OLIY MATEMATIKADAN INDIVIDUAL TOPSHIRIQLAR TO'PLAMI

Uch qismdan iborat

Fizika matematika fanlari doktori
professor A.P.Ryabushkoning
umumiy tahriri ostida

1 Qism

Belorussiya Xalq ta'limi vazirligi
tomonidan texnik-muxandis oliy o'quv yurtlari
uchun o'quv qo'llanma sifatida
tavsiya etilgan

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Mualliflar: A.P. Ryabushko, V.V. Barxatov, V.V. Derjavets, I.E. Yurut
Retsenzentlar: Moskva energetika instituti Oliy matematika kafedrasi, Minsk radiotexnika instituti Oliy matematika kafedrasi mudiri, f.-m.f.d., prof. L.A.Cherkas

Tarjimonlar: Toshkent davlat texnika universiteti Amaliy matematika kafedrasi mudiri A.B. Axmedov, dotsentlar: G. Shodmonov, A. Abdukarimov, D.N. Shamsiyev, E.E. Esonov

O'quv qo'llanma olij texnika o'quv yertlarining bakalavr 1-2 bosqich talabalari bilim savijasini nazorat qilish, mustaqil ta'limni to'la joriy etish, yoshlarning erishgan iqtidori asosida bilim va ko'nikmalarini rivojlantirishga mo'ljallangan.

Taqrizchilar

Irrigasija va meliorasiya instituti
"Oliy matematika" kafedrasi mudiri,
Xudojarov B. t.f.d.,prof.

Abu Rayxon Beruni nomli
Toshkent davlat texnika univeresiteti
"Amaliy matematika"kafedrasi
Abduraxmonov A. f.-m.f.n., dos.

Kitobning ruscha nashriga so'zboshi

Qo'lingizdagi ushbu kitob, "Oliy matematikadan individual topshiriqlar" nomli o'quv qo'llanmalar majmuasining birinchi qismi bo'lib, u oliy o'quv yurtlarining texnik – muhandislik mutaxassislari uchun mo'ljallangan 380-450 soatlik dastur asosida yozildi. Shuningdek, mazkur majmuadan, oliy matematika fanini o'qitish uchun ajratilgan soatlar anchagina kam bo'lgan boshqa yo'nalishdagi mutaxassislar tayyorlaydigan oliy o'quv yurtlarining talabalari ham foydalanishlari mumkin. (Buning uchun taqdim etilayotgan materiallardan keraklilarini tanlab olinishi lozim).

Tavsiya etilayotgan ushbu o'quv qo'llanma, auditoriyada amaliy mashg'ulotlar va mustaqil (nazorat) ishlarni o'tkazish uchun hamda oliy matematikaning barcha bo'limlari bo'yicha IUT (individual uy topshiriq)larni bajarish uchun mo'ljallangandir.

O'quv majmuaning birinchi qismida chiziqli va vektorlar algebrasi, analitik geometriya hamda bir argumentli funksiyaning differensial hisobiga bag'ishlangan mavzular bo'yicha ma'lumotlar keltirilgan.

Kitobning yaxshilanishi borasidagi bebaho ko'rsatma va maslahatlarini ayamaganliklari uchun mualliflar jamoasi, mazkur majmuaning taqrizchilari bo'lgan Moskva energetika instituti, Oliy matematika kafedrasining jamoasiga (kafedra mudiri, RAN FA muxbir a'zosi, fizika-matematika fanlari doktori, professor S.I. Poxojayev) Minsk radiotexnika institutining Oliy matematika kafedrasining mudiri, fizika-matematika fanlari doktori, professor L.A. Cherkasga hamda shu kafedraning dotsentlari, fizika-matematika fanlari nomzodlari L.A. Kuznetsov, P.A. Shmelyov, A.A. Karpuklarga, o'zlarining minnatdorchiliklarini bildiradilar.

Kitob borasidagi barcha fikr – mulohazalaringizni quyidagi manzilga yuborishingizni iltimos qilamiz: 220048, Minsk, Masherov shoh ko'chasi, 11, "Visheyshaya shkola" nashriyoti.

Mualliflar

Kitobning o'zbekcha nashriga so'zboshi

Respublikamizda olib borilayotgan demokratik islohotlar**,** iqtisodiyotning turli sohalariga bozor munosabatlarining jadal kirib kelishi, jahon bozorida raqobatbardosh mahsulotlarni ishlab chiqarish strategiyasi va "Kadrlar tayyorlash milliy dasturi" asosida ta'lim sohasida ustivor yo'nalishlarni belgilash, chuqur bilimga ega barkamol avlodni tarbiyalash, yoshlarni o'z ustida mustaqil ishlashga o’rgatish va tashabbusini oshirish - eng muhim ahamiyatga ega bo’lgan vazifalardan biri sifatida qaralmoqda.

E’tiboringizga havola etilayotgan ushbu kitob texnik muhandislik yo’nalishidagi oliy o’quv yurtlarida oliy matematikadan amaliyot darslarida keng qo’llanilib kelinmoqda.

O’quv qo’llanma uch qismdan iborat bo’lib birinchi qismi chiziqli va vektor algebrasi, analitik geometriya, matematik analiz bo’limlaridan auditoriyada amaliy mashg’ulotlar va mustaqil (nazorat) ishlarni o’tkazish uchun hamda individual uy topshiriqilarni bajarish uchun mo’ljallangandir.

Talabalarning iqtidori e’tiborga olingan holda, Respublikamiz ta’lim tizimida ommalashib borayotgan mustaqil bilim va ko’nikmalarni rivojlantirishga kitobda keng o’rin berilgan.

Kitobda keng o’quvchilar ommasiga yaxshi tanish bo’lgan umumbashariy terminlar tarjima qilinmasdan asli ko’rinishida ishlatilishi maqsadga muvofiq deb topildi.

O’ylaymizki, ushbu o’quv qo’lanmadan, oliy matematikaga nisbatan kam soat ajratilgan oliy o’quv yurtlari ham o’z extiyojlariga muvofiq samarali foydalana oladilar.

Tarjimonlar

USLUBIY KO'RSATMALAR

Tavsiya etilayotgan qo'llanmaning shakli, undan foydalanish uslubi, hamda talabaning ko'nikmalari va bilimlarini baholash mezonlarini tavsiflab chiqamiz.

Oliy matematika kursi bo'yicha barcha ma'lumotlar boblarga taqsimlangan bo'lib, ularning har birida masala va misollarni yechish uchun zarur bo'ladigan nazariy bilimlar (asosiy ta'riflar, tushunchalar, teoremalar va formulalar) keltirilgan.

Ushbu ma'lumotlar yechilgan mashqlar yordamida mustahkamlanadi. (Mashqlar yechishning boshlanishi - ► va oxiri - ◄ belgilar yordamida aniqlanadi.) So'ngra auditoriya topshiriqlari (**AT**) va o'tkazilayotgan mashg'ulotlarda 10-15 daqiqaga mo'ljallangan mustaqil (kichik-nazorat) ishlar uchun javoblari bilan birgalikda masala va misollar tanlab olingan. Nihoyat 30 variantdan iborat haftalik individual uy topshiriqlari (**IUT**), namunaviy misollar yechimi bilan birgalikda berilgan. **IUT** ma'lum qismining javoblari ham keltirilgan. Har bobning nihoyasida amaliy ahamiyatga molik, darajasi yuqori qiyinchilikka ega bo'lgan qo'shimcha topshiriqlar joylashtirilgan.

Ilovalarda muhim mavzular bo'yicha bir va ikki soatga mo'ljallangan (har biri 30 variantlik) nazorat ishlari keltirilgan.

AT topshiriqlarining raqamlanishi ikki qismdan iborat bo'lib, birinchi qismi bobni aniqlasa, ikkinchi qismi ushbu bobdagi topshiriqning tartib raqamini belgilaydi, masalan **AT 2.1** shifri ikinchi bobga tegishli birinchi topshiriqni anglatadi.

IUT uchun ham boblar bo'yicha raqamlash kiritilgan. Masalan **5.2 IUT** belgisi beshinchi bobdagi ikkinchi **IUT** ekanligini ta'kidlaydi. Har bir **IUT** ning ichida esa quyidagicha raqamlash kiritilgan: birinchi son topshiriqdagi masalaning tartib raqamiga tegishli bo'lsa, ikkinchisi variantning tartib raqamini bildiradi. Shunday qilib, **5.2:16-IUT** shifri talabaning **5.2-IUT** dan 16 variantdagi topshiriqlarini bajarishini belgilab, ushbu variantda **1.16, 2.16, 3.16, 4.16** masalalar borligini ta'kidlaydi. **IUT** bo'yicha variantlarni tanlab olishda oldingi topshiriqdan keyingisiga o'tganida tasodifiy yoki boshqa usulda almashtirish usulini qo'llash mumkin. Bundan tashqari ixtiyoriy talabaga **IUT** berilishida bir xil turdagi masalalarni har xil variantlardan olish mumkin. Masalan, **3.1;1.2;2.4;3.6-IUT** shifri talaba **3.1-IUT** dan birinchi masalani 1 - variantdan, ikinchisini 4 - variantdan, uchinchisini 6 - variantdan yechishini ta'kidlaydi. Bu ko'rinishdagi kombinatsion usul yordamida 30 ta variantdan keng qamrovli ko'p variantlar hosil qilishi ta'minlanadi.

IUT larni ba'zi oliy texnika o'quv yurtlari (Belorussiya qishloq xo'jaligini mexanizatsiyalash instituti, Belorussiya politexnika instituti, Uzoq sharq politexnika instituti v.b) ning o'quv jarayonida qo'llanilishi, **IUT** ni har bir haftalik auditoriya topshiriqlaridan keyin alohida har safar berish o'rniga, ikki haftada bir marta , ikki haftalik auditoriya mashg'ulotlari mazmuniga mos ravishda berish maqsadga muvofiq ekanligini ko'rsatdi. Ushbu qo'llanmaga muvofiq, talabalar bilan ishlashni tashkil etish bo'yicha umumiy tavsiyalarni beramiz.

1. Oliy o'quv yurtlarining 25 talik guruhlari har haftada ikkita auditoriya mashg'ulotlari, talabalar erkin qatnashadigan maslahat darslari rejalashtiriladi va haftalik **IUT** beriladi. Ushbu tadbirlarni samarali tashkil etish maqsadida, talabalar bilimini, xato va kamchiliklarini aniqlash va tuzatish yo'llarini ko'rsatgan holda, tizimli baholash uchun kafedra tomonidan oldindan tayyorlangan professor-o'qituvchilarga **IUT** ning javoblar varaqasi va yechimlar majmuasi beriladi (talabalar mustasno). Javoblar varaqasi har bir topshiriqlar uchun tayyorlansa, yechimlar majmuasi faqat yechish usulini, amallar ketma-ketligi va hisoblashlardagi ko'nikmalarning to'g'riligini tekshirish uchun zarur bo'lgan muhim masala va variantlar ishlab chiqiladi. Kafedra tomonidan yechimlar varaqasi qaysi **IUT** lar uchun zarurligini belgilanadi. Yechimlar varaqasi (bitta variant bitta varaqda joylashadi) talabalar tomonidan bajarilgan topshiriqlar bajarilishida o'z - o'zini nazorat qilish uchun, talabalar o'rtasida o'zaro nazorat tashkil etishda ishlatiladi. Lekin, ko'pchilik hollarda echimlar varaqasi yordamida o'qituvchi usulning to'g'riligini tekshirsa, talabalar o'zining hisob-kitoblari to'g'riligini nazoratdan o'tkazishi mumkin. Ushbu usullar 25 talabaning **IUT** larini 15-20 daqiqa davomida tekshirib baholash imkonini beradi.

2. Oliy o'quv yurtlarining 15 talik guruhlarida har haftada ikkita auditoriya mashg'ulotlari, guruhlar dars jadvalida mustaqil tayyorlanish uchun, o'qituvchi nazorati ostida haftalik yuklamaga kiritilgan ikki soatlik maslahat darslari rejalashtiriladi. Dars jarayonini ushbu taxlitda tashkil etish (Belorussiya qishloq xo'jaligini mexanizatsiyalash instituti), talabalarning mustaqil va ijodiy ishlashlari, bunda bilim sifatini o'qituvchilar tomonidan tezkor ravishda nazorat qilish darajasi sezilarli tarzda oshishi kuzatiladi. Yuqorida tavsiya etilgan usullar bu erda ham o'zining samarasini beradi. Lekin ushbu guruhlarda **AT** va **IUT** larni tekshirish tezlashadi va topshiriqlarni bajarishda nazariy bilimlarni nazorat qilish imkoni oshadi, o'zlashtirmovchi talabalardan mavjud qarzdorliklarni kamaytirish imkoniyati paydo bo'ladi. **IUT**, mustaqil va nazorat ishlari bo'yicha baholar jamlamasi yordamida o'quv jarayonini boshqarish, nazorat qilish , talabalar olgan bilimlarini sifatini baholash imkoni paydo bo'ladi.

Yuqorida aytilgan tadbirlarni amalga oshirish natijasida semestr mobaynida o'rganilgan bilimlar bo'yicha an'anaviy semestr (yillik) imtihonlardan voz kechish, hamda talabalar ko'nikmalari va bilimlarini baholash bo'yicha blokli-siklik (modulli-siklik) deb ataluvchi usuldan foydalanish mumkin bo'ladi. Ushbu usulning mohiyati quydagilardan iborat: Fanning semestrdagi (yillik) yuklamasi 3-5 ta blok (modul) larga bo'linadi va ularning har biri bo'yicha **AT, IUT** bajarilib, sikl yakunida ikki soatlik yozma nazorat o'tkazilib, bu yerda 2-3 ta nazariy savollar hamda 5-6 ta masala va misolllar beriladi. **AT, IUT** va yakuniy nazorat ballarining yig'indisi talabalarning har bir blok (modul) va semestr (o'quv yilida) hamma bloklar (modullar) bo'yicha olgan bilimlarini ham alohida ob'ektiv baholash imkonini beradi. Shunga o’xshash usul Belorussiya qishloq xo’jaligini mexanizatsiyalash institutida tadbiq qilingan.

Fikrimiz yakunida, ushbu qo’llanma o’rtacha imkoniyatli talabalarga mo’ljallanganligini va bu erdagi bilimlarni egallash oliy matematika fanidan qoniqarli va yaxshi ko’nikmalarga ega bo’lishlarini ta’minlashini ta’kidlashimiz mumkin. Iqtidorli va a’lo bahoga o’quvchi talabalar uchun rag’batlantirishning chora-tadbirlarini e’tiborga olgan holda alohida murakkab topshiriqlar (ta’limda individual yondashuv) tayyorlanishi zarur. Masalan, bu talabalarga, o’z ichiga ushbu qo’llanmadagi yuqori murakkablikka ega masalalar va nazariy mashqlar (ushbu maqsad uchun, xususan, har bir bob oxiridagi qo’shimcha topshiriqlar mo’ljallangan) butun semestr uchun ishlab chiqilishi kerak. O’qituvchi ushbu topshiriqlarni semestr boshida berib, ularning bajarilish ketma-ketligini belgilab (o’zining shaxsiy nazoratida), talabalarga oliy matematikadan ma’ruza va amaliyot darslarida erkin qatnashishga ruxsat berishi mumkin va hamma topshiriqlar muvaffaqiyatli bajarilgandan so’ng sessiyada a’lo baho qo’yiladi.

1.DETERMENANTLAR. MATRISALAR. CHIZIQLI ALQEBRAIK TENGLAMALAR SISTEMASI

1.1. Determinantlar va ularning xossalari.Determinantlarni hisoblash.

n-tartibli *determinant* deb kvadrat jadval ko'rinishida yoziluvchi va determinantning elementlari deb ataluvchi, berilgan a_{ij} $(i, j = \overline{1,n})$ sonlar orqali (ular hammasi n^2) quyida ko'rsatilgan qoidaga asosan hisoblanuvchi Δ_n songa aytiladi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \ldots\ldots & a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} & \ldots\ldots & a_{2n} \\ \ldots & \ldots & \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & a_{n3} & a_{n4} & \ldots\ldots & a_{nn} \end{vmatrix} \qquad (1.1)$$

Bu yerda *i* indeks (1.1) kvadrat jadvalning satr raqamini, *j*-esa ustun raqamini ko'rsatadi. Ularning kesishgan joyida a_{ij} element joylashadi. Bu jadvalning ixtiyoriy satr yoki ustunini qator deb ataymiz.

Determinantning asosiy diagonali deb, a_{11}, a_{22},...., a_{nn} elementlarning to'plamiga aytiladi.

a_{ij} *elementning* M_{ij} *minori* deb, n-tartibli Δ_n determinantning *i* – satri va *j* – ustunini o'chirishdan hosil bo'lgan (n-1)- tartibli Δ_{n-1} determinantga aytiladi.

Determinantning a_{ij} *elementining algebraik to'ldiruvchisi* A_{ij} quyidagi tenglik bilan aniqlanadi.

$$A_{ij} = (-1)^{i+j} M_{ij}.$$

Δ_n determinantning qiymati quyidagi qoida bo'yicha topiladi.

n=2 uchun

$$\Delta_2 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11} \cdot a_{22} - a_{12} . a_{21}. \qquad (1.2)$$

n=3 uchun

$$\Delta_3 = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \cdot A_{11} + a_{12} \cdot A_{12} + a_{13} \cdot A_{13}, \qquad (1.3)$$

bu yerda

$$A_{11} = (-1)^{1+1} M_{11} = (-1)^{1+1} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}; \; A_{12} = (-1)^{1+1} M_{12} = (-1)^{1+2} \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix};$$

$$A_{13} = (-1)^{1+1} M_{13} = (-1)^{1+3} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix};$$

A_{11}, A_{12}, A_{13} kattaliklar – algebraik to'ldiruvchilar, M_{11}, M_{12}, M_{13}–Δ_n determinantning a_{11}, a_{12}, a_{13} elementlariga mos keluvchi minorlardir. Bu minorlar Δ_3 determinantning mos satr va ustunini o'chirishdan hosil bo'lgan 2- tartibli determinantlardir. Masalan, M_{12} minorni topish uchun Δ_3 determinantdan birinchi satr va ikkinchi ustunni o’chirish kerak bo’ladi.

Ixtiyoriy n uchun

$$\Delta_n = \sum_{k=1}^{n} a_{ik} A_{ik} \,, \qquad (1.4)$$

bu yerda $A_{1k} = (-1)^{1+k} M_{1k}$, M_{1k} minorlar esa Δ_n determinantdan birinchi satr va *k*-ustunni o’chirishdan hosil bo’lgan (*n–1*) –tartibli determinantdir.

Masalan:

$$\Delta_2 = \begin{vmatrix} 3 & -2 \\ 1 & 5 \end{vmatrix} = 3 \cdot 5 - (-2) \cdot 1 = 17;$$

$$\Delta_3 = \begin{vmatrix} 4 & 7 & -2 \\ 3 & -1 & 5 \\ 5 & 0 & 7 \end{vmatrix} = 4 \begin{vmatrix} -1 & 5 \\ 0 & 7 \end{vmatrix} - 7 \begin{vmatrix} 3 & 5 \\ 5 & 7 \end{vmatrix} - 2 \begin{vmatrix} 3 & -1 \\ 5 & 0 \end{vmatrix} = 4(-7) - 7(21-25) - 2 \cdot 5 = -10;$$

$$\Delta_4 = \begin{vmatrix} -1 & 1 & 2 & 0 \\ 0 & 4 & 1 & -2 \\ 1 & -3 & -1 & 0 \\ 5 & 0 & 0 & 4 \end{vmatrix} = -1 \begin{vmatrix} 4 & 1 & -2 \\ -3 & -1 & 0 \\ 0 & 0 & 4 \end{vmatrix} - 1 \begin{vmatrix} 0 & 1 & -2 \\ 1 & -1 & 0 \\ 5 & 0 & 4 \end{vmatrix} + 2 \begin{vmatrix} 0 & 4 & -2 \\ 1 & -3 & 0 \\ 5 & 0 & 4 \end{vmatrix} - 0 \begin{vmatrix} 0 & 4 & 1 \\ 1 & -3 & -1 \\ 5 & 0 & 0 \end{vmatrix} =$$

$$= -(4 \cdot (-4) + 3 \cdot 4 - 2 \cdot 0) - (0 \cdot (-4) - 4 - 2 \cdot 5) + (0 \cdot (-12) - 4 \cdot 4 - 2 \cdot 15) = -74.$$

Eslatma. Agar determinantning elementlari biror funksiyalardan iborat bo’lsa, umuman olganda berilgan determinant ham funksiya bo’ladi (lekin, son bo’lishi ham mumkin). Masalan:

$$\begin{vmatrix} \cos x & \sin x \\ \sin x & \cos x \end{vmatrix} = \cos^2 x - \sin^2 x = \cos 2x; \quad \begin{vmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{vmatrix} = \cos^2 x + \sin^2 x = 1; \quad \begin{vmatrix} tgx & 2 \\ \frac{1}{2} & ctgx \end{vmatrix} = 1 - 1 = 0.$$

Δ_3 determinantni hisoblash *uchburchaklar qoidasi* (Sarryus qoidasi) bilan bir xildir.

$$\Delta_3 = a_{11} a_{22} a_{33} + a_{12} a_{23} a_{31} + a_{21} a_{32} a_{13} -$$
$$- (a_{13} a_{22} a_{31} + a_{12} a_{21} a_{33} + a_{23} a_{32} a_{11}) \,. \qquad (1.5)$$

Bu qoidaning sxematik yozuvi quyida keltirilgan.

Masalan:

$$\Delta_3 = \begin{vmatrix} 1 & 2 & -3 \\ 4 & 6 & 5 \\ 2 & -1 & 1 \end{vmatrix} = 1 \cdot 6 \cdot 1 + 2 \cdot 5 \cdot 2 + 4 \cdot (-1) \cdot (-3) - ((-3) \cdot 6 \cdot 2 + 2 \cdot 4 \cdot 1 + 5 \cdot (-1) \cdot 1) = 71$$

determinantning *asosiy xossalarini* ko’rib chiqamiz.

1) Determinantning ixtiyoriy qatorining elementlarining ularning algebraik to'ldiruvchilariga ko'paytmasining yig'indisi qatorning tartib raqamiga bog'liq emas va quyidagi determinantga teng:

$$\Delta_n = \sum_{k=1}^{n} a_{ik} A_{ik} = \sum_{k=1}^{n} a_{kj} A_{kj} \, . \qquad (1.6)$$

Bu tenglikni ((1.4) kabi) determinantlarni hisoblash qoidasi deb ham qabul qilish mumkin;

2) determinantda uning barcha satrlarini mos ustunlari bilan almashtirsak, determinantning qiymati o'zgarmaydi va aksincha;

3) agar determinantda ikkita parallel qatorlarning o'rinlarini almashtirsak, uning ishorasi teskarisiga o'zgaradi;

4) ikkita bir xil parallel qatorga ega bo'lgan determinantning qiymati nolga teng;

5) agar determinantning biror qatorining barcha elementlari umumiy ko'paytuvchiga ega bo'lsa, bu umumiy ko'paytuvchini determinant belgisidan tashqariga chiqarish mumkin. Bundan, biror qatorning barcha elementlarini λ songa ko'paytirsak, Δ_n determinantning qiymati ham shu λ songa ko'paytirilishi kelib chiqadi;

6) agar determinantning biror qatorining barcha elementlari nollardan iborat bo'lsa, u holda determinantning qiymati ham nolga teng bo'ladi;

7) ikkita parallel qatorning mos elementlari o'zaro proporsional bo'lsa, determinantning qiymati nolga teng;

8) determinantning biror qatorning barcha elementlarining unga parallel bo'lgan boshqa qatorning barcha mos elementlarining algebraik to'ldiruvchilariga ko'paytmasining yig'indisi nolga teng, ya'ni quyidagi tenglik o'rinlidir;

$$\sum_{k=1}^{n} a_{ik} A_{jk} = 0, \quad \sum_{k=1}^{n} a_{ki} A_{kj} = 0, \qquad (i \neq j);$$

9) agar determinantning biror qatorining barcha elementlari ikkita qo'shiluvchining ko'rinishida bo'lsa, u holda bunday determinant ikkita determinantning yig'indisidan iborat bo'ladi, ularning birinchisi mos qatorning birinchi qo'shiluvchisidan, ikkinchisi esa ikkinchi qo'shiluvchisidan iborat bo'ladi

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \ldots\ldots a_{1i} + b_{1i} \ldots\ldots a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} \ldots\ldots a_{2i} + b_{2i} \ldots\ldots a_{2n} \\ \ldots & \ldots & \ldots & \ldots\ldots\ldots\ldots\ldots\ldots \\ a_{n1} & a_{n2} & a_{n3} & a_{n4} \ldots a_{ni} + b_{ni} \ldots\ldots a_{nn} \end{vmatrix} =$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \ldots\ldots a_{1i} \ldots\ldots a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} \ldots\ldots a_{2i} \ldots\ldots a_{2n} \\ \ldots & \ldots & \ldots & \ldots\ldots\ldots\ldots \\ a_{n1} & a_{n2} & a_{n3} & a_{n4} \ldots a_{ni} \ldots\ldots a_{nn} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \ldots\ldots b_{1i} \ldots\ldots a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} \ldots\ldots b_{2i} \ldots\ldots a_{2n} \\ \ldots & \ldots & \ldots & \ldots\ldots\ldots\ldots \\ a_{n1} & a_{n2} & a_{n3} & a_{n4} \ldots b_{ni} \ldots\ldots a_{nn} \end{vmatrix};$$

Masalan:

$$\Delta_3=\begin{vmatrix} 2 & -1+2 & 4 \\ 7 & 3-1 & 3 \\ 4 & 2+3 & 5 \end{vmatrix}=\begin{vmatrix} 2 & -1 & 4 \\ 7 & 3 & 3 \\ 4 & 2 & 5 \end{vmatrix}+\begin{vmatrix} 2 & 2 & 4 \\ 7 & -1 & 3 \\ 4 & 3 & 5 \end{vmatrix};$$

10) determinantning biror qatorining barcha elementlariga unga parallel bo'lgan, boshqa qatorning barcha mos elementlarini ihtiyoriy λ songa ko'paytirib qo'shsa, determinantning qiymati o'zgarmaydi.

Masalan, determinantning ustunlari uchun bu xossa quyidagi tenglik ko'rinishida ifodalanadi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \ldots\ldots a_{1i} \ldots a_{1j} \ldots a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} \ldots\ldots a_{2i} \ldots a_{2j} \ldots a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & a_{n3} & a_{n4} \ldots a_{ni} \ldots a_{nj} \ldots a_{nn} \end{vmatrix}=\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \ldots\ldots a_{1i}+\lambda a_{1j} \ldots a_{1j} \ldots a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} \ldots\ldots a_{2i}+\lambda a_{2j} \ldots a_{2j} \ldots a_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ a_{n1} & a_{n2} & a_{n3} & a_{n4} \ldots a_{ni}+\lambda a_{nj} \ldots a_{nj} \ldots a_{nn} \end{vmatrix}.$$

Determinantni hisoblashning asosiy usullarini ko'rib chiqamiz.

1. *Determenant tartibini pasaytirishning samarali usuli*. 4-hossaga asosan n-tartibli determinant (*n-1*) – tartibli n – ta determinantni hisoblashga keltiriladi. Lekin tartibni pasaytirishning bu usuli samarali emas. Determinantni hisoblashning asosiy xossalaridan foydalanib, Δ_n determinantning biror qatorining bitta elementidan boshqa barcha elementlarini nolga aylantirib, $\Delta_n \neq 0$ determinantni har doim (*n-1*)-tartibli bitta determinantni hisoblashga olib kelishi mumkin.

1-misol. Determinantni hisoblang:

$$\Delta_4=\begin{vmatrix} 30 & -10 & 120 & 80 \\ -5 & 3 & -34 & -23 \\ 1 & 1 & 3 & -7 \\ -9 & 2 & 8 & -15 \end{vmatrix}.$$

►Determinantlarning 5-xossasiga asosan birinchi satrdan ko'paytuvchi 10 ni chiqaramiz, hosil bo'lgan satrni ketma-ket 3,1 va 2 sonlariga ko'paytirib, mos ravishda ikkinchi, uchinchi va to'rtinchi satrlarga qo'shamiz. U holda 10-xossaga asosan quyidagiga ega bo'lamiz.

$$\Delta_4=10\begin{vmatrix} 3 & -1 & 12 & 8 \\ 4 & 0 & 2 & 1 \\ 4 & 0 & 15 & 1 \\ -3 & 0 & 32 & 1 \end{vmatrix}.$$

Determinantlarning 1-xossasiga asosan ((1.6) tenglikning ikkinchisiga qarang) hosil qilingan determinantni ikkinchi ustun elementlari bo'yicha yoyish mumkin.

U holda

$$\Delta_4=10\begin{vmatrix} 4 & 2 & 1 \\ 4 & 15 & 1 \\ -3 & 32 & 1 \end{vmatrix}.$$

Sarryus qoidasi bo'yicha hisoblash mumkin bo'lgan uchinchi tartibli determinantni hosil qildik yoki buni ham yuqoridagi usul bilan bitta ikkinchi tartibli determinantni hisoblashga olib kelish mumkin.

Haqiqatan ham, hosil qilingan determinantning ikkinchi va uchinchi satrlaridan birinchi satrni ayirsak, quyidagiga ega bo'lamiz.

$$\Delta_4 = 10\begin{vmatrix} 4 & 2 & 1 \\ 0 & 13 & 0 \\ -7 & 30 & 0 \end{vmatrix} = 10\begin{vmatrix} 0 & 13 \\ -7 & 30 \end{vmatrix} = 10 \cdot 7 \cdot 13 = 910 \ . \blacktriangleleft$$

2. *Determinantni uchburchak ko'rinishiga keltirish.*

Asosiy diagonalning pastki yoki ustki qismida joylashgan barcha elementlari nollardan iborat bo'lgan determinant, uchburchak ko'rinishidagi determinant deb ataladi. Ma'lumki, bunday hollarda determinantning qiymati asosiy diagonal elementlarining ko'paytmasidan iborat. Har qanday Δ_n determinantni har doim uchburchak ko'rinishiga olib kelish mumkin.

2-misol. Determinantni hisoblang

$$\Delta_5 = \begin{vmatrix} 5 & 8 & 7 & 4 & -2 \\ -1 & 4 & 2 & 3 & 1 \\ 9 & 27 & 6 & 10 & -9 \\ 3 & 9 & 6 & 2 & -3 \\ 1 & 3 & 2 & 8 & -1 \end{vmatrix}$$

►Quyidagi amallarni bajaramiz. Determinantning beshinchi ustunini birinchi ustunga qo'shamiz. Ushbu ustunni 3 ga ko'paytirib ikkinchi ustunga, 2 ga ko'paytirib uchunchi ustunga, 8 ga ko'paytirib to'rtinchi ustunga qo'shamiz. Natijada berilgan determinantga teng kuchli bo'lgan, quyidagi uchburchak ko'rinishidagi determinantga ega bo'lamiz.

$$\Delta_5 = \begin{vmatrix} 3 & 2 & 3 & -12 & -2 \\ 0 & 7 & 4 & 11 & 1 \\ 0 & 0 & -12 & -62 & -9 \\ 0 & 0 & 0 & -22 & -3 \\ 0 & 0 & 0 & 0 & -1 \end{vmatrix} = -3 \cdot 7 \cdot 12 \cdot 22 = -5544 . \blacktriangleleft$$

Determinantlarni uchburchak shakliga keltirish, keyinchalik chiziqli tenglamalar sistemasini Jordan-Gauss usuli (uni Gauss usuli deb ham atashadi) bilan yechishda qo'llaniladi.

1.1–AT

1. Uchburchaklar qoidasidan (Sarryus qoidasidan) foydalanib determinantlarni hisoblang:

$$a)\begin{vmatrix} -1 & 3 & 2 \\ 2 & 8 & 1 \\ 1 & 1 & 2 \end{vmatrix}; \quad b)\begin{vmatrix} 3 & 4 & -5 \\ 8 & 7 & -2 \\ 2 & -1 & 8 \end{vmatrix}; \quad c)\begin{vmatrix} 1 & -2 & 1 \\ 3 & 1 & -5 \\ 4 & 2 & 5 \end{vmatrix}.$$

(Javob: *a*) -36 ; b) 0; c) 87.

2. Tartibini pasaytirish usuli bilan determinantlarni hisoblang:

$a)\begin{vmatrix} 15325 & 15323 & 37527 \\ 23735 & 23735 & 17417 \\ 23737 & 23737 & 17418 \end{vmatrix}$; b) $\begin{vmatrix} 2 & 4 & -1 & 2 \\ -1 & 2 & 3 & 1 \\ 2 & 5 & 1 & 4 \\ 1 & 2 & 0 & 3 \end{vmatrix}$.

(Javob: *a*)-22198; b) 16.)

3. Determinantlarni uchburchak shakliga keltirish usuli bilan hisoblang:

$a)\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 5 & 9 \\ 0 & 0 & 3 & 7 \\ -2 & -4 & -6 & 0 \end{vmatrix}$; $b)\begin{vmatrix} 1 & -2 & 5 & 9 \\ 1 & -1 & 7 & 4 \\ 1 & 3 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{vmatrix}$.

(Javob: *a*) 48 ; *b*) 20.)

4. Determinantlarni avval soddalashtirib so'ngra hisoblang:

$a)\begin{vmatrix} x^2+a^2 & ax & 1 \\ y^2+a^2 & ay & 1 \\ z^2+a^2 & az & 1 \end{vmatrix}$; $b)\begin{vmatrix} 7 & 8 & 5 & 5 & 3 \\ 10 & 11 & 6 & 7 & 5 \\ 5 & 3 & 6 & 2 & 5 \\ 6 & 7 & 5 & 4 & 2 \\ 7 & 10 & 7 & 5 & 0 \end{vmatrix}$.

(Javob: *a*) *a(x-y)(y-z)(x-z)* ; *b*) 5.)

Mustaqil ish

Determinantlarni hisoblang:

1. $\begin{vmatrix} 2 & 1 & 5 & 1 \\ 3 & 2 & 1 & 2 \\ 1 & 2 & 3 & -4 \\ 1 & 1 & 5 & 1 \end{vmatrix}$. **2.** $\begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{vmatrix}$. **3.** $\begin{vmatrix} 2 & 1 & 1 & 8 \\ 1 & -3 & -6 & 9 \\ 0 & 2 & 2 & -5 \\ 1 & 4 & 6 & 0 \end{vmatrix}$.

(Javob: 54.) (Javob: 160.) (Javob: -27.)

1.2. Matritsalar va ular ustida amallar.

Biror a_{ij} $(i=\overline{1,m}, \quad j=\overline{1,m})$ to'plamning mxn elementlaridan tuzilgan to'g'ri burchakli jadval matritsa deb ataladi va quyidagi ko'rinishda yoziladi

$$A=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \dots\dots a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} \dots\dots a_{2n} \\ \dots & \dots & \dots & \dots\dots \\ a_{m1} & a_{m2} & a_{m3} & a_{m4} \dots\dots a_{mn} \end{bmatrix} \text{ yoki } A=\begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \dots\dots a_{1n} \\ a_{21} & a_{22} & a_{23} & a_{24} \dots\dots a_{2n} \\ \dots & \dots & \dots & \dots\dots \\ a_{m1} & a_{m2} & a_{m3} & a_{m4} \dots\dots a_{mn} \end{pmatrix}. \quad (1.7)$$

Matritsaning elementlari 2 ta indeks bilan tartiblanadi. a_{ij} elementning birinchi *i* indeksi satr raqamini, *j* indeksi ustun raqamini belgilaydi. Ularning kesishish nuqtasida esa matritsaning a_{ij} elementi turadi. Matritsalar odatda lotin alifbosining katta harflari bilan belgilanadi. A, B, C,… . Agar matritsa m-satr va n-ustundan iborat bo'lsa, ta'rifga asosan uning o'lchami mxn bo'ladi. Zarur hollarda u $A_{m\text{x}n}$ kabi belgilanadi. Agar matritsaning a_{ij} elementlari sonlardan iborat bo'lsa, matritsa sonli matritsa deyiladi. Agar matritsaning a_{ij} elementlari funksiyalardan iborat bo'lsa, funksional matritsa, vektorlardan iborat bo'lsa, vektor matritsa deyiladi va hakozo.

A va B matritsalar o'zaro teng deyiladi, agar ularning barcha mos elementlari a_{ij} b_{ij} teng bo'lsa, ya'ni $a_{ij} = b_{ij}$. Bundan kelib chiqadiki, faqat bir xil o'lchamli matritsalarini o'zaro teng bo'lishi mumkin ekan.

Agar matritsaning ustunlari va satrlari soni teng, ya'ni $m=n$ bo'lsa, bunday matritsa kvadrat matritsa deyiladi. Agar $i=1$ bo'lsa, satr matritsa, $j=1$ bo'lsa, ustun matritsaga ega bo'lamiz. Ularni mos ravishda vektor-satr va vektor-ustun deb ham ataymiz.

Matritsalar ustida amallarning asosiylarini ko'rib chiqamiz.

1. *Matritsalarni qo'shish va ayirish.*

A va B matritsalarning yig'indisi (ayirmasi) deb, $A+B$ $(A\text{-}B)$ kabi belgilanuvchi shunday C matritsaga aytiladi va uning elementlari quyidagicha aniqlanadi: $c_{ij}= a_{ij} \pm b_{ij}$, bu erda a_{ij} va b_{ij} - A va B matritsalarning mos elementlari.

Masalan,

$$A=\begin{bmatrix}1 & 6\\ 2 & -4\\ -3 & 9\end{bmatrix}, B=\begin{bmatrix}-2 & 4\\ 3 & 7\\ 8 & -11\end{bmatrix} \text{ matritsalar uchin, } A+B=\begin{bmatrix}-1 & 10\\ 5 & 3\\ 5 & -2\end{bmatrix}, \ A-B=\begin{bmatrix}3 & 2\\ -1 & -11\\ -11 & 20\end{bmatrix}.$$

2. *Matritsani songa ko'paytirish.* A matritsa va λ sonning ko'paytmasi deb, λA kabi belgilanuvchi, o'lchami A matritsaning o'lchamiga teng bo'lgan shunday B matritsaga aytiladiki, uning elementlari quyidagicha aniqlanadi. $b_{ij}=\lambda\, a_{ij}$, bu erda a_{ij}-A matritsaning elementlari, ya'ni matritsani songa (sonni matritsaga) ko'paytirganda matritsaning barcha elementlari shu songa ko'paytirilishi kerak.

Masalan,

$\lambda=-2,\ A=\begin{bmatrix}3 & 0\\ 7 & -1\end{bmatrix}$ berilgan bo'lsin. U holda $\lambda A=-2\begin{bmatrix}3 & 0\\ 7 & -1\end{bmatrix}=\begin{bmatrix}-6 & 0\\ -14 & 2\end{bmatrix}$.

3. *Matritsalarni ko'paytirish.*

$A_{m\text{x}n}$ va $B_{n\text{x}p}$ matritsalar ko'paytmasi deb, shunday $C_{m\text{x}p}=A\cdot B$ (yoki qisqacha AB) matritsaga aytiladiki, uning elementlari quyidagicha aniqlanadi, $c_{ij}=\sum_{k=1}^{n} a_{ik}b_{kj}$, bu erda a_{ik}, b_{kj} - A va B matritsalarning elementlari. Bundan kelib chiqadiki, A matritsaning ustunlari soni B matritsaning satrlari soniga teng bo'lgandagina AB ko'paytma mavjud bo'ladi. AB matritsaning satrlar soni A matritsaning satrlari soniga, ustunlari esa, B matritsaning ustunlari soniga teng bo'ladi. AB ko'paytmaning mavjudligidan BA ko'paytmaning mavjudligi kelib chiqmaydi, va ko'paytma mavjud bo'lgan holda ham, quyidagi munosabat o'rinli bo'ladi. $AB\neq BA$. Agar $AB=BA$ bo'lsa, u holda A va B matritsalar o'rin almashinuvchi(yoki kommutirlanuvchi) matritsalar deyiladi. Ma'lumki, har doim $(AB)C=A(BC)$ munosabat o'rinlidir.

1-misol. Agar

$$A=\begin{bmatrix}4 & -5 & 8\\ 1 & 3 & -1\end{bmatrix},\qquad B=\begin{bmatrix}-1 & 5\\ -2 & -3\\ 3 & 4\end{bmatrix}$$ bo’lsa,

AB va *BA* ni toping.

► $AB=C_{2x2}=\begin{bmatrix}c_{11} & c_{12}\\ c_{21} & c_{22}\end{bmatrix}.$

Bu erda $c_{11}=4(-1)+(-5)(-2)+8\cdot 3=30;\quad c_{12}=4\cdot 5+(-5)(-3)+8\cdot 4=67;$
$c_{21}=1(-1)+3(-2)+(-1)3=-10;\quad c_{22}=1\cdot 5+3(-3)+(-1)4=-8.$

Natijada

$AB=\begin{bmatrix}30 & 67\\ -10 & -8\end{bmatrix}$. Endi *BA* ni topamiz. $BA=C_{3x3}=\begin{bmatrix}\tilde c_{11} & \tilde c_{12} & \tilde c_{13}\\ \tilde c_{21} & \tilde c_{22} & \tilde c_{23}\\ \tilde c_{31} & \tilde c_{32} & \tilde c_{33}\end{bmatrix}.$

Bu erda

$\tilde c_{11}=(-1)4+5\cdot 1=1;\quad \tilde c_{12}=(-1)(-5)+5\cdot 3=20;\quad \tilde c_{13}=(-1)8+5(-1)=-13;$

$\tilde c_{21}=(-2)4+(-3)1=-11;\quad \tilde c_{22}=(-2)(-5)+(-3)3=1;\quad \tilde c_{23}=(-2)8+(-3)(-1)=-13;$

$\tilde c_{31}=3\cdot 4+4\cdot 1=16;\quad \tilde c_{32}=3(-5)+4\cdot 3=-3;\quad \tilde c_{33}=3\cdot 8+4(-1)=20;$

va quyidagiga ega bo’lamiz.

$$BA=\begin{bmatrix}1 & 20 & -13\\ -11 & 1 & -13\\ 16 & -3 & 20\end{bmatrix}.$$

Shunday qilib, AB $\neq$ BA.◄

2-misol. $A=\begin{vmatrix}3 & 5\\ 1 & 2\end{vmatrix}$, $B=\begin{vmatrix}1 & -5\\ -1 & 2\end{vmatrix}$ matritsalar berilgan. *AB* va *BA* larni toping.

►Quyidagilarga egamiz:

$$AB=\begin{bmatrix}3\cdot 1+5(-1) & 3(-5)+5\cdot 2\\ 1\cdot 1+2(-1) & 1(-5)+2\cdot 2\end{bmatrix}=\begin{bmatrix}-2 & -5\\ -1 & -5\end{bmatrix}.$$

$$BA=\begin{bmatrix}1\cdot 3+(-5)\cdot 1 & 1\cdot 5+(\ 5)\cdot 2\\ (-1)\cdot 3+2\cdot 1 & (-1)\cdot 5+2\cdot 2\end{bmatrix}=\begin{bmatrix}-2 & -5\\ -1 & -5\end{bmatrix}.$$

Bundan kelib chiqadiki: *AB* = *BA*.◄

3-misol. Agar $A=\begin{bmatrix}1 & 3\\ -1 & 1\\ 2 & 5\end{bmatrix}$. $B=\begin{bmatrix}2 & -6 & 1\\ 1 & 3 & -1\end{bmatrix}$. $C=\begin{bmatrix}-1\\ 2\\ 4\end{bmatrix}$, bo’lsa

(AB)C va *A(BC)* ni toping.

►Quyidagiga egamiz:

$$AB=\begin{bmatrix}5&3&-2\\-1&9&-2\\9&3&-3\end{bmatrix}.\ (AB)C=\begin{bmatrix}-7\\11\\-15\end{bmatrix}.$$

$$BC=\begin{bmatrix}-10\\1\end{bmatrix}.\quad A(BC)=\begin{bmatrix}-7\\11\\-15\end{bmatrix}.$$

ya'ni (AB)C = A(BC). ◄

1.2–AT

1. *A* va *B* matritsalar berilgan,agar:

a) $A=\begin{bmatrix}1&8\\0&9\\-7&1\end{bmatrix}$. $B=\begin{bmatrix}-1&-7\\0&1\\6&-1\end{bmatrix}$.

b) $A=\begin{bmatrix}5&7&9&1\\4&3&-1&0\\-2&4&4&-3\end{bmatrix}$. $B=\begin{bmatrix}-4&1&-8&1\\0&1&-2&3\\2&5&11&7\end{bmatrix}$ bo'lsa, *A*+*B*, *2A*, *A*-*3B* larni toping.

2. *A* va *B* matritsalar berilgan,agar:

a) $A=\begin{bmatrix}1&0&2\\0&-1&3\\4&0&5\end{bmatrix}$, $B=\begin{bmatrix}2&7&1\\3&2&-4\\1&-3&5\end{bmatrix}$;

b) $A=\begin{bmatrix}1&1&0\\3&-1&5\end{bmatrix}$, $B=\begin{bmatrix}0&7\\3&4\\1&0\end{bmatrix}$;

c) $A=\begin{bmatrix}3\\4\\2\end{bmatrix}$, $B=\begin{bmatrix}5&-2&3\end{bmatrix}$;

d) $A=\begin{bmatrix}2&2\\1&2\end{bmatrix}$, $B=\begin{bmatrix}3&4\\2&3\end{bmatrix}$;

e) $A=\begin{bmatrix}0&0\\0&\alpha\end{bmatrix}$, $B=\begin{bmatrix}\beta&\gamma\\0&0\end{bmatrix}$ bo'lsa *AB* va *BA* ni toping.

(Javob: *a*) $AB=\begin{bmatrix}4&1&1\\0&-11&19\\13&13&29\end{bmatrix}$, $BA=\begin{bmatrix}6&-7&30\\-13&-2&-8\\21&3&18\end{bmatrix}$;

b) $AB=\begin{bmatrix}3&11\\2&17\end{bmatrix}$, $BA=\begin{bmatrix}21&-7&35\\15&-1&20\\1&1&0\end{bmatrix}$;

C) $BA=[13]$. $AB=\begin{bmatrix} 15 & -6 & 9 \\ 20 & -8 & 12 \\ 10 & -4 & 6 \end{bmatrix}$;

d) $AB=BA=\begin{bmatrix} 10 & 14 \\ 7 & 10 \end{bmatrix}$;

e) $AB=\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $BA=\begin{bmatrix} 0 & \alpha\gamma \\ 0 & 0 \end{bmatrix}$);

3. *A* matritsa bilan o'rin almashinuvchi (kommutirlanuvchi) barcha *B* kvadrat matritsalarni toping. Agar :

a) $A=\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}$; *b)* $A=\begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$ bo'lsa, $AB = BA$ tenglik bajarilishini tekshiring.

(Javob*: a)* $B=\begin{bmatrix} 3b & -b \\ b & 2b \end{bmatrix}$; *b)* $B=\begin{bmatrix} a+b & 5a \\ a & b \end{bmatrix}$,

bu erda *a,b*–ixtiyoriy sonlar (parametrlar) .)

4. *A, B* va *C* matritsalar berilgan. *A(BC), (AB)C* matritsalarni toping va *(AB)C = A(BC)* tenglikni ko'rsating.

a) $A=\begin{bmatrix} 3 & 1 \\ 5 & -6 \\ 7 & -8 \end{bmatrix}$, $B=\begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $C=\begin{bmatrix} 5 & 14 \\ -2 & -30 \end{bmatrix}$;

b) $A=\begin{bmatrix} 4 & 2 & 9 & -7 \\ 8 & 3 & 11 & 0 \end{bmatrix}$, $B=\begin{bmatrix} 4 \\ 3 \\ -2 \\ 8 \end{bmatrix}$, $C=\begin{bmatrix} -1 & 9 & 3 & 6 \end{bmatrix}$.

(Javob: a) $ABC=\begin{bmatrix} 43 & 96 \\ 18 & 758 \\ 28 & 1030 \end{bmatrix}$; *b)* $ABC=\begin{bmatrix} 52 & -468 & -156 & -312 \\ -19 & 171 & 57 & 114 \end{bmatrix}$).

Mustaqil ish

1. $A=\begin{bmatrix} 1 & -1 & 3 \\ 2 & 0 & 2 \end{bmatrix}$, $B=\begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}$, $C=\begin{bmatrix} 0 & 1 & 2 & 0 \\ -1 & 2 & 0 & 0 \\ 1 & 2 & 1 & 0 \end{bmatrix}$ matritsalar berilgan.*AB, BA, AC, CA, BC, CB* ko'paytmalarning ma'noga ega bo'ladiganlarini toping. (Javob: $BA=\begin{bmatrix} -2 & 0 & -2 \\ 3 & -1 & 5 \end{bmatrix}$, $AC=\begin{bmatrix} 4 & 5 & 5 & 0 \\ 2 & 6 & 6 & 0 \end{bmatrix}$.)

2. Agar:

$$A=\begin{bmatrix}1&4&7\\2&5&-8\\-3&6&9\end{bmatrix},\quad B=\begin{bmatrix}-2&1&-1\\1&0&2\\4&-1&0\end{bmatrix}$$ bo'lsa ,berilgan A va B matritsalar uchun $(A+3B)^2$ ni hisoblang.(Javob: $\begin{bmatrix}96&12&8\\-18&54&-8\\51&85&111\end{bmatrix}$.)

3. Agar :

$$A=\begin{bmatrix}5&9&7\\0&3&-2\end{bmatrix},\quad B=\begin{bmatrix}5&9\\0&3\\0&2\end{bmatrix},\quad C=\begin{bmatrix}3&4\\-1&0\end{bmatrix}$$ bo'lsa , *(AB)C* va *A(BC)* ni toping.

(Javob : $\begin{bmatrix}-11&100\\-5&0\end{bmatrix}$).

1.3 Teskari matritsalar. Elementar almashtirishlar. Matritsaning rangi. Kroneker – Kapelli teoremasi.

n-tartibli kvadrat matritsa

$$\text{A}=\begin{bmatrix}a_{11}&a_{12}&\cdots&a_{1n}\\a_{21}&a_{22}&\cdots&a_{2n}\\\multicolumn{4}{c}{\dots\dots\dots\dots\dots\dots}\\a_{n1}&a_{n2}&\cdots&a_{nn}\end{bmatrix}\qquad(1.8)$$

xosmas matritsa deyiladi, agar uning determinanti

$$\det\ \text{A}=\begin{vmatrix}a_{11}&a_{12}&\cdots&a_{1n}\\a_{21}&a_{22}&\cdots&a_{2n}\\\multicolumn{4}{c}{\dots\dots\dots\dots\dots\dots}\\a_{n1}&a_{n2}&\cdots&a_{nn}\end{vmatrix}\neq 0\qquad(1.9)$$

bo'lsa, det A =0 bo'lgan holda, A matritsa *xos* matritsa deyiladi.

Faqat xosmas *A* kvadrat matritsalar uchun A^{-1} teskari matritsa tushunchasi kiritiladi. A^{-1} matritsa xosmas A kvadrat matritsa uchun teskari matritsa deyiladi, agar $AA^{-1}=A^{-1}A=E$ bo'lsa , bu erda *E* –n tartibli birlik matritsa ya'ni

$$E=\begin{bmatrix}1&0&\cdots&0\\0&1&\cdots&0\\\multicolumn{4}{c}{\dots\dots\dots\dots\dots}\\0&0&\cdots&1\end{bmatrix}.\qquad(1.10)$$

Ma'lumki, A matritsa uchun quyidagi formula bilan aniqlanuvchi yagona A^{-1} teskari matritsa mavjud.

$$A^{-1} = \frac{A^*}{\det A},\ A^* = \begin{bmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{bmatrix} \qquad (1.11)$$

A^* matritsa *biriktirilgan* matritsa deb ataladi va uning elementlari transponirlangan A^t matritsaning

$$A^T = \begin{bmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ \cdots & \cdots & \cdots & \cdots \\ a_{1n} & a_{2n} & \cdots & ann \end{bmatrix} \qquad (1.12)$$

A_{ij} algebraik to'ldiruvchilaridan iborat bo'ladi.

1-misol. *A* matritsa berilgan.

$$\text{a)}\ A = \begin{bmatrix} -1 & 2 \\ 1 & 3 \end{bmatrix}; \quad b)\ A = \begin{bmatrix} 2 & -4 & 1 \\ 1 & -5 & 3 \\ 1 & -1 & 1 \end{bmatrix}.$$

Uning xosmas ekanligiga ishonch hosil qilib, unga teskari matritsa A^{-1} ni toping va $A \cdot A^{-1} = A^{-1} \cdot A = E$ tenglik o'rinli ekanligini tekshiring.

