}8 & 0 \end{Vmatrix} = \begin{Vmatrix} -12{,}4 & 0 & -6{,}2 \\ 0 & 5{,}8 & -6{,}2 \\ -12{,}4 & -5{,}8 & 0 \end{Vmatrix};$$

$$\text{g)}\ \begin{Vmatrix} -12{,}4 & 0 & -6{,}2 \\ 0 & 5{,}8 & -6{,}2 \\ -12{,}4 & -5{,}8 & 0 \end{Vmatrix}\cdot\begin{Vmatrix} 4 \\ 3 \\ 4 \end{Vmatrix} = \begin{Vmatrix} -74{,}4 \\ -7{,}4 \\ -67 \end{Vmatrix}$$

$$\text{h)}\ \begin{Vmatrix} 24 \\ 12 \\ -24 \end{Vmatrix} - \begin{Vmatrix} -74{,}4 \\ -7{,}4 \\ -67 \end{Vmatrix} = \begin{Vmatrix} 98{,}4 \\ 19{,}4 \\ 43 \end{Vmatrix}$$

i)
$$\frac{1}{8528{,}5}\begin{Vmatrix} -601 & 210 & 391 \\ -210 & 388 & 218 \\ -391 & 218 & 659 \end{Vmatrix} \cdot \begin{Vmatrix} 98{,}4 \\ 19{,}4 \\ 43 \end{Vmatrix} = \frac{1}{8528{,}5}\begin{Vmatrix} -59138{,}4 & +4074 & +16813 \\ -20664 & +7527{,}2 & +9374 \\ -38474{,}4 & +4229{,}2 & +28337 \end{Vmatrix} =$$

$$= \frac{1}{8528{,}5}\begin{Vmatrix} -45201 \\ -8528 \\ -57141 \end{Vmatrix} = \begin{Vmatrix} -5{,}3 \\ -1 \\ -6{,}7 \end{Vmatrix}$$

10. Sxema daraxtining shoxobchalaridagi toklarni hisoblaymiz:

$$I_\alpha = C_0(\underline{J} - M_\beta I_\beta) = \begin{Vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix} \cdot \left(\begin{Vmatrix} 4 \\ 3 \\ 4 \end{Vmatrix} - \begin{Vmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{Vmatrix} \cdot \begin{Vmatrix} -5{,}3 \\ -1 \\ -6{,}7 \end{Vmatrix} \right) = \begin{Vmatrix} 1{,}7 \\ 4{,}4 \\ 8{,}3 \end{Vmatrix}$$

$$\text{a)}\begin{Vmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{Vmatrix} \begin{Vmatrix} -5{,}3 \\ -1 \\ -6{,}7 \end{Vmatrix} = \begin{Vmatrix} 5{,}7 \\ -1{,}4 \\ -4{,}3 \end{Vmatrix}; \qquad \text{b)}\begin{Vmatrix} 4 \\ 3 \\ 4 \end{Vmatrix} - \begin{Vmatrix} 5{,}7 \\ -1{,}4 \\ -4{,}3 \end{Vmatrix} = \begin{Vmatrix} -1{,}7 \\ 4{,}4 \\ 8{,}3 \end{Vmatrix};$$

$$\text{v)}\begin{Vmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix} \cdot \begin{Vmatrix} -1{,}7 \\ 4{,}4 \\ 8{,}3 \end{Vmatrix} = \begin{Vmatrix} 1{,}7 \\ 4{,}4 \\ 8{,}3 \end{Vmatrix};$$

$$I_\beta = \begin{Vmatrix} -5{,}3 \\ -1 \\ -6{,}7 \end{Vmatrix} \begin{matrix} I_1 \\ I_3 \\ I_4 \end{matrix} \qquad I_\alpha = \begin{Vmatrix} 1{,}7 \\ 4{,}4 \\ 8{,}3 \end{Vmatrix} = \begin{matrix} I_2 \\ I_5 \\ I_6 \end{matrix} \qquad I = \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} \begin{Vmatrix} -5{,}3 \\ 1{,}7 \\ -1 \\ -6{,}7 \\ 4{,}4 \\ 8{,}3 \end{Vmatrix} \begin{matrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{matrix}$$

Bazisli tugunning toklarini tekshirish:

$$I_6 + I_5 - I_2 = 8{,}3 + 4{,}4 - 1{,}7 = 11 \text{ va } J_1 + J_2 + J_3 = 4 + 3 + 4 = 11;$$

2- tugun toklarini tekshirish:

$$-I_3 + I_2 + I_4 + J_1 = 1 + 1{,}7 - 6{,}7 + 4 = 0;$$

3- tugun toklarini tekshirish:

$$-I_4 - I_5 + I_1 + J_2 = 6{,}7 - 4{,}4 - 5{,}3 + 3 = 0;$$

4- tugun toklarini tekshirish:

$$I_3 - I_1 - I_6 + J_3 = -1 + 5{,}3 - 8{,}3 + 4 = 0.$$

2- BOB. BIR FAZALI SINUSOIDAL TOK CHIZIQLI ELEKTR ZANJIRLARINI HISOBLASH

2.1. Asosiy ma’lumotlar

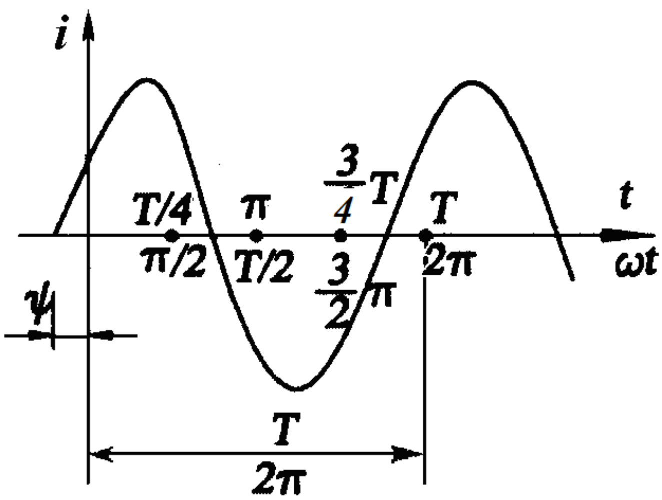
Yo‘nalishi va kattaligi davriy ravishda o‘zgarib turuvchi EYK, tok va kuchlanish *o‘zgaruvchan* EYK, *tok va kuchlanish* deyiladi. *Vaqt bo‘yicha sinusoidal o‘zgaruvchan EYK, tok va kuchlanish sinusoidal EYK, tok va kuchlanish deb ataladi*. Masalan, sinusoidal tok quyidagicha ifodalanadi:

$$i = I_m \sin\left(\omega t + \psi_i\right),$$

bunda i – tokning oniy qiymati, I_m – uning maksimal (amplituda) qiymati, ω – burchak chastota, ψ_i – boshlang‘ich faza.

2.1- rasmda sinusoidal tokning to‘lqin grafigi keltirilgan. *Davr T* – bu muayyan vaqt oralig‘i bo‘lib, tokning o‘zgarishi takrorlanadi, ya’ni to‘liq bitta to‘lqin o‘zgarishi kuzatiladi. Bir sekunddagi davrlar soni *chastota f* deb ataladi, $f = 1/T$.

$\omega = 2\pi f = 2\pi/T$ – o‘zgaruvchan tokning *burchak chastotasi*, rad/s yoki s^{-1}.



2.1- rasm.

Sinusoidal tok, EYK va kuchlanishning ta’sir etuvchi (effektiv) va o‘rtacha qiymatlari. Sinusoidal o‘zgaruvchan kattalikning juda keng ishlatiladigan qiymati bu uning ta’sir etuvchi qiymatidir. U *effektiv* yoki *o‘rtacha kvadratik qiymat* deb ham ataladi. Masalan, tokning ta’sir etuvchi qiymati quyidagicha aniqlanadi:

$$I=\sqrt{\frac{1}{T}\int_0^T i^2dt}=\sqrt{\frac{1}{T}\int_0^T I_m^2\sin^2(\omega t+\psi_i)\,dt}=\sqrt{\frac{I_m^2}{2T}\int_0^T\left[1-\cos(2\omega t+2\psi_i)\right]dt}=$$

$$=\sqrt{\frac{I_m^2}{2T}(T-0)}=\frac{I_m}{\sqrt{2}}=0{,}707\,I_m.$$

Sinusoidal o‘zgaruvchan kattaliklarning o‘rta qiymati deb, uning yarim davr ichidagi o‘rtacha qiymati tushuniladi. Demak, sinusoidal tokning o‘rtacha qiymati:

$$I_{\text{o'rta}}=\frac{1}{T/2}\int_0^{T/2} I_m\sin\omega t\,dt=\frac{2}{\pi}I_m\,.$$

Sinusoidal EYK va kuchlanishlar uchun ham xuddi shunday tengliklar o‘rinli:

$$E_{\text{o'rta}}=\frac{2}{\pi}E_m,\;\;U_{\text{o'rta}}=\frac{2}{\pi}U_m\,.$$

Sinusoidal tok zanjirlarining tahlil etishda amplituda va shakl koeffitsiyentlaridan keng foydalaniladi.

Amplituda koeffitsiyenti davriy o‘zgaruvchi funksiya amplitudasini uning ta’sir etuvchi qiymatiga nisbati bilan aniqlanadi. Sinusoidal tok uchun:

$$k_\alpha=\frac{I_m}{I}=\sqrt{2}.$$

Shakl koeffitsyienti davriy o‘zgaruvchi funksiyaning ta’sir etuvchi qiymatini uning o‘rtacha qiymatiga nisbati bilan aniqlanadi. Sinusoidal tok uchun:

$$k_{\text{sh}}=\frac{I}{I_{\text{o'rta}}}=\frac{\frac{I_m}{\sqrt{2}}}{\frac{2}{\pi}I_m}=\frac{\pi}{2\sqrt{2}}=1{,}11.$$

Sinusoidal tok zanjirlarida rezistor, induktiv g‘altak va kondensator. Rezistiv elementdan tarkib topgan oddiy elektr zanjir sxemasi 2.2- rasm, *a* da keltirilgan.

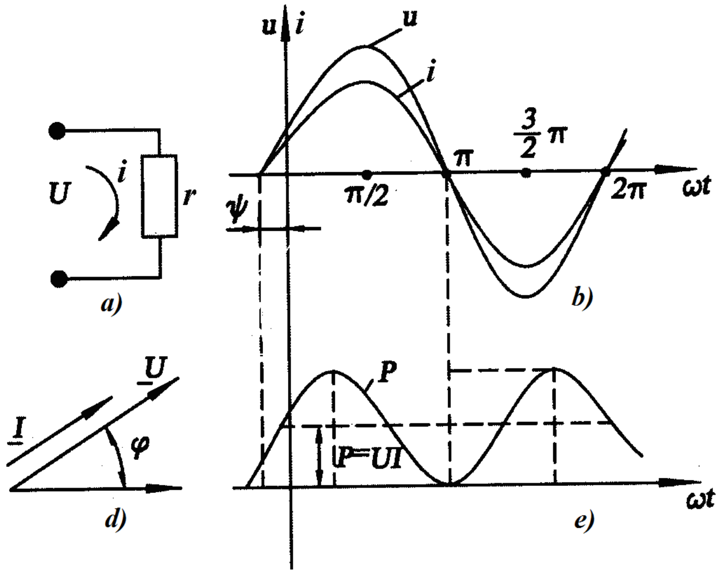
Rezistiv elementli zanjirning bir qismi uchun Om qonuni kuchlanish va tokning ta’sir etuvchi qiymatlari orqali quyidagicha yoziladi:

$$I=U/r.$$

Rezistiv elementli zanjirda tok va kuchlanish oniy qiymatlarining to‘lqin diagrammasi 2.2- rasm, *b* da, vektor diagrammasi esa 2.2- rasm, *d* da keltirilgan.

Ushbu zanjir uchun oniy quvvat quyidagi formula yordamida hisoblanadi:

$$p = ui = U_m I_m \sin^2(\omega t + \psi_u) = U_m I_m \left[\frac{1 - \cos 2(\omega t + \psi_u)}{2}\right] =$$
$$= UI - UI\cos 2(\omega t + \psi_u).$$



2.2- rasm.

Quvvatning *T* davrdagi o‘rta qiymati:

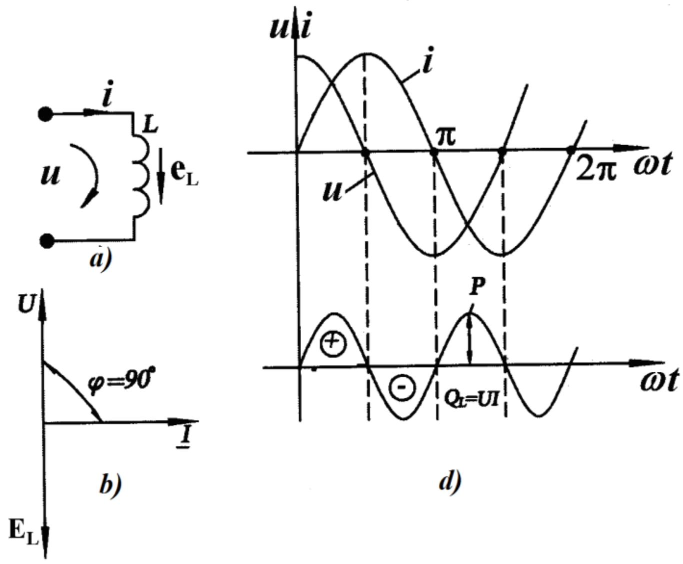
$$P = \frac{1}{T}\int_0^t p\,dt = \frac{1}{T}UI\int_0^T dt - \frac{1}{T}UI\int_0^T \cos 2(\omega t + \psi_u)dt = UI = rI^2 = \frac{U^2}{r}.$$

Oniy quvvatning o‘zgarish grafigi 2.2- rasm, *e* da keltirilgan.

Induktiv elementli elektr zanjirda sinusoidal tok. Induktiv qarshilik. Amalda har qanday induktiv g‘altak aktiv qarshilik *r* va induktivlik *L* ga ega. Induktiv g‘altak sxemada ko‘pincha ketma-ket ulangan rezistor (aktiv qarshilik) va induktiv elementlari bilan tavsiflanadi. Sxemada faqat induktiv element *L* ni ajratib olamiz va *r*=0 deb hisoblab, ideal induktiv g‘altakdagi jarayonni tahlil qilamiz.

Agar induktiv elementdan $i = I_m \sin\omega t$ tok o‘tsa, unda o‘zinduksiya hodisasiga asosan EYK hosil bo‘ladi (2.3- rasm, *a*):

$$e_L = -L(di/dt) = -\omega L I_m \cos\omega t = -\omega L I_m \sin(\omega t + 90^\circ) = -E_m \sin(\omega t + 90^\circ).$$



2.3- rasm.

Kuchlanish u_{ab} ning yo‘nalishi tokning musbat yo‘nalishi bilan bir xil. Induktivlikdagi kuchlanish:

$$u_{ab} = u_L = -e_L = \omega L I_m \sin\left(\omega t + 90^\circ\right) = U_m \sin\left(\omega t + 90^\circ\right).$$

Bu ifodadan $U_m = \omega L\, I_m$.

ωL ko‘paytma x_L deb belgilanadi va induktiv qarshilik deb ataladi, ya’ni:

$x_L = \omega L,$ uning o‘lchami $[x_L] = [\omega]\cdot[L] = [(1/\mathrm{s})\cdot\Omega\cdot \mathrm{s}] = [\Omega]$.

Tok va kuchlanish oniy qiymatlarining to‘lqin diagrammasi 2.3- rasm, *b* da, vektor diagrammasi esa 2.3- rasm, *d* da keltirilgan.

Rezistor va induktiv g‘altakdan iborat zanjir. O‘zgaruvchan tok ta’siridagi real induktiv g‘altak aktiv va induktiv qarshilikdan iborat.Induktiv g‘altakning ekvivalent sxemasi 2.4- rasm, *a* da, tok va kuchlanish vektor diagrammasi 2.4- rasm, *b* da, qarshiliklar uchburchagi esa 2.4- rasm, *d* da keltirilgan.

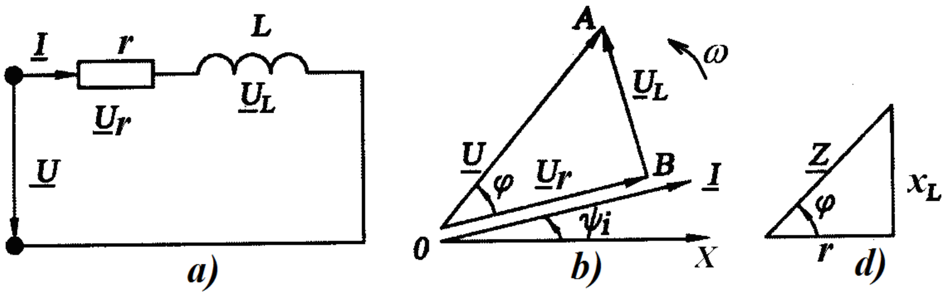
G‘altakdan $i = I_m \sin(\omega t + \psi_i)$ tok o‘tmoqda deb faraz qilaylik. Kirxgofning ikkinchi qonuniga binoan oniy qiymatlar uchun:

$$u = u_r + u_L = ir + L\, di/dt,$$

bunda u_r – induktiv g‘altakning aktiv qarshiligidagi kuchlanish;

u_L–induktivlikdagi kuchlanish:

$$u_L = L\frac{di}{dt} = L\frac{d\left[I_m \sin(\omega t + \psi_i)\right]}{dt} = \omega L I_m \cos(\omega t + \psi_i) = U_{Lm} \sin(\omega t + \psi_i + 90^\circ).$$

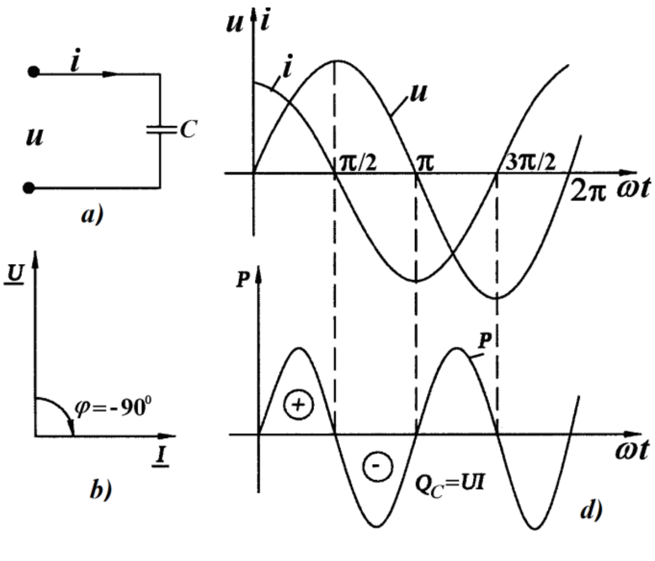


2.4- rasm.

Sig‘im elementli elektr zanjirdagi sinusoidal tok. Sig‘im qarshiligi (2.5- rasm). Agar kondensatorga berilgan kuchlanish vaqt bo‘yicha o‘zgarmasa, unda kondensator qoplamalaridan birida yig‘ilgan zaryad $q = CU$, ikkinchisidagi esa $q = -CU$ bo‘ladi, bu yerda C – kondensatorning sig‘imi. Zaryadlar miqdori o‘zgarmas bo‘lib, kondensatordan tok o‘tmaydi, chunki $i = dq/dt = 0$.

Kondensatorning davriy zaryadlanishi esa zanjirdan mos ravishda davriy tok o‘tishiga sabab bo‘ladi:

$$i = \frac{dq}{dt} = \frac{d}{dt}\left(CU_m \sin\omega t\right) = \omega C U_m \cos\omega t = I_m \sin\left(\omega t + 90^o\right).$$



2.5- rasm.

Kondensatorda tokning musbat yo‘nalishi kuchlanishning musbat yo‘nalishi bilan bir xil (2.5-rasm, *a*). Kondensatordan o‘tayotgan tok faza jihatdan kuchlanishga nisbatan 90^o oldinda bo‘ladi (2.5-rasm, *b, d*). Tokning amplituda qiymati kuchlanish amplitudasining sig‘im qarshiligiga nisbati bilan aniqlanadi:

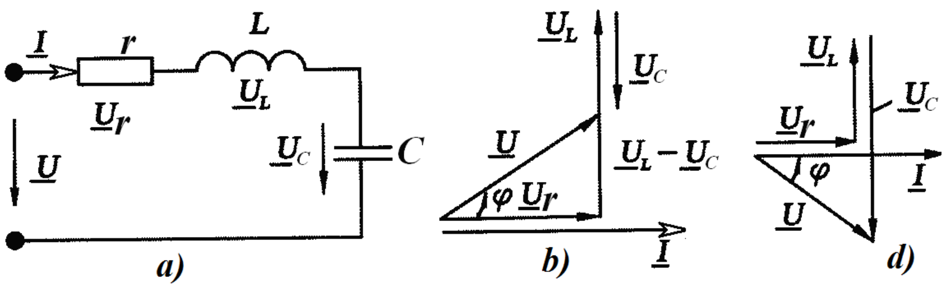
$$I_m = \frac{U_m}{x_C} = \frac{U_m}{1/\omega C} = \omega C U_m, \text{ bu yerda, } x_C = \frac{1}{\omega C} = [\Omega] \text{ – sig‘im qarshiligi.}$$

Rezistor, induktiv g‘altak va kondensator ketma-ket ulangan sinusoidal tok zanjiri. *r, L* va C elementlari ketma-ket ulangan zanjir (2.6- rasm, *a*) dan $i=I_m sin\omega t$ sinusoidal tok o‘tganda uning kirish qismalarida elementlarda pasaygan sinusoidal kuchlanishlarning algebraik yig‘indisiga teng bo‘lgan kuchlanish hosil bo‘ladi. Kirxgofning 2- qonuniga ko‘ra:

$$u = u_r + u_L + u_C \text{ yoki } \underline{U} = \underline{U}_r + \underline{U}_L + \underline{U}_C.$$

r qarshilikdagi kuchlanish faza jihatdan tok bilan mos, *L* induktivlikdagi kuchlanish tokdan 90^o oldinda, *C* sig‘imdagi kuchlanish esa tokdan 90^o orqada bo‘ladi (2.6- rasm, *b*). Om qonunidan foydalanib quyidagini yozishimiz mumkin:

$$u = ri + L\frac{di}{dt} + \frac{1}{C}\int idt = rI_m \sin \omega t + \omega L I_m \cos\omega t - \frac{I_m}{\omega C}\cos\omega t =$$
$$= rI_m \sin \omega t + \left(\omega L - \frac{1}{\omega C}\right) I_m \cos\omega t = rI_m \sin \omega t + xI_m \cos\omega t.$$



2.6- rasm.

Bu tenglama *kuchlanishlar oniy qiymatlari uchun Kirxgofning 2- qonunining trigonometrik shakli* deb ataladi. Undagi $x = x_L - x_C = \omega L - \dfrac{1}{\omega C}$ kattalik zanjirning *reaktiv qarshiligi* deb ataladi. Agar zanjirda $x_L > x_C$ bo‘lganda, $x > 0$ va $\varphi > 0$

(2.6- rasm, *b*) va zanjir induktiv xarakterga, $x_L < x_C$ bo‘lganda esa $x<0$ va $\varphi<0$ (2.6- rasm, *d*), zanjir sig‘im xarakterga, $x_L = x_C$ bo‘lganda $x=0$ va $\varphi = 0$ va zanjir aktiv xarakterga ega bo‘ladi.

Tok va kuchlanishlarning ta’sir etuvchi qiymatlari uchun:

$U = \sqrt{r^2 + x^2}\, I = zI$, bu yerda $z = \sqrt{r^2 + x^2}$ –zanjirning to‘la qarshiligi.

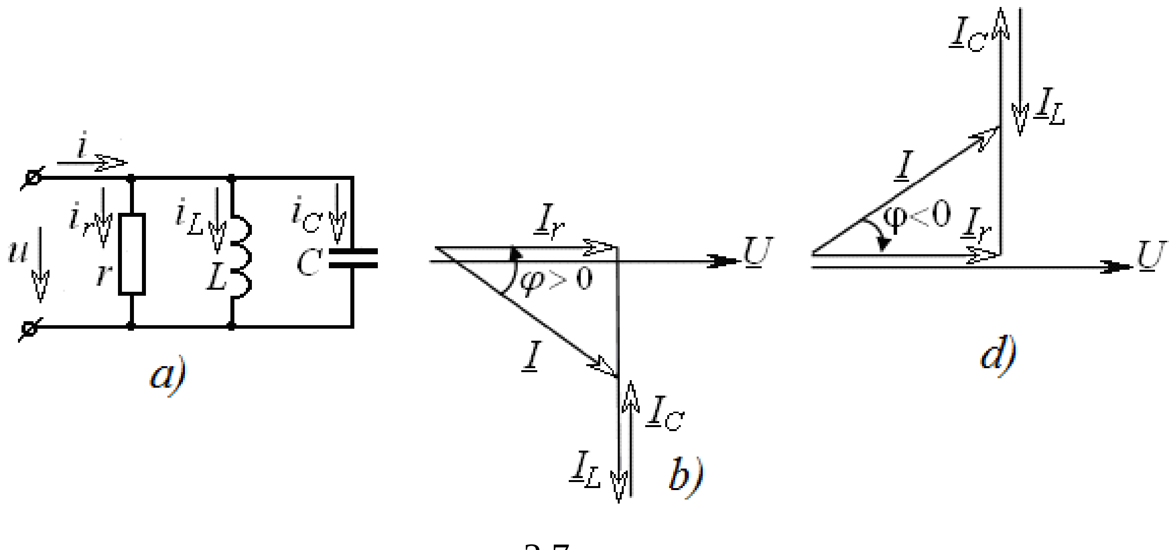
Rezistor, induktiv g‘altak va kondensator parallel ulangan sinusoidal tok zanjiri. *r, L* va C elementlari parallel ulangan zanjir (2.7- rasm, *a*) $u = U_m \sin\omega t$ sinusoidal kuchlanish manbayiga ulansa, undan o‘tadigan sinusoidal tok Kirxgofning 1- qonuniga ko‘ra zanjirning har bir elementidan o‘tayotgan toklarning algebraik yig‘indisiga teng, ya’ni

$$i = i_r + i_L + i_C .$$

r qarshilikdagi tok i_r kuchlanish bilan faza jihatdan mos, induktivlikdagi tok i_L 90^o ga orqada, sig‘imdagi tok i_C 90^o ga oldinda bo‘ladi. Zanjirdagi umumiy tok:

$$I_m \sin(\omega t - \varphi) = \frac{1}{r} U_m \sin\omega t - \frac{1}{\omega L} U_m \cos\omega t + \omega C U_m \cos\omega t =$$

$$= U_m \left[\frac{1}{r}\sin\omega t - \left(\frac{1}{\omega L} - \omega C \right)\cos\omega t \right] = U_m \left[g \sin\omega t - b\cos\omega t \right].$$



2.7- rasm.

Oxirgi tenglama toklar oniy qiymatlari uchun Kirxgof 1- qonunining trigonomet- rik shakli hisoblanadi. $b = b_L - b_c$ – zanjirning *reaktiv o‘tkazuvchanligi* deb ataladi. Agar $b_L > b_C$ bo‘lganda, $b > 0$ va $\varphi > 0$ bo‘lib, zanjir induktiv xarakterga (2.7- rasm, *b*), $b_L < b_C$ bo‘lganda, $b < 0$ va $\varphi < 0$ (2.7- rasm, *d*), zanjir sig‘im xarakterga, $b_L = b_C$ bo‘lganda esa $b = 0$ va $\varphi = 0$ bo‘lib zanjir aktiv xarakterga ega bo‘ladi.

I_m va φ quyidagi munosabatlar yordamida aniqlanadi:

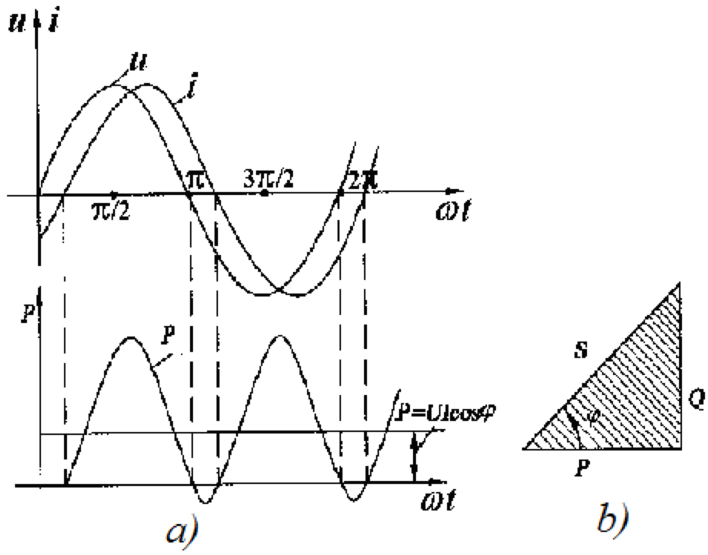
$$I_m = \sqrt{g^2 + b^2}U_m = \gamma U_m,\ \mathrm{tg}\varphi = \frac{b}{g},\ \varphi = \mathrm{arctg}\frac{b}{g},$$

bu yerda $\gamma = \sqrt{g^2 + b^2}$ – zanjirning to‘la o‘tkazuvchanligi.

Sinusoidal tok zanjirida quvvat. Zanjirdagi oniy quvvat:

$$p = ui = U_m I_m \sin\omega t \cdot \sin(\omega t - \varphi) = UI[\cos\varphi - \cos(2\omega t - \varphi)].$$

Oniy quvvat ikkita: doimiy $(UI\cos\varphi)$ va ikkilangan chastota bilan o‘zgaruvchi sinusoidal $(UI\cos\varphi(2\omega t - \varphi))$ tashkil etuvchilardan iborat. Induktiv xarakterli $\varphi > 0$ zanjirdagi tok, kuchlanish va quvvat oniy qiymatlarining grafigi 2.8- rasm, *a* da keltirilgan.



2.8- rasm.

Rezistorda iste’mol qilinayotgan aktiv quvvat oniy quvvatning bir davr mobaynidagi o‘rtacha qiymatiga teng:

$$P = \frac{1}{T}\int_0^T p dt = UI = I^2 R. \qquad (2.2)$$

Aktiv quvvatni quyidagicha ifodalash mumkin:

$$P = zI^2 \cos\varphi = rI^2, \;\; P = \gamma U^2 \cos\varphi = gU^2.$$

Kuchlanish va tokning berilgan qiymatlarida aktiv quvvatning maksimal qiymati zanjirning to'la quvvati deb ataladi.

$$S = UI \;\; [\mathrm{V \cdot A}].$$

Aktiv quvvat ifodasidan: $\cos\varphi = P / S$.

Elektr zanjirini hisoblashda va amaliyotda reaktiv quvvat tushunchasidan foydalaniladi:

$$Q = UI \sin\varphi = I^2 x = U^2 b \quad [\mathrm{var}].$$

Reaktiv quvvat manba bilan iste'molchi o'rtasidagi energiya almashuvi tezligini tavsiflaydi va reaktiv tok iste'molining o'lchovi hisoblanadi. Zanjir induktiv xarakterga ega, ya'ni $\varphi > 0$ bo'lganda reaktiv quvvat musbat, sig'im xarakterga ega, ya'ni $\varphi < 0$ bo'lganda esa manfiy bo'ladi. Aktiv, reaktiv va to'la quvvatlar o'zaro quyidagicha bog'langan (2.8- rasm, *b*):

$$S^2 = P^2 + Q^2, \;\; \sin\varphi = \frac{Q}{S}, \quad \mathrm{tg}\varphi = \frac{Q}{P}.$$

Om va Kirxgof qonunlarining kompleks shakli. Om va Kirxgof qonunlarining kompleks shaklini hosil qilish uchun *r, L* va C elementlari ketma-ket ulangan zanjir uchun (2.6- rasm):

$$u_r + u_L + u_C = u \;\text{ yoki }\; ri + L\frac{di}{dt} + \frac{1}{C}\int i dt = u,$$

bu yerda

$$i = I_m \sin(\omega t + \psi_i),$$
$$u_r = rI_m \sin(\omega t + \psi_i),$$
$$u_L = \omega L I_m \sin(\omega t + \psi_i + 90^o),$$
$$u_C = \frac{1}{\omega C} I_m \sin(\omega t + \psi_i - 90^o).$$

Yuqoridagi tenglamalarning kompleks shakli:

$$\underline{I}_m = I_m e^{j\psi_i},$$
$$\underline{U}_{rm} = rI_m e^{j\psi_i} = r\underline{I}_m,$$

$$\underline{U}_{Lm} = \omega L I_m e^{j(\psi_i + 90^0)} = \omega L I_m e^{j\psi_i} e^{j90^0} =$$
$$= \omega L I_m e^{j\psi_i}(\cos 90^0 + j\sin 90^0) = j\omega L I_m e^{j\psi_i} = j\omega L \underline{I}_m,$$
$$\underline{U}_{Cm} = \frac{1}{\omega C} I_m e^{j(\psi_i - 90^0)} = \frac{1}{\omega C} I_m e^{j\psi_i} e^{-j90^0} = -j\frac{1}{\omega C}\underline{I}_m.$$

Ushbu hosil qilingan tenglamalardan ko‘rinib turibdiki, sinusoidal kattaliklarni kompleks sonlar bilan almashtirishda *differensiallash amali* $j\omega$ *bilan, integrallash amali esa* $1/\, j\omega$ *bilan almashtiriladi*.

Ko‘rilayotgan zanjir uchun Kirxgof qonunining kompleks shakli quyidagicha yoziladi:

$$\underline{U}_{rm} + \underline{U}_{Lm} + \underline{U}_{cm} = \underline{U}_m \text{ yoki } r\underline{I}_m + j\omega L\underline{I}_m - j\frac{1}{\omega C}\underline{I}_m = \underline{U}_m,$$

bundan

$$\underline{I}_m = \frac{\underline{U}_m}{r + j\left(\omega L - \dfrac{1}{\omega C}\right)} = \frac{\underline{U}_m}{\underline{Z}} \text{ yoki ta'sir etuvchi qiymatlar uchun } \underline{I} = \underline{U}/\underline{Z}.$$

Oxirgi tenglik *Om qonunining kompleks shakli* deb ataladi.

$$\underline{Z} = r + j\left(\omega L - \frac{1}{\omega C}\right) = r + jx = z\cos\varphi + jz\sin\varphi = \sqrt{r^2 + x^2}\, e^{j\varphi} = ze^{j\varphi}$$

– *zanjirning kompleks qarshiligi* deb ataladi. Bunda kompleks qarshilikning haqiqiy qismi – aktiv qarshilik, mavhum qismi esa reaktiv qarshilikka teng bo‘ladi.

To‘la kompleks qarshilikka teskari bo‘lgan kattalik *to‘la kompleks o‘tkazuvchanlik* deb ataladi:

$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{1}{ze^{\pm j\varphi}} = \gamma e^{\mp j\varphi} = \gamma\cos\varphi \mp j\gamma\sin\varphi = g \mp jb,$$

bunda $\gamma = \sqrt{g^2 + b^2}$, $\varphi = \mathrm{arctg}\dfrac{b}{g}$ - mos ravishda to‘la kompleks o‘tkazuvchanlikning moduli va argumenti.

Sinusoidal tok zanjirlari uchun Kirxgof qonunlari kompleks tok va kuchlanishlar orqali quyidagicha ifodalanadi:

Zanjirning istalgan tugunidagi kompleks toklarning algebraik yig‘indisi nolga teng (Kirxgofning 1 - qonuni):

$$\sum_{k=1}^{n} \underline{I}_{\kappa} = 0.$$

Zanjirning istalgan berk konturida kompleks EYK larning algebraik yig‘indisi shu kontur kompleks qarshiliklarida pasaygan kompleks kuchlanishlarning algebraik yig‘indisiga teng (Kirxgofning 2 - qonuni):

$$\sum_{k=1}^{n} \underline{E}_k = \sum_{q=1}^{m} \underline{I}_q \underline{Z}_q \,.$$

Kompleks quvvat. To‘la quvvatning kompleks ko‘rinishi:

$$\tilde{S} = \underline{U}\,\overset{*}{\underline{I}} = UIe^{\pm j\varphi} = UI\cos\varphi \pm jUI\sin\varphi = P \pm jQ\,,$$

bunda $\dot{\underline{I}}$ – qo‘shma kompleks tok. Masalan, agar $\underline{I} = I_m e^{-j\varphi}$ bo‘lsa, u holda bu tokning qo‘shmasi $\dot{\underline{I}} = I_m e^{+j\varphi}$ ga teng bo‘ladi. $\tilde{S}$ *kompleks to‘la quvvat* deb ataladi. Uning haqiqiy qismi aktiv quvvatga, mavhum qismi esa reaktiv quvvatga teng, ya’ni:

$$P = \mathrm{Re}\left|\underline{U}\,\overset{*}{\underline{I}}\right| = \mathrm{Re}\left|\tilde{S}\right|; \quad Q = \mathrm{J}m\left|\underline{U}\,\overset{*}{\underline{I}}\right| = \mathrm{Jm}\left|\tilde{S}\right|.$$

Sinusoidal tok zanjirlarida quvvatlar balansi. Energiyaning saqlanish qonunidan ma’lumki, har qanday zanjirda aktiv quvvatlar balansi saqlanadi; manbalar uzatayotgan aktiv quvvatlar yig‘indisi iste’molchilar qabul qilayotgan aktiv quvvatlar yig‘indisiga teng:

$\sum\limits_{k=1}^{m} P_{km} = \sum\limits_{k=1}^{n} I_k^2 r_k$, bu yerda *m*- manbalar soni, *n*- iste’molchilar soni.

Zanjirdagi manbalar uzatayotgan reaktiv quvvatlar yig‘indisi sxemadagi induktiv g‘altak va kondensatorlar iste’mol qilayotgan reaktiv quvvatlar yig‘indisiga teng:

$$\sum_{k=1}^{m} Q_{km} = \sum_{k=1}^{n} I_k^2 x_k .$$

Kompleks quvvatlar balansi:

$$\sum_{k=1}^{m} \tilde{S}_{km} = \sum_{k=1}^{n} \tilde{S}_k .$$

Zanjir elementlarining kompleks qarshiliklar yig‘indisi teng bo‘lsa–da, ular modullarining yig‘indisi umumiy holda teng bo‘lmasligi mumkin. Shuning uchun to‘la quvvatlar balansi har doim ham saqlanmasligi mumkin, ya’ni:

$$\sum_{k=1}^{m} \underline{S}_{km} \neq \sum_{k=1}^{n} \underline{S}_k .$$

Sinusoidal tok zanjirlarini kompleks usulida hisoblash. Sinusoidal tok zanjirlarini hisoblashda o‘zgarmas tok zanjirlarini hisoblashda qo‘llanilgan barcha usullardan foydalaniladi. Tuzilgan tengliklardagi tok, kuchlanish, EYK va qarshiliklar kompleks shaklda yoziladi.

2.2. Rezistor, induktiv g‘altak va kondensatordan iborat bo‘lgan sinusoidal tok zanjirlarini hisoblash

2.1- masala. Bir jinsli magnit maydonda 3000 ayl/min tezlik bilan ramka aylanmoqda. Ramkada induksiyalangan EYK ning burchak va xususiy chastotalarini aniqlansin.

2.2- masala. Kuchlanishi $u = 141 \sin 500t$ V bo‘lgan generatorga qarshiligi $r = 10\ \Omega$ bo‘lgan reostat ulangan. Zanjirdagi tokning oniy va ta’sir etuvchi qiymatlari, to‘la va aktiv quvvatlar aniqlansin hamda tok, kuchlanish va oniy quvvatning egri chiziqlari chizilsin.

2.3- masala. Kuchlanishi $u = 141\sin 500t$ V bo‘lgan generatorga $L = 0{,}02$ H induktivlik ulangan. Zanjirdagi tokning oniy qiymat ifodasini yozing va ta’sir etuvchi qiymatini aniqlang. Zanjirning to‘la, aktiv va reaktiv quvvatlarini aniqlang hamda kuchlanish, tok va quvvat oniy qiymatlari o‘zgarishlarining vaqt diagrammasini tasvirlang. Magnit maydon energiyasi maksimal qiymatini aniqlang.

2.4- masala. Kuchlanishi $u = 141\sin 500t$ V generatorga sig‘imi $C = 200$ mkF bo‘lgan kondensator ulangan. Zanjirdagi tokning oniy qiymati ifodasini yozing va uning ta’sir etuvchi qiymatini aniqlang. Zanjirning to‘la, aktiv va reaktiv quvvatlarini aniqlang hamda kuchlanish, tok va oniy quvvatlarning vaqt diagrammalarini chizing. Elektr maydon energiyasining maksimal qiymatini aniqlang.

2.5- masala. Passiv ikki qutblikning qismalaridagi kuchlanish $u = 170\sin\omega t$ V, undan o‘tadigan tok esa $i = 5\sin(\omega t + 30^\circ)$ A. Zanjirning to‘la, aktiv va reaktiv quvvatlarini hamda quvvat koeffitsiyentini aniqlang.

Yechish. To‘la quvvat:

$$S = UI = \frac{170}{\sqrt{2}} \cdot \frac{5}{\sqrt{2}} = 425 \text{ VA}.$$

Kuchlanish va tok orasidagi faza siljish burchagi:

$$\varphi = \psi_u - \psi_i = 0 - 30^\circ = -30^\circ.$$

Zanjirning quvvat koeffitsiyenti:

$$\cos\varphi = \cos(-30^\circ) = 0{,}865 \text{ yoki } \cos\varphi = \frac{P}{S} = 0{,}865.$$

Aktiv quvvat:

$$P = S\cos\varphi = 425 \cdot \cos(-30^\circ) = 368 \text{ W}.$$

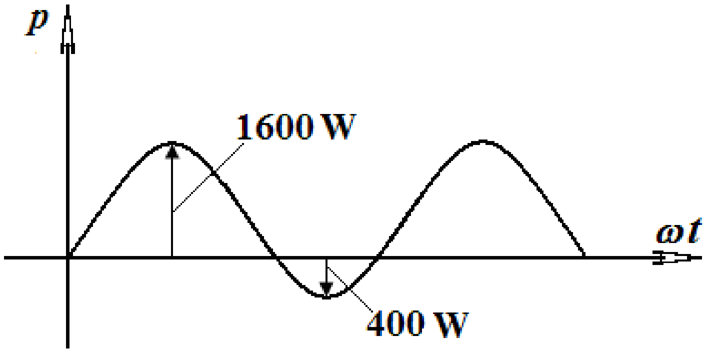
Reaktiv quvvat:

$$Q = S\sin\varphi = 425 \cdot \sin(-30^\circ) = -212{,}5 \text{ var}.$$

2.6- masala. Passiv ikkiqutblik kirishidagi kuchlanish va tok ma'lum:

$u = 311\sin(\omega t + 30^\circ)$ V, $i = 7{,}07\sin(\omega t - 60^\circ)$ A. Zanjirning to'la, aktiv va reaktiv quvvatlarini hamda quvvat koeffitsiyentini toping.

2.7- masala. Passiv ikkiqutblikdagi oniy quvvat o'zgarishi egri chizig'i 2.9-rasmda tasvirlangan. Zanjirning to'la, aktiv quvvatlari va quvvat koeffitsiyenti aniqlansin.



2.9- rasm.

Yechish. Oniy quvvatning o'rtacha qiymati aktiv quvvatga teng. Shuning uchun oniy quvvat o'zgarishi egri chizig'iga ko'ra:

$$P = p_{o'r} = \frac{1600 - 400}{2} = 600 \text{ W}.$$

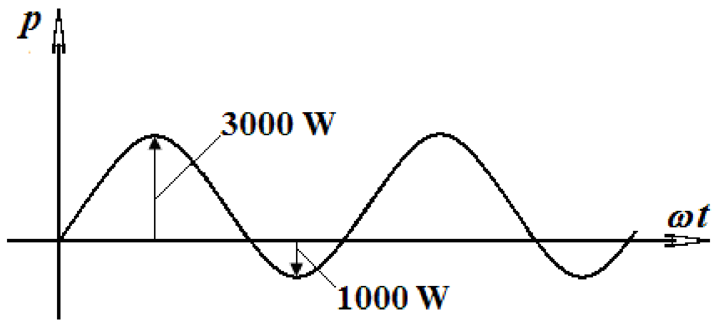
Ko'rilayotgan holat uchun to'la quvvat:

$$S = \frac{1600 + 400}{2} = 1000 \text{ VA}.$$

Quvvat koeffitsiyenti:

$$\cos\varphi = \frac{P}{S} = \frac{600}{1000} = 0{,}6.$$

2.8- masala. 2.10- rasmda quvvatning oniy qiymatlarii o'zgarish egri chizig'i keltirilgan. Yuklamaning quvvat koeffitsiyentini aniqlang.



2.10- rasm.

2.9- masala. G‘altakning induktivligi L ni va aktiv qarshiligi r ni aniqlash uchun $f_1 = 50$ Hz va $f_2 = 100$ Hz chastotada o‘lchashlar o‘tkazildi. Toklar mos ravishda 5 A va 3 A. Agar ikki holatda ham g‘altak qismalaridagi kuchlanish 120 V bo‘lsa, uning aktiv qarshiligi va induktivligini aniqlang.

Yechish. Ikkala chastota uchun g‘altakning to‘la qarshiliklarini aniqlaymiz:

$$Z' = \frac{U}{I'} = \frac{120}{5} = 24\ \Omega \text{ va } Z'' = \frac{U}{I''} = \frac{120}{3} = 40\ \Omega\ .$$

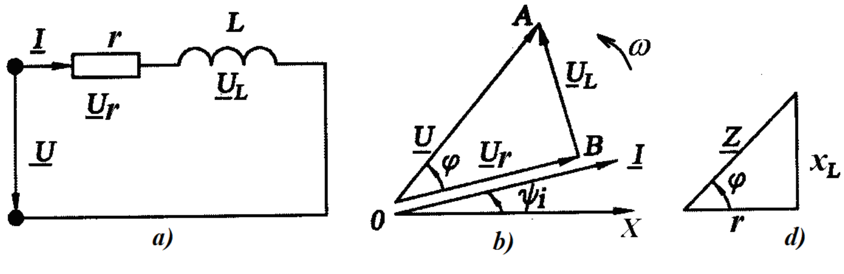
va quyidagi ikkita tenglamani tuzamiz:

$$24 = \sqrt{r^2 + (2\pi \cdot 50L)^2} \text{ va } 40 = \sqrt{r^2 + (2\pi \cdot 100L)^2}\ .$$

Ularni birgalikda yechib, quyidagilarni hosil qilamiz:

$$r = 14{,}87\ \Omega,\ L = 0{,}06\ \text{H}.$$

2.10- masala. Kuchlanishi 220 V bo‘lgan sanoat tarmog‘iga aktiv qarshiligi 6 Ω va induktivligi 50 mH bo‘lgan induktiv g‘altak ulangan (2.11- rasm, *a*). G‘altakdan o‘tuvchi tokning oniy qiymati ifodasini yozing. Induktiv g‘altak uchun tok va kuchlanishlar vektor diagrammasini hamda qarshiliklar uchburchagini quring.



2.11- rasm.

Yechish. G‘altakning induktiv qarshiligi (f=50 Hz):

$$x_L = \omega L = 314 \cdot 50 \cdot 10^{-3} = 15{,}7\ \Omega.$$

G‘altakning to‘la qarshiligi:

$$z = \sqrt{r^2 + x_L^2} = \sqrt{6^2 + 15{,}7^2} = 16{,}8\,\Omega.$$

Zanjirdagi tok:

$$I = U/Z = 220/16{,}8 = 13{,}1\ \text{A}.$$

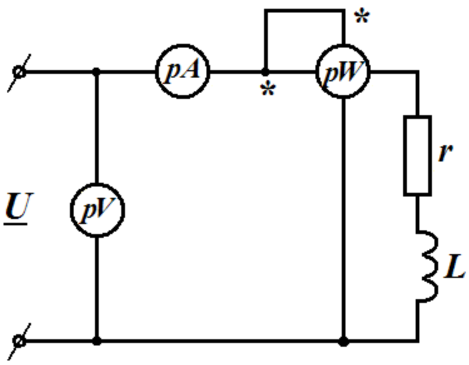
G‘altakning aktiv qarshiligidagi kuchlanish tushuvi: $U_r = Ir = 13{,}1 \cdot 6 = 78{,}6$ V, induktiv qarshiligidagi kuchlanish tushuvi esa $U_L = Ix_L = 13{,}1 \cdot 15{,}7 = 205$ V.

Vektor diagrammani qurish uchun abssissa o‘qining musbat yo‘nalishi bo‘yicha soat mili harakatiga qarama-qarshi yo‘nalishda ψ_i burchak ostida ma’lum masshtabda *(*m_u = 55 V/sm; m_i = 3 A/sm;*)* uzunligi 4,33 sm ga teng tok vektorini quramiz (2.11- rasm). Shu vektorning boshlanish nuqtasidan unga parallel ravishda belgilangan masshtabda uzunligi $\underline{U}_r$ bo‘lgan kuchlanish vektorini quramiz. $\underline{U}_L$ kuchlanish vektori $\underline{U}_r$ vektorining oxiridan tok vektoriga nisbatan 90^0 ilgarilagan ravishda quriladi. $\underline{U}_r$ vektorning boshi bilan $\underline{U}_L$ vektorning oxirini birlashtirib $\underline{U}$ kuchlanish vektorini hosil qilamiz. $\underline{I}$ tok va $\underline{U}$ kuchlanish vektorlari orasidagi burchakni o‘lchab $\varphi = -69^0$ ni topamiz.

Shunday qilib, $i = 13{,}1\sqrt{2}\sin\left(314 - 69^0\right)$A.

Qarshiliklar uchburchagining katetlari ma’lum masshtabda r va x_L, gipotenuzasi esa z ga teng (2.11- rasm, d).

2.11- masala. 2.12- rasmdagi zanjirda tajribalar o‘tkazilganda asboblar $U = 100$ V, $I = 4$ A, $P = 112$ W qiymatlarni ko‘rsatdi. Agar chastota 50 Hz bo‘lsa, r va L aniqlansin.



2.12- rasm

Yechish.

$$r=\frac{P}{I^2}=\frac{112}{4^2}=7\ \Omega;\ Z=\frac{U}{I}=\frac{100}{4}=25\ \Omega;$$

$$X_L=\sqrt{Z^2-r^2}=\sqrt{25^2-7^2}=24\ \Omega;$$

$$L=\frac{X_L}{\omega}=\frac{24}{2\pi\cdot 50}=\frac{24}{314}=0{,}076\ \text{H}.$$

2.12- masala. Induktiv g‘altak avval kuchlanishi 30 V bo‘lgan o‘zgarmas tok manbayiga, keyin esa chastotasi 50 Hz, kuchlanishi 30 V bo‘lgan sinusoidal tok manbayiga ulangan. Birinchi holatda o‘lchangan tok 3 A ni, ikkinchi holatda esa 5 A ni ko‘rsatgan. r va L aniqlansin.

2.13- masala. EYK $E=5\text{V}$ va burchak chastotasi $\omega=10^3\ \text{rad/s}$ bo‘lgan sinusoidal tok manbayiga $R=4\ \Omega$ aktiv qarshilik va kondensator ketma-ket ulangan. Agar zanjirdagi tok $I=1\ \text{A}$ bo‘lsa, kondensatorning sig‘imi qancha bo‘lgan?

Yechish. Zanjirning kompleks qarshiligi $\underline{Z}=R-j\dfrac{1}{\omega C}$; uning moduli $Z=\sqrt{R^2+\left(\dfrac{1}{\omega C}\right)^2}$. Om qonuniga ko‘ra $I=\dfrac{E}{Z}$, bundan $Z=\dfrac{E}{I}=\dfrac{5}{1}=5\Omega$.

Binobarin: $\dfrac{1}{\omega C}=\sqrt{Z^2-R^2}=\sqrt{5^2-4^2}=3\Omega$ va bundan

$$C=\frac{1}{\omega x_C}=\frac{1}{10^3\cdot 3}=0{,}333\cdot 10^{-3}=333{,}3\ \text{mkF}.$$

2.14- masala. Aktiv qarshiligi $r=6\ \Omega$, induktivligi $L=8\ \text{mH}$ bo‘lgan induktiv g‘altak kuchlanishi $u = 100\sqrt{2}\sin(1000t)$ V bo‘lgan manbaga ulangan. Tokning oniy qiymati ifodasi yozilsin, zanjirning to‘la, aktiv va reaktiv quvvatlari hamda g‘altakning aktiv va reaktiv tashkil etuvchi kuchlanishlari, ularning oniy qiymat ifodalari aniqlansin. Vektor diagramma qurilsin.

Yechish. Shoxobchalanmagan zanjirdagi tok ifodasini yozamiz:

$$i = I_m \sin(\omega t + \psi_u - \varphi),$$

bunda $I_m = \dfrac{U_m}{\sqrt{r^2 + X_L^2}} = \dfrac{100\sqrt{2}}{\sqrt{6^2 + (1000 \cdot 0{,}008)^2}} = 10\sqrt{2}$ A;

$$\varphi = \operatorname{arctg}\frac{\omega L}{R} = \operatorname{arctg}\frac{8}{6} = 53^\circ 08'.$$

Binobarin:

$$i = 10\sqrt{2}\sin(1000t - 53^\circ 08') \text{ A}.$$

Zanjirning aktiv, reaktiv va to‘la quvvatini aniqlaymiz:

$$P = I^2 r = 10^2 \cdot 6 = 600 \text{ W},$$
$$Q = I^2 X_L = 10^2 \cdot 8 = 800 \text{ var},$$
$$S = \sqrt{P^2 + Q^2} = 1000 \text{ VA}.$$

G‘altak kuchlanishining aktiv va reaktiv tashkil etuvchilarini topamiz:

$$U_a = rI = 6 \cdot 10 = 60 \text{ V},$$
$$U_r = X_L I = 8 \cdot 10 = 80 \text{ V}.$$

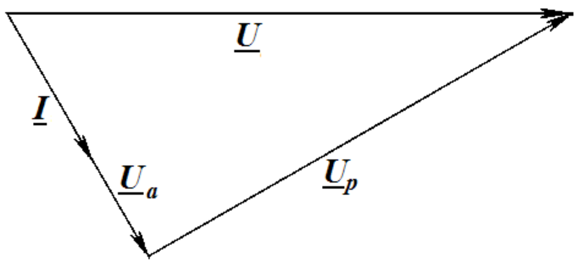
Kuchlanish aktiv tashkil etuvchisining fazasi tok fazasi bilan mos keladi:

$$u_a = 60\sqrt{2}\sin(1000t - 53^\circ 08') \text{ V}.$$

Kuchlanishning reaktiv tashkil etuvchisining fazasi tokka nisbatan $+90^\circ$ ga ilgarilab o‘zgaradi:

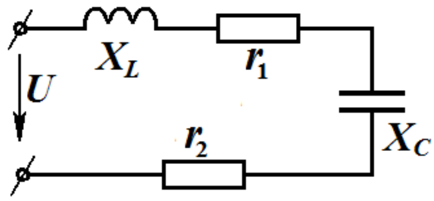
$$u_r = u_L = 80\sqrt{2}\sin(1000t + 36^\circ 52') \text{ V}.$$

Effektiv qiymatlar uchun tuzilgan vektor diagramma 2.13- rasmda tasvirlangan.



2.13- rasm

2.15- masala. Parametrlari $r_1 = r_2 = X_L = 4\ \Omega$, $X_C = 10\ \Omega$, $u = 170\sin(\omega t + 30^\circ)$ V bo‘lgan zanjir (2.14- rasm) uchun $I, \varphi, U_a, U_p, P, Q, S$ aniqlansin va tokning oniy qiymat ifodasi yozilsin.



2.14- rasm.

Yechish.

$$I = \frac{\frac{U_m}{\sqrt{2}}}{\sqrt{(r_1 + r_2)^2 + (X_L - X_C)^2}} = \frac{120}{\sqrt{8^2 + (-6)^2}} = 12\ \text{A};$$

$$\varphi = \operatorname{arctg}\frac{X_L - X_C}{r_1 + r_2} = \operatorname{arctg}\frac{-6}{8} = -36^\circ 52';$$

$$U_a = (r_1 + r_2)I = 8 \cdot 12 = 96\ \text{V};$$
$$U_r = U_L - U_C = (X_L - X_C)I = -6 \cdot 12 = -72\ \text{V};$$

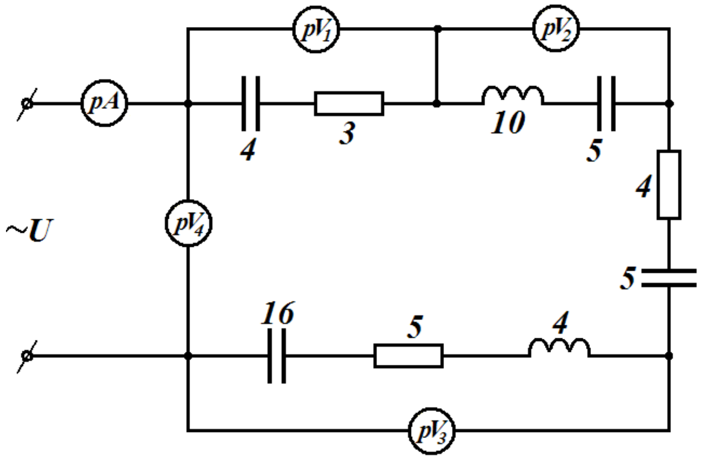
$$P = I^2(r_1 + r_2) = 12^2 \cdot 8 = 1152\ \text{W};$$
$$Q = I^2(X_L - X_C) = 12^2 \cdot (-6) = -864\ \text{var};$$

$$S = \sqrt{P^2 + Q^2} = 1440\ \text{VA};$$

$$i = I_m \sin(\omega t + \psi_u - \varphi) = 12\sqrt{2}\sin(\omega t + 30^\circ - (-36^\circ 52')) =$$
$$= 12\sqrt{2}\sin(\omega t + 66^\circ 52') \text{ A.}$$

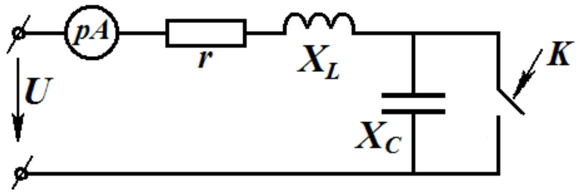
2.16- masala. Aktiv qarshiligi $r = 10\,\Omega$, induktivligi $L = 0{,}01\,\text{H}$ bo‘lgan induktiv g‘altak $C = 50\,\text{mkF}$ sig‘imli kondensator orqali $u = 170\sin(1000t)$ V kuchlanishli manbaga ulangan. $I, \varphi, U_r,\ U_L, U_C, P,\ Q, S$ lar aniqlansin.

2.17- masala. Zanjirda (2.15- rasm) ampermetr ko‘rsatishi 5 A. Voltmetrlar ko‘rsatishlari topilsin. Qarshiliklar qiymatlari 2.15- rasmda Ω larda ko‘rsatilagan.



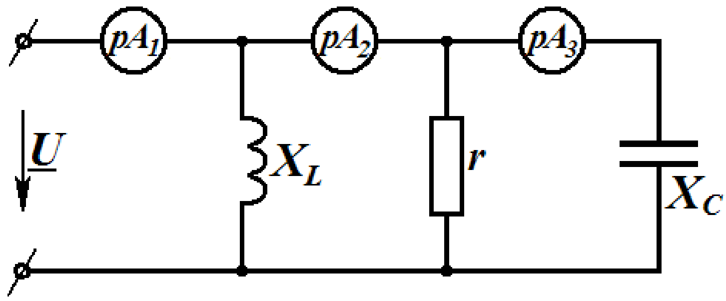
2.15- rasm.

2.18- masala. Zanjirda (2.16- rasm) $r = 7\,\Omega$, $U = 125$ V. X_L va X_C qarshiliklarning qanday qiymatlarida kalit ulangan va uzilgan holatlarda ham ampermetr ko‘rsatishi bir xil va 5 A ga teng bo‘ladi?



2.16- rasm.

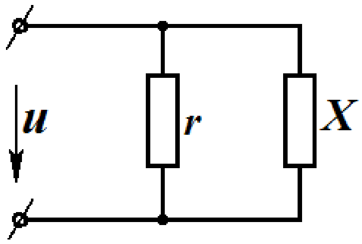
2.19- masala. Zanjirdagi (2.17- rasm) ampermetrlar ko‘rsatishlarini aniqlang. Berilgan: $U = 100$ V, $r = X_L = X_C = 10\ \Omega$.



2.17- rasm.

2.20- masala. Zanjirdagi (2.17- rasm) ampermetr ko‘rsatishini aniqlang. Berilgan $U = 12$ V, $r = 4\ \Omega$, $X_L = 1{,}5\ \Omega$, $X_C = 3\ \Omega$.

2.21- masala. Agar $u = 170\sin\omega t$ V, $i = 5\sqrt{2}\sin(\omega t - 30^\circ)$ A bo‘lsa, zanjirdagi (2.18- rasm) r va X qarshiliklarni aniqlang.

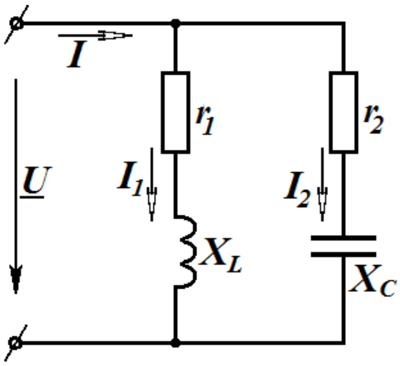


2.18- rasm.

2.22 -masala. Zanjirda (2.18- rasm) $u = 170\sin(\omega t + 30^\circ)$ V, $i = 5\sqrt{2}\sin(\omega t - 15^\circ)$ A bo‘lsa, r va X qarshiliklarni aniqlansin.

2.23- masala. Zanjirda (2.18- rasm) $u = 170\sin(\omega t - 30^\circ)$ V, $i = 5\sqrt{2}\sin(\omega t + 15^\circ)$ A bo‘lsa, r va X qarshiliklarni aniqlansin.

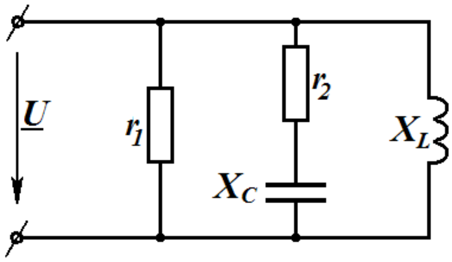
2.24- masala. Zanjirdagi (2.19- rasm) I_1, I_2, I toklar va zanjirning ekvivalent parametrlari (r_{ekv}, X_{ekv}) aniqlansin. Berilgan: $\underline{U} = 100$ V, $r_1 = 8\,\Omega$, $X_L = 6\ \Omega$, $r_2 = 24\ \Omega$, $X_C = 7\ \Omega$.



2.19- rasm.

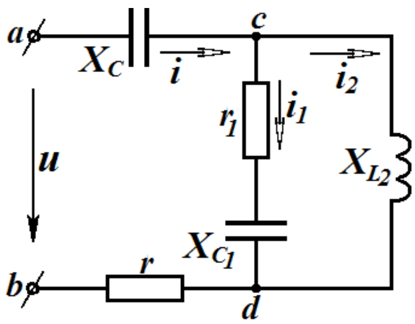
2.25- masala. Zanjir(2.19- rasm)dagi I tok va berilayotgan kuchlanish aniqlansin hamda aktiv va reaktiv qarshiliklardan iborat ketma-ket ulangan ekvivalent zanjirning parametrlari topilsin. Berilgan zanjirning parametrlari: $r_1 = 12\ \Omega$, $X_L = 16\ \Omega$, $r_2 = 12\ \Omega$, $X_C = 5\ \Omega$, $I_1 = 4\ \text{A}$.

2.26- masala. Zanjir (2.20- rasm) shoxobchalaridagi toklar va umumiy tok aniqlansin hamda tok va kuchlanishlar vektor diagrammasi qurilsin. Quyidagilar berilgan: $U = 220\ \text{V}$, $r_1 = 50\ \Omega$, $r_2 = 24\ \Omega$, $X_C = 32\ \Omega$, $X_L = 40\ \Omega$.



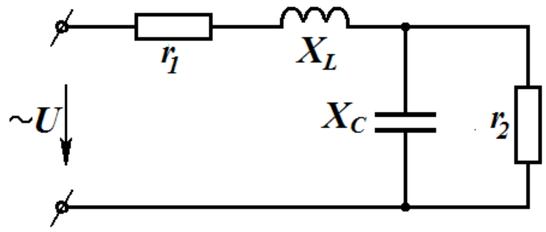
2.20- rasm.

2.27- masala. Zanjir (2.21- rasm) shoxobchalaridagi toklarning ta'sir etuvchi qiymatlari aniqlansin va ularning oniy qiymat ifodalari yozilsin hamda tok va kuchlanishlarning vektor diagrammasi qurilsin. Quyidagilar berilgan: $u = 80\sin(\omega t)\ \text{V}$, $r_1 = r = 2\ \Omega$, $X_{L2} = X_{C1} = 4\ \Omega$, $X_C = 14\ \Omega$.



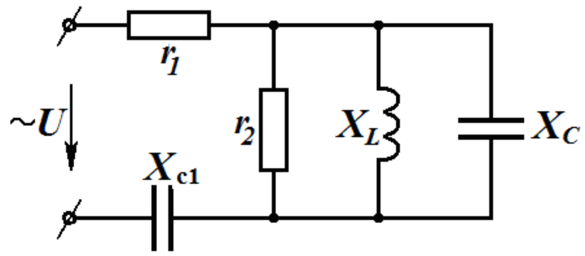
2.21- rasm.

2.28- masala. Zanjir (2.22- rasm)dagi toklar aniqlansin. Quyidagilar berilgan: $U = 220$ V, $r_1 = X_L = 50\ \Omega$, $r_2 = X_C = 100\ \Omega$.



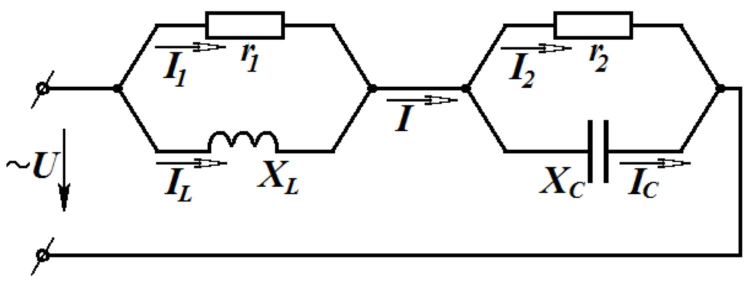
2.22- rasm.

2.29- masala. Zanjir (2.23- rasm)dagi kirish tok aniqlansin Quyidagilar berilgan: $U = 127$ V, $r_1 = r_2 = X_L = X_C = X_{C1} = 10\,\Omega$.



2.23- rasm.

2.30- masala. Zanjir (2.24- rasm)dagi kirish tok aniqlansin Quyidagilar berilgan: $U = 127$ V, $r_1 = X_C = 20\,\Omega$, $r_2 = X_L = 10\,\Omega$.



2.24- rasm.

2.3. Sinusoidal tok zanjirlarini kompleks usulda hisoblash

2.31- masala. Yuklama kuchlanishi va tokining effektiv qiymatlari kompleks ko‘rinishda berilgan: $\underline{U} = 120e^{j83^\circ 08'}$ V, $\underline{I} = 2{,}4e^{j30^\circ}$ A. Yuklama qarshiligi, quvvat koeffitsiyenti, to‘la, aktiv va reaktiv quvvatlar topilsin.

Yechish. Ta’sir etuvchi qiymatlar uchun Om qonunidan foydalanib yuklamaning kompleks qarshiligini topamiz:

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{120e^{j83^\circ 08'}}{2{,}4e^{j30^\circ}} = 50e^{j53^\circ 08'} = 30 + j40\ \Omega.$$

Bu kompleks sonning haqiqiy qismi aktiv qarshilikka, mavhum qismi esa – reaktiv qarshilikka teng, ya’ni: $r = 30\ \Omega$, $x_L = 40\ \Omega$.

Zanjirning quvvat koeffitsiyenti: $\cos\varphi = \dfrac{r}{Z} = \dfrac{30}{50} = 0{,}6\ (\varphi = 53^\circ 08')$.

To‘la quvvatni aniqlaymiz:

$$\tilde{S} = \underline{U}\dot{\underline{I}} = 120e^{j83^\circ 08'} \cdot 2{,}4e^{-j30^\circ} = 288e^{j53^\circ 08'} = 172{,}8 + j230{,}4\ \text{VA}.$$

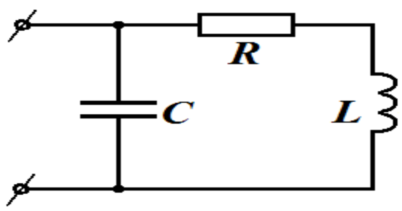
Bu kompleks sonning moduli to‘la quvvatga, haqiqiy tashkil etuvchisi aktiv quvvatni, mavhum tashkil etuvchisi esa reaktiv quvvatni aks ettiradi, ya’ni:

$S = 288\ \text{VA}$, $P = 172{,}8\ \text{W}$, $Q = 230{,}4\ \text{var}$.

2.32- masala. Agar yuklamadagi kuchlanish va tok mos ravishda:

a) $\underline{U} = 220$ V, $\underline{I} = 4 - j3$ A; b) $\underline{U} = 80 + j60$ V, $\underline{I} = 10$ A; d) $\underline{U} = 220$ V, $\underline{I} = 4 - j3$ A bo‘lsa, kompleks tekislikda ularning vektor diagrammalarini quring hamda yuklama kompleks qarshiligi va o‘tkazuvchanligini aniqlang.

2.33- masala. Zanjirning (2.25- rasm) kirish kompleks qarshiligini $\omega = 0$; 200; 400 va 1000 rad/s chastotalarda aniqlang. Bunda: $R = 40\ \Omega$, $L = 0{,}1$ H, $C = 31{,}3$ mkF.



2.25- rasm.

2.34- masala. 2.18- masala kompleks usulda yechilsin. Vektor diagramma qurilsin.

2.35- masala. 2.19- masala kompleks usulda yechilsin. Vektor diagramma qurilsin.

2.36- masala. 2.20- masala kompleks usulda yechilsin. Vektor diagramma qurilsin.

2.37- masala. 2.21- masala kompleks usulda yechilsin. Vektor diagramma qurilsin.

2.38- masala. 2.22- masala kompleks usulda yechilsin. Vektor diagramma qurilsin.

2.39- masala. 2.23- masala kompleks usulda yechilsin. Vektor diagramma qurilsin.

2.40- masala. 2.24- masala kompleks usulda yechilsin. Vektor diagramma qurilsin.

2.41- masala. 2.25- masalani kompleks usulda yechamiz.

Yechish. Birinchi shoxobchadagi tokning boshlang‘ich fazasini nol deb qabul qilamiz: $\underline{I}_1 = 4$ A.

Kirish kuchlanishi ta’sir etuvchi kompleks qiymatini aniqlaymiz:

$$\underline{U} = \underline{Z}_1 \underline{I}_1 = (r_1 + jX_L)\underline{I}_1 = (12 + j16) \cdot 4 = 80e^{j53^\circ 08'} \text{ V.}$$

Ikkinchi shoxobchadagi tokning ta’sir etuvchi qiymati kompleksini topamiz:

$$\underline{I}_2 = \frac{\underline{U}}{\underline{Z}_2} = \frac{\underline{U}}{(r_2 - jX_C)} = \frac{80e^{j53^\circ 08'}}{12 - j5} = \frac{80e^{j53^\circ 08'}}{13e^{-j22^\circ 40'}} = 6{,}15e^{j75^\circ 48'} =$$
$$= 1{,}51 + \text{j}5{,}95 \text{ A.}$$

Umumiy tok:

$$\underline{I} = \underline{I}_1 + \underline{I}_2 = 4 + 1{,}51 + \text{j}5{,}95 = 5{,}51 + \text{j}5{,}95 = 8{,}11e^{j47^\circ 10'} \text{ A.}$$

Kirish qarshiligi:

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{80e^{j53^\circ 08'}}{8{,}11e^{j47^\circ 10'}} = 9{,}86e^{j5^\circ 58'} = 9{,}85 + j0{,}985 \ \Omega.$$

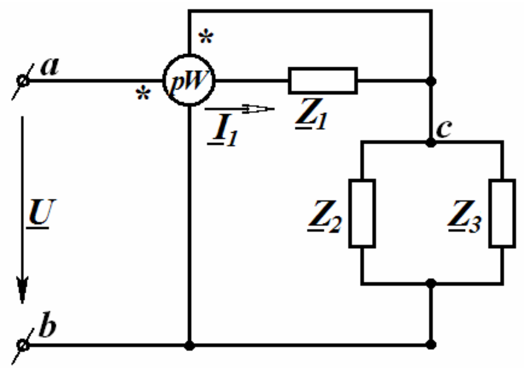
2.42- masala. 2.27- masala kompleks usulda yechilsin.

2.43- masala. 2.28- masala kompleks usulda yechilsin.

2.44- masala. 2.29- masala kompleks usulda yechilsin.

2.45- masala. 2.30- masala kompleks usulda yechilsin.

2.46- masala. Zanjirdagi (2.27- rasm) vattmetr ko‘rsatishi topilsin. Berilgan: $\underline{U} = 120e^{-j30^\circ}$ V, $\underline{Z}_1 = 20e^{j30^\circ}$ Ω, $\underline{Z}_2 = \underline{Z}_3 = 40e^{j30^\circ}$ Ω.



2.27- rasm.

Yechish. Kompleks tok $\underline{I}_1$ ni aniqlaymiz:

$$\underline{I}_1 = \frac{\underline{U}}{\underline{Z}} = \frac{\underline{U}}{\underline{Z}_1 + 0{,}5\underline{Z}_2} = \frac{120e^{-j30^\circ}}{20e^{j30^\circ} + 20e^{j30^\circ}} = \frac{120e^{-j30^\circ}}{40e^{j30^\circ}} = 3e^{-j60^\circ} \text{ A.}$$

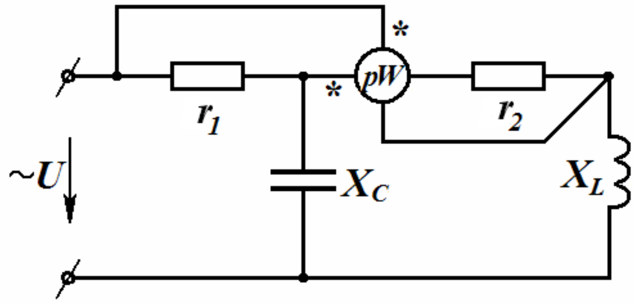
Kompleks kuchlanish $\underline{U}_{cb}$:

$$\underline{U}_{cb} = 0{,}5\underline{Z}_2\underline{I}_1 = 20e^{j30^\circ} \cdot 3e^{-j60^\circ} = 60e^{-j30^\circ} \text{ V.}$$

Vattmetr ko‘rsatishi:

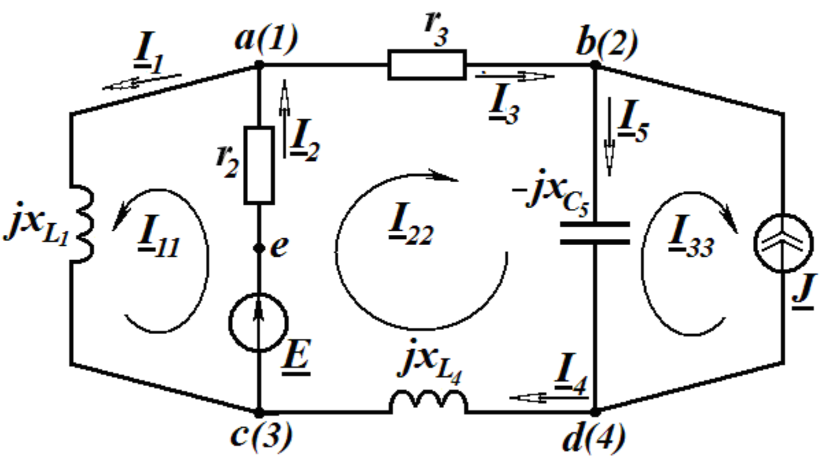
$$P_W = \mathrm{Re}\left[60e^{-j30^\circ} \cdot 3e^{j60^\circ}\right] = \mathrm{Re}\left[180e^{j30^\circ}\right] = 180\cos 30^\circ = 156 \text{ W.}$$

2.47- masala. Agar zanjir (2.28- rasm) parametrlari $U = 220$ V, $r_1 = X_C = 40$ Ω, $r_2 = X_L = 20$ Ω bo‘lsa, unga ulangan vattmetr ko‘rsatishini aniqlang.



2.28- rasm.

2.48- masala. Zanjir (2.29- rasm) shoxobchalaridagi toklarni kontur toklar usulida aniqlang. Berilgan: $r_2 = X_{L_1} = X_{C_5} = 10\,\Omega$, $r_3 = X_{L_4} = 5\,\Omega$, $\underline{E} = 100\,\text{V}$, $J = 10\,\text{A}$.



2.29- rasm.

Yechish. Berilgan zanjir uchun tenglamalar tuzishga kirishishdan avval tok manbayini ekvivalent EYK manbayiga almashtiramiz (2.29- rasm):

$$E' = -J(-jx_{C_5}) = -10\cdot(-j10) = j100\,.$$

Bog‘lanmagan konturlar toklari yo‘nalishlarini rasmda ko‘rsatilganidek tanlab tenglamalar sistemasini tuzamiz:

$$\left.\begin{aligned} \underline{I}_{11}\underline{Z}_{11} + \underline{I}_{22}\underline{Z}_{12} + \underline{I}_{33}\underline{Z}_{13} = \underline{E}_{11} \\ \underline{I}_{11}\underline{Z}_{21} + \underline{I}_{22}\underline{Z}_{22} + \underline{I}_{33}\underline{Z}_{23} = \underline{E}_{22} \end{aligned}\right\},$$

ularga koeffitsiyentlarni qo‘yamiz:

$$\underline{Z}_{11} = r_2 + jX_{L_1} = 10 + j10 = 10\sqrt{2}e^{j45^\circ}\ \Omega,$$

$$\underline{Z}_{22} = r_2 + r_3 + jX_{L_4} - jX_{C_5} = 15 - j5 = 15{,}8e^{-j18^\circ 26'}\ \Omega,$$

$$\underline{Z}_{12} = \underline{Z}_{21} = r_2 = 10\,\Omega,\ \ \underline{Z}_{13} = 0,$$

$$Z_{23} = -jX_{C_5} = -j10 = 10e^{-j90^\circ}\ \Omega,$$

$$\underline{E}_{11} = \underline{E} = 100\ \text{V},\ \underline{E}_{22} = \underline{E} + E' = 100 + j100\ \text{V}\ .$$

Natijada tenglama quyidagi ko‘rinishga keladi:

$$(10 + j10)\underline{I}_{11} + 10\underline{I}_{22} = 100,$$
$$10\underline{I}_{11} + (15 - j5)\underline{I}_{22} = 100 + j100.$$

Kontur toklar kompleks qiymatlarini aniqlovchilar (determinantlar) yordamida topamiz:

$$\underline{I}_{11} = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 100 & 10 \\ 100 + j100 & 15 - j5 \end{vmatrix}}{\begin{vmatrix} 10 + j10 & 10 \\ 10 & 15 - j5 \end{vmatrix}} = \frac{500 - j1500}{100 + j100} = (-5 - j10)\ \text{A},$$

$$\underline{I}_{22} = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 10 + j10 & 100 \\ 10 & 100 + j100 \end{vmatrix}}{100 + j100} = \frac{-1000 + j2000}{100 + j100} = (5 + j15)\ \text{A}.$$

Shoxobchalardagi toklarni mos kontur toklarning algebraik yig‘indisi sifatida topamiz:

$$\underline{I}_1 = \underline{I}_{11} = -5 - j10 = 11{,}2e^{-j126^\circ 34'}\ \text{A},$$
$$\underline{I}_2 = \underline{I}_{11} + \underline{I}_{22} = j5 = 5e^{j90^\circ}\ \text{A},$$
$$\underline{I}_3 = \underline{I}_4 = \underline{I}_{22} = 5 + j15 = 15{,}8e^{j71^\circ 31'}\ \text{A},$$
$$\underline{I}_5 = \underline{I}_{22} + \underline{I}_{33} = 15 + j15 = 15\sqrt{2}\, e^{j45^\circ}\ \text{A}.$$

Yechimlarni tekshirish uchun Kirxgofning 2-qonuni bo‘yicha tenglama tuzamiz:

$$r_2\underline{I}_2 + R_3\underline{I}_3 + (-jX_{C_5})\underline{I}_5 + jX_{L_4}\underline{I}_4 = E_1.$$

Bu tenglamaga son qiymatlarni qo‘ygandan so‘ng quyidagi ayniyatni hosil qilamiz:

$$10j5 + 5(5 + j15) + (-j10)(15 + j15) + j5(5 + j15) = 100 \ \text{yoki}\ 100 = 100.$$

Agar topografik vektor diagramma qurish zaruriyati tug‘ilsa, u holda avval zanjirning tugunlaridagi va oraliq nuqta e dagi kompleks potensiallarni aniqlash lozim bo‘ladi. $\varphi_d = 0$ deb qabul qilinadi va quyidagilar hisoblanadi:

$$\underline{\varphi}_e = \underline{\varphi}_d + \underline{E} = 100\ \text{V},$$
$$\underline{\varphi}_a = \underline{\varphi}_e - r_2\underline{I}_2 = 100 - 10 \cdot j5 = 100 - j50 = 112e^{-j26^\circ 30'}\ \text{V},$$
$$\underline{\varphi}_b = \underline{\varphi}_a - r_3\underline{I}_3 = 100 - j50 - 5(5 + j15) = 75 - j125 = 146e^{-j59^\circ}\ \text{V},$$
$$\underline{\varphi}_c = \underline{\varphi}_b - (-jX_{C5})\underline{I}_5 = 75 - j125 - (-j10)(15 + j15) = -75 + j25\ \text{V},$$
$$\underline{\varphi}_d = \underline{\varphi}_c - jX_{L4}\underline{I}_4 = -75 + j25 - j5(5 + j15) = 0.$$

2.49- masala. Zanjir (2.29- rasm) shoxobchalaridagi toklarni tugun potensiallar usulida aniqlang.

Yechish. $\varphi_4 = 0$ deb qabul qilib quyidagi kanonik tenglamalar sistemasini tuzamiz:

$$\left.\begin{aligned} \underline{Y}_{11}\underline{\varphi}_1 + \underline{Y}_{12}\underline{\varphi}_2 + \underline{Y}_{13}\underline{\varphi}_3 &= \underline{J}_{11}, \\ \underline{Y}_{21}\underline{\varphi}_1 + \underline{Y}_{22}\underline{\varphi}_2 + \underline{Y}_{23}\underline{\varphi}_3 &= \underline{J}_{22}, \\ \underline{Y}_{31}\underline{\varphi}_1 + \underline{Y}_{32}\underline{\varphi}_2 + \underline{Y}_{33}\underline{\varphi}_3 &= \underline{J}_{33}, \end{aligned}\right\}$$

bunda $\underline{Y}_{11} = \dfrac{1}{jX_{L_1}} + \dfrac{1}{r_2} + \dfrac{1}{r_3} = -j0{,}1 + 0{,}1 + 0{,}2 = 0{,}316e^{-j18^\circ 20'}$ S – 1- tugunga birikkan shoxobchalar o‘tkazuvchanliklarining komplekslari;

$$\underline{Y}_{22} = \frac{1}{Z_3} + \frac{1}{Z_5} = \frac{1}{r_3} + \frac{1}{-jX_{C_5}} = 0{,}2 + j0{,}1 = 0{,}244e^{j26^\circ 30'}\ S -$$

2- tugunga birikkan shoxobchalar o‘tkazuvchanliklarining komplekslari;

$\underline{Y}_{33} = \dfrac{1}{Z_4} + \dfrac{1}{Z_5} = \dfrac{1}{-jX_{L_4}} + \dfrac{1}{-jX_{C_5}} = -j0{,}2 + j0{,}1 = -j0{,}1\,S$ – 3- tugunga birikkan shoxobchalar o‘tkazuvchanliklarining komplekslari;

$\underline{Y}_{12} = \underline{Y}_{21} = -\dfrac{1}{Z_3} = -\dfrac{1}{r_3} = -0{,}2\text{ S}$ – 1- va 2- tugunlarni tutashtiruvchi shoxobchalar o‘tkazuvchanliklarining “–” ishora bilan olingan yig‘indisi;

$\underline{Y}_{13} = \underline{Y}_{31} = 0$; $\underline{Y}_{23} = \underline{Y}_{32} = -\dfrac{1}{Z_5} = -\dfrac{1}{-jX_{C_5}} = -\dfrac{1}{j10} = -j0{,}1\text{ S}$ – 2- va 3- tugunlarni tutashtiruvchi shoxobchalar o‘tkazuvchanliklarining “–” bilan olingan yig‘indisi; $\underline{J}_{11} = \dfrac{\underline{E}}{r_2} = \dfrac{100}{10} = 10\text{ A}$ – 1- tugunning hisoblangan tugun toki;

$\underline{J}_{22} = \underline{J} = 10\text{ A}$ – 2- tugunning hisoblangan tugun toki; $\underline{J}_{33} = -\underline{J} = -10\text{ A}$ – 3- tugunning hisoblangan tugun toki.

Topilgan koeffitsiyentlarni yuqoridagi tenglamalar sistemasiga qo‘yamiz:

$$\left.\begin{aligned}&(0{,}3-j0{,}1)\underline{\varphi}_1-0{,}2\underline{\varphi}_2=10,\\&-0{,}2\underline{\varphi}_1+(0{,}2+j0{,}1)\underline{\varphi}_2-j0{,}1\underline{\varphi}_3=10,\\&-j0{,}1\underline{\varphi}_2-j0{,}1\underline{\varphi}_3=-10,\end{aligned}\right\}$$

bu tenglamalar sistemasining yechimlari:

$$\underline{\varphi}_1=100-j50\ \text{V},\quad \underline{\varphi}_2=75-j125\ \text{V},\quad \underline{\varphi}_3=-75+j25\ \text{V}.$$

Shoxobchalardagi toklarni Om qonuni bo‘yicha topamiz:

$$\underline{I}_1=\frac{\underline{\varphi}_1-\underline{\varphi}_4}{\underline{Z}_1}=\frac{100-j50}{j10}=-5-j10\ \text{A},$$

$$\underline{I}_2=\frac{\underline{\varphi}_4-\underline{\varphi}_1+\underline{E}}{\underline{Z}_2}=\frac{-100+j50+100}{10}=j5\ \text{A},$$

$$\underline{I}_3=\frac{\underline{\varphi}_1-\underline{\varphi}_2}{\underline{Z}_3}=\frac{100-j50-75+j125}{5}=5+j15\ \text{A},$$

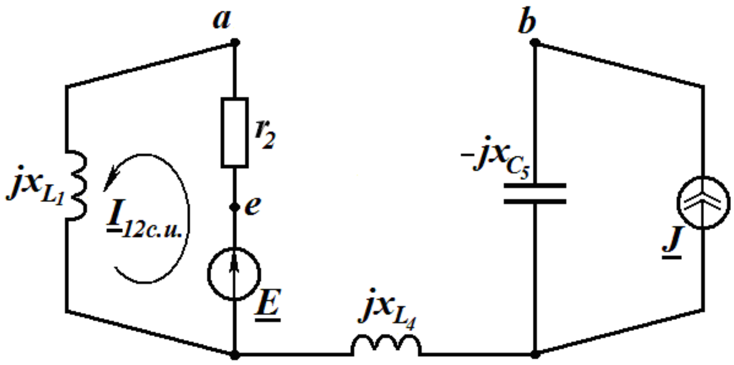
$$\underline{I}_4=\frac{\underline{\varphi}_3-\underline{\varphi}_4}{\underline{Z}_4}=\frac{-75+j25}{j5}=5+j15\ \text{A},$$

$$\underline{I}_5=\frac{\underline{\varphi}_2-\underline{\varphi}_3}{\underline{Z}_5}=\frac{75-j125+75-j25}{-j10}=15+j15\ \text{A}.$$

2.50- masala. Zanjirning (2.29- rasm) uchinchi shoxobchasidagi I_3 tokni ekvivalent generator usulida toping.

Yechish. Qarshiligi $r_3=5\ \Omega$ bo‘lgan shoxobchani uzib, 2.30- rasmda tasvirlangan zanjirni hosil qilamiz. Bu zanjir uzilgan shoxobchaga nisbatan ekvivalent generator vazifasini bajaradi. Bu ekvivalent generatorning kompleks EYK *a* va *b* nuqtalar orasidagi kompleks kuchlanish $U_{ab\ \text{s.ish.}}$ ga teng. Ushbu kuchlanishni aniqlash uchun dastlab tok $I_{12\ \text{s.ish.}}$ ni topamiz:

$$I_{12\ \text{s.ish.}}=\frac{\underline{E}}{r_2+jX_{L1}}=\frac{100}{10+j10}=5-j5=5\sqrt{2}e^{-j45^\circ}\ \text{A}.$$



2.30- rasm.

Yuqoridagi zanjir uchun quyidagi tenglamani yozish mumkin:

$$\underline{\varphi}_a = \underline{\varphi}_b - (-jX_{C_5})\underline{J} + jX_{L_1}\underline{I}_{12\text{s.ish.}},$$

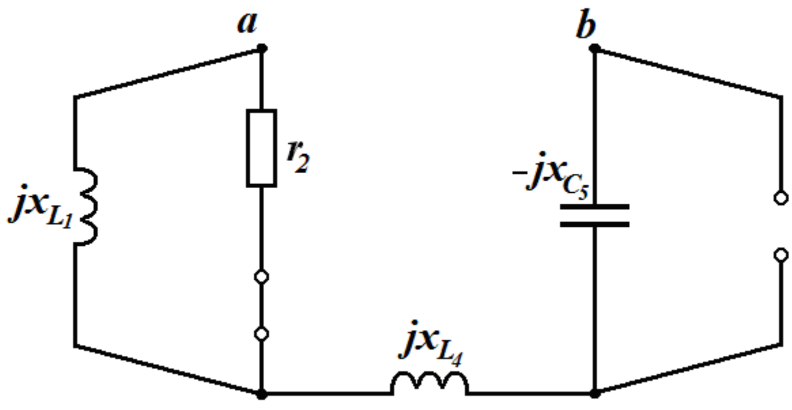
bundan

$$\underline{U}_{ab\ \text{s.ish.}} = \underline{\varphi}_a - \underline{\varphi}_b = jX_{L_1}\underline{I}_{12\text{s.ish.}} - (-jX_{C_5})\underline{J} =$$

$$= j10(5 - j5) - (-j10)\cdot 10 = 50 + j150 = 158e^{j71^\circ 34'}\ \text{V}.$$

Ekvivalent generatorning kompleks ichki qarshiligi 2.31- rasmdagi aktiv ikkiqutblikdan ularning energiya manbalari (ichki qarshiliklari qoldirilib) fikran olib tashlangan va 2.31- rasmda tasvirlangan passiv ikki qutblik kompleks kirish qarshiligiga teng.

2.32- rasmdagi sxema uchun



2.31- rasm, *a*

$$Z_{kir\,ab} = \frac{r_2\, jX_{L_1}}{r_2 + jX_{L_1}} + jX_{L_4} - jX_{C_5} = \frac{10\cdot j10}{10 + j10} + j5 - j10 = 5\,\Omega.$$

Nihoyat $\underline{I}_3$ tokni aniqlaymiz:

$$\underline{I}_3 = \frac{\underline{U}_{ab\ \text{s.ish.}}}{\underline{Z}_{kir\ \ ab} + \underline{Z}_3} = \frac{50 + j150}{5+5} = 5 + j15 = 15{,}8e^{j71^\circ 34'}\ \text{A}.$$

2.51- masala. Zanjir (2.29- rasm)ning $\underline{I}_5$ tokini ekvivalent generator usulida aniqlang.

2.52- masala. Zanjir (2.29- rasm) shoxobchalaridagi toklarni ustma-ustlash usulida aniqlang.

2.53- masala. Zanjir (2.24- rasm)dagi R_2 qarshilikdan o‘tadigan tok ekvivalent generator usulida aniqlansin.

2.54- masala. 2.27- masala kontur toklar usulida yechilsin.

2.55- masala. 2.29- rasmda tasvirlangan zanjirdagi $\underline{Z}_4$ kompleks qarshilik necha bo‘lganda shu shoxobchadagi aktiv quvvat maksimal bo‘ladi? Shu quvvat aniqlansin.

2.56- masala. Agar tok va kuchlanishning oniy qiymat ifodalari mos ravishda $i = 141\sin\ (314t + 60^\circ)$ A, $u = 7{,}07\sin(314t + 30^\circ)$ V ko‘rinishda berilgan bo‘lsa, aktiv, reaktiv va to‘la quvvatlar aniqlansin.

Yechish. Tok va kuchlanishning kompleks ta’sir etuvchi qiymatlarini yozib olamiz:

$$\underline{I} = \frac{141}{\sqrt{2}}e^{j60^0} = 100e^{j60^0}\text{A}; \quad \underline{U} = \frac{7{,}07}{\sqrt{2}}e^{j30^0} = 5e^{j30^0}\text{V}.$$

Kompleks to‘la quvvat:

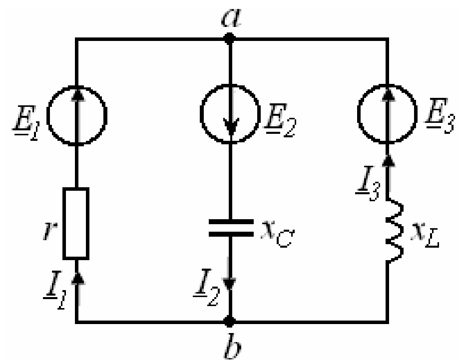
$$\tilde{S} = \underline{U}\,\overset{*}{\underline{I}} = 100e^{j60^0} \cdot 5e^{-j30^0} = 500e^{j30^0} = 500\cos 30^\circ + j500\sin 30^\circ =$$

$$= 433 + j250\ \text{VA}.$$

Shunday qilib, S = 500 VA, P = 433 W, Q = 250 var.

2.57- masala. Agar elektr zanjirda (2.33- rasm) $\underline{E}_1 = 120$ V; $\underline{E}_2 = 120e^{-j120^0}$V; $\underline{E}_3 = 120e^{-j240^0}$V; $r = x_L = x_C = 10\,\Omega$ bo‘lsa, shoxobchalardagi toklar aniqlansin.

Yechish. Ushbu zanjirni ikki tugun usuli yordamida hisoblash osonroq. Buning uchun shoxobchalarning kompleks o‘tkazuvchanliklarini aniqlaymiz:



2.32- rasm.

$$\underline{Y}_1 = \frac{1}{r} = 0{,}1 \text{ S};$$

$$\underline{Y}_2 = \frac{1}{-jx_C} = j0{,}1 \text{ S};$$

$$\underline{Y}_3 = \frac{1}{jx_L} = -j0{,}1 \text{ S}.$$

Ikki tugun orasidagi kompleks kuchlanishni topamiz:

$$\underline{U}_{ab} = \frac{\underline{E}_1\underline{Y}_1 + \underline{E}_2\underline{Y}_2 + \underline{E}_3\underline{Y}_3}{\underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3} =$$

$$= \frac{120 \cdot 0{,}1 + 120e^{-j120^0} \cdot j0{,}1 + 120e^{-j240}(-j0{,}1)}{0{,}1 + j0{,}1 - j0{,}1} = 328 \text{ V}.$$

Om qonuni bo‘yicha shoxobchalardagi toklar:

$$\underline{I}_1 = (\underline{E}_1 - \underline{U}_{ab})\underline{Y}_1 = (120 - 328) \cdot 0{,}1 = -20{,}8 \text{ A},$$

$$\underline{I}_2 = (-\underline{E}_2 + \underline{U}_{ab})\underline{Y}_2 = \left(-120e^{-j120^0} + 328\right) \cdot j0{,}1 = -40e^{j105^\circ} \text{ A},$$

$$\underline{I}_3 = (\underline{E}_3 - \underline{U}_{ab})\underline{Y}_3 = \left(120e^{-j240^0} - 328\right) \cdot (-j0{,}1) = -40e^{j75^\circ} \text{ A}.$$

2.58- masala. Induktivligi $L = 100$ mH bo‘lgan ideal induktiv g‘altak $R = 10\,\Omega$ aktiv qarshilikli rezistor bilan ketma-ket ulangan. Induktiv g‘altak qismlaridagi o‘zinduksiya EYK $e_L = 100\sin(314t + 30°)$ V bo‘lsa, zanjirdan o‘tayotgan tok va unga berilayotgan kuchlanishning oniy qiymatlari aniqlansin.

Yechish. Masalani kompleks usulda yechamiz. Tokning oniy qiymatini, kuchlanish va EYK oniy qiymatlarini bog‘laydigan tenglamadagi kompleks amplitudali algebraik tenglamalar bilan almashtiramiz. EYK e_L g‘altakdan o‘tayotgan tok i bilan quyidagicha bog‘langan: $e_L = -L\dfrac{di}{dt}$ yoki kompleks ko‘rinishda $E_{Lm} = I_m(-j\omega L)$, bunda $\omega L = X_L$ – g‘altakning induktiv qarshiligi.

Tokning amplituda qiymati:

$$I_m = \frac{E_{Lm}}{-j\omega L} = \frac{100e^{j30^\circ}}{-j314\cdot 100\cdot 10^{-3}} = 3.18e^{j120^\circ}\,\text{A}.$$

Zanjir kirishiga berilayotgan kuchlanishning oniy qiymati

$$u = u_R + u_L = iR + L\frac{di}{dt}$$

yoki:

$$\underline{U}_m = \underline{I}_m R + \underline{I}_m j\omega L = \underline{I}_m(R + j\omega L) = \underline{I}_m Z e^{j\varphi} = \underline{I}_m \underline{Z},$$

bunda $\underline{Z}$ – zanjirning kompleks qarshiligi; Z – kompleks qarshilikning moduli; φ – argumenti.

$$Z = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + (314\cdot 100\cdot 10^{-3})^2} = 33\Omega;\ \varphi = \operatorname{arctg}\frac{\omega L}{R} = \operatorname{arctg}\frac{31{,}4}{10} = 72^\circ 20';$$

$$\underline{U}_m = 3{,}18e-^{j90^0}\cdot 33e^{j72^\circ 20'} = 105e^{j162^\circ 20'}\text{V}.$$

Tok va kuchlanish amplitudalarining kompleks qiymatlaridan ularning oniy qiymat ifodalariga o‘tamiz:

$$i = 3{,}18\sin(\omega t + 120^\circ)\,\text{A},\ u = 105\sin(\omega t + 162^\circ 20')\ \text{V}.$$

2.59- masala. Induktiv g‘altakning parametrlarini aniqlash uchun ikkita tajriba o‘tkaziladi: a) o‘zgarmas tokda: U_1=100 V, I_1=1 A; va b) o‘zgaruvchan tokda: f =50 Hz; U_2=100 V; I_2=0,6 A. Induktiv g‘altakning qarshiligi va induktivligi aniqlansin.

Yechish. O‘zgarmas tokda o‘tkazilgan birinchi tajribada: $X_L=\omega L$=0, chunki ω=0. Shu sababli $R = \dfrac{U}{I} = \dfrac{100}{1} = 100\ \Omega$.

Ikkinchi tajribada: $Z=\frac{U}{I}=\frac{100}{0,6}=166,66\ \Omega$, ya'ni to'la qarshilik: $Z=\sqrt{R^2+(x_L)^2}=166,66\ \Omega$. Induktiv qarshilik:

$$x_L=\sqrt{Z^2-R^2}=\sqrt{166,66^2-100^2}=133,33\ \Omega.$$

Induktivlik esa: $L=\frac{X_L}{\omega}=\frac{133,33}{314}=0,42\ \text{H}$.

2.60- masala. Elektr motor kuchlanishi $U_{\text{nom}}=220\ \text{V}$ va chastotasi $f=50\,\text{Hz}$ bo'lgan manbaga ulangan. Motorning quvvati $P_{\text{nom}}=20\,\text{kW}$ va quvvat koeffitsiyenti $\cos\varphi_1=0,6$. Motor $\cos\varphi_2=0,9$ quvvat koeffitsiyenti bilan ishlashi uchun unga qanday sig'imli kondensator ulash kerak bo'ladi?

Yechish. $\cos\varphi=0,6$ da motor toki: $I_1=P/(U\cos\varphi_1)=20\cdot10^3/(220\cdot0,6)=152\ \text{A}$,

$$\varphi_1=\arccos 0,6=53^{\circ}10' \text{ va } \sin\varphi_1=0,8.$$

Motor tokining reaktiv tashkil etuvchisi: $I_{r1}=I_1\sin\varphi_1=152\cdot0,8=121,6\ \text{A}$,

$$\varphi_2=\arccos 0,9=26^{0}50' \text{ va } \sin\varphi_2=0,436.$$

Motor $\cos\varphi_2=0,9$ quvvat koeffitsiyenti bilan ishlashini ta'minlovchi kondensator toki: $I_2=P/(U\cos\varphi_2)=20\cdot10^3/(220\cdot0,9)=101\ \text{A}$.

Shu tokning reaktiv tashkil etuvchisi: $I_{r2}=I_2\sin\varphi_2=101\cdot0,436=44\ \text{A}$.

Kondensator ulangandan keyingi tokning qiymati:

$$I_C=I_{r1}-I_{r2}=121,6-44=77,6\ \text{A}.$$

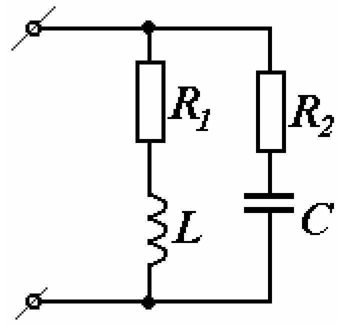
Sig'im qarshiligi: $x_C=\frac{1}{\omega C}=\frac{U}{I_C}=\frac{220}{77,6}=2,83\,\Omega.$

Kondensator sig'imi: $C=\frac{1}{x_C\omega}=\frac{1}{x_C 2\pi f}=\frac{1}{2,83\cdot2\cdot3,14\cdot50}=1130\ \text{mkF}.$

2.61- masala. Aktiv qarshiligi R_L=50 Ω bo'lgan induktiv g'altakka sig'imi $C=3,18\ \text{mkF}$ bo'lgan kondensator ketma-ket ulangan, manba kuchlanishi chastotasi f=500 Hz. Agar g'altak qismalaridagi kuchlanish pasayishi U_{ab}=315 V va zanjirdagi tokning fazasi kuchlanishdan 45° ga orqada qolsa, g'altak induktivligi L, zanjirga berilayotgan kuchlanish va undagi tok aniqlansin.

2.62- masala. 2.34- rasmdagi zanjirda R_1=9 Ω *;* R_2=3,0 Ω*; L*=1,28 mH*; f*=500 Hz . Kondensator sig‘imining qanday qiymatida induktivlikli shoxobchasidagi tok sig‘imli shoxobchadagi tokdan uch barobar katta bo‘ladi? Zanjirning kirish kompleks qarshiligi aniqlansin.

2.63- masala. Elektr zanjirning quvvat koeffitsiyentini oshirish uchun unga kondensator ulanadi. Agar zanjirning aktiv qarshiligi *R*=10 Ω va unga ketma-ket ulangan g‘altak induktivligi *L=100* mH bo‘lsa, zanjirning quvvat koeffitsiyenti cosφ=0,865 bo‘lishi uchun unga qanday sig‘imli kondensator yuklamaga: a) ketma-ket; b) parallel ulanishi lozim?

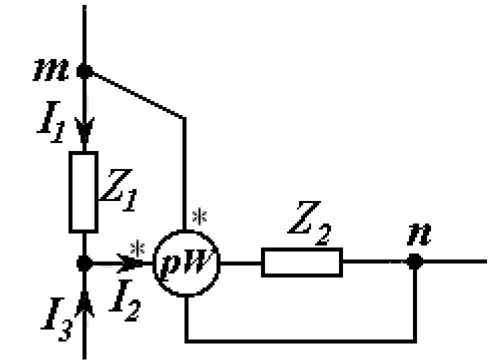


2.34-rasm

2.64- masala. 2.35- rasmda keltirilgan elektr zanjirning bir qismi uchun

$I_1 = 10ej37^o\,\text{A};\quad \underline{I}_3 = 8ej15^o\,\text{A};\quad Z_1 = 2.\Omega;\quad Z_2 = 1{,}8e - j44^o\,\Omega$

bo‘lganda vattmetr ko‘rsatishi aniqlansin. Jadvalda keltirilgan parametrlar uchun vattmetr ko‘rsatishi aniqlansin.

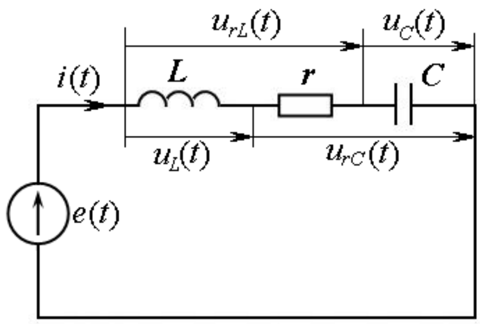


2.35-rasm

№	I_1 , A	I_2 , A	Z_1, Ω	Z_2, Ω
1	$12e^{j45}$	$10e^{j35}$	$3{,}2e^{j30}$	$1{,}8e^{-j45}$
2	$16e^{j60}$	$14e^{j40}$	$4{,}4e^{j30}$	$3{,}6e^{-j60}$
3	$18e^{j75}$	$16e^{j50}$	$5{,}2e^{j25}$	$4{,}8e^{-j75}$
4	$20e^{j15}$	$18e^{j60}$	$6{,}3e^{j35}$	$5{,}2e^{-j82}$
5	$24e^{j25}$	$22e^{j70}$	$8{,}4e^{j30}$	$6{,}3e^{-j75}$
6	$26e^{j40}$	$21e^{j75}$	$10e^{j32}$	$8{,}2e^{-j30}$
7	$18e^{j45}$	$21e^{j75}$	$12e^{j35}$	$9{,}6e^{-j45}$
8	$14e^{j60}$	$12e^{j25}$	$15e^{j45}$	$10{,}2e^{-j60}$
9	$8e^{j15}$	$6e^{j10}$	$18{,}2e^{j60}$	$12{,}3e^{-j7}$

Sinusoidal tok zanjirlarini oniy qiymatlar bo‘yicha hisoblash

2.65- masala. Zanjir (2.36- rasm) uchun $i(t)$ tok va $u_r(t)$, $u_C(t)$, $u_L(t)$, $u_{rL}(t)$, $u_{rC}(t)$ kuchlanishlarning oniy qiymatlarini hamda zanjir iste’mol qilayotgan aktiv P va reaktiv Q quvvatlar aniqlansin. Quyidagilar berilgan: $e(t)$=20sin100t V; r = 4 Ω; L=70 mH; C=2500 mkF.



2.36- rasm.

Yechish: Zanjirning reaktiv va to‘la qarshiliklarini aniqlaymiz:

$$X_L = \omega L = 100\cdot 70\cdot 10^{-3} = 7\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{100\cdot 2{,}5\cdot 10^{-3}} = 4\,\Omega$$

$$z = \sqrt{r^2 + (x_L - x_C)^2} = \sqrt{4^2 + (7-4)^2} = 5\ \Omega.$$

Manba kuchlanishi va zanjir toki orasidagi faza siljish burchagini aniqlaymiz:

$$\varphi = \mathrm{arctg}(x_L - x_C)/r = \mathrm{arctg}(3/4) = 37°.$$

Zanjir tokining amplituda qiymatini topamiz:

$$I_m = E_m/z = 20/5 = 4\ \text{A}.$$

Hosil bo‘lgan qiymatlardan foydalanib, oniy tok ifodasini yozamiz:

$$i(t) = I_m \sin(\omega t - \varphi) = 4\sin(100t - 37^0)\ \text{A}.$$

Aktiv qarshilikdagi kuchlanishni Om qonuni asosida topamiz:

$$u(t) = ri(t) = 16\sin(100t - 37°)\ \text{V}.$$

Induktivlik g‘altakdagi kuchlanishni topamiz:

$$u_L(t)=L\,di(t)/dt=70\cdot10^{-3}\cdot100\cdot4\cos(100t-37^0)=28\sin(100t+53^0)\ \text{V}.$$

Sig‘imdagi kuchlanishni quyidagi formuladan topamiz:

$$u_C=\left[\int i(t)dt\right]/C=4\left[-\cos(100t-37^0)\right]/0{,}25=16\sin(100t-127^0)\ \text{V}.$$

Ketma-ket ulangan aktiv va induktiv qarshiliklardagi kuchlanish amplitudasi:

$$U_{mrL}=I_mZ_{rL}=I_m\sqrt{r^2+X_L^2}=4\sqrt{4^2+7^2}=32\ \text{V},$$

faza siljish burchagi esa: $\varphi_{rL}=\text{arctg}(x_L/r)=\text{arctg}1{,}75=60^o$.

Bu kuchlanishning oniy qiymati quyidagiga teng: $u_{rL}(t)=32\sin(100t+23^0)$ V.

Xuddi shunga o‘xshash, ketma-ket ulangan aktiv va sig‘im qarshiliklardagi kuchlanishning amplitudasi:

$$U_{mrC}=I_mZ_{rC}=I_m\sqrt{r^2+x_C^2}=4\cdot5{,}66=22{,}64\ \text{V},$$

faza siljish burchagi esa: $\varphi_{rC}=\text{arctg}(-4/4)=-45^0$.

Bu kuchlanishining oniy qiymati quyidagi ko‘rinishga ega:

$$u_{rC}(t)=22{,}64\sin(100t-82^0)\ \text{V}.$$

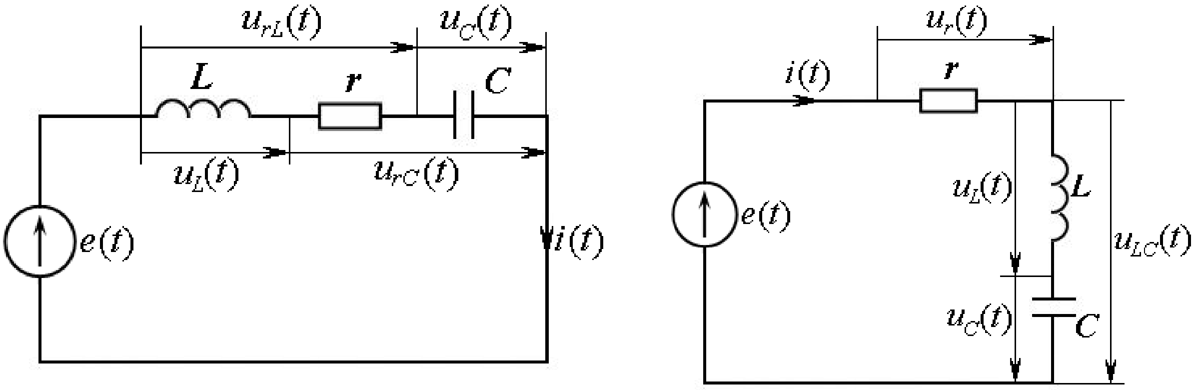
Zanjirda ajralayotgan o‘rtacha(aktiv) quvvat quyidagicha hisoblanadi:

$$P=\frac{I_m^2}{2}r=\frac{4^2}{2}4=32\ \text{W}.$$

Zanjirdagi reaktiv quvvat esa: $Q=\dfrac{U_mI_m}{2}\sin\varphi=48$ var.

2.1- vazifa. *R, L, C* ketma-ket ulangan konturni hisoblash.

2.37-rasmda tasvirlangan sxemalar uchun 2.1-jadvalda keltirilgan qiymatlardan foydalanib, quyidagi kattaliklarning oniy qiymatlari ifodasi topilsin: $e(t)$; $i(t)$, $u_r(t)$, $u_L(t)$, $u_C(t)$; $u_{rL}(t)$, $u_{rC}(t)$, $u_{LC}(t)$. Vektor diagramma qurilsin, aktiv, reaktiv va to‘la quvvatlar hisoblansin.



a) *b*)

2.37- rasm. 2.1- vazifaga sxema variantlari.

2.1-*jadval*

2.1. variant	r, Ω	L, mH	C, mkF	Kuchlanish va toklar
				$e(t)$, V
1	2	40	2000	$10\sin 100t$
2	5	14	1000	$10\sin 500t$
3	6	30	2500	$35\sin 200t$
4	4	5	500	$56\sin 1000t$
				$i(t)$, A
5	4	30	2500	$2{,}121\sin(200t - 45^\circ)$
6	4	30	2000	$3{,}354\sin(100t + 26^\circ)$
7	8	5	625	$2{,}910\sin(400t + 14^\circ)$
8	8	40	2000	$2{,}233\sin(100t + 7^\circ)$
				$u_L(t)$, V
9	4	7,5	625	$10{,}91\sin(400t + 104^\circ)$
10	8	30	1250	$14{,}55\sin(200t + 76^\circ)$
11	4	8	400	$46{,}57\sin(500t + 104^\circ)$
12	3	30	2500	$31{,}2\sin(200t - 53^\circ)$
				$u_C(t)$, V
13	3	10	250	$113{,}1\sin(500t - 45^\circ)$
14	3	60	400	$41{,}6\sin(50t - 56^\circ)$
15	2	6	250	$73{,}54\sin(1000t - 135^\circ)$

16	5	10	500	$31{,}38\sin(400t - 79^\circ)$
				$u_r(t)$, V
17	2	40	5000	$7{,}07\sin(100t - 45^\circ)$
18	6	40	1250	$16{,}64\sin(100t + 34^\circ)$
19	4	20	1250	$8{,}87\sin(100t + 56^\circ)$
				$u_{LC}(t)$, V
20	4	3	200	$17{,}89\sin(1000t - 64^\circ)$
21	4	35	1250	$9\sin(200t + 53^\circ)$
22	6	20	1000	$4{,}6\sin(200t - 81^\circ)$
23	6	2	125	$35{,}35\sin(1000t - 45^\circ)$
24	3	60	2500	$7{,}76\sin(100t + 56^\circ)$
				$u_{rL}(t)$, V
25	4	15	1250	$24{,}25\sin(200t + 51^\circ)$
26	5	12	500	$65{,}26\sin(500t + 29^\circ)$
27	4	14	250	$86{,}04\sin(500t + 74^\circ)$
				$u_{rC}(t)$, V
28	8	25	312	$54{,}88\sin(400t - 59^\circ)$
29	4	5	500	$15{,}37\sin(400t - 15^\circ)$
30	2	25	625	$25{,}45\sin(400t - 135^\circ)$

2.4. Elektr zanjirlarda rezonans

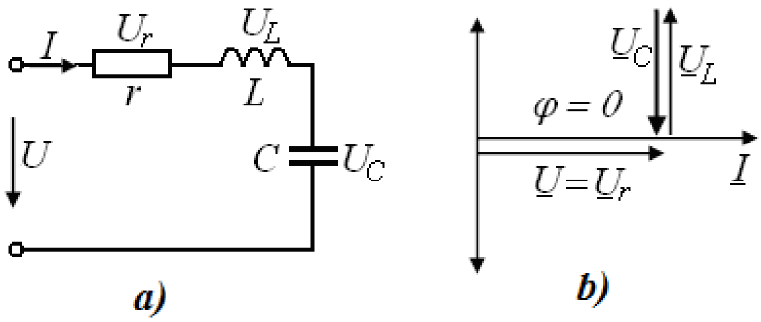
2.4.1. Kuchlanishlar rezonansi

r, *L* va *C* elementlari ketma-ket ulangan zanjirni ko‘rib chiqamiz (2.39- rasm, a), zanjirning kompleks qarshiligi:

$$\underline{Z} = r + j\left(\omega L - \frac{1}{\omega C}\right).$$

Kuchlanishlar rezonansi shartiga ko‘ra:

$X = \omega L - 1/\omega C = 0$ yoki $\omega L = 1/\omega C$, bundan $\omega_p = 1/\sqrt{LC}$ – rezonans chastotasi.



2.39- rasm.

Kuchlanishlar rezonansi rejimida zanjirning to‘la qarshiligi minimal bo‘lib, aktiv qarshilikka teng bo‘ladi:

$$Z = \sqrt{r^2 + (x_L - x_C)^2} = r.$$

Zanjirdagi tok bu paytda maksimal qiymatga erishadi:

$$I = \frac{U}{z} = \frac{U}{r}.$$

r qarshilikdagi kuchlanish pasayishi manba kuchlanishiga teng bo‘ladi:

$$U_r = Ir = U.$$

Rezonans rejimidagi induktiv yoki sig‘im qarshilik rezonans zanjir (kontur)ining *to‘lqin qarshiligi* deb ataladi:

$$\rho = \omega_p L = \frac{1}{\omega_p C} = \sqrt{\frac{L}{C}}.$$

Agar zanjirda $\rho >> r$ shart bajarilsa, u holda reaktiv elementlardagi kuchlanishlar manba kuchlanishidan bir necha marta katta bo‘ladi.

U_{Lp} yoki U_{Cp} ni U dan necha marta katta bo‘lishi konturning aslliligi bilan tavsiflanadi:

$$Q = \frac{U_{Lp}}{U} = \frac{U_{Cp}}{U} = \frac{I_p \omega_p L}{I_p r} = \frac{\omega_p L}{r} = \frac{\rho}{r}.$$

$d = 1/Q = r/\rho$ – zanjir (kontur) ning so‘nishi.

Rezonans rejimida zanjirdagi energetik munosabatlar ham o‘ziga xos ayrim xususiyatlarga ega. Zanjirdagi tok $i = I_m \sin \omega_p t$ bo‘lsa, kondensatordagi kuchlanish $U_C = \frac{1}{C}\int idt = U_{cm} \cos \omega_p t$ bo‘ladi.

Zanjirning elektr va magnit maydonlaridagi energiya:

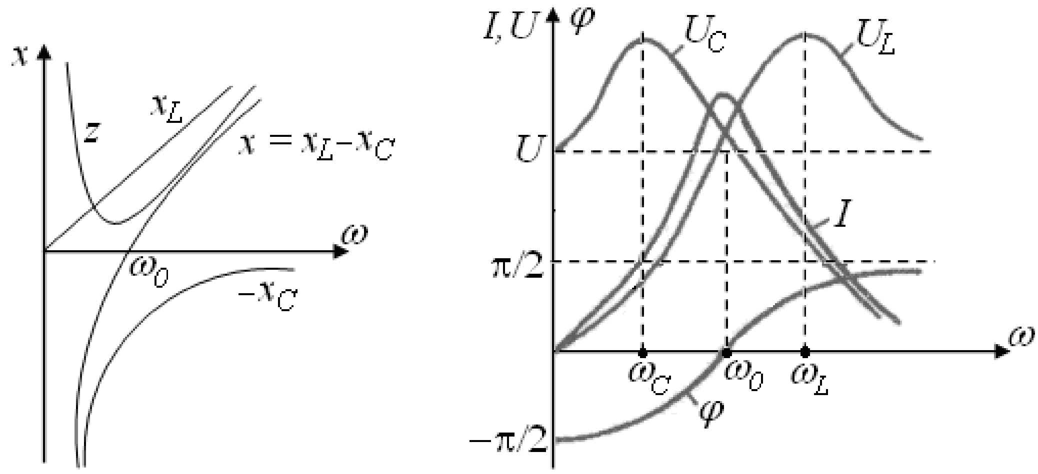
$$W = \frac{Li^2}{2} + \frac{Cu_c^2}{2} = \frac{LI_m^2}{2}\sin^2 \omega_p t + \frac{CU_{cm}^2}{2}\cos^2 \omega_p t = \frac{LI_m^2}{2} + \frac{CU_{cm}^2}{2} = \text{const}.$$

$$U_{cm} = \frac{1}{\omega_p C} I_m = \sqrt{\frac{L}{C}} I_m \text{ bo‘lganligi uchun } \frac{CU_{cm}^2}{2} = \frac{LI_m^2}{2}.$$

2.40- rasm, *a* da $x_L(\omega)$, $x_C(\omega)$, $x(\omega)$, $Z(\omega)$ va *b* da $I(\omega)$, $U_L(\omega)$, $U_C(\omega)$, $\varphi(\omega)$ grafiklari keltirilgan. $\omega = 0$ bo‘lganda $I(0) = 0$, chunki kondensator o‘zgarmas tokni o‘tkazmaydi.

$U_L(\omega)$ va $U_C(\omega)$ grafiklaridan ko‘rinib turibdiki, reaktiv elementlardagi kuchlanishlarning maksimal qiymatlari rezonans chastotasidan farq qilgan chastotalarga to‘g‘ri keladi. $U_L(\omega)$ kuchlanish $\omega_L > \omega_p$ da, $U_C(\omega)$ kuchlanish esa $\omega_L < \omega_p$ da maksimumga erishadi. Funksiyalarning ekstremumlarini topish sharti $dU_L / d\omega = 0$ va $dU_C / d\omega = 0$ dan:

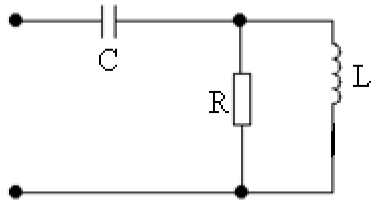
$$\omega_L = \omega_p \sqrt{\frac{2}{2-d^2}}\,,\ \omega_C = \omega_p \sqrt{\frac{2-d^2}{2}}.$$



a) *b)*

2.40- rasm.

2.66- masala. Berilgan zanjir uchun (2.41- rasm) rezonans chastotasining ifodasi topilsin.



2.41- rasm.

Yechish. Rezonans shartiga ko‘ra: $\varphi = \varphi_u - \varphi_i = 0$, $x_{kir} = 0$, ya’ni kuchlanish va tok orasidagi faza siljish burchagi va, binobarin, zanjirning kirish qarshiligi nolga teng bo‘lishi kerak. Zanjirning kompleks qarshiligi $\underline{Z}_{\text{kir}} = \frac{1}{j\omega C} + \frac{R \cdot j\omega L}{R + j\omega L}$.