►*a*) *A* berilgan: det $A = \begin{vmatrix} -1 & 2 \\ 1 & 3 \end{vmatrix} = -5 \neq 0;$ Endi algebraik to'ldiruvchilarni topamiz: $A_{11} = 3,\ A_{12} = -1,\ A_{21} = -2,\ A_{22} = -1.$ Shunday ekan:

$$A^{-1} = -\frac{1}{5}\begin{bmatrix} 3 & -2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} -3/5 & 2/5 \\ 1/5 & 1/5 \end{bmatrix},\quad AA^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A^{-1}A;$$

b) A matritsaning determinantini va uning algebraik to'ldiruvchilarni hisoblaymiz:

$\det A = -8 \neq 0$; $A_{11} = -2, A_{12} = 2, A_{13} = 4, A_{21} = 3, A_{22} = 1, A_{23} = -2, A_{31} = -7, A_{32} = -5, A_{33} = -6.$

U holda

$$A^{-1} = -\frac{1}{8}\begin{bmatrix} -2 & 3 & -7 \\ 2 & 1 & -5 \\ 4 & -2 & -6 \end{bmatrix}, \qquad AA^{-1} = A^{-1} = A^{-1}A = E\ .◄$$

Matritsaning rangi tushunchasini kiritamiz. *A* matritsada *k*-satr va *k*-ustunni ajratamiz, bu erda *k* - *m* va *n* sonlarining kichigiga teng yoki undan kichik sondir. *A* matritsaning *k* satr va *k* ustun elementlarining kesishishidan hosil bo'lgan elementlardan tuzilgan *k* - tartibli determinant *A* matritsaning *minori* yoki *A matritsadan kelib chiqqan determinant* deyiladi. Masalan

$\begin{bmatrix} 7 & -1 & 4 & 5 \\ 1 & 8 & 1 & 3 \\ 4 & -2 & 0 & -6 \end{bmatrix}$ matritsa uchun *k* =2 da quyidagi determinantlar

$\begin{vmatrix} 7 & -1 \\ 1 & 8 \end{vmatrix},\ \begin{vmatrix} 1 & 3 \\ 0 & -6 \end{vmatrix},\ \begin{vmatrix} -1 & 5 \\ -2 & -6 \end{vmatrix}$ berilgan *matritsadan kelib chiqqan determinantlar* bo'ladi.

A matritsaning *rangi* deb, undan *kelib chiqqan*, noldan farqli determinantlarning eng yuqori tartibiga aytiladi.Agar *matritsadan kelib chiqqan determinant k* -chi tartibli barcha determenantlar nolga teng bo'lsa, u holda A matritsaning *rangi* < *k* bo'ladi.

1–teorema. Agar:

1) *ixtiyoriy ikkita parallel qatorlarning o'rnini almashtirilsa*;

2) *qatorning har bir hadini noldan farqli biror λ≠0 ko'paytuvchiga ko'paytirilsa;*

3) *qatorning hadlariga unga parallel bo'lgan biror ko'paytuvchiga ko'paytirilgan boshqa qatorning mos elementlarini qo'shsa, matritsaning rangi o'zgarmaydi.*

1-3 almashtirishlar *elementar* almashtirishlar deb ataladi. Agar biror matritsa, ikkinchisidan elementar almashtirishlar yordamida hosil qilinsa, bu matritsalar *ekvivalent* matritsalar deb ataladi. *Ekvivalent* matritsalar A ~B kabi belgilanadi.

Tartibi berilgan matritsaning rangiga teng bo'lgan noldan farqli har qanday minor *matritsaning bazis minori* deb ataladi.

Matritsaning *rangini* topishning asosiy *usullarini* ko'rib chiqamiz.

1. Birlar va nollar usuli.

Elementar almashtirishlar yordamida ihtiyoriy matritsani har bir qatori faqat nollardan yoki bitta bir va qolganlari nollardan iborat bo'lgan matritsa ko'rinishga keltirish mumkin.

Hosil qilingan matritsaga ekvivalent bo'lgani uchun undagi birlar soni berilgan matritsaning rangiga teng bo'ladi.

2 –misol. Matritsaning rangini toping.

$$A=\begin{bmatrix} 1 & 1 & 2 & 3 & -1 \\ 2 & -1 & 0 & -4 & -5 \\ -1 & -1 & 0 & -3 & -2 \\ 6 & 3 & 4 & 8 & -3 \end{bmatrix}.$$

► *A* matritsaning 3-chi ustunini ½ ga ko'paytiramiz. So'ngra hosil bo'lgan birinchi satrni 2 ga ko'paytiramiz va uni 4-chi satrdan ayiramiz. Endi uchinchi ustun uchta nol va bitta birdan iborat bo'lib qoladi. Birinchi satrda birinchi, ikkinchi, to'rtinchi va beshinchi elementlarni osongina nolga aylantiramiz va quyidagiga ega bo'lamiz.

$$A\sim\begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 2 & -1 & 0 & -4 & -5 \\ -1 & -1 & 0 & -3 & -2 \\ 4 & 1 & 0 & 2 & -1 \end{bmatrix}.$$

Oxirgi matritsada to'rtinchi satrni 2-chi va 3-chi satrga qo'shsak, ikkinchi ustunda yana ikkita nollar hosil qilamiz, so'ngra to'rtinchi satrda to'rtinchi satr va ikkinchi ustun kesishgan joydagi birdan boshqa hammasini nol qilamiz. Natijada quyidagiga ega bo'lamiz. Natijada quyidagiga ega bo'lamiz.

$$A\sim\begin{bmatrix}0&0&1&0&0\\6&0&0&-2&-6\\3&0&0&-1&-3\\0&1&0&0&0\end{bmatrix}\sim\begin{bmatrix}0&0&1&0&0\\0&0&0&0&0\\3&0&0&-1&-3\\0&1&0&0&0\end{bmatrix}\sim\begin{bmatrix}0&0&1&0&0\\0&0&0&0&0\\0&0&0&1&0\\0&1&0&0&0\end{bmatrix}.$$

Uchta birni hosil qildik. Demak, *rang*A=3.

Bazis minor sifatida, masalan birinchi, uchinchi, to'rtinchi satrlar va ikkinchi, uchinchi va to'rtinchi ustunlar kesishishidagi (oxirgi matritsada satrlar va ustunlar kesishishida birlar turibdi) uchinchi tartibli determinantni olish mumkin.

$$\begin{vmatrix}1&2&3\\-1&0&-3\\3&4&8\end{vmatrix}\neq 0.$$ ◄

2.*O'rab oluvchi minorlar usuli. k*-tartibli M_k minorni o'z ichiga oluvchi *(k+1)* tartibli M_{k+1} minor, M_k ni *o'rab oluvchi* minor deb ataladi. Agar A matritsada $M_k\neq 0$ mavjud bo'lib, uni o'rab oluvchi barcha minorlar $M_{k+1}=0$ bo'lsa, u holda *rang*A=k bo'ladi.

3–m**isol**. Matritsaning rangini toping

$$A=\begin{bmatrix}1&-3&5&4\\2&-6&4&3\\3&-9&3&2\end{bmatrix}.$$

►Quyidagiga egamiz:

$M_2=\begin{vmatrix}-3&5\\-6&4\end{vmatrix}\neq 0;$ M_2 minorni o'rab oluvchi minorlar ikkita bo'lib, ularning har biri nolga tengdir.

$$M_3=\begin{vmatrix}1&-3&5\\2&-6&4\\3&-9&3\end{vmatrix},\quad M_3^*=\begin{vmatrix}-3&5&4\\-6&4&3\\-9&3&2\end{vmatrix}.$$

Shuning uchun *rang*A=2. Yuqoridagi M_2 minorni bazis minor sifatida qabul qilish mumkin.◄

2–teorema (Kroneker-Kapelli) *$x_1,x_2,...,x_n$, n-noma'lumga nisbatan m-ta chiziqli algebraik tenglamalar sistemasi*

$$\left.\begin{array}{l}a_{11}x_1+a_{12}x_2+\cdots+a_{1n}x_n=b_1\\a_{21}x_1+a_{22}x_2+\cdots+a_{2n}x_n=b_2\\\dots\dots\dots\dots\dots\dots\dots\dots\dots\\a_{m1}x_1+a_{m2}x_2+\cdots+a_{mn}x_n=b_m\end{array}\right\}\qquad(1.13)$$

birgalikda bo'lishi uchun (yechimga ega bo'lishi uchun) (1.13) sistemaning asosiy matritsasining rangi

$$A=\begin{bmatrix}a_{11}&a_{12}&\cdots&a_{1n}\\a_{21}&a_{22}&\cdots&a_{2n}\\\dots&\dots&\dots&\dots\\a_{m1}&a_{m2}&\cdots&a_{mn}\end{bmatrix}\qquad(1.14)$$

va (1.13) sistemaning kengaytirilgan matritsasining rangi

$$B=\left[\begin{array}{cccc|c} a_{11} & a_{12} & \cdots & a_{1n} & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & b_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & b_m \end{array}\right]=\left[\begin{array}{c|c} & b_1 \\ A & b_2 \\ & \cdots \\ & b_m \end{array}\right] \qquad (1.15)$$

teng bo'lishi zarur va yetarlidir, ya'ni rangA= rangB=r.

Agar *rang*A= *rangB* va *r=n* bo'lsa, u holda (1.13) sistema yagona yechimga;

agar *rang*A= *rangB* va *r<n* bo'lsa, u holda (1.13) sistema ihtiyoriy *n-r* parametrga bog'liq cheksiz ko'p yechimga ega bo'ladi.

Agar (1.13) sistemaning barcha ozod hadlari b_i $(i=\overline{1,m})$ nolga teng bo'lsa, (1.13) sistema *bir jinsli* sistema deb ataladi. Ozod hadlardan hech bo'lmaganda bittasi noldan farqli bo'lsa, u holda sistema *bir jinsli bo'lmagan* sistema deb ataladi. Bir jinsli tenglamalar sistemasi uchun hamma vaqt *rang*A= *rangB*, *va* ular har doim birgalikda bo'ladi.

4–Misol. Tenglamalar sistemasi birgalikda yoki birgalikda emasligini ko'rsating

$$\left.\begin{array}{l} 4x_1+3x_2-3x_3-x_4=4 \\ 3x_1+x_2-3x_3-2x_4=1 \\ 3x_1+x_2-x_4=0 \\ 5x_1+4x_2-2x_3-x_4=3 \end{array}\right\}$$

►Berilgan matritsaning kengaytirilgan matritsasini yozamiz hamda asosiy va kengaytirilgan matritsalarning rangini topamiz. Quyidagiga egamiz.

Asosiy va kengaytirilgan matritsalarning rangini birdaniga topish uchun ozod had turgan ustunni matritsaning boshqa ustunlari bilan almashtirmaymiz

$$B\sim\left[\begin{array}{cccc|c} 4 & 3 & -3 & -1 & 4 \\ 3 & -1 & 3 & -2 & 1 \\ 3 & 1 & 0 & -1 & 0 \\ 5 & 4 & -2 & 1 & 3 \end{array}\right].$$

B matritsaning ikkinchi ustunini 3 ga ko'paytiramiz va birinchi ustundan ayiramiz, hamda ikkinchi ustunni to'rtinchi ustunga qo'shamiz. Natijada uchinchi satrda ikkinchi ustundan boshqa barchasida nollarni hosil qilamiz. U holda ikkinchi usutunning qolgan barcha elementlarini nolga aylantirish oson.

Quyidagiga ega bo'lamiz

$$B\sim\left[\begin{array}{cccc|c} -5 & 0 & -3 & 2 & 4 \\ 6 & 0 & 3 & -3 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ -7 & 0 & -2 & 5 & 3 \end{array}\right].$$

Endi ikkinchi satrni birinchi va to'rtinchiga qo'shamiz, so'ngra hosil bo'lgan matritsada birinchi ustunni to'rtinchi bilan qo'shamiz.

U holda

$$B\sim\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 5 \\ 6 & 0 & 3 & 3 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 4 \end{array}\right].$$

Bu matritsaning uchinchi ustunini unga teng bo'lgan to'rtinchi ustundan ayiramiz va birinchi ustunga qo'shamiz. Hosil qilingan birinchi ustunni 5 ga ko'paytiramiz va beshinchi ustundan ayiramiz. U holda

$$B \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 9 & 0 & 3 & 0 & -44 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 4 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

rangA=3, *rangB=4* ekanligini hosil qildik, bundan *rangA≠ rangB*, ya'ni berilgan tenglamalar sistemasi birgalikda emas. ◄

1.3–AT

1. Agar

$$a)\ A = \begin{bmatrix} 3 & 1 & -5 \\ 1 & 2 & 4 \\ 3 & 2 & -1 \end{bmatrix}; \qquad b)\ A = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 3 & 1 & -7 \\ 2 & 7 & 6 & 1 \\ 1 & 2 & 2 & 1 \end{bmatrix}$$

bo'lsa, berilgan A matritsaga teskari matritsa A^{-1} ni toping.

$$\left(\text{Javob } a)\ A^{-1} = \frac{1}{3}\begin{bmatrix} -10 & -9 & 14 \\ 13 & 12 & -17 \\ -4 & -3 & 5 \end{bmatrix}; \quad b)\ A^{-1} = \frac{1}{15}\begin{bmatrix} -7 & 3 & -13 & 41 \\ -13 & -3 & 8 & -16 \\ 18 & 3 & -3 & 6 \\ -3 & -3 & 3 & -6 \end{bmatrix}. \right)$$

2. Agar

$$\text{a)}\ A = \begin{bmatrix} -8 & 1 & -7 & -5 & -5 \\ -2 & 1 & -3 & -1 & -1 \\ 1 & 1 & -1 & 1 & 1 \end{bmatrix}, \qquad \text{b)}\ A = \begin{bmatrix} 1 & 0 & 1 & -1 \\ 2 & 1 & 3 & -2 \\ -3 & -1 & -4 & 3 \\ 4 & -1 & 3 & -4 \\ 1 & 1 & 2 & -1 \end{bmatrix},$$

$$\text{c)}\ A = \begin{bmatrix} -1 & 4 & 2 & 0 \\ 1 & 8 & 2 & 1 \\ 2 & 7 & 1 & -4 \end{bmatrix}$$

bo'lsa, elementar almashtirishlar yoki o'rab oluvchi minorlar usuli yordamida A matritsaning rangini toping va biror bazis minorini ko'rsating. (Javob: *a)2; b)2: c)5.*

3. Asosiy matritsa A va kengaytirilgan matritsa B ni bilgan holda ularga mos keluvchi chiziqli tenglamalar sistemasini yozing va uning birgalikda emasligini Kroneker-Kapelli teoremasi yordamida ko'rsating.

$$\text{a)}\ A = \begin{bmatrix} 1 & -1 & 1 & -2 \\ 1 & -1 & 2 & -1 \\ 5 & -5 & 8 & -7 \end{bmatrix}, \qquad B = \left[\begin{array}{c|c} & 1 \\ A & 2 \\ & 3 \end{array}\right];$$

b) $A=\begin{bmatrix}3 & -1 & 1\\ 1 & -5 & 1\\ 2 & 4 & 0\\ 2 & 1 & 3\\ 5 & 0 & 4\end{bmatrix},\quad B=\left[\begin{array}{c|c} A & \begin{matrix}6\\12\\-6\\3\\9\end{matrix}\end{array}\right].$

(Javob: *a) rangA=2, rangB=3*,ya'ni sistema birgalikda emas;
b) rangA= rangB=3, ya'ni sistema birgalikda.)

Mustaqil ishlar

1. 1) $A=\begin{bmatrix}3 & 5 & -2\\ 1 & -3 & 2\\ 6 & 7 & -3\end{bmatrix}$ matritsaga teskari matritsa A^{-1} ni toping.

2) $A=\begin{bmatrix}3 & 1 & 2\\ 7 & 3 & 5\\ 15 & 7 & 11\\ 11 & 5 & 8\end{bmatrix}$ bo'lsa, elementar almashtirishlar yordamida *A* matritsaning rangini toping va biror bazisli minorini ko'rsating.

$\left(\text{Javob}:1)\ A^{-1}=\frac{1}{10}\begin{bmatrix}-5 & 1 & 4\\ 15 & 3 & -8\\ 25 & 9 & -14\end{bmatrix};\ 2)\ rang\ A=2.\right)$

2. 1) Agar $A=\begin{bmatrix}2 & 1 & 3\\ 1 & 0 & 2\\ 1 & 0 & 4\end{bmatrix}$ bo'lsa, A matritsa uchun A^{-1} ni toping va $AA^{-1}=E$ ekanligini ko'rsating.

2) Agar $A=\begin{bmatrix}1 & -1 & -1 & 5 & 1\\ -2 & 0 & 1 & 1 & 2\\ -3 & 1 & 2 & -4 & 1\end{bmatrix}$ bo'lsa, A matritsaning rangini toping va uning biror bazisli minorini ko'rsating.

$\left(\text{Javob}:1)\ A^{-1}=-\frac{1}{2}\begin{bmatrix}0 & -4 & 2\\ -2 & 5 & -1\\ 0 & 1 & -1\end{bmatrix};\ 2)\ rang\ A=2.\right)$

3. 1) $A=\begin{bmatrix}0 & 2 & -1\\ -2 & -1 & 2\\ 3 & -2 & -1\end{bmatrix}$ bo'lsa, *A* matritsa uchun A^{-1} ni toping.

2) $A=\begin{bmatrix}4 & -1 & 2 & 0\\ -1 & 1 & 1 & 1\\ 1 & 3 & 2 & -1\\ 0 & 4 & 3 & 0\end{bmatrix}$ bo'lsa, *A* matritsaning rangini toping va biror bazisli minorini ko'rsating

$$\left(\text{Javob}: 1)\ A^{-1}=\begin{bmatrix}5 & 4 & 3\\ 4 & 3 & 2\\ 7 & 6 & 4\end{bmatrix};\ 2)\ rang\ \ A=3.\right)$$

Chiziqli algebraik tenglamalarni echish usullari.

Maritsa usuli. Aytaylik (1.13) sistema uchun m=n va (1.14) ko'rinishdagi asosiy matritsa xosmas bo'lsin, ya'ni det≠0 bo'lsin. U holda A matritsa uchun (1.11) formula bilan aniqlanuvchi yagona A^{-1} teskari matritsa mavjud bo'ladi.

Ozod hadlar va noma'lumlar uchun ustun-matritsalarni kiritamiz.

$$X=\begin{bmatrix}x_1\\ x_2\\ \vdots\\ x_n\end{bmatrix};\ \tilde{B}=\begin{bmatrix}b_1\\ b_2\\ \vdots\\ b_n\end{bmatrix}\ . \qquad (1.16)$$

U holda (1.13) sistemani matritsaviy formada quyidagicha yozish mumkin. $AX=B$. Bu matritsaviy tenglamani chapdan A^{-1} ga ko'paytirib, quyidagini hosil qilamiz.

$A^{-1}\,AX= A^{-1}B$, bundan $EX=X= A^{-1}B$. Shunday qilib, X matritsaviy yechim A^{-1} va B larning ko'paytmasi yordamida oson topiladi.

1– misol.

$$\left.\begin{aligned}2x-4y+z&=3\\ x-5y+3z&=-1\\ x-\ y+\ z&=1\end{aligned}\right\}$$

tenglamalar sistemasini matritsa usuli bilan yeching.

►Quyidagiga egamiz:

$$A=\begin{bmatrix}2 & -4 & 1\\ 1 & -5 & 3\\ 1 & -1 & 1\end{bmatrix},\ X=\begin{bmatrix}x\\ y\\ z\end{bmatrix},\ \tilde{B}=\begin{bmatrix}3\\ -1\\ 1\end{bmatrix}, \qquad \det\ A=-8\,.$$

Teskari matritsa quyidagiga teng bo'ladi.

$$A^{-1}=-\frac{1}{8}\begin{bmatrix}-2 & 3 & -7\\ 2 & 1 & -5\\ 4 & -2 & -6\end{bmatrix}$$

(§1.3 dagi 2-misolga qarang). Noma'lumlarni topamiz.

$$X=-\frac{1}{8}\begin{bmatrix}-2 & 3 & -7\\ 2 & 1 & -5\\ 4 & -2 & -6\end{bmatrix}\begin{bmatrix}3\\ -1\\ 1\end{bmatrix}=-\frac{1}{8}\begin{bmatrix}-2\cdot3+3(-1)-7\cdot1\\ 2\cdot3+1(-1)-5\cdot1\\ 4\cdot3-2(-1)-6\cdot1\end{bmatrix}=-\frac{1}{8}\begin{bmatrix}-16\\ 0\\ 8\end{bmatrix}=\begin{bmatrix}2\\ 0\\ -1\end{bmatrix},$$

ya'ni berilgan sistemaning yechimi. $x=2,\ y=0,\ z=-1$◄

Kramer formulalari. Agar (1.13) sistema uchun m=n va $detA\neq0$ bo'lsa, u holda $x_i(i=1,\overline{n})$ noma'lumlarni hisoblash uchun quyidagi Kramer formulalari o'rinlidir.

$$x_i = \frac{\Delta_n^{(i)}}{\Delta_n} \qquad (i = \overline{1,n}), \qquad (1.17)$$

Bu erda $\Delta_n = \det A$, $\Delta_n^{(i)}$ esa Δ_n-da *i*-chi ustunni berilgan matritsaning ozod hadlardan iborat ustunga almashtirishdan hosil bo'lgan *n*-chi tartibli determinantdir.

2. Misol. Tenglamalar sistemasini Kramer formulalari yordamida yeching.

$$\left.\begin{aligned} 2x_1 - x_2 - 3x_3 &= 3 \\ 3x_1 + 4x_2 - 5x_3 &= -8 \\ 2x_1 + 7x_3 &= 17 \end{aligned}\right\}$$

►Δ_3 ni hisoblaymiz.

$$\Delta_3 = \det A = \begin{vmatrix} 2 & -1 & -3 \\ 3 & 4 & -5 \\ 0 & 2 & 7 \end{vmatrix} = 56 - 18 + 20 + 21 = 79 .$$

Δ_3 da birinchi, ikkinchi va uchinchi ustunlarni ketma-ket ozod hadlar ustuni bilan almashtirib, quyidagilarni hosil qilamiz.

$$\Delta_3^{(1)} = \begin{vmatrix} 3 & -1 & -3 \\ -8 & 4 & -5 \\ 17 & 2 & 7 \end{vmatrix} = 395, \qquad x_1 = \frac{\Delta_3^{(1)}}{\Delta_3} = \frac{395}{79} = 5 .$$

$$\Delta_3^{(2)} = \begin{vmatrix} 2 & 3 & -3 \\ 3 & -8 & -5 \\ 0 & 17 & 7 \end{vmatrix} = -158, \qquad x_2 = \frac{\Delta_3^{(2)}}{\Delta_3} = -\frac{158}{79} = -2 .$$

$$\Delta_3^{(3)} = \begin{vmatrix} 2 & -1 & 3 \\ 3 & 4 & -8 \\ 0 & 2 & 17 \end{vmatrix} = 237, \quad x_3 = \frac{\Delta_3^{(3)}}{\Delta_3} = \frac{237}{79} = 3 .$$◄

Noma'lumlarni ketma-ket yo'qotishning Jordan-Gauss usuli. Agar (1.13) sistemaning asosiy matritsasi A $r \le n$ rangga ega bo'lsa, u holda bu sistemaning kengaytirilgan matritsasi B ni har doim elementar almashtirishlar yordamida satrlari va ustunlarini o'rinlashtirib, quyidagi ko'rinishga keltirish mumkin.

$$\left[\begin{array}{cccccccc|c} 1 & \tilde{a} & \cdots & \tilde{a}_{1r} & \tilde{a}_{1r+1} & \cdots & \tilde{a}_{1n} & & \tilde{b}_1 \\ 0 & 1 & \cdots & \tilde{a}_{2r} & \tilde{a}_{2r+1} & \cdots & \tilde{a}_{2n} & & \tilde{b}_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots \\ 0 & 0 & \cdots & 1 & \tilde{a}_{rr+1} & \cdots & \tilde{a}_{rn} & & \tilde{b}_r \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & & \tilde{b}_{r+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots \\ 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & & \tilde{b}_m \end{array}\right] \qquad (1.18)$$

(1.18) matritsa berilgan sistemaga ekvivalent bo'lgan sistemaning kengaytirilgan matritsasi bo'ladi (ya'ni berilgan sistema bilan bir xil yechimga ega bo'ladi).

$$\left.\begin{array}{r} x_1 + \tilde{a}_{12}\, x_2 + \cdots + \tilde{a}_{1r}\, x_r + \tilde{a}_{1r+1}\, x_{r+1} + \cdots + \tilde{a}_{1n}\, x_n = \tilde{b}_1 \\ x_2 + \cdots + \tilde{a}_{2r}\, x_r + \tilde{a}_{2r+1}\, x_{r+1} + \cdots + \tilde{a}_{2n}\, x_n = \tilde{b}_2 \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ x_r + \tilde{a}_{rr+1}\, x_{r+1} + \cdots + \tilde{a}_{rn}\, x_n = \tilde{b}_r \\ 0 = \tilde{b}_{r+1} \\ \cdots\cdots\cdots \\ 0 = \tilde{b}_m \end{array}\right\} . \qquad (1.19)$$

Agar $\overline{b_{r=1}},....,\overline{b_m}$ sonlaridan hech bo'lmaganda bittasi noldan farqli bo'lsa, u holda (1.19) sistema va asosiy sistema (1.13) lar birgalikda bo'lmaydi. Agar $\overline{b_{r=1}} = ... = \overline{b_m} = 0$ bo'lsa, u holda (1.13) sistema birgalikda bo'ladi, (1.19) sistemadan $x_r,,x_{r-1},\ldots,x_2,x_1$ bazis noma'lumlarni $x_{r+1},\ldots,x_n$, ozod noma'lumlar orqali yaqqol ko'rinishda ketma-ket ifodalash mumkin, ya'ni (1.13) sistemani yechish mumkin. Agar $r=n$ bo'lsa, bu sistemaning yechimi yagona bo'ladi.

3-misol.Jordan-Gaussning ketma-ket yo'qotish usuli yordamida berilgan sistemaning birgalikda ekanligini tekshiring, birgalikda bo'lgan holda yeching.

$$\left.\begin{array}{r} 2x_1 + 3x_2 + 11x_3 + 5x_4 = 2 \\ x_1 + x_2 + 5x_3 + 2x_4 = 1 \\ 3x_1 + 3x_2 + 9x_3 + 5x_4 = -2 \\ 2x_1 + x_2 + 3x_3 + 2x_4 = -3 \\ x_1 + x_2 + 3x_3 + 4x_4 = -3 \end{array}\right\}$$

► Kengaytirilgan B matritsani tuzamiz va satrlar ustida zarur elementar almashtirishlarni bajaramiz.

$$B = \left[\begin{array}{cccc|c} 2 & 3 & 11 & 5 & 2 \\ 1 & 1 & 5 & 2 & 1 \\ 3 & 3 & 9 & 5 & -2 \\ 2 & 1 & 3 & 2 & -3 \\ 1 & 1 & 3 & 4 & -3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 5 & 2 & 1 \\ 1 & 1 & 3 & 4 & -3 \\ 2 & 3 & 11 & 5 & 2 \\ 2 & 1 & 3 & 2 & -3 \\ 3 & 3 & 9 & 5 & -2 \end{array}\right] \sim$$

$$\sim -\frac{1}{2}\left[\begin{array}{cccc|c} 1 & 1 & 5 & 2 & 1 \\ 0 & 0 & -2 & 2 & -4 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & -1 & -7 & -2 & -5 \\ 0 & 0 & -6 & -1 & -5 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 5 & 2 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & -7 & 7 \\ 0 & 0 & -6 & -1 & -5 \end{array}\right] \sim$$

$$\sim \left[\begin{array}{cccc|c} 1 & 1 & 5 & 2 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -7 & 7 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 5 & 2 & 1 \\ 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Oxirgi matritsada berilgan sistemaga mos keluvchi sistema hosil bo'ladi.

$$\left.\begin{array}{r} x_1 + x_2 + 5x_3 + 2x_4 = 1 \\ x_2 + x_3 + x_4 = 0 \\ x_3 + x_4 = 2 \\ x_4 = -1 \end{array}\right\}.$$

Bundan, pastdan yuqoriga harakatlanib, ketma-ket topamiz.

$x_4 = -1, \quad x_3 = 2 + x_4 = 2 - 1 = 1, \; x_2 = -x_3 - x_4 = -1 + 1 = 0,$

$x_1 = 1 - x_2 - 5x_3 - 2x_4 = 1 - 5 + 2 = -2.$

Shunday qilib, berilgan sistema birgalikda va uning yechimi yagona ekanligi kelib chiqadi. (r= n =4): x_1=-2; x_2=0,….

Tekshirish yordamida topilgan yechimning to'g'riligiga ishonch hosil qilish mumkin. ◄

4–misol. Jordan-Gauss usuli yordamida berilgan sistema ikki parametrga bog'liq cheksiz ko'p echimga ega ekanligini ko'rsating va bu yechimlarni toping.

$$\left.\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 5 \\ x_2 + x_3 + x_4 &= 3 \\ x_1 + x_2 \qquad &= 2 \end{aligned}\right\}$$

►Kengaytirilgan B matritsani tuzamiz va satrlarning elementar almashtirishlari yordamida *rang*A va *rangB* larni topamiz*:*

$$B = \left[A \middle| \tilde{B} \right] = \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 0 & 1 & 1 & 1 & 3 \\ 1 & 1 & 0 & 0 & 2 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & -1 & -1 & -1 & -3 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 1 & 1 & 5 \\ 0 & 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

Ko'rinib turibdiki, *rang*A = *rangB*=2<*n*=4.Shuning uchun sistema birgalikda va ikkita parametrga bog'liq *(n-r =4-2=2*) cheksiz ko'p yechimga ega.

Berilgan B matritsaga ekvivalent bo'lgan oxirgi matritsaga, berilgan sistemaga ekvivalent

$$\left.\begin{aligned} x_1 + 2x_2 + x_3 + x_4 &= 5 \\ x_2 + x_3 + x_4 &= 3 \end{aligned}\right\}$$

sistema mos keladi. $\Delta_2 = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1 \neq 0$ bo'lganligi sababli bazis noma'lumlar sifatida x_1 va x_2 ni olamiz, x_3 va x_4 larni ozod noma'lumlar (parametrlar) deb qabul qilamiz.U holda oxirgi sistemaning ikkinchi tenglamasidan quyidagini hosil qilamiz: $x_2 = 3 - x_3 - x_4$. Bu ifodani birinchi tenglamaga qo'yib x_1 ni topamiz. $x_1 = 5 - 2(3 - x_3 - x_4) - x_3 - x_4 = -1 + x_3 + x_4$. ◄

I z o h: Bazis noma'lumlar sifatida x_1, x_3, yoki x_1, x_4, yoki x_2, x_3, yoki x_2, x_4 larni qabul qilishimiz mumkin edi, ammo x_3, x_4 larni qabul qila olmaymiz, chunki ularning oldilaridagi koeffitsiyntlardan tuzilgan determinant nolga teng $\left(\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0\right)$, shuning uchun x_3 va x_4 larni x_1 va x_2 orqali ifodalab bo'lmaydi.

1.4–AT

1. Kroneker- Kapelli teoremasi yordamida sistemaning birgalikda ekanligini ko'rsating, sistemani matritsa ko'rinishida yozing va uni matritsa usulida eching:

a) $\begin{cases} 2x_1 - x_2 = -1 \\ x_1 + 2x_2 - x_3 = -2 \\ x_2 + x_3 = -2 \end{cases}$; b) $\begin{cases} 4x_1 + 2x_2 - x_3 = 0 \\ x_1 + 2x_2 + x_3 = 1 \\ x_2 - x_3 = -3 \end{cases}$.

(Javob: *a)x_1= x_2= x_3=-1; b) x_1=1, x_2=-1, x_3=2.)*

2. Kramer formulalaridan foydalanib, tenglamalar sistemasini eching:

a) $\begin{cases} 2x_1 + x_2 - x_3 = 0 \\ x_2 + 4x_3 = -6 \\ x_1 + x_3 = 1 \end{cases}$; b) $\begin{cases} 2x_1 + 3x_2 + 8x_4 = 0 \\ x_2 - x_3 + 3x_4 = 0 \\ x_3 + 2x_4 = 1 \\ x_1 + x_4 = -24 \end{cases}$.

(Javob: *a)x_1=1, x_2=-2, x_3=0; b) x_1=-19, x_2=26, x_3=11,x_4=-5.)*

3. Tenglamalar sistemasini Jordan-Gauss usulida eching:

a) $\begin{cases} 3x_1 - 2x_2 + x_3 - x_4 = 0, \\ 3x_1 - 2x_2 - x_3 + x_4 = 0, \\ x_1 - x_2 + 2x_3 + 5x_4 = 0; \end{cases}$

b) $\begin{cases} 4x_1 + 2x_2 - 3x_3 + 2x_4 = 3, \\ 2x_1 + 3x_2 - 2x_3 + 3x_4 = 2, \\ 3x_1 + 2x_2 - 3x_3 + 4x_4 = 1. \end{cases}$

(Javob: *a)x_1=14t, x_2=21t, x_3=x_4=t(t-ixtiyoriy son);*
b) x_1=-10t+10, x_2=t, x_3=-16t, x_4=4-5t (t- ixtiyoriy son).)

4. Tenglamalar sistemasi birgalikda ekanligini tekshiring, agar birgalikda bo’lsa, uni yeching.

$$\left.\begin{aligned} 2x_1 + 5x_2 - 8x_3 &= 8, \\ 4x_1 + 3x_2 - 9x_3 &= 9, \\ 2x_1 + 3x_2 - 5x_3 &= 7, \\ x_1 + 8x_2 - 7x_3 &= 12, \end{aligned}\right\}.$$

(Javob: *x_1=3, x_2=2, x_3=1.)*

5. Bir jinsli tenglamalar sistemasini yeching.

$$\left.\begin{aligned} x_1 + 2x_2 + 4x_3 - 3x_4 &= 0 \\ 3x_1 + 5x_2 + 6x_3 - 4x_4 &= 0 \\ 4x_1 + 5x_2 - 2x_3 + 3x_4 &= 0 \\ 3x_1 + 8x_2 + 24x_3 - 19x_4 &= 0 \end{aligned}\right\}$$

(Javob*: x_1=8x_3-7x_4, x_2=-6x_3+5x_4.)*

Mustaqil ish

1. Tenglamalar sistemasini matritsalar usuli bilan yeching va yechimni tekshiring.

$$\left.\begin{aligned} 2x_1 - x_2 + 5x_3 &= 4, \\ 3x_1 - x_2 + 5x_3 &= 0, \\ 5x_1 + 2x_2 + 13x_3 &= 2. \end{aligned}\right\}.$$

2. Tenglamalar sistemasini Kramer formulalari bo'yicha yeching va yechimni tekshiring.

$$\left.\begin{array}{l} x_1 - 2x_2 - x_3 = -2, \\ 2x_1 - x_2 = -1, \\ x_1 + x_3 = -2. \end{array}\right\}.$$

3. Tenglamalar sistemasini Jordan-Gauss usuli bilan yeching va yechimni tekshiring.

$$\left.\begin{array}{l} x_1 - 4x_2 + 3x_3 = -22, \\ 2x_1 + 3x_2 + 5x_3 = 12, \\ 3x_1 - x_2 - 2x_3 = 0. \end{array}\right\}.$$

1.5. I– BOBGA INDIVIDUAL UY VAZIFALARI.

1.1–IUT

1. Berilgan Δ determinant uchun a_{i2}, a_{3j} elementlarning minorlari va algebraik to'ldiruvchilarni toping. Δ determinantni: a) i- satr elementlari bo'yicha yoyib; b) j-ustun elementlari bo'yicha yoyib; c) i- satr elementlarini nolga aylantirib, hisoblang.

1.1. $\begin{vmatrix} 1 & 1 & -2 & 0 \\ 3 & 6 & -2 & 5 \\ 1 & 0 & 6 & 4 \\ 2 & 3 & 5 & -1 \end{vmatrix}$, $i = 4$, j = 1 .

1.2. $\begin{vmatrix} 2 & 0 & -1 & 3 \\ 6 & 3 & -9 & 0 \\ 0 & 2 & -1 & 3 \\ 4 & 2 & 0 & 6 \end{vmatrix}$, $i = 1$, j = 3 .

1.3. $\begin{vmatrix} 2 & 7 & 2 & 1 \\ 1 & 1 & -1 & 0 \\ 3 & 4 & 0 & 2 \\ 0 & 5 & -1 & -3 \end{vmatrix}$, $i= 4$, j = 1 .

1.4. $\begin{vmatrix} 4 & -5 & -1 & -5 \\ -3 & 2 & 8 & -2 \\ 5 & 3 & 1 & 3 \\ -2 & 4 & -6 & 8 \end{vmatrix}$, $i= 1$, j = 3 .

1.5 . $\begin{vmatrix} 3 & 5 & 3 & 2 \\ 2 & 4 & 1 & 0 \\ 1 & -2 & 2 & 1 \\ 5 & 1 & -2 & 4 \end{vmatrix}$, $i= 2$, j = 4 .

1.6. $\begin{vmatrix} 3 & 2 & 0 & -5 \\ 4 & 3 & -5 & 0 \\ 1 & 0 & -2 & 3 \\ 0 & 1 & -3 & 4 \end{vmatrix}$, $i= 1$, j = 2 .

1.7. $\begin{vmatrix} 2 & -1 & 2 & 0 \\ 3 & 4 & 1 & 2 \\ 2 & -1 & 0 & 1 \\ 1 & 2 & 3 & -2 \end{vmatrix}$, $i= 2$, j = 3.

1.8. $\begin{vmatrix} 3 & 2 & 0 & -2 \\ 1 & -1 & 2 & 3 \\ 4 & 5 & 1 & 0 \\ -1 & 2 & 3 & -3 \end{vmatrix}$, $i=3$, j = 1.

1.9. $\begin{vmatrix} 0 & -1 & 2 & 0 \\ 3 & 4 & 1 & 2 \\ 2 & -1 & 0 & 1 \\ 1 & 2 & 3 & -2 \end{vmatrix}$, $i = 4$, $j = 3$.

1.10. $\begin{vmatrix} 0 & -2 & 1 & 7 \\ 4 & -8 & 2 & -3 \\ 10 & 1 & -5 & 4 \\ -8 & 3 & 2 & -1 \end{vmatrix}$, $i = 4$, $j = 2$.

1.11. $\begin{vmatrix} 5 & -3 & 7 & -1 \\ 3 & 2 & 0 & 2 \\ 2 & 1 & 4 & -6 \\ 3 & -2 & 9 & 4 \end{vmatrix}$, $i = 3$, $j = 4$.

1.12. $\begin{vmatrix} 4 & -1 & 1 & 5 \\ 0 & 2 & -2 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 1 & -2 \end{vmatrix}$, $i = 1$, $j = 2$.

1.13. $\begin{vmatrix} 1 & 8 & 2 & -3 \\ 3 & -2 & 0 & 4 \\ 5 & -3 & 7 & -1 \\ 3 & 2 & 0 & 2 \end{vmatrix}$, $i = 1$, $j = 4$.

1.14. $\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & 3 & 2 \\ 3 & 0 & 2 & 1 \\ 3 & -1 & 4 & 3 \end{vmatrix}$, $i = 2$, $j = 4$.

1.15. $\begin{vmatrix} 3 & 1 & 2 & 3 \\ 4 & -1 & 2 & 4 \\ 1 & -1 & 1 & 1 \\ 4 & -1 & 2 & 5 \end{vmatrix}$, $i = 1$, $j = 3$.

1.16. $\begin{vmatrix} 3 & 1 & 2 & 0 \\ 5 & 0 & -6 & 1 \\ -2 & 2 & 1 & 3 \\ -1 & 3 & 2 & 1 \end{vmatrix}$, $i = 3$, $j = 2$.

1.17. $\begin{vmatrix} 1 & -1 & 0 & 3 \\ 3 & 2 & 1 & -1 \\ 1 & 2 & -1 & 3 \\ 4 & 0 & 1 & 2 \end{vmatrix}$, $i = 3$, $j = 1$.

1.18. $\begin{vmatrix} 5 & 0 & 4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & 1 & 1 \end{vmatrix}$, $i = 2$, $j = 4$.

1.19. $\begin{vmatrix} 6 & 2 & -10 & 4 \\ -5 & -7 & -4 & 1 \\ 2 & 4 & -2 & -6 \\ 3 & 0 & -5 & 2 \end{vmatrix}$, $i = 2$, $j = 3$.

1.20. $\begin{vmatrix} -1 & -2 & 4 & 1 \\ 2 & 3 & 0 & 6 \\ 2 & -2 & 1 & 4 \\ 3 & 1 & -2 & -1 \end{vmatrix}$, $i = 4$, $j = 3$.

1.21. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & -4 & 3 \\ 3 & -4 & -1 & 2 \\ 4 & 3 & -2 & -1 \end{vmatrix}$, $i = 1$, $j = 2$.

1.22. $\begin{vmatrix} -1 & 1 & -2 & 3 \\ 1 & 2 & 2 & 3 \\ -2 & 3 & 1 & 0 \\ 2 & 3 & -2 & 0 \end{vmatrix}$, $i = 3$, $j = 2$.

1.23. $\begin{vmatrix} -1 & 2 & 0 & 4 \\ 2 & -3 & 1 & 1 \\ 3 & -1 & 2 & 4 \\ 2 & 0 & 1 & 3 \end{vmatrix}$, $i = 4$, $j = 4$.

1.24. $\begin{vmatrix} 4 & 1 & 2 & 0 \\ -1 & 2 & 1 & -1 \\ 3 & -1 & 2 & 1 \\ 5 & 0 & 4 & 2 \end{vmatrix}$, $i = 3$, $j = 2$.

1.25. $\begin{vmatrix} 4 & 3 & -2 & -1 \\ -2 & 1 & -4 & 3 \\ 0 & 4 & 1 & -2 \\ 5 & 0 & 1 & -1 \end{vmatrix}$, $i = 2$, $j = 3$.

1.26. $\begin{vmatrix} 3 & -5 & 1 & 2 \\ 0 & 1 & -1 & -2 \\ 3 & 1 & -3 & 0 \\ 1 & 2 & -1 & 2 \end{vmatrix}$, $i = 4$, $j = 1$.

1.27. $\begin{vmatrix} 2 & -2 & 0 & 3 \\ 3 & 2 & 1 & -1 \\ 1 & 1 & -2 & 1 \\ 3 & 4 & -4 & 0 \end{vmatrix}$, $i = 3$, $j = 4$.

1.28. $\begin{vmatrix} 6 & 0 & -1 & 1 \\ 2 & -2 & 0 & 1 \\ 1 & 1 & -3 & 3 \\ 4 & 1 & -1 & 2 \end{vmatrix}$, $i = 1$, $j = 2$.

1.29. $\begin{vmatrix} -1 & -2 & 3 & 4 \\ 2 & 0 & 1 & -1 \\ 3 & -3 & 1 & 0 \\ 4 & 2 & 1 & -2 \end{vmatrix}$, $i = 4$, $j = 4$.

1.30. $\begin{vmatrix} -4 & 1 & 2 & 0 \\ 2 & -1 & 2 & 3 \\ -3 & 0 & 1 & 1 \\ 2 & 1 & -2 & 3 \end{vmatrix}$, $i = 2$, $j = 2$.

2. Ikkita A va B matritsalar berilgan. Quyidagilarni toping: a) AB ; b) BA ; c) A^{-1}; g) $A\,A^{-1}$; d) $A^{-1}A$.

2.1. $A = \begin{bmatrix} 2 & -1 & -3 \\ 8 & -7 & -6 \\ -3 & 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -1 & -2 \\ 3 & -5 & 4 \\ 1 & 2 & 1 \end{bmatrix}$.

2.2. $A = \begin{bmatrix} 3 & 5 & -6 \\ 2 & 4 & 3 \\ -3 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 8 & -5 \\ -3 & -1 & 0 \\ 4 & 5 & -3 \end{bmatrix}$.

2.3. $A = \begin{bmatrix} 2 & 1 & -1 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 6 & 0 \\ 2 & 4 & -6 \\ 1 & 2 & 1 \end{bmatrix}$.

2.4. $A = \begin{bmatrix} -6 & 1 & 11 \\ 9 & 2 & 5 \\ 0 & 4 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 & 1 \\ 0 & 2 & 7 \\ 1 & -3 & 2 \end{bmatrix}$.

2.5. $A = \begin{bmatrix} 3 & 1 & 2 \\ -1 & 0 & 2 \\ 1 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 & 2 \\ 2 & 1 & 1 \\ 3 & 7 & 1 \end{bmatrix}$.

2.6. $A = \begin{bmatrix} 2 & 3 & 2 \\ 1 & 3 & -1 \\ 4 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 & -1 \\ 3 & 1 & 2 \\ 5 & 3 & 0 \end{bmatrix}$.

2.7. $A = \begin{bmatrix} 6 & 7 & 3 \\ 3 & 1 & 0 \\ 2 & 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 & 5 \\ 4 & -1 & -2 \\ 4 & 3 & 7 \end{bmatrix}$.

2.8. $A = \begin{bmatrix} -2 & 3 & 4 \\ 3 & -1 & -4 \\ -1 & 2 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 3 & 1 \\ 0 & 6 & 2 \\ 1 & 9 & 2 \end{bmatrix}$.

2.9. $A = \begin{bmatrix} 1 & 7 & 3 \\ -4 & 9 & 4 \\ 0 & 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 5 & 2 \\ 1 & 9 & 2 \\ 4 & 5 & 2 \end{bmatrix}$.

2.10. $A = \begin{bmatrix} 2 & 6 & 1 \\ 1 & 3 & 2 \\ 0 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -3 & 2 \\ -4 & 0 & 5 \\ 3 & 2 & -3 \end{bmatrix}$.

2.11. $A = \begin{bmatrix} 6 & 9 & 4 \\ -1 & -1 & 1 \\ 10 & 1 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 4 & 3 \\ 0 & 5 & 2 \end{bmatrix}$.

2.12. $A = \begin{bmatrix} 1 & 0 & 3 \\ 3 & 1 & 7 \\ 2 & 1 & 8 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 & 4 \\ -3 & 0 & 1 \\ 5 & 6 & -4 \end{bmatrix}$.

2.13. $A = \begin{bmatrix} 5 & 1 & -2 \\ 1 & 3 & -1 \\ 8 & 4 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 5 & 5 \\ 7 & 1 & 2 \\ 1 & 6 & 0 \end{bmatrix}$

2.14. $A = \begin{bmatrix} 2 & 2 & 5 \\ 3 & 3 & 6 \\ 4 & 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & -1 \end{bmatrix}.$

2.15. $A = \begin{bmatrix} 1 & -2 & 5 \\ 3 & 0 & 6 \\ 4 & 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 1 & 1 \\ 2 & 3 & 3 \\ 1 & -2 & -1 \end{bmatrix}.$

2.16. $A = \begin{bmatrix} 5 & 4 & 2 \\ 1 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 1 & -5 \\ 3 & -7 & 1 \\ 1 & 2 & 2 \end{bmatrix}.$

2.17. $A = \begin{bmatrix} 3 & 1 & 0 \\ 4 & 3 & 2 \\ 2 & 2 & -7 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 7 & 0 \\ 5 & 3 & 1 \\ 1 & -6 & 1 \end{bmatrix}.$

2.18. $A = \begin{bmatrix} 8 & -1 & -1 \\ 5 & -5 & -1 \\ 10 & 3 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 2 & 5 \\ 3 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}.$

2.19. $A = \begin{bmatrix} 3 & -7 & 2 \\ 1 & -8 & 3 \\ 4 & -2 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 5 & -3 \\ 2 & 4 & 1 \\ 2 & 1 & -5 \end{bmatrix}.$

2.20. $A = \begin{bmatrix} 3 & -1 & 0 \\ 3 & 5 & 1 \\ 4 & -7 & 5 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 0 & 2 \\ 1 & -8 & 4 \\ 3 & 0 & 2 \end{bmatrix}.$

2.21. $A = \begin{bmatrix} 2 & -1 & -4 \\ 4 & -9 & 3 \\ 2 & -7 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & -4 \\ 5 & -6 & 4 \\ 7 & -4 & 1 \end{bmatrix}.$

2.22. $A = \begin{bmatrix} 8 & 5 & -1 \\ 1 & 5 & 3 \\ 1 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & -7 & -6 \\ 3 & 2 & -1 \\ 0 & 1 & 2 \end{bmatrix}.$

2.23. $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & -4 & 1 \\ 4 & -3 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & -4 \\ 2 & 5 & -3 \\ 4 & -3 & 2 \end{bmatrix}.$

2.24. $A=\begin{bmatrix} 5 & -8 & -4 \\ 7 & 0 & -5 \\ 4 & 1 & 0 \end{bmatrix},\quad B=\begin{bmatrix} 1 & 5 & 5 \\ 1 & 2 & 1 \\ 2 & -1 & -3 \end{bmatrix}.$

2.25. $A=\begin{bmatrix} 1 & 2 & 1 \\ 1 & -2 & 4 \\ 3 & -5 & 3 \end{bmatrix},\quad B=\begin{bmatrix} 7 & 5 & 1 \\ 5 & 3 & -1 \\ 1 & 2 & 3 \end{bmatrix}.$

2.26. $A=\begin{bmatrix} -3 & 4 & 2 \\ 1 & -5 & 3 \\ 0 & 1 & 2 \end{bmatrix},\quad B=\begin{bmatrix} 1 & 4 & 4 \\ 1 & 3 & 2 \\ -4 & 1 & 2 \end{bmatrix}.$

2.27. $A=\begin{bmatrix} -3 & 4 & 0 \\ 4 & 5 & 1 \\ -2 & 3 & 3 \end{bmatrix},\quad B=\begin{bmatrix} 1 & 7 & -1 \\ 0 & 2 & 6 \\ 2 & -1 & 1 \end{bmatrix}.$

2.28. $A=\begin{bmatrix} -3 & 4 & -3 \\ 1 & 2 & 3 \\ 3 & 0 & -1 \end{bmatrix},\quad B=\begin{bmatrix} 2 & -2 & 0 \\ 5 & 4 & 1 \\ 1 & -1 & 2 \end{bmatrix}.$

2.29. $A=\begin{bmatrix} -1 & 0 & 2 \\ 2 & 3 & 2 \\ 3 & 7 & 1 \end{bmatrix},\quad B=\begin{bmatrix} 3 & 0 & 1 \\ -3 & 1 & 7 \\ 1 & 3 & 2 \end{bmatrix}.$

2.30. $A=\begin{bmatrix} 1 & 1 & -4 \\ 2 & -4 & 6 \\ 1 & 2 & -1 \end{bmatrix},\quad B=\begin{bmatrix} 0 & -1 & 1 \\ 2 & 5 & 0 \\ 1 & -1 & 2 \end{bmatrix}.$

Namunaviy variantni yechish.

1. Berilgan Δ determinant uchun a_{12}, a_{32} elementlarning minorlari va algebraik to'ldiruvchilarni toping. Δ determinantni: a) birinchi satr elementlari bo'yicha yoyib; b) ikkinchi ustun elementlari bo'yicha yoyib; c) birinchi satr elementlarini nolga aylantirib, hisoblang.

$$\Delta_4=\begin{vmatrix} -3 & 2 & 1 & 0 \\ 2 & -2 & 1 & 4 \\ 4 & 0 & -1 & 4 \\ 3 & 1 & -1 & 4 \end{vmatrix}.$$

►Quyidagilarni topamiz:

$$M_{12}=\begin{vmatrix}2 & 1 & 4\\ 4 & -1 & 2\\ 3 & -1 & 4\end{vmatrix}= -8-16+6+12+4-16=-18.$$

$$M_{32}=\begin{vmatrix}-3 & 1 & 0\\ 2 & 1 & 4\\ 3 & -1 & 4\end{vmatrix}= -12+12-12-8=-20.$$

a_{12}, a_{32} elementlarning algebraik to'ldiruvchilari mos ravishda quyidagilarga teng:

$$A_{12}=(-1)^{1+2}M_{12}=-(-18)=18.$$

$$A_{32}=(-1)^{3+2}M_{32}=-(-20)=20.$$

a) Δ determinantni birinchi satr elementlari bo'yicha yoyib hisoblaymiz:

$$\Delta_4=a_{11}A_{11}+a_{12}A_{12}+a_{13}A_{13}+a_{14}A_{14}=$$

$$=-3\begin{vmatrix}-2 & 1 & 4\\ 0 & -1 & 2\\ 1 & -1 & 4\end{vmatrix}-2\begin{vmatrix}2 & 1 & 4\\ 4 & -1 & 2\\ 3 & -1 & 4\end{vmatrix}+1\begin{vmatrix}2 & -2 & 4\\ 4 & 0 & 2\\ 3 & 1 & 4\end{vmatrix}=$$

$= -3(8+2+4-4)-2(-8-16+6+12+4-16)+(16-12-4+32)=38;$

b) Δ determinantni ikkinchi ustun elementlari bo'yicha yoyib hisoblaymiz:

$$\Delta_4=-2\begin{vmatrix}2 & 1 & 4\\ 4 & -1 & 2\\ 3 & -1 & 4\end{vmatrix}-2\begin{vmatrix}-3 & 1 & 0\\ 4 & -1 & 2\\ 3 & -1 & 4\end{vmatrix}+1\begin{vmatrix}-3 & 1 & 0\\ 2 & 1 & 4\\ 4 & -1 & 2\end{vmatrix}=$$

$= -2(-8+6-16+12+4-16)-2(12+6-6-16)+(-6+16-12-4)=38;$

c) Δ determinantni birinchi satr elementlarini no'lga aylantirib, hisoblaymiz. Determinantlarning 10- xossasidan foydalanamiz (1.1 ga qarang). Determinantning uchinchi ustunini 3 ga ko'paytiramiz va birinchi ustunga qo'shamiz, so'ngra uchinchi ustunini -2 ga ko'paytiramiz va ikkinchi ustunga qo'shamiz.U holda birinchi satrning bitta elementidan boshqa barcha elementlari nollardan iborat bo'ladi. Hosil bo'lgan determinantni birinchi satr elementlari bo'yicha yoyib hisoblaymiz:

$$\Delta_4=\begin{vmatrix}-3 & 2 & 1 & 0\\ 2 & -2 & 1 & 4\\ 4 & 0 & -1 & 4\\ 3 & 1 & -1 & 4\end{vmatrix}=\begin{vmatrix}0 & 0 & 1 & 0\\ 5 & -4 & 1 & 4\\ 1 & 2 & -1 & 2\\ 0 & 3 & -1 & 4\end{vmatrix}=\begin{vmatrix}5 & -4 & 4\\ 1 & 2 & 2\\ 0 & 3 & 4\end{vmatrix}=\begin{vmatrix}0 & -14 & -6\\ 1 & 2 & 2\\ 0 & 3 & 4\end{vmatrix}=-(-56+18)=38.$$

Determinantlarning 10 – xossasidan foydalanib uchinchi tartibli determinantning birinchi ustunida no'llarni hosil qildik.◄

2. Ikkita A va B matritsalar berilgan.

$$A=\begin{bmatrix}-4 & 0 & 1\\ 2 & -1 & 3\\ 3 & 2 & 2\end{bmatrix},\quad B=\begin{bmatrix}1 & 2 & -3\\ 2 & 0 & 1\\ -2 & 1 & 3\end{bmatrix}$$

Quyidagilarni toping: *a*) AB ; b) BA ; c) A^{-1} ; d) AA^{-1}; e) $A^{-1}A$.

►*a)* *A* matritsaning ustunlar soni *B* matritsaning satrlar soniga teng, shuning uchun *AB* ko'paytma ma'noga ega bo'ladi. Elementlari $c_{ij}=a_{i1}b_{1j}+$

$a_{i2}b_{2j} + a_{i3}b_{3j} + \ldots + a_{in}b_{nj}$ formula bilan aniqlanuvchi. $C = AB$ matritsani topamiz.

$$C = AB = \begin{bmatrix} -4 & 0 & 1 \\ 2 & -1 & 3 \\ 3 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & 1 \\ -2 & 1 & 3 \end{bmatrix} =$$

$$= \begin{bmatrix} -4+0-2 & -8+0+1 & 12+0+3 \\ 2-2-6 & 4+0+3 & -6-1+9 \\ 3+4-4 & 6+0+2 & -9+2+6 \end{bmatrix} = \begin{bmatrix} -6 & -7 & 15 \\ -6 & 7 & 2 \\ 3 & 8 & -1 \end{bmatrix};$$

b) BA matritsani hisoblaymiz:

$$BA = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 0 & 1 \\ -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} -4 & 0 & 1 \\ 2 & -1 & 3 \\ 3 & 2 & 2 \end{bmatrix} =$$

$$= \begin{bmatrix} -4+4-9 & 0-2-6 & 1+6-6 \\ -8+0+3 & 0+0+2 & 2+0+2 \\ 8+2+9 & 0-1+6 & -2+3+6 \end{bmatrix} =$$

$$= \begin{bmatrix} -9 & -8 & 1 \\ -5 & 2 & 4 \\ 19 & 5 & 7 \end{bmatrix}.$$ Demak, $AB \neq BA$;

c) A matritsaga teskari matritsa A^{-1} quyidagi formula bilan aniqlanadi (1.11 formulaga qarang).

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix},$$

bu erda $\det A = \begin{vmatrix} -4 & 0 & 1 \\ 2 & -1 & 3 \\ 3 & 2 & 2 \end{vmatrix} = 8 + 4 + 3 + 24 = 39 \neq 0.$

Bundan ko'rinadiki, A xosmas matritsa, demak unga teskari matritsa A^{-1} mavjud. Quyidagilarni topamiz:

$A_{11} = \begin{vmatrix} -1 & 3 \\ 2 & 2 \end{vmatrix} = -8,$ $A_{12} = -\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = 5,$ $A_{23} = -\begin{vmatrix} -4 & 0 \\ 3 & 2 \end{vmatrix} = 8,$

$A_{21} = -\begin{vmatrix} 0 & 1 \\ 2 & 2 \end{vmatrix} = 2,$ $A_{22} = \begin{vmatrix} -4 & 1 \\ 3 & 2 \end{vmatrix} = -11,$ $A_{33} = \begin{vmatrix} -4 & 0 \\ 2 & -1 \end{vmatrix} = 4.$

$A_{31} = \begin{vmatrix} 0 & 1 \\ -1 & 3 \end{vmatrix} = 1$ $A_{32} = -\begin{vmatrix} -4 & 1 \\ 3 & 2 \end{vmatrix} = 14.$

$A_{13} = \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = 7,$

U holda:

$$A^{-1}=\frac{1}{39}\begin{bmatrix}-8 & 2 & 1\\ 5 & -11 & 14\\ 7 & 8 & 4\end{bmatrix}\begin{bmatrix}-\frac{8}{39} & -\frac{2}{39} & \frac{1}{39}\\ -\frac{5}{39} & -\frac{11}{39} & \frac{14}{39}\\ \frac{7}{39} & \frac{8}{39} & \frac{4}{39}\end{bmatrix};$$

d) U holda:

$$AA^{-1}=\begin{bmatrix}-4 & 0 & 1\\ 2 & -1 & 3\\ 3 & 2 & 2\end{bmatrix}\begin{bmatrix}-\frac{8}{39} & -\frac{2}{39} & \frac{1}{39}\\ -\frac{5}{39} & -\frac{11}{39} & \frac{14}{39}\\ \frac{7}{39} & \frac{8}{39} & \frac{4}{39}\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}=E;$$

e) yoki:

$$A^{-1}A=\frac{1}{39}\begin{bmatrix}-8 & 2 & 1\\ 5 & -11 & 14\\ 7 & 8 & 4\end{bmatrix}\begin{bmatrix}-4 & 0 & 1\\ 2 & -1 & 3\\ 3 & 2 & 2\end{bmatrix}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}.$$

Demak, teskari matritsa to’g’ri topilgan.◄

1.2– IUT

1. Tenglamalar sistemasi birgalikda ekanligini tekshiring, agar birgalikda bo’lsa uni :

a) Kramer formulalari bo’yicha;

b) teskari matritsa yordamida (matritsalar usulida) ;

c) Gauss usulida yeching.

1.1. $\begin{cases}2x_1+x_2+3x_3=7,\\ 2x_1+3x_2+x_3=1,\\ 3x_1+2x_2+x_3=6.\end{cases}$

1.2. $\begin{cases}2x_1-x_2+2x_3=3,\\ x_1+x_2+2x_3=-4,\\ 4x_1+x_2+4x_3=-3.\end{cases}$

1.3. $\begin{cases}3x_1-x_2+x_3=12,\\ x_1+2x_2+4x_3=6,\\ 5x_1+x_2+2x_3=3.\end{cases}$

1.4. $\begin{cases}2x_1-x_2+3x_3=-4,\\ x_1+3x_2-x_3=11,\\ x_1-2x_2+2x_3=-7.\end{cases}$

1.5. $\begin{cases}3x_1-2x_2+4x_3=12,\\ 3x_1+4x_2-2x_3=6,\\ 2x_1-x_2-x_3=-9.\end{cases}$

1.6. $\begin{cases}8x_1+3x_2-6x_3=-4,\\ x_1+x_2-x_3=2,\\ 4x_1+x_2-3x_3=-5.\end{cases}$

1.7. $\begin{cases}4x_1+x_2-3x_3=9,\\ x_1-x_2-x_3=-2,\\ 8x_1+3x_2-6x_3=12.\end{cases}$

1.8. $\begin{cases}2x_1+3x_2+4x_3=33,\\ 7x_1-5x_2=24,\\ 4x_1+x_3=39.\end{cases}$

1.9. $\begin{cases} 2x_1 + 3x_2 + 4x_3 = 12, \\ 7x_1 - 5x_2 + x_3 = -33, \\ 4x_1 + x_3 = -7. \end{cases}$

1.10. $\begin{cases} x_1 + 4x_2 - x_3 = 6, \\ 5x_2 + 4x_3 = -20, \\ 3x_1 - 2x_2 + 5x_3 = -22. \end{cases}$

1.11. $\begin{cases} 3x_1 - 2x_2 + 4x_3 = 21, \\ 3x_1 + 4x_2 - 2x_3 = 9, \\ 2x_1 - x_2 - x_3 = 10. \end{cases}$

1.12. $\begin{cases} 3x_1 - 2x_2 - 5x_3 = 5, \\ 2x_1 + 3x_2 - 4x_3 = 12, \\ x_1 - 2x_2 + 3x_3 = -1. \end{cases}$

1.13. $\begin{cases} 4x_1 + x_2 + 4x_3 = 19, \\ 2x_1 - x_2 + 2x_3 = 11, \\ x_1 + x_2 + 2x_3 = 8. \end{cases}$

1.14. $\begin{cases} 2x_1 - x_2 + 2x_3 = 0, \\ 4x_1 + x_2 + 4x_3 = 6, \\ x_1 + x_2 + 2x_3 = 4. \end{cases}$

1.15. $\begin{cases} 2x_1 - x_2 + 2x_3 = 8, \\ x_1 + x_2 + 2x_3 = 11, \\ 4x_1 + x_2 + 4x_3 = 22. \end{cases}$

1.16. $\begin{cases} 2x_1 - x_2 - 3x_3 = -9, \\ x_1 + 5x_2 + x_3 = 20, \\ 3x_1 + 4x_2 + 2x_3 = 15. \end{cases}$

1.17. $\begin{cases} 2x_1 - x_2 - 3x_3 = 0, \\ 3x_1 + 4x_2 + 2x_3 = 1, \\ x_1 + 5x_2 + x_3 = -3. \end{cases}$

1.18. $\begin{cases} -3x_1 + 5x_2 + 6x_3 = -8, \\ 3x_1 + x_2 + x_3 = -4, \\ x_1 - 4x_2 - 2x_3 = -9 \end{cases}$

1.19. $\begin{cases} 3x_1 + x_2 + x_3 = -4, \\ -3x_1 + 5x_2 + 6x_3 = 36, \\ x_1 - 4x_2 - 2x_3 = -19. \end{cases}$

1.20. $\begin{cases} 3x_1 - x_2 + x_3 = 11, \\ 5x_1 + x_2 + 2x_3 = 8, \\ x_1 + 2x_2 + 4x_3 = 16. \end{cases}$

1.21. $\begin{cases} 3x_1 - x_2 + x_3 = 9, \\ 5x_1 + x_2 + 2x_3 = 11, \\ x_1 + 2x_2 + 4x_3 = 19. \end{cases}$

1.22. $\begin{cases} 2x_1 + 3x_2 + x_3 = 4, \\ 2x_1 + x_2 + 3x_3 = 0, \\ 3x_1 + 2x_2 + x_3 = 1. \end{cases}$

1.23. $\begin{cases} 2x_1 + 3x_2 + x_3 = 12, \\ 2x_1 + x_2 + x_3 = 16, \\ 3x_1 + 2x_2 + x_3 = 8. \end{cases}$

1.24. $\begin{cases} x_1 - 2x_2 + 3x_3 = 14, \\ 2x_1 + 3x_2 - 4x_3 = -16, \\ 3x_1 - 2x_2 - 5x_3 = -8. \end{cases}$

1.25. $\begin{cases} 3x_1 + 4x_2 - 2x_3 = 11, \\ 2x_1 - x_2 - x_3 = 4, \\ 3x_1 - 2x_2 + 4x_3 = 11. \end{cases}$

1.26. $\begin{cases} x_1 + 5x_2 - 6x_3 = -15, \\ 3x_1 + x_2 + 4x_3 = 13, \\ 2x_1 - 3x_2 + x_3 = 9. \end{cases}$

1.27. $\begin{cases} 4x_1 - x_2 = -6, \\ 3x_1 + 2x_2 + 5x_3 = -14, \\ x_1 - 3x_2 + 4x_3 = -19. \end{cases}$

1.28. $\begin{cases} 5x_1 + 2x_2 - 4x_3 = -16, \\ x_1 + 3x_3 = -6, \\ 2x_1 - 3x_2 + x_3 = 9. \end{cases}$

1.29. $\begin{cases} x_1+4x_2-x_3=-9, \\ 4x_1-x_2+5x_3=-2, \\ 3x_2-7x_3=-6. \end{cases}$

1.30. $\begin{cases} 7x_1+4x_2-x_3=13, \\ 3x_1+2x_2+3x_3=3, \\ 2x_1-3x_2+x_3=-10. \end{cases}$

2. Tenglamalar sistemasi birgalikda ekanligini tekshiring, agar birgalikda bo'lsa uni :

a) Kramer formulalari bo'yicha;

b*)* teskari matritsa yordamida (matritsalar usulida) ;

c) Gauss usulida yeching.

2.1. $\begin{cases} 3x_1+2x_2-4x_3=8, \\ 2x_1+4x_2-5x_3=11, \\ x_1-2x_2+x_3=1. \end{cases}$

2.8. $\begin{cases} 5x_1-9x_2-4x_3=6, \\ x_1-7x_2-5x_3=1, \\ 4x_1-2x_2+x_3=2. \end{cases}$

2.2. $\begin{cases} x_1+x_2+x_3=1, \\ x_1-x_2+2x_3=-5, \\ 2x_1+3x_3=-2. \end{cases}$

2.9. $\begin{cases} x_1-5x_2+x_3=3, \\ 3x_1+2x_2-x_3=7, \\ 4x_1-3x_2=1. \end{cases}$

2.3. $\begin{cases} 2x_1-x_2+4x_3=15, \\ 3x_1-x_2+x_3=8, \\ 5x_1-2x_2+5x_3=0. \end{cases}$

2.10. $\begin{cases} 5x_1-5x_2-4x_3=-3, \\ x_1-x_2+5x_3=1, \\ 4x_1-4x_2-9x_3=0. \end{cases}$

2.4. $\begin{cases} 3x_1-3x_2+2x_3=2, \\ 4x_1-5x_2+2x_3=1, \\ x_1-2x_2=5. \end{cases}$

2.11. $\begin{cases} 7x_1-2x_2-x_3=2, \\ 6x_1-4x_2-5x_3=3, \\ x_1+2x_2+4x_3=5. \end{cases}$

2.5. $\begin{cases} 3x_1+2x_2-4x_3=8, \\ 2x_1+4x_2-5x_3=1, \\ 5x_1+6x_2-9x_3=2. \end{cases}$

2.12. $\begin{cases} 4x_1-3x_2+x_3=3, \\ x_1+x_2-x_3=4, \\ 3x_1-4x_2+2x_3=2. \end{cases}$

2.6. $\begin{cases} 3x_1+x_2+2x_3=-3, \\ 2x_1+2x_2+5x_3=5, \\ 5x_1+3x_2+7x_3=1. \end{cases}$

2.13. $\begin{cases} 3x_1+x_2+2x_3=1, \\ 2x_1+2x_2-3x_3=9, \\ x_1-x_2+x_3=2. \end{cases}$

2.7. $\begin{cases} 4x_1-7x_2-2x_3=0, \\ 2x_1-3x_2-4x_3=6, \\ 2x_1-4x_2+2x_3=2. \end{cases}$

2.14. $\begin{cases} 6x_1+3x_2-5x_3=0, \\ 9x_1+4x_2-7x_3=3, \\ 3x_1+x_2-2x_3=5. \end{cases}$

2.15. $\begin{cases} 8x_1 - x_2 + 3x_3 = 2, \\ 4x_1 + x_2 + 6x_3 = 1, \\ 4x_1 - 2x_2 - 3x_3 = 7. \end{cases}$

2.16. $\begin{cases} 2x_1 + 3x_2 + 4x_3 = 5, \\ x_1 + x_2 + 5x_3 = 6, \\ 3x_1 + 4x_2 + 9x_3 = 0. \end{cases}$

2.17. $\begin{cases} 2x_1 - 3x_2 - 4x_3 = 1, \\ 7x_1 - 9x_2 - x_3 = 3, \\ 5x_1 - 6x_2 + 3x_3 = 7. \end{cases}$

2.18. $\begin{cases} 5x_1 + 6x_2 - 2x_3 = 2, \\ 2x_1 + 3x_2 - x_3 = 9, \\ 3x_1 + 3x_2 - x_3 = 1. \end{cases}$

2.19. $\begin{cases} 3x_1 + x_2 - 2x_3 = 6, \\ 5x_1 - 3x_2 + 2x_3 = 4, \\ -2x_1 + 5x_2 - 4x_3 = 0. \end{cases}$

2.20. $\begin{cases} 2x_1 + x_2 + x_3 = 2, \\ 5x_1 + x_2 + 3x_3 = 4, \\ 7x_1 + 2x_2 + 4x_3 = 1. \end{cases}$

2.21. $\begin{cases} x_1 - 2x_2 - 3x_3 = 3, \\ x_1 + 3x_2 - 5x_3 = 0, \\ 2x_1 + x_2 - 8x_3 = 4. \end{cases}$

2.22. $\begin{cases} x_1 - 4x_2 - 2x_3 = 0, \\ 3x_1 - 5x_2 - 6x_3 = 2, \\ 4x_1 - 9x_2 - 8x_3 = 1. \end{cases}$

2.23. $\begin{cases} 4x_1 + x_2 - 3x_3 = 1, \\ 3x_1 + x_2 - x_3 = 2, \\ x_1 - 2x_3 = 5. \end{cases}$

2.24. $\begin{cases} 3x_1 - 5x_2 + 3x_3 = 4, \\ x_1 + 2x_2 + x_3 = 8, \\ 2x_1 - 7x_2 + 2x_3 = 1. \end{cases}$

2.25. $\begin{cases} x_1 - 2x_2 + 3x_3 = 6, \\ 2x_1 + 3x_2 - 4x_3 = 2, \\ 3x_1 + x_2 - x_3 = 5. \end{cases}$

2.26. $\begin{cases} 5x_1 - x_2 - 2x_3 = 1, \\ 3x_1 - 4x_2 + x_3 = 7, \\ 2x_1 + 3x_2 - 3x_3 = 4. \end{cases}$

2.27. $\begin{cases} 2x_1 + 8x_2 - 7x_3 = 0, \\ 2x_1 - 5x_2 + 6x_3 = 1, \\ 4x_1 + 3x_2 - x_3 = 7. \end{cases}$

2.28. $\begin{cases} 3x_1 + 4x_2 + x_3 = 2, \\ x_1 + 5x_2 - 3x_3 = 4, \\ 2x_1 - x_2 + 4x_3 = 5. \end{cases}$

2.29. $\begin{cases} 2x_1 - 3x_2 + 2x_3 = 5, \\ 3x_1 + 4x_2 - 7x_3 = 2, \\ 5x_1 + x_2 - 5x_3 = 9. \end{cases}$

2.30. $\begin{cases} 4x_1 - 9x_2 + 5x_3 = 2, \\ 7x_1 - 4x_2 + x_3 = 11, \\ 3x_1 + 5x_2 - 4x_3 = 5. \end{cases}$

3. Bir jinsli chiziqli algebraik tenglamalar sistemasini yeching.

3.1. $\begin{cases} x_1 + x_2 + x_3 = 0, \\ 2x_1 - 3x_2 + 4x_3 = 0, \\ 4x_1 - 11x_2 + 10x_3 = 0. \end{cases}$

3.2. $\begin{cases} 3x_1 - x_2 + 2x_3 = 0, \\ x_1 + x_2 + x_3 = 0, \\ x_1 + 3x_2 + 3x_3 = 0. \end{cases}$

3.3. $\begin{cases} x_1 + 3x_2 + 2x_3 = 0, \\ 2x_1 - x_2 + 3x_3 = 0, \\ 3x_1 - 5x_2 + 4x_3 = 0. \end{cases}$

3.4. $\begin{cases} 4x_1 - x_2 + 10x_3 = 0, \\ x_1 + 2x_2 - x_3 = 0, \\ 2x_1 - 3x_2 + 4x_3 = 0. \end{cases}$

3.5. $\begin{cases} 2x_1 + 5x_2 + x_3 = 0, \\ 4x_1 + 6x_2 + 3x_3 = 0, \\ x_1 - x_2 - 2x_3 = 05. \end{cases}$

3.6. $\begin{cases} 3x_1 - x_2 - 3x_3 = 0, \\ 2x_1 + 3x_2 + x_3 = 0, \\ x_1 + x_2 + 3x_3 = 0. \end{cases}$

3.7. $\begin{cases} x_1 - x_2 + 2x_3 = 0, \\ 2x_1 + x_2 - 3x_3 = 0, \\ 3x_1 + 2x_3 = 0. \end{cases}$

3.8. $\begin{cases} 2x_1 - x_2 - 5x_3 = 0, \\ x_1 + 2x_2 - 3x_3 = 0, \\ 5x_1 + x_2 + 4x_3 = 0. \end{cases}$

3.9. $\begin{cases} 5x_1 - 5x_2 + 4x_3 = 0, \\ 3x_1 + x_2 + 3x_3 = 0, \\ x_1 + 7x_2 - x_3 = 0. \end{cases}$

3.10. $\begin{cases} x_1 + 3x_2 - x_3 = 0, \\ 2x_1 + 5x_2 - 2x_3 = 0, \\ x_1 + x_2 + 5x_3 = 0. \end{cases}$

3.11. $\begin{cases} 2x_1 + x_2 + 3x_3 = 0, \\ 3x_1 - x_2 + 2x_3 = 0, \\ x_1 + 3x_2 + 4x_3 = 0. \end{cases}$

3.12. $\begin{cases} x_1 - 2x_2 - x_3 = 0, \\ 2x_1 + 3x_2 + 2x_3 = 0, \\ 3x_1 - 2x_2 + 5x_3 = 0. \end{cases}$

3.13. $\begin{cases} 2x_1 + x_2 - x_3 = 0, \\ 3x_1 - 2x_2 + 4x_3 = 0, \\ x_1 - 5x_2 + 3x_3 = 0. \end{cases}$

3.14. $\begin{cases} 4x_1 + x_2 + 3x_3 = 0, \\ 8x_1 - x_2 + 7x_3 = 0, \\ 2x_1 + 4x_2 - 5x_3 = 0. \end{cases}$

3.15. $\begin{cases} x_1 + 4x_2 - 3x_3 = 0, \\ 2x_1 + 5x_2 + x_3 = 0, \\ x_1 - 7x_2 + 2x_3 = 0. \end{cases}$

3.16. $\begin{cases} x_1 - 2x_2 + x_3 = 0, \\ 3x_1 + x_2 + 2x_3 = 0, \\ 2x_1 - 3x_2 + 5x_3 = 0. \end{cases}$

3.17. $\begin{cases} x_1 + 2x_2 + 3x_3 = 0, \\ 2x_1 - x_2 - x_3 = 0, \\ 3x_1 + 3x_2 + 2x_3 = 0. \end{cases}$

3.18. $\begin{cases} 3x_1 + 2x_2 = 2, \\ x_1 - x_2 + 2x_3 = 0, \\ 4x_1 - 2x_2 + 5x_3 = 5. \end{cases}$

3.19. $\begin{cases} 2x_1 - x_2 + 3x_3 = 0, \\ x_1 + 2x_2 - 5x_3 = 0, \\ 3x_1 + x_2 + x_3 = 0. \end{cases}$

3.20. $\begin{cases} 3x_1 + 2x_2 - x_3 = 0, \\ 2x_1 - x_2 + 3x_3 = 0, \\ 4x_1 + 3x_2 + 4x_3 = 0. \end{cases}$

3.21. $\begin{cases} x_1 - 3x_2 - 4x_3 = 0, \\ 5x_1 - 8x_2 - 2x_3 = 0, \\ 2x_1 + x_2 - x_3 = 0. \end{cases}$

3.22. $\begin{cases} 3x_1 + 5x_2 - x_3 = 0, \\ 2x_1 + 4x_2 - 3x_3 = 0, \\ x_1 - 3x_2 + x_3 = 0. \end{cases}$

3.23. $\begin{cases} 3x_1 - 2x_2 + x_3 = 0, \\ 2x_1 - 3x_2 + 2x_3 = 0, \\ 4x_1 + x_2 - 4x_3 = 0. \end{cases}$

3.24. $\begin{cases} 7x_1 + x_2 - 3x_3 = 0, \\ 3x_1 - 2x_2 + 3x_3 = 0, \\ x_1 - x_2 + 2x_3 = 0. \end{cases}$

3.25. $\begin{cases} x_1 + 2x_2 - 4x_3 = 0, \\ 2x_1 - x_2 - 3x_3 = 0, \\ x_1 + 3x_2 + x_3 = 0. \end{cases}$

3.26. $\begin{cases} 7x_1 - 6x_2 + x_3 = 0, \\ 4x_1 + 5x_2 = 0, \\ x_1 - 2x_2 + 3x_3 = 0. \end{cases}$

3.27. $\begin{cases} 5x_1 - 4x_2 + 2x_3 = 0, \\ 3x_2 - 2x_3 = 0, \\ 4x_1 + x_2 - 3x_3 = 0. \end{cases}$

3.28. $\begin{cases} 6x_1 + 5x_2 - 4x_3 = 0, \\ x_1 + x_2 - x_3 = 0, \\ 3x_1 + 4x_2 + 3x_3 = 0. \end{cases}$

3.29. $\begin{cases} 8x_1 + x_2 - 3x_3 = 0, \\ x_1 + 5x_2 + x_3 = 0, \\ 4x_1 - 7x_2 + 2x_3 = 0. \end{cases}$

3.30. $\begin{cases} x_1 + 7x_2 - 3x_3 = 0, \\ 3x_1 - 5x_2 + x_3 = 0, \\ 3x_1 + 4x_2 - 2x_3 = 0. \end{cases}$

4. Bir jinsli chiziqli algebraik tenglamalar sistemasini yeching.

4.1. $\begin{cases} 5x_1 - 3x_2 + 4x_3 = 0, \\ 3x_1 + 2x_2 - x_3 = 0, \\ 8x_1 - x_2 + 3x_3 = 0. \end{cases}$

4.2. $\begin{cases} 5x_1 - 6x_2 + 4x_3 = 0, \\ 3x_1 - 3x_2 + x_3 = 0, \\ 2x_1 - 3x_2 + 3x_3 = 0. \end{cases}$

4.3. $\begin{cases} x_1 + 2x_2 - 5x_3 = 0, \\ 2x_1 - 4x_2 + x_3 = 0, \\ 3x_1 - 2x_2 - 4x_3 = 0. \end{cases}$

4.4. $\begin{cases} x_1 + x_2 + x_3 = 0, \\ 2x_1 - 3x_2 + 4x_3 = 0, \\ 3x_1 - 2x_2 + 5x_3 = 0. \end{cases}$

4.5. $\begin{cases} x_1 + 2x_2 + 4x_3 = 0, \\ 5x_1 + x_2 + 2x_3 = 0, \\ 4x_1 - x_2 - 2x_3 = 0. \end{cases}$

4.6. $\begin{cases} 3x_1 - x_2 + x_3 = 0, \\ 2x_1 + 3x_2 - 4x_3 = 0, \\ 5x_1 + 2x_2 - 3x_3 = 0. \end{cases}$

4.7. $\begin{cases} x_1 - 2x_2 + x_3 = 0, \\ 3x_1 + 3x_2 + 5x_3 = 0, \\ 4x_1 + x_2 + 6x_3 = 0. \end{cases}$

4.8. $\begin{cases} 2x_1 + x_2 - 3x_3 = 0, \\ x_1 + 2x_2 - 4x_3 = 0, \\ x_1 - x_2 + x_3 = 0. \end{cases}$

4.9. $\begin{cases} 2x_1 - x_2 + 2x_3 = 0, \\ 4x_1 + x_2 + 5x_3 = 0, \\ 2x_1 + 2x_2 + 3x_3 = 0. \end{cases}$

4.10. $\begin{cases} 4x_1 + x_2 + 4x_3 = 0, \\ 3x_1 - 2x_2 - x_3 = 0, \\ 7x_1 - x_2 + 3x_3 = 0. \end{cases}$

4.11. $\begin{cases} 3x_1 - 2x_2 + x_3 = 0, \\ 2x_1 + 3x_2 - 5x_3 = 0, \\ 5x_1 + x_2 - 4x_3 = 0. \end{cases}$

4.12. $\begin{cases} 5x_1 + x_2 + 2x_3 = 0, \\ 3x_1 + 2x_2 - 3x_3 = 0, \\ 2x_1 - x_2 + x_3 = 0. \end{cases}$

4.13. $\begin{cases} x_1 + 2x_2 - 5x_3 = 0, \\ x_1 - 2x_2 - 4x_3 = 0, \\ 2x_1 - 9x_3 = 0. \end{cases}$

4.14. $\begin{cases} x_1 - 3x_2 + 5x_3 = 0, \\ x_1 + 2x_2 - 3x_3 = 0, \\ 2x_1 - x_2 + 2x_3 = 0. \end{cases}$

4.15. $\begin{cases} 2x_1 - x_2 + 2x_3 = 0, \\ 3x_1 + 2x_2 - 3x_3 = 0, \\ 5x_1 + x_2 - x_3 = 0. \end{cases}$

4.16. $\begin{cases} 2x_1 - x_2 + 2x_3 = 0, \\ x_1 - 3x_2 - 3x_3 = 0, \\ x_1 + 2x_2 + x_3 = 0. \end{cases}$

4.17. $\begin{cases} x_1 - 3x_2 - 2x_3 = 0, \\ 3x_1 - x_2 + 4x_3 = 0, \\ 2x_1 - 2x_2 + x_3 = 0. \end{cases}$

4.18. $\begin{cases} 5x_1 + x_2 - 2x_3 = 0, \\ 3x_1 - x_2 + x_3 = 0, \\ 2x_1 + 2x_2 - 3x_3 = 0. \end{cases}$

4.19. $\begin{cases} 3x_1 + 2x_2 - 3x_3 = 0, \\ 2x_1 - 3x_2 + x_3 = 0, \\ 5x_1 - x_2 - 2x_3 = 0. \end{cases}$

4.20. $\begin{cases} 4x_1 - x_2 + 5x_3 = 0, \\ 2x_1 - 3x_2 + 2x_3 = 0, \\ 2x_1 + 2x_2 + 3x_3 = 0. \end{cases}$

4.21. $\begin{cases} x_1 + 5x_2 + x_3 = 0, \\ 2x_1 - 3x_2 - 7x_3 = 0, \\ 3x_1 + 2x_2 - 6x_3 = 0. \end{cases}$

4.22. $\begin{cases} 3x_1 + 4x_2 - x_3 = 0, \\ x_1 - 5x_2 + 2x_3 = 0, \\ 4x_1 - x_2 + x_3 = 0. \end{cases}$

4.23. $\begin{cases} 2x_1 + 4x_2 - 3x_3 = 0, \\ x_1 - 3x_2 + 2x_3 = 0, \\ 3x_1 + x_2 - x_3 = 0. \end{cases}$

4.24. $\begin{cases} 7x_1 - 6x_2 - x_3 = 0, \\ 3x_1 - 3x_2 + 4x_3 = 0, \\ 4x_1 - 3x_2 - 5x_3 = 0. \end{cases}$

4.25. $\begin{cases} 5x_1 - 3x_2 + 2x_3 = 0, \\ 2x_1 + 4x_2 - 3x_3 = 0, \\ 3x_1 - 7x_2 + 5x_3 = 0. \end{cases}$

4.26. $\begin{cases} x_1 - 8x_2 + 7x_3 = 0, \\ 3x_1 + 5x_2 - 4x_3 = 0, \\ 4x_1 - 3x_2 + 3x_3 = 0. \end{cases}$

4.27. $\begin{cases} 5x_1 + 8x_2 - 5x_3 = 0, \\ 7x_1 + 5x_2 - x_3 = 0, \\ 2x_1 - 3x_2 + 4x_3 = 0. \end{cases}$

4.28. $\begin{cases} 5x_1 + x_2 - 6x_3 = 0, \\ 4x_1 + 3x_2 - 7x_3 = 0, \\ x_1 - 2x_2 + x_3 = 0. \end{cases}$

4.29. $\begin{cases} 2x_1 - x_2 + 4x_3 = 0, \\ 7x_1 - 5x_2 + 3x_3 = 0, \\ 5x_1 - 4x_2 - x_3 = 0. \end{cases}$

4.30. $\begin{cases} 2x_1 + 2x_2 - x_3 = 0, \\ 5x_1 + 4x_2 - 6x_3 = 0, \\ 3x_1 + 2x_2 - 5x_3 = 0. \end{cases}$

Namunaviy variantlarni yechish.

1. Bir jinsli bo’lmagan chiziqli algebraik tenglamalar sistemasi berilgan.

$$\left.\begin{aligned} x_1 + 5x_2 - \ x_3 &= 3, \\ 2x_1 + 4\,x_2 - 3x_3 &= 2, \\ 3x_1 - \ \ x_2 - \ 3x_3 &= -7. \end{aligned}\right\}$$

ushbu sistemaning birgalikda ekanligini tekshiring, agar birgalikda bo'lgan holda uni :

a) Kramer formulalari bo'yicha;

b*)* teskari matritsa yordamida (matritsalar usulida) ;

c) Gauss usulida yeching.

►Sistemaning birgalikda ekanligini Kroneker – Kappeli teoremasi bo'yicha tekshiramiz. Elementar almashtirishlar yordamida berilgan sistema matritsasining rangini

$$A = \begin{bmatrix} 1 & 5 & -1 \\ 2 & 4 & -3 \\ 3 & -1 & -3 \end{bmatrix} \text{va kengaytirilgan } B = \left[\begin{array}{ccc|c} 1 & 5 & -7 & 3 \\ 2 & 4 & -3 & 2 \\ 3 & -1 & -3 & -7 \end{array}\right]$$

matritsaning rangini topamiz. Buning uchun *B* matritsaning birinchi satrini -2 ga ko'paytirib ikkinchisiga qo'shamiz, so'ngra birinchi satrini -3 ga ko'paytirib uchinchisiga qo'shamiz, ikkinchi va uchinchi ustunlarning o'rinlarini almashtiramiz. Natijada quyidagiga ega bo'lamiz:

$$B = \left[\begin{array}{ccc|c} 1 & 5 & -1 & 3 \\ 2 & 4 & -3 & 2 \\ 3 & -1 & -3 & -7 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 5 & -1 & 3 \\ 0 & -6 & -1 & -4 \\ 0 & -16 & 0 & -16 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -1 & 5 & 3 \\ 0 & -1 & -6 & -4 \\ 0 & 0 & -16 & -16 \end{array}\right].$$

Bundan ko'rinib turibdiki, *rang A = rang B = 3* (ya'ni matritsaning rangi noma'lumlar soniga teng). Demak, berilgan sistema birgalikda va yagona yechimga ega.

a) Kramer formulalari bo'yicha (1.17) bo'yicha yechimlarni topamiz:

$$x_1 = \frac{\Delta_3^1}{\Delta_3}; \qquad x_2 = \frac{\Delta_3^2}{\Delta_3}; \qquad x_3 = \frac{\Delta_3^3}{\Delta_3}.$$

bu erda:

$$\Delta_3 = \begin{bmatrix} 1 & 5 & -1 \\ 2 & 4 & -3 \\ 3 & -1 & -3 \end{bmatrix} = \text{-}16\,; \quad \Delta_3^1 = \begin{bmatrix} 3 & 5 & -1 \\ 2 & 4 & -3 \\ -2 & -1 & -3 \end{bmatrix} = 64\,;$$

$$\Delta_3^2 = \begin{bmatrix} 1 & 3 & -1 \\ 2 & 2 & -3 \\ 3 & -7 & -3 \end{bmatrix} = \text{-}16; \quad \Delta_3^3 = \begin{bmatrix} 1 & 5 & 3 \\ 2 & 4 & 2 \\ 3 & -1 & -7 \end{bmatrix} = 32.$$

Bundan:

$x_1 = 64 / (-16) = -4$, $x_2 = -16 / (-16) = 1$, $x_3 = 32 / (-16) = -2$.◄

b) Sistemaning yechimini teskari matritsa yordamida topish uchun, tenglamalar sistemasini $AX = \bar{B}$ matritsa shaklida yozib olamiz. Sistemaning matritsa shaklidagi yechimi quyidagi ko'rinishda bo'ladi. $X = A^{-1}\bar{B}$. (1.11) formuladan foydalanib A^{-1} teskari matritsani topamiz. (Δ = *det A* = -16 $\neq$ 0 bo'lgani uchun teskari matritsa mavjud).

$$A_{11} = \begin{vmatrix} 4 & 3 \\ -1 & -3 \end{vmatrix} = -15, \quad A_{21} = -\begin{vmatrix} 5 & -1 \\ -1 & -3 \end{vmatrix} = 16, \quad A_{31} = \begin{vmatrix} 5 & -1 \\ 4 & -3 \end{vmatrix} = -11.$$

$$A_{12} = -\begin{vmatrix} 2 & -3 \\ 3 & -3 \end{vmatrix} = -3, \quad A_{22} = \begin{vmatrix} 1 & -1 \\ 3 & -3 \end{vmatrix} = 0, \quad A_{32} = -\begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 1.$$

$$A_{13} = \begin{vmatrix} 2 & 4 \\ 3 & -1 \end{vmatrix} = -14, \quad A_{23} = -\begin{vmatrix} 1 & 5 \\ 3 & -2 \end{vmatrix} = 16, \quad A_{33} = \begin{vmatrix} 1 & 5 \\ 2 & -3 \end{vmatrix} = -6.$$

$$A^{-1} = \frac{1}{-16}\begin{bmatrix} -15 & 16 & -11 \\ -3 & 0 & 1 \\ -14 & 16 & 6 \end{bmatrix}.$$

Sistemaning yechimi:

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \frac{1}{-16}\begin{bmatrix} -15 & 16 & -11 \\ -3 & 0 & 1 \\ -14 & 16 & -6 \end{bmatrix}\begin{bmatrix} 3 \\ 2 \\ 7 \end{bmatrix} = \begin{bmatrix} (-46+32+77)/(-16) \\ (-9-7)/(-16) \\ (-42+32+42)/(-16) \end{bmatrix} = \begin{bmatrix} -4 \\ 1 \\ -2 \end{bmatrix}.$$

Shunday qilib, *:* $x_1 = -4$, $x_2 = 1$, $x_3 = -2$;

c) Sistemani Gauss usuli bilan yechamiz. Ikkinchi va uchinchi tenglamalardan x_1 ni chiqarib tashlaymiz. Buning uchun birinchi tenglamani 2 ga ko'paytiramiz va ikkinchi tenglamadan ayiramiz, so'ngra birinchi tenglamani 3 ga ko'paytiramiz va uchinchi tenglamadan ayiramiz.Natijada quyidagiga ega bo'lamiz:

$$\left.\begin{aligned} x_1 + 5x_2 - x_3 &= 3, \\ -6x_2 - x_3 &= -4, \\ -16x_2 &= -16. \end{aligned}\right\}.$$

Hosil qilingan sistemadan yechimlarni topamiz: $x_1 = -4$, $x_2 = 1$, $x_3 = -2$;◄.

2. Bir jinsli bo'lmagan chiziqli algebraik tenglamalar sistemasi berilgan:

$$\left.\begin{aligned} 2x_1 - 3x_2 + x_3 &= 2, \\ 3x_1 + x_2 - 3x_3 &= 1, \\ 5x_1 - 2x_2 - 2x_3 &= 4. \end{aligned}\right\},$$

ushbu sistemaning birgalikda ekanligini tekshiring, agar birgalikda bo'lgan holda uni :

a) Kramer formulalari bo'yicha;

b*)* teskari matritsa yordamida (matritsalar usulida) ;

c) Gauss usulida yeching.

►Sistemaning birgalikda ekanligini Kroneker – Kappeli teoremasi yordamida tekshiramiz.

$$B = \left[\begin{array}{ccc|c} 2 & -3 & 1 & 2 \\ 3 & 1 & -3 & 1 \\ 5 & -2 & -2 & 4 \end{array}\right].$$

Kengaytirilgan matritsada uchinchi va birinchi ustunlarning o'rinlarini almashtiramiz, birinchi satrini 3 ga ko'paytiramiz va ikkinchi satrga qo'shamiz, ikkinchi satrdan uchinchi satrni ayiramiz, natijada quyidagiga ega bo'lamiz.

$$B=\left[\begin{array}{ccc|c} 2 & -3 & 1 & 2 \\ 3 & 1 & -3 & 1 \\ 5 & -2 & -2 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & 2 & 2 \\ -3 & 1 & 3 & 1 \\ -2 & -2 & 5 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & 2 & 2 \\ 0 & -8 & 9 & 7 \\ 0 & -8 & 9 & 8 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & -3 & 2 & 2 \\ 0 & -8 & 9 & 7 \\ 0 & 0 & 0 & 1 \end{array}\right].$$

Bundan ko'rinib turibdiki, *rangA=2*, *rangB=3* . Kroneker – Kappeli teoremasiga asosan *rang A* $\neq$ *rang B* ekanligidan berilgan sistemaning birgalikda emas ekanligini kelib chiqadi.◄

3. Bir jinsli chiziqli algebraik tenglamalar sistemasini yeching.

$$\left.\begin{array}{r} 2x_1 - 4x_2 + 5x_3 = 0, \\ x_1 + x_2 - 3x_3 = 0, \\ 3x_1 - x_2 + 2x_3 = 0. \end{array}\right\}.$$

Sistemaning determinanti

$$\Delta_3 = \begin{vmatrix} 2 & -4 & 5 \\ 1 & 2 & -3 \\ 3 & -1 & 2 \end{vmatrix} = 11 \neq 0,$$

shuning uchun sistema nollardan iborat yagona yechimga ega bo'ladi. $x_1 = x_2 = x_3 = 0.$

4. Bir jinsli chiziqli algebraik tenglamalar sistemasini yeching.

$$\left.\begin{array}{r} 3x_1 + 4x_2 - x_3 = 0, \\ x_1 - 3x_2 + 5x_3 = 0, \\ 4x_1 + x_2 + 4x_3 = 0. \end{array}\right\}.$$

Sistemaning determinanti

$$\Delta_3 = \begin{vmatrix} 3 & 4 & -1 \\ 1 & -3 & 5 \\ 4 & 1 & 4 \end{vmatrix} = 0.$$

Uning uchun berilgan tenglamalar sistemasi cheksiz ko'p yechimga ega. *n=3*,

rangA=2 bo'lgani uchun sistemaning ihtiyoriy ikkita tenglamasini olamiz (masalan, birinchi va ikkinchisini) va uning yechimini topamiz. Quyidagiga ega bo'lamiz:

$$\left.\begin{array}{r} 3x_1 + 4x_2 - x_3 = 0 \\ x_1 - 3x_2 + 5x_3 = 0 \end{array}\right\}$$

x_1 va x_2 noma'lumlar oldidagi koeffitsiyentlardan tuzilgan determinant noldan farqli bo'lgani uchun bazis noma'lumlar sifatida x_1 va x_2 ni olamiz (boshqa noma'lumlar juftini ham olishimiz mumkin) x_3 qatnashgan hadlarni tenglamaning o'ng tomoniga o'tkazamiz:

$$\left.\begin{array}{r} 3x_1 + 4x_2 = x_3 \\ x_1 - 3x_2 = -5x_3 \end{array}\right\}.$$

Bu sistemani Kramer formulalari (1.17) yordamida yechamiz.

$$x_1 = \frac{\Delta_2^{(1)}}{\Delta_2}, \qquad x_2 = \frac{\Delta_2^{(2)}}{\Delta_2}.$$

Bu erda:

$$\Delta_2=\begin{vmatrix}3 & 4\\ 1 & -3\end{vmatrix}=-9-4=-13\ ,\quad \Delta_2^{(1)}=\begin{vmatrix}x_3 & 4\\ -5x_3 & -3\end{vmatrix}=-3x_3+20x_3=17x_3,$$

$$\Delta_2^{(2)}=\begin{vmatrix}3 & x_3\\ 1 & -5x_3\end{vmatrix}=-16x_3,$$

Bundan $x_1=-\dfrac{17x_3}{13}, x_2=\dfrac{16x_3}{13}$. Agar $x_3=13k$ (k - ixtiyoriy proportsionallik koeffitsienti) bo'lsa, u holda algebraik tenglamalar sistemasining echimi $x_1=-17k$, $x_2=16k$, $x_3=13k$ ekanligi kelib chiqadi.

1 BOBga qo'shimcha masalalar

1. $\begin{vmatrix}1 & x_1 & x_1^2\\ 1 & x_2 & x_2^2\\ 1 & x_3 & x_3^2\end{vmatrix}=(x_2-x_1)(x_3-x_1)(x_3-x_2)$. ekanligini isbotlang.

2. n-tartibli determinantni hisoblang:

a) $\begin{vmatrix}1 & a & 0 & \dots\dots & 0 & 0\\ 1 & 1+a & a & \dots\dots & 0 & 0\\ 0 & 1 & 1+a & \dots\dots & 0 & 0\\ \dots & \dots & \dots & \dots & \dots & \dots\\ 0 & 0 & 0 & \dots\dots & 1+a & a\\ 0 & 0 & 0 & \dots\dots & 1 & 1+a\end{vmatrix}$; b) $\begin{vmatrix}x+1 & 1 & 1 & \dots & 1 & 1\\ -1 & x & 0 & \dots & 0 & 0\\ 0 & -1 & x & \dots & 0 & 0\\ \dots & \dots & \dots & \dots & \dots & \dots\\ 0 & 0 & 0 & \dots & x & 0\\ 0 & 0 & 0 & \dots & -1 & x\end{vmatrix}$;

c) $\begin{vmatrix}a+1 & x & x & \dots & x & x\\ 1 & a & x & \dots & x & x\\ 1 & 0 & a & \dots & x & x\\ \dots & \dots & \dots & \dots & \dots & \dots\\ 1 & 0 & 0 & \dots & a & x\end{vmatrix}$; d) $\begin{vmatrix}x & a & a & \dots & a\\ b & x & 0 & \dots & 0\\ b & 0 & x & \dots & 0\\ \dots & \dots & \dots & \dots & \dots\\ b & 0 & 0 & \dots & x\end{vmatrix}$;

e) $\begin{vmatrix}1 & 2 & 0 & 0 & 0 & 0\\ 3 & 8 & 0 & 0 & 0 & 0\\ -2 & 3 & 2 & -1 & 0 & 0\\ 7 & 2 & 3 & 2 & 0 & 0\\ 5 & -1 & 3 & 5 & 7 & -5\\ 2 & 3 & 7 & 2 & 2 & -1\end{vmatrix}$ f) $\begin{vmatrix}1 & 2 & 3 & 1 & 5\\ 0 & 1 & 0 & 5 & 1\\ 2 & 1 & 2 & 3 & 2\\ 0 & 3 & 0 & 1 & 3\\ 3 & 2 & 1 & 3 & 4\end{vmatrix}$;

(Javob: a) 1; b) $(x^{n+1}-1)(x-1)$; c) $a^n+(a-x)^{n-1}$); d) $x^n-(n-1)\,a\,\mathrm{b}\,x^{n-2}$;e) 42; f) 168;)

3. Quyidagi tenglamalar sistemasini t parametrning mumkin bo'lgan barcha qiymatlarida eching:

$$\left.\begin{aligned} 2x - y + 3z &= -7 \\ x + 2y - 6z &= t \\ tx + 5y - 15z &= 8 \end{aligned}\right\}$$

(Javob: $t \neq -1$ va $t \neq 5$ bo'lganda sistema birgalikda emas, $t = 5$, bo'lsa, u holda $x = -9/5$, $y = (15a + 17)/5$, $z = a$; agar $t = -1$, bo'lsa u holda $x = -3$, $y = 3a + 1$, $z = a$, bu erda a - ixtiyoriy son.)

4. λ ning qanday qiymatlarida bir jinsli tenglamalar sistemasi noldan farqli echimga ega bo'ladi?

$$\left.\begin{aligned} -\lambda x_1 + x_2 + \ldots + x_n &= 0, \\ x_1 - \lambda x_2 + \ldots + x_n &= 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ x_1 + x_2 + \ldots - \lambda x_n &= 0 \end{aligned}\right\}$$

(Javob: $\lambda = n - 1$, $\lambda = -1$.)

5. Agar n-tartibli A va B kvadrat matritsalarning birortasi maxsuslikka ega bo'lsa, ularning ko'paytmasi AB ham maxsus matritsa bo'lishini ko'rsating.

6. $\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}^2$ ni hisoblang $\left(\text{Javob}: \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} \right).$

7. Matritsaviy tenglamar sistemasini eching:

$$\left.\begin{aligned} X + Y &= \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \\ 2X + 3Y &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned}\right\} \quad \left(\text{Javob: } X = \begin{bmatrix} 2 & 3 \\ 0 & 2 \end{bmatrix}, \quad Y = \begin{bmatrix} -1 & -2 \\ 0 & -1 \end{bmatrix}. \right)$$

8. Berilgan sistemada,chiziqli bog'liq bo'lmagan tenglamalar sonini aniqlang va uning umumiy echimini toping.

$$\left.\begin{aligned} x_1 + x_2 - x_3 - 2x_4 - 2x_5 &= 0, \\ 2x_1 + 3x_2 - 2x_3 - 5x_4 - 4x_5 &= 0, \\ x_1 - x_2 - x_3 - 2x_5 &= 0, \\ x_1 - 2x_2 - x_3 + x_4 - 2x_5 &= 0. \end{aligned}\right\}$$

(Javob: $x_1 = x_3 + 2x_5$, $x_2 = x_4$.)

9. $x^2 + y^2 + 3xy + x + 4y = 0$ chiziq tenglamasini kanonik ko'rinishga keltiring va unga mos koordinatalar sistemasini almashtirishni ko'rsating.

$\left(\text{Жавоб } -\frac{5}{2}x'^2 + \frac{1}{2}y'^2 = 1, \ x = -2 + \frac{x' + y'}{\sqrt{2}}, \ y = 1 + \frac{x' - y'}{\sqrt{2}}. \right)$

10. $9x^2 + 6xy + y^2 - x - 2y - 14 = 0$ tenglama bilan aniqlanuvchi chiziqning parabola bo'lishini ko'rsating.

11. $$\begin{vmatrix} 1 & 1 & 1 \\ tg\alpha & tg\beta & tg\gamma \\ tg^2\alpha & tg^2\beta & tg^2\gamma \end{vmatrix} = \frac{\sin(\alpha-\beta)\sin(\beta-\gamma)\sin(\gamma-\alpha)}{\cos^2\alpha\ \cos^2\beta\ \cos^2\gamma}$$

tenglikning to'g'riligini isbotlang:

12. Tenglamani eching.

$$\text{a)} \begin{vmatrix} 1 & 3 & x \\ 4 & 5 & -1 \\ 2 & -1 & 5 \end{vmatrix} = 0 \ ; \qquad \text{b)} \begin{vmatrix} 3 & x & -4 \\ 2 & 1 & 3 \\ x+10 & 1 & 1 \end{vmatrix} = 0.$$

(Javob: a) x=-3;b) $x_1 = -10$, $x_2 = 2$.)

13. Tengsizlikni eching:

$$\text{a)} \begin{vmatrix} 3 & -2 & 1 \\ 1 & x & -2 \\ -1 & 2 & -1 \end{vmatrix} < 1; \quad \text{b)} \begin{vmatrix} 2 & x+2 & -1 \\ 1 & 1 & -2 \\ 5 & -3 & x \end{vmatrix} > 0$$

(Javob: a) x>3,5; b) -6< x <-4.)

14. Agar

$$\left.\begin{aligned} a_1x + b_1y + c_1z + d_1 &= 0, \\ a_2x + b_2y + c_2z + d_2 &= 0, \\ a_1x + b_3y + c_3z + d_3 &= 0, \\ a_4x + b_4y + c_4z + d_4 &= 0 \end{aligned}\right\}$$

tenglamalar sistemasi birgalikda bo'lsa, u holda

$$\cdot\begin{vmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{vmatrix} = 0$$

bo'lishini isbotlang.