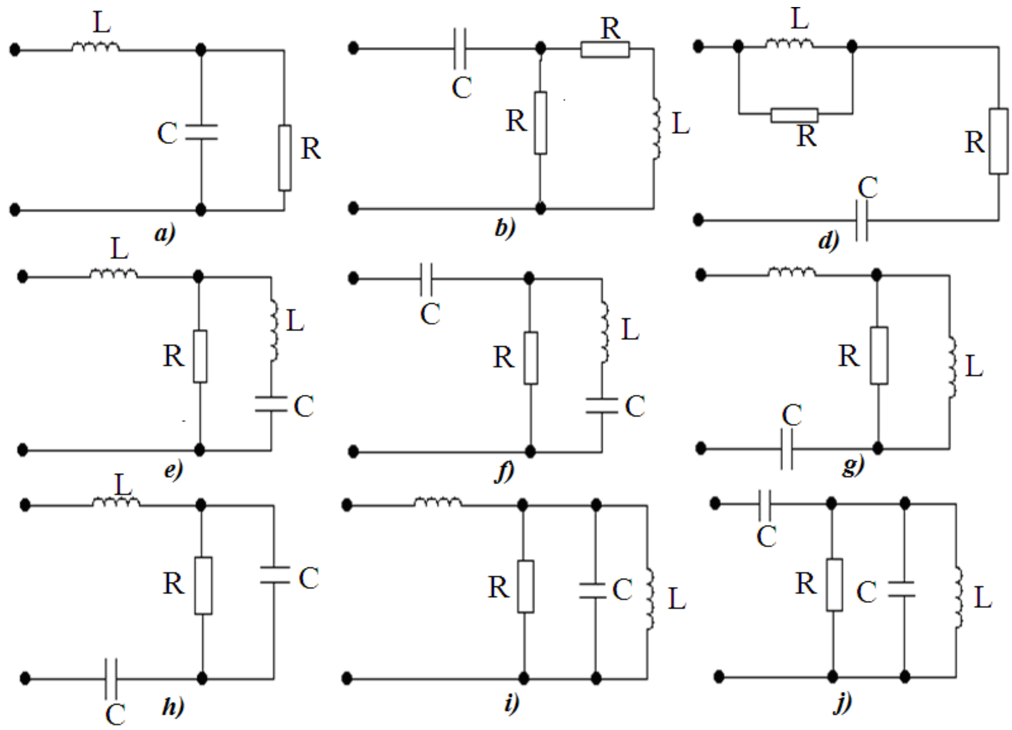
Uning reaktiv tashkil etuvchisini topamiz:

$$X_{\text{kir}} = \frac{-R^2 - \omega^2 L^2 + R^2 \cdot LC}{\omega C\left(R^2 + \omega^2 L^2\right)}.$$

$x_{\text{kir}} = 0$ shartga ko‘ra rezonans chastotasi quyidagiga teng:

$$\omega = \frac{R}{\sqrt{L\left(R^2C - L\right)}}.$$

2.67- masala. Keltirilgan zanjirlar uchun (2.42- rasm) rezonans chastotalarining ifodalari topilsin.

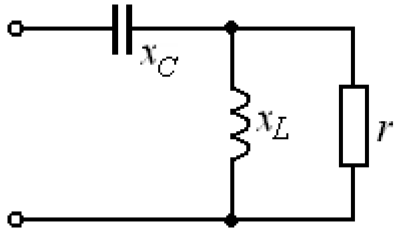


2.42- rasm.

2.68- masala. Sig'imi C=10^{-6} F bo'lgan real kondensatorning $\omega = 10^4$ rad/s chastotada aslligi $Q = \omega C / g = 2000$. (g – aktiv o'tkazuvchanligi). Rezistor va ideal kondensatordan iborat parallel va ketma-ket ulangan zanjir parametrlarini toping.

2.69- masala. Real induktiv g'altakning chastotasi $\omega = 10^3$ rad/s va tok kuchi $I = 1$ A bo'lganda quvvat isrofi P=4 W. Agar g'altak induktivligi L= 0,1 H bo'lsa, uning assiligi aniqlansin.

2.70- masala. Agar kuchlanish rezonansi rejimida ishlayotgan ikki qutblikning to'la kirish qarshiligi z_{kir}=20 Ω va sig'im qarshiligi x_C=40 Ω bo'lsa, u holda uning induktiv x_L va aktiv r qarshiliklari qiymatlarini aniqlang (2.43- rasm).



2.43- rasm.

Yechish. Ikki qutblikning kirish kompleks qarshiligi quyidagiga teng:

$$\underline{Z}_{\text{kir}} = -jx_C + \frac{jx_L r}{r + jx_L} = \frac{x_L^2 r}{r^2 + x_L^2} + j\left(-x_C + \frac{x_L r^2}{r^2 + x_L^2}\right).$$

Kuchlanish rezonansi shartiga ko‘ra zanjirning reaktiv qarshiligi nolga teng. Bu shartdan foydalanib quyidagi tenglamalar sistemasini tuzishimiz mumkin:

$$\left.\begin{aligned} Z_{\text{kir}} &= \frac{x_L^2 r}{r^2 + x_L^2} = 20, \\ x &= -x_C + \frac{x_L r^2}{r^2 + x_L^2} = 0. \end{aligned}\right\}.$$

Bu sistemani x_L va r ga nisbatan yechib, quyidagilarni topamiz: x_L=50 **Ω**, r=100Ω.

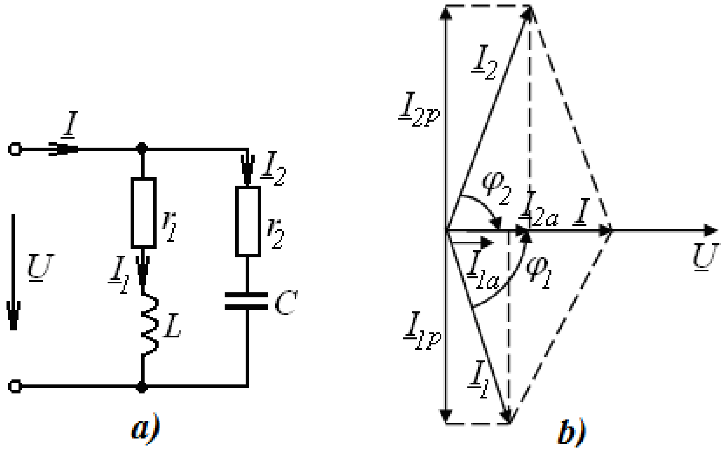
2.4.2. Toklar rezonansi

Reaktiv elementlar o‘zaro parallel ulangan zanjir (2.44- rasm, *a*) da ma’lum shartlar bajarilganda toklar rezonansi yuz beradi.

Rezonans rejimida manba toki va kuchlanish orasidagi faza siljish burchagi nolga teng bo‘ladi, reaktiv o‘tkazuvchanlik esa $b = b_L - b_C = 0$ yoki $b_L = b_C$ bo‘ladi. Rezonans rejimi uchun qurilgan vektor diagrammadan ko‘rinib turibdiki (2.44- rasm, *b*), reaktiv elementlardagi toklar modul qiymatlari jihatdan o‘zaro teng, yo‘nalishlari esa qarama-qarshi. Bunda $|I_L| = |I_C|$ toklar manba tokidan bir necha marotaba ortib ketishi mumkin. *Shuning uchun ham bu rezonans zanjirdagi toklar rezonansi deb ataladi*.

Zanjir reaktiv o‘tkazuvchanligining nolga tenglik shartidan:

$$\frac{\omega_r L}{r_1^2+(\omega_r L)^2}=\frac{1/\omega_r C}{r_2^2+(1/\omega_r C)^2},\ \text{bundan}\ \omega_r=\sqrt{1/LC}\sqrt{\frac{(L/C)-r_1^2}{(L/C)-r_2^2}}.$$



2.44- rasm.

Ideallashtirilgan holat $r_1=r_2=0$ uchun zanjirning kirish qarshiligi cheksiz katta qiymatga teng bo‘lib, tok nolga teng. Energiya manbadan zanjirga uzatilmaydi hamda kondensator va induktiv g‘altak maydonlarida davriy ravishda to‘planib almashinib turadi.

$\omega_r C=1/\omega_r L=\sqrt{C/L}=\gamma-$ *zanjirning to‘lqin o‘tkazuvchanligi* deb ataladi.

Induktiv g‘altak yoki kondensatordagi tokning manba tokidan necha marta katta ekanligini ko‘rsatuvchi kattalik *zanjir (kontur) ning aslligi* deb ataladi:

$$Q=\frac{I_{Cr}}{I}=\frac{I_{Lr}}{I}=\frac{\omega_r CU}{gU}=\frac{1}{g}\sqrt{\frac{C}{L}}=\frac{\gamma}{g}.$$

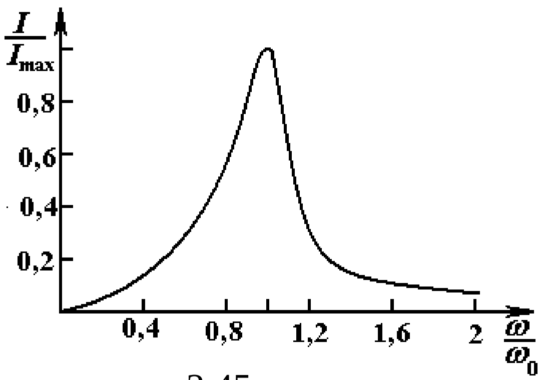
2.71- masala. Ketma-ket rezonans zanjirida R=2 Ω, C=100 mkF*;* L=40mH. Rezonans chastota, kontur aslligi, o‘tkazish kengligi, o‘tkazish kengligining kontur aslligiga bog‘lanishini aniqlang. Rezonans egri chizig‘ini $I/I_{\max}=f(n)$ nisbiy birliklarda quring, bunda $n=\omega/\omega_0$.

Yechish. Konturning rezonans chastotasi $\omega_0=\frac{1}{\sqrt{LC}}=500\,\text{rad/s}$. Kontur aslligi $Q=\omega_0 L/R=10$. Rezonans egri chizig‘ini quyidagi tenglamadan qurish mumkin:

$$I=\frac{U}{\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}}=\frac{U}{R\sqrt{1+\frac{1}{R^2}\left(n\omega_o L-\frac{1}{n\omega_o C}\right)^2}}=\frac{I_{\max}}{\sqrt{1+Q^2\left(n-\frac{1}{n}\right)^2}},$$

bunda $\omega/\omega_0 = n$ yoki nisbiy birliklarda (2.45- rasm):

$$\frac{I}{I_{\max}}=\frac{1}{\sqrt{1+Q^2\left(n-\frac{1}{n}\right)^2}}.$$



2.45- rasm.

O‘tkazish kengligining ta’rifi bo‘yicha so‘nggi tenglamadan

$$\sqrt{2}=\sqrt{1+Q^2\left(n-\frac{1}{n}\right)^2};\qquad 1=\pm Q\left(n_{1.2}-\frac{1}{n_{1.2}}\right)$$

yoki

$$\frac{1}{Q}=n_2-\frac{1}{n_2};\qquad -\frac{1}{Q}=n_1-\frac{1}{n_1},$$

bundan

$$n_1\cdot n_2=1 \text{ yoki } \omega_0=\sqrt{\omega_1\cdot\omega_2};$$

$$n_2-n_1=\frac{\omega_2-\omega_1}{\omega_0}=\frac{1}{Q},$$

bunda ω_1 va ω_2 – o‘tkazish kengligining chegaraviy chastotalari. Oxirgi tenglamalarni birgalikda yechamiz:

$$\omega_1=\frac{\omega_o}{2Q}\left[\sqrt{1+4Q}-1\right]\approx 135 \text{ rad/s},$$

$$\omega_2=\frac{\omega_o}{2Q}\left[1+\sqrt{1+4Q}\right]\approx 185 \text{ rad/s}.$$

2.72- masala. Parallel kontur (2.46- rasm, *a*) rezonans rejimiga sozlangan. $I_2 = 14{,}1$ A, $U = 100$ V va $I_2 = I_3$. Birinchi va uchinchi ampermetrlarning ko‘rsatishlarini va kontur parametrlarini aniqlang.

Yechish. Masalani vektor diagramma qurish usulida yechamiz. Kuchlanish vektorini haqiqiy o‘qqa qo‘yamiz (2.46-rasm, *b*). Induktiv shoxobchadagi tok kuchlanishga nisbatan 90^0 orqada qolishini, $\underline{I}_2$ va $\underline{I}_3$ toklar teng bo‘lganligi hamda rezonans rejimida induktivlikdan o‘tadigan tok $\underline{I}_1$ va $\underline{I}_2$ reaktiv tashkil etuvchisi $\underline{I}_2 = I_{2r}$ ga teng bo‘lishini e’tiborga olib, I_2 tok vektori kuchlanish vektoriga nisbatan $+45^0$ burchak ostida quriladi.

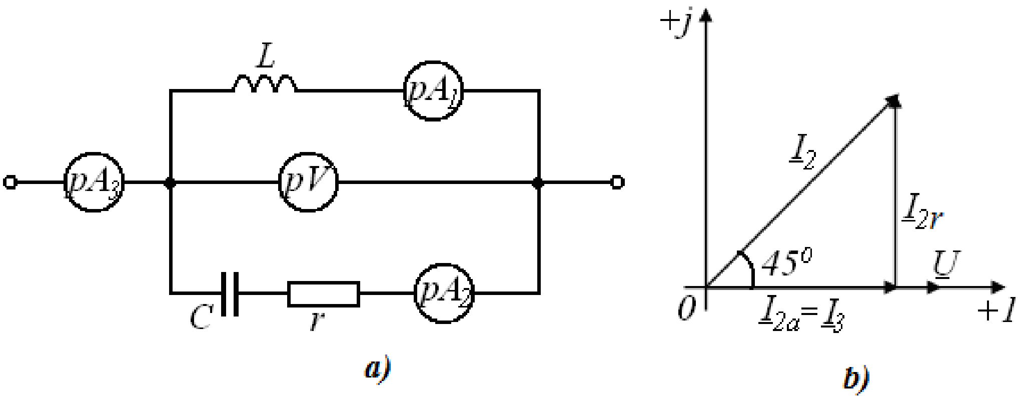
Demak, $I_3 = I_2 \cos 45^0 = 14{,}1 \cos 45^0 = 10$ A, $I_1 = I_2 \sin 45^0 = 14{,}1 \sin 45^0 = 10$ A.

Induktiv qarshilik $x_L = U / I_1 = 100/10 = 10\ \Omega$.

C va r dan tashkil topgan ikkinchi shoxobchaning to‘la qarshiligi $z_2 = U / I_2 = 100/14{,}1 = 7{,}09\ \Omega$; bundan

$$r = z_2 \cos 45^0 = 7{,}09 \cdot 0{,}705 = 5\ \Omega;$$

$$x_C = z_2 \sin 45^0 = 7{,}09 \cdot 0{,}705 = 5\ \Omega.$$



2.46- rasm.

2.73- masala. Induktiv g‘altakning 100 V kuchlanishda va 1 A tokda hamda f chastotada quvvat isrofi 10 W ga teng. G‘altakning berilgan chastotada aslligini hamda ketma-ket va parallel ulanishda ekvivalent almashish sxemalari parametrlari aniqlansin.

Yechish. G‘altakning ketma-ket va parallel ulanishdagi aslliklari:

$$Q_{\text{kk}} = \frac{\omega L}{R_{\text{kk}}}; \quad Q_{\text{par}} = \frac{R_{\text{par}}}{\omega L} = \frac{1}{\omega L \cdot g_{\text{par}}},$$

bunda, R_{kk}, R_{par} - mos ravishda ketma-ket va paralel almashlash sxemalari qarshiliklari.

Kontur parametrlari:

$R_{\text{kk}} = 10\ \Omega$; $\omega L = 99{,}5\ \Omega$; $Q = 9{,}95$; $R_{\text{par}} = 100\ \Omega$;

chunki $P = I^2 R = UI\cos\varphi$ va $Z = \frac{U}{I} = \frac{100}{1} = 100\ \Omega$; $R = \frac{P}{I^2} = \frac{10}{1^2} = 10\ \Omega$,

bulardan

$$\omega L = \sqrt{Z^2 - R^2} = \sqrt{100^2 - 10^2} = 99{,}5\ \Omega.$$

2.74-masala. O‘zaro ketma-ket ulangan aktiv qarshilik $r = 10\ \Omega$, induktivligi $L = 0{,}01$ H va aktiv qarshiligi $r_{\text{k}} = 0$ bo‘lgan induktiv g‘altak hamda sig‘imi $C = 1$ mkF isrofsiz kondensatordan iborat konturning rezonans burchak chastotasi ω_0 ni, xususiy chastotasi f_0 ni, xarakteristik qarshiligi ρ ni, aslligi Q ni va so‘nish koeffitsiyenti d ni aniqlang. Agar kuchlanish $u = \sqrt{2}\sin\omega_0 t$ V bo‘lsa, tok ta’sir etuvchi qiymatini, g‘altakdagi va kondensatordagi kuchlanishlar pasayishlarini toping.

Yechish. Konturning burchak chastotasini aniqlaymiz:

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-2} \cdot 10^{-6}}} = 10^4 \text{ rad/s};$$

$$f_0 = \frac{\omega_o}{2\pi} = \frac{10^4}{2 \cdot 3{,}14} = 1590 \text{ Hz}.$$

Xarakteristik qarshiligi:

$$\rho = \sqrt{\frac{L}{C}} = \sqrt{\frac{10^{-2}}{10^{-6}}} = 100\ \Omega.$$

Asllik:

$$Q = \frac{\rho}{R} = \frac{100}{10} = 10.$$

So‘nish koeffitsiyenti:

$$d = \frac{1}{Q} = \frac{1}{10} = 0{,}1\,.$$

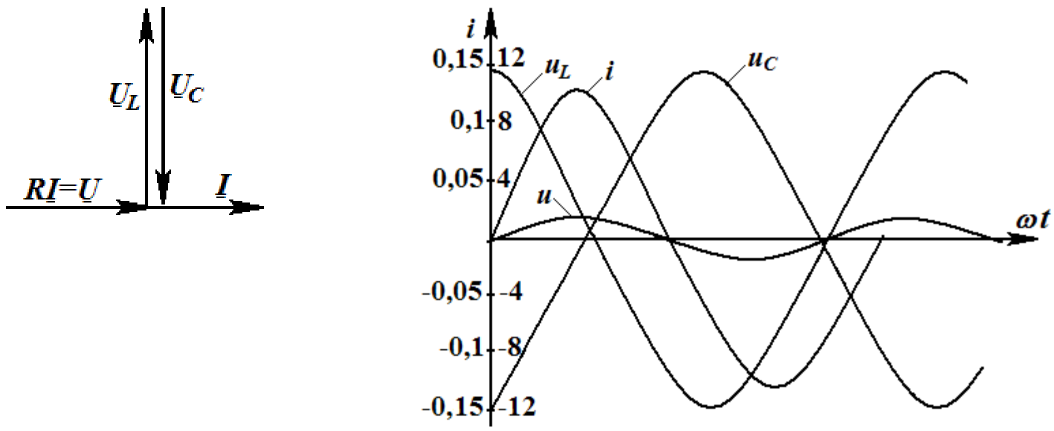
Tokning ta'sir etuvchi qiymati:

$$I_0 = \frac{U}{R} = \frac{1}{10} = 0{,}1 \text{ A.}$$

G'altakdagi va kondensatordagi kuchlanishlar pasayishlari:

$$U_L = U_C = UQ = 1 \cdot 10 = 10 \text{ V.}$$

Vektor va vaqt diagrammalari 2.47- rasmda tasvirlangan.



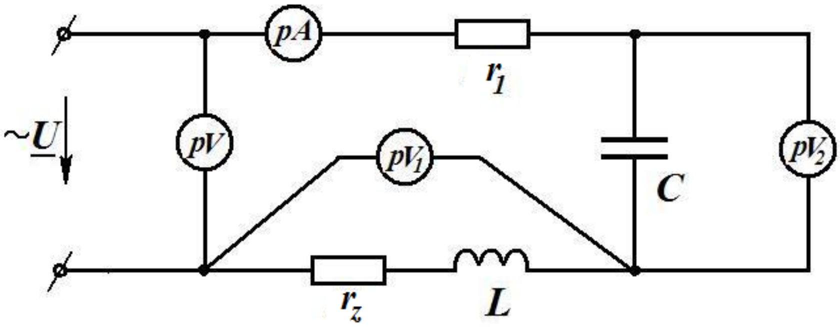
a) *b)*

2.47- rasm.

2.75-masala. Oldingi masaladagi zanjirda kondensator qisqa tutashtirilganda induktivlikdan o'tadigan tok va uning qismalaridagi kuchlanishning oniy qiymatlari ifodalari yozilsin.

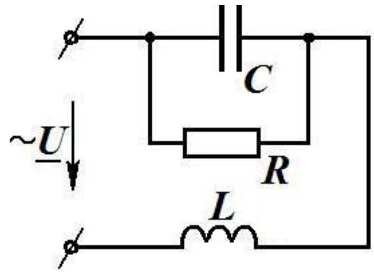
2.76- masala. Induktiv g'altak L va kondensator C ketma-ket ulangan. Zanjir kirishidagi kuchlanish $U = 100$ V. Kondensatordagi kuchlanish $U_C = 80$ V. Zanjirda kuchlanishlar rezonansi bo'lganda, g'altak qismalaridagi kuchlanish aniqlansin.

2.77- masala. Agar zanjirga (2.48- rasm) ulangan o'lchash asboblari $U = 50$ V, $U_1 = 125$ V, $U_2 = 120$ V, $I = 5$ A ni ko'rsatgan bo'lsa, uning r_z, L, C, r_1 parametrlari aniqlansin. Sinusoidal tok chastotasi $f = 50$ Hz.



2.48- rasm.

2.78- masala. Sig‘imi $C = 2\,\text{mkF}$ bo‘lgan kondensator va induktivligi $L = 0{,}01\,\text{H}$ bo‘lgan induktiv g‘altak ketma-ket ulangan. Kondensatorga qanday aktiv qarshilik parallel ulanganda (2.49- rasm) kuchlanish rezonansi $\omega = 5000\ \text{rad/s}$ chastotada sodir bo‘ladi?



2.49- rasm.

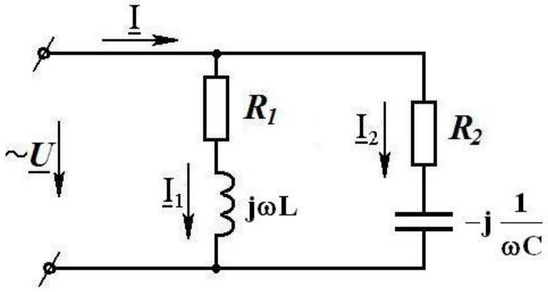
2.79- masala. Agar zanjirdagi (2.49- rasm) $L = 10\,\text{mH},\ C = 0{,}5\,\text{mkF},$ $r = 100\ \Omega$ bo‘lsa, rezonans burchak chastotasini aniqlang.

2.80- masala. Zanjirda (2.49- rasm) kuchlanish rezonansi sodir etilgan. Agar $u = 10\sqrt{2}\sin(10^4 t + 30°)\ \text{V},\ L = 5\,\text{mH},\ r = 100\,\Omega$ bo‘lsa, zanjirning aktiv quvvati aniqlansin.

2.81- masala. Zanjir (2.50- rasm) parametrlari $r_1 = 8\,\Omega,\ r_2 = 6\,\Omega,$ $L = 1\,\text{mH},\ C = 10\,\text{mkF}$ bo‘lsa, tok rezonansi sodir bo‘lishi uchun burchak chastota ω_0 va chastota f_0 topilsin. Agar zanjirga berilayotgan kuchlanishning ta’sir etuvchi qiymati 100 V bo‘lsa, zanjir shoxobchalaridagi toklar aniqlansin.

Yechish. Rezonans burchak chastotasi

$$\omega_0=\frac{1}{\sqrt{LC}}\cdot\sqrt{\frac{L/C-R_1^2}{L/C-R_2^2}}=\frac{1}{\sqrt{10^{-3}\cdot10\cdot10^{-6}}}\sqrt{\frac{\frac{10^{-3}}{10\cdot10^{-6}}-8^2}{\frac{10^{-3}}{10\cdot10^{-6}}-6^2}}=10^4\sqrt{\frac{100-64}{100-36}}=7500\ \frac{\text{rad}}{\text{s}}\,.$$



2.50- rasm.

Chastota $f_0=\dfrac{\omega_0}{2\pi}=1194$ Hz.

Shoxobchalardagi toklar komplekslarini topamiz:

$$\underline{I}_1=\frac{\underline{U}}{\underline{Z}_1}=\frac{\underline{U}}{R_1+j\omega_0 L}=\frac{100}{8+j7500\cdot10^{-3}}=\frac{100}{10{,}95e^{j43^\circ10'}}=$$

$$=9{,}14e^{-j43^\circ10'}=(6{,}64-j6{,}24)\ A.$$

$$\underline{I}_2=\frac{\underline{U}}{\underline{Z}_2}=\frac{\underline{U}}{R_2-j\dfrac{1}{\omega_0 C}}=\frac{100}{6-j\dfrac{1}{7500\cdot10\cdot10^{-6}}}=\frac{100}{6-j13{,}3}=$$

$$=6{,}85e^{j65^\circ45}=(2{,}82+j6{,}24)\ \text{A}.$$

Zanjirning shoxobchalanmagan qismidagi tok

$$\underline{I}=\underline{I}_1+\underline{I}_2=6{,}64-j6{,}24+2{,}82+j6{,}24=9{,}46\ \text{A}.$$

Shuni qayd etamizki, bu tokni $\underline{I}=\dfrac{\underline{U}}{r_{\text{rez}}}$ formuladan aniqlash ham mumkin, bunda

$$r_{\text{rez}}=\frac{r_1r_2+\rho^2}{r_1+r_2}=\frac{8\cdot6+100}{8+6}=10{,}57\ \Omega,$$

bu yerda $\rho=\sqrt{\dfrac{L}{C}}=\sqrt{\dfrac{1\cdot10^{-3}}{10\cdot10^{-6}}}=10$.

Natijada $\underline{I}=\dfrac{100}{10{,}57}=9{,}46$ A.

2.82-masala. Zanjir (2.50- rasm) parametrlari $r_2 = 50\,\Omega$, $L = 5\,\text{mH}$, $C = 5\,\text{mkF}$, kirish kuchlanish $\underline{U} = 10\,\text{V}$. Agar tok rezonansi $\omega = 4000\,\text{rad/s}$ chastotada sodir etilgan bo‘lsa, r_1 ni toping.

Yechish. Ikkkinchi shoxobchaning reaktiv o‘tkazuvchanligini aniqlaymiz.

$$b_2 = \frac{x_2}{r_2^2 + x_2^2} = \frac{-\dfrac{1}{\omega C}}{r_2^2 + \left(-\dfrac{1}{\omega C}\right)^2} = \frac{-\dfrac{1}{4\cdot 10^3\cdot 5\cdot 10^{-6}}}{50^2 + (-\dfrac{1}{4\cdot 10^3\cdot 5\cdot 10^{-6}})^2} = -0{,}01\,\text{S}.$$

Toklar rezonansi sodir bo‘lganda ikkala shoxobchaning reaktiv o‘tkazuvchanligi nolga teng bo‘lishi kerak:

$$b_1 + b_2 = 0,$$

binobarin, $b_1 = -b_2 = -(-0{,}01) = 0{,}01\,\text{S}.$

Shunday qilib, $b_1 = \dfrac{\omega L}{r_1^2 + (\omega L)^2} = 0{,}01\,\text{S}.$

yoki
$$\frac{4\cdot 10^3\cdot 5\cdot 10^{-3}}{r_1^2 + (4\cdot 10^3\cdot 5\cdot 10^{-3})^2} = 0{,}01\,\text{S},$$

bundan $r_1 = 40\,\Omega.$

Rezonans holatda zanjirning kirish qarshiligi

$$Z_{\text{rez}} = r_{\text{rez}} = \frac{r_1 r_2 + \dfrac{L}{C}}{r_1 + r_2} = \frac{40\cdot 50 + \dfrac{5\cdot 10^{-3}}{5\cdot 10^{-6}}}{40 + 50} = 33{,}33\,\Omega.$$

Zanjirning aktiv qarshiligi

$$\underline{P} = \frac{U^2}{r_{rez}} = \frac{10^2}{33{,}33} \approx 3\,\Omega.$$

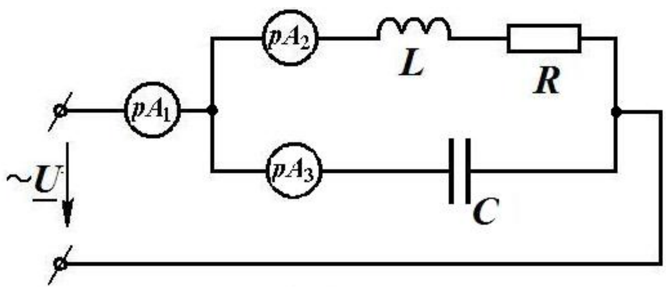
2.83-masala. Zanjir (2.50-rasm) parametrlari $r_1 = 40\ \Omega$, $L = 5\,\text{mH}$, $C = 5\,\text{mkF}$, $u = 100\sqrt{2}\sin 4000t\ \text{V}$ Tok rezonansi $\omega = 4000\ \text{rad/s}$ burchak chastotada sodir etilishi uchun r_2 necha bo‘lishi kerak? Zanjir shoxobchalaridagi toklarning oniy qiymat ifodalari yozilsin va vektor diagramma qurilsin.

2.84- masala. Zanjirda (2.50- rasm) $\omega = 4000\,\text{rad/s}$ burchak chastotada toklar rezonansi sodir etildi. Agar $r_1 = 40\,\Omega$, $r_2 = 50\,\Omega$, $C = 5\,\text{mkF}$ va $u = 100\sqrt{2}\sin 4000t$ V bo‘lsa, L ning qiymatini aniqlang. Shoxobchadagi toklarning oniy qiymat ifodalarini yozing va vektor diagrammasini chizing.

2.85- masala. Zanjirda (2.50- rasm) $\omega = 4000\,\text{rad/s}$ burchak chastotada toklar rezonansi sodir etildi. Agar $r_1 = 40\,\Omega$, $r_2 = 50\,\Omega$, $L = 20\,\text{mH}$ va $u = 100\sqrt{2}\sin(4000t + 30^\circ)$ V bo‘lsa, C ning qiymatini aniqlang. Shoxobchadagi toklarning ta’sir etuvchi qiymatlarini aniqlang.

2.86- masala. Zanjirda (2.51- rasm) toklar rezonansi sodir etildi. Agar zanjirdagi ampermetrlar ko‘rsatishlari $I_1 = 4\,\text{A}$, $I_2 = 5\,\text{A}$ bo‘lsa, uchinchi ampermetrning ko‘rsatishini aniqlang.

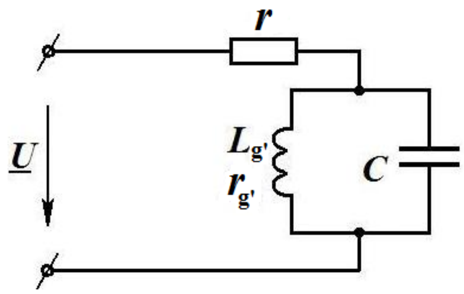
2.87- masala. Elektr motor quvvati $P = 2$ kW, quvvat koeffitsiyenti $\cos\varphi = 0{,}8$. Nominal kuchlanish $U = 220\,\text{V}$, chastotasi $f = 50\,\text{Hz}$ bo‘lsa, quvvat koeffitsiyenti $\cos\varphi = 1{,}0$ bo‘lishi uchun, unga parallel ulangan kondensator necha bo‘lishi kerak?



2.51- rasm.

2.88- masala. Induktivligi $L_{g'} = 2{,}5 \cdot 10^{-3}$ H va aktiv qarshiligi $r_{g'} = 1000\ \Omega$ bo‘lgan g‘altak hamda sig‘imi $C = 120\,\text{pF} = 120 \cdot 10^{-12}$ F bo‘lgan kondensator parallel rezonansli konturni tashkil qilib, $r = 21\,\text{k}\Omega$ rezistor orqali kuchlanishi $U = 66\,\text{V}$ bo‘lgan sinusoidal kuchlanish manbayiga ulangan. Tok rezonansi rejimini ta’minlaydigan manba chastotasini, bu rejimda konturning to‘la qarshiligini va barcha toklarni aniqlang.

Yechish. Tok rezonansi shartini $(b_L = b_C)$ ta'minlaydigan manba chastotasini $r_{g'} = r_1$ va $r_2 = 0$ larni e'tiborga olib hisoblaymiz (2.52- rasm):



2.52- rasm.

$$f_{\text{rez}} = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{(L/C)-r_1^2}{(L/C)-r_2^2}} = \frac{1}{2\cdot 3{,}14\sqrt{2{,}5\cdot 10^{-3}\cdot 120\cdot 10^{-12}}}\times$$

$$\times\sqrt{\frac{(2{,}5\cdot 10^{-3}/120\cdot 10^{-12})-(1000)^2}{2{,}5\cdot 10^{-3}/120\cdot 10^{-12}-(21\cdot 10^3)^2}} = 290{,}4\cdot 10^3\cdot 0{,}976 = 284\cdot 10^3 \text{ Hz.}$$

Rezonans konturini tashkil qiluvchi shoxobchalarning o'tkazuvchanliklari:

$b_L = b_C = \omega_r C = 2\pi f_r C = 2\cdot 3{,}14\cdot 284\cdot 10^3\cdot 120\cdot 10^{-12} = 2{,}14\cdot 10^{-4}$ S.

Kontur o'tkazuvchanligi esa $b = b_L - b_C = 2{,}14\cdot 10^{-4} - 2{,}14\cdot 10^{-4} = 0.$

Konturining q aktiv o'tkazuvchanligi uning g'altakdan iborat shoxobchasining aktiv o'tkazuvchanligiga teng, chunki ikkinchi shoxobchaning aktiv qarshiligi $r_2 = 0$ shuning uchun

$$q = q_1 = \frac{r_{g'}}{r_{g'}^2 + (\omega L)^2} = \frac{1000}{1000^2 + (178\cdot 10^4\cdot 2{,}5\cdot 10^{-3})^2} = 0{,}48\cdot 10^{-4} \text{ S.}$$

Konturning to'la o'tkazuvchanligi $\gamma = \sqrt{q^2 + b^2} = q = 0{,}48\cdot 10^{-4}$ S.

Rezonans konturining to'la qarshiligini to'la o'tkazuvchanlik orqali hisoblaymiz: $Z_k = 1/\gamma = 1/0{,}48\cdot 10^{-4} = 20{,}8\cdot 10^3$ Ω.

Konturning hisoblangan to'la qarshiligi zanjir uchun dastlab berilgan r qarshilikdan 10 baravar kichik. Shu sababli konturdagi kuchlanish quyidagiga teng:

$$U_k = \frac{U}{r + Z_{\text{k}}}\cdot Z_{\text{k}} = 66\cdot\frac{21}{210 + 21} = 6 \text{ V.}$$

Induktiv g‘altakdagi tokning aktiv va reaktiv tashkil etuvchilari:

$$I_{1a} = Uq_1 = 6{,}0 \cdot 0{,}48 \cdot 10^{-4} = 0{,}284 \cdot 10^{-3}\,\text{A},$$
$$I_{1r} = Ub_1 = 6{,}0 \cdot 2{,}14 \cdot 10^{-4} = 1{,}26 \cdot 10^{-3}\,\text{A}.$$

Shu shoxobchadagi umumiy tok

$$I_1 = \sqrt{I_{1a}^2 + I_{1r}^2} = \sqrt{(0{,}284 \cdot 10^{-3})^2 + (1{,}26 \cdot 10^{-3})^2} = 1{,}29 \cdot 10^{-3}\ \text{A}.$$

Ikkinchi shoxobchadagi tok

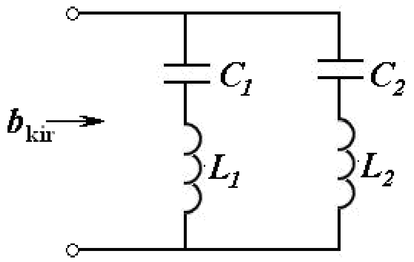
$$I_{2a} = Uq_2 = U \cdot 0 = 0,\ \ I_{2p} = Ub_2 = Ub_1 = I_{1p} = 1{,}26 \cdot 10^{-3}\,\text{A}$$

va shuning uchun $I_2 = 1{,}26 \cdot 10^{-3}$ A.

Zanjirning umumiy toki:

$$I = \sqrt{(I_{1a} + I_{2a})^2 + (I_{1r} - I_{2r})^2} = \sqrt{(0{,}284 \cdot 10^{-3})^2 + 0} = 0{,}284 \cdot 10^{-3}\,\text{A}.$$

2.89- masala. Reaktiv ikkiqutblik (2.53- rasm) uchun rezonans chastotalari aniqlansin va x_{kir} (ω) grafigi tuzilsin. Sxema elementlarining parametrlari quyidagicha: L_1=10 mH; L_2=20 mH; C_1=4 mkF; C_2=1 mkF.



2.53- rasm

Yechish. Ikkiqutblikning kirish reaktiv o‘tkazuvchanligini aniqlaymiz:

$$b_{\text{kir}} = 1/x_1 + 1/x_2;$$

bunda $x_1 = x_{L1} - x_{C1} = \omega L_1 - 1/\omega C_1$, $x_2 = x_{L2} - x_{C2} = \omega L_2 - 1/\omega C_2$ – mos ravishda ikkiqutblikning birinchi va ikkinchi shoxobchalarning reaktiv qarshiliklari.

Shoxobchalar qarshilik qiymatlarini formulaga qo‘yib reaktiv o‘tkazuvchanlik ifodasini topamiz:

$$b_{\text{kir}} = \frac{\omega C_1}{1 - \omega^2 L_1 C_1} + \frac{\omega C_2}{1 - \omega^2 L_2 C_2}$$

yoki

$$b_{\text{kir}} = \frac{\omega\left[(C_1 + C_2) - C_1 C_2 (L_1 + L_2)\omega^2\right]}{(1-\omega^2 L_1 C_1)(1-\omega^2 L_2 C_2)}.$$

Kuchlanishlar rezonansini aniqlash uchun bu ifodaning maxrajini nolga tenglashtiramiz

$$\omega_{01} = \frac{1}{\sqrt{L_1 C_1}}; \quad \omega_{02} = \frac{1}{\sqrt{L_2 C_2}} .$$

Bu ifodalarga element parametrlarini qo‘yib, kuchlanish rezonanslari chastotalarini topamiz:

$$\omega_{01} = \frac{1}{\sqrt{10^{-2} \cdot 4 \cdot 10^{-6}}} = \frac{10^4}{2} = 500 \text{ rad/s},$$

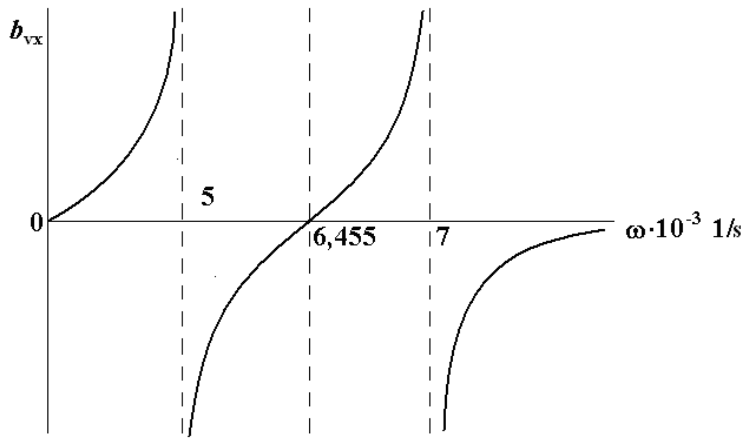
$$\omega_{02} = \frac{1}{\sqrt{2 \cdot 10^{-2} \cdot 10^{-6}}} = \frac{10^4}{\sqrt{2}} = 700 \text{ rad/s}.$$

Bu chastotalarda kirish o‘tkazuvchanligi cheksiz qiymatga teng (va, binobarin, kirish qarshiligi nolga teng) bo‘ladi.

Kirish o‘tkazuvchanligining suratini nolga tenglashtirib, tok rezonanslari chastotasini aniqlaymiz:

$$\omega_{03} = \sqrt{(C_1 + C_2)/C_1 C_2 (L_1 + L_2)} = 10^4 \sqrt{5/12} = 6455 \text{ rad/s}.$$

Bu chastotada kirish o‘tkazuvchanligi nolga teng bo‘ladi, zanjirdan tok o‘tmaydi. Kirish o‘tkazuvchanlikning chastotaviy xarakteristikasini $b_{\text{kir}}=f(\omega)$ 2.54- rasmda keltirilgan.

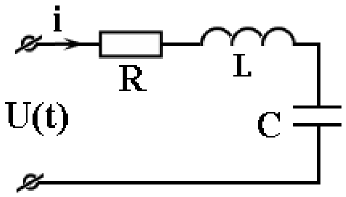


2.54- rasm. Zanjirning chastotaviy xarakteristikasi.

2.4.3. Ketma-ket ulangan tebranma konturning chastotaviy xarakteristi-kasi

2.4.3.1. Asosiy nazariy ma’lumotlar

Ketma-ket ulangan tebranma konturda (2.55-rasm) $x = x_{L0} - x_{C0} = 0$, ya’ni $\omega L = \dfrac{1}{\omega C}$ shart bajarilganda kuchlanish rezonansi paydo bo‘ladi.



2.55- rasm.

Rezonans chastotasi $\omega_0 = \dfrac{1}{\sqrt{LC}}$ yoki $f_0 = \dfrac{\omega_0}{2\pi} = \dfrac{1}{2\pi\sqrt{LC}}$. (2.3)

Konturning to‘lqin qarshiligi $\rho = \omega_0 L = \dfrac{1}{\omega_0 C} = \sqrt{\dfrac{L}{C}}$. (2.4)

Rezonansdagi kontur qarshiligi $Z_0 = R$.

Konturning o‘z aslligi $Q = \dfrac{\rho}{R} = \dfrac{U_{L0}}{U} = \dfrac{U_{C0}}{U}$. (2.5)

Yuklamali konturning aslligi $Q_{\text{yuk}} = \dfrac{\rho}{R + R_{\text{yuk}} + R_i}$. (2.6)

Konturning so‘nishi $d = \dfrac{1}{Q}$. (2.7)

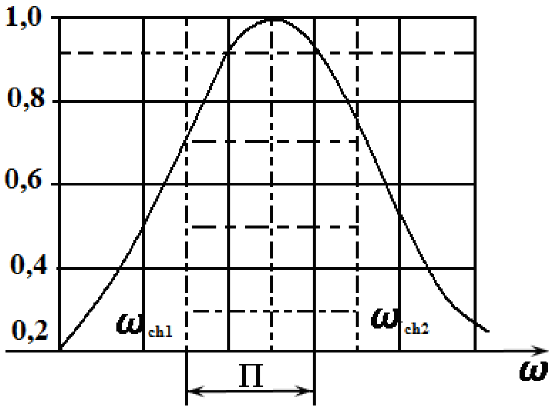
Absolut nosozlik $\Delta\omega = \omega - \omega_0$ yoki $\Delta f = f - f_0$. (2.8)

Nisbiy nosozlik $\delta = \dfrac{\Delta\omega}{\omega_0} = \dfrac{\Delta f}{f_0}$. (2.9)

Umumlashgan nosozlik $\xi = \dfrac{X}{R} = \dfrac{\omega L - \dfrac{1}{\omega C}}{R} = Q\left(\dfrac{\omega}{\omega_0} - \dfrac{\omega_0}{\omega}\right) = Q\nu$. (2.10)

Nosozlik faktori $\nu = \frac{\omega}{\omega_0} - \frac{\omega_0}{\omega} = \frac{f}{f_0} - \frac{f_0}{f}$. (2.11)

O‘tkazishning absolut polosasi (2.56- rasm), ya’ni ma’lum chastotali tokni o‘tkazish:



2.56- rasm.

$$\Pi = \omega_{42} - \omega_{41} = \frac{\omega_0}{Q} = d\omega_0, \frac{\text{rad}}{\text{s}} \text{ yoki } \Pi = \frac{f_0}{Q} = df_0, \text{Hz}. \quad (2.12)$$

O‘tkazishning nisbiy polosasi $\Pi_{\text{nis}} = \frac{\Pi}{\omega_0}$. (2.13)

Yuklamali kontur uchun $\Pi = \frac{f_0}{Q_{\text{yuk}}}$; $\Pi_{nis} = \frac{1}{Q_{\text{yuk}}}$. (2.14)

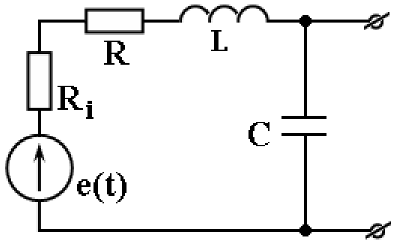
Kuchlanish bo‘yicha uzatishning kompleks koeffitsiyenti:

aktiv qarshilikda $K_{UR}(j\omega) = \frac{\underline{U}_R}{\underline{U}} = \frac{1}{\sqrt{1+\xi^2}} e^{-j\text{arctg}\xi}$: (2.15)

induktivlikda $K_{UL}(j\omega) = \left(\frac{\underline{U}_L}{\underline{U}}\right) = \frac{Q}{\sqrt{1+\xi^2}} \frac{\omega}{\omega_0} e^{j\left(\frac{\pi}{2}\text{arctg}\xi\right)}$. (2.16)

sig‘imda $K_{UC}(j\omega) = \frac{\underline{U}_C}{\underline{U}} = \frac{Q}{\sqrt{1+\xi^2}} \frac{\omega}{\omega_0} e^{j\left(\frac{\pi}{2}+\text{arctg}\xi\right)}$ (2.17)

2.90-masala. Ketma-ket ulangan tebranma kontur (2.57-rasm) sinusoidal kuchlanish manbayiga ulangan. Kontur rezonansga sozlangan. Zanjir parametrlari: $R = 9\ \Omega$, $L = 80$ mkH, $C = 2000$ pF, $E = 3$ V, $R_i = 1\,\Omega$. Rezonans chastotasi, to‘lqin qarshiligi, asllik va o‘tkazish polosasi, kontur elementlaridagi tok va kuchlanishlar aniqlansin.



2.57- rasm.

Yechish. Konturning rezonans chastotasini aniqlaymiz:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{6.28\sqrt{80\cdot 10^{-6}\cdot 2000\cdot 10^{-12}}} = 400 \text{ kHz}.$$

Konturning to‘lqin qarshiligi $\rho = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{80\cdot 10^{-6}}{2000\cdot 10^{-12}}} = 200\ \Omega$.

Yuklangan konturning aslligi $Q_{\text{yuk}} = \dfrac{\rho}{R+R_i} = \dfrac{200}{9+1} = 20$.

O‘tkazish polosasini aniqlaymiz: $\Pi = \dfrac{f_0}{Q_{yuk}} = \dfrac{400\cdot 10^3}{20} = 20\cdot 10^3 = 20 \text{ kHz}$.

Rezonans rejimida kontur elementlaridagi tok va kuchlanishni hisoblaymiz:

$$I_0 = \frac{E}{R+R_i} = \frac{3}{9+1} = 0{,}3 \text{ A}.$$

R qarshilikdagi kuchlanish pasayishi $U_{R0} = I_0 R = 0{,}3\cdot 9 = 2{,}7$ V.

Reaktiv elementlardagi kuchlanish pasayishlari $U_{C0} = U_{L0} = QE = 20\cdot 3 = 60$ V.

Sig‘imdagi kuchlanishning kompleks uzatish koeffitsiyentlari AChX va FChX larni hisoblaymiz. (2.10) ni e’tiborga olib (2.17) dan quyidagini hosil qilamiz:

$$K_{UC}(f) = \frac{f_0}{f}\frac{Q}{\sqrt{1+\left(\dfrac{f}{f_0}-\dfrac{f_0}{f}\right)}}; \qquad \psi(f) = -\frac{\pi}{2} - \text{arctg}\left(\frac{f}{f_0}-\frac{f_0}{f}\right)Q.$$

2.91-masala. Agar ketma-ket ulangan tebranma konturning rezonans chastotasi $f_0 = 1$ MHz va isroflar qarshiligi $0{,}5\ \Omega$ bo‘lganda o‘tkazish polosasi $\Pi = 6$ kHz bo‘lsa, uning sig‘imi va kompleks uzatish koeffitsiyenti aniqlansin, sig‘imdagi AChX va FChX lari qurilsin.

Yechish. Konturning kerak bo‘lgan aslligini aniqlaymiz:

$$Q = \frac{f_0}{\Pi} = \frac{10^6}{6 \cdot 10^3} \approx 167 .$$

Kondensator sig‘imini $\omega_0 = \dfrac{1}{\sqrt{LC}}$ formuladan aniqlaymiz:

$$C = \frac{1}{\omega_0^2 L} = \frac{1}{\omega_0 \rho} = \frac{1}{\omega_0 QR} = \frac{1}{2\pi f_0 QR} = \frac{1}{6{,}28 \cdot 10^6 \cdot 167 \cdot 0{,}5} = 2 \cdot 10^{-9}\,\text{F} = 2000\ \text{pF}.$$

AChX va FChX larni hisoblaymiz. (2.16) formuladan induktivlikdagi kompleks uzatish koeffitsiyentidan foydalanamiz. (2.10) ni hisobga olib quydagini yozamiz:

$$K_{UL} = \frac{f}{f_0} \frac{Q}{\sqrt{1 + \left(\dfrac{f}{f_0} - \dfrac{f_0}{f}\right)^2 Q^2}}; \qquad \psi(f) = \frac{\pi}{2} - \text{arctg}\left(\frac{f}{f_0} - \frac{f_0}{f}\right) Q .$$

Funksiya qiymatlarini quydagi chastotalarda topamiz.

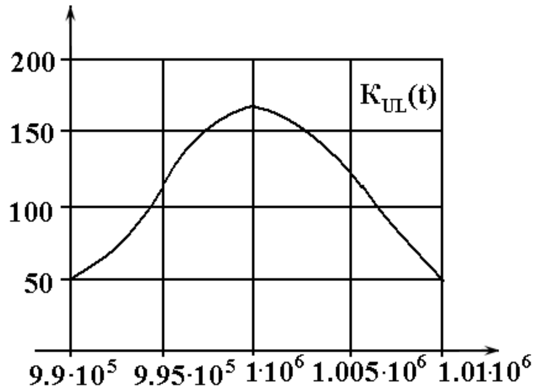
$$f = 0; \quad f_0 = 1\,\text{MHz} \quad f_{41} = f_0 - \frac{\Pi}{2} = (1 - 0.003) \cdot 10^6 = 0{,}997\ \text{MHz};$$

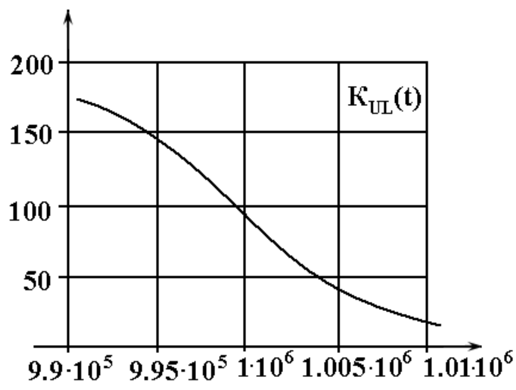
$$f_{42} = f_0 + \frac{\Pi}{2} = (1 + 0{,}003) \cdot 10^6 = 1{,}003\ \text{MHz}\ \text{va h.k.}, \quad f \to \infty\ \text{gacha.}$$

AChX maksimumga ega bo‘lgan chastotani aniqlaymiz.

$$f_{0L} = f_0 \sqrt{1 + \frac{1}{2Q^2}} = 1{,}00001 \cdot 10^6\ \text{Hz}.$$

AChX va FChX larni grafik tasviri 2.58- rasmda keltirilgan.





2.58- rasm.

2.92- masala. Ketma-ket ulangan tebranma konturda rezonans vaqtida kondensatordagi kuchlanish 120 V, isrof qarshilikli induktiv g‘altakda esa 130 V. Kontur kirishidagi kuchlanishni vektor diagrammasidan foydalanib aniqlang.

2.93-masala. Agar *R*, *L*, *C* dan iborat ketma-ket ulangan tebranma konturning aslligi 50, to‘lqin qarshiligi $2\,\text{k}\Omega$ va rezonans chastotasi $1\,\text{MHz}$ bo‘lsa, uning parametrlarini toping.

2.94- masala. Ketma-ket ulangan tebranma konturning rezonans chastotasi $465\,\text{kHz}$, qarshilik isrofi $4\,\Omega$, o‘tkazish polosasi $\Pi = 4\,\text{kHz}$ bo‘lsa, g‘altak induktivligini aniqlang, induktivlikdagi kuchlanish bo‘yicha AChX va FChXlarni quring.

Bitta reaktiv elementli zanjirlarning chastotaviy xarakteristikasi

Elektr zanjirlar tahlilining chastotaviy usullari. Asosiy nazariy ma’lumotlar

Zanjir chastotaviy xarakteristikalari uning kompleks funksiyalarining komponentlaridir.

Zanjirning kompleks funksiyasi (ZKF):

$$K(j\omega) = \frac{\underline{A}_{\text{chiq}}(\omega)}{\underline{A}_{\text{kir}}(\omega)} = \frac{A_{\text{chiq}}(\omega)}{A_{\text{kir}}(\omega)} e^{j[\psi_{\text{chiq}}(\omega) - \psi_{\text{kir}}(\omega)]} = K(\omega)e^{j\psi(\omega)}.$$

Amplituda-chastotaviy xarakteristika (AChX):

$$K(\omega) = |K(j\omega)| == \frac{A_{\text{chiq}}(\omega)}{A_{\text{kir}}(\omega)}.$$

Faza-chastotaviy xarakteristika (FChX):

$$\psi(\omega) = \arg K(j\omega) = \psi_{\text{chiq}}(\omega) - \psi_{\text{kir}}(\omega).$$

Amplituda-faza-chastotaviy xarakteristika (AFChX) (zanjirning kompleks funksiyasi):

$$K(j\omega) = K(\omega)e^{j\psi(\omega)} = P(\omega) - jQ(\omega),$$

bunda $P(\omega)$ - haqiqiy chastotaviy xarakteristika;

$Q(\omega)$ - mavhum chastotaviy xarakteristika.

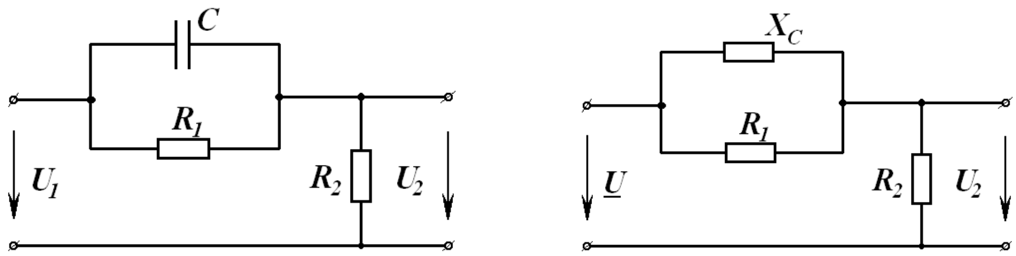
Oddiy zanjirlarning kompleks funksiyalarini bevosita Om qonunidan aniqlash mumkin.

Zanjirning o'tkazish polosasi deb, signal quvvati 2 marta kamaygan, tok (kuchlanishning) amplitudasi esa (ta'sir etuvchi qiymati) maksimal qiymatga nisbatan $\sqrt{2}$ marta kamaygan chastota diapazoni chegaralariga aytiladi.

O'tkazish polosasi $\mathrm{rad/s}$, s^{-1} yoki Hz bilan o'lchanadi.

2.95-masala. Differensiallovchi *RC*-kontur uchun (2.59-rasm, *a*) kuchlanish bo'yicha kompleks uzatish koeffitsiyentini aniqlang. Amplituda-chastotaviy-xarakteristika (AChX) va faza-chastotaviy xarakteristikalar (FChX) ifodalarini toping va ularni quring.

Yechish. 1. Zanjirning kompleks almashlash sxemasini tasvirlaymiz (2.59-rasm, *b*).



2.59- rasm.

2. Zanjir chiqishidagi kompleks kuchlanishni aniqlaymiz:

$$\underline{U}_2 = \frac{\underline{U}_1}{R_2 + \dfrac{R_1/j\omega C}{R_1 + 1/j\omega C}} R_2 = \underline{U}_1 \frac{R_2 + j\omega C R_1 R_2}{R_1 + R_2 + j\omega C R_1 R_2}.$$

Hosil bo'lgan ifodaning surat va maxrajidagi $j\omega$ qatnashmagan hadlarni qavsdan tashqariga chiqaramiz:

$$\underline{U}_2 = \frac{\underline{U}_1 R_2}{R_1 + R_2} \cdot \frac{1 + j\omega C R_1}{1 + j\omega C R_1 R_2/(R_1 + R_2)}.$$

Binobarin,

$$K_U(j\omega) = \frac{\underline{U}_2}{\underline{U}_1} = \frac{R_2}{R_1 + R_2} \cdot \frac{1 + j\omega C R_1}{1 + j\omega C R_1 R_2/(R_1 + R_2)}.$$

Quyidagi belgilashlarni kiritamiz:

$$R_1/(R_1+R_2)=k;\quad R_1C=\tau_1;\quad \frac{R_2R_1C}{R_1+R_2}=\tau_2;\quad k\tau_1=\tau_3;\quad k<1.$$

τ kattalik zanjirning *vaqt doimiysi* deyiladi va sekundlarda o‘lchanadi; k – o‘zgarmas tokdagi, ya’ni $\omega=0$ chastotada kuchlanish bo‘yicha kuchaytirish koeffitsiyenti. Kiritilgan belgilarni hisobga olsak, quyidagini hosil qilamiz:

$$K_U(j\omega)=k\frac{1+j\omega\tau_1}{1+j\omega\tau_2}.$$

Bu ifodada $j\omega$ o‘rniga p qo‘ysak, avtomatika nazariyasida qo‘llaniladigan, zanjirning uzatish funksiyasi deb nomlangan ifodani olamiz.

3. AChX va FChX larning analitik ifodalarini olish uchun kompleks funksiyaning darajali shaklini yozamiz:

$$K_U(j\omega)=\frac{k\sqrt{1+(j\omega\tau_1)^2}\cdot e^{j\mathrm{arctg}\,\omega\tau_1}}{\sqrt{1+(j\omega\tau_2)^2}\cdot e^{j\mathrm{arctg}\,\omega\tau_2}}=k\frac{\sqrt{1+(j\omega\tau_1)^2}}{\sqrt{1+(j\omega\tau_2)^2}}\cdot e^{j(\mathrm{arctg}\,\omega\tau_1-\mathrm{arctg}\,\omega\tau_2)}.$$

So‘nggi ifodadan AChX va FChX uchun mos ravishda uzatish funksiyalarining analitik ifodalarini olish uchun kompleks funksiyaning darajali shaklini yozamiz:

$$K_U(\omega)=\left|K_U(j\omega)\right|=k\sqrt{\frac{1+(\omega\tau_1)^2}{1+(\omega\tau_2)^2}},$$

$$\psi(\omega)=\arg K(j\omega)=\mathrm{arctg}(\omega\tau_1)-\mathrm{arctg}(\omega\tau_2).$$

4. AChX va FChX larni ikkita nuqta koordinatalari bo‘yicha quramiz. Buning uchun $K_U(\omega)$ va $\psi(\omega)$ larni ikkita chegaraviy qiymatlar uchun hisoblaymiz:

$$\omega=0 \text{ da: } K_U(0)=\frac{R_2}{R_1+R_2}=k;\quad \psi(0)=0;$$

$$\omega=\infty \text{ da: } K_U(\infty)=1;\quad \psi(\infty)=0;$$

$K_U(\omega)$ AChX grafigi (2.60- rasm, *a*) $\dfrac{R_2}{R_1+R_2}=k$ dan 1 gacha monoton o‘sib boruvchi egri chiziqdir.

$\psi(\omega)$ FChX grafigini ikkita grafik yig‘indisi sifatida qurish mumkin.

$\psi(\omega)$ funksiyaning ekstremumini topamiz:

$$\omega_m = \frac{1}{\sqrt{\tau_1\tau_2}} = \frac{1}{\tau_1\sqrt{k}} = \frac{1}{R_1C}\sqrt{\frac{R_1+R_2}{R_2}} = \sqrt{\omega_1\omega_2}\,;$$

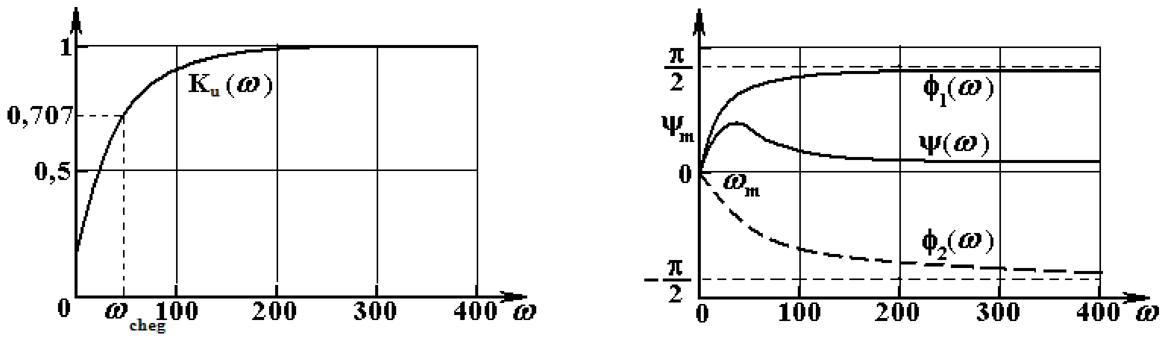
bunda $\omega_1 = \dfrac{1}{\tau_1}$; $\omega_2 = \dfrac{1}{\tau_2}$.

ω_m ni $\psi(\omega)$ funksiyaga qo‘yamiz:

$$\psi_m = \operatorname{arctg}\sqrt{\frac{R_1+R_2}{R_2}} - \operatorname{arctg}\sqrt{\frac{R_2}{R_1+R_2}}\,.$$

AChX va FChX grafiklar 2.60- rasmda tasvirlangan.

$j=\sqrt{-1}$; $\tau_1 = 0{,}1$; $\tau_2 = 0{,}02$; $k = 0{,}2$ $\quad K_U(\omega) = \left\| \dfrac{k(1+j\omega\tau_1)}{(1+j\omega\tau_2)} \right\|$



2.60- rasm.

2.96- masala. Kontur toklar usulidan foydalanib integrallovchi zanjirning kompleks uzatish koeffitsiyenti aniqlansin (2.61- rasm, *a*). Zanjir parametrlari $R_1 = 10\,\text{k}\Omega$; $R_2 = 30\,\text{k}\Omega$; $C = 500\,\text{pF}$.

Yechish. 1. Zanjirni kompleks almashlash sxema bilan tasvirlaymiz (4.61- rasm, *b*). Berilgan sxema ikkita bog‘lanmagan konturdan iborat. Birinchi konturdagi I_{11} tok manba orqali o‘tadi (rasmda keltirilmagan). Kontur toklar yo‘nalishini bir xil qilib olamiz.

2. Bog‘lanmagan konturlar uchun kontur qarshiliklar matritsasini tuzamiz:

$$Z(j\omega)=\begin{vmatrix} R_1+\dfrac{1}{j\omega C} & \dfrac{1}{j\omega C} \\ -\dfrac{1}{j\omega C} & R_2+\dfrac{1}{j\omega C} \end{vmatrix}$$

Yuqoridagi qayd etilgan ma’lumotlarga ko‘ra τ_1, τ_2 va k ni hisoblaymiz:

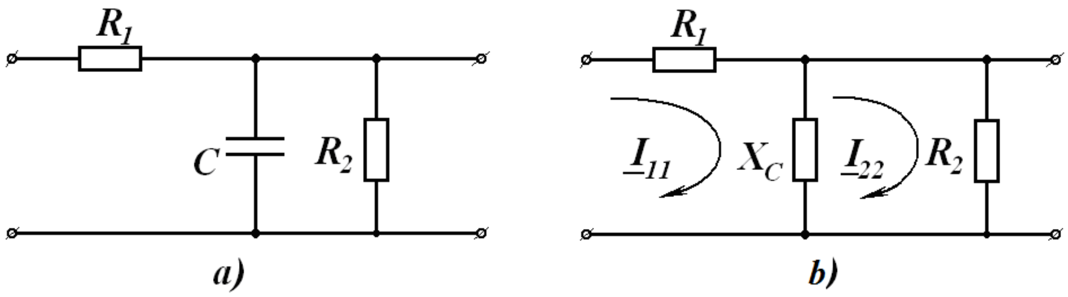
$$j:=\sqrt{-1};\ \tau_1=5\cdot10^{-6};\ \tau_2=9{,}673\cdot10^{-6};\ k=0{,}636;$$

$$K(\omega)=k\frac{1+j\omega\tau_1}{1+j\omega\tau_2};\ K_U(\omega)=\left[\frac{k\sqrt{1+(\omega\tau_1)^2}}{\sqrt{1+(\omega\tau_2)^2}}\right],$$

$$\varphi_1(\omega)=(\operatorname{arctg}(\omega\tau_1))\cdot\left(\frac{180}{\pi}\right);\ \varphi_2(\omega)=(\operatorname{arctg}(\omega\tau_2))\cdot\left(\frac{180}{\pi}\right);$$

$$\psi(\omega)=\varphi_1(\omega)+\varphi_2(\omega);$$

$$\omega=0{,}5...1000000.$$



2.61- rasm.

3. Kompleks uzatish koeffitsiyentini aniqlaymiz, bunda kompleks funksiya $\underline{Z}_{uz}(\omega)=\dfrac{\underline{U}_N}{\underline{I}_1}$ bo‘lsa, kontur toklar usulidan foydalanib quyidagini hosil qilamiz:

$$K_U(j\omega)=Z_{\text{yuk}}\frac{\Delta_{1N}}{\Delta},$$

bunda $Z_{\text{yuk}}=R_2$ – yuklama qarshiligi; Δ_{1N} va Δ - mos ravishda aniqlovchi va kontur qarshiliklarining algebraik to‘ldiruvchisi:

$$\Delta_{1N}=\Delta_{12}=(-1)^3\left(-\frac{1}{j\omega C}\right)=\frac{1}{j\omega C};$$

$$\Delta=\left(R_1+\frac{1}{j\omega C}\right)\left(R_2+\frac{1}{j\omega C}\right)-\left(\frac{1}{j\omega C}\right)^2=\frac{R_1+R_2+R_1R_2j\omega C}{j\omega C}.$$

Hosil bo‘lgan ifodalarni qo‘yib, quyidagini olamiz:

$$K_U(j\omega)=\frac{R_2}{R_1+R_2+j\omega CR_1R_2}=\frac{R_2}{R_1+R_2}\cdot\frac{1}{1+j\omega\tau\frac{R_2}{R_1+R_2}}=k\frac{1}{1+j\omega\tau k},$$

yoki

$$K_U(j\omega)=k\frac{1}{1+j\omega\tau k}=\frac{k}{\sqrt{1+(\omega\tau k)^2}}e^{-j\mathrm{arctg}(\omega\tau k)} \quad (2.18)$$

bunda $k=\dfrac{R_2}{R_1+R_2}$; $\tau=R_1C$.

4. $K_U(\omega)$ ni chastotalarning chegaraviy $\omega=0$ va $\omega=\infty$ lardagi qiymatlarini hisoblaymiz:

$$K_U(0)=k=\frac{R_2}{R_1+R_2}=\frac{30\cdot10^3}{(10+30)\cdot10^3}=0{,}75;\quad K_U(\infty)=0.$$

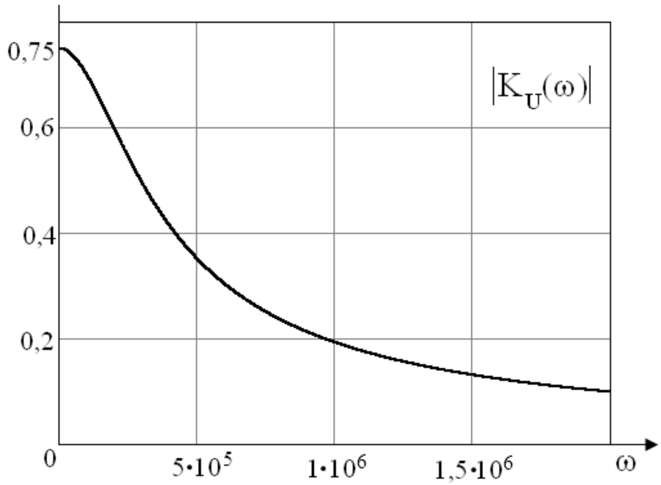
Hosil bo‘lgan natijalarni quyidagicha tushintirish mumkin: chastota nol bo‘lganda (o‘zgarmas tok rejimi) sig‘im qarshilik cheksiz katta bo‘ladi, undan o‘tadigan tok nolga teng bo‘ladi, ya’ni shoxobchani uzilgan deb hisoblash mumkin. Bunda zanjirni uzatish funksiyasi $K_U(0)=K_{U\max}=0{,}75$ bo‘lgan kuchlanishning rezistiv bo‘lgichi deb qarash mumkin. Chastota oshgan sari sig‘im qarshilik kamayadi. Agar $\omega\to\infty$, bo‘lsa unda $X_C=0$ va R_2 qarshilikni shuntlaydi. Bu holda $R_2X_C/(R_2+X_C)=0$; $\underline{U}_2=0$; $K_U(\omega\to\infty)=0$; Natijada, AChX grafigi 2.62- rasmda keltirilgan.

5. O‘tkazish polosasini aniqlaymiz. Ta’rif bo‘yicha

$$K_U(\omega_{\text{cheg}})=\frac{K_{U\text{mak}}}{\sqrt{2}}=\frac{R_2}{\sqrt{2}(R_1+R_2)}=0{,}533.$$

Shuning uchun (2.18) dan quyidagi kelib chiqadi:

$$\frac{R_2}{\sqrt{(R_1+R_2)^2+(\omega_{\text{cheg}}CR_1R_2)^2}}=\frac{R_2}{(R_1+R_2)\sqrt{2}}. \quad (2.19)$$



2.62- rasm.

(2.19) tenglamani o‘zgartirishdan so‘ng, quyidagini hosil qilamiz:

$$\sqrt{(R_1+R_2)^2+(\omega_{\text{cheg}}CR_1R_2)^2}=\sqrt{2(R_1+R_2)^2},$$

$$(R_1+R_2)^2+(\omega_{\text{cheg}}CR_1R_2)^2=2(R_1+R_2)^2,$$

$$(\omega_{\text{cheg}}CR_1R_2)^2=(R_1+R_2)^2,$$

bundan

$$\omega_{\text{cheg}}=\frac{R_1+R_2}{R_1R_2C}=0{,}267\cdot10^6\ \text{Hz},,$$

yoki

$$f_{\text{cheg}}=\frac{\omega_{\text{cheg}}}{2\pi}=\frac{0{,}267\cdot10^6}{6{,}28}=0{,}0425\cdot10^6\ \text{Hz}=42{,}5\ \text{kHz}.$$

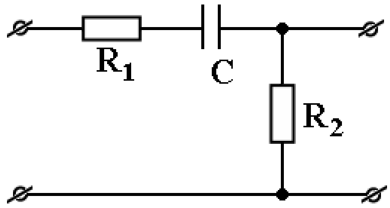
Binobarin, zanjirning o‘tkazish polosasi:

$\Pi=0{,}267\cdot10^6$ Hz yoki $\Pi=42{,}5$ kHz.

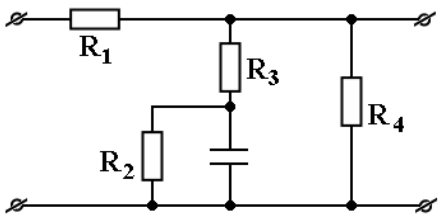
Bu zanjir o‘tkazish polosasi 42,5 kΩ li quyi chastotali filtr bo‘lgani sababli $f>f_{\text{cheg}}$ da signallar juda katta so‘nishga ega.

2.97- masala. Differensiallovchi konturning (2.63- rasm) kuchlanish bo‘yicha kompleks koeffitsiyentini, AChX, FChX, ω_{cheg} ifodalarini aniqlang va ularning grafiklarini quring. Zanjir parametrlari $R_1=2$ kΩ, $R_2=3$ kΩ, $C=0{,}1$ mkF.

2.98- masala. Integrallovchi konturning (2.64- rasm) kompleks uzatish koeffitsiyentini aniqlang. Chastota $\omega=0$ va $\omega\to\infty$ o‘zgarganda AChX ni hisoblang va grafigini quring.



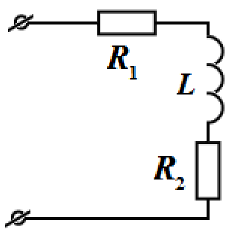
2.63- rasm.



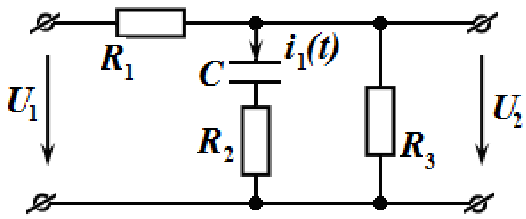
2.64- rasm.

2.99- masala. 2.65- rasmda keltirilgan zanjir uchun kirish kompleks o‘tkazuvchanligini aniqlang. AChX, FChX, $P(\omega)$, $Q(\omega)$larni hisoblang va shu xarakteristikalarni quring. Zanjir parametrlari $R_1 = 100\ \Omega$, $R_2 = 200\ \Omega$, $L = 0{,}1\ \text{H}$.

2.100- masala. Zanjirning (2.66- rasm) kompleks uzatish o‘tkazuvchanligini aniqlang. $\omega = 0$ va $\omega \to \infty$ uchun AChXni quring. Zanjir parametrlari $R_1 = 10\ \text{k}\Omega$, $R_2 = 20\ \text{k}\Omega$, $R_3 = 30\ \text{k}\Omega$, $C = 0{,}1\ \text{mkF}$.



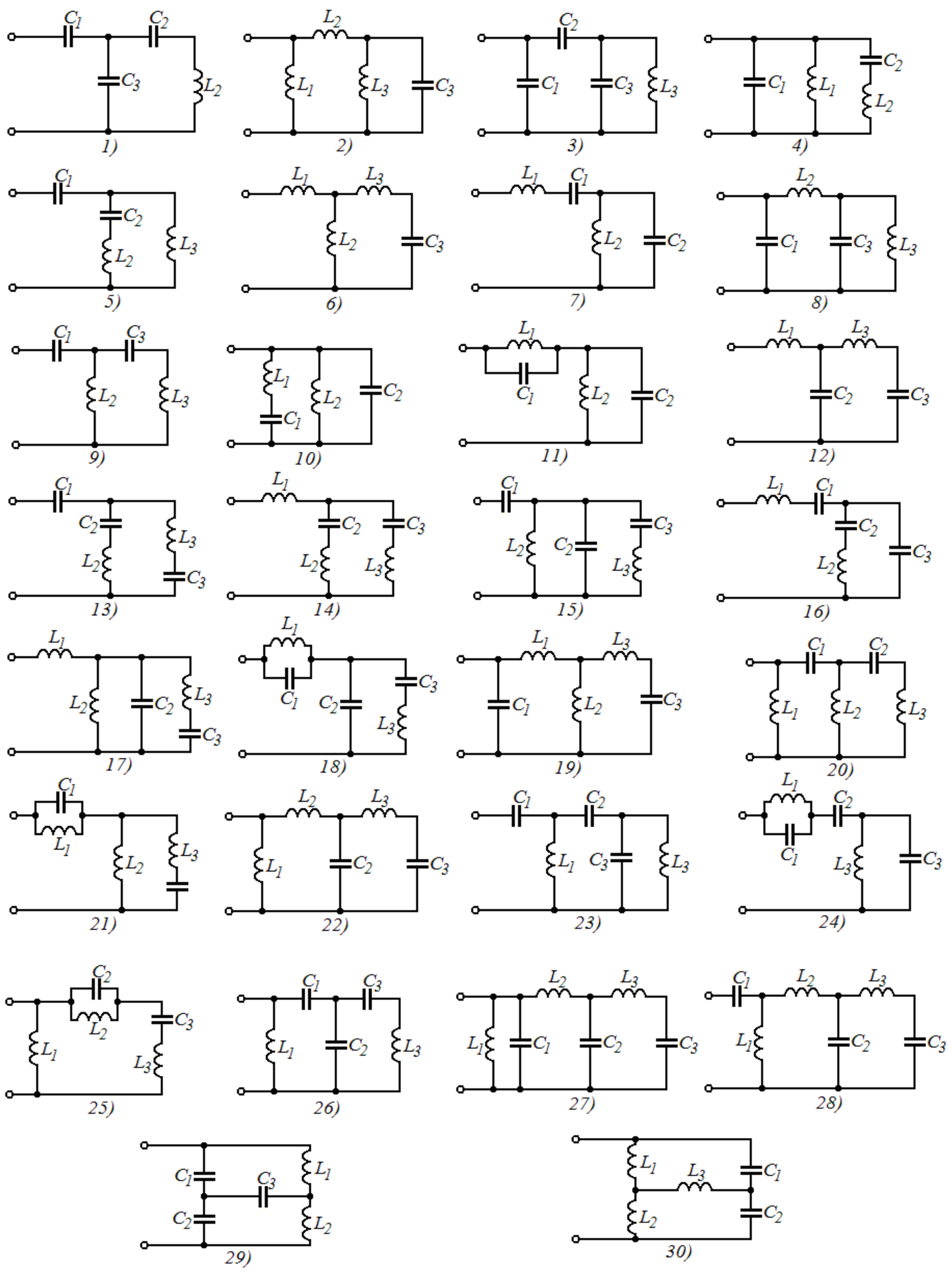
2.65–rasm.



2.66- rasm.

2.4- vazifa. Rezonans sxemalarini hisoblash

Zanjirlar (2.67- rasm) uchun rezonans chastotalar aniqlansin va kirish qarshiligining (yoki kirish o‘tkazuvchanligining) chastotaviy xarakteristikasi chizilsin. Sxema parametrlari 2.2- jadvalda (bunda L_0=1 mH, C_0=1 mkF) keltirilgan. Sxema nomeri (2.67- rasm) jadvalda berilgan nomerlarga mos keladi.



2.67- rasm. 2.4-vazifaga sxema variantlari.

2.2- *jadval*

Variant	L_1/L_0	L_2/L_0	L_3/L_0	C_1/C_0	C_2/C_0	C_3/C_0
1	5	8	4	3	6	8
2	30	50	20	30	60	30
3	8	10	6	60	30	60
4	10	20	40	1	3	5
5	8	3	6	10	50	30
6	30	50	20	7	4	9
7	9	6	5	3	6	4
8	8	10	6	60	40	80
9	30	60	40	6	8	4
10	70	80	30	50	30	40
11	60	40	60	4	7	3
12	5	7	3	50	30	40
13	6	3	4	7	5	9
14	100	150	300	80	50	30
15	6	7	3	70	60	40
16	50	30	70	6	8	4
17	30	50	90	9	16	7
18	8	5	4	7	5	4
19	250	600	400	60	50	90
20	70	60	76	80	50	60
21	9	5	6	7	4	3
22	80	50	70	40	30	90
23	7	5	2	6	5	1
24	60	80	30	60	50	20
25	80	50	30	60	40	80
26	6	4	9	7	2	8
27	90	70	40	80	40	20
28	8	4	5	40	20	70
29	70	30	50	7	10	3
30	8	10	7	9	4	5

2.5. O‘zaro induktiv bog‘langan zanjirlarni hisoblash

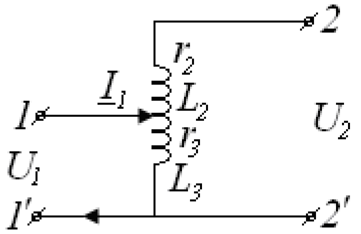
2.101- masala. Induktiv bog‘langan zanjir (masalan, avtotransformator)ning (2.68- rasm) 1-1′ qismalariga sinusoidal kuchlanish ulangan. Agar $U_1 = 10$ V $r_2 = r_3 = 3\ \Omega$; $\omega L_2 = \omega L_3 = 4\ \Omega$; $\omega M = 2\ \Omega$; bo‘lsa, yuklama ulanmagan chiqish qismalaridagi kuchlanish topilsin.

Yechish. Kirish kuchlanishi $\underline{U}_1 = U_1 = 10$ V. Kirish tokini hisoblaymiz:

$$\underline{I}_1 = \frac{\underline{U}_1}{(r_3 + j\omega L_3)} = \frac{10}{(3 + j4)} = 1{,}2 - j1{,}6 \text{ A.}$$

Chiqish $\underline{U}_2$ kuchlanishi zanjirning 2 qismasidan 2′ qismasigacha bo‘lgan kuchlanishlar yig‘indisidan iborat:

$$\underline{U}_2 = j\omega M \underline{I}_1 + \underline{U}_1 = j2(1{,}2 - j1{,}6) + 10 = 13{,}2 + j2{,}4 \text{ V.}$$

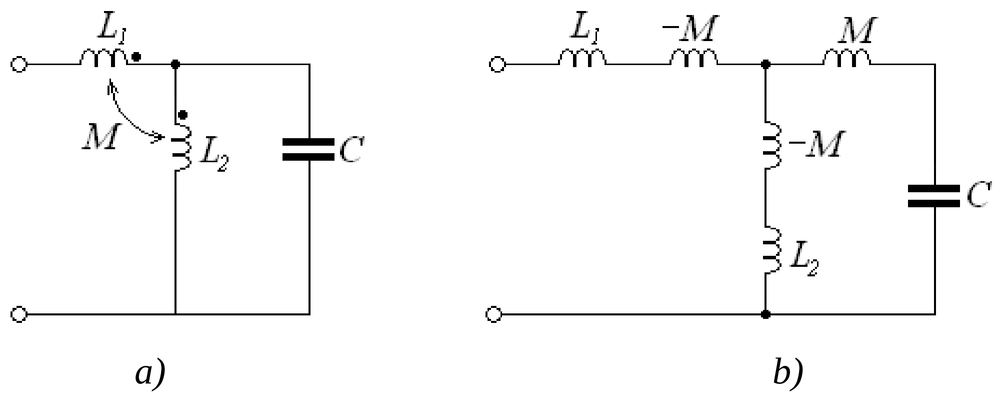


2.68- rasm.

Agar L_2 induktivlikning pastki qismasi L_3 induktivlikning yuqori qismasi bilan bir xil nomlangan bo‘lsa, u holda L_2 va L_3 elementlardan o‘tuvchi toklar bir xil nomlangan qismalarga nisbatan har xil yo‘nalishda bo‘ladi. Shuning uchun $j\omega M \underline{I}_1$ had $\underline{U}_2$ kuchlanish ifodasiga minus ishora bilan qo‘shilar edi va bu holda:

$$\underline{U}_2 = -j\omega M \underline{I}_1 + \underline{U}_1 = -j2(1{,}2 - j1{,}6) + 10 = 6{,}8 - j2{,}4 \text{ V.}$$

2.102- masala. Parametrlari x_{L1} = 10 Ω, x_{L2} = 5 Ω, x_M = 5 Ω, x_C = 10 Ω bo‘lgan zanjirning (2.69- rasm, *a*) kompleks kirish qarshiligini aniqlang.



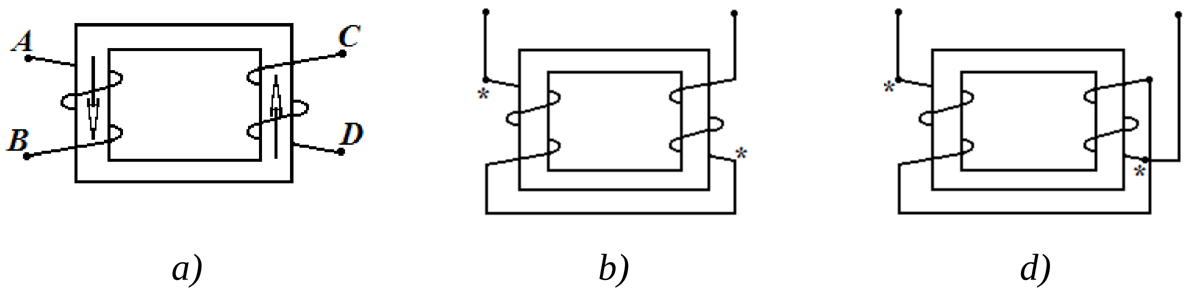
2.69- rasm.

Yechish. Bir xil nomlangan qismalar bitta tugunga ulanganligini hisobga olib zanjirni induktiv bog‘lanishsiz ko‘rinishga keltiramiz (2.69- rasm, *b*).

Hosil bo‘lgan zanjir kirish qarshiligining kompleks qiymatini aniqlaymiz:

$$\underline{Z}_{\text{kir}} = jx_{L1} - jx_M + \frac{(jx_M - jx_C)(jx_{L2} - jx_M)}{jx_M - jx_C + jx_{L2} - jx_M} = j5 = 5e^{j90^0}\ \Omega\ .$$

2.103- masala. Zanjirning (2.70- rasm, *a*) bir xil nomlangan qismalarini aniqlang. Ketma-ket mos hamda ketma-ket qarama-qarshi ulangan sxemani tasvirlang.

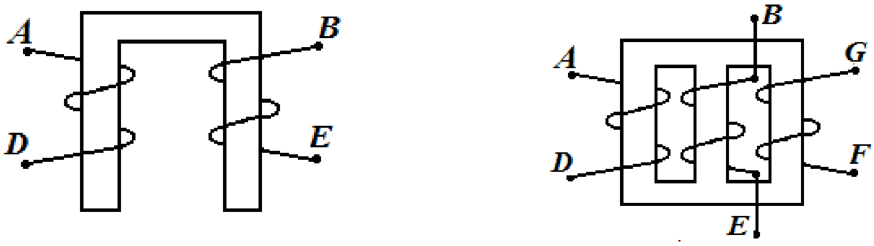


2.70- rasm.

Yechish. Faraz qilaylik *AB* chulg‘amdan tok *A* qismadan *B* qismaga yo‘nalgan bo‘lsin. *A* qismani * bilan belgilaymiz. O‘ng qo‘l qoidasiga ko‘ra *AB* chulg‘am hosil qiladigan magnit oqim yo‘nalishini aniqlaymiz. Magnit o‘tkazgichning chap o‘zagidan oqim yuqoridan pastga yo‘nalgan. Bu oqim *CD* chulg‘amni pastdan yuqoriga qarab kesib o‘tadi va uning o‘zaro oqimidir. *CD* chulg‘amdan shunday yo‘nalishdagi tok o‘tkazilishi kerakki, u hosil qilgan o‘zmagnit oqimining yo‘nalishi o‘zaro oqim yo‘nalishi bilan bir xil bo‘lsin. O‘ng qo‘l qoidasi bo‘yicha tokni *D* qismadan *C* qismaga yo‘naltirish kerak. Binobarin, bir xil nomlangan qismalar *A* va *D* hamda *B* va *C* bo‘ladi. Bu qismalarning har qanday jufti bir xil belgilanishi kerak. 2.70- rasmda bir xil nomlangan qismalar yulduzchalar bilan belgilangan.

Ketma-ket mos ulangan chulg‘amlar 2.70- rasm, *b* da, ketma-ket qarama-qarshi ulanganlari esa 2.70- rasm, *d* da tasvirlangan.

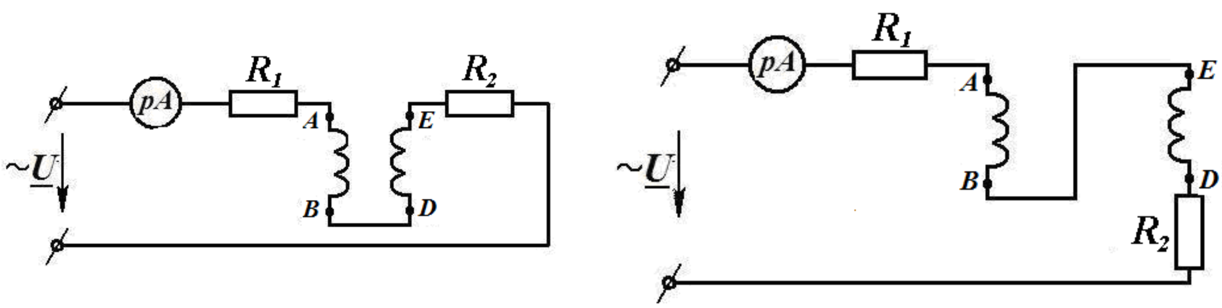
2.104- masala. 2.71- rasmda tasvirlangan ferromagnit o‘zaklarda joylashgan chulg‘amlarning bir xil nomlangan qismalarini aniqlang.



2.71- rasm.

2.5.1. Induktiv bog‘langan zanjirlarning ekvivalent sxemalari

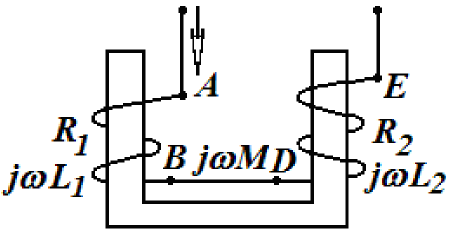
2.105- masala. Zanjir 2.72- rasm, *a* bo‘yicha ulanganda ampermetr ko‘rsatishi 2 A, 2.72- rasm, *b* bo‘yicha ulanganda esa 4 A bo‘lgan. Induktiv bog‘langan chulg‘amlarning bir xil nomlangan qismalarini aniqlang. Ikkala holda ham zanjirga berilayotgan kuchlanish bir xil.

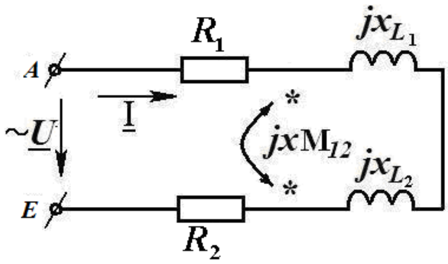


a) *b)*

2.72- rasm.

2.106- masala. Zanjirdagi (2.73- rasm, *a*) tok va har bir chulg‘amdagi kuchlanish aniqlansin. Vektor diagramma qurilsin. Berilgan: $R_1 = 2\,\Omega$, $R_2 = 3\,\Omega$, $L_1 = 6\,\text{mH}$, $L_2 = 10\,\text{mH}$, $K_\text{b} = 25{,}8\%$, $u = 184\sin 1000\,t$ V.





a) *b)*

2.73- rasm.

Yechish. 2.73- rasm, *a* dagi chulg‘amlardan o‘tadigan tokni va u hosil qiladigan magnit oqim yo‘nalishlarini o‘ng qo‘l qoidasi bilan belgilab ko‘ramizki, o‘z va o‘zaro magnit oqimlar qarama-qarshi. Bu holat 2.73- rasm, *b* da tasvirlangan. Bunda birinchi va ikkinchi chulg‘amlarning, o‘zinduksiya kompleks qarshiliklari

$$jX_{L1} = j\omega L_1 = j10^3 \cdot 6 \cdot 10^{-3} = j6\ \Omega;$$
$$jX_{L2} = j\omega L_2 = j10^3 \cdot 10 \cdot 10^{-3} = j10\ \Omega.$$

va g‘altaklarning o‘zaro induksiya kompleks qarshiligi:

$$jX_{M12} = j\omega M_{12} = j\omega K_b \cdot \sqrt{L_1 L_2} = jK_b\sqrt{X_{L1} X_{L2}} = j2\ \Omega.$$

Kirxgofning ikkinchi qonuni asosida tenglama tuzamiz:

$$\underline{U} = \underline{I}R_1 + \underline{I}jX_{L_1} - \underline{I}\,jX_{M_{12}} + \underline{I}R_2 + \underline{I}\,jX_{L_2} - \underline{I}\,jX_{M_{12}}.$$

Bu tenglamadan tok kompleksini topamiz:

$$\underline{I} = \frac{\underline{U}}{r_1 + r_2 + jX_{L_1} + jX_{L_2} - 2\,jX_{M_{12}}} = \frac{130}{13e^{j67°20'}} = 10e^{-j67°20'}\ \text{A.}$$

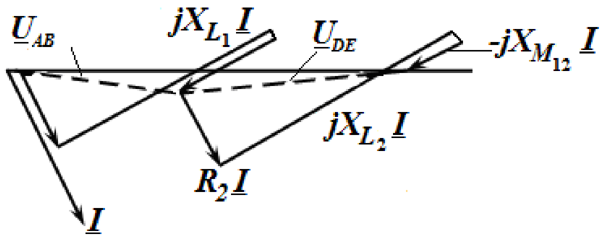
Tokning oniy qiymati

$$i = 10\sqrt{2}\sin(1000\,t - 67°20')\ \text{A.}$$

Birinchi va ikkinchi chulg‘amlardagi kuchlanishlarning komplekslari:

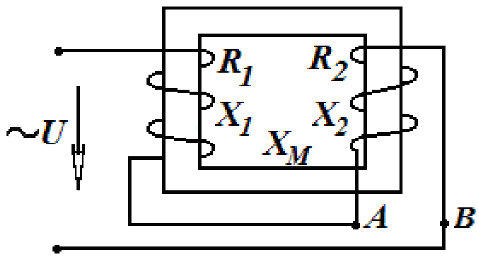
$$\underline{U}_{AB} = \underline{I}r_1 + \underline{I}\,jX_{L_1} - \underline{I}\,jX_{M_{12}} = 44{,}8e^{-j3°50'}\ \text{V},$$
$$\underline{U}_{DE} = \underline{I}r_2 + \underline{I}\,jX_{L_2} - \underline{I}\,jX_{M_{12}} = 85{,}5e^{j2°10'}\ \text{V}.$$

Zanjirning vektor diagrammasi 2.74- rasmda keltirilgan.



2.74- rasm.

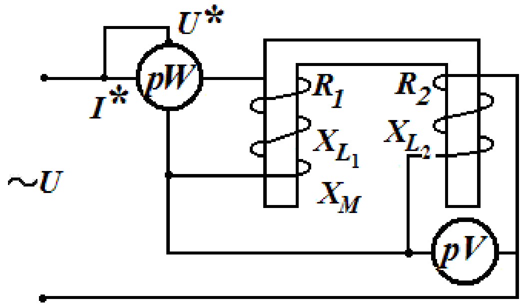
2.107- masala. Zanjirdagi (2.75- rasm) tok hamda har bir chulg‘amdagi kuchlanish pasayishi aniqlansin, bunda $R_1 = 2\,\Omega$, $R_2 = 4\,\Omega$, $X_{L1} = 2\,\Omega$, $X_{L2} = 8\,\Omega$, $U = 127\,\text{V}$. Vektor diagramma qurilsin. Magnit o‘tkazgich ferromagnit materialdan yasalgan deb qabul qilinsin.



2.75- rasm.

2.108- masala. 2.75- rasmdagi zanjir kuchlanishi $U = 24\,\text{V}$ bo‘lgan o‘zgarmas tok manbayiga ulangan bo‘lsa, undagi tok topilsin.

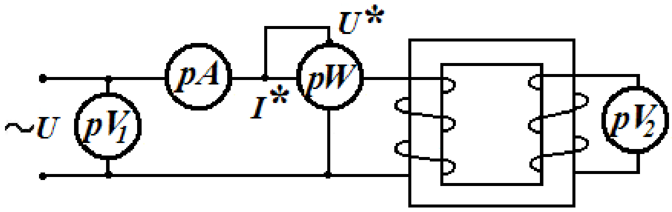
2.109- masala. Zanjirga ulangan (2.76- rasm) vattmetr va voltmetr ko‘rsatishini aniqlang, bunda $R_1 = R_2 = 30\,\Omega$, $X_{L_1} = 20\,\Omega$, $X_{L_2} = 40\,\Omega$, $X_M = 10\,\Omega$, $U = 220\,\text{V}$.



2.76- rasm.

2.110- masala. Transformator (2.77- rasm) chulg‘amlarining induktivliklari L_1, L_2 va M aniqlansin, bunda $f = 50\,\text{Hz}$ chastotada asboblar quyidagilarni ko‘rsatgan: $U_1 = 130\,\text{V}$; $I_1 = 5\,\text{A}$; $P = 250\,\text{W}$; $U_2 = 173\,\text{V}$.

Voltmetr qarshiliklarini cheksiz katta deb oling.



2.77- rasm.

Yechish. Zanjirda birinchi chulg‘amdan ikkinchisiga energiya uzatilmagani uchun vattmetr birinchi chulg‘amdagi quvvatni ko‘rsatadi.

$P = I_1^2 r_1,$

bundan $r_1 = \dfrac{P}{I_1^2} = \dfrac{250}{5^2} = 10\,\Omega.$

Birinchi chulg‘amning kirish qarshiligi $Z_1 = \dfrac{U_1}{I_1} = \dfrac{130}{5} = 26\,\Omega.$

Birinchi chulg‘amning to‘la qarshiligi

$X_{L1} = \sqrt{Z_1^2 - r_1^2} = \sqrt{26^2 + 10^2} = 24\,\Omega.$

Shu chulg‘amning induktivligi

$L_1 = \dfrac{X_{L1}}{2\pi \cdot f} = \dfrac{24}{314} = 76{,}5$ mH.

Ikkinchi chulg‘amda tok bo‘lmagani uchun uning kuchlanishi quydagicha topiladi:

$U_2 = I_1 X_m = I_1 \omega M$,

bundan $M = \dfrac{U_2}{\omega I_1} = \dfrac{173}{314 \cdot 5} = 0{,}11\,\text{H} = 110$ mH.

Zanjirdagi ikkala chulg‘amdan bir xil magnit oqim o‘tgani uchun magnit bog‘lanishning koeffitsiyenti $K_b = 1$.

$M = K_b \sqrt{L_1 L_2}$ formuladan

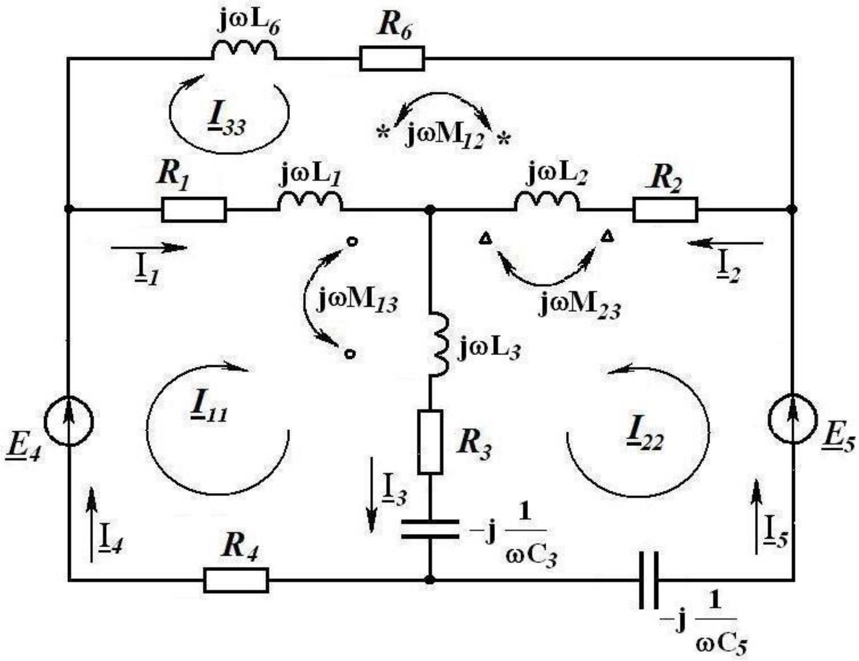
$$L_2 = \frac{M^2}{K_b^2 L_1} = \frac{0{,}11^2}{1 \cdot 0{,}0765} = 0{,}158\ \text{H} = 158\ \text{mH}.$$

2.111- masala. Zanjir uchun (2.78- rasm) Kirxgofning ikkinchi qonuni bo‘yicha tenglamalar tuzilsin va kontur toklar usulidan foydalanib kanonik tenglamalar sistemasining koeffitsiyentlari aniqlansin.

Yechish. Zanjirning uchta bog‘lanmagan konturlari uchun belgilangan tok yo‘nalishlarini hisobga olib, pastdagi chap kontur uchun

$$R_1 \cdot \underline{I}_1 + j\omega L_1 \underline{I}_1 + j\omega M_{12} \underline{I}_2 + j\omega M_{13} \underline{I}_3 + j\omega L_3 \underline{I}_3 + j\omega M_{13} I_1 +$$

$$+ j\omega M_{23} \underline{I}_2 + R_3 \underline{I}_3 - j\frac{1}{\omega C_3} \underline{I}_3 + R_4 \underline{I}_4 = \underline{E}_4.$$



2.78- rasm.

L_1 g‘altakdagi L_2 g‘altakdan o‘zaro induksiya tufayli paydo bo‘layotgan kuchlanish kompleksining ishorasiga e’tibor beramiz (tenglamaning chap tomonidagi uchinchi hadi). L_1 g‘altak bo‘yicha yurish yo‘nalishi va L_2 g‘altakdan o‘tadigan tok yo‘nalishi bir xil nomlangan qismalarga (yulduzchalarga) nisbatan mos kelgani uchun $j\omega M_{12} \underline{I}_2$ had oldida “+” ishora qo‘yiladi. Xuddi shu qoida bo‘yicha o‘zaro induksiyani hisobga oluvchi tenglamaning hadlari oldidagi ishoralar qo‘yilgan.

Past o‘ng tomondagi kontur uchun:

$$r_2\underline{I}_2 + j\omega L_2\underline{I}_2 + j\omega M_{12}\underline{I}_1 + j\omega M_{23}\underline{I}_3 + j\omega L_3\underline{I}_3 + j\omega M_{13}\underline{I}_1 +$$
$$+ j\omega M_{23}\underline{I}_2 + r_3\underline{I}_3 - j\frac{1}{\omega C_3}\underline{I}_3 - j\frac{1}{\omega C_5}\underline{I}_5 = \underline{E}_5.$$

Yuqoridagi kontur uchun:

$$j\omega L_6\underline{I}_6 + r_6\underline{I}_6 + r_2\underline{I}_2 + j\omega L_2\underline{I}_2 + j\omega M_{12}\underline{I}_1 + j\omega M_{23}\underline{I}_3 - j\omega L_1\underline{I}_1 -$$
$$- j\omega M_{12}\underline{I}_2 - j\omega M_{13}\underline{I}_3 - r_1\underline{I}_1 = 0.$$

Bu tenglamada L_1 dagi 2 va 3 o‘zaro induksiyadan paydo bo‘lgan kuchlanish kompleksi oldidagi qo‘yilgan ishora shu sababdanki, L_1 g‘altak bo‘yicha yurish yo‘nalishi hamda 2 va 3 g‘altaklarning bir xil joylashgan qismlaridagi yo‘nalishi qarama-qarshidir.

2.78- rasmdagi zanjir uchun kontur toklar usulidan foydalanib, quyidagi kanonik tenglamalar sistemasini tuzish mumkin:

$$Z_{11}\underline{I}_{11} + Z_{12}\underline{I}_{22} + Z_{13}\underline{I}_{33} = \underline{E}_{11},$$
$$Z_{21}\underline{I}_{11} + Z_{22}\underline{I}_{22} + Z_{23}\underline{I}_{33} = \underline{E}_{22},$$
$$Z_{31}\underline{I}_{11} + Z_{32}\underline{I}_{22} + Z_{33}\underline{I}_{33} = \underline{E}_{33}.$$

Agar bog‘lanmagan konturlar uchun kontur toklar yo‘nalishini 2.78- rasmda ko‘rsatilganidek olsak, kanonik tenglamalar sistemasining koeffitsiyentlari quyidagicha bo‘ladi:

$$Z_{11} = r_1 + r_3 + r_4 + j(\omega L_1 + \omega L_3 - \frac{1}{\omega C_3} + 2\omega M_{13}),$$

$$Z_{22} = r_2 + r_3 + j(\omega L_2 + \omega L_3 - \frac{1}{\omega C_3} - \frac{1}{\omega C_5} + 2\omega M_{23}),$$
$$Z_{33} = r_1 + r_2 + r_6 + j(\omega L_1 + \omega L_2 + \omega L_6 - 2\omega M_{12}).$$

Birinchi konturning kompleks qarshiligida o‘zaro induksiya kompleks qarshiligi $2j\omega M_{13}$ “+” ishora bilan kirgan, chunki L_1 va L_3 g‘altaklar birinchi kontur toki yo‘nalishi bo‘yicha mos ulangan. L_1 va L_2 g‘altaklar uchinchi kontur toklar yo‘nalishiga qarama-qarshi ulangan, shu sababli bu g‘altaklarning o‘zaro qarshilik kompleksining ikkilangani kompleks qarshilik Z_{33} tarkibiga “–“ ishora bilan kirgan.

Konturlarning o‘zaro qarshiligini yozib chiqamiz:

$$Z_{12} = Z_{21} = r_3 + j\omega L_3 - j\frac{1}{\omega C_3} + j\omega M_{13} + j\omega M_{12} + j\omega M_{23},$$
$$Z_{13} = Z_{31} = -r_1 - j\omega L_1 + j\omega M_{12} + j\omega M_{23} - j\omega M_{13},$$
$$Z_{23} = Z_{32} = r_2 + j\omega L_2 - j\omega M_{12} - j\omega M_{13} + j\omega M_{23}.$$

Konturlarning o‘zaro qarshiliklar komplekslariga, yondosh shoxobchalarning qarshiliklar komplekslaridan tashqari o‘zaro qarshiliklar kompleksi kirgan. Ularning ishoralari yondosh konturlarning kontur toklari yo‘nalishini magnit bog‘langan elementlarning bir nomli qismalariga nisbatan olinadi: mos ulanganda “+” ishora, qarama-qarshi ulanganda “–” ishora qo‘yiladi.

2.112- masala. Zanjir (2.79- rasm) shoxobchalaridagi toklar va vattmetrlar ko‘rsatishini aniqlang. Chulg‘amlarning aktiv qarshiligidagi quvvat isroflarini vattmetrlar ko‘rsatishlari bilan solishtiring. Bir chulg‘amdan ikkinchisiga magnit maydon orqali o‘tadigan energiya intensivligi (tezligi) ni, ya’ni quvvatlarni aniqlang. Berilgan: $U = 120\,\text{V},\ X_1 = 8\,\Omega,\ X_2 = 6\,\Omega,\ X_M = 6\,\Omega,\ r_1 = 6\,\Omega,\ r_2 = 8\,\Omega.$

Yechish. Kirxgof qonunlari bo‘yicha tenglamalar tuzamiz:

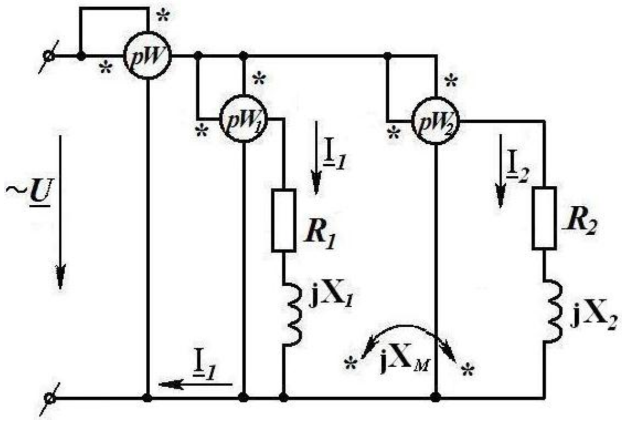
$$\underline{I}_1 r_1 + \underline{I}_1 jX_1 + \underline{I}_2 jX_м = \underline{U},$$
$$\underline{I}_1 jX_м + \underline{I}_2 r_2 + \underline{I}_2 jX_2 = \underline{U},$$
$$\underline{I} = \underline{I}_1 + \underline{I}_2,$$

yoki

$$\underline{I}_1 Z_1 + \underline{I}_2 Z_м = \underline{U},$$
$$\underline{I}_2 Z_м + \underline{I}_2 Z_2 = \underline{U},$$
$$\underline{I}_1 + \underline{I}_2 = \underline{I},$$

bunda

$$Z_1 = r_1 + jX_1 = 6 + j8 = 10e^{j53^\circ 08'}\ \Omega,$$
$$Z_2 = r_2 + jX_2 = 8 + j6 = 10e^{j36^\circ 52'}\ \Omega,$$
$$Z_м = jX_м = 6e^{j90^\circ}\ \Omega.$$



2.79- rasm.

Birinchi ikkita tenglamani birgalikda yechib quyidagini hosil qilamiz:

$$\underline{I}_1=\frac{\begin{vmatrix}\underline{U} & Z_M\\ \underline{U} & Z_2\end{vmatrix}}{\begin{vmatrix}Z_1 & Z_M\\ Z_M & Z_2\end{vmatrix}}=U\,\frac{Z_2-Z_M}{Z_1Z_2-Z_M^2}=120\cdot\frac{8+j6-j6}{10e^{j53^\circ 08'}\cdot 10e^{j36^\circ 52'}-(j6)^2}=$$

$$=9e^{-j70^\circ 12'}=(3{,}05-j8{,}5)\ \text{A},$$

$$\underline{I}_2=\frac{\begin{vmatrix}Z_1 & \underline{U}\\ Z_M & \underline{U}\end{vmatrix}}{\begin{vmatrix}Z_1 & Z_M\\ Z_M & Z_2\end{vmatrix}}=U\,\frac{Z_1-Z_M}{Z_1Z_2-Z_M^2}=120\cdot\frac{6+j8-j6}{36+j100}=7{,}15e^{-j51^\circ 47'}=$$

$$=(4{,}42-j5{,}62)\ \text{A}.$$

Zanjirning shoxobchalanmagan qismidagi tokning kompleksi

$$\underline{I}=\underline{I}_1+\underline{I}_2=3{,}05-j8{,}5+4{,}42-j5{,}62=7{,}47-j14{,}12=16e^{-j62^\circ 08'}\ \text{A}.$$

Eslatib o‘tamizki, magnit bog‘langan chulg‘amlar parallel ulanganda umumiy tok kompleksi quyidagi formuladan topiladi:

$$\underline{I}=\underline{U}\cdot\frac{Z_1+Z_2\pm 2Z_M}{Z_1\cdot Z_2-Z_M^2},$$

bunda suratdagi “–” ishora g‘altaklar mos ulanganda, “+” ishora esa qarama-qarshi ulanganda qo‘yiladi.

Vattmetrlarning ko‘rsatishlarini aniqlaymiz:

$$P_{W_1}=\text{Re}\left[\underline{U}\,\overset{*}{\underline{I}}_1\right]=\text{Re}\left[120(3{,}05+j8{,}5)\right]=120\cdot 3{,}05=366\ \text{W},$$

$$P_{W_2}=\text{Re}\left[\underline{U}\,\overset{*}{\underline{I}}_2\right]=\text{Re}\left[120(4{,}42+j5{,}62)\right]=120\cdot 4{,}42=530{,}4\ \text{W},$$

$$P_{W}=\text{Re}\left[\underline{U}\,\overset{*}{\underline{I}}\right]=\text{Re}\left[120(7{,}47+j14{,}12)\right]=120\cdot 7{,}47=896{,}4\ \text{W}.$$

Chulg‘amlarning aktiv qarshiliklaridagi quvvat isroflari

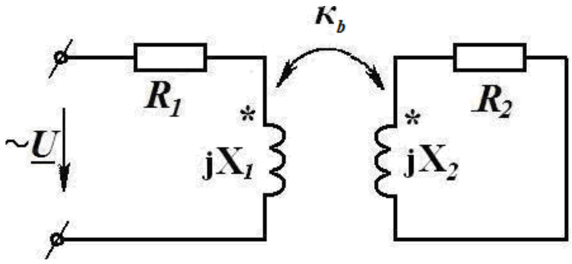
$$P_1 = I_1^2 r_1 = 9{,}0^2 \cdot 6 = 486\,\text{W},$$
$$P_2 = I_2^2 r_2 = 7{,}15^2 \cdot 8 = 410{,}4\,\text{W}.$$

Birinchi vattmetrning ko‘rsatishini birinchi chulg‘amning aktiv qarshiligidagi isroflar bilan solishtirib ko‘rsak, bu shoxobchaning quvvatni qabul qilishi $(P_1 = 486\,\text{W})$ energiya manbalaridan $(P_{W1} = 366\,W)$ qabul qilnadigan quvvatdan $120\,\text{W}$ ga ko‘p va P_2 dagi quvvat P_{W2} dan $120\ \text{W}$ ga kam.

Binobarin, birinchi shoxobcha bu energiyani ikkinchi chulg‘amdan magnit maydon orqali qabul qiladi. Ya’ni energiya ikkinchi chulg‘amdan birinchi chulg‘amga magnit orqali o‘tadi.

2.113- masala. 2.112- masala magnit bog‘langan chulg‘amlarning qarama-qarshi ulangan holati uchun yechilsin.

2.114- masala. O‘zaksiz transformatorning (2.80- rasm) ikkilamchi chulg‘amidagi tok kompleksi $\underline{I}_2 = 4\,\text{A}$. Zanjir elementlarining qarshiliklari $r_1 = 5\,\Omega,\ r_2 = 7\,\Omega,\quad X_1 = 20\,\Omega,\ X_2 = 24\,\Omega.$ Chulg‘amlarning magnit bog‘lanish koeffitsiyenti $K_b = 46\%.$ Transformatorning birinchi chulg‘amiga berilayotgan kuchlanish kompleksi aniqlansin.



2.80- rasm.

Yechish. Kirxgofning ikkinchi qonuni bo‘yicha tenglamalar tuzamiz:

$$\left.\begin{aligned} \underline{U} &= r_1\underline{I}_1 + jX_1\underline{I}_1 - jX_M\underline{I}_2, \\ 0 &= r_2\underline{I}_2 + jX_2\underline{I}_2 - jX_M\underline{I}_1 \end{aligned}\right\} \qquad (1)$$

yoki

$$\left.\begin{aligned} \underline{U} &= Z_1\underline{I}_1 - Z_M\underline{I}_2, \\ 0 &= -Z_M\underline{I}_1 + Z_2\underline{I}_2 \end{aligned}\right\} \qquad (1')$$

bunda

$$Z_1 = r_1 + jX_1 = 5 + j20 = 20{,}6e^{j76^\circ}\ \Omega,$$
$$Z_2 = r_2 + jX_2 = 7 + j24 = 25e^{j73^\circ 45'}\ \Omega,$$
$$Z_M = jX_M = jK_{cs}\sqrt{X_1X_2} = j0{,}46\sqrt{20\cdot 24} = j10 = 10e^{j90^\circ}\ \Omega.$$

(1′) sistemaning ikkinchi tenglamasidan $\underline{I}_1$ ni aniqlaymiz:

$$\underline{I}_1 = \underline{I}_2\frac{Z_2}{Z_M} = 4\cdot\frac{25e^{j73^\circ 45'}}{10e^{j90^\circ}} = 10e^{-j16^\circ 15'}\ \text{A}.$$

Bu tokning qiymatini (1′) sistemaning birinchi tenglamasiga qo‘yamiz:

$$\underline{U} = 10e^{-j16^\circ 15'}\cdot 20{,}6e^{j76^\circ} - 4\cdot 10e^{j90^\circ} = 206e^{j59^\circ 45'} - j40 =$$
$$= 104 + j178 - j40 = 104 + j138 = 173e^{j53^\circ}\ \text{V}.$$

2.115- masala. Zanjirga (2.80- rasm) berilayotgan kuchlanish necha bo‘lganda ikkinchi chulg‘amga magnit maydon orqali uzatilayotgan energiya tezligi 56 J/s ni tashkil etadi?

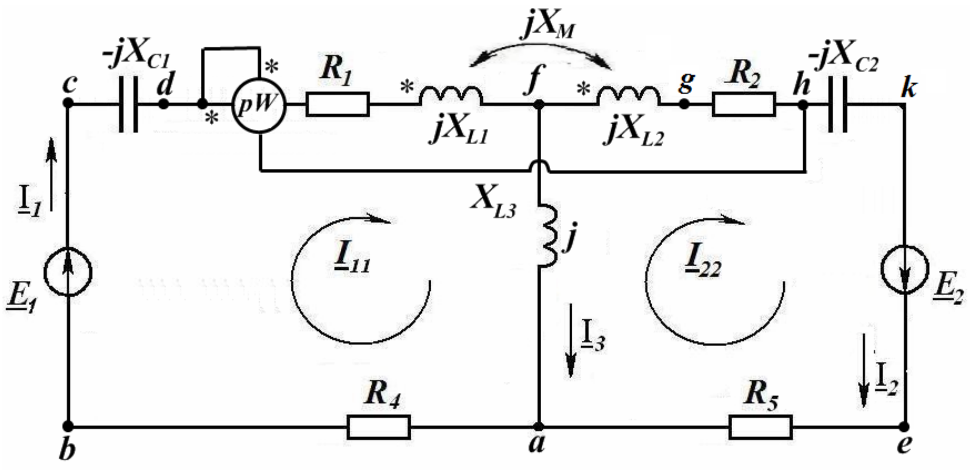
2.116-masala. Zanjir shoxobchalaridagi (2.81-rasm, *a*) toklarni kontur toklar usulida aniqlang. Natijalarni Kirxgofning ikkinchi qonuni bo‘yicha tekshiring. Vattmetr ko‘rsatishini aniqlang. Zanjir quyidagi parametrlarga ega: $r_1 = r_2 = r_4 = r_5 = 10\ \Omega$, $X_{L1} = X_{L3} = X_M = 20\,\Omega$, $X_{C1} = X_{L2} = 40\ \Omega$, $X_{C2} = 60\,\Omega$, $E_1 = E_2 = 200\,\text{V}$. Shu bilan birga ikkinchi generator EYK ining fazasi birinchi generator EYK fazasiga nisbatan chorak davrga ilgarilaydi.

Yechish. Bog‘lanmagan kontur toklar yo‘nalishini 2.81- rasmga ko‘ra belgilab, tenglamalarning kanonik sistemasini tuzamiz:

$$\left.\begin{aligned} Z_{11}\underline{I}_{11} + Z_{12}\underline{I}_{22} = E_{11};\\ Z_{21}\underline{I}_{11} + Z_{22}\underline{I}_{22} = E_{22};\end{aligned}\right\},$$

bunda koeffitsiyentlar quyidagilarga teng:

$$Z_{11} = r_1 + r_4 + j(X_{L1} + \frac{X_{L3}}{2} - X_{C1}) = 10 + 10 + j(20 + 10 - 40) = (20 - j10)\,\Omega,$$
$$Z_{22} = r_2 + r_5 + j(X_{L2} + \frac{X_{L3}}{2} - X_{C2}) = 10 + 10 + j(40 + 10 - 60) = (20 - j10)\,\Omega,$$
$$Z_{12} = Z_{21} = -j\frac{X_{L3}}{2} + jX_M = -j10 + j20 = j10\,\Omega.$$



2.81- rasm.

Shartga ko‘ra $E_{11} = \underline{E}_1 = 200\,\text{V}$, $E_{22} = j\underline{E}_2 = j200\,\text{V}$. Koeffitsiyentlarni kanonik tenglamalar sistemasiga qo‘yib kontur toklarni aniqlaymiz:

$$\underline{I}_{11} = \frac{\begin{vmatrix} 200 & j10 \\ j200 & 20-j10 \end{vmatrix}}{\begin{vmatrix} 20-j10 & j10 \\ j10 & 20-j10 \end{vmatrix}} = (10+j5)\ \text{A}, \quad \underline{I}_{22} = \frac{\begin{vmatrix} 20-j10 & 200 \\ j10 & j200 \end{vmatrix}}{\begin{vmatrix} 20-j10 & j10 \\ j10 & 20-j10 \end{vmatrix}} = j5\ \text{A}.$$

Shoxobchalardagi haqiqiy toklar komlekslari quyidagicha bo‘ladi:

$$\underline{I}_1 = \underline{I}_{11} = 10 + j5 = 11{,}2e^{j26^\circ 30'}\ \text{A},$$
$$\underline{I}_2 = \underline{I}_{22} = j5 = 5e^{j90^\circ}\ \text{A},$$
$$\underline{I}_3 = \underline{I}_{11} - \underline{I}_{22} = 10\ \text{A}.$$

Yechimlarni Kirxgofning ikkinchi qonuni bo‘yicha tekshirib chiqamiz. Chap kontur tenglamasi

$\underline{I}_1[r_1 + r_4 + j(X_{L1} - X_{C1})] + \underline{I}_2 jX_M + \underline{I}_3 j\dfrac{X_{L3}}{2} = E$ ga son qiymatlarini qo‘ygandan so‘ng quyidagi ayniyatga aylanadi:

$$(10+j5)(20-j20) + j5\cdot j20 + 10\cdot j10 = 200$$

yoki $200 + j100 - j200 + 100 - 100 + j100 = 200; \quad 200 = 200.$

Quvvatlar balansi tenglamasini tuzamiz:

$$\sum \underline{E}\overset{*}{\underline{I}} = \sum I^2 Z + \sum \tilde{S}_M,$$

bunda $\sum \tilde{S}_M$ – magnit bog‘langan uchastkaning reaktiv quvvati.

$$\sum \tilde{S}_M = \pm j2Q_M = \pm 2Z_M I_1 I_2 \cos(\psi_{i1} - \psi_{i2}).$$

“+” ishora magnit bog‘langan uchastkalarning mos ulanishiga, “–” qarama-qarshi ulanishga to‘g‘ri keladi.

Quvvatlar balansi tenglamasiga son qiymatlarini qo‘yib quyidagi ayniyatni olamiz:

$$200(10-j5)+j200(-j5)=125(20-j20)+25(20-j20)+100\cdot j10+$$
$$+2\cdot j20\cdot 11{,}2\cdot 5\cdot\cos\left(26^\circ 30'-90^\circ\right)$$

yoki $3000-j1000=3000-j1000.$

Topologik diagramma qurish uchun avval quyidagilarni ($\varphi_a=0$ da) aniqlaymiz:

$$\underline{\varphi}_b=\underline{\varphi}_a-\underline{I}_1 r_4=-(10+j5)\cdot 10=-100-j50=-112e^{j26^\circ 30'}=112e^{-j153^\circ 30'}\ \text{V},$$
$$\underline{\varphi}_c=\underline{\varphi}_b+\underline{E}_1=-100-j50+200=100-j50=112e^{-j26^\circ 30'}\ \text{V},$$
$$\underline{\varphi}_d=\underline{\varphi}_c-\underline{I}_1\left(-jX_{C1}\right)=100-j50-(10+j5)(-j40)=100-j50+j400-200=$$
$$=-(100-j350)=-363e^{-j74}=363e^{j106^\circ}\ \text{V},$$
$$\underline{\varphi}_e=\underline{\varphi}_d-\underline{I}_1 r_1=-100+j350-(10+j5)\cdot 10=-100+j350-100-j50=$$
$$=-(200-j300)=-361e^{-j56^\circ 20'}=361e^{j123^\circ 40'}\ \text{V},$$
$$\underline{\varphi}_f=\underline{\varphi}_a+\underline{I}_3 j\frac{X_{L3}}{2}=10\cdot j10=j100\ \text{V}$$

yoki

$$\underline{\varphi}_f=\underline{\varphi}_e-\underline{I}_1 jX_{L1}-\underline{I}_2 jX_M=-200+j300-(10+j5)\cdot j20-j5\cdot j20=$$
$$=-200+j300-j200+100+100=100e^{j90^\circ}\ \text{V},$$
$$\underline{\varphi}_g=\underline{\varphi}_f-\underline{I}_2 X_{L2}-\underline{I}_1 jX_M=j100-j5\cdot j40-(10+j5)\cdot j20=$$
$$=j100+200-j200+100=300-j100=316e^{-j18^\circ 30'}\ \text{V},$$
$$\underline{\varphi}_h=\underline{\varphi}_g-\underline{I}_2 r_2=300-j100-j5\cdot 10=300-j150=336e^{-j26^\circ 30'}\ \text{V},$$
$$\underline{\varphi}_k=\underline{\varphi}_h-\underline{I}_2\left(-jX_{C2}\right)=300-j150-j5(-j60)=-j150=150e^{-j90^\circ}\ \text{V},$$
$$\underline{\varphi}_e=\underline{\varphi}_k+\underline{E}_2=-j150+j200=j50=50e^{j90^\circ}\ \text{V},$$
$$\underline{\varphi}_a=\underline{\varphi}_e-\underline{I}_2 r_2=j50-j5\cdot 10=0.$$

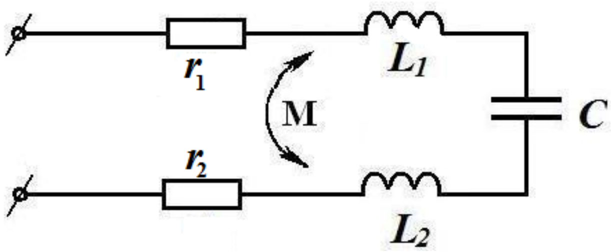
Vattmetr ko‘rsatishini aniqlaymiz:

$$P_W=\text{Re}\left[\underline{U}_{dh}\ \overset{*}{\underline{I}}_1\right]=\text{Re}\left[640e^{j128^\circ 40'}\left(-11{,}2e^{-j26^\circ 30'}\right)\right]=6980\ \text{W}.$$

2.117- masala. Zanjirga ulangan ampermetr (2.81- rasm, *a*) voltmetr bilan almashtirilgan. Uning ko‘rsatishini aniqlang.