15. Berilgan tenglamalar sistemasini tekshiring va uning λ parametrning qiymatlariga bog'liq bo'lgan umumiy echimini toping.

$$\left.\begin{aligned} 5x_1 - 3x_2 + 2x_3 + 4x_4 &= 3, \\ 4x_1 - 2x_2 + 3x_3 + 7x_4 &= 1, \\ 3x_1 - 6x_2 - x_3 - 5x_4 &= 9, \\ 7x_1 - 3x_2 + 7x_3 + 17x_4 &= \lambda. \end{aligned}\right\}$$

(Javob: $\lambda \neq 0$ da sistema birgalikda emas. $\lambda = 0$ bo'lganda sistema birgalikda va uning umumiy echimi $x_1 = (-5x_3 - 13x_{4-3})/2$, $x_2 = (-7x_3 - 19x_4 - 7)/2$.)

16. λ ning qanday qiymatdarida berilgan tenlamalar sistemasi echimga ega yoki birgalikda emas ekanini ko'rsating.

$$\left.\begin{aligned}\lambda x_1+x_2+x_3+x_4&=1,\\ x_1+\lambda x_2+x_3+x_4&=1,\\ x_1+x_2+\lambda x_3+x_4&=1,\\ x_1+x_2+x_3+\lambda x_4&=1.\end{aligned}\right\}$$

(Javob: agar $\lambda=-3,$ bo'lsa, u holda sistema birgalikda emas; agar $\lambda\neq 1\ \ \lambda\neq -3$ bo'lsa, u holda $x_1=x_2=x_3=x_4=1\ (\lambda+3)$; agar $\lambda=1$ bo'lsa, u holda echim $\sum_{i=1}^{4}x_i=1$ bitta tenglama bilan aniqlanadi.)

17. λ ning barcha qiymatlarida tenglamalar sistemasining echimlarini toping.

$$\left.\begin{aligned}\lambda x_1+x_2+x_3&=0,\\ x_1+\lambda x_2+x_3&=0,\\ x_1+x_2+\lambda x_3&=0.\end{aligned}\right\}$$

(Javob: agar $(\lambda+2)(\lambda-1)\neq 0$ bo'lsa, $x_1=x_2=x_3=0;$ agar $\lambda=-2$ bo'lsa,u holda echim $x_1=x_2=x_3$, agar $\lambda=1$ bo'lsa,u holda echim $x_1+x_2+x_3=0$ bitta tenglama bilan aniqlanadi)

18. Chiziqli tenglamalar sistemasining ikkita echimi yig'indisi ham uning echimi bo'lishining zaruriy va etarli shartini toping.

(Javob: Sistemaning bir jinsli bo'lishligi.)

19. Chiziqli tenglamalar sistemasi ikkita echimlarining ko'paytmasi va $\lambda\neq 1$ son ham uning echimi bulishining zaruriy va etarli shartini toping.

(Javob: Sistemaning bir jinsli bo'lishligi.)

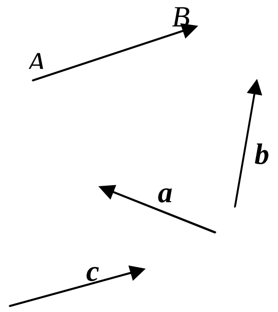
20. Qanday shart bajarilganda berilgan bir jinsli bo'lmagan chiziqli tenglamalar sistemasining ixtiyoriy echimlarining chiziqli kombinatsiyasi bu sistemaning echimi bo'ladi?

(Javob:Chiziqli kombinatsiya koeffitsentlarining yig'indisi birga teng bo'lganda.)

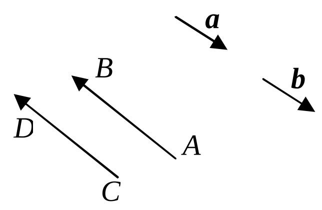
2. Vektor algebrasi.

2.1. Vektorlar. Vektorlar ustidagi chiziqli amallar. Vektorning o'qdagi proektsiyasi (izi). Vektorning koordinatalari.

Yo'naltirilgan kesma *vektor* deb ataladi. Agar vektorning boshi A nuqtada bo'lib, uchi (oxiri) B nuqtada bo'lsa, uni $\overrightarrow{AB}$ kabi belgilanadi. Agarda vektorning boshi va uchi (oxiri) ko'rsatilmagan bo'lsa, u holda uni lotin alfavitining kichik harflari bo'lgan $\vec{a}$, $\vec{b}$, $\vec{c}$ lar bilan belgilanadi. Chizmada vektorning yo'nalishi strelka orqali tasvirlanadi (2.1-rasm).



2.1-rasm



2.2-rasm

Biror $\overrightarrow{AB}$ vektorga qarama-qarshi yo'nalgan vektorni $\overrightarrow{BA}$ kabi belgilanadi. Agar vektorning boshi va uchi (oxiri) ustma-ust tushadigan bo'lsa, uni nol-vektor deb atalib, $\vec{0}$ kabi belgilanadi. Mazkur vektorning yo'nalishi noaniq bo'lib, unga ixtiyoriy yo'nalishni yozish mumkin. *Vektorning moduli yoki uzunligi* deb, uni tashkil qiluvchi kesmaning uzunligiga aytiladi. $\overrightarrow{AB}$ va $\vec{a}$ vektorlarning modullarini mos ravishda $|\overrightarrow{AB}|$ va $|\vec{a}|$ kabi belgilanadi.

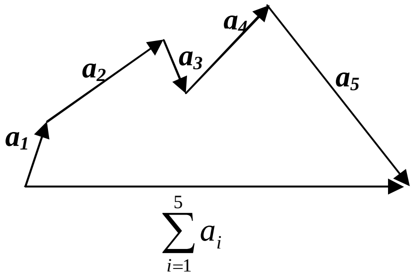
Bitta yoki parallel to'g'ri chiziqlarda yotuvchi vektorlarni *kollinear* vektorlar deb atalib, bitta tekislikda yotuvchi vektorlarni esa, komplanar vektorlar deb ataladi.

Agar ikkita vektor o'zaro kollinear bo'lib, bir xil yo'nalishda hamda bir xil uzunlikda bo'lsalar, u holda ularni teng vektorlar deb yuritiladi. 2.2-rasmda o'zaro teng vektorlar juftligi $\overrightarrow{AB}\ va\ \overrightarrow{CD}$, $\vec{a}\,va\,\vec{b}$ tasvirlangan. $\overrightarrow{AB} = \overrightarrow{CD}\ \ va\ \ \vec{a} = \vec{b}$. Vektorlarning tengligi ta'rifidan, ularni o'zlarini o'zlariga (tengliklarini buzmagan holda) parallel qilib ko'chirish mumkinligi kelib chiqadi. Bu xildagi vektorlarni erkin (ozod) vektorlar deb yuritiladi.

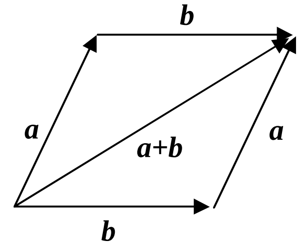
Vektorlarning ustida ularni songa *ko'paytirish* hamda ularni *qo'shish* kabi chiziqli amallari aniqlangan.

Biror $\vec{a}$ vektorning $\alpha \neq 0$ songa ko'paytmasi deb, $\alpha\vec{a}$ kabi vektorga aytiladi va uning moduli $|\alpha| \cdot |\vec{a}|$ bo'ladi, agar $\alpha > 0$ bo'lsa, uning yo'nalishi bilan bir xilda bo'ladi, agar $\alpha < 0$ bo'ladigan bo'lsa, $\vec{a}$ vektorga qarama-qarshi yo'nalishda bo'ladi.

Chekli sondagi $\vec{a}_i$ (i=$\overline{1,n}$) vektorlarning *yig'indisi* deb, $\vec{a}_1+\vec{a}_2+...+\vec{a}_n=\sum_{i=1}^{n} a_i$ vektorga aytiladi va uning boshi $\vec{a}_1$ vektorning boshida bo'lib, uchi esa ketma-ket qo'shiluvchi vektorlardan tuzilgan siniq chiziqning oxirgi $\vec{a}_n$ vektorining uchida joylashgan bo'ladi. (2.3-rasm). Ushbu qoidani siniq chiziqlarning tutashuvi qoidasi deb yuritiladi. Agar vektorlarning soni ikkita bo'lsa, ularning yig'indisi parallelogramm qoidasiga teng kuchli bo'ladi (2.4-rasm).



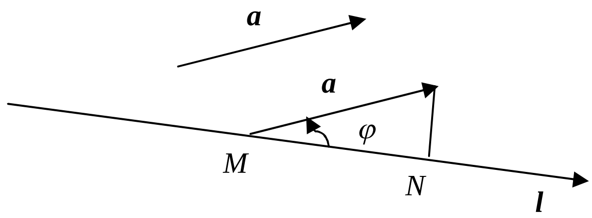
2.3-rasm



2.4-rasm

Musbat yo'nalishi berilgan biror l to'g'ri chiziqni *l o'qi* deb yuritiladi.

Biror $\vec{a}$ vektorning l o'qdagi *proeksiyasi (izi)* deb, $|\vec{a}|$ bilan $\vec{a}$ vektorning l o'qning musbat yo'nalishi bilan tashkil etgan burchagi $\varphi\,(0\leq\varphi\,(\leq \pi)$ ning kosinusiga bo'lgan ko'paytmasiga aytiladi, hamda $np_l\vec{a}$ kabi belgilanadi. Demak, ta'rifga binoan, $np_l\vec{a}=|\vec{a}|\cdot\cos\varphi$ (2.5-rasm).



2.5-rasm

Chizmadan ko'rinib turibdiki, $\vec{a}$ vektorning l o'qdagi proeksiyasi geometrik jihatdan MN kesmaning uzunligi bilan xarakterlanar ekan. Bu erda, agar $0\leq\ \varphi\leq\frac{\pi}{2}$ bo'lsa, kesmaning uzunligi "+" ishora bilan olinadi, agar $\frac{\pi}{2}<\varphi\leq\pi$ bo'lsa, "-" ishora bilan olinadi. Xususan, agar $\varphi=\frac{\pi}{2}$ bo'lsa, MN kesma nuqtaga aylanadi va $np_l\vec{a}=0$ bo'ladi.

Biror $\vec{a}$ vektorning Ox, Oy va Oz koordinata o'qlardagi proeksiyalari *uning koordinatalari* deb atalib, ular mos ravishda x, y, z lar bilan belgilanadi. $\vec{a}=(x;\ y;\ z)$ kabi yozuv, $\vec{a}$ vektorning koordinatalari $x;\ y;\ z$ lardan iborat ekanligini anglatadi.

Ikkita vektor teng bo'lishi uchun ularning mos koordinatalarining tengligi zarur va yetarlidir.

Agar $M_1(x_1;\ y_1;\ z_1)$ va $M_2(x_2;\ y_2;\ z_2)$ bo'lsa, u holda $\overrightarrow{M_1M_2}=(x_2-x_1;\ y_2-y_1;\ z_2-z_1)$.

Agar λ_i ixtiyoriy sonlar bo’lganda, $\vec{a}=\sum_{i=1}^{n} a_i$ kabi ifodani $\vec{a}_i$ *vektorlarning chiziqli kombinatsiyasi* deb yuritiladi. Agar $\vec{a}=(x_i;\ y_i;\ z_i)$ $(i=\overline{1,n})$ kabi bo’lsa, u holda: $\vec{a}=(\sum_{i=1}^{n}\lambda_i x_i;\ \sum_{i=1}^{n}\lambda_i y_i;\ \sum_{i=1}^{n}\lambda_i z_i)$.

Vektorlar ustidagi chiziqli amallar ham aynan sonlar ustidagi qo’shish va ko’paytirish amallariga o’xshash xossalarga egadir. Masalan,

$\vec{a}+\vec{b}=\vec{b}+\vec{a}$, $(\alpha+\beta)\vec{a}=\alpha\vec{a}+\beta\vec{a}$, $\alpha(\vec{a}+\vec{b})=\alpha\vec{a}+\alpha\vec{b}$,

$\vec{a}+(-1)\vec{a}=\vec{a}-\vec{a}=0$, $1\vec{a}=\vec{a}$, $0\cdot\vec{a}=0$ va hokazo.

Agar n dona $\vec{a}_1$ vektorlar uchun $\sum_{i=1}^{n}\lambda_i\vec{a}_I=0$ kabi tenglik faqat $\lambda_i=0$ bo’lganda o’rinli bo’ladigan bo’lsa, u holda $\vec{a}_i$ vektorlar sistemasini *chiziqli bog’lanmagan* sistema deb ataladi. Agar, λ_i sonlardan hech bo’lmaganda biri noldan farqli bo’lgan holda $\sum_{i=1}^{n}\lambda_i\vec{a}_i=0$ kabi tenglik o’rinli bo’lsa, u holda $\vec{a}_i$ vektorlar sistemasini *chiziqli bog’lanishdagi* sistema deyiladi. Masalan, ixtiyoriy kollinear vektorlar, uchta komplanar vektorlar hamda uch o’lchovli fazodagi to’rtta va undan ko’proq vektorlar chiziqli bog’langan vektorlardir.

Fazodagi uchta tartiblangan va o’zaro chiziqli bog’lanmagan $\vec{e}_1$, $\vec{e}_2$ va $\vec{e}_3$ kabi vektorlarni *bazis* vektorlar deyiladi. Tartiblangan har qanday uchta nokomplanar vektorlar har doim bazis vektorlardir. Fazodagi har qanday $\vec{a}$ vektorni $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$ bazis vektorlar bo’yicha yoyib yozish mumkin, ya’ni, bazis vektorlarning chiziqli kombinatsiyasi shaklida ifodalash mumkin: $\vec{a}=x\vec{e}_1+y\vec{e}_2+z\vec{e}_3$. Bu erdagi x, y va z lar $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$, bazislardagi $\vec{a}$ vektorning koordinatalaridir. Agar bazis vektorlar o’zaro perpendikulyar hamda birlik vektorlar bo’lsa, ularni *ortonormallashgan* bazis vektorlar deb ataladi. U xildagi bazis vektorlarni $\vec{i}$, $\vec{j}$ va $\vec{k}$ deb belgilanadi.

1-misol. $\vec{a}$, $\vec{b}$, va $\vec{c}$ vektorlar berilgan (2.6 a-rasm). Ularning chiziqli kombinatsiyalari $-2\vec{a}+\frac{1}{3}\vec{b}+4\vec{c}$ ni chizmada tasvirlansin.

► Tekislikda *0* nuqtani ixtiyoriy tanlab, undan $-2\vec{a}$ ni olamiz (2.6 b-rasm). Uning uchiga $\frac{1}{3}\vec{b}$ vektorni qo’yamiz va $\frac{1}{3}\vec{b}$ vektorning uchiga $4\vec{c}$ vektorni qo’yamiz. Agar *0* nuqta bilan $4\vec{c}$ vektorni birlashtirsak, natijada qaralayotgan vektorlarning chiziqli kombinatsiyalari hosil bo’ladi.◄

2.6- rasm

2-misol. $\vec{a}$=(2;-1; 8), $\vec{e}_1$=(1; 2; 3), $\vec{e}_2$=(1; -1; -2) va $\vec{e}_3$=(1; -6; 0) kabi vektorlar $\vec{i}$, $\vec{j}$, $\vec{k}$ ortonormallashgan bazis vektorlarda berilgan. Ulardan keyingi uchtasi bazis vektorlar ekanligi isbotlanib, $\vec{a}$ vektorning shu bazisdagi koordinatalari aniqlansin.

► Agar $\vec{e}_1$, $\vec{e}_2$ va $\vec{e}_3$ vektorlarning koordinatalaridan tuzilgan determinant nolga teng bo'lmasa, $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$ vektorlar chiziqli bog'lanmagan bo'lib, ular bazis vektorlardir, ya'ni:

$$\Delta = \begin{vmatrix} 1 & 2 & 3 \\ 1 & -1 & -2 \\ 1 & -6 & 0 \end{vmatrix} = -19 \neq 0.$$

Demak, qaralayotgan $\vec{e}_1$, $\vec{e}_2$ va $\vec{e}_3$ vektorlar bazis vektorlardir.

$\vec{a}$ vektorning $\vec{e}_1$, $\vec{e}_2$, $\vec{e}_3$ bazislardagi koordinatalarini x, y, z deb belgilaymiz. U holda: $\vec{a}$ = (x; y; z) = x$\vec{e}_1$ + y$\vec{e}_2$ + z$\vec{e}_3$. Masalaning shartiga binoan, $\vec{a} = 2\vec{i} - \vec{j} + 8\vec{k}$, $\vec{e}_1 = \vec{i} + 2\vec{j} + 3\vec{k}$, $\vec{e}_2 = \vec{i} - \vec{j} - 2\vec{k}$, $\vec{e}_3 = \vec{i} - 6\vec{j}$ bo'lgangi uchun, vektorlarning tenglik shartidan foydalanib, $2\vec{i} - \vec{j} + 8\vec{k} = (x+y+z)\vec{i} + (2x-y-6z)\vec{j} + (3x-2y)\vec{k}$ ni yozamiz. Bu tenglikdan esa

$$\begin{cases} x + y + z = 2 \\ 2x - y - 6z = -1 \\ 3x - 2y = 8 \end{cases}$$

kabi sistema hosil bo'ladi. Uning yechimi: *x = 2; y = -1; z = 1.*

Demak: $\vec{a} = 2\vec{e}_1 - \vec{e}_2 + \vec{e}_3$ = (2; -1; 1).◄

2.1- AT

1. Berilgan $\vec{a}$ va $\vec{b}$ vektorlarga nisbatan, quyidagi vektorlarning chiziqli kombinatsiyalari tuzilsin:

a) $2\vec{a} + \vec{b}$ b) $\vec{a} - 3\vec{b}$ c) $\frac{1}{3}\vec{a} + \frac{1}{2}\vec{b}$ d) $-3\vec{a} - \frac{1}{2}\vec{b}$.

2. Agar $\overrightarrow{AB} = \vec{c}$, $\overrightarrow{BC} = \vec{a}$ va $\overrightarrow{CA} = \vec{b}$ vektorlar ABC uchburchakning tomonlarini tashkil etsalar, uchburchakning medianalari bilan ustma-ust tushadigan $\overrightarrow{AM}$, $\overrightarrow{BN}$ va $\overrightarrow{CR}$ vektorlarni $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar orqali ifodalansin.

(Javob: $\overrightarrow{AM} = -\frac{1}{2}a + c$ yoki $\overrightarrow{AM} = \frac{1}{2}(c-b)$, $\overrightarrow{BN} = a + \frac{1}{2}b$ yoki $\overrightarrow{BN} = \frac{1}{2}(a-c)$, $\overrightarrow{SR} = b + \frac{1}{2}c$ yoki $\overrightarrow{CR} = \frac{1}{2}(b-a)$.)

3. Uchburchakli *SABC* piramidada $\overrightarrow{SA} = \vec{a}$, $\overrightarrow{SB} = \vec{b}$ va $\overrightarrow{SC} = \vec{c}$ bo'lib, agar *ABC* uchburchak massasining markazi *0* nuqtada bo'lsa, u holda $\overrightarrow{SO}$ vektor aniqlansin.

(Javob: $\overrightarrow{SO} = \frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$.)

4. 4. Asoslarining uzunliklari *AD* = 4 va *BC* = 2 bo'lgan *ABCD* to'g'ri burchakli trapetsiyada ∠*D* = 45° bo'lsa, $\overrightarrow{AD}$, $\overrightarrow{AB}$, $\overrightarrow{BC}$ va $\overrightarrow{AC}$ vektorlarning $\overrightarrow{CD}$ vektor bilan berilgan *l* o'qdagi proektsiyalari topilsin. (Javob: $pr_l\overrightarrow{AD} = 2\sqrt{2}$, $pr_l\overrightarrow{AB} = -\sqrt{2}$, $pr_l\overrightarrow{BC} = \sqrt{2}$, $pr_l\overrightarrow{AC} = 0$.)

5. Agar $\vec{a}$ vektor *Ox* va *Oy* koordinata o'qlari bilan mos ravishda α=60° va β=120° burchaklar tashkil etadigan bo'lib, $|\vec{a}|$=2 bo'lsa, $\vec{a}$ vektorning koordinatalari hisoblansin. (Javob: $\vec{a} = (1, -1, \sqrt{2})$ yoki $\vec{a} = (1, -1, -\sqrt{2})$.)

6. Agar $\vec{a}$=(3, -2, 6) va $\vec{b}$=(-2, 1, 0) vektorlar berilgan bo'lsalar, $2\vec{a}$ - $\frac{1}{3}\vec{b}$, $\frac{1}{3}\vec{a}$ - $\vec{b}$ va $2\vec{a}$ + $3\vec{b}$ vektorlarning koordinatalari topilsin.

7. (Javob: (20/3, -13/3, 12); (3,-5/3,2); (0,-1,12).)

8. $\vec{a}$=(2; -3; 6) va $\vec{b}$=(-1; 2; -2) vektorlar hosil qilgan bissektrisa bo'ylab yo'nalgan $\vec{e}$ birlik vektorning koordinatalari aniqlansin.

(Javob: e = $\left(-\frac{1}{\sqrt{42}}, \frac{5}{\sqrt{42}}, \frac{4}{\sqrt{42}}\right)$.)

9. Biror vektorlar bazisida $\vec{a}$=(1,1,2), $\vec{e}_1$=(2, 2, -1), $\vec{e}_2$ = (0, 4, 8), $\vec{e}_3$ = (-1, -1, 3) kabi vektorlar berilgan bo'lsa, $\vec{e}_1$, $\vec{e}_2$ va $\vec{e}_3$ vektorlar, vektorlar bazisini tashkil etishliklarini tekshirib, bu bazisdagi $\vec{a}$ vektorning koordinatalari aniqlansin. (Javob: $\vec{a}$=(1,0, 1))

Mustaqil ish.

1. $\vec{a}$=(3, -5, 8) va $\vec{b}$=(-1, 1, -4) vektorlarga qurilgan parallelogramning diagonallarining uzunliklari topilsin. (Javob: $|\vec{a}+\vec{b}|$=6, $|\vec{a}-\vec{b}|$=14.)

2. $\overrightarrow{AB}$ = (2, 6, -4) va $\overrightarrow{AC}$ = (4, 2, -2) vektorlar, ABC uchburchakning tomonlarini aniqlaydigan bo'lsa, C uchidan tushirilgan mediana bilan ustma-ust tushadigan $\overrightarrow{CD}$ vektorning uzunligi hisoblansin. (Javob: $|\overrightarrow{CD}|=\sqrt{10}$.)

3. $\vec{a}$=(-3, 0, 4) va $\vec{b}$=(5, 2, 14) vektorlar tashkil etgan burchak bissektrisasi bo'ylab yo'nalgan $\vec{s}$ vektorning koordinatalari topilsin. (Javob: $\vec{s}$= λ(-2, 1, 13), $\lambda > 0$.)

2.2. Kesmani berilgan nisbatda bo'lish.
Vektorlarning skalyar ko'paytmasi va uning tadbiqlari.

Berilgan M_1M_2 kesmani biror M nuqta orqali bo'lingandagi nisbat deb, shunday bir λ songa aytiladiki, uning uchun har doim $\overrightarrow{M_1M} = \lambda\overrightarrow{MM_2}$ kabi tenglik o'rinli bo'ladi. Agar *M(x; y; z)*, $M_1(x_1; y_1; z_1)$ va $M_2(x_2; y_2; z_2)$ kabi bo'lsa, ularning koordinatalari bilan λ son orasidagi bog'lanish quyidagicha beriladi:

$$x=\frac{x_1+\lambda x_2}{1+\lambda},\ y=\frac{y_1+\lambda y_2}{1+\lambda},\ z=\frac{z_1+\lambda z_2}{1+\lambda}.$$

Agar λ > 0 bo'lsa, M_1M_2 kesmani *ichki* bo'lish deb, aksincha λ <0 bo'lsa, *tashqi* bo'linish deb ataladi. Agar λ = 1 bo'lsa, *M* nuqta M_1M_2 kesmaning o'rtasi bo'ladi. ($\lambda \neq -1$)

1-misol. Bir jinsli sterjenning uchlari $M_1(3;-5;8)$ va $M_2(7;13;-6)$ nuqtalarda bo'lsa, sterjen massasi markazining koordinatalari topilsin.

► Bir jinsli sterjen massasining markazi *S(x; y; z)* uning o'rtasida joylashganligi uchun λ = 1 bo'ladi, shuning uchun:

$$x=\frac{x_1+x_2}{2}=\frac{3+7}{2}=5,\ y=\frac{y_1+y_2}{2}=\frac{-5+13}{2}=4,\ z=\frac{z_1+z_2}{2}=\frac{8-6}{2}=1.\ ◄$$

Berilgan *ikkita* $\vec{a}$ va $\vec{b}$ *vektorlarning skalyar ko'paytmasi* deb, $\vec{a}\,\vec{b}$ = c kabi belgilanadigan shunday bir songa aytiladiki, u son berilgan vektorlar modullarining ular orasidagi burchak kosinusiga bo'lgan ko'paytmasidan iboratdir:

$$\vec{a}\,\vec{b} = |\vec{a}|\,|\vec{b}|\cdot\cos(\vec{a}\,\hat{}\,\vec{b}),$$

bu erda $(\vec{a}\,\hat{}\,\vec{b})$ burchak, $\vec{a}$ va $\vec{b}$ vektorlar yo'nalishidagi kichik burchak bo'lib, u har doim $0 \le (\vec{a}\,\hat{}\,\vec{b}) \le \pi$ kabi bo'ladi.

Skalyar ko'paytmaning asosiy xossalarini sanab o'tamiz:

1) $\vec{a}\,\vec{b} = \vec{b}\cdot\vec{a}$;
2) $(\lambda\vec{a})\,\vec{b} = \lambda\,(\vec{a}\cdot\vec{b}) = \vec{a}\,(\lambda\vec{b})$;
3) $\vec{a}\,(\vec{b}+\vec{s}) = \vec{a}\,\vec{b} + \vec{a}\cdot\vec{s}$;
4) $\vec{a}\,\vec{b} = |\vec{a}|\cdot pr_{\vec{a}}\,\vec{b} = |\vec{b}|\,pr_{\vec{b}}\,\vec{a}$;
5) $\vec{a}\,\vec{a} = |\vec{a}|^2$;
6) $\vec{a}\,\vec{b} = 0 \Leftrightarrow \vec{a} \perp \vec{b}$.

Agar $\vec{a}=x_1\vec{i} + y_1\vec{j} + z_1\vec{k} = (x_1;\, y_1;\, z_1)$ va $\vec{b}=x_2\vec{i} + y_2\vec{j} + z_2\vec{k} = (x_2;\, y_2;\, z_2)$ bo'lsa, $\vec{a}\cdot\vec{b} = x_1x_2 + y_1y_2 + z_1z_2$ dir, hamda $|\vec{a}|=\sqrt{x_1^2 + y_1^2 + z_1^2}$, $|\vec{b}|=\sqrt{x_2^2 + y_2^2 + z_2^2}$ kabi bo'ladi.

Agar $\alpha =(\vec{a}\,\hat{}\,Ox)$, $\beta = (\vec{a}\,\hat{}\,Oy)$, $\gamma =(\vec{a}\,\hat{}\,Oz)$ bo'lib, $\vec{a} = (x_1;\, y_1;\, z_1)= x_1\vec{i} + y_1\vec{j} + z_1\vec{k}$ kabi bo'lsa, quyidagi formulalar o'rinli bo'ladi:

$$\cos\alpha = \frac{\vec{a}\cdot\vec{i}}{|\vec{a}|} = \frac{x_1}{\sqrt{x_1^2+y_1^2+z_1^2}}, \quad \cos\beta = \frac{\vec{a}\cdot\vec{j}}{|\vec{a}|} = \frac{y_1}{\sqrt{x_1^2+y_1^2+z_1^2}},$$

$$\cos\gamma = \frac{\vec{a}\cdot\vec{k}}{|\vec{a}|} = \frac{z_1}{\sqrt{x_1^2+y_1^2+z_1^2}}, \quad \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1.$$

Bu erdagi $\cos\alpha$, $\cos\beta$ va $\cos\gamma$ lar $\vec{a}$ *vektorning yo'naltiruvchi kosinuslari deb yuritiladi.*

Agarda $\vec{F}$ kuch ta'sirida biror moddiy jism, $\vec{S}$ vektor bilan ifodalanuvchi yo'l bo'ylab harakatlanadigan bo'lsa, u holda uning bajargan ishi A ni quyidagicha hisoblanadi:

$$A = \vec{F}\,\vec{S} = |\vec{F}|\cdot|\vec{S}|\cos(\vec{F}\,\hat{}\,\vec{S}).$$

2-misol. Agar moddiy jism $\vec{F}_1=(3;\,-4;\,5)$, $\vec{F}_2=(2;\,1;\,-4)$ va $\vec{F}_3=(-1;\,6;\,2)$ kuchlar ta'siri ostida $M_1(4;2;-3)$ nuqtadan $M_2(7;\,4;\,1)$ nuqtaga tomon to'g'ri chiziqli harakat qiladigan bo'lsa, teng ta'sir qiluvchi $\vec{F}$ kuchning bajargan ishi hisoblansin.

► $\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = (4;3;3)$ hamda $\overrightarrow{M_1M_2} = \vec{S} = (3;2;4)$ bo'lganligi uchun $A = \vec{F}\cdot\vec{S} = = 4\cdot3 + 3\cdot2 + 3\cdot4 = 30$. ◄

2.2.–AT

1. *ABCD* parallelogramning ikkita uchi *A(2,-3,-5)* va *B(-1,3,2)* nuqtalarda bo'lib, uning diagonallarining kesishish nuqtasi *E(4, -1, 7)* nuqtada bo'lsa, uning qolgan ikki uchining koordinitalari topilsin. (*Javob: C(6,1,19), D (9,-5,12)).*

2. Uchlari *A(-1,8,-3)* va *B(9,-7,-2)* nuqtalarda bo'lgan *AB* kesma, M_1, M_2, va M_4 nuqtalar yordamida 5 ta bir xil kesmalarga ajratilgan bo'lsa, M_1 va M_3 nuqtalarning koordinatalari topilsin. (*Javob:* $M_1(1,5,-2)$, $M_3(5,-1,0)$).

3. Berilgan *B(2,0,2)* va *D(5,-2,0)* nuqtalar orqali uchta bir xil bo'laklarga bo'lingan *AB* kesmaning uchlari *A* va *B* nuqtalarning koordinatalari aniqlansin. (Javob: *A(-1, 2, 4), B(8, -4, -2).)*

4. Agar $\varphi = (\vec{a}\wedge\vec{b}) = \frac{2\pi}{3}$ bo'lib, $|\vec{a}|$=3, $|\vec{b}|$=4 bo'lsa, quyidagilar hisoblansin: $\vec{a}^2$, $\vec{b}^2$, $(\vec{a}+\vec{b})^2$, $(\vec{a}-\vec{b})^2$, $(3\vec{a}-2\vec{b})\ (\vec{a}+2\vec{b})$. (Javob: 9; 16; 13; 37; -61).

5. Agar $\overrightarrow{OA} = \vec{a}$ va $\overrightarrow{OB} = \vec{b}$ vektorlar uchun $|\vec{a}|$=2, $|\vec{b}|$=4 bo'lib, $(\vec{a}\wedge\vec{b})$ =60° bo'lsa, AOB uchburchakning $\overrightarrow{OM}$ medianasi bilan $\overrightarrow{OA}$ tomoni orasidagi φ burchak aniqlansin. (Javob*:* cos $\varphi = \frac{2}{\sqrt{7}}$, $\varphi \approx 41°$).

6. Jismga ta'sir etayotgan kuch F, $|F| = 15$, uni o'z yo'nalishiga $\varphi = 60^0$ burchak ostida *4 m* masofaga siljitsa, natijada bajarilgan ish miqdori aniqlansin (Javob:30dj).

7. $\vec{a}$ = (4, -2, -4) va $\vec{b}$= (6,-3,2) vektorlar berilgan. Quyidagilar hisoblansin: $\vec{a}\ \vec{b}$; $\vec{a}^2$; $\vec{b}^2$; $(\vec{a}+\vec{b})^2$; $(\vec{a}-\vec{b})^2$; $(2\vec{a}-3\vec{b})\ (\vec{a}+2\vec{b})$. (Javob:22; 36; 49; 129; 41; –200).

8. Agar ABC uchburchakning uchlari A(-1,-2,4), B(-4,-2,0) va C(3,-2,1) nuqtalarda bo'lsa, uning B uchidagi tashqi burchagi hisoblansin. (*Javob:* $\frac{3\pi}{4}$).

9. Moddiy jism, $\vec{F}$= (5,4,3) kuch ta'sirida $\vec{C} = (2,1,-2)$ vektorning boshidan uchiga tomon ko'chgan bo'lsa, u kuchning bajargan ishi A ni hamda $\vec{F}$ kuchning yo'nalishi bilan siljish yo'nalishlari orasidagi φ burchak aniqlansin.

(*Javob:* A=8, $\cos\varphi \approx 0.38$, $\varphi \approx 1.18$ radian yoki $\varphi \approx$67° 40.)

Mustaqil ish.

1. To'rtburchakning uchlari A(1;-2;2), B(1;4;0), C(-4;1;1) va D(-5;-5;3) kabi nuqtalarda bo'lsa, uning diagonallari orasidagi φ burchak topilsin. (Javob: φ= 90°)

2. $\vec{a} = \alpha\vec{i} - 3\vec{j} + 2\vec{k}$ bilan $\vec{b} = \vec{i} + 2\vec{j} - \alpha\vec{k}$ vektorlar α ning qanday qiymatida o'zaro perpendikulyar bo'ladi? (Javob*:* α = -6.)

3. 3. Agar $\vec{a}\ \vec{b} = 3$ va $\vec{a} = (2; 1; -1)$ *kabi bo'lsa*, $\vec{a}$ vektorga kollinear bo'lgan $\vec{b}$ vektorning koordinatalari topilsin. (Javob*:* $\vec{b}$ = (1; $\frac{1}{2}$; - $\frac{1}{2}$).)

2.3. Vektorlarning vektor va aralash ko'paytmalari hamda ularning tatbiqlari.

Agar uchta $\vec{a}$, $\vec{b}$, $\vec{c}$ kabi tartibdagi nokomplanar vektorlar bitta O nuqtaga qo'yilib, $\vec{c}$ vektorning uchidan qaralganda $\vec{a}$ vektordan $\vec{b}$ vektorgacha bo'lgan eng qisqa burilish soat strelkasiga teskari yo'nalishda bo'ladigan bo'lsa, $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlar *o'ng* sistema tashkil etadigan vektorlar, aks holda *chap* sistema tashkil etadigan vektorlar deb ataladi. (2.7 a va b rasmlar).

Berilgan $\vec{a}$ va $\vec{b}$ *vektorning vektor ko'paytmasi* deb, $\vec{c} = \vec{a} \times \vec{b}$ kabi belgi bilan yoziladigan va quyidagi uchta shartni qanoatlantiruvchi $\vec{c}$ vektorga aytiladi. 1) $|\vec{c}| = |\vec{a}|\cdot|\vec{b}|\sin(\vec{a}\wedge\vec{b})$;

2) $\vec{c} \perp \vec{a}$, $\vec{c} \perp \vec{b}$;

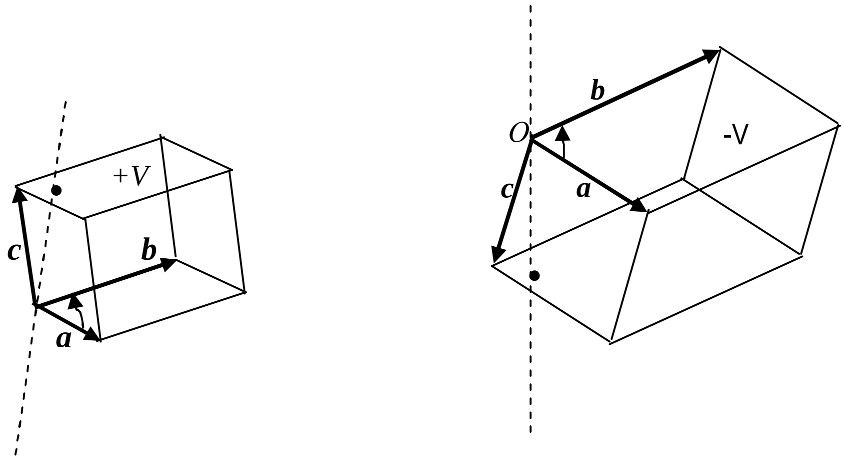
3) $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlar o'ng sistema tashkil etadi (2.8 – rasm)

*Vektor ko'paytmaning asosiy xossalari*ni sanab o'tamiz:

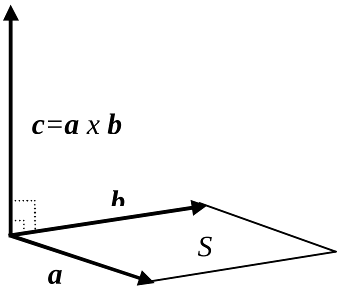
1) $\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$; 2) $(\lambda\vec{a}) \times \vec{b} = \lambda(\vec{a} \times \vec{b}) = \vec{a} \times (\lambda\vec{b})$;

3) $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$; 4) $\vec{a} \times \vec{b} = 0 \Leftrightarrow \vec{a} \parallel \vec{b}$;

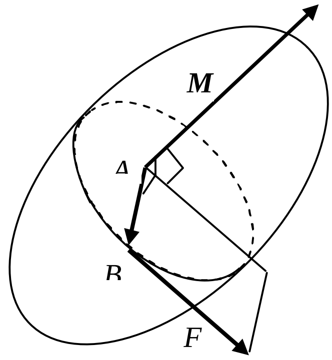
a *b*



2.7-rasm



2.8 – rasm



2.9 – rasm

5) Vektor ko'paytmaning moduli $\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallelogrammning yuziga teng, ya'ni: $|\vec{a} \times \vec{b}| = S$ (2.8 – rasmga qaralsin.)

Agar $\vec{a} = (x_1, y_1, z_1)$ va $\vec{b} = (x_2, y_2, z_2)$ kabi bo'lsalar, u holda ularning vektor ko'paytmasini ifodalaydigan vektorning koordinatalari quyidagicha aniqlanadi:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} = \left(\begin{vmatrix} y_1 & z_1 \\ y_2 & z_2 \end{vmatrix}, \begin{vmatrix} x_1 & z_1 \\ x_2 & z_2 \end{vmatrix}, \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} \right).$$

Vektor ko'paytma yordamida, biror A *nuqtada biriktirilgan jismning ixtiyoriy* B *nuqtasiga qo'yilgan* $\vec{F}$ *kuchning aylanma momenti* $\vec{M}$ ni hisoblash mumkin, ya'ni: $\vec{M} = \overrightarrow{AB} \times \vec{F}$ (2.9 – rasm).

1-misol. Berilgan *A(-1, 2, 4)* nuqtaga qo'yilgan $\vec{F} = (3, 2, 1)$ kuchning koordinata boshi 0 ga nisbatan aylanma momenti $\vec{M}$ ning koordinatalari topilsin:

► $\vec{M} = \overrightarrow{OA} \times \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 2 & 4 \\ 3 & 2 & 1 \end{vmatrix}$ = (-6, 13, -8). ◄

Uchta $\vec{a}$, $\vec{b}$ *va* $\vec{c}$ *vektorlarning aralash ko'paytmasi* deb, $(\vec{a} \cdot \vec{b}) \cdot \vec{c}$ ($|a \times b| \cdot c$) kabi songa aytiladi.

Aralash ko'paytma quyidagi xossalarga ega:

1) $(\vec{a} \cdot \vec{b}) \cdot \vec{c} = \vec{a} \cdot (\vec{b} \cdot \vec{c})$. Shuning uchun aralash ko'paytmani $\vec{a}\,\vec{b}\,\vec{c}$ kabi belgilash mumkin.

2) $\vec{a}\,\vec{b}\,\vec{c} = \vec{b}\,\vec{c}\,\vec{a} = \vec{c}\,\vec{a}\,\vec{b} = -\vec{b}\,\vec{a}\,\vec{c} = -\vec{c}\,\vec{b}\,\vec{a} = -\vec{a}\,\vec{c}\,\vec{b}$;

3) Aralash ko'paytma, geometrik jihatdan shu vektorlarga qurilgan parallelopipedning hajmiga teng, ya'ni: $\vec{a}\,\vec{b}\,\vec{c} = \pm V$. Bu erda, agar $\vec{a}, \vec{b}, \vec{c}$ vektorlar o'ng sistema tashkil etsalar "+" ishora olinib, agar chap sistemaga bo'ysunsalar "-" ishora olinadi (2.7 – rasmga qaralsin).

4) $\vec{a} \cdot \vec{b}\,\vec{c} = 0 \Leftrightarrow \vec{a}, \vec{b}, \vec{c}$ lar komplanar vektorlar;

Vektorlar $\vec{a} = (x_1, y_1, z_1)$, $\vec{b} = (x_2, y_2, z_2)$, $\vec{c} = (x_3, y_3, z_3)$ berilgan. U holda:

$$\vec{a}\,\vec{b}\,\vec{c} = \begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$$

2 – Misol. Agar $\vec{a}$ = (1, 3, 1), $\vec{b}$ = (-2, 4, -1) va $\vec{c}$ = (2, 4, -6) vektorlar berilgan bo'lsalar, u holda ularning komplanar vektorlar yoki nokomplanar vektorlar ekanliklari aniqlanib, agar nokomplanar vektorlar bo'lsa, ularning o'ng sistema yoki chap sistema tashkil etishlari ko'rsatilsin hamda ularga qurilgan parallelopipedning hajmi hisoblansin.

►$\vec{a}\,\vec{b}\,\vec{c} = \begin{vmatrix} 1 & 3 & 1 \\ -2 & 4 & -1 \\ 2 & 4 & -6 \end{vmatrix}$ = - 78 ekanligidan ko'rinib turibdiki, vektorlar nokomplanar bo'lib, chap sistema tashkil etadi hamda V=78 ga teng.◄

2.3 – AT

1. Agar $\vec{a} \perp \vec{b}$ bo'lib, $|\vec{a}|$=3, $|\vec{b}|$=4 bo'lsa, quyidagilar hisoblansin: $|\vec{a} \cdot \vec{b}|$; $|(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})|$; $|(3\vec{a} - \vec{b}) \cdot (\vec{a} - 2\vec{b})|$. (*Javob*: 12; 24; 60.)

2. $\vec{a}$ = *(3,-1,-2)* va $\vec{b}$ = *(1,2,-1)* vektorlar berilgan. Quyidagi vektorlarning koordinatalari topilsin: $\vec{a}\vec{b}$;$(2\vec{a}+\vec{b})\vec{b}$;$(2\vec{a}-\vec{b})(2\vec{a}+\vec{b})$. (Javob:*(5,1,7);(10,2,14); (20,4,28)*).

3. Agar ABC uchburchakning uchlari *A(1, 2, 0)*, *B(3, 0, 3)* va *C(5, 2, 6)* nuqtalarda bo'lsa, uning yuzi hisoblansin. (*Javob*: $2\sqrt{13}$).

4. $\vec{F}$ = (2, 2, 9) kuch *A(4, 2, -3)* nuqtaga qo'yilgan bo'lsa, uning *B(2, 4, 0)* nuqtaga nisbatan $\vec{M}$ momentining yo'naltiruvchi kosinuslari hamda $\vec{M}$ ning qiymati hisoblansin. (Javob: $|\vec{M}|$=*28*; $\cos\alpha = \frac{3}{7}$, $\cos\beta = \frac{6}{7}$, $\cos\gamma = -\frac{2}{7}$)

5. Uchlari *A (2, 0, 4), B (0, 3, 7), C (0, 0, 6)* va *S (4, 3, 5)* nuqtalarda bo'lgan piramidaning hajmi *V* ni hamda *ACS* yoqqa tushirilgan balandligi *N* ni hisoblansin. (Javob: $V = 2; N = \frac{2}{\sqrt{3}}$.)

6. *A(1, 2, -1), B(4, 1, 5), C(-1, 2, 1)* va *D(2,1, 3)* nuqtalar bitta tekislikda yotadimi? (*Javob*: yotadi.)

7. Quyida berilgan vektorlarning komplanar yoki nokomplanar ekanliklari tekshirilsin: a)$\vec{a}=(2,3,1)$, $\vec{b}=(1,-1,3)$, $\vec{c}=(-1,9,-11)$,b)$\vec{a}=(3,2,-1)$, $\vec{b}=(2,1,2)$, $\vec{c}=(3,-1,2)$ (Javob: a) komplanar; b) nokomplanar).

8. $\vec{a}=(3, 4, 0)$, $\vec{b}=(0, -4, 1)$ va $\vec{c}=(0, 2, 5)$ vektorlarning o'ng yoki chap sistema tashkil etishliklari tekshirilsin. (Javob*:* chap sistema tashkil etadi).

Mustaqil ish.

1. 1) Agar $|\vec{a}| = 10$, $|\vec{b}| = 2$ va $\vec{a}\cdot\vec{b} = 12$ bo'lsa, $|\vec{a}\times\vec{b}|$ ni hisoblansin. (Javob: 16)

2. 2) $\vec{a}=(0, -1, 1)$ va $\vec{b}=(1, 1, 1)$ vektorlarga qurilgan parallelogrammning yuzi hisoblansin. (Javob: 6)

3. $\vec{F}= (3, 2, -4)$ kuch *A(2, -1, 1)* nuqtaga qo'yilgan bo'lsa, u kuchning koordinata boshi *O* nuqtaga nisbatan aylanma momenti $\vec{M}$ ning koordinatalari topilsin. (Javob*:* $\vec{M} = (2, 11, 7)$).

4. Berilgan $\vec{a}=(7,6,1)$, $\vec{b}=(4,0,3)$ va $\vec{c}=(3,6,4)$ vektorlarga qurilgan uchburchakli prizmaning hajmi *V* hisoblansin. (Javob*: V=24*).

2.4. 2-bobga doir individual uy topshiriqlari (IUT)

2.1–IUT

1. $\vec{a}= \alpha\vec{m} + \beta\vec{n}$ va $\vec{b}= \gamma\vec{m} + \delta\vec{n}$ vektorlar berilgan bo'lib, bu erda $|\vec{m}|$=k, $|\vec{n}|$=l, $(\vec{m}\wedge\vec{n}) = \varphi$ kabi bo'lsa, quyidagilar hisoblansin:

a) $(\lambda\vec{a} + \mu\vec{b})(\nu\vec{a} + \vec{b})$; b) $\text{np}_{\vec{b}}(\nu\vec{a} + \tau\vec{b})$; c) $\cos(\vec{a}\wedge\tau\vec{b})$.

1.1. α=-5, β=-4, γ=3, δ=6, $k = 3$, $l = 5$ $= \frac{5\pi}{3}$, $\lambda = -2$, $\mu = \frac{1}{3}$, $\nu = 1$, $\tau = 2$.
(Javob*:*a) 2834.)

1.2. α = -2, β = 3, γ = 4, δ = -1, k =1, l =3, φ=π, $\lambda = 3$, $\mu = 2$, $\nu = -2$, $\tau = 4$.
(Javob*:* a) -950.)

1.3. α = 5, β = -2, γ = -3, δ = -1, k =4, l =5, $\varphi = \frac{4\pi}{3}$, $\lambda = 2$, $\mu = 3$, $\nu = -1$, $\tau = 5$.
(Javob: a) -1165.)

1.4. α = 5, β = 2, γ = -6, δ = -4, k = 3, l = 2, $\varphi = \frac{5\pi}{3}$, $\lambda = -1$, $\mu = \frac{1}{2}$, $\nu = 2$, $\tau = 3$.
(Javob: a) 416.)

1.5. α = 3, β = -2, γ = -4, δ = 5, , k =2, l = 3, $\varphi = \frac{\pi}{3}$, $\lambda = 2$, $\mu = -3$,$\nu = 5$, $\tau = 1$.
(Javob: a) 750.)

1.6. $\alpha = 2, \beta = -5, \gamma = -3, \delta = 4, k = 2, l = 4, \varphi = \frac{2\pi}{3}, \lambda = 3, \mu = -4, \nu = 2, \tau = 3.$ (Javob: a) -2116.)

1.7. $\alpha = 3, \beta = 2, \gamma = -4, \delta = -2, k = 2, l = 5, \varphi = \frac{4\pi}{3}, \lambda = 1, \mu = -3, \nu = 0, \tau = -\frac{1}{2}.$ (Javob: a) 165.)

1.8. $\alpha = 5, \beta = 2, \gamma = 1, \delta = -4, k = 3, l = 2, \varphi = \pi, \lambda = 1, \mu = -2, \nu = 3, \tau = -4.$ (Javob: a) -583.)

1.9. $\alpha = -3, \beta = -2, \gamma = 1, \delta = 5, k = 3, l = 6, \varphi = \frac{4\pi}{3}, \lambda = -1, \mu = 2, \nu = 1, \tau = 1.$ (Javob: a) 1287.)

1.10. $\alpha = 5, \beta = -3, \gamma = 4, \delta = 2, k = 4, l = 1, \varphi = \frac{2\pi}{3}, \lambda = 2, \mu = -\frac{1}{2}, \nu = 3, \tau = 0.$ (Javob: a) 2337.)

1.11. $\alpha = -2, \beta = 3, \gamma = 3, \delta = -6, k = 6, l = 3, \varphi = \frac{5\pi}{3}, \lambda = 3, \mu = -\frac{1}{3}, \nu = 1, \tau = 2.$ (Javob: a) -936.)

1.12. $\alpha = -2, \beta = -4, \gamma = 3, \delta = 1, k = 3, l = 2, \varphi = \frac{7\pi}{3}, \lambda = -\frac{1}{2}, \mu = 3, \nu = 1, \tau = 2.$ (Javob: a) 320.)

1.13. $\alpha = 4, \beta = 3, \gamma = -1, \delta = 2, k = 4, l = 5, \varphi = \frac{3\pi}{2}, \lambda = 2, \mu = -3, \nu = 1, \tau = 2.$ (Javob: a) 352.)

1.14. $\alpha = -2, \beta = 3, \gamma = 5, \delta = 1, k = 2, l = 5, \varphi = 2\pi, \lambda = -3, \mu = 4, \nu = 2, \tau = 3.$ (Javob: a) 1809.)

1.15. $\alpha = 4, \beta = -3, \gamma = 5, \delta = 2, k = 4, l = 7, \varphi = \frac{4\pi}{3}, \lambda = -3, \mu = 2, \nu = 2, \tau = -1.$ (Javob: a) -5962.)

1.16. $\alpha = -5, \beta = 3, \gamma = 2, \delta = 4, k = 5, l = 4, \varphi = \pi, \lambda = -3, \mu = \frac{1}{2}, \nu = -1, \tau = 1.$ (Javob: a) 3348.)

1.17. $\alpha = 5, \beta = -2\ \gamma = 3, \delta = 4, k = 2, l = 5, \varphi = \frac{\pi}{2}, \lambda = 2, \mu = 3, \nu = 1, \tau = -2.$ (Javob: a) -2076.)

1.18. $\alpha = 7, \beta = -3, \gamma = 2, \delta = 6, k = 3, l = 4, \varphi = \frac{5\pi}{3}, \lambda = 3, \mu = -\frac{1}{2}, \nu = 2, \tau = 1.$ (Javob: a) 1728.)

1.19. $\alpha = 4, \beta = -5, \gamma = -1, \delta = 3, k = 6, l = 3, \varphi = \frac{2\pi}{3}, \lambda = 2, \mu = -5, \nu = 1, \tau = 2.$ (Javob: a) 1044.)

1.20. $\alpha = 3, \beta = -5, \gamma = -2, \delta = 3, k = 6, l = 3, \varphi = \frac{3.\pi}{2}, \lambda = 4, \mu = 5, \nu = 1, \tau = -2.$ (Javob: a) 1994.)

1.21. $\alpha = -5, \beta = -6, \gamma = 2, \delta = 7, k = 2, l = 7, \varphi = \pi, \lambda = -2, \mu = 5, \nu = 1, \tau = 3.$ (Javob: a) 29 767.)

1.22. $\alpha = -7, \beta = 2, \gamma = 4, \delta = 6, k = 2, l = 9, \varphi = \frac{\pi}{3}, \lambda = 1, \mu = 2, \nu = -1, \tau = 3.$ (Javob: a) 20 758.)

1.23. $\alpha = 5, \beta = 4, \gamma = -6, \delta = 2, k = 2, l = 9, \varphi = \frac{2\pi}{3}, \lambda = 3, \mu = 2, \nu = 1, \tau = -\frac{1}{2}.$ (Javob: a) 2751.)

1.24. α=- 5, β = -7, γ = - 3, δ = 2, k = 2, l = 11, $\varphi = \frac{3\pi}{2}$, λ = - 3, μ = 4, ν = -1, τ = 2. (Javob: a) 38 587.)