2.5.2. Induktiv bog‘langan zanjirlarda rezonanslar

2.118- masala. Zanjirning (2.82- rasm) parametrlari $U = 100$ V, $f = 500\,\text{Hz}$, $r_1 = 12\,\Omega$, $r_2 = 8\,\Omega$, $L_1 = L_2 = 2M = 0{,}02\,\text{H}$. G‘altaklar qarama-qarshi ulangan. Kondensator sig‘imi necha bo‘lganda zanjirda rezonans rejimi sodir bo‘ladi va kondensatordagi kuchlanish pasayishi qancha bo‘ladi?



2.82- rasm.

Yechish. Kuchlanish rezonansida zanjirning reaktiv qarshiligi nolga teng bo‘ladi:

$$jX_1 + jX_2 - j2X_M - jX_C = 0$$

yoki $\omega(L_1 + L_2 - 2M) - \dfrac{1}{\omega C} = 0,$

bundan $C = \dfrac{1}{\omega^2(L_1 + L_2 - 2M)} = \dfrac{1}{(2\pi \cdot 500)^2 \cdot (0{,}02 + 0{,}02 - 0{,}02)} = 5{,}06\,\text{mkF}.$

Rezonans holatidagi tokning ta’sir etuvchi qiymati:

$$I = \frac{U}{r_1 + r_2} = \frac{100}{20} = 5\,\text{A}.$$

Kondensator qismalaridagi kuchlanishning ta’sir etuvchi qiymati:

$$U_C = IX_C = I\frac{1}{\omega C} = 5 \cdot \frac{1}{2\pi \cdot 500 \cdot 5{,}06 \cdot 10^{-6}} = 314\,\text{V}.$$

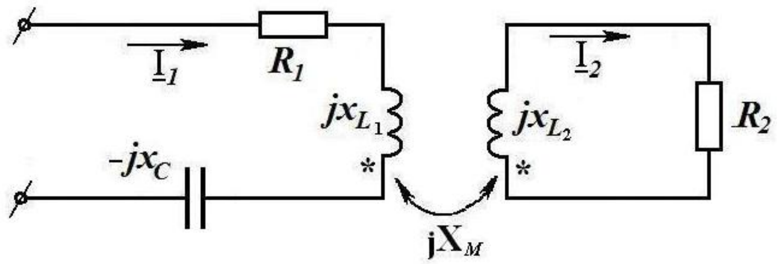
2.119- masala. Avvalgi masala zanjirida kondensator sig‘imi $C = 5{,}06\,\text{mkF}$ va g‘altaklar mos ulansa, qanday chastotada rezonans sodir etiladi?

2.120- masala. Zanjirdagi (2.84- rasm) kondensator X_C qarshiligi qanday bo‘lganda kirish tokining fazasi kirish kuchlanish fazasiga mos tushadi? Zanjir parametrlari:

$X_{L_1} = 10\,\Omega,\ X_{L_2} = 20\,\Omega,\ X_M = 10\,\Omega,\ r_1 = 2\,\Omega,\ r_2 = 10\,\Omega,\ \underline{U}_1 = 100\,\text{V}.$

Yechish. Kirxgofning ikkinchi qonuni bo‘yicha tenglamalar tuzamiz:

$$\left.\begin{aligned} &\underline{I}_1(r_1 + jX_{L1} - jX_C) - \underline{I}_2 jX_M = \underline{U}_1, \\ &-\underline{I}_1 jX_M + \underline{I}_2(r_2 + jX_{L2}) = 0. \end{aligned}\right\}$$



2.84- rasm.

Ikkinchi tenglamadan $\underline{I}_2$ tokni ifodalaymiz:

$$\underline{I}_2 = \underline{I}_1 \frac{jX_M}{r_2 + jX_{L_2}},$$

va uni birinchi tenglamaga qo‘yib, quyidagini hosil qilamiz:

$$\underline{I}_1 Z_K = \underline{U},$$

bunda

$$Z_K = r_1 + jX_{L_1} - jX_C + \frac{X_M}{r_2 + jX_{L_2}} = r_1 + \frac{r_2 \cdot X_M^2}{r_2^2 + X_M^2} + j\left(X_{L_1} - X_C - \frac{X_{L_2} \cdot X_M^2}{r_2^2 + X_{L_2}^2}\right).$$

Shart bo‘yicha $J_M Z_K = 0$, ya’ni rezonansda qarshilikning reaktiv tashkil etuvchisi nolga teng.

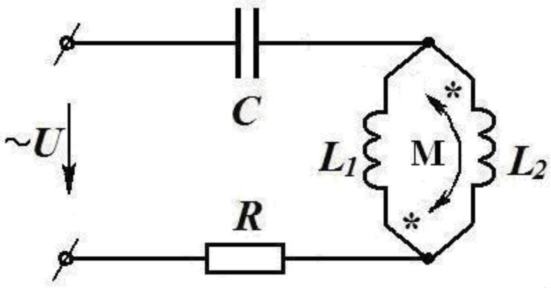
$$X_C = X_{L_1} - \frac{X_{L_2} \cdot X_M^2}{r_2^2 + X_{L_2}^2} = 10 - \frac{20 \cdot 10^2}{10^2 + 20^2} = 6\,\Omega.$$

Tok komplekslarini aniqlaymiz:

$$\underline{I}_1 = \frac{\underline{U}}{Z_K} = \frac{100}{2+2} = 25\,\text{A},$$

$$\underline{I}_2 = \underline{I}_1 \frac{jX_M}{r_2 + jX_{L_2}} = 25 \cdot \frac{j10}{10 + j20} = 10 + j5 = 11{,}2e^{j26^\circ 30'}\ \text{A}.$$

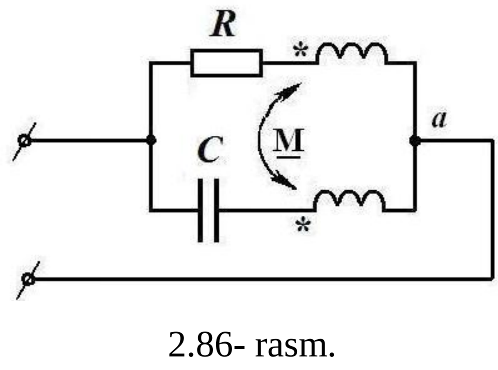
2.121- masala. Zanjirni (2.85- rasm) kuchlanish rezonasiga rostlash uchun kondensator sig‘imi necha bo‘lishi kerak? Zanjir uchun berilganlar:
$L_1 = 15\,\text{mH},\ L_2 = 45\,\text{mH},\ M = 15\,\text{mH},\ \omega = 1000\,\text{Hz}.$



2.85- rasm.

2.122- masala. 2.121- masala zanjirdagi g‘altaklar mos ulangan holati uchun yechilsin.

2.123- masala. Zanjirni (2.86- rasm) toklar rezonansi shartiga $\omega = 1000\,\text{Hz}$ chastotada rostlash uchun kondensator sig‘imi necha bo‘lishi kerak? G‘altaklar *a* tugunga bir xil nomlangan qismalari bilan ulangan. Bog‘lanish koeffitsiyenti 50%. Berilgan: $L_1 = L_2 = 20\,\text{mH},\ R = 10\ \Omega.$



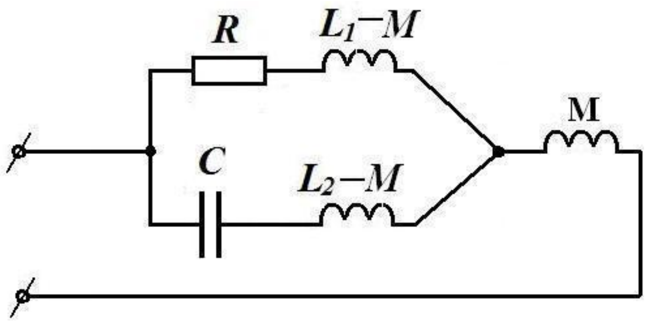
2.86- rasm.

Yechish. Ajratishni bajarib, 2.87- rasmdagi zanjirni hosil qilamiz. Zanjir uchun toklar rezonansi shartidan quyidagilarni topamiz:

$$\frac{\omega(L_1 - M)}{[\omega(L_1 - M)]^2 + r^2} = -\frac{1}{\omega(L_2 - M) - \dfrac{1}{\omega C}},$$

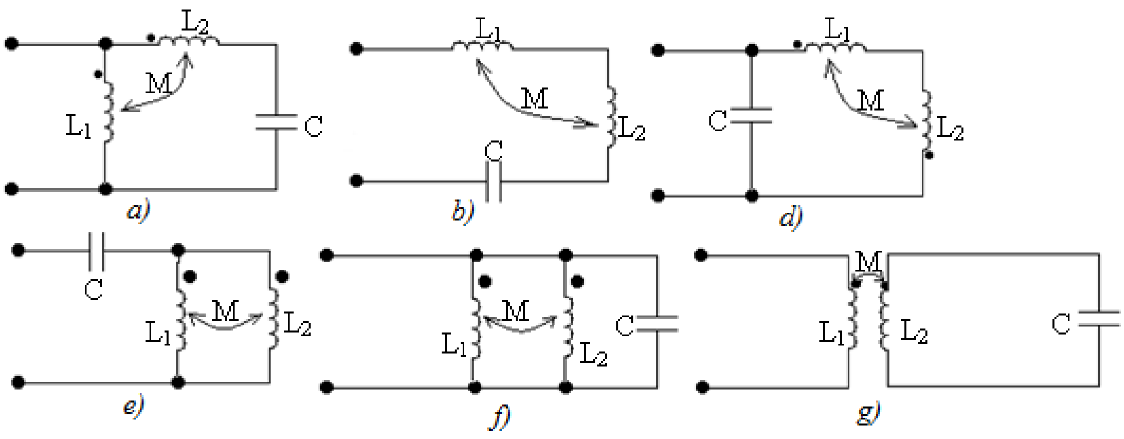
bundan

$$X_C = \frac{1}{\omega C} = 30\,\Omega \text{ yoki } C = 33{,}3\,\text{mkF.}$$



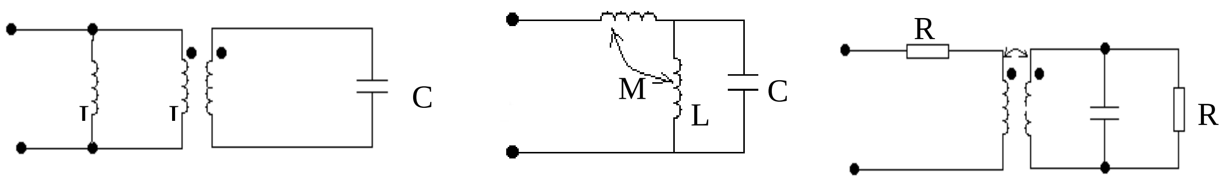
2.87- rasm.

2.124- masala. Quyida keltirilgan zanjirlar uchun (2.88- rasm) rezonans chastotalari ifodalari topilsin (M>0).



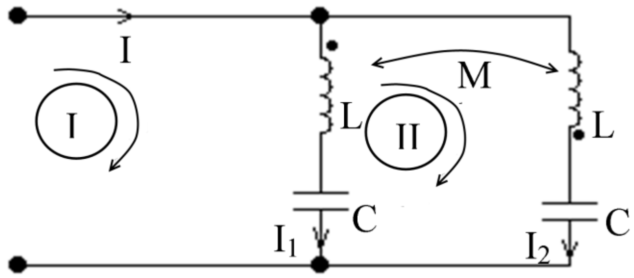
2.88- rasm.

2.125- masala. Quyida keltirilgan zanjirlar uchun (2.89- rasm) $L = 10^{-3}$ H, $C = 10^{-5}$ F, $M = 5\cdot10^{-4}$ H , $R = 1\Omega$ parametrlarda rezonans chastota aniqlansin.



2.89- rasm.

2.126- masala. Keltirilgan zanjir uchun (2.90- rasm) L = 10^{-3} H , C = 10^{-5} F, M= $5\cdot 10^{-4}$ H; R = 1 Ω. Rezonans chastotasi aniqlansin.



2.90- rasm.

Yechish. Toklarning shartli musbat yo‘nalishi va kontur bo‘ylab yurishini tanlab olamiz (2.90- rasm). Ikkinchi kontur II uchun Kirxgofning ikkinchi qonuniga ko‘ra induktiv bog‘langanlik yo‘nalisnini e’tiborga olib quydagi tenglamani yozamiz:

$$-I_1\left(j\omega L+\frac{1}{j\omega C}\right)+\underline{I}_2 j\omega M+\underline{I}_2\left(j\omega L+\frac{1}{j\omega C}\right)-I_1 j\omega M=0,$$

bundan $\underline{I}_1$ = $\underline{I}_2$ ni hosil qilamiz.

Birinchi kontur I uchun Kirxgofning ikkinchi qonuniga ko‘ra quydagi tenglamani yozamiz:

$$-\underline{U}+\underline{I}_1\left(j\omega L+\frac{1}{j\omega C}\right)-\underline{I}_2 j\omega M=0,$$

bundan:

$$\underline{I}=\frac{\underline{U}}{j\omega L+\dfrac{1}{j\omega C}-j\omega M},\ \text{chunki}\ \underline{I}_1=\underline{I}_2;$$

Oxirgi ifodadan rezonans chostatasi shartini ifodalash mumkin:

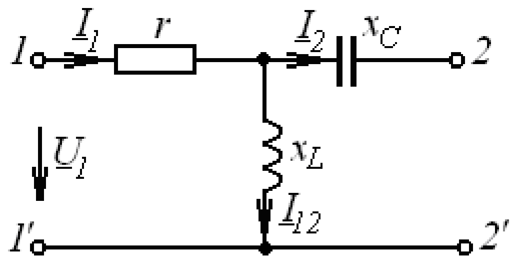
b_{kir} = 0; ya’ni $\omega_1=0$ va $\omega_2=\dfrac{1}{\sqrt{C(L-M)}}$ ga masala shartida berilgan qiymatlarni qo‘yamiz.

$$\omega_2=1{,}4\cdot 10^4\ \text{Hz}.$$

2.6. Chiziqli to‘rtqutbliklar

2.6.1. Istalgan yuklama uchun to‘rtqutblikning kirish qarshiligi

2.127- masala. 2.91- rasmda keltirilgan to‘rtqutblikning *T*-simon almashlash sxemasi uchun $\underline{A}$, $\underline{B}$, $\underline{C}$, $\underline{D}$ koeffitsiyentlarni toping va $\underline{A}\underline{D}-\underline{B}\underline{C}=1$ munosabatni tekshirib ko‘ring. Quyidagilar berilgan: r = 100 Ω, x_L = 200 Ω, x_C = 100 Ω.



2.91- rasm.

Yechish. To‘rtqutblik koeffitsiyentlarini topish uchun uning salt ish va qisqa tutashish rejimlari uchun tenglamalardan foydalanamiz:

$$\underline{A}=\frac{\underline{U}_{10}}{U_{2n}}=\frac{\underline{U}_1}{(\underline{U}_1\cdot jx_L)/(r+jx_L)}=\frac{r+jx_L}{jx_L}=\frac{100+j200}{j200}=1-j0{,}5\,.$$

$\underline{B}$ koeffitsiyentni aniqlash uchun avval ikkinchi konturga tegishli qisqa tutashish tokini topamiz:

$$\underline{I}_{2q}=\frac{\underline{U}_{1q}}{r+\dfrac{x_L x_C}{jx_L-jx_C}}\cdot\frac{jx_L}{jx_L-jx_C}=\frac{\underline{U}_{1q}\,jx_L}{x_L x_C+jr(x_L-x_C)}\,.$$

$\underline{I}_{2q}$ tokning topilgan ifodasidan foydalanib $\underline{B}$ koeffitsiyentni aniqlaymiz:

$$\underline{B} = \frac{\underline{U}_{1q}}{\underline{I}_{2q}} = \frac{x_L x_C + jr(x_L - x_C)}{jx_L} = 50 - j100\ \Omega,$$

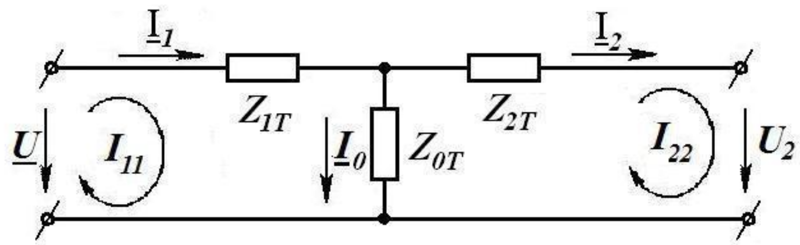
$$\underline{C} = \frac{\underline{I}_{10}}{\underline{U}_{20}} = \frac{\underline{I}_{10}}{jx_L \underline{I}_{10}} = -j0{,}005\ S,$$

$$\underline{D} = \frac{\underline{I}_{1q}}{\underline{I}_{2q}} = \frac{\underline{I}_{1q}}{\underline{I}_{1q} \dfrac{jx_L}{jx_L - jx_C}} = \frac{x_L - x_C}{x_L} = 0{,}5.$$

Endi $\underline{A}\underline{D} - \underline{B}\underline{C} = 1$ munosabatni tekshirib ko‘ramiz:

$$\underline{A}\underline{D} - \underline{B}\underline{C} = (1 - j0{,}5) \cdot 0{,}5 - (50 - j100) \cdot (-j0{,}005) = 1.$$

2.128- masala. To‘rtqutblikning T-simon almashlash sxemasi (2.92- rasm) orqali $\|A\|$ shaklli tenglamalar koeffitsiyentlarini ifodalang.



2.92- rasm.

Yechish. Kirxgof qonunlaridan foydalanib to‘rtqutblikning T- simon sxemasi uchun quyidagi tenglamalarni yozish mumkin:

$$\left.\begin{aligned} \underline{U}_1 &= Z_{1T}\underline{I}_1 + Z_{2T}\underline{I}_2 + \underline{U}_2, \\ \underline{I}_1 &= \underline{I}_0 + \underline{I}_2. \end{aligned}\right\} \qquad (1)$$

(1′) tenglamalar sistemasini to‘rtqutblikning $\|A\|$ shaklli tenglamalari

$$\left.\begin{aligned} \underline{U}_1 &= A_{11}\underline{U}_2 + A_{12}\underline{I}_2 \\ \underline{I}_1 &= A_{21}\underline{U}_2 + A_{22}\underline{I}_2 \end{aligned}\right\}.$$

(1′)

bilan solishtirib ko‘ramizki (1) sistemaning ikkinchi tenglamasida $\underline{I}_0$ ni $\underline{I}_2$ yoki $\underline{U}_2$ ni almashlash sxemaning parametrlari orqali ifodalash lozim, buning uchun shu sistemaning birinchi tenglamasida $\underline{I}_1$ tokdan qutilish kerak.

$\underline{I}_0 = \dfrac{\underline{U}_2 + \underline{Z}_{2T}\underline{I}_2}{\underline{Z}_{0T}}$ bo‘lgani uchun,

$$\underline{I}_1 = \frac{\underline{U}_2}{\underline{Z}_{0T}} + \frac{\underline{Z}_{2T}}{\underline{Z}_{0T}}\underline{I}_2 + \underline{I}_2 = \frac{1}{\underline{Z}_{0T}}\underline{U}_2 + \left(1 + \frac{\underline{Z}_{2T}}{\underline{Z}_{0T}}\right)\underline{I}_2 .$$

(2)

(2) ni (1′) - sistemaning ikkinchi tenglamasi bilan solishtiramiz:

$$A_{21} = \frac{1}{\underline{Z}_{0T}}, \quad A_{22} = 1 + \frac{\underline{Z}_{2T}}{\underline{Z}_{0T}}.$$

(2) tenglamani (1) ning birinchi tenglamasiga qo‘ygandan so‘ng

$$\underline{U}_1 = \underline{Z}_{1T}\left[\frac{1}{\underline{Z}_{0T}}\underline{U}_2 + \left(1 + \frac{\underline{Z}_{2T}}{\underline{Z}_{0T}}\right)\underline{I}_2\right] + \underline{Z}_{2T}\underline{I}_2 + \underline{U}_2 =$$

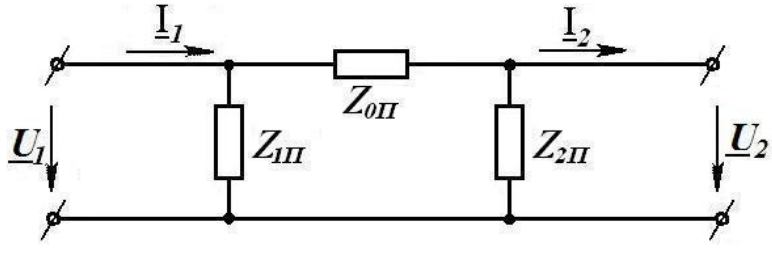
$$= \left(1 + \frac{\underline{Z}_{1T}}{\underline{Z}_{0T}}\right)\underline{U}_2 + \left(\underline{Z}_{1T} + \underline{Z}_{2T} + \frac{\underline{Z}_{1T}\underline{Z}_{2T}}{\underline{Z}_{0T}}\right)\underline{I}_2 .$$

(3)

(3) ni (1′) sistemaning birinchi tenglamasi bilan solishtirib, quyidagini hosil qilamiz:

$$A_{11} = 1 + \frac{\underline{Z}_{1T}}{\underline{Z}_{0T}}, \quad A_{12} = \underline{Z}_{1T} + \underline{Z}_{2T} + \frac{\underline{Z}_{1T}\underline{Z}_{2T}}{\underline{Z}_{0T}}.$$

2.129- masala. To‘rtqutblikning П-simon sxema parametrlari orqali uning $\|A\|$ shaklli tenglamasidagi koeffitsiyentlarni ifodalang (2.93- rasm).



2.93- rasm.

Yechish. Qisqa tutashish va salt ishlash usulidan foydalanamiz. To‘rtqutblikning $\|A\|$ shaklli tengalamalarini yozamiz:

$$\left.\begin{aligned} \underline{U}_1 &= A_{11}\underline{U}_2 + A_{12}\underline{I}_2\ , \quad (1)\\ \underline{I}_1 &= A_{21}\underline{U}_2 + A_{22}\underline{I}_2\ . \quad (2)\end{aligned}\right\}$$

Salt ishlash rejimida

$$A_{11} = \frac{\underline{U}_1}{\underline{U}_2}\bigg|_{|I_2=0|} = \frac{\underline{U}_{1\text{s.i.}}}{\underline{U}_{2\text{s.i.}}} \qquad (3)$$

$$A_{21} = \frac{\underline{I}_1}{\underline{U}_2}\bigg|_{|I_2=0|} = \frac{\underline{I}_{1\text{s.i.}}}{\underline{U}_{2\text{s.i.}}} \qquad (4)$$

Qisqa tutashish rejimida $\|A\|$ shaklli tenglamalaridan

$$A_{12} = \frac{\underline{U}_1}{\underline{I}_2}\bigg|_{|U_2=0|} = \frac{\underline{U}_{1\text{q.t.}}}{\underline{I}_{2\text{q.t.}}} \qquad (5)$$

$$A_{22} = \frac{\underline{I}_1}{\underline{I}_2}\bigg|_{|U_2=0|} = \frac{\underline{I}_{1\text{s.i.}}}{\underline{I}_{2\text{s.i.}}} \qquad (6)$$

П-simon to‘rtqutblikning salt ishlash rejimida

$$\underline{U}_{2\text{s.i.}} = \frac{\underline{U}_{1\text{s.i.}}}{Z_{O\Pi} + Z_{2\Pi}} \cdot Z_{2\Pi},$$

$$\underline{I}_{1s.i.} = \frac{\underline{U}_{1\text{s.i.}}}{Z_{1\Pi}} + \frac{\underline{U}_{1\text{s.i.}}}{Z_{O\Pi} + Z_{2\Pi}}.$$

Oxrigi ifodalarni va (3) ni e’tiborga olib, quyidagini hosil qilamiz:

$$A_{11} = \frac{Z_{O\Pi} + Z_{2\Pi}}{Z_{2\Pi}} = 1 + \frac{Z_{O\Pi}}{Z_{2\Pi}};$$

(4) ni e’tiborga olib esa:

$$A_{21} = \frac{Z_{1\Pi} + Z_{2\Pi} + Z_{O\Pi}}{Z_{1\Pi}Z_{2\Pi}} = \frac{1}{Z_{1\Pi}} + \frac{1}{Z_{2\Pi}} + \frac{Z_{O\Pi}}{Z_{1\Pi}Z_{2\Pi}}.$$

П-simon sxemadagi qisqa tutashish rejimidan:

$$\underline{I}_{1\text{q.t.}} = \frac{\underline{U}_{1\text{q.t.}}}{Z_{1\Pi}} + \frac{\underline{U}_{1\text{q.t.}}}{Z_{O\Pi}} = \frac{\underline{U}_{1\text{q.t.}}}{\dfrac{Z_{1\Pi}Z_{O\Pi}}{Z_{O\Pi} + Z_{1\Pi}}}, \quad \underline{I}_{2\text{q.t.}} = \frac{\underline{U}_{1\text{q.t.}}}{Z_{O\Pi}}.$$

(5) ni e’tiborga olib quyidagini hosil qilamiz:

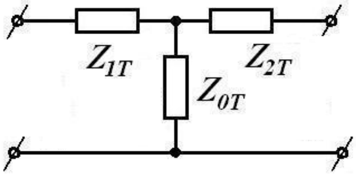
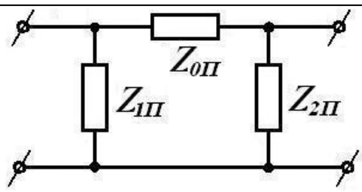
$$A_{12}=\frac{\underline{U}_{1\text{q.t.}}}{\underline{I}_{2\text{q.t.}}}=\frac{\underline{U}_{1\text{q.t.}}}{\dfrac{\underline{U}_{1\text{q.t.}}}{Z_{O\Pi}}}=Z_{O\Pi};$$

(6) ni e’tiborga olib esa:

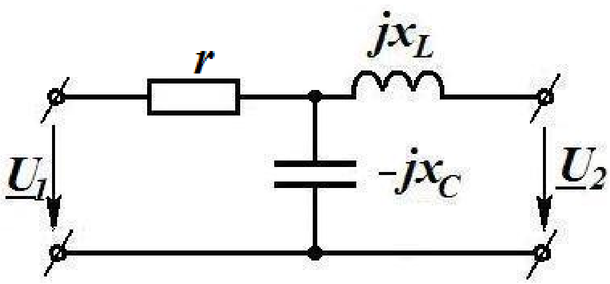
$$A_{22}=\frac{\underline{I}_{1\text{q.t.}}}{\underline{I}_{2\text{q.t.}}}=\frac{\dfrac{\underline{U}_{1k.t.}\left(Z_{1\Pi}+Z_{O\Pi}\right)}{Z_{1\Pi}Z_{O\Pi}}}{\dfrac{\underline{U}_{1\text{q.t.}}}{Z_{O\Pi}}}=1+\frac{Z_{O\Pi}}{Z_{1\Pi}}.$$

2.128- va 2.129- masalalar natijalari 2.4- jadvalda keltirilgan.

2.4-*jadval*

Koeffitsiyent / To‘rtqutblik	A_{11}	A_{12}	A_{21}	A_{22}
	$1+\frac{\underline{Z}_{1T}}{\underline{Z}_{0T}}$	$\underline{Z}_{1T}+\underline{Z}_{2T}+\frac{\underline{Z}_{1T}\underline{Z}_{2T}}{\underline{Z}_{0T}}$	$\frac{1}{\underline{Z}_{0T}}$	$1+\frac{\underline{Z}_{2T}}{\underline{Z}_{0T}}$
	$1+\frac{Z_{O\Pi}}{Z_{2\Pi}}$	$Z_{O\Pi}$	$\frac{Z_{1\Pi}+Z_{2\Pi}+Z_{O\Pi}}{Z_{1\Pi}Z_{2\Pi}}$	$1+\frac{Z_{O\Pi}}{Z_{1\Pi}}$

2.130- masala. T-simon to‘rtqutblik (2.94- rasm) zanjir parametrlari $r=X_C=1\Omega,\ X_L=2\,\Omega$ bo‘lsa, $A_{11},\ A_{12},\ A_{21},\ A_{22}$ koeffitsiyentlar aniqlansin. $A_{11}A_{22}-A_{12}A_{21}=1$ munosabat o‘rinliligi ko‘rsatilsin. Agar $\underline{U}_1=10$ V bo‘lsa, ikkilamchi qismalardagi kuchlanish aniqlansin.



2.94- rasm.

Yechish. 2.4- jadvaldagi formulalardan foydalanamiz:

$$A_{11} = 1 + \frac{\underline{Z}_{1T}}{\underline{Z}_{0T}} = 1 + \frac{R}{-jX_C} 1 + \frac{1}{-j} = 1 + j = \sqrt{2}e^{j45^\circ}\ \Omega,$$

$$A_{12} = \underline{Z}_{1T} + \underline{Z}_{2T} + \frac{\underline{Z}_{1T}\underline{Z}_{2T}}{\underline{Z}_{0T}} = 1 + j2 + \frac{j2}{j2-1} = j2 - 1 = \sqrt{5}e^{j116^\circ 30'}\ \Omega,$$

$$A_{21} = \frac{1}{\underline{Z}_{0T}} = \frac{1}{-j} = j = e^{j90^\circ}\ \frac{1}{\Omega},$$

$$A_{22} = 1 + \frac{\underline{Z}_{2T}}{\underline{Z}_{0T}} = 1 + \frac{j2}{-j} = 1 - 2 = -1.$$

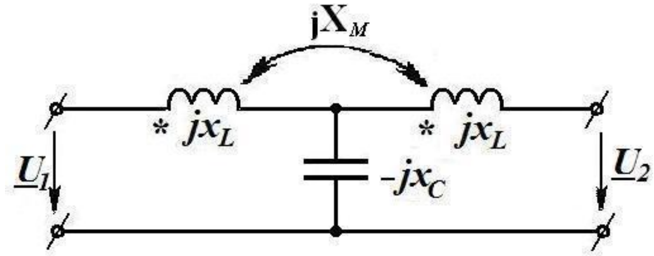
Tekshirish:

$$A_{11}A_{22} - A_{12}A_{21} = (1+j)(-1) - (j2-1)j = 1.$$

Salt ishlash rejimidagi $\underline{U}_2$ ni aniqlaymiz:

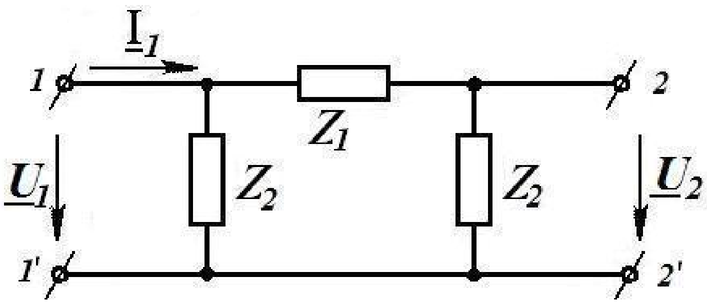
$$\underline{U}_{2|I_2=0|} = \underline{U}_{2\text{s.ish}} = \frac{\underline{U}_1}{A_{11}} = \frac{10}{\sqrt{2}e^{j45^\circ}} = 5\sqrt{2}e^{-j45^\circ}\ \text{V}.$$

2.131-masala. Agar T-simon to‘rtqutblikda (2.95-rasm) $X_L = 10\,\Omega$, $X_M = 5\,\Omega$, $X_C = 25\ \Omega$ bo‘lsa, $A_{11}, A_{12}, A_{21}, A_{22}$ koeffitsiyentlar aniqlansin, $A_{11}A_{22} - A_{12}A_{21} = 1$ munosabat o‘rinliligini tekshiring. Agar $\underline{U}_1 = 40\,\text{V}$ bo‘lsa, ikkilamchi qismalardagi kuchlanish topilsin.



2.95- rasm.

2.132-masala. П-simon simmetrik to‘rtqutblikning (2.96- rasm) parametrlari $Z_1 = j10\,\Omega$, $Z_2 = -j5\,\Omega$. Umumlashgan koeffitsiyentlar aniqlansin. Agar $2-2'$ ochiq qismalaridagi kuchlanish $\underline{U}_{2\text{s.ish}} = 10\,\text{V}$ bo‘lsa, $\underline{U}_1$ va $\underline{I}_1$ aniqlansin.



2.96- rasm.

2.133-masala. 2.132-masala kuchlanishning ta’sir etuvchi qiymati o‘zgarmas qoldirib, chastotasi ikki baravar kamayganda yechilsin. Sxema parametrlari (L va C) o‘zgarmagan holda qolgan.

2.134- masala. To‘rtqutblikning koeffitsiyentlar matritsasi ma’lum

$\|A\| = \begin{Vmatrix} 1+j & j2-1 \\ j & -1 \end{Vmatrix}$. Shu to‘rtqutblikning birlamchi va ikkilamchi qismalari

tomonidagi salt ishlash va qisqa tutashish parametrlari aniqlansin. $\dfrac{Z_{1\text{s.i.}}}{Z_{1\text{q.t.}}} = \dfrac{Z_{2\text{s.i.}}}{Z_{\text{q.t.}}}$

munosabat to‘g‘riligi ko‘rib chiqilsin.

Yechish. 2.4- jadvalda keltirilgan formulalardan quyidagilarni topamiz:

$$Z_{1\text{s.is}} = \frac{A_{11}}{A_{21}} = \frac{1+j}{j} = 1-j = \sqrt{2}e^{-45^\circ}\ \Omega;$$

$$Z_{1\text{q.t.}} = \frac{A_{12}}{A_{22}} = \frac{j2-1}{-1} = 1-j2 = \sqrt{5}e^{-j63^\circ30'}\ \Omega;$$

$$Z_{2\text{s.ish.}} = \frac{A_{22}}{A_{21}} = \frac{-1}{j} = j = e^{j90^\circ}\ \Omega;$$

$$Z_{2\text{q.t.}} = \frac{A_{12}}{A_{11}} = \frac{j2-1}{1+j} = 0{,}5 + j1{,}5 = \sqrt{2{,}5}e^{j71^\circ35'}\ \Omega.$$

Nisbatlarni hisoblaymiz:

$$\frac{Z_{1\text{s.ish}}}{Z_{1\text{q.t.}}} = \frac{(1-j)}{(1-j2)}\frac{(1+j2)}{(1+j2)} = \frac{1+j2-j+2}{5} = \frac{3+j}{5},$$

$$\frac{Z_{2\text{s.ish.}}}{Z_{2\text{q.t.}}} = \frac{j}{0{,}5+j1{,}5} = \frac{j(0{,}5-j1{,}5)}{2{,}5} = \frac{3+j}{5}.$$

Binobarin, $\dfrac{Z_{1\text{s.i.}}}{Z_{1\text{q.t.}}}=\dfrac{Z_{2\text{s.i.}}}{Z_{\text{q.t.}}}$ munosabat bajarildi.

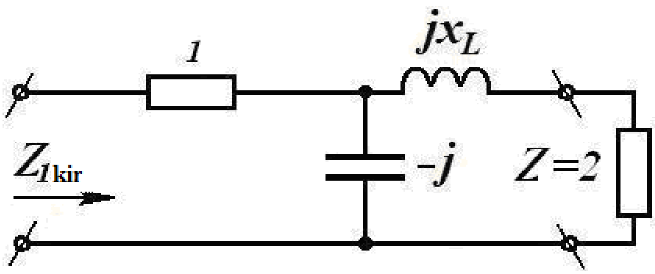
2.135- masala. 2.130- masalada (2.94- rasm) keltirilgan to‘rtqutblik uchun chiqish qismalariga $Z_{\text{yuk}}=2\,\Omega$ yuklama qarshilik ulanganda birinchi qismalarga nisbatan kirish qarshiligi aniqlansin. Kirish qismalarga $Z_0=1\Omega$ qarshilik ulanganda ikkinchi qismalariga nisbatan chiqish qarshilik aniqlansin.

Yechish. 2.4- jadvalda keltirilgan formula bo‘yicha, 2.130- masalada hisoblan- gan koeffitsiyentlardan foydalanib, quyidagini topamiz:

$$Z_{1\text{kir}}=\frac{A_{11}Z_{\text{yuk}}+A_{12}}{A_{21}Z_{\text{yuk}}+A_{22}}=\frac{(1+j)2+j2-1}{j2-1}=1{,}4-j1{,}2=1{,}85e^{-j40^\circ 35'}\ \Omega\,.$$

Zanjirning kirish qarshiligi odatdagi usulda topilganda ham xuddi shunday natija olinadi (2.97- rasm):

$$Z_{1\text{kir}}=1+\frac{(2+j2)(-j)}{2+j2-j}=1{,}4-j1{,}2=1{,}85e^{-j40^\circ 35'}\ \Omega.$$



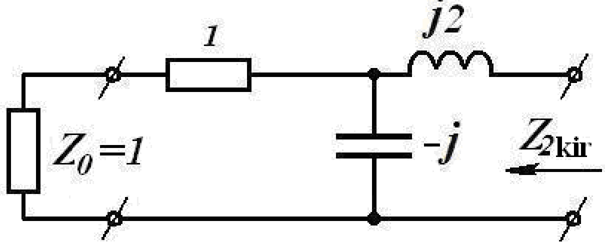
2.97- rasm.

2.4- jadvalda keltirilgan formuladan foydalanamiz:

$$Z_{2\text{kir}}=\frac{A_{22}Z_0+A_{12}}{A_{21}Z_0+A_{11}}=\frac{-1\cdot 1+j2\cdot 1}{j\cdot 1+1\cdot j}=\frac{j2-2}{1+j2}=\frac{(j2-2)(1-j2)}{5}=(0{,}4+j1{,}2)\ \Omega;$$

2.98- rasm asosida odatdagi usul bo‘yicha

$$Z_{2\text{kir}}=(0{,}4+j1{,}2)\ \Omega.$$



2.98- rasm.

2.136- masala. To‘rtqutblikning umumlashgan uchta kompleks koeffitsiyentlari ma’lum: $A_{11} = 1 + j,\ A_{12} = (4 + j4)\ \Omega,\ A_{22} = 0{,}5.$ Kirish qarshiligi aktiv bo‘lishi uchun, uning chiqish qismalariga qanday aktiv qarshilik ulanishi lozim?

Yechish. Dastlab to‘rtqutblikning to‘rtinchi koeffitsiyentini topamiz:

$$A_{21} = \frac{A_{11}A_{22} - 1}{A_{12}} = \frac{(1 + j)0{,}5 - 1}{4 + j4} = \frac{-0{,}5 + j0{,}5}{4 + j4} = j0{,}125\,\mathrm{S}.$$

Ixtiyoriy yuklama $Z_{\text{yuk}} = r_{\text{yuk}}$ uchun to‘rtqutblikning kirish qarshiligi ifodasini yozamiz:

$$Z_{1\text{kir}} = \frac{A_{11}Z_{\text{yuk}} + A_{12}}{A_{21}Z_{\text{yuk}} + A_{22}} = \frac{(1 + j)r_{\text{yuk}} + 4 + j4}{j0{,}125r_{\text{yuk}} + 0{,}5} = \frac{(4 + r_{\text{yuk}}) + j(4 + r_{\text{yuk}})}{0{,}5 + j0{,}125r_{\text{yuk}}}.$$

Bu ifodani maxrajidagi soniga qo‘shma kompleks bo‘lgan kompleks songa ko‘paytirib, quyidagini hosil qilamiz:

$$Z_{1\text{kir}} = \frac{2 + r_{\text{yuk}} + 0{,}125\, r_{\text{yuk}}^2 + j(2 - 0{,}125\, r_{\text{yuk}}^2)}{0{,}25 + 0{,}125^2 \cdot r_{\text{yuk}}^2}.$$

Masala shartiga ko‘ra $Z_{1\text{kir}} = r_{1\text{kir}}$, binobarin, $J_t Z_{1\text{kir}} = 0$, ya’ni

$$\frac{2 - 0{,}125\, r_{\text{yuk}}^2}{0{,}25 + 0{,}125^2 \cdot r_{\text{yuk}}^2} = 0,$$

bundan

$$Z_{\text{yuk}} = r_{\text{yuk}} = \sqrt{\frac{2}{0{,}125}} = 4\,\Omega.$$

2.137- masala. To‘rtqutblikning uchta kompleks koeffitsiyentlari ma’lum: $A_{12} = (4 + j4)\,\Omega,\ A_{21} = j0{,}125\,\Omega,\ A_{22} = 0{,}5$. Shu to‘rtqutblikning chiqish qismalariga qanday sof reaktiv qarshilik ulanganda, uning kirish o‘tkazuvchanligi nolga teng bo‘ladi?

2.138- masala. Simmetrik to‘rtqutblikning kompleks koeffitsiyentlari $A_{11} = (1 - j0{,}5)\,\Omega,\ A_{12} = -j5\,\Omega$. Shu to‘rtqutblikka qanday sof reaktiv qarshilik ulanganda kirish tokining fazasi kirish kuchlanishi fazasi bilan to‘g‘ri keladi?

2.139- masala. Agar to‘rtqutblik koeffitsiyentlari $A_{11} = (1 - j0{,}5)\,\Omega$, $A_{21} = (0{,}2 - j0{,}05)\,\Omega$ bo‘lsa, yuklama qarshiligining qanday qiymatida to‘rtqutblikning kirish qarshiligi yuklama qarshiligiga teng bo‘ladi?

2.140- masala. Nosimmetrik to‘rtqutblikning to‘g‘ri uzatishi bo‘yicha salt ishlash va qisqa tutashish tajribalarida hamda teskari uzatishi bo‘yicha salt ishlash tajribalarida quyidagi natijalar olingan:

$$U_{1\text{s.ish.}} = 100\ \text{V},\ I_{1\text{s.ish.}} = 7{,}07\ \text{A},\ P_{1\text{s.ish.}} = 500\ \text{W},\ \varphi_{1\text{s.ish.}} < 0;$$
$$U_{1\text{.q.t.}} = 100\ \text{V},\ I_{1\text{.q.t.}} = 4{,}46\ \text{A},\ P_{1\text{.q.t.}} = 200\ \text{W},\ \varphi_{1\text{.q.t.}} < 0;$$
$$U_{2\text{s.ish.}} = 100\ \text{V},\ I_{2\text{s.i.sh}} = 10\ \text{A},\ P_{2\text{s.ish.}} = 0,\ \varphi_{2\text{s.ish.}} > 0.$$

Shu to‘rtqutblikning $\|A\|$ shakldagi koeffitsiyentlari va T-simon almashlash sxemasining parametrlari aniqlansin.

Yechish. Sinalgan to‘rtqutblikning to‘g‘ri uzatishi bo‘yicha salt ishlash tajribasidan olingan natijalar asosida kirish qarshiligining moduli va argumentini aniqlaymiz:

$$Z_{1\text{s.ish.}} = \frac{U_{1\text{s.ish.}}}{I_{1\text{s.ish.}}} = \frac{100}{7{,}07} = 14{,}14\,\Omega,$$
$$\varphi_{1\text{s.ish.}} = \arccos\frac{P_{1\text{s.ish.}}}{U_{1\text{s.ish.}}\, I_{1\text{s.ish.}}} = \arccos\frac{500}{100\cdot 7{,}07} = 45^\circ.$$

Demak, kirish qarshiligining kompleksi

$$\underline{Z}_{1\text{s.ish.}} = 14{,}14 e^{-j45^\circ} = (10 - j10)\,\Omega.$$

Shunga o‘xshash

$$\underline{Z}_{1\text{q.t.}} = 10 - j20 = 22{,}4 e^{-j63^\circ 30'}\,\Omega;\quad \underline{Z}_{2\text{s.ish.}} = j10 = 10 e^{j90^\circ}\,\Omega.$$

Salt ishlash va qisqa tutashish tajribasi parametrlari orasidagi munosabatlardan quyidagini aniqlaymiz:

$$\underline{Z}_{2\text{q.t.}} = 15{,}81 e^{j71^\circ 30'} = (5 + j15)\,\Omega.$$

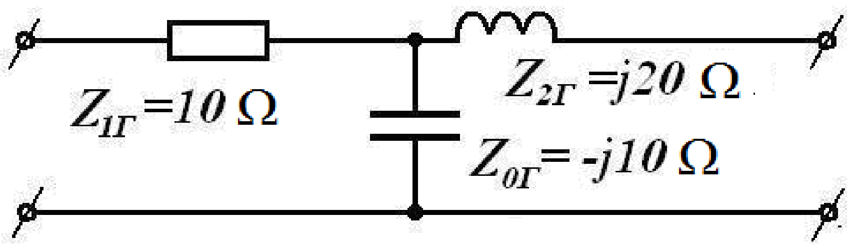
2.4-jadvalda keltirilgan formulalar hamda salt ishlash va qisqa tutashish parametrlariga ko‘ra to‘rtqutblikning koeffitsiyentlarini aniqlaymiz:

$$A_{11}=\sqrt{\frac{\underline{Z}_{1\text{s.ish.}}\cdot\underline{Z}_{1\text{q.t.}}}{\underline{Z}_{2\text{q.t.}}(\underline{Z}_{1\text{s.ish.}}-\underline{Z}_{1\text{q.t.}})}}=\sqrt{\frac{14{,}14e^{-j45^\circ}\cdot 22{,}4e^{-j63^\circ30'}}{15{,}81e^{j71^\circ30'}(10-j10-10+j20)}}=$$
$$=\pm(1+j),$$
$$A_{12}=A_{11}\underline{Z}_{2\text{q.t.}}=\pm(1+j)(5+j15)=\pm22{,}4e^{j126^\circ30'}\ \Omega,$$
$$A_{21}=\frac{A_{11}}{\underline{Z}_{1\text{s.ish.}}}=\frac{\pm(1+j)}{10-j10}=\pm j0{,}1\,\text{S},$$
$$A_{22}=\frac{A_{12}}{\underline{Z}_{1\text{q.t.}}}=\frac{\pm22{,}4e^{j126^\circ30'}}{22{,}4e^{-j63^\circ30'}}=\pm(-1).$$

T-simon almashlash sxemasining parametrlarini aniqlaymiz:

$$Z_{0T}=\frac{1}{A_{21}}=\frac{1}{\pm j0{,}1}=\mp j10\,\Omega,$$
$$Z_{1T}=\frac{A_{11}-1}{A_{21}}=\frac{\pm(1+j)-1}{\pm j0{,}1}=(10;\ -10-j20)\,\Omega,$$
$$Z_{2T}=\frac{A_{22}-1}{A_{21}}=\frac{\pm(-1)-1}{\pm j0{,}1}=(j20;\ 0)\,\Omega.$$

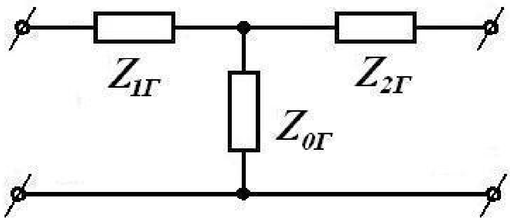
Hisoblangan parametrlarning yuqoridagi ishorasiga mos keladigan T-simon almashlash sxemasi 2.99- rasmda tasvirlangan.



2.99- rasm.

Pastki ishoralarga mos keladigan sxemani yaratib bo‘lmaydi, chunki chap yelkadagi aktiv qarshilik manfiy bo‘lishi kerak ($Z_{1T}=(-10-j20)\,\Omega$).

2.141-masala. Agar simmetrik to‘rtqutblikning T-simon almashlash sxemasi (2.100- rasm) parametrlari $Z_{1T}=Z_{2T}=10\,\Omega$, $Z_{0T}=-j10\,\Omega$ bo‘lsa, uning xarakteristik parametrlarini aniqlang.



2.100- rasm.

Yechish. Avval to‘rtqutblikning $\|A\|$ shakldagi koeffitsiyentlarini aniqlaymiz:

$$A_{11} = 1 + \frac{Z_{1T}}{Z_{0T}} = 1 + \frac{10}{-j10} = 1 + j = 1{,}41e^{j45^\circ},$$

$$A_{12} = Z_{1T} + Z_{2T} + \frac{Z_{1T}\,Z_{2T}}{Z_{0T}} = 10 + 10 + \frac{100}{-j10} = (20 + j10)\,\Omega,$$

$$A_{21} = \frac{1}{Z_{0T}} = \frac{1}{-j10} = j0{,}1\,\text{S},$$

$$A_{22} = A_{11} = 1 + j.$$

Simmetrik to‘rtqutblikning kirish va chiqish qismalariga nisbatan simmetrik qarshiligi bir xil:

$$Z_{1C} = Z_{2C} = Z_C = \sqrt{\frac{A_{12}}{A_{21}}} = \sqrt{\frac{20 + j10}{j0{,}1}} = 14{,}95e^{-j31^\circ 45'}\ \Omega.$$

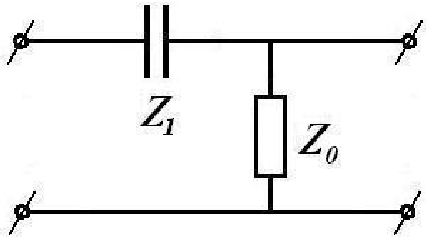
To‘rtqutblikning uzatish o‘lchovini (mera peredachi) aniqlaymiz:

$$\varGamma = \ln\left(\sqrt{A_{11}\cdot A_{22}} + \sqrt{A_{12}\cdot A_{21}}\right) = \ln\left(1 + j + \sqrt{(20 + j10)\,j0{,}1}\right) =$$

$$= \ln 2{,}9e^{j51^\circ 45'} = \ln 2{,}9e^{j0{,}9},$$

ya’ni $a = \ln 2{,}9 = 1{,}06$ Nep, $b = 0{,}9\ \text{rad} = 51^\circ 45'$.

2.142- masala. To‘rtqutblikning T-simon almashlash sxemasi (2.101- rasm) parametrlari $Z_1 = -j100\,\Omega$, $Z_0 = 100\,\Omega$ bo‘lsa, uning xarakteristik parametrlari aniqlansin.



2.101- rasm.

2.143- masala. To‘rtqutblikning xarakteristik parametrlari ma’lum $Z_{1C} = 119e^{-j67^\circ 30'}\ \Omega$, $Z_{2C} = 84e^{-j22^\circ 30'}\ \Omega$, $\Gamma = 0{,}76 - j0{,}574$. П-simon almashlash sxemaning parametrlari va $A_{11}, A_{12}, A_{21}, A_{22}$ koeffitsiyentlar aniqlansin.

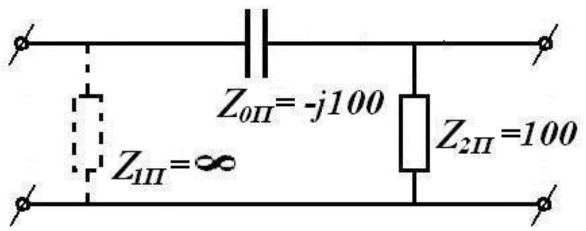
Yechish. Avval $\mathrm{sh}\Gamma$ va $\mathrm{ch}\Gamma$ larni aniqlaymiz:

$$\mathrm{sh}\Gamma = \mathrm{sh}(0{,}76 - j0{,}574) = \mathrm{sh}0{,}76\cos(-0{,}574) + j\mathrm{ch}0{,}76\sin(-0{,}574) =$$
$$= 0{,}84 \cdot 0{,}84 + j1{,}31 \cdot (-0{,}542) = 0{,}707 - j0{,}707 = e^{-j45^\circ};$$
$$\mathrm{ch}\Gamma = \mathrm{ch}(0{,}76 - j0{,}574) = \mathrm{ch}0{,}76\cos(-0{,}574) + j\mathrm{sh}0{,}76\sin(-0{,}574) =$$
$$= 1{,}31 \cdot 0{,}84 + j0{,}84 \cdot (-0{,}542) = 1{,}1 - j0{,}452 = 1{,}19e^{-j22^\circ 30'}.$$

2.4- jadvaldagi formulalardan foydalanib quyidagilarni aniqlaymiz:

$$A_{11} = \sqrt{\frac{\underline{Z}_{1C}}{\underline{Z}_{2C}}}\,\mathrm{ch}\Gamma = \sqrt{\frac{119e^{-j67^\circ 30'}}{84e^{-j22^\circ 30'}}} \cdot 1{,}19e^{-j22^\circ 30'} = 1{,}414e^{-j45^\circ};$$
$$A_{12} = \sqrt{\underline{Z}_{1C}\underline{Z}_{2C}}\,\mathrm{sh}\Gamma = \sqrt{119e^{-j67^\circ 30'} \cdot 84e^{-j22^\circ 30'}} \cdot e^{-j45^\circ} = -j100\,\Omega;$$
$$A_{21} = \frac{\mathrm{sh}\Gamma}{\sqrt{\underline{Z}_{1C}\underline{Z}_{2C}}} = \frac{e^{-j45^\circ}}{100e^{-j45^\circ}} = 0{,}01\,\mathrm{S};$$
$$A_{22} = \sqrt{\frac{\underline{Z}_{2C}}{\underline{Z}_{1C}}}\,\mathrm{ch}\Gamma = \frac{\mathrm{ch}\Gamma}{\sqrt{\underline{Z}_{1C}/\underline{Z}_{2C}}} = \frac{1{,}19e^{-j22^\circ 30'}}{1{,}19e^{-j22^\circ 30'}} = 1.$$

To‘rtqutblikning П-simon sxemasidagi (2.102-rasm) kompleks qarshiliklarini aniqlaymiz:



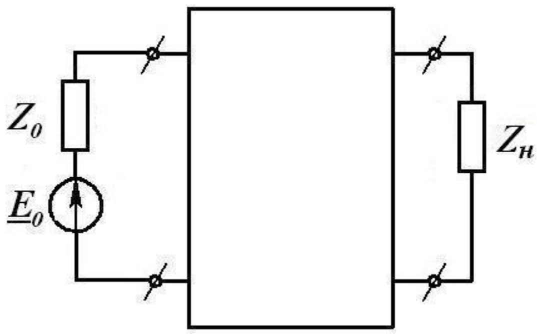
2.102- rasm.

$$\underline{Z}_{0\Pi} = A_{12} = -j100\,\Omega;$$
$$\underline{Z}_{1\Pi} = \frac{A_{12}}{A_{22} - 1} = \frac{-j100}{1 - 1} = -j\infty;$$
$$\underline{Z}_{2\Pi} = \frac{A_{12}}{A_{11} - 1} = \frac{-j100}{1 - j - 1} = 100\,\Omega.$$

2.144-masala. Simmetrik to‘rtqutblikning xarakteristik parametrlari berilgan: $\underline{Z}_C = (12{,}7 - j7{,}85)\,\Omega$, $\Gamma = 1{,}06 + j0{,}9$. $\|A\|$ shakldagi koeffitsiyentlar A_{11}, A_{12}, A_{21}, A_{22} aniqlansin.

2.145- masala. To‘rtqutblikning xarakteristik parametrlari berilgan: $\underline{Z}_C = 17{,}8e^{-j54°15'}$ Ω, $\underline{Z}_C = 12{,}6e^{j80°48'}$ Ω, $\Gamma = 0{,}985 - j1{,}27$. $\|A\|$ shakldagi koeffitsiyentlar $A_{11},\ A_{12},\ A_{21},\ A_{22}$ aniqlansin.

2.146- masala. Generatordan ($\underline{E} = 130$ V, ichki qarshiligi $\underline{Z}_0 = R_0 = 1\ \Omega$) yuklamaga ($\underline{Z}_{\text{yuk}} = R_{\text{yuk}} = 25\,\Omega$) eng katta quvvat uzatish uchun generator va yuklama orasiga simmetrik muvofiqlashtiruvchi to‘rtqutblik o‘rnatilgan (2.103-rasm). Bunday to‘rtqutblikning $A_{11},\ A_{12},\ A_{21},\ A_{22}$ koeffitsiyentlari topilsin, uning T-simon almashlash sxemasining parametrlari hisoblansin hamda yuklama generatorga bevosita to‘g‘ridan-to‘g‘ri va to‘rtqutblik orqali ulanganda uning quvvati aniqlansin.



2.103- rasm.

Yechish. To‘rtqutblikning koeffitsiyentlarini quyidagi tenglamalarni birgalikda yechib topamiz:

$$\underline{Z}_{1\,\text{kir}} = \frac{A_{11}\underline{Z}_{\text{yuk}} + A_{12}}{A_{21}\underline{Z}_{\text{yuk}} + A_{22}}, \tag{1}$$

$$\underline{Z}_{2\,\text{kir}} = \frac{A_{22}\underline{Z}_0 + A_{12}}{A_{21}\underline{Z}_0 + A_{11}}, \tag{2}$$

$$A_{11} = A_{22}, \tag{3}$$

$$A_{11}\cdot A_{22} - A_{12}\cdot A_{21} = 1. \tag{4}$$

Birinchi tenglama – generator ichki qarshiligining ($\underline{Z}_0 = R_0 = 1\ \Omega$) ($\underline{Z}_{\text{yuk}} = R_{\text{yuk}} = 25\,\Omega$) qarshilik bilan yuklangan to‘rtqutblikning kirish qarshiligiga muvofiqlashtirish shartini ifodalaydi.

Ikkinchi tenglama – to‘rtqutblikning birlamchi qismalariga qarshilik ($\underline{Z}_0 = R_0 = 1\ \Omega$) ulangan holda, to‘rtqutblik yuklama qarshiligining ($\underline{Z}_{\text{yuk}} = R_{\text{yuk}} = 25\,\Omega$) ikkilamchi qismalar tomonidan kirish qarshiligiga muvofiqlashtirish shartini ifodalaydi.

Uchinchi tenglama – to‘rtqutblikning simmetriklik shartini, to‘rtinchi tenglama esa koeffiitsiyentlar orasidagi bog‘lanishni ifodalaydi.

Tenglamalarni birgalikda yechish quyidagilarni beradi:

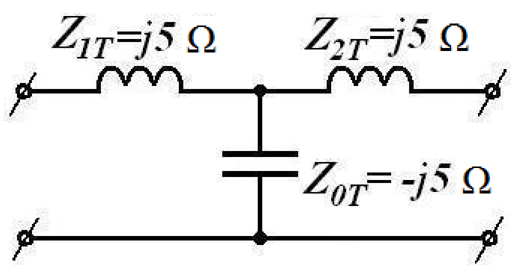
$$A_{11} = A_{22} = 0;$$

$$A_{12} = -\frac{1}{A_{21}} = \pm j5\ \Omega.$$

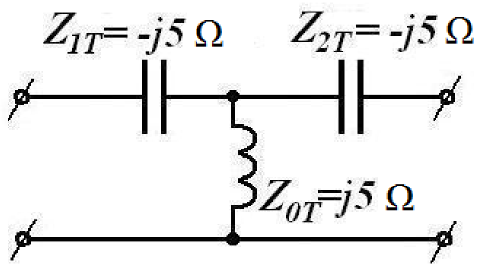
T-simon almashlash sxemasining kompleks qarshiliklarini aniqlaymiz:

$$\underline{Z}_{0T} = \frac{1}{A_{21}} = \mp j5\,\Omega;\ \underline{Z}_{1T} = \underline{Z}_{2T} = \frac{A_{11}-1}{A_{21}} = \pm j5\,\Omega.$$

Sxemaning elementlari reaktiv qarshiliklardan iborat bo‘lgani uchun, uni ikkita usul bilan bajarish mumkin (2.104- rasm, *a*, *b*).



a)



b)

2.104- rasm.

Generatorga bevosita ulangan yuklamaning quvvati:

$$P'_{\text{yuk}} = I^2 R_{\text{yuk}} = \left(\frac{130}{1+25}\right)^2 \cdot 25 = 5^2 \cdot 25 = 625\ \text{W}.$$

Generatorga muvofiqlashtirilgan to‘rtqutblik orqali ulangan yuklamadagi quvvatni aniqlash uchun avval $\underline{I}_1$ va $\underline{I}_2$ toklarni hisoblaymiz:

$$\underline{I}_1 = \frac{\underline{E}}{R_0 + Z_{\text{kir}}} = \frac{130}{1+1} = 65 \text{ A};$$

$$\underline{I}_2 = \underline{I}_1 \frac{Z_{0T}}{Z_{0T} + Z_{2T} + Z_{\text{yuk}}} = 65 \cdot \frac{-j5}{-j5 + j5 + 25} = -13 \text{ A};$$

$$P''_{\text{yuk}} = \underline{I}_2^2 R_{\text{yuk}} = 13^2 \cdot 25 = 4225 \text{ W}.$$

2.147- masala. Ichki qarshiligi $\underline{Z}_{\text{ich}} = (1 + j2)\ \Omega$ bo‘lgan generator bilan $\underline{Z}_{\text{yuk}} = (100 - j200)\,\Omega$ yuklama orasiga, yuklamaga maksimal quvvat uzatish maqsadida o‘rnatilgan ideal transformatorning transformatsiya koeffitsiyentini aniqlang.

Yechish. Generatordan yuklamaga uzatiladigan quvvat maksimal bo‘lishi uchun, ular orasidagi ideal transformatorning kirish qarshiligi generatorning ichki qarshiligiga qo‘sh kompleks bo‘lishi lozim, ya’ni

$$\underline{Z}_{1\text{kir}} = Z_0^* = (1 - j2)\ \Omega.$$

Ideal transformatorda energiya isrofi va sochma magnit oqim bo‘lmagani sababli

$$\underline{U}_1 \underline{I}_1^* = \underline{U}_2 \underline{I}_2^*,$$

$$\frac{\underline{U}_1}{\underline{U}_2} = \frac{U_1}{U_2} = \frac{w_1}{w_2} = n,$$

bundan $\underline{U}_1 = n\underline{U}_2$; (1)

$$\frac{\underline{I}_1^*}{\underline{I}_2^*} = \frac{\underline{I}_1}{\underline{I}_2} = \frac{I_1}{I_2} = \frac{w_2}{w_1} = \frac{1}{n},$$

bundan $\underline{I}_1 = \frac{1}{n}\underline{I}_2$ (2).

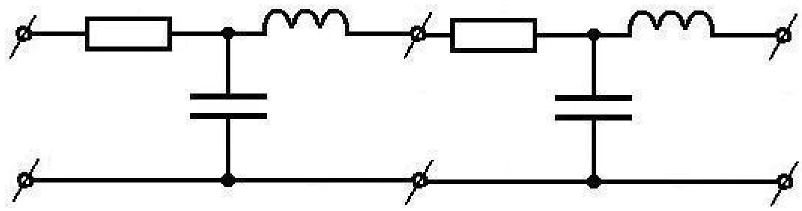
Ideal transformatorning kirish qarshiligi:

$$\underline{Z}_{1\text{kir}} = \frac{\underline{U}_1}{\underline{I}_1} = \frac{n\underline{U}_2}{\frac{1}{n}\underline{I}_2} = Z_{\text{yuk}}\, n^2;$$

bundan $n = \frac{w_1}{w_2} = \sqrt{\frac{\underline{Z}_{1\text{kir}}}{\underline{Z}_{\text{yuk}}}} = \sqrt{\frac{1 - j2}{100 - j200}} = \frac{1}{10}.$

2.148-masala. Ichki qarshiligi $\underline{Z}_{\text{ich}} = (1 + j3)\,\Omega$ bo‘lgan generator va maksimal quvvat ajralib turadigan $\underline{Z}_{\text{yuk}} = (25 + j50)\,\Omega$ yuklama orasiga ulangan ideal transformatorning transformatsiya koeffitsiyenti hamda yuklamaga qo‘shimcha ketma-ket ulangan $\underline{Z}_{\text{qo'sh}}$ qarshilik topilsin.

2.149-masala. Parametrlari 2.130-masalada berilgan ikkita to‘rtqutblik 2.105-rasmda tasvirlangan sxemalar bo‘yicha kaskad ravishda ulangan. Ulanishning har bir holati uchun $\|A\|$ shakldagi koeffitsiyentlar aniqlansin.



2.105- rasm.

Yechish. 2.105-rasmda tasvirlangan kaskad ravishda ulangan to‘rtqutbliklarni ko‘rib chiqamiz. Tarkibiy to‘rtqutbliklar mos matritsalarining ko‘paytmasi kaskad usulida ulangan ekvivalent to‘rtqutblikning matritsasini beradi, ya’ni:

$$\|A\|_{\text{ekv}} = \|A'\| \cdot \|A''\| = \begin{Vmatrix} A'_{11} & A'_{12} \\ A'_{21} & A'_{22} \end{Vmatrix} \cdot \begin{Vmatrix} A''_{11} & A''_{12} \\ A''_{21} & A''_{22} \end{Vmatrix} = \begin{Vmatrix} A'_{11}A''_{11} + A'_{12}A''_{21} & A'_{11}A''_{12} + A'_{12}A''_{22} \\ A'_{21}A''_{11} + A'_{22}A''_{21} & A'_{21}A''_{12} + A'_{22}A''_{22} \end{Vmatrix} =$$

$$= \begin{Vmatrix} A^e_{11} & A^e_{12} \\ A^e_{21} & A^e_{22} \end{Vmatrix}.$$

a) 2.130-masalada yechimidan kelib chiqadiki, har bir to‘rtqutblikning $\|A\|$ matritsasi

$$\|A'\| = \|A''\| = \begin{Vmatrix} 1+j & j2-1 \\ j & -1 \end{Vmatrix}.$$

Ekvivalent to‘rtqutblikning koeffitsiyentlarini aniqlaymiz:

$$A_{11\text{ekv}} = A'_{11}A''_{11} + A'_{12}A''_{21} = (1+j)(1+j) + (j2-1)\,j = j-2;$$
$$A_{12\text{ekv}} = A'_{11}A''_{12} + A'_{12}A''_{22} = (1+j)(j2-1) + (j2-1)(-1) = (-2-j)\,\Omega;$$
$$A_{21\text{ekv}} = A'_{21}A''_{11} + A'_{22}A''_{21} = j(1+j) + (-1)\,j = -1\,S;$$
$$A_{22\text{ekv}} = A'_{21}A''_{12} + A'_{22}A''_{22} = j(j2-1) + (-1)(-1) = -1-j.$$

Tekshirib ko‘ramiz:

$$A_{11\text{ekv}}A_{22\text{ekv}} - A_{12\text{ekv}}A_{21\text{ekv}} = (j-2)(-1-j) - (-2-j)(-1) = 1.$$

b) 2.106- rasmda tasvirlangan sxemada birinchi to‘rtqutblikning chiqish qisma- lari ikkinchi to‘rtqutblikning chiqish qismalariga ulangan. Bu holda ikkinchi to‘rtqutblikning A''_{11} va A''_{22} koeffitsiyentlari joy almashadi:

$$\|A\|_{\text{ekv}} = \begin{Vmatrix} 1+j & j2-1 \\ j & -1 \end{Vmatrix} \cdot \begin{Vmatrix} -1 & j2-1 \\ j & 1+j \end{Vmatrix} = \begin{Vmatrix} -3-j2 & j2-6 \\ -j2 & -3-j2 \end{Vmatrix}.$$

Tekshirish:

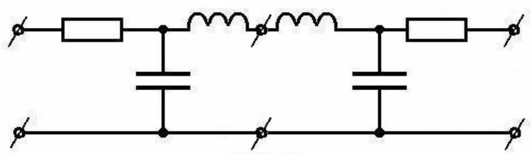
$$A_{11\text{ekv}}A_{22\text{ekv}} - A_{12\text{ekv}}A_{21\text{ekv}} = (-3-j2)^2 - (-j2)(j2-6) = 9 + j12 - 4 - 4 - j12 = 1.$$

d) To‘rtqutbliklar 2.107- rasm bo‘yicha ulanganda, quyidagiga ega bo‘lamiz:

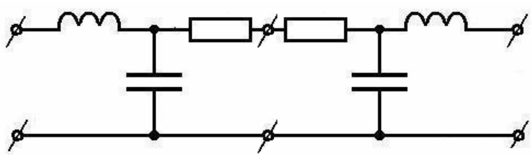
$$\|A\|_{\text{ekv}} = \begin{Vmatrix} -1 & j2-1 \\ j & 1+j \end{Vmatrix} \cdot \begin{Vmatrix} 1+j & j2-1 \\ j & -1 \end{Vmatrix} = \begin{Vmatrix} -3-j2 & 2-j4 \\ -2+j2 & -3-j2 \end{Vmatrix}.$$

Tekshirish:

$$A_{11\text{ekv}}A_{22\text{ekv}} - A_{12\text{ekv}}A_{21\text{ekv}} = (-3-j2)^2 - (-2+j2)(2-j4) = 1.$$



2.106- rasm.

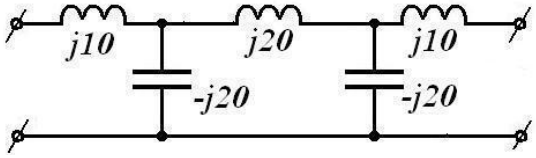


2.107- rasm.

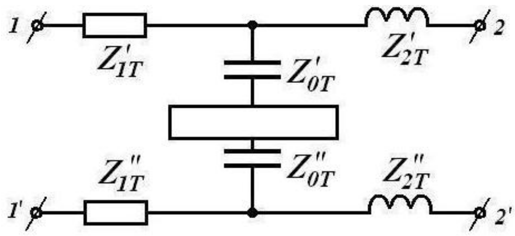
2.150- masala. 2.108- rasmda tasvirlangan to‘rtqutblikning A_{11}, A_{12}, A_{21}, A_{22} koeffitsiyentlari, salt ishlash va qisqa tutashish parametrlari hamda xarakteristik parametrlari aniqlansin.

Ko‘rsatma: berilgan to‘rtqutblikni ikkita simmetrik to‘rtqutbliklarning kaskad ulanishidan iborat deb qarash lozim.

2.151-masala. 2.130- masalada berilgan ikkita bir xil to‘rtqutbliklar ketma-ket ulangan. Hosil bo‘ladigan ekvivalent to‘rtqutblikning ulanishi va koeffitsiyentlari aniqlansin.



2.108- rasm.



2.109- rasm.

Yechish. Ketma-ket ulangan to‘rtqutbliklarning sxemasi 2.109-rasmda tasvirlangan.

To‘rtqutbliklar ketma-ket ulanganda hosil bo‘ladigan ekvivalent to‘rtqutblikning $\|Z\|_{\text{ekv}}$ matritsasi tarkibiy to‘rtqutbliklar matritsalarining yig‘indisiga teng bo‘ladi, ya’ni

$$\|Z\|_{\text{ekv}} = \|Z'\| + \|Z''\|.$$

Har bir to‘rtqutbliklar matritsasini 2.4- jadvalda keltirilgan formulalardan foydalanib topamiz:

$$\|Z'\| = \|Z''\| = \begin{Vmatrix} \dfrac{A_{11}}{A_{21}} & \dfrac{\|A\|}{A_{21}} \\ \dfrac{1}{A_{21}} & \dfrac{A_{22}}{A_{21}} \end{Vmatrix} = \begin{Vmatrix} \dfrac{1+j}{j} & \dfrac{1}{j} \\ \dfrac{1}{j} & \dfrac{-1}{j} \end{Vmatrix} = \begin{Vmatrix} 1-j & -j \\ -j & j \end{Vmatrix}.$$

Ekvivalent to‘rtqutbliklarning $Z_{11},\ Z_{12},\ Z_{21},\ Z_{22}$ koeffitsiyentlarini aniqlaymiz. Tarkibiy to‘rtqutbliklar bir xil bo‘lgani uchun

$$Z_{11\text{ekv}} = 2Z'_{11} = 2Z''_{11} = (2 - j2)\ \Omega;$$

$$Z_{12\text{ekv}} = 2Z'_{12} = 2Z''_{12} = -j2\ \Omega;$$

$$Z_{21\text{ekv}} = 2Z'_{21} = 2Z''_{21} = -j2\ \Omega;$$

$$Z_{22\text{ekv}} = 2Z'_{22} = 2Z''_{22} = j2\ \Omega.$$

2.4- jadvaldan foydalanib ekvivalent to‘rtqutblikning $\|A\|_{\text{ekv}}$ matritsasini aniqlaymiz:

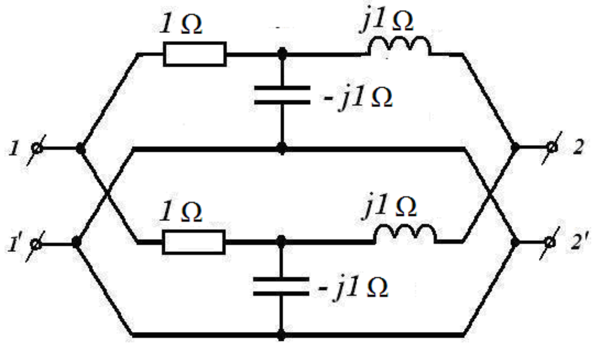
$$\|A\|_{\text{ekv}} = \begin{Vmatrix} \dfrac{Z^e_{11}}{Z^e_{21}} & \dfrac{\|Z^e\|}{Z^e_{21}} \\ \dfrac{1}{Z^e_{21}} & \dfrac{Z^e_{22}}{Z^e_{21}} \end{Vmatrix} = \begin{Vmatrix} \dfrac{2-j2}{-j2} & \dfrac{(2-j2)j2-(-j2)(-j2)}{-j2} \\ \dfrac{1}{-j2} & \dfrac{j2}{-j2} \end{Vmatrix} = \begin{Vmatrix} 1+j & -2+j4 \\ j0{,}5 & -1 \end{Vmatrix}.$$

Tekshirish: $A_{11\text{ekv}}A_{22\text{ekv}} - A_{12\text{ekv}}A_{21\text{ekv}} = (1+j)(-1) - j0{,}5(-2+j4) = 1.$

Masala yechimidan ma’lum bo‘ldiki, ikkita bir xil to‘rtqutblik ketma-ket ulanganda A_{11} va A_{22} koeffitsiyentlar o‘zgarmas qoladi, A_{12} va A_{21} koeffitsiyentlar esa 2 baravar o‘zgaradi.

2.152- masala. 2.130- masalada berilgan ikkita bir xil to‘rtqutbliklar parallel ulangan. Ulanish sxemasini va ekvivalent to‘rtqutblikning A_{11}, A_{12}, A_{21}, A_{22} koeffitsiyentlari aniqlansin.

Yechish. Berilgan to‘rtqutbliklarning parallel ulangan sxemasi 2.110-rasmda tasvirlangan.



2.110- rasm.

To‘rtqutbliklar parallel ulanganda $\|\mathrm{Y}\|$ shakl qo‘llaniladi. Ekvivalent to‘rtqutblikning matritsasi $\|\mathrm{Y}\|_{\mathrm{ekv}}$ tarkibiy to‘rtqutbliklarning matritsalari yig‘indisiga teng, ya’ni:

$$\|\mathrm{Y}\|_{\mathrm{ekv}} = \|\mathrm{Y}'\| + \|\mathrm{Y}''\|.$$

Tarkibiy to‘rtqutbliklar $\|\mathrm{Y}'\| = \|\mathrm{Y}''\|$ matritsasini 2.4- jadvalda keltirilgan $\|A\|$ shaklli matritsalardan chiqarmiz:

$$\|\mathrm{Y}'\| = \|\mathrm{Y}''\| = \begin{Vmatrix} \dfrac{A_{22}}{A_{12}} & \dfrac{-|A|}{A_{12}} \\ \dfrac{-1}{A_{12}} & \dfrac{A_{11}}{A_{12}} \end{Vmatrix} = \begin{Vmatrix} \dfrac{-1}{j2-1} & \dfrac{-1}{j2-1} \\ \dfrac{-1}{j2-1} & \dfrac{1+j}{j2-1} \end{Vmatrix} = \begin{Vmatrix} 0{,}2+j0{,}4 & 0{,}2+j0{,}4 \\ 0{,}2+j0{,}4 & 0{,}2-j0{,}6 \end{Vmatrix}.$$

Ekvivalent to‘rtqutblikning matritsasi:

$$\|\mathrm{Y}\|_{\mathrm{ekv}} = \|\mathrm{Y}'\| + \|\mathrm{Y}''\| = \begin{Vmatrix} 0{,}4+j0{,}8 & 0{,}4+j0{,}8 \\ 0{,}4+j0{,}8 & 0{,}4-j1{,}2 \end{Vmatrix}.$$

2.4- jadvalda keltirilgan formulalardan foydalanib,
$\|\mathrm{Y}\|_E$ ekvivalent to‘rtqutblikdan $\|A\|_{\mathrm{ekv}}$ matritsaga o‘tamiz:

$$\|A\|_{ekv}=\begin{Vmatrix}\frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}}\\ -\frac{|Y|}{Y_{21}} & -\frac{Y_{11}}{Y_{21}}\end{Vmatrix}=\begin{Vmatrix}\frac{-0,4+j1,2}{0,4+j0,8} & \frac{-1}{0,4+j0,8}\\ \frac{-(0,4+j0,8)(0,4-j1,2)+(0,4+j0,8)^2}{0,4+j0,8} & \frac{-0,4-j0,8}{0,4+j0,8}\end{Vmatrix}=\begin{Vmatrix}1+j & j-0,5\\ j2 & -1\end{Vmatrix}.$$

Tekshirish: $A_{11ekv}A_{22ekv}-A_{12ekv}A_{21ekv}=(1+j)(-1)-j2(j-0,5)=1.$

Shunga e'tibor qilamizki, ikkita bir xil to‘rtqutblikni parallel ulaganda, A_{11} va A_{22} koeffitsiyentlar o‘zgarmaydilar, A_{12} va A_{21} koeffitsiyentlar esa 2 marta o‘zgaradi, chunonchi A_{12} (o‘lcham birligi **Ω**) 2 marta kamayadi, A_{21} esa (o‘lcham birligi *S*) 2 marta oshadi.

2.6.2. Filtr parametrlarini hisoblash

2.5-*jadval*

Quyi chastotali filtr	Yuqori chastotali filtr	Kesimli filtr	To‘suvchi filtr
$L=\frac{k}{\pi\ f_C}$	$L=\frac{k}{4\pi\ f_C}$	$L_1=\frac{k}{\pi\,(f_2-f_1)}$ $L_2=\frac{k(f_2-f_1)}{4\pi\ f_1\ f_2}$	$L_1=\frac{k(f_2-f_1)}{\pi\ f_1\ f_2}$ $L_2=\frac{k}{4\pi(f_2-f_1)}$
$C=\frac{1}{\pi\ f_C k}$	$C=\frac{1}{4\pi\ f_C k}$	$C_1=\frac{f_2-f_1}{4\pi\,k\ f_1\ f_2}$ $C_2=\frac{1}{4\pi\,k(f_2-f_1)}$	$C_1=\frac{1}{4\pi\,k(f_2-f_1)}$ $C_2=\frac{f_2-f_1}{\pi\,k\ f_1\ f_2}$
$f_C=\frac{1}{\pi\sqrt{LC}}$	$f_C=\frac{1}{4\pi\sqrt{LC}}$	$f_{1,2}=\frac{1}{2\pi}\cdot(\sqrt{\frac{1}{L_1C_2}+\frac{1}{L_1C_1}}$ $\mp\sqrt{\frac{1}{L_1C_2}})$	$f_{1,2}=\frac{1}{8\pi}(\sqrt{\frac{1}{L_2C_1}+\frac{16}{L_1C_1}}$ $\mp\sqrt{\frac{1}{L_2C_1}})$

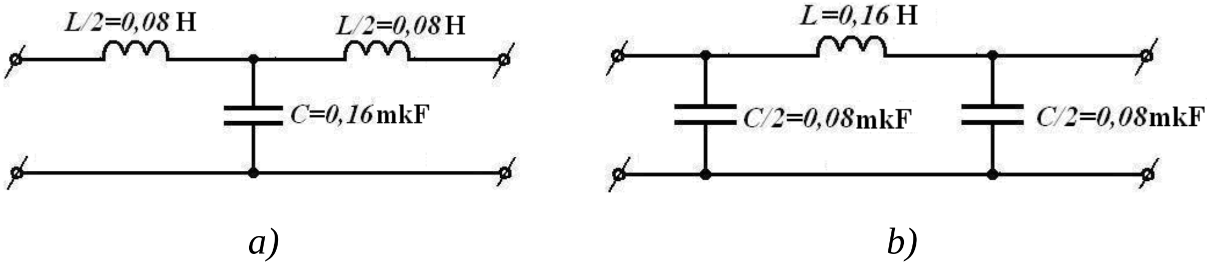
2.153-masala. Yuklama qarshiligi $R=1000\,\Omega$ va siklik chastotasi $f_C=2000\,\text{Hz}$ bo‘lgan k turidagi quyi chastotali filtr elementlari aniqlansin. T va П-simon filtrlar sxemasi tasvirlansin.

Yechish. 2.5- jadvalda keltirilgan formulalardan

$$L=\frac{k}{\pi\ f_C}=\frac{1000}{3{,}14\cdot 2000}=0{,}16\,\text{H},$$

$$C=\frac{1}{\pi\ f_C k}=\frac{1}{3{,}14\cdot 2000\cdot 1000}=0{,}16\cdot 10^{-6}=0{,}16\ \text{mkF}.$$

T va P-simon filtrlar sxemasi 2.111- rasm, *a* va *b* larda tasvirlangan.



2.111- rasm.

2.154- masala. k turidagi quyi chastotali filtr induktivligi $L=0{,}01\,\text{H}$ g‘altakdan va sig‘imlari $C=0{,}5\,\text{mkF}$ bo‘lgan kondensatorlardan yig‘ilgan. Shu filtrning siklik chastotasini va nominal xarakteristik qarshiligini aniqlang.

2.155- masala. 2.154- masalada berilgan filtrdan nechta bo‘g‘inlar (zveno) kaskad holda yig‘ilganda $f=10\,\text{kHz}$ bo‘lgan chastotada so‘nishi $10\,\text{Nep}$ bo‘ladi?

Yechish. 2.5-jadvaldagi formulalardan foydalanib $f=10\,\text{kHz}$ chastotada bir bo‘g‘inda (zvenoda) qanday so‘nish bo‘lishini aniqlaymiz:

$$\text{ch}\,\frac{a}{2}=\frac{f}{f_C}=\frac{10000}{3180}=3{,}62.$$

Giperbolik funksiya jadvallaridan $a/2=1{,}81,\ a=3{,}62.$

So‘nish qiymati $a=10\,\text{Nep}$ bo‘lishi uchun kaskad ravishda $10/3{,}62=2{,}76\approx 3$ bo‘g‘in ulanishi kerak?

2.156- masala. k turidagi quyi chastotali filtr $f_C=100\,\text{Hz}$ chastotaga ega va qarshiligi $R=1000\,\Omega$ bo‘lgan yuklama uchun mo‘ljallangan. Quyidagilarni aniqlash kerak: 1) filtr (L va S) parametrlari; 2) $f = 50\,\Omega$ chastotada neperlarda, detsibellarda hamda kirish va chiqish kuchlanishlarning ta'sir etuvchi nisbatlarida ifodalangan so'nishi; 3) filtr elementlari ko'rsatilgan T- va P-simon sxemasini tasvirlang.

Yechish. 1) Filtr parametrlarini 2.5-jadvalda keltirilgan formulalar yordamida aniqlaymiz:

$$L = \frac{k}{4\pi\ f_C} = \frac{1000}{4 \cdot 3{,}14 \cdot 100} = 0{,}8\,\text{H}.$$

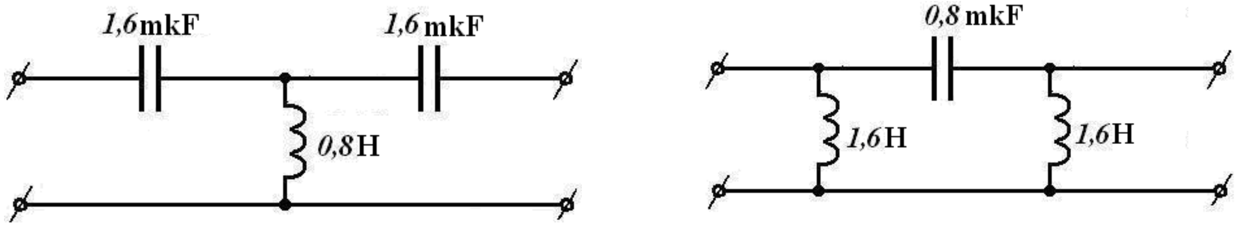
$$C = \frac{1}{4\pi\ f_C k} = \frac{1}{4 \cdot 3{,}14 \cdot 100 \cdot 1000} = 0{,}8 \cdot 10^{-6} = 0{,}8\,\text{mkF}.$$

2) Filtrning o'z so'nishini aniqlaymiz:

$$\text{ch}\frac{a}{2} = \frac{f_C}{f} = \frac{100}{50} = 2; \quad \frac{a}{2} = 1{,}32, \quad a = 2{,}64\,\text{Nep} = 22{,}9\,\text{db};$$

$$\frac{U_1}{U_2} = e^a = e^{2{,}64} = 14.$$

T va П-simon filtr sxemalari 2.112- rasm, *a* va *b* da tasvirlangan.



a) *b)*

2.112- rasm.

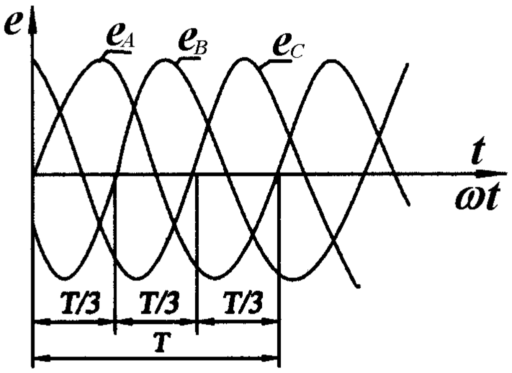
3- BOB. UCH FAZALI ZANJIRLARNING SIMMETRIK VA NOSIMMETRIK REJIMLARIGA DOIR MISOLLAR YECHISH

3.1. Asosiy ma'lumotlar

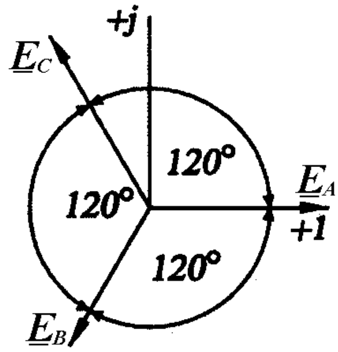
Uch fazali simmetrik sistema. Bir xil chastotali, amplitudali va fazalari bo'yicha o'zaro 120° ga siljigan sinusoidal EYK lar tizimi uch fazali simmetrik EYK lar tizimi deb ataladi (3.1- rasm).

Bu tizimda EYK lar kompleks qiymatlarini quyidagicha yozish mumkin:

$$\underline{E}_A = E; \underline{E}_B = Ee^{-j120^0}; \underline{E}_C = Ee^{-j240^0} = Ee^{j120^0}. \qquad (3.1)$$



a)



b)

3.1- rasm.

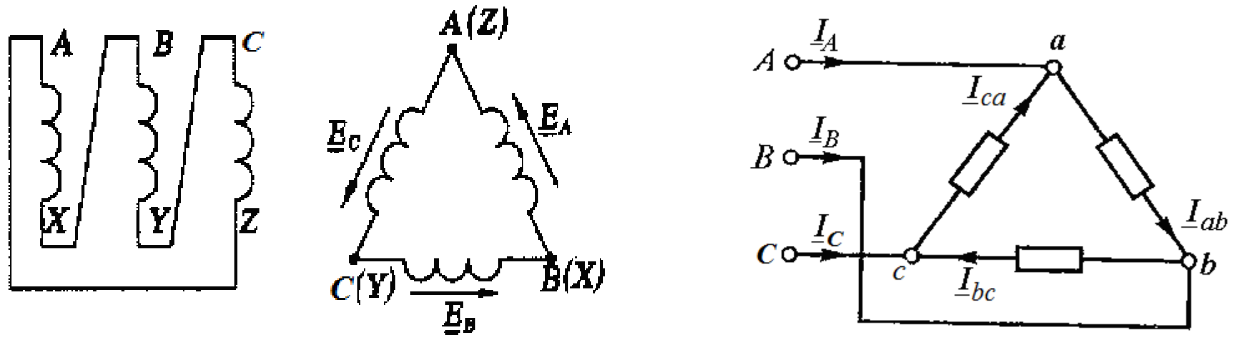
Bu tizimning asosiy xususiyati shundan iboratki, vaqtning istalgan paytida EYK lar oniy qiymatlarining algebraik yig'indisi nolga teng bo'ladi, ya'ni

$$e_A + e_B + e_C = 0 \qquad (3.2)$$

yoki $\underline{E}_A + \underline{E}_B + \underline{E}_C = 0$. Bunga ushbu tenglik orqali ishonch hosil qilish mumkin:

$$\underline{E}_A + \underline{E}_B + \underline{E}_C = E + Ee^{-j120^0} + Ee^{j120^0} =$$
$$= E\left(1 - 0{,}5 - j\frac{\sqrt{3}}{2} - 0{,}5 + j\frac{\sqrt{3}}{2}\right) = 0.$$

Uch fazali zanjirlarning ulanish sxemalari. Uch fazali tizimning generatorida faza chulg'amlarini uchburchak sxemasida ular o'zaro mos ketma-ket ulanadi (3.2- rasm, *a* va *b*). Uchburchak sxemasida ulanganda chulg'amlar berk kontur hosil qilsa-da, konturdagi tok nolga teng, chunki EYK larning geometrik yig'indisi nolga teng bo'ladi.

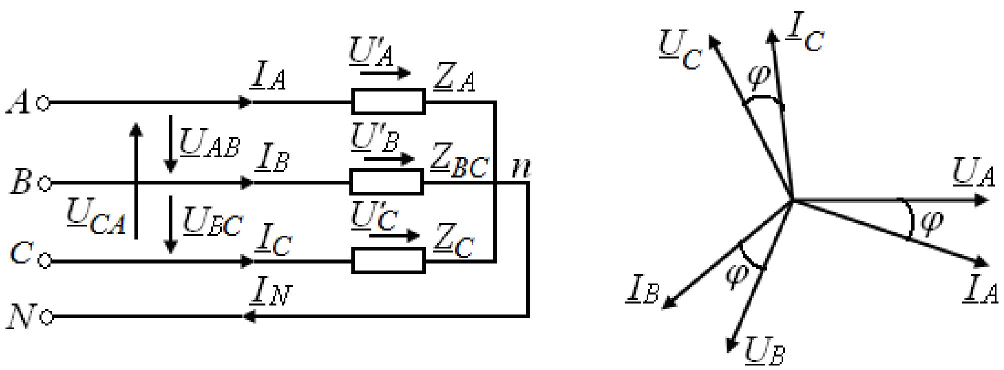


a) b) d)

3.2- rasm.

Uchburchak sxemasida faza kuchlanishi unga mos liniya kuchlanishiga teng, ya'ni $U_F = U_L$. Yuklama fazalaridagi $\underline{I}_{ab}$, $\underline{I}_{bc}$, $\underline{I}_{ca}$ toklar *faza toklari* deb, manbadan yuklamaga ulangan simlardan o'tuvchi $\underline{I}_A, \underline{I}_B, \underline{I}_C$ toklar *liniya toklari* deb ataladi (3.2- rasm, *d*).

Yuklama yulduz usulida ulangan zanjirda simmetrik rejim. Yulduz usulda ulangan simmetrik zanjirda (3.3- rasm, *a*) $\underline{Z}_A = \underline{Z}_B = \underline{Z}_C = ze^{j\varphi}$.



a) b)

3.3- rasm.

Yuklama qarshiliklari induktiv xarakterga (φ >0) ega deb, kuchlanishlar va toklarning vektor diagrammasini quramiz (3.3- rasm, *b*). Simmetrik rejimda $\underline{U}_A$, $\underline{U}_B$, $\underline{U}_C$ faza kuchlanishlari simmetrik vektorlar sistemasini hosil qiladi:

$$\underline{U}_A = U_F,\ \underline{U}_B = U_F e^{-j120^0},\ \underline{U}_C = U_F e^{j120^0}.$$

Simmetrik rejimda liniya kuchlanishlarining qiymatlari teng va bir-biridan o'zaro 120° ga siljigan bo'ladi.

$$\underline{U}_{AB} = \underline{U}_A - \underline{U}_B,\ \underline{U}_{BC} = \underline{U}_B - \underline{U}_C,\ \underline{U}_{CA} = \underline{U}_C - \underline{U}_A,$$

yoki

$$\underline{U}_{AB} = \underline{U}_A - \underline{U}_B = U_F - U_F e^{-j120^0} = U_F(1 - e^{-j120^0}) =$$
$$= U_F\left(1 + 0{,}5 + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}U_F\left(\frac{\sqrt{3}}{2} + j\frac{1}{2}\right) = \sqrt{3}U_F e^{j30^0}.$$

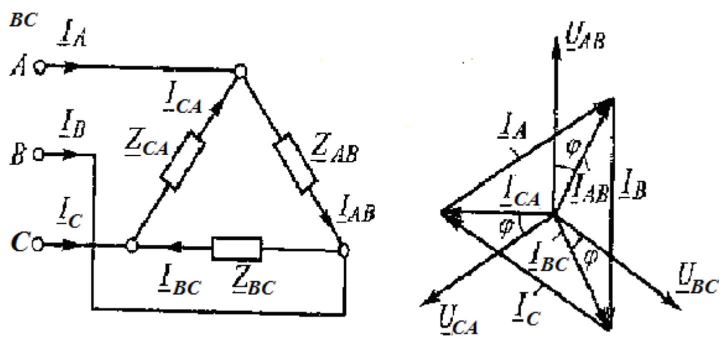
Xuddi shunga o‘xshash,

$$\underline{U}_{BC} = \sqrt{3}\underline{U}_B e^{j30^0} = \sqrt{3}U_F e^{-j90^0},$$
$$\underline{U}_{CA} = \sqrt{3}\underline{U}_C e^{j30^0} = \sqrt{3}U_F e^{j150^0}.$$

Yuklama uchburchak usulda ulangan zanjirda simmetrik rejim. Yuklama uchburchak ulangan uch fazali zanjirda $\underline{Z}_A = \underline{Z}_B = \underline{Z}_C = ze^{j\varphi}$ shart bajarilganida va u simmetrik EYK (kuchlanish) lar sistemasidan ta’minlanganida zanjirda simmetrik rejim yuzaga keladi (3.4- rasm, *a*). Kuchlanishlarning topografik diagrammasi 3.4- rasm, *b* da keltirilgan. Unga ko‘ra kuchlanishlar simmetrik yulduz hosil qiladi.

Kirxgofning birinchi qonuniga ko‘ra liniya toklari faza toklari orqali quyidagicha ifodalanadi:

$$\underline{I}_A = \underline{I}_{AB} - \underline{I}_{CA},\ \underline{I}_B = \underline{I}_{BC} - \underline{I}_{AB},\ \underline{I}_C = \underline{I}_{CA} - \underline{I}_{BC}.$$



3.4- rasm.

$\underline{I}_A$ liniya tokini faza toklari orqali ifodalaganda:

$$\underline{I}_A = \underline{I}_{AB} - \underline{I}_{CA} = I_F - I_F e^{j120^0} = I_F(1 - e^{j120^0}) = \sqrt{3}I_F e^{-j30^0},$$

boshqa liniya toklari uchun

$$\underline{I}_B = \underline{I}_{BC} - \underline{I}_{AB} = \sqrt{3}I_F e^{-j150^0}, \underline{I}_C = \sqrt{3}I_F e^{j90^0}.$$

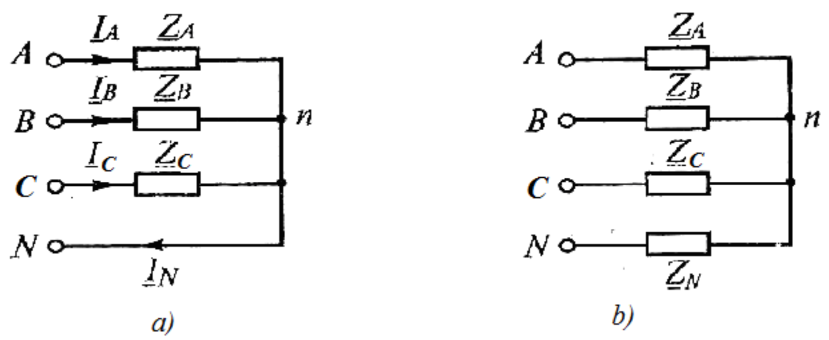
Simmetrik uch fazali sistemaning quvvatlari. Uch fazali zanjirlarni hisoblashda liniya kuchlanishi va toklaridan foydalanish qulayroq. Yulduz ulangan sxema uchun $U_F = U_L/\sqrt{3}$, $I_F = I_L$, uchburchak ulangan sxema uchun esa $U_F = U_L$, $I_F = I_L/\sqrt{3}$ bo‘lganligi sababli ulanish sxemasidan qat’iy nazar simmetrik uch fazali zanjirning aktiv quvvati $P = \sqrt{3}U_L I_L \cos\varphi$ ifoda bilan aniqlanadi.

Simmetrik uch fazali zanjirning reaktiv va to‘la quvvatlari quyidagicha topiladi:

$$Q = \sqrt{3}U_L I_L \sin\varphi, \quad S = \sqrt{3}U_L I_L.$$

Nosimmetrik uch fazali zanjirlar.

Yulduz usulda ulangan neytral simli zanjir. Yuklama qarshiliklari $(\underline{Z}_A \neq \underline{Z}_B \neq \underline{Z}_C)$ yulduz usulda ulangan to‘rt simli $(Z_N = 0)$ zanjir simmetrik kuchlanishlar sistemasidan ta’minlanayotgan bo‘lsin (3.5-rasm, *a*). Qarshiligi nolga teng bo‘lgan neytral simning mavjudligi fazalarning o‘zaro bog‘liqsizligini ta’minlaydi va shuning uchun $\underline{U}_A = U_F$; $\underline{U}_B = U_F e^{-j120^0}$; $\underline{U}_C = U_F e^{j120^0}$ bo‘lganda iste’molchilar faza toklari $\underline{I}_A = \frac{\underline{U}_A}{\underline{Z}_A}$, $\underline{I}_B = \frac{\underline{U}_B}{\underline{Z}_B}$, $\underline{I}_C = \frac{\underline{U}_C}{\underline{Z}_S}$ tengliklar yordamida, neytral simdagi tok esa Kirxgofning 1- qonuniga ko‘ra $\underline{I}_N = \underline{I}_A + \underline{I}_B + \underline{I}_C$ formula bilan topiladi.



3.5- rasm.

Agar neytral simning qarshiligini hisobga olish lozim bo‘lsa, u holda neytralning siljish kuchlanishi aniqlanadi (3.5- rasm, *b*):

$$\underline{U}_N = \underline{I}_N \underline{Z}_N = \frac{\underline{U}_A \underline{Y}_A + \underline{U}_B \underline{Y}_B + \underline{U}_C \underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C + \underline{Y}_N},$$

bu yerda $\underline{Y}_A = \frac{1}{\underline{Z}_A}$, $\underline{Y}_B = \frac{1}{\underline{Z}_B}$, $\underline{Y}_C = \frac{1}{\underline{Z}_C}$, $\underline{Y}_N = \frac{1}{\underline{Z}_N}$ – fazalar va neytral sim kompleks o‘tkazuvchanliklari.

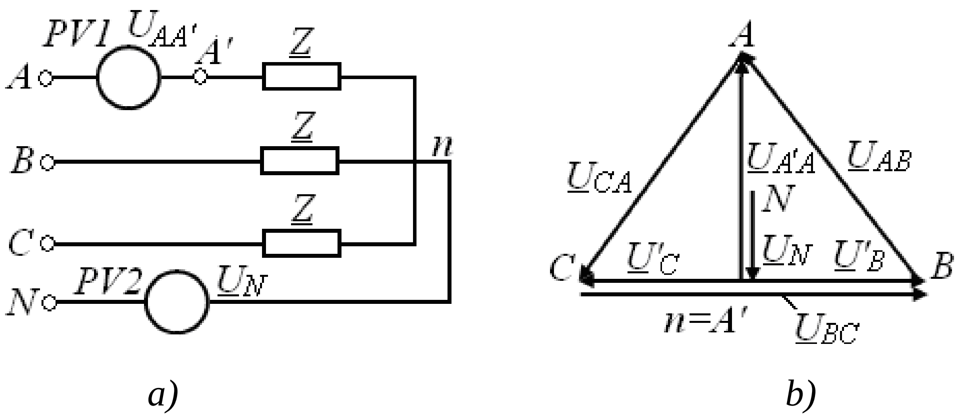
Yulduz usulda ulangan neytral simsiz zanjir. Neytral simsiz yulduz sxemada faza kuchlanishlarining o‘zaro bog‘liqsizligi ta’minlanmaydi. Bir fazada yuklama qiymatining o‘zgarishi boshqa fazalarda tok va kuchlanishlarning o‘zgarishiga sabab bo‘ladi. Masalan, *A* fazada liniya simining uzilishi qolgan ikkita faza qarshiliklari o‘zaro ketma-ket ulanib, $\underline{U}_{BC}$ liniya kuchlanishi ta’siri ostida qolishiga sabab bo‘ladi (3.6-rasm, *a*). $\underline{Z}_B = \underline{Z}_C = \underline{Z}$ bo‘lganida $\underline{Z}_B$ va $\underline{Z}_C$ qarshiliklarda $\underline{U}_{BC}$ liniya kuchlanishi teng bo‘linib pasayadi (3.6- rasm, *b*). Bunda neytralning siljish kuchlanishi $\underline{U}_N = \underline{U}_{nN} = -\frac{\underline{E}_A}{2}$, *A* faza simining uzilgan nuqtalari orasidagi kuchlanish esa $\underline{U}_{AA'} = \underline{U}_{AN'} = \underline{E}_A - \underline{U}_N = \frac{3}{2}\underline{E}_A$ ga teng bo‘ladi.

Oxirgi tenglikni ikki tugun usulini qo‘llab aniqlashimiz ham mumkin, ya’ni:

$$\underline{U}_N = \frac{\sum \underline{E}\underline{Y}}{\sum \underline{Y}} = \frac{\underline{E}_A \cdot 0 + \underline{E}_B\underline{Y}_B + \underline{E}_C\underline{Y}_C}{0 + \underline{Y}_B + \underline{Y}_C} = \frac{\underline{Y}_B(\underline{E}_B + \underline{E}_C)}{2\underline{Y}_B} = -\frac{\underline{E}_A}{2},$$

$$\underline{U}_{AA'} = \underline{E}_A - \underline{U}_N = \underline{E}_A + \frac{\underline{E}_A}{2} = \frac{3}{2}\underline{E}_A.$$

Shunday qilib, yulduz sxemali simmetrik uch fazali zanjirda bitta liniya simining uzilishi o‘sha faza yuklamasidagi kuchlanishni nolgacha, boshqa ikkita faza yuklamalaridagi kuchlanishlarni $\frac{U_F}{\frac{U_L}{2}} = U_F : \frac{\sqrt{3}U_F}{2} = \frac{2}{\sqrt{3}}$ martagacha, ya’ni 16% gacha kamaytirib yuboradi.



3.6- rasm.

Yuklamasi uchburchak ulangan zanjir. Nosimmetrik $(\underline{Z}_{AB} \neq \underline{Z}_{BC} \neq \underline{Z}_{CA})$ yuklamali uch fazali zanjirda faza toklari Om qonuniga binoan aniqlanadi:

$$\underline{I}_{AB} = \frac{\underline{U}_{AB}}{\underline{Z}_{AB}},\ \underline{I}_{BC} = \frac{\underline{U}_{BC}}{\underline{Z}_{BC}},\ \underline{I}_{CA} = \frac{\underline{U}_{CA}}{\underline{Z}_{CA}}.$$

Liniya toklari *A*, *B* va *C* tugunlar uchun tuzilgan Kirxgofning 1- qonunidan foydalanib topiladi:

$$\underline{I}_A = \underline{I}_{AB} - \underline{I}_{CA},\ \ \underline{I}_B = \underline{I}_{BC} - \underline{I}_{AB},\ \ \underline{I}_C = \underline{I}_{CA} - \underline{I}_{BC}.$$

Hisoblash natijalarini $\underline{I}_A + \underline{I}_B + \underline{I}_C = 0$ tenglama yordamida tekshirib ko‘rish mumkin. Shuni ta’kidlash joizki, nosimmetrik rejimda $I_L \neq \sqrt{3} I_F$.

Nosimmetrik uch fazali zanjirning quvvati. Nosimmetrik uch fazali zanjirda fazalar quvvati o‘zaro teng bo‘lmaydi. Yulduz sxemada ulangan uch fazali zanjirning kompleks quvvati fazalar kompleks quvvatlarining algebraik yig‘indisiga teng, ya’ni:

$$\underline{S} = \underline{S}_A + \underline{S}_B + \underline{S}_C = \underline{U}_A \overset{*}{\underline{I}}_A + \underline{U}_B \overset{*}{\underline{I}}_B + \underline{U}_C \overset{*}{\underline{I}}_C.$$

Zanjirning aktiv va reaktiv quvvatlari:

$$P = P_A + P_B + P_C = U_A I_A \cos\varphi_A + U_B I_B \cos\varphi_B + U_C I_C \cos\varphi_S;$$

$$Q = Q_A + Q_B + Q_C = U_A I_A \sin\varphi_A + U_B I_B \sin\varphi_B + U_C I_C \sin\varphi_C.$$

Yuklama uchburchak sxemada ulangan nosimmetrik uch fazali zanjir quvvati ifodalari yuqorida qayd etilganidek, faza quvvatlarining yig‘indisi ko‘rinishida aniqlanadi. Uch fazali zanjirda ham $S = \sqrt{P^2 + Q^2}$ tenglik o‘rinli.

Tarmoqlangan uch fazali zanjirlarni hisoblash. Yulduz va uchburchak sxemalarda ulangan hamda liniya qarshiliklari hisobga olinishi kerak bo‘lgan bir nechta iste’molchiga ega uch fazali zanjirlarni hisoblashda ekvivalent o‘zgartirish usullaridan, xususan, qarshiliklar uchburchagini unga ekvivalent bo‘lgan qarshiliklar yulduziga va aksincha o‘zgartirishlardan keng foydalaniladi. Bu almashtirishlarga ko‘ra:

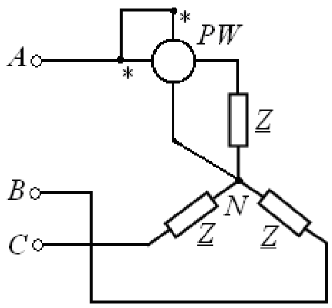
$$\underline{Z}_A = \frac{\underline{Z}_{AB}\underline{Z}_{CA}}{\underline{Z}_{AB} + \underline{Z}_{BC} + \underline{Z}_{CA}},\ \underline{Z}_B = \frac{\underline{Z}_{BC}\underline{Z}_{AB}}{\underline{Z}_{AB} + \underline{Z}_{BC} + \underline{Z}_{CA}},\ \underline{Z}_C = \frac{\underline{Z}_{CA}\underline{Z}_{BC}}{\underline{Z}_{AB} + \underline{Z}_{BC} + \underline{Z}_{CA}}.$$

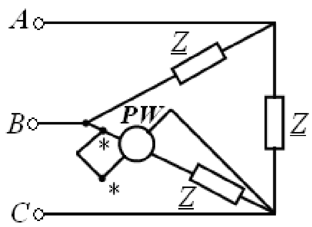
Simmetrik yuklamada $Z_{Y} = \frac{Z_{\Delta}}{3}$.

$\underline{Z}_{AB} = \underline{Z}_{A} + \underline{Z}_{B} + \frac{\underline{Z}_{A}\underline{Z}_{B}}{\underline{Z}_{C}}$, $\underline{Z}_{BC} = \underline{Z}_{B} + \underline{Z}_{C} + \frac{\underline{Z}_{B}\underline{Z}_{C}}{\underline{Z}_{A}}$,

$\underline{Z}_{CA} = \underline{Z}_{C} + \underline{Z}_{A} + \frac{\underline{Z}_{C}\underline{Z}_{A}}{\underline{Z}_{B}}$. Simmetrik yuklamada $Z_{\Delta} = 3Z_{Y}$.

Simmetrik uch fazali zanjirda aktiv quvvatni o‘lchash. Ma’lumki, simmetrik uch fazali zanjirda faza quvvatlari o‘zaro teng, shuning uchun ham zanjir quvvatini aniqlash uchun bitta fazadagi aktiv quvvatni o‘lchash kifoya. Zanjirning aktiv quvvati $P = 3P_{W} = 3P_{F}$ bo‘ladi. 3.7- rasm, *a* da yulduz ulangan va neytral nuqtasidan foydalanish imkoniyati bo‘lgan zanjirning *A* fazasiga vattmetrni ulash sxemasi keltirilgan. Iste’molchi uchburchak ulanganda vattmetrni ulash sxemasi esa 3.7- rasm, *b* da ko‘rsatilgan.





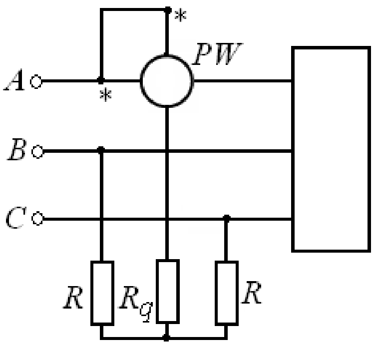
a) *b)*

3.7- rasm.

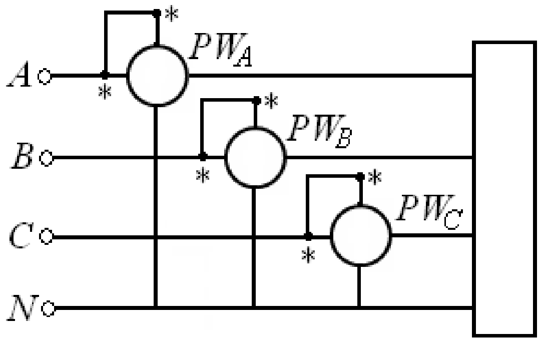
Agar yulduz ulangan zanjirda neytral nuqtadan foydalanish imkoniyati bo‘lmasa, u holda uni sun’iy ravishda hosil qilinadi. Bu nuqta uchta rezistorning yulduz ulanishidan hosil bo‘ladi (3.8-rasm). Bunda $R_q+R_W = R$, bu yerda R_W – vattmetr kuchlanish chulg‘amining qarshiligi. Zanjirning aktiv quvvati $P = 3P_{W}$ ifoda bilan aniqlanadi.

Nosimmetrik uch fazali zanjirda aktiv quvvatni o‘lchash. Nosimmetrik uch fazali zanjirlarda fazalarning aktiv quvvati o‘zaro teng emas. Shuning uchun to‘rt simli uch fazali zanjirda har bir fazadagi aktiv quvvatni o‘lchash lozim bo‘ladi (3.9- rasm). Istalgan xarakterli yuklamada uch fazali zanjirning aktiv quvvati uchala vattmetr ko‘rsatishlarining yig‘indisiga teng:

$$P = P_A + P_B + P_C = U_A I_A \cos\varphi_A + U_B I_B \cos\varphi_B + U_C I_C \cos\varphi_C \,.$$

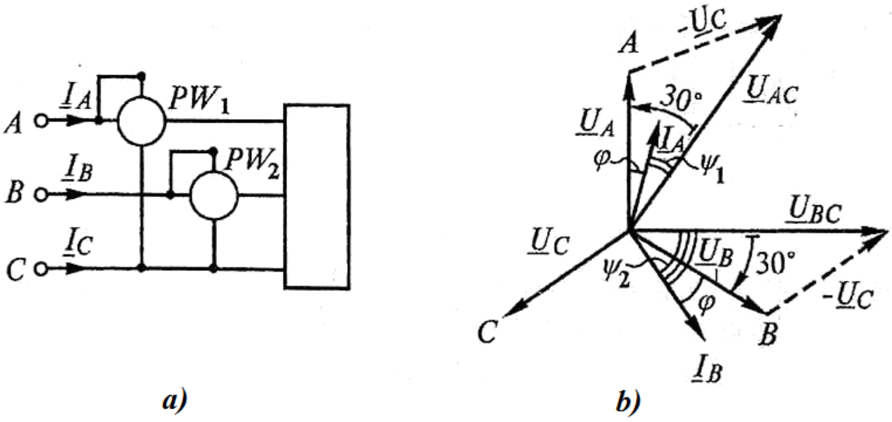


3.8- rasm.



3.9- rasm.

Amalda uch simli uch fazali zanjirlarda aktiv quvvat ko‘pchilik hollarda ikkita vattmetr yordamida o‘lchanadi (3.10- rasm, *a*). Ikkala vattmetr ko‘rsatishining yig‘indisi zanjir aktiv quvvatiga teng.



3.10- rasm.

Simmetrik yuklamada ikkita vattmetr ko‘rsatishi asosida zanjir reaktiv quvvatini va faza siljish burchagini aniqlash mumkin:

$$P_1 - P_2 = UI(\cos(30^0 - \varphi) - \cos(30^0 + \varphi)) = UI \sin\varphi \,,$$

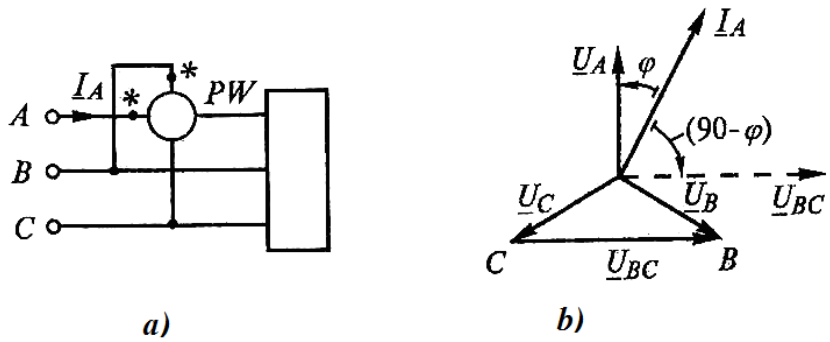
$$Q = \sqrt{3} UI \sin\varphi = \sqrt{3}(P_1 - P_2) \,,$$

$$\mathrm{tg}\varphi = \frac{Q}{P} = \frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2} \,, \; \varphi = \mathrm{arctg}\frac{\sqrt{3}(P_1 - P_2)}{P_1 + P_2} \,.$$

Reaktiv quvvatni o‘lchash. 3.11- rasm, *a* da uch fazali simmetrik zanjir reaktiv quvvatini vattmetr yordamida o‘lchash sxemasi keltirilgan. Vattmetrlar ko‘rsatkichi $\underline{U}_{BC}$ va $\underline{I}_A$ vektorlarning skalyar ko‘paytmasi bilan aniqlanadi, ya’ni:

$$P_W = U_{BC} I_A \cos\left(\underline{U}_{BC} \hat{\ } \underline{I}_A\right) = U_{BC} I_A \cos(90^\circ - \varphi) = U_{BC} I_A \sin\varphi.$$

$\underline{U}_{BC}$ va $\underline{I}_A$ vektorlar orasidagi faza siljish burchagi 3.11- rasm, *b* dagi vektor diagrammadan aniqlanadi.



3.11- rasm

Zanjirning reaktiv quvvati:

$$Q = \sqrt{3} U_{BC} I_A \sin\varphi = \sqrt{3} P_W .$$

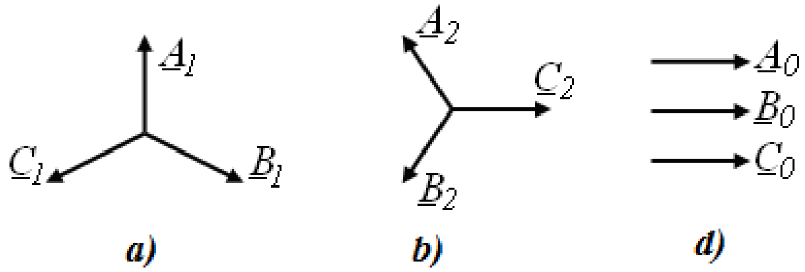
Simmetrik tashkil etuvchilar usuli. Nosimmetrik uch fazali zanjirlarda tok yoki kuchlanishlarni simmetrik tashkil etuvchilar yig‘indisi ko‘rinishida ifodalash asosida hisoblash usuli *simmetrik tashkil etuvchilar usuli* deb ataladi. Ushbu usulning mohiyati quyidagicha:

1. Istalgan uch fazali nosimmetrik kuchlanishlar sistemasi nol, to‘g‘ri va teskari deb ataluvchi ketma-ketlikli tashkil etuvchilar ko‘rinishida ifodalanadi.

2. Kuchlanishning har bir tashkil etuvchisidan zanjirda hosil bo‘lgan toklar aniqlanadi.

3. Ustma-ustlash prinsipi asosida zanjirdagi toklar topiladi.

To‘g‘ri ketma-ketlikli tashkil etuvchilar modullari teng va o‘zaro 120^0 ga siljigan A_1, B_1 va C_1 vektorlardan iborat bo‘lib, ketma-ketlik tartibi A_1, B_1, C_1 ko‘rinishida bo‘ladi (3.12- rasm, a). Bunda $\underline{B}_1 = a^2 \underline{A}_1$, $\underline{C}_1 = a\underline{A}_1$.



3.12- rasm.

Teskari ketma-ketlikli tashkil etuvchilar modullari teng va o‘zaro 120^0 ga siljigan A_2, B_2, C_2 vektorlardan iborat bo‘lib, ketma-ketlik tartibi A_2, C_2, B_2 ko‘rinishida bo‘ladi (3.12- rasm, *b*). Bunda $\underline{B}_2 = a\underline{A}_2,\ \underline{C}_2 = a^2\underline{A}_2$.

Nol ketma-ketlikli tashkil etuvchilar modullari teng va faza jihatidan mos bo‘lgan 3 ta bir xil vektordan iborat (3.12- rasm, *d*). Bunda $\underline{A}_0 = \underline{B}_0 = \underline{C}_0$.

$\underline{A},\ \underline{B},\ \underline{C}$ nosimmetrik vektorlar sistemasini uchta simmetrik tashkil etuvchilar yig‘indisi ko‘rinishida yozamiz:

$$\left.\begin{aligned} &\underline{A} = \underline{A}_1 + \underline{A}_2 + \underline{A}_0, \\ &\underline{B} = \underline{B}_1 + \underline{B}_2 + \underline{B}_0 = a^2\underline{A}_1 + a\underline{A}_2 + \underline{A}_0, \\ &\underline{C} = \underline{C}_1 + C_2 + \underline{C}_0 = a\underline{A}_1 + a^2\underline{A}_2 + \underline{A}_0. \end{aligned}\right\} \qquad (3.5)$$

Agar $\underline{A},\ \underline{B},\ \underline{C}$ nosimmetrik kattaliklar (tok yoki kuchlanish) berilgan bo‘lsa, u holda nol, to‘g‘ri va teskari tashkil etuvchilar quyidagilarga teng:

$$\underline{A}_0 = \frac{1}{3}(\underline{A} + \underline{B} + \underline{C}). \qquad (3.6)$$

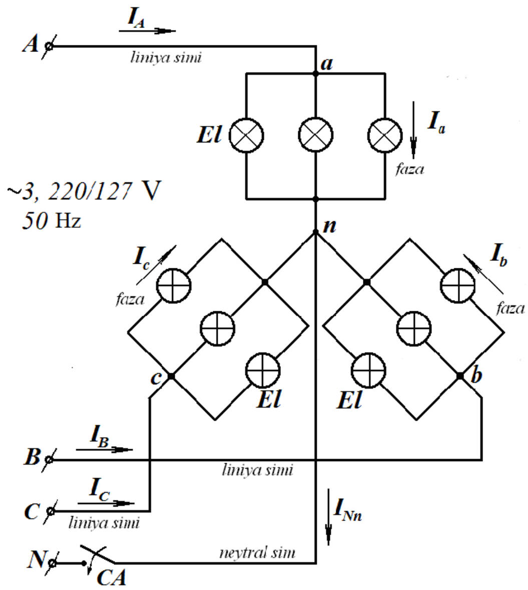
$$\underline{A}_1 = \frac{1}{3}(\underline{A} + a\underline{B} + a^2\underline{C}), \qquad (3.7)$$

$$\underline{A}_2 = \frac{1}{3}(\underline{A} + a^2\underline{B} + aC). \qquad (3.8)$$

3.2. Simmetrik uch fazali zanjirlar

3.1- masala. Nominal kuchlanishi $U_{\text{n}} = 220\,\text{V}$ va quvvati $P_{\text{n}} = 25\,\text{W}$ bo‘lgan 3 tadan cho‘g‘lanma lampalar yulduz sxemasida birlashtirilib, fazalararo (liniya) kuchlanishi 220 V, faza kuchlanishi esa 127 V li uch fazali elektr energiya manbasiga (3.13- rasm) ulangan. Liniya va faza toklarini hamda neytral simdan o‘tuvchi I_{Nn} tokni aniqlang. Kuchlanishlarning topografik va toklarning vektor diagrammasini yasang.

Yechish. Uch fazali sistema simmetrik bo‘lgani uchun faqat bir faza uchun fazadan o‘tuvchi tokni hisoblaymiz.



3.13- rasm.

Buning uchun dastlab lampaning qarshiligini topamiz:

$$R_{EL} = \frac{U^2_{\text{nom}}}{P_{\text{nom}}} = \frac{220^2}{25} = \frac{48400}{25} = 1936\ \Omega.$$

Uchta bir xil ulangan lampalar o‘zaro parallel ulangani uchun ularning umumiy qarshiligi:

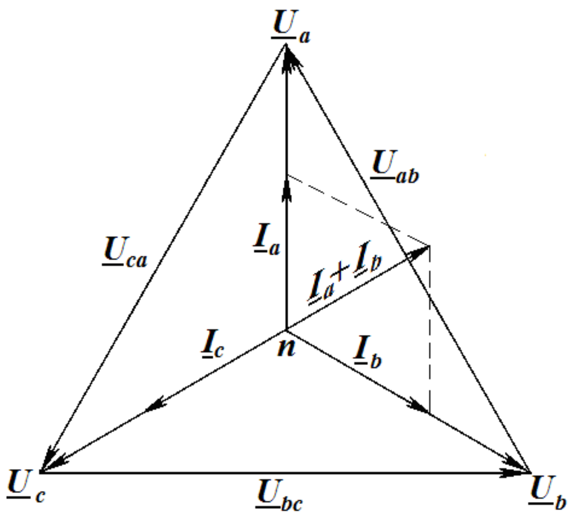
$$R_{\text{um}} = \frac{R_{EL}}{3} = \frac{1936}{3} = 645{,}3\ \Omega\,.$$

Fazadan o‘tuvchi tok $I_F = \dfrac{U_F}{R_{\text{um}}} = \dfrac{127}{645{,}3} = 0{,}196\ \text{A}.$

Kuchlanish va toklarning vektor diagrammasini quyidagi masshtabda yasaymiz:

$$m_U = 44\ \text{V/sm},$$
$$m_I = 0{,}1\ \text{A/sm}.$$

Dastlab $\underline{U}_a$, $\underline{U}_b$ va $\underline{U}_s$ faza kuchlanishlar vektorlarini $2\pi/3$ rad (120°) o‘zaro siljish burchagi bilan yasaymiz (3.14- rasm). $\underline{U}_{ab}$, $\underline{U}_{bc}$ va $\underline{U}_{ca}$ fazalararo kuchlanish vektorlari $\underline{U}_{ab} = \underline{U}_a - \underline{U}_b$, $\underline{U}_{bc} = \underline{U}_b - \underline{U}_c$ va $\underline{U}_{ca} = \underline{U}_c - \underline{U}_a$ tegishli faza vektorlarining ayirmasi, ular teng tomonli uchburchak hosil qiladi. $\underline{I}_a$, $\underline{I}_b$ va $\underline{I}_c$ faza toklari mos $\underline{U}_a$, $\underline{U}_b$ va $\underline{U}_c$ faza kuchlanishi vektorlari bilan ustma-ust bo‘ladi. Vektor diagrammadan neytral simdan o‘tuvchi tok $\underline{I}_{Nn} = \underline{I}_a + \underline{I}_b + \underline{I}_c = 0$ va fazalararo kuchlanishlar faza kuchlanishidan $\sqrt{3}$ marta katta ekanligini ko‘rish mumkin.