1.25. α = 5, β = - 8, γ = - 2, δ = 3, k = 4, l = 3, $\varphi = \frac{4\pi}{3}$, λ = 2, μ = −3, ν = 1, τ = 2. (Javob: a) 1048.)

1.26. α = -3, β = 5, γ = 1, δ = 7, k = 4, l = 6, $\varphi = \frac{5\pi}{3}$, λ = -2, μ = 3, ν = 3, τ = -2. (Javob: a) -2532.)

1.27. α = -3, β = 4, γ = 5, δ = -6, k = 4, l = 5, φ = π, λ = 2, μ = 3, ν = -3, τ = -1. (Javob: a) 21 156.)

1.28. α = 6, β = -7, γ = -1, δ = -3, k = 2, l = 6, $\varphi = \frac{4\pi}{3}$, λ = 3, μ = −2, ν = 1, τ = 4. (Javob: a) -12 200.)

1.29. α = 5, β = 3, γ = -4, δ = -2, k = 6, l = 3, $\varphi = \frac{5\pi}{3}$, λ = -2, $\mu = -\frac{1}{2}$, ν = 3, τ = 2. (Javob: a) -2916.)

1.30. α = 4, β = -3, γ = -2, δ = 6, k = 4, l = 7, $\varphi = \frac{\pi}{3}$, λ = 2, $\mu = -\frac{1}{2}$, ν = 3, τ = 2. (Javob: a) -801.)

2. Berilgan *A, B, C* nuqtalarning koordinatalariga nisbatan ko'rsatilgan vektorlar uchun quyidagilar aniqlansin: a) $|\vec{a}|$; b) $\vec{a}\cdot\vec{b}$; c) $np_{\vec{a}}\vec{c}$; d) kesmani berilgan α:β kabi nisbatda bo'luvchi *M* nuqtaning koordinatalari.

2.1. *A (4, 6, 3), B (-5, 2, 6), C (4, -4, -3),* $a = 4\overrightarrow{CB} - \overrightarrow{AC}$, $b = \overrightarrow{AB}$, $c = \overrightarrow{CB}$, $d = \overrightarrow{AC}$, $l = AB$, $\alpha = 5$, $\beta = 4$. (Javob*: a)* $\sqrt{4216}$*; b) 314; d)* $(-1, \frac{34}{9}, \frac{14}{3})$*).*

2.2. *A (4, 3, -2), B (-3, -1, 4), C (2, 2, 1),* $a = -5\overrightarrow{AC} + 2\overrightarrow{CB}$, $b = \overrightarrow{AB}$, $c = \overrightarrow{AC}$, $d = \overrightarrow{CB}$, $l = BC$, $\alpha = 2$, $\beta = 3$. (Javob: *a)* $\sqrt{82}$*; b) -50; d)* $(-1, \frac{1}{5}, \frac{14}{5})$*).*

2.3. *A (-2, -2, 4), B (1, 3, -2), C (1, 4, 2),* $a = 2\overrightarrow{AC} - 3\overrightarrow{BA}$, $b = \overrightarrow{BC}$, $c = \overrightarrow{BC}$, $d = \overrightarrow{AC}$, $l = BA$, $\alpha = 2$, $\beta = 1$. (Javob: *a)* $\sqrt{1750}$*; b) -53;d)* $(-1, -\frac{1}{3}, 2)$*).*

2.4. *A (2, 4, 3), B (3, 1, -4), C (-1, 2, 2),* $a = 2\overrightarrow{BA} + 4\overrightarrow{AC}$, $b = \overrightarrow{BA}$, $c = b$, $d = \overrightarrow{AC}$, $l = BA$, $\alpha = 1$, $\beta = 4$. (Javob*: a)* $\sqrt{300}$*; b) 78; d)* $(\frac{14}{5}, \frac{8}{5}, -\frac{13}{5})$*).*

2.5. *A (2, 4, 5), B (1, -2, 3), C (-1, -2, 4),* $a = 3\overrightarrow{AB} - 4\overrightarrow{AC}$, $b = \overrightarrow{BC}$, $c = b$, $d = \overrightarrow{AB}$, $l = AB$, $\alpha = 2$, $\beta = 3$. (Javob*: a)* 11*; b) -20; d)* $(\frac{8}{5}, \frac{8}{5}, \frac{21}{5})$*).*

2.6. *A (-1, -2, 4), B (-1, 3, 5), C (1, 4, 2),* $a = 3\overrightarrow{AC} - 7\overrightarrow{BC}$, $b = \overrightarrow{AB}$, $c = b$, $d = \overrightarrow{AC}$, $l = AC$, $\alpha = 1$, $\beta = 7$. (Javob*: a)* $\sqrt{410}$*; b) 70;d)* $(-\frac{3}{4}, -\frac{5}{4}, \frac{15}{4})$*).*

2.7. *A (1, 3, 2), B (-2, 4, -1), C (1, 3, -2),* $a = 2\overrightarrow{AB} + 5\overrightarrow{CB}$, $b = \overrightarrow{AC}$, $c = b$, $d = \overrightarrow{AB}$, $l = AB$, $\alpha = 2$, $\beta = 4$. (Javob: *a)* $\sqrt{491}$*; b) 4; d)* $(0, \frac{10}{3}, 1)$*).*

2.8. *A (2, -4, 3), B (-3, -2, 4), C (0, 0, -2), B* $a = 3\overrightarrow{AC} - 4\overrightarrow{CB}$, $b = c = \overrightarrow{AB}$, $d = \overrightarrow{CB}$, $l = AC$, $\alpha = 2$, $\beta = 1$. (Javob*: a)* $\sqrt{1957}$*; b) -29; d)* $(\frac{2}{3}, -\frac{4}{5}, -\frac{1}{3})$*).*

2.9. *A (3, 4, -4), B (-2, 1, 2), C (2, -3, 1),* $a = 5\overrightarrow{CB} + 4\overrightarrow{AC}$, $b = c = \overrightarrow{BA}$, $d = \overrightarrow{AC}$, $l = BA$, $\alpha = 2$, $\beta = 5$. (Javob*: a)* $\sqrt{1265}$*; b) -294;d)* $(-\frac{4}{7}, \frac{13}{7}, 2.7)$*).*

2.10. *A (0, 2, 5), B (2, -3, 4), C (3, 2, -5), a = -3*$\overrightarrow{AB}$ *+ 4*$\overrightarrow{CB}$*, b = c =* $\overrightarrow{AC}$*, d =* $\overrightarrow{AB}$*, l = AC, α = 3, β = 2.* (Javob*: a)* $\sqrt{1646}$*; b) -420; d) (* $\frac{9}{5}$*, 2 , −1)).*

2.11. *A (-2, -3, -4), B (2, -4, 0), C (1, 4, 5), a = 4*$\overrightarrow{AC}$ *- 8*$\overrightarrow{BC}$*, b = c =* $\overrightarrow{AB}$*, d =* $\overrightarrow{BC}$*, l = AB, α = 4, β = 2.* (Javob*: a)* $\sqrt{1777}$*; b) 80; d) (*$\frac{2}{3}$*, -* $\frac{11}{3}$ *, -* $\frac{4}{3}$*)).*

2.12. *A (-2, -3, -2), B (1, 4, 2), C (1, -3, 3), a = 2*$\overrightarrow{AC}$ *– 4*$\overrightarrow{BC}$*, b = c =* $\overrightarrow{AB}$*, d =* $\overrightarrow{AC}$*, l = BC, α = 3, β = 1.* (Javob: *a)* $\sqrt{856}$*; b) 238; d) (1, -* $\frac{5}{4}$ *,* $\frac{11}{4}$*)).*

2.13. *A(5, 6, 1), B(-2, 4, -1), C(3, -3, 3), a = 3*$\overrightarrow{AB}$ *- 4*$\overrightarrow{BC}$*, b = c =* $\overrightarrow{AC}$*, d =* $\overrightarrow{AB}$*, l = BC, α = 3, β = 2.* (Javob*: a)* $\sqrt{2649}$*; b) -160; d) (1, -* $\frac{1}{5}$ *,* $\frac{7}{5}$*)).*

2.14. *A (10, 6, 3), B (-2, 4, 5), C (3, -4, -6), a = 5*$\overrightarrow{AC}$ *- 2*$\overrightarrow{CB}$*, b = c =* $\overrightarrow{BA}$*, d =* $\overrightarrow{AC}$*, l = CB, α = 1, β = 5.* (Javob*: a)* $\sqrt{9470}$*; b) -298; d) (*$\frac{13}{6}$*, -* $\frac{8}{3}$ *, - 25,6)).*

2.15. *A (3, 2, 4), B (-2, 1, 3), C (2, -2, -1), a = 4*$\overrightarrow{BC}$ *- 3*$\overrightarrow{AC}$*, b =* $\overrightarrow{BA}$*, c =* $\overrightarrow{AC}$*, d =* $\overrightarrow{BC}$*, l = AC, α = 2, β = 4.* (Javob: *a)* $\sqrt{362}$*; b) 94;d) (* $\frac{8}{3}$*,* $\frac{2}{3}$ *,* $\frac{7}{3}$*)).*

2.16. *A (-2, 3, -4), B (3, -1, 2), C (4, 2, 4), a = 7*$\overrightarrow{AC}$ *+ 4*$\overrightarrow{CB}$*, b = c =* $\overrightarrow{AB}$*, d =* $\overrightarrow{CB}$*, l = AB, α = 2, β = 5.* (Javob*: a)* $\sqrt{4109}$*; b) 554; d) (-* $\frac{4}{7}$*,* $\frac{13}{7}$ *, -* $\frac{16}{7}$*)).*

2.17. *A(4, 5, 3), B(-4, 2, 3), C(5, -6, -2), a = 9*$\overrightarrow{AB}$ *- 4*$\overrightarrow{BC}$*, b = c =* $\overrightarrow{AC}$*, d =* $\overrightarrow{AB}$*, l = BC, α = 5, β = 1.* (Javob*: a)* $\sqrt{12\,089}$*; b) -263; d) (*$\frac{7}{2}$*, -* $\frac{14}{3}$ *, -* $\frac{7}{6}$*)).*

2.18. *A (2, 4, 6), B (-3, 5, 1), C (4, -5, -4), a = - 6*$\overrightarrow{BC}$ *+ 2*$\overrightarrow{BA}$*, b = c =* $\overrightarrow{CA}$*, d =* $\overrightarrow{BA}$*, l = BC, α = 1, β = 3.* (Javob*: a)* $\sqrt{5988}$*; b) 986; d) (-* $\frac{5}{4}$*,* $\frac{5}{2}$ *, -* $\frac{1}{4}$*)).*

2.19. *A (-4, -2, -5), B (3, 7, 2), C (4, 6, -3), a = 9*$\overrightarrow{BA}$ *+ 3*$\overrightarrow{BC}$*, b = c =* $\overrightarrow{AC}$*, d =* $\overrightarrow{BC}$*, l = BA, α = 4, β = 3.* (Javob*: a)* $\sqrt{16\,740}$*; b) -1308; d) (-1,* $\frac{13}{7}$ *, −2)).*

2.20. *A (5, 4, 4), B (-5, 2, 3), C (4, 2, -5), a = 11*$\overrightarrow{AC}$ *- 6*$\overrightarrow{AB}$*, b =* $\overrightarrow{BC}$*, c =* $\overrightarrow{AB}$*, d =* $\overrightarrow{AC}$*, l =BC, α = 3, β = 1.* (Javob*: a)* $\sqrt{11\,150}$*; b) 1185; d) (*$\frac{7}{4}$*, 2 , −3)).*

2.21. *A (3, 4, 6), B (-4, 6, 4), C (5, -2, -3), a = -7*$\overrightarrow{BC}$ *+ 4*$\overrightarrow{CA}$*, b =* $\overrightarrow{BA}$*, c =* $\overrightarrow{CA}$*, d =* $\overrightarrow{BC}$*, l = BA, α = 5, β = 3.* (Javob*: a)* $\sqrt{18\,666}$*; b) -487; d) (* $\frac{3}{8}$*,* $\frac{19}{4}$ *,* $\frac{21}{4}$*)).*

2.22. *A (-5, -2, -6), B (3, 4, 5), C (2, -5, 4), a = 8*$\overrightarrow{AC}$ *- 5*$\overrightarrow{BC}$*, b = c =* $\overrightarrow{AB}$*, d =* $\overrightarrow{BC}$*, l = AC, α = 3, β = 4.* (Javob*: a)* $\sqrt{11\,387}$*; b) 1549; d) (-2, -* $\frac{23}{7}$ *, -* $\frac{12}{7}$*)).*

2.23. *A (3, 4, 1), B (5, -2, 6), C (4, 2, -7), a = -7*$\overrightarrow{AC}$ *+ 5*$\overrightarrow{AB}$*, b = c =* $\overrightarrow{BC}$*, d =* $\overrightarrow{AC}$*, l = AB, α = 2, β = 3.* (Javob*: a)* $\sqrt{6826}$*; b) -1120; d) (*$\frac{19}{5}$*,* $\frac{8}{5}$ *, 3)).*

2.24. *A (4, 3, 2), B (-4, -3, 5), C (6, 4, -3), a = 8*$\overrightarrow{AC}$ *- 5*$\overrightarrow{BC}$*, b = c =* $\overrightarrow{BA}$*, d =* $\overrightarrow{AC}$*, l = BC, α = 2, β = 5.* (Javob: *a)* $\sqrt{1885}$*; b) -434; d) (-* $\frac{8}{7}$*, −1 ,* $\frac{19}{7}$*)).*

2.25. *A (-5, 4, 3), B (4, 5, 2), C (2, 7, -4), a = 3*$\overrightarrow{BC}$ *+ 2*$\overrightarrow{AB}$*, b = c =* $\overrightarrow{CA}$*, d =* $\overrightarrow{AB}$*, l = BC, α = 3, β = 4.* (Javob: *a)* $\sqrt{608}$*; b) -248; d) (*$\frac{22}{7}$*,* $\frac{41}{7}$ *, -* $\frac{4}{7}$*)).*

2.26. *A (6, 4, 5), B (-7, 1, 8), C (2, -2, -7), a = 5*$\overrightarrow{CB}$ *- 2*$\overrightarrow{AC}$*, b =* $\overrightarrow{AB}$*, c =* $\overrightarrow{CB}$*, d =* $\overrightarrow{AC}$*, l = AB, α = 3, β = 2.* (Javob*: a)* $\sqrt{11\,899}$*; b) 697; d) (-*$\frac{9}{5}$*,* $\frac{11}{5}$ *,* $\frac{34}{5}$*)).*

2.27. *A (6, 5, -4), B (-5, -2, 2), C (3, 3, 2),* $a = 6\overrightarrow{AB} - 3\overrightarrow{CB}$, $b = c = \overrightarrow{AC}$, $d = \overrightarrow{CB}$, $l = BC$, $\alpha = 1$, $\beta = 5$. (Javob*: a)* $\sqrt{3789}$*; b) 396; d)* $(-\frac{11}{3}, -\frac{7}{6}, 2)$*).*

2.28. *A (-3, -5, 6), B (3, 5, -4), C (2, 6, 4),* $a = 4\overrightarrow{AC} - 5\overrightarrow{BA}$, $b = \overrightarrow{CB}$, $c = \overrightarrow{BA}$, $d = \overrightarrow{AC}$, $l = BA$, $\alpha = 4$, $\beta = 2$. (Javob*: a)* $\sqrt{14\,700}$*; b) 470; d)* $(-1, -\frac{5}{3}, \frac{8}{3})$*).*

2.29. *A (3, 5, 4), B (4, 2, -3), C (-2, 4, 7),* $a = 3\overrightarrow{BA} - 4\overrightarrow{AC}$, $b = \overrightarrow{AB}$, $c = \overrightarrow{BA}$, $d = \overrightarrow{AC}$, $l = BA$, $\alpha = 2$, $\beta = 5$. (Javob: *a)* $\sqrt{539}$*; b) -85; d)* $(\frac{26}{7}, \frac{20}{7}, -1)$*).*

2.30. *A (4, 6, 7), B (2, -4, 1), C (-3, -4, 2),* $a = 5\overrightarrow{AB} - 2\overrightarrow{AC}$, $b = c = \overrightarrow{BC}$, $d = \overrightarrow{AB}$, $l = AB$, $\alpha = 3$, $\beta = 4$. (Javob*: a)* $\sqrt{1316}$*; b) -40; d)* $(\frac{22}{7}, \frac{12}{7}, \frac{31}{7})$*).*

3. Berilgan $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlarning bazis vektorlarni tashkil etishliklari aniqlanib, $\vec{d}$ vektorning bu bazisdagi koordinatalari topilsin.

3.1. ***a****=(5, 4, 1),****b****= (-3, 5, 2), **c**=(2, -1, 3), **d**=(7, 23, 4).* (Javob: *(3, 2,-1)).*
3.2. ***a****=(2, -1, 4), **b**=(-3, 0, -2), **c**=(4, 5, -3), **d**=(0, 11, -14).* (Javob: *(-1, 2, 2)).*
3.3. ***a****=(-1, 1, 2), **b** =(2, -3, -5), **c**=(-6, 3, -1), **d**=(28, -19, -7).*(Javob*: (2, 3, -4)).*
3.4. ***a****=(1, 3, 4), **b**=(-2, 5, 0), **c**=(3, -2, -4), **d**=(13, -5, -4).* (Javob*: (2, -1, 3)).*
3.5. ***a****=(1, -1, 1), **b=**(-5, -3, 1), **c=**(2, -1, 0), **d=**(-15, -10, 5).* (Javob*: (2, 3, -1)).*
3.6. ***a****=(3, 1, 2), **b**=(-7, -2, -4), **c**=(-4, 0, 3), **d**=(16, 6, 15).* (Javob*: (2, -2, 1)).*
3.7. ***a****=(-3, 0, 1), **b**=(2, 7, -3), **c**=(-4, 3, 5), **d**=(-16, 33, 13).* (Javob*: (2, 3, 4)).*
3.8. ***a****=(5, 1, 2), **b**=(-2, 1, -3), **c**=(4, -3, 5), **d**=(15, -15, 24).* (Javob: *(-1, 28, 4)).*
3.9. ***a****=(0, 2, -3), **b**=(4, -3, -2), **c**=(-5, -4, 0), **d**=(-19, -5, -4).* (Javob: *(2, -1, 3)).*
3.10. ***a****= (3, -1, 2), **b**=(-2, 3, 1), **c** =(4, -5, -3), **d**=(-3, 2, -3).* (Javob*: (-1, 2, 1)).*
3.11. ***a****=(5, 3, 1), **b**=(-1, 2, -3), **c**=(3, -4, 2), **d**=(-9, 34, -20).* (Javob: *(2, 4, -5)).*
3.12. ***a****=(3, 1, -3), **b**=(-2, 4, 1), **c**=(1, -2, 5), **d**=(1, 12, -20).* (Javob: *(2, 1, -3)).*
3.13. ***a****=(6, 1, -3), **b**=(-3, 2, 1), **c**=(-1, -3, 4), **d**=(15, 6, -17).* (Javob: *(1, -2, -3)).*
3.14. ***a****=(4, 2, 3), **b**=(-3, 1, -8), **c**=(2, -4, 5), **d**=(-12, 14, -31).* (Javob: *(0, 2, -3)).*
3.15. ***a****=(-2, 1, 3), **b**=(3, -6, 2), **c**=(-5, -3, -1), **d**=(31, -6, 22).* (Javob: *(3, 4, -5)).*
3.16. ***a****=(1, 3, 6), **b**=(-3, 4, -5), **c**=(1, -7, 2), **d**=(-2, 17, 5).* (Javob: *(12, 1, -1)).*
3.17. ***a****=(7, 2, 1), **b**=(5, 1, -2), **c**=(-3, 4, 5), **d**=(26, 11, 1).* (Javob: *(2, 3, 1)).*
3.18. ***a****=(3, 5, 4), **b**=(-2, 7, -5), **c**=(6, -2, 1), **d**=(6, -9, 22).* (Javob*: (2, -3, -1)).*
3.19. ***a****=(5, 3, 2), **b**=(2, -5, 1), **c**=(-7, 4, -3), **d**=(36, 1, 15).* (Javob: *(5, 2, -1)).*
3.20. ***a****=(11, 1, 2), **b**=(-3, 3, 4), **c**=(-4, -2, 7), **d**=(-5, 11, -15).* (Javob: *(-1, 2, -3)).*
3.21. ***a****=(9, 5, 3), **b**=(-3, 2, 1), **c**=(4, -7, 4), **d**=(-10, -13, 8).* (Javob*: (-1, 3, 2)).*
3.22. ***a****=(7, 2, 1), **b**=(3, -5, 6), **c**=(-4, 3, -4), **d**=(-1, 18, -16).* (Javob*: (2, -1, 3)).*
3.23. ***a****=(1, 2, 3), **b**=(-5, 3, -1), **c**=(-6, 4, 5), **d**=(-4, 11, 20).* (Javob*: (3, -1, 2)).*
3.24. ***a****=(-2, 5, 1), **b**=(3, 2, -7), **c**=(4, 3, 2), **d**=(-4, 22, -13).* (Javob*: (3, 2, -1)).*
3.25. ***a****=(3, 1, 2), **b**=(-4, 3, -1), **c**=(2, 3, 4), **d**=(14, 14, 20).* (Javob*: (2, 0, 4)).*
3.26. ***a****=(3, -1, 2), **b**=(-2, 4, 1), **c**=(4, -5, -1), **d**=(-5, 11, 1).* (Javob*: (-1, 5, 2)).*
3.27. ***a****=(4, 5, 1), **b**=(1, 3, 1), **c**=(-3, -6, 7), **d**=(19, 33, 0).* (Javob: *(3, 4, -1)).*
3.28. ***a****=(1, -3, 1), **b**=(-2, -4, 3), **c**=(0, -2, 3), **d**=(-8, -10, 13).* (Javob*: (-2, 3, 2)).*
3.29. ***a****=(5, 7, -2), **b**=(-3, 1, 3), **c**=(1, -4, 6), **d**=(14, 9, -1).* (Javob*: (2, -1, 1)).*
3.30. ***a****=(-1, 4, 3), **b**=(3, 2, -4), **c**=(-2, -7, 1), **d**=(6, 20, -3).* (Javob*: (1, 1, -2)).*

1. $\vec{a} = -\vec{m} + 6\vec{n}$ va $\vec{b} = 3\vec{m} + 4\vec{n}$ vektorlar berilgan bo'lib, u erda $|\vec{m}| = 2$, $|\vec{n}| = 5$ va $(\vec{m}\wedge\vec{n}) = \frac{2\pi}{3}$ bo'lsa, quyidagilar topilsin: a) $\vec{a}\,\vec{b}$; b) $np_{\vec{b}}(4\vec{a} - 5\vec{b})$; c) $\cos((2\vec{b} - \vec{a})\wedge 4\vec{b})$

► a) $\vec{a}\cdot\vec{b} = (-\vec{m} + 6\vec{n})(3\vec{m} + 4\vec{n}) = -3\vec{m}^2 + 14|\vec{m}|\cdot|\vec{n}|\cos\frac{2\pi}{3} + 24\vec{n}^2 =$
$-3\,2^2 + 14\cdot 2\,5\cdot(-\frac{1}{2}) + 24\,5^2 = 518$;

b) $\vec{c} = 4\vec{a} - 5\vec{b} = -19\vec{m} + 4\vec{n}$ deb belgilaymiz. U holda:
$np_{\vec{b}}\vec{c} = \frac{\vec{c}\cdot\vec{b}}{|\vec{b}|}$; hamda, $\vec{c}\cdot\vec{b} = (-19\vec{m} + 4\vec{n})(3\vec{m} + 4\vec{n}) = -57\vec{m}^2 - 64|\vec{m}|\cdot|\vec{n}|\cos\frac{2\pi}{3} + +16\vec{n}^2 = -148$ va $|\vec{b}| = \sqrt{(3\vec{m} + 4\vec{n})^2} = \sqrt{9\vec{m}^2 + 24|\vec{m}|\cdot|\vec{n}|\cos\frac{2\pi}{3} + 16\vec{n}^2} = \sqrt{316}$ bo'lganligi uchun, $np_{\vec{b}}(4\vec{a} - 5\vec{b}) = -\frac{148}{\sqrt{316}}$ ni hosil qilamiz.

c) Agar $\vec{d} = 2\vec{b} - \vec{a} = 7\vec{m} + 2\vec{n}$, $\vec{e} = 4\vec{b} = 12\vec{m} + 16\vec{n}$ deb belgilash kiritsak, u holda: $\cos(\vec{d}\wedge\vec{e}) = \frac{\vec{d}\cdot\vec{e}}{|\vec{d}|\cdot|\vec{e}|}$ kabi yoziladi.

$\vec{d}\cdot\vec{e} = (7\vec{m} + 2\vec{n})(12\vec{m} + 16\vec{n}) = 84\vec{m}^2 + 136|\vec{m}|\cdot|\vec{n}|\cos\frac{2\pi}{3} + 32\vec{n}^2 = 456$ va $|\vec{d}| = \sqrt{(7\vec{m} + 2\vec{n})^2} = \sqrt{49\vec{m}^2 + 28|\vec{m}|\cdot|\vec{n}|\cos\frac{2\pi}{3} + 4\vec{n}^2} = \sqrt{156}$ hamda $|\vec{e}| = \sqrt{(12\vec{m} + 16\vec{n})^2} = \sqrt{144\vec{m}^2 + 384|\vec{m}|\cdot|\vec{n}|\cos\frac{2\pi}{3} + 256\vec{n}^2} = \sqrt{5056}$
bo'lgani uchun pirovardida quyidagini hosil qilamiz:

$\cos((2\vec{b} - \vec{a})\wedge 4\vec{b}) = \frac{456}{\sqrt{788736}} \approx 0{,}5$.◄

2. Berilgan A (-5,1,6), B(1,4,3) va C(6,3,9) nuqtalarga nisbatan quyidagilar hisoblansin:

a) $\vec{a} = 4\overrightarrow{AB} + \overrightarrow{BC}$ vektorning moduli;

b) $\vec{a}$ va $\vec{b} = \overrightarrow{BC}$ vektorlarning skalyar ko'paytmasi;

c) $\vec{c} = \vec{b}$ vektorning $\vec{d} = \overrightarrow{AB}$ vektorlardagi proeksiyasi;

d) l = AB kesmani 1:3 nisbatda bo'ladigan M nuqtaning koordinatalari.

► a) $\overrightarrow{AB} = (6, 3, -3)$ va $\overrightarrow{BC} = (5, -1, 6)$ bo'lganligidan, $4\overrightarrow{AB} + \overrightarrow{BC} = (29,11,-6)$ ni va undan $|4\overrightarrow{AB} + \overrightarrow{BC}| = \sqrt{29^2 + 11^2 + (-6)^2} = \sqrt{998}$ ni hosil qilamiz.

b) $\vec{a} = (29, 11, -6)$ va $\vec{b} = (5, -1, 6)$ ekanligidan, $\vec{a}\,\vec{b} = 29\,5 + 11\cdot(-1) + (-6)\,6 = = 98$ ni topamiz.

c) $np_{\vec{d}}\vec{s} = \frac{\vec{s}\cdot\vec{d}}{|\vec{d}|}$, $\vec{d} = (6, 3, -3)$ va $\vec{s}\cdot\vec{d} = 30 - 3 - 18 = 9$ hamda $|\vec{d}| = \sqrt{36 + 9 + 9}$ bo'lganligi uchun $np_{\overrightarrow{AB}}\overrightarrow{BC} = \frac{9}{\sqrt{54}}$ ni topamiz.

d) $\lambda = \frac{1}{3}$ bo'lganligi uchun: $x_M = \frac{-5+\frac{1}{3}\cdot 1}{1+\frac{1}{3}} = -\frac{7}{2}$, $y_M = \frac{1+4\cdot\frac{1}{3}}{1+\frac{1}{3}} = \frac{7}{4}$ hamda $z_M = \frac{6+\frac{1}{3}\cdot 3}{1+\frac{1}{3}} = \frac{21}{4}$ larni va M $(-\frac{7}{2}, \frac{7}{4}, \frac{21}{4})$ larni hosil qilamiz.◄

3. Berilgan $\vec{a}$ = (3; -1; 0), $\vec{b}$ = (2; 3; 1) va $\vec{c}$ = (-1; 4; 3) vektorlar bazis tashkil etishliklarini ko'rsatib, $\vec{d}$ = (2; 3; 7) vektorning ushbu bazisdagi koordinatalari aniqlansin.

► $$\vec{a}\cdot\vec{b}\cdot\vec{c} = \begin{vmatrix} 3 & -1 & 0 \\ 2 & 3 & 1 \\ -1 & 4 & 3 \end{vmatrix} = 22 \neq 0$$

bo'lganligi uchun, $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar bazis vektorlardir va $\vec{d}$ vektor ular orqali chiziqli bog'lanishdadir: $\vec{d} = \alpha\vec{a} + \beta\vec{b} + \gamma\vec{c}$ yoki koordinatalar shaklida esa,

$$\begin{cases} 3\alpha + 2\beta - \gamma = 2, \\ -\alpha + 3\beta + 4\gamma = 3, \\ \beta + 3\gamma = 7. \end{cases}$$

Ushbu sistemani Kramer formulalari yordamida yechamiz, ya'ni:

$$\Delta = 22,\ \Delta(\alpha) = \begin{vmatrix} 2 & 2 & -1 \\ 3 & 3 & 4 \\ 7 & 1 & 3 \end{vmatrix} = 66,\ \Delta(\beta) = \begin{vmatrix} 3 & 2 & -1 \\ -1 & 3 & 4 \\ 0 & 7 & 3 \end{vmatrix} = -44,$$

$$\Delta(\gamma) = \begin{vmatrix} 2 & 2 & 2 \\ 3 & 3 & 3 \\ 7 & 1 & 7 \end{vmatrix} = 66,\ \alpha = \frac{\Delta(\alpha)}{\Delta} = 3,\ \beta = \frac{\Delta(\beta)}{\Delta} = -2,\ \gamma = \frac{\Delta(\gamma)}{\Delta} = 3,$$

Shu boisdan, $\vec{d}$ = (3; -2; 3) =$3\vec{a} - 2\vec{b} + 3\vec{c}$ deb yoza olamiz. ◄

2.2-IUT

1. Berilgan $\vec{a}$, $\vec{b}$ va $\vec{c}$ vektorlar orqali quyidagilar hisoblansin: a) Uch vektorning aralash ko'paytmasi; b) Vektor ko'paytmaning moduli; c) Ikki vektorning skalyar ko'paytmasi; d) Ikkita vektorning kolleniarligi yoki ortogonalligi tekshirilsin; e) Uchta vektorning komplanarligi tekshirilsin.

1.1. ***a****=2****i*** *– 3****j*** *+* ***k****,* ***b****=****j*** *+ 4****k****,* ***c****=5****i*** *+ 2****j*** *– 3****k****; a)* ***a****, 3****b****,* ***c****; b) 3****a****, 2****c****; c)* ***b****, -4****c****; d)* ***a****,* ***c****; e)* ***a****, 2****b****, 3****c****.* (Javob*: a) -261; b)* $\sqrt{19116}$*; c) 40).*

1.2. ***a****=3****i*** *+ 4****j*** *+* ***k****,* ***b****=****i*** *- 2****j*** *+ 7****k****,* ***c****=3****i*** *- 6****j*** *+ 21****k****; a) 5****a****, 2****b****,* ***c****; b)* ***4b,*** *2****c****; c)* ***a****,* ***c****; d)* ***b****,* ***c****; e) 2****a****, -3****b****, 3****c****.* (Javob*: a) 0; b)* 0*; c) 6).*

1.3. ***a****=2****i*** *– 4****j*** *- 2****k****,* ***b****=7****i*** *+ 3****j****,* ***c****=3****i*** *+ 5****j*** *– 7****k****; a)* ***a****, 2****b****, 3****c****; b) 3****a****, -7****b****; c)* ***c****; -2****a****; d)* ***a****,* ***c****; e) 3****a****, 2****b****, 3****c****.* (Javob*: a) -1840; b)* $\sqrt{612108}$*; c) 0).*

1.4. ***a****=-7****i*** *+ 2****k****,* ***b****=2****i*** *- 6****j*** *+ 4****k****,* ***c****=****i*** *- 3****j*** *+ 2****k****; a)* ***a****, - 2****b****, -7****c****; b) 4****b****, 3****c****; c) 2****a****, -7****c****; d)* ***b****,* ***c****; e) 2****a****, 4****b****, 3****c****.* (Javob*: a) 0; b)* 0*; c) 42).*

1.5. ***a****=-4****i*** *+ 2****j*** *-* ***k****,* ***b****=3****i*** *+ 5****j*** *- 2****k****,* ***c****=****j*** *+ 5****k****; a)* ***a****, 6****b****, 3****c****; b) 2****b****,* ***a****; c)* ***a****, - 4****c****; d)* ***a,*** ***b****; e)* ***a****, 6****b****, 3****c****.* (Javob*: a) -2538; b)* $\sqrt{3192}$*; c) 12).*

1.6. ***a****=3****i*** *– 2****j*** *+* ***k****,* ***b****=2****j*** *- 3****k****,* ***c****=-3****i*** *+ 2****j*** *-* ***k****; a)* ***a****, - 3****b****, 2****c****; b) 5****a,*** *3****c****, c) - 2****a****, 4****b****; d)* ***a****,* ***c****; e) 5****a****, 4****b****, 3****c****.* (Javob*: a) 0; b)* 0*; c) 56).*

1.7. ***a****=4****i*** *–* ***j*** *+ 3****k****,* ***b****=2****i*** *+ 3****j*** *- 5****k****,* ***c****=7****i*** *+ 2****j*** *+ 4****k****; a) 7****a****, - 4****b****, 2****c****; b) 3****a****, 5****c****; c) 2****b****, 4****c****; d)* ***b****,* ***c****; e) 7****a****, 2****b****, 5****c****.* (Javob*: a) -4480; b)*$\sqrt{78750}$*; c) 0).*

1.8. ***a****=4****i*** *+ 2****j*** *– 3****k****,* ***b****=2****i*** *+* ***k****,* ***c****=-12****i*** *- 6****j*** *+ 9****k****; a) 2****a****, 3****b****,* ***c****; b) 4****a,*** *3****b****; c)* ***b****, -4****c****; d)* ***a****,* ***c****; e) 2****a****, 3****b****, -4****c****.* (Javob*: a) 0; b)* $\sqrt{17280}$*; c) 60).*

1.9. $\boldsymbol{a}=-\boldsymbol{i}+5\boldsymbol{k}$, $\boldsymbol{b}=-3\boldsymbol{i}+2\boldsymbol{j}+2\boldsymbol{k}$, $\boldsymbol{c}=-2\boldsymbol{i}-4\boldsymbol{j}+\boldsymbol{k}$; a) $3\boldsymbol{a}, -4\boldsymbol{b}, 2\boldsymbol{c}$; b) $7\boldsymbol{a}, -3\boldsymbol{c}$; c) $2\boldsymbol{b}, 3\boldsymbol{a}$; d) $\boldsymbol{b}, \boldsymbol{c}$; e) $7\boldsymbol{a}, 2\boldsymbol{b}, -3\boldsymbol{c}$. (Javob: a) -1680; b) $\sqrt{219177}$; c) 78).

1.10. $\boldsymbol{a}=6\boldsymbol{i}-4\boldsymbol{j}+6\boldsymbol{k}$, $\boldsymbol{b}=9\boldsymbol{i}-6\boldsymbol{j}+9\boldsymbol{k}$, $\boldsymbol{c}=\boldsymbol{i}-8\boldsymbol{k}$; a) $2\boldsymbol{a}, -4\boldsymbol{b}, 3\boldsymbol{c}$; b) $3\boldsymbol{b}, -9\boldsymbol{c}$; c) $3\boldsymbol{a}, -5\boldsymbol{c}$; d) $\boldsymbol{a}, \boldsymbol{b}$; e) $3\boldsymbol{a}, -4\boldsymbol{b}, -9\boldsymbol{c}$. (Javob: a) 0; b) $\sqrt{6488829}$; c) 630).

1.11. $\boldsymbol{a}=5\boldsymbol{i}-3\boldsymbol{j}+4\boldsymbol{k}$, $\boldsymbol{b}=2\boldsymbol{i}-4\boldsymbol{j}-2\boldsymbol{k}$, $\boldsymbol{c}=3\boldsymbol{i}+5\boldsymbol{j}-7\boldsymbol{k}$; a) $\boldsymbol{a}, -4\boldsymbol{b}, 2\boldsymbol{c}$; b) $-2\boldsymbol{b}, 4\boldsymbol{c}$; c) $-3\boldsymbol{a}, 6\boldsymbol{c}$; d) $\boldsymbol{b}, \boldsymbol{c}$; e) $\boldsymbol{a}, -2\boldsymbol{b}, 6\boldsymbol{c}$. (Javob: a) -464; b) $\sqrt{127488}$; c) 504).

1.12. $\boldsymbol{a}=-4\boldsymbol{i}+3\boldsymbol{j}-7\boldsymbol{k}$, $\boldsymbol{b}=4\boldsymbol{i}+6\boldsymbol{j}-2\boldsymbol{k}$, $\boldsymbol{c}=6\boldsymbol{i}+9\boldsymbol{j}-3\boldsymbol{k}$; a) $-2\boldsymbol{a}, \boldsymbol{b}, -2\boldsymbol{c}$; b) $4\boldsymbol{b}, 7\boldsymbol{c}$; c) $5\boldsymbol{a}, -3\boldsymbol{b}$; d) $\boldsymbol{b}, \boldsymbol{c}$; e) $-2\boldsymbol{a}, 4\boldsymbol{b}, 7\boldsymbol{c}$. (Javob: a) 0; b) 0; c) -240).

1.13. $\boldsymbol{a}=-5\boldsymbol{i}+2\boldsymbol{j}-2\boldsymbol{k}$, $\boldsymbol{b}=7\boldsymbol{i}-5\boldsymbol{k}$, $\boldsymbol{c}=2\boldsymbol{i}+3\boldsymbol{j}-2\boldsymbol{k}$; a) $2\boldsymbol{a}, 4\boldsymbol{b}, -5\boldsymbol{c}$; b) $-3\boldsymbol{b}, 11\boldsymbol{c}$; c) $8\boldsymbol{a}, -6\boldsymbol{c}$; d) $\boldsymbol{a}, \boldsymbol{c}$; e) $8\boldsymbol{a}, -3\boldsymbol{b}, 11\boldsymbol{c}$. (Javob: a) 4360; b) $33\sqrt{682}$; c) 0).

1.14. $\boldsymbol{a}=-4\boldsymbol{i}-6\boldsymbol{j}+2\boldsymbol{k}$, $\boldsymbol{b}=2\boldsymbol{i}+3\boldsymbol{j}-\boldsymbol{k}$, $\boldsymbol{c}=-\boldsymbol{i}+5\boldsymbol{j}-3\boldsymbol{k}$; a) $5\boldsymbol{a}, 7\boldsymbol{b}, 2\boldsymbol{c}$; b) $-4\boldsymbol{b}, 11\boldsymbol{a}$; c) $3\boldsymbol{a}, -7\boldsymbol{c}$; d) $\boldsymbol{a}, \boldsymbol{b}$; e) $3\boldsymbol{a}, 7\boldsymbol{b}, -2\boldsymbol{c}$. (Javob: a) 0; b) 0; c) 672).

1.15. $\boldsymbol{a}=-4\boldsymbol{i}+2\boldsymbol{j}-3\boldsymbol{k}$, $\boldsymbol{b}=-3\boldsymbol{j}+5\boldsymbol{k}$, $\boldsymbol{c}=6\boldsymbol{i}+6\boldsymbol{j}-4\boldsymbol{k}$; a) $5\boldsymbol{a}, -\boldsymbol{b}, 3\boldsymbol{c}$; b) $-7\boldsymbol{a}, 4\boldsymbol{c}$; c) $3\boldsymbol{a}, 9\boldsymbol{b}$; d) $\boldsymbol{a}, \boldsymbol{c}$; e) $3\boldsymbol{a}, -9\boldsymbol{b}, 4\boldsymbol{c}$. (Javob: a) -1170; b) $56\sqrt{638}$; c) 567).

1.16. $\boldsymbol{a}=-3\boldsymbol{i}+8\boldsymbol{j}$, $\boldsymbol{b}=2\boldsymbol{i}+3\boldsymbol{j}-2\boldsymbol{k}$, $\boldsymbol{c}=8\boldsymbol{i}+12\boldsymbol{j}-8\boldsymbol{k}$; a) $4\boldsymbol{a}, -6\boldsymbol{b}, 5\boldsymbol{c}$; b) $-7\boldsymbol{a}, 9\boldsymbol{c}$; c) $3\boldsymbol{b}, -8\boldsymbol{c}$; d) $\boldsymbol{b}, \boldsymbol{c}$; e) $4\boldsymbol{a}, -6\boldsymbol{b}, 9\boldsymbol{c}$. (Javob: a) 0; b) $252\sqrt{917}$; c) -1632).

1.17. $\boldsymbol{a}=2\boldsymbol{i}-4\boldsymbol{j}-2\boldsymbol{k}$, $\boldsymbol{b}=-9\boldsymbol{i}+2\boldsymbol{k}$, $\boldsymbol{c}=3\boldsymbol{i}+5\boldsymbol{j}-7\boldsymbol{k}$; a) $7\boldsymbol{a}, 5\boldsymbol{b}, -\boldsymbol{c}$; b) $-5\boldsymbol{a}, 4\boldsymbol{b}$; c) $3\boldsymbol{b}, -8\boldsymbol{c}$; d) $\boldsymbol{a}, \boldsymbol{c}$; e) $7\boldsymbol{a}, 5\boldsymbol{b}, -\boldsymbol{c}$. (Javob: a) - 10430; b) $\sqrt{40389}$; c) 984).

1.18. $\boldsymbol{a}=9\boldsymbol{i}-3\boldsymbol{j}+\boldsymbol{k}$, $\boldsymbol{b}=3\boldsymbol{i}-15\boldsymbol{j}+21\boldsymbol{k}$, $\boldsymbol{c}=\boldsymbol{i}-5\boldsymbol{j}+7\boldsymbol{k}$; a) $2\boldsymbol{a}, -7\boldsymbol{b}, 3\boldsymbol{c}$; b) $-6\boldsymbol{a}, 4\boldsymbol{c}$; c) $5\boldsymbol{b}, 7\boldsymbol{a}$; d) $\boldsymbol{b}, \boldsymbol{c}$; e) $2\boldsymbol{a}, -7\boldsymbol{b}, 4\boldsymbol{c}$. (Javob: a) 0; b) $\sqrt{3365604}$; c) 3255).

1.19. $\boldsymbol{a}=-2\boldsymbol{i}+4\boldsymbol{j}-3\boldsymbol{k}$, $\boldsymbol{b}=5\boldsymbol{i}+\boldsymbol{j}-2\boldsymbol{k}$, $\boldsymbol{c}=7\boldsymbol{i}+4\boldsymbol{j}-\boldsymbol{k}$; a) $\boldsymbol{a}, -6\boldsymbol{b}, 2\boldsymbol{c}$; b) $-8\boldsymbol{b}, 5\boldsymbol{c}$; c) $-9\boldsymbol{a}, 7\boldsymbol{c}$; d) $\boldsymbol{a}, \boldsymbol{b}$; e) $\boldsymbol{a}, -6\boldsymbol{b}, 5\boldsymbol{c}$. (Javob: a) 1068; b) $\sqrt{478400}$; c) -315).

1.20. $\boldsymbol{a}=-9\boldsymbol{i}+4\boldsymbol{j}-5\boldsymbol{k}$, $\boldsymbol{b}=\boldsymbol{i}-2\boldsymbol{j}+4\boldsymbol{k}$, $\boldsymbol{c}=-5\boldsymbol{i}+10\boldsymbol{j}-20\boldsymbol{k}$; a) $-2\boldsymbol{a}, 7\boldsymbol{b}, 5\boldsymbol{c}$; b) $-6\boldsymbol{b}, 7\boldsymbol{c}$; c) $9\boldsymbol{a}, 4\boldsymbol{c}$; d) $\boldsymbol{b}, \boldsymbol{c}$; e) $-2\boldsymbol{a}, 7\boldsymbol{b}, 4\boldsymbol{c}$. (Javob: a) 0; b) $\sqrt{52611300}$; c) 6660).

1.21. $\boldsymbol{a}=2\boldsymbol{i}-7\boldsymbol{j}+5\boldsymbol{k}$, $\boldsymbol{b}=-\boldsymbol{i}+2\boldsymbol{j}-6\boldsymbol{k}$, $\boldsymbol{c}=3\boldsymbol{i}+2\boldsymbol{j}-4\boldsymbol{k}$; a) $-3\boldsymbol{a}, 6\boldsymbol{b}, -\boldsymbol{c}$; b) $5\boldsymbol{b}, 3\boldsymbol{c}$; c) $7\boldsymbol{a}, -4\boldsymbol{b}$; d) $\boldsymbol{b}, \boldsymbol{c}$; e) $7\boldsymbol{a}, -4\boldsymbol{b}, 3\boldsymbol{c}$. (Javob: a) 2196; b) $\sqrt{126900}$; c) 1288).

1.22. $\boldsymbol{a}=7\boldsymbol{i}-4\boldsymbol{j}-5\boldsymbol{k}$, $\boldsymbol{b}=\boldsymbol{i}-11\boldsymbol{j}+3\boldsymbol{k}$, $\boldsymbol{c}=5\boldsymbol{i}+5\boldsymbol{j}+3\boldsymbol{k}$; a) $3\boldsymbol{a}, -7\boldsymbol{b}, 2\boldsymbol{c}$; b) $2\boldsymbol{b}, 6\boldsymbol{c}$; c) $-4\boldsymbol{a}, -5\boldsymbol{c}$; d) $\boldsymbol{a}, \boldsymbol{c}$; e) $-4\boldsymbol{a}, 2\boldsymbol{b}, 6\boldsymbol{c}$. (Javob: a) 28728; b) $\sqrt{870912}$; c) 0).

1.23. $\boldsymbol{a}=4\boldsymbol{i}-6\boldsymbol{j}-2\boldsymbol{k}$, $\boldsymbol{b}=-2\boldsymbol{i}+3\boldsymbol{j}+\boldsymbol{k}$, $\boldsymbol{c}=3\boldsymbol{i}-5\boldsymbol{j}+7\boldsymbol{k}$; a) $6\boldsymbol{a}, 3\boldsymbol{b}, 8\boldsymbol{c}$; b) $-7\boldsymbol{b}, 6\boldsymbol{a}$; c) $-5\boldsymbol{a}, 4\boldsymbol{c}$; d) $\boldsymbol{a}, \boldsymbol{b}$; e) $-5\boldsymbol{a}, 3\boldsymbol{b}, 4\boldsymbol{c}$. (Javob: a) 0; b) 0; c) -560).

1.24. $\boldsymbol{a}=3\boldsymbol{i}-\boldsymbol{j}+2\boldsymbol{k}$, $\boldsymbol{b}=-\boldsymbol{i}+5\boldsymbol{j}-4\boldsymbol{k}$,$=6\boldsymbol{i}-2\boldsymbol{j}+4\boldsymbol{k}$; a) $4\boldsymbol{a}, -7\boldsymbol{b}, -2\boldsymbol{c}$; b) $6\boldsymbol{a}, -4\boldsymbol{c}$; c) $-2\boldsymbol{a}, 5\boldsymbol{b}$; d) $\boldsymbol{a}, \boldsymbol{c}$; e) $6\boldsymbol{a}, -7\boldsymbol{b}, -2\boldsymbol{c}$. (Javob: a) 0; b) 0; c) 160).

1.25. $\boldsymbol{a}=-3\boldsymbol{i}-\boldsymbol{j}-5\boldsymbol{k}$, $\boldsymbol{b}=2\boldsymbol{i}-4\boldsymbol{j}+8\boldsymbol{k}$, $\boldsymbol{c}=3\boldsymbol{i}+7\boldsymbol{j}-\boldsymbol{k}$; a) $2\boldsymbol{a}, -\boldsymbol{b}, 3\boldsymbol{c}$; b) $-9\boldsymbol{a}, 4\boldsymbol{c}$; c) $5\boldsymbol{b}, -6\boldsymbol{c}$; d) $\boldsymbol{b}, \boldsymbol{c}$; e) $2\boldsymbol{a}, 5\boldsymbol{b}, -6\boldsymbol{c}$. (Javob: a) 0; b) $\sqrt{2519424}$; c) 900).

1.26. $\boldsymbol{a}=-3\boldsymbol{i}+2\boldsymbol{j}+7\boldsymbol{k}$, $\boldsymbol{b}=\boldsymbol{i}-5\boldsymbol{k}$, $\boldsymbol{c}=6\boldsymbol{i}+4\boldsymbol{j}-\boldsymbol{k}$; a) $-2\boldsymbol{a}, \boldsymbol{b}, 7\boldsymbol{c}$; b) $5\boldsymbol{a}, -2\boldsymbol{c}$; c) $3\boldsymbol{b}, \boldsymbol{c}$; d) $\boldsymbol{a}, \boldsymbol{c}$; e) $-2\boldsymbol{a}, 3\boldsymbol{b}, 7\boldsymbol{c}$. (Javob: a) 1260; b) $10\sqrt{2997}$; c) 33).

1.27. $\boldsymbol{a}=3\boldsymbol{i}-\boldsymbol{j}+5\boldsymbol{k}$, $\boldsymbol{b}=2\boldsymbol{i}-4\boldsymbol{j}+6\boldsymbol{k}$, $\boldsymbol{c}=\boldsymbol{i}-2\boldsymbol{j}+3\boldsymbol{k}$; a) $-3\boldsymbol{a}, 4\boldsymbol{b}, -5\boldsymbol{c}$; b) $6\boldsymbol{b}, 3\boldsymbol{c}$; c) $\boldsymbol{a}, 4\boldsymbol{c}$; d) $\boldsymbol{b}, \boldsymbol{c}$; e) $-3\boldsymbol{a}, 4\boldsymbol{b}, -5\boldsymbol{c}$. (Javob: a) 0; b) 0; c) 80).

1.28. $\boldsymbol{a}=4\boldsymbol{i}-5\boldsymbol{j}-4\boldsymbol{k}$, $\boldsymbol{b}=5\boldsymbol{i}-\boldsymbol{j}$, $\boldsymbol{c}=2\boldsymbol{i}+4\boldsymbol{j}-3\boldsymbol{k}$; a) $\boldsymbol{a}, 7\boldsymbol{b}, -2\boldsymbol{c}$; b) $-5\boldsymbol{a}, 4\boldsymbol{b}$; c) $8\boldsymbol{c}, -3\boldsymbol{a}$; d) $\boldsymbol{a}, \boldsymbol{c}$; e) $-3\boldsymbol{a}, 4\boldsymbol{b}, 8\boldsymbol{c}$. (Javob: a) 2114; b) $20\sqrt{857}$; c) 0).

1.29. $\boldsymbol{a}=-9\boldsymbol{i}+4\boldsymbol{k}$, $\boldsymbol{b}=2\boldsymbol{i}-4\boldsymbol{j}+6\boldsymbol{k}$, $\boldsymbol{c}=3\boldsymbol{i}-6\boldsymbol{j}+9\boldsymbol{k}$; a) $3\boldsymbol{a}, -5\boldsymbol{b}, -4\boldsymbol{c}$; b) $6\boldsymbol{b}, 2\boldsymbol{c}$; c) $-2\boldsymbol{a}, 8\boldsymbol{c}$; d) $\boldsymbol{b}, \boldsymbol{c}$; e) $3\boldsymbol{a}, 6\boldsymbol{b}, -4\boldsymbol{c}$. (Javob: a) 0; b) 0; c) -144).

1.30. ***a****=5****i*** *- 6****j*** *- 4****k****,* ***b****=4****i*** *+ 8****j*** *- 7****k****,* ***c****=3****j*** *– 4****k****; a) 5****a****, 3****b****, -4****c****; b) 4****b****, a; c) 7****a****, -2****c****; d)* ***a****,* ***b****; e) 5****a****, 4****b****, -2****c****.* (Javob: *a) 11 940; b)* $4\sqrt{9933}$*; c) 28).*

2. Agar piramidaning uchlari A, B, C, D nuqtalarda bo'lsa, quyidagilar hisoblansin:

a) ko'rsatilgan yoqning yuzi;

b) l qirraning o'rtasidan va piramidaning ikki uchi orqali o'tuvchi kesimning yuzi;

c) ABCD piramidaning hajmi.