3.14- rasm.

3.2- masala. Agar 3.1- masaladagi *a* fazada bitta lampa, *b* fazada ikkita va *c* fazada uchta lampa bo‘lsa, faza toklarini hisoblang. Kuchlanish va toklarning vektor diagrammasini yasang.

Yechish. *a* fazadan o‘tuvchi tok

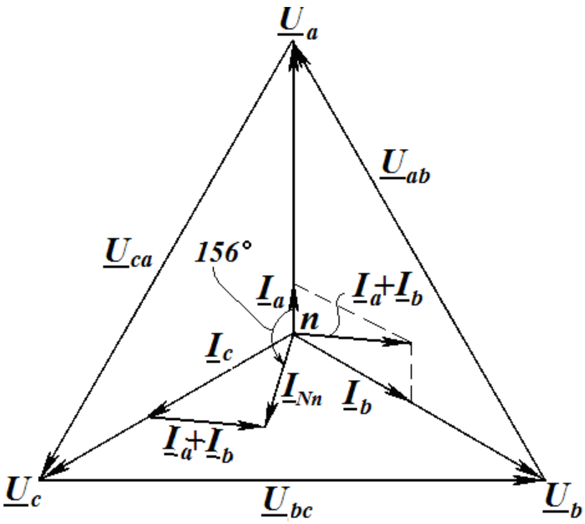
$$I_a = \frac{U_F}{R_{EL}} = \frac{127}{1936} = 0{,}065 \text{ A}.$$

b va *c* faza toklari

$$I_b = 0{,}13 \text{ A}, \quad I_s = 0{,}196 \text{ A}.$$

Vektor diagrammani $m_U = 44$ V/sm, $m_I = 0{,}1$ A/sm masshtabda yasaymiz (3.15-rasm). *Nn* neytral simdan o‘tuvchi tokni $\underline{I}_{Nn} = \underline{I}_a + \underline{I}_b + \underline{I}_c$ ifoda bo‘yicha vektor diagrammadan topamiz.

Demak, neytral simdagi tok $\underline{I}_{Nn} = 0{,}105$ A, uning fazasi esa 156° bo‘lib, fazalarning har xil yuklanishida ham neytral sim faza kuchlanishlarining simmetriyasini ta’minlaydi.



3.15- rasm.

3.3- masala. Agar 3.2- masalada *Nn* neytral sim uzilsa (3.13- rasmda *CA* kalit ochiq), fazalar tokini va kuchlanishlarini toping, vektor diagrammasini yasang.

Yechish. *CA* kalit ochiq bo‘lsa, *Nn* neytral simdan tok o‘tmaydi. Iste’molchilar *n* nuqtasi *N* manba neytraliga nisbatan siljiydi. Natijada $\underline{U}_a$, $\underline{U}_b$ va $\underline{U}_C$ faza kuchlanishlari har xil qiymatga ega bo‘ladi. $\underline{U}_{Nn}$ – neytralning siljish kuchlanishini quyidagi formula yordamida hisoblash mumkin:

$$\underline{U}_{Nn} = \frac{\underline{U}_a\underline{Y}_a + \underline{U}_b\underline{Y}_b + \underline{U}_C\underline{Y}_C}{\underline{Y}_a + \underline{Y}_b + \underline{Y}_C},$$

bunda $\underline{Y}_a = \dfrac{1}{\underline{Z}_a}$, $\underline{Y}_b = \dfrac{1}{\underline{Z}_b}$ va $\underline{Y}_C = \dfrac{1}{\underline{Z}_C}$ – fazalarning kompleks o‘tkazuvchanliklari, bizning masalada esa,

$$\underline{Y}_a = G_a = \frac{1}{R_a} = \frac{1}{1936} = 5{,}16\cdot10^{-4}\ \text{S},$$

$$\underline{Y}_b = G_b = \frac{1}{R_b} = \frac{1}{968} = 10{,}32\cdot10^{-4}\ \text{S}$$

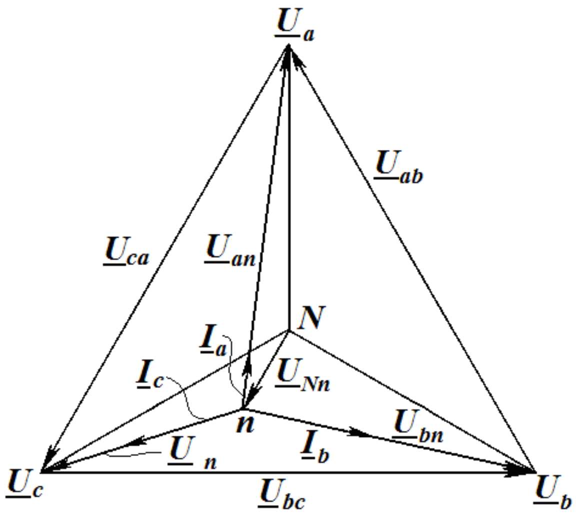
$$\underline{Y}_C = G_C = \frac{1}{R_C} = \frac{1}{645{,}3} = 15{,}5\cdot10^{-4}\ \text{S}.$$

$$\underline{U}_{Nn} = \frac{127e^{j0^\circ}\cdot5{,}16\cdot10^{-4} + 127e^{-j120^\circ}\cdot10{,}32\cdot10^{-4} + 127e^{j120^\circ}\cdot15{,}5\cdot10^{-4}}{5{,}16\cdot10^{-4} + 10{,}32\cdot10^{-4} + 15{,}5\cdot10^{-4}} =$$

$$= \frac{655{,}32\cdot10^{-4}e^{j0^\circ} + 1310{,}64\cdot10^{-4}e^{-j120^\circ} + 1968{,}5\cdot10^{-4}e^{j120^\circ}}{30{,}98\cdot10^{-4}} =$$

$$= -31{,}76 + j18{,}26 = 36{,}63\cdot e^{-j150^\circ10'}.$$

Tok va kuchlanishlarning vektor diagrammasi 3.16-rasmda keltirilgan. Undan berilgan masshtabda $(m_U = 44 \text{ V/sm}, \; m_I = 0{,}1 \text{ A/sm})\, U_{an} = 167\,\text{V}$, $U_{bn} = 140{,}8\,\text{V}$ $U_{cn} = 101\,\text{V}$ ni topish mumkin



3.16- rasm.

$\underline{U}_{Nn}$ vektorni grafik usulda ham aniqlash mumkin. Bu holda tajriba natijalaridan foydalaniladi.

O‘lchangan U_{a_n}, U_{b_n} va U_{c_n} faza kuchlanishlarini tanlangan kuchlanish masshtabi bo‘yicha *a*, *b* va *c* nuqtalardan sirkul yordamida o‘tkazilgan yoylarning kesishgan nuqtasi iste’mol neytrali *n* bo‘ladi.

Topilgan nosimmetrik kuchlanish qiymatlaridan foydalanib faza toklari hisoblanadi:

$$I_a = \frac{U_{a_n}}{R_{EL}} = \frac{167}{1936} = 0{,}086\text{ A}, \quad I_b = \frac{U_{b_n}}{R_b} = \frac{140{,}8}{968} = 0{,}145\text{ A},$$

$$I_c = \frac{U_{c_n}}{R_{\text{um}}} = \frac{101}{645{,}3} = 0{,}156\text{ A}.$$

Bu toklarning vektorlari $\underline{I}_a$, $\underline{I}_b$ va $\underline{I}_c$ tegishli faza kuchlanish vektorlari bilan ustma-ust joylashadi.

3.4-masala. Agar 3.1-masaladagi cho‘g‘lanma lampalar uchburchak usulda birlashtirilib, $U_{\text{nom}} = 220\text{ V}$ kuchlanishli uch fazali elektr energiya manbayiga ulangan bo‘lsa (3.17- rasm), liniya va faza toklarini aniqlang. Kuchlanish va toklarning vektor diagrammasini yasang.

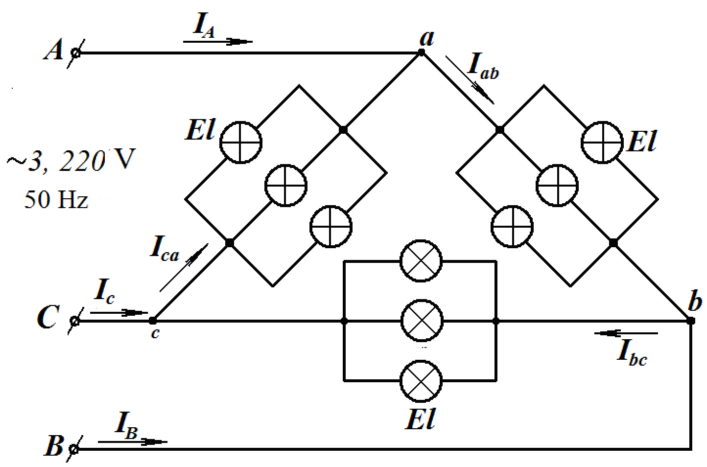
Yechish. I_{ab}, I_{bc} va I_{ca} faza toklarini hisoblaymiz, $\sim 3,\ 220\ \text{V},\ 50\ \text{Hz}$.

$$I_{ab} = \frac{U_{ab}}{R_{ab}} = \frac{220}{645{,}3} = 0{,}34\ \text{A},\quad I_{bc} = \frac{U_{bc}}{R_{bc}} = \frac{220}{645{,}3} = 0{,}34\ \text{A},\quad I_{ca} = \frac{U_{ca}}{R_{ca}} = \frac{220}{645{,}3} = 0{,}34\ \text{A}.$$

Kuchlanish va faza toklarining vektor diagrammalarini $m_U = 44\ \text{V/sm},\ m_I = 0{,}17\ \text{A/sm}$ masshtablarda yasaymiz.

$\underline{I}_a$, $\underline{I}_b$ va $\underline{I}_c$ liniya toklari faza toklaridan $\sqrt{3}$ marta katta bo‘ladi:

$$I_{\text{liniya}} = I_A = I_B = I_C = \sqrt{3} I_{\text{faza}} = \sqrt{3} \cdot 0{,}34 = 0{,}588\ \text{A}.$$



3.17- rasm.

Har bir liniya toki vektorini

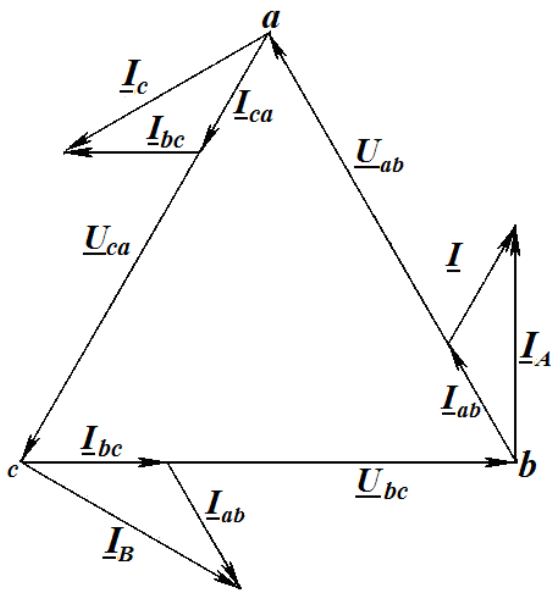
$$\underline{I}_A = \underline{I}_{ab} - \underline{I}_{ca},\quad \underline{I}_B = \underline{I}_{bc} - \underline{I}_{ab},\quad \underline{I}_C = \underline{I}_{ca} - \underline{I}_{bc}$$

ifodalardan foydalanib, tegishli faza tok vektorlarining ayirmasi sifatida grafik usuldan foydalanib topamiz (3.18- rasm). Toklarning faza siljish burchaklarini transportir bilan o‘lchab,

$$\underline{I}_A = 0{,}58\, e^{j0^\circ},\quad \underline{I}_B = 0{,}58\, e^{-j120^\circ},\quad \underline{I}_C = 0{,}58\, e^{j120^\circ}$$

yozamiz.

Xulosa qilib aytganda, iste’molchilar yulduz va uchburchak sxemalarida ulanganda $P_\Delta = 3P_Y$ ekanligiga ishonch hosil qilish mumkin.



3.18- rasm.

Yulduz sxemada quvvat

$$P_Y = \sqrt{3} U_L\, I_L \cos\varphi = \sqrt{3} \cdot 220 \cdot 0{,}197 \cong 75\,\text{W}.$$

Uchburchak sxemada quvvat

$$P_\Delta = \sqrt{3} U_L\, I_L \cos\varphi = \sqrt{3} \cdot 220 \cdot 0{,}589 \cong 225\,\text{W}.$$

Demak, $P_\Delta = 3P_Y$ bo‘ladi.

Ikkala holda ham quvvat koeffitsiyenti $\cos\varphi = 1$.

3.5-masala. Uchta bir xil rezistor uchburchak usulida simmetrik liniya kuchlanishiga ulangan. Agar rezistorlar liniya kuchlanishiga yulduzcha usulida qayta ulansa, liniya toklari necha marta o‘zgaradi?

Yechish. Uchburchak usulida ulanganda rezistorlar liniya kuchlanishi U_l ta’sirida bo‘lib, ulardagi toklar $I_l/\sqrt{3}$ ga teng. Yulduzcha usulida ulanganda rezistorlar $U_l/\sqrt{3}$ kuchlanish ta’sirida bo‘ladi va oqibatda ulardagi tok $\sqrt{3}$ marta kamayadi hamda $I_l/3$ ga teng bo‘ladi, bunda I_l – avvalgi liniya toki kattaligi. Yulduzcha usulda ulanganda liniyadagi va yuklama fazalaridagi toklar teng bo‘ladi. Shunday qilib, yulduzcha usulida ulangan sxemadagi liniya toklar uchburchak bo‘yicha ulangan sxemaga nisbatan 3 marta kam bo‘ladi.

3.3. Nosimmetrik uch fazali zanjirlar

3.6-masala. Quyida berilgan uch fazali kuchlanishlarning simmetrik tashkil etuvchilarini hisoblang. $\underline{U}_{AB} = (-120 + j90)$ V, $\underline{U}_{BC} = 120$ V, $\underline{U}_{CA} = -j90$ V.

Yechish. Nol ketma-ketlikni hisoblaymiz:

$$\underline{U}_0 = \frac{\underline{U}_{AB} + \underline{U}_{BC} + \underline{U}_{CA}}{3} = \frac{-120 + j90 + 120 - j90}{3} = 0.$$

Demak, nol ketma-ketlik nolga teng.

Simmetrik tashkil etuvchilarning to‘g‘ri ketma-ketligini quyidagi formulaga ko‘ra hisoblaymiz:

$$\underline{U}_1 = \frac{\underline{U}_{AB} + a\underline{U}_{BC} + a^2\underline{U}_{CA}}{3}.$$

Bu ifodada $\underline{U}_{BC}$ oldidagi operator $a = e^{j120^\circ} = -0{,}5 + j0{,}866$ bo‘lgani uchun, $\underline{U}_{BC} = 120e^{j0^\circ}$ ni a ga ko‘paytiramiz.

$\underline{U}_{CA}$ ni operator a^2 ga ko‘paytiramiz, bunda $a^2 = e^{-j120^\circ} = -0{,}5 - j0{,}866.$

$$\underline{U}_1 = \frac{-120 + j90 + (-0{,}5 + j0{,}866)\cdot 120 + (-0{,}5 - j0{,}866)(-j90^\circ)}{3} = (-86 + j79{,}6) \text{ V}$$

bo‘ladi.

Simmetrik tashkil etuvchilarning teskari ketma-ketligini hisoblaymiz:

$$\underline{U}_2 = \frac{\underline{U}_{AB} + a^2\underline{U}_{BC} + a\underline{U}_{CA}}{3} = \frac{-120 + j90 + (-0{,}5 - j0{,}866)120 + (-0{,}5 + j0{,}866)(-j90^\circ)}{3} =$$
$$= (-34 + j10{,}4) \text{ V}.$$

Liniya kuchlanishlarining yig‘indisi uch fazali elektr zanjirlarda hamma vaqt nolga teng, ya’ni

$$\underline{U}_{C_л} = \frac{\underline{U}_{AB} + \underline{U}_{BC} + \underline{U}_{CA}}{3} = 0,$$

demak, liniya kuchlanishlari nol ketma-ketlikka ega emas.

Uch fazali elektr zanjirlarda liniya kuchlanishlari qancha nosimmetrik bo‘lsa, simmetrik tashkil etuvchilari tarkibidagi teskari ketma-ketlik tashkil etuvchilari shuncha katta bo‘ladi. Shuning uchun liniya kuchlanishlarining teskari ketma-ketlik qiymatining to‘g‘ri ketma-ketlik qiymatiga nisbati $\frac{U_2}{U_1} \cdot 100\%$ asimmetriya o‘lchami bo‘lib, u *asimmetriya ko‘rsatkichi* deb ataladi.

Elektrotexnik qurilmalarni ishlatish qoidalari bo‘yicha asimmetriya koeffitsiyenti 5% dan ko‘p bo‘lmasligi shart.

Simmetrik toklarning to‘g‘ri ketma-ketlik kattaliklarining geometrik yig‘indisi nolga teng.

Simmetrik toklarning teskari ketma-ketlik kattaliklarining geometrik yig‘indisi ham nolga teng.

Neytral simdan o‘tuvchi tok esa uchala liniya toklarining geometrik yig‘indisiga yoki ular komplekslarining algebraik yig‘indisiga teng bo‘ladi.

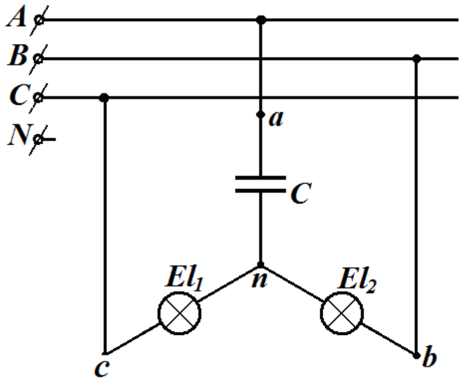
3.4. Yulduz va uchburchak sxemalarda ulangan uch fazali zanjirlar

3.7- masala. Ikkita El_1 va El_2 cho‘g‘lanma lampa va kondensator *C* yulduz usulida ulangan (3.19- rasm). Bu holda *C* kondensator ulangan manba fazasini shartli *A* faza deb qabul qilinsa, El_1 lampa El_2 lampaga nisbatan yorug‘roq yonsa, El_1 lampa *B* fazaga ulangan, El_2 lampa ulangan faza esa manbaning *S* fazasi bo‘ladi. Shuni isbotlang.

Yechish. Masalani yechishni soddalashtirish maqsadida faza iste’molchi-larining o‘tkazuvchanlik modullarini o‘zaro teng deb qabul qilamiz, ya’ni

$$\underline{Y}_A = \frac{1}{\underline{Z}_A} = jY, \;\; \underline{Y}_B = \underline{Y}_C = Y,$$

bu erda $\underline{Z}_A = -jX_C$; X_C – sig‘imning qarshiligi [Ω].



3.19- rasm.

Bu hol uchun liniya kuchlanishlarini simmetrik deb qabul qilamiz.

Iste’molchilar neytral nuqtasi n ning manba neytral nuqtasi N ga nisbatan siljish kuchlanishi

$$\underline{U}_{nN} = \frac{\underline{U}_A \underline{Y}_a + \underline{U}_B \underline{Y}_b + \underline{U}_C \underline{Y}_C}{\underline{Y}_a + \underline{Y}_b + \underline{Y}_s + \underline{Y}_{nN}} = -0{,}2\underline{U}_f + j0{,}6\underline{U}_f .$$

Manbaning “B” fazasiga ulangan lampa El_1 kuchlanishi

$$\underline{U}_b = \underline{U}_B - \underline{U}_{nN} = -0{,}3\underline{U}_f - j1{,}47\underline{U}_f .$$

Uning moduli

$$u_b = \sqrt{(0{,}3U_f)^2 + (1{,}47U_f)^2} = 1{,}49U_f .$$

“ C ” fazaga ulangan lampa El_2 kuchlanishi esa

$$\underline{U}_s = \underline{U}_S - \underline{U}_{nN} = -0{,}3\underline{U}_f + j0{,}27\underline{U}_f .$$

Uning moduli

$$u_s = \sqrt{(0{,}3U_f)^2 + (0{,}27U_f)^2} = 0{,}4U_f .$$

Demak, El_1 lampa El_2 lampaga nisbatan yorug‘roq yonadi. Ushbu ko‘rib chiqilgan masalani quyidagi berilgan qiymatlar uchun hisoblaymiz. Berilgan $U_f = 240$ V.

Yechish. Demak, tegishli faza kuchlanishlari

$$\underline{U}_a = U_f\, e^{j0^\circ} = 240e^{j0^\circ},$$

$$\underline{U}_b = U_f\, e^{-j120^\circ} = 240e^{-j120^\circ} = 240(\cos 120^\circ - j\sin 120^\circ) = 240(-0{,}5 - 0{,}86) = (-120 - j208)\ \text{V},$$

$$\underline{U}_c = U_f\, e^{j120^\circ} = 240(\cos 120^\circ + j\sin 120^\circ) = (-120 + j208)\ \text{V}.$$

Iste’molchi neytrali n ning manba neytrali N ga nisbatan siljish kuchlanishini hisoblaymiz:

$$\underline{U}_{nN} = \frac{\underline{U}_A jY + \underline{U}_A e^{-j120^\circ} Y + \underline{U}_A e^{j120^\circ} Y}{jY + Y + Y + 0} =$$

$$= \frac{U_f\, jY + U_f(\cos 120^\circ - j\sin 120^\circ)Y + U_f(\cos 120^\circ + j\sin 120^\circ)Y}{jY + 2Y} =$$

$$= \frac{U_f\, jY + U_f(-0{,}5 - j0{,}87)Y + U_f(-0{,}5 + j0{,}87)Y}{2Y + jY} =$$

$$= \frac{-U_f Y + U_f\, jY}{2Y + jY} = \frac{-U_f(Y - jY)(2Y - jY)}{(2Y + jY)(2Y - jY)} =$$

$$= \frac{-U_f(Y^2 - j3Y^2)}{5Y^2} = U_f(0{,}2 + j0{,}6).$$

Demak, $\underline{U}_{nN} = U_f(0{,}2 + j0{,}6)$.

Tegishli faza kuchlanishlari

"*B*" faza kuchlanishi

$$\underline{U}_b = \underline{U}_B - \underline{U}_{nN} = U_f(-0{,}5 - j0{,}87) - U_f(0{,}2 + j0{,}6) =$$
$$= U_f(-0{,}3 - j1{,}47).$$

Kuchlanishning moduli

$$U_b = \sqrt{(0{,}3U_f)^2 + (1{,}47U_f)^2} = 1{,}44U_f = 357{,}6 \text{ V}.$$

"*C*" faza kuchlanishi

$$\underline{U}_c = \underline{U}_C - \underline{U}_{nN} = U_f(-0{,}5 + j0{,}87) - U_f(0{,}2 + j0{,}6) =$$
$$= U_f(-0{,}3 + j0{,}27).$$

"*C*" faza kuchlanishining moduli

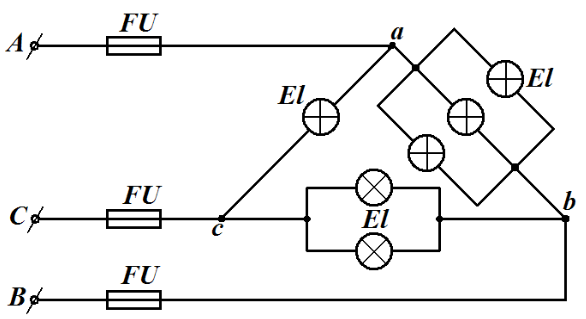
$$U_c = \sqrt{(0{,}3U_f)^2 + (0{,}27U_f)^2} = 0{,}4U_f = 96 \text{ V}..$$

Demak, iste'molchida "*B*" faza kuchlanishi "*C*" faza kuchlanishidan katta ($u_b > u_c$).

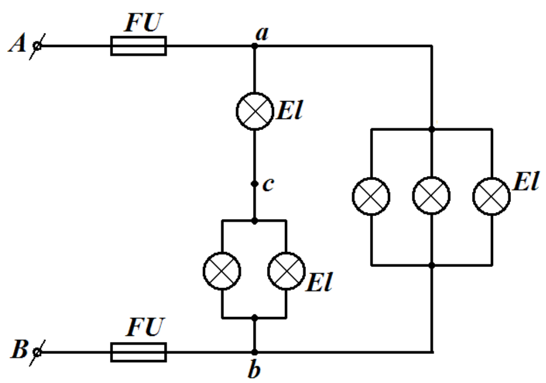
El_1 lampa El_2 lampaga nisbatan yorug'roq yonadi va fazalar ketma-ketligi *A, B, C* tartibda bo'ladi.

Ushbu masalani $U_f = 127$ V uchun hisoblang.

3.8-masala. Uch fazali elektr zanjirida cho'g'lanma lampalarning nominal quvvati va kuchlanishi bir xil (3.20-rasm). Agar "*C*" fazasida saqlagich kuyib qolsa, iste'molchi har bir fazasidagi kuchlanishlar U_{ab}, U_{bc} va U_{ca} ni hisoblang.



3.20- rasm.



3.21- rasm.

Yechish. Masala sharti bo‘yicha elektr zanjir sxemasini chizamiz (3.21-rasm). Umumiy qarshilik

$$R_{\text{um}\,acb} = 1{,}5R_{EL}\,.$$

ab fazada esa

$$R_{\text{um}\,ab} = \tfrac{1}{3}R_{EL}\,.$$

Toklarning qiymatlari

acb shoxobcha uchun

$$I = \frac{U}{1{,}5R};$$

Bunda

$$U_{ac} = IR = \frac{U}{1{,}5R}\cdot R = \frac{2}{3}U.$$

$$U_{cb} = I\cdot R/2 = \frac{U}{1{,}5R}\cdot\frac{R}{2} = \frac{1}{3}U.$$

Ikkinchi *ab* shoxobcha uchun $U_{ab} = 220$ V.

Demak, $U_{ab} = 220$ V,

$U_{bc} = \frac{1}{3}U_{ab}$,

$U_{ca} = \frac{2}{3}U_{ab}$ bo‘ladi.

3.9-masala. Uch fazali ta’minlash manbayiga uchta bir xil rezistorli yuklama $r_A = r_B = r_C$ yulduzcha usulida ulangan (3.22-rasm, *a*). Ta’minlash manbayining faza kuchlanishlar simmetrik va rubilnikning qayta ulanishlarida o‘zgarmasdir. Berilgan: $U_l = 380\,\text{V}$ va $I_A = I_B = I_C = \sqrt{3}\,\text{A}$. Quyidagi rejimlar uchun kuchlanishlarning topografik vektor diagrammalarini va toklarning vektor diagrammalarini quring.

1. Simmetrik rejim (1- va 3- rubilniklar tutashtirilgan, 2- rubilnik uzilgan);

2. Rubilniklarning holati 1- banddagidek, lekin r_A rezistor qarshiligi xuddi shunday bo‘lgan sig‘im bilan almashtirilgan;

3. 1 va 2 - rubilniklar uzilgan, 3 - rubilnik esa tutashtirilgan;

4. Barcha 1-, 2-, 3- rubilniklar uzilgan;

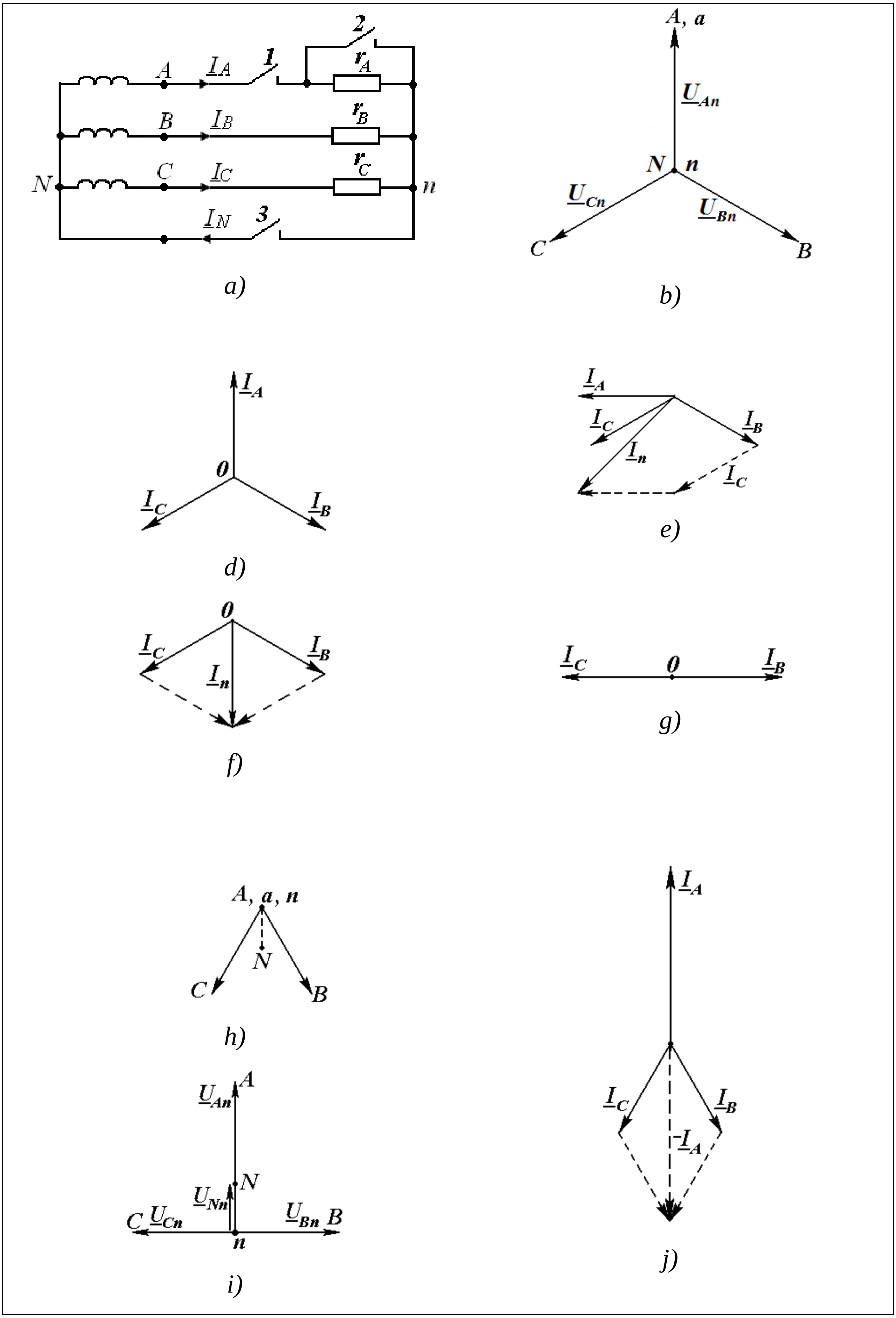
5. 1- va 2 - rubilniklar tutashtirilgan, 3 - rubilnik uzilgan.

Yechish. 1. Simmetrik rejimda manbadagi va yuklamadagi faza kuchlanishlar bir xil va teng $380/\sqrt{3}=220$ V; kuchlanishlarning topografik diagrammasi va toklarning diagrammasi 3.22- rasm, *b* va *d* larda tasvirlangan. Faza toklar vektorlarining yo'nalishi kuchlanishlar vektorlarining yo'nalishiga (aktiv yuklamada) mos. Neytral o'tkazgichda tok yo'q.

2. r_A rezistorni sig'imli qarshilik bilan almashtirilganda yuklamadagi faza kuchlanishlar o'zgarmaydi. I_B va I_C toklar o'zgarmaydi, I_A dagi tokning son qiymati o'zgarmaydi, lekin uning fazasi kuchlanish fazasiga nisbatan 90^0 ga ilgarilaydi. Bu hol uchun kuchlanshining topografik diagrammasi avvaldagidek (3.22- rasm, *b*), toklarning vektor diagrammalari 3.22- rasm, *e* da tasvirlangan. Neytral o'tkazgichdagi (simdagi) tok $\underline{I}_N=\underline{I}_A+\underline{I}_B+\underline{I}_C$, ya'ni $\underline{I}_N=\sqrt{2}$ A. Shuni aytish joizki, agar 3-rubilnik ajratilsa, u holda $\underline{I}_N=0$ hamda N va n nuqtalarning potensiallari turlicha bo'ladi, faza kuchlanishlar $\underline{U}_{An},\underline{U}_{Bn},\underline{U}_{Cn}$ mos ravishdagi manba $\underline{U}_{AN},\underline{U}_{BN},\underline{U}_{CN}$ kuchlanishlariga teng bo'lmaydi.

3. Agar 1-rubilnik ajratilsa *a* nuqtaning potensiali n nuqtaning potensialiga teng bo'ladi. Topografik diagrammada boshqa o'zgarishlar bo'lmaydi (3.22- rasm, *b*). Bu holdagi toklarning vektor diagrammasi 3.22- rasm, *f* da ko'rsatilgan. Bunda $I_N=1$ A.

4. Agar 3-rubilnikni qo'shimcha ravishda ajratsak, N va n nuqtalarni potensiallari har xil bo'lib qoladi. B va C fazalardagi rezistorlar ketma-ket ulanib qoladi va natijada har bir qarshilikda liniya kuchlanishi $\underline{U}_{BC}$ ning yarmiga to'g'ri keladi. Topografik diagrammada n va a nuqtalar BS kesmaning o'rtasida bo'ladi (3.22- rasm, *g*).



2
1
A
$\underline{I}_A$
r_A
B
$\underline{I}_B$
r_B
C
$\underline{I}_C$
r_C
N
n
$\underline{I}_N$
3
a)
A, a
$\underline{U}_{An}$
N
n
$\underline{U}_{Cn}$
$\underline{U}_{Bn}$
C
B
b)
$\underline{I}_A$
0
$\underline{I}_C$
$\underline{I}_B$
d)
$\underline{I}_A$
$\underline{I}_C$
$\underline{I}_B$
$\underline{I}_n$
$\underline{I}_C$
e)
0
$\underline{I}_C$
$\underline{I}_B$
$\underline{I}_n$
f)
$\underline{I}_C$
0
$\underline{I}_B$
g)
A, a, n
N
C
B
h)
$\underline{I}_A$
$\underline{I}_C$
$\underline{I}_B$
$-\underline{I}_A$
j)
$\underline{U}_{An}$
A
N
$\underline{U}_{Nn}$
C
$\underline{U}_{Cn}$
$\underline{U}_{Bn}$
B
n
i)

3.22- rasm.

Topografik diagrammadan neytral N va n nuqtalar va ajratilgan A fazaning uchlari orasidagi kuchlanishlarni topami:

$$U_{Nn} = U_{AN} / 2 = 100{,}0 \text{ V},$$
$$U_{An} = (3/2)U_{AN} = 329{,}9 \text{ V}.$$

Rezistorlar r_B va r_C dagi kuchlanishlar $\frac{U_l/\sqrt{3}}{U_l/\sqrt{2}} = \frac{2}{\sqrt{3}}$ marta kamayadi, xuddi shu miqdorda rezistorlardagi toklar ham $I_B = I_S = \sqrt{3}/2$ marta kamayadi. Toklarning vektor diagrammasi (3.22- rasm, *h*)da tasvirlangan.

5. 1- va 2-rubilniklar tutashganda va 3- rubilnik ajratilganda A, a va n nuqtalardagi potensiallar bir xil bo‘ladi (3.22- rasm, *i*). r_B va r_C rezistorlardagi kuchlanishlar liniya kuchlanishiga teng: $\underline{U}_{Ba} = \underline{U}_{BA}$ va $\underline{U}_{Cn} = \underline{U}_{CA}$. Shuning uchun I_B va I_C toklar simmetrik rejimga nisbatan $\sqrt{3}$ barobar katta bo‘ladi. I_A tokni vektor diagrammadan (3.22- rasm, *j*) topamiz. $I_A = -(I_B + I_C)$; $I_A = 3$ A.

3.10-masala. Uchta bir xil qarshilik uchburchak usulida ulangan (3.23-rasm, *a*). Manbaning liniya kuchlanishlari simmetrik sistemasining qiymati $U_{AB} = U_{BC} = U_{CA} = 220$ V 1- va 2- rubilniklar ajralganda o‘zgarmaydi. 1, 2- rubilniklar qo‘shilganda liniya toklari $I_l = 1$ A.

Zanjir kuchlanishlarning topografik va toklarning vektor diagrammalarini quyidagi rejimlar uchun qurilsin:

1. Simmetrik rejim (1- va 2- rubilniklar tutashtirilganda);

2. 1- rubilnik ajratilgan, 2- rubilnik tutashtirilgan;

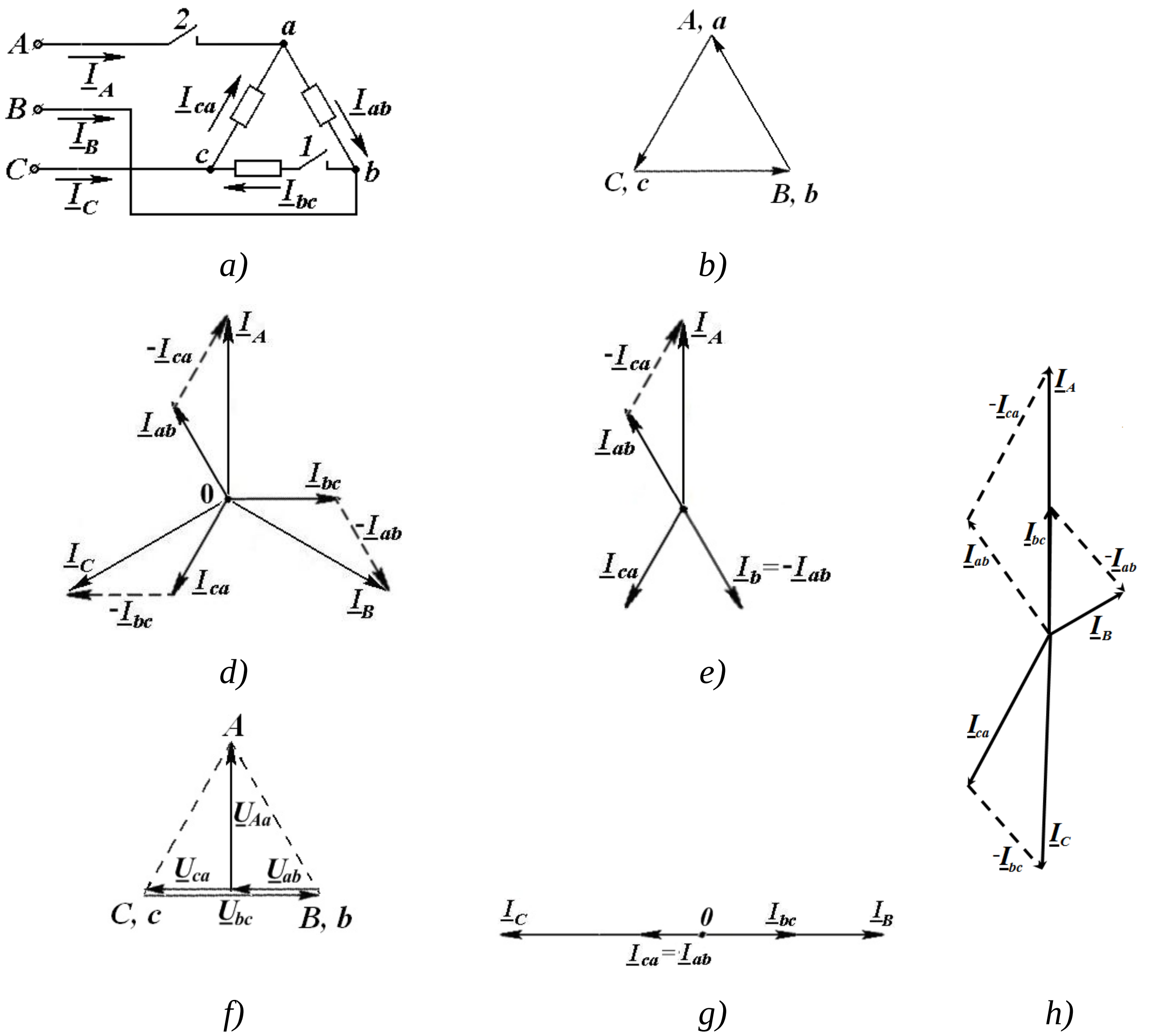
3. 2- rubilnik ajratilgan, 1- rubilnik tutashtirilgan;

4. 1- va 2- rubilniklar tutashtirilgan va *BC* fazadagi rezistor qarshiligi xuddi shunday bo‘lgan kondensator bilan almashtirilgan.

Yechish. 1. Simmetrik rejim zanjir kuchlanishining va toklarning topografik diagrammalari 3.23-rasm, *b* da keltirilgan. Yuklama fazalaridagi toklar liniyadagiga nisbatan $\sqrt{3}$ marta kam: $I_{ab} = I_{bc} = I_{ca} = I/\sqrt{3}$ A. Faza toklarining vektorlari I_{ab}, I_{bc}, I_{ca} yo‘nalishlari bo‘yicha U_{ab}, U_{bc} va U_{ca} (aktiv yuklama) vektorlar yo‘nalishiga mos. Liniya toklari faza toklari ayirmasiga tengdir:

$I_A = I_{ab} - I_{ca}$; $I_B = I_{bc} - I_{ab}$; $I_C = I_{ca} - I_{bc}$.

2. 1- rubilnik ajratilganda tok $I_{bc} = 0$, “*b*” nuqtaning potensiali *s* nuqtaning potensiali bilan bir xil. I_{ab} va I_{ca} o‘zgarmasdan qoladi, shu sababli $\underline{I}_A = \underline{I}_{ab} - \underline{I}_{ca}$ avvalgi qiymatda bo‘ladi. I_B va I_C toklar o‘zgaradi: $I_B = -I_{ab}$; $I_C = -I_{ca}$. Toklarning vektor diagrammasi 3.23- rasm, *e* da keltirilgan.



3.23- rasm.

3. 2- rubilnik ajratilganda va 1- rubilnik tutashtirilganda *sa* va *ab* shoxobchada rezistorlar ketma-ket ulangan bo‘ladi. Bu rezistorlardagi kuchlanishlar pasayishi U_{bc} liniya kuchlanishini yarmiga teng bo‘ladi. Topografik diagramma 3.23-rasm, *f* da *a* nuqta *bc* kesmaning o‘rtasida joylashadi. *A* fazaning ajratilgan qismalari

orasidagi kuchlanish teng:

$U_{Aa} = U_{AB}\cos 30^0 = 220\sqrt{3}/2 = 190{,}5\,\text{V}$. *sa* va *ab* shoxobchalardagi kuchlanish simmetrik rejimga nisbatan 2 marta kamayadi. Shoxobchadagi toklar ham shuncha marta kamayadi: $I_{ca} = I_{ab} = 0{,}5(1/\sqrt{3}) = 0{,}289\,\text{A}$. I_B va I_C toklarni vektor diagrammadan aniqlaymiz (3.23- rasm, *g*).

$$I_C = I_{ca} - I_{bc};\; I_B = I_{bc} - I_{ab};\; I_C = I_B = 0{,}866\,\text{A}.$$

4. Zanjir kuchlanishlarining topografik diagrammasi birinchi band kabi bo‘ladi. Toklarning vektor diagrammasi 3.23- rasm, *h* da keltirilgan. Undan quyidagilarni aniqlaymiz:

$$I_A = 1\,\text{A}.\; I_B = 2I_{bc}\cos 75^\circ = 2(1/\sqrt{3})\cdot 0{,}259 = 0{,}299\,\text{A}.$$

$$I_C = 2I_{ca}\cos 15^\circ = 2(1/\sqrt{3})\cdot 0{,}966 = 1{,}15\,\text{A}.$$

3.11-masala. Uch fazali asinxron dvigatelning stator chulg‘amlari nosimmetrik kuchlanishlar sistemasi $\underline{U}_A = 400\,\text{V}$, $\underline{U}_B = -j300\,\text{V}$, $\underline{U}_C = j300\,\text{V}$ ga ulangan. Dvigatelning neytral nuqtasi izolatsiyalangan. Dvigatel fazalarining to‘g‘ri va teskari ketma-ketlikli toklarga qarshiliklari $\underline{Z}_1 = 6 + j6\,\Omega$, $\underline{Z}_2 = 0{,}3 + j1{,}1\,\Omega$. Liniya toklari, dvigatel iste’mol qilayotgan aktiv, reaktiv va to‘la quvvatlar topilsin.

Yechish. 1. Faza kuchlanishlarining simmetrik tashkil etuvchilari quyidagicha topiladi:

$$\underline{U}_0 = \frac{1}{3}(\underline{U}_A + \underline{U}_B + \underline{U}_C) = \frac{400 - j300 + j300}{3} \approx 133\ \text{V},$$

$$\underline{U}_1 = \frac{1}{3}(\underline{U}_A + a\underline{U}_B + a^2\underline{U}_C) = \frac{1}{3}\left[400 + e^{j120^0}(-j300) + e^{j240^0}\,j300\right] = 307\ \text{V},$$

$$\underline{U}_2 = \frac{1}{3}(\underline{U}_A + a^2\underline{U}_B + a\underline{U}_C) = \frac{1}{3}\left[400 + e^{j240^0}(-j300) + e^{j120^0}\,j300\right] = -40{,}7\ \text{V}.$$

2. Toklarning simmetrik tashkil etuvchilari Om qonuniga asosan quyidagicha aniqlanadi:

$$\underline{I}_0 = \frac{\underline{U}_0}{\underline{Z}_0} = \frac{133}{\infty} = 0,\quad \underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}_1} = \frac{307}{6 + j6} = 36{,}2e^{-j45^0}\ \text{A},\ \underline{I}_2 = \frac{\underline{U}_2}{\underline{Z}_2} = \frac{-40{,}7}{0{,}3 + j1{,}1} = -35{,}7e^{-j75^0}\ \text{A}.$$

3. Liniya toklari:

$$\underline{I}_A = \underline{I}_1 + \underline{I}_2 = 36{,}2e^{-j45^0} - 35{,}7e^{-j75^0} = 19{,}2e^{-j29^0} \text{ A},$$
$$\underline{I}_B = a^2\underline{I}_1 + a\underline{I}_2 = 36{,}2e^{-j45^0}e^{j240^0} - 35{,}7e^{-j75^0}e^{j120^0} = 69e^{-j160^0} \text{ A},$$
$$\underline{I}_C = a\underline{I}_1 + a^2\underline{I}_2 = 36{,}2e^{-j45^0}e^{j120^0} + 35{,}7e^{-j75^0}e^{j240^0} = 51e^{j29^0} \text{A}.$$

4. Aktiv quvvat:

$$P = \mathrm{Re}\left|3\underline{U}_0\overset{*}{\underline{I}}_0 + 3\underline{U}_1\overset{*}{\underline{I}}_1 + 3\underline{U}_2\overset{*}{\underline{I}}_2\right| = 3\underline{U}_1\underline{I}_1\cos\varphi_1 + 3\underline{U}_2\underline{I}_2\cos\varphi_2 =$$
$$= 3\cdot 307\cdot 36{,}2\cos 45^0 + 3\cdot 40{,}7\cdot 35{,}7\cos 75^0 = 24{,}7 \text{ kW}.$$

5. Reaktiv quvvat:

$$Q = \mathrm{Im}\left|3\underline{U}_1\overset{*}{\underline{I}}_1 + 3\underline{U}_2\overset{*}{\underline{I}}_2\right| = 3\underline{U}_1\underline{I}_1\sin\varphi_1 + 3\underline{U}_2\underline{I}_2\sin\varphi_2 =$$
$$= 3\cdot 307\cdot 36{,}2\sin 45^0 + 3\cdot 40{,}7\cdot 35{,}7\sin 75^0 = 27{,}8 \text{ kvar}.$$

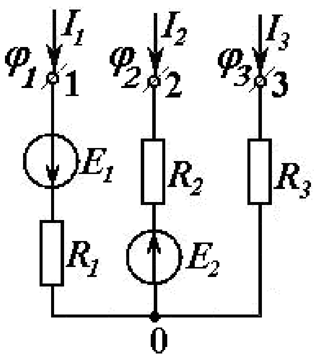
6. To‘la quvvat va quvvat koeffitsiyenti:

$$S = \sqrt{P^2 + Q^2} = \sqrt{24{,}7^2 + 27{,}8^2} = 37{,}2 \text{ kVA},$$
$$\cos\varphi = \frac{P}{S} = \frac{24{,}7}{37{,}2} = 0{,}68.$$

3.12- masala. Quyidagi uch fazali (3.24- rasm) yulduzcha usulida ulangan sxemada $E_1 = 10$ V, $E_2 = 25$ V, $R_1 = 2$ kΩ, $R_2 = 8$ kΩ, $R_3 = 12$ kΩ, $\varphi_1 = -5$ V, $\varphi_2 = 16$ V, $\varphi_3 = 28$ V. “0” tugun potensiali va I_1, I_2, I_3 toklar hisoblansin.

Yechish. Om qonuniga asosan shoxobchalardagi toklarni tugun potensiallari orqali aniqlaymiz:

$$I_1 = (\varphi_1 - \varphi_0 + E_1)g_1;$$
$$I_2 = (\varphi_2 - \varphi_0 - E_2)g_2;$$
$$I_3 = (\varphi_3 - \varphi_0)g_3.$$



3.24- rasm.

Bu yerda g_1, g_2 va g_3 – shoxobchalar o‘tkazuvchanliklari. Kirxgofning birinchi qonuni bo‘yicha $I_1 + I_2 + I_3 = 0$, bundan

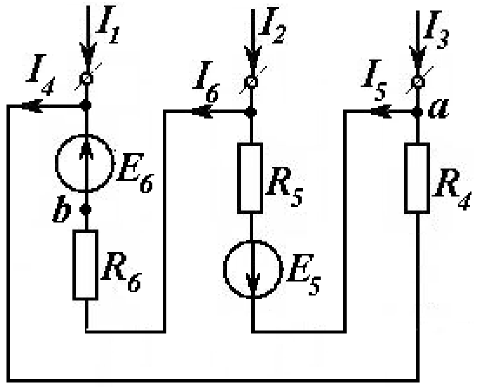
$$\varphi_0 = \frac{\varphi_1 g_1 + \varphi_2 g_2 + \varphi_3 g_3 + E_1 g_1 - E_2 g_2}{g_1 + g_2 + g_3} = 5{,}25\,\text{V}.$$

Topilgan φ_0 qiymatini yuqoridagi toklar ifodalariga qo‘yib hisoblaymiz:

$I_1 = -0{,}125\,\text{mA}; I_2 = -1{,}78\,\text{mA}; I_3 = 1{,}895\,\text{mA}$.

3.13-masala. Uchburchak ko‘rinishida ulangan (3.25-rasm) quyidagi sxemada $I_1 = 10\,\text{mA}$; $I_2 = -20\,\text{mA}$, qarshilklar $R_4 = 5\,\text{k}\Omega, R_5 = 3\,\text{k}\Omega, R_6 = 2\,\text{k}\Omega$

va EYKlar $E_5 = 20\,\text{V}, E_6 = 40\,\text{V}$ bo‘lsa, I_4, I_5, I_6 toklar va, shuningdek, “*a*” va “*b*” nuqtalari orasidagi potensiallar farqini aniqlang.



3.25- rasm.

Yechish. Uch fazali zanjir uchun Kirxgofning birinchi qonuniga ko‘ra $I_1 + I_2 + I_3 = 0$, bundan I_3 tokni hisoblaymiz:

$$I_3 = -I_1 - I_2 = 10\,\text{mA}.$$

I_4, I_5 va I_6 toklarni hisoblash uchun Kirxgofning birinchi qonuni bilan ikkita va ikkinchi qonuni bilan bitta tenglama tuzamiz:

$I_3 + I_4 = I_5; I_6 + I_1 = I_4; I_4 R_4 + I_5 R_5 + I_6 R_6 = E_6 - E_5,$

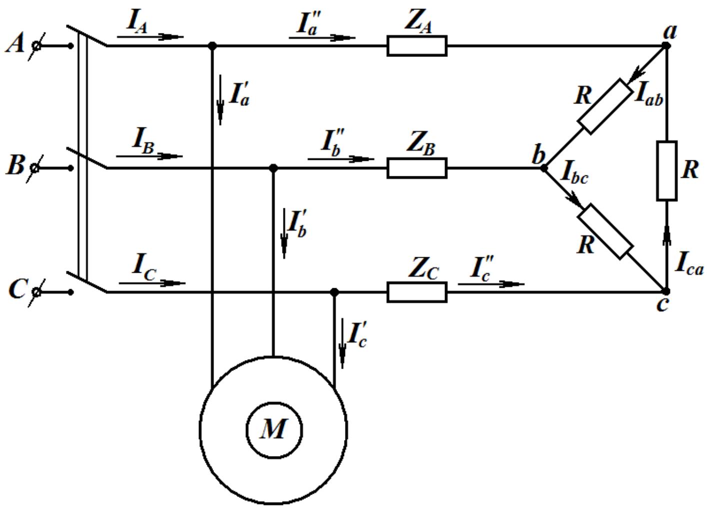
bundan $I_4 = \dfrac{E_6 - E_5 + I_1 R_6 - I_3 R_5}{R_4 + R_5 + R_6} = 1\,\text{mA};$

$$I_5 = 11\,\text{mA}; I_6 = -9; \;\; \varphi_a = \varphi_b - I_6 R_6 - I_4 R_4 = \varphi_b - 23\,\text{V}; \; U_{ab} = -23\,\text{V}.$$

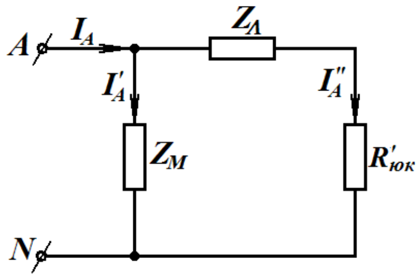
3.14-masala. Agar elektr sxemasi 3.26-rasmda keltirilgan uch fazali elektr zanjirida manba kuchlanishi $220/127$ V, elektr motorning kuchlanishi $U_M = 380/220$ V, quvvati $P_2 = 2{,}8$ kW, quvvat koeffitsiyenti $\cos\varphi = 0{,}8$, foydali ish koeffitsiyenti $\eta = 0{,}75$, elektr uzatish liniyasi simlarining to‘la qarshiligi $\underline{Z}_l = 5 + j10\ \Omega$, shu liniya oxirida uchburchak usulida ulangan cho‘g‘lanma lampalarning qarshiligi $R_{EL} = 30\ \Omega$ bo‘lsa, elektr zanjirining simmetrik ish rejimida va yuklamaning bir fazasi uzilgan holatida, ya’ni nosimmetrik rejim uchun barcha shoxobchalardagi toklarni hisoblang. Nosimmetrik yuklamali rejim uchun liniya toklarining simmetrik tashkil etuvchilarini toping. Kuchlanishlar va toklarning vektorlar diagrammasini quring.

Yechish. 1. Elektr motorning manbaga ulash sxemasini tanlaymiz. Manbaning liniya kuchlanishi elektr motor kuchlanishining kichik qiymatiga teng bo‘lgani uchun, motor chulg‘amlarini uchburchak sxemada birlashtiramiz
(3.26-rasm).

2. Aktiv R qarshilikli uchburchak usulida ulangan iste’molchini unga ekvivalent bo‘lgan simmetrik yulduz sxemasiga almashtiramiz. Bu holda $R' = \dfrac{R}{3} = 10\ \Omega$ bo‘ladi va yuklama liniya qarshiliklari Z_A, Z_B va Z_C bilan o‘zaro ketma-ket hamda elektr motorga uning tegishli kompleks qarshiliklari bilan o‘zaro parallel ulanib qoladi (3.27-rasm). 3.27-rasmda ekvivalent zanjir sxemasining bir fazasi keltirilgan.



3.26- rasm. Elektr zanjir sxemasi.



3.27- rasm. Bir fazali ekvivalent elektr zanjir sxemasi.

3. 3.27- rasmdagi elektr motor I'_A va yuklama I''_A toklarini hisoblaymiz.

a) Elektr motor toki

$$I'_A = \frac{P_2}{\sqrt{3}U \cdot \eta \cdot \cos\varphi} = \frac{2800}{\sqrt{3} \cdot 220 \cdot 0{,}75 \cdot 0{,}8} = 12{,}25 \text{ A.}$$

Bu tokning vektori tegishli faza kuchlanishi $U_F = 127$ V dan $\varphi = \arccos 0{,}8 = 37^\circ$ ga orqada qoladi.

Kompleks shakldа tegishli faza toklarining ifodasi quyidagicha bo‘ladi:

$$\underline{I}'_A = 12{,}25e^{-j37^\circ} = 9{,}8 - j7{,}39 \text{ A,}$$
$$\underline{I}'_B = \underline{I}'_A e^{-j120^\circ} = 12{,}25e^{-j157^\circ} = -11{,}3 - j4{,}78 \text{ A,}$$
$$\underline{I}'_C = \underline{I}'_A e^{j120^\circ} = 12{,}25e^{j83^\circ} = 1{,}5 + j12{,}17 \text{ A.}$$

b) Zanjirning aktiv qarshilikli yuklama liniya toklari esa quyidagicha hisoblanadi:

$$\underline{I}''_A = \frac{\underline{U}_A}{\underline{Z}_A} = \frac{\underline{U}_A}{\underline{Z}_L + R'_{\text{yuk}}} = \frac{127e^{j0^\circ}}{18e^{j33^\circ 40'}} = 7{,}06e^{-j33^\circ 40'} = 5{,}86\text{-}j3{,}97 \text{ A.}$$

Bu ifodadagi $\underline{Z}_A$ liniyaning va iste’molchining qarshiligi.

$$\underline{Z}_A = \underline{Z}_L + R'_{\text{yuk}} = 5 + j10 + 10 = 15 + j10 = 18e^{j33^\circ 40'}.$$

$$\underline{I}''_B = \underline{I}''_A e^{-j120^\circ} = 7{,}06e^{-j153^\circ 40'} = -6{,}3 - j3{,}09 \text{ A,}$$
$$\underline{I}''_C = \underline{I}''_A e^{j120^\circ} = 7{,}06e^{j86^\circ 20'} = 0{,}44 + j7{,}06 \text{ A.}$$

d) Liniya simlaridan o‘tuvchi toklarni hisoblaymiz:

$$\underline{I}_A = \underline{I}'_A + \underline{I}''_A = 15{,}68 - j11{,}36 = 19{,}3e^{-j36^\circ} \text{ A},$$
$$\underline{I}_B = \underline{I}'_B + \underline{I}''_B = -17{,}6 - j7{,}87 = 19{,}3e^{-j156^\circ} \text{ A},$$
$$\underline{I}_C = \underline{I}'_C + \underline{I}''_C = 2{,}94 + j19{,}23 = 19{,}3e^{j84^\circ} \text{ A}.$$

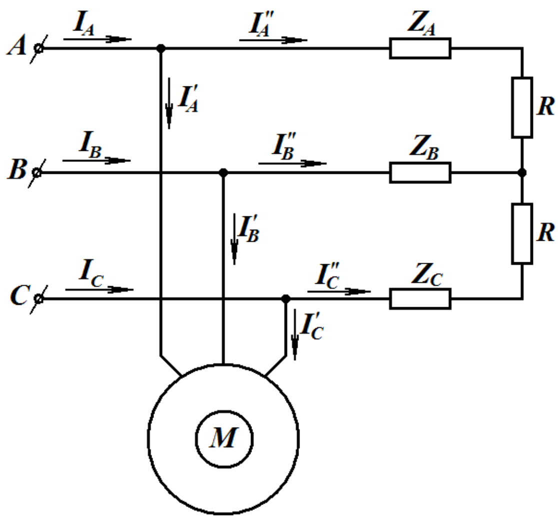
e) uchburchak sxemasida ulangan lampalarning toklari:

$$\underline{I}_{ab} = \frac{\underline{I}''_A e^{j30^\circ}}{\sqrt{3}} = 4{,}07e^{-j3^\circ 40'} = 4{,}07 - j0{,}267 \text{ A},$$
$$\underline{I}_{bc} = \underline{I}_{ab} e^{-j120^\circ} = -2{,}27 - j3{,}39 \text{ A},$$
$$\underline{I}_{ca} = \underline{I}_{ab} e^{j120^\circ} = 4{,}07e^{-j116^\circ 20'} = -1{,}8 + j3{,}66 \text{ A}.$$

4. Iste’molchining bir fazasi (*ac*) uzilgan holati uchun zanjirning elektr sxemasi 3.28-rasmdagi ko‘rinishga keltiriladi. Bu zanjirni quyidagi tartibda hisoblaymiz.

1. Elektr motorning faza toklari $\underline{I}'_A$, $\underline{I}'_B$ va $\underline{I}'_C$ qiymatlari o‘zgarmaydi, ya’ni:

$$\underline{I}'_A = 12{,}25e^{-j30^\circ}, \quad \underline{I}'_B = 12{,}25e^{-j150^\circ}, \quad \underline{I}'_C = 12{,}25e^{j90^\circ}.$$



3.28- rasm. Iste’molchining bir fazasi uzilgan elektr zanjirning sxemasi.

2. Yuklama faza toklarini aniqlash uchun uning qismalarida kuchlanishni va fazalar o‘tkazuvchanligini topamiz.

Liniya kuchlanishi $\underline{U}_A$ haqiqiy sonlar o‘qi yo‘nalishida bo‘lganligi va faza kuchlanishi $\underline{U}_{AB}$ undan 30° ga ilgarilanganligi sababli quyidagicha ifodalanadi:

$$\underline{U}_{AB} = 220e^{j30^\circ} = 190 + j110 \text{ V}.$$

Boshqa liniyalar kuchlanishi quyidagicha bo‘ladi:

$$\underline{U}_{BC} = \underline{U}_{AB}e^{-j120^\circ} = 220e^{-j90^\circ} = -j220 \text{ V},$$
$$\underline{U}_{CA} = \underline{U}_{AB}e^{j120^\circ} = 220e^{j150^\circ} = -190 + j110 \text{ V}.$$

Faza qarshiliklari quyidagicha ifodalanadi:

$$\underline{Z}_A = \underline{Z}_C = \underline{Z}_Л + R = 35 + j10 = 36{,}5e^{j16^\circ}\ \Omega,$$
$$\underline{Z}_B = \underline{Z}_Л = 5 + j10 = 11{,}2e^{j63^\circ30'}\ \Omega.$$

Tegishli faza o‘tkazuvchanliklari:

$$\underline{Y}_A = \underline{Y}_C = 1/\underline{Z}_A = 1/36{,}5e^{j16^\circ} = 0{,}028e^{-j16^\circ} = 0{,}027 - j0{,}0077 \text{ S},$$
$$\underline{Y}_B = 1/\underline{Z}_B = 1/11{,}2e^{j63^\circ30'} = 0{,}089e^{-j63^\circ30'} = 0{,}04 - j0{,}08 \text{ S}.$$

Yuklamaning faza kuchlanishlarini hisoblaymiz:

$$\underline{U}'_A = \frac{\underline{U}_{AB}\underline{Y}_B - \underline{U}_{CA}\underline{Y}_C}{\underline{Y}_A + \underline{Y}_B + \underline{Y}_C} = \frac{220e^{j30^\circ}\cdot 0{,}089e^{-j63^\circ30'} - 220e^{j150^\circ}\cdot 0{,}028e^{-j16^\circ}}{2\cdot(0{,}027 - j0{,}0077) + 0{,}04 - j0{,}08} =$$
$$= \frac{16{,}35 - j10{,}81 + 4{,}28 - j4{,}44}{0{,}094 - j0{,}0954} = \frac{20{,}63 - j15{,}25}{0{,}094 - j0{,}0954} = \frac{25{,}6e^{-j36^\circ30'}}{0{,}1339e^{-j45^\circ20'}} =$$
$$= 191e^{j8^\circ50'} = 189 + j29{,}4 \text{ V}.$$

$$\underline{U}'_B = \underline{U}'_A - \underline{U}_{AB} = 189 + j29{,}4 - 190 - j110 = 80{,}6e^{-j90^\circ} \text{ V},$$
$$\underline{U}'_C = \underline{U}'_A + \underline{U}_{CA} = 189 + j29{,}4 - 190 + j110 = 139{,}4e^{j90^\circ} \text{ V}.$$

3. Yuklama toklari:

$$\underline{I}''_A = \frac{\underline{U}'_A}{\underline{Z}_A} = \frac{191e^{j8^\circ50'}}{36{,}5e^{j16^\circ}} = 5{,}2e^{-j7^\circ10'} = 5{,}17 - j0{,}66 \text{ A},$$

$$\underline{I}''_B = \frac{\underline{U}'_B}{\underline{Z}_B} = \frac{80{,}6e^{-j90^\circ}}{11{,}2e^{j63^\circ30'}} = 7{,}2e^{-j153^\circ30'} = -6{,}44 - j3{,}22 \text{ A},$$

$$\underline{I}''_C = \frac{\underline{U}'_C}{\underline{Z}_C} = \frac{139{,}4e^{j90^\circ}}{36{,}5e^{j16^\circ}} = 3{,}92e^{j74^\circ} = 1{,}27 + j3{,}88 \text{ A}.$$

4. Umumiy liniya toklarini hisoblaymiz:

$$\underline{I}_A = \underline{I}'_A + \underline{I}''_A = 14{,}97 - j8{,}05 = 17e^{-j28^\circ15'} \text{ A},$$
$$\underline{I}_B = \underline{I}'_B + \underline{I}''_B = -17{,}74 - j8 = -19{,}5e^{-j155^\circ45'} \text{ A},$$
$$\underline{I}_C = \underline{I}'_C + \underline{I}''_C = 2{,}77 + j16{,}06 = 16{,}2e^{j80^\circ10'} \text{ A}.$$

5. Nosimmetrik toklarning simmetrik tashkil etuvchilariga yoyamiz:

Toklarning nol tashkil etuvchilari:

a) Analitik usul

$$\underline{I}_{A0} = \underline{I}_{B0} = \underline{I}_{C0} = \frac{\underline{I}_A + \underline{I}_B + \underline{I}_C}{3} = \frac{14{,}97 - j8{,}05 - 17{,}74 - j8 + 2{,}77 + j16{,}05}{3} = 0.$$

Toklarning to‘g‘ri ketma-ketlik tashkil etuvchilari

$$\underline{I}_{A_1} = \frac{\underline{I}_A + a\underline{I}_B + a^2\underline{I}_C}{3} = 14{,}41 - j9{,}95 = 17{,}5e^{-j34^\circ 40'} \text{ A},$$

$$\underline{I}_{B_1} = \underline{I}_{A_1}e^{-j120^\circ} = 17{,}5e^{-j154^\circ 40'} \text{ A},$$

$$\underline{I}_{C_1} = \underline{I}_{A_1}e^{j120^\circ} = 17{,}5e^{j85^\circ 20'} \text{ A}.$$

Toklarning teskari ketma-ketlik tashkil etuvchilari

$$\underline{I}_{A_2} = \frac{\underline{I}_A + a^2\underline{I}_B + a\underline{I}_C}{3} = 0{,}573 - j5{,}683 = 5{,}683e^{j84^\circ 15'} \text{ A},$$

$$\underline{I}_{B_2} = \underline{I}_{A_2}e^{j120^\circ} = 5{,}683e^{j204^\circ 15'} \text{ A},$$

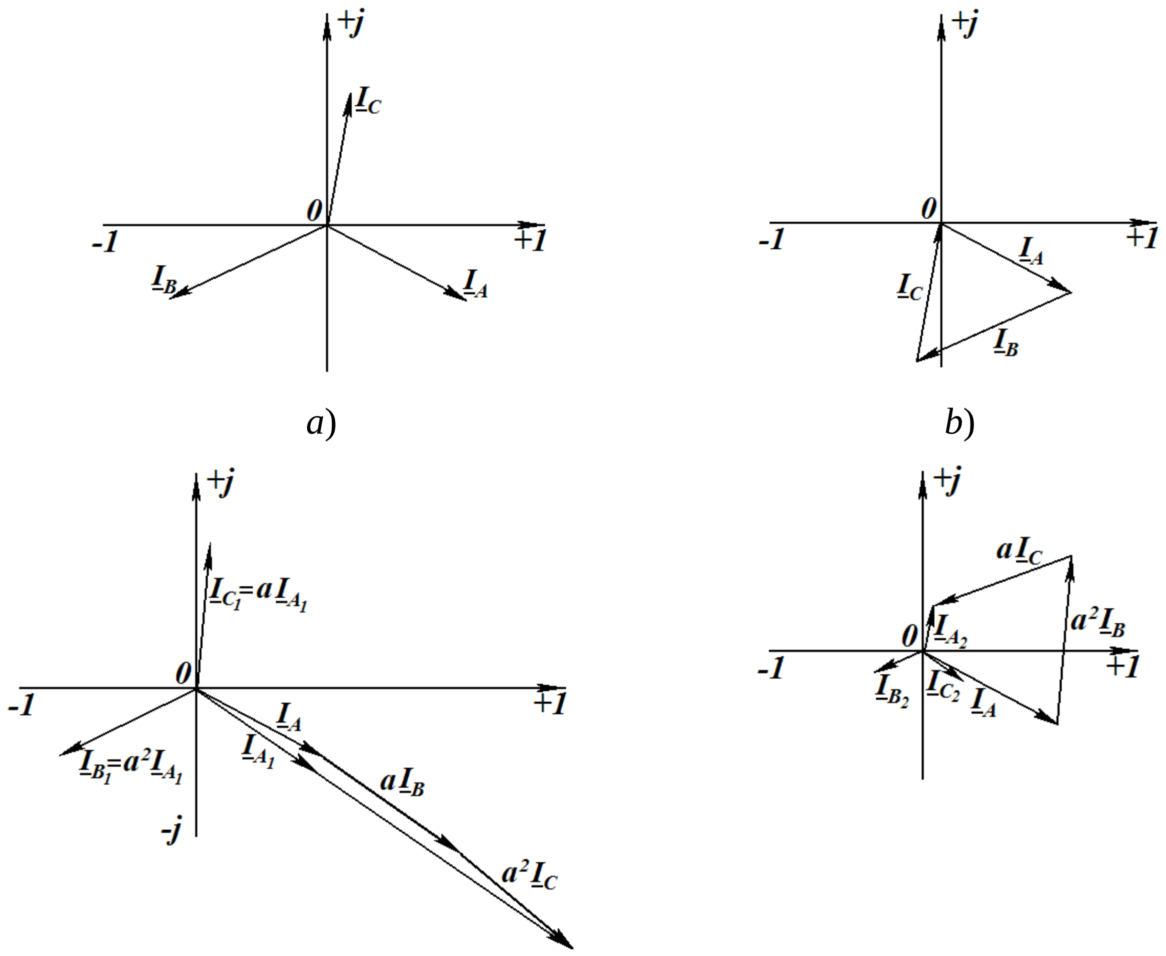
$$\underline{I}_{C_2} = \underline{I}_{A_2}e^{-j120^\circ} = 5{,}683e^{-j35^\circ 45'} \text{ A}.$$

b) Grafik usul.

Kompleks sonlar tekisligida $\underline{I}_A$, $\underline{I}_B$ va $\underline{I}_C$ hamda $a\underline{I}_A$, $a^2\underline{I}_A$, $a\underline{I}_C$ va $a^2\underline{I}_C$ tok vektorlarini yasaymiz (3.29- rasm, *a*, *b*). So‘nggi vektorlar operator $a = e^{j120^\circ}$ ga ko‘paytirilganda $\underline{I}_A$, $\underline{I}_B$ va $\underline{I}_C$ vektorlarga nisbatan 120° ga ilgarilaydi, a^2 ga ko‘paytirilganda esa, 120° ga orqada qoladi. $\underline{I}_A$, $\underline{I}_B$ va $\underline{I}_C$ tok vektorlarini geometrik qo‘shib va hosil bo‘lgan yig‘indi vektorning 1/3 qismini olib, nol ketma-ketlik $\underline{I}_{A_0}$, $\underline{I}_{B_0}$ va $\underline{I}_{C_0}$ tashkil etuvchilarini topamiz (3.29- rasm, *b*), xuddi shunday amallarni $\underline{I}_A$, $a\underline{I}_B$ va $a^2\underline{I}_C$ vektorlar ustida bajarib, toklarning to‘g‘ri ketma-ketlik tashkil etuvchilarining $\underline{I}_{A_1}$, $\underline{I}_{B_1}$ va $\underline{I}_{C_1}$ qiymatlari hisoblanadi (3.29- rasm, *d*). $\underline{I}_A$, $a^2\underline{I}_B$ va $a\underline{I}_C$ vektorlar yordamida toklarning teskari ketma-ketlik tashkil etuvchilarining $\underline{I}_{A_2}$, $\underline{I}_{B_2}$ va $\underline{I}_{C_2}$ vektoralri aniqlanadi (3.29- rasm, *e*).

Shu o‘rinda uch simli uch fazali sistemada liniya (fazalararo) kuchlanishlar va toklar tarkibida uchga karrali garmonikalar bo‘lmasligini qayd etish joiz.

Endi quyida (3.29- rasm, *a*, *b*, *d*, *e*) toklarning vektorlar diagrammasi keltirilgan.



3.29- rasm. Toklarning vektorlar diagrammasi. *a* – $\underline{I}_A$, $\underline{I}_B$ va $\underline{I}_C$ toklarning to‘g‘ri, *b* – nol, *d* – to‘g‘ri va *e* – teskari ketma-ketlik tashkil etuvchilarini grafik usulda aniqlash.

3.1-vazifa Boshiga uch fazali asinxron elektr motori, oxiriga uchburchak sxemali yuklama ulangan elektr zanjiri berilgan (3.30- rasm). Sxema parametrlari 3.1- jadvalda berilgan.

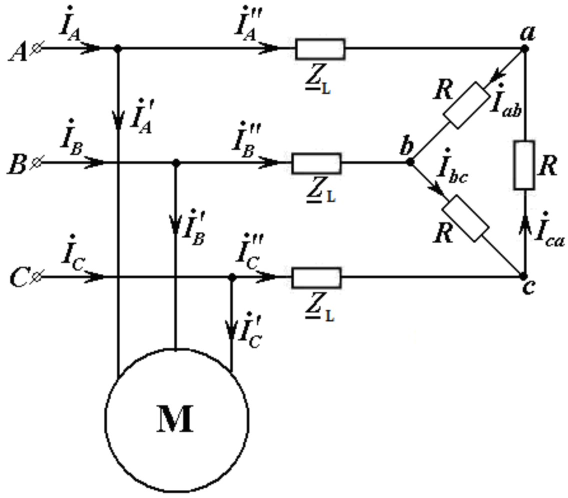
1. Elektr motorning ulanish sxemasi aniqlansin.

2. Barcha shoxobchalardagi toklar aniqlansin.

3. Kuchlanishlarning topografik vektor diagrammasi va toklarning vektor diagrammasi tuzilsin.

4. Yuklamaning biror fazasi uzilgan holda, sxema shoxobchalaridagi toklar topilsin.

5. 4- bandda hisoblangan toklarni simmetrik tashkil etuvchilari analitik va grafik usullarda aniqlansin.



3.30- rasm. 3.1- vazifaga elektr zanjir sxemasi.

3.1- *jadval*

Variant №	Manba		Elektr motor parametrlari					Liniya		Yuklama
	U_1, V	U_2, V	U_1, V	U_2, V	P_n, kW	η, %	$\cos\varphi$	r_L, Ω	x_L, Ω	R, Ω
1	380	220	380	220	1,2	62	0,9	0,8	1,6	21
2	380	220	220	127	2,4	66	0,83	1,5	3	24
3	220	127	220	127	2,5	70	0,85	1,2	2,4	27
4	220	127	380	220	2,6	65	0,86	1,4	2,8	32
5	380	220	380	220	3,4	68	0,87	1,3	2,6	34
6	380	220	220	127	4,6	69	0,8	0,9	1,8	40
7	220	127	220	127	3,6	71	0,78	0,85	1,7	52
8	220	127	380	220	5,2	80	0,81	0,77	1,8	33
9	380	220	380	220	3,8	78	0,88	0,79	1,6	18
10	380	220	220	127	2,3	70	0,9	0,86	1,7	20
11	220	127	220	127	4,2	75	0,84	0,95	1,9	32
12	220	127	380	220	2,8	78	0,9	1,25	2,5	25
13	380	220	380	220	3,7	74	0,85	1,35	2,7	23
14	380	220	220	127	3,4	73	0,87	0,98	2	36
15	220	127	220	127	4,5	78	0,73	0,96	1,98	35
16	220	127	380	220	4,6	78	0,9	0,8	1,7	31
17	380	220	380	220	4,8	79	0,83	1,5	3	41
18	380	220	220	127	5,6	81	0,85	1,2	1,6	45
19	220	127	220	127	5	80	0,86	1,4	3	42
20	220	127	380	220	4	72	0,87	1,3	2,4	38
21	380	220	380	220	3,1	68	0,8	0,9	2,8	37
22	380	220	220	127	2,1	62	0,78	0,85	2,6	25
23	220	127	220	127	6,2	82	0,81	0,77	1,8	26
24	220	127	380	220	6	81	0,88	0,79	1,7	28
25	380	220	380	220	7,2	85	0,9	0,86	1,8	45
26	380	220	220	127	6,5	74	0,84	0,95	1,6	41
27	220	127	220	127	4,4	65	0,9	1,25	1,7	37
28	220	127	380	220	4,8	68	0,85	1,35	1,9	20
29	380	220	220	127	2,9	60	0,87	0,98	2,5	26
30	220	127	380	220	3,9	65	0,73	0,96	2,7	30

4- BOB. DAVRIY NOSINUSOIDAL TOK CHIZIQLI ELEKTR ZANJIRLARINI HISOBLASH

4.1. Asosiy ma'lumotlar

Nosinusoidal tok va kuchlanishlarni Fure trigonometrik qatoriga yoyish. Fure teoremasiga ko'ra davriy o'zgaruvchi signallar (EYK, kuchlanish, tok) doimiy va chastotasi asosiy chastotaga nisbatan butun songa farq qiluvchi bir qancha sinusoidal (garmonik) o'zgaruvchi tashkil etuvchilarning yig'indisidan iborat. Nosinusoidal davriy funksiyaning *asosiy garmonika* deb ataluvchi birinchi garmonikasining chastotasi nosinusoidal funksiyaning davri bilan aniqlanadi, qolgan garmonik tashkil etuvchilar esa asosiy garmonikadan chastotasi 2, 3, 4 va hokazo marta katta bo'lgan *yuqori garmonikalar* deb ataladi.

Davri *2π* ga yoki vaqt bo'yicha esa *T* ga teng bo'lgan Dirixle shartiga bo'ysunuvchi *f(x)* funksiya Fure qatoriga quyidagicha yoyiladi:

$$\begin{aligned} f(x) = A_0 &+ A'_{1m}\sin x + A'_{2m}\sin 2x + A'_{3m}\sin 3x + \ldots + A'_{km}\sin kx + \ldots + \\ &+ A''_{1m}\cos x + A''_{2m}\cos 2x + A''_{3m}\cos 3x + \ldots + A''_{km}\cos kx + \ldots \end{aligned} \tag{1}$$

bu yerda

$A_0 = \dfrac{1}{2\pi}\int_0^{2\pi} f(x)dx$ – qatorning doimiy tashkil etuvchisi,

$A'_{km} = \dfrac{1}{\pi}\int_0^{2\pi} f(x)\sin kx dx$ – *k*-garmonika sinus tashkil etuvchisining amplitudasi,

$A''_{km} = \dfrac{1}{\pi}\int_0^{2\pi} f(x)\cos kx dx$ – *k*-garmonika kosinus tashkil etuvchisining amplitudasi.

Bunda (1) Fure qatorini umumiy holda quyidagicha yozish mumkin:

$$f(x) = A_0 + A_{1m}\sin(x+\psi_1) + A_{2m}\sin(2x+\psi_2) + \ldots =$$

$$= A_0 + \sum_{k=1}^{\infty} A_{km}\sin(kx+\psi_k),$$

bu yerda

$A_{km} = \sqrt{{A'_{km}}^2 + {A''_{km}}^2}$ – Fure qatori *k*- garmonikasining amplitudasi. Agar *k* – toq son bo‘lsa, u holda u toq garmonikalarni, *k* – juft son bo‘lsa, u juft garmonikalarni belgilaydi. $\psi_k = \mathrm{arctg}\frac{A''_{km}}{A'_{km}}$ esa *k* - garmonikaning boshlang‘ich fazasi. ψ_k ni aniqlashda A'_{km} va A''_{km} larning ishorasiga e’tibor berish kerak bo‘ladi.

Davriy egri chiziqlarning simmetriya xususiyatlari. Agar davriy nosinusoidal funksiya doimiy va bir qancha sinus tashkil etuvchilarga yoyilsa, unda uning o‘rtacha qiymati doimiy tashkil etuvchisiga teng bo‘ladi, ya’ni:

$$\frac{1}{2\pi}\int_0^{2\pi} f(x)dx = \frac{1}{2\pi}\int_0^{2\pi}\left[A_0 + A_{1m}\sin(x+\psi_1) + A_{2m}\sin(2x+\psi_2) + \ldots +\right]dx = A_0.$$

4.1-rasm, *a*, *b* va 4.2-rasmlarda uchta nosinusoidal egri chiziq ko‘rsatilgan. Ular maxsus xususiyatlarga ega. Agar 4.1- rasmda berilgan funksiyaning yarim davrini *x* o‘qi bo‘yicha chapga ko‘chirilsa, unda *x* o‘qiga nisbatan simmetrik funksiyaga ega bo‘lamiz, chunki uzuq chiziq bilan chizilgan funksiyaning *x* o‘qiga nisbatan aks tasvirini olsak, u *f(x)* funksiya bilan ustma-ust tushadi, ya’ni $f(x) = -f(x+\pi)$ shart bajariladi. Bunday funksiyalar toq va abssissa o‘qiga nisbatan simmetrik funksiyalar bo‘lib, ular Fure qatoriga yoyilsa, unda doimiy tashkil etuvchi A_0 va juft garmonikalar bo‘lmaydi,

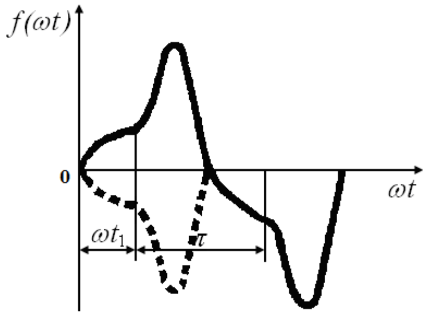
$$A_0 = A'_{2m} = A''_{2m} = A'_{4m} = A''_{4m} = 0,$$

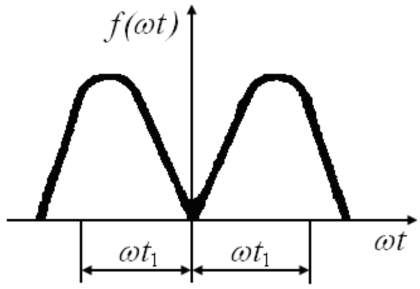
ya’ni uning tarkibida faqat quyidagi tashkil etuvchilar mavjud bo‘ladi:

$$f(x) = A'_{1m}\sin x + A''_{1m}\cos x + A'_{3m}\sin 3x + A'_{1m}\sin 3x + \ldots$$

yoki

$$f(x) = A_{1m}\sin(x+\psi_1) + A_{3m}\sin(3x+\psi_3) + A_{5m}\sin(5x+\psi_5) + \ldots .$$





4.1- rasm.

Abssissa o‘qiga nisbatan simmetrik egri chiziqqa sinusoidal kuchlanish manbayiga ulangan ferromagnit o‘zakli g‘altakdan o‘tayotgan tok egri chizig‘i misol bo‘la oladi.

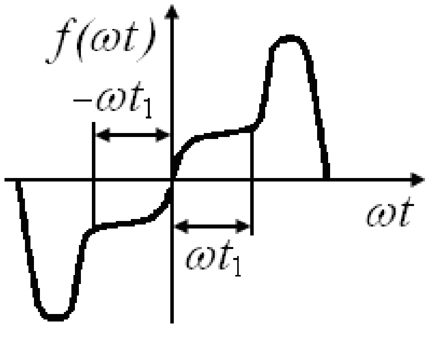
4.1- rasm, *b* da keltirilgan davriy egri chiziq esa ordinata o‘qiga nisbatan simmetrik bo‘lib, unda *f(–x)=f(x)* shart bajariladi. Bu funksiyalar Fure qatoriga yoyilganda, unda qatorning sinus tashkil etuvchilari $(A_1^{'} = A_2^{'} = A_3^{'} = 0)$ bo‘lmaydi, faqat doimiy va boshlang‘ich fazasi nolga teng bo‘lgan kosinus tashkil etuvchilari bo‘ladi, ya’ni:

$$f(x) = A_0 + A_{1m}^{''} \cos x + A_{2m}^{''} \cos 2x + A_{3m}^{''} \cos 3x + \ldots.$$

Koordinata boshiga nisbatan simmetrik funksiyalarda (4.2-rasm) $-f(-x) = f(x)$ shart bajariladi. Bunday funksiyalar toq funksiyalar bo‘lib, ular Fure qatoriga yoyilsa, unda doimiy va kosinus tashkil etuvchilari bo‘lmaydi, ya’ni:

$$\psi_k = 0; A_0 = 0; A_{km}^{''} = 0; A_{km} = A_{km}^{'}.$$

$$f(x) = A_{1m}^{'} \sin x + A_{2m}^{'} \sin 2x + A_{3m}^{'} \sin 3x + A_{4m}^{'} \sin 4x + \ldots.$$



4.2- rasm.

Simmetriya holatlari mavjudligi funksiya Fure qatoriga yoyilganda uning tarkibida qaysi tashkil etuvchilar bo‘lishini oldindan bilish imkonini beradi (4.1- jadval).

Simmetriya shartlari va Fure qatori koeffitsiyentlari

4.1- *jadval*

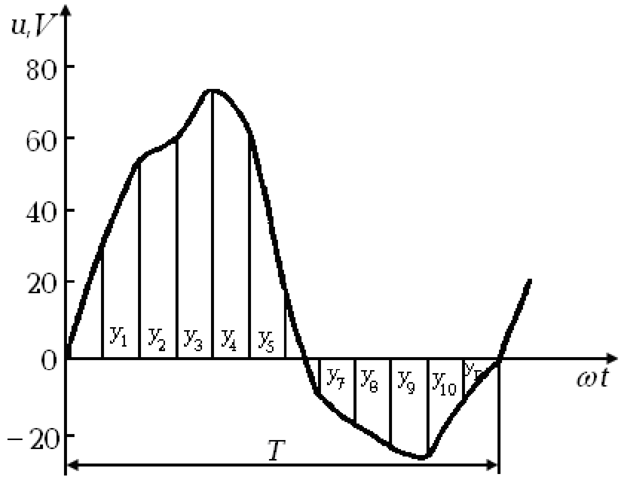
№	Simmetriya turi (nisbatan)	Matematik sharti	Koeffitsiyentlari		
			A_0	A_k'	A_k''
1	Ordinata o‘qiga (juft funksiya)	*f(x) = f(–x)*	bor	bor	nol
2	Koordinata boshiga (toq funksiya)	*f(x) =–f(–x)*	nol	nol	bor
3	Abssissa o‘qiga (toq funksiya)	*f(x) = –f(x+π)*	nol	faqat toq hadlari bor	faqat toq hadlari bor
4	1- va 2- holatlar bir paytda	*f(x) = f(–x)*, *f(x) = –f(x+π)*	nol	faqat toq hadlari bor	nol
5	2- va 3- holatlar bir paytda	*f(x) = –f(–x)*, *f(x) = –f(x+π)*	nol	nol	faqat toq hadlari bor

Davriy funksiyalarni grafoanalitik usulda trigonometrik qatorlarga yoyish. Fure qatori garmonikalarini grafoanalitik usulda aniqlash, odatda, funksiya grafigi (masalan, tok yoki kuchlanish ossillogrammmasi) berilganda qo‘llaniladi. Bunda berilgan aniq integrallar cheklangan sonli yig‘indilar bilan almashtiriladi. Avval ossillogrammadan nosinusoidal funksiyaning davri *(T)* ni aniqlaymiz va uni *n* ta bir xil kesmalarga bo‘lamiz (4.3- rasm). $\Delta x = 2\pi / n$, bunda *n*=12, 24 *yoki* 36 ga teng qilib olinishi maqsadga muvofiq. Bo‘linish nuqtalaridan y_1, y_2, y_3, ..., y_p (bu yerda *p* – ordinatalar tartib raqami) ordinatalar o‘tkazamiz. Ta’rifga asosan doimiy tashkil etuvchi:

$$A_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)dx \approx \frac{1}{2\pi}\sum_{p=1}^{p=n} f_p(x)\Delta x = \frac{1}{2\pi}\sum_{p=1}^{p=n} f_p(x)\frac{2\pi}{n},$$

yoki

$$A_0 = \frac{1}{n}\sum_{p=1}^{n} f_p(x),$$



4.3- rasm.

bu yerda *r*-har haddagi *n* sonining indeksi bo‘lib, uning qiymati (*n–1*) gacha o‘zgaradi. $f_p(x)$ berilgan funksiyaning $x = (p-0{,}5)\Delta x$ miqdordagi qiymati, ya’ni *p* – intervalning o‘rtasiga to‘g‘ri keladigan qiymat.

k - garmonika sinus tashkil etuvchisining amplitudasi:

$$A'_{km} = \frac{1}{\pi}\int_0^{2\pi} f(x)(\sin kx)dx \approx \frac{2}{n}\sum_{p=1}^{p=n} f_p(x)\frac{2\pi}{n}\sin kx\frac{2\pi}{n},$$

yoki

$$A'_{km} = \frac{2}{n}\sum_{p=1}^{p=n} f_p(x)\sin\left(kp\frac{2\pi}{n}\right).$$

k – garmonika kosinus tashkil etuvchisining amplitudasi:

$$A''_{km} = \frac{2}{n}\sum_{p=1}^{p=n} f_p(x)\cos\left(kp\frac{2\pi}{n}\right),$$

bu yerda $\sin_p kx$ va $\cos_p kx$ lar sin*kx* va cos*kx* funksiyalarning $x = (p-0{,}5)\Delta x$ dagi tegishli qiymatlaridir, ya’ni *p* – intervalning o‘rtasiga to‘g‘ri keluvchi amplitudalar. Mazkur usul yordamida hisoblashda ko‘pincha bir davrni 24 yoki 18 qismlarga bo‘linadi. Har qanday funksiyani Fure qatoriga grafoanalitik yoyishda, albatta, uning asosiy xususiyatlarini, ya’ni bu funksiya qaysi o‘qqa yoki koordinata boshiga nisbatan simmetrikligini aniqlab olishimiz kerak bo‘ladi. Funksiyada u yoki bu simmetriya xususiyatining bo‘lishi bizga oldindan qanday tashkil etuvchilarni aniqlash lozimligini ko‘rsatib beradi (4.1- jadval).

Nosinusoidal tok va kuchlanishlarning maksimal, effektiv (ta’sir etuvchi) va o‘rtacha qiymatlari. Davriy o‘zgaruvchan nosinusoidal kattalik (tok yoki kuchlanish) $f(\omega t)$ o‘zining garmonik tashkil etuvchilaridan tashqari yana uchta qiymatlari bilan tavsiflanadi:

1) bir davr ichidagi eng katta, ya’ni maksimal qiymati A_m.

2) davrdagi o‘rta kvadratik qiymati, ya’ni effektiv (ta’sir etuvchi) qiymati:

$$A = \sqrt{\frac{1}{T}\int_0^T f^2(\omega t)dt};$$

3) modulining o‘rtacha qiymati:

$$A_{\text{o'r}} = \frac{1}{T}\int_0^T |f(\omega t)|dt.$$

Elektron voltmetrlarning ko‘pchiligi o‘lchanayotgan kattalik modulining o‘rtacha qiymatini o‘lchaydi:

$$U_{\text{o'r}} = \frac{1}{T}\int_0^T |u|dt = \frac{2}{\pi}\int_0^{2\pi} |u|dt.$$

Tok va EYK uchun:

$$I_{\text{o'r}} = \frac{1}{T}\int_0^T |i|dt; \quad E_{o'r} = \frac{1}{T}\int_0^T |e|dt.$$

Nosinusoidal tokning effektiv qiymati:

$$I = \sqrt{\frac{1}{T}\sum\int_0^T I_{km}^2 \sin^2(k\omega t + \psi_k)dt} = \sqrt{I_0^2 + I_1^2 + I_2^2 + I_3^2 + \dots} \ .$$

Xuddi shunday ravishda davriy nosinusoidal kuchlanishning effektiv qiymati:

$$U = \sqrt{U_0^2 + U_1^2 + U_2^2 + U_3^2 + \dots} \ .$$

Nosinusoidal davriy funksiyalar shakllarini tavsiflovchi koeffitsiyentlar. Nosinusoidal davriy tok va kuchlanishlarni baholashda uchta koeffitsiyentdan foydalaniladi.

1. *Shakl koeffitsiyenti* – funksiya effektiv qiymatini uning o‘rtacha qiymatiga nisbati bilan aniqlanadi:

$k_{\text{sh}} = A/A_{\text{o'r}}$, sinusoida uchun $k_{\text{sh}} = \frac{\pi}{2}\Big/\sqrt{2} = 1{,}11$.

2. *Amplituda koeffitsiyenti* – funksiyaning maksimal qiymatini uning effektiv qiymatiga nisbati bilan aniqlanadi:

$k_a = \dfrac{A_{\max}}{A}$, sinusoida uchun $k_a = \sqrt{2} = 1{,}41$.

3. *Buzilish koeffitsiyenti* nosinusoidal funksiya birinchi (asosiy) garmonikasi effektiv qiymatining funksiyaning effektiv qiymatiga nisbatiga teng:

$k_b = \dfrac{A_1}{A}$, $k_b < 1$sinusoida uchun $k_b = 1$.

Masalan, kuchlanish va tok uchun,

$$k_b = \frac{U_1}{\sqrt{U_0^2 + U_1^2 + U_2^2 + U_3^2 + \ldots}}, \; k_b = \frac{I_1}{\sqrt{I_0^2 + I_1^2 + I_2^2 + I_3^2 + \ldots}}.$$

Buzilish koeffitsiyenti 1 dan qancha kam farq qilsa, bu funksiya sinusoidaga shuncha yaqin. Bu koeffitsiyent sanoat korxonalarida 5% dan oshmasligi kerak.

Davriy nosinusoidal tok zanjirlarida quvvatlar. Nosinusoidal tokning *aktiv quvvati* deb oniy quvvatining bir davr mobaynidagi o‘rtacha qiymatiga aytiladi:

$$P = \frac{1}{T}\int_0^T uidt.$$

Agar nosinusoidal kuchlanish va tok Fure qatorlariga yoyilgan bo‘lsa, ya’ni:

$$u = U_0 + U_{1m}\sin(\omega t + \psi_1) + U_{2m}\sin(2\omega t + \psi_2) + \\ + U_{3m}\sin(3\omega t + \psi_3) + \ldots,$$

$$i = I_0 + I_{1m}\sin(\omega t + \psi_1 - \varphi_1) + I_{2m}\sin(2\omega t + \psi_2 - \varphi_2) + \\ + I_{3m}\sin(3\omega t + \psi_3 - \varphi_3) + \ldots$$

bo‘lsa u holda ular ko‘paytmalarini integrallab, effektiv qiymatlarini hisobga olib quyidagini hosil qilamiz:

$$P = U_0 I_0 + U_1 I_1 \cos\varphi_1 + U_2 I_2 \cos\varphi_2 + \ldots = U_0 I_0 + \sum_{k=1}^{\infty} U_k I_k \cos\varphi_k,$$

ya’ni nosinusoidal tokning aktiv quvvati doimiy tashkil etuvchi va har bir garmonika aktiv quvvatlarining yig‘indisiga teng bo‘ladi.

Sinusoidal tok reaktiv quvvatining ta’rifiga ko‘ra har bir garmonika reaktiv quvvatlarining algebraik yig‘indisiga teng:

$$Q = \sum_{k=1}^{\infty} U_k I_k \sin\varphi_k .$$

Nosinusoidal tok zanjirining to‘liq quvvati *S* nosinusoidal kuchlanish va toklar effektiv qiymatlarining ko‘paytmasiga teng:

$$S = UI,$$

bunda $U = \sqrt{U_0^2 + U_1^2 + U_2^2 + ...}, \; I = \sqrt{I_0^2 + I_1^2 + I_2^2 + ...}$.

Sinusoidal rejimdan farqli ravishda davriy nosinusoidal tok zanjirlarida aktiv va reaktiv quvvatlar kvadratlarining yig‘indisi to‘liq quvvat kvadratiga teng bo‘lmaydi, ya’ni T^2 ga farq qiladi:

$$P^2 + Q^2 = S^2 - T^2.$$

T kattalik *buzilish quvvati* deb ataladi va kuchlanish hamda tok egri chiziqlari shakllarining sinusoidadan chetlanish darajasi bilan tavsiflanadi.

Davriy nosinusoidal tok chiziqli elektr zanjirlarini hisoblash. Avvalo, EYKning doimiy tashkil etuvchisi ta’siridan hosil bo‘lgan tok va kuchlanishlarni hisoblaymiz. Bunda o‘zgarmas tokda induktiv elementdagi kuchlanish tushuvi hamda sig‘im elementidagi tok nolga tengligini unutmaslik zarur:

$$\omega_0 = 0; \; \omega_0 L = 0; \; U_L = \omega_0 L I_0 = 0; \; x_C = \frac{1}{\omega_0 C} = \infty; \; I_0 = \frac{U_C}{x_C} = 0.$$

Bundan keyin birinchi (asosiy) garmonika ta’siridan hosil bo‘lgan tok va kuchlanishlarni hisoblaymiz, so‘ngra ikkinchi, uchinchi va h.k. garmonikalar ta’siridan hosil bo‘lgan tok va kuchlanishlar alohida hisoblanadi. Hisoblashda induktiv qarshilik chastotaga to‘g‘ri proporsiyada o‘sib borishini e’tiborga olish kerak. Masalan, *k*- garmonika uchun induktiv qarshilik $x_{L_1} = k\omega L = kx_L$, ya’ni birinchi garmonikadagi x_{L_1} qarshilikka qaraganda *k* marta katta bo‘ladi.

Sig‘im qarshiligi esa chastota ortishi bilan kamayadi. *k*- garmonika uchun sig‘im qarshiligi x_{C_k} birinchi garmonikadagi x_{C_1} qarshiligidan *k* marta kichikdir, ya’ni:

$$x_{C_k} = \frac{1}{k\omega C}.$$

R, L, C elementlari ketma-ket ulangan zanjirda *k*- garmonika toki I_k kompleks shaklda quyidagicha yoziladi:

$$\underline{I}_k = \frac{\underline{U}_k}{R + jk\omega L + \dfrac{1}{jk\omega C}} = \frac{\underline{U}_k}{R + j\left(k\omega L - \dfrac{1}{k\omega C}\right)} = \frac{\underline{U}_k}{Z_k} = \frac{\underline{U}_k}{\underline{Z}_k e^{j\varphi_k}},$$

bu yerda

$$Z_k = \sqrt{R^2 + \left(k\omega L - \frac{1}{k\omega C}\right)^2}, \quad \mathrm{tg}\varphi_k = \frac{k\omega L - \dfrac{1}{k\omega C}}{R}.$$

Nosinusoidal tok va kuchlanishlarni ekvivalent sinusoidalar bilan almashtirish. Tok va kuchlanishning shakli bizni qiziqtirmagan hollarda nosinusoidal kuchlanish va toklarni ularga ekvivalent bo‘lgan sinusoidalar bilan almashtirish usuli qo‘llaniladi.

Ekvivalent kuchlanish va tok sinusoidalari orasidagi φ_e burchak siljishining qiymati ekvivalent sinusoidal tokning aktiv quvvati nosinusoidal tokning aktiv quvvatiga tenglik shartidan olinadi:

$$\cos\varphi_{\text{ekv}} = \frac{P}{UI}.$$

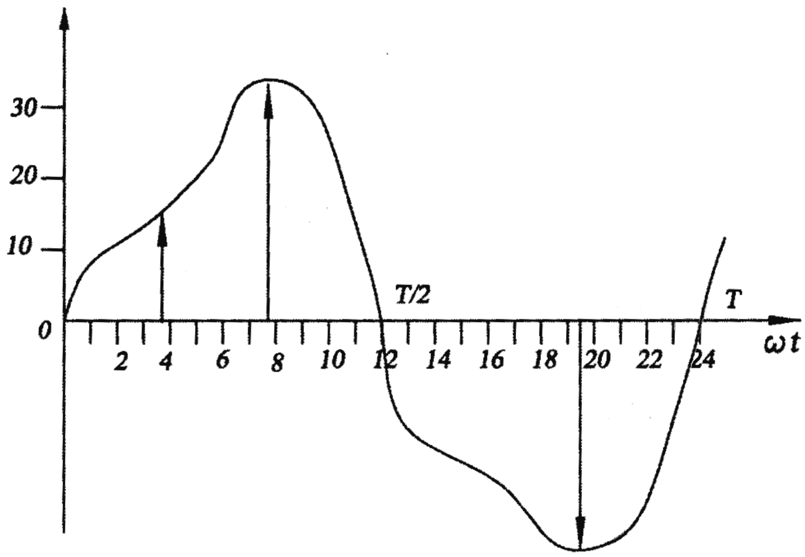
Nosinusoidal tok zanjirlarida rezonans hodisalari. Nosinusoidal kuchlanish va tokli zanjirlarda har bir garmonik tashkil etuvchilar uchun alohida rezonans bo‘lishi mumkin.

R, L, C elementlari ketma-ket ulangan tebranish konturidagi har bir garmonika toki:

$$I_k = \frac{U_k}{\sqrt{r^2 + \left(k\omega L - \dfrac{1}{k\omega C}\right)^2}}.$$

4.2. Davriy nosinusoidal tokli chiziqli elektr zanjirlar

4.1- masala. 4.4- rasmda keltirilgan $f(\omega t)$ funksiyaning birinchi va uchinchi garmonikalari aniqlansin.



4.4- rasm.

Yechish. $f(\omega t)$ funksiyaning davrini $n = 24$ ta teng bo‘lakka bo‘lamiz, unda uning birinchi yarim davri bo‘laklarining ordinatalari quyidagi qiymatlarga ega:

r	1	2	3	4	5	6	7	8	9	10	11	12
$f(\omega t)$	7	11	13,5	15,4	17,4	20,5	25,4	32,5	27,7	19,2	10	5

Egri chiziq abssissa o‘qiga nisbatan simmetrik bo‘lganligi uchun Fure qatorining tarkibida uning doimiy tashkil etuvchisi bo‘lmaydi, ya’ni $A_0 = 0$ va qator faqat toq garmonikalardan iborat bo‘ladi. Sinus va kosinus tashkil etuvchilari birinchi garmonikalarining amplitudalari mos ravishda quyidagi formulalar bilan aniqlanadi:

$$A'_{1m} = \frac{4}{n}\sum_{p=1}^{n/2} f_p(\omega t)\sin p\omega t,$$

$$A'_{1m}=\frac{4}{24}(7\cdot\sin 7^0 30' + 11\cdot\sin 22^0 30' + 13{,}5\cdot\sin 37^0 30' + 15{,}4\cdot\sin 52^0 30' +$$
$$+17{,}4\cdot\sin 67^0 30' + 20{,}5\cdot\sin 82^0 30' + 25{,}4\cdot\sin 97^0 30' + 32{,}5\cdot\sin 112^0 30' +$$
$$+21{,}7\cdot\sin 127^0 30' + 19{,}2\cdot\sin 142^0 30' + 10\sin 157^0 30' + 5\sin 172^0 30' = 25{,}3.$$

Shunga o‘xshash:

$$A''_{1m}=\frac{4}{24}\sum_{p=1}^{n/2} f_p(\omega t)\cos p\omega t = -5{,}23\,.$$

Birinchi garmonikaning amplitudasi $A_{1m}=\sqrt{\left(A'_1\right)^2+\left(A''_1\right)^2}=\sqrt{25{,}3^2+(-5{,}23)^2}=25{,}83$,

boshlang‘ich fazasi esa $\psi_1 = arctg\dfrac{A''_{1m}}{A'_{1m}} = -11^0 10'$.

Uchinchi garmonika sinus va kosinus tashkil etuvchilarining amplitudalari:

$$A'_{3m}=\frac{4}{24}\sum_{p=1}^{12} f_p(x)\sin_p 3x \approx 3{,}47,\quad A'_{3m}=\frac{4}{24}\sum_{p=1}^{12} f_p(x)\cos_p 3x \approx ?$$

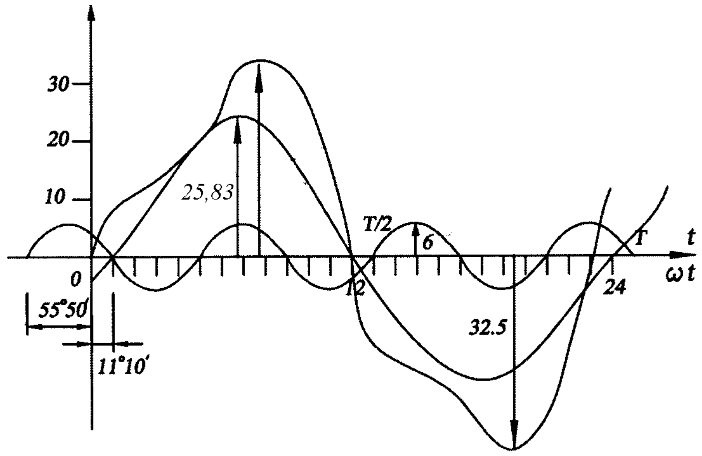
Uchinchi garmonikaning amplitudasi va boshlang‘ich fazasi:

$$A_{3m}=\sqrt{\left(A'_3\right)^2+\left(A''_3\right)^2}=6,\quad \psi_3 = arctg\frac{A''_{3m}}{A'_{3m}}\approx 55^0 50'.$$

Demak, uchinchi garmonika bilan chegaralangan Fure qatorini quyidagicha yozamiz:

$$f(\omega t) = A_{1m}\sin(\omega t+\psi_1) + A_{3m}\sin(3\omega t+\psi_3)\approx$$
$$= 25{,}83\sin(\omega t - 11^0 10') + 6\sin(3\omega t + 55^0 50').$$

4.5-rasmda birinchi va uchinchi garmonikalarning grafigi hamda ularning yig‘indisidan hosil bo‘lgan umumiy nosinusoidal funksiya keltirilgan. Uni berilgan nosinusoidal egri chiziq bilan solishtirish mumkin. Garmonik tashkil etuvchilarni dasturli mikrokalkulatorlar va shaxsiy kompyuterlarda standart programmalardan foydalanib tez hisoblash mumkin.



4.5- rasm.

4.2- masala. Tokning oniy qiymati $i = 100 + 282\sin\omega t + 141(3\omega t + 30^0) + 70{,}7\sin(5\omega t - 20^0)$ A ga teng. Tokning effektiv qiymati aniqlansin.

Yechish.

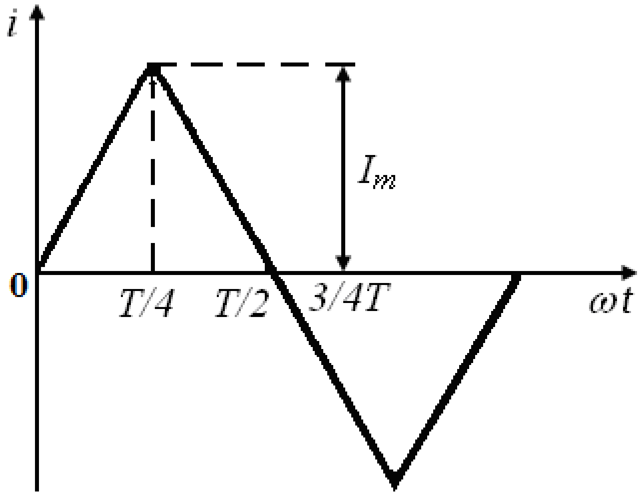
$$I = \sqrt{I_0^2 + \left(\frac{I_{m1}}{\sqrt{2}}\right)^2 + \left(\frac{I_{m2}}{\sqrt{2}}\right)^2 + \left(\frac{I_{m5}}{\sqrt{2}}\right)^2} = \sqrt{100^2 + \frac{282^2}{2} + \frac{141^2}{2} + \frac{70{,}7^2}{2}} = 250 \text{ A.}$$

Bir davr uchun tokning o‘rtacha qiymati:

$$I_{\text{o'r}} = \frac{1}{T}\int_0^T idt = \frac{1}{T}\int_0^T \left|\left(I_0 + i_1 + i_2 + i_3 + i_4 + ...\right)\right| dt.$$

Ma’lumki, sinusoidal tokning bir davrdagi o‘rtacha qiymati nolga teng, shuning uchun $I_{\text{o'r}} = I_0$.

4.3- masala. Nosinusoidal tokning o‘zgarish grafigi 4.6- rasmda tasvirlangan. Amplituda va shakl koeffitsiyentlari aniqlansin.



4.6- rasm.

Yechish. Tok egri chizig‘i abssissa o‘qiga nisbatan simmetrik bo‘lganligi sababli uning effektiv va modul bo‘yicha o‘rtacha qiymatlarini aniqlash uchun funksiyani 0 dan $t = T/4$ gacha oraliqda integrallaymiz. Bu oraliqda tok $i = \frac{4I_m}{T}t$ qonuniyat bilan o‘zgaradi va uning effektiv qiymati:

$$I = \sqrt{\frac{1}{T}\int_0^T i^2 dt} = \sqrt{\frac{4}{T}\int_0^{T/4}\left(I_m \frac{4t}{T}\right)^2 dt} = \frac{I_m}{\sqrt{3}}.$$

Tok modulining o‘rtacha qiymati:

$$I_{\text{o'r}} = \frac{4}{T}\int_0^T |i| dt = \frac{4}{T}\int_0^T \frac{I_m \cdot 4t}{T} dt = \frac{I_m}{2}.$$

Shakl koeffitsiyenti:

$$k_{\text{sh}} = \frac{I}{I_{\text{o'r}}} = \frac{I_m \cdot 2}{\sqrt{3}I_m} = 1{,}15.$$

Amplituda koeffitsiyenti:

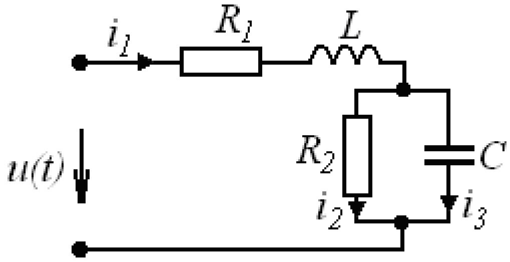
$$k_a = \frac{I_m}{I} = \sqrt{3} = 1{,}73.$$

4.4- masala. Elektr zanjiri $u(t) = 50 + 141\sin(\omega t + 30^0) + 70{,}7\sin(3\omega t - 60^0)$ V kuchlanish manbayiga ulangan (4.7- rasm). Zanjir shoxobchalaridagi oniy toklar ifodalarini va ularning effektiv qiymatlarini aniqlang. Bunda

$R_1 = 20\,\Omega$ $R_2 = 5\,\Omega$; $\omega L = 3\,\Omega$; $\frac{1}{\omega C} = 9\,\Omega.$

Yechish. Manba kuchlanishining doimiy tashkil etuvchisidan zanjirda hosil bo‘lgan toklarni hisoblaymiz: $U_{(0)} = 50\,\text{V}$.

$$I_{1(0)} = I_{2(0)} = \frac{U_{(0)}}{R_1 + R_2} = \frac{50}{20+5} = 2\ \text{A},\ I_{3(0)} = 0.$$



4.7- rasm.

Zanjirni birinchi garmonika uchun hisoblaymiz:

$$u(t) = 141\sin(\omega t + 30^0)\ \text{V} \text{ yoki kompleks shaklda } \underline{U}_{(1)} = 100e^{j30^0}\ \text{V}.$$

Parallel uchastkaning kompleks qarshiligi:

$$Z_{23(1)} = \frac{\underline{Z}_2\underline{Z}_{3(1)}}{\underline{Z}_2 + \underline{Z}_{3(1)}} = \frac{R_2\left(-j\dfrac{1}{\omega C}\right)}{R_2 - j\dfrac{1}{\omega C}} = \frac{5\cdot(-j9)}{5-j9} = 4{,}37e^{-j29^0} = 3{,}83 - j2{,}12\,\Omega.$$

Birinchi garmonika uchun zanjirning ekvivalent qarshiligi:

$$\underline{Z}_{(1)} = \underline{Z}_{1(1)} + \underline{Z}_{23(1)} = R_1 + j\omega L + \underline{Z}_{23(1)} = 20 + j3 + 3{,}83 - j2{,}12 = 23{,}9e^{j2^0}\ \Omega.$$

Zanjirning qismidagi tokning birinchi garmonikasi effektiv qiymatining kompleksi:

$$\underline{I}_{1(1)} = \frac{\underline{U}_{(1)}}{\underline{Z}_{(1)}} = \frac{100e^{j30^0}}{23{,}9e^{j2{,}12^0}} = 4{,}184e^{j28^0}\ \text{A}.$$

Parallel uchastkadagi kuchlanish:

$$\underline{U}_{23(1)} = \underline{I}_{1(1)}\underline{Z}_{23(1)} = 4{,}184e^{j28^0}\cdot 4{,}37e^{-j29^0} = 18{,}28e^{-j1^0}\ \text{V}.$$

Ikkinchi va uchinchi shoxobchadagi toklar:

$$\underline{I}_{2(1)} = \frac{\underline{U}_{23(1)}}{R_2} = \frac{18{,}28e^{-j1^0}}{5} = 3{,}66e^{-j1^0}\,\text{A},$$

$$\underline{I}_{3(1)} = \frac{\underline{U}_{23(1)}}{-j\dfrac{1}{\omega C}} = \frac{18{,}28e^{-j1^0}}{-j9} = 2{,}03e^{j89^0}\,\text{A}.$$

Birinchi garmonika toklarining oniy qiymatlari:

$$i_{1(1)} = 4{,}184\sqrt{2}\sin(\omega t + 28^0),\ \text{A};\ i_{2(1)} = 3{,}66\sqrt{2}\sin(\omega t - 1^0)\ \text{A};$$

$$i_{3(1)} = 2{,}03\sqrt{2}\sin(\omega t + 89^0)\ \text{A}.$$

Manba kuchlanishining uchinchi garmonikasidan zanjirda hosil bo‘lgan toklarni topamiz. Uchinchi garmonika kuchlanishi:

$$u_{(3)} = 70{,}7\sin(3\omega t - 60^0)\ \text{V},\ \underline{U}_{(3)} = 50e^{-j60^0}\text{V}.$$

Uchinchi garmonika uchun zanjirning kompleks qarshiliklari:

$$\underline{Z}_{23(3)} = \frac{\underline{Z}_2\underline{Z}_{3(3)}}{\underline{Z}_2 + \underline{Z}_{3(3)}} = \frac{R_2\left(-j\dfrac{1}{3\omega C}\right)}{R_2 - j\dfrac{1}{3\omega C}} = \frac{5\cdot(-j3)}{5 - j3} = 2{,}57e^{-j59^0} = 1{,}32 - j2{,}2\ \Omega,$$

$$\underline{Z}_{(3)} = R_1 + j3\omega L + \underline{Z}_{23(3)} = 20 + j9 + 1{,}32 - j2{,}2 = 21{,}32 + j6{,}8 = 22{,}38e^{j18^0}\ \Omega.$$

Tok va kuchlanishlar:

$$\underline{I}_{1(3)} = \frac{\underline{U}_{(3)}}{\underline{Z}_{(3)}} = \frac{50e^{-j60^0}}{22{,}38e^{j18^0}} = 2{,}23e^{-j78^0}\ \text{A},$$

Parallel ulangan shoxobchalardagi kuchlanish:

$$\underline{U}_{23(3)} = \underline{I}_{1(3)}\cdot\underline{Z}_{23(3)} = 2{,}23e^{-j78^0}\cdot 2{,}57e^{-j59^0} = 5{,}73e^{-j137^0}\ \text{V},$$

Ikkinchi va uchinchi shoxobchalardagi toklar:

$$\underline{I}_{2(3)} = \frac{\underline{U}_{23(3)}}{R_2} = \frac{5{,}73e^{-j137^0}}{5} = 1{,}14e^{-j137^0}\text{A},$$

$$\underline{I}_{3(3)} = \frac{\underline{U}_{23(3)}}{-j\dfrac{1}{3\omega C}} = \frac{5{,}73e^{-j137^0}}{-j3} = 1{,}91e^{-j47^0}\ \text{A}.$$

Uchinchi garmonika toklarining oniy qiymatlari:

$i_{1(3)} = 2{,}23\sqrt{2}\sin(3\omega t - 78^0)$ A,
$i_{2(3)} = 1{,}14\sqrt{2}\sin(3\omega t - 137^0)$ A,
$i_{3(3)} = 1{,}91\sqrt{2}\sin(3\omega t - 47^0)$ A.

Shoxobchadagi oniy toklar:

$$i_1 = I_{1(0)} + i_{1(1)} + i_{1(3)} = 2 + 4{,}184\sqrt{2}\sin(\omega t + 28^0) + 2{,}23\sqrt{2}\sin(3\omega t - 78^0) \text{ A},$$

$$i_2 = I_{2(0)} + i_{2(1)} + i_{2(3)} = 2 + 3{,}66\sqrt{2}\sin(\omega t - 1^0) + 1{,}14\sqrt{2}\sin(3\omega t - 137^0) \text{ A},$$

$$i_3 = I_{3(0)} + i_{3(1)} + i_{3(3)} = 2{,}03\sqrt{2}\sin(\omega t + 89^0) + 1{,}91\sqrt{2}\sin(3\omega t - 47^0) \text{ A}.$$

Toklarning effektiv qiymatlari:

$$I_1 = \sqrt{I_{1(0)}^2 + I_{1(1)}^2 + I_{1(3)}^2} = \sqrt{2^2 + 4{,}184^2 + 2{,}23^2} = 5{,}146 \text{ A},$$

$$I_2 = \sqrt{I_{2(0)}^2 + I_{2(1)}^2 + I_{2(3)}^2} = \sqrt{2^2 + 3{,}66^2 + 1{,}146^2} = 4{,}32 \text{ A},$$

$$I_3 = \sqrt{I_{3(1)}^2 + I_{3(3)}^2} = \sqrt{2{,}03^2 + 1{,}91^2} = 2{,}79 \text{ A}.$$

4.3. Nosinusoidal tok va kuchlanishlarni ekvivalent sinusoidalar bilan almashtirish

4.5- masala. Berilgan nosinusoidal tok va kuchlanish ekvivalent sinusoidalar bilan almashtirilsin va ular orasidagi φ_{ekv} faza siljish burchagi topilsin.

$U = 25{,}9\sin(\omega t - 11^0 40') + 6\sin(3\omega t + 53^0 50')$ V,
$i = 3\sin(\omega t - 40^0) + 0{,}9\sqrt{2}\sin(3\omega t + 125^0)$ A.

Yechish.

$U_1 = \dfrac{25{,}9}{\sqrt{2}} = 18{,}3$ V, $U_3 = \dfrac{6}{\sqrt{2}} = 4{,}26$ V,

$I_1 = \dfrac{3}{\sqrt{2}} = 2{,}13$A, $I_3 = 0{,}9$A,

$\varphi_1 = -11^0 40' - (-40^0) = 28^0 20'$, $\varphi_3 = -71^0 10'$.

$P = 18{,}3 \cdot 2{,}13\cos 28^0 20' + 4{,}26 \cdot 0{,}9\cos(-71^0 10') = 35{,}5$ W.

Ekvivalent kuchlanish: $U = \sqrt{U_1^2 + U_3^2} = \sqrt{18{,}3^2 + 4{,}26^2} = 18{,}79$ V.

Ekvivalent tok: $I = \sqrt{2{,}13^2 + 0{,}9^2} = 2{,}31$ A .

$$S = UI = 18{,}79 \cdot 2{,}31 = 43{,}4 \text{ VA}.$$

Kuchlanish ekvivalent sinusoidasining effektiv qiymati *U*=18,79 V. Tok ekvivalent sinusoidasining effektiv qiymati esa *I=2,31* A.

Quvvat koeffitsiyenti $\cos\varphi_{\text{ekv}} = \dfrac{P}{U\,I} = \dfrac{35{,}5}{18{,}79 \cdot 2{,}31} = 0{,}818.$

Ekvivalent kuchlanish va tok orasidagi faza siljish burchagi:

$$\varphi_{\text{ekv}} = \arccos 0{,}818 = 35^\circ.$$

4.4. Nosinusoidal tok zanjirlarida rezonans hodisalari

4.6-masala. Ikki yarim davrli to‘g‘rilagich yordamida ω chastotali sinusoidal tokdan hosil bo‘lgan nosinusoidal kuchlanish to‘rtqutblikning 1-1' qismlariga ulangan (4.8-rasm). L_1C_1-ketma-ket va L_2C_2 -parallel konturlar ikkinchi garmonika chastotasi 2ω ga rezonansiga sozlangan, ya’ni

$$2\omega L_1 = \frac{1}{2\omega C_1} \text{ va } 2\omega L_2 = \frac{1}{2\omega C_2}.$$

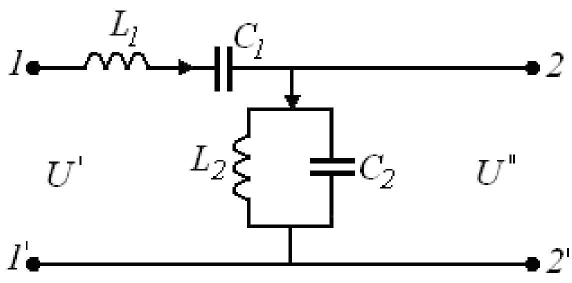
Agar $\omega L_1 = \omega L_2 = 10\,\Omega,\ U_m = 1000\,\text{V}$ bo‘lsa, to‘rtqutblikning 2-2' qismlarida paydo bo‘ladigan kuchlanish effektiv qiymatini aniqlang.

Yechish. Chiqish kuchlanishda ikkinchi garmonika ajraladi, chunki bu garmonika uchun L_1C_1 ketma-ket kontur qarshiligi va L_2C_2 parallel kontur o‘tkazuvchanligi nolga teng. Shu bilan birga boshqa garmonikalar uchun mos qarshiliklar va o‘tkazuvchanliklar ma'lum qiymatga ega bo‘lib, garmonika tartib raqami oshishi bilan o‘sib boradi.

4.8- rasmdan ko‘rinib turibdiki, salt yurish rejimida

$$U_k^{''} = \frac{U_k^{'} Z_2}{\left(\underline{Z}_1 + \underline{Z}_2\right)},$$

bu yerda $\underline{Z}_1 = j\left(k\omega L_1 - 1/k\omega C_1\right),\ \underline{Z}_2 = \dfrac{L_2 / C_2}{j\left(k\omega L_2 - 1/k\omega C_2\right)}.$



4.8- rasm.

$U^{'}$ kuchlanishni quyidagi formula bo‘yicha yozamiz:

$$U^{'}(\omega t)=\frac{4U_m}{\pi}\left(\frac{1}{2}+\frac{1}{1\cdot 3}\cos 2\omega t-\frac{1}{3\cdot 5}\cos 4\omega t+...\right).$$

Chiqish $U_1^{"}$ kuchlanish tarkibida doimiy tashkil etuvchisi bo‘lmaydi, ya’ni $U_{(0)}^{"}=0,$ chunki L_1C_1 ketma-ket konturda kondensator mavjud. Shu bilan birga $U^{"}$ kuchlanish tarkibida birinchi va yuqori toq garmonikalar ham bo‘lmaydi, chunki bu garmonikalar kirish kuchlanishi $U^{'}$ tarkibida ham mavjud emas.

Ikkinchi garmonika uchun: $\underline{Z}_{1(2)}=0$ va $\underline{Z}_{2(2)}=\infty$ bo‘lganligi sababli, to‘rtqutblikning kirish va chiqish qismalaridagi kuchlanish, $U_{(2)}^{"}=U_{(2)m}^{'}=4U_m/3\pi\approx 425\,\text{V}$ ga teng bo‘ladi.

To‘rtinchi garmonika uchun:

$$\underline{Z}_{1(4)}=j30\,\Omega,\ \underline{Z}_{2(4)}=-j13\,\Omega,\ U_{1m(4)}^{"}=4U_m/15\pi\approx 85\,\text{V}.$$

Shuning uchun

$$U_{4m}^{"}=85\cdot(13/17)=65\,\text{V}.$$

Oltinchi garmonika uchun:

$$\underline{Z}_{1(6)}=j53\,\Omega,\ \underline{Z}_{2(6)}=-j7{,}5\,\Omega,\ U_{6m}^{"}=4U_m/35=36\,\text{V},U_{6m}^{"}=36\cdot(7{,}5/45{,}5)=6\,\text{V}.$$

Sakkizinchi va undan yuqori bo‘lgan garmonikalarni e'tiborga olmasa ham bo‘ladi, chunki ularning maksimal qiymatlari juda kichik. Shunday qilib, to‘rtqutblikning 2-2' qismlaridagi kuchlanish:

$$U^{"}=\sqrt{\left(425^2+65^2+6^2\right)/2}=304\,\text{V}.$$

Ikkinchi garmonikaning effektiv qiymati:

$$U_{(2)}^{"}=425/\sqrt{2}=300\,\text{V}.$$

5-BOB. CHIZIQLI ELEKTR ZANJIRLARIDA O‘TKINCHI JARAYONLARNI HISOBLASH

5.1. Asosiy ma’lumotlar

Elektr zanjirlarning ishlashida ikkita rejim farqlanadi: *o‘tkinchi* va *barqaror*.

O‘tkinchi jarayonlarni hisoblashda *klassik, operator, holat o‘zgaruvchan-liklari* va *Dyuamel integrali usullari* qo‘llaniladi.

5.2. O‘tkinchi jarayonlarni hisoblashning klassik usuli

Elektr zanjirlardagi o‘tkinchi jarayonlarni *klassik usul* bilan tahlil etishda tok va kuchlanishlarning oniy qiymatlari uchun tenglamalar sistemasi tuziladi. Bu tenglamalar bevosita Kirxgof qonunlari yoki zanjirni hisoblashning boshqa usullari, misol uchun kontur toklar usuli yoki tugun potensiallari usuli bo‘yicha tuziladi. Bunda zanjir elementlaridagi tok va kuchlanishlar orasidagi quyidagi munosabatlardan foydalaniladi:

$$U_R = iR;\ U_L = L\frac{di}{dt};\ U_C = \frac{1}{C}\int idt;$$

$$i = \frac{U_R}{R},\ i = \frac{1}{L}\int U_L dt;\ i = C\frac{dU_C}{dt}. \tag{5.1}$$

Umumiy holda mujassamlangan *R, L, C* elementlardan iborat chiziqli elektr zanjirlar uchun tenglama integro-differensial bo‘ladi. Bu tenglamani qayta differensiallab quyidagi o‘zgarmas koeffitsiyentli chiziqli nojins (bir jinslimas) tenglama olish mumkin:

$$a_n\frac{d^n y(t)}{dt^n} + a_{n-1}\frac{d^{n-1}y(t)}{dt^{n-1}} + ... + \frac{dy(t)}{dt} + a_0 y(t) = b_m\frac{d^m f(t)}{dt^m} +$$

$$+ b_{m-1}\frac{d^{m-1}f(t)}{dt} + ... + b_1\frac{df(t)}{dt} + b_0 f(t), \tag{5.2}$$

bunda a_n –zanjir sxemasiga va undagi elementlarning parametrlariga bog‘liq o‘zgarmas koeffitsiyentlar; $y(t)$–chiqish kattaligi (tok yoki kuchlanish);

$f(t)$–zanjirga ta’sir qiluvchi tashqi ta’sir EYK yoki tok manbayi.

Differensial tenglamaning hosila darajasi zanjir darajasini belgilaydi. Misol uchun bu tartib birinchi bo‘lsa, zanjir *birinchi tartibli* deb ataladi va h.k.

(5.2) tenglamaning yechimi quyidagi ko‘rinishda izlanadi:

$$y(t) = y_{er}(t) + y_b(t) \tag{5.3}$$

Bunda $y_{er}(t)$ – erkin tashkil etuvchi bo‘lib, quyidagi bir jinsli tenglamaning yechimi hisoblanadi:

$$a_n \frac{d^n y(t)}{dt^n} + a_{n-1} \frac{d^{n-1} y(t)}{dt^{n-1}} + \ldots + a_1 \frac{dy(t)}{dt} + a_0 y(t) = 0 \tag{5.4}$$

$y_b(t)$ – (5.2) ni xususiy echimi bo‘lgan ta’sir etuvchining tashkil etuvchisi.

$y_{er}(t)$ – energiya saqlovchi, L va C elementlarning energiya tarqaltishi yoki yig‘ishini xarakterlaydigan tashkil etuvchi.

Bir jinsli differensial tenglamalarning yechimi quydagi ko‘rinishga ega

$$y_0(t) = A_1 e^{P_1 t} + A_2 e^{P_2 t} + \ldots + A_n e^{P_n t} = \sum_{k=1}^{n} A_k e^{P_k t}\ , \tag{5.5}$$

bunda p_k

$$a_n p^n + a_{n-1} p^{n-1} + \ldots + a_1 p + a_0 = 0 \tag{5.6}$$

xarakteristik tenglamaning ildizlari;

A_k – integrallash doimiylari.

Passiv elektr zanjirlar tegishli xarakteristik tenglamasining $p_к$ ildizlari har doim haqiqiy yoki manfiy bo‘lgan qo‘shma kompleks kattaliklardan iborat bo‘ladi. Tenglamalar ildizining xarakteri erkin tashkil etuvchining ko‘rinishini belgilaydi:

a) ildizlar manfiy, haqiqiy va teng emas, ya’ni $p_1 \neq p_2$ holda – erkin tashkil etuvchi

$$-i_{er}(t) = A_1 e^{-p_1 t} + A_2 e^{-p_2 t};$$

b) ildizlar manfiy, haqiqiy va teng $p_1 = p_2 = a$; erkin tashkil etuvchi–

$$i_{er}(t) = (A_1 + tA_2) e^{-pt};$$

v) ildizlar qo‘shma kompleksli $p_{12} = -b \pm j\omega$; erkin tashkil etuvchi –

$$i_{er}(t) = (A_1 \cos \omega t + A_2 \sin \omega t) e^{-bt};$$

Integrallash doimiysi va xarakteristik tenglamalar ildizlari zanjir tartibiga bog‘liq. Integrallash doimiylari izlanayotgan funksiyalarning boshlang‘ich shartlaridan va uning hosilalaridan topiladi.

Kommutatsiya qonunlari. Boshlang‘ich shartlar.

Kommutatsiyaning birinchi qonuni:

$$i_L(0_-) = i_L(0_+)\,. \tag{5.7}$$

Kommutatsiyaning ikkinchi qonuni:

$$u_L(0_-) = u\ \ (0_+)\,. \tag{5.8}$$

Boshlang‘ich shartlar – tok va kuchlanishlarning $t = 0$ dagi qiymatlaridir.

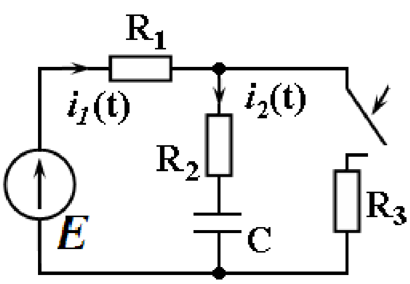
Bog‘lanmagan boshlang‘ich shartlar – kommutatsiya qonunlari bo‘yicha sakrab o‘zgarmaydigan kattaliklar. Ular kommutatsiyadan avvalgi barqaror rejimni hisoblash asosida aniqlanadi:

$$i_L(0_-) = i_L(0_-)\,.$$

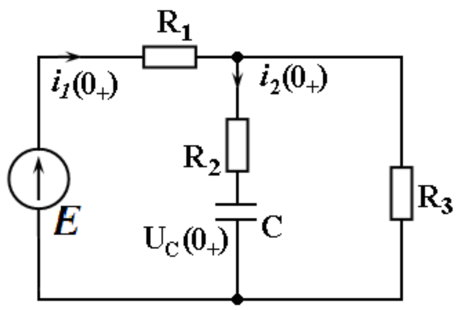
Bog‘langan boshlang‘ich shartlar – kommutatsiya onida $(t = 0_+)$ sakrab o‘zgaradigan tok va kuchlanishlarning qiymati. Ular kommutatsiya qonunlarini e’tiborga olgan holda Kirxgof qonunlari asosida kommutatsiyadan keyin hosil bo‘lgan sxemadan aniqlanadi.

Boshlang‘ich shartlarni hisoblash misoli. Bog‘lanmagan boshlang‘ich shartlarni (5.1- rasm) kommutatsiyaning ikkinchi qonunidan aniqlanadi va bunda quyidagi formula qo‘llaniladi:

$$u_c(0_-) = u_c(0_+) = E \tag{5.9}$$



5.1- rasm.



5.2- rasm.

Bog‘langan boshlang‘ich shartlarni kommutatsiyadan keyingi sxemadan (5.2–rasm), $U_{C0}(0_+)$ qiymat ma’lumligini hisobga olib, Kirxgof qonunlari asosida tenglamalar yozib topiladi, ya’ni

$$\left.\begin{aligned} i_1(0_+) - i_2(0_+) - i_3(0_+) = 0; \\ i_1(0_+)R_1 + i_3(0_+)R_3 = E; \\ i_1(0_+)R_3 - i_2(0_+)R_2 - u_c(0_+) = 0. \end{aligned}\right\} \tag{5.10}$$

(5.10) dagi birinchi tenglamadan $i_1(0_+)$ ifodasini ikkinchi tenglamaga qo‘yamiz:

$$\left.\begin{aligned} i_2(0_+)R_1 + i_3(0_+)(R_1 + R_3) = E; \\ -i_2(0_+)R_2 + i_3(0_+)R_3 = E. \end{aligned}\right\} \tag{5.11}$$

(5.11) tenglamalar sistemasi matritsa shaklida quyidagi ko‘rinishga ega:

$$\begin{vmatrix} R_1 & R_1 + R_3 \\ -R_2 & R_3 \end{vmatrix} \begin{vmatrix} i_2(0_+) \\ i_3(0_+) \end{vmatrix} = \begin{vmatrix} E \\ E \end{vmatrix}. \tag{5.12}$$

(5.12) tenglamalar sistemasining yechimi Kramer formulalariga ko‘ra quyidagi ko‘rinishga ega:

$$i_2(0_+) = \frac{\Delta i_2}{\Delta}; \quad i_3(0_+) = \frac{\Delta i_3}{\Delta} \ . \tag{5.13}$$

Bunda

$$\Delta = \begin{vmatrix} R_1 & R_1 + R_3 \\ -R_2 & R_3 \end{vmatrix} = R_1R_3 + R_1R_2 + R_2R_3;$$

$$\Delta i_2 = \begin{vmatrix} E & R_1 + R_3 \\ E & R_3 \end{vmatrix} = -ER_1;$$

$$\Delta i_3 = \begin{vmatrix} R_1 & E \\ -R_2 & E \end{vmatrix} = -E(R_1 + R_2);$$

$$i_2(0_+) = \frac{-ER_1}{\Delta}; \quad i_3(0_+) = \frac{E(R_1 + R_2)}{\Delta};$$

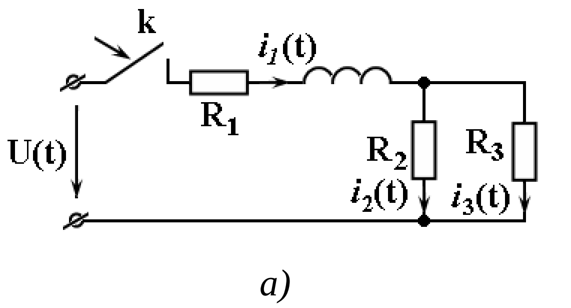
$$i_1(0_+) = i_2(0_+) + i_3(0_+) = \frac{ER_2}{\Delta}.$$

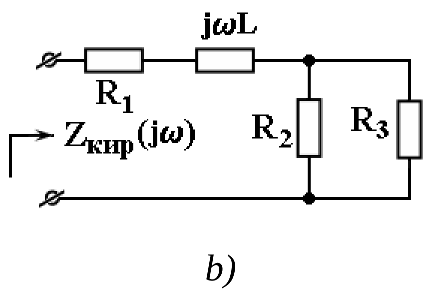
Kommutatsiyadan keyingi sxemaning kirish kompleks qarshiligi bo'yicha xarakteristik tenglama ildizlarini aniqlash. EYK manbayi bo'lgan zanjir uchun kirish kompleks qarshiligi ifodasi yoziladi va uni nolga tenglashtiriladi yoki tok manbasi bo'lgan zanjir uchun kompleks o'tkazuvchanlik ifodasi yoziladi va uni nolga tenglashtiriladi hamda $j\omega$ ni p ga almashtiriladi, keyin esa $Z_{\text{kir}}(p)=0$ yoki $Y_{\text{kir}}(p)=0$ tenglamadan p aniqlanadi.

Quyidagi zanjir uchun (5.3- rasm, *a*) xarakteristik tenglama ildizlarni topamiz.

Ko'rilayotgan sxemadan (5.3- rasm, *a*) kompleks almashlash sxemasiga o'tamiz (5.3- rasm, *b*) va kirish qarshiligi ifodasini yozamiz:

$$Z_{\text{kir}}(j\omega)=R_1+j\omega L+\frac{R_2R_3}{R_2+R_3}\quad . \tag{5.14}$$





5.3- rasm.

$j\omega$ ni p bilan almashtiramiz va $Z_{\text{kir}}(p)$ ni nolga tenglashtiramiz:

$$Z_{\text{kir}}(p)=R_1+j\omega L+\frac{R_2R_3}{R_2+R_3}=0. \tag{5.15}$$

Bundan tenglamaning ildizi

$$p_1=-\frac{R_1+\dfrac{R_2R_3}{R_2+R_3}}{L},\quad \frac{\text{rad}}{\text{s}}. \tag{5.16}$$

O'tkinchi jarayonlarni hisoblash tartibi. Kommutatsiyadan keyingi sxema uchun kommutatsiya qonunlari bo'yicha tok yoki kuchlanishi sakrab o'zgarmaydigan kattaliklarning differensial tenglamalari tuziladi.

Tuzilgan tenglamalarning yechimlari barqaror $i_{\text{bar}}(t)$ va erkin $i_{\text{er}}(t)$ tashkil etvchilar yig‘idisi sifatida topiladi:

$$\begin{aligned} i(t) &= i_{\text{bar}}(t) + i_{\text{er}}(t) \\ u(t) &= u_{\text{bar}}(t) + u_{\text{er}}(t) \end{aligned}; \qquad (5.17)$$

Zanjirning barqaror, ya’ni o‘tkinchi jarayon tomom bo‘lgandan keyingi rejimi $(t \to \infty)$ $i_{\text{bar}}(t)$ yoki $u_{\text{bar}}(t)$ ifodalari topiladi.

Xarakteristik tenglamalarning p_n ildizlari asosida erkin tashkil etuvchilar ifodalari yoziladi. Integrallash doimiylar funksiyaning boshlang‘ich shartlari va uni hosilalari asosida hisoblanadi.

Yechim tekshiriladi va tegishli o‘tkinchi jarayonlar grafiklari quriladi.

O‘tkinchi jarayonlarning sifat tahlili. Reaktiv elementi bitta bo‘lgan zanjirlardagi o‘tkinchi jarayonlarning sifat tahlilini differensial tenglamalarni yechmasdan bajarish mumkin. Bunda kommutatsiya qonunlarini e’tiborga olib Kirxgof qonunlaridan foydalaniladi. Buning uchun ikkita holat qo‘llaniladi:

1) birinchi tartibli zanjirlarda erkin tashkil etuvchilar eksponensial qonun $Ae^{-\frac{t}{\tau}}$ bo‘yicha so‘nadi;

2) barqaror tashkil etuvchi qonun zanjirga ta’sir etayotgan kuchlanishni xarakteri bilan aniqlanadi.

Birinchi tartibli zanjirlardagi o‘tkinchi jarayonlarni hisoblash tartibi:

– o‘tkinchi tok va kuchlanishlarning qiymatlari $t(0_-)$ va $t(0_+)$ uchun aniqlanadi;

– o‘tkinchi tok va kuchlanishlarning qiymatlari $(t = \infty)$ uchun aniqlanadi;

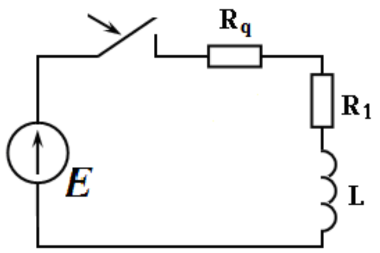
– vaqt doimiylsi τ ni aniqlanadi va o‘tkinchi tok, kuchlanishlarning ifodalari yoziladi;

– tok va kuchlanishlarning grafiklari quriladi.

5.1- masala. Elektromagnit relening chulg‘ami EYK manbayiga *E*=12 V $R_{\text{q}} = 5\ \Omega$ qo‘shimcha qarshilik orqali ulangan (5.4- rasm). Agar rele chulg‘amlarining parametrlari $R_1 = 15\ \Omega$, $L = 0{,}4\,\text{H}$ bo‘lsa, relening ishga tushirish vaqtini toping. $i_{\text{ish}} = 0{,}12\ \text{A}$.

Yechish. Kommutatsiyadan keyingi sxema uchun differensial tenglama tuziladi.

$$L\frac{di(t)}{dt}+(R_{\text{q}}+R_1)i(t)=E.$$



5.4- rasm.

O‘tkinchi tokni $i(t)=i_{\text{bar}}(t)+i_{\text{er}}(t)$ ko‘rinishda izlaymiz. Barqaror $i_{\text{b}}(t)$ tokni $(t\to\infty)$ kommutatsiyadan keyingi sxema uchun aniqlaymiz.

$$i_{\text{bar}}=\frac{E}{R_{\text{q}}+R_1}=\frac{12}{20}=0{,}6\ \text{A}.$$

O‘tkinchi tokning erkin tashkil etuvchisi

$$i_{\text{er}}(t)=Ae^{-\frac{t}{\tau}}.$$

bunda, A integral doimiysini boshlang‘ich shartdan aniqlaymiz:

$$i_L(0_-)=i_L(0_+)=i_{\text{bar}};$$

$$i(t)\big|_{t=0}=i_{\text{bar}}\big|_{t=0}+i_{\text{er}}\big|_{t=0}=\frac{E}{R_q+R_1}+A=0,$$

bundan $A=-0{,}6\,\text{A}$.

Elektr zanjirning vaqt doimiysini aniqlaymiz:

$$\tau=\frac{L}{R_{\text{q}}+R_1}=\frac{0{,}4}{20}=0{,}02\,\text{s}.$$

O‘tkinchi jarayon tokini aniqlaymiz:

$$i(t)=0{,}6-0{,}6e^{-\frac{t}{0{,}02}}=0{,}6(1-e^{-50t})\text{A}.$$

Relening ishlga tushish vaqtini ushbu tenglamadan aniqlaymiz:

$$i_{\text{ish}} = 0{,}12 = 0{,}6(1 - e^{-\frac{t_{ish}}{0{,}02}}).$$

Quyidagi o‘zgarishlardan so‘ng

$$0{,}2 = 1 - e^{-\frac{t_{\text{ish}}}{0{,}02}};\ e^{-\frac{t_{\text{ish}}}{0{,}02}} = 0{,}8; \qquad -\frac{t_{\text{ish}}}{0{,}02} = \ln 0{,}8;$$

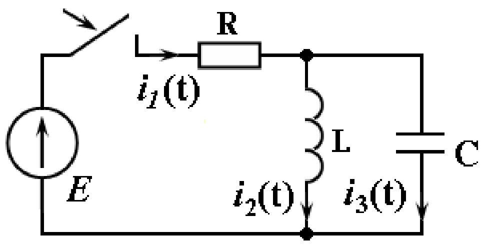
$$-\frac{t_{\text{ish}}}{0{,}02} = -0{,}223; \rightarrow t_{\text{ish}} = 4{,}46 \text{ ms}.$$

5.2-masala. L va C elemnetlar parallel tebranma konturi EYK $E = 125\,\text{V}$ bo‘lgan manbaga ulangan (5.5- rasm). Quyidagi uchta hol uchun kondensatordagi $U_C(t)$ kuchlanish o‘zgarish qonuniyati aniqlansin:

1) $R_1 = 250\ \Omega;\ L = 667\,\text{mH};\ C = 2\,\text{mkF};$

2) $R_1 = 100\ \Omega;\ R_2 = 40\ \Omega;\ L = 667\text{mH}\ \ C = 5\,\text{mkF};$

3) $R_1 = 100\ \Omega;\ \ L = 40\text{mH};\ C = 5\,\text{mkF};$



5.5- rasm.

Yechish. Kondensatordagi kuchlanishni o‘tkinchi jarayondagi ifodasini barqaror va erkin tashkil etuvchilar sifatida yozamiz:

$$U_C(t) = U_{C\text{bar}}(t) + U_{C\text{er}}(t)$$

Barqaror kuchlanishni aniqlaymiz $U_{C\text{bar}}(t)$. Barqaror rejimda $t \rightarrow \infty$ kondensator induktiv g‘altak bilan qisqa shuntlangan, ya’ni $U_{C\text{bar}}(t) = 0$. Xarakteristik tenglama ildizlarini p_n aniqlaymiz. Buning uchun zanjirning kompleks shakldagi kirish qarshiligini topib, $j\omega$ ni p bilan almashtiramiz va hosil bo‘lgan ifodani nolga tenglashtiramiz:

$$Z(p)=R+\frac{pL\frac{1}{pC}}{pL+\frac{1}{pC}}=0 \text{ yoki } RLCp^2+Lp+R=0\,.$$

Bundan

$$p_{1,2}=-\frac{1}{2RC}\pm\sqrt{\frac{1}{4R^2C^2}-\frac{1}{LC}}\,.$$

Berilgan uchta parametrlar uchun $p_{1,2}$ ildizlarni hisoblaymiz:

1) $p_{1,2}=-\frac{1}{2\cdot 250\cdot 2\cdot 10^{-6}}\pm\sqrt{\left(\frac{1}{2\cdot 250\cdot 2\cdot 10^{-6}}\right)^2-\frac{1}{667\cdot 10^{-3}\cdot 2\cdot 10^{-6}}}=(-1000\pm 500)\frac{\text{rad}}{\text{s}}$

$p_1=-500\ \frac{\text{rad}}{\text{s}}$; $p_2=-1500\ \frac{\text{rad}}{\text{s}}$

Ildizlar manfiy va har-xil, shu bois kondensatordagi kuchlanishning erkin tashkil etuvchisi

$$U_{Cer}(t)=A_1e^{-500t}+A_2e^{-1500t}$$

2) $p_{1,2}=-\frac{1}{2\cdot 250\cdot 5\cdot 10^{-6}}\pm\sqrt{\left(\frac{1}{2\cdot 100\cdot 5\cdot 10^{-6}}\right)^2-\frac{1}{40\cdot 10^{-3}\cdot 1\cdot 10^{-6}}}=-5\cdot 10^{-3}\,\frac{\text{rad}}{\text{s}};$

Ildizlar haqiqiy va teng, binobarin,

$$U_{Cer}(t)=A_1e^{-500t}+A_2e^{-500t};$$

3) $p_{1,2}=-\frac{1}{2\cdot 100\cdot 5\cdot 10^{-6}}\pm\sqrt{\left(\frac{1}{2\cdot 100\cdot 5\cdot 10^{-6}}\right)^2-\frac{1}{40\cdot 10^{-3}\cdot 5\cdot 10^{-6}}}=(1000\pm j2000)\frac{\text{rad}}{\text{s}}$

Ildizlar qo‘shma kompleks, bu holda

$$U_{\text{cer}}(t)=(A_1\cos 2000t+A_2\sin 2000t)\cdot e^{-1000t}\,.$$

Integral doimiylari A_1 va A_2 ni boshlang‘ich shartlardan aniqlaymiz:

$$U_c(0_-)=U_c(0_+)=0;$$

$$i_2(0_-)=i_2(0_+)=0;$$

Kommutatsiyadan bevosita avval vaqt uchun $t(0_+)$ Kirxgof qonunlari asosida tuzilgan tenglamalarni yechamiz:

$$i_1(0_+)=i_2(0_+)+i_3(0_+);$$

$$E = i_1(0_+)R + U_c(0_+);$$

Bundan keyin $U_c(t)$ va $i_3(t) = C\frac{dU_c(t)}{dt}$ ifodalaridan A_1 va A_2 larni topamiz.

$t = (0_+)$ vaqt uchun $i_3(0_+) = \frac{E}{R}$

Ildizlarning uchta holati uchun $U_c(t)$ ni hisoblaymiz:

1) ildizlar haqiqiy va teng emas (5.6- rasm): $p_1 = -500$; $p_2 = -150$,

bunda

$$U_c(t) = U_{bar}(t) + U_{er}(t) = 0 + A_1 e^{-500t} + A_2 e^{-1500t};$$

$$i_3(t) = C\frac{dU_c(t)}{dt} = 2\cdot 10^{-6}(-500A_1 e^{-500t} - 1500A_2 e^{-1500t}).$$

Kommutatsiyadan bevosita keyingi vaqt $t = (0_+)$ da

$$U_C(0_+) = A_1 + A_2 = 0;$$

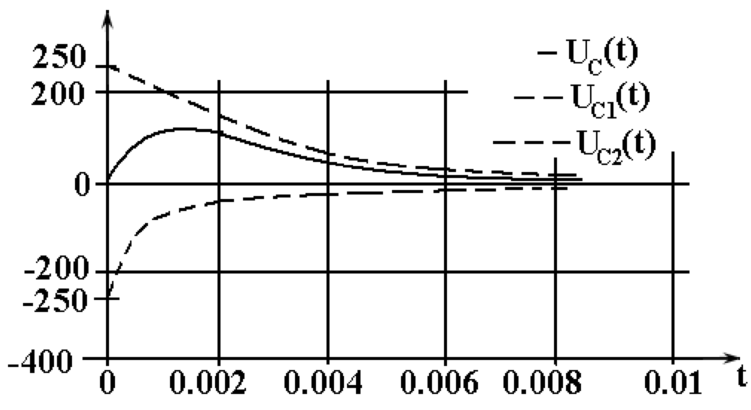
$$i_3(0_+) = 10^{-3} A_1 - 3\cdot A_2 = \frac{E}{R} = \frac{125}{250} = 0{,}5\ \text{A}..$$

binobarin,

$$A_1 = -A_2 = 250\ \text{V};$$

$$U_C(t) = 250e^{-500t} - 250e^{-1500t}\ \text{V}.$$

$U_c(t)$ kuchlanishning grafigi va uning tashkil etuvchilari 5.6- rasmda tasvirlangan.



5.6- rasm.

2) ildizlar teng va haqiqiy: $p_1 = 5000$; $p_2 = -5000$; bu hol uchun

$$U_c(t) = U_{cbar}(t) + U_{cer}(t) = A_1 e^{-5000t} + A_2 e^{-5000t};$$

$$i_3(t) = 10^{-6}(-500A_1 + A_2 - 500A_2 t)e^{-5000t};$$

$t = (0_+)$ da

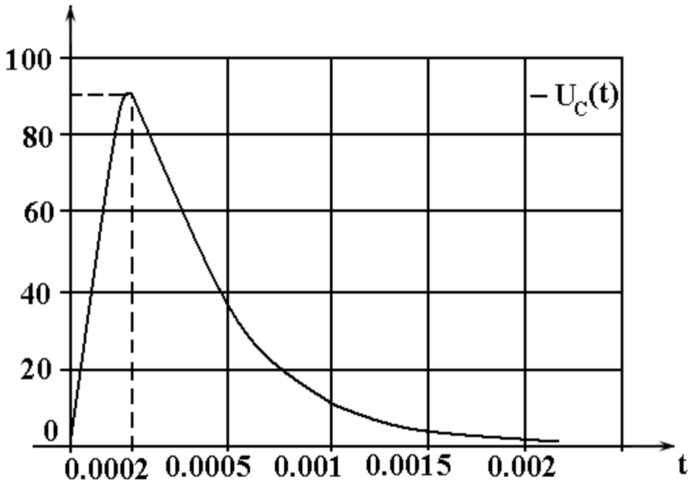
$$U_c(0_+) = A_1 = 0;$$

$$i_3(0_+) = 10^{-6} A_2 = \frac{E}{R} = 1{,}25 \text{ A}.$$

bundan

$$A_2 = 1{,}25 \cdot 10^6 \text{ V/s} \quad \text{va} \quad U_C(t) = 1{,}25 \cdot 10^6 \, e^{-5000t} \text{ V}$$

$U_c(t)$ kuchlanishning grafigi 5.7- rasmda keltirilgan.



5.7- rasm

3) Ildizlar qo‘sh kompleksli $p_{1,2} = (-1000 \pm j2000)\dfrac{\text{rad}}{\text{s}}$, bu holda

$$U_c(t) = U_{cbar}(t) + U_{cer}(t) = 0 + (A_1 \cos 2000 \cdot t + A_2 \sin 2000 \cdot t)e^{-1000t};$$

$$i_3(t) = 5 \cdot 10^{-6}(-A_1 \sin 2000t + A_2 \cos 2000t) \cdot 2000e^{-1000t};$$

$t = (0_+)$ da

$$U_c(0_+) = A_1 + 0 = A_1 = 0;$$

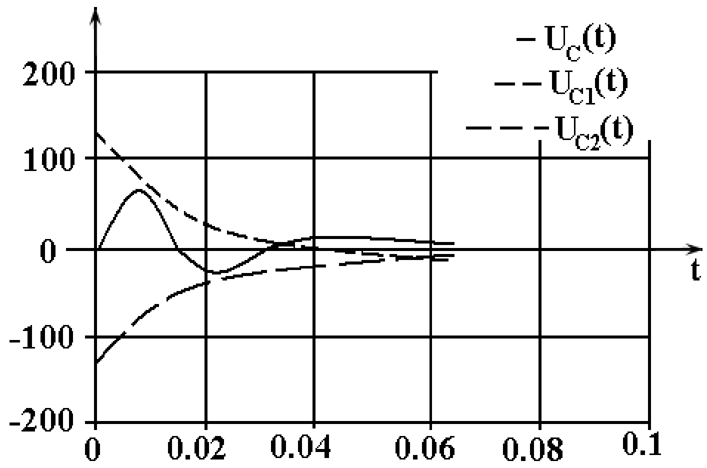
$$i_3(0_+) = 5 \cdot 10^{-6} \cdot 2000 A_2 = \frac{E}{R} = 1{,}25 \text{ A},$$

bundan

$$A_2 = 125 \text{ V} \quad \text{va}$$

$$U_c(t) = 125e^{-1000t} \sin 200t \text{ V}.$$

$U_c(t)$ grafigi 5.8- rasmda tasvirlangan.



5.8- rasm.

5.3- masala. Parametrlari $R_1 = 30\ \Omega$; $R_2 = 20\ \Omega$; $C = 50\,\text{mkF}$; $E = 50\,\text{V}$ bo‘lgan integrallovchi zanjir R_2 qarshilikka ulangan (5.9- rasm). O‘tkinchi jarayonda kondensatordagi kuchlanish va shoxobchalardagi toklar o‘zgarishini sifat nuqtayi nazardan tahlil qiling.

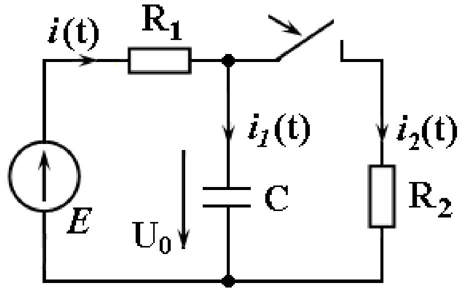
Yechish. O‘tkinchi toklar va kuchlanishlarning qiymatini $t = (0_-)$ va $t = (0_+)$ lar uchun aniqlaymiz. $t = (0_-)$ dagi ekvivalent sxemadan (5.10- rasm) topiladi. $U_c(0_-) = E$; $t = (0_+)$ dagi ekvivalent sxema 5.11- rasmda keltirilgan.

Bu sxema uchun Kirxgof qonunlari asosida tenglamalar sistemasini tuzamiz:

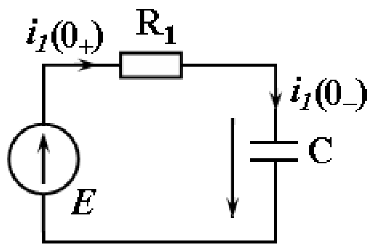
$$i(0_+) - i_1(0_+) - i_2(0_+) = 0;\quad i(0_+) = 0;$$

$$i(0_+)R_1 + U_c(0_+) = E;\quad i_2(0_+) = \frac{E}{R_2} = \frac{100}{20} = 5\,\text{A};$$

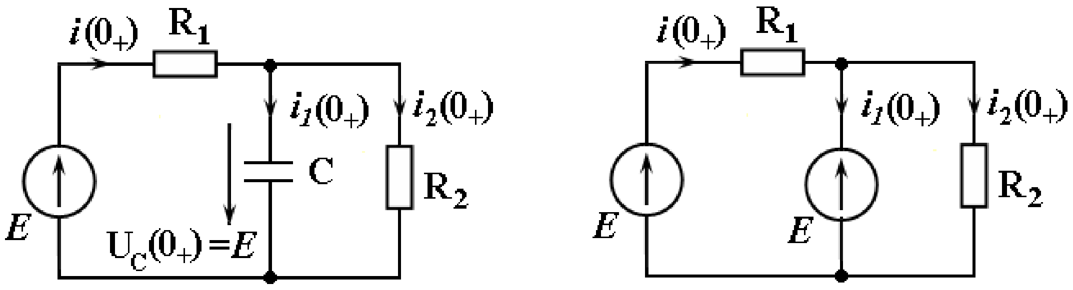
$$i_2(0_+)R_2 - U_c(0_+) = 0;\quad i_1(0_+) = i(0_+) - i_2(0_+) = -5\,\text{A}.$$



5.9- rasm.



5.10- rasm.



a) *b)*

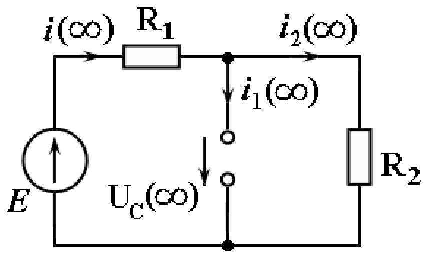
5.11- rasm.

$t \to \infty$ uchun o‘tkinchi toklar va kuchlanishlarni aniqlaymiz. $t \to \infty$ uchun ekvivalent sxema 5.12- rasmda keltirilgan.

$$i(\infty) = i_2(\infty) = \frac{E}{R_1 + R_2} = 2 \text{ A}.$$

$$i_1(\infty) = 0 \,.$$

$$U_c(\infty) = \frac{E}{R_1 + R_2} R_2 = 40 \text{ V}.$$



5.12- rasm.

(5.13) dan foydalanib τ ni aniqlaymiz:

$$\tau = R_{\text{ekv}} C = \frac{R_1 R_2}{R_1 + R_2} C = \frac{20 \cdot 30}{20 + 30} \cdot 50 \cdot 10^{-6} = 6 \cdot 10^{-4} \text{ s}.$$

O‘tkinchi toklar va kuchlanishlar uchun analitik ifodalarni yozamiz

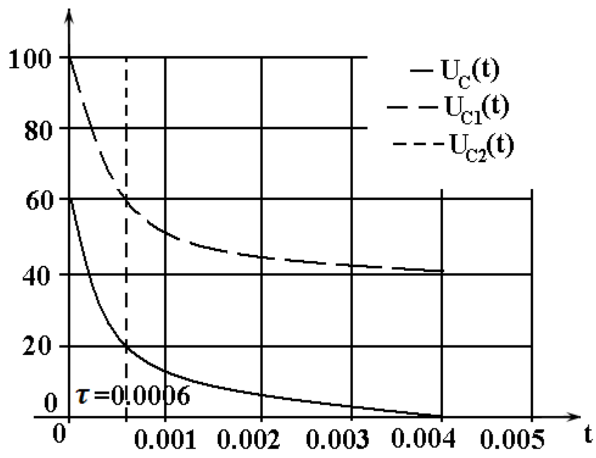
$$U_c(t) = 40 + 60 e^{-1667t} \text{ V};$$

$$i(t) = 2(1 - e^{-1667t}) \text{ A};$$

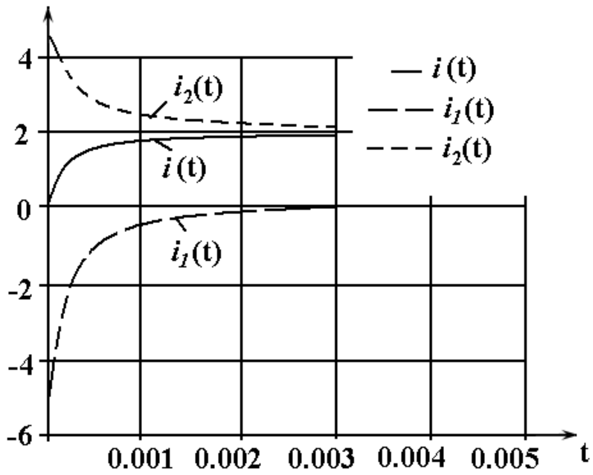
$$i_1(t) = -5^{-1667t} \text{ A};$$

$$i_2(t) = 2 + 3e^{-1667t}) \text{ A}.$$

Kuchlanish va toklarning grafiklari 5.13- va 5.14- rasmlarda keltirilgan.



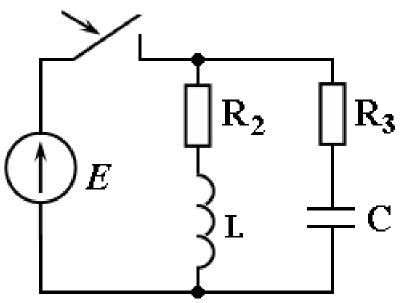
5.13- rasm.



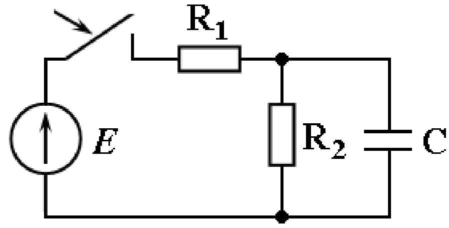
5.14- rasm.

5.4-masala. 5.15-rasmdagi sxema uchun kalit ulangandan so‘ng o‘tkinchi jarayon tenglamalar sistemasi Kirxgof qonuni bo‘yicha yozilsin. Agar $E = 50\,\text{mV}$, $R_2 = R_3 = 10\ \Omega$ bo‘lsa, shoxobchalardagi $i_1(t),\ i_2(t),\ i_3(t)$ toklarni va kondensatordagi kuchlanish $U_c(t)$ ning boshlang‘ich qiymatlari aniqlansin.

5.5- masala. Zanjirdagi (5.16- rasm) kondensator necha voltgacha zaryadlanganda kalit ulangandan so‘ng o‘sha zahoti barqaror rejim paydo bo‘ladi? Berilgan: $R_1 = 20\ \Omega$; $R_2 = 30\ \Omega$; $C = 20\,\text{mkF};$; $E = 20\,\text{V}$.



5.15- rasm.

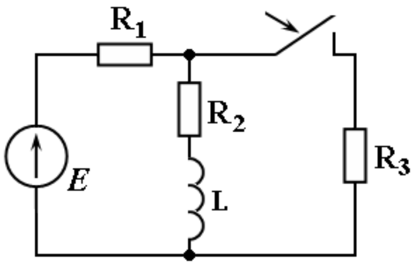


5.16- rasm.

5.6- masala. Aktiv qarshiligi $r_z = 2{,}75\ \Omega$, induktivligi $L = 0{,}55\,\text{H}$, induktiv g‘altak o‘zgarmas EYK $E = 100\,\text{mV}$ manbaga ulangan. O‘tkinchi tok ifodasini yozing.

5.7- masala. Sig‘imi $C = 100\,\text{mkF}$ li kondensator 100 V gacha zaryadlangan, u qarshiligi $r_z = 20\ \Omega$ va induktivligi $L = 100\,\text{mH}$ bo‘lgan g‘altakka razryadlanadi. Kondensatorning $U_c(t)$ kuchlanish o‘zgarish qonuniyatini aniqlansin. Agar shu g‘altak bilan qarshiligi $R = 43{,}3\ \Omega$ bo‘lgan rezistor ulansa, $U_c(t)$ o‘zgarish qonuniyati topilsin.

5.8- masala. Sxemasi 5.17- rasmda tasvirlangan elektr zanjirda agar $E = 120\ \text{mV}$; $R_1 = R_2 = R_3 = 4\ \Omega$; $L = 0{,}1\ \text{H}$ bo‘lsa, kalit ulangandan so‘ng barcha shoxobchalardagi toklar aniqlansin.



5.17- rasm.

5.9-masala. Kuchlanishi o‘zgarmas $U = 100\,\text{V}$ bo‘lgan manbaga qarshiligi $r_q = 20\ \Omega$ bo‘lgan induktiv g‘altak ulangan. $t_1 = 0{,}04\,\text{s}$ da g‘altakdagi kuchlanish $U_q(t_1) = 60\,\text{V}$. G‘altak induktivligi aniqlansin.

5.10- masala. RC zanjir o‘zgarmas $U = 200\,\text{V}$ kuchlanish manbayiga ulangan. Zanjirdagi tok $i(0_+) = 0{,}04\,\text{A}$. Zanjirning o‘zgarmas doimiysi $\tau = 0{,}25\,\text{s}$. Kondensatorning sig‘imi aniqlansin.

5.3. O‘tkinchi jarayonlarni hisoblashning operator usuli

Elektr zanjir nazariyasida chiziqli differensial va integro-differensial tenglamalarni yechish uchun Laplas o‘zgartirishlariga asoslangan *operator usuli* qo‘llaniladi. Bu usulning mohiyati shundaki, haqiqiy t o‘zgaruvchilar funksiyasi kompleks $p = \delta + j\omega$ o‘zgaruvchilarga shunday o‘zgartiladiki, integro-differensial tenglamalar o‘rniga algebraik tenglamalar hosil bo‘ladi. Bu tenglamalar yechilgandan so‘ng haqiqiy t o‘zgaruvchilarga qayta o‘tiladi.

Haqiqiy t o‘zgaruvchilar funksiyasidan kompleks p o‘zgaruvchiga o‘tish *Laplasning to‘g‘ri o‘zgartirishi* yordamida bajariladi:

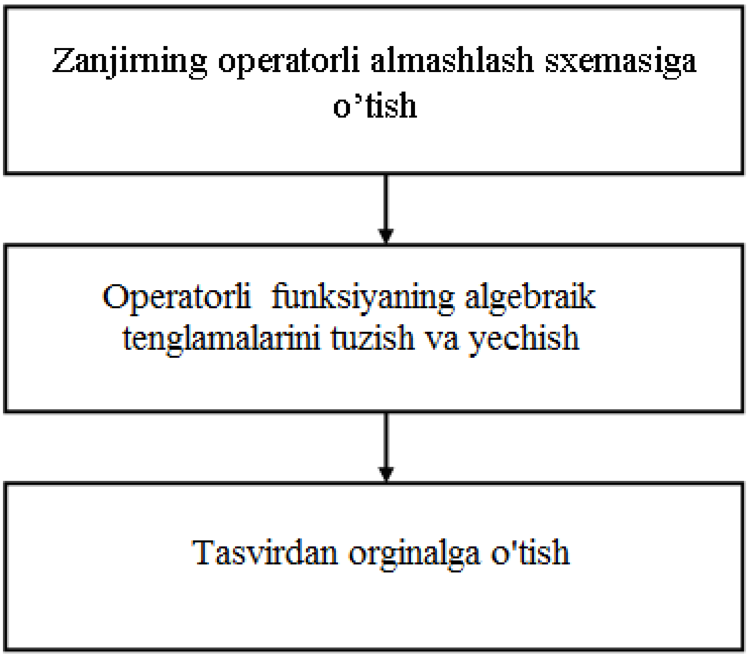
$$F(p) = \int_{0}^{\infty} f(t)e^{-pt}dt\,. \tag{5.18}$$

Kompleks p o‘zgaruvchidan haqiqiy t o‘zgaruvchiga o‘tish *Laplasning teskari o‘zgartirishi* yordamida bajariladi:

$$f(t) = \frac{1}{2\pi j}\int_{\delta - j\omega}^{\delta + j\omega} F(p)e^{pt}dp\,. \tag{5.19}$$

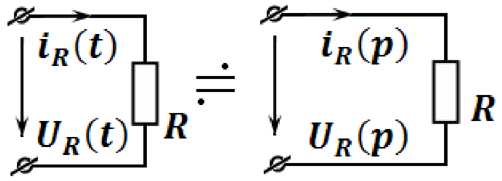
$f(t)$ funksiya original funksiya deyiladi, $F(p)$ funksiya esa *original funksiyaning Laplas bo‘yicha tasvir funksiyasi* yoki qisqacha *tasvir funksiya* deyiladi.

Laplas o‘zgartirishlarining xossalari bilan birga yoyish teoremasidan ham foydalanamiz, tasvir funksiyadan haqiqiy funksiyaga o‘tishni tezlashtirish uchun jadvallardan foydalaniladi. Shunday qilib, operator usuli quyidagi uchta bosqichdan iborat:



Elementlarning ekvivalent operator sxemalari

Aktiv qarshilik. 5.18- rasmda aktiv qarshilikning operator ko‘rinishidagi almashlash sxemasi tasvirlangan:

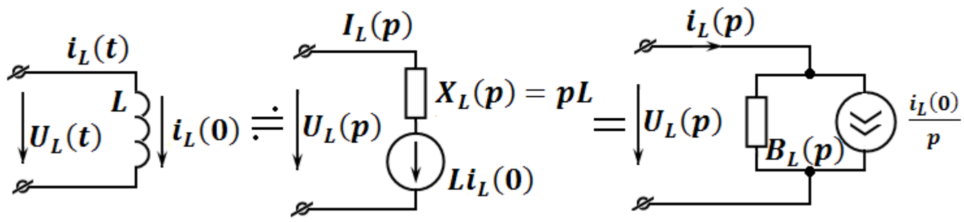


5.18- rasm.

Zanjirning bu qismi uchun operator kuchlanish va tok orasidagi bog‘lanish quyidagi ko‘rinishga ega:

$$U_R(p) = RI_R(p). \tag{5.20}$$

Induktivlik. Noli bo‘lmagan boshlang‘ich shartlarda induktivlikdan iborat zanjir uchastkasining almashlash sxemasi 5.19- rasmda tasvirlangan.



5.19- rasm.

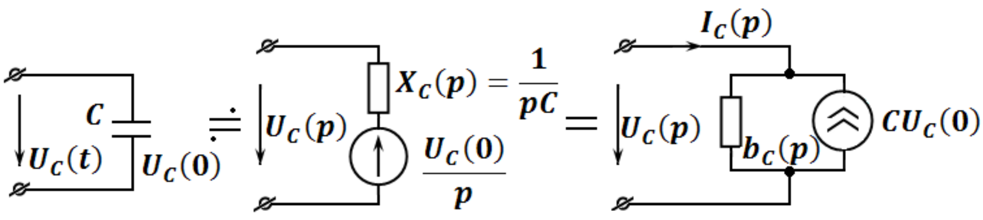
Operator kuchlanish va tok orasidagi bog‘lanish quydagicha yoziladi:

$$U_L(p) = pLI(p) - Li(p). \tag{5.21}$$

Sig‘im. Noli bo‘lmagan boshlang‘ich shartlarda sig‘imli zanjir uchun operator almashlash sxemasi 5.20- rasmda tasvirlangan.

Operator kuchlanish va tok orasidagi boshlanish quydagicha yoziladi:

$$U_c(p) = \frac{1}{pc}I(p) + \frac{U_c(0)}{p} \tag{5.22}$$



5.20- rasm.

Nol bo‘lmagan boshlang‘ich shartlarda induktivlik va sig‘im uchun ekvivalent operator sxemalarda ichki EYK deb atladigan qo‘shimcha EYK $Li(0)$ va $\frac{U_c(0)}{p}$ manbalari paydo bo‘ladi. Ular g‘altakning magnit maydonida va kondensatorning elektr maydonida kommutatsiya onida energiya zahiralanganini ko‘rsatadi. Shunday qilib, integro-differensial tenglamalardan algebraik tenglamalarga o‘tganda boshlang‘ich shartlar avtomatik ravishda hisobga olinadi.

Hisoblash tartibi:

1) bog‘lanmagan boshlang‘ich shartlar aniqlanadi;

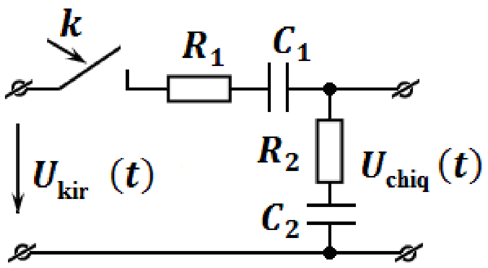
2) kommutatsiyadan keyingi zanjir uchun ekvivalent operator sxema tuziladi;

3) ekvivalent sxemani hisoblashning qulay usuliga ko‘ra operator shaklidagi tenglamalar sistemasi tuziladi va uni noma’lum operator kattalikka nisbatan yechiladi;

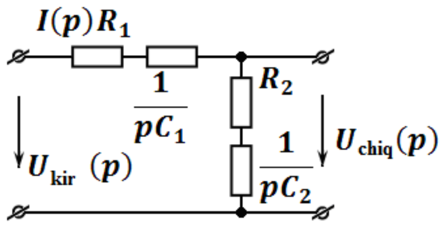
4) noma’lum kattaliklarning originalini topish uchun moslik jadvallaridan yoki yoyish formulalaridan foydalaniladi.

5.11- masala. Elektr zanjirlarni va avtomatik boshqarish sistemalarini korreksiyalashda ishlatiladigan o‘zgarmas tokning integro-differensial konturning (5.21- rasm) chiqish kuchlanishi $U_{\text{chiq}}(t)$ aniqlansin. Berilgan: $U_{\text{kir}}(t) = 100$ V; $R_1 = R_2 = 250\ \Omega$; $C_1 = 20$ mkF; $C_2 = 4$ mkF.

Yechish: Bog‘lanmagan boshlang‘ich shartlarni, ya’ni sig‘imlardagi kuchlanishlarni aniqlaymiz. Kommutatsiyadan avval kalit uzilgan edi, demak, sig‘imlardagi kuchlanish nolga teng bo‘lgan: $U_{C_1} = 0$; $U_{C_2} = 0$;



5.21- rasm.



5.22- rasm.

Kommutatsiyadan keyingi ekvivalent operator sxema tuziladi (5.22- rasm).

$U_{\text{chiq}}(p) = I(p)Z_2(p)$ - kuchlanish topiladi, bunda

$$I(p) = \frac{U_{\text{chiq}}(p)}{Z(p)}; \; Z_2(p) = R_2 + \frac{1}{pC_2}.$$

Zanjirning operator qarshiligi topiladi:

$$Z(p) = \left(R_1 + R_2 + \frac{1}{pC_1} + \frac{1}{pC_2} \right) = (R_1 + R_2) + \frac{C_1 + C_2}{pC_1C_2} = \left(500 + \frac{30 \cdot 10^4}{p} \right) \; \Omega;$$

Tok ifodasi:

$$I(p) = \frac{\frac{100}{p}}{500 + \frac{30 \cdot 10^4}{p}} = \frac{100}{500p + 30 \cdot 10^4} = \frac{0{,}2}{p + 600} \quad \frac{\text{A}}{\text{s}}.$$

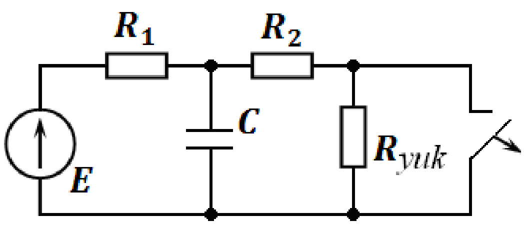
Chiqish kuchlanishi:

$$U_{chiq}(p) = I(p)\left(R_2 + \frac{1}{pC_2} \right) = \frac{0{,}2}{p + 600}\left(250 + \frac{25 \cdot 10^4}{p} \right) = \left(\frac{50}{p + 600} + \frac{5 \cdot 10^4}{p(p + 600)} \right) \text{V}.$$

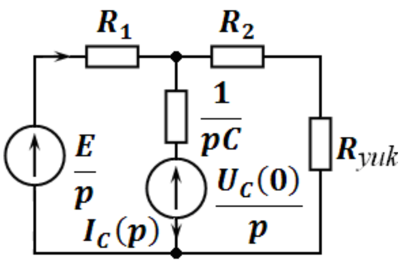
Bu tasvirlangan funksiya uchun Laplasning o‘zgartirish jadvalidan original topiladi.

$$U_{\text{chiq}}(t) = 50e^{-600t} + \frac{5 \cdot 10^4}{600}(1 - e^{-600t}) = 50e^{-600t} + 83{,}3 - 83{,}3e^{-600t} = (83{,}3 - 33{,}3e^{-600t}) \text{ V}.$$

5.12-masala. Quyi chastotali passiv RC - filtrga yuklama R_{yuk} qarshiligi ulangan (5.23-rasm). Berilgan: $E = 40\,\text{V}$; $C = 1800\,\text{pF}$; $R_1 = R_2 = 2\,\text{k}\Omega$; $R_{\text{yuk}} = 3\,\text{k}\Omega$. Kondensatordan o‘tadigan o‘tkinchi tok aniqlansin.



5.23- rasm.



5.24- rasm.

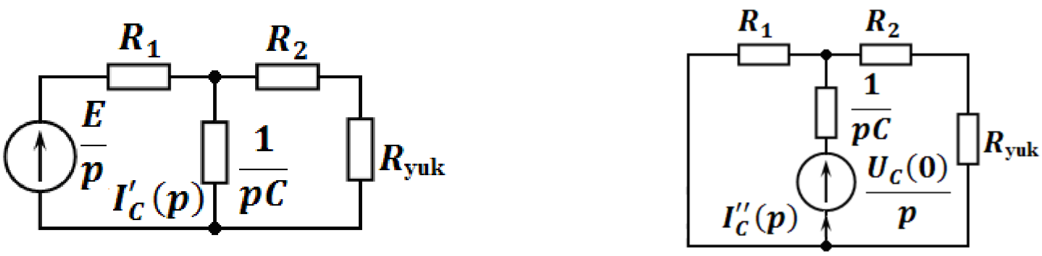
Yechish: Bog‘lanmagan boshlang‘ich shartlarni sig‘imdagi kuchlanish bo‘yicha topamiz:

$$U_c(0)=\frac{E}{R_1+R_2}R_2=\frac{40}{4\cdot 10^3}\cdot 2\cdot 10^3\ \text{V}.$$

Kommutatsiyadan keyingi operator almashlash sxemasini tuzamiz (5.24- rasm). Sxemada ikkita manba paydo bo‘lgani sababli, operator $I_C(p)$ tokni ustma-ustlash usuli bo‘yicha, ya’ni ustlash $I_C'(p)$ va $I_C''(p)$ toklarning algebraik yig‘indisi sifatida topamiz.

Ustlash toklarga mos bo‘lgan operator sxemalar 5.25- rasmda tasvirlangan.

Izlanayotgan tok $I_C(p)=I_C'(p)+I_C''(p);$



a)　　*b*)

5.25- rasm.

Birinchi ustlash tokini 5.25- rasm, *a* dan topamiz:

$$I_C'(p)=\frac{\dfrac{E}{p}}{\left[R_1+\dfrac{(R_1+R_{yuk})\dfrac{1}{pC}}{R_2+R_{yuk}+\dfrac{1}{pC}}\right]}\cdot\frac{R_2+R_{yuk}}{\left[R_2+R_{yuk}+\dfrac{1}{pC}\right]}.$$

Tegishli o‘zgartirishlardan so‘ng,

$$I_C'(p)=\frac{E}{R_1}\cdot\frac{1}{p+\dfrac{R_1+R_2+R_{yuk}}{R_1(R_2+R_{yuk})C}}.$$

Ikkinchi ustlash tokini topamiz (5.25- rasm, *b*):

$$I_C''=\frac{\dfrac{U_C(0)}{p}}{\dfrac{1}{pC}+\dfrac{R_1+R_2+R_{yuk}}{R_1(R_2+R_{yuk})C}}=\frac{U_C(0)(R_1+R_2+R_{yuk})}{R_1(R_2+R_{yuk})}\cdot\frac{1}{p+\dfrac{R_1+R_2+R_{yuk}}{R_1(R_2+R_{yuk})C}}.$$

Izlanayotgan tok:

$$I_C(p)=\frac{E}{R_1}\cdot\frac{1}{p+\frac{R_1+R_2+R_{\text{yuk}}}{R_1(R_2+R_{\text{yuk}})C}}-\frac{U_C(0)(R_1+R_2+R_{\text{yuk}})}{R_1(R_2+R_{\text{yuk}})}\cdot\frac{1}{p+\frac{R_1+R_2+R_{\text{yuk}}}{R_1(R_2+R_{\text{yuk}})C}}=$$

$$\left[\frac{E}{R_1}-\frac{U_C(0)(R_1+R_2+R_{\text{yuk}})}{R_1(R_2+R_{\text{yuk}})}\right]\cdot\frac{1}{p+\frac{R_1+R_2+R_{\text{yuk}}}{R_1(R_2+R_{\text{yuk}})C}}.$$

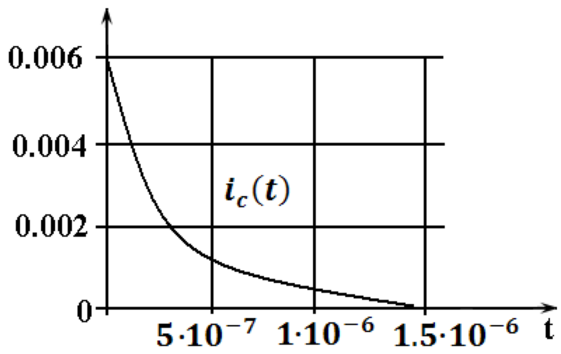
$I_C(p)$ tasvir funksiyaning originali moslik jadvallaridan topiladi:

$$i_C(t)=\left[\frac{E}{R_1}-\frac{U_C(0)(R_1+R_2+R_{\text{yuk}})}{R_1(R_2+R_{\text{yuk}})}\right]e^{-\frac{R_1+R_2+R_{\text{yuk}}}{R_1(R_2+R_{\text{yuk}})C}t}.$$

Formulaga son qiymatlarini qo‘yib quydagini olamiz:

$$i_c(t)=6\cdot10^{-3}e^{-\frac{t}{2{,}57\cdot10^{-6}}}=6\cdot10^{-3}e^{-3{,}89\cdot10^{6}t}\ \text{A}.$$

$i_c(t)$ tokning grafigi 5.26- rasmda tasvirlangan.



5.26- rasm.

5.13- masala. Parallel tebranma kontur (5.27- rasm) $E=125\,\text{V}$ kuchlanishli o‘zgarmas tok manbayiga ulangan. Agar *a)* $R=250\ \Omega$; $L=667\,\text{mH}$; $C=2\,\text{mkF}$ va *b)* $R=100\ \Omega$; $L=40\,\text{mH}$; $C=5\,\text{mkF}$ bo‘lsa, kondensatordagi kuchlanish aniqlansin.

Yechish. Bog‘lanmagan boshlang‘ich shartlar – kondensatordagi kuchlanish $U_C(0)=0$ va g‘altakdagi tok $i_L(0)=0$.

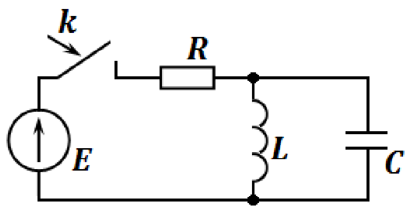
Kommutatsiyadan keyingi ekvivalent operator almashlash sxemasini tuzamiz (5.28- rasm).

Kondensatordagi kuchlanishning tasvir funksiyasini topamiz.

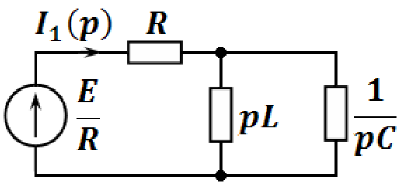
$$U_C(p) = I_1(p)\frac{pL\dfrac{1}{pC}}{pL + \dfrac{1}{pC}},$$

bunda

$$I_1(p) = \frac{E(p)}{Z(p)} = \frac{E}{p\left[R + \dfrac{pL\dfrac{1}{pC}}{pL + \dfrac{1}{pC}}\right]} = \frac{E(p^2LC + 1)}{p(p^2RLC + pL + R)}$$



5.27- rasm.



5.28- rasm.

Kondensatordagi kuchlanish

$$U_C(p) = \frac{E}{RC\left(p^2 + \dfrac{1}{RC}p + \dfrac{1}{LC}\right)} = \frac{F_1(p)}{F_2(p)},$$

bunda

$$F_1(p) = \frac{E}{RC}, \qquad F_2(p) = p^2 + \frac{1}{RC}p + \frac{1}{LC}$$

Topilgan $U_C(p)$ kasrli-ratsional tasvir funksiyaning originali yoyish teoremasiga ko‘ra aniqlanadi:

$$U_c(p) = \sum_{i=1}^{m}\frac{A_i}{p - p_i}$$

bunda p_i – tasvir $U_C(p)$ funksiyaning qutblari;

A_i – tasvir funksiya $U_C(p)$ komponentlarining boshlang‘ich qiymatlari.

Oddiy qutb p_i lardagi komponentlarning boshlang‘ich qiymatlari:

$$A_i = \frac{F(p_i)}{F_2'(p_i)}$$

bunda

$$F_2'(p_i) = \left.\frac{dF_2(p)}{dp}\right|_{p=p_1}$$

Birinchi variantning berilgan qiymatlar uchun:

$$F_1(p) = \frac{125}{(250 \cdot 2 \cdot 10^{-6})} = 0.25 \cdot 10^6;$$

$$F_2(p) = p^2 + \frac{1}{250 \cdot 2 \cdot 10^{-6}} p + \frac{1}{667 \cdot 10^{-3} \cdot 2 \cdot 10^{-6}} =$$

$$= p^2 + 2000p + 0.75 \cdot 10^6 = 0;$$

$U_C(p)$ tasvir funksiyaning qutblari:

$$p_1 = -500 \ \frac{\text{rad}}{\text{s}}; \ \ p_1 = -1500 \ \frac{\text{rad}}{\text{s}}.$$

F_2' hosila hamda uning p_1 va p_2 lardagi qiymatlari hisoblanadi:

$$F_2'(p) = 2p + 2000;$$

$$F_2'(p_1) = 2(-500) + 2000 = 1000;$$

$$F_2'(p_2) = 2(-1500) + 2000 = -1000.$$

A_i komponentlarning boshlang‘ich qiymatlarini hisoblaymiz:

$$A_1 = \frac{F_1(p_1)}{F_2'(p_1)} = \frac{0{,}25 \cdot 10^6}{1000} = 250 \text{ V};$$

$$A_2 = \frac{F_1(p_1)}{F_2'(p_1)} = \frac{0{,}25 \cdot 10^6}{-1000} = -250 \text{ V},$$

binobarin,

$$U_C(t) = A_1 e^{p_1 t} + A_2 e^{p_2 t} = 250(e^{-500t} - e^{-1500t}) \text{ V}.$$

$U_C(t)$ ning grafigi 5.29- rasmda tasvirlangan:

b) ikkinchi variantning son qiymatlari uchun

$$F_1(p) = \frac{125}{100 \cdot 5 \cdot 10^{-6}} = 0{,}25 \cdot 10^6; \ \ F_2(p) = p^2 + 2000p + 5 \cdot 10^6.$$

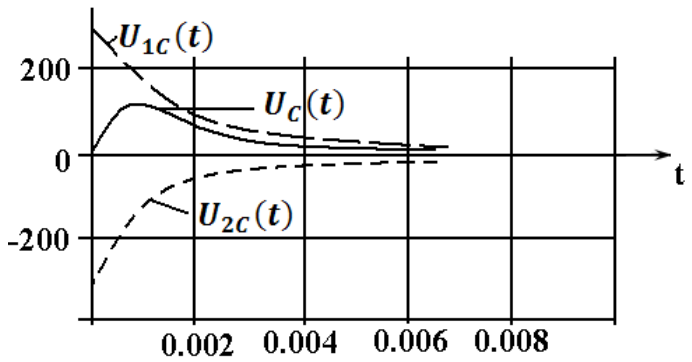
Qutblari uchun esa $p_{1,2} = (-1000 \pm j2000)\dfrac{\text{rad}}{\text{s}}$

$F_2(p)$ ning hosilalari topiladi va p_1, p_2 larda qiymatlari hisoblanadi:

$$F_2' = 2p + 2000;$$

$$F_2'(p_1) = 2(-1000 + j2000) + 2000 = j4000;$$

$$F_2^{'}(p_2) = 2(-1000 - j2000) + 2000 = -j4000;$$



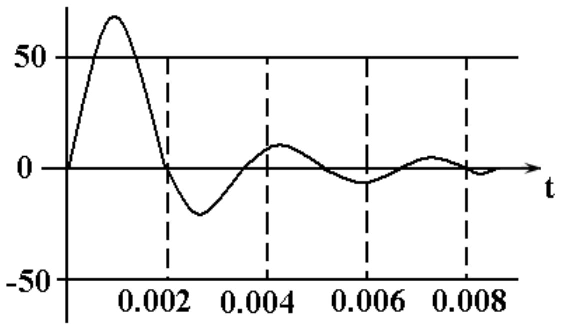
5.29- rasm.

Yoyish teoremasidan quyidagini aniqlaymiz:

$$U_c(t) = \sum_{i=1}^{2} \frac{F_1(p_i)}{F_2^{'}(p_i)} e^{p_i t} = \frac{2{,}5 \cdot 10^5}{j4 \cdot 10^3} e^{(-1000+j2000)t} + \frac{2{,}5 \cdot 10}{-j4 \cdot 10^3} e^{(-1000-j2000)t} =$$

$$-j62{,}5e^{-(1000-j2000)t} + j62{,}5e^{-(1000+j2000)t} = -62{,}5e^{-1000t} \cdot e^{j2000t}$$
$$+62{,}5e^{-1000t} \cdot e^{-j2000t} = j62{,}5e^{-1000t}(e^{-j2000t} - e^{j2000t}) =$$
$$j62{,}5e^{-1000t}(\cos 2000t - j\sin 2000t - \cos 2000t - j\sin 2000t) =$$
$$125e^{-1000t} \cdot \sin 2000t.$$

Bu variant uchun $U_C(t)$ning grafigi 5.30- rasmda tasvirlangan.



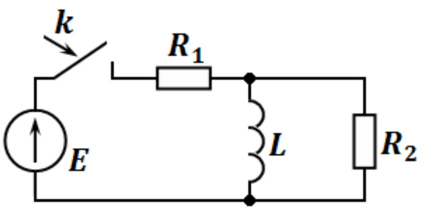
5.30- rasm.

Mustaqil yechish uchun masalalar

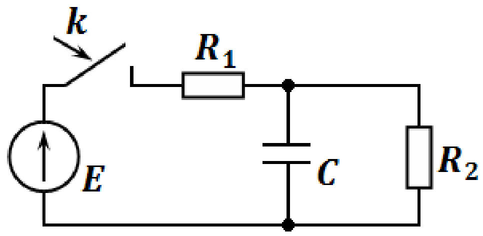
5.14- masala. 5.31- rasmda keltirilgan elektr zajiri sxemasida agar

R_1=20 Ω; R_2=30 Ω; L=0,3 H; E=120 mV bo‘lsa, shoxobchalardagi toklar aniqlansin.

5.15- masala. 5.32- rasmda keltirilgan elektr zajiri sxemasida agar R_1=40 Ω; R_2=10 Ω; C=25 mF; E=10 V bo‘lsa, shoxobchalardagi toklar aniqlansin.



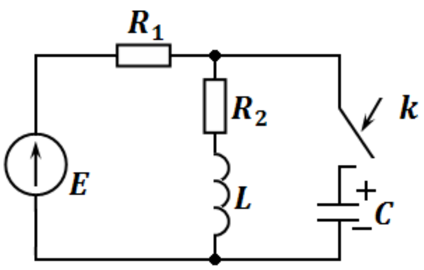
5.31- rasm.



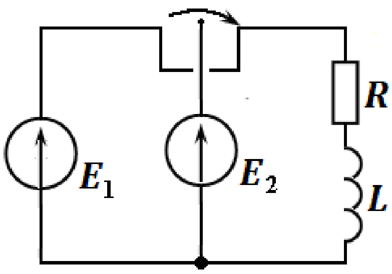
5.32- rasm.

5.16- masala. Elektr zanjir sxemasida (5.33- rasm) sig‘imi $C = 2{,}5\,\text{mF}$ bo‘lgan kondensator kuchlanishi $U_0 = 20\text{ V}$ gacha zaryadlangan edi. Agar E=60 V; R_1=400 Ω; R_2=800 Ω; L=0.2 H; bo‘lsa, shoxobchalardagi toklar aniqlansin.

5.17- masala. Ketma-ket ulangan qarshilik $R = 2\ \Omega$ va kondensator $C = 50$ mkF kuchlanish manbayi $U = 100e^{-5t}\text{ V}$ ga ulangan. Kondensatordagi o‘tkinchi tok va kuchlanish o‘zgarishini aniqlang.



5.33- rasm.



5.34- rasm.

5.18- masala. Zanjir (5.34- rasm) kirishidagi tok kommutatsiyadan keyingi quyidagi uchta holat uchun operator usulida hisoblansin:

$E_1 = 100\text{ V}, \quad E_2 = 0;$

$E_1 = 100\ \text{V}, \quad E_2 = -100\ \text{V};$

$E_1 = 100\ \text{V}, \quad E_2 = 100\sqrt{2}\sin(100t + 45)\ \text{V};$

Bunda $R = 100\ \Omega$, $L = 1$ H.

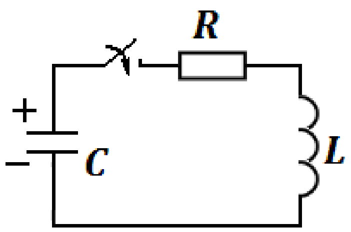
5.19- masala. Kondensator ketma-ket ulangan RL zanjirga razryadlanadi. Razryadlanish xarakterini quyidagi uchta holat uchun aniqlang.

R=50 Ω; L=1 mH; C=10 mkF;

R=20 Ω; L=1 mH; C=10 mkF;

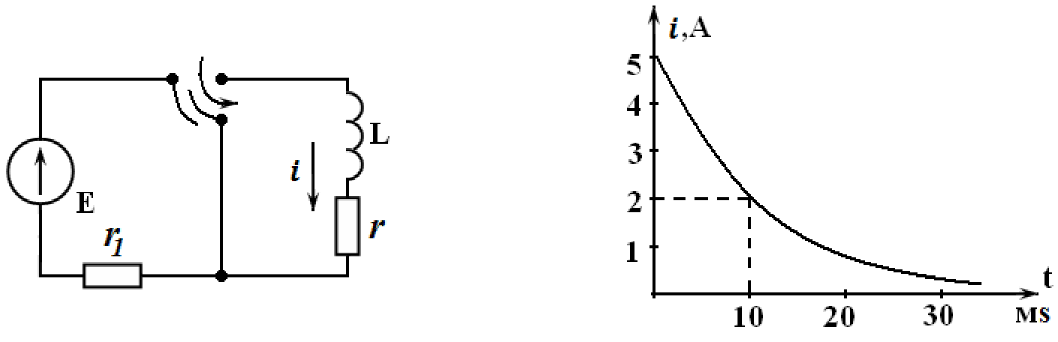
R=5 Ω; L=1 mH; C=10 mkF.

5.20- masala. 5.35- rasmda keltirilgan elektr zanjirida $U_c(0)$=150 V; R=20Ω; L=0.2 H; C=108 mkF bo‘lsa, u holda kalit ulangan vaqtda $U_C(t)$, $i(t)$ aniqlansin.



5.35- rasm.

5.21-masala. Elektr mashina qo‘zg‘atish cho‘lg‘amining kuchlanishi $E = 200\ \text{V}$li va ichki qarshiligi $r = 20\ \Omega$ bo‘lgan EYK manbayidan zanjir uzulmasdan qisqa tutashtirilgan (5.36-rasm). Chulg‘am induktivligi $L = 0{,}2$ H va qarshiligi $r = 20$ Ω, chulg‘amdagi tok aniqlansin va $i(t)$ grafik qurilsin.



a) b)

5.36- rasm.

Yechish. Cho‘lg‘amdagi kommutatsiyadan avvalgi tokni aniqlaymiz:

$$i(0) = E/(r + r_1) = 200/(20 + 20) = 5\ \text{A}.$$

Kommutatsiyadan keyingi holat uchun differensial tenglama bir jinsli bo‘ladi:

$$Ldi/dt + ri = 0\ . \qquad (5.23)$$

Xarakteristik tenglamani kommutatsiyadan keyingi sxemaning kompleks qarshiligidagi ifodasidan $j\omega \to p$ ga almashtirib topamiz: $Lp + r = 0$; U bitta ildizga ega, $p_1 = -r/L = -100\,\dfrac{\text{rad}}{\text{s}}$.

Kommutatsiyadan so‘ng barqaror tok yo‘q, shuning uchun (5.23) tenglama yechimini

$$i = Ae^{p1t} \qquad (5.24)$$

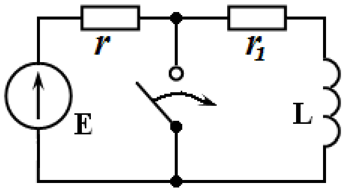
ko‘rinishda topamiz.

Integral doimiysi A ni aniqlash uchun (5.24) ning yechimini $t = 0$ da topamiz: $i(0) = A$. Kommutatsiyaning birinchi qonuni bo‘yicha induktivlikdagi tok sakrab o‘zgarmaydi, binobarin $i(0) = 5\,\text{A}$, bundan $A = i(0) = 5\,\text{A}$. Shunday qilib, qo‘zg‘atish cho‘lg‘amdagi tok $i = 5e^{-100t}$ A.

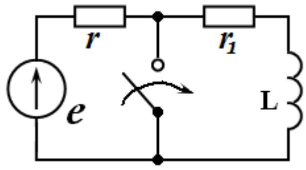
Tok o‘zgarishi grafigi 5.36- rasm, *b* da keltirilgan.

5.22-masala. 5.37-rasmdagi sxemada kalit ulanadi. Agar sxema parametrlari: $E = 90$ V; $r_1 = 30\ \Omega$; $r = 15\,\Omega$; $L = 0{,}1$ H bo‘lsa, $i(t)$ aniqlansin va uning grafigi qurilsin.

5.23-masala. 5.38-rasmdagi sxemada: $e = 50\sqrt{2}\sin(100t + 30^0)$ V, $r_1 = 20\,\Omega$, $r = 30\ \Omega$, $L = 0{,}5\,\text{H}$. Kalit ulangandan so‘ng i tok aniqlansin.



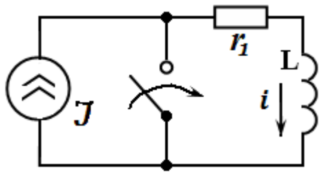
5.37- rasm.



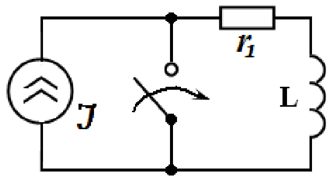
5.38- rasm.

5.24- masala. 5.39- rasmdagi sxema uchun kalit ulangandan so‘ng *i* tok aniqlansin. Berilgan: $J = 6$ A, $r = 10$ Ω, $L = 0{,}25$ H.

5.25- masala. Agar 5.40- rasmdagi sxemada $J(t) = 0{,}5\sin(1000t + 53^\circ 13')$ A, $r = 120\,\Omega$, $L = 0{,}1\,\text{H}$ bo‘lsa, kalit ulangandan so‘ng i tok aniqlansin.



5.39- rasm.



5.40- rasm.

5.26- masala. Agar 5.41- rasmdagi sxema uchun $E = 150$ V, $r_1 = r_2 = r_3 = 100\,\Omega$, $L = 0{,}1\,\text{H}$ ekanligi ma’lum bo‘lsa, kalit ulangandan so‘ng barcha shoxobchalardagi toklarni toping.

Yechish. Bog‘lanmagan boshlang‘ich shart – induktivlikdagi tokni aniqlaymiz:

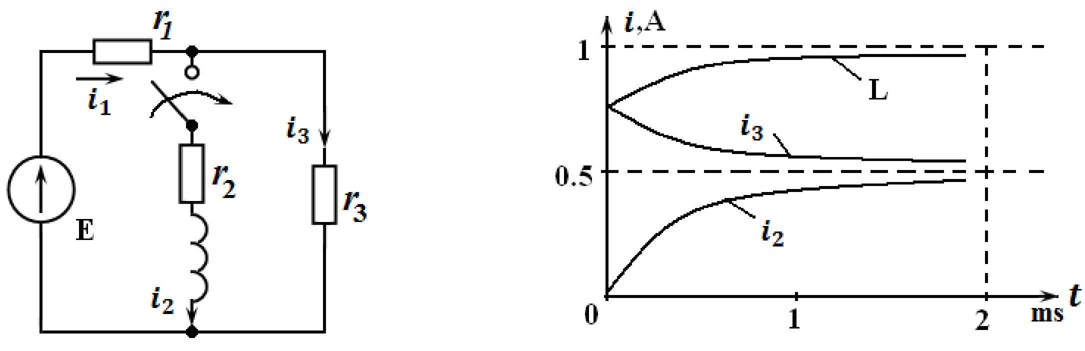
$$i_2(0_-) = 0.$$

Kommutatsiyadan keyingi sxema uchun Kirxgofning tenglamalarini tuzamiz:

$$-i_1 + i_2 + i_3 = 0,$$

$$r_1 i_1 + r_3 i_3 = E,$$

$$r_2 i_2 + L\frac{di_2}{dt} - r_3 i_3 = 0,$$



a) *b)*

5.41- rasm.

Olingan tenglamalarni bitta o‘zgaruvchi, masalan, i_2 ga nisbatan o‘zgartiramiz. Buning uchun tenglamalardan i_1 va i_3 larni ozod etamiz. Natijada tok uchun differensial tenglama hosil qilamiz:

$$L\frac{di_2}{dt}+(r_2+\frac{r_1r_3}{r_1+r_3})i_2=E\frac{r_3}{r_1+r_3}.$$

Bu tenglamaning yechimini quyidagi ko‘rinishda qidiramiz:

$$i_1=i_{1\text{bar}}+i_{1\text{er}};\quad i_2=i_{2\text{bar}}+i_{2\text{er}};\; i_3=i_{3\text{bar}}+i_{3\text{er}}.$$

Barqaror rejimdagi toklar:

$$i_{1\text{bar}}=\frac{E}{r_1+r_2r_3/(r_2+r_3)}=1\,\text{A};\quad i_{2\text{bar}}=i_{1\text{bar}}\frac{r_3}{r_2+r_3}=0{,}5\,\text{A};\quad i_{3\text{bar}}=i_{1\text{bar}}\frac{r_2}{r_2+r_3}=0{,}5\,\text{A}.$$

$L_p+(r_2+\frac{r_1r_3}{r_1+r_3})=0$ xarakteristik tenglama bitta ildizga ega, ya’ni

$$p_1=-\frac{r_2+r_1r_3/(r_1+r_3)}{L}=-1500\,\text{rad/s},$$

bundan erkin tashkil etuvchi toklar:

$$i_{1\text{er}}=Ae^{p_1t};\quad i_{2\text{er}}=Be^{p_1t};\quad i_{3\text{er}}=Ce^{p_1t},$$

O‘tkinchi toklarning umumiy ifodasi:

$$i_1=1+Ae^{p_1t};\; i_2=0{,}5+Be^{p_1t},\quad i_3=0{,}5+Ce^{p_1t}.$$

Xususiy holda $t=0$ uchun

$$i_1(0)=1+A;\;\; i_2(0)=0{,}5+B;\;\; i_3(0)=0{,}5+C. \qquad (5.25)$$

Integral doimiylarini aniqlash uchun, toklarning boshlang‘ich qiymatlarini topamiz. Kommutatsiya qonuni bo‘yicha bog‘lanmagan boshlang‘ich shart induktivlikdagi tokdir.

$$i_2(0)=i_2(0_-)=0.$$

Boshqa, $i_1(0)$ va $i_3(0)$ toklarning boshlang‘ich qiymatlarini, ya’ni bog‘langan boshlang‘ich shartlarni $t=0$ vaqt onida yozilgan Kirxgofning ikkita birinchi tenglamasidan aniqlaymiz:

$$\begin{aligned}&-i_1(0)+i_2(0)+i_3(0)=0;\\&r_1i_1(0)+r_3i_3(0)=E;\\&i_1(0)=i_3(0)=E/(r_1+r_3)=0{,}75\,\text{A}.\end{aligned} \qquad (5.25)$$

tenglamalardan integral doimiylarini topamiz:

$$A=-0{,}25\,\text{A};\quad B=-0{,}5\,\text{A};\quad C=0{,}25\,\text{A}.$$

Shunday qilib, toklarning ifodalari quyidagicha:

$i_1 = 1 - 0{,}25e^{-150\Omega}\,\text{A}$; $i_2 = 0{,}5 - 0{,}5e^{-150\Omega}\,\text{A}$; $i_3 = 0{,}5 + 0{,}25e^{-150\Omega}\,\text{A}$.

Toklarning vaqt bo‘yicha o‘zgarish grafiklari 5.41- rasm, *b* da keltirilgan.

5.27- masala. 5.42- rasmdagi sxemaning parametrlari $E = 120\,\text{V}$, $J = 6\,\text{A}$, $r_1 = r_2 = r_3 = 10\ \Omega$, $L = 0{,}05$ H. Birinchi shoxobchadagi kalit ajratilganda ikkinchi va uchinchi shoxobchadagi toklar aniqlansin.

Yechish. EYK E va tok J manbalaridan induktivlikdagi kommutatsiyadan avvalgi tokni aniqlaymiz:

$$i_3(a) = i_3'(0_-) + i_3\ (0_-) = 4 + 2 = 6\,\text{A}.$$

Zanjirning kommutatsiyadan keyingi sxemasi uchun Kirxgofning qonunlari asosida tenglamalar tuzamiz:

$$-J - i_2 + i_3 = 0;$$

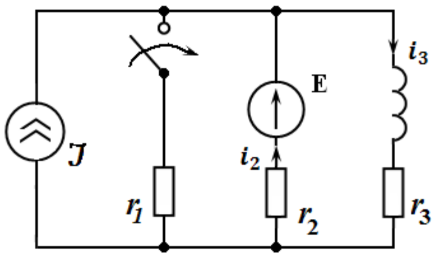
$$r_2 i_2 + L\frac{di_3}{dt} + r_3 i_3 = E \cdot$$

Bu ikkita tenglamadan i_2 tokni chiqarib tashlab, i_3 tok uchun differensial tenglama hosil qilamiz:

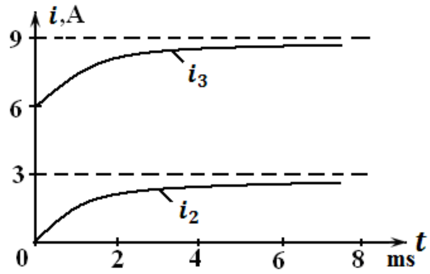
$$L\frac{di_3}{dt} + (r_2 + r_3)i_3 = E + r_2 J \cdot \qquad (5.26)$$

Izlanayotgan toklar:

$$i_2 = i_{\text{bar}} + i_{2\text{ekv}};\quad i_3 = i_{3\text{bar}} + i_{3\text{ekv}}.$$



a)



b)

5.42- rasm.

Barqaror rejimdagi tokni ustmalash usulida topamiz:

$$i_{2б} = E/(r_2 + r_3) - Jr_3/(r_2 + r_3) = 3\text{ A}.$$
$$i_{3б} = E/(r_2 + r_3) + Jr_2/(r_2 + r_3) = 9\text{ A}.$$

$Lp + (r_2 + r_3) = 0$ xarakteristik tenglama quyidagi ildizga ega:

$$P_1 = -(r_2 + r_3)/L = -400\text{ rad/s}.$$

Shunga ko‘ra tokning erkin tashkil etuvchilari:

$$i_{2\text{er}} = Ae^{p_1 t};\quad i_{3\text{er}} = Be^{P_1 t}.$$

Demak, toklar:

$$i_2 = 3 + Ae^{P_1 t};\ i_3 = 9 + Be^{P_1 t}\text{ A}. \qquad (5.27)$$

Integral doimiylarini aniqlash uchun (5.27) dan foydalanamiz. $t = 0$ da:

$$i_2(0) = 3 + A; \quad i_3(0) = 9 + B.$$

Bog‘langan boshlang‘ich shartni – induktivlikdagi $i_3(0)$ tokni kommutatsiya qonunidan aniqlaymiz:

$$i_3(0) = i_3(0_-) = 6\text{ A}.$$

Bog‘langan boshlang‘ich $i_2(0)$ shart, ya’ni boshlang‘ich qiymatni $t = 0$ vaqt uchun

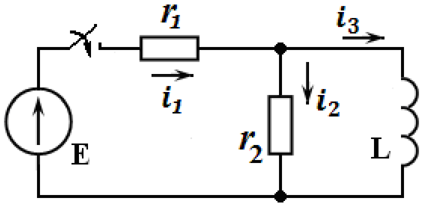
$$-J - i_2 + i_3 = 0.$$

tenglamadan aniqlaymiz, bundan $i_2(0) = i_3(0) = -J = 0$ va integral doimiylari: $A = i_2(0) - 3 = -3\text{ A}$; $B = i_3(0) - 9 = -3\text{ A}$.

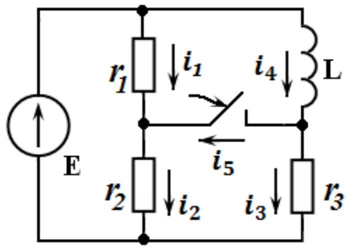
Izlanayotgan toklar. $i_2 = 3 - 3e^{-400t}\text{ A}$; $i_3 = 9 - 3e^{-400t}\text{ A}$. Bu ifodalarning grafiklari 5.42- rasm, *b* da tasvirlangan.

5.28- masala. 5.43- rasmdagi sxema uchun shoxobchalardagi kommutatsiyadan keyingi toklarni toping. Berilgan: $E = 36\text{ V}$; $r_1 = 12\,\Omega$; $r_2 = 24\,\Omega$; $L = 0{,}02\text{ H}$.

5.29- masala. 5.44- rasmdagi sxemada kommutatsiyadan so‘ng barcha toklar topilsin. Berilgan: $E = 90\text{ V}$; $r_1 = 10\ \Omega$; $r_2 = 30\ \Omega$; $r_2 = 15\ \Omega$; $L = 0{,}04\text{ H}$.

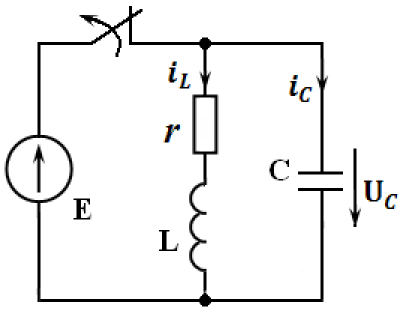


5.43- rasm.



5.44- rasm.

5.30- masala. 5.45- rasmda tasvirlangan zanjirda $C = 0{,}5\,\text{mkF}$, $L = 1\ \text{H}$, $r = 40\ \Omega$, $E = 40\,\text{V}$ bo‘lsa, induktivlikdagi tok va kuchlanish aniqlansin.



5.45- rasm.

Yechish. Kommutatsiya qismlariga nisbatan elektr zanjirning kompleks qarshiligini aniqlaymiz:

$$Z(j\omega) = \frac{(r + j\omega L)\dfrac{1}{j\omega C}}{r + j\omega L + \dfrac{1}{j\omega C}}.$$

Elektr zanjirning xarakteristik tenglamasini tuzamiz:

$$Z(p) = \frac{(r + pL)\dfrac{1}{pC}}{r + pL + \dfrac{1}{pC}} = 0.$$

Berilgan parametrlarni bu tenglamaga qo‘yib, quyidagi qo‘shma kompleks ildizlarni topamiz:

$$p_1 = \alpha + j\omega_c = -20 + j40;$$
$$p_2 = \alpha - j\omega_c = -20 - j40;$$

bunda $\alpha = -20$ – so‘nish koeffitsiyenti; $\omega_c = 40$ – kontur tebranishlarining burchak chastotasi.

Xarakteristik tenglamaning ildizlari kompleks bo‘lgani uchun induktivlikdagi tok ifodasi quyidagi ko‘rinishga ega bo‘ladi:

$$i_L = i_{LЭЭР} = Ae^{\beta t}\sin(\omega_c t + \psi_i),$$

bu yerda A, ψ_i – integral doimiylari, A – tokning amplitudasi, ψ_i – boshlang‘ich faza. Induktivlikdagi kuchlanishning ifodasi:

$$U_L = Ldi_L/dt = LAe^{\beta t}[\omega_c\cos(\omega_c + \psi_i) + \beta\sin(\omega_c + \psi_i)] =$$
$$= LA\sqrt{\omega_c^2 + \beta^2}\cos(\omega_c + \psi_i + \varphi),$$

bunda $\mathrm{tg}\varphi = \beta/\omega_C = -2$; bundan faza siljishi $\varphi = -63^\circ 30'$.

Bu tenglamalardan integral doimiylarini aniqlash uchun kommutatsiya qonunlaridan foydalanamiz. Kommutatsiyaning birinchi qonuniga ko‘ra:

$$i_L(0_+) = A\sin\psi = 1,$$

$$u_L(0_+) = LA(\beta\sin\psi_1 + \omega_c\cos\psi_1) = 0.$$

Ikkinchi tenglama bo‘yicha $tg\psi_1 = -\omega_c/\beta = 2,$ buning asosida $\psi_1 = 63^\circ 30'$ (yoki $\psi_1 = 1{,}108$ rad). Bundan keyin birinchi tenglamadan:

$$A = \frac{1}{\sin 63^\circ 30'} = 1{,}117 \ .$$

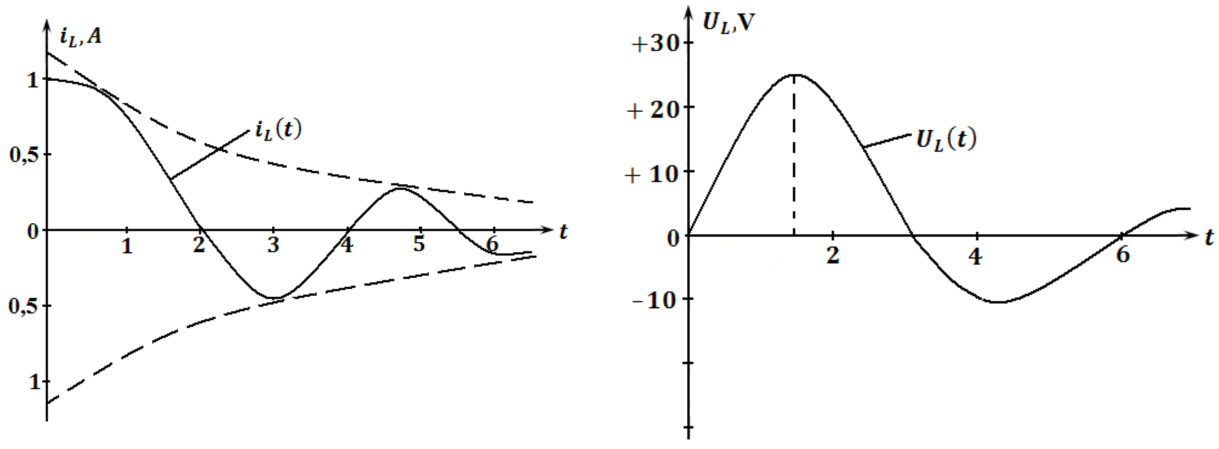
Shunday qilib, induktivlikdan o‘tadigan tok quyidagi ifodaga ega:

$$i_L = 1{,}117e^{-20t}\sin(4ot + 63^\circ 30')\ \text{A}.$$

Parametrlarni ifodalarga qo‘yib induktivlikdagi kuchlanishni topamiz:

$$u_L = 1{,}117e^{-20t}[40\cos(40t + 63^0 30^0] - 20\sin(40t + 63^0 30') = 50e^{-20t}\sin 40t\ \text{V}.$$

Tok va kuchlanishning vaqt bo‘yicha o‘zgarish grafiklari 5.46- rasmda keltirilgan.



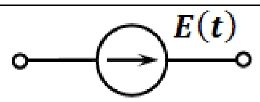
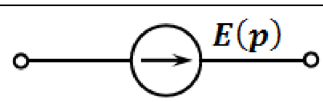
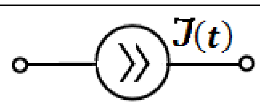
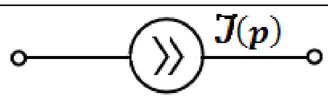
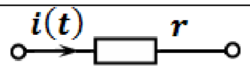
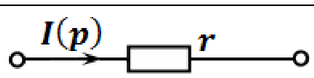
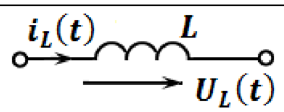
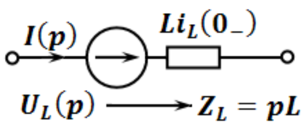
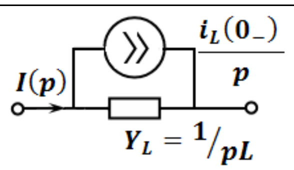
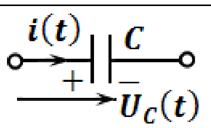
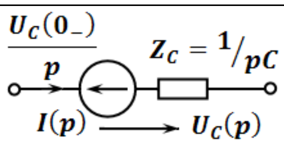
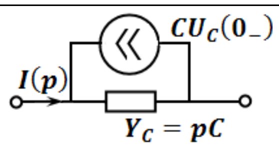
a) *b)*

5.46- rasm.

b) O'tkinchi jarayonlarni operator usulda hisoblash.

Zanjir elementlarining operatorli almashlash sxemalari

5.1-*jadval*

Zanjir elementi	Original sxema	Operatorli almashlash sxemasi
Kuchlanish manbayi		
Tok manbayi		
Qarshilik		
Induktivlik		 
Sig'im		 

Operatorli almashlash sxemasini hisoblash bosqichida o‘zgarmas tok zanjirlarining ma’lum bo‘lgan barcha hisoblash usullarini qo‘llash mumkin:

– kirxgof qonunlarini va har qanday ekvivalent o‘zgartirishlarini;

– kontur toklar va tugun potensiallari usullarini;

– ustma-ustlash va ekvivalent generator usullarini.

Bunda to‘g‘ri va teskari qo‘yilgan masalalarni yechish mumkin, chunki operatorli almashlash sxemasi zanjir tok va kuchlanishlarining tasvir funksiyasini aniqlashga imkon beradi.

Zanjirning original funksiyasini uning ekvivalent operatorli funksiyasidan aniqlash bosqichida quyidagi usullardan foydalanish joiz:

– ma’lumotlarda keltiriladigan operator moslik jadvalidan foydalanish (5.1-jadval);

– operator moslik jadvalidan foydalanish;

– yoyish formulalarini qo‘llash.

Agar $F(p)$ operatorli tasvir funksiya surat $F_1(p)$ pomlinomining m darajasi maxraj $F_2(p)$polinomining n darajasidan katta bo‘lmagan kasrli - ratsional funksiya bilan ifodalangan bo‘lsa,

$$F(p) = \frac{F_1(p) a_m p^m + a_{m-1} p^{m-1} + ... + a_1 p + a_0}{F_2(p) b_n p^n + b_{n-1} p^{n-1} + ... + b_1 p + в_0},$$

unda originalni quyidagi yoyish formulasidan aniqlash mumkin:

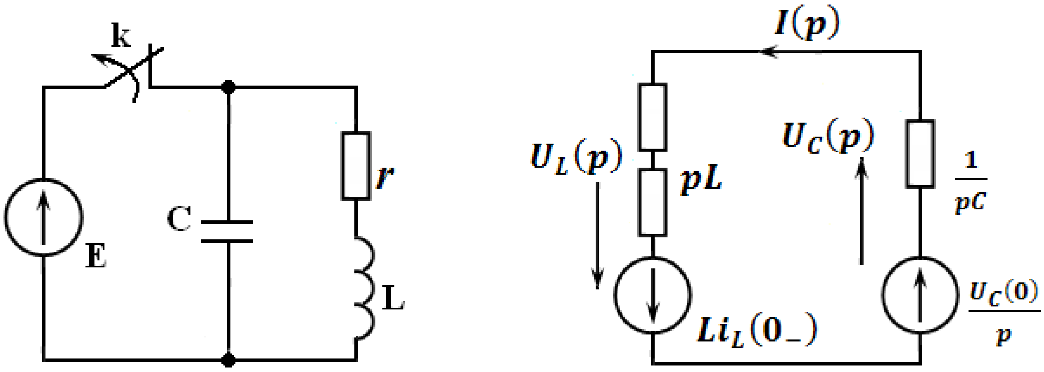
$$f(t) = \sum_{k=1}^{n} \frac{F_1(p_k)}{F_2^{'}(p_k)} e^{p_k t},$$

Bunda $p_k - F_2(p) = 0$ tenglamaning ildizlari, $F_2^{'}(p) = \frac{dF_2}{dp}_{p=p_к}$ – maxrajning $p = p_k$ dagi hosilasi.

5.31- masala. 5.47- rasmda keltirilgan zanjirning parametrlari quyidagi qiymatlarga ega: $E = 40\,\text{V}$; $r = 40\ \Omega$; $L = 1\,\text{H}$; $C = 1/300\ \text{F}$. Zanjirdagi o‘tkinchi jarayonni operator usulda hisoblang.

Yechish. Masalani yechishni kalit uzilgandan so‘ng original sxemaga mos bo‘lgan operator sxemani tuzishdan boshlaymiz.

Operator sxema (5.47- rasm, *b*) original sxemadan shu bilan farq qiladiki, unda induktivlik 5.1- jadval bo‘yicha qarshiligi pL va kuchlanishi $Li_L(0_-) = LE/r = 1\cdot 40/40 = 1\,\text{V}$ bo‘lgan manba bilan, sig‘im esa qarshiligi $(pC)^{-1}$ va kuchlanishi $\dfrac{u_C(0_-)}{p} = \dfrac{40}{p}$ V bo‘lgan manba bilan almashtirilgan.



a) *b)*

5.47- rasm.

Sxemani hisoblash uchun Kirxgofning ikkinchi qonuni bo‘yicha tenglama tuzamiz:

$$I(p)(r + pL + 1/pC) = U_c(0_-)/p + Li_L(0_-)$$

va undan operator tokni topamiz:

$$I(p) = \frac{U_c(0_-)/p + Li_L(0_-)}{r + pL + 1/pC} = \frac{p+40}{p^2 + 40p + 300} = \frac{p+40}{(p+10)(p+30)}.$$

Sig‘imdagi hamda induktivlikdagi kuchlanishlarni topamiz:

$$u_C(p) = I(p)/pC - u_C(0_-)/p = \frac{-(40p+1300)}{p^2+40p+300} = \frac{-(40p+1300)}{(p+10)(p+30)};$$

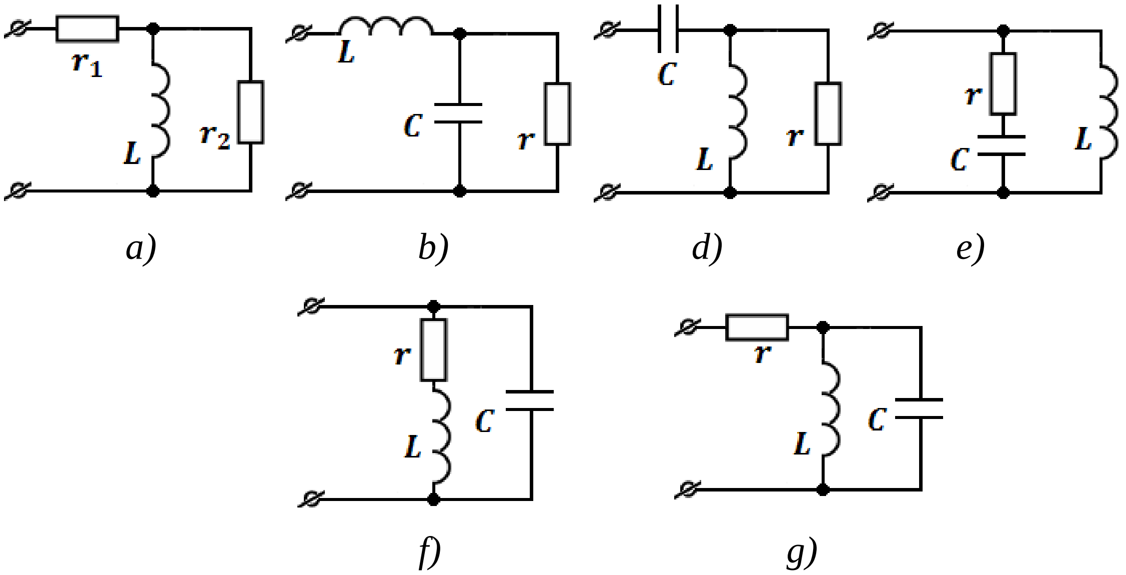
$$u_L(p) = I(p)/(pL) - i_L(0_-)L = \frac{-300}{p^2+40p+300} = \frac{-300}{(p+10)(p+30)}.$$

Hosil qilingan ifodalardan ko‘rinib turibdiki, kuchlanish va tokning mahrajlari bir xil, ya’ni $p_1 = -10$ rad/s , $p_2 = -30$ rad/s.

Endi yoyish formulalaridan foydalanib quyidagilarni hosil qilamiz:

$$i(t) = 1{,}5e^{-10t} - 0{,}5e^{-30t}\ \text{A},$$
$$u_C(t) = -45e^{-10t} + 5e^{-30t}\ \text{V},$$
$$u_L(t) = -15(e^{-10t} - e^{-30t})\ \text{V}.$$

5.32- masala. 5.48- rasmda keltirilgan elektr zanjirlarning operator qarshiligini ifodalang.



5.48- rasm.

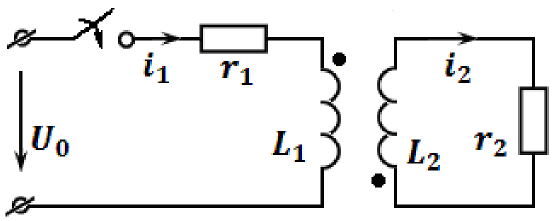
5.33- masala. Havo transformatori o‘zgarmas tok manbayiga ulanganda(5.49- rasm), $i_1(t)$ va $i_2(t)$ larni ifodalang bunda birlamchi va ikkilamchi konturlarning parametrlari r_1, L_1, r_2, L_2.

Uchta holni ko‘rib chiqing.

1) $r_1 / L_1 = r_2 / L_2$;

2) $L_1 L_2 = M^2$;

3) $r_1 = r_2 = r$; $M = L_1 = L_2 = L$.



5.49- rasm.

Yechish. 5.49- rasmdagi sxema uchun quyidagi tenglamalarni tuzamiz.

$$r_1 i_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = u_0;$$

$$r_2 i_2 + M \frac{di_2}{dt} + L_2 \frac{di_2}{dt} = 0$$

yoki operator shaklida:

$$(r_1 + pL_1)I_1(p) + MpI_2(p) = u_0 / p;$$
$$MpI_1(p) + (r_2 + L_2 p)I_2(p) = 0.$$

Quyidagi belgilashlarni kiritamiz: $\delta_1 = \frac{r_1}{L_1}$, $\delta_2 = \frac{r_2}{L_2}$, $\delta = 1 - k^2$, $\mathrm{k}^2 = \frac{M^2}{L_1 L_2}$. Sodda o‘zgartirishlardan so‘ng,

$$I_1(p) = \frac{U_0}{pL_1}\frac{\delta_e + p}{[\delta p^2 + (\delta_1 + \delta_2)p + \delta_1\delta_2]},$$

$$I_2(p) = -\frac{U_0 M}{L_1 L_2[\delta p^2 + (\delta_1 + \delta_2)p + \delta_1\delta_2]}.$$

Yoyish formulasidan foydalanib, toklarning oniy qiymat ifodalarini topamiz:

$$i_1(t) = \frac{u_0}{r_1} + \frac{u_0}{\beta L_1}[(1 + \frac{\zeta p_2}{\delta_2})e^{p_1 t} - (1 + \frac{\delta p_1}{\delta_1})e^{p_2 t}],$$
$$i_2(t) = -\frac{u_0 M}{\beta L_1 L_2}(e^{p_1 t} - e^{p_2 t}),$$

bunda $\beta = \sqrt{(\delta_1 - \delta_2)^2 + 4k^2\delta_1\delta_2}$, $p_{1,2} = -[-(\delta_1 + \delta_2) + b]/2\delta.$

Xususiy hollarda $i_1(t)$ va $i_2(t)$ soddaroq formulalar bilan ifodalanadi.

Masalan, $\delta_1 = \delta_2 = \delta$ da:

$$i_1(t) = \frac{u_0}{r_1} - \frac{u_0}{2r_1}[e^{-\frac{\delta}{1+k}t} + e^{-\frac{\delta}{1-k}t}]; \quad i_2(t) = \frac{u_0}{2\sqrt{r_1 r_2}};$$

$k = 1$, ya’ni $M^2 = L_1 L_2$ da:

$$i_1(t) = \frac{u_0}{r_1} - \frac{u_0}{r_1}\frac{\delta_2}{\delta_1 + \delta_2}e^{p_1 t}, \quad i_2(t) = -\frac{u_0}{M(\delta_1 + \delta_2)}e^{p_1 t}.$$

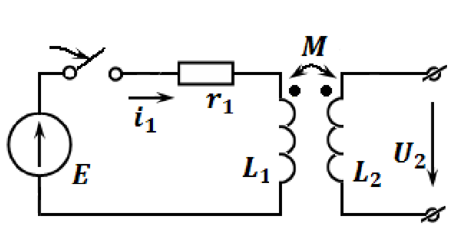
Agar $r_1 = r_2 = r$ va $L_1 = L_2 = L$ bo‘lsa,

$$i_1(t) = \frac{u_0}{r}(1 - \frac{1}{2}e^{-\frac{\delta}{2}t}); \quad i_2(t) = -\frac{u_0}{2r}e^{-\frac{\delta}{2}t}.$$

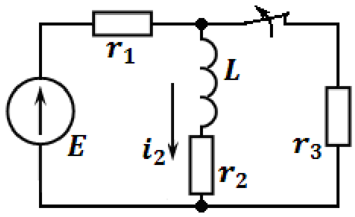
5.34-masala. Transformatorning birlamchi zanjiridagi i_1 tokni va ikkilamchi chulg‘amidagi U_2 kuchlanishni kalit ulangandan so‘ng toping (5.50- rasm).

Berilgan: $E = 30\,\mathrm{V}$; $r_1 = 150\,\Omega$; $L_2 = 0{,}2\,\mathrm{H}$; $M = 0{,}4\,\mathrm{H}$.

5.35- masala. 5.51- rasmdagi sxema uchun uchinchi shoxobchadagi kalit ochilganda i_2 tok aniqlansin. Berilgan $E = 24\,\mathrm{V}$, $r_1 = r_3 = 6\,\Omega$, $r_2 = 12\,\Omega$, $L = 0{,}05\,\mathrm{H}$.



5.50- rasm.



5.51- rasm.

5.4. O‘ram (svertka) integrali usuli bo‘yicha o‘tkinchi va barqaror jarayonlarni hisoblash va tahlil qilish

O‘ram (svertka) integralining (Dyuamel integrali) xususiyatlari. Agar nol boshlang‘ich shartlardagi zanjirga beriladigan ta’sir signali murakkab shaklga ega bo‘lsa, o‘tkinchi jarayonlardagi tok va kuchlanishlarni hisoblash uchun Dyuamel integralidan foydalaniladi.

5.2- jadvalda Dyuamel integralining yozilish shakllari keltirilgan.

5.2- *jadval*

Dyuamel (o‘ram) integralining yozilish shakllari	
$f_{\text{chiq}}(t) = f_{\text{kir}}(0)h(t) + \int\limits_0^t f'_{\text{kir}}(t-\tau)h(\tau)d(\tau)$	(5.28)
$f_{\text{chiq}}(t) = f_{\text{kir}}(0)h(t) + \int\limits_0^t f'_{\text{kir}}(\tau)h(t-\tau)d(\tau)$	(5.29)
$f_{\text{ckir}}(t) = f_{\text{kir}}(0)h(t) + \int\limits_0^t f'_{\text{kir}}(t-\tau)h'(\tau)d(\tau)$	(5.30)
$f_{\text{chiq}}(t) = f_{\text{kir}}(0)h(t) + \int\limits_0^t f_{\text{kir}}(\tau)h'(t-\tau)d(\tau)$	(5.31)
$f_{\text{chiq}}(t) = \dfrac{d}{dt}\int\limits_0^t f_{\text{kir}}(t-\tau)h(\tau)d\tau$	(5.32)
$f_{\text{chiq}}(t) = \int\limits_0^t f_{\text{kir}}(\tau)k(t-\tau)d\tau$	(5.33)
$f_{\text{chiq}}(t) = \dfrac{d}{dt}\int\limits_0^t f_{\text{kir}}(\tau)h(t-\tau)d\tau$	(5.34)
$f_{\text{chiq}}(t) = \int\limits_0^t f_{\text{kir}}(t-\tau)k(\tau)d\tau$	(5.35)

$f_{\text{chiq}}(t)$ bilan belgilangan o‘tkinchi jarayondagi tok va kuchlanishni aniqlash uchun jadvalda berilgan sakkizta shakldan yozilishi sodda bo‘lgani tanlanadi. Dyuamel integraliga kirish $f_{\text{kir}}(t)$ signalidan tashqari vaqt xarakteristikalari ham kiradi: o‘tish funksiyasi (xarakteristika) *h(t)* va impulsli o‘tish funksiyasi (xarakteristika) *k(t)*. Vaqt xarakteristikalari – bu namunaviy *1(t)* yoki *δ(t)* ta’sirlar sxemadagi nol boshlang‘ich shartlarda bo‘lgan zanjirning reaksiyasidir. Vaqt xarakteristikalar o‘tkinchi *h(t)* hisoblanadi.

Agar kirishga ta’sir qiladigan signal *1(t)* bo‘lsa, vaqt xarakteristikalar o‘tkinchi *h(t)* hisoblanadi yoki agar ta’sir signali *δ(t)* funksiya bo‘lsa, vaqt xarakteristikasi impulsli *k(t)* hisoblanadi.

O‘tish xarakteristikasi quyidagicha aniqlanadi.

$$h(t)=\frac{f_{\text{chiq}}(t)}{f_{\text{kir}}(t)}=\frac{f_{\text{chiq}}(t)}{1(t)1(B;A)} \qquad (5.36)$$

(5.36) formuladan shu narsa kelib chiqadiki, agar zanjir $E=1\cdot 1(t)B$ EYK yoki $I=1\cdot 1(t)\,A$ tok manbayiga ulangan bo‘lsa, o‘tkinchi xarakteristika $i(t)$ yoki $u(t)$ kuchlanishga mos bo‘ladi.

Impulsli xarakteristika *k(t)* quyidagi munosabatdan aniqlanadi:

$$k(t)=h'(t)+\delta(t)h(0). \qquad (5.37)$$

5.3- jadvalda namunaviy ta’sir grafiklari hamda vaqt xarakteristikalarining turlari keltirilgan.

Elektr zanjirlarning o‘tkinchi va impuls xarakteristikalarini qurish. O‘tkinchi va impuls xarakteristikalarini qurishning klassik usuli.

O‘tkinchi xarakteristikalarni klassik usulda hisoblash uchun:

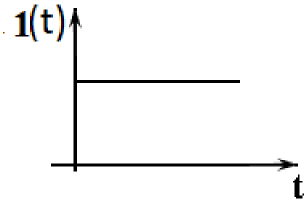
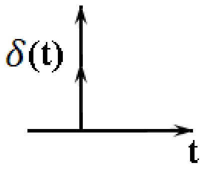
1) zanjir kirishiga birlamchi pog‘onali ta’sir – kuchlanish (tok) $1\cdot 1(t)B,A$ beriladi;

2) quyidagi berilgan munosabatlardan foydalanib $h(t)$ ni aniqlash uchun formula yoziladi:

$$h(t)=\frac{f_{\text{chiq}}(t)}{f_{\text{kir}}(t)}=\frac{f_{\text{chiq}}(t)}{1\cdot 1(B;A)};$$

3) klassik usuldan foydalanib $f_{\text{chiq}}(t)$, ya'ni izlanayotgan xarakteristikaning turiga ko'ra, o'tkinchi rejimning tok yoki kuchlanishi topiladi.

5.3- *jadval*

Ta'sir turi	Reaksiya	Vaqt xarakteristikalarining turlari
	$h(t)$– o'tkinchi	$h_u(t) = \dfrac{U_{\text{chiq}}}{1 \cdot 1(t)B}$; $h_i(t) = \dfrac{i_{\text{chiq}}(t)}{1 \cdot 1(t)A}$ $Y(t) = \dfrac{i_{\text{chiq}}(t)}{1 \cdot 1(t)B}$; $Z(t) = \dfrac{U_{\text{chiq}}(t)}{1 \cdot 1(t)A}$.
	$k(t)$–impulsli	$K_u(t)$; $K_i(t)$; $K_Y(t)$; $K_Z(t)$.

4) $h(t)$ hisoblanadi va uning grafigi quriladi;

5) sxema bo'yicha $t(0_+)$ va $t(\infty)$ lardagi hisoblar tekshiriladi va analitik $h(t)\ _{t(0_+)}$ ва $h(t)\ _{t\to\infty}$ ifodalar bilan solishtiriladi.

Impulsli xarakteristikalarni hisoblash uchun impulsli va o'tkinchi xarakteristikalarni bog'laydigan (5.37) formuladan foydalaniladi:

$k(t) = h'(t) + \delta(t)h(0)$.

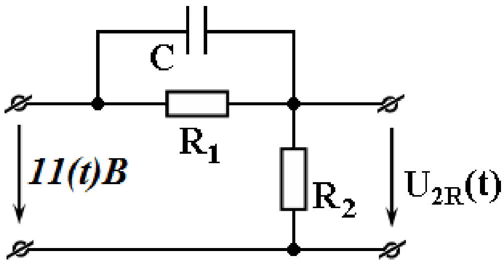
O'tkinchi va impulsli xarakteristikalarni klassik usul bilan hisoblashga misol.

Differensiallovchi RC - kontur uchun (5.52- rasm) R_2 qarshilikdagi kuchlanish bo'yicha o'tkinchi va impulsli xarakteristikalarni aniqlang.

5.52- rasmda tasvirlangan zanjir kirishiga birlamchi pog'onali $1 \cdot 1(t)B$ kuchlanish beriladi.

(5.36) munosabatdan va 5.3- jadvaldan foydalanib $h(t)$ ni aniqlash uchun quyidagi formula yoziladi:

$$k_{UR_2}(t)=\frac{U_{R_2}(t)}{1\cdot 1(t)B}$$



5.52- rasm.

R_2 dagi o‘tkinchi rejimdagi kuchlanish klassik usulda aniqlanadi:

$$U_{R_2}(t)=U_{U_{2bar}}(t)+U_{R_{2er}}(t);$$

$$U_{R_{2bar}}(t)=\frac{R_2}{R_1+R_2}1\cdot 1(t);\quad U_{R_{2er}}(t)=Ae^{p_1t};$$

$$U_{R_2}(t)=\left(\frac{R_2}{R_1+R_2}1\cdot 1(t)+Ae^{p_1t}\right)B.$$

A boshlang‘ich qiymat quyidagi ifodadan aniqlanadi:

$$U_{R_2}(0_+)+\frac{R_2\cdot 1\cdot 1(t)}{R_1+R_2}+A=1\cdot 1(t)-\frac{R_2\cdot 1\cdot 1(t)}{R_1+R_2}=\frac{R_1}{R_1+R_2}1\cdot 1(t);$$

Ildizni $p_1=\frac{1}{\tau}$ ifodadan topamiz, bunda $\tau=\frac{R_1\cdot R_2}{R_1+R_2}C.$

O‘tkinchi rejimdagi R_2 qarshilikdagi kuchlanish quyidagi ko‘rinishga ega:

$$U_{R_2}(t)=\left(\frac{R_2}{R_1+R_2}1\cdot 1(t)+\frac{R_1}{R_1+R_2}1\cdot 1(t)e^{-\frac{R_1+R_2}{R_1R_2}t}\right)B.$$

Kuchlanish bo‘yicha o‘tkinchi xarakteristikani aniqlaymiz:

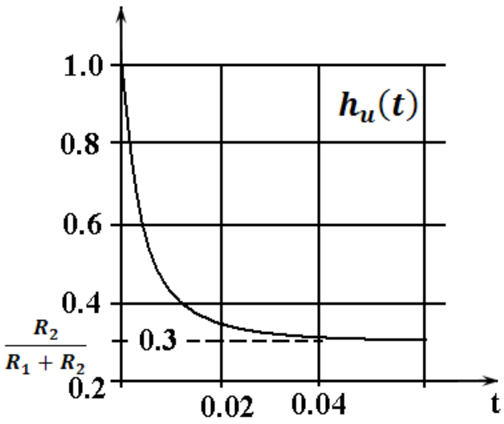
$$h_{U_{R2}}(t)=\frac{U_{R_2}(t)}{1\cdot 1(t)B}=\frac{R_2}{R_1+R_2}+\frac{R_1}{R_1+R_2}e^{-\frac{R_1+R_2}{R_1R_2}t}$$

Kuchlanish bo‘yicha $h_{U_{R2}}(t)$ o‘tkinchi xarakteristikaning grafigi 5.53- rasmda tasvirlangan.

Impuls xarakteristika (5.37) o‘tish xarakteristika bilan bog‘lanish formulasidan aniqlanadi:

$$h_{U_{R2}}(t) = h'_{U_{R2}}(t) + h_{U_{R2}}(0)\delta(t) = -\frac{1}{R_2C}e^{-\frac{R_1+R_2}{R_1 \cdot R_2}t} + \delta t, \frac{1}{\text{s}}$$

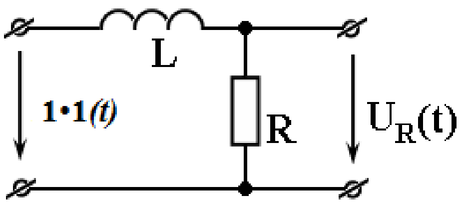
Impuls xarakteristikaning o‘lchami $\frac{1}{s}$, fizik jihatdan esa u o‘tish xarakteristikaning o‘zgarish tezligini xarakterlaydi.



5.53- rasm.

O‘tkinchi jarayonlarni Dyuamel integrali usulida yechish uchun masalalar

5.36- masala. Integrallovchi RL – zanjirning 5.54- rasm R qarshiligidagi kuchlanishni aniqlang. Kirish kuchlanishi $U(t) = Ue^{-\alpha t}$ V.



5.54- rasm.

Yechish. 5.2- jadvaldan Dyuamel integralining analitik ifodasini tanlanadi:

$$f_{\text{chiq}}(t) = f_{,}(t)h(0) + \int_0^t U(\tau)h'(t-\tau)d\tau, \qquad (5.38)$$

Dyuamel integraliga kiruvchi kattaliklarni aniqlashtirilib quydagilar olinadi:

$$U_R(t) = U(t)h_{UR}(0) + \int_0^t U(\tau)h'(t-\tau)d\tau. \qquad (5.39)$$

R qarshilikdagi kuchlanish bo‘yicha o‘tkinchi xarakteristika quyidagicha aniqlanadi:

$$h_{U_R}(t) = \frac{U_R(t)}{1 \cdot 1(t)}.$$

O‘tkinchi h_{U_R} xarakteristikani hisoblash uchun 5.54- rasmdan foydalanamiz.

$$U_R(t) = (1 - e^{-\frac{R}{L}t}) \cdot 1 \cdot 1(t) = (1 - e^{-\frac{t}{\tau}}) \cdot 1 \cdot 1(t) B,$$

bunda $\tau = \frac{L}{R}$, binobarin,

$$h_{U_R}(t) = 1 - e^{-\frac{t}{\tau}} = 1 - e^{-\frac{R}{L}t}$$

Dyuamel integraliga kiradigan komponentlarni hisoblaymiz:

$$U(t) = Ue^{-\alpha t}; \quad U(\tau) = Ue^{-\alpha t};$$

$$h_{U_R}(0) = 0; \quad h_{U_R}(t) = \frac{R}{L} e^{-\frac{R}{L}t};$$

$$h'_{U_R}(t - \tau) = \frac{R}{L} e^{-\frac{R}{L}(t-\tau)}.$$

Izlanayotgan kuchlanish (5.37) ifoda asosida integrallab topiladi:

$$U_R(t) = \int_0^t Ue^{-a\tau} \cdot \frac{R}{L} e^{-\frac{R}{L}t} \cdot e^{\frac{R}{L}\tau} d\tau = \frac{UR}{L} e^{-\frac{R}{L}t} \int_0^t e^{\left(\frac{R}{L}-a\right)} d\tau =$$

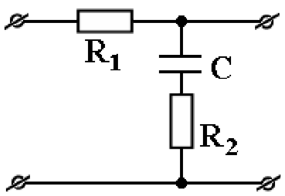
$$= \frac{UR}{L\left(\frac{R}{L} - a\right)} e^{-\frac{R}{L}t} \cdot e^{\left(\frac{R}{L}-a\right)\tau} \Bigg|_0^t = \frac{U}{1 - \frac{a \propto}{R}} \cdot e^{-\frac{R}{L}t} \left(e^{\frac{R}{L}t} \cdot e^{-at} - 1\right) =$$

$$= \frac{U}{1 - \frac{aL}{R}} \left(e^{-at} - e^{-\frac{R}{L}t}\right) B \tag{5.40}$$

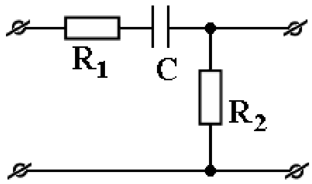
Mustaqil yechish uchun masalalar

5.37- masala. Integrallovchi RC–kontur uchun (5.55- rasm) kuchlanish bo‘yicha o‘tkinchi va impuls xarakteristikalarni aniqlang. Berilgan: R_1=1 kΩ; R_2=3 kΩ; C=0,5 mkF.