2.1. *A (3, 4, 5), B (1, 2, 1), C (-2, -3, 6), D (3, -6, -3); a) ACD; b)* l*=AB, C* va *D.* (Javob*: a)* $\sqrt{2114}$*; b)* $\frac{\sqrt{4426}}{2}$*; c) 42.)*

2.2. *A (-7, -5, 6), B (-2, 5, -3), C (3, -2, 4), D (1, 2, 2); a) VCD; b)* l*=CD, A* va *B.* (Javob*: a)* $\sqrt{1350}$*; b)* $\frac{\sqrt{8937}}{2}$*; c)* $\frac{77}{3}$*.)*

2.3. *A (1, 3, 1), B (-1, 4, 6), C (-2, -3, 4), D (3, 4, -4); a) ACD; b)* l*=BC, A* va *D.* (Javob*: a)*$\frac{\sqrt{891}}{2}$*; b)* $\frac{3\sqrt{2}}{2}$*; c) 3.)*

2.4. *A (2, 4, 1), B (-3, -2, 4), C (3, 5, -2), D (4, 2, -3); a) ABD; b)* l*=AC, B* va *D.* (Javob*: a)* $\sqrt{395}$*; b)* $\frac{\sqrt{205}}{2}$*; c)* $\frac{25}{3}$*.)*

2.5. *A (-5, -3, -4), B (1, 4, 6), C (3, 2, -2), D (8, -2, 4); a) ACD; b)* l*=BC, A* va *D.* (Javob: *a)* $\frac{\sqrt{6137}}{2}$*; b)* $\frac{\sqrt{7289}}{2}$*; c)* $\frac{304}{3}$*.)*

2.6. *A (3, 4, 2), B (-2, 3, -5), C (4, -3, 6), D (6, -5, 3); a) ABD; b)* l*=BD, A* va *C.* (Javob*: a) 8*$\sqrt{26}$*; b)* $\frac{\sqrt{1826}}{2}$*; c) 40.)*

2.7. *A (-4, 6, 3), B (3, -5, 1), C (2, 6, -4), D (2, 4, -5); a) ACD; b)* l*=AD, B* va *C.* (Javob: *a)* $\sqrt{94}$*; b)* $\frac{\sqrt{1554}}{2}$*; c)* $\frac{100}{3}$*.)*

2.8. *A (7, 5, 8), B (-4, -5, 3), C (2, -3, 5), D (5, 1, -4); a) BCD; b)* l*=BC, A* va *D.* (Javob*: a)* $\sqrt{1150}$*; b)* $\sqrt{4101}$*; c)* $\frac{202}{3}$*.)*

2.9. *A (3, -2, 6), B (-6, -2, 3), C (1, 1, -4), D (4, 6, -7); a) ABD; b)* l*=BD, A* va *C.* (Javob*: a)* $\sqrt{5040}$*; b)* $\sqrt{212}$*; c) 52.)*

2.10. *A (-5, -4, -3), B (7, 3, -1), C (6, -2, 0), D (3, 2, -7); a) BCD; b)* l*=AD, B* va *C.* (Javob*: a)*$\frac{\sqrt{1422}}{2}$*; b)* $\sqrt{504}$*; c) 44.)*

2.11. *A (3, -5, -2), B (-4, 2, 3), C (1, 5, 7), D (-2, -4, 5); a) ACD; b)* l*=BD, A* va *C.* (Javob*: a)* $\frac{\sqrt{6986}}{2}$*; b)* $\sqrt{1261}$*; c)* $\frac{202}{3}$*.)*

2.12. *A (7, 4, 9), B (1, -2, -3), C (-5, -3, 0), D (1, -3, 4); a) ABD; b)* l*=AB, C* va *D.* (Javob*: a)* $\sqrt{1179}$*; b)* 17*; c) 50.)*

2.13. *A (-4, -7, -3), B (-4, -5, 7), C (2, -3, 3), D (3, 2, 1); a) BCD; b)* l*=BC, A* va *D.* (Javob*: a)* $\sqrt{276}$*; b)* $\sqrt{1393}$*; c)* $\frac{148}{3}$*.)*

2.14. *A (-4, -5, -3), B (3, 1, 2), C (5, 7, -6), D (6, -1, 5); a) ACD; b)* l*=BC, A* va *D.* (Javob*: a)* $\sqrt{7281}$*; b)* $\sqrt{2726}$*; c) 46.)*

2.15. *A (5, 2, 4), B (-3, 5, -7), C (1, -5, 8), D (9, -3, 5); a) ABD; b)* l*=BD, A* va *C.* (Javob*: a) 2*$\sqrt{266}$*; b)* $\frac{\sqrt{1405}}{2}$*; c)* $\frac{286}{3}$*.)*

2.16. *A (-6, 4, 5), B (5, -7, 3), C (4, 2, -8), D (2, 8, -3); a) ACD; b)* l*=AD, B* va *C.* (Javob*: a)* $2\sqrt{251}$*; b)* $25\frac{\sqrt{38}}{2}$*; c)* 150*.)*

2.17. *A (5, 3, 6), B (-3, -4, 4), C (5, -6, 8), D (4, 0, -3); a) BCD; b)* l*=BC, A* va *D.* (Javob*: a)* $\sqrt{2294}$*; b)* $2\sqrt{406}$*; c)* $\frac{332}{3}$*.)*

2.18. *A (5, -4, 4), B (-4, -6, 5), C (3, 2, -7), D (6, 2, -9); a) ABD; b)* l*=BD, A* va *C.* (Javob*: a)* $\sqrt{4140}$*; b)* $\sqrt{405}$*; c)* $\frac{82}{3}$*.)*

2.19. *A (-7, -6, -5), B (5, 1, -3), C (8, -4, 0), D (3, 4, -7); a) BCD; b)* l*=AD, B* va *C.* (Javob: *a)* $\frac{\sqrt{158}}{2}$*; b)* $\frac{\sqrt{2266}}{2}$*; c)* $\frac{86}{3}$*.)*

2.20. *A (7, -1, -2), B (1, 7, 8), C (3, 7, 9), D (-3, -5, 2); a) ACD; b)* l*=BD, A* va *C.* (Javob*: a)* $\sqrt{5957}$*; b)* $\sqrt{1361}$*; c)* $\frac{124}{3}$*.)*

2.21. *A (5, 2, 7), B (7, -6, -9), C (-7, -6, 3), D (1, -5, 2); a) ABD; b)* l*=AB, C* va *D* .(Javob*: a)* $\sqrt{3194}$*; b)19* $\frac{\sqrt{2}}{2}$*; c)* 76*.)*

2.22. *A (-2, -5, -1), B (-6, -7, 9), C (4, -5, 1), D (2, 1, 4); a) BCD; b)* l*=BC, A* va *D.* (Javob: *a)* $\sqrt{1802}$*; b)* $\frac{\sqrt{2142}}{2}$*; c)* $\frac{226}{3}$*.)*

2.23. *A (-6, -3, -5), B (5, 1, 7), C (3, 5, -1), D (4, -2, 9); a) ACD; b)* l*=BC, A* va *D.* (Javob*: a)*$\frac{\sqrt{24101}}{2}$ *b)* $\sqrt{2969}$*; c)* $\frac{4}{3}$*.)*

2.24. *A (7, 4, 2), B (-5, 3, -9), C (1, -5, 3), D (7, -9, 1); a) ABD; b)* l*=BD, A* va *C.* (Javob*: a)* $\sqrt{11161}$*; b)* $\frac{\sqrt{5629}}{2}$*; c)* 186*.)*

2.25. *A (-8, 2, 7), B (3, -5, 9), C (2, 4, -6), D (4, 6, -5); a) ACD; b)* l*=AD, B* va *C.* (Javob: *a)* $\sqrt{584}$*; b)*$\frac{\sqrt{9754}}{2}$*; c)* $\frac{296}{3}$*.)*

2.26. *A (4, 3, 1), B (2, 7, 5), C (-4, -2, 4), D (2, -3, -5); a) ACD; b)* l*=AB, C* va *D.* (Javob*: a)* $\sqrt{1666}$*; b)* $\frac{\sqrt{9746}}{2}$*; c)* $\frac{80}{3}$*.)*

2.27. *A (-9, -7, 4), B (-4, 3, -1), C (5, -4, 2), D (3, 4, 4); a) BCD; b)* l*=CD, A* va *B.* (Javob*: a)* $\sqrt{1346}$*; b)* $\frac{\sqrt{13250}}{2}$*; c)* 120*.)*

2.28. *A (3, 5, 3), B (-3, 2, 8), C (-3, -2, 6), D (7, 8, -2); a) ACD; b)* l*=BD, A* va *C.* (Javob*: a)* $\frac{\sqrt{785}}{2}$*; b)* $\frac{\sqrt{58}}{2}$*; c)* $\frac{26}{3}$*.)*

2.29. *A (4, 2, 3), B (-5, -4, 2), C (5, 7, -4), D (6, 4, -7); a) ABD; b)* l*=AD, B* va *C.* (Javob*: a)* $\sqrt{3086}$*; b)*$\sqrt{501}$*; c)* $\frac{178}{3}$*.)*

2.30. *A (-4, -2, -3), B (2, 5, 7), C (6, 3, -1), D (6, -4, 1); a) ACD; b)* l*=BC, A* va *D.* (Javob*: a)* $\sqrt{1469}$*; b)*$\sqrt{1964}$*; c)* 116*.)*

3. Agar $\vec{F}$ kuch A nuqtaga qo'yilgan bo'lsa, a) $\vec{F}$ kuchning A nuqtadan B nuqtaga tomon to'g'ri chiziq bo'ylab ko'chgandagi bajargan ishi; b) $\vec{F}$ kuchning B nuqtaga nisbatan momentining moduli hisoblansin:

3.1. ***F****=(5, -3, 9), A(3, 4, -6), B(2, 6, 5).* (Javob: *a) 88; b)* $\sqrt{6746}$*).*

3.2. ***F****=(-3, 1, -9), A(6, -3, 5), B(9, 5, -7).* (Javob: *a) 107; b)* $\sqrt{8298}$*).*

3.3. ***F****=(2, 19, -4), A(5, 3, 4), B(6, -4, -1).* (Javob: *a) 111; b)* $\sqrt{16254}$*).*

3.4. ***F****=(-4, 5, -7), A(4, -2, 3), B(7, 0, -3). (*Javob*: a) 40; b)* $\sqrt{2810}$*).*

3.5. ***F****=(4, 11, -6), A(3, 5, 1), B(4, -2, -3). (*Javob*: a) 49; b)* $\sqrt{9017}$*).*

3.6. ***F****=(3, -5, 7), A(2, 3, -5), B(0, 4, 3). (*Javob*: a) 45; b)* $\sqrt{2819}$*).*

3.7. ***F****=(5, 4, 11), A(6, 1, -5), B(4, 2, -6). (*Javob*: a) 17; b)* $\sqrt{683}$*).*

3.8. ***F****=(-9, 5, 7), A(1, 6, -3), B(4, -3, 5). (*Javob*: a) 16; b)* $\sqrt{23614}$*).*

3.9. ***F****=(6, 5, -7), A(7, -6, 4), B(4, 9, -6). (*Javob*: a) 127; b)* $\sqrt{20611}$*).*

3.10. ***F****=(-5, 4, 4), A(3, 7, -5), B(2, -4, 1). (*Javob*: a) 15; b)* $\sqrt{8781}$*).*

3.11. ***F****=(4, 7, -3), A(5, -4, 2), B(8, 5, -4). (*Javob*: a) 93; b) 15*$\sqrt{3}$*).*

3.12. ***F****=(2, 2, 9), A(4, 2, -3), B(2, 4, 0). (*Javob*: a) 27; b) 28).*

Berilgan A nuqtaga qo'yilgan $\vec{P}$, $\vec{Q}$ va $\vec{R}$ kuchlarga nisbatan

a)kuchlarga teng ta'sir etuvchi kuchning qo'yilgan nuqtasidan B nuqtaga tomon to'g'ri chiziqli harakati natijasida bajargan ishi;

b) kuchlarga teng ta'sir etuvchi kuchning B nuqtaga nisbatan momentining miqdori hisoblansin

3.13. ***P****=(9, -3, 4),* ***Q****=(5, 6, -2),* ***R****=(-4, -2, 7), A(-5, 4, -2), B(4, 6, -5) (*Javob: *a) 65; b)* $\sqrt{12883}$*).*

3.14. ***P****=(5, -2, 3),* ***Q****=(4, 5, -3),* ***R****=(-1, -3, 6), A(7, 1, -5), B(2, -3, -6) (*Javob: *a) 46; b) 2*$\sqrt{521}$*).*

3.15. ***P****=(3, -5, 4),* ***Q****=(5, 6, -3),* ***R****=(-7, -1, 8), A(-3, 5, 9), B(5, 6, -3) (*Javob*: a) 100; b)* $\sqrt{1306}$*).*

3.16. ***P****=(-10, 6, 5),* ***Q****=(4, -9, 7),* ***R****=(5, 3, -3), A(4, -5, 9), B(4, 7, -5) (*Javob*: a) 126; b) 2*$\sqrt{3001}$*).*

3.17. ***P****=(5, -3, 1),* ***Q****=4, 2, -6),* ***R****=(-5, -3, 7), A(-5, 3, 7) , B(3, 8, -5 (*Javob*: a) 4; b)* $\sqrt{12389}$*).*

3.18. *.****P****=(-5, 8, 4),* ***Q****=(6, -7, 3),* ***R****=(3, 1, -5), A(2, -4, 7), B(0, 7, 4) (*Javob: *a) 8; b) 4*$\sqrt{197}$*).*

3.19. *.****P****=(7, -5, 2),* ***Q****=(3, 4, -8),* ***R****=(-2, -4, 3), A(-3, 2, 0), B(6, 4, -3) (*Javob*: a) 71; b)* $\sqrt{4171}$*).*

3.20. ***P****=(3, -4, 2),* ***Q****=(2, 3, -5),* ***R****= (-3, -2, 4), A(5, 3, -7), B(4, -1, -4) (*Javob*: a) 13; b)* $\sqrt{195}$*).*

3.21. ***P****=(4, -2, -5),* ***Q****=(5, 1, -3),* ***R****=(-6, 2, 5), A(-3, 2, -6), B(4, 5, -3) (*Javob: *a) 15; b) 2*$\sqrt{262}$*).*

3.22. ***P****=(7, 3, -4),* ***Q****=(9, -4, 2),* ***R****=(-6, 1, 4), A(-7, 2, 5), B(4, -2, 1) (*Javob*: a) 122; b)* $\sqrt{3108}$*).*

3.23. ***P****=(9, -4, 4),* ***Q****=(-4, 6, -3),* ***R****=(3, 4, 2), A(5, -4, 3), B(4, -5, 9) (*Javob: *a) 4; b)* $\sqrt{4126}$*).*

3.24. ***P****=(6, -4, 5),* ***Q****=(-4, 7, 8),* ***R****=(5, 1, -3), A(-5, -4, 2), B(7, -3, 6) (*Javob*: a) 128; b)* $\sqrt{10181}$*).*

3.25. ***P****=(5, 5, -6),* ***Q****=(7, -6, 6),* ***R****=(-4, 3, 4), A(-9, 4, 7), B(8, -1, 7) (*Javob: *a) 126; b) 10*$\sqrt{105}$*).*

3.26. ***P****=(7, -6, 2),* ***Q****=(-6, 2, -1),* ***R****=(1, 6, 4), A(3, -6, 1), B(6, -2, 7) (*Javob*: a) 44; b)* $\sqrt{77}$*).*

3.27. ***P***=(4, -2, 3), ***Q***=(-2, 5, 6), ***R***=(7, 3, -1), A(-3, -2, 5), B(9, -5, 4)
(Javob*: a) 82; b)* $\sqrt{21150}$*).*

3.28. ***P***=(7, 3, -4), ***Q***=(3, -2, 2), ***R***=(-5, 4, 3), A(-5, 0, 4), B(4, -3, 5)
(Javob*: a) 31; b)* $4\sqrt{230}$*).*

3.29. ***P***=(3, -2, 4), ***Q***=(-4, 4, -3), ***R***=(3, 4, 2), A(1, -4, 3), B(4, 0, -2)
(Javob*: a) 15; b)* $5\sqrt{89}$*).*

3.30. ***P***=(2, -1, -3), ***Q***=(3, 2, -1), ***R***=(-4, 1, 3), A(-1, 4, -2), B(2, 3, -1)
(Javob*: a) 0; b)* $\sqrt{66}$*).*

Namunaviy variantning yechilishi.

1. $\vec{a} = 4\vec{i} + 4\vec{k}, \vec{b} = -\vec{i} + 3\vec{j} + 2\vec{k}$ va $\vec{c} = 3\vec{i} + 5\vec{j}$ vektorlar berilgan. Quyidagilar bajarilsin:

a) $\vec{a}, \vec{b}$ va $5\vec{c}$ vektorlarning aralash ko'paytmasi hisoblansin;

b) $3\vec{c}$ va $\vec{b}$ vektorlarning vektor ko'paytmasining moduli topilsin;

c) $\vec{a}$ va $3\vec{b}$ vektorlarning skalyar ko'paytmasi hisoblansin;

d) $\vec{a}$ va $\vec{b}$ vektorlarning kollinearligi yoki ortogonalligi tekshirilsin;

e) $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlarning komplanarligi aniqlansin;

►a) $5\vec{c} = 15\vec{i} + 25\vec{j}$ bo'lganligidan,

$$(\vec{a} \times \vec{b})\cdot 5\vec{c} = \begin{vmatrix} 4 & 0 & 4 \\ -1 & 3 & 2 \\ 15 & 25 & 0 \end{vmatrix} = -100 - 180 - 200 = -480;$$

b) $3\vec{s} = 9\vec{i} + 15\vec{j}$ $bo'lganligi$ $uchun$,

$$3\vec{s} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 9 & 15 & 0 \\ -1 & 3 & 2 \end{vmatrix} = 30\vec{i} + 27\vec{k} + 15\vec{k} - 18\vec{j} = 30\vec{i} - 18\vec{j} + 42\vec{k}$$ hamda

$$\overline{3\vec{s}} \times \vec{b} = \sqrt{30^2 + (-18)^2 + 42^2} = \sqrt{2988};$$

c) $3\vec{b} = -3\vec{i} + 9\vec{j} + 6\vec{k}$ bo'lganligidan, $\vec{a}\ 3\vec{b}$ = 4 · (-3) + 0 9 + 4 6 = 12;

d) $\vec{a} = (4; 0; 4)$ va $\vec{b}$=(-1; 3; 2) dan, $\frac{4}{-1} \neq \frac{0}{3} \neq \frac{4}{2}$ bo'lganligi uchun $\vec{a}$ va $\vec{b}$ vektorlar kollinear emas.

$\vec{a} \cdot \vec{b} = 4 \cdot (-1) + 0 \cdot 3 + 4 \cdot 2 = 4 \neq 0$ bo'lganligi uchun $\vec{a}$ va $\vec{b}$ vektorlar ortogonal ham emaslar.

$$e)\ \vec{a}\vec{b}\vec{c} = \begin{vmatrix} 4 & 0 & 4 \\ -1 & 3 & 2 \\ 3 & 5 & 0 \end{vmatrix} = -20 - 36 - 40 \neq 0,\ ya'ni,$$

$\vec{a}, \vec{b}$ va $\vec{c}$ nokomplanar vektorlar ◄

2. Piramidaning uchlari A(2; 3; 4), B(4; 7; 3), C(1; 2; 2) va D(-2; 0; -1) nuqtalardadir.

a) *ABC* yoq yuzi hisoblansin;

b) *AB, AC* va *AD* qirralar orqali o'tuvchi kesim yuzasi topilsin;

c) *ABCD* piramidaning hajmi hisoblansin;

►a) $S_{\Delta ABC}=\frac{1}{2}\left|\overrightarrow{AB}\cdot\overrightarrow{AC}\right|$ kabi bo'lganligidan va $\overrightarrow{AB}$=(2; 4;-1), $\overrightarrow{AC}$=(-1;-1;-2) bo'lganligi uchun

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & 4 & -1\\ -1 & -1 & -2\end{vmatrix}=-9\vec{i}+5\vec{j}+2\vec{k},\text{ hamda}$$

$$S_{\Delta ABC}=\frac{1}{2}\cdot\sqrt{(-9)^2+5^2+2^2}=\frac{1}{2}\sqrt{110};$$

b) *AB, BC* va *AD* qirralarning o'rtalari mos ravishda *K(3;5;3,5),M(1,5;2,5;3)* va *N(0;1,5;1,5)* nuqtalarda yotadi. Shuning uchun,

$S_{\text{kesim}}=\frac{1}{2}\left|\overrightarrow{KM}\cdot\overrightarrow{KN}\right|$ deb yozamiz. $\overrightarrow{KM}$=(-1,5; -2,5; -0,5) va $\overrightarrow{KN}$=(-3; -3,5; -2) bo'lganidan,

$$\overrightarrow{KM}\times\overrightarrow{KN}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -1{,}5 & -2{,}5 & -0{,}5\\ -3 & -3{,}5 & -2\end{vmatrix}=3{,}25\vec{i}-1{,}5\vec{j}-2{,}25\vec{k}$$ ni hosil qilamiz. U holda

$S_{\text{kesim}}=\frac{1}{2}\sqrt{3{,}25^2+1{,}5^2+2{,}25^2}=\frac{1}{2}\sqrt{17{,}875}$ ga teng bo'ladi.

c) $V_{\text{piramida}}=\frac{1}{6}\left|(\overrightarrow{AB}\cdot\overrightarrow{AC})\cdot\overrightarrow{AD}\right|$ bo'lganligi uchun hamda $\overrightarrow{AD}$=(-4; -3; -5) ni inobatga olsak,

$$(\overrightarrow{AB}\times\overrightarrow{AC})\cdot\overrightarrow{AD}=\begin{vmatrix}2 & 4 & -1\\ -1 & -1 & -2\\ -4 & -3 & -5\end{vmatrix}=11\text{ unda }V=\frac{11}{6}$$ ni hosil qilamiz.◄

3. $\vec{F}$=(2, 3, -5) kuch A (1, -2, 2) nuqtaga qo'yilgan bo'lsa:

a) $\vec{F}$ kuch A (1, -2, 2) nuqtadan B (1, 4, 0) nuqtaga to'g'ri chiziqli harakat qilib ko'chgandagi uning bajargan ishi;

b) $\vec{F}$ kuchning B nuqtaga nisbatan momentining moduli hisoblansin.

►a) A = $\vec{F}\cdot\vec{S}$ hamda $\vec{S}=\overrightarrow{AB}$ = (0,6,-2) bo'lganligidan, $\vec{F}\ \overrightarrow{AB}$= 2·0+3·6+(-5)·(-2) = 28, demak: A = 28.

b) Kuchning momenti formulasiga ko'ra, $\vec{M}\quad\overrightarrow{BA}\ \vec{F}$ dan hamda $\overrightarrow{BA}$ =(0, -6, 2) bo'lganligidan:

$$\overrightarrow{BA}\times\vec{F}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 0 & -6 & 2\\ 2 & 3 & -5\end{vmatrix}=24\vec{i}+4\vec{j}+12\vec{k}\text{ demak }\left|\vec{M}\right|=\sqrt{24^2+4^2+12^2}=4\sqrt{46}$$ ◄

2.5. 2 – bobga doir qo'shimcha topshiriqlar.

1. Berilgan $\vec{a}=2\vec{i}-\vec{j}+3\vec{k}$, $\vec{b}=\vec{i}-3\vec{j}+2\vec{k}$ va $\vec{s}=3\vec{i}+2\vec{j}-4\vec{k}$ vektorlarga nisbatan, $\vec{x}$ vektor uchun $\vec{x}\cdot\vec{a}$=-5, $\vec{x}\cdot\vec{b}$ = -11 va $\vec{x}\cdot\vec{s}$ =20 kabi shartlar bajariladigan bo'lsa, u holda $\vec{x}$ vektorning koordinatalari topilsin. (Javob: $\vec{x}=2\vec{i}+3\vec{j}-2\vec{k}$)

2. Agar $\vec{x}$ vektor, Oz o'qiga va $\vec{a}$ = (8, -15, 3) vektorga perpendikulyar bo'lib, Ox o'q bilan o'tkir burchak hosil qiladigan bo'lsa, $|\vec{x}|$ = 51 ekanligini inobatga olib, $\vec{x}$ vektorning koordinatalari aniqlansin. (Javob: $\vec{x}$ = (45, 24, 0).)

3. To'g'ri kanalning qirg'oqlari bo'ylab o'zgarmas tezlikda harakatlanayotgan ikkita traktor ikkita arqon yordamida barka (kema)ni tortib ketmoqda. Agarda, arqonlarning tortishish kuchlari $|\vec{F}_1|$ = 800 H va $|\vec{F}_2|$ = 960 H kabi bo'lib, ular orasidagi burchak 60° bo'lsa, u holda barkaning harakatiga suvning qarshiligini hamda arqonlar bilan harakat yo'nalishi orasidagi α va β burchaklar aniqlansin. (Bu erda barka (kema)ning harakatini qirg'oq bo'ylab parallel deb olinadi.) (Javob: $|\vec{S}|$ ≈ 1530 H, α ≈33°, β ≈27°).

4. C(-1,4,-2) nuqtaga qo'yilgan uchta $\vec{F}$= (2,-1,-3), $\vec{Q}$=(3,2,-1) va $\vec{P}$= (-4,1,3) kuchlarga teng ta'sir etuvchi kuchning A(2,3,-1) nuqtaga nisbatan momentining miqdorini hamda yo'naltiriuvchi kosinuslarini aniqlansin. (Javob:$\sqrt{66}$, $\cos\alpha=\frac{1}{\sqrt{66}}$, $\cos\beta= -\frac{4}{\sqrt{66}}$, $\cos\gamma= -\frac{7}{\sqrt{66}}$)

5. Tetraedrning uchta uchlari A (2, 1, -1), B (3, 0, 1) va C (2, -1, 3) nuqtalarda bo'lib, uning hajmi V = 5 ga teng bo'lsa, hamda uning to'rtinchi D uchi Oy o'qida yotadigan bo'lsa, u D nuqtaning koordinatalari topilsin. (Javob: D_1 (0, 8, 0), D_2 (0, -7, 0).)

6. Agar rombning tomonlari umumiy uchdan chiqadigan $\vec{a}$ va $\vec{b}$ vektorlarda yotadigan bo'lsa, rombning diagonallari o'zaro perpendikulyar ekanligi isbotlansin.

7. Ikkita o'zaro perpendikulyar ortlar bo'yicha yoyilgan $\overrightarrow{AB} = 5\vec{a} + 2\vec{b}$, $\overrightarrow{BC} = 2\vec{a} - 4\vec{b}$, $\overrightarrow{CA} = -7\vec{a} + 2\vec{b}$, vektorlar berilgan. ABC uchburchakning $\overrightarrow{AM}$ medianasi va $\overrightarrow{AD}$,balandligi aniqlansin. (Javob: $|\overrightarrow{AM}| = 6$, $|\overrightarrow{AD}| = 12\sqrt{5}/5$.)

8. Agar $\vec{a}$, $\vec{b}$ va $\vec{s}$ vektorlar uchun $\vec{a} \times \vec{b} + \vec{b} \times \vec{s} + \vec{s} \times \vec{a} = 0$ kabi tenglik o'rinli bo'lsa, u vektorning komplanarliklari isbotlansin.

9. ABCD trapeksiyada asoslari $|\overrightarrow{AD}| : |\overrightarrow{BC}|$ = λ nisbatda bo'lib, $\overrightarrow{AC} = \vec{a}$, $\overrightarrow{BD} = \vec{b}$ vektorlar orqali $\overrightarrow{AB}$, $\overrightarrow{BC}$, $\overrightarrow{CD}$, $\overrightarrow{DA}$ ifodalansin. (Javob: $\overrightarrow{AB} = \lambda\frac{a+b}{1+\lambda}$, $\overrightarrow{BC} = \frac{\vec{a}+\vec{b}}{1+\lambda}$, $\overrightarrow{CD} = \frac{\lambda b - a}{1+\lambda}$, $\overrightarrow{DA} = -\frac{\lambda(a+b)}{1+\lambda}$.)

10. OABC tetraedr berilgan. Boshi $\overrightarrow{OA}$ qirraning o'rtasida bo'lib, uchi ABC uchburchak medianalarining kesishish nuqtasida bo'lgan $\overrightarrow{EF}$ vektorni $\overrightarrow{OA}$, $\overrightarrow{OB}$ va $\overrightarrow{OC}$ vektorlar orqali ifodalansin. (Javob: $\overrightarrow{EF} =(2\overrightarrow{OB}+2\overrightarrow{OC}-\overrightarrow{OA})/6$)

11. To'rtta $\vec{a}$=(1, 2, 3), $\vec{b}$=(2, -2, 1), $\vec{c}$=(4, 0, 3) va $\vec{d}$=(16, 10, 18) kabi vektorlar berilgan. Agar $\vec{x}$ vektor, $\vec{d}$ vektorning $\vec{a}$ va $\vec{b}$ vektorlar tekislikdagi $\vec{s}$ vektorga parallel bo'lgan yo'nalish bo'yicha proeksiyasi bo'lsa, u $\vec{x}$ vektorning koordinatalari topilsin. (Javob: $\vec{x}$= (-4, 10, 3).)

12. ABC to'g'ri burchakli uchburchakning to'g'ri burchagi uchidan $\overrightarrow{AB}$ gipotenuzaga $\overrightarrow{CH}$ perpendikulyar tushirilgan. Agar katetlarning uzunliklari $|\overrightarrow{BC}|$ =a va $|\overrightarrow{CA}|$ =b bo'lsa, $\overrightarrow{CH}$ vektorni $\overrightarrow{CA}$ va $\overrightarrow{CB}$ vektorlar orqali ifodalansin. (Javob: $\overrightarrow{CH} = (a^2\overrightarrow{CA} + b^2\overrightarrow{CB}) / (a^2 + b^2)$.)

13. Ikkita A (1, 2, 3) va B (7, 2, 5) nuqtalar berilgan. AB kesma yotgan to'g'ri chiziq ustida shunday bir M nuqta aniqlansinki, B va M nuqtalar, A nuqtaning har xil tomonida joylashgan bo'lib, AM kesma esa, AB kesmadan ikki marta uzunroq bo'ladigan bo'lsin. (Javob: M (-11, 2, -1).)

14. Ikkita $\vec{a}$ = (-3, 0, 4) va $\vec{b}$ = (5, -2, -14) vektorlar bitta O nuqtadan chiquvchi vektorlar bo'lsa, shunday bir $\vec{e}$ birlik vektorning koordinatalari topilsinki, u o'z navbatida

O nuqtadan chiqib, $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchakni teng ikkiga bo'ladigan bo'lsin. (Javob: $\vec{e}$ = (- $\frac{2}{\sqrt{6}}$, $\frac{1}{\sqrt{6}}$, - $\frac{1}{\sqrt{6}}$).)

15. Trapetsiyaning uchta ketma – ket keladigan uchlari A (-3,-2,-1),B (1,2,3) va C(9,6,4) nuqtalardadir. Agar AD asos uzunligi 15 ga teng bo'lsa, uning to'rtinchi D uchining, diagonallarining kesishish nuqtasi M ning hamda yon tomonlarining kesishish nuqtasi N ning koordinatalari topilsin. (*Javob*: D ($\frac{31}{3}$, $\frac{14}{3}$, $\frac{2}{3}$), M($\frac{9}{2}$, 3, $\frac{17}{8}$), N(7, 8, 9).)

16. Kubning bir uchidan chiquvchi uchta yon yoqlarining diagonallari bo'ylab yo'nalgan uchta kuchning qiymatlari mos ravishda 1, 2 va 3 ga teng. Ushbu kuchlarga teng ta'sir etuvchi kuchning qiymati hamda uning tashkil etuvchi kuchlar bilan hosil qilgan burchaklari aniqlansin. (Javob:5,$arccos\frac{7}{10}$, $arccos\frac{8}{10}$, $arccos\frac{9}{10}$).

17. Ikkita $\vec{a}$ = (8, 4, 1) va $\vec{b}$ = (2, -2, 1) kabi vektorlar berilgan. Shunday bir $\vec{c}$ vektor aniqlansinki, u $\vec{a}$ va $\vec{b}$ vektorlar bilan komplanar vektor bo'lib, $\vec{a}$ vektorga perpendikulyar bo'lsin hamda $\vec{b}$ vektor bilan o'tmas burchak tashkil etib uzunligi $|\vec{b}|$ ga teng bo'lsin. (Javob: $\vec{c}$ = (-$\frac{5}{\sqrt{2}}$, $\frac{11}{\sqrt{2}}$, - $\frac{4}{\sqrt{2}}$).)

18. Ikkita $\vec{a} = 7\vec{i} + 6\vec{j} - 6\vec{k}$ bilan $\vec{b} = 6\vec{i} + 2\vec{j} + 9\vec{k}$ vektorlarni kubning qirralari deb qarash mumkin ekanligini ko'rsatib, uning uchinchi qirrasini ifodalaydigan vektor aniqlansin. (Javob: $\pm(6\vec{i} - 9\vec{j} - 2\vec{k})$.)

19. Uchta $\vec{a}$ = (8, 4, 1), $\vec{b}$ = (2, -2, 1) va $\vec{c}$ = (4, 0, 3) kabi vektor berilgan. Shunday bir $\vec{d}$ birlik vektor topilsinki, u $\vec{a}$ va $\vec{b}$ vektorlarga perpendikulyar bo'lib, tartiblangan $\vec{a}, \vec{b}, \vec{c}$ va $\vec{a}, \vec{b}, \vec{d}$ vektorlar bir xil orientatsiyalangan bo'lsin.

(Javob: d = (- $\frac{1}{3\sqrt{2}}$, $\frac{1}{3\sqrt{2}}$, $\frac{4}{3\sqrt{2}}$).)

20. Biror O nuqtaga qo'yilgan uchta $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = \vec{b}$ va $\overrightarrow{OC} = \vec{c}$ vektorlar berilgan. O nuqtadan chiqib, $\overrightarrow{OA}$, $\overrightarrow{OB}$ va $\overrightarrow{OC}$ vektorlar bilan o'zaro teng o'tkir burchaklar tashkil etadigan $\overrightarrow{OD} = \vec{d}$ vektor topilsin. (*Javob*: $\vec{d} = \pm (|\vec{a}|(\vec{b} \times \vec{c}) + |\vec{b}|(\vec{c} \times \vec{a}) + |\vec{c}|(\vec{a} \times \vec{b}))$.)

3. TEKISLIKLAR VA TO'G'RI CHIZIQLAR

3.1. TEKISLIK

Asosiy teorema.To'g'ri burchakli Dekart koordinatlari sistemasidagi har qanday tekislik tenglamasi

$$Ax + By + Cz + D = 0 \qquad (3.1)$$

kabi ko'rinishga keltiriladi (bu erda *A,B,C* va *D* lar berilgan sonlar bo'lib, $A^2 + B^2 + C^2 < \neq 0$ shartni qanoatlantiradi) va aksincha, (3.1) tenglama, har doim biror tekislikning tenglamasi bo'ladi. Mazkur (3.1) tenglama, tekislikning umumiy tenglamasi deb yuritiladi. Tenglamadagi *A,B,C* koeffitsiyentlar, shu tenglama bilan ifodalanadigan tekislikka perpendikulyar bo'lgan $\vec{n}$ vektorining koordinatalari bo'lib ushbu vektor tekislikning normal vektori deb ataladi, hamda u fazodagi koordinatalar sistemasiga nisbatan tekislikning joylashishini (orientatsiyasini) belgilaydi.

Tekisliklarning turli xil usullarda berilishi va ularga mos tenglamalarning ko'rinishlari mavjud.

1. *Berilgan nuqta va normal vektor orqali aniqlangan tekislik tenglamasi.* Agar tekislik $M_0(x_0; y_0; z_0)$ nuqtadan o'tib, $\vec{n} = (A; B; C)$ vektoriga perpendikulyar bo'ladigan bo'lsa, u holda uning tenglamasi quyidagicha yoziladi:

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0\,. \qquad (3.2)$$

2. *Tekislikning kesmalardagi tenglamasi.*Agar tekislik koordinata o'qlari Ox, Oy, Oz larni mos ravishda $M_1(a; 0; 0)$, $M_2(0; b; 0)$ *va* $M_3(0; 0; c)$nuqtalarda kesib o'tadigan bo'lsa, u holda uning tenglamasini quyidagicha yozish mumkin:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1\,. \qquad (3.3)$$

Bu erda: a≠0; b≠ 0; c≠ 0.

3. *Uchta nuqta bo'yicha tekislik tenglamasi.* Agar tekislik bitta to'g'ri chiziqda yotmaydigan $M_i(x_i; y_i; z_i)$ $(i = \overline{1,3})$ nuqtalaridan o'tadigan bo'lsa, u holda uning tenglamasini quyidagicha yozish mumkin:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0\,. \qquad (3.4)$$

Ushbu determenantni birinchi satr elementlari buyichajchib (3.2) ga kelamiz.

Yuqorida ko'rib o'tilgan (3.2) – (3.4) tenglamalarni (3.1) ko'rinishga keltirish mumkin.

Tekislikka nisbatan eng sodda masalalarni qarab o'tamiz:

1. Umumiy tenglamalari $A_1x + B_1y + C_1z + D_1 = 0$ va $A_2x + B_2y + C_2z + D_2 = 0$ bo'lgan tekisliklar orasidagi φ burchak quyidagi formulalar orqali aniqlanadi:

$$\cos\varphi = \cos(\overrightarrow{n_1}\hat{}\overrightarrow{n_2}) = \frac{(\overrightarrow{n_1}\cdot\overrightarrow{n_2})}{|\overrightarrow{n_1}|\cdot|\overrightarrow{n_2}|} = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} * \sqrt{A_2^2 + B_2^2 + C_2^2}}, \qquad (3.5)$$

bu erda, $\overrightarrow{n_1}=(A_1, B_1, C_1)$bilan $\overrightarrow{n_2}=(A_2, B_2, C_2)$ vektorlar berilgan tekisliklarning normal vektorlaridir.

(3.5) formula yordamida berilgan *tekisliklarning perpendikulyarlik* $(\overrightarrow{n_1}\cdot\overrightarrow{n_2}) = 0$ yoki $A_1A_2 + B_1B_2 + C_1C = 0$ *va parallellik*

$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} \neq \frac{D_1}{D_2}$ *shartlari* yqoridagi ko'rinishga ega bo'ladi.

2. Berilgan $M_0(x_0, y_0, z_0)$ nuqtadan umumiy tenglamasi $Ax + By + Cz + D = 0$ kabi bo'lgan *tekislikkacha bo'lgan masofa d* quyidagi formula bilan aniqlanadi:

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}.$$

3.1–AT

1. Quyida berilganlarga binoan:

a) $M_0(7, -3,5)$ nuqtadan o'tib, *Oxz* tekislikka parallel,

b) A(-3, 1, -2) nuqtadan va *Oz* o'qidan o'tuvchi,

c) $M_1(4,0, -2)$ va $M_2(5,1,7)$ nuqtalardan o'tuvchi hamda *Ox* o'qiga parallel,

d) *V(2, 1, -1)* nuqtadan o'tuvchi va $\vec{n} = (1, -2,3)$ normal vektorga ega,

e) C*(3, 4, -5)* nuqtadan o'tib, $\vec{a} = (3,1, -1)$va $\vec{b} = (1, -2,1)$vektorlariga parallel tekislikning tenglamasi tuzilib, tekislik chizmada tasvirlansin.

(Javob:$a)\ y + 3 = 0, b)x + 3y = 0, c)9y - z - 2 = 0, d)x - 2y + 3z + 3 = 0,\ e)x + 4y + 7z + 16 = 0.$)

2. Uchlari *A(5,4,3), B(2,3,-2), C(3,4,2)* va D*(-1,2,1)* nuqtalarda bo'lgan tetraedrning biror yon yoqining tenglamasi tuzilsin. Hosil qilingan tenglamaning to'g'riligi tekshirilsin.

3. Quyida berilganlarga binoan:

a)$M_1(1,1,1)$ va $M_2(2,3,4)$ nuqtalardan o'tuvchi hamda *2x-7y+5z+9 = 0* tekislikka perpendikulyar:

b) $M_0(7, -5,1)$ nuqtadan o'tib, koordinata o'qlaridan bir xil musbat kesmalar kesadigan tekislikning tenglamasi tuzilsin

(Javob: a) $31x + y - 11z - 21 = 0$, b) $x + y + z - 3 = 0$.)

4. Umumiy tenglamalari *x-2y+2z-3=0* va *3x–4y+5=0* bo'lgan tekisliklar orasidagi burchak hisoblansin.

(Javob: $cos\varphi = \frac{11}{15}$, $\varphi \approx 45^0 51'$.)

5. Umumiy tenglamalari *3x + 6y + 2z – 15* = 0 va *3x + 6y + 2z + 13 = 0* bo'lgan tekisliklar orasidagi masofa hisoblansin (javob: 4.)

6. Umumiy tenglamalari 3x – y + 7z – 4 = 0 va 5x + 3y – 5z + 2 = 0 bo'lgan tekisliklar orasidagi ikki yoqli burchaklarni teng ikkiga bo'luvchi tekisliklarning tenglamalari tuzilsin.

(Javob: $x + 2y - 6z + 3 = 0$, $4x + y + z - 1 = 0$.)

Mustaqil ish.

1. Berilgan *P(1,0,2)* nuqtadan o'tib, *2x–y+3z–1=0* va *3x+6y+3–5=0* tekisliklarga perpendikulyar bo'lgan tekislik tenglamasi tuzilsin. (Javob: *7x–y-5 z +3=0*).

2. Absissa *Ox* va ordinata *Ou* o'qlaridan *a=3* va *b=-2* kesmalar kesib, $\vec{S} = (2,1,-1)$ vektorga parallel bo'lgan tekislik tenglamasi tuzilsin.(Javob: *2x–3y+z–6=0)*

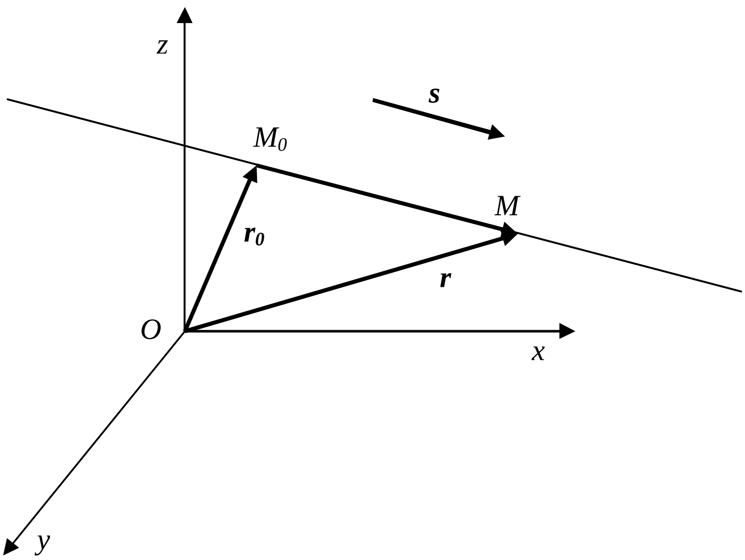
3. Umumiy tenglamasi *2x–2y+4z–5=0* bo'lgan tekislikka perpendikulyar bo'lib, *Ox* va *Ou* o'qlaridan mos ravishda *a=-2 va b=2/3* birlikdagi kesma kesadigan tekislik tenglamasi tuzilsin. (Javob: *x – 3y – 2z + 2 = 0.*)

4. *4x–3y–z–7=0* tekislikka nisbatan *R(-3,1,-9)* nuqtaga simmetrik bo'lgan Q nuqtaning koordinatalari topilsin. (Javob*: Q(1, -2, -10).)*

3.2. Fazodagi to'g'ri chiziq. To'g'ri chiziq va tekislik.

Fazodagi to'g'ri chiziqning berilishiga binoan, uning turli hildagi tenglamalarini qarash mumkin.

1. *To'g'ri chiziqning vektor – parametrik tenglamasi.*



3.1-rasm

Faraz qilamiz, to'g'ri chiziq $M_0(x_0, y_0, z_0)$ nuqtadan o'tib, $\vec{s} = (m; n; p)$ vektorga parallel bo'lsin; *M(x,y,z)* esa, to'g'ri chiziqning ixtiyoriy nuqtasi bo'lsin. Agar $\vec{r_0}$ bilan $\vec{r}$ lar M_0 va M (3.1-rasm) nuqtalarning radius vektorlari bo'ladigan bo'lsa, vektorlarni qo'shish qoidasiga ko'ra, quyidagi

$$\vec{r} = \vec{r_0} + (t \cdot \vec{s}) \quad (-\infty < t < +\infty) \qquad (3.6)$$

tenglik o'rinli bo'ladi.

Mazkur tenglama *to'g'ri chiziqning vektor – parametrik tenglamasi* deb yuritiladi, $\vec{s}$ - *to'g'ri chiziqning yo'naltiruvchi vektori* deb ataladi hamda t ni o'zgaruvchi *parametr* deyiladi.

2. *To'g'ri chiziqning parametrik tenglamalari*.

(3.6) tenglamadan quyidagi uchta tenglamani hosil qilamiz,

$$\left.\begin{aligned} x &= x_0 + mt, \\ y &= y_0 + nt, \\ z &= z_0 + pt, \end{aligned}\right\} \tag{3.7}$$

ular *to'g'ri chiziqning parametrik tenglamalari* deyiladi.

3. *To'g'ri chiziqning kanonik (eng sodda) tenglamasi*.

Agar (3.7) tenglamalarning har birini t ga nisbatan yechib, hosil bo'lgan ifodalarni tenglashtirsak,

$$\frac{x-x_0}{m} = \frac{y-y_0}{n} = \frac{z-z_0}{p} \tag{3.8}$$

to'g'ri chiziqning kanonik tenglamasini hosil qilamiz. Ta'kidlash lozimki, (3.6) – (3.8) tenglamalardan biri orqali boshqa tenglamalarni hosil qilish mumkin.

4. *Fazodagi berilgan ikkita nuqtadan o'tuvchi to'g'ri chiziq tenglamasi* .

Agar to'g'ri chiziq $M_1(x_1; y_1; z_1)$ *va* $M_2(x_2; y_2; z_2)$kabi berilgan nuqtalardan o'tadigan bo'lsa, tenglamasi quyidagicha yoziladi:

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1} = \frac{z-z_1}{z_2-z_1} \ . \tag{3.9}$$

5. *Fazodagi to'g'ri chiziqning umumiy tenglamasi ikkita*

$$\left.\begin{aligned} A_1x + B_1y + C_1z + D_1 = 0 \\ A_2x + B_2y + C_2z + D_2 = 0 \end{aligned}\right\}, \qquad \begin{aligned} \vec{n}_1 = (A_1, B_1, C_1), \\ \vec{n}_2 = (A_2, B_2, C_2), \end{aligned} \tag{3.10}$$

kabi kesishuvchi tekisliklar fazodagi to'g'ri chiziqni beradi. Bu erda: $\overrightarrow{n_1} = (A_1; B_1; C_1)$ *va* $\overrightarrow{n_2} = (A_2; B_2; C_2)$. $\overrightarrow{n_1} \neq \overrightarrow{n_2}$. Mazkur (3.10) ifoda fazodagi to'g'ri chiziqning umumiy tenglamasi deb yuritiladi. To'g'ri chiziqning yo'naltiruvchi $\vec{s}$ vektori

$$\vec{s} = \overrightarrow{n_1} \times \overrightarrow{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix}$$

formula orqali aniqlanadi.To'g'ri chiziqda yotuvchi belgilangan $M_0(x_0; y_0; z_0)$ni esa, (3.10) sistemani yechib aniqlanadi. Natijada, (3.10) tenglamani (3.8) kabi kanonik tenglamaga keltiriladi.

1-misol. Agar to'g'ri chiziq

$$\left.\begin{aligned} x - y + 2z + 4 = 0, \\ 3x + y - 5z - 8 = 0 \end{aligned}\right\}$$

umumiy tenglama bilan berilgan bo'lsa, uning kanonik tenglamasi yozilsin.

► $\vec{s} = \overrightarrow{n_1} \times \overrightarrow{n_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & 2 \\ 3 & 1 & -5 \end{vmatrix} = (3; 11; 4)$ ni aniqlaymiz. Agar berilgan sistemada z = 0 deb olsak, undan x = 1 va u = 5 larni topamiz, ya'ni, to'g'ri chiziqda yotuvchi nuqta $M_0(1; 5; 0)$ dan iboratdir. Kanonik tenglama esa, quyidagicha yoziladi:

$$\frac{x-1}{3} = \frac{y-5}{11} = \frac{z}{4}. ◀$$

Fazodagi ikkita to’g’ri chiziqning o’zaro joylashuvi holatlarini ko’rib chiqamiz:

Agar ikkita to’g’ri chiziq fazoda qaralayotgan bo’lsa, ular yoki kesishishadi yoki parallel, yoki ustma-ust tushadilar, ya’ni har qanday holatda ham ular biror burchak tashkil etadilar va u to’g’ri chiziqlarning yo’naltiruvchi vektorlari $\overrightarrow{s_1}$ bilan $\overrightarrow{s_2}$ orasidagi burchakka teng bo’ladi. Agar to’g’ri chiziqlar o’zlarining kanonik tenglamalari bo’lgan quyidagi

$$\frac{x-x_1}{m_1} = \frac{y-y_1}{n_1} = \frac{z-z_1}{p_1} \quad va \quad \frac{x-x_2}{m_2} = \frac{y-y_2}{n_2} = \frac{z-z_2}{p_2}, \tag{3.11}$$

tenglamalar orqali berilgan bo’lsa, ular orasidagi φ burchak quyidagicha aniqlanadi:

$$cos\varphi = cos(\overrightarrow{s_1}\wedge\overrightarrow{s_2}) = \frac{(\overrightarrow{s_1}\cdot\overrightarrow{s_2})}{|\overrightarrow{s_1}|\cdot|\overrightarrow{s_2}|} = \frac{m_1m_2+n_1n_2+p_1p_2}{\sqrt{m_1^2+n_1^2+p_1^2} * \sqrt{m_2^2+n_2^2+p_2^2}} \tag{3.12}$$

Shuningdek, *to’g’ri chiziqlarning o’zaro perpendikuliyarlik sharti* quyidagicha yoziladi:

$$\overrightarrow{s_1}\cdot\overrightarrow{s_2} = 0 \quad yoki \quad m_1m_2 + n_1n_2 + p_1p_2 = 0.$$

Mazkur *to’g’ri chiziqlarning o’zaro parallellik sharti* esa, $\frac{m_1}{m_2} = \frac{n_1}{n_2} = \frac{p_1}{p_2}$ kabi bo’ladi.

O’zaro parallel bo’lmagan to’g’ri chiziqlarning kesishishining zarur va etarli sharti quyidagicha yoziladi:

$$\overrightarrow{M_1M_2} \cdot \overrightarrow{S_1} \cdot \overrightarrow{S_2} = 0 \text{ yoki} \begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ m_1 & n_1 & p_1 \\ m_2 & n_2 & p_2 \end{vmatrix} = 0 \ . \tag{3.13}$$

Berilgan $M_1(x_1, y_1, z_1)$ nuqtadan, $M_0(x_0, y_0, z_0)$ nuqtadan o’tuvchi va $\vec{s} = (m, n, p)$ yo’naltiruvchi vektorga ega bo’lgan (3.8) to’g’ri chiziqqacha bo’lgan masofa h quyidagicha aniqlanadi.

$$h = \frac{|\vec{s}\times\overrightarrow{M_0M_1}|}{|\vec{s}|}. \tag{3.14}$$

Endi to’g’ri chiziq bilan tekislikning o’zaro joylashuvini ko’rib o’tamiz.

$Ax + By + Cz + D = 0$ tekislik bilan (3.8) to’g’ri chiziq kesishishi, parallel bo’lishi, yoki to’g’ri chiziq tekislikda yotishi ham mumkin.

To’g’ri chiziqning kanonik tenglamasi bo’lgan (3.8) dan uning parametrik tenglamasi (3.7) ga o’tib, x,y,z larning qiymatlarini tekislik tenglamasiga qo’ysak, noma’lum t parametriga nisbatan,

$$(Am + Bn + Cp)t + (Ax_0 + By_0 + Cz_0 + D) = 0 \tag{3.15}$$

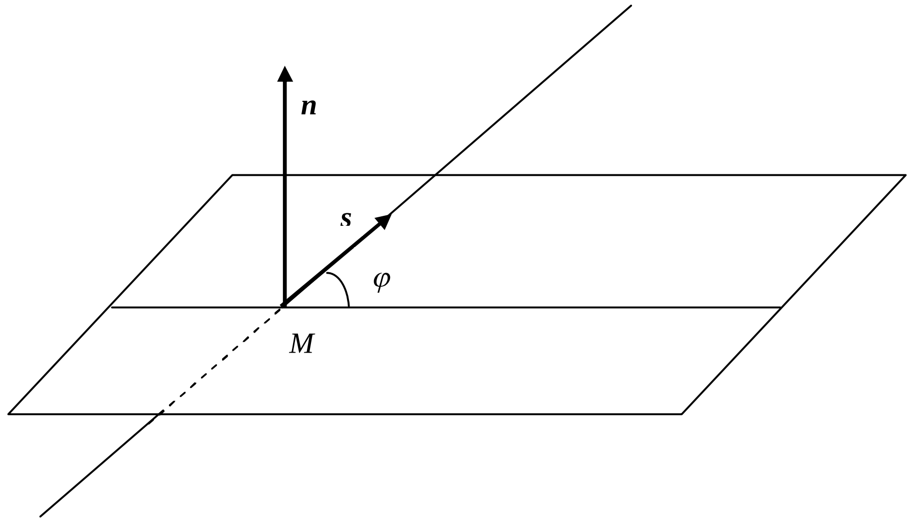
tenglama xosil bo’ladi.