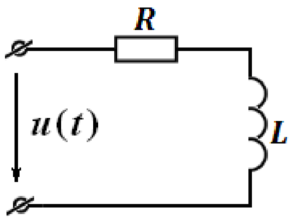
5.38- masala. Differensiallovchi RC–kontur (5.56- rasm) uchun o‘tkinchi va impulsli o‘tkazuvchanlik va kuchlanish bo‘yicha o‘tkinchi koeffitsiyent aniqlansin, bunda R_1=2 kΩ; R_2=3 kΩ; C=0,1 mkF.



5.55- rasm.



5.56- rasm.



5.57- rasm.

5.39- masala. Integrallovchi kontur uchun (5.57- rasm) kuchlanish bo‘yicha o‘tkinchi va impuls xarakteristika aniqlansin va uning grafigi qurilsin. Berilgan:

R_1=3 kΩ; R_2=2 kΩ; R_3=2 kΩ; R_4=4 kΩ; C=0.2 mkF.

5.40- masala. Parametrlari $R = 20\,\Omega,\ \ L = 10\,\text{mH}$ bo‘lgan, ketma-ket ulangan RL- elementlariga $U(t) = 4e^{-400t}$ V kuchlanish ta’sir qilsa, ustlash integralidan foydalanib zanjirdagi tokni aniqlang.

Chiziqli elektr zanjirlardagi o‘tkinchi jarayonlarni Dyuamel integrali usulida hisoblash ketma-ketligi.

1. Tok yoki kuchlanish bo‘yicha izlanayotgan konkret funksiyasining o‘tkinchi xarakteristikasini aniqlaymiz.

2. EYK yoki tok manbasi funksiyasi ko‘rinishi bo‘yicha interval sonlarini va har bir intervaldagi funksiya hosilalarini aniqlaymiz.

3. Intervallar bo‘yicha boshlang‘ich funksiyaning yozamiz.

4. O‘zgaruvchilar $h(t-\tau), h(t-t_1), h(t-t_2)$ larni almashtiramiz va ularni izlanayotgan funksiyalarga qo‘yamiz.

5. Izlanayotgan funksiyani intervallar bo‘yicha grafigini quramiz.

5.41- masala. 5.58- rasmdagi zanjirga 5.59- rasmda tasvirlangan EYK manbayi qo‘shilgan. Sxema parametrlari: $R = 100\ \Omega$, $L = 0{,}1\,\text{H}$. Induktivlik $i_L(t)$ tokining funksiyasi aniqlansin va uni grafigi qurilsin.

Yechish. O‘tkinchi xarakteristika $h(t)$, ya’ni birlamchi manba ulanishiga bo‘lgan sxemaning reaksiyasini aniqlaymiz. Sxemaning reaksiyasi deb izlanayotgan funksiya, ya’ni $i_L(t)$. Birlamchi manba – bu, kuchlanishi $e(t) = 1\,\text{V} = \text{const}$ bo‘lgan o‘zgarmas EYK manbayidir, yoki tok kuchi $i_0(t) = 1\,\text{A} = \text{const}$ bo‘lgan o‘zgarmas tok manbayidir.

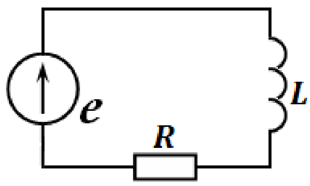
Binobarin, 5.58- rasmdagi sxema $e(t) = 1\,\text{V} = \text{const}$ kuchlanish qo‘shilganda o‘tkinchi jarayonni, ya’ni $h(t) = i_L(t)$ aniqlash kerak.

Masalani klassik usulda yechamiz. Izlanayotgan funksiyaning umumiy ko‘rinishini yozamiz:

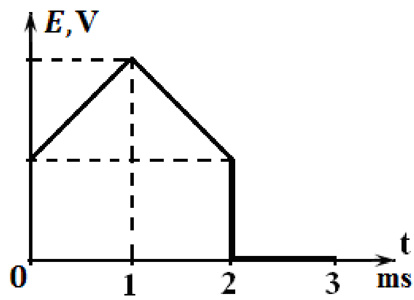
$$i_L(t) = A_1 e^{p_1 t} + i_L(t). \qquad (5.41)$$

Bog‘lanmagan boshlang‘ich shartni aniqlaymiz. $i_L(0) = 0,$ chunki kommutatsiyagacha manba qo‘shilmagan edi.

Bog‘langan boshlang‘ich shartni aniqlash kerak emas, chunki izlanayotgan $i_L(t)$ funksiya uchun $i_L(0)$ aniqlangan. Xarakteristik tenglamaning p_1 ildizini kommutatsiyadan keyingi sxemadan aniqlaymiz (5.60- rasm).



5.58- rasm.



5.59- rasm.

Xarakteristik tenglamani tuzish uchun sxemadan manbani olib tashlaymiz va o‘rniga uning ichki qarshiligini qoldiramiz. U ko‘rilayotgan holda nolga teng. Operator qarshilik $Z(p) = R + Lp$ ni nolga tenglashtirsak, xarakteristik tenglama hosil bo‘ladi:

$$Lp + R = 0,$$

$$p_1 = -\frac{R}{L} = \frac{100}{0,1} = -1000 \text{ rad/s}.$$

Izlanayotgan $i_L(t)$ funksiyani barqaror turg‘un tashkil etuvchisi $i_{LT}(t)$ ni aniqlaymiz. Kommutatsiyadan keyin sxemaga o‘zgarmas EYK manbayi ulangani uchun, turg‘un tok

$$i_{LT}(t) = i_{LT}(0) = \frac{1}{R} = 0,01 \text{ A}.$$

Integral doimiysi A_1 ni quyidagi tenglamadan aniqlaymiz:

$$i_L(0) = A_1 + i_{LT}(0), \text{ bunda } A_1 = i_L(0) - i_{LT}(0) = -i_{LT}(0),$$

binobarin, $A_1 = -0,01$.

Topilgan parametrlarni, ya’ni A_1, p_1 va i_{LT} larni (5.41) tenglamaga qo‘yamiz:

$$h(t) = -0,01e^{-1000t} + 0,01.$$

O‘tkinchi $h(t)$ xarakteristika topilgandan so‘ng, 5.59- rasmda berilgan $e(t)$ funksiyani tahlil etamiz. Funksiyaga ko‘ra u nechta intervalga ega ekanligini, va har bir intervaldagi hosilalarini aniqlaymiz. $e(t)$ funksiya grafigidan ko‘rinib turibdiki, u uchta intervalga va har birida turlicha qonuniyatga ega.

I interval $0 \le t < t_1 = 1\,\text{ms} = 10^{-3}\text{s}$;

II interval $t_1 \le t < t_2 = 2\,\text{ms} = 2 \cdot 10^{-3}\text{s}$;

III interval $t_2 \le t < \infty$.

I intervalda $e_1(t)$ funksiya chiziqli qonun bilan o‘zgaradi. $e_1(t) = kt = b$;

Funksiya hosilasi $e_1'(t) = k$; bunda, $k = \dfrac{\Delta e}{\Delta t}$;

I intervalda $e_1'(t) = \dfrac{\Delta e}{\Delta t} = \dfrac{20-10}{1-10^{-3}} = \dfrac{10}{10^{-3}} = 10^4$ V/s ;

II intervalda $e_{11}'(t) = \dfrac{\Delta e_{II}}{\Delta t_{II}} = \dfrac{20-10}{(2-1)\cdot 10^{-3}} = -10^4$ V/s ;

III intervalda funksiya orttirmaga ega emas: $e_{III}'(t) = 0$.

Dyuamel integrali bilan izlanayotgan $i_L(t)$ funksiya ifodalarining soni intervallar soniga teng, ya'ni uchga teng.

I intervalda

$$i_{LI}(t) = e(0)h(t) + \int_0^t e'_I(\tau)h(t-\tau)d'\tau \qquad (5.42)$$

II intervalda

$$i_{LII}(t) = e(0)h(t) + \int_0^t e'_I(\tau)h(t-\tau)d(\tau) + [e_{II}(t_1) - e_I(t_1)]h(t-t_1) + \int_{t_1}^t e'_{II}(\tau)h(t-\tau)d\tau \qquad (5.43)$$

III intervalda

$$i_{LIII}(t) = e(0)h(t) + \int_0^{t_1} e_1^1(\tau)h(t-\tau)d\tau + [e_{II}(t_1) - e_1(t_1)] \cdot h(t-t_1) +$$
$$\int_{t_1}^t e_{II}^1(\tau)h(t-\tau)d\tau + [e_{III}(t_2) - e_{II}(t_2)]h(t-t_2) + \int_0^t e_{III}^1(\tau)h(t-\tau)d\tau \qquad (5.44)$$

5.59- rasmdagi grafikdan ko'ramizki, $e(0) = 10$ V.

Funksiya $h(t-\tau)$ ni aniqlash uchun o'tkinchi xarakteristikada o'zgaruvchilarni almashtiramiz. Quyidagini e'tiborga olib, t ning o'rniga $t-\tau$ ni qo'yamiz:

$$h(t-\tau) = -0{,}01e^{-1000(t-\tau)} + 0{,}01;$$

$$0{,}01e^{-1000(t-\tau)} = -0{,}01e^{-1000t} \cdot e^{1000\tau}.$$

Topilgan barcha ifodalarni (5.42) ga qo'yib $i_{LI}(t)$ ifoda uchun quyidagini hosil qilamiz:

$$i_{LI}(t) = 10(-0{,}01e^{-1000t} + 0{,}01) + \int_0^t 10^4(-0{,}01e^{-1000t} \cdot e^{1000\tau} + 0{,}01)d\tau.$$

Yig'indi integralini integrallar yig'indisi ko'rinishida ifodalaymiz, bunda

$$i_{LI}(t) = -0{,}1e^{-1000t} + 0{,}01 + \int_0^t (-100e^{-1000t}e^{1000\tau}d\tau + \int_0^t 100d\tau.$$

Ko'paytuvchi e^{-1000t} ni bog'lanmagan o'zgaruvchi bo'lgani uchun integral ostidan chiqaramiz, chunki τ bo'yicha integrallanmoqda, bunda

$$i_{LI}(t) = -0{,}1e^{-1000t} + 0{,}1 - 100e^{-1000t} \cdot \frac{1}{1000} e^{1000\tau}\Big|_0^t + 100\tau\Big|_0^t =$$
$$= -0{,}1e^{-1000t} + 0{,}1 - 0{,}1e^{-1000t}(e^{1000t} - e^0) + 100t = \qquad (5.45)$$
$$= -0{,}1e^{-1000t} + 0{,}1 - 0{,}1 + 0{,}1e^{-1000t} + 100t \text{ A}$$

Ikkinchi uchastkaga mos bo‘lgan $i_{LII}(t)$ ni aniqlash uchun (5.43) ifodaga va $e(t)$ funksiya grafigi (5.59- rasm) ga murojaat qilamiz. Grafikdan ko‘rinib turibdiki, t_1 onda EYK sakrashi yo‘q, ya’ni $e_I(t_1) = e_{II}(t_1)$.

Binobarin (5.43) ifodadagi $[e_{II}(t_1) - e(t_1)] - h(t - t_1)$ yig‘indi nolga teng. Shunga ko‘ra izlanayotgan $i_{LII}(t)$ funksiyani (5.43) ga asoslanib hisoblash mumkin:

$$i_{LII}(t) = 10(-0{,}001e^{-1000t} + 0{,}01) + \int_0^{10^{-3}} 10^4(-0{,}01e^{-1000t} \cdot e^{1000\tau})d\tau +$$
$$+ \int_0^{10^{-3}} 10^4 \cdot 0{,}01 d\tau + \int_{10^{-3}}^{t} (-10^4)(-0{,}01e^{-1000\tau} \cdot e^{1000\tau})dt + \int_{10^{-3}}^{t} (-10^4) \cdot 0{,}01 d\tau =$$
$$= -0{,}1e^{-1000t} + 0{,}1 - 100e^{-1000t} \cdot \frac{1}{1000} e^{1000\tau}\Big|_0^{10^{-3}} + 100\tau\Big|_0^{10^{-3}} +$$
$$+ 100e^{-1000t} \cdot \frac{1}{1000} e^{1000\tau}\Big|_{10^{-3}}^{t} - 100\tau\Big|_{10^{-3}}^{t} = +0{,}1 - 0{,}1e^{-1000t} \cdot e^1 +$$
$$+ 0{,}1 + 0{,}1e^{-1000t} \cdot e^{1000t} - 0{,}1e^{-1000t} \cdot e^1 - 100t + 0{,}1 =$$
$$= 0{,}4 + 0{,}2 \cdot 2{,}72e^{-1000t} - 100t = 0{,}4 - 0{,}544e^{-1000t} - 100t \text{ A.}$$

$$i_{LII}(t) = 0{,}4 - 0{,}544e^{-1000t} - 100t \quad \text{A.} \qquad (5.46)$$

Hisoblashlarni tekshirib chiqamiz. Induktivlikdagi tok kommutatsiya qonuniga bo‘ysingani uchun $i_{LI}(t_1)$ funksiyaning qiymati $i_{LII}(t_1)$ qiymatiga teng bo‘lishi kerak:

$$i_{LI}(t_1) = 100 \cdot 10^{-3} = 0{,}1 \text{ A};$$
$$i_{LII}(t_1) = 0{,}4 - 0{,}544e^{-1} - 0{,}1 = 0{,}4 - 0{,}2 - 0{,}1 = 0{,}1 \text{ A}.$$

Binobarin, $i_{LI}(t_1) = i_{LII}(t_1) = 0{,}1$ A. Funksiya $i_{LIII}(t)$ umumiy holda (5.44) ifoda bilan aniqlanadi. $e(t)$ funksiyaning grafigidan ko‘rinib turibdiki, uchinchi intervalda $e_{III}(t)=0$ va uning hosilasi $e'_{III}(t)=0$. Lekin, ikkinchi va uchinchi intervallar orasida EYK sakrashi mavjud:

$$e_{III}(t_2)-e_{II}(t_2)=0-10=-10 \text{ V.}$$

Binobarin, (5.45) formulada quyidagi qo‘shiluvchi mavjud bo‘ladi:

$$[e_{III}(t_2)-e_{II}(t_2)]h(t-t_2).$$

$h(t-t_2)$ funksiyani aniqlash uchun $h(t)$ o‘tkinchi xarakteristika o‘zgaruvchilar $t_2=2\cdot 10^{-3}$ s vaqtdagi almashtirshlarni bajaramiz:

$$h(t-t_2)=-0{,}01e^{-1000(t-t_2)}+0{,}01=-0{,}01e^{-1000t}\cdot e^{2}+0{,}01=$$
$$=-0{,}01e^{-1000t}\cdot 7{,}381+0{,}01=-0{,}07389\cdot e^{-1000t}+0{,}01.$$

Hosil bo‘lgan $h(t-t_2)$ ifodani (5.45)ga qo‘yamiz:

$$i_{LIII}(t)=-0{,}1e^{-1000t}+0{,}1+\int_0^{10^{-3}}10^4(-0{,}01e^{-1000t}\cdot d\tau)+$$
$$+\int_0^{10^{-3}}10^4\cdot 0{,}01\cdot d\tau+\int_{10^{-3}}^{2\cdot 10^{-3}}(-10^4)\cdot(-0{,}01e^{-1000t}\cdot e^{1000\tau}d\tau)+$$
$$+\int_{10^{3}}^{2\cdot 10^{-3}}(-10^4)\cdot 0{,}01d\tau+(-10)(-0{,}07389e^{-1000t}+0{,}01)=-0{,}1e^{-1000t}+0{,}1-$$
$$-100e^{-1000t}\cdot\frac{1}{1000}(e^{1000 10^{-3}}-e^{0})+100\cdot 10^{-3}+100e^{-1000t}\cdot\frac{1}{1000}(e^{1000 10^{-3}}-e^{0})+100\cdot 10^{-3}+$$
$$+100e^{-1000t}\cdot\frac{1}{1000}(e^{1000 2\cdot 10^{-3}}-e^{1000 10^{-3}})-100(2\cdot 10^{-3}-10^{-3})+0{,}7389e^{-1000t}-0{,}1=$$
$$=-0{,}1e^{-1000t}+0{,}1-0{,}1e^{-1000t}\cdot e^{1}+0{,}1e^{-1000t}+0{,}1+0{,}1e^{-1000t}\cdot e^{2}-0{,}1+0{,}7389e^{-1000t}-0{,}1=$$
$$=(-0{,}2718+0{,}46707+0{,}7389)\cdot e^{-1000t}=0{,}93417e^{-1000t}\text{A}$$

$$i_{LIII}(t)=0{,}93417\cdot e^{-1000t}\text{A}. \qquad (5.47)$$

Endi II va III chegaralarda hisoblashlarni tekshirib chiqamiz, ya’ni $i_{LII}(t_2)$ tok $i_{LIII}(t_2)$ tokka teng bo‘lishi kerak:

$$i_{LII}(t_2)=0{,}4-0{,}544e^{-1000 2\cdot 10^{-3}}-100\cdot 2\cdot 10^{-3}=0{,}4-0{,}544e^{-2}-0{,}2=$$
$$=0{,}4-0{,}544\cdot 0{,}1353-0{,}2=0{,}4-0{,}0736-0{,}2=0{,}1266 \text{ A.}$$

$$i_{LIII}(t_2)=0{,}93417\cdot e^{-1000 2\cdot 10^{-3}}=0{,}1264 \text{ A.}$$

Hisoblashlar xatoligi

$$\delta = \frac{0{,}1266 - 0{,}1264}{0{,}1260} \cdot 100\% = 0{,}14\%.$$

Intervallardagi hisoblangan funksiyaning grafigini quramiz (5.61- rasm).

Shunday qilib, hisoblangan funksiya:

$i_{LI}(t) = 100t$ A;

$i_{LII}(t) = 0{,}4 - 0{,}544e^{-1000t} - 100t$ A;

$i_{LIII}(t) = 0{,}934e^{-1000t}$ A.

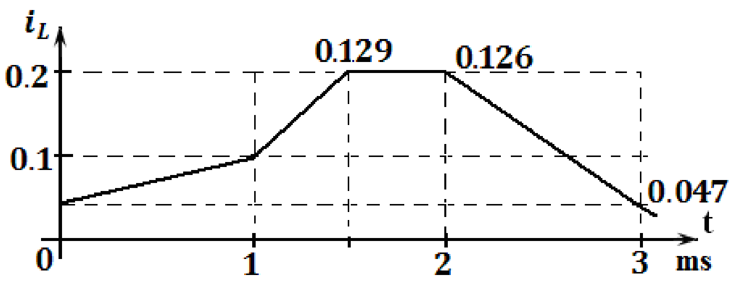
$i_{LI}(t)$ grafigini qurish uchun ikkita nuqtadagi qiymatlarni hisoblash yetarli: $i_{LI}(t = 1\,\text{ms}) = 0{,}1$ A, chunki $i_{LI}(t)$ to‘g‘ri chiziq.

$i_{LII}(t)$ grafigini qurish uchun $i_{LII}(t_1) = 0{,}1$ A va $i_{LII}(t_2 = 2) = 0{,}126$ A ikkita nuqta yetarli emas, shuning uchun yana bitta nuqtadagi qiymatini hisoblaymiz:

$i_{LII}(t = 1{,}5\,\text{ms}) = 0{,}4 - 0{,}54 \cdot 0{,}233 - 0{,}15 = 0{,}4 - 0{,}121 - 0{,}15 = 0{,}129$ A.

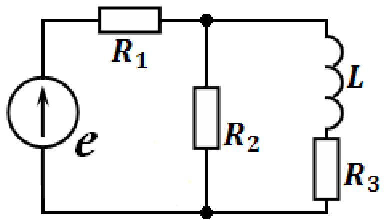
$i_{LIII}(t)$ grafikni qurish uchun $i_{LIII}(t = 3\,\text{ms})$ ni hisoblash kerak:

$i_{LIII}(t = 3\,\text{ms}) = 0{,}934e^{-1000 \cdot 3 \cdot 10^{-3}} = 0{,}934e^{-3} = 0{,}934 \cdot 0{,}0498 = 0{,}047$ A.

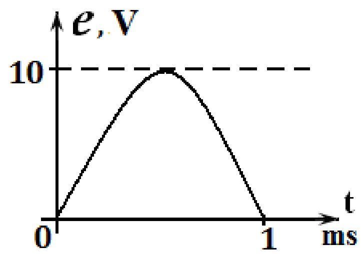


5.61- rasm.

5.42- masala. 5.62- rasm, *a* da tasvirlangan sxemaga sinusoidal qonun bo‘yicha o‘zgaradigan EYK (5.62- rasm, *b*) manbayi qo‘shiladi. Sxema parametrlari $R_1 = R_2 = R_3 = 50\ \Omega$; $L = 0{,}1$ H. Induktivlikdagi tok $i_1(t)$ ni aniqlang va uni grafigini quring.



a)



b)

5.62- rasm.

Yechish. 1. Sxemaga kuchlanishi birlamchi $e(t) = 1\,\text{V} = \text{const}$ bo‘lgan EYK manbasi ulanganda son jihatdan induktivlikdagi $i_L(t)$ tokka teng bo‘lgan o‘tkinchi $h(t)$ xarakteristikani aniqlaymiz.

O‘tkinchi jarayonni hisoblash uchun klassik usuldan foydalanamiz:

$i_L(t) = A_1 e^{p_1 t} + i_{LT}(t)$.

Bog‘lanmagan boshlang‘ich shart $i_L(0) = 0$.

Bog‘langan boshlang‘ich shartni hisoblaymiz, chunki $i_1(t)$ aniqlanmoqda.

Xarakteristik tenglamaning p_1 ildizini aniqlaymiz, buning uchun sxemaning (5.63- rasm) xarakteristik tenglamasini tuzamiz:

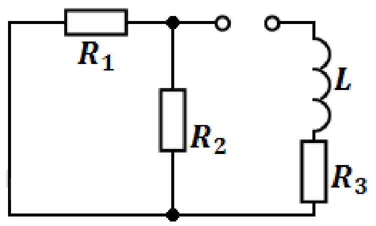
$$\frac{R_1 \cdot R_2}{R_1 + R_2} + R_3 + Lp = 0, \quad \frac{\mathrm{R}_1 R_2}{R_1 + R_2} + R_3 = R_{\text{EYK}} \frac{50 \cdot 50}{100} + 50 = 75\,\Omega;$$

$$p_1 = -\frac{R_{\text{EYK}}}{L} = -\frac{75}{0,1} = -750 \text{ rad/s.}$$

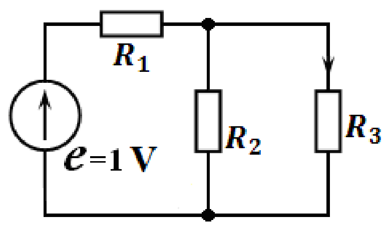
Turg‘un tok tashkil etuvchisi $i_{LT}(t)$ni o‘zgarmas tok almashlash sxemasi bo‘yicha aniqlaymiz:

$$I_1 = \frac{E}{R_1 + \dfrac{R_2 \cdot R_3}{R_2 + R_3}}; \qquad i_{LT} = I_1 \cdot \frac{R_2}{R_2 + R_3} = 0,0067 \text{ A};$$

Integral doimiysi $A_1 = i_{LT}(0) = -0,0067$ A.



5.63- rasm.



5.64- rasm.

O‘tkinchi xarakteristika quyidagiga teng:

$$h(t) = -0{,}0067e^{-750t} + 0{,}0067. \tag{5.48}$$

2. EYK manbasi funksiyasidan ko‘rinib turibdiki, u ikki intervaldan iborat:

I intervalda: $0 \le t \le t_1 = 1 \cdot 10^{-3}\text{s}; \quad e_1(t) = E_m \sin \omega t.$

5.62- rasm, *b* dan ko‘rinib turibdiki, $E_m = 10$ V, $T/2 = 10^{-3}$ s, demak,

$T = 2 \cdot 10^{-3}$ s, u holda $\omega = \dfrac{2\pi}{T}$ bo‘lgani uchun, $\omega = \dfrac{2\pi}{2 \cdot 10^{-3}} = \pi \cdot 10^3 \text{rad/s}$.

Binobarin $e_1(t) = 10\sin(\pi \cdot 10^3 t) = 10\sin(3142t)$ V.

Birinchi intervaldagi EYKning hosilasi

$$e_1'(t) = 10 \cdot 3142 \cdot \cos(3142t) = 3{,}14 \cdot 10^3 \cos(3142t).$$

II intervalda (5.62- rasm, *b* ga qarang).

$e_{II}(t) = 0$; binobarin $e_{II}'(t) = 0$. Intervallar bo‘yicha izlanayotgan $i_L(t)$ tok funksiyasining umumiy ko‘rinishi:

$$i_{LI}(t) = e(0)h(t) + \int_0^t e_1'(\tau)h(t-\tau)d\tau; \tag{5.49}$$

$$\begin{gathered} i_{LII}(t) = e(0)h(t) + \int_0^{t_1} e_1'(\tau)h(t-\tau)d\tau + [e_1(t) - e_1(t_1)]h(t-t_1) + \\ + \int_{t_1}^{t} e_{11}'(\tau)h(t-\tau)d\tau. \end{gathered} \tag{5.50}$$

$e(0) = 0$ bo‘lgani uchun, (5.49) va (5.50) ifodalardagi $e(0)h(t)$ qo‘shiluvchi nolga teng bo‘ladi.

Bundan tashqari, ikkinchi intervaldagi EYK funksiyasining hosilasi nolga teng: $e_{II}'(t) = 0$ bo‘lgani uchun, (5.50) ifodadagi oxirgi qo‘shiluvchi ham nolga teng bo‘ladi.

Nihoyat, intervallar orasida EYK ning sakrashlari yo‘q, shuning uchun $e_I(t_1) - e_{II}(t_1) = 0$. Yuqoridagilarni e’tiborga olib intervallardagi $i_L(t)$ tok quyidagicha ifodalanadi:

$$i_{LI}(t) = \int_0^t e_I'(\tau) h(t-\tau) d\tau; \tag{5.51}$$

$$i_{LII}(t) = \int_0^{t_1} e_I'(\tau) h(t-\tau) d\tau. \tag{5.52}$$

3. Intervallardagi funksiyani aniqlaymiz:

$$i_{LI}(t) = \int_0^t 3{,}14 \cdot 10^4 \cos(3142\tau) \cdot (-0{,}0067 e^{-750(t-\tau)} + 0{,}0067) d\tau =$$

$$= \int_0^t (-210{,}4 \cos(3142\tau) e^{-750(t-\tau)}) d\tau + \int_0^t 210{,}4 \cos(3142\tau) d\tau =$$

$$= -210{,}4 e^{-750t} [\frac{e^{750\tau}}{750^2 + 3142^2} [750 \cos(3142\tau) + 3142 \sin(3142\tau)]] \Big|_0^t +$$

$$+ \frac{210{,}4}{3142} \sin(3142\tau) \Big|_0^t = -210{,}4 e^{-750t} [\frac{e^{750t}}{104{,}347 \cdot 10^5} (750 \cos(3145t) +$$

$$+ 3142 \sin(3142t)) - \frac{e^0}{104{,}347 \cdot 10^5} \cdot 750] + 0{,}06696 \sin(3142t) =$$

$$= 0{,}01512 \cos(3142t) - 0{,}063238 \sin(3142t) + 0{,}01512 e^{-750t} +$$

$$+ 0{,}06684 \sin(3142t) = -0{,}01512 \cos(3142t) + 0{,}0036025 \sin(3142t) +$$

$$+ 0{,}01512 e^{-750t}.$$

$$i_{LI}(t) = -0{,}01512 \cos(3142t) + 0{,}0036025 \sin(3142t) + 0{,}01512 e^{-750t} \text{ A.}$$

Ikkinchi intervalda induktivlikdagi tok funksiyasi (5.52) ifodadan aniqlanadi, u (5.51) faqat dan integralning yuqori chegarasi bilan farq qiladi. Binobarin, $i_{LI}(t)$ hisoblashlaridan foydalanamiz, faqat integralni yangi chegaralarini e’tiborga olamiz:

$$i_{LII}(t)=\int_0^{t_1} e_I'(\tau)h(t-\tau)d\tau=-210{,}4e^{-750t}\left[\frac{e^{750t}}{750^2+3142^2}\right]\cdot$$

$$750\cos(3142\tau)+3142\sin(3142\tau))]\Big|_0^{t_1=1\cdot10^{-3}}+210{,}4\frac{1}{3142}\sin(3142\tau)\Big|_0^{t_1=-10^{-3}}=$$

$$=-210{,}4e^{-750t}[\frac{e^{0,75}}{104{,}35\cdot10^5}(750^{-1}+3142)-\frac{e^0}{104{,}35\cdot10^5}\cdot750]+0{,}06696\sin3{,}142=$$

$$=2{,}0125\cdot10^{-5}\cdot e^{-750t}\cdot e^{0,75}+0{,}01512e^{-750t}+0=0{,}03195e^{-750t}+0{,}01512e^{-750t}=$$

$$=0{,}048e^{-750t}\text{ A.}$$

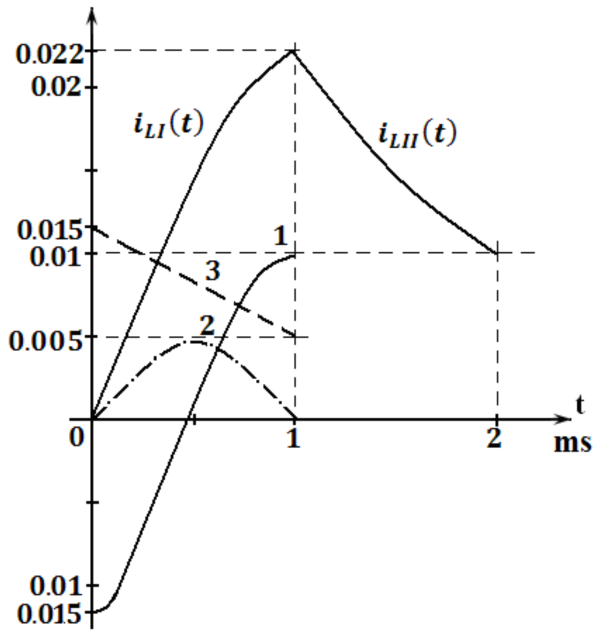
$$i_{LII}(t)=0{,}0471e^{-750t}\text{ A.}$$

4. $i_{LI}(t)$ va $i_{LII}(t)$ funksiyalarning grafigini qurish uchun ularni yozib olamiz:

$$i_{LI}(t)=-0{,}015\cos3142t+0{,}0036\sin3142t+0{,}015e^{-750t}\text{A.}$$

$$i_{LII}(t)=0{,}0471e^{-750t}\text{A.}$$

Vaqt oralig‘i $0\le t\le 2\text{ms}$ da $i_L(t)$ grafigini intervallar bo‘yicha quramiz. sin3142*t* va cos3142*t* funksiyalar davri 2 ms. Vaqt o‘qi va tok o‘qi masshtablarini tanlab olamiz (5.65- rasm).

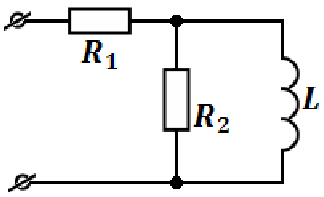


5.65- rasm.

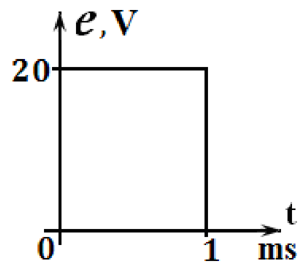
Funksiya - $0{,}015\cos(3142t)$ grafigi 5.65- rasmdagi 1-egri chizig‘ini va $0{,}0036\sin3142t$ funksiya grafigi 2-egri chizig‘ini hamda eksponenta grafigi

$0{,}015e^{-750t}$ 3-egri chizig‘ini quramiz. Birinchi interval $0 \le t \le t_1$ dagi funksiyalarni qo‘shamiz va natijada $i_{LI}(t)$ funksiyani $1\,\text{ms} \le t < 2\,\text{ms}$ intervalda quramiz.

5.43-masala. 5.66-rasmdagi sxemaga 5.67- rasmdagi EYK manbayi qo‘shilgan. Sxema parametrlari $R_1 = R_2 = 100\ \Omega$, $L = 0{,}2\,\text{H}$. Induktivlikdagi $i_L(t)$ tok funksiyasini va uning grafigini quring.

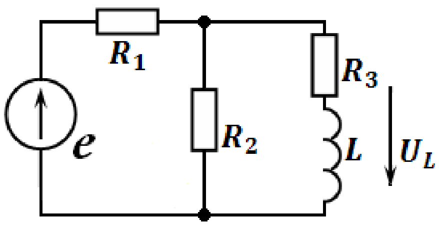


5.66- rasm.

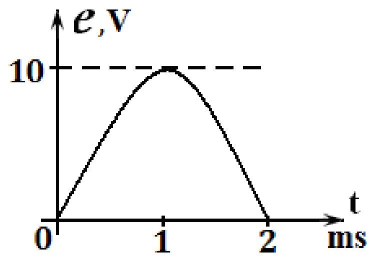


5.67- rasm.

5.44- masala. Parametrlari $R_1 = R_2 = R_3 = 100\ \Omega$; $L = 0{,}15\,\text{H}$ bo‘lgan elektr sxemaga (5.68- rasm) sinusoidal qonun bo‘yicha o‘zgaradigan EYK manbayi (5.69- rasm) qo‘shilgan. Induktivlikdagi $U_L(t)$ kuchlanish funksiyasini aniqlang va uning grafigini quring.

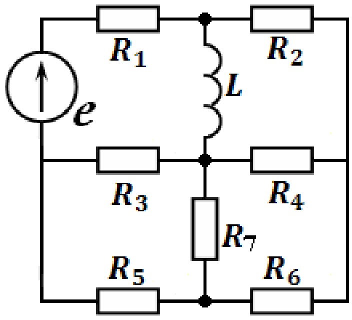


5.68- rasm.

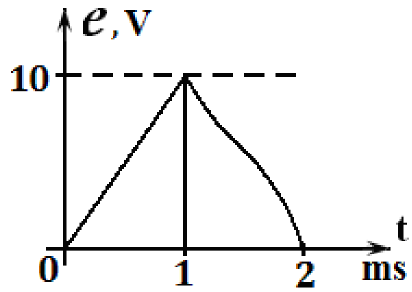


5.69- rasm.

5.45- masala. Parametrlari $R_1 = R_2 = R_3 = 100\ \Omega$; $R_5 = R_6 = R_7 = 200\ \Omega$; $L = 0{,}2$ H bo‘lgan 5.70- rasmdagi elektr sxemaga, 5.71- rasmda keltirilgan qonun bo‘yicha o‘zgaradigan EYK manbayi qo‘shilgan. Induktivlikdagi $i_L(t)$ tok funksiyasi aniqlansin.

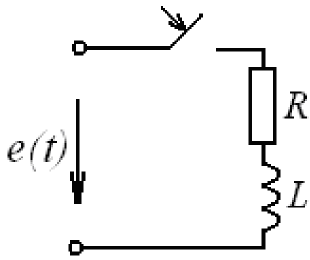


5.70 – rasm.



5.71 – rasm.

5.46- masala. Parametrlari $R=100\ \Omega$ va $L=1\ \text{H}$ bo‘lgan zanjir sinusoidal kuchlanish $(U=220\ \text{V},\ f=50\ \text{Hz})$ manbayiga ulangan (5.72- rasm). Kuchlanishning boshlang‘ich fazalari $\psi_1=0$ va $\psi_2=72^\circ30'$ bo‘lgan onda ulangan ikki holat (rejim) uchun o‘tkinchi tok ifodalarini toping.



5.72- rasm.

Yechish. Turg‘un tok oniy qiymati ifodasini kompleks usulda aniqlaymiz:

$$\underline{I}_m=\frac{\underline{U}_m}{\underline{Z}}=\frac{220\sqrt{2}}{100+j314\cdot 1}=0{,}94e^{-72^\circ 20'}.$$

Uning oniy qiymati:

$$i_T=I_m\sin(\omega t+\psi-\varphi)=0{,}94\sin(\omega t+\psi-72^\circ 20')\ A.$$

Ulanishning birinchi rejimi uchun $(\psi_1=0)$:

$$i_{T1}=0{,}94\sin(\omega t-72^\circ 20')\ \text{A}.$$

va ikkinchi rejim uchun $(\psi_2=72^\circ 30')$:

$$i_{T2}=0{,}94\sin\omega t\ \text{A}.$$

Tokning erkin tashkil etuvchisi quyidagicha aniqlanadi:

$$i_{\text{er}}=Ae^{-\frac{t}{\tau}}.$$

Integral doimiysi A ni kommutatsiyaning birinchi qonunidan aniqlaymiz:

$$i(0)=0=i_T+i_{\text{er}}=I_m\sin(0+\psi-\varphi)+Ae^{-\frac{0}{\tau}}=I_m\sin(\psi-\varphi)+A,$$

binobarin, $A=-I_m\sin(\psi-\varphi)$, $i_{\text{er}}=-I_m\sin(\psi-\varphi)e^{-\frac{t}{\tau}}$, bunda $\tau=\dfrac{L}{R}=\dfrac{1}{100}=0{,}01\,\text{s}$.

Mos ravishda ikkala rejim uchun:

$$i_{\text{er}1}=-0{,}94\sin(-72^\circ20')e^{-100t}=0{,}896e^{-100t},$$
$$i_{\text{er}2}=-0{,}94\sin0^\circ e^{-100t}=0.$$

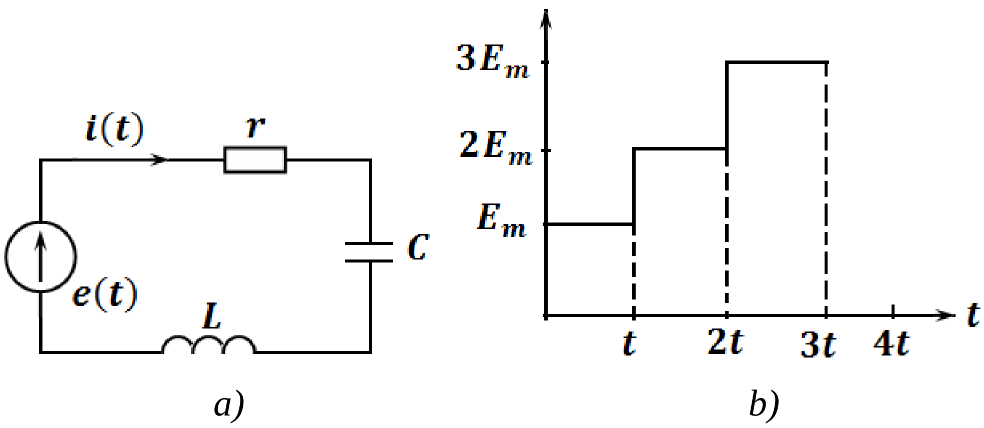
Demak, ko‘rilayotgan ikki holat uchun o‘tkinchi toklar quyidagiga teng:

$$i_1=0{,}94\sin(\omega t-72^\circ20')+0{,}86e^{-100t},$$
$$i_2=0{,}94\sin\omega t.$$

Ikkinchi rejimda erkin tok yo‘qligi zanjirda o‘tkinchi jarayon bo‘lmasligini ko‘rsatadi va o‘tkinchi tok faqat turg‘un tokdan iborat bo‘ladi.

5.47- masala. Ketma-ket ulangan tebranma konturga (5.73- rasm, *a*) 5.73- rasm, *b* da tasvirlangan pog‘onali kuchlanish berilgan. Kontur elementlarining parametrlari quyidagi qiymatlarga ega:

$r=400\,\Omega$; $L=0{,}1\,\text{H}$; $C=2{,}5\,\text{mkF}$; $E_m=10\,\text{V}$; $t_u=0{,}5\,\text{ms}$.



5.73- rasm. Zanjir sxemasi va unga berilayotgan kuchlanish.

Yechish. Sig‘imdagi kuchlanish uchun konturning o‘tkinchi xarakteristikasini hisoblaymiz. O‘tkinchi xarakteristikani hisoblash uchun kontur kirishiga birlamchi pog‘onali $e(t)=1(t)$ funksiya beriladi deymiz va C sig‘imdagi kuchlanishni bu holda quyidagicha yozish mumkin:

$$h_C = h_{Cer} + h_{Cbar}$$

Ko‘rilayotgan sxema uchun sig‘imdagi kuchlanish manba kuchlanishiga teng, ya’ni $h_{Cbar} = 10\,\text{V}.$.

Kuchlanish erkin tashkil etuvchisini quyidagi ko‘rinishda izlaymiz.

$$h_{Cerk} = A_1 e^{p_1 t} + A_2 e^{p_2 t}.$$

bunda p_1, p_2 - xarakteristik tenglamaning ildizlari.

Ildizlar qiymatini xarakteristik tenglamadan topamiz:

$$Z_{kir}(p) = r + pL + \frac{1}{pC} = 0$$

bundan

$$p^2 + \frac{pr}{L} + \frac{1}{LC} = 0$$

ildizlar

$$p_{1,2} = -\frac{r}{2L} + \sqrt{\left(\frac{r}{2L}\right)^2 - \frac{1}{LC}},$$

bu qiymatlarni qo‘ysak,

$$p_0 = p_2 = p = 2000\,\text{rad/s}.$$

Xarakteristik tenglama ildizlari manfiy, haqiqiy va bir-biriga teng bo‘lgani uchun o‘tkinchi jarayon so‘nuvchi chegaraviy rejimga to‘g‘ri keladi:

$$h_C(t) = 1 - (1 - pt)e^{pt}.$$

Ildiz qiymatini qo‘ygandan so‘ng

$$h_C = 1 - (1 + 2000t)e^{-2000t}.$$

Sig‘imdagi kuchlanishning o‘tkinchi xarakteristikasi 5.74- rasmda keltirilgan.

O‘tkinchi jarayonni hisoblash uchun kirish kuchlanishini bo‘laklangan funksiya ko‘rinishda ifodalaymiz:

$$e(t) = \begin{cases} E_m, & 0 < t < t_u \quad \text{uchun;} \\ 2E_m, & t_u < t < 2t_u, \text{uchun;} \\ 3E_m, & 2t_u < t < 3t_u \quad \text{uchun;} \end{cases}$$

$$0, \quad 3t_u < t < \infty \qquad \text{uchun.}$$

Funksiya $e(t)$ da uchastkalarning chegaralarida uzilishlar mavjud, ular tenglamalar tuzilayotganda Dyuamel integrali yordamida hisobga olinadi.

$0 < t < t_u$ dagi birinchi uchastkada $e(t)$ funksiya uzluksizligi $t = 0$ da sinadi, shu sababli sig‘imdagi kuchlanishni quyidagicha tasvirlash mumkin:

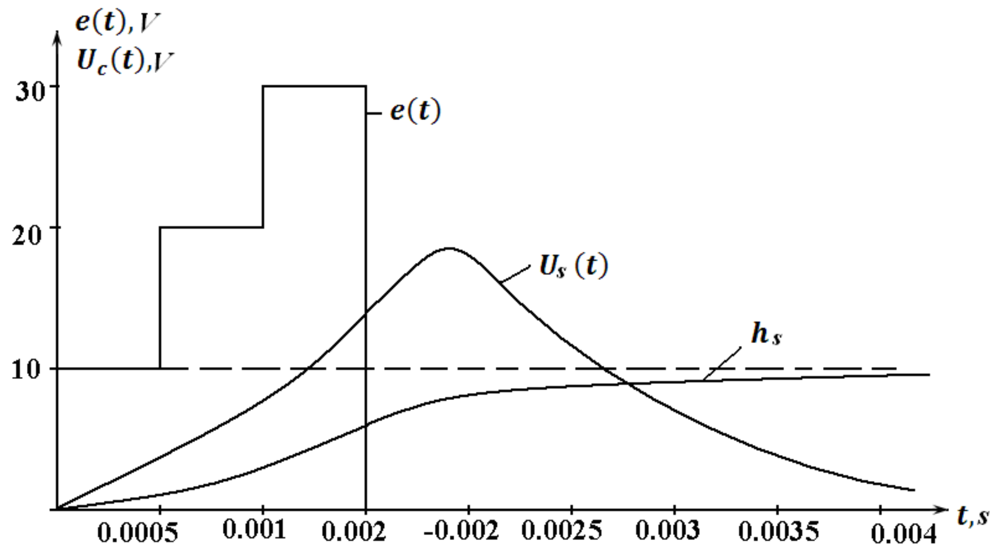
$$U_C = e(0_+)\cdot h_c(t) + \int_0^t e^{'}(\tau)\cdot h_s\cdot(t-\tau)d\tau = E_m\left[1-(1+2000t)e^{-2000t}\cdot\varsigma(t)\right]\text{V}.$$

bu ifodada hosila $e'(\tau)$=0, shu sababli sig‘imdagi kuchlanish $t = 0$ da faqat pog‘onali $e(t)$ o‘zgarishiga ega.

$t_u < t < 2t_u$ ikkinchi uchastkada $e(t)$ funksiya $t = t_u$ da sinishga ega, shu sababli sig‘imdagi kuchlanish quyidagicha ifodalanadi:

$$U_C = E_m\left[1-(1+2000t)e^{-2000t}\cdot 1(t)\right] + E_m\left\{1-\left[1-2000(t-t_u)e^{t-t_u}\right]\cdot 1(t-t_u)\right\}$$

Bu ifodada hosila $e'(\tau) = 0$ nolga tengliligi hamda sig‘imdagi kuchlanish $t = t_u$ da faqat $e(t)$ ning sakrab o‘zgarishi hisobga olingan.



5.74- rasm.

Shunga o‘xshash uchastkadagi U_c kuchlanishni $2t_u < t < 3t_u$ da topamiz:

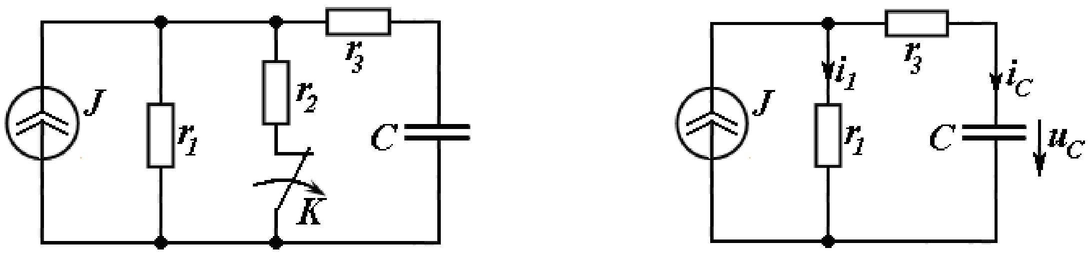
$$U_C = E_m\left[1-(1+2000t)e^{-2000t}\cdot 1(t)\right]+$$
$$E_m\left\{1-\left[1-2000\left(t-t_u\right)e^{t-t_u}\right]\cdot 1(t-t_u)\right\}+$$
$$E_m\left\{1-\left[1-2000\left(t-2t_u\right)e^{t-2t_u}\right]\cdot 1(t-2t_u)\right\}.$$

.Oxirgi to‘rtinchi $3t_u < t < \infty$ intervalda kirish kuchlanishi $3E_m$ dan 0 gacha o‘zgaradi, ya’ni $-3E_m$ orttirmaga ega, shu sababli sig‘imdagi kuchlanish quyidagicha ifodalanadi:

$$U_C = E_m\left[1-(1+2000t)e^{-2000t}\cdot 1(t)\right]+$$
$$E_m\left\{1-\left[1-2000\left(t-t_u\right)e^{t-t_u}\right]\cdot 1(t-t_u)\right\}+$$
$$E_m\left\{1-\left[1-2000\left(t-2t_u\right)e^{t-2t_u}\right]\cdot 1(t-2t_u)\right\}-$$
$$E_m\left\{1-\left[1-2000\left(t-3t_u\right)e^{t-3t_u}\right]\cdot 1(t-3t_u)\right\}.$$

Uchastkalarni bo‘lish uchun sig‘imdagi kuchlanishlarning formulalariga birlamchi kechikish funksiyasi kiritilgan, bular argumentning manfiy qiymatlarida nolga tengdir. $U_c(t)$ grafigi 5.74- rasmda keltirilgan.

5.48- masala. Elektr zanjir (5.75- rasm, *a*, *b*) sig‘imidagi kuchlanishni $U_c(t)$ kalit ajratilgandan so‘ng aniqlang. Zanjir elementlarining parametrlari quyidagicha: J=1 A; $r_1 = r_2 = r_3 = 100\ \Omega$; C=10 mkF.



a) *b)*

5.75- rasm. Dastlabki *(a)* va hisobiy *(b)* sxemalar.

Yechish. 5.75- rasmdagi kalit ajratilganda sig‘imdagi boshlang‘ich va turg‘un kuchlanishni aniqlaymiz. Sig‘imdagi boshlang‘ich kuchlanish parallel ulangan r_1 va r_2 qarshiliklardagi kuchlanish bilan aniqlanadi:

$$u_C(0) = J\frac{r_1 r_2}{r_1 + r_2} = 1\cdot 50 = 50\ \text{V}.$$

Sig‘imdagi turg‘un kuchlanish r_3 qarshilikdan o‘tadigan tok yordamida aniqlanadi:

$$u_{C\,tur} = J\,r_1 = 1 \cdot 100 = 100 \text{ V}.$$

Shunday qilib, sig‘imdagi kuchlanish jarayoni $u_C(0)$ dan u_{Ctur} gacha bo‘ladigan qayta zaryadlanishga bog‘liq. Bu holda boshlang‘ich va keyingi rejimlarda sig‘im *C* dan tok o‘tmaydi.

Kalit ajratilgandan so‘ng sxema 5.75- rasm, *b* dagi ko‘rinishga o‘tadi. Bu sxemaga Kirxgof qonunlari bo‘yicha tenglamalar tuzamiz:

$$i_1 + i_C = J\,,$$

$$-i_1 r_1 + i_C r_3 + U_C = 0\,.$$

Bu tenglamalarga sig‘imdagi tok ifodasi $i_C = C\dfrac{dU_C}{dt}$ ni qo‘yib, birinchi darajali differensial tenglamani hosil qilamiz:

$$C(r_1 + r_3)\frac{dU_C}{dt} + U_C = r_1\,J$$

Tenglamaning yechimini $U_C = U_{C\,\text{erk}} + U_{S\,t}$ ko‘rinishda izlaymiz.

Sig‘im kuchlanishining erkin tashkil etuvchisini quyidagi ko‘rinishda aniqlaymiz:

$$U_{\text{Cerk}} = Ae^{pt}\,,$$

bunda *r* quyidagi xarakteristik tenglamani ildizi:

$$C(r_1 + r_3)\,p + 1 = 0\,.$$

Ildiz $p = -\dfrac{1}{C(r_1 + r_3)\,p} = -500\,\text{rad/s}\,.$

Sig‘imdagi kuchlanishni erkin va turg‘un tashkil etuvchilarning yig‘indisi sifatida aniqlaymiz:

$$U_C = Y_{\text{Cerk}} + U_{\text{Cm}} = Ae^{-500t} + 1$$

Integral doimiysi *A* ni kommutatsiyaning ikkinchi qonuni bo‘yicha topamiz, ya’ni *t=0* da sig‘imdagi kuchlanish:

$$U_C(0_-) = U_c(0_+) = 50\,\text{V}\,.$$

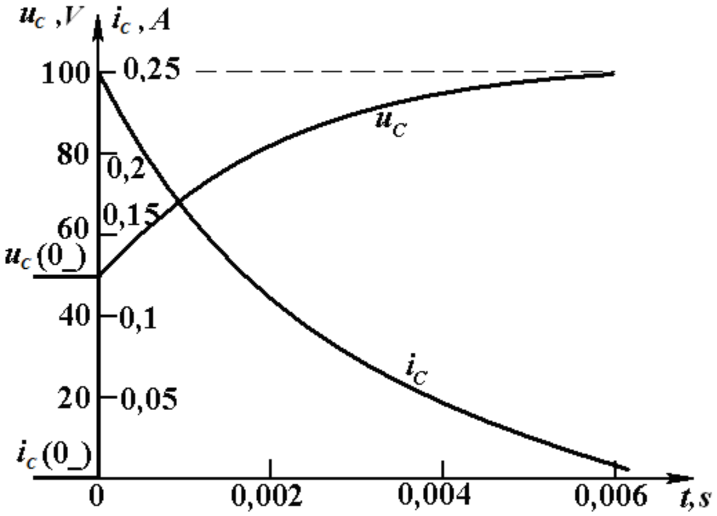
Bu ifodadan $A = U_C(0\_) - U_{Cm} = -50$ V.

Shunday qilib, sig‘imdagi kuchlanishning ifodasi

$$U_C = 100 - 50e^{-500t} \text{ V}.$$

Sig‘imdan o‘tadigan tok $i_C = C\dfrac{dU_C}{dt} = 0{,}25e^{-500t}$ A.

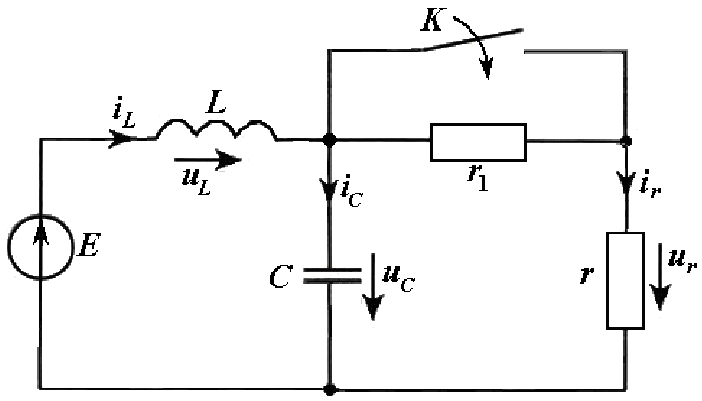
Sig‘imdagi kuchlanish va tok o‘zgarishi 5.76- rasmda tasvirlangan.



5.76- rasm.

Ikkinchi darajali o‘zgarmas tok elektr zanjirlardagi o‘tkinchi jarayonlarni hisoblash.

5.49- masala. Elektr zanjirda (5.77- rasm) kalit K ulangandan so‘ng barcha shoxobchalardagi toklar hamda sig‘im va induktivlikdagi kuchlanish ifodalari aniqlansin. Agar zanjir elementlarining parametrlari *E*=180 V; *C*=10 mkF; *L*=0,1 H; $r_1 = 20\ \Omega$, $r = 40\ \Omega$ bo‘lsa, sig‘imdagi tok va kuchlanishning vaqt bo‘yicha o‘zgarish egri chizig‘ini tasvirlang.



5.77- rasm.

Yechish. Avval zanjir shoxobchalaridagi tok yo‘nalishlarini belgilaymiz (5.77- rasm). Kalit ulanmasdan sxema elementlaridagi boshlang‘ich shartlarni aniqlaymiz. Kommutatsiyadan avval induktivlik L dan o‘tayotgan tok

$$i_L(0_-) = i_r(0_-) = \frac{E}{r_1 + r} = \frac{180}{20 + 40} = 3\,\text{A}.$$

Kommutatsiyadan avval sig‘imdan o‘tadigan tok nolga teng $i_C(0_-) = 0$;

Induktivlikda kuchlanish bo‘lmaydi, chunki undan o‘zgarmas tok o‘tadi shuning uchun $U_L(0_-) = 0$.

Sig‘imdagi kuchlanish manba kuchlanishiga teng:

$$U_C = E = 180\,\text{V}.$$

2. Kirxgof qonunlaridan foydalanib, kalit K qo‘shilgandan keyingi holat uchun tenglamalar tuzamiz:

$$U_C + U_L = E,$$

$$i_L = i_C + i_r,$$

$$U_C = U_r; \quad i_r = \frac{U_C}{r}.$$

3. Bunda Kirxgofning ikkinchi qonunidan foydalanib, kuchlanish U_c uchun differensial tenglama tuzamiz:

$$u_C + u_L = u_C + L\frac{di_L}{dt} = E.$$

Tenglamadagi i_L tokni quyidagicha ifodalab:

$$i_L = i_C + i_r = C\frac{du_C}{dt} + \frac{u_C}{r},$$

u_C kuchlanish uchun differensial tenglamani hosil qilamiz:

$$u_C + \frac{L}{r}\frac{du_C}{dt} + LC\frac{d^2u_C}{dt^2} = E.$$

Bu tenglamaning o‘ng va chap tomonini LC ga bo‘lib, u_C ga nisbatan ikkinchi tartibli differensial tenglamani hosil qilamiz:

$$LC\frac{d^2u_C}{dt^2}+\frac{1}{rC}\frac{du_C}{dt}+\frac{u_C}{LC}=\frac{E}{LC}.$$

4. Bu differensial tengamaning yechimini quyidagi ko‘rinishda qidiramiz:

$$u_C=u_{C\text{ er}}+u_{C\text{ ct}},$$

bunda $u_{C\text{ er}}$ – kuchlanishning erkin tashkil etuvchisi bo‘lib, bir jinsli differensial tenglama yechimidan aniqlanadi: $\frac{d^2u_{C\text{ er}}}{dt^2}+\frac{1}{rC}\frac{du_{C\text{ er}}}{dt}+\frac{u_{C\text{ er}}}{LC}=0$; $u_{C\text{ ct}}$ – sig‘im kuchlanishining turg‘un tashkil etuvchisi, u kommutatsiyadan keyingi barqaror rejimdagi kuchlanishga teng: $u_{C\text{ ct}}=E$.

Bir jinsli differensial tenglamaning yechimi kuchlanishning erkin tashkil etuvchisi ifodasini beradi:

$$u_{C\text{ er}}=A_1p_1e^{p_1t}+A_2p_2e^{p_2t},$$

Bunda A_1, A_2 - integral doimiylari; p_1, p_2 esa $p^2+\frac{1}{rC}p+\frac{1}{LC}=0$ xarakteristik tenlgamaning ildizlari. Ularni topamiz:

$$p_{1,2}=-\frac{1}{2rC}\pm\sqrt{\frac{1}{(2rC)^2}-\frac{1}{LC}};$$

$$p_1=-2000\text{ rad/s};\quad p_2=-500\text{ rad/s}.$$

Ildizlar manfiy, haqiqiy va xar xil qiymatli bo‘lgani uchun o‘tkinchi jariyon aperiodik xarakterga ega:

$$U_c=A_1e^{p_1t}+A_2e^{p_2t}=A_1e^{-2000t}+A_2e^{-500t}+180.$$

Integral doimiylari A_1 va A_2 ni aniqlashga o‘tamiz. Buning uchun kommutatsiya qonunlaridan foydalanamiz, chunonchi

$$A_1+A_2=0.$$

Ikkinchi tengamani tuzish uchun kalit K bevosita ulangandan so‘ng sig‘imdan o‘tadigan tok i_c ni topamiz, ya’ni $i_C(0_+)$ ni aniqlaymiz. Zanjir toklari uchun $t=0_+$ shartni e’tiborga olib Kirxgof tenglamalarini tuzamiz:

$i_C(0_+)=i_L(0_+)+i_r(0_+),$

bunda $i_L(0_+) = i_L(0_-) = 3\,\text{A}; \quad i_r(0_+) = \dfrac{i_C(0_+)}{r} = \dfrac{180}{40} = 4{,}5\,\text{A}$.

U holda $i_C(0_+)$ tok:

$i_C(0_+) = 3 - 4{,}5 = -1{,}5\,\text{A}$,

ya’ni sig‘imdagi tok 0 dan 1,5 A gacha sakrab o‘zgaradi.

Kommutatsiyadan keyin sig‘imdan o‘tadigan tokni hisoblash uchun quyidagi formuladan foydalanamiz:

$$i_C = C\frac{du_C}{dt} = C(A_1 p_1 e^{p_1 t} + A_2 p_2 e^{p_2 t}).$$

Bu formulaga *t=0* ni qo‘yib, integral doimiylarini aniqlash uchun ikkinchi tenglamani hosil qilamiz:

$$A_1 p_1 + A_2 p_2 = \frac{i_C(0_+)}{C} = \frac{1{,}5}{10^{-5}} = 1{,}5 \cdot 10^5.$$

Shunday qilib, integral doimiylarini aniqlash uchun ikkita tenglama hosil bo‘ladi:

$$\begin{cases} A_1 + A_2 = 0; \\ A_1 p_1 + A_2 p_2 = 1{,}5 \cdot 10^5. \end{cases}$$

Sistemani yechimlari $A_1 = -A_2 = 1{,}5 \cdot \dfrac{10^5}{p_1 - p_2} = 100.$

Natijada sig‘imdagi kuchlanish quyidagicha bo‘ladi:

$$U_C = \left[100(e^{-2000t} - e^{-500t}) + 4{,}5\right]\text{V}.$$

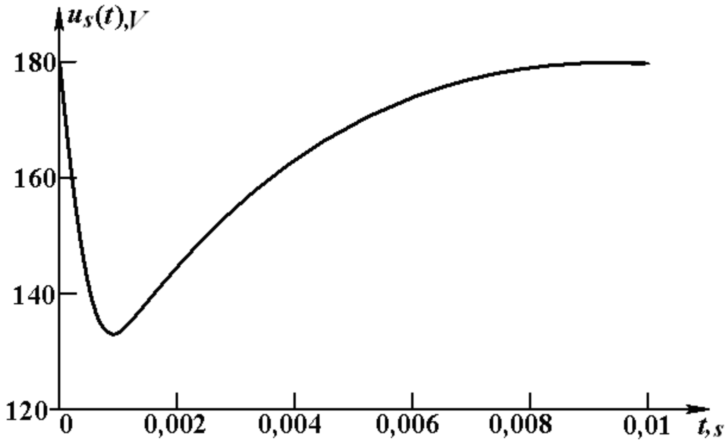
Sig‘imdan o‘tadigan tok:

$$i_C = C\frac{dU_C}{dt} = (-2e^{-2000t} + 0{,}5e^{-500t})\ \text{A}.$$

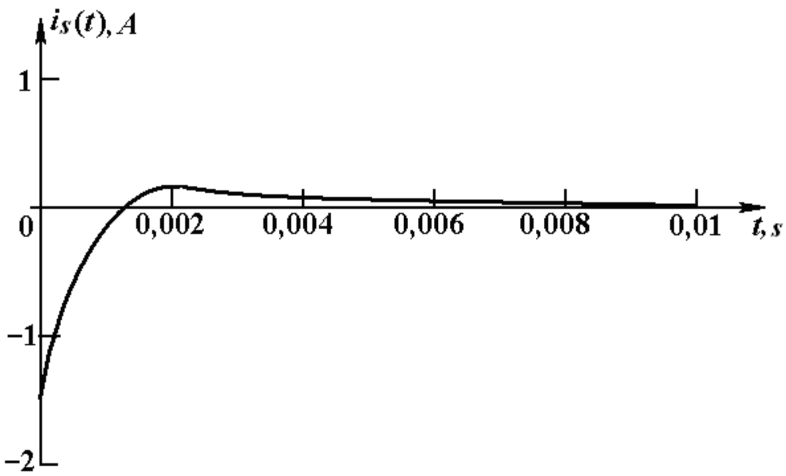
Sig‘im va qarshalikdagi tokni yig‘indisi induktivlikdagi tokni beradi:

$$i_L = i_C + i_r = (0{,}5e^{-2000t} - 2e^{-500t} + 4{,}5)\ \text{A}.$$

Kuchlanish va toklarning grafiklari 5.78- rasmda keltirilgan.



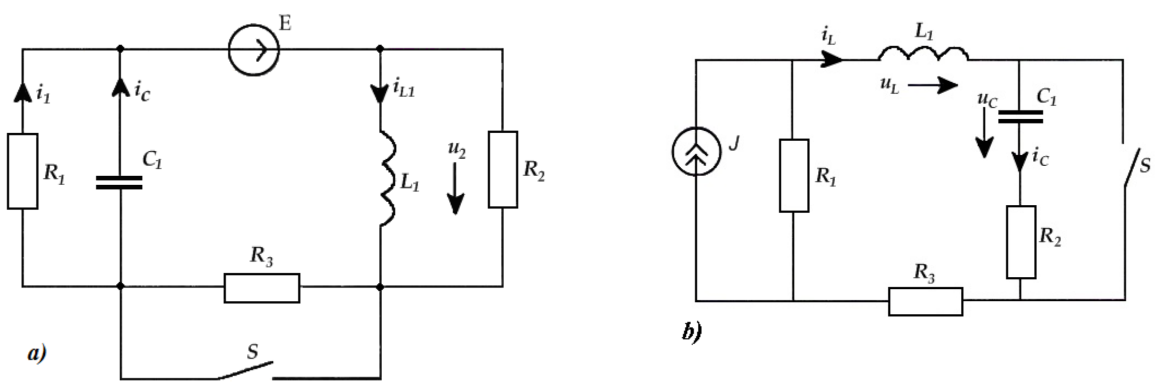
a)

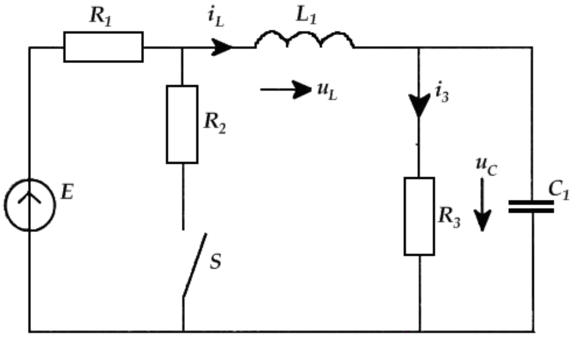


b)

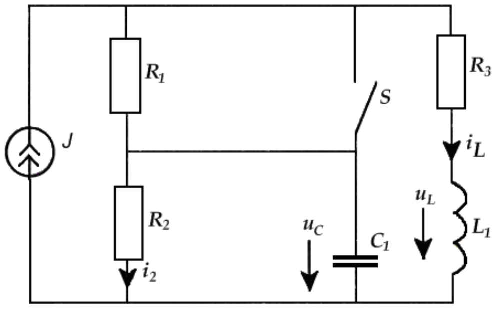
5.78- rasm . Sig‘imdagi kuchlanish va tok grafiklari.

5.2-vazifa. Berilgan zanjirlarda (5.79-rasm) 5.4-jadvalda ko‘rsatilgan kattaliklarning kommutatsiyadan keyingi oniy qiymat ifodalarini aniqlang. Variantlarda sxemani, elementlar parametrini va kommutatsiya turini tanlash 5.4-jadvalda ko‘rsatilgan. Hisoblashni klassik va operator usullarida bajaring.

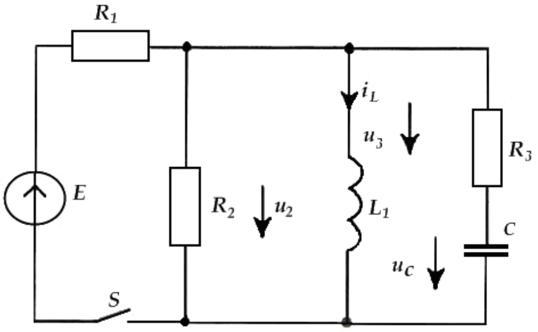



R1
iL
L1
uL
i3
R2
E
uC
C1
R3
S

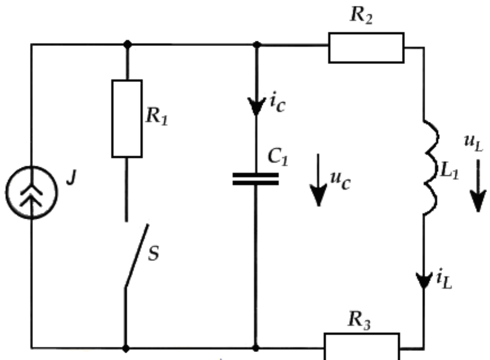
d)


R1
R3
S
J
iL
uL
R2
uC
C1
L1
i2

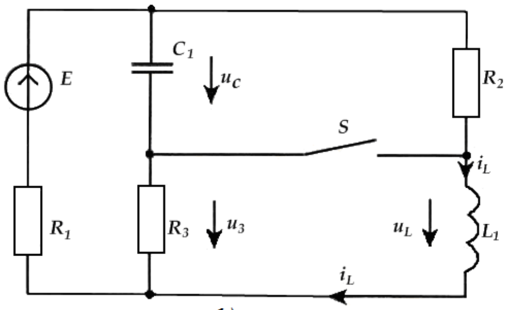
e)


R1
iL
u3
R3
E
R2
u2
L1
C
uC
S

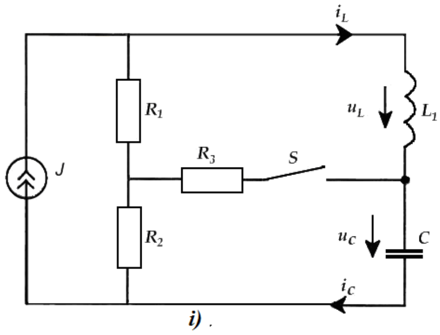
f)


R2
iC
R1
C1
uL
J
uC
L1
S
iL
R3

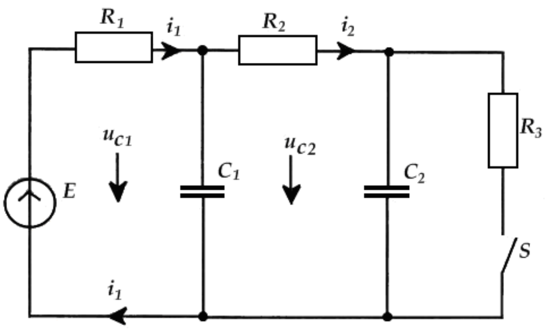
g)


C1
uC
E
R2
S
iL
R1
R3
u3
uL
L1
iL

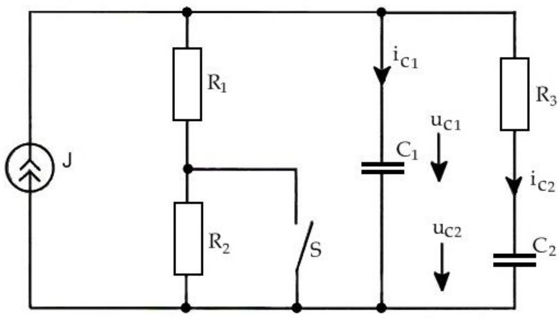
h)


iL
R1
uL
L1
R3
S
J
R2
uC
C
iC

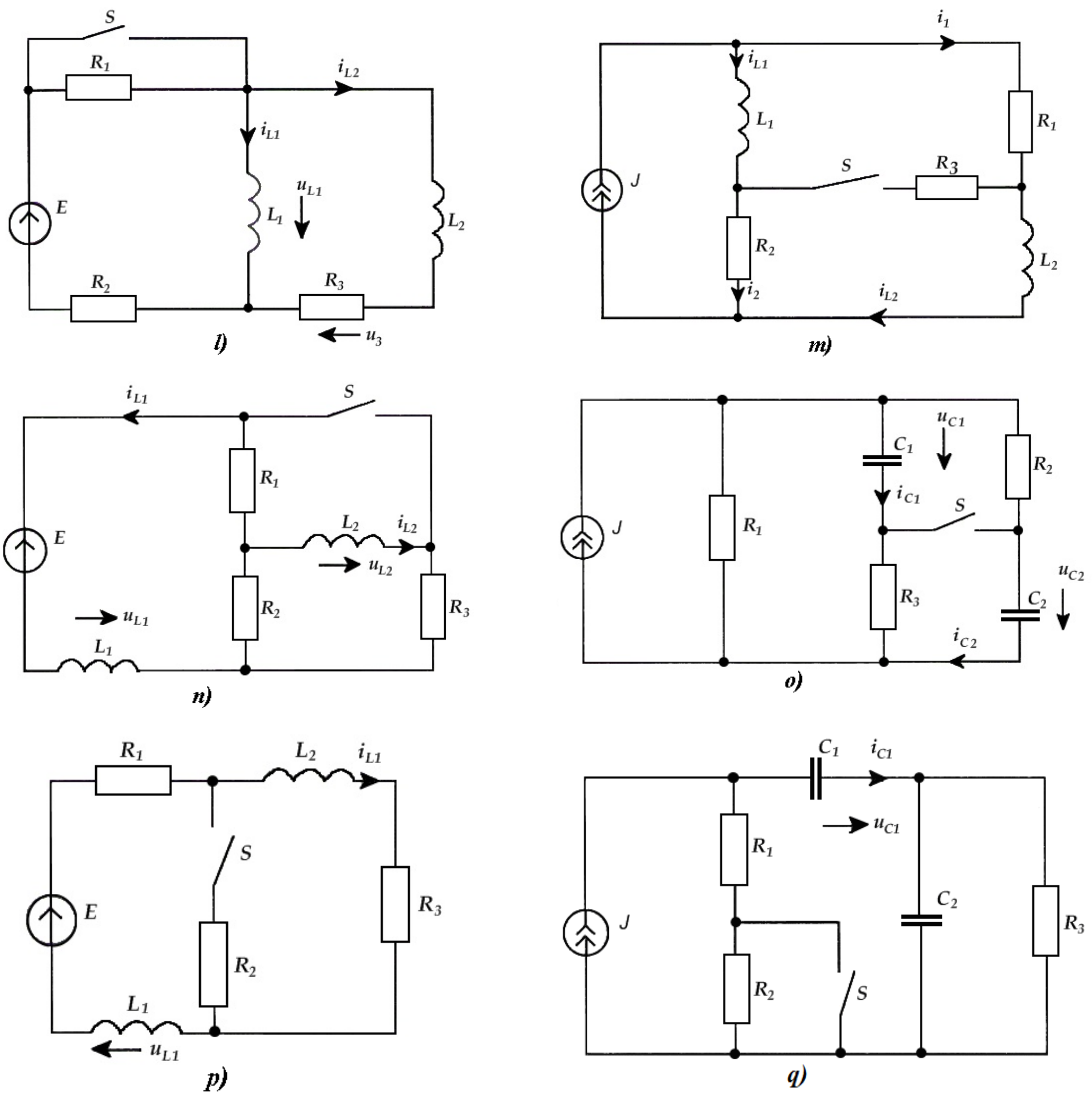
i)


R1
i1
R2
i2
R3
uC1
uC2
E
C1
C2
S
i1

j)


R1
iC1
R3
uC1
J
C1
iC2
uC2
R2
S
C2

k)



5.79- rasm. 5.2-vazifaga zanjir sxemalari.

5.4- *jadval*

Vari-ant	sxema	E, V	J, A	L_1, mH	L_2, mH	C_1, mkF	C_2, mkF	R_1, Ω	R_2, Ω	R_3, Ω	Toping	Kal it S
1	a	120		1		10		4	4	8	i_c, i_1	U
2	b	-	5	1		10		20	20	10	u_c, i_c	A
3	d	120		10		10		100	500	500	i_L, u_L	U
4	e	-	10	1		500		2	2	6	u_c, i_2	A
5	f	120		1		10		1	3	3	i_L, u_2	U
6	g	-	30	5		4		20	10	20	u_c, i_c	U
7	h	60		2		5		10	10	10	i_L, u_L	A
8	i	-	30	0.2		10		5	5	10	u_c, i_c	U
9	j	120				10	50	10	20	10	u_{c1}, i_1	A
10	k	-	5			20	40	10	20	30	u_{c1}, i_{c1}	U
11	l	60		10	30			4	4	8	i_{L1}, u_{L1}	U
12	m	-	12	1	4			4	6	8	i_{L1}, i_1	U
13	n	120		2	8			4	4	8	u_{L1}, i_{L1}	A
14	o	-	10			10	30	6	4	2	u_{c1}, i_{c1}	A
15	p	80		1	5			8	4	4	u_{L1}, i_{L1}	U
16	q	-	2			10	20	5	15	20	u_{c1}, i_{c1}	A
17	a	100		1		0.5		10	5	5	u_2, i_{L1}	A
18	b	-	2	1		1		40	20	10	u_L, i_L	A
19	d	100		10		1		40	60	60	u_c, i_3	A
20	e	-	4	1		10		4	4	2	u_L, i_L	U
21	f	120		5		1		2	4	2	u_c, u_3	A
22	g	-	4	10		1		30	20	10	u_L, i_L	U
23	h	60		1		0.2		20	15	5	u_c, u_3	A
24	i	-	6	1		0.5		2	4	4	i_L, u_L	A
25	j	80				20	10	5	15	20	u_{c2}, i_{c2}	A
26	k	-	8			4	2	10	20	10	u_{c2}, u_3	U
27	l	60		5	15			5	15	5	u_3, i_{L2}	U
28	m	-	12	4	2			2	4	6	i_{L2}, i_2	A
29	n	30		6	1			4	6	10	u_{L2}, i_{L2}	U
30	o	-	4			20	10	40	20	60	u_{c2}, i_{c2}	A

6-BOB. O‘ZGARMAS VA O‘ZGARUVCHAN TOK NOCHIZIQ ELEKTR VA DOIMIY MAGNIT OQIMLI MAGNIT ZANJIRLARINI HISOBLASH

6.1. Asosiy ma’lumotlar

*O‘zgaruvchan tok nochiziq zanjirlar*dagi barqaror jarayonlar grafik, analitik va grafoanalitik usullar yordamida hisoblanadi.

Zanjirlar tarkibidagi nochiziq elementlarning xarakteristikalari kesim-chiziqli funksiya bilan ifodalanadigan masalalar *grafik* va *grafoanalitik* usullar bilan yechilishi maqsadga muofiq. Bularda xarakteristikaning chiziqli uchastkalarida masala chiziqli ravishda yechiladi va bir chiziqli uchastkadan boshqa chiziqli uchastkaga o‘tish koordinatasi egilish burchaklari aniqlanadi.

Analitik usullar qo‘llanilganda nochiziq elementlarning xarakteristikalari eng sodda bo‘lgan funksiyalar – ko‘p hollarda polinomlar bilan ifodalanadi (approksimatsiyalanadi). Eng sodda analitik usul zanjir tenglamasini (mumkin bo‘lgan holda) bevosita yechishdan iborat. Bunda *garmonik balans usuli*, *garmonik chiziqlashtirish*, hamda *iteratsion usullardan* keng qo‘llaniladi.

Masala garmonik balans usulda yechilganda izlanayotgan kattalik noma’lum koeffitsientlarga ega bo‘lgan davriy funksiya bilan ifodalanadi va zanjir tenglamasiga qo‘yiladi. Noma’lumlar tenglamalarning o‘ng va chap qismidagi garmonik tashkil etuvchi koeffitsientlar o‘zaro tenglashtirib aniqlanadi.

Garmonik chiziqlashtirish usulining mohiyati shundaki, nochiziq elementga garmonik signal zanjir ta’sirida faqat asosiy garmonika inobatga olinadi, yuqori garmonikalar esa tashlab yuboriladi va yechish uchun kompleks usul qo‘llanishi mumkin.

Nochiziq element nochiziq kompleks qarshilik bilan, ya’ni nochiziq element kuchlanishi va toki birinchi garmonikalarining nisbatlari bilan xarakterlanadi. Shuni e’tiborga olish kerakki, nochiziq kompleks qarshilik tokning amplituda va chastotasiga bog‘liq. Masalan, nochiziq rezistorning volt-amper xarakteristikasi

$$u(i) = ai + bi^3 \tag{6.1}$$

polinom bilan ifodalansa, nochiziq kompleks aktiv qarshilik

$$\underline{Z}_H = R_H = a + \frac{3}{4} b\, I_m^2 \tag{6.2}$$

bilan ifodalanadi, bunda I_m – sinusoidal tokning amplitudasi.

Nochiziq elementning qarshiligi aniqlangandan so‘ng, zanjir tenglamasi algebraik usullar bilan yechiladi.

Zanjir tenglamalarida yetarli darajada birinchi yaqinlashtirishi tashlab yuborish mumkin bo‘lganda *interatsion usul* qo‘llaniladi.

Noma’lum $x(t)$ ga nisbatan zanjir tenglamasi

$$f\,(x, dx/dt, d^2x/dt^2, ...) = 0 \tag{6.3}$$

avval ikki qismga bo‘linadi, bunda o‘ng qismdagi yetarli darajada kichik bo‘lgan

$$f_1(x, dx/dt, ...) = Ef_2(x, dx/dt, ...) \tag{6.4}$$

tashlab yuboriladi, bunda E – kichik parametr. Kichik hadlar tashlab yuborilgan (E parametr nolga tenglashtiriladi) tenglamani

$$f\,(x, dx/dt, ...) = 0 \tag{6.5}$$

yechib, nolli $x_0(t)$ *yaqinlashganlik yechim* topiladi. Topilgan $x_0(t)$ yechim (6.4) tenglamaning o‘ng tomoniga qo‘yiladi, bunda ikkinchi darajali kichik hadlar, agar ular mavjud bo‘lsa, tashlab yuboriladi va $x(t)$ yechim

$$(x, dx/dt, ...) = Ef_2(x_0, dx_0/dt, ...) \tag{6.6}$$

tenglamadan topiladi. Bu yechim *birinchi yaqinlashtirilgan yechim* hisoblanadi. Keyingi yaqinlashtirilgan yechimlarini topish uchun yuqoridagi amallarni qaytarish lozim bo‘ladi.

O‘zgaruvchan tok nochiziq zanjirlarini tok va kuchlanishning ta’sir etuvchi qiymatlari bo‘yicha hisoblash magnit o‘tkazgichlardagi gisterezisni va belgilangan toklarni ixtiyoriy istalgan qiymatini e’tiborga olishga imkon beradi. Nosinusoidal tok va kuchlanishlar ekvivalent sinusoidalar bilan almashtiriladi. Gisterezis va uyurmaviy toklarning isrofi eksperimentdan olingan natijalar bo‘yicha, ya’ni bevosita vattmetr ko‘rsatishidan yoki po‘latdagi solishtirma isroflarning egri chiziqlaridan aniqlanadi. Bunda tok va kuchlanishning ekvivalent sinusoidalarini kompleks usulda yozish va vektor diagrammalardan foydalanish mumkin.

Magnit o‘tkazgichdagi F_{m} oqimning amplituda qiymati ma’lum bo‘lsa, unga mos bo‘lgan kuchlanishning ta’sir etuvchi qiymati quyidagi formula bilan topiladi:

$$\underline{U}_F = j4{,}44\, fw\underline{\Phi}_m, \tag{6.7}$$

bunda f – chastota; w – o‘ramlar soni.

Nochiziq zanjirlarga doir masalalarda izlanayotgan kattaliklarni aniqlash uchun kubik tenglamalarni yechishga to‘g‘ri keladi.

$$x^3 + 3px + 2q = 0 \tag{6.8}$$

tenglama bitta haqiqiy ildizga ega

$$x = \sqrt[3]{-q + \sqrt{q^2 + p^3}} + \sqrt[3]{-q - \sqrt{q^2 + p^3}} \tag{6.9}$$

Quyidagi tenglama

$$y^3 + ay^2 + by + c = 0 \tag{6.10}$$

ya’ni o‘zgaruvchan

$$x = y + a/3, \text{ ni} \tag{6.11}$$

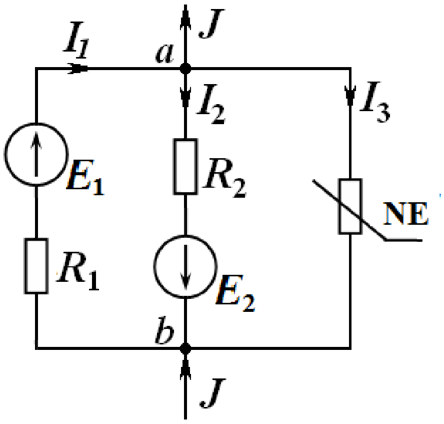
kiritib, (6.8) tenglama ko‘rinishiga keltiriladi, bunda

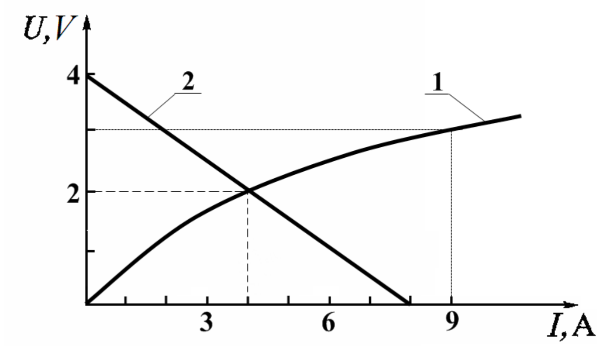
$$2q = \frac{2}{27}a^3 - \frac{1}{3}ab + c;\ 3p = b - \frac{1}{3}a^2. \tag{6.12}$$

6.2. O‘zgarmas tok nochiziq zanjirlarini hisoblash

6.2.1. Nochiziq zanjirlarni ekvivalent generatorlar usulida hisoblash

6.1-masala. Nochiziq element (NE) 6.1-rasmda ko‘rsatilganidek ulangan. NE ning voltmetr xaraktiristikasi 6.1- rasm, *b* da (egri chiziq) tasvirlangan. Sxema parametrlari E_1=18 V; E_2=6 V; J=1 A; R_1=3 Ω; R_2=6 Ω. Barcha toklar aniqlansin.





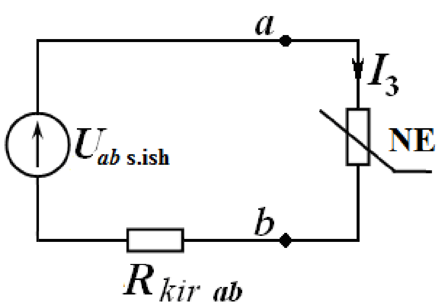
a) *b)*

6.1- rasm.

Yechish. NE dagi tokni ekvivalent generator usulida aniqlaymiz. Sxemada NE ni fikran uzamiz va ekvivalent generator parametrlari $U_{ab.\text{s.ish.}}$ va $R_{\text{kir}\,ab}$ larni aniqlaymiz. Ularni ikki tugun usuli bo‘yicha topamiz:

$$\mathrm{U}_{\text{ab s.ish.}} = \frac{E_1G_1 - E_2G_2 - J}{G_1 + G_2} = 8\ \mathrm{V}; \quad R_{\text{kir}\,ab} = \frac{R_1R_2}{R_1 + R_2} = 2\ \Omega.$$

Generator sxemasi 6.2- rasmdagi ko‘rinishga keltiriladi.



6.2- rasm.

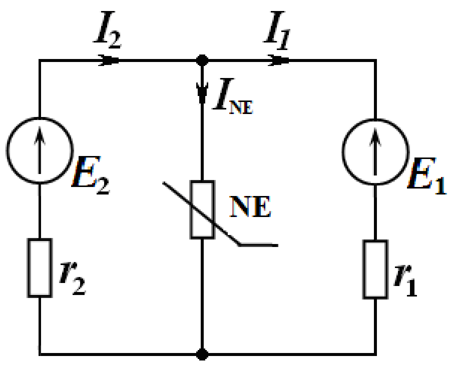
NE ishchi rejimini uning VAX 6.1-rasm, *b* da (1 egri chiziq)

$U_{\text{NE}} = U_{ab\,\text{s.ish.}} - I_3 R_{\text{kir}\,ab} = 8 - 2I_3$ tenglama bo‘yicha qurilgan yuklama chizig‘i bilan kesishgan nuqtadan foydalanib topamiz. Natijada quyidagilarni hosil qilamiz: I_3=3 A, $U_{ne}=U_{ab}=2$ V; Boshqa shoxobchalardagi toklarni Om qonunidan aniqlaymiz:

$$\text{I}_1 = (E_1 - U_{ab})G_1 = 5{,}33\,\text{A};$$
$$\text{I}_2 = (E_2 - U_{ab})G_2 = 1{,}33\,\text{A}.$$

Haqiqatan ham, Kirxgofning birinchi qonuni bo‘icha $I_1 - I_2 - I_2 - J = 0$.

6.2- masala. Nochiziq element (NE) ichki qarshiliklari r_1 va r_2 bo‘lgan bir-biriga parallel ulangan ikkita EYK $E_1 = 11{,}56\,\text{V}$ va $E_2 = 16{,}56\,\text{V}$ manbalarning qismalariga ulangan (6.3- rasm). Agar r_1=2,5 Ω, r_2=6 Ω, $R_{\text{NE}} = 30\,\Omega$ bo‘lsa, shoxobchalardagi toklarni aniqlang.



6.3- rasm.

Yechish. Kirxgofning birinchi qonuni bo‘yicha

$$I_2 = I_1 + I_{\text{NE}} \quad . \tag{6.13}$$

Kirxgofning ikkinchi qonuni bo‘yicha ikkita tenglama tuzamiz:

$$E_2\text{-}E_1 = I_1 r_1 + I_2 r_2; \tag{6.14}$$

$$E_2 = I_{\text{NE}} R_{\text{NE}} + I_2 r_2 \tag{6.15}$$

(6.13) – (6.15) tenglamalarga berilgan qiymatlarni qo‘yamiz va sistemani yechib quyidagilarni topamiz:

$$I_1\text{=0,3 A; } I_2\text{=0,71 A; } I_{NE} = 0{,}41\,\text{A}.$$

E_1 – manba istimolchi sifatida E_2 – generator sifatida ishlaydi, ya’ni quvvatlar balansi tuzulganda E_1 EYK quvvati manfiy bo‘ladi:

$$E_2 I_2 \text{-} E_1 I_1 = I_1^2 r_1 + I_2^2 r_2 + I_3^2 R_{NE};$$
$$16{,}56 \cdot 0{,}71 - 11{,}56 \cdot 0{,}3 = (0{,}3)^2 \cdot 2{,}5 + (0{,}71)^2 \cdot 6 + (0{,}41)^2 \cdot 30.$$

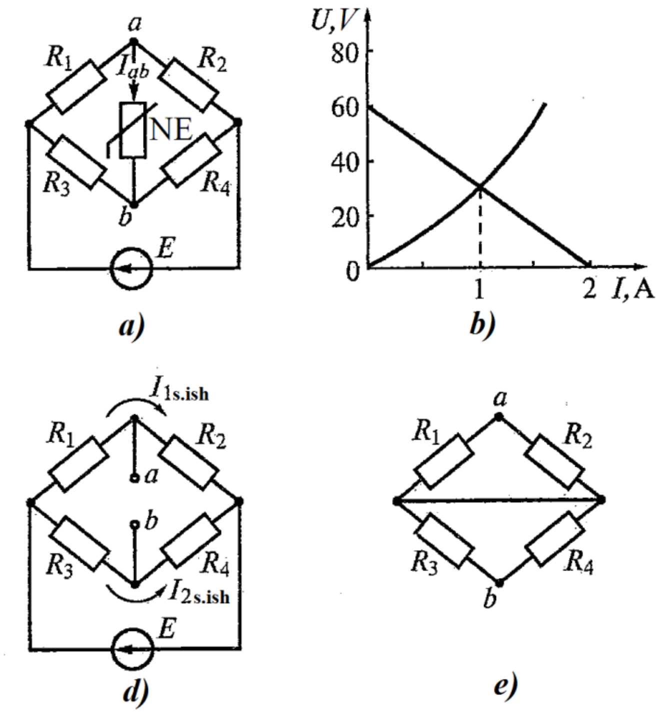
8,29 W≈8,29 W.

Manba qismlaridagi kuchlanish pasayishi

$$U = I_{NE} R_{NE} = 0.41 \cdot 30 = 12.3 \text{ V}.$$

6.3- masala. 6.4- rasm, *a* da keltirilgan zanjirdagi nochiziq elementdan o'tayotgan tok topilsin. Bunda $R_1 = 30\ \Omega$, $R_2 = 60\ \Omega$, $R_3 = 20\ \Omega$, $R_4 = 20\ \Omega$, $E = 360\ \text{V}$ bo'lib, NE VAXsi 6.4- rasm, *b* da berilgan.

Yechish. Ekvivalent generator usulidan foydalanmiz.



6.4- rasm.

1. $U_{ab.\text{s.ish.}}$ kuchlanishni topish uchun 6.4- rasm, *d* dagi sxemadan foydalanamiz.

$$I_{1\text{s.ish}} = \frac{E}{R_1 + R_2} = \frac{360}{30 + 60} = 4 \text{ A}; \qquad I_{2\text{s.ish.}} = \frac{E}{R_3 + R_4} = \frac{360}{20 + 20} = 9 \text{ A};$$

$$U_{\text{abs.ish.}} = I_{2\text{s.ish.}} R_3 - I_{1\text{s.ish.}} R_1 = 9 \cdot 20 - 4 \cdot 30 = 60 \text{ V}.$$

2. Ekvivalent generatorning ichki qarshiligini topish uchun 6.4-rasm, *e* dagi sxemadan foydalanamiz:

$$R_{\text{ekv}} = R_{\text{ich}} = \frac{R_1 \cdot R_2}{R_1 + R_2} + \frac{R_3 \cdot R_4}{R_3 + R_4} = 30\,\Omega.$$

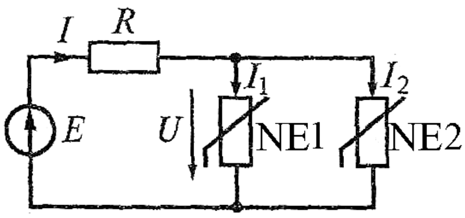
3. Ekvivalent generator tashqi xarakteristikasini qurish uchun zanjirdagi qisqa tutashish tokini quyidagicha aniqlaymiz:

$$I_{\text{q.t}} = \frac{E_{\text{ekv}}}{R_{\text{ekv}}} = \frac{U_{\text{abs.ish}}}{R_{\text{ich}}} = \frac{60}{30} = 2\text{ A}.$$

6.4- rasm, *b* da ekvivalent generator tashqi xarakteristikasi $E_{\text{ekv}} = 60\text{ V}$ da $I = 0$ va qisqa tutashuvda, ya'ni $U = 0$ da $I_{\text{q.t}} = 2$ A ($I_{\text{q.t}}$ – qisqa tutashish rejimidagi tok) nuqtalar asosida qurilgan. Bu xaratkeristikani NE VAX bilan kesishgan nuqtasining absissasi izlanayotgan tok qiymatiga teng bo'ladi, ya'ni $I_{ab} = 1\text{ A}$.

6.2.2. Nochiziq elektr zanjirlarini analitik usulda hisoblash

6.4-masala. 6.5-rasmda keltirilgan zanjir uchun quyidagilar berilgan: $E = 30\text{ V}$; $R = 20\,\Omega$ NE1 va NE2 larning VAXlari mos ravishda $I_1(U) = a_1 U + a_2 U^2$ va $I_2(U) = b_1 U + b_2 U^2$ tenglamalar ko'rinishida berilgan, bu yerda $a_1 = 0{,}1$ A/V; $a_2 = 0{,}003$ A/V^2; $b_1 = 0{,}04$ A/V; $b_2 = 0{,}002$ A / ^{2} Nochiziq elementlardagi kuchlanish va tok topilsin.



6.5- rasm.

Yechish. Berilgan zanjir uchun Kirxgofning birinchi va Om qonunlari asosida quyidagilarni yozamiz:

$$I = I_1 + I_2, \quad I = \frac{E - U}{R},$$

chunki Kirxgofning qonuniga ko‘ra $IR + U = E$.

Yuqoridagi 1-tenglamaga $I_1(U)$ va $I_2(U)$ ning ifodalarini qo‘yib va ikkala tenglamaning o‘ng tomonlarini o‘zaro tenglashtirib quyidagini hosil qilamiz:

$$[(E - U / R)] = (a_2 + b_2)U^2 + (a_1 + b_1)U,$$

yoki berilgan qiymatlarni qo‘ysak: $0{,}005U^2 + 0{,}1U - 1{,}5 = 0.$ Bundan $U = 10$ V.

O‘zaro parallel ulangan nochiziq elementlardagi kuchlanishning topilgan qiymatini ularning VAXlari tenglamalariga qo‘yib, toklarni topamiz:

$$I_1 = 0{,}1 \cdot 10 + 0{,}003 \cdot 10^2 = 0{,}4 \text{ A},$$
$$I_2 = 0{,}04 \cdot 10 + 0{,}002 \cdot 10^2 = 0{,}6 \text{ A}.$$

6.2.3. Nochiziq elektr zanjirlarini chiziqlashtirish (linearizatsiyalash) usuli yordamida hisoblash

6.5-masala. 6.6-rasm, *a* dagi sxema uchun nochiziq element va chiziqli qarshilikdagi kuchlanish pasayishlari hamda zanjirdan o‘tayotgan tok topilsin. Quyidagilar berilgan: $U = 230$ V; $R_1 = 190\,\Omega$, NE VAXsi 6.6-rasm, b da keltirilgan.

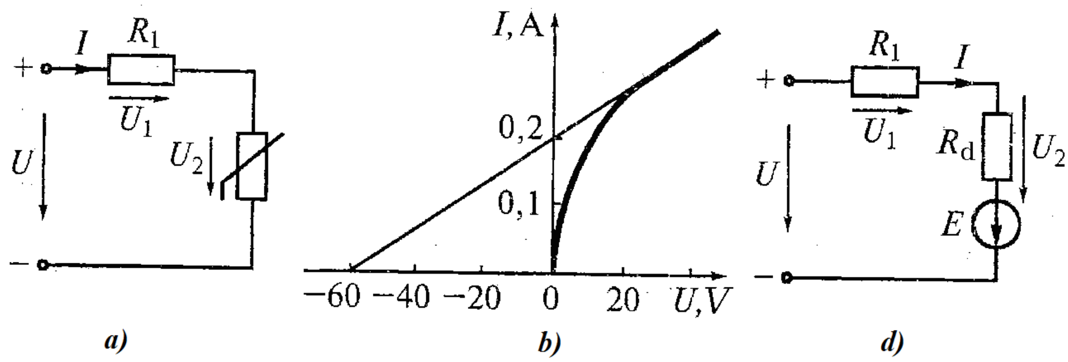
Yechish. NE VAX ishchi qismini to‘g‘ri chiziq bilan almashtiramiz (6.6-rasm, *b*). Natijada quyidagini hosil qilamiz:

$$U_2 = E + IR_\text{d}$$

bu yerda, $E = -60$ V; $R_d = \dfrac{\Delta U}{\Delta I} = \dfrac{60}{0.2} = 300\ \ \Omega.$

Yuqoridagi almashtirish natijasida 6.6- rasm, *d* da ko‘rsatilgan chiziqli zanjirni hosil qilamiz. Kirxgofning 2 -qonuniga ko‘ra

$$U = U_1 + U_2 = IR_1 + E + IR_\text{d} = 190I - 60 + 300I = 490I - 60.$$



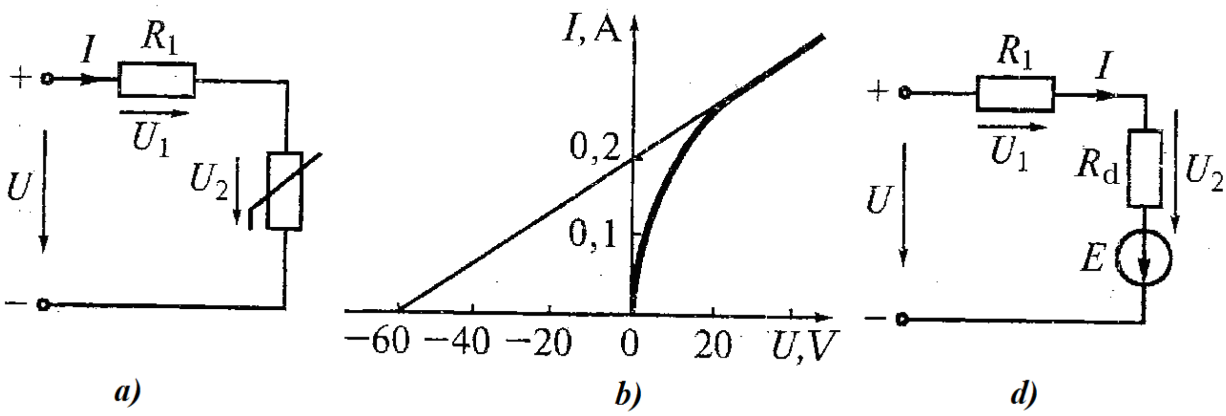
6.6- rasm.

Bundan

$$I = \frac{U+60}{490} = \frac{230+60}{490} \approx 0{,}6;\quad U_1 = IR_1 = 0{,}6 \cdot 190 = 114\ \text{V};$$
$$U_2 = -60 + I \cdot 300 = -60 + 0{,}6 \cdot 300 = 120\ \text{V}.$$

6.2.4. Nochiziq elektr zanjirlarini iteratsiya usuli yordamida hisoblash

6.6- masala. 6.7- rasm, *a* da berilgan sxemadagi ikkita nochiziq elementning VAXlari 6.7- rasm, *b* da keltirilgan. Ulardagi tok masshtabi $m_I = 0{,}1$ A/mm,



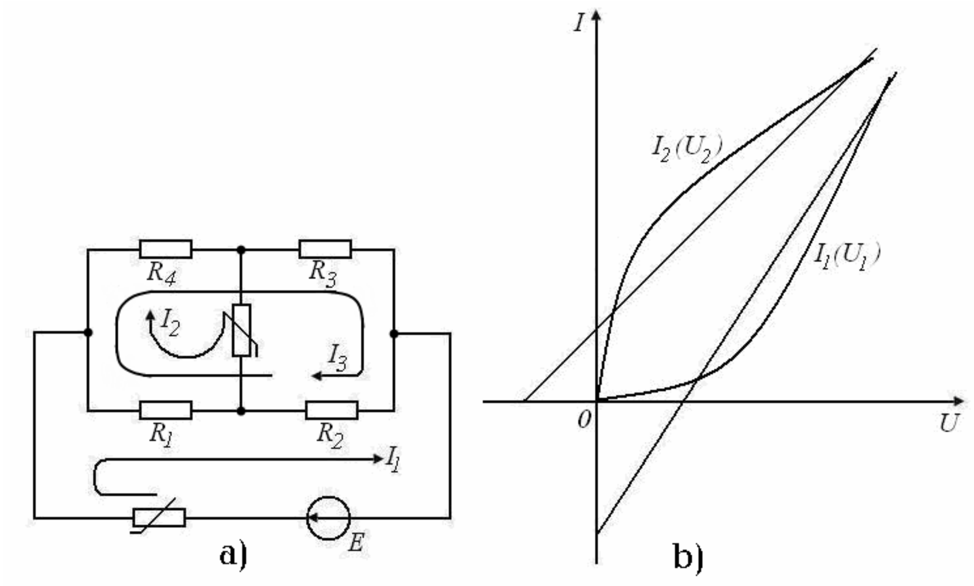
kuchlanish masshtabi esa $m = 0{,}5$ V/mm.

Agar E=20 V, R_1=4 Ω, R_2=1 Ω, R_3=3 Ω, R_4=2 Ω bo‘lsa, nochiziq elementlardagi tok va kuchlanishlarni iteratsiya usuli yordamida aniqlang.

Yechish. Zanjirdagi kontur toklar shunday tanlanganki, nochiziq elementlardagi toklar bevosita kontur toklarga teng. Sxema uchun Kirxgofning 2 - qonuni asosida tenglamalar sistemasini quyidagicha tuzamiz:

$$(R_1+R_2)I_{11}-R_1I_{22}-(R_1+R_2)I_{33}=E-U_1;$$
$$-R_1I_{11}+(R_1+R_4)I_{22}+(R_1+R_4)I_{33}=-U_2;$$
$$-(R_1+R_2)I_{11}+(R_1+R_4)I_{22}+(R_1+R_2+R_3+R_4)I_{33}=0.$$

bu yerda I_{11}, I_{22}, I_{33} – kontur toklar.



6.7- rasm.

Oxirgi tenglamadan I_{33} tokni topib, uni 1- va 2- tenglamalarga qo‘yamiz va natijada quyidagi 2 ta tenglamani hosil qilamiz:

$$\frac{(R_1+R_2)(R_3+R_4)}{\sum R}I_{11}-\frac{R_1R_3-R_2R_4}{\sum R}I_{22}=E-U,$$
$$-\frac{R_1R_3-R_2R_4}{\sum R}I_{11}+\frac{(R_1+R_4)(R_2+R_3)}{\sum R}I_{22}=-U_2.$$

Chiziqli qarshiliklar va EYK qiymatlarini qo‘yamiz:

$$2{,}5I_{11}-I_{22}=20-U_1,$$
$$-I_{11}+2{,}4I_{22}=-U_2.$$

Iteratsiya usuliga ko‘ra yechimga birinchi yaqinlashishni topish uchun nochiziq elementlar VAX larini chiziqli qarshiliklar va EYK lar bilan almashtiramiz va grafiklardan $E_1=6\,\text{V}$ va $R_{d1}=3\ \Omega$, $E_2=25\,\text{V}$, $R_{d2}=4\ \Omega$ ekanligiga ishonch hosil qilamiz. Linearizatsiyalangan NE larda tok va kuchlanishlar quyidagi tenglamalar bilan bog‘langan:

$$U_1 = 3I_1 + 6;$$
$$U_2 = 4I_2 - 2{,}5.$$

Bu ifodalarni Kirxgofning 2- qonuni asosida tuzilgan tenglamalarga qo‘yib toklarning 1- taqribiy qiymatlarini topamiz: $I_1 = 2{,}7$ A, $I_2 = 0{,}81$ A.

Toklarning topilgan qiymatlariga VAX lardan $U_1 = 0{,}8$ V va $E_1 = 6$ V kuchlanishlar mos keladi. Ularni yana tenglamalar sistemasiga qo‘yib, I_1 va I_2 toklarning yangi qiymatlarini topamiz va h.k. Ketma-ket yaqinlashish natijalari quyidagi jadval ko‘rinishida berilgan:

Yaqinlashish tartibi	I_1	U_1	I_2	U_2
1	2,7	16	0,81	0,8
2	2,46	15,4	0,8	0,722
3	2,37	15,2	0,78	0,646
4	2,31	15	0,72	0,593
5	2,3	14,9	0,712	0,588

Jadvaldagi toklar va kuchlanishlar qiymatlaridan ko‘rinib turibdiki, 4- yaqinlashishdan keyin ularning qiymatlari qaytirila boshladi. Shunday qilib, berilgan zanjir nochiziq elementlaridagi tok va kuchlanishlar quyidagiga teng:

$$U_1 = 14{,}9\,\text{V}, \quad I_1 = 2{,}3\text{A},$$
$$U_2 = 0{,}588\,\text{V}, \quad I_2 = 0{,}715\text{A}.$$

6.2.5. Shoxobchalangan magnit zanjirlarini hisoblash

Shoxobchalangan magnit zanjirlarini hisoblash tartibini 6.8- rasm, *a* da keltirilgan zanjir misolida ko‘rib chiqamiz.

6.7- masala. Berilgan magnit zanjiri shoxobchalaridagi magnit oqimlarining qiymatlari aniqlansin. Quyidagilar berilgan:

$I_1 = 2$ A, $W_1 =$ 500 o‘ram, $l_1 = 30$ sm, $S_1 = 4$ mm^2,

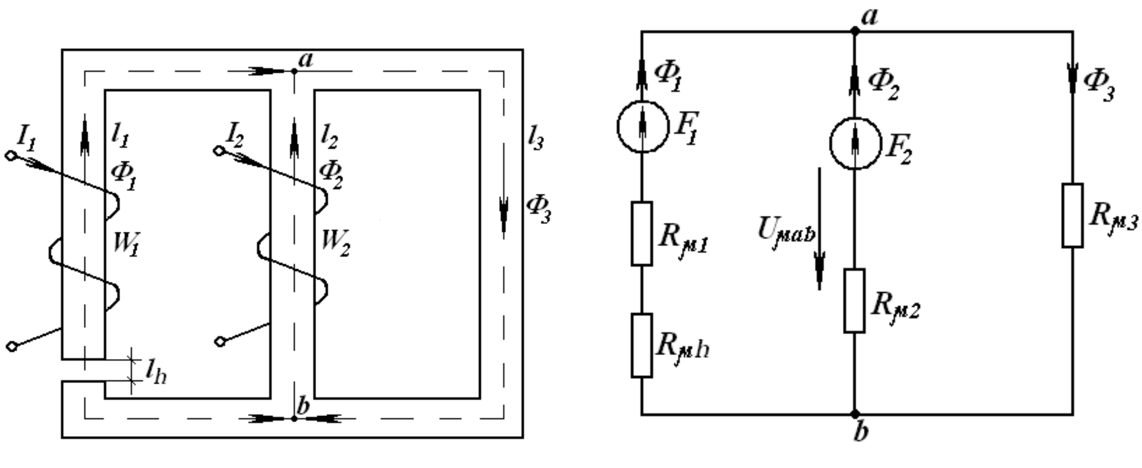
$I_2 = 5$ A, $W_2 =$ 150 o‘ram, $l_2 = 20$ sm, $S_2 = 6$ mm^2,

$l_3 = 30$ sm, $S_3 = 4$ mm^2, $l_h = 1$ mm.

Ferromagnit o‘zakning magnit xossasi quyidagi jadval ko‘rinishda berilgan va magnitlanish egri chizig‘i bilan ifodalanadi:

V, Tl	0	0,22	0,75	0,93	1,02	1,14	1,28	1,47	1,53	1,57
N, A/m	0	20	40	60	80	120	200	400	600	800

Yechish. 1. Berilgan zanjirni quyidagi ekvivalent sxema ko‘rinishida tasvirlaymiz (6.8- rasm, *b*).



a) *b)*

6.8- rasm.

2. $U_{\mu ab}$ kuchlanishni o‘zak qismlaridagi MYK va magnit qarshiligi orqali aniqlaymiz:

$U_{\mu ab} = F_1 - H_1 l_1 - H_h l_h$ – o‘zakning chap qismi uchun,

$U_{\mu ab} = F_2 - H_2 l_2$ – o‘zakning o‘rtadagi qismi uchun,

$U_{\mu ab} = H_3 l_3$ – o‘zakning o‘ng qismi uchun.

3. Har bir o‘zak qismi uchun magnit induksiyasiga ixtiyoriy qiymatlar berib

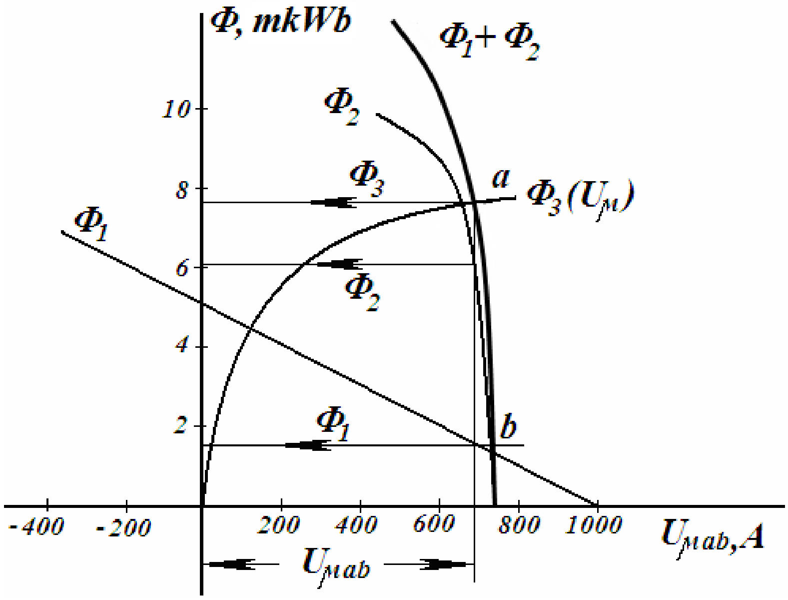
$\varPhi = BS$ formula yordamida o‘zak qismlaridagi magnit oqimlarini hisoblaymiz.

4. Magnitlanish egri chizig‘idan zanjirning ferromagnit qismidagi kuchlanganliklarni aniqlaymiz. Havo oralig‘idagi kuchlanganlikni $H_h = 8 \cdot 10^3 \, \frac{\text{A}}{\text{m}}$ formula yordamida hisoblaymiz.

5. O‘zakning har bir qismi uchun magnit kuchlanishlarni 2- bandda keltirilgan formulalardan foydalanib topamiz.

Hisoblash natijalarini jadval ko‘rinishida yozamiz.

6. Bitta koordinatalar sistemasiga $\varPhi_1 = f(U_{\mu ab})$, $\varPhi_2 = f(U_{\mu ab})$, $\varPhi_3 = f(U_{\mu ab})$ egri chiziqlar grafiklarini quramiz hamda $\varPhi_1$ va $\varPhi_2$ grafiklar ordinatalarini o‘zaro qo‘shish natijasida $\varPhi_1 + \varPhi_2 = f(U_{\mu ab})$ ni hosil qilamiz (6.9- rasm).



6.9- rasm.

7. Magnit zanjir uchun Kirxgofning 1- qonuni $\varPhi_1 + \varPhi_2 = \varPhi_3$ bajariladigan *a* nuqtani topamiz va undan abssissa o‘qiga *ab* perpendikular tushiramiz. Bu perpendikularning abssissa o‘qi bilan kesishgan nuqtasi berilgan zanjir uchun $U_{\mu ab}$ kuchlanishning haqiqiy qiymatiga teng bo‘ladi. To‘g‘ri chiziqning magnit oqimlari grafiklari bilan kesishgan nuqtalari esa magnit oqimlarining haqiqiy qiymatlarini beradi:

$\Phi_1 = 1{,}3$ mkWb, $\Phi_2 = 6$ mkWb, $\Phi_3 = 7{,}3$ mkWb.

6.3. O‘zgaruvchan tok nochiziq zanjirlarini hisoblash

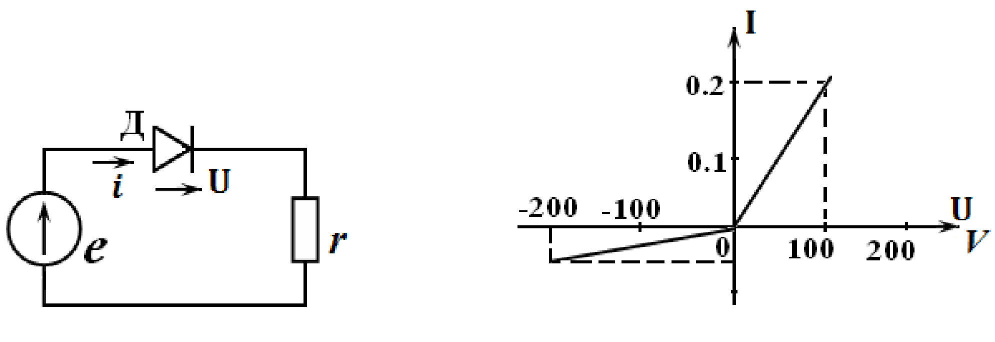
6.3.1. Hisoblashning grafik va grafo-analitik usullari

6.8-masala. Sinusoidal tok manbasiga EYK $e = 150\sin\omega t$, qarshiligi $r = 1$ kΩ li rezistor va volt-amper xarakteristikasi 6.10-rasm, *b* da tasvirlangan diod *D* orqali ulangan. Tok $i(t)$ grafigi qurilsin, zanjirdagi tokning o‘rta qiymati I_0 va ta’sir etuvchi I qiymati hamda manbaning *R* quvvati va rezistordagi R_r quvvat isrofi topilsin.

Yechish. Hisoblashda diodning ekvivalent r_ekv qarshiligi tokning to‘g‘ri va teskari yo‘nalishida har xil bo‘lishini e’tiborga olamiz. Zanjirning ekvivalent sxemasi 6.10- rasm, *a* da keltirilgan.

Diodning ekvivalent qarshiligi 6.10-rasm, *b* dagi xarakteristikadan aniqlanadi: to‘g‘ri yo‘nalishda $100/0{,}2 = 500\,\Omega$ va teskari yo‘nalishda $200/0{,}5 = 400\,\Omega$

Zanjirdagi to‘g‘ri yo‘nalishdagi i_1 va teskari yo‘nalishdagi i_2 toklarning amplituda qiymati $I_m = E_m/(r + r_\text{ekv})$ va mos ravishda to‘g‘ri yo‘nalishda 0,1 A va teskari yo‘nalishda 0,03 A ga teng. $i(t)$ grafigi 6.10- rasm, *b* da keltirilgan.



a) b)

6.10- rasm.

Tokning o‘rta qiymati $I_0 = I_{01} - I_{02}$, bunda $I_{01} = I_{m1}/\pi$; $I_{02} = I_{m2}/\pi$.

Shunday qilib, $I_0 = (0{,}1 - 0{,}03)/\pi = 0{,}0222$ A.

Tokning ta’sir etuvchi qiymati

$$I = \sqrt{\frac{1}{2\pi}\left(\int_{m1}^{\pi} I_{m1}^2 \sin^2 \omega t \cdot d(\omega t) + \int_{\pi}^{2\pi} I_{m2}^2 \sin^2 \omega t \cdot d(\omega t)\right)} = \frac{1}{2}\sqrt{I_{m1}^2 + I_{m2}^2} = \frac{1}{2}\sqrt{0{,}1^2 + 0{,}03^2} = 0{,}05 \text{ A.}$$

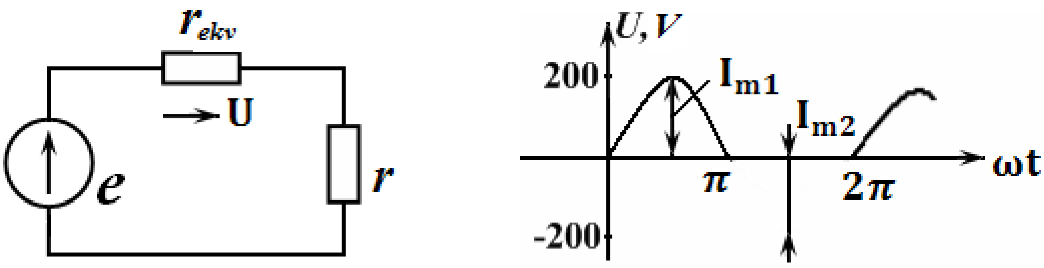
Manbaning quvvati

$$P = \frac{1}{2\pi}\int_0^{2\pi} eid(\omega t) = \frac{1}{2\pi}\left(\int_0^{\pi} ei_1 d(\omega t) + \int_{\pi}^{2} ei_2 d(\omega t)\right) = \frac{E_m}{4}(I_{m1} + I_{m2}) = \frac{150}{4}(0{,}1 + 0{,}03) = 4{,}9 \text{ W.}$$

Rezistordagi quvvat isroflari

$$P_r = rI^2 = 1000 \cdot 0{,}052^2 = 2{,}7 \text{ W.}$$

6.9- masala. Qarshiligi $r = 1$ kΩ rezistordan va unga ketma-ket ulangan ideal dioddan iborat zajirga (6.11- rasm, *a*) $u = 200\sin\omega t$ V kuchlanish (6.11- rasm, *b*) berilgan. Zanjirdagi tokning o‘rta va ta’sir etuvchi qiymatlarini hamda rezistordagi quvvat isroflarini toping.

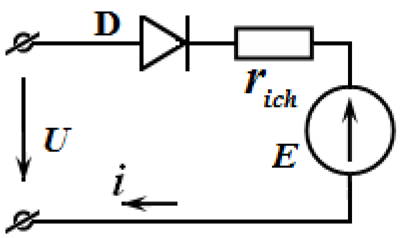


a) *b)*

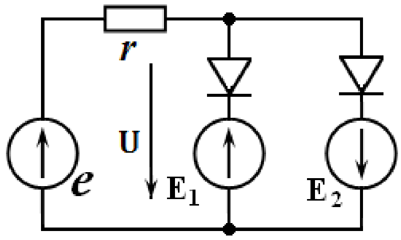
6.11- rasm.

6.10-masala. EYK *E*=12V va ichki qarshiligi $r = 6$ Ω bo‘lgan akkumulyator ideal diod orqali amplitudasi 24 V li sinusoidal kuchlanish manbasiga ulangan (6.12-rasm). Tokning maksimal va o‘rta qiymatlari hamda dioddagi maksimal teskari kuchlanish aniqlansin.

6.11- masala. 6.13- rasmda tasvirlangan zanjir uchun $r = 100$ Ω, $e = 20\sin 1000\,t$ V, $E_1 = 5$ V, $E_2 = 10$ V. Diodlar ideal. Kuchlanish U ning musbat va manfiy impulslarining front uzunligini aniqlang.

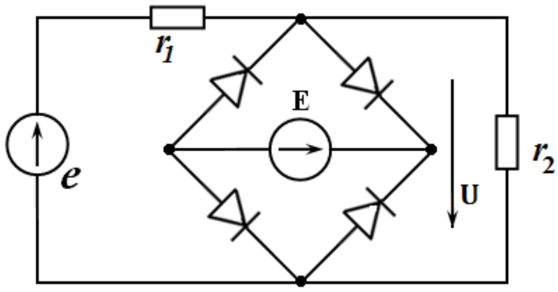


6.12- rasm.



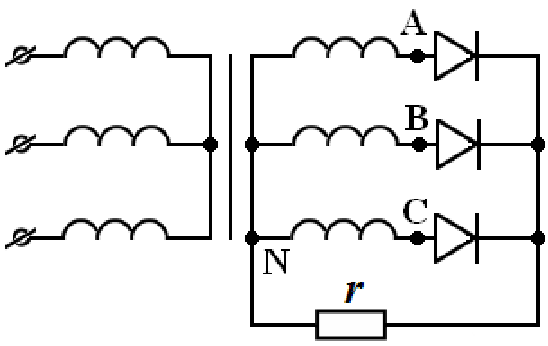
6.13- rasm.

6.12- masala. 6.14- rasmda tasvirlangan zanjirda $r_1 = 100\ \Omega$, $r_2 = 400\ \Omega$, $e = 100\sin 1000t$; $E = 20\,\text{V}$. Diodlar ideal. Kuchlanish U ni impulslar frontining davomiyligi aniqlansin.



6.14- rasm.

6.13- masala. To‘g‘rilagich o‘rta no‘qtali uch fazali sxema bo‘yicha yig‘ilgan (6.15- rasm). Yuklama qarshiligi $R = 20\ \Omega$, ikkilamchi faza kuchlanishi 220 V. Diodlar ideal. Tokning o‘rta va ta’sir etuvchi qiymati hamda rezistordagi quvvat isroflarini aniqlang.



6.15- rasm.