Quyidagi uch hol bo’lishi mumkin:

1. Agar $Am + Bn + Cp \neq 0$ bo’lsa, (3.15) tenglama

$$t = -(Ax_0 + By_0 + Cz_0 + D)/(Am + Bn + Cp)$$

yagona yechimga ega bo’ladi, uni keltirib (3.7) parametrik tenglamalariga qo’yadigan bo’lsak, tekislik bilan to’g’ri chiziqning kesishishi nuqtasi M ning koordinatalari aniqlangan bo’ladi (3.2-rasm).



(3.2-rasm).

2. Agar $Am + Bn + Cp = 0$ bo'lib,

$$Ax_0 + By_0 + Cz_0 + D \neq 0 \qquad (3.16)$$

bo'lsa, (3.15)tenglama yechimga ega bo'lmaydi va to'g'ri chiziq tekislik bilan umumiy nuqtaga ega emas. Shu boisdan, (3.16) shart, *to'g'ri chiziq bilan tekislikning o'zaro paralellik sharti* bo'ladi.

3. Agar

$$Am + Bn + Cp = 0, \; Ax_0 + By_0 + Cz_0 + D = 0 \qquad (3.17)$$

bo'lsa, t parametrning har qanday qiymati (3.15) tenglamaning yechimi bo'ladi, ya'ni to'g'ri chiziqning har qanday nuqtasi tekislikda yotadi. Odatda (3.17) tengliklarni to'g'ri chiziqning tekislikda yotishlik shartlari deb yuritiladi.

Ta'rifga ko'ra, to'g'ri chiziq bilan tekislik orasidagi burchak deb, to'g'ri chiziq bilan uning tekislikdagi ortogonal proeksiyasi orasidagi φ burchakka aytiladi va u burchak quyidagicha hisoblanadi:

$$|cos(\vec{n}\wedge\vec{s})| = \sin\varphi = \frac{|Am+Bn+Cp|}{\sqrt{A^2+B^2+C^2}\;\sqrt{m^2+n^2+p^2}} \;. \qquad (3.18)$$

3.2. –AT

1. Berilgan $M_0(2{,}0{,}-3)$ nuqtadan o'tib, a) $\vec{s} = (2,-3{,}5)$ vektorga parallel bo'lgan va b) $\left.\begin{matrix} 2x - y + 3z - 11 = 0, \\ 5x + 4y - z + 8 = 0, \end{matrix}\right\}$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziqlarning kanonik tenglamalari tuzilsin.

(javob: a) $\frac{x-2}{2} = \frac{y}{-3} = \frac{z+3}{5}$; b)$\frac{x-2}{11} = \frac{y}{-17} = \frac{z+3}{-13}$.)

2. Quyida keltirilgan tenglamalar bilan berilgan to'g'ri chiziqlar va tekisliklarning o'zaro joylashishlari aniqlansin. Agar ular kesishadigan bo'lsalar, kesishish nuqtasining koordinatalari topilsin:

a)$\frac{x+1}{2} = \frac{y-3}{4} = \frac{z}{3}$ va $3x - 3y + 2z - 5 = 0$;

b)$\frac{x-13}{8} = \frac{y-1}{2} = \frac{z-4}{3}$ va $x + 2y - 4z + 1 = 0$;

c)$\frac{x-7}{5} = \frac{y-4}{1} = \frac{z-5}{4}$ va $3x - y + 2z - 5 = 0$.

(javob: a) parallel b) tekislikda yotgan to'g'ri chiziq c) M(2, 3, 1) nuqtada kesishuvchi).

3. $M_1(5,4,6)$ va $M_2(-2,-17,-8)$ nuqtalardan o'tuvchi to'g'ri chiziqqa nisbatan R(2,-5, 7) nuqtaga simmetrik bo'lgan Q nuqtaning koordinatalari aniqlansin. (Javob: *Q(4,-1, -3).)*.

4. $\left.\begin{matrix} x-2y+3=0 \\ 3y+z-1=0 \end{matrix}\right\}$ to'g'ri chiziq bilan $2x+3y-z+1=0$ tekislik orasidagi burchak topilsin. (Javob: Sinφ = 5/7 , $\varphi \approx 45^0 36'$.).

Mustaqil ish.

1. $\frac{x-2}{5}=\frac{y-3}{1}=\frac{z+1}{2}$ to'g'ri chiziqdan o'tib, $x+4y-3z+7=0$ tekislikka perpendikulyar bo'lgan tekislik tenglamasi tuzilsin. (Javob: $11x-17y-19z+10=0$. *)*

2. $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z}{2}\, va\, \frac{x-7}{3}=\frac{y-1}{4}=\frac{z-3}{2}$ to'g'ri chiziqlar orasidagi masofa aniqlansin. (Javob: d = 3.)

3. $\frac{x+2}{-1}=\frac{y-3}{2}=\frac{z-4}{3}\, va\, \frac{x}{3}=\frac{y+4}{2}=\frac{z-3}{5}$ to'g'ri chiziqlarning kesishmasliklari isbotlansin.

3.3. Tekislikdagi to'g'ri chiziq.

Asosiy teorema. *Tekislikda kiritilgan to'g'ri burchakli Oxu Dekart koordinatalari sistemasida har qanday to'g'ri chiziq, x va u noma'lumlarga nisbatan birinchi darajali*

$$Ax+By+C=0 \qquad (3.19)$$

kabi algebrik tenglama bilan beriladi va aksincha har qanday (3.19) kabi tenglama to'g'ri chiziqni ifodalaydi (bu erda: $A^2+B^2>0$, A, B, C *lar haqiqiy sonlardir).*

To'g'ri chiziqqa perpendikulyar bo'lgan $\vec{n}=(A,B)$ vektorini *to'g'ri chiziqning normal vektori* deb ataladi hamda (3.19) ni to'g'ri chiziqning umumiy tenglamasi deyiladi.Agar (3.19) tenglamada B ≠ 0 bo'lsa, uni

$$y=kx+b(k=tg\alpha) \qquad (3.20)$$

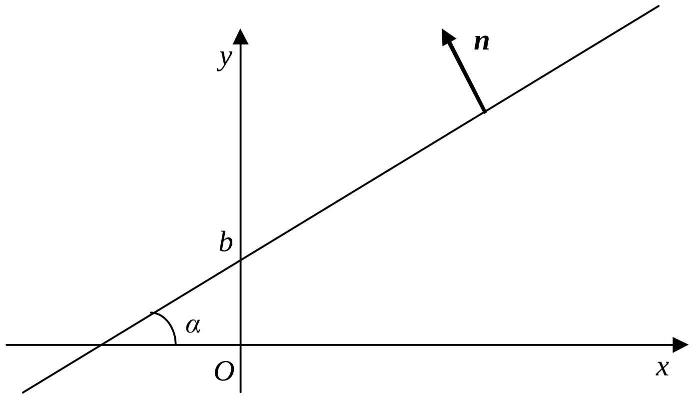
ko'rinishida yozish mumkinki, uni *to'g'ri chiziqning k burchak koeffitsientli tenglamasi* deyiladi. Bu erda, α - to'g'ri chiziqning Ox o'qining musbat yo'nalishi bilan tashkil etgan hamda soat strelkasiga qarshi yo'nalishga ega burchak bo'lib, to'g'ri chiziqning og'ish burchagi deb yuritiladi, *b* esa, to'g'ri chiziqning Oy o'qidan kesib o'tgan kesmasining qiymatini belgilaydigan sondir (3.3-rasm). Tekislikdagi to'g'ri chiziqning boshqa xildagi tenglamalari ham mavjuddir:

1) Berilgan $M_0(x_0,y_0)$ *nuqta va k burchak koeffitsientga nisbatan tenglama*

$$y-y_0=k(x-x_0) \qquad (3.21)$$

2) To'g'ri chiziqning parametrik tenglamalari

$$\left.\begin{aligned} x &= x_0 + mt, \\ y &= y_0 + nt, \end{aligned}\right\} \qquad (3.22)$$



3.3-rasm

ko'rinishda bo'lib, bu erda, $\vec{s} = (m, n)$, to'g'ri chiziqning yo'naltiruvchi vektori hamda $M_0(x_0; y_0)$ nuqta to'g'ri chiziqda yotadi.

3) (3.22) tengliklardan, *to'g'ri chiziqning kanonik tenglamasi* hosil bo'ladi:

$$\frac{x - x_0}{m} = \frac{y - y_0}{n} \qquad (3.23)$$

4)
$$\frac{x}{a} + \frac{y}{b} = 1 \qquad (3.24)$$

ni *to'g'ri chiziqning kesmalardagi tenglamasi* deb yuritiladi.

Tekislikdagi ikkita to'g'ri chiziqlarning o'zaro joylashishlarini qarab o'tamiz.

1. Agar to'g'ri chiziqlar o'zlarining umumiy tenglamalari $A_1x + B_1y + C_1 = 0$ va $A_2x + B_2y + C_2 = 0$ tenglamalari bilan berilgan bo'lsa, ular orasidagi burchak quyidagicha aniqlanadi:

$$\cos\varphi = \frac{(\overrightarrow{n_1} \cdot \overrightarrow{n_2})}{|\overrightarrow{n_1}| \cdot |\overrightarrow{n_2}|} = \frac{A_1A_2 + B_1B_2}{\sqrt{A_1^2 + B_1^2}\sqrt{A_2^2 + B_2^2}}. \qquad (3.25)$$

Ularning perpendikulyarlik va parallellik shartlari mos ravishda quyidagicha yoziladi:

$$A_1A_2 + B_1B_2 = 0, \qquad (3.26)$$

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} \neq \frac{C_1}{C_2}. \qquad (3.27)$$

2. Agar to'g'ri chiziqlar $y_1 = k_1x + b_1$ va $y_2 = k_2x + b_2$ kabi tenglamalar bilan berilsa, ular orasidagi φ burchak,

$$tg\varphi = \frac{k_2 - k_1}{1 + k_1k_2} \qquad (3.28)$$

formula bilan hisoblanadi, hamda ularning parallellik va perpendikulyarlik shartlari mos ravishda, $k_1 = k_2$ va $k_1k_2 = -1$ bo'ladi.

Shuningdek, $M_0(x_0;\ u_0)$ nuqtadan (3.19) to'g'ri chiziqqacha bo'lgan d masofa,

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} \qquad (3.29)$$

formula bilan aniqlanadi.

3.3–AT

1. Berilgan tenglamalar bilan ifodalanadigan to'g'ri chiziqlari chizlib, ularning burchak koeffitsiyentlari hamda koordinata o'qlaridan kesgan kesmalari aniqlansin:

a) $2x - y + 3 = 0$; $b)$ $5x + 2y - 8 = 0$;

c) $3x + 8y + 16 = 0$; $d)$ $3x - y = 0$.

2. Agar teng yonli trapetsiyaning asoslari 10 va 6 ga teng bo'lib, yon tomonlari katta asos bilan 60^0 li burchak tashkil etadigan bo'lsa, yon tomonlari bo'yicha o'tadigan to'g'ri chiziqlarning tenglamalari yozilsin (bu erda, katta asos Ox o'q bo'ylab yotadi hamda trapetsiyaning simmetriya o'qi esa Oy o'qida yotadi deb hisoblansin). (Javob:$y = 0$, $y = 2\sqrt{3}$, $y = \sqrt{3x} + 5\sqrt{3}$, $y = -\sqrt{3}\,x + 5\sqrt{3}$.)

3. Agar $\vec{F} = (m, n)$ kuch $M_0(x_0, u_0)$ nuqtaga qo'yilgan bo'lsa, shu kuch yo'nalishdagi to'g'ri chiziq tenglamasi yozilsin. (Javob: $nx - my + my_0 - nx_0 = 0$.)

4. Berilgan *A(3, –1)* nuqtadan o'tib, a) *Ox* absissa o'qiga, b) *Oy* ordinata o'qiga; c) koordinata tekisligi birinchi chorak burchagi bissektrisasiga; d) $y = 3x + 9$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziqlarning tenglamalari yozilsin. (Javob*:* a) *y = -1;* b) *x = 3*; c) *y = x – 4*; d) *y = 3x – 10*.)

5. Berilgan *A(-1,3)* va *B(4,5)* nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi yozilsin. (*Javob*: $2x - 5y + 17 = 0$.)

6. Yorug'likning nuri $y = \frac{2}{3}x - 4$ to'g'ri chiziq bo'ylab yo'nalayotgan bo'lsa, u nurning *Ox* o'qi bilan uchrashish nuqtasi *M* ning koordinatalari hamda qaytgan nurning tenglamasi topilsin. (Javob: *M(6, 0)*, $y = -\frac{2}{3}x + 4$.)

7. *A(-2,3)* nuqtadan o'tuvchi $2x - 3y + 8 = 0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasi tuzilsin (Javob: $3x + 2y = 0$.)

8. Tomonlaridan biri $x - 2y - 7 = 0$ to'g'ri chiziqda yotadigan, hamda uchlardan biri *A(2,-5)* nuqtada bo'lgan kvadratning yuzi hisoblansin. (Javob: 5.)

Mustaqil ish

1. Berilgan *R(5, 2)* nuqtadan o'tib, koordinata o'qlaridan o'zaro teng kesmalar hosil qiluvchi to'g'ri chiziq tenglamasi yozilsin. *(*Javob: *x + y – 7 = 0*.)

2. Tenglamasi $12x + 5y - 52 = 0$ bo'lgan to'g'ri chiziqqa parallel va undan 2 birlik masofada uzoqlikda joylashgan to'g'ri chiziq tenglamasi topilsin. (Javob: $12x + 5y - 26 = 0$ yoki $12x + 5y - 78 = 0$.)

3. Shunday to'g'ri chiziq tenglamasi tuzilsinki, u M_0*(4,-3)* nuqtadan o'tib, koordinata o'qlari bilan yuzi 3 ga teng bo'lgan uchburchak hosil qilsin. (Javob*:* $\frac{x}{2} + \frac{u}{3} = 1$ yoki $\frac{x}{4} + \frac{u}{3/2} = -1$.)

4. Koordinata boshidan o'tib, $y = 2x + 5$ to'g'ri chiziq bilan 45^0 li burchak tashkil etadigan to'g'ri chiziq tenglamasi yozilsin.(Javob: $3x + y = 0$.)

5. $3x + 4y - 2 = 0$ va $8x + 6y + 5 = 0$ to'g'ri chiziqlar tashkil qilgan burchaklardan kichigi hisoblansin, hamda $A(13/14; -1)$ nuqta ushbu burchak bissektrisasida yotishligi isbotlanib, chizmasi chizilsin. (Javob: $cos\varphi = 24/25 = 0{,}96$, $\varphi \approx 16^0 15'$.)

3.4. 3 – bobga individual uy vazifalari

3.1–IUT

1. $A_1(x_1, y_1), A_2(x_2, y_2), A_3(x_3, y_3)$ va $A_4(x_4, y_4)$ nuqtalar berilgan.
Quyidagi tenglamalar tuzilsin:
a) $A_1A_2A_3$ tekislik;
b) A_1A_2 to'g'ri chiziq;
c) $A_1A_2A_3$ tekislikka perpendikulyar bo'lgan A_4M to'g'ri chiziq;
d) A_1A_2 to'g'ri chiziqqa parallel A_3N to'g'ri chiziq;
e) A_4 nuqtadan o'tib, A_1A_2 to'g'ri chiziqqa perpendikulyar bo'lgan tekislik;
hisoblansin:
f) A_1A_4 to'g'ri chiziq bilan $A_1A_2A_3$ tekislik orasidagi burchak sinusi;
g) Oxy koordinata tekisligi bilan $A_1A_2A_3$ tekislik orasidagi burchakning kosinusi.

1.1. $A_1(3,1,4)$, $A_2(-1,6,1)$, $A_3(-1,1,6) A_4(0,4,-1)$.
1.2. $A_1(3,-1,2)$, $A_2(-1,0,1)$, $A_3(1,7,3)$, $A_4(8,5,8)$.
1.3. $A_1(3,5,4)$, $A_2(5,8,3)$, $A_3(1,2,-2)$, $A_4(-1,0,2)$.
1.4. $A_1(2,4,3)$, $A_2(1,1,5)$, $A_3(4,9,3)$, $A_4(3,6,7)$.
1.5. $A_1(9,5,5)$, $A_2(-3,7,1)$, $A_3(5,7,8)$, $A_4(6,9,2)$
1.6. $A_1(0,7,1)$, $A_2(2,-1,5)$, $A_3(1,6,3)$, $A_4(3,-9,8)$.
1.7. $A_1(5,\ 5,4)$, $A_2(1,-1,4)$, $A_3(3,5,1)$, $A_4(5,8,-1)$.
1.8. $A_1(6,1,1)$, $A_2(4,6,6)$, $A_3(4,2,0)$, $A_4(1,2,6)$.
1.9. $A_1(7,5,3)$, $A_2(9,4,4), A_3(4,5,7), A_4(7,9,6)$.
1.10. $A_1(6,8,2)$, $A_2(5,4,7) A_3(2,4,7) A_4(7,3,7)$.
1.11. $A_1(4,2,5)$, $A_2(0,7,1)$, $A_3(0,2,7)$, $A_4(1,5,0)$.
1.12. $A_1(4,4,10)$, $A_2(7,10,2)$, $A_3(2,8,4)$, $A_4(9,6,9)$.
1.13. $A_1(4,6,5)$, $A_2(6,9,4)$, $A_3(2,10,10)$, $A_4(7,5,9)$.
1.14. $A_1(3,5,4)$, $A_2(8,7,4), A_3(5,10,4), A_4(4,7,8)$.
1.15. $A_1(10,9,6)$, $A_2(2,8,2)$, $A_3(9,8,9)$, $A_4(7,10,3)$.
1.16. $A_1(1,8,2)$, $A_2(5,2,6)$, $A_3(5,7,4)$, $A_4(4,10,9)$.
1.17. $A_1(6,6,5)$, $A_2(4,9,5)$, $A_3(4,6,11)$, $A_4(6,9,3)$.
1.18. $A_1(7,2,2)$, $A_2(-5,7,-7)$, $A_3(5,-3,1)$, $A_4(2,3,7)$.
1.19. $A_1(8,-6,4)$, $A_2(10,5,-5)$, $A_3(5,6,-8)$, $A_4(8,10,7)$.
1.20. $A_1(1,-1,3)$, $A_2(6,5,8)$, $A_3(3,5,8)$, $A_4(8,4,1)$.
1.21. $A_1(1,-2,7)$, $A_2(4,2,10)$, $A_3(2,3,5)$, $A_4(5,3,7)$.
1.22. $A_1(4,2,10)$, $A_2(1,2,0)$, $A_3(3,5,7)$, $A_4(2,-3,5)$.
1.23. $A_1(2,3,5)$, $A_2(5,3,-7)$, $A_3(1,2,7)$, $A_4(4,2,0)$.

1.24. $A_1(5,3,7)$, $A_2(-2,3,5)$, $A_3(4,2,10)$, $A_4(1,2,7)$.
1.25. $A_1(4,3,5)$, $A_2(1,9,7)$, $A_3(0,2,0)$, $A_4(5,3,10)$.
1.26. $A_1(3,2,5)$, $A_2(4,0,6)$, $A_3(2,6,5)$, $A_4(6,4,-1)$.
1.27. $A_1(2,1,6)$, $A_2(1,4,9)$, $A_3(2,-5,8)$, $A_4(5,4,2)$.
1.28. $A_1(2,1,7)$, $A_2(3,3,6)$, $A_3(2,-3,9)$, $A_4(1,2,5)$.
1.29. $A_1(2,-1,7)$, $A_2(6,3,1)$, $A_3(3,2,8)$, $A_4(2,-3,7)$.
1.30. $A_1(0,4,5)$, $A_2(3,-2,1)$, $A_3(4,5,6)$, $A_4(3,3,2)$.

2. Quyidagi masalalar yechilsin.

2.1. Berilgan $M(-2{,}7{,}3)$ nuqtadan o'tib, $x-4y+5z-1=0$ tekislikka parallel bo'lgan tekislikning koordinata o'qlaridan kesgan kesmalarining qiymatlari aniqlansin $\left(\text{Javob: } -\frac{1}{15}, \frac{4}{15}, -\frac{1}{3}\right)$.

2.2. Uchlari $M_1(1{,}5{,}6)$ va $M_2(-1{,}7{,}10)$ nuqtalarda bo'lgan M_1M_2 kesmaning o'rtasidan o'tib shu kesmaga perpendikulyar bo'lgan tekislik tenglamasi tuzilsin (Javob: $x-y-2z+22=0$).

2.3. $M(2{,}0,-0{,}5)$ nuqtadan $4x-4y+2z+17=0$ tekislikkacha bo'lgan masofa topilsin (Javob: $d=4$).

2.4. $A(2,-3{,}5)$ nuqtadan o'tib, Oxy tekislikka paralel bo'lgan tekislik tenglamasi tuzilsin (Javob: $z-5=0$).

2.5. $A(2{,}5,-1)$ nuqta va Ox o'qi orqali o'tuvchi tekislik tenglamasi tuzilsin (Javob: $y+5z=0$).

2.6. $A(2{,}5,-1)$ va $B(-3{,}1{,}3)$ nuqtalardan o'tuvchi hamda Oy o'qiga parallel bo'lgan tekislik tenglamasi tuzilsin (Javob: $4x+5z-3=0$).

2.7. $A(3{,}4{,}0)$ nuqta va $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z+1}{2}$ to'g'ri chiziq orqali o'tuvchi tekislik tenglamasi tuzilsin (Javob: $y-z-4=0$).

2.8. O'zaro parallel bo'lgan $\frac{x-3}{2}=\frac{y}{1}=\frac{z-1}{2}$ va $\frac{x+1}{2}=\frac{y-1}{1}=\frac{z}{2}$ to'g'ri chiziqlar orqali o'tadigan tekislik tenglamasi tuzilsin. (Javob: $x+2y-2z-1=0$).

2.9. $A(3{,}2,-5)$ nuqta bilan Ox o'qi orqali o'tuvchi tekislikning $3x-y-7z+9=0$ tekislik bilan kesishishidan hosil bo'lgan to'g'ri chiziqning umumiy tenglamasi tuzilsin. ($Javob$: $3x-y-7z+9=0{,}5y+2z=0$).

2.10. Agar tekislik $M(6,-10{,}1)$ nuqtadan o'tib, Ox va Oz o'qlardan mos ravshida $a=-3$ va $c=2$ kesmalar kesadigan bo'lsa, uning kesmalar bo'yicha tenglamasi tuzilsin $\left(\text{Javob: } \frac{x}{-3}+\frac{y}{-4}+\frac{z}{2}=1\right)$

2.11. $A(2{,}3,-4)$ nuqtadan o'tib, $\vec{a}=(4{,}1,-1)$ va $\vec{b}=(2,-1{,}2)$ vektorlarga parallel bo'lgan tekislik tenglamasi tuzilsin (Javob: $x-10y-6z+4=0$).

2.12. $A(1{,}1{,}0)$ va $B(2,-1,-1)$ nuqtalardan o'tuvchi hamda $5x+2y+3z-7=0$ tekislikka perpendikulyar bo'lgan tekislik tenglamasi tuzilsin.
(Javob: $x+2y-3z-3=0$).

2.13. Koordinata boshidan o'tib, $2x-3y+z-1=0$ va $x-y+5z+3=0$ tekisliklarga perpendikulyar bo'lgan tekislik tenglamasi tuzilsin

(Javob: $14x + 9y - z = 0$).

2.14. $A(3,-1,2)$ va $B(2,1,4)$ nuqtalardan o'tuvchi hamda $\vec{a} = (5,-2,-1)$ vektorga parallel bo'lgan tekislikning tenglamasi tuzilsin. (Javob: $2x + 9y - 8z + +19 = 0$).

2.15. Agar $A(5,-2,3)$ va $B(1,-3,5)$ nuqtalar $\overrightarrow{AB}$ vektorning boshi va oxiri bo'lsa, koordinata boshidan o'tib, $\overrightarrow{AB}$ vektorga perpendikulyar bo'lgan tekislik tenglamasi tuzilsin. (Javob: $4x + y - 2z = 0$)

2.16. $M(2,-3,3)$ nuqtadan o'tib, $3x + y - 3z = 0$ tekislikka parallel bo'lgan tekislikning koordinata o'qlaridan kesgan kesmalarning qiymatlari topilsin. (Javob: $-2,-6,2.$)

2.17. Uchlari $M_1(2,3,-4)$ va $M_2(-1,2,-3)$ nuqtalarda bo'lgan M_1M_2 kesmaga perpendikulyar bo'lib, $M(1,-1,2)$ nuqtadan o'tuvchi tekislik tenglamasi tuzilsin. (Javob: $3x + y - z = 0$).

2.18. $\frac{x}{6} = \frac{y-3}{8} = \frac{z-1}{9}$ to'g'ri chiziqning $x + 3y - 2z - 1 = 0$ tekislikka parallel ekanligi hamda $x = t + 7, y = t - 2, z = 2t + 1$ to'g'ri chiziqning ushbu tekislikda yotishi ko'rsatilsin.

2.19. Koordinata tekisligi Oxz ga parallel bo'lib, $A(3,-4,1)$ nuqtadan o'tuvchi tekislikning umumiy tenglamasi yozilsin (Javob: $y + 4 = 0$).

2.20. Ordinata o'qi bo'yicha hamda $M(3,-5,2)$ nuqtadan o'tuvchi tekislik tenglamasi tuzilsin. (Javob: $2x - 3z = 0$).

2.21. $M(1,2,3)$ va $N(-3,4,-5)$ nuqtalardan o'tib Oz o'qiga parallel bo'lgan tekislik tenglamasi tuzilsin (Javob: $x + 2y - 5 = 0$)

2.22. $M(2,3,-1)$ nuqtadan hamda $x = t - 3, y = 2t + 5, z = -3t + 1$ to'g'ri chiziqdan o'tuvchi tekislik tenglamasi tuzilsin (Javob: $10x + 13y + 12z - 47 = 0$).

2.23. $M(4,-3,1)$ nuqtaning $x - 2y - z - 15 = 0$ tekislikdagi proeksiyasi topilsin. (Javob: $M_1(5,-5,0)$).

2.24. $x - 4y + z - 1 = 0$ bilan $2x + by + 10z - 3 = 0$ tekisliklar b ning qanday qiymatida o'zaro perpendikulyar bo'ladilar? (Javob: $b = 3$).

2.25. Koordinata o'qlaridan 0 dan farqli bir xil o'lchamdagi kesmalar kesadigan hamda $M(2,-3,-4)$ nuqtadan o'tadigan tekislik tenglamasi tuzilsin.
(Javob: $x + y + z + 5 = 0$).

2.26. $\frac{x}{3} = \frac{y-5}{n} = \frac{z+5}{6}$ to'g'ri chiziq bilan $ax + 2y - 2z - 7 = 0$ tekislik, n va a ning qanday qiymatlarida o'zaro perpendikulyar bo'ladi? (Javob: $a = -1, n = -6$).

2.27. Agar tekislik $A(2,3,-1)$ va $B(1,1,4)$ nuqtalardan o'tib, $x - 4y + 3z + 2 = 0$ tekislikka perpendikulyar bo'lsa, uning tenglamasi tuzilsin. (Javob: $7x + 4y + 3z - 23 = 0$).

2.28. Ikkita $x + 5y - z + 7 = 0$ va $3x - y + 2z - 3 = 0$ tekisliklarga perpendikulyar bo'lgan va koordinata boshidan o'tadigan tekislik tenglamasi tuzilsin (Javob: $9x - 5y - 16z = 0$).

2.29. Ikkita $M(2,3,-5)$ va $N(-1,1,-6)$ nuqtalardan o'tib, $\vec{a} = (4,4,3)$ vektorga parallel bo'lgan tekislik tenglamasi tuzilsin. (Javob: $2x - 5y + 4z + 31 = 0$).

2.30. Ikkita $3x - 5y + cz - 3 = 0$ bilan $x - 3y + 2z + 5 = 0$ tekisliklar ***c*** ning qanday qiymatiga o'zaro perpendikulyar bo'ladi? (Javob: $c = -9$).

3. Quyidagi masalalar yechilsin.

3.1. $\frac{x-1}{6} = \frac{y+2}{2} = \frac{z}{-1}$ to'g'ri chiziq va $\begin{cases} x - 2y + 2z - 8 = 0 \\ x + 6z - 6 = 0 \end{cases}$ to'g'ri chiziqlarning o'zaro parallel ekanliklari isbotlansin.

3.2. $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z-3}{3}$ to'g'ri chiziq bilan $2x + y - z = 0$ tekislikning o'zaro parallel ekanligi hamda $\frac{x-2}{2} = \frac{y}{-1} = \frac{z-4}{3}$ to'g'ri chiziqning esa, ushbu tekislikda yotishi isbotlansin.

3.3. Koordinata o'qlari bilan mos ravishda $60^0, 45^0$va120^0 li burchak tashkil etuvchi hamda $M(1,-3,3)$ nuqtadan o'tuvchi to'g'ri chiziq, tenglamasi tuzilsin. (Javob: $\frac{x-1}{1} = \frac{y+3}{\sqrt{2}} = \frac{z-3}{-1}$.)

3.4. $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-1}{6}$ to'g'ri chiziqning $\begin{cases} 2x + y - 4z + 2 = 0 \\ 4x - y - 5z + 4 = 0 \end{cases}$ to'g'ri chiziqqa perpendikulyar ekanligi isbotlansin.

3.5. Uchlari $A(3,6,-7), B(-5,1,-4)$va $C(0,2,3)$ nuqtalarda bo'lgan uchburchakning S uchidan tushirilgan medianasining parametrik tenglamalari tuzilsin (Javob: $x = 2t, y = -3t + 2, z = 17t + 3.$)

3.6. Quyidagi $\frac{x+2}{3} = \frac{y-1}{n} = \frac{z}{1}$ to'g'ri chiziq, n ning qanday qiymatida $\begin{cases} x + y - z = 0 \\ x - y - 5z - 8 = 0 \end{cases}$ to'g'ri chiziqqa parallel bo'ladi? (Javob: $n = -2$).

3.7. $\frac{x-1}{1} = \frac{y+1}{-2} = \frac{z}{6}$ to'g'ri chiziq va $2x + 3y + z - 1 = 0$ tekislikning kesishish nuqtasi topilsin (Javob: $M(2,-3,6)$).

3.8. $P(3,1,-1)$ nuqtaning $x + 2y + 3z - 30 = 0$ tekislikdagi proeksiyasi topilsin (Javob: $P_1(5,5,5)$).

3.9. Ikkita $3x - 5y + cz - 3 = 0$, $x + 3y + 2z + 5 = 0$ tekislik *c* ning qanday qiymatida o'zaro perpendikulyardir? (Javob: $c = 6$).

3.10. $ax + 3y - 5z + 1 = 0$ tekislik *a* ning qanday qiymatida $\frac{x-1}{4} = \frac{y+2}{3} = \frac{z}{1}$ to'g'ri chiziqqa parallel bo'ladi? (Javob: $a = -1$).

3.11. m va S ning qanday qiymatlarida $\frac{x-2}{m} = \frac{y+1}{4} = \frac{z-5}{-3}$ to'g'ri chiziq bilan $3x - 2y + cz + 1 = 0$ tekislik o'zaro perpendikulyar bo'ladi? (Javob: $m = -6$, $c = 1,5$.

3.12. Koordinata boshidan o'tuvchi va $x = 2t + 5$, $y = -3t + 1$, $z = -7t - 4$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziq tenglamasi tuzilsin (Javob: $\frac{x}{2} = \frac{y}{-3} = \frac{z}{-7}$).

3.13. $A(0,0,2)$, $B(4,2,5)$, va $C(12,6,11)$ nuqtalarning bitta to'g'ri chiziqda yotishi yoki yotmasliklari tekshirilsin (Javob: yotadi).

3.14. Berilgan $M(2,-5,3)$ nuqtadan o'tib $\begin{cases} 2x - y + 3z - 1 = 0 \\ 5x + 4y - z - 7 = 0 \end{cases}$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziq tenglamasi tuzilsin.
(Javob: $\frac{x-2}{-11} = \frac{y+5}{17} = \frac{z-3}{13}$).

3.15. Berilgan $M(2,-3,4)$ nuqtadan o'tib $\frac{x+2}{1} = \frac{y-3}{-1} = \frac{z+1}{1}$ va $\frac{x+4}{2} = \frac{y}{1} = \frac{z-4}{-3}$ to'g'ri chiziqlarga perpendikulyar bo'lgan to'g'ri chiziq tenglamasi tuzilsin
(Javob: $\frac{x-2}{2} = \frac{y+3}{5} = \frac{z-4}{3}$).

3.16. $ax + by + 6z - 7 = 0$ tekislik, a va b ning qanday qiymatlarida $\frac{x-2}{2} == \frac{y+5}{-4} = \frac{z+1}{-9}$ to'g'ri chiziqqa perpendikulyar bo'ladi? (Javob: $a = 4,\ b = -8$).

3.17. $\frac{x}{6} = \frac{y-3}{-8} = \frac{z-1}{-9}$ to'g'ri chiziqning $x + 3y - 2z + 1 = 0$ tekislikka parallel ekanligi va $x = t + 7, y = t - 2,\ z = 2t + 1$ to'g'ri chiziqning esa,ushbu tekislikda yotishi ko'rsatilsin.

3.18. $k(-3,1,-2)$ nuqtadan hamda Oz o'q orqali o'tuvchi tekislik tenglamasi tuzilsin (Javob: $x + 3y = 0$).

3.19. $\frac{x}{1} = \frac{y-1}{-2} = \frac{z}{3}$, $\begin{cases} 3x + y - 5z + 1 = 0 \\ 2x + 3y - 8z + 3 = 0 \end{cases}$ to'g'ri chiziqlar o'zaro perpendikulyar ekanligi ko'rsatilsin.

3.20. d ning qanday qiymatida $\begin{cases} 3x - y + 2z - 6 = 0 \\ x + 4y - z + d = 0 \end{cases}$ to'g'ri chiziq Oz o'qini kesadi? (Javob: $d = 3$).

3.21. Ushbu $\begin{cases} x = 2t + 5 \\ y = -t + 2 \\ z = pt - 7 \end{cases}$ va $\begin{cases} x + 3y + z + 2 = 0 \\ x - y - 3t - 2 = 0 \end{cases}$ to'g'ri chiziqlar p ning qanday qiymatida parallel bo'ladi. (Javob: $p = -5$).

3.22. . $\frac{x-7}{5} = \frac{y-1}{1} = \frac{z-5}{4}$ to'g'ri chiziq bilan $3x - y + 2z - 8 = 0$ tekislikning kesishish nuqtasi topilsin. (Javob: $M(2,0,1)$).

3.23. Berilgan $A(2,-5,3)$ nuqtadan o'tib Oxz tekislikka parallel bo'lgan tekislikning tenglamasi tuzilsin. (Javob: $y + 5 = 0$).

3.24. Berilgan $M(5,3,2)$ nuqtadan va Oy o'qi orqali o'tadigan tekislik bilan $x + 2y - z + 5 = 0$ tekislikda kesishishidan hosil bo'lgan to'g'ri chiziqning umumiy tenglamasi tuzilsin. (Javob: $x + 2y - z + 5 = 0,\ 2x - 5z = 0$).

3.25. b va d ning qanday qiymatlarida $\begin{cases} x - 2y + z - 9 = 0 \\ 3x + by + z + d == 0 \end{cases}$ to'g'ri chiziq Oxy tekislikda yotadi? (Javob: $b = -6,\ d = -27$).

3.26. Berilgan $M_0(2,3,3)$ nuqtadan o'tib, $\vec{a} = (-1,-3,1)\ va\ \vec{b} = (4,1,6)$ vektorlarga parallel bo'lgan tekislik tenglamasi tuzilsin.
(Javob: $19x - 10y - 11z + 25 = 0$).

3.27. Ox o'qiga parallel bo'lib $E(3,4,5)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi tuzilsin. $\left(\text{Javob: } \frac{x-3}{1} = \frac{y-4}{0} = \frac{z-5}{0}\right)$.

3.28. Shunday bir to'g'ri chiziq tenglamasi tuzilsinki, u $M(2,3,1)$ nuqtadan o'tib, $\frac{x+1}{2}=\frac{y}{-1}=\frac{z-2}{3}$ to'g'ri chiziqqa perpendikulyar bo'lsin.

(Javob: $\frac{x-2}{3}=\frac{y-3}{3}=\frac{z-1}{-1}$).

3.29. Berilgan $M(1,-5,3)$ nuqtadan o'tuvchi hamda $\frac{x}{2}=\frac{y-2}{3}=\frac{z+1}{-1}$ va $x=3t+1,\ y=-t-5,\ z=2t+3$ to'g'ri chiziqlarga perpendikulyar bo'lgan to'g'ri chiziqning kanonik tenglamasi yozilsin. (Javob: $\frac{x-1}{5}=\frac{y+5}{-7}=\frac{z-3}{-11}$).

3.30. $M(4,3,10)$ nuqtaga $\frac{x-1}{2}=\frac{y-2}{4}=\frac{z-3}{5}$ to'g'ri chiziqqa nisbatan simmetrik nuqta topilsin (Javob: $M_1(2,9,6)$).

Namunaviy variantning yechilishi.

1. To'rtta $A_1(4,7,8)$, $A_2(-1,13,0)$, $A_3(2,4,9)$, va $A_4(1,8,9)$ nuqtalar berilgan.

a) $A_1A_2A_3$ tekislikning;

b) A_1A_2 to'g'ri chiziqning;

c) $A_1A_2A_3$ tekislikka perpendikulyar bo'lgan A_4M to'g'ri chiziqning;

d) A_1A_2 to'g'ri chiziqqa parallel bo'lgan A_4N to'g'ri chiziqning tenglamalari tuzilsin.

e) A_1A_4 to'g'ri chiziq bilan $A_1A_2A_3$ tekislik orasidagi burchakning sinusi;

f) Oxy koordinata tekisligi bilan $A_1A_2A_3$ tekislik orasidagi burchakning kosinusi hisoblansin.

►**a)** $A_1A_2A_3$ tekislik tenglamasi (3.4) formula orqali aniqlaymiz:

$\begin{vmatrix} x-4 & y-7 & z-8 \\ -5 & 6 & -8 \\ -2 & -3 & 1 \end{vmatrix}=0$ bundan esa $6x-7y-9z+97=0$ ni hosil qilamiz.

b) A_1A_2 to'g'ri chiziqning tenglamasini yozish uchun berilgan ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasi ((3.9) formulaga qaralsin) dan foydalanamiz:

$$\frac{x-4}{5}=\frac{y-7}{-6}=\frac{z-8}{8};$$

c) A_4M to'g'ri chiziq $A_1A_2A_3$ tekislikka perpendikulyar bo'lganligi shartiga binoan, to'g'ri chiziqning yo'naltiruvchi vektori $\vec{S}$ uchun $A_1A_2A_3$ tekislik normal vektori $\vec{n}=(6,-7,-9)$ ni olish mumkin u holda A_4M to'g'ri chiziq tenglamasini (3.8) ni hisobga olgan holda quyidagicha yozamiz:

$$\frac{x-1}{6}=\frac{y-8}{-7}=\frac{z-9}{-9};$$

d) A_4N to'g'ri chiziq A_1A_2 to'g'ri chiziqqa parallel bo'lganligi bois ularning yo'naltiruvchi vektorlari $\vec{s_1}$ va $\vec{s_2}$ ni, $\vec{s_1}=\vec{s_2}=(5,-6,8)$ deb yozish mumkin. Natijada, A_4N to'g'ri chiziq tenglamasi quyidagicha bo'ladi:

$$\frac{x-1}{5}=\frac{y-8}{-6}=\frac{z-9}{8};$$

e) Yuqorida keltirilgan (3.18) formulaga binoan:

$$sin\varphi = \frac{|16\cdot5+(-7)\cdot(-6)+(-9)\cdot8|}{\sqrt{6^2+(-7)^2+(-9)^2}\sqrt{5^2+(-6)^2+8^2}} = \frac{34}{\sqrt{11}\sqrt{166}} \approx 0{,}8;$$

f) (3.5) formulaga binoan:

$$cos\varphi = \frac{\overrightarrow{n_1}\cdot\overrightarrow{n_2}}{|\overrightarrow{n_1}|\cdot|\overrightarrow{n_2}|} = \frac{0\cdot6+0\cdot(-7)+1\cdot(-9)}{\sqrt{1}\cdot\sqrt{6^2+(-7)^2+(-9)^2}} = -\frac{9}{\sqrt{166}} \approx -0{,}7.$$

2. Berilgan $M(4{,}3{,}1)$va $N(-2{,}0,-1)$ nuqtalardan o'tuvchi, hamda $A(1{,}1,-1)$ va $B(-3{,}1{,}0)$ nuqtalardan o'tuvchi to'g'ri chiziqqa parallel bo'lgan tekislik tenglamasi tuzilsin. ◄

► (3.9) formulaga binoan AB to'g'ri chiziq tenglamasi $\frac{x-1}{-4} = \frac{y-1}{0} = \frac{z+1}{1}$ kabi yoziladi.

Tekislik $M(4{,}3{,}1)$ nuqtadan o'tganligi uchun uning tenglamasi $A(x-4)+B(y-3)+C(z-1)=0$ kabi bo'ladi. Mazkur tekislik, $N(-2{,}0,-1)$ nuqtadan o'tgani uchun $A(-2-4)+B(0-3)+C(-1-1)=0$ yoki $6A+3B+2C=0$ shart o'rinli bo'ladi. Shuningdek, tekislik AB to'g'ri chiziqqa parallel bo'lganligidan, $-4A+0B+1C=0$ yoki $4A-C=0$ shart bajariladi. $\begin{cases}6A+3B+2C=0\\ 4A-C=0\end{cases}$ sistemani yechib, $C=4A$, $B=-\frac{14}{3}A$ ni topamiz. U holda: $A(x-4)-\frac{14}{3}A(y-3)+4A(z-1)=0$ va bu erda $A\neq0$ ekanligini nazarda tutib, $3(x-4)-14(y-3)+12(z-1)=0$ yoki $3x-14y+12z+18=0$ ni topamiz.◄

3. $M_1(6,-4,-2)$ nuqtaning $x+y+z-3=0$ tekislikka nisbatan simmetrik bo'lgan $M_2(x_2,y_2,z_2)$ nuqtasining koordinatalari topilsin.

► M_1 va M_2 nuqtalardan o'tib, berilgan tekislikka perpendikulyar bo'lgan to'g'ri chiziqning parametrik tenglamalarini yozib olamiz: $x=6+t$; $y=-4+$ $+t$; $t=-2+t$. Bu to'g'ri chiziq bilan berilgan tekislikning tenglamalarini birgalikda yechib, $t=1$ va u orqali, M_1M_2 to'g'ri chiziqning berilgan tekislikning kesishish nuqtasi $M(7,-3,-1)$ ni aniqlaymiz. Ushbu aniqlangan nuqta, M_1M_2 kesmaning o'rtasidagi nuqta bo'lganligi sababli, $7=\frac{6+x_2}{2}$; $-3=\frac{-4+y_2}{2}$ va $-1=\frac{-2+z_2}{2}$ ni yozish mumkin (§2.2 dagi 1-misolga qaralsin) va undan esa, M_2 nuqtaning koordinatalari $x_2=8$; $y_2=-2$ va $z_2=0$ larni aniqlaymiz.◄

3.2-IUT

1. ABC uchburchakning uchlari $A(x_1,y_1)$, $B(x_2,y_2)$ va $C(x_3,y_3)$ lar berilgan. Quyidagilani topish lozim:

a) AB tomon tenglamasi,

b) CH balandlik tenglamasi,

c) AM mediana tenglamasi,

d) AM mediana bilan CH balandlikning kesishish nuqtasi N,

e) C uchidan o'tib, AB tomonga parallel bo'lgan to'g'ri chiziq tenglamasi,
f) C nuqtadan AB to'g'ri chiziqqacha bo'lgan masofa.

1.1. $A(-2,4)$, $B(3,1)$, $C(10,7)$,
1.2. $A(-3,-2)$, $B(14,4)$ $C(6,8)$,
1.3. $A(1,7)$ $B(-3,-1)$, $C(11,-3)$,
1.4. $A(1,0)$, $B(-1,4)$, $C(9,5)$,
1.5. $A(1,-2)$, $B(7,1)$ $C(3,7)$,
1.6. $A(-2,-3)$, $B(1,6)$, $C(6,1)$,
1.7. $A(-4,2)$, $B(-6,6)$, $C(6,2)$,
1.8. $A(4,-3)$, $B(7,3)$, $C(1,10)$,
1.9. $A(4,-4)$, $B(8,2)$, $C(3,8)$,
1.10. $A(-3,-3)$ $B(5,-7)$, $C(7,7)$,
1.11. $A(1,-6)$, $B(3,4)$, $C(-3,3)$,
1.12. $A(-4,2)$, $B(8,-6)$, $C(2,6)$,
1.13. $A(-5,2)$, $B(0,-4)$, $C(5,7)$,
1.14. $A(4,-4)$, $B(6,2)$, $C(-1,8)$,
1.15. $A(-3,8)$, $B(-6,2)$, $C(0,-5)$,
1.16. $A(6,-9)$, $B(10,-1)$, $C(-4,1)$,
1.17. $A(4,1)$, $B(-3,-1)$, $C(7,-3)$,
1.18. $A(-4,2)$ $B(6,-4)$, $C(4,10)$,
1.19. $A(3,-1)$, $B(11,3)$, $C(-6,2)$,
1.20. $A(-7,-2)$, $B(-7,4)$, $C(5,-5)$,
1.21. $A(-1,-4)$, $B(9,6)$, $C(-5,4)$,
1.22. $A(10,-2)$, $B(4,-5)$, $C(-3,1)$,
1.23. $A(-3,-1)$, $B(-4,-5)$, $C(8,1)$,
1.24. $A(-2,-6)$, $B(-3,5)$, $C(4,0)$,
1.25. $A(-7,-2)$, $B(3,-8)$, $C(-4,6)$,
1.26. $A(0,2)$, $B(-7,-4)$, $C(3,2)$,
1.27. $A(7,0)$, $B(1,4)$, $C(-8,-4)$,
1.28. $A(1,-3)$, $B(0,7)$, $C(-2,4)$,
1.29. $A(-5,1)$, $B(8,-2)$, $C(1,4)$,
1.30. $A(2,5)$, $B(-3,1)$, $C(0,4)$,

2. Quyidagi masalalar yechilsin.

2.1. Absissa o'qidan 3 birlikdagi kesma kesib o'tadigan hamda $3x-2y-7=0$ va $x+3y-6=0$ to'g'ri chiziqlarning kesishish nuqtasidan o'tuvchi to'g'ri chiziq tenglamasi topilsin (Javob: $x=3$).

2.2. $B(2,-3)$ va $C(-5,1)$ nuqtalardan o'tuvchi to'g'ri chiziqdagi $A(-8,12)$ nuqtaning proeksiyasi topilsin. (Javob: $A_1(-12,5)$.

2.3. Agar uchburchak ABC ning ikkita uchi $A(-4,4)$ va $B(4,-12)$ nuqtalarda bo'lib, uning balandliklarining kesishish nuqtasi $M(4,2)$ bo'lsa, uning uchinchi S uchining koordinatalari aniqlansin (Javob: $C(8,4)$).

2.4. Ordinata o'qidan 2 ga teng kesma hosil qiluvchi va $2y - x - 3 = 0$ chiziqqa parallel bo'lgan to'g'ri chiziq tenglamasi tuzilsin.(Javob: $x - 2y + 4 = 0$).

2.5. $A(2,-3)$ nuqta bilan $2x - y = 5$ va $x + y = 1$ to'g'ri chiziqlarning kesishish nuqtasidan o'tuvchi to'g'ri chiziq tenglamasi topilsin (Javob: $x = 2$).

2.6. Uchlari $A(3,6)$, $B(5,2)$, $C(-1,-3)$ va $D(-5,5)$ nuqtalarda bo'lgan ABCD to'rtburchakning trapetsiya ekanligi ko'rsatilsin.

2.7. $B(2,5)$, va $C(1,0)$ nuqtalardan o'tuvchi to'g'ri chiziqqa perpendikulyar va $A(3,1)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi yozilsin.
(Javob: $x + 5y - 8 = 0$).

2.8. $M(-3,-2)$va $N(1,6)$ nuqtalardan o'tuvchi to'g'ri chiziqqa parallel bo'lib, $A(-2,1)$ nuqtadan o'tadigan to'g'ri chiziq tenglamasi tuzilsin.
(Javob: $2x - -y + 5 = 0$).

2.9. $M(2,-1)$ nuqtaga, $x - 2y + 3 = 0$ to'g'ri chiziqqa nisbatan simmetrik bo'lgan nuqta aniqlansin $\left(\text{Javob: } M_1\left(-\frac{4}{5},\frac{23}{5}\right)\right)$.

2.10. Uchlari $A(-1,-3)$, $B(3,5)$, $C(5,2)$va $D(3,-5)$ nuqtalarda bo'lgan ABCD to'rtburchakning diagonallarining kesishish O nuqtasi topilsin $\left(\text{Javob: } O\left(3,\frac{1}{3}\right)\right)$.

2.11. $6x - 4y + 5 = 0$ va $2x + 5y + 8 = 0$ to'g'ri chiziqlarning kesishish nuqtasidan o'tib, Ox o'qiga parallel bo'lgan to'g'ri chiziq tenglamasi yozilsin (Javob: $y = -1$).

2.12. Agar ABC uchburchakning AB tomon tenglamasi $4x + y = 12$ va BH va AM balandlik tenglamalari mos ravishda $5x - 4y = 12$ va $x + y = 6$ bo'lsa, uning qolgan ikki tomonining tenglamalari tuzilsin.
(Javob: $7x - 7y - 16 = 0$, $4x + 5y - 28 = 0$).

2.13. ABC uchburchakning ikkita $A(-6,2)$va $B(2,-2)$ uchlari berilgan hamda uning balandliklarining kesishish nuqtasi $H(1,2,)$ nuqtadadir AC tomon bilan BH balandlikning kesishish nuqtasi M ning koordinatalari topilsin (Javob: $M\left(\frac{10}{17},\frac{62}{17}\right)$.

2.14. ABC uchburchakning uchlari $A(-4,2)$, $B(3,-5)$ va $C(5,0)$ nuqtalarda bo'lsa, A va B uchlardan tushirilgan balandliklarning tenglamasi tuzilsin (Javob: $3x + 5y + 2 = 0$, $9x + 2y - 28 = 0$).

2.15. Uchlari $A(2,3)$, $B(0,-3)$, $C(6,-3)$ nuqtalarda bo'lgan ABC uchburchak tomonlarning o'rtalaridan o'tkazilgan perpendikulyarlarning kesishish nuqtasining koordinatalari topilsin (Javob: $M\left(3,-\frac{2}{3}\right)$.

2.16. ABC uchburchakning AB, AC va BC tomonlarning tenglamalari mos ravishda $2x - y - 3 = 0$, $x + 5y - 7 = 0$ va $3x - 2y + 13 = 0$ bo'lsa, A uchidan tushirilgan balandlik tenglamasi topilsin ($Javob$: $2x + 3y - 7 = 0$).

2.17. Uchburchakning uchlari $A(3,1), B(-3,-1)$ va $C(5,-12)$ nuqtalarda bo'lsa, C uchidan tushirilgan medianasining tenglamasi tuzilib, uning uzunligi ham hisoblansin $\left(\text{Javob}: 2x + y + 2 = 0, d = \frac{54}{\sqrt{17}} \approx 13{,}1\right)$.

2.18. Koordinata boshidan hamda $2x + 5y - 8 = 0$ va $2x + 3y + 4 = 0$ to'g'ri chiziqlarning kesishish nuqtasidan o'tuvchi to'g'ri chiziq tenglamasi tuzilsin $(\text{Javob}: 6x + 11y = 0)$.

2.19. $3x + 5y - 15 = 0$ to'g'ri chiziqning koordinata o'qlari bilan kesishish nuqtalardan shu to'g'ri chiziqqa o'tkazilgan perpendikulyarning tenglamalari tuzilsin. $(\text{Javob}: 5x - 3y - 25 = 0{,}5x - 3y + 9 = 0)$.