6.14-masala. Magnitlanish egri chizig‘i $H = 100\ \text{B} + 200\,\text{B}^3$ polinom bilan approksimatsiyalangan po‘lat magnit o‘tkazgichga o‘ramlar soni 500 ta bo‘lgan g‘altak o‘rnatilgan, bunda magnit kuchlanganlik – amper/metr, magnit induksiya o‘lchov birligi Teslada. Magnit o‘tkazgichning ko‘ndalang kesimi $S = 10^3\text{m}^3$, magnit liniyaning o‘rta uzunligi $l = 0{,}5\,\text{m}$. G‘altak garmonik kuchlanish manbayi $u = 110\sqrt{2}\cos\omega\text{t}, \text{V}$ ga ulangan. Agar *a) f*=50 Hz; *b) f*=100 Hz bo‘lsa, g‘altakdagi tokning oniy va ta’sir etuvchi qiymatlari aniqlansin.

6.15-masala. Parametrlari 6.14-masalada berilgan g‘altakka parallel ravishda sig‘imi 4 mkF li kondensator ulangan (6.16-rasm). Zanjir chastotasi 50 Hz li sinusoidal kuchlanish manbasiga qo‘shilgan. Kuchlanishning qaysi qiymatida zanjirda birinchi garmonika chastotasida tok rezonansi sodir etiladi? Bu rejimda barcha shoxobchalardagi toklarning ta’sir etuvchi qiymati topilsin.

Yechish. Berilgan N(V) egri chiziq bo‘yicha $i_l(\psi)$ ifodasini tuzamiz, bunda ψ – magnit ilashlilik va

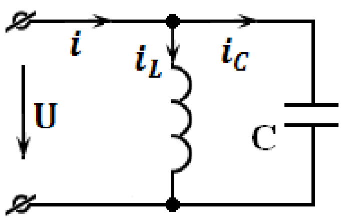
$$B = \frac{\psi}{\omega S} = \frac{\psi}{500 \cdot 10^{-3}} = 2\psi; \quad H = \frac{\omega\, i}{L} = \frac{500}{0{,}5} i_L = 1000 i_L.$$

Bu ifodalarni 6.14- masaladagi magnitlanish egri chizig‘i ifodasiga qo‘yib, quyidagini olamiz:

$$i_L = 0.2\psi + 1.6\psi^3 \qquad (6.16)$$

bunda tok amperlarda, magnit ilashlilik – veberlarda. Tok rezonansida konturdagi umumiy i ning 1 - garmonikasi nolga teng bo‘lishi kerak.

$$i_1 = i_{L1} + i_{C1} = 0 \qquad (6.17)$$



6.16- rasm.

Sinusoidal u kuchlanishda g‘altakning magnit ilashiligi ham sinusoidal, chunki $u = d\psi / dt$ va nolli boshlang‘ich faza bilan tanlab olinishi mumkin

$$\psi = \psi_m \sin\omega\, t. \qquad (6.18)$$

G‘altakdagi tokni (6.16) tenglamadan aniqlaymiz.

$$i_L = (0{,}2\psi_m + \frac{3}{4}1{,}6\psi_m^3)\sin\omega\, t - \frac{1}{4}1{,}6\psi_m^3 \sin 3\psi\, t = \\ = (0{,}2\psi_m + 1{,}2\psi_m^3)\sin\omega\, t - 0{,}4\psi_m^3 \sin 3\omega\, t. \qquad (6.19)$$

Kondensatordagi tok:

$$i_C = CdU/dt = Cd^2\psi/dt^2 = -\omega^2 C\psi_m \sin\omega\, t = -0{,}394\psi_m \sin\omega\, t\,. \qquad (6.20)$$

Rezonans (6.17) shartidan quyidagini olamiz:

$$(0{,}2\psi_m + 1{,}2\psi_m^3)\sin\omega\, t - 0{,}394\;\psi_m \sin\omega \mathrm{t} = 0\,,$$

bunda: *a)* $\psi_m = 0$ – ma'lum yechimi yoki *b)* $1{,}2\psi_m^2 + 0{,}2 - 0{,}394 = 0$ va $\psi_m = 0{,}402\,\mathrm{Wb}$.

Kuchlanish $u = d\psi/dt = \omega\psi_m \cos\omega\, t,$ shuning uchun tok rezonansidagi kuchlanishning ta'sir etuvchi qiymati:

$$U = \omega\psi_m / \sqrt{2} = 314 \cdot 0{,}402/\sqrt{2} = 89\,\mathrm{V}\,.$$

G'altakdagi tokning oniy qiymati (6.19) aniqlanadi:

$$i_L = 0{,}1584 \sin\omega\,\mathrm{t} - 0{,}026 \sin 3\omega\,\mathrm{t},\,\mathrm{A}.$$

G'altakdagi tokning ta'sir etuvchi qiymati:

$$I_L = \sqrt{0{,}1584^2/2 + 0{,}026^2/2} = 0{,}114\,\mathrm{A}.$$

Kondensatordagi tokning ta'sir etuvchi qiymati:

$$I_C = 0{,}394\psi_m / \sqrt{2} = 0{,}394 \cdot 0{,}402/\sqrt{2} = 0{,}112\,\mathrm{A}.$$

Manba tokining oniy qiymati:

$$i = i_L + i_C = -0{,}4\psi_m^3 3\omega\, t = -0{,}026 \sin 3\omega\,\mathrm{t},\,\mathrm{A}.$$

Manba tokining ta'sir etuvchi qiymati:

$$I = 0{,}026/\sqrt{2} = 0{,}0184\,\mathrm{A}.$$

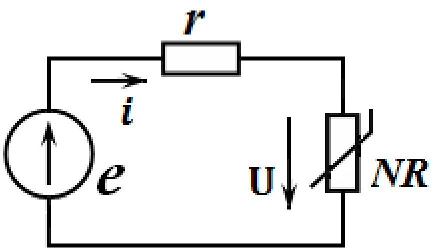
6.16-masala. O'ramlar soni 100 ta bo'lgan g'altak po'lat magnit o'tkazgichga o'rnatilgan va unga sig'imi 8 mkF li kondensator ketma-ket ulangan. Shu zanjir sinusoidal tok $i = I_m \sin 1000t$ manbayiga qo'shilgan. Magnit oqimni g'altak tokiga bog'lanishi $\Phi = 2 \cdot 10^{-3} i - 10^{-3} i^3$ polinom bilan approksimatsiyalangan, bunda oqim – veberda, tok – amperda. Tok amplitudasini o'zgartirib birinchi garmonika chastotasida rezonansga sozlangan. Rezonans rejim tokining qanday amplitudasida sodir bo'lishligini aniqlang, bunda zanjir kirishidagi, g'altakdagi, kondensatordagi kuchlanishlarni toping.

6.17-masala. Zanjir ketma-ket ulangan chiziqli va nochiziq rezistorlardan (NR) hamda EYK $e = E_m \sin\omega t$ manbayidan iborat bo'lib, bunda

$E_m = 120$ V; $\omega = 314$ rad/s. (6.17-rasm) chiziqli rezistor qarshiligi $r = 40\,\Omega$, nochiziq rezistorning volt-amper xarakteristikasi

$$u(i) = ai + bi^3 \qquad (6.21)$$

ifoda bilan approksimatsiyalangan, bunda $a = 50\,\Omega; b = 40$ V/A^3 Tokning birinchi garmonikasini: 1) garmonik balans; 2) garmonik chiziqlashtirish usullari yordamida aniqlang.



6.17- rasm.

Yechish. 1. *Garmonik balans usuli.* Kirgofning ikkinchi qonuni bo‘yicha zanjirning tenglamasi:

$$(r + a)i + b\,i^3 = e. \qquad (6.22)$$

Tokni garmonik funksiya $i = I_m \sin(\omega\,\mathrm{t} - \varphi)$ bilan ifodalaymiz. Tok amplitudasi I_m ni va EYK ga nisbatan faza siljishi φ ni aniqlash kerak.

Keyingi o‘zgartirishlarni soddalashtirish maqsadida $\omega t - \varphi = 0$ deb belgilaymiz. Unda EYK $e = E_m \sin(\theta + \varphi)$.

Izlanayotgan tokning umumiy ifodasini zanjir tenglamasi (6.22) ga qo‘yamiz va EYK ifodasidagi sinus va kosinus tashkil etuvchilarini ajratamiz va $\sin^3\alpha = \frac{1}{4}(3\sin\alpha - \sin 3\alpha)$ ni e’tiborga olib quyidagini hosil qilamiz:

$$(r + a)I_m \sin\theta + \frac{1}{4} b\,I_m^3 (3\sin\theta - \sin 3\theta) = E_m \cos\varphi \cdot \sin\theta + E_m \sin\varphi \cdot \cos\theta.$$

Bu tenglikning chap va o‘ng qismlaridagi bir xil nomlangan trigonometrik funksiyalardagi koeffitsiyentlarni tenglashtirib, quyidagilarni olamiz:

$$\theta = E_m \sin\varphi; \qquad (6.23)$$

$$(r+a)I_m + \frac{3}{4} b I_m^3 = E_m \cos\varphi. \qquad (6.24)$$

(6.23) tenglamadan kelib chiqadiki, $\sin\varphi = 0$; $\varphi = 0$; $\cos\varphi = 1$. (6.24) tenglamaga berilgan qiymatlarni qo‘yib, quyidagini olamiz:

$$(40+50)I_m + \frac{3}{4} \cdot 40 I_m^3 = 120,$$

va uni $I_m^3 + 3I_m - 4 = 0$ ko‘rinishga keltirib, (6.9) formula bo‘yicha yechamiz, bunda $p = 1, q = -2$;

$$I_m = \sqrt[3]{2+\sqrt{4+1}} + \sqrt[3]{2-\sqrt{4+1}} = 1{,}62 - 0{,}62 = 1\,\text{A}, \qquad i = 1\sin\omega t, \text{A}.$$

2. *Garmonik chiziqlashtirish usuli.* Tok ifodasi $i = I_m \sin(\omega\,\text{t} - \varphi) = I_m \sin\theta$ bo‘lganda I_m va φ ni aniqlash kerak.

Nochiziq rezistor 1-garmonikasining qarshiligini aniqlaymiz,

$$R_\text{N}(I_m) = U_{1m}(I_m)/I_m,$$

bunda (6.21) ifoda bo‘yicha $U_{1m} = (50I_m + 30I_m^3)$ nochiziq element kuchlanishi 1 – garmonikasining ampilitudasi.

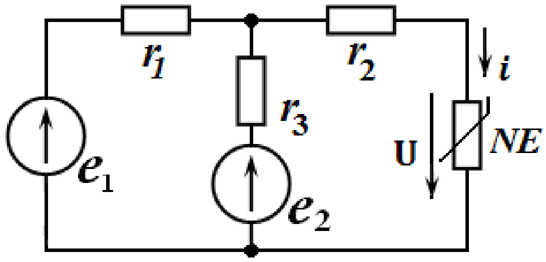
Kompleks shakldagi qarshilik $\underline{Z}_\text{N}(I_m) = R_\text{N}(I_m) = 50 + 30I_m^2$, chunki rezistordagi kuchlanish va tok birinchi garmonikalarining faza siljishlari mos keladi. Zanjir uchun Kirxgofning ikkinchi qonuni bo‘yicha kompleks shaklda tenglama tuzamiz: $[r + \underline{Z}_\text{N}(I_m)]\underline{I}_m = \underline{E}_m$, bunda $\underline{E}_m = \underline{E}_m$; $\underline{I}_m = I_m < -\varphi -$ kompleks kattaliklar oxirgi ifodaga son qiymatlarni qo‘ygandan so‘ng,

$$(40 + 50 + 30I_m^2)I_m e^{-j\varphi} = 120.$$

Bu ifoda tenglamaning o‘ng qismi haqiqiy bo‘lgani uchun $\varphi = 0$ va tenglama $I_m^3 + 3I_m - 4 = 0$ shaklga keltiriladi, bundan $I_m = 1\,\text{A}$.

6.18-masala. 6.17-rasmdagi zanjir uchun rezistor qarshiligi $r = 40\ \Omega$, $e = 300\sin\omega t\ \text{V}$, nochiziq elementning (NE) volt-amper xarakteristikasi $i(U) = 0{,}02U + 4\cdot 10^{-6}U^3$ polinom bilan approksimatsiyalangan, bunda tok – amperda, kuchlanish – voltda. Nochiziq rezistorning volt-amper xarakteristikasi $U(i) = 400i + 8000i^3$ (bunda kuchlanish voltda, tok amperda) nolinom bilan approksimatsiya qilingan. Garmonik balans va garmonik chiziqlashtirish usullari bo‘yicha i tokning birinchi garmonikasi aniqlansin.

6.19-masala. 6.18-rasmda keltirilgan zanjir uchun $r_1 = 600\,\Omega$, $r_2 = 300\,\Omega$, $r_3 = 300\,\Omega$ $e_1 = e_2 = 340\sin 314t$ V. Nochiziq rezistorning volt-amper xarakteristikasi $U(i) = 400i + 8000i^3$ polinom bilan approksimatsiyalangan, bunda kuchlanish – voltda, tok – amperda. Garmonik balans va garmonik chiziqlashtirish usullari bo‘yicha i tokning birinchi garmonikasi aniqlansin.



6.18- rasm.

6.20-masala. Elektr zanjir ketma-ket ulangan qarshiligi $r = 200\ \Omega$ chiziqli rezistordan va xarakteristikasi $i = a\psi + b\psi^3$, (bunda $a = 50$ A/Wb, $b = 4\cdot 10^7$ A/Wb^3) polinom bilan approksimatsiyalangan g‘altakdan hamda EYK $e = 19\sin\omega t$,V ($\omega = 104$ rad/s) manbayidan iborat. G‘altakdagi magnit ilashlilik sinusoidal $\psi = \psi_m \sin(\omega t - \eta)$ deb magnit ilashlilikning amplitudasi ψ_m, boshlang‘ich fazasi η va $i(t)$ tokni garmonik balans va garmonik chiziqlashtirish usullarida toping.

6.21-masala. Volt-amper xarakteristikasi $u(i) = ai + bi^3$, (bunda $a = 40\ \Omega$, $b = 1400$ V/A^3) polinom bilan approksimatsiyalangan nochiziq rezistor, induktivligi $L = 2$ mH isrofsiz g‘altak bilan ketma-ket ulangan va EYK manbasiga $L = 17\sin 1000t$ ga qo‘shilgan. Zanjirdagi tok sinusoidal $i = I_m \sin(1000t - \varphi)$ deb garmonik balans va garmonik chiziqlashtirish usullari yordamida tok amplitudasi I_m va faza siljishi φ ni aniqlang.

6.22-masala. Xarakteristikasi $i(\psi) = a\psi + b\psi^3$ ($a = 50$ A/ Wb, $b = 10^7$ A/Wb^3) bo‘lgan nochiziq g‘altak va qarshiligi $r = 200\,\Omega$ li rezistor ketma-ket ulanib EYK

$e = E_m \sin\omega t = 10\sqrt{2}\sin 10^4 t, \mathrm{V}$ manbayiga qo‘shilgan. $b\psi^3 << a\psi$ hisoblab, iteratsion usulda g‘altakdagi kuchlanishni va zanjirdagi tokning oniy qiymatini aniqlang.

Yechish. Kirxgofning ikkinchi qonuni bo‘yicha zanjir tenglamasini yozamiz:

$$d\psi / dt + ri(\psi) = e. \tag{6.25}$$

(6.25) ga berilgan $i(\psi)$ni qo‘yamiz va tenglamaning o‘ng tomoniga kichik hadni o‘tkazamiz:

$$e - d\psi / dt - ar\psi = br\psi^3. \tag{6.26}$$

Nolli chiziqli yaqinlashtirishni aniqlaymiz. Kichik had bo‘lgan $br\psi^3$ni tashlab yuboramiz va hosil bo‘lgan chiziqli differensial tenglamani kompleks usulda yechamiz. (6.26) tenglamani kompleks shaklda yozamiz:

$$(ar + j\omega)\underline{\psi}_m = \underline{E}_m,$$

$$\underline{\psi}_m = \frac{\underline{E}_m}{ar + j\omega} = \frac{10\sqrt{2}}{(50\cdot 200 + j10^4)} = 10^{-3}e^{-j45^\circ}\,\mathrm{Wb}.$$

va magnit ilashlilikning oniy qiymatini nolli yaqinlashtirishini ifodalaymiz:

$$\psi_0 = \Psi_{0m}\sin(\omega t - \varphi) = 10^{-3}\sin(10^4 t - 45^0)\ \mathrm{Wb}.$$

Birinchi yaqinlashtirishni topish uchun nol yaqinlashtirshni (6.26) tenglamaning o‘ng tomoniga yozamiz va guruhlashdan so‘ng quyidagini hosil qilamiz:

$$d\psi / dt + ar\psi = E_m\sin(\omega t) - \frac{3}{4}br\psi_{0m}^3\sin(\omega t - \alpha) + \frac{1}{4}br\psi_{0m}^3\sin 3(\omega t - \alpha). \qquad (6.27).$$

(6.27) tenglamani chiziqli zanjirning nosinusoidal tashqi ta’sirdagi (o‘ng tomoni) tenglamasi deb qabul qilinishi mumkin.

(6.27) tenglamaning 1- garmonikasi uchun kompleks shakldagi tenglamasini yozamiz:

$$(ar + j\omega)\underline{\psi}_{1m} = \underline{E}_m - \frac{3}{4}br\psi_{0m}^3 e^{-3\alpha}.$$

shundan so‘ng son qiymatlarni qo‘yib magnit ilashishlik

$$\psi_{1m} = 0{,}925\cdot 10^{-3} < -40^0 20^{'}\ \mathrm{Wb}$$

ni va kuchlanishni topamiz:

$$\underline{U}_{1m} = 1\omega\psi_{1m} = 9{,}25 < 49^0 40^{'}\,\text{V}.$$

(6.27) tenglamaning 3- garmonikasini kompleks shaklda yozamiz:

$$(ar + j3\omega)\psi_{3m} = \frac{1}{4} br\psi_{m0}^3 < -3\alpha\ .$$

Uning yechimi quyidagi ko‘rinishga ega:

$$\underline{\psi}_{3m} = 0{,}0162 \cdot 10^{-3} < +153^0 30'\ \text{Wb},$$

va kuchlanish bo‘yicha

$$\underline{U}_{3m} = j3\omega\psi_{3m} = 0{,}486 < -116^0 30'\,\text{V}.$$

Magnit ilashlilik oniy qiymatining ifodasi:

$$\psi(t) = 0{,}925 \cdot 10^{-3} \sin(10^4 t - 40^0 20') + 0{,}0162 \cdot 10^{-3} \sin(3 \cdot 10^4 t + 153^0 30').$$

G‘altakdagi kuchlanish

$$u = 9{,}25 \sin(10^4 t + 49^0 40') + 0{,}486 \sin(3 \cdot 10^4 t - 116^0 30')\ \text{V}.$$

Zanjirdagi tok Om qonuni bo‘yicha

$$i(t) = (e - u) / r\ .$$

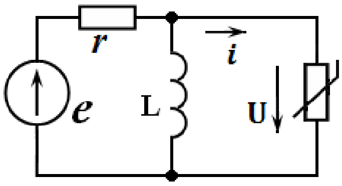
Tokning har bir tashkil etuvchisini kompleks usulda topamiz. Birinchi garmonika uchun $\underline{I}_{1m} = (\underline{E}_m - \underline{U}_{\Delta m}) / r = 0{,}054 < -40^0 50'$ A; 3- garmonika uchun $\underline{I}_{3m} = -\underline{U}_{3m} / r = -0{,}0024 < -116^0 30', \text{A}$.

Tokning oniy qiymati:

$$i(t) = 0{,}054 \sin(10^4 t - 40^0 50') - 0{,}0024 \sin(3 \cdot 10^4 t - 116^0 30'),\ \ \text{A}\ .$$

6.23-masala. Elektr zanjir ketma-ket ulangan EYK manbasi $e = 100 \sin 10^4 t, \text{V}$ induktivligi 40 mH bo‘lgan isrofsiz induktiv g‘altak va volt-amper xarakteristikasi $u(i) = ri + bi^3$, (bunda $r = 300\ \Omega$, b=2500 V/A^3) polinom bilan approksimatsiyalangan nochiziq rezistordan iborat. Agar $bi^3 << ri$ o‘rinli bo‘lsa, zanjir tokining oniy qiymatini iteratsion usulda aniqlang.

6.24-masala. 6.19-rasmda tasvirlangan zanjir uchun: $r = 400\,\Omega$, $L = 40\,\text{mH}$ nochiziq rezistorning volt-amper xarakteristikasi $i(u) = au + bu^3 = 0{,}002u + 210^{-7} u^3$, bunda tok – amperda, kuchlanish – voltda; EYK manbayi $e = 100 \sin 10^4 t$, V. Agar $bu^3 << au$ bo‘lsa, nochiziq rezistordagi kuchlanish va tokning oniy qiymati hamda manba toki iteratsion usulda topilsin.



6.19- rasm.

6.1-vazifa. Zanjir (6.20-rasm) ketma-ket ulangan chiziqli r va nochiziq rezistorlardan hamda EYK manbasi $e = E_m \sin \omega t$ dan iborat. Nochiziq rezistorning volt-amper xarakteristikasi

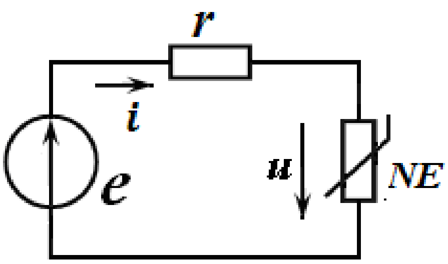
$$u(i) = ai + bi^3$$

ifodasi bilan approksimatsiyalangan. Zanjirning parametrlari 6.1-jadvalda keltirilgan.

1) tokning birinchi va uchinchi garmonikalarini: a) garmonik balans; b) garmonik chiziqlashtirish usullari yordamida aniqlang;

2) natijaviy volt-amper xarakteristikasini chizing;

3) tokning vaqt diagrammasini grafik usulda tuzing hamda a) va b) usullardan olingan diagrammalarni o‘zaro solishtiring.



6.20- rasm

6.1- *jadval*

Variant №	E_m, V	f, Hz	r, Ω	Nochiziq rezistor parametrlari	
				a, Ω	b, V/A^3
1	24	50	40	50	40
2	42	60	30	40	50
3	55	50	30	40	40
4	60	60	50	60	50
5	70	50	60	70	60
6	80	60	70	80	70
7	90	80	80	90	80
8	100	100	90	100	70
9	120	80	100	120	90
10	140	80	100	120	100
11	90	60	30	40	30
12	100	50	40	50	40
13	120	60	50	60	50
14	140	50	60	70	60
15	150	60	70	80	70
16	120	50	80	90	60
17	90	80	70	80	70
18	80	100	60	70	60
19	70	60	40	50	40
20	60	50	50	60	50
21	50	80	50	60	40
22	40	50	60	70	60
23	50	60	60	50	50
24	60	50	40	50	40
25	70	60	50	40	50
26	80	80	30	50	30
27	90	60	40	60	40
28	100	50	50	50	50
29	90	60	40	50	60
30	120	50	40	50	40

6.4. Tok va kuchlanishlarning ta’sir etuvchi qiymatlari bo‘yicha hisoblash

6.25- masala. O‘ramlar soni $\omega = 500\,\text{Hz}$li g‘altak transformatorli po‘lat magnit o‘tkazgichga o‘rnatilgan va kuchlanishi $U = 220\,\text{V}$ tarmoqqa (chastotasi 50 Hz) ulangan. G‘altakdagi tok $I = 10\,\text{A}$, aktiv quvvat esa $P = 1500\,\text{W}$. Chulg‘amning aktiv qarshiligi $r_m = 10\ \Omega$. Magnit o‘tkazgichdagi oqimning amplitudasi $\Phi_m = 10^{-3}$ Wb.

G‘altakning almashlash sxemasini tuzing va vektor diagrammasini quring.

Yechish. G‘altakning almashlash sxemasi 6.21- rasmda keltirilgan, bunda L_{ekv} –magnit o‘tkazgichdagi oqim tufayli mavjud bo‘lgan ekvivalent induktivlik; L_{soch} –sochilma induktivlik; r_{ekv} – magnit o‘tkazgichdagi isroflarni hisobga olgan ekvivalent aktiv qarshilik; r_{chol} – chulg‘am qarshiligi.

Almashlash sxema parametrlarini aniqlashdan avval, (6.7) formuladan, $\underline{\Phi}_m = \Phi_m$ ligini e’tiborga olib, magnit o‘tkazgichdagi oqim bilan bog‘langan $\underline{U}_F = j4{,}44 f\omega\underline{\Phi}_m = j111\ \text{V}$ kuchlanishning tashkil etuvchisini hisoblaymiz.

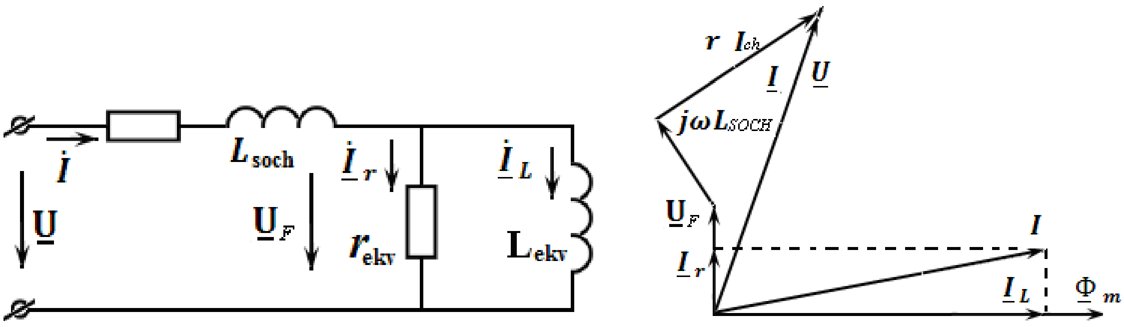
Joul – Lens qonuni bo‘yicha aktiv quvvatnining (isrofning) $P = r_m I^2 + U_F^2 / r_{\text{ekv}}$ ifodasini yozib magnit o‘tkazgichdagi isroflarning ekvivalent qarshiligini aniqlaymiz, bundan $r_{\text{ekv}} = 24{,}5\ \Omega$.

Magnit o‘tkazgichdagi isroflar tufayli o‘tadigan tokning tashkil etuvchisi:

$$\underline{I}_r = \underline{U}_F / r_{\text{ekv}} = j4{,}53\,\text{A}.$$

Ekvivalent induktiv qarshilik $x_{\text{ekv}} = \omega L_{\text{ekv}}$ ni aniqlash uchun, tokni topamiz $I_L = \sqrt{I^2 - I_r^2} = 8{,}9\,\text{A}$ va $x_{\text{ekv}} = U_F / I_L = 12{,}5\ \ \Omega$.

Sochma induktiv qarshilikni $x_{\text{soch}} = \omega L_{\text{soch}}$ almashlash sxemasi bo‘yicha g‘altakning kompleks qarshiligini hisoblaymiz (6.21- rasm, *a*):



a) *b)*

6.21- rasm.

$\underline{Z} = r_{\text{m}} + j\left(x_{\text{soch}} + \dfrac{jx_{\text{ekv}} r_{\text{ekv}}}{r_{\text{ekv}} + jx_{\text{ekv}}}\right) = 15{,}05 + j(x_{\text{soch}} + 9{,}9)$ va bunda to‘la qarshilikni Om qonuni bo‘yicha hisoblab $Z = U / I = 22\ \Omega$ ni olamiz. Shunday qilib,

$22^2 = 15{,}05^2 + (x_{\text{soch}} + 9{,}9)^2$, bundan $x_{\text{soch}} = 6{,}2\ \ \Omega$.

Almashlash sxemasi bo‘yicha qurilgan vektor diagramma 6.21- rasm, *b* da tasvirlangan.

6.26- masala. Po‘lat o‘zakli induktiv g‘altak kuchlanishi $U = 220$ V bo‘lgan sinusoidal tok manbayiga ulanganda, tok $I = 10$ A, aktiv quvvat $P = 1500$ W bo‘lgan. Chulg‘amning o‘zgarmas tokka bo‘lgan qarshiligi $r_{\text{m}} = 10\ \ \Omega$. Sochma oqimni hisobga olmasdan, g‘altakning parallel almashlash sxemasini tuzing va u uchun $\underline{I} = I$ ni e’tiborga olib, vektor diagrammasini quring.

6.27- masala. Po‘lat magnit o‘zakli g‘altak tok va kuchlanishlarining ta’sir etuvchi qiymati $U = 200I - 10I^3$ ifoda bilan approksimatsiyalanadi, bunda kuchlanish – voltlarda, tok – amperlarda. G‘altak sig‘imi 29 mkF li kondensator bilan ketma-ket ulangan bo‘lib, chastotasi 50 Hz li sinusoidal tok manbasiga ulangan.

Kuchlanish rezonansi sodir etiladigan tokning ta’sir etuvchi qiymati aniqlansin.

6.5. O‘zgaruvchan tokni nochiziq elementlar yordamida to‘g‘rilash

6.28- masala. Qarshiligi $R_{\text{yuk}} = 500\ \Omega$ chiziqli rezistor yuklama sifatida ballast rezistor $R = 100\ \Omega$ va ideal diod orqali EYKsi $e(t) = 100\sqrt{2}\sin 314t$ V bo‘lgan manbaga ulangan. Yuklamadagi kuchlanishning o‘zgarmas tashkil etuvchisini va ballast rezistordagi quvvat isrofi aniqlansin.

Yechish. Manba EYKning musbat yarim davrida yuklamadagi kuchlanishni hisoblaymiz:

$$u_{\text{yuk}} = \frac{R_{\text{yuk}}}{R + R_{\text{yuk}}} e(t) = \frac{500}{100+500} 100\sqrt{2}\sin 314t = 117{,}8 \sin \omega t \text{ V.} \qquad (0 < \alpha < \pi).$$

Zanjirdan o‘tadigan nosinusoidal tokning yarim davridagi tok quyidagicha o‘zgaradi.

$$i(t) = \frac{e(t)}{R_{\text{b}} + R_{\text{yuk}}} = \frac{100\sqrt{2}\sin \omega t}{100+500} = 0{,}235 \sin \omega t \text{ A.}$$

Yuklamadagi kuchlanishning o‘zgarmas tashkil etuvchisi:

$$u_{\text{yuk}} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} u_{\text{yuk}}^2 \sin^2 \omega t d(\omega t)} = \frac{1}{2} u_m = \frac{1}{2} \cdot 117{,}8 = 58{,}9 \text{ V.}$$

Musbat yarim davrida ballast R_b qarshiligidagi kuchlanish pasyishi:

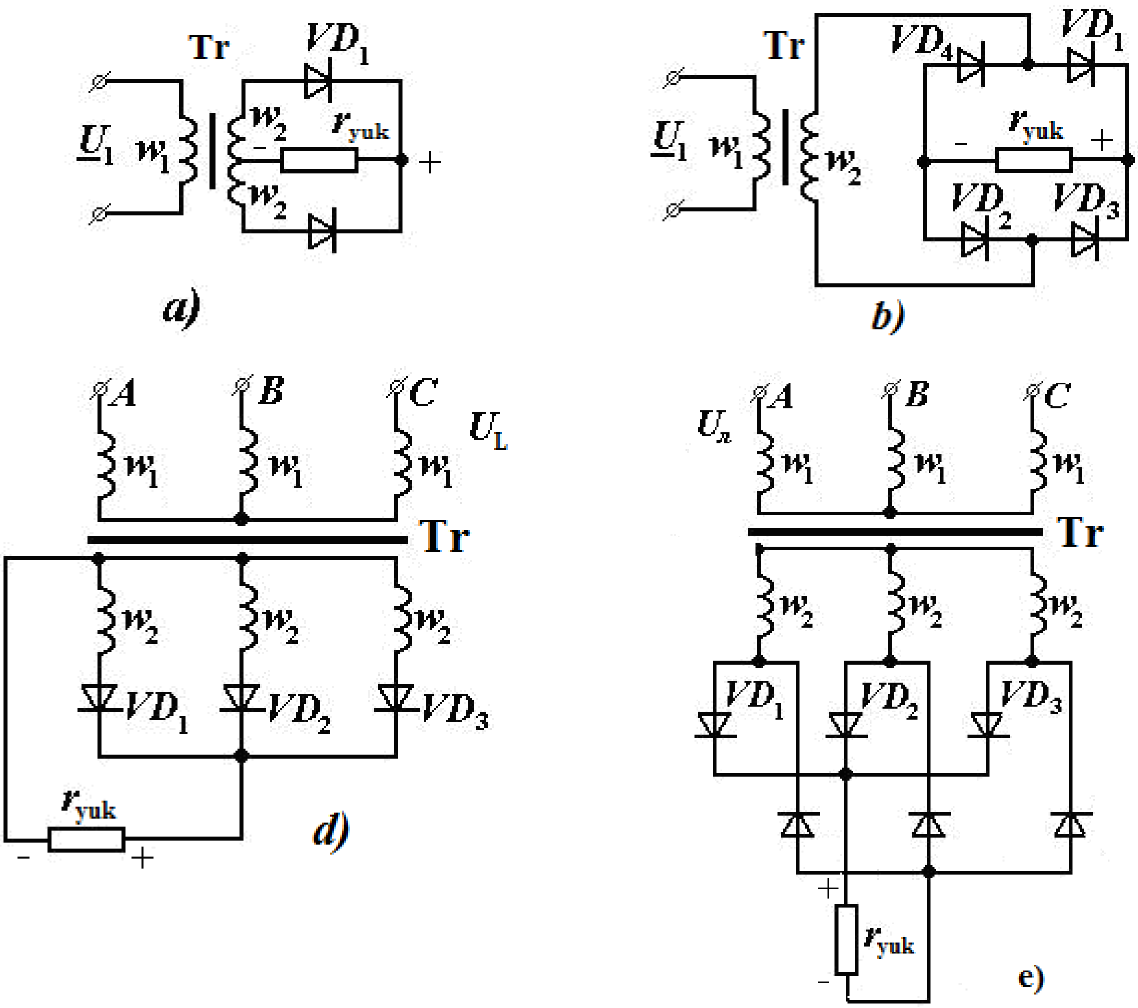
$$u_R = \frac{R_b}{R_b + R_{\text{yuk}}} e(t) = \frac{100}{100+500} \cdot 100\sqrt{2}\sin 314t = 23{,}57 \sin 314t \text{ V.}$$

Zanjirning ballast qarshiligidagi quvvat isrofi:

$$P = \frac{1}{2\pi}\int_0^{2\pi} u_b i d(\omega t) = \frac{1}{2\pi}\int_0^{2\pi} (23{,}57 \sin 314t \cdot 0{,}235 \sin 314t) d(\omega t) = 1{,}3 \text{ W.}$$

6.2- vazifa. Ikki yarim davrli yarim o‘tkazgichli to‘g‘rilagichlarni hisoblash.

Yarim o‘tkazgichli to‘g‘rilagichlarning asosiy sxemalari 6.22-rasmda berilgan. To‘g‘rilagich parametrlari 6.2-jadvalda keltirilgan. Diodlar ideal. Quyidagilar aniqlansin: har bir shoxobchadagni toklarning o‘rta, ta’sir etuvchi qiymatlari hamda rezistordagi aktiv quvvat. Toklar va chiqish kuchlanishining vaqt diagrammalari chizilsin.



6.22- rasm. Ikki yarim davrli to‘g‘rilagichlar sxemasi.

6.2- *jadval*

Variant №	Sxema №	Transformatsiya koeffitsiyenti	U_{lin}, kV	Yuklama qarshiligi r_{yuk}, Ω
1	a	16,7	10	10
2	b	12	0,38	86
3	v	3,63	10	4
4	g	3,63	10	8
5	a	16,7	10	6
6	b	16,7	10	10
7	v	3,63	10	12
8	g	3,63	10	10
9	a	1,73	0,38	8
10	b	1,73	0,38	6
11	v	3,63	10	4
12	g	12,5	10	12
13	a	12,5	10	24
14	b	1,73	0,22	36
15	v	3,63	10	12
16	g	3,63	10	8
17	a	1,73	0,22	10
18	b	1,73	0,22	10
19	v	3,63	10	12
20	g	3,63	10	36
21	a	16,7	10	24
22	b	16,7	10	12
23	v	4	10	8
24	g	4	10	6
25	a	1,73	0,38	10
26	b	1,73	0,38	12
27	v	3,63	10	24
28	g	3,63	10	36
29	a	1,73	0,38	12
30	b	1,73	0,38	10

6.6. Ferromagnit elementli o‘zgaruvchan tok elektr zanjirlarini hisoblash

6.5.1. Ferromagnit o‘zakli induktiv g‘altak almashlash sxemasi va vektor diagrammasi

6.29-masala. 6.23-rasm, *a* da keltirilgan ferromagnit o‘zakli induktiv g‘altak uchun quyidagilar berilgan: $U = 120$ V, $I = 0{,}3$ A, $P = 10$ W, $w = 200$ o‘ram, $f = 400$ Hz. Ferromagnit o‘zak yupqa po‘lat tunukalardan yig‘ilgan bo‘lib, $d_{\text{ich}} = 56\,mm, d_{\text{t}} = 83{,}5\,mm,$ qalinligi $b = 20\,mm,$ o‘zak ko‘ndalang kesimini po‘lat bilan to‘ldirish koeffitsiyenti $k_{\text{p}} = 0{,}9.$ Chulg‘amning o‘zgarmas tokka qarshiligi R=20 Ω. O‘zakdagi B_m va H_m qiymatlari hamda solishtirma quvvat isrofi topilsin. B_m va H_m larni hisoblashda chulg‘am qarshiligi hisobga olinmasin.

Yechish. Ferromagnit o‘zakdagi magnit oqimi:

$$\Phi_m = \frac{U}{4{,}44 fw} = \frac{120}{4{,}44 \cdot 400 \cdot 200} = 3{,}38 \cdot 10^{-4} \text{ Wb}.$$

Po‘lat o‘zakning aktiv yuzasi:

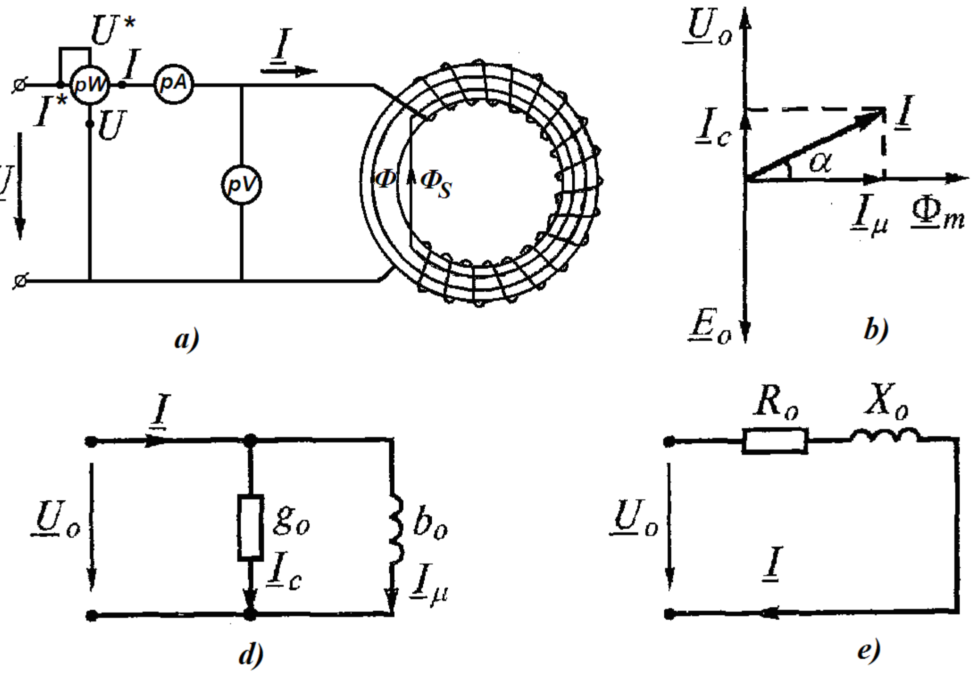
$$S_{\text{p}} = \frac{d_{\text{t}} - d_{\text{ich}}}{2} bk_{\text{p}} = \frac{83{,}5 - 56}{2} 20 \cdot 0{,}9 \cdot 10^{-6} = 2{,}4 \cdot 10^{-4}\ m^2.$$

O‘zakdagi magnit induksiyasining amplituda qiymati:

$$B_m = \frac{\Phi_m}{S_P} = \frac{3{,}38 \cdot 10^{-4}}{2{,}4 \cdot 10^{-4}} = 1{,}41 \text{ T}.$$

Magnit maydoni kuchlanganligi chiziqlarining o‘rtacha uzunligi:

$$l_{o'r} = \frac{d_{\text{t}} + d_{\text{ich}}}{2} \pi = \frac{83{,}5 + 56}{2} 3{,}14 \cdot 10^{-3} = 0{,}219\, m.$$



6.31- rasm.

Ferromagnit o‘zakdagi magnit maydoni kuchlanganligining amplituda qiymati:

$$H_m = \frac{Iw\sqrt{2}}{l_{o'r}} = \frac{0{,}3 \cdot 200\sqrt{2}}{0{,}219} = 387\,\frac{A}{m}.$$

O‘zakdagi solishtirma quvvat isrofini topish uchun avval po‘lat o‘zakning massasini topamiz:

$$m_{o'z} = \gamma_p S_p l_{o'r}$$

bu yerda $\gamma_p = 7{,}8 \cdot 10^3\,\frac{kg}{m^3}$ - po‘latning zichligi.

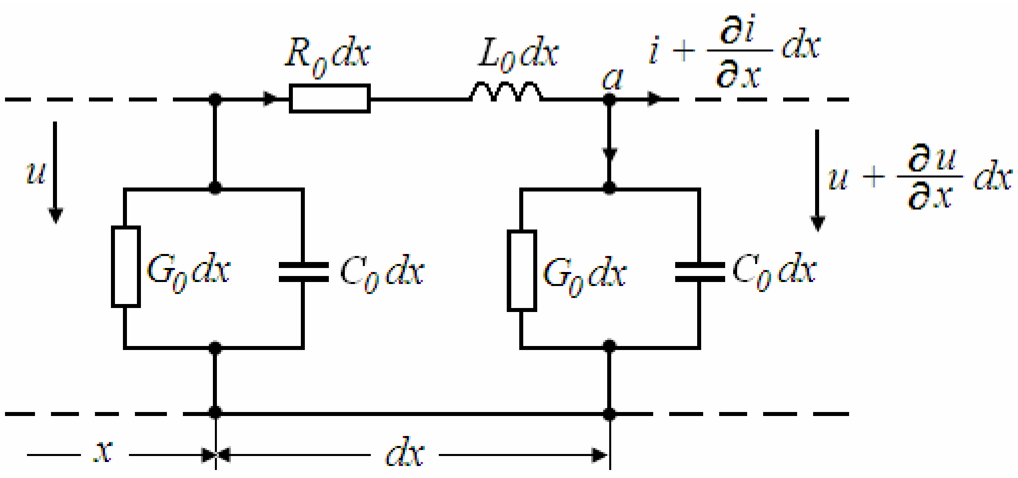
O‘zakdagi solishtirma quvvat isrofi:

$$p_{o'z} = \frac{P - RI^2}{m_{o'z}} = \frac{10 - 20 \cdot 0{,}3^2}{0{,}41} = 20\,\frac{\mathrm{W}}{\mathrm{kg}}.$$

7- BOB. TARQOQ PARAMETRLI ELEKTR ZANJIRLARNI HISOBLASH

7.1. Asosiy ma'lumotlar

Bir jinsli liniyaning differensial tenglamalari.



7.1- rasm.

R_0, L_0, G_0, C_0 parametrlar bir jinsli liniyaning *birlamchi parametrlari* deb ataladi. Shuni ta'kidlash lozimki, R_0 – liniya simlarining qarshiligi, G_0 esa liniya simlari orasidagi izolyatsiya o'tkazuvchanligi bo'lganligi sababli $R_0 \neq \frac{1}{G_0}$.

7.1- rasmdagi zanjir berk konturi uchun Kirxgofning 2- qonuni asosida quyidagi tenglamalarni tuzamiz:

$$-\frac{\partial u}{\partial x} = R_0 i + L_0 \frac{\partial i}{\partial t}, \qquad (7.1)$$

$$-\frac{\partial i}{\partial x} = G_0 u + C_0 \frac{\partial u}{\partial t}. \qquad (7.2)$$

(7.1) va (7.2) tenglamalar *bir jinsli liniyaning differensial (telegraf) tenglamalari* deb ataladi.Ulardagi «–» ishora tok va kuchlanishlarni liniya elementar uchastkasining boshidan oxiriga qarab *kamayib* borishini bildiradi.

Bir jinsli liniyaning kompleks shakldagi tenglamalarini tok va kuchlanishlarning ta'sir etuvchi qiymatlari uchun quyidagicha yoziladi:

$$-\frac{d\underline{U}}{dx}=(R_0+j\omega L_0)\underline{I}=\underline{Z}_0\underline{I}, \tag{7.3}$$

$$-\frac{d\underline{I}}{dx}=(G_0+j\omega C_0)\underline{U}=\underline{Y}_0\underline{U}, \tag{7.4}$$

bu yerda $\underline{Z}_0$, $\underline{Y}_0$ – liniyaning uzunlik birligidagi kompleks qarshiligi va o‘tkazuvchanligi bo‘lib, $\underline{Z}_0 \neq \frac{1}{\underline{Y}_0}$.

Liniyaning boshidagi kuchlanish $\underline{U}_1$ va tok $\underline{I}_1$ ma’lum bo‘lganda, uning istalgan nuqtasidagi kuchlanish va toki quyidagicha ifodalanadi:

$$\underline{U}=\frac{1}{2}(\underline{U}_1+\underline{I}_1\underline{Z}_T)e^{-\underline{\gamma}x}+\frac{1}{2}(\underline{U}_1-\underline{I}_1\underline{Z}_T)e^{\underline{\gamma}x}, \tag{7.5}$$

$$\underline{I}=\frac{1}{2}\left(\frac{\underline{U}_1}{\underline{Z}_T}+\underline{I}_1\right)e^{-\underline{\gamma}x}-\frac{1}{2}\left(\frac{\underline{U}_1}{\underline{Z}_T}-\underline{I}_1\right)e^{\underline{\gamma}x}. \tag{7.6}$$

bu yerda $\underline{Z}_T=\sqrt{\frac{\underline{Z}_0}{\underline{Y}_0}}=\sqrt{\frac{R_0+j\omega L_0}{G_0+j\omega C_0}}=Z_T e^{j\varphi_T}$ – liniyaning *to‘lqin* yoki *xarakteristik qarshiligi* bo‘lib, u liniyaning ixtiyoriy qismida bir xil qiymatga ega bo‘ladi. Agar

$$\frac{1}{2}(e^{\underline{\gamma}x}-e^{-\underline{\gamma}x})=\mathrm{sh}(\underline{\gamma}x)\ ,\ \frac{1}{2}(e^{\underline{\gamma}x}+e^{-\underline{\gamma}x})=\mathrm{ch}(\underline{\gamma}x) \tag{7.7}$$

ekanligini inobatga olsak, u holda yuqoridagi tenglamalarni giperbolik shaklda quyidagicha yoziladi:

$$\underline{U}=\underline{U}_1\mathrm{ch}(\underline{\gamma}x)-\underline{I}_1\underline{Z}_T\mathrm{sh}(\underline{\gamma}x), \tag{7.8}$$

$$\underline{I}=\underline{I}_1\mathrm{ch}(\underline{\gamma}x)-\frac{\underline{U}_1}{\underline{Z}_T}\mathrm{sh}(\underline{\gamma}x). \tag{7.9}$$

Bir jinsli liniya boshidagi kuchlanish va tok qiymatlari ma’lum bo‘lganda, liniyaning istalgan nuqtasidagi kuchlanish va tok qiymatlari (7.5), (7.6), (7.8) va (7.9) lardan topiladi.

Agar liniya oxiridagi kuchlanish $\underline{U}_2$ va tok $\underline{I}_2$ ma’lum bo‘lsa, u holda liniyaning oxiridan ixtiyoriy x' masofadagi kuchlanish va tok quyidagicha topiladi:

$$\underline{U}=\frac{1}{2}\left(\underline{U}_2+\underline{I}_2\underline{Z}_T\right)e^{\underline{\gamma}x'}+\frac{1}{2}\left(\underline{U}_2-\underline{I}_2\underline{Z}_T\right)e^{-\underline{\gamma}x'}, \tag{7.10}$$

$$\underline{I} = \frac{1}{2}\left(\underline{I}_2 + \frac{\underline{U}_2}{\underline{Z}_T}\right)e^{\gamma x'} + \frac{1}{2}\left(\underline{I}_2 - \frac{\underline{U}_2}{\underline{Z}_T}\right)e^{-\gamma x'}, \qquad (7.11)$$

yoki giperbolik funksiyalar orqali

$$\underline{U} = \underline{U}_2 \mathrm{ch}(\underline{\gamma} x') + \underline{I}_2 \underline{Z}_T \mathrm{sh}(\underline{\gamma} x'), \qquad (7.12)$$

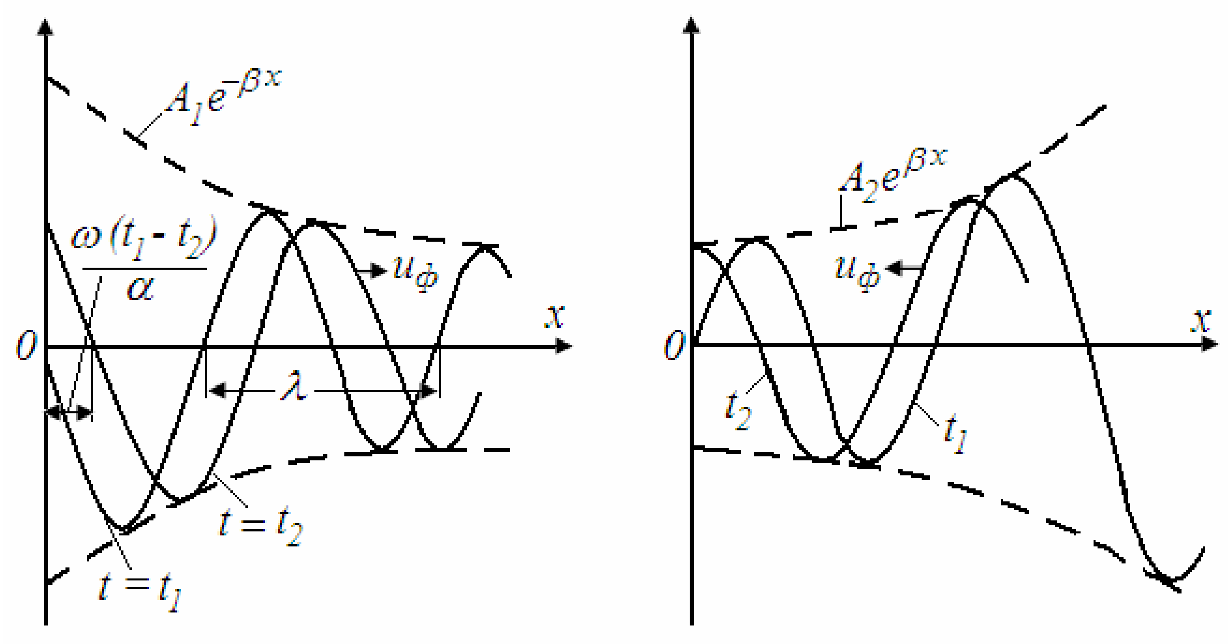
$$\underline{I} = -\frac{\underline{U}_2}{\underline{Z}_T} \mathrm{sh}(\underline{\gamma} x') + \underline{I}_2 \mathrm{ch}(\underline{\gamma} x'). \qquad (7.13)$$

Bir jinsli liniyaning kuchlanish va tokning oniy qiymatlaridagi tenglamalari quyidagicha ifodalanadi:

$$u = \sqrt{2}A_1 e^{-\alpha x} \sin(\omega t - \beta x + \psi_1) + \sqrt{2}A_2 e^{\alpha x} \sin(\omega t + \beta x + \psi_2) = u_T + u_a \qquad (7.14)$$

$$i = -\frac{\sqrt{2}}{Z_T} A_1 e^{-\alpha x} \sin(\omega t - \beta x + \psi_1 - \varphi) - \frac{\sqrt{2}}{Z_T} A_2 e^{\alpha x} \sin(\omega t + \beta x + \psi_2 - \varphi) = i_T - i_a, \quad (7.15)$$

Ushbu tenglamalardagi har bir qo‘shiluvchi yuguruvchi to‘lqin sifatida qaraladi, unda u_T va i_T – kuchlanish va tokning *tushuvchi to‘lqinlari* (7.2- rasm, *a*), u_a va i_a esa kuchlanish va tokning *aks to‘lqinlari* deb ataladi (7.2- rasm, *b*).



a) *b)*

7.2- rasm.

Tebranish fazasi siljishining tezligiga to‘lqinning fazaviy tezligi deb ataladi. Bunda to‘lqinning fazasi vaqt va koordinataga bog‘liq emas, ya’ni tushuvchi to‘lqin uchun:

$$\omega t - \beta x + \psi_1 = \text{const},$$

bundan $\dfrac{d}{dt}(\omega t - \beta x + \psi_1) = 0$ yoki $\dfrac{dx}{dt} = \vartheta_{\text{F}} = \dfrac{\omega}{\beta}$.

To‘lqinning tarqalish yo‘nalishi bo‘yicha tebranish fazasi 2π *ga farq qiluvchi eng yaqin ikki nuqta orasidagi masofa to‘lqin uzunligi deb ataladi,* ya’ni

$$\omega t - \beta(x + \lambda) + \psi_1 = \omega t - \beta x + \psi_1 - 2\pi\,.$$

bundan $\lambda = \dfrac{2\pi}{\beta}$.

Oxirgi tenglikdan β ni topib, uni fazaviy tezlik ifodasiga qo‘ysak:

$$\vartheta_{\Phi} = \frac{\omega}{\beta} = \frac{\lambda}{T},$$

ya’ni to‘lqin $t = T$ vaqt mobaynida $x = \lambda$ masofa yo‘l bosib ulguradi. *Agar liniyaning uzunligi undagi elektromagnit to‘lqin uzunligiga yaqin bo‘lsa, u holda bunday liniya uzun liniya deb ataladi.*

Bir jinsli liniyaning ikkilamchi parametrlari.

Bir jinsli liniyaning ikkilamchi yoki xarakteristik parametrlariga susayish koeffitsiyenti α, faza koeffitsiyenti β va to‘lqin qarshiligi Z_T kiradi. Bu parametrlar liniyaning birlamchi parametrlari R_0 , L_0 , G_0 , C_0 orqali quyidagicha ifodalanadi:

$$\underline{\gamma} = \sqrt{(R_0 + j\omega L_0)(G_0 + j\omega C_0)} = \sqrt{R_0G_0 - \omega^2 L_0C_0 + j\omega(L_0G_0 + C_0R_0} = \alpha + j\beta\,.$$

bundan

$$\alpha = \sqrt{\frac{1}{2}\left[R_0G_0 - \omega^2 L_0C_0 + \sqrt{(R_0^2 + \omega^2 L_0^2)(G_0^2 + \omega^2 C_0^2)}\right]}\,, \qquad (7.16)$$

$$\beta = \sqrt{\frac{1}{2}\left[\omega^2 L_0C_0 - R_0G_0 + \sqrt{(R_0^2 + \omega^2 L_0^2)(G_0^2 + \omega^2 C_0^2)}\right]}\,. \qquad (7.17)$$

Elektromagnit maydon nazariyasida quyidagi tenglik isbotlangan:

$$L_0 C_0 = \frac{\varepsilon\mu}{c^2}, \tag{7.18}$$

bu yerda $c = 3 \cdot 10^8$ m/s - yorug‘likning vakkumdagi tezligi, ε, μ - tokli o‘tkazgich atrofidagi muhitning nisbiy dielektrik va magnit singdiruvchanliklari.

Bir jinsli liniyani to‘rtqutblik ko‘rinishda tasvirlash.

(7.12) va (7.13) tenglamalarni $x' = l$ uchun yozamiz:

$$\left.\begin{aligned} \underline{U}_1 &= \underline{U}_2 \mathrm{ch}\underline{\gamma}l + \underline{I}_2 \underline{Z}_T \mathrm{sh}\underline{\gamma}l, \\ \underline{I}_1 &= \frac{\underline{U}_2}{\underline{Z}_T} \mathrm{sh}\underline{\gamma}l + \underline{I}_2 \mathrm{ch}\underline{\gamma}l. \end{aligned}\right\} \tag{7.19}$$

Bu tenglamalarni to‘rtqutblikning quyidagi tenglamalari bilan tenglashtiramiz:

$$\left.\begin{aligned} \underline{U}_1 &= \underline{A}\underline{U}_2 + \underline{B}\underline{I}_2, \\ \underline{I}_1 &= \underline{C}\underline{U}_2 + \underline{D}\underline{I}_2. \end{aligned}\right\} \tag{7.20}$$

Natijada, uzun liniya koeffitsiyentlari quyidagicha bo‘lgan simmetrik to‘rtqutblik bo‘lib qoladi:

$$\underline{A} = \mathrm{ch}\underline{\gamma}l, \ \underline{B} = \underline{Z}_T \mathrm{sh}\underline{\gamma}l, \ \underline{C} = \frac{\mathrm{sh}\underline{\gamma}l}{\underline{Z}_T}, \ \underline{D} = \underline{A} = \mathrm{ch}\underline{\gamma}l \, .$$

Bir jinsli liniyaning xossalari.

Signalni buzmaydigan liniyada susayish koeffitsiyenti va fazaviy tezlik signal chastotasiga bog‘liq bo‘lmaydi. Bunda liniya boshidagi signalning shakli uning oxiridagi shakli bilan bir xil bo‘ladi. Bu shart quyidagicha ifodalanadi:

$$\frac{R_0}{G_0} = \frac{L_0}{C_0}. \tag{7.21}$$

Bunda liniyaning ikkilamchi parametrlari quyidagicha aniqlanadi:

$$\alpha = \sqrt{R_0 G_0}, \ \beta = \omega\sqrt{L_0 C_0}, \ \vartheta_F = \frac{\omega}{\beta} \approx \frac{1}{\sqrt{L_0 C_0}}, \ Z_T = \sqrt{\frac{L_0}{C_0}}. \tag{7.22}$$

Isrofsiz liniya. Ayrim hollarda, ayniqsa, yuqori chastotali signallar liniya bo‘ylab uzatilganda, $\omega L_0 >> R_0$ va $\omega C_0 >> G_0$ bo‘lganligi sababli liniyadagi isroflar hisobga olmaslik darajada kichik bo‘ladi, ya’ni bunday liniyalarda

$$R_0 = 0, \ \ G_0 = 0. \tag{7.23}$$

(7.23) shart bajariladigan liniyalar *isrofsiz liniyalar* deb ataladi. Bunday liniyalar uchun uning ikkilamchi parametrlari quyidagilarga teng bo‘ladi:

$$\gamma = j\omega\sqrt{L_0C_0} = j\beta,\ \alpha = 0,\ \beta = \omega\sqrt{L_0C_0},\ Z_T = \sqrt{\frac{L_0}{C_0}},$$

$$\varphi_T = 0,\ \vartheta_F = \frac{\omega}{\beta} = \frac{1}{\sqrt{L_0C_0}},\ \lambda = \frac{2\pi}{\beta}. \qquad (7.24)$$

(7.24) tengliklarning tahlili shuni ko‘rsatadiki, liniyadagi susayish nolga teng, to‘lqin qarshilik aktiv bo‘lib chastotaga bog‘liq emas.

Isrofsiz liniyada tenglamalardagi kompleks argumentli giperbolik funksiyalar haqiqiy argumentli oddiy trigonometrik funksiyalar bilan almashadi:

$$\underline{U} = \underline{U}_1\cos\beta x - j\underline{I}_1\underline{Z}_T\sin\beta x, \qquad (7.25)$$

$$\underline{I} = \underline{I}_1\cos\beta x - j\frac{\underline{U}_1}{\underline{Z}_T}\sin\beta x, \qquad (7.26)$$

$$\underline{U} = \underline{U}_2\cos\beta x + j\underline{I}_2\underline{Z}_T\sin\beta x, \qquad (7.27)$$

$$\underline{I} = \underline{I}_2\cos\beta x + j\frac{\underline{U}_2}{\underline{Z}_T}\sin\beta x. \qquad (7.28)$$

Shuni ta’kidlash joizki, liniya bo‘ylab uzatilayotgan energiya elektr va magnit maydonlarining energiyalari yig‘indisidan iborat bo‘ladi. Agar isrofsiz liniya oxiriga qarshiligi liniyaning to‘lqin qarshiligiga teng yuklama ulansa, u holda liniyaning istalgan qismida $\frac{L_0 i^2}{2} = \frac{C_0 u^2}{2}$ shart bajariladi.

Turg‘un to‘lqinlar liniya oxirida qisqa tutashish, salt ish va reaktiv yuklama bo‘lganda yuzaga keladi.

Salt ish rejimida $I_2 = 0,\ Z_{\text{yuk}} = \infty$ va isrofsiz liniya uchun $\alpha = 0$ bo‘lganda, turg‘un to‘lqin tenglamalari quyidagi ko‘rinishda bo‘ladi:

$$u_0 = U_{2m}\cos\beta x\sin\omega t, \qquad (7.29)$$

$$i_0 = \frac{U_{2m}}{Z_T}\sin\beta x\cos\omega t. \qquad (7.30)$$

7.2. Bir jinsli liniyaning differensial tenglamalari

7.1- masala. Ikkita mis simdan iborat havo elektr uzatish liniyasi berilgan. O‘tkazgichlar ko‘ndalang kesimining radiusi $a = 10\,\text{mm},$ ular orasidagi masofa $b = 2718\,\text{mm}$. Bir jinsli liniyaning birlamchi parametrlari topilsin. Misning solishtirma qarshiligi $\rho = 0.018\,\text{mk}\Omega\cdot\text{m},$, havoning solishtirma o‘tkazuvchanligi $\sigma = 1.79\cdot10^{-20}\,\text{S/km}.$ Liniyaning birlamchi parametrlari chastotaga bog‘liq emas, sirt va o‘tkazgichlarning yaqinlik effektlari hisobga olinmasin.

Yechish. Liniyaning 1 km uzunlikdagi aktiv qarshiligi:

$$R_0 = \frac{2\rho}{\pi a^2} = \frac{2\cdot 0{,}018\cdot10^{-3}}{3{,}14\cdot10^{-4}} = 0{,}115\,\frac{\Omega}{\text{km}}.$$

Liniyaning induktivligi o‘ramlari soni birga teng bo‘lgan g‘altak induktivligi sifatida hisoblanadi:

$$L_0 = \frac{\mu\mu_0}{\pi}\cdot\ln(\frac{b}{a}) = \frac{4\pi\cdot10^{-4}}{\pi}\cdot\ln 271{,}8 = 2.24\,\text{mH/km}.$$

Ikkita o‘tkazgichli liniyaning uzunlik birligidagi aktiv o‘tkazuvchanligi va sig‘imi mos ravishda quyidagicha hisoblanadi:

$$C_0 = \frac{\varepsilon\varepsilon_0\pi}{\ln(\frac{b}{a})} = \frac{8{,}856\cdot10^{-9}\cdot3{,}14}{\ln 271{,}8} = 4{,}95\,\text{mkF/km}.$$

$$G_0 = \frac{\sigma\pi}{\ln\left(\frac{b}{a}\right)} = \frac{1.79\cdot10^{-20}\cdot3.14}{\ln 271.18} = 10^{-20}\,\frac{\text{S}}{\text{km}}.$$

7.2.1. Bir jinsli liniyaning ikkilamchi parametrlari

7.2-masala. KM - 4x2,52/9,4 rusumli standart koaksial kabel liniyaning birlamchi va ikkilamchi parametrlarini aniqlang. Kabellar o‘zaro shaybasimon polietilen bilan izolatsiyalangan. Chastota $f = 220\,\text{kHz}$, simlar diametri $d = 2{,}52\,\text{mm}$ va $D = 9{,}4\,\text{mm}$ izolatsiyaning dielektrik singdiruvchanligi $\varepsilon = 1{,}1$, dielektrik isroflar burchagining tangensi $\text{tg}\delta = 0{,}5\cdot10^{-4}$, harorat $t = 20^\circ C$.

Yechish. Koaksial kabel liniyaning birlamchi parametrlari quyidagicha topiladi:

$$R_0 = 8{,}35\sqrt{f}\,(\frac{1}{d}+\frac{1}{D})\cdot 10^{-2} = 8{,}35\sqrt{220\cdot 10^3}\,(\frac{1}{2{,}52}+\frac{1}{9{,}4})\cdot 10^{-2} = 19{,}7\ \ \Omega/\text{km},$$

$$L_0 = 2\ln\frac{D}{d}\cdot 10^{-4} = 2\ln\frac{9{,}4}{2{,}52}\cdot 10^{-4} = 2{,}63\cdot 10^{-4}\ \text{H/km,},$$

$$G_0 = \omega C_0 tg\delta = 2\pi\cdot 220\cdot 10^3\cdot 46{,}5\cdot 10^{-9}\cdot 0{,}5\cdot 10^{-4} = 3{,}2\cdot 10^{-6}\ \text{S/km},$$

$$C_0 = \frac{\varepsilon}{18\ln\frac{D}{d}}\cdot 10^{-6} = \frac{1{,}1\cdot 10^{-6}}{18\ln\frac{9{,}4}{2{,}52}} = 46{,}5\cdot 10^{-9}\ \text{F/km..}$$

Koaksial kabel liniyaning ikkilamchi parametrlarini hisoblashda, agar $R_0 = 19{,}7 << \omega L_0 = 364\ \Omega$ va $G_0 = 3{,}2\cdot 10^{-6} << \omega C_0 = 64000\cdot 10^{-6}$ S ekanligini hisobga olsak, u holda:

$$Z_T = \sqrt{\frac{L_0}{C_0}} = \sqrt{\frac{2{,}63\cdot 10^{-4}}{46{,}5\cdot 10^{-9}}} = 75\ \Omega,$$

$$\alpha = \frac{R_0}{2}\sqrt{\frac{C_0}{L_0}} + \frac{G_0}{2}\sqrt{\frac{L_0}{C_0}} = \frac{19{,}7}{2}\sqrt{\frac{46{,}5\cdot 10^{-9}}{2{,}63\cdot 10^{-4}}} + \frac{3{,}2\cdot 10^{-6}}{2}\sqrt{\frac{2{,}63\cdot 10^{-4}}{46{,}5\cdot 10^{-9}}} =$$

$$= 0{,}131\ \text{Np/km} = 1{,}14\ \text{dB/km},$$

$$\beta = \omega\sqrt{L_0 C_0} = 2\cdot 3{,}14\cdot 220\cdot 10^3\cdot\sqrt{2{,}63\cdot 10^{-4}\cdot 46{,}5\cdot 10^{-9}} =$$

$$= 4{,}83\ \text{rad/km.}$$

To‘lqin uzunligi va fazaviy tezlikni quyidagicha hisoblaymiz:

$$\lambda = \frac{2\pi}{\beta} = \frac{6{,}28}{4{,}83} = 1{,}3\ \text{m},\quad \vartheta_F = \frac{\omega}{\beta} = \frac{2\cdot 3{,}14\cdot 220\cdot 10^3}{4{,}83} = 286000\ \text{km/s.}$$

Hisoblash natijalarini taqqoslash maqsadida R_0 va G_0 lar qiymatlarini hisobga olgan holda liniyaning ikkilamchi parametrlarini topamiz:

$$\underline{Z}_T = \sqrt{\frac{R_0 + j\omega L_0}{G_0 + j\omega C_0}} = \sqrt{\frac{19{,}7 + j2\cdot 3{,}14\cdot 220\cdot 10^3\cdot 2{,}63\cdot 10^{-4}}{3{,}2\cdot 10^{-6} + j2\cdot 3{,}14\cdot 220\cdot 10^3\cdot 46{,}5\cdot 10^{-9}}} =$$

$$= 75{,}2e^{-j1^\circ 33'}\ \Omega,$$

$$\underline{\gamma} = \sqrt{(R_0 + j\omega L_0)(G_0 + j\omega C_0)} = \alpha + j\beta = 0{,}13 + j4{,}8\ \ km^{-1}.$$

Ushbu topilgan qiymatlarni avvalgi ($R_0 = 0,\ G_0 = 0$) qiymatlar bilan o‘zaro taqqoslab xatolik juda kichikligiga ishonch hosil qilamiz.

7.3- masala. Ikkita po‘lat simli liniyaning ikkilamchi parametrlari:

$$\underline{Z}_T = 1510e^{-j20^\circ 21'}\ \Omega, \quad \alpha = 13{,}6\ \text{mNp/km}, \quad \beta = 36{,}4\ \text{rad/km}.$$

Liniya uzunligi $l = 38\,\text{km}$, yuklama qarshiligi $Z_{\text{yuk}} = 1355e^{-j21{,}4}\,\Omega$ va liniyaga yuklama moslanmagan. Liniya boshiga kuchlanishi $U_1 = 10\,\text{V}$, chastotasi $f = 800$ Hz sinusoidal tok berilgan. Quyidagilar topilsin: 1) yuklamadagi kuchlanish va tok, liniya boshidagi tok, liniyaning yuklamaga ulangan holatdagi kirish qarshiligi; 2) yuklama iste’mol qilinayotgan quvvat, liniyaga berilayotgan quvvat va liniyaning FIK.

Yechish. Talab qilinayotgan kattaliklarni topishga kirishishdan avval hisoblashlarda kerak bo‘ladigan $\underline{\gamma}l,\ sh\underline{\gamma}l$ va $ch\underline{\gamma}l$ lar qiymatlarini aniqlaymiz:

$$\underline{\gamma}l = \alpha l + j\beta l = 13{,}6\cdot 10^{-3}\cdot 38 + j36{,}4\cdot 10^{-3}\cdot 38 = 0{,}505 + j1{,}385,$$
$$sh\underline{\gamma}l = sh(0{,}505 + j1{,}385) = sh\ 0{,}505\cos 1{,}385 + jch\ 0{,}505\sin 1{,}385 =$$
$$= 1{,}114e^{j85^\circ},$$

demak,

$$ch\underline{\gamma}l = ch(0{,}505 + j1{,}385) = ch\ 0{,}505\cos 1{,}385 +$$
$$+ jch\ 0{,}505\sin 1{,}385 = 0{,}56e^{j68^\circ}.$$

Liniya oxiridagi kuchlanish (7.8) formula yordamida $x = l$ va $\underline{I}_2 = \dfrac{\underline{U}_2}{Z_{\text{yuk}}}$ larni inobatga olib, quyidagicha aniqlanadi:

$$\underline{U}_2 = \frac{\underline{U}_1}{ch\underline{\gamma}l + \dfrac{\underline{Z}_T}{\underline{Z}_ю}sh\underline{\gamma}l} = \frac{10}{0{,}56e^{j68^\circ} + \dfrac{1510e^{-j20^\circ 21'}}{1355e^{j21{,}4^\circ}}1{,}114e^{j85^\circ}} = 5{,}78e^{-j52{,}25^\circ}\ \text{V}.$$

Yuklamadagi tok:

$$\underline{I}_2 = \frac{\underline{U}_2}{\underline{Z}_{\text{yuk}}} = \frac{5{,}78e^{-j52{,}25^\circ}}{1355e^{j21{,}4^\circ}} = 4{,}27e^{-j73{,}3^\circ} \text{ mA.}$$

Liniya boshidagi tokni (7.13) formula yordamida topamiz:

$$\underline{I}_1 = \underline{I}_2 ch\underline{\gamma}l + \frac{\underline{U}_2}{\underline{Z}_T} sh\underline{\gamma}l = 5{,}78e^{j32{,}83^\circ} \text{ mA.}$$

Yuklamaga ulangan liniyaning kirish qarshiligi:

$$\underline{Z}_{\text{kir}} = \frac{\underline{U}_1}{\underline{I}_1} = \frac{10}{5{,}78 10^{-3} e^{j32{,}83^\circ}} = 1732e^{-j32{,}83^\circ} \ \Omega.$$

Liniyaga berilayotgan quvvat:

$$\underline{P}_1 = \underline{U}_1 \underline{I}_1 \cos\varphi_1 = 48{,}6 \text{ mW,}$$

Liniya oxiridagi quvvat:

$$\underline{P}_2 = \underline{U}_2 \underline{I}_2 \cos\varphi_2 = 23{,}1 \text{ mW}.$$

Foydali ish koeffitsiyenti:

$$\eta = \frac{P_2}{P_1} = \frac{23{,}1}{48{,}6} = 0{,}475.$$

7.4-masala. 7.3-masalada keltirilgan liniya parametrlaridan foydalangan holda liniya oxiridan qaytgan to‘lqin akslanish koeffitsiyenti va liniyaning kirish qarshiligini toping.

Yechish. Akslanish koeffitsiyenti:

$$p = \frac{\underline{Z}_{\text{yuk}} - \underline{Z}_T}{\underline{Z}_{\text{yuk}} + \underline{Z}_T} = \frac{1355e^{j21^\circ 5'} - 1510e^{j20^\circ 21'}}{1355e^{j21^\circ 5'} + 1510e^{j20^\circ 21'}} = 0{,}385e^{j98^\circ 36'}.$$

Liniyaning kirish qarshiligini quyidagi formula yordamida topishimiz mumkin:

$$\underline{Z}_1 = \frac{\underline{U}_1}{\underline{I}_1} = \underline{Z}_T \frac{\underline{Z}_{\text{yuk}} + \underline{Z}_T th\underline{\gamma}l}{\underline{Z}_T + \underline{Z}_{\text{yuk}} th\underline{\gamma}l} = \underline{Z}_T th(\underline{\gamma}l + \underline{n}),$$

bu yerda $th\underline{n} = th\dfrac{\underline{Z}_{\text{yuk}}}{\underline{Z}_T}$ yoki $\underline{n} = \dfrac{1}{2}\ln\dfrac{\underline{Z}_T + \underline{Z}_{\text{yuk}}}{\underline{Z}_T - \underline{Z}_{\text{yuk}}}$ bo‘lib, $\underline{n}$ liniyaning to‘lqin qarshiligini yuklama bilan muvofiqlashtirilmaganligini bildiradi.

$\underline{Z}_1$ ni aniqlashda quyidagi ikki usuldan foydalanish mumkin:

1-usul. Avval $\underline{n}$ va $(\underline{\gamma}l+\underline{n})$ larni hisoblab topamiz, ya'ni:

$$\underline{n}=\frac{1}{2}\ln\frac{\underline{Z}_T+\underline{Z}_{\text{yuk}}}{\underline{Z}_T-\underline{Z}_{\text{yuk}}}=\frac{1}{2}\ln 2{,}6e^{j80{,}74^\circ}=\frac{1}{2}\ln 2{,}66+j\frac{1}{2}\frac{80{,}74^0}{57{,}3^0}=0{,}482+j0{,}711,$$

$$(\underline{\gamma}l+\underline{n})=0{,}505+j1{,}385+0{,}482+j0{,}711=0{,}987+j2{,}096==x+jy,$$

bu yerda $x=0{,}987\,\text{Np}$, $y=2{,}096\,\text{rad}$, ya'ni u ikkinchi chorakda joylashadi.

$$\left|th(\underline{\gamma}l+\underline{n})\right|=th(x+jy)=T$$

$$T=\sqrt{\frac{ch2x-\cos 2y}{ch2x+\cos 2y}}=\sqrt{\frac{ch1{,}974-\cos 4{,}192}{ch1{,}974+\cos 4{,}192}}=1{,}174,$$

$$tg\varphi_T=\pm\frac{\sin 2y}{\sin 2x}=\frac{\sin 4{,}192}{\sin 1{,}974}=-\frac{0{,}867}{3{,}516}=-0{,}247,$$

bundan,

$$\varphi_T=\text{arc tg}(-0{,}247)=-13^\circ 47'.$$

Yuqoridagi hisoblashlarga asoslanib $\underline{Z}_1$ ni topamiz:

$$\underline{Z}_1=1510e^{-j20^\circ 21'}1{,}147e^{-j13^\circ 47'}=1732e^{-j34^\circ 8'}\ \Omega.$$

2-usul. Giperbolik tangens funksiyani haqiqiy va mavhum qismlarga ajratib, liniyaning kirish qarshiligini topamiz:

$$th(x+jy)=\frac{sh2x}{ch2x+\cos 2y}+j\frac{\sin 2y}{ch2x+\cos 2y}=\frac{sh1{,}974}{ch1{,}974+\cos 4{,}192}+$$

$$+j\frac{\sin 4{,}192}{ch1{,}974+\cos 4{,}192}=1{,}114-j0{,}275=1{,}143e^{-j13^\circ 50'};$$

$$\underline{Z}_1=\underline{Z}_T th(x+jy)=1510e^{-j20^\circ 21'}1{,}143\cdot e^{-j13^\circ 50'}=1728e^{-j34^\circ 11'}\ \Omega.$$

Ta'kidlash lozimki, ko'rib chiqilgan ikkala usulning aniqlik darajasi deyarli bir xil bo'lsa-da, vaqt sarfiga ko'ra 2- usul tejamkorroq.

7.5- masala. Parametrlari R_0=2,5 Ω/km; G_0=1·10^{-6} Ω/km; L_0=1,9·10^{-3} H/km $^{-}$; G_0=8·10^{-9} F/km; bo'lgan aloqa liniya orqali diapazoni 100 Hz – 10 kHz (ovoz chastotalari) bo'lgan signal uzatilmoqda. Agar ruxsat etilgan so'nish $\alpha\ell\le 3{,}3\,\text{Np}$ bo'lsa, liniyaning eng katta uzunligi aniqlansin. Aniqlangan liniya uzunligi bo'yicha signalning uzatish vaqti qancha bo'ladi?

Yechish. Berilgan kattaliklarga ko‘ra taqsimlanish doimiyliklar $\gamma = \alpha + j\beta$ ni quyi va yuqori garmonik tashkil etuvchilari uchun aniqlaymiz. Hisoblangan natijalar 7.1- jadvalda keltirilgan.

7.1- *jadval*

f, Hz	*100*	*10·10³*
ω, Hz	*628*	*628·10³*
$Z_0 = R_0 + j\omega L_0, \quad \Omega/\text{km}$	*2,5+j1,193*	*2.5+j119,3*
$Y_0 = (G_0 + j\omega C)\cdot 10^{-6}\,\text{S/km}$	*1+5,024*	*1+j502,4*
$\gamma = \sqrt{Z_o Y_o} = (\alpha + j\beta)\cdot 10^{-3}\,\text{km}^{-1}$	*1,906+j2,99*	*2,94+j245*
$\alpha \cdot 10^{-3}, \text{Np/km}$	*1,906*	*2,94*
$\beta \cdot 10^{-3}, rad/km$	*2,99*	*245*
$u_\phi = \omega/\beta \cdot 10^{-3}, \text{km/s}$	*209,4*	*255*
$\Delta t = (1_{\max}/v_\phi)\cdot 10^{-3}, \text{s}$	*5,36*	*4,4*

So‘nishi 3,3 Np dan katta bo‘lmagan liniya uzunligini eng katta bo‘lgan so‘nish koeffitsiyentidan aniqlaymiz: $\alpha_{\max} = 2.94 \cdot 10^{-3}$ Np/km, uzatilayotgan signalning eng katta chastotasi $f_2 = 10 \cdot 10^3$ Hz ga mos keladi: $\alpha_{\max} l_{\max} = 3{,}3$ Np bundan $l_{\max} = 3{,}3/(2{,}94 \cdot 10^{-3}) = 1122$ km.

Signalning uzatish vaqtini quyi chastota bo‘yicha hisoblaymiz: $f_1 = 100$ Hz $\vartheta_{F1} = 209.4 \cdot 10^3 \text{km/s}, \quad \Delta t_1 = l_{\max}/\vartheta_{F1} = 5.36 \cdot 10^{-3}\text{s}$. Yuqori chastota uchun $f_2 = 10 \cdot 10^3$ Hz; $\vartheta_{F2} = 255 \cdot 10^3 \text{km/s}$; $\Delta t_2 = l_{\max}/\vartheta_{\phi 2} = 4.4 \cdot 10^{-3}\text{s}$.

Turli garmonikalarning liniya bo‘yicha uzatilish vaqti farqli bo‘lgani uchun, liniyadagi kirish va chiqish signallardagi garmonik tashkil etuvchilarning fazaviy munosabatlari turlicha, binobarin, chiqish signal kirish signaldan farqli bo‘ladi.

7.6- masala. Parametrlari $Z_T = 713e^{-j6}$ Ω, γ $(1{,}4 + \mathrm{j}14)\cdot 10^{-3}\,\mathrm{km}^{-1}$, $l = \lambda$ bo‘lgan liniyaning oxiri salt yurish rejimida ishlaydi. Agar liniya boshidagi kuchlanish $U_1(t)$=500sinωt, V bo‘lsa, liniya oxiridagi kuchlanish va boshidagi tok aniqlansin.

7.7- masala. Uzunligi l=55,5 km kabel liniyasi kirishidagi kuchlanish $U_1(t)$=500sin1000t. Qisqa tutashtirilganda liniya boshidagi tok $i_{1\mathrm{q.t}}$=1sin(1000t+21,5°) A, salt ishlashda esa $i_{1\mathrm{s.ish}}(t)$=1sin(1000t+68,5°) A. Agar liniya uzunligi $\lambda/2$ dan kichik bo‘lsa, liniyaning: birlamchi parametrlari R_0, G_0, L_0, C_0 hamda muvofiqlashtirilgan yuklamada liniya oxiridagi kuchlanish va tok aniqlansin.

Yechish. Liniyaning qisqa tutashtirilgan va salt ishlashidagi kirish qarshiliklari mos ravishda:

$$\underline{Z}_{\mathrm{kir.q.t}} = \frac{\underline{U}_{1m}}{\underline{I}_{1\mathrm{m.q.t}}} = \frac{500e^{j0^\circ}}{1e^{j21^\circ 30'}} = 500e^{-j21^\circ 30'}\ \Omega,$$

$$\underline{Z}_{\mathrm{kir.s.ish}} = \frac{\underline{U}_{1m}}{\underline{I}_{\mathrm{m.s.ish}}} = \frac{500e^{j0^\circ}}{1e^{j68^\circ 30'}} = 500e^{-j68^\circ 30'}\ \Omega$$

va

$$Z_T = \sqrt{Z_{\mathrm{kir.q.t}}\cdot Z_{\mathrm{kir.s.ish}}} = 500e^{-j45^\circ}\ \Omega.$$

$$\mathrm{th}\gamma l = \sqrt{Z_{\mathrm{kir.q.t}}/Z_{\mathrm{kir.s.ish}}} = \sqrt{e^{-j21^\circ 30'}/e^{-j68^\circ 30'}} = 1e^{j23^\circ 30'}.$$

Ma’lumki, $\mathrm{th}\gamma l = \dfrac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}} = \dfrac{e^{2\gamma l} - 1}{e^{2\gamma l} + 1}$, binobarin, $e^{2\gamma l} = \dfrac{1 + \mathrm{th}\gamma l}{1 - \mathrm{th}\gamma l} = \dfrac{1 + e^{j23{,}5^\circ}}{1 - e^{j23{,}5^\circ}}$.

Oxirgi ifodani ko‘rsatkich shakliga keltiramiz, bunda kompleks tekislikdagi vektorning ma’lum holatiga argumenti butun son 2π ga farq qiladigan cheksiz ko‘p kompleks sonlar mos kelishligini hisobga olamiz. Shu sababli $e^{2\gamma l} = e^{2\gamma l}e^{2\gamma\beta l} = 4{,}74e^{j(90^0 + 2n\pi)}$, binobarin,

$$2\alpha\ell = \ln 4{,}7 = 1{,}556; \quad 2\beta l = 90^0 + 2n\pi; \quad \alpha l \approx 0{,}78; \quad \beta l = 45^0 + n\pi.$$

Shart bo‘yicha liniya uzunligi $l < \dfrac{\lambda}{2}$. Bundan $\beta l < \pi$, ya’ni $\beta l = 45^\circ = 0{,}78$ rad, shu sababli $\alpha = \frac{\alpha l}{l} = \frac{0.78}{55.5} \approx 1.41 \cdot 10^{-2}\,\text{Nep/km.}$ $\beta = \frac{\beta l}{l} = \frac{0,78}{55,5} = 1,41 \cdot 10^{-2}$ rad/km va $\gamma = (1,41 + j1,41) \cdot 10^{-2}\ \text{km}^{-1}$, binobarin $u_2(t) = 229,36\sin(\omega t - 45^\circ)$ V.

Liniyaning birlamchi parametrlari:

$$R_0 + j\omega L_0 = \gamma Z_T = (1,41 + j1,41) \cdot 10^{-2} \cdot 500e^{-j45^\circ} = 2 \cdot 10^{-2} e^{j45^\circ} \cdot 500e^{-j45^\circ} =$$
$$= 10e^{j0^\circ}\ \Omega/\text{km},$$

bundan, $R = 10\ \frac{\Omega}{\text{km}}$, $L = 0$,

$$\frac{\gamma}{Z_T} = G_0 + j\omega C_0 = \frac{2 \cdot 10^{-2} e^{j45^\circ}}{500e^{-j45^\circ}} = 0,4 \cdot 10^{-4} e^{j90^\circ} = 40 \cdot 10^{-6}\ \frac{\Omega}{\text{km}},\ G_0 = 0,\ \ C_0 = 40 \cdot 10^{-6}\ \text{F}.$$

Muvofiqlashtirilgan rejimda

$$u_2(t) = U_{m1} e^{-2l} \sin(\omega t - \beta l)\ \ i_2(t) = \frac{U_{m1}}{|Z_T|} e^{-2l} \sin(\omega t - \beta l + 45^\circ).$$

Umumiy holdagi liniya $i_2(t) = 0,46\sin\omega t$ A.

7.8-masala. Liniyaning ikkilamchi parametrlari $Z_T = 500e^{-j45^o}$, $\gamma = 0,02e^{j45^\circ}\ \text{km}^{-1}$ va $l = \frac{\lambda}{8}$. Agar liniyaga berilayotgan kuchlanish $U_1(t) = 500\sin(\omega t)$ V va manbaning ichki qarshiligi nolga yaqin bo‘lsa, liniyaning yuklamasi $\underline{Z}$ qanday bo‘lganda unda eng katta quvvat ajraladi?

7.9-masala. Isrofsiz liniya parametrlari $L_0 = 2 \cdot 10^{-3}\ \text{H}/\text{km}$; $C_0 = 8 \cdot 10^{-9}\ \text{F}/\text{km}$. $R_0 = 0$; $G_0 = 0$; $l = 130$ km; Agar liniyaga berilayotgan kuchlanish $U_1(t) = 500\sin(2\pi \cdot 4167)t$ V va liniya oxirida $Z_{\text{yuk}}=Z_T$; b) $Z_{\text{yuk}}=\infty$; v) $Z_{\text{yuk}}=0$; bo‘lganda Z_T, v_f, λ aniqlansin va *a) – b)* holatlardagi yuklamadagi kuchlanish va tokning oniy qiymat ifodalari aniqlansin.

7.10-masala. Uzunligi l=62,5 km isrofsiz liniyaning to‘lqin qarshiligi Z_T=500 Ω da kirish qarshiligi Z_{kir}= – j134 Ω, liniyaning ishchi to‘lqin uzunligi λ=100 km. Liniya oxiriga ulangan yuklamaning moduli va xarakteri aniqlansin.

7.11-masala. Sirdaryo GRES – Toshkent elektr energiyaning uzatish liniyasining uzunligi 110 km bo‘lib, kuchlanishi $U_l = 500$ kV, chastotasi

$f = 50$ Hz. Liniyaning loyiha birlamchi parametrlari quydagicha: $R_0 = 0{,}08\ \Omega/\text{km}$, $L_0 = 1{,}336 \cdot 10^{-3}$ H/km, $C_0 = 8{,}6 \cdot 10^{-9}$ F/km. Har bir fazaning izolatsiya va tojli razryad isroflari yig'indisi 2500 W/km. Liniyaning ikkilamchi parametrlari, to'lqin uzunligini va fazaviy tezligi aniqlansin.

Yechish. Formula $P_0 = U_l^2 G_0$ dan o'tkazuvchanlikni topamiz:

$$G_0 = \frac{P_2}{U_f^2} = \frac{2500}{(500 \cdot \sqrt{3})^2 \cdot 10^6} = 2{,}99 \cdot 10^{-8}\ \text{S/km}.$$

Liniyaning har bir kilometridagi kompleks qarshiligi va o'tkazuvchanligi:

$$Z_0 = R_0 + j\omega L_0 = 0{,}08 + 2 \cdot 3{,}14 \cdot 50 \cdot 1{,}336 \cdot 10^{-3} = 0{,}427\, e^{j79^\circ 20'}\ \Omega/\text{km},$$

$$Y_0 = G_0 + j\omega C_0 = 2{,}99 \cdot 10^{-8} + j2 \cdot 3{,}14 \cdot 50 \cdot 8{,}6 \cdot 10^{-9} \approx 2{,}76 \cdot 10^{-6} e^{j90^\circ}\ \text{S/km}.$$

Liniyaning to'lqin qarshiligi xarakteristikalari:

$$Z_T = \sqrt{Z_0 / Y_0} = \sqrt{0{,}427 \cdot e^{j79^0 13'} / 2{,}45 \cdot 10^{-6} \mathring{a}^{+j90^0}} = 417{,}5 \cdot \mathring{a}^{-j5^0 23'}\ \Omega;$$

$$\gamma = \sqrt{Z_0 Y_0} = \sqrt{0{,}427\, e^{j79^\circ 20'} \cdot 2{,}70 \cdot 10^{-6} e^{j90^\circ}} = 1{,}073 \cdot 10^{-3}\, e^{j84^\circ 37'}\ 1/\ \text{km};$$

$$\alpha \approx 1{,}073 \cdot 10^{-3} \cos 84{,}6^\circ = 0{,}101 \cdot 10^{-3}\ \text{dB/km};$$

$$\beta \approx 1{,}073 \cdot 10^{-3} \sin 84{,}6^\circ = 1{,}068 \cdot 10^{-3}\ \text{rad/km};$$

$$\lambda = \frac{2\pi}{\beta} = 2 \cdot \frac{3{,}14}{1{,}068 \cdot 10^{-3}} = 5883\ \text{km},$$

$$c = \lambda f = 5883 \cdot 50 = 296000\ \text{km/s}.$$

7.12-masala. Oldingi masala natijalarida agar Toshkentda qabul qilanayotgan quvvat 300 MW, $U_2 = \sqrt{3} \cdot 280$ kV; cosφ=1 bo'lsa, quydagilarni aniqlansin. 1) Toshkentdagi (liniya oxiri)tok; 2) Sirdaryo GRES liniya boshidagi va oxiridagi kuchlanishlar orasidagi fazalar siljishi; 3) liniya FIK.

Yechish.1) Yuklamaning aktiv qarshiligida (cosφ=1) Toshkentdagi tok:

$$I_2 = \frac{P_2}{3U_{2af} \cos\varphi_2} = \frac{300 \cdot 10^6}{3 \cdot 28867 \cdot 1} = 346{,}4\ \text{A}.$$

2) Kompleks argument:

$$\gamma l = \alpha l + j\beta l = 0{,}101 \cdot 10^{-3} \cdot 110 + j1{,}068 \cdot 10^{-3} \cdot 110 = 0{,}011 + j0{,}1174.$$

dan giperbolik funksiyalarning jadvallaridan yoki mikrokalkulatordan foydalanib hisoblaymiz:

$$\alpha l = 0{,}011, \quad \beta l = j0{,}1174\,\text{rad.}$$

radiani gradusga aylantiramiz:

$$0{,}1174 \cdot \frac{180}{3{,}14} = 6{,}72^\circ;$$

$$e^{\alpha l} = e^{0{,}01} = 2{,}718^{0{,}011} = 1{,}011; \; e^{-\alpha l} = \frac{1}{1{,}011} = 0{,}9891;$$

$$e^{j\beta l} = e^{j6{,}72} = 0{,}99996 + j0{,}11701;$$

$$e^{\gamma l} = e^{\alpha l} \cdot e^{\beta l} = 1{,}011(0{,}99996 + j0{,}11701) = 1{,}0109 + j0{,}01183;$$

$$e^{\gamma l} = e^{-\alpha l} \cdot e^{-\beta l} = 0{,}9891(0{,}99996 - j0{,}11701) = 0{,}9890 - j0{,}1170;$$

$$\text{sh}(\gamma l) = \frac{e^{\gamma l} - e^{-\gamma l}}{2} = \frac{1{,}0109 + j0{,}1183 - 0{,}9890 + j0{,}1157}{2} = 0{,}1095 + j0{,}1179 =$$
$$0{,}1602 e^{j46{,}89^0};$$

$$\text{ch}(\gamma l) = \frac{e^{\gamma l} + e^{-\gamma l}}{2} = \frac{1{,}0109 + j0{,}1183 + 0{,}9890 - j0{,}1157}{2} = 0{,}9999 + j0{,}0026 =$$
$$= 0{,}9963 e^{j0{,}23^0}.$$

Sirdaryo GRES dagi faza kuchlanish va tokini quydagicha hisoblaymiz:

$$U_1 = U_2 \text{ch}\gamma l + I_2 Z_T \text{sh}\gamma l = \sqrt{3} \cdot 280 \cdot 10^3 \cdot 0{,}9963 e^{j0{,}23} +$$
$$+ 346{,}4 \cdot 417{,}5 e^{-j5{,}4^\circ} \cdot 0{,}1602 e^{j46{,}89^\circ} = 483179 e^{j0{,}23} + 23168{,}40 e^{j41{,}49} =$$
$$= 500252{,}9 e^{j1{,}88}$$
$$= 482607 e^{j0{,}23} + 149047{,}5 e^{-j5^\circ 23'} = 631082 - j11053 = 631178 e^{j1{,}003^\circ} \text{ V.}$$

$$I_1 = \frac{U_2}{Z_T} \text{sh}\gamma l + I_2 \text{ch}\gamma l = \frac{288675 \cdot 10^3}{397{,}6 \cdot e^{-j5{,}4^\circ}} \cdot 0{,}1602 e^{j46{,}89} + 346{,}41 \cdot 0{,}9909 e^{j0{,}14} =$$
$$= 352{,}23 e^{j12{,}53^\circ} \text{ A.}$$

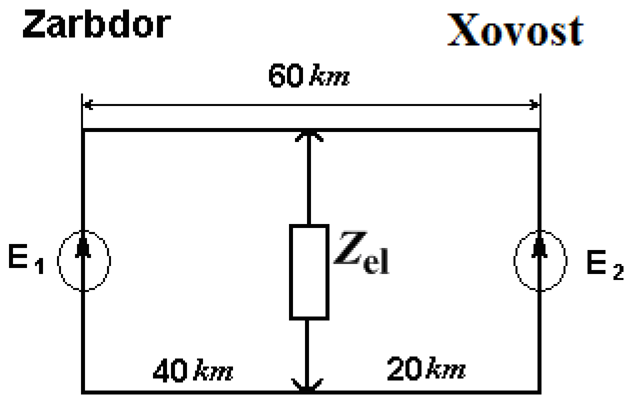
Ya`ni liniya boshidagi tokning faza siljishi kuchlanish fazasiga nisbatan $10{,}55^\circ$ ilgarilaydi.

3) GRES ning liniyaga berayotgan aktiv quvvat:

$$P_1 = 3U_1 I_1 \cos\varphi_1 = 3 \cdot 500252{,}9 \cdot 352{,}2 = \cos\left(-19{,}8^\circ - 12{,}53^\circ\right) = 311{,}84\,\text{MW.}$$

4) Liniyaning FIK $\eta = \frac{P_2}{P_1} \cdot 100\% = \frac{300}{311,84} \cdot 100\% = 96,2\ \%$.

7.13- masala. O‘zbekiston temir yo‘llariga tegishli magistral liniyadagi “Zarbdor” va “Xovost” tortuvchi nimstansiyalar orasidagi masofa $l = 60\ \text{km}$. Ular orasidagi kontakt tarmog‘i ikkila tomondan kuchlanishi 27,5 kV bo‘lgan manbalar orqali ta`minlanadi. Agar kontakt tarmog‘ining ikkilamchi parametrlari $\alpha = 0,465 \cdot 10^{-3}\ \frac{\text{Np}}{\text{km}}$; $\beta = 1,55 \cdot 10^{-3}\ \text{rad/km}$; $Z_T = 292 e^{j11^0 57'}$, nominal tok $I = 350\ \text{A}$, quvvat koeffitsiyenti $\cos\varphi = 0,8$ bo‘lgan “O‘zbekiston” rusumidagi elektrovoz birinchi nimstansiyadan 40 km uzoqlikda harakatlanayotgan bo‘lsa, elektrovozga tushayotgan kuchlanish va tok aniqlansin (7.3- rasm).

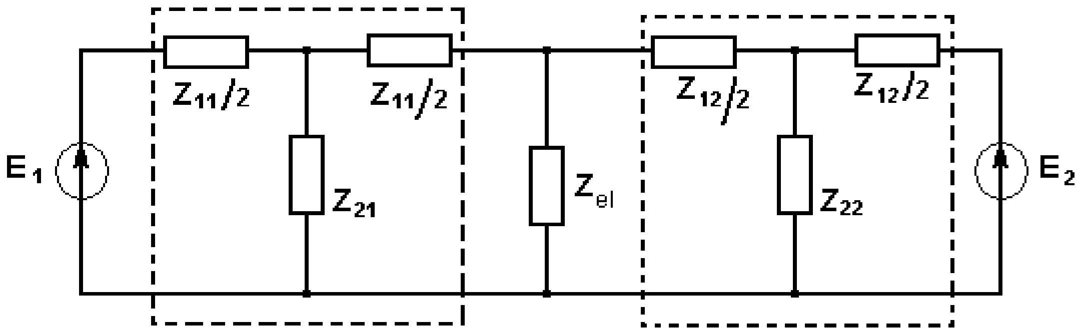


7.3- rasm.

Yechish. Kontakt tarmog‘iga tegishli uzatish liniyaning tarqalish koeffitsiyenti

$$\gamma = \alpha + j\beta = 0,465 \cdot 10^{-3} + j1,55 \cdot 10^{-3} = \sqrt{(0,465 \cdot 10^{-3})^2 + (1,55 \cdot 10^{-3})^2} \cdot e^{jarctg\frac{1,55 10^{-3}}{0,465 10^{-3}}} =$$
$$= 1,61 e^{j73,3}\ {}^{1}\!/_{\text{km}}.$$

Har bir nimstansiyadan elektrovozgacha bo‘lgan kontakt tarmog‘ining qarshiliklarini ekvivalent T-simon to‘rtqutbliklar bilan almashtirsak, zanjirni hisoblash sxemasi quyidagi ko‘rinishga ega bo‘ladi (7.4- rasm):



7.4- rasm.

Zarbdordan elektrovozgacha bo‘lgan 40 km uzunlikdagi kontakt tarmog‘ining qarshiligi ekvivalent simmetrik T-simon to‘rtqutblikka almashtiriladi.

$$\underline{Z}_{11} = \frac{2(\mathrm{ch}\underline{\gamma l} - 1)}{\mathrm{sh}\underline{\gamma l}} \cdot \underline{Z_T};$$

$$\underline{Z}_{21} = \frac{\underline{Z}_T}{\mathrm{sh}\underline{\gamma l}}$$

bunda, $\underline{Z}_{11,}\underline{Z}_{21}$ – shu T-simon to‘rtqutblikning mos ravishda bo‘ylama va ko‘ndalang qarshiligi.

$\mathrm{ch}\gamma l = \dfrac{e^{\gamma l} + e^{-\gamma l}}{2}$ va $\mathrm{sh}\gamma l = \dfrac{e^{\gamma l} - e^{-\gamma l}}{2}$ larni e’tiborga olib, ularga son qiymatlar qo‘yiladi:

$$\mathrm{ch}\gamma l = \mathrm{ch}(\alpha l + \beta l) = \mathrm{ch}(0{,}465 \cdot 10^{-3} \cdot 40 + j1{,}55 \cdot 10^{-3} \cdot 40) =$$

$$= \frac{e^{0{,}0186 + j0{,}062} + e^{-0{,}0186 - j0{,}062}}{2} = \frac{e^{0{,}0186 + j0{,}062} + e^{-0{,}0186 - j0{,}062}}{2} =$$

$$= \frac{1{,}02(\cos 3{,}55^0 + j\sin 3{,}55^0) + 0{,}98(\cos 3{,}55^o - j\sin 3{,}55^0)}{2} =$$

$$= \frac{0{,}999 + j0{,}06315 + 0{,}996 - j0{,}0607}{2} = 0{,}9975 - j0{,}0012;$$

$$\mathrm{sh}\gamma l + \mathrm{sh}(\alpha l + j\beta l) = \frac{0{,}999 + j0{,}06315 - 0{,}0607}{2} = 0{,}0015 + j0{,}0619 =$$

$$= 0{,}06192 e^{\mathrm{arctg}\frac{0{,}0612}{0{,}0015}} = 0{,}06192 e^{j88{,}60^0};$$

$$Z_{11}=\frac{2(\mathrm{ch}\gamma l-1)}{\mathrm{sh}\gamma l}\cdot Z_T=\frac{2(0{,}0{,}9975-j0{,}0012-1)}{0{,}06192e^{j88{,}6^0}}=\frac{2(-0{,}0025-j0{,}0012)}{0{,}06192e^{j88{,}6^0}}\cdot Z_T=$$

$$=\frac{2\cdot 0{,}00277e^{j\mathrm{arctg}\frac{0{,}0012}{0{,}0025}}}{0{,}06192e^{j88{,}6^0}}\cdot 292e^{j11{,}9^0}=\frac{2\cdot 0{,}00277e^{j(25{,}84+180^0)}}{0{,}06192e^{j88{,}6}}\cdot 292e^{j11{,}9^0}=26{,}125e^{+j128{,}94}\Omega;$$

$$Z_{21}=\frac{Z_c}{\mathrm{sh}\gamma l}=\frac{292e^{j11{,}9}}{0{,}066192e^{j88{,}60^0}}=4715{,}76e^{-j76{,}7}.$$

Xovost nimstansiyadan $l=20$ km uzunlikdagi kontakt tarmog‘i ekvivalent T-simon simmetrik to‘rtqutblik bilan almashtiriladi:

$$\mathrm{sh}\gamma l=\frac{e^{+\gamma l}-e^{-\gamma l}}{2}=\frac{1{,}0087+j0{,}0313-0{,}9902+j0{,}0370}{2}=0{,}00925-j0{,}031=$$
$$=0{,}04438\cdot e^{j73{,}38};$$

$$Z_{12}=\frac{2(\mathrm{ch}\gamma l-1)}{\mathrm{sh}\gamma l}\cdot Z_c=\frac{2(0{,}9994+j0{,}0003-1)}{0{,}0438e^{j73{,}38}}\cdot 292e^{j11{,}9^0}=\frac{2(0{,}00067)e^{-j26{,}56^0}}{0{,}0438e^{j73{,}38^0}}\cdot 292e^{j11{,}9^0}=$$
$$=4{,}46e^{-j88{,}004};$$

$$Z_{22}=\frac{Z_T}{\mathrm{sh}\gamma l}=\frac{292e^{j11{,}9^0}}{0{,}0438e^{j73{,}38}}=6666{,}6e^{-j61{,}48^0}.$$

7.4-rasmdagi zanjir biroz soddalashtiriladi: Elektrovoz qarshiligi Z_{el} ga nisbatan chap va o‘ng tomonda zanjir qismlari ekvivalent manbalar bilan almashtiriladi:

chap tomondagi ekvivalent manbaning ichki qarshiligi:

$$Z_{1.\mathrm{ich}}=\frac{Z_{11}/2\cdot Z_{21}}{Z_{11}/2+Z_{21}}+Z_{11}/2=\frac{\frac{26{,}125e^{j128{,}94}}{2}\cdot 475{,}76e^{-j76{,}7}}{\frac{26{,}125e^{j128{,}94}}{2}+475{,}76e^{-j76{,}7}}+\frac{26{,}125e^{j128{,}94}}{2}=$$

$$=\frac{13{,}0625e^{j128{,}94^0}\cdot 4715{,}76e^{-j76{,}7^0}}{13{,}0625e^{-j128{,}94^0}+4715{,}76e^{-j76{,}7^0}}+\frac{26{,}125e^{j128{,}94}}{2}=-5{,}967+j7{,}3933-8{,}20+10{,}16=$$
$$=-14{,}16+j17{,}39=22e^{j129{,}16^0};$$

Chap tomondagi shartli ekvivalent manbaning (7.4-rasm) EYK i:

$$\underline{E}_{1.\text{ekv}} = \frac{\underline{E}_1}{\underline{Z}_{11}/2 + \underline{Z}_{21}} \cdot \underline{Z}_{21} = \frac{27,5\cdot 10^3}{6475,85e^{-j76,64}} \cdot 4715,76e^{-j76,7^0} = 20,02e^{-j0,06^0} \text{ kV.}$$

O‘ng tomondagi shartli ekvivalent manbaning ichki qarshiligi:

$$\underline{Z}_{1.\text{ich}} = \frac{\underline{Z}_{12}/2 \cdot \underline{Z}_{22}}{\underline{Z}_{12}/2 + \underline{Z}_{22}} + \underline{Z}_{12}/2 = \frac{\dfrac{4,46e^{-j88,04}}{2} \cdot 6666,6e^{-j61,8}}{\dfrac{4,46e^{-j88,04}}{2} \cdot 6666,6e^{-j61,48}} + \frac{4,46e^{-j88,04^0} \cdot 4715,76e^{-j76,7^0}}{2}$$

$$= 2,61e^{-j88,4^0}.$$

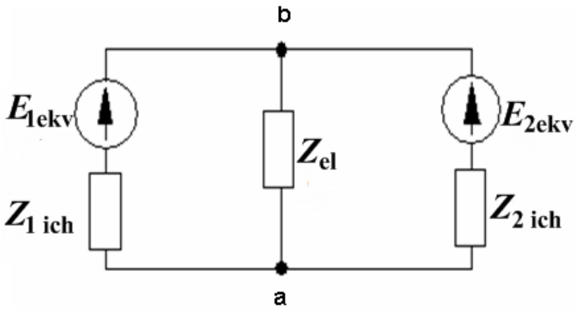
O‘ng tomondagi ekvivalent manbaning EYK i:

$$\underline{E}_{2.\text{ekv}} = \frac{\underline{E}_2}{\underline{Z}_{12}/2 + \underline{Z}_{22}} \cdot \underline{Z}_{22} = \frac{27,5\cdot 10^3}{6475,85e^{-j61,50^0}} \cdot 6666,6e^{-j61,48^0} = 27,49e^{+j0,02^0} \text{kV}$$

Shu o‘zgartirishlardan keyin zanjir quyidagi ko‘rinishga ega bo‘ladi (7.5-rasm). Izlanayotgan kuchlanish va tokni ikki tugun usulida aniqlaymiz.

Elektrovozning berilgan parametrlariga kompleks qarshiligi topiladi:

$$\underline{Z}_{\text{el}} = R_{\text{el}} + j\omega L_{\text{el}} = 105 + j314 \cdot 0,250 = \sqrt{105^2 + 78^2}\, e^{j arctg\frac{78}{105}} = 130,80e^{j36,6^0}\ \Omega;$$



7.5- rasm.

$$\underline{U}_{ab}=\frac{\underline{E}_{1\text{ekv}}\cdot\dfrac{1}{\underline{Z}_{1.ic}}+\underline{E}_{2\text{ekv}}\dfrac{1}{\underline{Z}_{2.\text{ich}}}}{\dfrac{1}{\underline{Z}_{1.\text{ich}}}+\dfrac{1}{\underline{Z}_{2.\text{ich}}}+\dfrac{1}{\underline{Z}_{\text{el}}}}=\frac{20{,}02e^{-j0{,}06^0}\cdot\dfrac{1}{22{,}42e^{j129{,}1^0}}+27{,}49e^{+j0{,}02^0}\cdot\dfrac{1}{2{,}619e^{-j88{,}4^0}}}{\dfrac{1}{22{,}40e^{j0{,}02^0}}+\dfrac{1}{2{,}459e^{-88{,}4^0}}+\dfrac{1}{89{,}65e^{j36{,}6^0}}}=$$

$$=\frac{0{,}892e^{-129{,}06^0}+10{,}51e^{j88{,}06^0}}{0{,}044e^{-0{,}02^0}+0{,}406e^{+j88{,}04^0}+0{,}0076e^{-36{,}6^0}}=\frac{11{,}0905e^{-89{,}9^0}}{0{,}3656e^{-80{,}17^0}}=26{,}8e^{-\text{j}9{,}73^0}\ \text{kV.}$$

Elektrovoz toki:

$$\underline{I}_{\text{el}}=\frac{\underline{U}_{ab}}{\underline{Z}_{\text{el}}}=\frac{26{,}8e^{-j9{,}73}\cdot 10^3}{89{,}65e^{j5{,}23^0}}=298{,}94e^{-\text{j}14{,}96^0}\ \text{A.}$$

Uzun uzatish liniyalarini hisoblash.

7.1-vazifa. Liniyaning 7.1- jadvalda berilgan R_0, L_0, C_0, G_0, chastotasi f, liniya uzunligi l, liniya oxiridagi kuchlanish U_2 hamda tok I_2 va yuklama qarshiligi Z_{yuk} bo‘yicha quyidagilarni hisoblang:

1. Liniya boshidagi kuchlanish U_1 va tok I_1, liniya boshidagi va oxiridagi aktiv P va to‘la S quvvatlar hamda liniyaning F.I.K.

2. 1- banddagi liniyani isrofsiz deb faraz qilib, ya’ni $R_0 = G_0 = L_0 = 0$ va liniya oxiridagi yuklama aktiv xarakterga ega va qiymat jihatidan kompleks qarshilikning moduliga teng deb, liniya boshidagi kuchlanish va elektromagnit to‘lqin uzunligi λ aniqlansin.

3. 2- banddagi liniya uchun kuchlanish effektiv qiymatini liniya bo‘ylab taqsimlanish grafigini quring.

7.1- *jadval*

Variant №	f, Hz	l, km	R_0, Ω/km	$C_0 \cdot 10^{-9}$, F/km	$L_0 \cdot 10^{-3}$, H/km	$G_0 \cdot 10^{-6}$, S/km	U_2, V	I_2, A	Z_{yuk}, Ω
1	50	56	0,2	20	30	0,625	27500	$300e^{j30^\circ}$	$10e^{j30^\circ}$
2	1000	250	5	4,8	2,08	0,7	50	-	$400e^{j15^\circ}$
3	10000	500	29	5,9	2,6	0,75	50	-	$300e^{j30^\circ}$
4	50	40	0,3	30	26	0,8	27500	$350e^{j45^\circ}$	-
5	1800	80	12	5,75	2,1	0,6	24,4	-	$500e^{j60^\circ}$
6	3000	120	20	5,6	3,1	0,85	39,3	-	$400e^{j15^\circ}$
7	50	42	0,4	45	36	0,84	27500	$400e^{j60^\circ}$	-
8	7000	300	25	6	3,6	0,9	20	$0{,}02e^{j30^\circ}$	-
9	50	35	0,5	50	28	0,68	6000	$200e^{j30^\circ}$	-
10	800	50	2,9	6,25	4,0	0,66	-	$0{,}5e^{j20^\circ}$	$500e^{j30^\circ}$
11	600	40	2,5	6,35	2,8	0,72	10	-	$300e^{j45^\circ}$
12	50	48	0,35	60	20	0,77	10000	$300e^{j45^\circ}$	-
13	950	55	5,6	6,8	3,0	0,85	10	$0{,}3e^{j45^\circ}$	-

14	900	60	8	6,6	1,6	0,84	20	-	$600e^{j30^\circ}$
15	8000	400	48,5	4,5	5,6	0,91	10	-	$800e^{j20^\circ}$
16	50	36	0,26	45	32	0,65	27500	$450e^{j60^\circ}$	-
17	1200	100	6,2	4,6	3,4	0,74	-	$0{,}12e^{j30^\circ}$	$300e^{j45^\circ}$
18	1600	110	10	4,8	3,2	0,67	60	$0{,}33e^{j45^\circ}$	-
19	50	40	0,45	50	40	0,73	10000	$250e^{j60^\circ}$	-
20	500	40	5	5,2	2,9	0,84	30	$0{,}7e^{j15^\circ}$	-
21	1100	90	25,5	5,6	3,4	0,64	18	$0{,}4e^{j45^\circ}$	-
22	200	30	2,1	4,9	4,2	0,79	22	-	$200e^{j45^\circ}$
23	50	50	0,6	35	42	0,69	27500	$350e^{j60^\circ}$	-
24	1300	120	6,5	6,2	4,6	0,8	20	$0{,}15e^{j45^\circ}$	-
25	1650	150	8,6	5,3	4,9	0,75	50	$0{,}12e^{j60^\circ}$	-
26	50	30	0,48	30	38	0,7	10000	$280e^{j30^\circ}$	-
27	50	46	0,55	40	29	0,72	35000	$600e^{j60^\circ}$	-
28	2500	180	12,5	4,8	5,3	0,66	40	$0{,}2e^{j15^\circ}$	-
29	50	42	0,33	50	35	0,88	27500	$500e^{j60^\circ}$	-
30	3600	200	18,6	5,9	6,2	0,76	30	$0{,}16e^{j45^\circ}$	-

8- BOB. ELEKTROMAGNIT MAYDONNI HISOBLASHGA DOIR MASALALAR YECHISH

8.1. Asosiy ma'lumotlar

Elektromagnit maydon ikkita asosiy vektor bilan xarakterlanad: *Elektr maydonning* kuchlanganligi *E* va *magnit maydonning induksiyasi B.*

Elektr maydonning kuchlanganligi qo'zg'almas nuqtaviy *q* zaryadga ta'sir qiladigan *F* kuch bo'yicha aniqlanadi:

$$F = qE. \tag{8.1}$$

Magnit maydonning induksiyasi o'zgarmas V tezlik bilan harakatlanadigan zaryadga ta'sir qiladigan kuch bilan aniqlanadi, ya'ni

$$F = q[\overline{V}\ \overline{B}], \tag{8.2}$$

yoki i tok o'tayotgan l uzunlikdagi chiziqli elementga ta'sir qilayotgan kuch bilan aniqlanadi, ya'ni

$$F = [(i\,\overline{l^0})\overline{B}]l, \tag{8.3}$$

bunda $\overline{l^0}$ – tok yo'nalishiga mos bo'lgan birlik vektor.

Dielektrik uchun

$$\overline{D} = \varepsilon_0 \overline{E} + \overline{P}, \tag{8.4}$$

bunda $\overline{D}$ va $\overline{P}$ – mos ravishda *elektr siljish* va qutblanish vektorlari; ε_0– *elektr singdiruvchilik*, o'zgarmas kattalik.

Chiziqli dielektrik uchun

$$\overline{P} = \varepsilon_0\, \chi_r \overline{E} = \varepsilon_0(\varepsilon_r - 1)\overline{E}, \tag{8.5}$$

$$\overline{D} = \varepsilon_r \varepsilon_0 \overline{E}, \tag{8.6}$$

bunda χ_r – nisbiy dielektrik ta'sirchanlik; ε_r – nisbiy dielektrik singdiruvchanlik.

Magnit muhit uchun

$$\overline{B} = \mu_0 \overline{H} + \overline{M}, \tag{8.7}$$

bu yerda $\overline{H}$ va $\overline{M}$ – magnit maydon kuchlanganligi va magnitlanganligi vektorlari; μ_0– magnit singdiruvchanlik, o'zgarmas kattalik.

Chiziqli magnit muhit uchun

$$\overline{M} = \chi \overline{H} , \quad (8.8)$$

bunda χ – magnit ta'sirchanlik;

$$\overline{B} = \mu_r \mu_0 \overline{H} , \quad (8.9)$$

bunda μ_r – muhitning nisbiy magnit singdruvchanligi.

Elektromagnit maydondagi nuqtaviy q_1 va q_2 zaryadlarning o'zaro ta'sir kuchi Kulon qonuni bo'yicha aniqlanadi:

$$F = \frac{q_1 q_2}{4\pi\varepsilon_r\varepsilon_0 R^2} , \quad (8.10)$$

bu yerda R – zaryadlar orasidagi masofa.

Nuqtaviy q zaryad kuchlanganligi:

$$E = \frac{q\overrightarrow{R^0}}{4\pi\varepsilon_2\varepsilon_0 R^2} , \quad (8.11)$$

bunda $\overrightarrow{R^0}$ – zaryad q dan kuzatish nuqtasiga yo'naltirilgan birlik vektor; R – zaryad q dan kuzatish nuqtasigacha bo'lgan masofa.

Potensialli elektr maydon uchun A va B nuqtalar orasidagi potensiallar ayirmasi:

$$\varphi_A - \varphi_B = \int_A^B E dl . \quad (8.12a)$$

Potensial

$$\varphi = -\int E dl + C , \quad (8.12b)$$

bunda dl – integrallash yo'lining elementi; C – integral doimiysi.

i_1 va i_2 toklar o'tayotgan ikkita cheksiz uzun parallel ingichka o'tkazgichning l uzunlikdagi kesmalariga ta'sir etayotgan kuch:

$$F = \mu_r \mu_0 \frac{i_1 i_2}{2\pi r} l , \quad (8.13)$$

bu yerda r – o'tkazgichlar orasidagi masofa.

i tok o‘tayotgan cheksiz uzun ingichka o‘tkazgichning induksiyasi:

$$B = \mu_r \mu_0 \frac{i}{2\pi r}[l^0 r^0], \tag{8.14}$$

bunda $r -$ tok o‘tkazgichning o‘qidan kuzatish nuqtasi M orasidagi masofa (8.1-rasm); l^0 – tok yo‘nalishi bilan mos bo‘lgan birlik vektor; r_0 – o‘tkazgich o‘qidan kuzatish nuqtasiga yo‘naltirilgan birlik vektor.

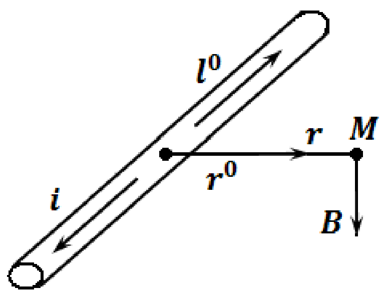
Elektromagnit maydonning integral shakldagi tenglamalari.

Gauss teoremasi

$$\oint \overline{D} d\overline{S} = \sum q \tag{8.15}$$

yoki

$$\oint \varepsilon_0 \overline{E} d\overline{S} = \sum (q + q_{\text{bog'}}) \,. \tag{8.16}$$



8.1- rasm.

Bir jinsli muhit uchun ($\varepsilon_r = \text{const}$)

$$\oint \overline{E} d\overline{S} = \frac{\sum q}{\varepsilon_0 \varepsilon_r} \,. \tag{8.17}$$

Qutblanish vektori P va bog‘langan $q_{\text{bog‘}}$ zaryad orasidagi bog‘lanish:

$$\oint PdS = -\sum q_{\text{bog‘}} \,, \tag{8.18}$$

(8-15) – (8.18) formulalarda $\sum q$ va $\sum q_{\text{bog‘}} - S$ sirt ichida joylashgan erkin va bog‘langan zaryadlarning algebraik yig‘indisi.

To‘la tok qonuni

$$\oint Hdl = \sum i = \int J_{to'la} dS \,, \tag{8.19}$$

bunda i – integrallash konturi bilan o‘rab olingan to‘la tok; $J_{to'la}$ - to‘la tokning zichligi;

$$J_{to'la} = J + \partial \overline{D} / \partial t\,, \tag{8.20}$$

J – o‘tkazuvchanlik tokining zichligi.

Elektromagnit induksiya (Faradey) qonuni:

$$e = \oint \overline{E} d\bar{l} = -\partial \overline{\Phi} / \partial t = -\frac{\partial}{\partial t} \int \overline{B} d\overline{S}\,. \tag{8.21}$$

Kompleks shaklda

$$E = \oint \overline{E} dl = j\omega \overline{\Phi}\,, \tag{8.22}$$

bunda e va E – EYK; $\overline{\Phi}$ – magnit oqim.

Magnit oqimning uzliksizlik tenglamasi

$$\oint \overline{B} d\overline{S} = 0\,. \tag{8.23}$$

To‘la tokning uzliksizlik tenglamasi

$$\oint \overline{J}_{\text{to'la}}\, d\overline{S} = 0 \tag{8.24}$$

yoki

$$\oint JdS = -\partial (\sum q) / \partial t\,. \tag{8.25}$$

Elektromagnit maydonning differensial shakldagi tenglamalari.

Gauss teoremasi

$$\text{div}\overline{D} = \rho\,, \tag{8.26}$$

$$\text{div}\varepsilon_0 \overline{E} = \rho + \rho_{bog'}\,. \tag{8.27}$$

Bir jinsli muhit uchun:

$$\text{div}E = \rho / \varepsilon_r \varepsilon_0\,. \tag{8.28}$$

Qutblanish vektori $\overline{P}$ va bog‘langan zaryadning hajmiy zichligi $\rho_{\text{bog‘}}$ orasidagi bog‘lanish:

$$\text{div}\overline{P} = -\rho_{\text{bog‘}}, \tag{8.29}$$

bunda ρ – zaryadning hajmiy zichligi.

To‘la tok qonuni

$$\mathrm{rot}\overline{H} = \overline{J}_{\mathrm{to'la}} = \overline{J} + \frac{\partial D}{\partial t}. \qquad (8.30)$$

Elektromagnit induksiya qonuni

$$rot\overline{E} = -\frac{\partial \overline{B}}{\partial t}. \qquad (8.31)$$

Magnit oqimning uzluksizligi tenglamasi:

$$\mathrm{div}\overline{B} = 0, \qquad (8.32)$$

va bir jinsli muhit uchun

$$\mathrm{div}\overline{H} = 0 \qquad (8.33)$$

To‘la tok uzliksizligining tenglamasi

$$div(\overline{J} + \partial\overline{D}/\partial t) = 0 \qquad (8.34)$$

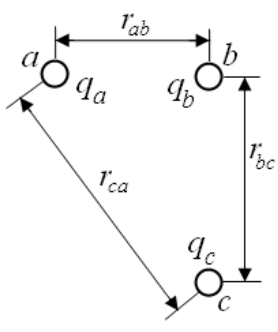
Elektr maydonning energiyasi $W_E = \int \frac{ED}{2} dV$. (8.35)

Magnit maydonning energiyasi $W_M = \int \frac{BH}{2} dV$. (8.36)

Potensial elektr maydon uchun $\overline{E} = -\mathrm{grad}\,\varphi$. (8.37)

8.2. Elektromagnit maydonga doir masalalar

8.1-masala. 8.2-rasmda tasvirlangan vakuumdagi nuqtaviy zaryadlarga ta’sir qiluvchi kuchni aniqlang. Berilgan: $q_a = 4\cdot 10^{-12}$C, $q_b = 15\cdot 10^{-12}$C, $q_c = 15\cdot 10^{-12}$C Zaryadlar orasidagi masofalar: $r_{ab} = 0{,}3$ mm; $r_{bc} = 0{,}4$ mm; $r_{ca} = 0{,}5$ mm.



8.2- rasm.

Yechish. Zaryad q_a ta’sir qiluvchi kuch $F_a = q_a E_a$, bunda E_a – elektr maydonning zaryad joylashgan o‘rnidagi kuchlanganlik, ya’ni

$$E_a = E_{ca} + E_{bc} = \frac{q_c r^0{}_{ca}}{4\pi\varepsilon_0 r_{ca}^2} + \frac{q_b r_{ba}^0}{4\pi\varepsilon_0 r_{ba}^2},$$

bunda r_{ca} – nuqta c dan a nuqtaga yo‘naltirilgan birlamchi radius- vektor; r_{ba} - *b* nuqtadan *a* nuqtaga yo‘naltirilgan birlamchi radius - vektor.

Dekart koordinata sistemasida:

$$E_a = -(\frac{q_c \cos\alpha_1}{4\pi\varepsilon_0 r_{ca}^2} + \frac{q_b}{4\pi\varepsilon_0 r_{ba}^2})i + (\frac{q_c \sin\alpha_1}{4\pi\varepsilon_0 r_{ca}^2})j,$$

bunda $\cos\alpha_1 = \frac{r_{ba}}{r_{ca}} = \frac{0,3}{0,5} = 0,6;\ \sin\alpha = \frac{r_{bc}}{r_{ca}} = \frac{0,4}{0,5} = 0,8$, i va j – Dekart koordinatalar sistemasida abssissa va ordinata o‘qlarining yo‘nalishini belgilaydigan vektorlar.

Bu holda $F_a = (-7,25i + 1,72j)$ mkN.

Shunga o‘xshash, $F_b = (6,0i + 14,3j)$ mkN,

$$F_c = (1,2i - 14,52j) \text{ mkN.}$$

8.2-masala. Bir xil zaryadlangan ikkita zoldrcha uzunligi 50 sm iplar bilan umumiy nuqtaga osib qo‘yilgan. Agar har bir zoldrirning massasi 16 mg va zaryadi $5\cdot10^{-8}$ C bo‘lsa, iplar orasidagi burchak aniqlansin.

8.3-masala. Vakuumdagi yassi metall plastina sirtining bir tomonida bir xil taqsimlangan zaryad $q_S = 10^{-10}$ C/m^2 mavjud. Plastina qirralaridan uzoqda bo‘lgan nuqtalarning *E* kuchlanganligini va potensialini aniqlang. Plastinadan *a* masofada joylashgan $q = 10^{-12}$ C sinov zaryadga ta’sir qiladigan kuchni toping.

Yechish. Cheksiz metall plastina bilan chegaralangan yarim fazoda, kuchlanganligi plastina yuzasiga perpendikular bo‘lgan bir jinsli elektr maydon mavjud. Plastinaning ikkinchi tomonida maydon mavjud emas. Gauss sirti sifatida plastinani kesib o‘tadigan va yon qirralari plastinaga perpendikular bo‘lgan paralelepiped olib, Gauss teoremasi (8.16) bo‘yicha quyidagini hosil qilamiz:

$$\varepsilon_0 E\Delta S = q_S \Delta S,$$

bunda ΔS – parallelepipedning kesim yuzasi. Parallelepipedning boshqa qirralaridan o‘tadigan oqim nolga teng.

Koordinata boshini q_s zaryad taqsimlangan sirtdan boshlab, uning o‘qini plastina sirtiga perpendikular bo‘yicha yo‘naltirib quyidagini olish mumkin:

$$E = E_y = q_S / \varepsilon_0 = 11{,}3 \text{ V/m}.$$

Sinov zaryadiga ta’sir etadigan kuch (8.1) bo‘yicha $F = qE = 11{,}3 \cdot 10^{-12}$ N va potensial (8.2) bo‘yicha $\varphi = -\int E_y d_y + const = -11{,}3y$ V. Agar plastinada $y = 0$ bo‘lsa, u holda potensial $\varphi = 0$ bo‘ladi.

8.4-masala. Kondensator plastinalari yuzasi $S = 25\,\text{sm}^2$ va ularning orasidagi masofa $d = 2\,\text{mm}$ bo‘lsa, maydon kuchlanganligi, elektr siljish, qutblanish, erkin va bog‘langan zaryadlar miqdori quyidagi ikki hol uchun aniqlansin: a) dielektrik – havo; b) dielektrik – tekstolit ($\varepsilon_r = 4$). Kondensatordagi o‘zgarmas kuchlanish – 1 kV.

Yechish. a) Bir jinsli muhit bilan to‘ldirilgan yassi kondensatorda maydon kuchlanganligi barcha joyda bir xil (chet effektni hisobga olmaymiz), plastina tekisligiga perpendikular va berilgan kuchlanishda

$$E = U / d = 5 \cdot 10^5 \text{V/m}.$$

Havo bilan to‘ldirilganda ($\varepsilon_r = 1$) elektr siljish $D = \varepsilon_0 E = 4{,}43 \cdot 10^{-6} \text{C/m}^2$, qutblanish $\overline{P} = \overline{D}$. Sirtda erkin zaryadlar metall plastina yuzasi bo‘yicha (8.15) ga ko‘ra bir xil taqsimlangan, ya’ni $q_s = D = 4{,}43$lek C/m^2. Plastinaning erkin zaryadi $q = q_s S = 11{,}1 \cdot 10^{-9}\text{C}$. Bog‘langan zaryad nolga teng. Sig‘im

$$C = \frac{q}{U} = \frac{q_s S}{U} = \frac{DS}{U} = \frac{\varepsilon_0 \varepsilon_r S}{d} = 11{,}1\,\text{pF}.$$

b) Berilgan kuchlanishda maydon kuchlanganligi har qanday bir jinsli dielektrikda avvalgidek qoladi, shuning uchun

$E = U / d = 5 \cdot 10^5$ V/m; $D = \varepsilon_0 \varepsilon_r E = 17{,}7 \cdot 10^{-6}$ C/m^2;

$P = (\varepsilon_r - 1)\varepsilon_0 E = 13{,}3 \cdot 10^{-6}$ C/m^2;

$q = q_s S = DS = 44{,}4 \cdot 10^{-9}$ C.

Bog‘langan zaryad va uning zichligini (8.18) ko‘ra topamiz. Kondensatorning musbat zaryadlangan plastinasi oldidagi bog‘langan zaryad

$$q_{\text{bog'}} = -PS = -35 \cdot 10^{-9}\,\text{C}.$$

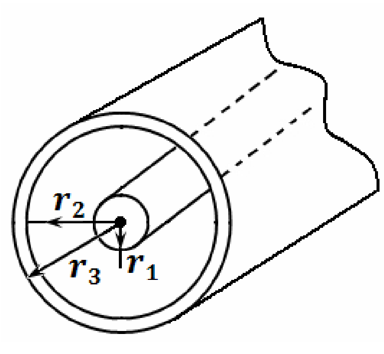
Sig‘im $C = \frac{q}{U} = \frac{\varepsilon_0 \varepsilon_r S}{d} = 44{,}4 \cdot 10^{-12} = 44{,}4\ \mathrm{pF}$.

8.5-masala. Kattaligi 1 C zaryadni joylashtirish mumkin bo‘lgan havodagi metall sharning radiusi necha bo‘lishi kerak? Shu sharning sig‘imi qancha bo‘ladi? Havoning teshilish kuchlanishi $E_{\mathrm{tesh}} = 30\ \mathrm{kV/sm.}$.

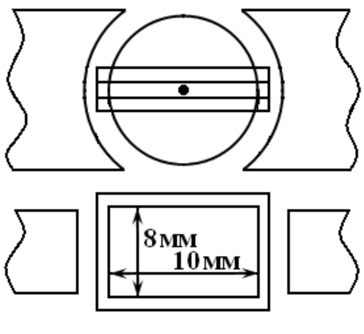
8.6-masala. $U = 10\ \mathrm{kV}$ kuchlanish ta’sirida turgan havoli silindrsimon kondensatorda (8.3-rasm) $r_1 = 2$ mm; $r_2 = 6$ mm bo‘lsa, uning maydon kuchlanganligi, potensial taqsimlanishi va sig‘imi hisoblansin. $E(r)$ va $\varphi(r)$ grafiklari qurilsin.

8.7-masala. Magnitoelektrik o‘lchash asbobining ramkasi o‘zgarmas magnitning havo tirqishida joylashgan. Tirqishdagi induksiya bir jinsli va (8000 Hz) 8,2 T. Ramka 100 o‘ramdan iborat bo‘lib, undan o‘tadigan tok 10 mA. Ramka o‘lchamlari 8.4-rasmda keltirilgan. Ramkaga ta’sir etuvchi aylanuvchi moment aniqlansin. Asbob shkalasining bir tekisliligi nima balan tushintiriladi?

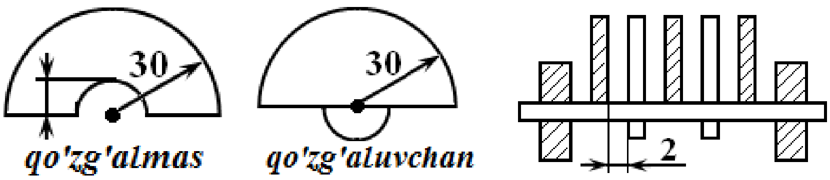
8.8-masala. Havo kondensatori qo‘zg‘almas va qo‘zaluvchan plastinalardan iborat. Kondensator sig‘imini qo‘zg‘aluvchan plastinalarning α burchak siljishiga bog‘lanishini aniqlang. 8.5-rasmda kondensatorning o‘lchamlar millimetrlarda berilgan. Chetki effektni hisobga olmang.



8.3- rasm.



8.4- rasm.



8.5- rasm.

8.9-masala. Radiusi $R_0 = 0{,}5$ sm bo‘lgan sharsimon elektron bulutning ichidagi va tashqarisidagi E maydon kuchlanganligi va φ potensialini hisoblang. Ikki hol uchun E va φ qiymatlarni koordinataga nisbatan bog‘lanishini quring: a) zaryadning hajmiy zichligi o‘zgarmas va $\rho = 2{,}5 \cdot 10^{-9}$ C/sm^3; b) zaryadning xajmiy zichligi $\rho = 2{,}5 \cdot 10^{-9} a/(a + R_0)$ C/sm^3, bunda $a = 0{,}2$ sm.

Yechish. Sharning maydon simmetriyasiga ko‘ra maydon kuchlanganligining radial tashkil etuvchisi $(E = E_R)$ faqat radiusga bog‘liq va binobarin, uning qiymati berilgan radiusning sferasida o‘zgarmasdir hamda kuchlanganlik vektori sfera sirtiga normaldir. Shuning uchun masalani Gauss teoremasidan (8.16) foydalanib yechish mumkin. Bunda integrallash sirti sifatida sfera olinadi. Sferik koordinatalar sistemasida $R < R_0$ bo‘lganda

$\varepsilon_0 E \cdot 4\pi R^2 = \dfrac{4}{3}\pi R^3 \rho,$ bundan $\vec{E} = \rho \vec{R} / 3\varepsilon_0 = -9{,}4 \cdot 10^3 R$ V/sm.

(8.12 b) formuladan potensialni topamiz:

$R = 0$ da $\varphi = 0$ deb qabul qilib, $C_1 = 0$ ni hosil qilamiz.

$R > R_0$ da esa quyidagiga ega bo‘lamiz:

$$\varepsilon_0 E \cdot 4\pi R^2 = \frac{4}{3}\pi R_0^3 \rho$$

bundan $E = \rho R_0^3 / 3\varepsilon_0 R^2 = -11{,}8 \cdot 10^{-2} / R^2$ V/sm,

potensial esa

$$\varphi = -\int E dR + C_2 = \frac{\rho R_0^3}{3\varepsilon_0 R} + C_2 = (-\frac{11{,}8 \cdot 10^2}{R} + C_2) \text{ V}.$$

$R = R_0$ bo‘lganda potensiallar teng bo‘lishi kerak, bundan integral doimiysi $C_2 = -\rho R_0^2 / 2\varepsilon_0 = 3{,}52 \cdot 10^3$ V.

8.10-masala. Sirtida $q = 20$ mkC zaryad bir tekis taqsimlangan $R_0 = 25$ sm radiusli metall sharning ichidagi va tashqarisidagi (havodagi) maydon kuchlanganligi va potensiali aniqlansin, koordinataga nisbatan E va φ bog‘lanishlarni toping, bunda cheksiz uzoq nuqtadagi potensial nolga teng deb oling.

Yechish. Masalani yechishda Gauss teoremasidan foydalanamiz. Shar ichida, ya'ni $R < R_0$ bo'lganda, R sharning markazidan boshlangan masofa (8.15) formulaga asosan $D \cdot 4\pi R^2 = 0,$ ya'ni $D = 0$; $E = 0$.

Shar tashqarisida, ya'ni $R > R_0$ da, $\varepsilon_0 E \cdot 4\pi R^2 = q$ va $E = E_R = q / 4\pi\varepsilon_0 R^2 = 18 \cdot 10^6 / R^2$ V/sm.

Potensialni hisoblaymiz:

$$\varphi = -\int E dR = \frac{18 \cdot 10^6}{R} + C .$$

Shardan cheksiz uzoq nuqtada $R \to \infty$ potensial $\varphi = 0$ bo'lgani uchun, o'zgarmas integral doimiysi $C = 0$ va $R = R_0$ da $\varphi_0 = 720$ kV.

8.11-masala. Zaryadning hajmiy zichligi $-2 \cdot 10^{-15}$C/sm^3 bo'lgan silindr shaklli elektron nurning 1) ichidagi; 2) tashqarisidagi maydon kuchlanganligi E va potensiali φ ni hisoblang. Nurning radiusi $r_0 = 1$mm. Nur uzunligi radiusdan ancha katta, hisoblarni nur uchlaridan uzoqdagi soha uchun bajaring.

Yechish. Maydonning silindrik simmetriyasidan kelib chiqib, maydonning kuchlanganligi faqat radius r ga bog'liq bo'lgan radial tashkil etuvchidan $(E = E_r)$ iborat. Binobarin, berilgan radiusli silindrda uning qiymati o'zgarmas va kuchlanganlik vektori silindr sirtiga normal yo'nalgan. Masalani yechish uchun Gauss teoremasidan foydalanib integrallash sirti sifatida nurning o'qi bo'yicha yo'nalgan va birlik uzunlikka ega silindr olinadi.

$r < r_0$ bo'lganda silindrik koordinata sistemasida $\varepsilon_0 E_1 \cdot 2\pi r = \pi r^2 \rho$, bundan $E_1 = \rho r / 2\varepsilon_0$; potensial esa

$$\varphi_1 = -\int E_1 dr = -\rho r^2 / 4\varepsilon_0 + C_1 .$$

$r = 0$ da potensialni $\varphi = 0$ deb, $C_1 = 0$ ni hosil qilamiz.

$r > r_0$ da esa, $\varepsilon_0 E_2 \cdot 2\pi r = \pi r_0^2 \rho$, bundan $E_2 = \rho r_0^2 / 2\varepsilon_0 r$.

Potensial esa

$$\varphi_2 = -\int E_2 dr = -\frac{\rho r_0^2}{2\varepsilon_0} \ln r + C_2 .$$

$r < r_0$ bo‘lganda, potensiallar teng, shuning uchun integral doimiysi quyidagiga

teng: $$C_2 = \frac{\rho r_0^{\,2}}{4\varepsilon_0}(2\ln r - 1).$$

Qiymatlarni qo‘yib, quyidagilarni hosil qilamiz:

$$E_1 = -11{,}3r \text{ mV/sm};$$

$$\varphi_1 = 0{,}565 \cdot 10^{-2} r^2 \text{ V};$$

$$E_2 = -\frac{0{,}113}{r} \text{ mV/sm};$$

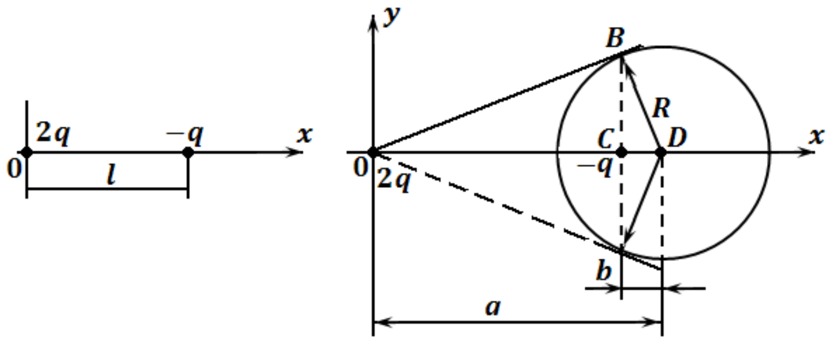
$$\varphi_2 = 0{,}565 \cdot 10^{-4}(1 + 2\ln 10r) \text{ V}.$$

8.3. Elekrostatik maydon masalalari

8.12-masala. Vakuumda $+2q$ va $-q$ nuqtaviy zaryadlar bir-biriga nisbatan $l = 10$ sm uzoqlikda joylashgan. (8.6- rasm, *a*).

1) cheksiz uzoqlashgan nuqtaning potensiali nolga teng deb, sferik shaklga ega ekvipotensial sirtning joylanishini, o‘lchamini va ekvipotensial sirtning potensialini toping.

2) $2q$ zaryad joylashgan nuqta orqali o‘tkazilgan sfera urinmalaridan o‘tuvchi tekislikda $-q$ zaryad yotishini isbotlang.



a) *b)*

8.6- rasm.

Yechish. 1) Tanlab olingan koordinata o‘qlari uchun zaryadlar joylashgan tekislikda har qanday nuqtaning potensiali quyidagiga teng.

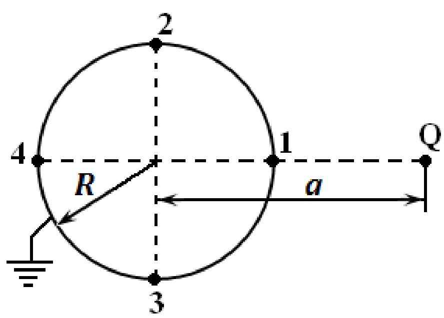
$$\varphi = \frac{2q}{4\pi\varepsilon_0\sqrt{x^2+y^2}} - \frac{q}{4\pi\varepsilon_0\sqrt{(x-l)^2+y^2}}.$$

Agar potensialni $\varphi = 0$ deb olsak, xy tekislikdagi ekvipotensial aylana tenglamasini beradi, ya'ni $(x-\frac{4}{3}l)^2 + y^2 = \frac{4}{9}l^2$. Binobarin, cheksiz uzoqlashgan nuqtaning potensialiga teng bo'lgan ekvipotensial aylana shaklini beradi (8.6- *b* rasm). Aylana radiusi $R = 2l/3 = 6{,}7$ sm, uning markazi koordinata boshidan $a = 4l/3 = 13{,}3$ sm uzoqlikda yotadi. Shuni e'tirof etish kerakki, boshqa ekvipotensiallar murakkabroq shaklga ega bo'ladi.

2) $2q$ zaryad joylashgan nuqtadan (koordinata boshidan) sferaga o'tkazilgan urinmalar va urinma nuqtasidan o'tuvchi tekislik o'tkazamiz. (8.6-rasm, *b*). OBD uchburchak BSD uchburchakka o'xshash bo'lganligi uchun $b/R = R/a$ yoki $b = R^2/a$; bundan tashqari, $b = a - OC$, shu sababli, $OC = a - R^2/a = l4/3 - l/3 = l$.

Shunday qilib $-q$ zaryad C nuqtada joylashgan va haqiqatdan urinmalar kesishgan nuqtadan o'tuvchi tekislikda yotadi.

8.13-masala. O'tkazgich materialdan yasalgan yerlangan va radiusi $R = 0{,}2$ m bo'lgan shar markazidan $a = 1$ m masofada nuqtaviy $q = 10^{-7}$ C zaryad joylashgan (8.7- rasm). Shar sirtidagi 1, 2, 3, va 4 nuqtalardagi induksiyalangan zaryadlar zichligi aniqlansin.



8.7- rasm.

Yechish. Shar sirti – potensiali $\varphi = 0$ bo'lgan ekvipotensial sirtdir. Shuning uchun hisoblashda sirti yerlangan sharni va nuqtaviy zaryad q ni har xil ishorali ikkita nuqtaviy zaryad bilan almashtirish mumkin. (8.12-masalaning yechimiga qarang). Bitta q zaryad berilgan bo‘lib, ikkinchi zaryad Q_1ning qiymatini va joylashishini yerlangan sharning potensialini nolga tenglashtirib aniqlash mumkin. Shar sirtidagi masofa har qanday ikkita nuqta uchun tuzilgan tenglamalardan quyidagicha topiladi:

$$Q = -QR/a = -0{,}2Q;\ (l-a) = b = R^2/a = 0{,}04\,\text{m},$$

bunda l –shu Q va Q_1 zaryadlar orasidagi masofa (8.8- rasm). Bulardan keyin masalani ikkita nuqtaviy zaryadlarning qiymatlari va joylashishiga ko‘ra yechamiz.

Shar sirtidagi zaryad zichligini quyidagicha hisoblaymiz (8.8-rasmga qarang):

$$q_s = \varepsilon_0 E_n = \frac{Q}{4\pi r_1^2}\cos\alpha_1 + \frac{Q_1}{4\pi r_2^2}\cos\alpha_2 .$$

$r_1, r_2, \cos\alpha_1$ va $\cos\alpha_2$ larni R va θ sferik koordinatalar sistemasi orqali ifodalaymiz:

$$q_s = \frac{Q}{4\pi}\left[\frac{R-a\ \cos\theta}{\sqrt{(\mathrm{R}^2+\mathrm{a}^2-2\mathrm{aR}\theta)^3}} - 0{,}2\frac{R-b\cos\theta}{\sqrt{(R^2+b^2-2bR\cos\theta)^3}}\right]$$

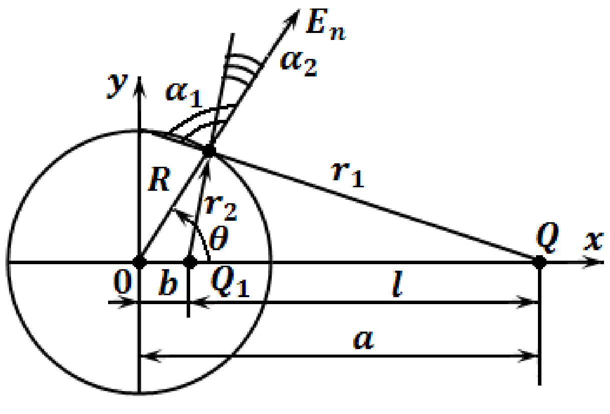
va berilgan qiymatlarda

$$q_s = 8\cdot 10^{-9}\left[\frac{0{,}2-\cos\theta}{\sqrt{(1{,}04-0{,}4\cos\theta)^3}} - \frac{0{,}04-0{,}008\cos\theta}{\sqrt{(0{,}0416-0{,}016\cos\theta)^3}}\right]\text{C/m}^2 .$$

1-nuqta uchun ($\cos\theta = 1$) $\mathrm{q_s} = -74{,}6\ \text{nC/m}^2$;

2- va 3- nuqtalar uchun ($\cos\theta = 0$) $\mathrm{q_s} = -36{,}1\ \text{nC/m}^2$;

4-nuqta uchun ($\cos\theta = -1$) $\mathrm{q_s} = -22{,}2\,\text{nC/m}^2$.

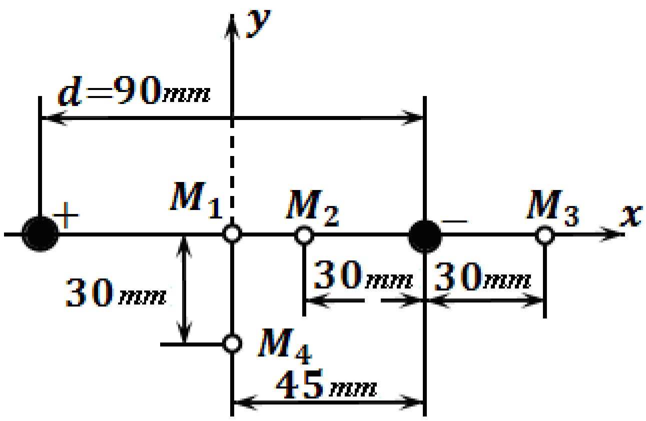


8.8- rasm.

8.14- masala. 8.13- masalani shar yerlanmagan holat uchun yeching. Cheksiz uzoq nuqtaning potensiali nol deb, shar potensialini aniqlang.

8.15-masala. 8.13-masaladagi nuqtaviy zaryadni sharga tortish kuchini aniqlang: a) yerlangan shar uchun; b) yerlanmagan shar uchun.

8.16-masala. Simlarining radiusi $r_0 = 2{,}5$ mm bo‘lgan, 10 kV o‘zgarmas kuchlanishli ikki simli liniya berilgan (8.9- rasm). Liniya yaqinidagi M_1, M_2, M_3 va M_4 nuqtalardagi maydon kuchlanganliklari va potensiallar hisoblansin. Liniya sig‘imi aniqlansin. $E(x,0)/E(0,0)$ va $\varphi(x,0)/(U/2)$ ifodalar topilsin.



8.9- rasm.

8.17-masala. Konstruktiv parametrlari $2S = 25$ mm, $r_0 = 10$ mm bo‘lgan ikki simli liniyaga 1 kV kuchlanishli o‘zgarmas tok manbayi ulangan. Simning sirtidagi zaryad taqsimlanishini toping va birlik uzunlikdagi sig‘imni aniqlang. Simlar o‘qidan o‘tadigan tekislikdagi x koordinata bo‘yicha maydon kuchlanganligi ifodasini toping.

Yechish. Ikkita metall simni ikkita zaryadlangan koordinatalari $a = \sqrt{S^2 - r_0^2} = 7{,}5$ mm bilan aniqlangan o‘q bilan almashtiramiz (8.10- rasm, *a*), bunda 2*a* – o‘tkazgichlarning elektr o‘qlari orasidagi masofa.

Birlik uzunlikdagi o‘tkazgichlarning sig‘imi $C_0 = \tau/(\varphi_1 - \varphi_2)$, bunda τ – o‘tkazgichning birlik uzunligidagi zaryadi; φ_1 va φ_2 – mos ravishda chap va o‘ng o‘tkazgichlarning potensiali.8.10- rasm, *b* dan ko‘rinib turibdiki, 1- va 2 nuqtalar uchun $$\varphi_1 = \frac{\tau}{2\pi\varepsilon_0}\ln\frac{a+S-r_0}{a-S+r_0}; \quad \varphi_2 = \frac{\tau}{2\pi\varepsilon_0}\ln\frac{a-S+r_0}{a+S-r_0}.$$

Sig‘im va zaryad

$$C_0 = \pi\varepsilon_0 / \ln\frac{a+S-r_0}{a-S+r_0} = 0{,}4 \text{ pF/sm}; \ \tau = C_0 U = 0{,}4\cdot 10^{-9} \ \text{ C/sm.}$$

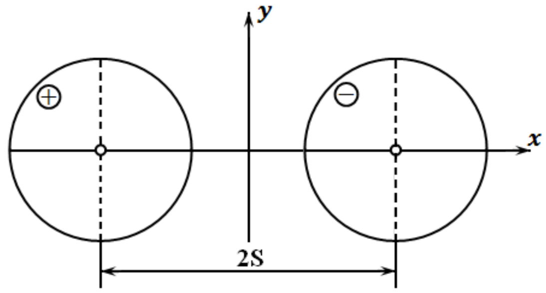
Sim yuzasidagi zaryad elektr maydonlarning chegaraviy shartiga ko‘ra quyidagicha hisoblanadi:

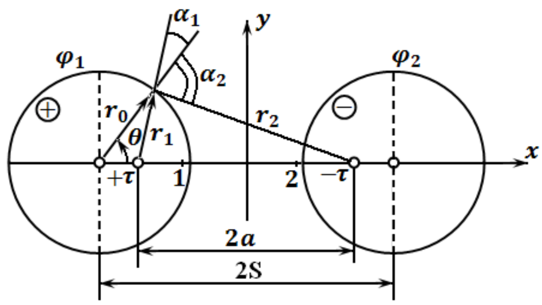
$$q_S = \varepsilon_0 E_n = \varepsilon_0\{\frac{\tau}{2\pi\varepsilon_0 r_1}\cos\alpha_1 + \frac{\tau}{2\pi\varepsilon_0 r_2}\cos\alpha_2\},$$

bunda $r_1^2 = r_0^2 + (S-a)^2 - 2r_0(S-a)\cos\Theta$; $r_2^2 = r_0^2 + (S+a)^2 - 2r_0(S+a)\cos\Theta$;

$$\cos\alpha_1 = \frac{r_0 - (S-a)\cos\Theta}{r_1}; \quad \cos\alpha_1 = \frac{(S+a)\cos\Theta - r_0}{r_2};$$

yoki $$q_S = \frac{\tau}{2\pi}\{\frac{r_0 - (S-a)\cos\Theta}{r_0^2 + (S-a)^2 - 2r_0(S-a)\cos\Theta} - \frac{r_0 - (S+a)\cos\Theta}{r_0^2 + (S+a)^2 - 2r_0(S+a)\cos\Theta}\}.$$





8.10- rasm.

Simlarning o‘qidan o‘tadigan tekislikdagi maydonning kuchlanganligi bitta tashkil etuvchidan iborat:

$$E_x = \frac{\tau}{2\pi\varepsilon_0}\{\frac{1}{0{,}75+x} + \frac{1}{0{,}75-x}\} \ \text{ V/sm.}$$

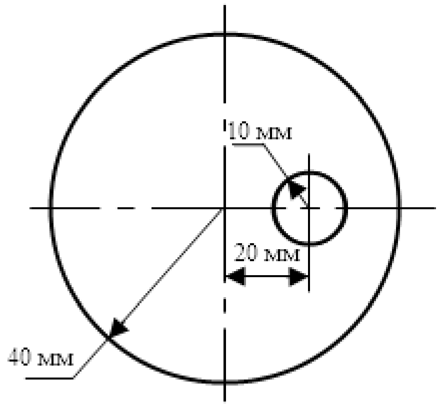
bunda x – santimetrda.

8.18- masala. Ikki simli uzatish liniyasi radiuslari mos ravishda 6 va 10 mm va oraliqlari 25 mm bo‘lgan ikkita silindrik mis simdan iborat. Sistema sig‘imini aniqlang.

8.19-masala. Havoda bir-birining ichida nokoaksial joylashgan ikkita silindrlar (8.11-rasm) orasidagi sig‘imni va teshilish kuchlanishni aniqlang. Havoning teshilish elektr mustahkamligi $E_{\text{tesh}} = 30\,\text{kV/sm}$.

8.20-masala. 8.12-rasm, *a* da tasvirlangan simlar sistemasining potensiali va zaryadlarini aniqlang hamda quyidagi holatlarda yechim qanday o‘zgarishini ko‘rsating: a) agar 1- va 2- simlarning rubilniklari qo‘shilsa, 3- simning rubilnigini qo‘shmasdan qoldirsa; b) agar 2- simni yerdan ajratib, keyin 1-simni manbadan uzsa; v) agar 3- simni yerga ulasa.

Berilgan: $U = 10\,\text{kV}$, simning radiusi $r_0 = 10\,\text{mm}$; $h_1 = h_3 = 3\,\text{m}$; $h = 4{,}5\,\text{m}$; $d = 2\,\text{m}$.



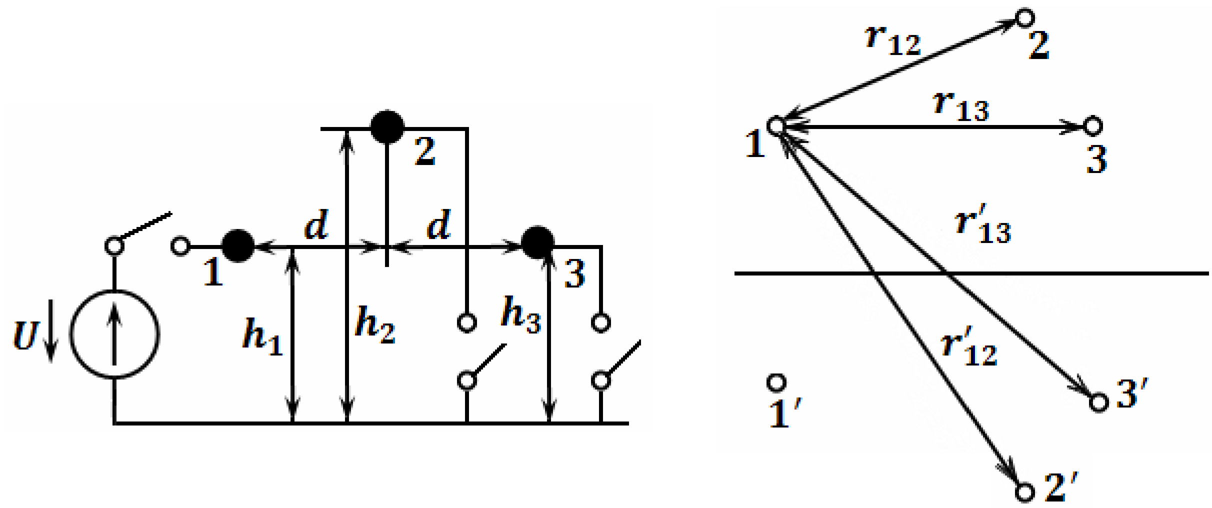
8.11- rasm.

Yechish. Berilgan simlar sistemasini ko‘zgu tasvirlar usuli bo‘yicha oltita simlar sistemasi bilan almashtiramiz (8.12- rasm, *b*).

1- va 2- simlarning potensiallari masala shartida berilgan: $\varphi_1 = 10\,\text{kV}$; $\varphi_2 = 0$; zaryad $\tau_3 = 0$.

Boshqa potensiallar va zaryadlarni zaryadlangan simlar uchun quyida yozilgan potensial koeffitsientli Maksvell formulalaridan aniqlanadi:

$$\left.\begin{aligned} \varphi_1 &= \alpha_{11}q_1 + \alpha_{12}q_2 + \alpha_{13}q_3, \\ \varphi_2 &= \alpha_{21}q_1 + \alpha_{22}q_2 + \alpha_{23}q_3, \\ \varphi_3 &= \alpha_{31}q_1 + \alpha_{32}q_2 + \alpha_{33}q_3. \end{aligned}\right\}$$



a) *b)*

8.12- rasm.

Bu sistemada q ni τ ga almashtirib va berilgan shartlarga ko‘ra quyidagini yozamiz:

$$10^4 = \alpha_{11}\tau_1 + \alpha_{12}\tau_2, \tag{8.38}$$

$$0 = \alpha_{21}\tau_1 + \alpha_{22}\tau_2, \tag{8.39}$$

$$\varphi_3 = \alpha_{31}\tau_1 + \alpha_{32}\tau_2, \tag{8.40}$$

bunda $\alpha_{11} = \alpha_{33} = \dfrac{1}{2\pi\varepsilon_0}\ln\dfrac{h_1}{r_0} = \dfrac{1}{2\pi\varepsilon_0}\ln 600 = 115\cdot 10^9$ m/F;

$\alpha_{22} = \dfrac{1}{2\pi\varepsilon_0}\ln\dfrac{h_2}{r_0} = \dfrac{1}{2\pi\varepsilon_0}\ln 900 = 123\cdot 10^9$ m/F;

$\alpha_{12} = \alpha_{21} = \alpha_{23} = \alpha_{32} = \dfrac{1}{2\pi\varepsilon_0}\ln\dfrac{r'_{12}}{r_{12}} = 20\cdot 10^9$ m/F;

$\alpha_{13} = \alpha_{31} = \dfrac{1}{2\pi\varepsilon_0}\ln\dfrac{r'_{13}}{r_{13}} = 10{,}6\cdot 10^9$ m/F – mazkur Maksvell formulalari asosida ikki simli liniyaning potensialini $\varphi = \dfrac{\tau}{2\pi\varepsilon_0}\ln\dfrac{r_-}{r_+}$ e’tiborga olib yozilgan.

(8.38) va (8.39) ni birgalikda yechib: $\tau_1 = 80{,}0$ mkC/mm $\tau_2 = -12$ mkC/mm (8.40) tenglamadan $\varphi_3 = 605$ V topamiz.

Qayta ulashlardan so‘ng $\tau'_1 = \tau_1$; $\tau'_2 = \tau_2$; $\varphi_2 = 0$. Bundan keyin potensial koeffitsiyentli tenglamalar sistemasidan quyidagilarni aniqlaymiz: $\varphi'_1 = 9950$ V;

$\varphi_2' = -88{,}0$ V; $\tau_3' = -4{,}76$ mkC/mm.

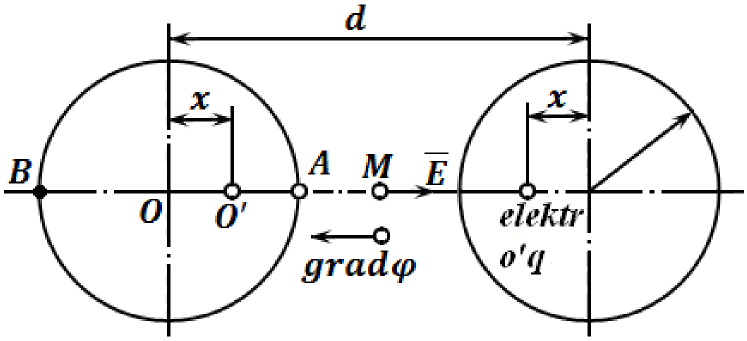
8.21- masala. Diametrlari 10 mm ikkita simlar havoda bir-biriga parallel joylashgan (8.13- rasm). Ular orasidagi masofa 20 mm. Har bir simning bir metrdagi zaryadi $1 \cdot 10^{-8}$ C. Chapdagi sim musbat, o‘ngdagi – manfiy. Zaryadlangan simlar sirtidagi eng katta va eng kichik zaryadlar zichligini toping.

Yechish. Elektr o‘qlar holatini topamiz: $x = \frac{d}{2} - \sqrt{(\frac{d}{2})^2 - r^2} = 1{,}35$ mm.

Metall sirtidagi zaryadlar zichligi: $\delta = D = \varepsilon_a E$.

Binobarin, kuchlanganlik qayerda katta bo‘lsa, zichlik ham o‘sha yerda katta bo‘ladi.

Agar musbat zaryad hosil qiladigan maydon kuchlanganligi yo‘nalishi shu zaryaddan chiqish yo‘nalishiga mos bo‘lsa va musbat zaryad hosil qilayotgan maydon kuchlanganligi zaryad tomonga yo‘nalgan bo‘lsa, ma’lumki, eng katta maydon kuchlanganligi *A* nuqtada, eng kichigi esa *B* nuqtada bo‘ladi.



8.13- rasm.

A nuqtadagi kuchlanganlik ikkila zaryad kuchlanganliklarining yig‘indisi bo‘ladi, *B* - nuqtada esa ayirmasiga teng bo‘ladi, ya’ni:

$$E_A = \frac{Q}{2\pi\varepsilon_a l(r-x)} + \frac{Q}{2\pi\varepsilon_a l(d-r-x)};$$

$$E_B = \frac{Q}{2\pi\varepsilon_a l}(\frac{1}{r+x} - \frac{1}{d-r-x}).$$

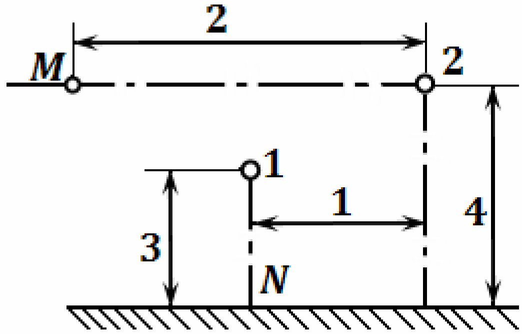
Bundan

$$D_A = \delta_A = \varepsilon_a E_A = \frac{10^{-8}}{2\pi}\left(\frac{1}{0{,}005-0{,}00135} + \frac{1}{0{,}02-0{,}005-0{,}00135}\right) = 0{,}544 \text{ mkC/m}^2;$$

$$D_B = \delta_B = \varepsilon_a E_B = \frac{10^{-8}}{2\pi}\left(\frac{1}{0{,}005-0{,}00135} - \frac{1}{0{,}02-0{,}005-0{,}00135}\right) = 0{,}186\ \text{mkC/m}^2.$$

Shunday qilib, A nuqtadagi zaryad zichligi B nuqtanikidan 2,92 marta kattadir.

8.22-masala. 1-simning birlik uzunligidagi zaryadi $\tau_2 = 2\cdot 10^{-9}$ C/m, 2-simning birlik uzunligidagi zaryadi $\tau_2 = 10^{-9}$ C/m (8.14- rasm). Yer potensialini nolga teng deb, M nuqta potensialini aniqlang.



8.14- rasm.

Yechish.

$$\varphi_M = \frac{\tau_1}{2\pi\varepsilon_a}\ln\frac{b_1M}{a_1M} + \frac{\tau_2}{2\pi\varepsilon_a}\ln\frac{b_{2M}}{a_{2M}} = \frac{2\cdot 10^{-9}}{2\pi\cdot 8{,}86\cdot 10^{-12}}\ln\frac{\sqrt{7^2+1^2}}{\sqrt{2}} -$$
$$-\frac{10^{-9}}{2\pi\cdot 8{,}86\cdot 10^{-12}}\ln\frac{\sqrt{8^2+2^2}}{2} = 30{,}6\ \text{V}.$$

8.23-masala: Har birining og‘irligi $P = 2\cdot 10^{-2}$ N dan bo‘lgan ikkita shar havo $(\varepsilon_r = 1)$ da uzunligi $l = 2$ m dan bo‘lgan ingichka ipak ipga osib qo‘yilgan (8.15- rasm). Sharlar bir xil ishorali va teng $q = 5\cdot 10^{-8}$ C zaryadlar bilan zaryadlanganda, o‘zaro itarishib, bir-biridan qanday masofaga uzoqlashadi?

Yechish. Zaryadlangan sharlar muvozanatda bo‘lganda har bir sharga ta’sir etuvchi kuchlarning geometrik yig‘indisi nolga teng bo‘lishi kerak.

8.15- rasmdagi chizmadan ko‘rinadiki, sharning R og‘irlik kuchining $F_1 = P\text{tg}\alpha$ tashkil etuvchisi zaryadlarning o‘zaro itarishish Kulon kuchi

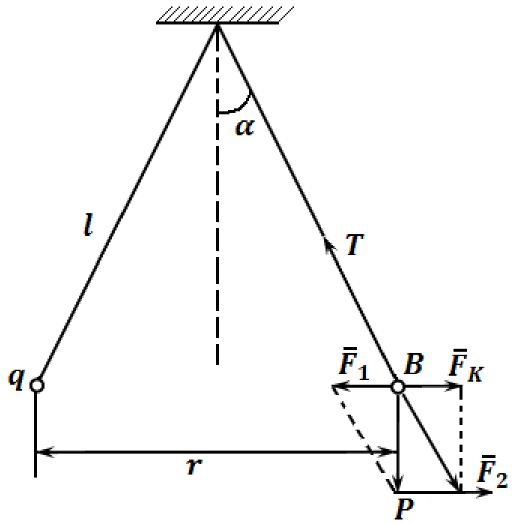
$$F_K = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\varepsilon_r r^2}$$

bilan muvozanatlashadi, ya’ni

$$P\text{tg}\alpha = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{\varepsilon_r r^2};\qquad (8.41)$$

bunda α juda kichik burchak bo‘lgani uchun ΔOAB dan:

$$\text{tg}\alpha \approx \sin\alpha = \frac{OB}{AB} = \frac{r/2}{l} = \frac{r}{2l}.$$



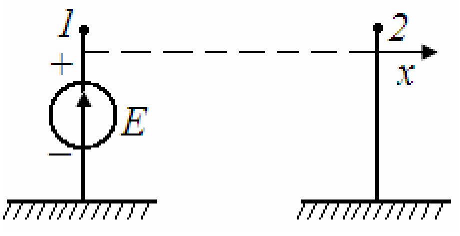
8.15- rasm.

Buni (8.41) ga qo‘yilsa: $$P\frac{r}{2l} = \frac{1}{4\varepsilon_0\varepsilon_r}\frac{q^2}{r^2},$$

Bundan izlanayotgan r masofa quyidagiga teng bo‘ladi:

$$r = \sqrt[3]{\frac{2lq^2}{4\pi\varepsilon_0\varepsilon_r P}} = \sqrt[3]{\frac{2\cdot 2\cdot 25\cdot 10^{-16}}{4\pi\cdot 8{,}85\cdot 10^{-12}\cdot 1\cdot 2\cdot 10^{-2}}} = 0{,}165\ m.$$

8.24- masala. Oraliq masofasi 2 sm bo‘lgan ikkita yassi elektrod vakuumda joylashgan (8.16- rasm). O‘ngdagi elektrod yerlangan, chapdagisi esa EYKi 220 V bo‘lgan batareyaning musbat qismasiga ulangan, manfiy qismasi esa yerlangan. Elektrodlar orasiga zichligi (bunda a = 30 kV/sm^3) qonuniyat bilan o‘zgaradigan hajmiy zarad joylashtirilgan. x – chap plastinadan boshlanadigan masofa. Elektrodlar orasidagi fazoda potensialning o‘zgarish qonuniyatini aniqlang.



8.16- rasm.

Yechish. Elektrodlarning o‘lchami ular orasidagi masofadan ancha katta deb hisoblaymiz va x o‘qini 8.16-rasmdagidek yo‘naltiramiz. Masala shartiga ko‘ra potensial faqat x o‘qi bo‘ylab o‘zgaradi, y va z o‘qlari bo‘ylab esa o‘zgarmaydi. Binobarin,

$$\frac{d^2\varphi}{dx^2}=-\frac{\rho}{\varepsilon_0}=ax.$$

Yuqoridagi ifodani x bo‘yicha ikki marta integrallab quyidagi ifodalarni hosil qilamiz: $\dfrac{d\varphi}{dx}=\dfrac{ax^2}{2}+C_1,\ \ \varphi=\dfrac{ax^3}{6}+C_1x+C_2\cdot$

Chegaraviy shartlardan foydalanib, integrallash doimiylarini aniqlaymiz:

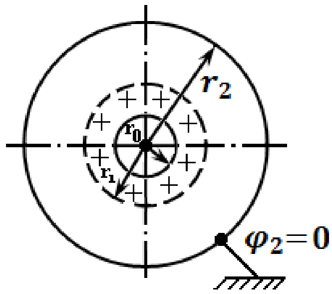
x=0 da φ=200 = C_2;

x=2 sm da $\quad \varphi=200+2C_1+\dfrac{30\cdot10^3\cdot8}{6}=0 \quad C_1=-20100\ \text{V/sm}.$

Binobarin,

$$\varphi=\frac{30\cdot10^3x^3}{6}-20100x+200=5000x^3-20100x+200\ \text{V}.$$

8.25-masala. Havo izolatsiyali silindrik kondensatorning radiusi r_0 bo‘lgan ichki elektrodining atrofida hajmiy zichligi $\rho\ \left[\text{C/sm}^3\right]$ toj zaryad mavjud (8.17-rasm). Tojning radiusi r_1. Tashqi elektrodning radiusi r_2. Ichki elektrodning potensiali φ_0, tashqi elektrodning potensiali esa nolga teng. Hajmiy zaryadlar egallagan 1- joyni I soha va erkin zaryadlar bo‘lmagan 2-joyni II soha. 2- joydagi φ potensialni aniqlaydigan formulani keltirib chiqaring.



8.17- rasm.

Yechish. I soha uchun quydagini yozish mumkin:

$$\frac{1}{r}\cdot\frac{d}{dr}\left(r\frac{d\varphi_I}{dr}\right)=-\frac{\rho}{\varepsilon_0}. \tag{8.42}$$

(8.42)ni ikki martta integrallash quyidagini beradi:

$$\varphi_1=-\frac{\rho r^2}{4\varepsilon_0}+C_1\ln r+C_2. \tag{8.43}$$

II soha uchun

$$\frac{1}{r}\cdot\frac{d}{dr}\left(r\frac{d\varphi_{II}}{dr}\right)=0.$$

Bundan
$$\varphi_{II}=C_3\ln r+C_4. \tag{8.44}$$

To‘rtta integral doimiy C_1, C_2, C_3, C_4 larni aniqlash uchun to‘rtta tenglama tuzamiz:

a) $r=r_0$ da $\varphi_I=\varphi_0$, shuning uchun

$$\varphi_0=-\frac{\rho r_0^2}{4\varepsilon_0}+C_1\ln r_0+C_2;$$

b) $r=r_1$ da $\varphi_I=\varphi_{II}$, binobarin,

$$-\frac{\rho r_1^2}{4\varepsilon_0}+C_1\ln r_1+C_2=C_3\ln r_1+C_4;$$

d) $r=r_2$ da $\varphi_{II}=0$, u holda $0=C_3\ln r_2+C_4$;

e) $r=r_1$ da D elektr siljish vektorining normal tashkil etuvchilari teng:

$$\varepsilon_0\left|\frac{d\varphi_1}{dr}\right|_{r=r_1}=\varepsilon_0\left|\frac{\varphi_{II}}{dr}\right|_{r=r_1},$$

yoki

$$C_3 = C_1 - \frac{\rho r_1^2}{2\varepsilon_0}.$$

a), b), d), e) tenglamalarni birgalikda yechib, quyidagini aniqlaymiz:

$$C_1 = \frac{\dfrac{\rho}{4\varepsilon_0}(r_1^2 - r_0^2) + \dfrac{\rho r_1^2}{2\varepsilon_0}\ln\dfrac{r_2}{r_1} - \varphi_0}{\ln\dfrac{r_2}{r_0}}$$

Keyin g) dan C_3, d) dan C_4 va a) dan C_2 aniqlanadi.

Aniqlangan integral doimiylarini (8.43) va (8.44) ga qo‘yib, φ_1 va φ_2 aniqlanadi.

8.26-masala. Qandaydir fazo sohasi dekart sistemasida faqat x koordinataga bog‘liq elektrostatik maydon mavjud: $\varphi = 5x^3 - 60x^2$. Shu maydondagi erkin zaryadlar zichligini toping.

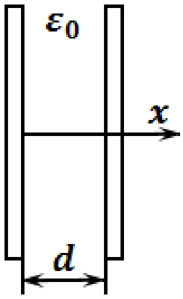
Yechish. Ma’lumki, maydonni ifodalaydigan Puasson tenglamasini quyidagicha yozish mumkin: $\dfrac{d^2\varphi}{dx^2} = -\dfrac{\rho_{\text{er}}}{\varepsilon_a}$. Potensialni x bo‘yicha ikki marta differensiallaymiz:

$$\frac{d\varphi}{dx} = 15x^2 - 120x\,; \quad \frac{d^2\varphi}{dx^2} = 30x - 120\,.$$

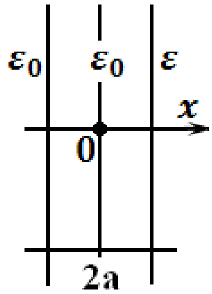
Binobarin $\rho_{\text{er}} = (-30x + 120)\varepsilon_a$

8.27-masala. Dekart sistemasida faqat x koordinataga bog‘liq elektrostatik maydon mavjud: $\varphi = 3x^3 + 40x^2 + 25$ V shu maydondagi erkin zaryadlar zichligini toping.

8.28-masala. Vakuumda joylashgan yassi elektrodlar orasidagi potensial x yo‘nalish bo‘yicha $\varphi = ax^3 + bx + c$ (8.18- rasm), bunda $a = -6{,}28 \cdot 10^8$ V/m^3, $b = -9{,}42 \cdot 10^5$ V/m^2; $c = -12 \cdot 10^2$ V/m. Elektrodlar tomoni $l = 0{,}1\,\text{m}$ bo‘lgan kvadratlardan iborat bo‘lib, ularning orasidagi masofa $d = 0{,}5 \cdot 10^{-2}\,\text{m}$. Chetli effektni e’tiborga olmasdan, elektrodlar orasidagi yig‘ilgan hajmiy zaryad topilsin.



8.18- rasm.



8.19- rasm.

8.29-masala. Havoning cheksiz uzun qatlami bir tekis zaryadlangan. Hajmiy zaryad $\rho = -10^{-4}/(12\pi)$ C/m^3. Zaryadlangan qatlamning qalinligi 2a=10 mm, eni $h >> a$. (8.19-rasm). Maydon kuchlanganligi modulini va potensialning o‘zgarish grafigini x koordinata bo‘yicha $0 < x < \infty$ oraliqda quring. $x = 0$ da $\varphi = 0$ deb qabul qiling.

8.30-masala. Dielektrikdan yasalgan shar ($\varepsilon_r = 3$) zaryadlangan va havoda joylashgan. Zaryadning hajmiy zaryadi shar markazidan R masofada $\rho = kR$ funksiya bo‘yicha o‘zgaradi, bunda $k = 1/(10\pi)$ C/m^4. Sharning radiusi $R_0 = 1$ sm. Shar markazidan boshlab R masofa bo‘yicha maydon kuchlanganligi E shar ichida va tashqarisida qanday funksiya bo‘yicha o‘zgaradi? $R = \infty$ da $\varphi = 0$ deb qabul qiling.

8.4. O‘zgarmas tok elektr maydoni masalalari

8.31-masala. Uzunligi 1 km koaksial kabelning sizilma toki aniqlansin. Kabelning tomiri bilan qobig‘i orasi o‘tkazuvchanligi $\gamma = 10^{-8}$ Ω^{-1}m^{-1} bo‘lgan dielektrik bilan to‘ldirilgan. Tomir radiusi r_1, qobiq radiusi $r_2 = er_1$, bunda e – natural logarifmlar asosi. Tomir bilan qobiq orasidagi kuchlanish 10 kV.

Yechish. Sizilma tok $I = UG$. O‘tkazuvchanlik:

$$G = \frac{2\pi\gamma l}{\ln(r_2/r_1)} = \frac{2\pi \cdot 10^{-8} \cdot 10^3}{1} = 6{,}28 \cdot 10^{-5} \text{ S.}$$

Mukammal bo‘lmagan izolatsiyaning sizilma toki:

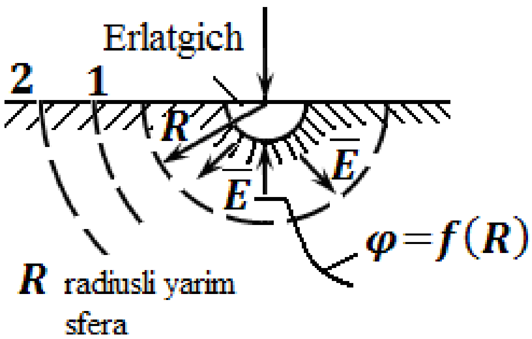
$$I = 10^4 \cdot 6{,}28 \cdot 10^{-5} = 0{,}628 \text{ A/km}.$$

8.32- masala. Yerlatgich maydon masalasini ko‘rib chiqamiz. Yerlatgichdan o‘tadigan tok I yer qatlamidan o‘tib boshqa yerlatgich atrofida yig‘iladi. Yer qaytuvchi o‘tkazgich vazifasini o‘taydi. Agar yerga metall yarim sfera joylashtirilsa, undan o‘tadigan tokni I deb, tok yig‘iladigan ikkinchi elektrodni esa juda uzoqda deb qabul qilinsa, unda radiusi R bo‘lgan yarim sferaning sirtidan o‘tadigan tokning zichligi $\delta = I/2\pi R^2$ (sfera sirti $4\pi R^2$, yarim sferaniki $2\pi R^2$). Maydon kuchlanganligi $E = \delta/\gamma = I/(2\pi\gamma R^2)$ ga teng. Yer sirtidagi ikki nuqta orasidagi kuchlanishni topamiz (1 va 2 nuqtalar):

$$U_{12} = \int_{R_1}^{R_2} E dR = \frac{I}{2\pi\gamma} \int_{R_1}^{R_2} \frac{dR}{R^2} = -\frac{I}{2\pi\gamma}\left(\frac{1}{R_2} - \frac{1}{R_1}\right) = \frac{I}{2\pi\gamma}\left(\frac{1}{R_1} - \frac{1}{R_2}\right).$$

8.20-rasmda yer yuzidagi potensial o‘zgarish egri chizig‘i tasvirlangan. Inson qadami orasidagi masofada $R_1 = 22\,\text{m}$, $R_2 = 23\,\text{m}$, ya’ni 1 va 2 nuqtalar orasidagi kuchlanishni topamiz. Yerlatgichdan o‘tadigan tok $I = 1000\,\text{A}$, yerning o‘tkazuvchanligi $\gamma = 10^{-2}$ $\Omega^{-1}\cdot\text{m}^{-1}$ deb olamiz.

$$U_{12} = \frac{I}{2\pi\gamma}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{10^3}{2\pi\cdot 10^{-2}}\left(\frac{1}{22} - \frac{1}{23}\right) \approx 31{,}9\,\text{m}.$$



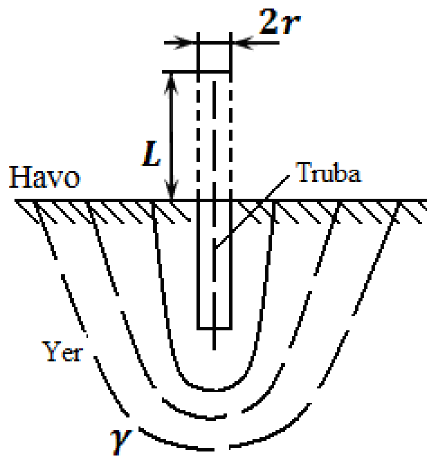
8.20- rasm.

Aks tasvirlash usuli

8.33-masala. Yer yuzasiga perpendikular qoqilgan r radiusli uzunligi l bo‘lgan po‘lat trubali yerlagichning o‘tkazuvchanligini aniqlang. Ikkinchi elektrod cheksiz uzoqlikda joylashgan, yerning o‘tkazuvchanligi $\gamma \cdot L / r >> 1$.

Yechish. Yerlagich maydonining tasviri 8.21-rasmda keltirilgan. Yerga qoqilgan l uzunlikdagi truba, rasmda xuddi shu uzunlikdagi havodagi truba bilan qo‘shilgan. Yerlagichning o‘tkazuvchanligi $2l$ uzunlikdagi truba yarmining o‘tkazuvchanligiga teng:

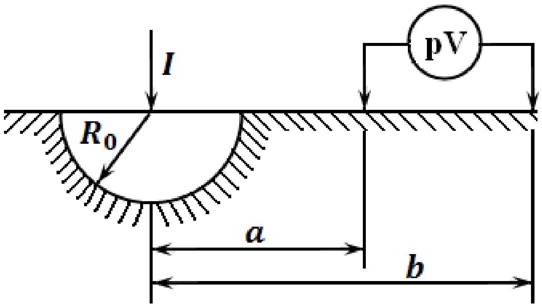
$$G \approx \frac{2\pi\gamma l}{\ln\frac{2l}{r} - 0,307}.$$



8.21- rasm.

8.34- masala. Agar yerga qoqilgan yarim sferik yerlatgichdan $I = 105$ A tok o‘tayotgan bo‘lib, uning atrofidagi maksimal qadam kuchlanish $U = 50$ V bo‘lsa, yerga qoqilgan yarim sferik yerlatgichning R radiusini toping. Odam qadamini 0,8 m deb olinsin. Tuproqning solishtirma o‘tkazuvchanligi $\gamma = 5 \cdot 10^{-2}$ S/m.

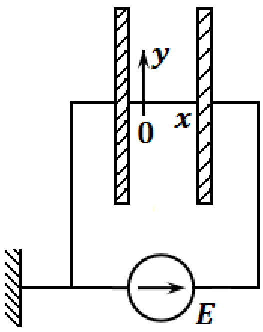
8.35-masala. Tuproqning solishtirma qarshiligini aniqlash uchun unga radiusi $R_0 = 25$ sm metall yarim sfera o‘rnatilgan (8.22-rasm). Yerlagichdan o‘tadigan tok $I = 5$ A. Yarim sferadan $a = 50$ sm, $b = 100$ sm, uzoqliklarda qoqilgan zondlar orasidagi kuchlanish $U = 40$ V bo‘lsa, ikkinchi elektrod ancha uzoqda joylashgan deb qabul qilib, tuproqning o‘tkazuvchanligini aniqlang.



8.22- rasm.

8.36-masala. Koaksial kabel 100 kV o‘zgarmas kuchlanishga ulangan. Uning ichki tomirining radiusi $r_1 = 5\,\text{mm}$, tashqi qobig‘ining radiusi $r_2 = 20\,\text{mm}$. Uzunligi $l = 1\,\text{km}$ kabel izolatsiyasining notekis qizishi tufayli solishtirma o‘tkazuvchanlik tashqi qobiqdan ichki tomirga qarab $\gamma = \gamma_0(1 + r_1 / r)$ bo‘yicha o‘zgaradi, bunda $r_1 < r < r_2$, $\gamma = 10^{-10}\,\text{S/m}$. Izolatsiya o‘tkazuvchanligini, kabelning nomukammal izolatsiyasidan o‘tadigan tokni va izolatsiya qizishiga sarf bo‘ladigan quvvatni aniqlang.

8.37-masala. Yassi kondensator (8.23-rasm) nojins nomukammal, dielektrik singdiruvchanligi $\varepsilon_a = \varepsilon_0\varepsilon_{r1}(1 + kx^2)$ bo‘lgan (bunda ε_{r1} va $k-$ o‘zgarmas kattaliklar) dielektrik bilan to‘ldirilgan. Solishtirma o‘tkazuvchanlik $\gamma = \text{const}$. Kondensator o‘zgarmas U kuchlanishga ulangan. Elektrodlar orasidagi masofa d. Elektrodlar orasidagi hajmiy zaryad zichligi qonuniyatini aniqlang.



8.23- rasm.

8.5. Magnit maydon masalalari

Asosiy tenglamalar:

$$rotH = J\,, \qquad (8.45\text{ a})$$

$$\oint Hdl = \int JdS = I\,, \qquad (8.45\text{ b})$$

$$div\ \ B = 0\,, \qquad (8.46\text{ a})$$

$$\oint BdS = 0\,, \qquad (8.46\text{ b})$$

bunda l – integrallash konturi; S – integrallash yuzasi; J – o‘tkazuvchanlik tokining zichligi.

Vektorli potensial A quyidagi ifoda bilan kiritiladi:

$$rotA = B. \qquad (8.47)$$

Magnit oqim:

$$\Phi = \int BdS = \oint Adl\,. \qquad (8.48)$$

Yassi parallel maydonda 1 va 2 nuqtalar orasidagi sistemaning birlik uzunligiga to‘g‘ri keladigan magnit oqim:

$$\Phi_0 = A_1 - A_2\,. \qquad (8.49)$$

Magnit induksiya liniyalar oilasining tenglamasi:

$$A = \text{const}\,. \qquad (8.50)$$

Yassi meridian maydonda magnit induksiya liniyalari oilasining tenglamasi:

$$rA = \text{const}\,. \qquad (8.51)$$

Bir jinsli muhitda vektor potensial uchun Puasson tenglamasi:

$$\Delta^2 \vec{A} = -\mu_r \mu_0 \vec{J}\,. \qquad (8.52)$$

Chegaraviy shartlar:

$$H_{2t} - H_{1t} = J_S\,, \qquad (8.53)$$

bunda H_t – muhitlarning ajratish chegarasidagi urinma bo‘yicha yo‘nalgan maydon kuchlanganlik vektorining tashkil etuvchisi; J_S – tokning sirt zichligi.

$$B_{1n} = B_{2n}\,, \qquad (8.54)$$

bunda B_n – muhitlar chegarasiga normal bo‘lgan induksiya vektorining tashkil etuvchisi.

(8.54) o‘rniga vektor potensial uchun chegaraviy shartdan foydalanish mumkin:

$$A_{1t} = A_{2t}. \qquad (8.55)$$

Chiziqli muhit uchun ustma-ustlash usuliga ko‘ra

$$B = B_1 + B_2 + ... = \sum_k B_k , \qquad (8.56)$$

$$A = A_1 + A_2 + ... = \sum_k A_k , \qquad (8.57)$$

bunda B_k va A_k mos toklar qo‘zg‘atgan yig‘indilar.

Magnit oqimlar va ularni qo‘zg‘atuvchi toklar orasidagi bog‘lanish

$$\left.\begin{array}{l} \Phi_1 = m_{11}I_1 + m_{12}I_2 + m_{13}I_3 + \dots \\ \Phi_2 = m_{21}I_2 + m_{22}I_2 + m_{23}I_3 + \dots \\ \Phi_3 = m_{31}I_1 + m_{32}I_2 + m_{33}I_3 + \dots \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \end{array}\right\} \qquad (8.58)$$

bunda m_{KK} – konturlarning o‘zinduksiyasi; m_{iK} – konturlarning o‘zaro induktivligi.

Chiziqli muhitlardagi magnit maydonlarni hisoblashda aks tasvir usuli qo‘llaniladi. Ayniqsa, bu usul turli magnit singdiruvchanliklarga ega ikkita muhitning yassi yoki silindrik sirt chegarali masalalarni yechishda qo‘l keladi.

Agar chegara yassi bo‘lsa, har bir o‘tkizgichdagi I tokning muhitdagi magnit maydoni shu tok va chegaraning boshqa tomonida xuddi shunday masofa uzoqligida joylashga fikriy (fiktiv) tok I bilan aniqlanadi. Bunda butun fazoning magnit singdiruvchanligi tokli o‘tkazgich joylashgan muhitning magnit singdiruvchanligiga teng deb olinadi. Tok

$$I_1 = I(\mu_{2r} - \mu_{1r})(\mu_{2r} + \mu_{1r}) = m_1 \cdot I , \qquad (8.59)$$

bunda μ_{2r} – boshqa muhitning magnit singdiruvchanligi.

Ikkinchi muhitdagi maydon I tok qayerda joylashgan bo‘lsa, xuddi shu joydagi fikriy I_2 tok bilan aniqlanadi, butun fazoning magnit singdiruvchanligi μ_{2r} ga teng.

Tok quyidagiga teng:

$$I_2 = I \cdot 2\mu_{1r} / (\mu_{1r} + \mu_{2r}) = m_2 \cdot I . \qquad (8.60)$$

Toklar bo‘lmagan fazoda, magnit maydonni ifodalash uchun skalyar potensial kiritilishi mumkin, u H magnit kuchlanganlik bilan quyidagicha bog‘langan:

$$H = -\mathrm{qrad}\varphi_M \, . \qquad (8.61)$$

Nojins muhitda skalyar potensial quyidagi tenglamaga to‘g‘ri keladi:

$$\nabla^2 \varphi_M = \mathrm{div} M \, , \qquad (8.62)$$

bunda M – magnitlanish vektori.

Bir jinsli muhitda skalyar potensial Laplas tenglamasiga to‘g‘ri keladi.

$$\nabla^2 \varphi_M = 0 \, . \qquad (8.63)$$

O‘zgarmas magnit maydon bilan zaryadi bo‘lmagan elektrostatik maydon orasida shaklan o‘xshashlik (analogiya) mavjud. Analogik parametrlar quyidagicha:

Magnit maydon	H	φ_M	B	μ_r	μ_0	F
Elektrostatik maydon	E	φ	D	ε_r	ε_0	F_D

bunda Φ_D – elektr siljish vektorining oqimi.

O‘zgarmas magnit muhitda magnit yopishqoqlik bo‘lmasa, o‘zgaruvchan maydon masalalari o‘zgarmas maydon masalalari kabi yechiladi.

Birinchi yaqinlashtirishda magnit yopishqoqlik quyidagi tenglama bilan hisobga olinadi:

$$\tau dM / dt = (\mu_r - 1)H - M \, , \qquad (8.64)$$

bunda τ – magnit yopishqoqlikning o‘zgarmas doimiyligi.

Sinusoidal maydondagi yopishqoqlik tenglamasi:

$$j\omega\tau \underline{M} = (\mu_r - 1)\underline{H} - \underline{M} \, . \qquad (8.65)$$

Yopishqoqlikni hisobga oladigan kompleks magnit singdiruvchanlik:

$$\underline{\mu_r} = \mu_r^1 - j\mu_r^{11} = (\mu_r + j\omega\tau)/(1 + j\omega\tau) \, . \qquad (8.66)$$

Magnit maydonning energiyasini quyidagicha aniqlash mumkin:

1) maydonning barcha V hajmi bo‘yicha integrallash yordamida maydon

vektorlari orqali:

$$W = \int \frac{1}{2} BHdV \,, \tag{8.67}$$

2) tokli konturlarning magnit oqimlari va toklar orqali:

$$W = \sum \frac{1}{2} I_K \Phi_K \,, \tag{8.68}$$

3) toklar oqayotgan *V* hajmning potensiallari va tok zichligi vektorlari orqali:

$$W = \int \frac{1}{2} AJdV \,, \tag{8.69}$$

Magnitlangan jism sababli g'altakning oqim ilashish (oqim ilashish teoremasi),

$$\Psi = \frac{\mu_0}{I} \int H_I MdV, \tag{8.70}$$

bunda integrallash magnitlanishga ega bo'lgan magnitlangan jismning hajmi bo'yicha bajariladi; $H_1 -$magnitlangan jism olib chiqilgan maydonning kuchlanganligi, lekin bunda g'altakdan I tok o'tadi deb hisoblanadi.

Bulardan tashqari, masala yechishda ko'p qo'llaniladigan quyidagi formulalarni keltiramiz.

Parallel toklarning o'zaro ta'sir kuchi, ya'ni Amper qonuni F o'tkazgichlardan o'tayotgan toklarning kuchlari I_1, I_2 ga, o'tkazgichning uzunligi *l* ga to'g'ri mutanosib bo'lib, ular orasidagi masofa r_0 ga teskari mutanosibdir, ya'ni

$$F = \frac{\mu_0 \mu}{4\pi} \cdot \frac{2I_1 I_2}{r_0} l \,,$$

bunda $\mu_0 = 4\pi \cdot 10^{-7} \frac{\text{H}}{\text{m}} = 12.56 \cdot 10^{-7} \frac{\text{H}}{\text{m}}$.

Konturning magnit momenti R_m konturdan o'tayotgan tok kuchi *I* ning, konturning yuzi *S* ga ko'paytmasiga teng.

$$P_m = I \cdot S \,.$$

Induksiyasi *B* bo'lgan magnit maydondagi *I* tokli yopiq konturga ta'sir qiluvchi juft kuchning aylantiruvchi momenti *M* quyidagi formuladan aniqlanadi.

$$M = P_m \cdot B \sin \alpha \,,$$

bunda P_m – tokli konturning magnit momenti, α – magnit maydon induksiya vektori $\overline{B}$ ning yo‘nalishi bilan kontur tekisligiga tushirilgan normal $\overline{n}$ orasidagi burchak.

Bir jinsli ($\overline{B} = \text{const}$) magnit maydondagi tokli o‘tkazgichga ta’sir qiluvchi amper kuchi F_A o‘tkazgichdan o‘tayotgan tokning kuchi I ga, o‘tkazgichning uzunligi l ga, magnit maydonning induksiyasi B ga va o‘tkazgich bilan magnit induksiya chiziqlari orasidagi burchak α ning sinusiga to‘g‘ri mutanosibdir, ya’ni

$$F_A = IlB\sin\alpha .$$

Bir jinsli ($\overline{B} = \text{const}$) magnit maydonda harakatlanayotgan zaryadli zarraga ta’sir qiluvchi Lorens kuchi F_L zarraning zaryadi q ga, tezligi v ga, magnit maydon induksiyasi B ga va $\overline{v}$ bilan $\overline{B}$ vektorlar orasidagi burchak α ning sinusiga to‘g‘ri mutanosibdir.

$$F_\text{L} = qvB\sin\alpha .$$

Cheksiz uzun, to‘g‘ri bo‘lgan I tokli o‘tkazgich hosil qilgan magnit maydonning induksiyasi B va kuchlanganligi H quyidagi formulalardan aniqlanadi:

$$B = \mu_0\mu\frac{I}{2\pi R}, \qquad H = \frac{B}{\mu_0\mu} = \frac{I}{2R} .$$

I tok o‘tayotgan solenoid (toroid) o‘zagida hosil bo‘lgan magnit maydonning induksiyasi B va kuchlanganligi H quyidagiga teng:

$$B = \mu_0\mu \cdot In = \mu_0\mu \cdot I\frac{N}{t}, \quad H = \frac{B}{\mu_0\mu} = I \cdot n = I\frac{w}{t},$$

bunda $n = \dfrac{w}{t}$ - solenoid (toroid)ning uzunlik birligiga mos keladigan o‘ramlar soni, l – solenoid o‘zagining uzunligi, w – o‘ramlar soni va Iw – amper o‘ramlar soni.

Berilgan sirt orqali o‘tayotgan magnit induksiya oqimi F magnit maydon induksiyasi B ning yuzi S hamda sirtga tushirilgan normal $\overline{n}$ bilan $\overline{B}$ vektor orasidagi burchak α kosinusining ko‘paytmasiga teng:

$$\Phi = BS\cos\alpha .$$

Bir jinsli ($\bar{B}=\text{const}$) magnit maydonda $\bar{v}$ tezlik bilan harakatlanayotgan uzunligi l ga teng bo‘lgan o‘tkazgich uchlarida hosil bo‘lgan induksion EYK quyidagi formuladan aniqlanadi:

$$\varepsilon_i = Blv \sin\alpha ,$$

bunda α – magnit maydon induksiyasi kuch chiziqlari bilan $\bar{v}$ tezlik vektori orasidagi burchak.

Uzunligi l, ko‘ndalang kesim yuzasi S, o‘ramlar soni w bo‘lgan solenoidning induktivligi L quyidagi formuladan aniqlanadi:

$$L = \mu_0\mu \cdot n^2 V = \mu_0\mu\left(\frac{w}{t}\right)^2 l \cdot S = \mu_0\mu\frac{w^2}{l}S ,$$

bunda $n = \frac{w}{t}$ – solenoidning uzunlik birligiga mos kelgan o‘ramlar soni, $V = l \cdot S$ – solenoid hajmi, μ – solenoid o‘zagining nisbiy magnit singdiruvchanligi, μ_0 – magnit doimiysi.

Muhitning nisbiy magnit singdiruvchanligi μ muhitdagi magnit maydonning induksiyasi B vakuumdagi induksiyasi B_0 dan necha martaga farq qilishini ifodalaydi:

$$\mu = \frac{B}{B_0} .$$

Bir jinsli ($\bar{B}=\text{const}$) magnit maydonning energiyasi W_m quyidagi formuladan aniqlanadi:

$$W_\text{m} = \frac{B^2}{2\mu_0\mu} \cdot V = \frac{\mu_0\mu \cdot H^2}{2} \cdot V = \frac{H \cdot B}{2} \cdot V ,$$

bunda V – magnit maydon egallagan fazoning hajmi.

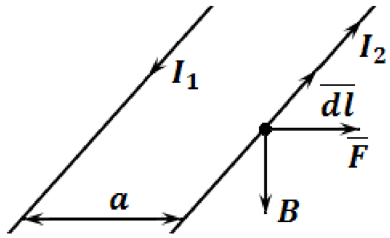
Magnit maydon energiyasining zichligi

$$W_\text{m} = \frac{H \cdot B}{2} .$$

Elektromagnit maydon to‘liq energiyasi zichligi elektr W_e va magnit W_m maydonlar energiyalari zichliklarining yig‘indisig teng bo‘ladi:

$$W = W_e + W_m = \frac{D^2}{2\varepsilon_0\varepsilon} + \frac{B^2}{2\mu_0\mu} = \frac{\varepsilon_0\varepsilon E^2}{2} + \frac{\mu_0\mu \cdot H^2}{2} = \frac{ED}{2} + \frac{HB}{2}.$$

8.38- masala. Trolleybus kontakt o‘tkazgichlari orasidagi masofa *a*=50 sm (8.24- rasm). Simlardan oqadigan tok $I_1 = 1000\,\text{A}$. Uzunligi 1 m bo‘lgan o‘tkazgichlarning o‘zaro itarish kuchlarini hisoblang.



8.24- rasm.

Yechish. Kuchni hisoblash uchun $F = \frac{I_1 I_2 \mu_0 dl}{2\pi \cdot a}$ formuladan foydalanamiz. Ikkinchi o‘tkazgichning $\overline{dl}$ elementi va chap o‘tkazgichning $\overline{B}$ induksiyasi orasidagi burchak 90^0 ga teng. Shuning uchun $[\overline{dl} \cdot \overline{B}]$ vektor ko‘paytmaning moduli $dlB\sin 90^0 = dlB$ ga teng.

Birinchi o‘tkazgichdagi tok ikkinchi o‘tkazgich joylashgan nuqtalarda hosil qilgan magnit induksiyasi, to‘la tok qonuni bo‘yicha $B = \frac{\mu_0 I_1}{2\pi \cdot a}$ ga teng. Bundan

$$F = I_2 dlB = \frac{I_1 I_2 \mu_0 \mu \cdot dl}{2\pi \cdot a}$$

yoki

$$F = \frac{1000 \cdot 1000 \cdot 1.256 \cdot 10^{-6} \cdot 1 \cdot 1}{2 \cdot 3.14 \cdot 0.5} = 0.4\,\text{N}.$$

Kuch ta’sirida o‘tkazgichlar bir - biridan itariladi.

8.39- masala. Ikkita uzun parallel kabel sim bir-biridan $r_0 = 15\,\text{mm}$ masofada joylashgan. Agar kabel simlar qisqa tutushganda ulardan $I = 30\,\text{kA}$ tok o‘tsa, simning har $l = 1\,\text{m}$ ga ta’sir qiluvchi *F* kuchni toping.

8.40-masala. Liniyada qisqa tutashuv yuz berganda taqsimlash qurilmasining shinalari orqali $i = 60\,\text{kA}$ tok kuchi bo‘lsa, bir-biridan $r_0 = 10sm$ masofada turgan ikkita parallel o‘tkazgichlarning har $l = 1{,}2$ m uzunligiga ta’sir qiluvchi *F* kuch nimaga teng bo‘ladi?

8.41-masala. Ikkita parallel sim izolyatorlarga mahkamlangan. Simlar orasidagi masofa $r_0 = 4\,\text{sm}$. Simlar $U = 120\,\text{V}$ kuchlanish ostida. Qo‘shni izolyatorlar orasidagi masofa $l = 2\,\text{m}$. $P = 24\,\text{kW}$ quvvat iste’mol qiluvchi elektrodvigatelga tok o‘tganda, simlar izolyatorga qanday kuch bilan ta’sir qiladi? Simlar og‘irligini hisobga olmang.

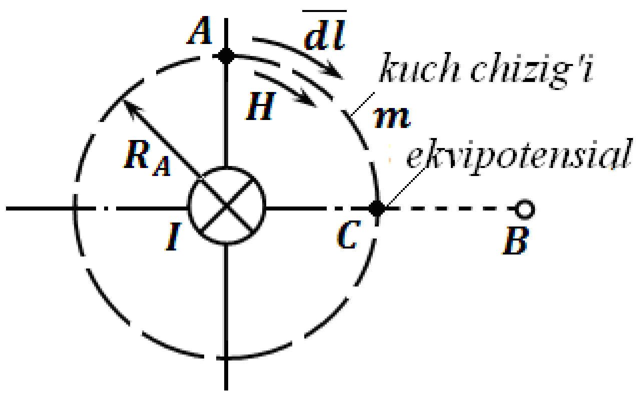
8.42-masala. Magnit maydon kuch chiziqlari yo‘nalishiga $\alpha = 30^0$ burchak ostida joylashgan $l = 0{,}25\,\text{m}$ uzunlikdagi to‘g‘ri o‘tkazgichga $F_A = 3\,\text{N}$ kuch ta’sir qiladi. Agar o‘tkazgichdagi tok kuchi $I = 30\,\text{A}$ bo‘lsa, magnit maydon induksiyasi *B* topilsin.

Yechish. Amper qonuniga binoan magnit maydondagi tokli o‘tkazgichga $F_l = IlB\sin\alpha$ kuch ta’sir qiladi, bunda α – magnit kuch chiziqlari bilan o‘tkazgich orasidagi burchak. Bundan

$$B = \frac{F_A}{Il\sin\alpha} = \frac{3}{30 \cdot 0.25 \cdot \sin 30^0} = 0.8\ \text{T}$$

8.43-masala. Elektr motordan $I = 25\,\text{A}$ tok o‘tganda uning yakori chulg‘ami simiga $F_A = 1.2\,\text{N}$ kuch ta’sir qiladi. Agar simning uzunligi $l = 20\,\text{sm}$ bo‘lsa, sim joylashgan magnit maydonning induksiyasi *B* ni toping.

8.44-masala. $I = 10\ \text{A}$ chiziqli o‘zgarmas tokning magnit maydonda joylashgan *A* va *B* nuqtalar orasidagi skalyar magnit potensiali (magnit kuchlanish) ayirmasini toping (8.25- rasm).



8.25- rasm.

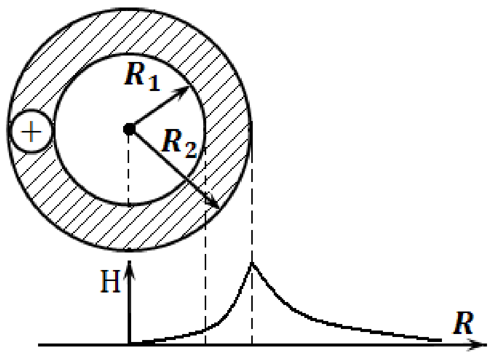
Yechish.

$$U_{MAB}=\int_A^B \overline{H}\cdot\overline{dl}=\int_{AmC}\overline{H}\cdot\overline{dl}+\int_{CnB}\overline{H}\cdot\overline{dl}\,;$$

$$\int_{AmC}\overline{H}dl=H\int dl=\frac{I}{2\pi R_A}\cdot\frac{2\pi R_A}{4}=\frac{I}{4};\qquad \int_{CnB}\overline{H}\overline{dl}=0.$$

Chunki shu uchastkada $\overline{H}$ va $\overline{dl}$ orasidagi burchak 90^0. Binobarin, $U_{MAB}=I/4=4\,\text{A}$.

8.45-masala. Ichki radiusi r_1 va tashqi radiusi r_2 bo‘lgan truba yoqalab o‘zgarmas I tok o‘tadi (8.26- rasm). Truba ichidagi, jismidagi va tashqarisidagi magnit kuchlanganlikni aniqlovchi formulalarni keltirib chiqaring.



8.26- rasm

Yechish. Ko‘rsatilgan sohalardagi kuchlanganliklarni to‘la tok qonuni asosida topamiz. Agar truba o‘qi atrofida radiusi $r\le r_1$ bo‘lgan aylana chizilsa, bu aylana tokni qamrab olmaydi. Shuning uchun $r\le r_1$ da N=0, ya’ni trubaning ichida magnit maydon bo‘lmaydi. Trubadagi tok zichligi

$$\delta=\frac{I}{\pi\left(r_2^2-r_1^2\right)};$$

Radiusi $r_1\le r\le r_2$ bo‘lgan aylana $\delta\pi\left(r^2-r_1^2\right)=\delta\cdot S_{\text{kesim}}$ tokni o‘rab (qamrab) oladi. Shuning uchun bu oraliqdagi r ni o‘zgarishida

$$H=\frac{I\left(r^2-r_1^2\right)}{2\pi\cdot r\left(r_2^2-r_1^2\right)}$$

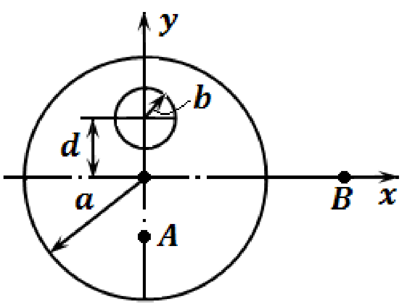
Truba tashqarisida $r\ge r_2$ da kuchlanganlik H giperbolik qonun $H=I/2\pi\cdot r$ bo‘yicha o‘zgaradi. $H=f(r)$ grafik 8.26- rasmda tasvirlangan.

8.46-masala. Radiusi a=20 mm bo‘lgan to‘g‘ri chiziqli silindrik o‘tkazgichda radiusi $b = 5\,\text{mm}$ li silindrik aylana kesimli teshik mavjud. Silindrlarning o‘qlari orasidagi masofa $d = 10\,\text{mm}$ (8.27-rasm) o‘tkazgich bo‘ylab “biz tomondan” zichligi $\delta = 5 \cdot 10^3\,\text{A/m}^2$ li o‘zgarmas tok o‘tadi. Teshik ichidagi hamda A va B nuqtalardagi magnit maydon kuchlanganliklarini toping. A va B nuqtalarning koordinatalari: $x_A = 0$; $y_A = -10\,\text{mm}$; $x_B = 40\,\text{mm}$; $y_B = 0$.

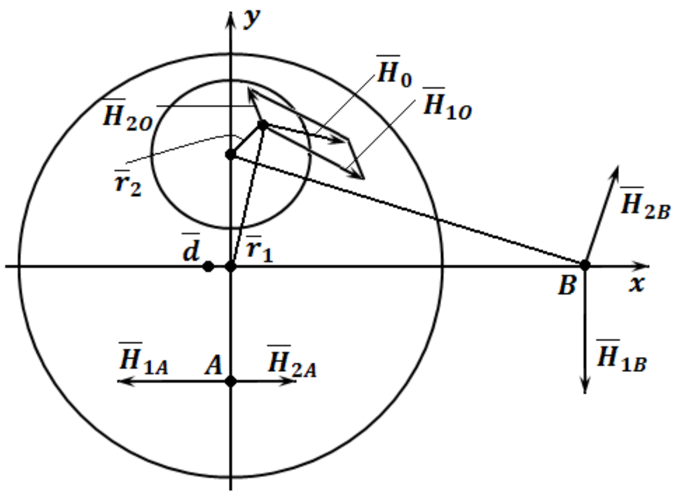
Yechish. Silindrik o‘tkazgichning ichidagi va tashqarisidagi magnit maydonni ustma-ustlash usulida topish mumkin. Radiusi a bo‘lgan monolit silindrik o‘tkazgichdan zichligi δ li tokning maydonidan (kuchlanganlik $\overline{H}_1$) hamda b radiusli silindr yoqalab teskari tomonga o‘tadigan xuddi shu zichlikka ega tokning maydonidan (kuchlanganlik $\overline{H}_2$) (8.28- rasm), silindrik teshikning ichida joylashgan ixtiyoriy O‘ nuqtadagi magnit maydonning kuchlanganligi

$$\overline{H}_0 = \overline{H}_{10} + 2\overline{H}_{20} = \frac{\delta}{2}\left[\overline{k}\left(\overline{r}_2 - \overline{r}_1\right)\right],$$

bunda $\overline{H}_{10} = -\left[\overline{\delta} \cdot \overline{r}_1\right]/2$; $\overline{H}_{20} = \left[\overline{\delta} \cdot \overline{r}_2\right]/2$.



8.27- rasm.



8.28- rasm.

Chunki $\overline{r}_1 - \overline{r}_2 = \overline{d} = \overline{jd}$ (21.1.a - rasm) da $\overline{H} = \frac{\delta}{2}\left[\overline{k}\left(-\overline{j}\right)d\right] = \overline{i}\frac{\delta d}{2} = \overline{i} \cdot 25 \cdot 10^2\,\text{A/м}$.

Nuqta A dagi kuchlanganlik:

$$\overline{H}_{1A} = -\overline{i} \cdot \frac{\delta \cdot r_{1A}}{2} = -\overline{i} \cdot 25 \cdot 10^2, \text{A/m};$$

$$\overline{H}_{2A} = \bar{i} \cdot \frac{I}{2\pi \cdot r_{2A}} = \bar{i} \cdot \frac{\delta\pi \cdot b^2}{2\pi \cdot r_{2A}} = \bar{i} \cdot 3.13 \cdot 10^2, \mathrm{A/m};$$

bunda r_{1A}, r_{2A} – A nuqtadan birinchi va ikkinchi silindr o‘qigacha bo‘lgan masofa. Shunday qilib, $\overline{H}_A = \overline{H}_{1A} + \overline{H}_{2A} = -i \cdot 21.87 \cdot 10^2, \mathrm{m}$. B nuqtadagi kuchlanganliklarning modullari

$$H_{1B} = \frac{\delta\pi \cdot a^2}{2\pi \cdot r_{1B}} = 25 \cdot 10^2 \ \mathrm{A/m};$$

$$H_{2B} = \frac{\delta\pi \cdot b^2}{2\pi \cdot r_{2B}} = 1.51 \cdot 10^2 \ \mathrm{A/m},$$

bunda $r_{1B,}$ r_{2B} – B nuqtadan birinchi va ikkinchi silindrlar o‘qigacha bo‘lgan masofa.

Vektor shaklda

$$\overline{H}_{1B} = \bar{j} \cdot 25 \cdot 10^2, \mathrm{A/m};$$

$$\overline{H}_{2B} = \bar{i} \cdot H_{2B} \sin\alpha + \bar{j} \cdot H_{2B} \cos\alpha = \left(\bar{i} \cdot 1.51 \sin 14.5^0 + \bar{j} \cdot 1.51 \cos 14.5^0\right) \cdot 10^2 =$$
$$= \left(\bar{i} \cdot 0.377 + \bar{j} \cdot 1.46\right) \cdot 10^2, \mathrm{A/m};$$

Natijada quyidagini hosil qilamiz:

$$\overline{H}_B = \overline{H}_{1B} + \overline{H}_{2B} = \left(-\bar{j} \cdot 23.54 + \bar{i} \cdot 0.377\right) \cdot 10^2, \mathrm{A/m}.$$

8.47-masala. To‘g‘ri chiziqli radusi $a = 30\,\mathrm{mm}$ li silindrik o‘tkazgichda dumaloq kesimli silindrik teshik mavjud (8.27-rasm). O‘tkazgich yoqalab o‘zgarmas $I = 700\,\mathrm{A}$ li tok o‘tadi. O‘tkazgich o‘qidagi magnit maydonning kuchlanganligi $\overline{H}_1 = \bar{i} \cdot 600\ \mathrm{A/m}$, teshik o‘qidagi magnit maydonning kuchlanganligi $\overline{H}_2 = \bar{i} \cdot 2400\ \mathrm{A/m}$. Silindrik teshikni radiusi va o‘tkazgich hamda teshik o‘qlari orasidagi masofani toping.

8.6. O‘tkazuvchi muhitdagi elektr maydon masalalari

8.48-masala. Ichki simining diametri 10 mm va tashqi simining diametri 20 mm bo‘lgan koaksial kabel izolatsiyasining solishtirma elektr o‘tkazuvchanligi $\gamma = 10^{-8}\ \frac{\mathrm{S}}{\mathrm{m}}$ Kabelning 1 km uzunligiga to‘g‘ri keladigan izolatsiya qarshiligini aniqlang.

Yechish. Koaksial kabel silindrsimon kondensatorga o‘xshash. Uning sig‘imi $C = \dfrac{2\pi\varepsilon_a l}{\ln\dfrac{r_2}{r_1}}$ formula yordamida hisoblanadi. Yuqorida ko‘rib chiqilgan analogiyadan foydalanib, 1 km uzunlikdagi kabel izolatsiyasining o‘tkazuvchanligini topamiz:

$$\mathrm{G} = \frac{2\pi\gamma l}{\ln\dfrac{r_2}{r_1}} = \frac{2\pi \cdot 10^{-8} \cdot 10^3}{\ln\dfrac{20}{10}} = 9 \cdot 10^{-5}\ \mathrm{S}.$$

Izolatsiya qarshiligi:

$$R = \frac{1}{G} = \frac{1}{9 \cdot 10^{-5}} \approx 1{,}1 \cdot 10^4\ \Omega.$$

8.49-masala. Diametri 10 mm, uzunligi 1 m va o‘zaro 20 sm masofada joylashgan ikkita parallel sim solishtirma elektr o‘tkazuvchanligi $\gamma = 0.03\ \dfrac{\mathrm{S}}{\mathrm{m}}$ bo‘lgan suvga botirilgan. Simlar kuchlanishi 110 V bo‘lgan manbaga ulanganda tok va iste’mol qilinadigan quvvatni toping.

Yechish. Ikkita simli liniyadagi singari simlar orasidagi o‘tkazuvchanlik:

$$G = \frac{2\pi\gamma l}{\ln\dfrac{r_2}{r_1}} = \frac{2 \cdot 3{,}14 \cdot 0{,}03 \cdot 1}{\ln\dfrac{0{,}2}{0{,}01}} = 0{,}0625\,\mathrm{S}.$$

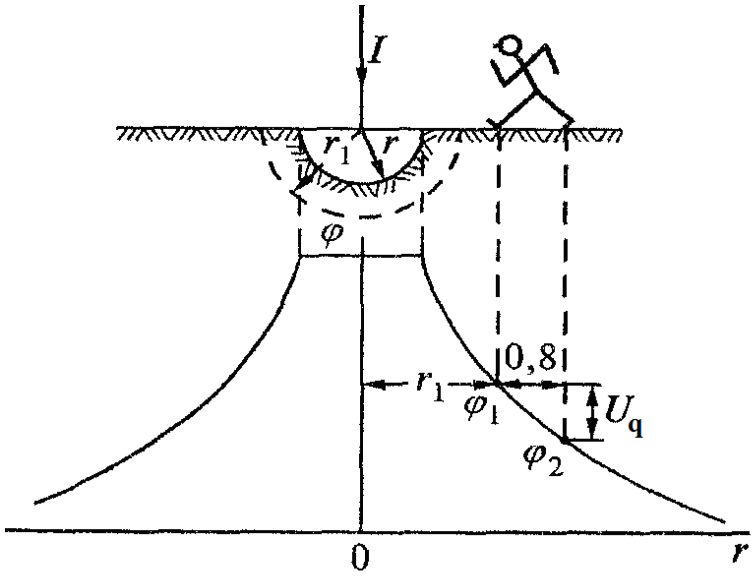
Zanjirdagi tok:

$$I = UG = 110 \cdot 0{,}0625 = 6{,}875\ \mathrm{A}.$$

Quvvat esa:

$$P = UI = 110 \cdot 6{,}875 = 756{,}25\,\mathrm{W}.$$

8.50-masala. Yarim shar shaklidagi yerlatgich elektr maydonini hisoblang (8.29-rasm).



8.29- rasm.

Yechish. Yarim shar sig‘imi $C = 2\pi\varepsilon_a r.$ Yuqorida qayd etilgan analogiyaga ko‘ra yerlatgich o‘tkazuvchanligi $G = 2\pi\gamma r.$

Qarshilik esa $R = \dfrac{1}{G} = \dfrac{1}{2\pi\gamma r}.$

Yerlatgichdan yerga tok yoyilib o‘tishdagi kuchlanish:

$$U = IR = \frac{I}{2\pi\gamma r}.$$

Yerlatgichdan r_1 masofada joylashgan nuqtalardagi tok zichligi:

$$\delta = \frac{I}{2\pi r_1^2}.$$

Bu nuqtalardagi maydon kuchlanganligi:

$$E = \frac{\delta}{\gamma} = \frac{I}{2\pi\gamma r_1^2}.$$

Yerlatgich bilan nuqtalar orasidagi kuchlanish:

$$U = \varphi_{r_1} - \varphi_r = \int_r^{r_1} E dr = \frac{I}{2\pi\gamma}\left(\frac{1}{r_1} - \frac{1}{r}\right).$$

Inson qadami (0,8 m)ga mos keluvchi kuchlanish:

$$U_K = \int_r^{r_1+0,8} E dr = \frac{I}{2\pi\gamma}\left(\frac{1}{r_1} - \frac{1}{r+0,8}\right).$$

Yerlatgich qoqilgan yer atrofidagi potensial:

$$\varphi = \frac{I}{2\pi\gamma r_1} = U\frac{r}{r_1}.$$

O‘zgarmas tokning magnit maydoni

Magnit maydondagi mexanik kuchlar

8.51-masala. Bir jinsli magnit maydonining kuch chiziqlariga perpendikular holda uzunligi $l = 0.25\,\mathrm{m}$ va toki $I = 100\,\mathrm{A}$ bo‘lgan to‘g‘ri o‘tkazgich joylashtirilgan. Magnit induksiya *B*=1T O‘tkazgichga ta’sir etayotgan kuch aniqlansin.

Yechish. Elektromagnit kuch: $F = IBl\sin\alpha = 100\cdot 1\cdot 0,25\sin 90^0 = 25$ N.

8.52- masala. Magnit induksiyasi *B*=1,25 T bo‘lgan bir jinsli maydonda unga perpendikular bo‘lgan relslar $l = 0.5\,\mathrm{m}$ oraliq masofada o‘rnatilgan. Uzunligi 4 m relslar bo‘yicha siljiydigan o‘tkazgichga $m = 1\,\mathrm{kg}$ massali yuk osilgan (8.30-rasm). Yukning yo‘l oxiridagi tezligi $\vartheta = 50m/s$ bo‘lsa, zanjirdan qanday tok o‘tadi?

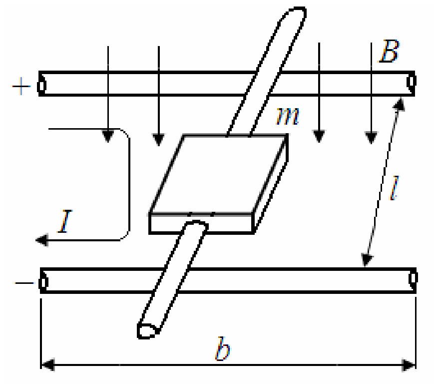
Yechish. Yukning yo‘l oxiridagi kinetik energiyasi:

$$W = mv^2/2 = 1\cdot 50^2/2 = 1250\,\mathrm{J}.$$

O‘tkazgichni magnit maydoni kesib o‘tganda quyidagi formula bilan aniqlanadigan ish bajariladi:

$\frac{m\vartheta^2}{2} = I\Phi$. Bu shartdan zanjirdan o‘tayotgan tokni topamiz:

$$I = \frac{mv^2}{2\Phi}\cdot\frac{1}{BS} = 1250\frac{1}{1,25\cdot 0,5\cdot 4} = 500\,\mathrm{A}.$$



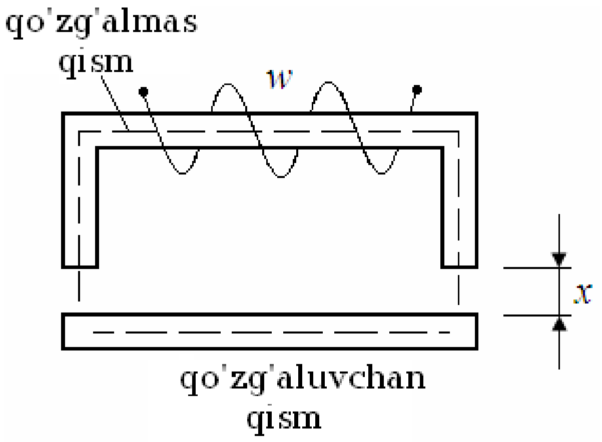
8.30- rasm.

8.53-masala. Yassi elektromagnit magnitlovchi chulg‘am o‘ralgan qo‘zg‘almas qism va qo‘zg‘aluvchi yakordan iborat (8.31-rasm). Magnit o‘tkazgichning o‘lchamlari va g‘altakning o‘ramlar soni ma’lum bo‘lsa, elektromagnitning tortish kuchini aniqlang.

Yechish. Sistema magnit maydonining energiyasi:

$$W_{sist.} = W_{m.o'.} + W_{h.o.} = \frac{BH_{m.o'.}}{2} V_{m.o'.} + \frac{BH_{h.o.}}{2} V_{h.o.}.$$

bunda, $H_{m.o'.}$ va $H_{h.o.}$ - mos ravishda magnit o‘tkazgich va havo oralig‘idagi magnit maydon kuchlanganliklari, $V_{m.o'.}$ va $V_{h.o.}$ - mos ravishda magnit o‘tkazgich va havo oralig‘ining hajmlari.



8.31- rasm.

Magnit o‘tkazgichdagi maydon kuchlanganligi havo oralig‘idagi kuchlanganlikdan ko‘p marta kam bo‘lganligi uchun deyarli barcha energiya havo oralig‘ida to‘planadi. Shuning uchun:

$$W_{sist.} \approx W_{h.o.} = \frac{BH_{h.o.}}{2} Sx.$$

x, S – havo oralig‘ining mos ravishda kengligi va ko‘ndalang kesimining yuzi.

Elektromagnitning tortish kuchi: $F = \frac{BH_{\text{h.o.}}}{2} S = \frac{B_2}{2\mu_0} S = \frac{B_2}{2\mu_0} S = \frac{\mu_0 H^2_{\text{h.o.}}}{2} S$

Javoblar

1.3. 3 A. **1.4**. $R_5 = 4\ \Omega$. **1.5**. $R_{AB} = 10\ \Omega$, $R_{CD} = 6{,}15\ \Omega$. **1.6**. $R_{AB}=7/11R;\ R_{BC}=0;\ R_{SD}=10/11R$;

1.7. $R_{AE} = \frac{1}{3}R$. **1.8**. $R_{AB} = 4\ \Omega$; a)3,75 Ω; b)3,75 Ω; d)4 Ω; e)4 Ω. **1.11**. J: E=24 V; r_{ich}=0,5 Ω;

1.12. I=3,2 A; R=75,6 Ω; P=800 W. **1.13**. R_{min}=500 Ω; R_{max}=27500 Ω; $P_{yuk.min}$=0,25 W; $P_{yuk.max}$=2.5 W; $P_{p.min}$=2.75 W; $P_{r.max}$=5 W. **1.14**. 5 ta element. **1.15**. 110 V. **1.16**. 120 V. **1.17**. 50 V; 0,4 Ω. **1.18**. 6 V. **1.19**. 116 V. **1.26**. $U_{1\text{-}2}$=0,1 V; $U_{1\text{-}3}$ =1 V; $U_{1\text{-}4}$=10 V. **1.27**. -4V.

1.29. $a)\ R_{1s.ish.} = 120\Omega;\ R_{2.q.t} = 72\Omega;\ b)\ R_{1s.ish.} = 20\ \Omega;\ \ R_{2q.t.} = 18\ \Omega;$

$v)\ R_{1s.i.sh.} = 838\ \Omega;\ R_{2q.t} = 200\Omega$. **1.30**. J: 0,3 A; 0,2 A; 0,15 A; 0,1 A; 0,15 A.

1.35. 10 A, 9 A, 1 A, 8 A, 7 A, 17 A, 10 A, 5 A, 5 A.

1.41. $I_1 = 0;\ I_3 = 1\ \text{A};\ I_4 = 3\ \text{A};\ I_5 = 1\ \text{A}.$

1.42. $I_2 = 2{,}67\ \text{A};\ I_3 = 6\ \text{A};\ I_4 = 6{,}67\ \text{A};\ I_5 = 2\ \text{A};\ I_6 = 8{,}67\ \text{A}.$

1.43. $I_1 = 0{,}67\ \text{A},\ I_2 = 1{,}67\ \text{A},\ I_3 = 2{,}33\ \text{A},\ I_4 = 4\ \text{A}.$

1.44. 5,2 A; 2,375 A; 3,35 A; 2,81 A; 0,975 A.

1.48. 0,8 A; 4 A; 11 A. **1.52**. J: 40 A; 60 A. **1.58**. J: $I_3 = 13\ \text{A}$.

1.60. 2870 V, 287 A, 123 A. **1.74**. $\frac{J}{2}$. **1.75**. 1 A. **1.76**. 7 Ω; 28 W. **1.77**. 1 A.

2-bob

2.1. 314 rad/s, 50 Hz.

2.2. J: 14,1sin500t (A); 10 A; 1000 V·A; 1000 W; 1000(1 − cos1000t) W.

2.3.

$i = 14{,}1\sin(500t - \pi/2)\ \text{A};\ I = 10\ \text{A};\ S = 10000\ \text{VA};\ Q = 1000\ \text{var};$

$P = 0;\ p = 1000\cos(1000t + \pi/2)\ \text{W};\ W_{\text{Mmax}} = 2\ \text{J}.$

2.4. $i = 14{,}1\sin(500t + \pi/2)\ \text{A};\ I = 10\ \text{A};\ S = 1000\ \text{V}\cdot\text{A};\ Q = -1000\ \text{var};$

$P = 0;\ p = 1000\sin 1000t\ \text{W};\ W_{\text{Э}\max} = 2\ \text{J}.$

2.8. 0.5; **2.12.** 6 Ω; 25,5 mΩ.

2.16. 8,5 A, −45°, 85 V, 85 V, 170 V, 722 W, −722 var, 1020 VA.

2.17. 25 V, 25 V, 65 V, 100 V. **2.18.** 24 Ω, 48 Ω. **2.19.** 10 A, $10\sqrt{2}$ A, 10 A. **2.20.** 5 A, 5 A, 4 A. **2.21.** 20,7; $X_L = 12\ \Omega$. **2.22.** 17 Ω, 17 Ω.

2.23. 16,98 Ω, −16,98 Ω.

2.24. 10 A, 4 A, 12,8 A, $r_{ekv} = 7{,}23\,\Omega$, $X_{ekv} = 2{,}98\,\Omega$.

2.25. 8,11 *A*, 80 V, 9,85 Ω, 0,985 Ω.

2.26. 4,4 A, 5,5 A, 5,5 A, 7,79 A.

2.27. $I = 4\ \text{A};\ i = 4\sqrt{2}\sin(\omega t + 45^\circ)\ \text{A};$

$I_1 = 8\ \text{A};\ i_1 = 8\sqrt{2}\sin(\omega t + 135^\circ)\ \text{A};$

$I_2 = 8{,}95\ \text{A};\ i_1 = 8{,}95\sqrt{2}\sin(\omega t - 18^\circ 30')\ \text{A}.$

2.28. 2,2 A, 1,56 A, 1,56 A.

2.29. 5,66 A. **2.30.** $I = 10\ \text{A},\ I_1 = I_C = 4{,}47\ \text{A},\ I_2 = I_L = 8{,}94\ \text{A}.$

2.47. 645 Ω. **2.55.** $(10 + j5)\ \Omega$; 625 Ω.

2.61. $L = 47{,}6$ mH; $\underline{I} = 2e^{-j71^\circ 30'}$ A; $\underline{U}_{ac} = 141e^{-j26^\circ 30'} 50\sqrt{2}e^{-j71^\circ 30'}$ V.

2.62. S=26,5 mkF; $\underline{Z}_k = (4{,}3 + j5{,}88)\,\Omega$. **2.63.** a) C=*120* mkF; b) C=75 mkF.

2.64. $P = \mathrm{Re}[U_{mn} \overset{*}{I}_2] = 630\ \mathrm{W}$. **2.75.** $i = 0{,}01\sqrt{2}\sin(\omega_0 t - 84^\circ 16')$ A; $u_2 = \sqrt{2}\sin(\omega_0 t + 5^\circ 44')$ V.
2.76. 128 V. **2.77.** $7\,\Omega$; 0,0765 H; 133 mkF; $3\,\Omega$.
2.78. 100 Ω. **2.79.** 10^4 s. **2.80.** 2 W.
2.83. $i_1 = 3{,}16\sin(4000t - 26°30')$ A, $i_2 = \sqrt{2}\sin(4000t + 45°)$ A,
$i = 3\sqrt{2}\sin 4000t$ A.
2.84. $L' = 20$ H, $i_1 = 1{,}58\sin(4000t - 63°30')$ A,
$i_2 = \sqrt{2}\sin(4000t + 45°)$ A, $i = 2{,}12\sin 4000t$ A.
$L'' = 5$ H, $i_1 = 3{,}16\sin(4000t - 26°30')$ A,
$i_2 = \sqrt{2}\sin(4000t + 45°)$ A, $i = 3\sqrt{2}\sin 4000t$ A.
2.85. $C = 5$ mkF; $I_1 = 1{,}12$ A; $I_2 = 1$ A; $I = 1{,}5$ A. **2.86.** 3 A. **2.87.** 99 mkF.
2.92. $U_{\text{kir}} = 50$ V. **2.93.** $R = 40\ \Omega$, $L = 0{,}32$ mH, $C = 80$ pF, $\Pi = (0{,}99 - 1{,}01)$ rad/s.
2.94. $L = 159$ mH. **2.97.** $\omega_{\text{cheg}} = 2\cdot 10^3\ \dfrac{\text{rad}}{\text{s}}$.

2.98. $K_u(0) = \dfrac{R_4(R_2 + R_3)}{R_4(R_2 + R_3) + R_1(R_2 + R_3 + R_4)}$;

$K_{u(\infty)} = \dfrac{R_3 R_4}{R_3 R_4 + R_1(R_3 + R_4)}$.

2.99. $Y_{\text{kir}} = \dfrac{1}{\sqrt{300^2 + (0{,}1\omega)^2}}\, e^{-jarctg\left(\frac{0{,}1\omega}{300}\right)}$. **2.100.** $Y_{\text{uz}}(0) = 0$; $Y_{\text{uz}} = (\infty) = 0{,}0273$ ms.
2.104. *a)* A va E; *b)* A va E; A va F; B va G. **2.105.** A va E. **2.107.** 20 A; 56.6 V; 113 V. **2.108.** 4 A.
2.109. J: 145 W; 128 V.
2.113. $\underline{I}_1 = 16{,}2e^{-j13^\circ 52'}$ A; $\underline{I}_2 = 17{,}25e^{-j3^\circ 25'}$ A; $\underline{I} = 33{,}25e^{-j8^\circ 30'}$ A;
$P_{W1} = 1890$ W; $P_{W2} = 2070$ W; $P = 3960$ W;
$P_1 = 1570$ W; $P_2 = 2390$ W; $P_{12M} = 320$ W.
2.115. 122 V. **2.117.** $100\sqrt{2}$ V. **2.119.** 290 rad/s. **2.121.** 200 mkF. **2.122.** 66,*7* mkF.
2.131. 0,5; j22,5; j0,0333; 0,5; 80 V. **2.132.** -1; $j10$; 0; -1; $10e^{j180^0}$; 0.
2.133. 0,5; j5; j0,15; 0,5; 5 V; j1,5 A. **2.137.** $Z_{\text{yuk}} = j4\,\Omega$. **2.138.** $Z_{\text{yuk}} = j10\,\Omega$.
2.139. $Z_{1\text{kir}} = Z_{\text{yuk}} = (3{,}9 - j3)\,\Omega$.
2.142. $Z_{1C} = 119e^{-j67^\circ 30'}\ \Omega$, $Z_{2C} = 84e^{-j22^\circ 30'}\ \Omega$, $\Gamma = 0{,}76 - j0{,}574$.
2.144. $A_{11} = A_{22} = 1 + j$, $A_{12} = (20 + j10)\ \Omega$, $A_{21} = j0{,}1$ S.
2.145. $A_{11} = 1 + j$, $A_{12} = (j20 - 10)\ \Omega$, $A_{21} = j0{,}1\ S$, $A_{22} = -1$.
2.148. $n = \dfrac{1}{5}$; $\underline{Z}_{\text{qo'sh}} = -j125\ \Omega$.

2.150. $A_{11} = A_{22} = -0{,}5$; $A_{12} = j15\ \Omega$; $A_{21} = j0{,}05$ S;
$Z_{\text{s.ish}} = j10\ \Omega$; $Z_{\text{q.t}} = -j30\ \Omega$; $Z_C = 10\sqrt{3}\ \Omega$; $\Gamma = j\dfrac{\pi}{3}$.
2.154. 3180 rad/s; 100 Ω.

5-bob

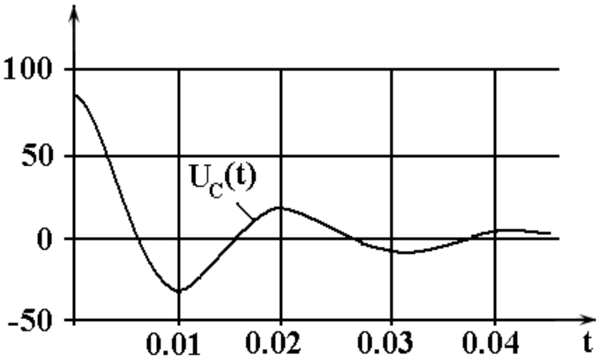
5.4
$$\left.\begin{aligned} &i_1(t)=i_2(t)+i_3(t),\\ &E=i_3(t)R_3+\frac{1}{C}\int i_3(t)dt,\\ &E=i_2(t)R_2+L\frac{di_2}{dt}. \end{aligned}\right\}$$

Kommutatsiyaning birinchi qonuni bo'yicha induktivlikdagi tok sakrab o'zgarmaydi, bundan $i_2(0_+)=0.$ Kommutatsiyaning ikkinchi qonuni bo'yicha kondensatordagi kuchlanish sakrab o'zgarmaydi, ya'ni $U_2(0_+)=0.$ Shuning uchun
$$i_2(0_+)=i_3(0_+)=\frac{E}{R_2}=5\,\text{mA}.$$
.

5.5. $U_C=132$ V.

5.6. $i(t)=40\left(1-e^{\frac{t}{0.2}}\right)$mA $t=138$ ms.

5.7. 1. $U_C(t)=100\cos(300t-18.43)e^{-100t}$ V. 2. $U_C(t)=U(1+\alpha t)\cdot e^{-\alpha t}$ - bu kondensatordagi kuchlanish o'zgarishini ifodasi, bunda $\alpha=316.5\ \frac{1}{s}$, $R_3=43.3\ \Omega.$ chegaraviy aperiodik jarayon o'zgarishiga mos keladi.



5.17-rasm

5.8. $i_1(t)=\left(20+2.5e^{-60t}\right)$mA; $i_2(t)=\left(10+5e^{-60t}\right)$mA; $i_3(t)=\left(10-2.5e^{-60t}\right)$mA.

5.9. L=0.5 H; **5.10.** C=50 mkF. **5.14.** $i_1(t)=\left(6-3.6e^{-40t}\right)$mA; $i_2(t)=6\left(1-e^{-40t}\right)$mA; $i_3(t)=2.4e^{-40t}$ mA;

5.15. $i_1(t)=\left(0.2+0.05e^{-5\cdot10^6 t}\right)$A; $i_2(t)=0.2\left(1-e^{-5\cdot10^6 t}\right)$A; $i_3(t)=0.25e^{-5\cdot10^6 t}$ A;

5.16.
$$i_1(t)=\left(0.05+0.1e^{-2000t}-0.05e^{-3000t}\right)\text{A};$$
$$i_2(t)=\left(0.05-0.1e^{-2000t}+0.1e^{-3000t}\right)\text{A};$$
$$i_3(t)=\left(0.2e^{-2000t}-0.15e^{-3000t}\right)\text{A}.$$

5.17. $i(t)=\left(0.1e^{-10t}-0.05e^{-5\cdot t}\right)$A; $U_C(t)=200\left(e^{-5t}-e^{-10t}\right)$V.

5.18. 1 va 2 shartlar uchun $I(p)=\dfrac{E_2(p)+Li(0)}{R+pL}$;

1) $i(t)=e^{-100t}$A.

2) $i(t)=\left(-1+2e^{-100t}\right)$A.

Uchinchi shart uchun $I(p)=\dfrac{E_2(p)+jLi(0)}{R+pL}=\dfrac{200+j(p+100)}{(p-j100)(p+100)}$;

$i(t)=\left(\sin 100t+e^{-100t}\right)$A.

5.19. 1) aperiodik; 2) kritik (chegaraviy); 3) tebranma.

5.20. $U_C=154e^{-50t}\sin(209\cdot t+76^0 30')$ V; $i(t)=-3.5e^{-50t}\sin(209\cdot t)$ A.

5.22. $2e^{-150t}$A. **5.23.** $0{,}5e^{-60t}$A; **5.24.** $6e^{-40t}$A. **5.25.** $0{,}4e^{-1200t}$A. **5.28.** $3-2e^{-400t}$A; e^{-400t}; $3-3e^{-400t}$A. **5.29.** $1{,}5e^{-125t}$A; $3-0{,}5e^{-125t}$A; $6-1e^{-125t}$ A; $9-3e^{-125t}$A.

5.32. a) $Z(p)=\dfrac{r_1r_2+pL(r_1+r_2)}{r_2+pL}$; b) $Z(p)=\dfrac{r+pL+p^2rLC}{rCp+1}$;

d) $Z(p)=\dfrac{r+pL+p^2rLC}{(r+pL)Cp}$; e) $Z(p)\dfrac{pL(prC+1)}{Cp(r+pL)^{+1}}$; f) $Z(p)=\dfrac{r+pL}{(r+pL)pC+1}$;

g) $Z(p)=\dfrac{r+pL+p^2rLC}{p^2LC+1}$; **5.34.** $0{,}2-0{,}2e^{-750t}$A; $60e^{-750t}$ V. **5.35.** 1,33-0,53e^{-300t} A.

5.37. $h_u(t)=1-0.25\cdot e^{-500t}$; $k_u(t)=\left[125e^{-500t}+0.75\delta(t)\right]\dfrac{1}{s}$

5.38. $K_u(t)=\left[-0.4\cdot 10^3e^{-2\cdot 10^3\cdot t}+0.2\delta(t)\right]\dfrac{\text{mS}}{\text{s}}$; $Y(t)=0.2e^{-2\cdot 10^3 t}$mS; $h_u(t)=0.6e^{-2\cdot 10^3 t}$

5.39. $h_u(t)=0.4-0.093\cdot e^{-3840t}$; $k_u(t)=\left[357\cdot e^{-3840t}-0.307\delta(t)\right]\dfrac{1}{s}$

5.40. $i(t)=2(e^{-2000t}-e^{-4000t})$ mA

5.44. $i_{CI}(t)=0.05-0.05e^{667t}$, A; $i_{CII}(t)=-0.182e^{-667t}+0.05$, A; $i_{CIII}(t)=0.1087e^{-667t}$, A

5.45. $i_{LI}(t)=0{,}1168e^{-642t}-0{,}1168+75{,}1t$, A; $t\in\left[0;10^{-3}\right]$s;

$i_{LII}(t)=-0.327e^{-642t}+0.267-75{,}1t$, A; $t\in\left[10^{-3};2\cdot 10^{-3}\right]$s;

$i_{LIII}(t)=0.0947e^{-642t}$, A; $t\in\left[2\cdot 10^{-3};+\infty\right]$s.

6-bob.

6.9. 0,0638 A; 0,1 A; 10 W. **6.10.** 2 A; 0,44 A; 36 V. **6.11.** 0,25 ms; 0,52 ms. **6.12.** 0,25 ms.

6.13. 12,9 A; V, 05 A ; 3400 W.

6.14. *a)* $0{,}2455\sin\omega t-0{,}0488\sin 3\omega t$, A, 0,177 A

b) $0{,}0678\sin\omega t-0{,}0061\sin 3\omega t$, A; 0,048 A.

6.16. 1 A; 53 V; 103 V; 88 V. **6.18.** 0.21sinωt A. **6.19.** 0.21sinωt A.

6.20. 10^{-3} Wb; 32^0; $0.608\sin(10^4t-\eta)-0{,}01\sin(10^4-\eta)$.

6.21. 0,2 A; $15^0 40^1$.

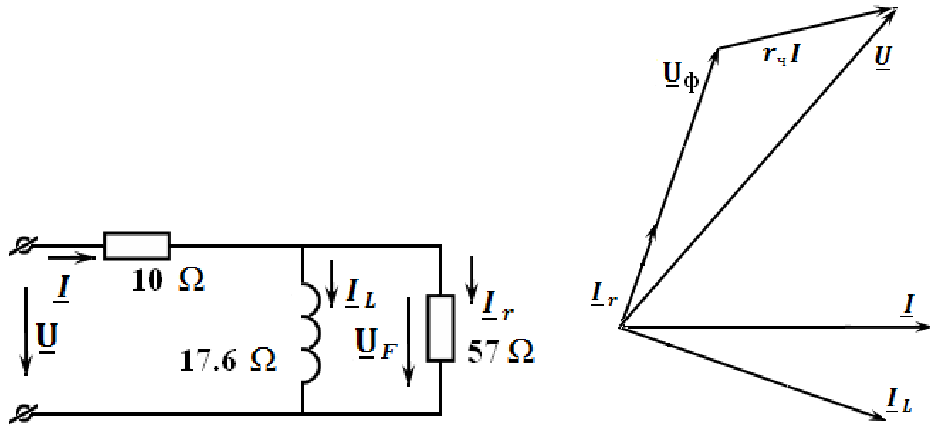
6.23. $0{,}184\sin(10^4t-45^0 40')+0{,}004\sin(3\cdot 10^4t+124^0 30')$, A.

6.24. $0{,}109\sin(10^4t+27^0)+0{,}003\sin(3\cdot 10^4t-101^0)$,

$45{,}6\sin(10^4 t + 27^0) + 1{,}2\sin(3\cdot 10^4 t + 97^0 30'),$

$0{,}157\sin(10^4 t - 19^0) + 0{,}003\sin(3\cdot 10^4 t - 82^0 30)\text{A}.$

6.26.



a) b)

Almashlash sxema va vektor diagramma.

6.27. 3 A.

7-bob.

7.6. $U_1(t)$=417.4sin(ωt+5.65^0)V; $i_1(t)$=0.395sin((ωt+6^0)A; **7.8.** Z_{yuk}=465+j193, P_{yukmax}=53.46W.

7.9. *a)* $U_2(t)$=500sin(ωt-60^0) V; $i_2(t)$=lsin(ωt-60^0) A; *b)* $U_2(t)$=1000sinωt; $i_2(t)$=0;

7.10. $\mathrm{Z_{yuk}} = \text{-j}500\sqrt{3}\ \Omega$.

8-bob.

8.2. 62^0 40^1. **8.5.** 54,7 m; 6100 pF.

8.6. $E = \dfrac{U}{r l_n \dfrac{r_2}{r_1}} = \dfrac{9100}{r}\cdot$ V/sm, $\varphi = \dfrac{U}{l_n \dfrac{r_2}{r_1}} \cdot l_n \dfrac{r_2}{r} = 9100 \ln \dfrac{r_2}{r_1}$, V, $C = \dfrac{2\pi\varepsilon_0}{l_n \dfrac{r_2}{r_1}} = 2{,}52\,\text{pF}$, $r = r_2$ da $\varphi = 0$

8.8. $36.5\dfrac{\alpha}{180^0}$ pF, $\alpha -$ graduslarda. **8.14.** -0,0348 mkC/m^2; 0,0038 mkC/m^2; 0,0038mC/m^2; 0,0178 mkC/m^2; 900V. **8.15.** a) 19,5 mkN; b) 1,5 mkN.

8.16. 620 V/sm; 700V/sm; 350V/sm; 435 V/sm; 0; 0,98 kV; 1,95kV; 0; 7,80 pF/m; $E(x{,}0)/E(0{,}0) = 20{,}4/(20{,}4 - x^2)$; $\dfrac{\varphi(x_1 0)}{U/2} = \dfrac{1}{3{,}58}\ln\left|\dfrac{x-4{,}5}{x+4{,}5}\right|$, bunda *x*-santimetrda.

8.18. 26,6 pF/m. **8.19.** 0,524 pF/sm; 21,8 kV. **8.27.** $\rho_{\text{erk}} = -18x - 80$ **8.28.** $q = 5\cdot 10^{-9}\,\text{C}$.

8.29. $\varphi_1 = 1{,}5\cdot 10^5 x^2$V; $E_1 = 3\cdot 10^5 x$ V/m; $\varphi_2 = -15\cdot 10^2 x(2{,}5\cdot 10^3 - \text{x})$ V; $E_2 = 15\cdot 10^2$ V/m.

8.30. $\varphi_1 = -10^3 R^3 + 10^3 B$; $\varphi_2 = 9/R$ $E_1 = R^0 \cdot 3\cdot 10^8 \cdot R^2$ V/m;; $E_2 = R^0 \cdot 9/R^2$ V/m..

8.34. $R_0 = 193\,\text{sm}$. **8.35.** $\gamma = 1{,}99\cdot 10^{-2}$ S/m. **8.37.** $\rho = -\dfrac{2U\varepsilon_0\varepsilon_{r1}k}{d}$ **8.39.** 12 kN

8.40. 8.64 kN. **8.41.** $4\cdot 10^{-3}$ N. **8.43.** 20 T. **8.47.** $b = 0.9\,sm$, $d = 1.8\,\text{sm}$..

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