2.20. Agar to'rtburchak tomonlarining tenglamalari $x - y = 0, x + 3y = 0$, $x - y - 4 = 0$ va $3x + y - 12 = 0$ kabi bo'lsalar, uning diagonallarining tenglamalari tuzilsin. $(\text{Javob}: y = 0, x = 3)$,

2.21. ABC uchburchakning uchlari $A(4,6), B(-4,0)$va $C(-1,-4)$ nuqtalarda bo'lsa, uning CM medianasi bilan CK balandligi tenglamasi tuzilsin. $\left(\text{Javob}: 7x - y + 3 = 0\ (CM), 4x + 3y + 16 = 0\ (CK)\right)$.

2.22. $P(5,2)$ nuqtadan o'tib, a) koordinata o'qlaridan bir xil uzunlikdagi kesma kesib o'tadigan, b) Ox parallel bo'lgan, c) Oy o'qiga parallel bo'lgan to'g'ri chiziq tenglamalari tuzilsin. $(\text{Javob}: x + y - 7 = 0, y = 2, x = 5)$.

2.23. $A(-2,3)$ nuqtadan o'tib, Ox o'qi bilan a) 45^0, b) 90^0, c) 0^0 li burchaklar tashkil etadigan to'g'ri chiziq tenglamalari tuzilsin $(\text{Javob}: x - y + 5 = 0, x + 2 = 0, y - 3 = 0)$.

2.24. Abssissasi 3 ga teng bo'lgan S nuqta, $A(-6,-6), B(-3,-1)$ nuqtalar bilan bitta to'g'ri chiziqda yotadigan bo'lsa, uning ordinatasi qanchaga teng bo'ladi? $(\text{Javob}: y = 9)$.

2.25. $2x - 5y - 1 = 0$ bilan $x + 4y - 7 = 0$ to'g'ri chiziqlarning kesishish nuqtasidan, hamda uchlari $A(4,-3)$ va $B(-1,2)$ nuqtalarda bo'lgan kesmani $\lambda = \frac{2}{3}$ nisbatda bo'luvchi nuqtadan o'tuvchi to'g'ri chiziq tenglamasi tuzilsin. $(\text{Javob}: 2x - y - 5 = 0)$.

2.26. Agar rombning ikkita tomonining tenglamalari $2x - 5y - 1 = 0$ va $2x - -5y - 34 = 0$ bo'lib, diagonallaridan birining tenglamasi $x + 3y - 6 = 0$ bo'lsa, uning ikkinchi diagonalining tenglamasi topilsin $(\text{Javob}: 3x - y - 23 = 0)$.

2.27. Uchlari $A(-3,1), B(7,5)$, va $C(5,-3)$ nuqtalarda bo'lgan ABC uchburchak medianalarining kesishish nuqtasi E ning koordinatalari aniqlansin. $\left(\text{Javob}: E(3,1)\right)$.

2.28. $A(-1,1)$ nuqtadan o'tib, $2x + 3y = 6$ to'g'ri chiziq bilan 45^0 burchak tashkil etadigan to'g'ri chiziqlarning tenglamalari yozilsin. $(\text{Javob}: x - 5y + 6 = 0, \quad 5x + y + 4 = 0)$.

2.29. Agar ABC uchburchakning bir uchi $A(2,3)$ nuqtada bo'lib, ikkita balandliklarining tenglamalari $2x - 3y + 1 = 0$ va $x + 2y + 1 = 0$ bo'lsa, uning AB va AC tomonlarining tenglamalari tuzilsin. $\left(Javob: 2x - y - 1 = 0(AB),\ 3x + 2y - 12 = 0(AC)\right)$.

2.30. Agar parallelogrammning ikkita tomonining tenglamalari $x - 2y = 0$, $x - y - 1 = 0$ bo'lib, diagonallarining kesishish nuqtasi $M(3, -1)$ bo'lsa, uning qolgan ikkita tomonining tenglamasi tuzilsin.
$(Javob: x - y - 7 = 0, \quad x - 2y - 10 = 0)$.

Namunaviy variantning yechilishi.

1. ABC uchburchakning uchlari $A(4;3)$; $B(-3;-3)$ va $C(2;7)$ nuqtalarda bo'lsa, quyidagilar aniqlansin:
a) AB tomon tenglamasi;
b) CN balandlik tenglamasi;
c) AM mediana tenglamasi;
d) AM mediana bilan CN balandlikning tenglamasi kesishish nuqtasi;
e) C uchidan o'tib AB tomonga parallel bo'lgan to'g'ri chiziq tenglamasi;
f) C nuqtadan AB tomongacha bo'lgan masofa.

►**a)** AB tomon tenglamasini (3.9) formuludan foydalanib tuzamiz:

$$\frac{x-4}{-3-4} = \frac{y-3}{-3-3}, \quad \text{yoki } 6x - 7y - 3 = 0;$$

b) AB tomon tenglamasini y ga nisbatan yechib, AB to'g'ri chiziqning burchak koeffitsiyenti $k_1 = \frac{6}{7}$ ni topamiz. AB bilan CH to'g'ri chiziqlarning perpendikulyarlik shartidan foydalanib (3.28 formulaga qaralsin), CH balandlikning burchak koeffitsiyenti $k_2 = -\frac{7}{6}$ ekanligini aniqlaymiz.U holda, (3.21.) tenglamadan foydalanib, CA balandlikning tenglamasini yozamiz:

$$y - 7 = -\frac{7}{6}(x - 2), \text{ yoki } 7x + 6y - 56 = 0;$$

c) BC tomon o'rtasi bo'lgan $M(x, y)$ nuqtaning koordinatalari,$x = \frac{-3+2}{2} = -\frac{1}{2}$ va $y = \frac{-3+7}{2} = 2$ (§2.2 ga qaralsin) larni topib, A va M nuqtalardan o'tuvchi mediana tenglamasini tuzamiz:

$$\frac{x-4}{-\frac{1}{2}-4} = \frac{y-3}{2-3} = 2x - 9y + 19 = 0;$$

d) BC bilan CN balandliklarning tenglamalarini birgalikda yechib, ularning kesishish nuqtasi bilan N ning koordinatalarini aniqlaymiz,

$$\begin{cases} 7x + 6y - 56 = 0 \\ 2x - 9y + 19 = 0 \end{cases} \text{ sistemani yechib } N\left(\frac{26}{5}, \frac{49}{15}\right) \text{ ni topamiz;}$$

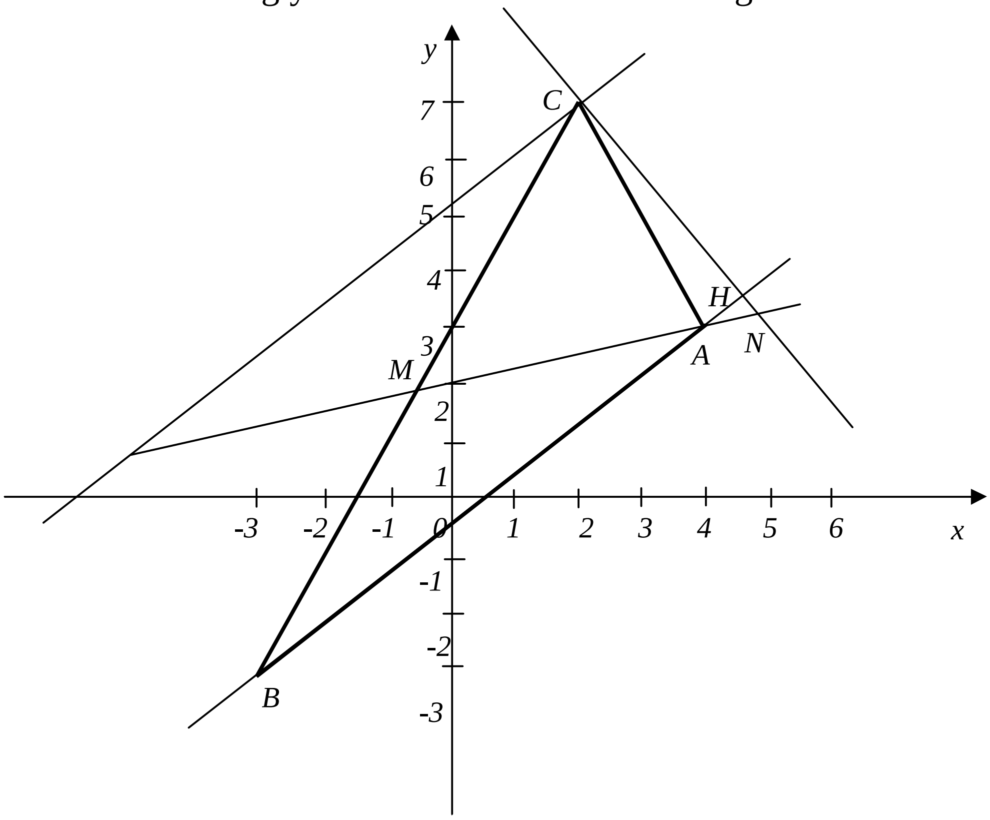
e) C uchdan o'tib AB tomonga parallel bo'lgan to'g'ri chiziqning burchak koeffitsiyentini ham $k_1 = 6/7$ bo'ladi. U holda, (3.21) tenglamaga ko'ra hamda C nuqtaning koordinatalariga binoan, CD to'g'ri chiziq tenglamasini tuzamiz:

$$y - 7 = \frac{6}{7}(x - 2), \text{ yoki } 6x - 7y + 37 = 0;$$

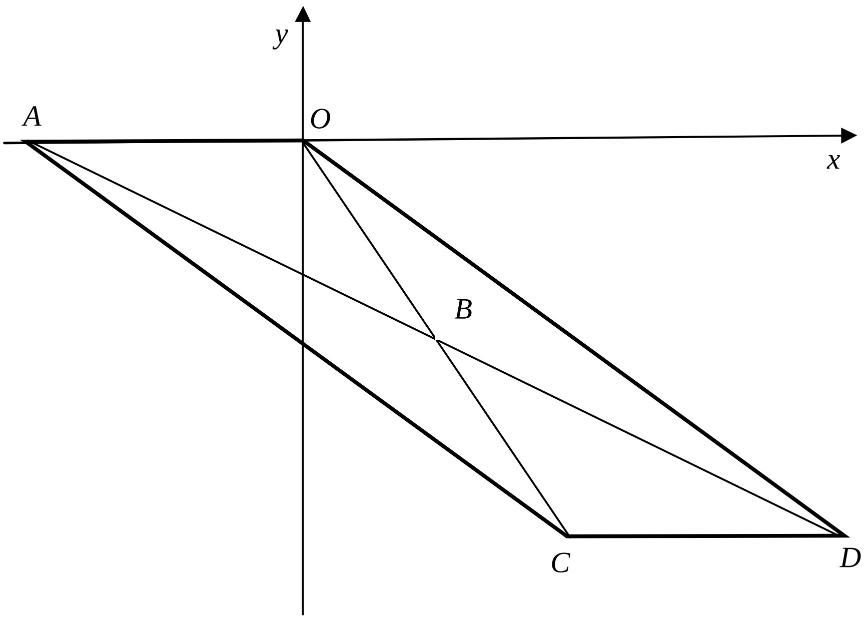
f) C nuqtadan AB tomonlarga bo'lgan d masofani (3.29) formuladan foydalanib aniqlaymiz:

$$d = |DH| = \frac{|6 \cdot 2 - 7 \cdot 7 - 3|}{\sqrt{6^2 + (-7)^2}} = \frac{40}{\sqrt{85}} \approx 4{,}4.$$

Ushbu masalaning yechimi 3.4-rasmda aks ettirilgan. ◀



3.4-rasm



3.5-rasm

2. 3.5-rasm $OACD$ parallelogrammning ikkita uchi $O(0{,}0)$va $A(-2{,}0)$ nuqtalarda bo’lib, diagonallarining kesishishi nuqtasi $B(2,-2)$ bo’lsa, parallelogramm tomonlarining tenglamalari yozilsin.

► OA tomon tenglamasining $y = 0$ ekanligi ma'lum. $B(2,-2)$ nuqta AD (3.5-rasm) diagonalning o'rtasi bo'lganligidan, $D(x,y)$ uchining koordinatalarini kesmani teng ikkiga bo'lishi formulasidan foydalanib topamiz:

$$2 = \frac{-2+x}{2}, \quad -2 = \frac{0+y}{2}, \quad \text{yoki } x = 6, y = -4.$$

Demak, $B(6,-4)$ endi, OA va CD tomonlar parallel bo'lganliklari uchun CD tomon tenglamasi $u = -4$ ekanligini topamiz.

OD tomon tenglamasi: $\frac{x-0}{6-0} = \frac{y-0}{-4-0}$, yoki $2x + 3y = 0$.

AC tomonning tenglamasini tuzish uchun uning $A(-2,0)$ nuqtadan o'tib, OD tomonga parallel ekanligidan foydalanamiz (3.21. tenglamaga qaralsin),

$$y - 0 = -\frac{2}{3}(x+2) \text{ yoki } 2x + 3y + 4 = 0. ◄$$

3.5. 3-bobga qo'shimcha masalalar.

1. $x - 7y = 1$ va $x + y = -7$ to'g'ri chiziqlar tashkil qilgan burchak ichida yotadigan $A(1,1)$ nuqtadan o'tuvchi burchak bissektrisasining tenglamasi yozilsin ($Javob$: $3x - y + 17 = 0$).

2. Agar $ABCD$ parallelogramning AB, BC, CD va DA tomonlari mos ravishda $P(3,0), Q(6,6), R(5,9)$va $S(-5,4)$ nuqtalardan o'tib, diagonallari $M(1,6)$ nuqtada kesishadigan bo'lsa, uning tomonlarini tenglamalari tuzilsin.($Javob$: $x + 2y - 3 = 0, 2x - y - 6 = 0, x + 2y - 23 = 0, 2x - y + 14 = 0$).

3. Rombning bir tomonining tenglamasi $x + 3y - 8 = 0$ bo'lib, diagonalining tenglamasi $2x + y + 4 = 0$ bo'lsa,hamda $A(-9,-1)$ nuqta berilgan tomonga parallel ekanligi ma'lum bo'lsa qolgan tomonlarining tenglamalari tuzilsin. ($Javob$: $x + 3y + 12 = 0, 3x - y - 4 = 0, 3x - y + 16 = 0$).

4. Agar ABC uchburchak AB va AC tomonlarining tenglamalari mos ravshida $2x + 3y - 6 = 0$ va $x + 2y - 5 = 0$ bo'lib, B uchidagi burchak $\frac{\pi}{4}$ ga teng bo'lsa, A uchidan BC tomonga tushirilgan balandlik tenglamasi yozilsin. ($Javob$: $x - 5y + 23 = 0$).

5. Agar ABC uchburchakning bir uchi $A(2,-4)$ nuqtada bo'lib, uning ikkita burchaklari bissektrisalarining tenglamalari $x + y - 2 = 0$ va $x - 3y - 6 = 0$ bo'lsa, uchburchak tomonlarining tenglamalari tuzilsin

($Javob$: $x + 7y - 6 = 0$, $x - y - 6 = 0$, $7x + y - 10 = 0$).

6. ABC uchburchakning bir uchi $A(-4,2)$ nuqtada bo'lib, ikkita medianalarning tenglamalari $3x - 2y + 2 = 0$ va $3x + 5y - 12 = 0$ bo'lsa,uning tomonlari tenglamalari tuzilsin.

($Javob$: $2x + y - 8 = 0$, $x - 3y + 10 = 0$, $x + 4y - 4 = 0$).

7. Uchlari $A(-3,-1), B(1,-5), C(9,3)$ nuqtalarda bo'lgan uchburchakning AB va AC tomonlari A uchidan boshlab $\lambda = 3$ nisbatda bo'lingan. Bo'linish nuqtalari bilan ularga qarama-qarshi uchlarni birlashtiruvchi to'g'ri chiziqlar bilan mediananing bir nuqtada kesishishlari isbotlansin.

8. Radiusu $R = 5$ bo'lgan aylanaga $3x + 4y - 30 = 0$ va $3x - 4y + 12 = 0$ to'g'ri chiziqlar urilib o'tadilar ushbu urinma to'g'ri chiziqlar hamda urinish nuqtalariga o'tkazilgan aylana radiuslari hosil qilgan to'rtburchakning yuzasi hisoblansin. $(Javob: S \approx 1{,}68)$.

9. Ikkita $A(-3{,}8)$ va $B(2{,}2)$ nuqtalar berilgan Ox o'qida shunday bir M nuqta topilsinki, AMB siniq chiziq eng qisqa uzunlikka ega bo'ladigan bo'lsin. $\left(Javob: M(1{,}0)\right)$.

10. $\frac{x+3}{1} = \frac{y+1}{2} = \frac{z+1}{1}$ va $\begin{cases} x = 3z - 4 \\ y = z + 2 \end{cases}$ to'g'ri chiziqlarning kesishish nuqtasi A topilsin. $\left(Javob: A(-1{,}3{,}1)\right)$.

11. Berilgan $P(7{,}9{,}7)$ nuqtadan $\frac{x-2}{4} = \frac{y-1}{3} = \frac{z}{2}$ to'g'ri chiziqqacha bo'lgan masofa topilsin $\left(Javob: \sqrt{22}\right)$.

12. O'zaro kesishmaydigan $\frac{x-9}{4} = \frac{y+2}{-3} = \frac{z}{1}$ va $\frac{x}{-2} = \frac{y+7}{9} = \frac{z-2}{2}$ to'g'ri chiziqlar orasidagi eng qisqa masofa aniqlansin. $(Javob: 7)$.

13. Uchlari $A(4{,}1{,}-2), B(2{,}0{,}0)\ va\ C(-2{,}3{,}-5)$ nuqtalardagi uchburchakning B uchidan qarama-qarshi tomonga tushirilgan balandlik tenglamasi tuzilsin ($Javob$: $\frac{x-2}{74} = \frac{y}{57} = \frac{z}{-110}$).

14. Qirrasining uzunligi 1 bo'lgan kubning uchidan, shu uchidan o'tmaydigan diagonaligacha bo'lgan masofa aniqlansin. $\left(Javob: d = \sqrt{\frac{2}{3}}\right)$.

15. Oxy koordinata tekisligida shunday bir M nuqta aniqlansinki, ushbu M nuqtadan $A(-1{,}2{,}5)$ va $B(1{,}-16{,}10)$ nuqtalargacha bo'lgan masofalarning yig'indisi eng kichik bo'lsin ($Javob$: $M(3{,}-4{,}0)$).

16. Biror $M(x, y, z)$ nuqta, $M_0(15{,}-24{,}-16)$ nuqtadan $\vec{S} = (-2{,}2{,}1)$ vektor yo'nalishi bo'ylab, $v = 12$ tezlik bilan tekis va to'g'ri chiziqli harakat qilayotgan bo'lsa, M nuqtaning harakat traektoriyasi $3x + 4y + 7z - 17 = 0$ tekislikni kesib o'tishi ko'rsatilsin, kesishish nuqtasi M_1 ning koordinatalari aniqlansin. $\left(Javob: M_1(-25{,}16{,}4)\right)$.

17. Tenglamalari $\frac{x-1}{2} = \frac{y+2}{-3} = \frac{z-5}{4}$ va $\frac{x-7}{3} = \frac{y-2}{2} = \frac{z-1}{-2}$ bo'lgan to'g'ri chiziqlarning bitta tekislikda yotishi isbotlansin, hamda u tekislikning tenglamasi tuzilsin. $(Javob: 2x - 16y - 13z + 31 = 0)$.

18. $C(3{,}-4{,}-2)$ nuqtaning, o'zaro parallel bo'lgan $\frac{x-5}{13} = \frac{y-6}{1} = \frac{z+3}{-4}$ va $\frac{x-2}{13} = \frac{y-3}{1} = \frac{z+3}{-4}$ to'g'ri chiziqlardan o'tuvchi tekislikdagi proeksiyasi topilsin. $\left(Javob: C_1(2{,}-3{,}-5)\right)$.

19. Oxy tekislikdagi $M(4{,}-3)$ nuqta orqali to'g'ri chiziq shunday o'tkazilsinki, uning koordinata o'qlari va o'zi bilan tashkil etgan uchburchakning yuzi 3 ga teng bo'lsin. $(Javob: 3x + 2y - 6 = 0\ yoki\ 3x + 8y + 12 = 0)$.

20. Kvadrat shaklidagi yer maydonining chegarasini saqlanib qolgan ustunlar yordamida tiklash lozim bo'lsin: ulardan biri, maydonning o'rtasida joylashgan,

qolgan ikkitasi chegaraning qarama-qarshi tomonlarida joylashgan. Shuningdek, markazidagi ustun $M_1(1{,}6)$ nuqtada bo'lib, qolgan ikkitasi esa, $A(5{,}9)$, va $B(3{,}0)$ nuqtalardadir. Maydonning chegarasini ifodalaydigan to'g'ri chiziqlarning tenglamalari tuzilsin.
$(Javob: x + 2y - 23 = 0, x + 2y - 3 = 0, 2x - y - 6 = 0, 2x - y + 14 = 0)$.

21. $x = x_0 + lt$, $y = y_0 + mt$, $z = z_0 + nt$ to'g'ri chiziq orqali o'tib, $Ax + By + Cz + D = 0$ tekislikka perpendikulyar bo'lgan tekislikning tenglamasi quyidagicha bo'lishi isbotlansin : $\begin{vmatrix} x - x_0 & y - y_0 & z - z_0 \\ l & m & n \\ A & B & C \end{vmatrix} = 0$

22. $3x + 12y - 3z - 5 = 0$ va $3x - 4y + 9z + 7 = 0$ tekisliklarga parallel bo'lib, $\frac{x+5}{2} = \frac{y-3}{-4} = \frac{z+1}{3}$ va $\frac{x-3}{-2} = \frac{y+1}{3} = \frac{z-2}{4}$ to'g'ri chiziqlarni kesib o'tadigan to'g'ri chiziqning parametrik tenglamalari tuzilsin.
$(Javob: x = 8t - 3, \quad y = -3t - 1, \quad z = -4t + 2.)$

4. CHIZIQLAR VA SIRTLAR

4.1. IKKINCHI TARTIBLI EGRI CHIZIQLAR

Ikkinchi tartibli egri chiziq deb tekislikdagi Dekart koordinatalari x va y lardan iborat bo'lgan shunday $M(x,y)$ nuqtalar to'plamiga aytiladiki, x va y lar o'z navbatida

$$a_{11}x^2 + 2a_{12}xy + a_{22}y^2 + 2a_1x + 2a_2y + a_0 = 0, \qquad (4.1)$$

2-darajali algebraik tenglamalarni qanoatlantiradilar. Bu yerdagi $a_{11}, a_{12}, a_{22}, a_1, a_2$ va a_0 lar haqiqiy o'zgarmas sonlardir. Yuqoridagi (4.1) ni *ikkinchi tartibli egri chiziqlarning umumiy tenglamasi* deb ataladi.

Ushbu tenglamaning xususiy hollarini qarab o'tamiz.

1. Markazi $C(x_0, y_0)$ nuqtada bo'lib, radiusi R bo'lgan *aylana*

$$(x-x_0)^2 + (y-y_0)^2 = R^2 \qquad (4.2)$$

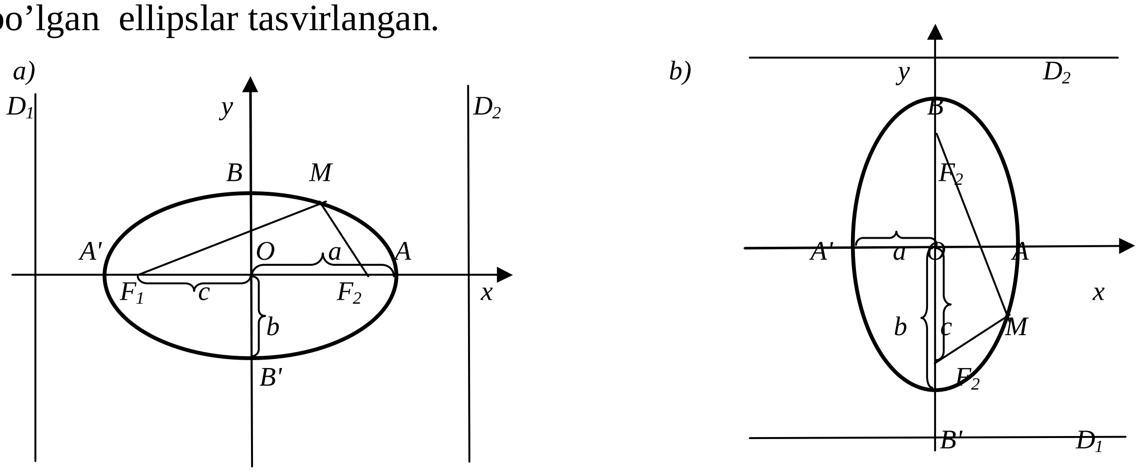
tenglama bilan beriladi.

2. Markazi koordinata boshida bo'lib, uchlari Ox va Oy o'qlarining mos ravishda A, A' va B, B' nuqtalarida joylashgan hamda yarim o'qlari a va b lardan iborat bo'lgan *ellips*,

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad (4.3)$$

eng sodda (kanonik) tenglama bilan aniqlanadi.

Quyidagi $4.1a$) rasmda katta yarim o'qi a va kichik yarim o'qi b $(a > b)$ bo'lgan ellips, $4.1b$)rasmda esa, katta yarim o'qi b va kichik yarim o'qi a $(a < b)$ bo'lgan ellipslar tasvirlangan.



4.1 rasm

Ellipslardagi F_1 va F_2 lar, ularning *fokus nuqtalaridir*. Ta'rifga ko'ra, ellipsda yotuvchi har qanday $M\ (x,y)$ nuqta uchun $a > b$ bo'lganda $F_1M + F_2M = 2a$ shart va $(a < b)$ bo'lganda esa $F_1M + F_2M = 2b$ shart o'rinli bo'ladi. Agar $c = OF_1 = OF_2$ deb belgilash kiritilsa, birinchi holda $b^2 = a^2 - c^2$, ikkinchi holda esa,

$a^2 = b^2 - c^2$, kabi tengliklar bajariladi. D_1 va D_2 to'g'ri chiziqlar *ellipsning direktrisalari* deb atalib ularning tenglamalari $x = \pm\frac{a}{\varepsilon} = \pm\frac{a^2}{c}$ (agar $a > b$ bo'lsa) va $x = \pm\frac{b}{\varepsilon} = \pm\frac{b^2}{c}$ (agar $a < b$ bo'lsa) ko'rinishida aniqlanadi. Koordinata o'qlari ellipsning simmetriya o'qlari deb yuritiladi.

Ta'rifga binoan fokuslar orasidagi F_1F_2 masofaning katta yarim o'qi uzunligiga bo'lgan nisbati *ellipsning ekstsentrisiteti* (uni biz ε bilan belgilaymiz) deb ataladi:

$$\varepsilon = \frac{c}{a}(a > b) \qquad \text{va} \qquad \varepsilon = \frac{c}{b}(a < b)$$

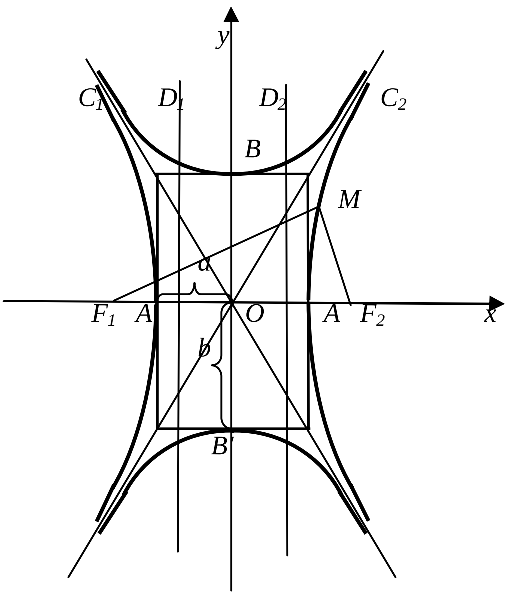
Har ikkala holda ham har doim $0 \le \varepsilon < 1$ kabi tengsizlik o'rinli bo'ladi.

3. Markazi koordinata boshida bo'lib uchlari Ox o'qidagi A va A' nuqtalarida joylashgan hamda haqiqiy yarim o'qi *a*, mavhum yarim o'qi *b* bo'lgan *giperbolaning kanonik* (*eng sodda*) *tenglamasi*

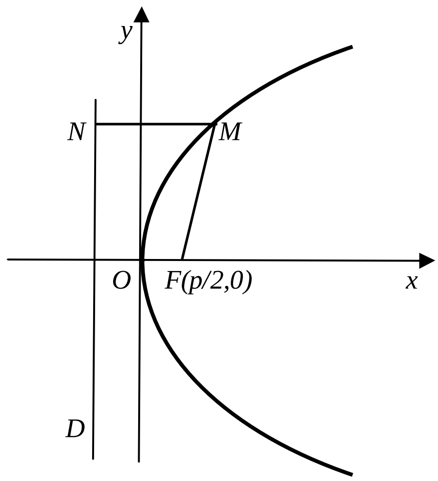
$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \qquad (4.4)$$

ko'rinishida aniqlanadi.

Quyidagi 4.2 –*rasm*da, asimptotalari C_1 va C_2 ($y = \pm\frac{b}{a}x$) to'g'ri chiziqlar, ekstsentristeti $\varepsilon = \frac{c}{a}$, direktrisalari D_1 va D_2 $\left(x = \pm\frac{a}{\varepsilon}\right)$ hamda fokus nuqtalari $F_1(-c;0)$ va $F_2(c;0)$ bo'lgan giperbola tasvirlangan. Giperbola uchun har doim $b^2 = c^2 - a^2$ kabi tenglik o'rinli bo'ladi, shuning uchun, $\varepsilon = \sqrt{1 + b^2/a^2} > 1$ bo'ladi.



4.2- rasm



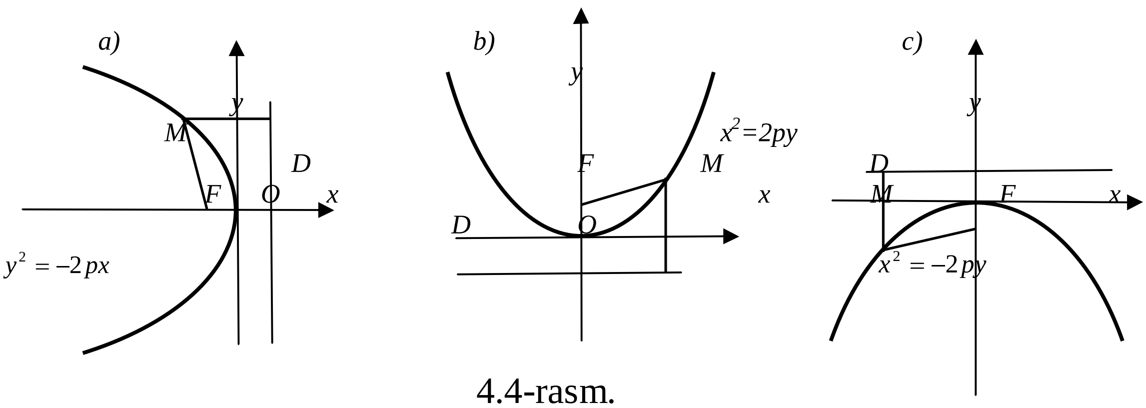
4.3- rasm

Giperbolaning tarifiga binoan unda yotuvchi har qanday M nuqta uchun $|F_1M - F_2M| = 2a$ shart o'rinlidir. Kanonik tenglamasi

$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad (4.5)$$

bo'lgan giperbola (4.4) ga *qo'shma giperbola* deb yuritiladi. Uning uchlari Oy o'qining B va B' nuqtalarida joylashgan bo'lib asimptotalari esa, (4.4) giperbolaning asimptotalari bilan ustma-ust tushadi $(4.2-\text{расм})$. Shuningdek, koordinata o'qlari giperbolada ham uning simmetriya o'qlari bo'ladi.

4. Uchi koordinata boshida bo'lib, Ox o'qiga nisbatan simmetrik joylashgan *parabolaning kanonik (eng sodda) tenglamasi* $y^2=2px$ ko'rinishda beriladi. U 4.3-rasmda tasvirlangan. Paraboladagi $F(p/2;0)$ nuqta, uning *fokusi*, $x=-p/2$ tenglama bilan berilgan D to'g'ri chiziq esa, *direktrisasi* deb ataladi. Paraboladagi ixtiyoriy M nuqta uchun $FM=MN$ tenglik o'rinlidir. Ox o'qi parabolaning simmetriya o'qi bo'lib, $p>0$ son esa, uning *parametri* deb yuritiladi. $y^2=-2px$, $x^2=2py$, va $x^2=-2py$ lar ham parabolaning *kanonik tenglamalaridir*. Ular 4.4 *a-c* – rasmlarda tasvirlangan.



4.4-rasm.

Eslatma. Quyidagi $\frac{(x-x_0)^2}{a^2}\pm\frac{(y-y_0)^2}{b^2}=1$ hamda $(y-y_0)^2=2py(x-x_0)$ kabi tenglamalar ham mos ravishda ellips, giperbola hamda parabolalarning tenglamalaridir. Ular Oxy koordinatalar sistemasiga nisbatan shunday parallel ko'chirilganki, natijada ellips, giperbolalarning markazlari hamda parabolaning uchi $C(x_0:y_0)$ nuqtada joylashadi. Ellips, giperbola hamda parabolalar uchun quyida keltirilgan ajoyib umumiy hossa har doim o'rinlidir, ya'ni: ularning ixtiyoriy $M(x;y)$ nuqtasidan fokusgacha bo'lgan masofaning, shu nuqtadan, tanlangan fokus nuqtaning direktrisasigacha bo'lgan masofaga nisbati o'zgarmas miqdor bo'lib, u miqdor ekstsentrisitet ε ga tengdir. Parabolaning ekstsentrisiteti 1 ga tengdir. Ushbu xossani, 2-tartibli egri chiziqlarning ta'rifi sifatida qabul qilish mumkin.

1-misol. $A(1,0)$ nuqta hamda tenglamasi $x=2$ bo'lgan chiziq dekart koordinatalari sistemasida berilgan. Shunday egri chiziq tenglamasi tuzilsinki, uning har bir $M(x,y)$ nuqtasi: a) berilgan to'g'ri chiziqqa nisbatan A nuqtaga ikki marta yaqin bo'lsin; b) berilgan to'g'ri chiziqqa nisbatan A nuqtadan ikki marta uzoqda bo’lsin; v) ham to’g’ri chiziqqacha, ham A nuqtagacha bir xil masofalarda joylashgan bo’lsin.

► a) shart bo’yicha $2MA = MN$ (4.5-rasm), hamda $N(2, y)$ bo’lganligidan, quyidagilarni hosil qilamiz:

$$2\sqrt{(x-1)^2 + y^2} = \sqrt{(x-2)^2},$$

$$4\left(x^2 - 2x + 1 + y^2\right) = x^2 - 4x + 4, \quad 3x^2 + 4y^2 - 4x = 0,$$

$$3\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) + 4y^2 = \frac{4}{3},$$

$$3\left(x - \frac{2}{3}\right)^2 + 4y^2 = \frac{4}{3} \quad , \quad \frac{\left(x - \frac{2}{3}\right)^2}{\frac{4}{9}} + \frac{y^2}{\frac{1}{3}} = 1.$$

Shunday qilib, ellipsning tenglamasi hosil qilindi. Bu erda $A(1{,}0)$ nuqta, o’ng fokus nuqta bilan ustma –ust tushadi, $x = 2$ esa, o’ng direktrisadir.

b) Shartga binoan, $MA = 2MN$ (4.6-rasm). Demak :

$$\sqrt{(x-1)^2 + y^2} = 2\sqrt{(x-2)^2}$$

$$x^2 - 2x + 1 + y^2 = 4x^2 - 16x + 16,$$

$$3x^2 - y^2 - 14x + 15 = 0, \quad 3\left(x^2 \frac{14}{3}x + \frac{49}{9}\right) - y^2 = \frac{49}{3} - 15 = \frac{4}{3} \quad ,$$

$$\frac{\left(x - \frac{7}{3}\right)^2}{\frac{4}{9}} - \frac{y^2}{\frac{4}{3}} = 1.$$

Bu giperbolaning tenglamasidir. $A(1;0)$ nuqta chap fokus bilan ustma-ust tushadi , $x = 2$ esa, chap direktrisadir.

v) Shartga binoan, $MA = MN$ (4.7-rasm) . Demak : $\sqrt{(x-1)^2 + y^2} = \sqrt{(x-2)^2}$,

$$x^2 - 2x + 1 + y^2 = x^2 - 4x + 4,$$

$$y^2 = -2x + 3 \text{ yoki } \; y^2 = -2\left(x - \frac{3}{2}\right).$$

Bu esa, parabolaning tenglamasi bo’lib $A(1;0)$ nuqta fokus bilan ustma-ust tushadi , $x = 2$ direktrisadir.◄

Agar (4.1) umumiy tenglama, ellips yoki giperbola yoki parabolaning tenglamasini ifodalagan bo’lsa, u holda koordinata o’qlarini koordinata boshi atrofida $tg2\alpha = 2a_{12}/(a_{11} - a_{22})$ tenglama orqali aniqlanadigan α burchakka burish hamda bu o’qlarni parallel ko’chirish yordamida har doim yangi koordinatalar sistemasida ularning kanonik tenglamalarini hosil qilish mumkin bo’ladi.

Ayniqsa, agar (4.1) tenglamada $a_{12} = 0$ bo’lsa, uni kanonik shaklga keltirish yanada osonlashadi. Bu holda, to’la kvadrat ajratish usulidan foydalaniladi.

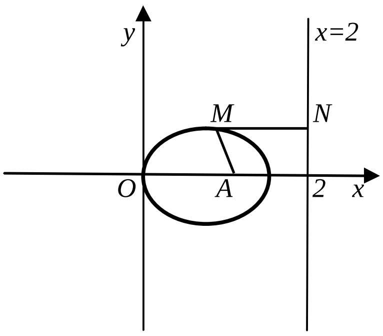
2-misol. $4x^2 + 9y^2 + 32x - 54y + 109 = 0$ tenglama bilan berilgan egri chiziq tenglamasini kanonik shaklga keltirilib, uni chizmada tasvirlansin.

► *x* va *u* lar qatnashgan hadlarni to’la kvadratgacha to’ldiramiz :

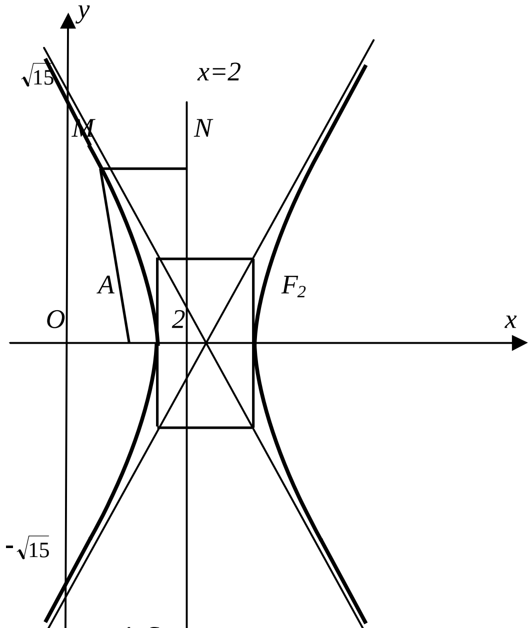
$$4\left(x^2+8x+16\right)+9\left(y^2-6y+9\right)=64+81-109=36,$$

$$4(x+4)^2+9(y-3)^2=36 \text{ yoki } \frac{(x-4)^2}{9}+\frac{(y-3)^2}{4}=1$$

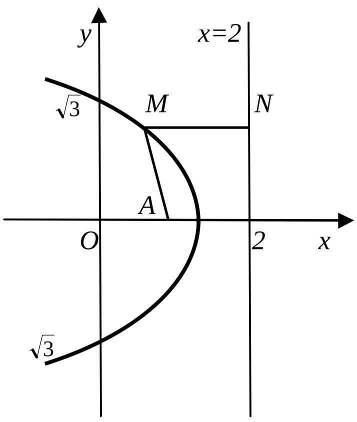
Bu esa, markazi $C(-4;3)$ nuqtada bo'lib, katta va kichik yarim o'qlari mos ravishda $a=3$ va $b=2$ bo'lgan ellipsning kanonik tenglamasidir (4.8-rasm).◄



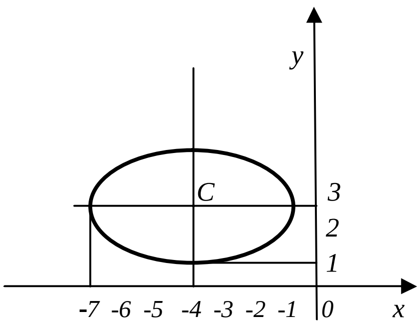
4.5- rasm



4.6- rasm



4.7- rasm



4.8- rasm

4.1- AT

1. Kanonik tenglamasi $\frac{x^2}{25}+\frac{y^2}{9}=1$ bo'lgan ellipsning fokus nuqtalarining koordinatalari, eksentrisiteti hamda direktrisalarining tenglamalari aniqlanib, chizmasi chizilsin.

$\left(\text{javob}: F_1(-4{,}0), F_2(4{,}0), E=0{,}8, x=\pm 25/4.\right)$

2. Giperbolaning $\frac{x^2}{36}-\frac{y^2}{64}=1$ kanonik tenglamasi berilgan. Uning yarim o'qlari, fokuslari, ekstsentrisiteti, asimptotalarining tenglamalari hamda direktrisasi aniqlanib, chizmasi chizilsin.

3. Parabolaning $x^2 = 6y$ kanonik tenglamasiga ko'ra, uning direktrisasi, fokusi aniqlanib, chizmasi keltirilsin.

4. Agar ellipsda, quyidagilar ma'lum bo'lsa:

a) kichik o'qi 24 va fokuslar orasidagi masofasi 10;

b) fokuslar orasidagi masofa 6, ekstsentrisitet $\varepsilon = \frac{3}{5}$;

v) fokuslar orasidagi masofa 4, direktrisalari orasidagi masofa 5;

g) direktrisalari orasidagi masofa 32, $\varepsilon = 0{,}5$ bo'lsa, u holda uning kanonik tenglamasi tuzilsin.

5. Agar giperbolada quyidagilar berilgan bo'lsa:

a) uchlari orasidagi masofa 8, fokuslar orasidagi masofa 10 ga teng bo'lsa;

b) haqiqiy yarim o'qi 5 ga teng bo'lib, uchlari markazi bilan fokuslar orasidagi masofani teng ikkiga bo'lsa;

v) haqiqiy o'qi 6 ga teng bo'lib, giperbola $A(9,-4)$ nuqtadan o'tsa;

g) $P(-5,2)$ va $Q(2\sqrt{5},2)$ nuqtalar giperbolada yotsa, u holda giperbolaning kanonik tenglamalari tuzilsin.

6. Agar quyidagilar ma'lum bo'lsa:

a) parabolaning fokusi $F(0,2)$ va uchi $O(0,0)$ nuqtalarda bo'lsa;

b) parabola Ox o'qiga nisbatan simmetrik bo'lib, $O(0,0)$ va $M(1,-4)$ nuqtalardan o'tadigan bo'lsa;

v) parabola Ox o'qiga nisbatan simmetrik bo'lib, $O(0,0)$ va $N(6,-2)$ nuqtalardan o'tadigan bo'lsa, u holda uning kanonik tenglamalari tuzilsin.

7. To'la kvadrat ajratish va koordinata boshini ko'chirish yordamida tenglamalari quyida keltirilgan egri chiziqlarning turi, o'lchamlari hamda tekislikdagi joylashishlari aniqlansin va chizmalari chizilsin.

a) $x^2 + y^2 - 4x + 6y + 4 = 0;$

b) $2x^2 + 5y^2 + 8x - 10y - 17 = 0;$

v) $x^2 - 6y^2 - 12x + 36y - 18 = 0;$

g) $x^2 - 8x + 2y + 18 = 0.$

Mustaqil ish

1. Agar aylananing diametrlaridan birining uchlari $A(3,9)$ va $B(7,3)$ nuqtalarda bo'lsa, uning tenglamasi tuzilsin. $\left(\text{Javob}: (x-5)^2 + (y-6)^2 = 13.\right)$

2. Agar giperbolaning uchlari $\frac{x^2}{225} + \frac{y^2}{144} = 1$ ellipsning fokus nuqtalarida, fokus nuqtalari esa, ellips uchlarida joylashgan bo'lsa, uning kanonik tenglamasi tuzilsin. $\left(\text{Javob}: \frac{x^2}{81} - \frac{y^2}{144} = 1.\right)$

3. Agar $M(x, y)$ nuqta, harakatlanish jarayoninig ixtiyoriy momentida ham $A(8,4)$ nuqtadan, ham ordinata o'qidan teng uzoqlikda joylashadigan bo'lsa, ushbu nuqta harakat traektoriyasining tenglamasi tuzilsin. $\left(\text{Javob}: (y-4)^2 = 16(x-4) - \text{parabola}\right)$.

4. Agar $M(x, y)$ nuqta, harakatlanish jarayoninig ixtiyoriy momentida $A(5,0)$ nuqtadan $5x - 16 = 0$ to'g'ri chiziqqa nisbatan 1,25 marta uzoqlikda joylashgan bo'lsa, harakat traektoriyasining tenglamasi yozilsin. $\text{Javob}: \frac{x^2}{16} - \frac{y^2}{9} = 1.)$

5. Gorizontga nisbatan o'tkir burchak ostida otilgan raketa parabola yoyi bo'yicha harakatlanib, start joyidan 60 km masofada qulagan. Start joyini koordinata boshida, qulash joyini esa, Ox ning musbat yarmida, hamda raketaning eng katta balandligi 18 *km* deb olinadigan bo'lsa, raketaning parabolik traektoriyasining tenglamasi yozilsin va parametri aniqlansin. $\left(\text{Javob}: (x-30)^2 = -50(y-18), \, p = 25 \, km.\right)$

4.2 . IKKINCHI TARTIBLI SIRTLAR

Ikkinchi tartibli sirtlar deb, fazodagi shunday nuqtalar to'plamiga aytiladiki, ularning Dekart koordinatalari bo'lgan *x*, *u* va *z* lar quyidagi ikkinchi darajali algebraik tenglamani qanoatlantiradi:

$$a_{11}x^2 + a_{22}y^2 + a_{33}z^2 + 2a_{12}xy + 2a_{13}xz + 2a_{23}yz + 2a_1x + 2a_2y + 2a_3z + a_0 = 0,$$

bu yerdagi $a_{11}, a_{22}, \ldots, a_0$ koefisiyentlar o'zgarmas haqiqiy sonlar bo'lib, mazkur tenglama *ikkinchi tartibli sirtning umumiy tenglamasi* deyiladi. ikkinchi tartibli sirtlarning 9 ta sinfi mavjud bo'lib, ularning *kanonik tenglamalarini* umumiy tenglamadan hosil qilish uchun koordinata o'qlarini o'zgartirish usulidan (koordinata o'qlarini parallel ko'chirish hamda ularni koordinata boshi atrofida burish) foydalaniladi.

Mazkur o'zgartirishlar natijasida quyidagi kanonik tenglamalarni hosil qilamiz:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{b^2} = 1 \qquad \text{(ellipsoidlar)}, \qquad (4.6)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \qquad \text{(bir pallali giperboloidlar)}, \qquad (4.7)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -1 \qquad \text{(ikki pallali giperboloidlar)}, \qquad (4.8)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0 \qquad \text{(ikkinchi tartibli konuslar)}, \qquad (4.9)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2z \qquad \text{(elliptik paraboloidlar)}, \qquad (4.10)$$

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 2z \qquad \text{(giperbolik paraboloidlar)}, \qquad 4.11)$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{(elliptik silindrlar)}, \qquad (4.12)$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \qquad \text{(giperbolik silindrlar),} \qquad (4.13)$$

$$x^2=2py \qquad \text{(parabolik silindrlar).} \qquad (4.14)$$

Bu yerdagi *a*, *b*, *s*, *r* parametrlar, o'zgarmas musbat sonlar bo'lib, ular o'z navbatida sirtlarning hossalarini ma'lum ma'noda tavsiflaydilar.

Ikkinchi tartibli sirtning umumiy tenglamasidan kanonik tenglamani hosil qilish ancha murakkab jarayondir, ammo *xu*, *xz*, *uz* hadlar qatnashmagan hollarda $(a_{12}=a_{13}=a_{23}=0)$, umumiy tenglamadan kanonik tenglamani hosil qilish uchun aynan ikkinchi tartibli egri chiziqlardagiga o'xshash to'la kvadrat ajratish va koordinata o'qlarini parallel ko'chirish usulidan foydalanamiz.

1-misol. $x^2-2y^2+4z^2+2x-12y-8z-3=0$; tenglamani kanonik ko'rinishga keltirib, mazkur tenglama bilan berilgan sirtning turi, xossalari hamda uning *Oxyz* koordinatalar sistemasiga nisbatan joylashganligi aniqlansin.

► Tenglama tarkibidagi *x*, *y*, *z* larga nisbatan hadlarda to'la kvadratlar ajratib, quyidagini hosil qilamiz:

$$\left(x^2+2x+1-1\right)+2\left(y^2+6y+9-9\right)+4\left(z^2-2z+1-1\right)-3=0,$$

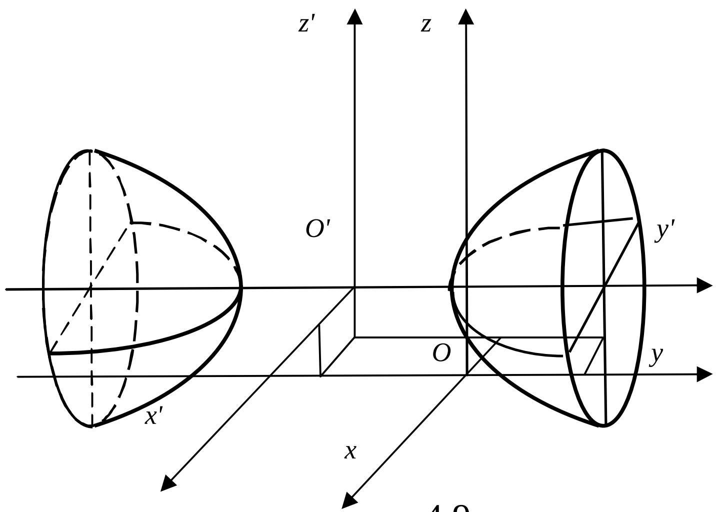
$$(x+1)^2-2(y+3)^2+4(z-1)^2=3+1-18+4=-10,$$

$$\frac{(x+1)^2}{10}-\frac{(y+3)^2}{5}+\frac{(z-1)^2}{\frac{5}{2}}=1.$$

Agar $x'=x+1$, $y'=y+3$ va $z'=z-1$ formulalar orqali koordinata o'qlarini parallel ko'chiriladigan bo'lsa, yangi sistemaning koordinata boshi $O'(-1;-3;1)$ nuqtada bo'lib, sirtning kanonik tenglamasi

$$\frac{x'^2}{10}-\frac{y'^2}{5}+\frac{z'^2}{\frac{5}{2}}=-1$$

ko'rinishda yoziladi. Mazkur tenglamadan ko'rinmoqdaki, qaralayotgan sirt ikki



4.9- rasm

pallali giperboloid bo'lib, u yangi $O'y'$ o'q bo'yicha cho'zinchoq hamda uning markazi $O'(-1;-3;1)$ nuqtada bo'lar ekan $\left(a = \sqrt{10},\ b = \sqrt{5},\quad c = \sqrt{5/2}\ \right)(4.9 - \textit{расм})$. ◄

Yuqorida sanab o'tilgan barcha (4.6)- (4.14) kabi sirtlarning shakllari va xossalarini *parallel kesimlar usuli* deb ataluvchi usul yordamida aniqlash mumkin bo'ladi. Mazkur usulning mohiyati shundaki, sirtlarni koordinata tekisliklariga parallel tekisliklar bilan kesiladi, undan keyin esa, kesimlarda hosil bo'lgan chiziqlarning shakli va xossalariga ko'ra, sirtning shakli va xossalari haqidagi ma'lumotlar hosil qilinadi.

2-misol. Bir pallali giperboloid $\frac{x^2}{16}+\frac{y^2}{4}-\frac{z^2}{9}=1$ ning shakli va xossalari aniqlanib, uning chizmasi chizilsin.

► Sirtni, tenglamasi $z=h$ bo'lgan gorizontal tekisliklar bilan kesamiz.

$$\begin{cases} \frac{x^2}{16}+\frac{y^2}{4}=1+\frac{h^2}{9} \\ z=h \end{cases}$$

tenglamalar sistemasidan ko'rish mumkinki, bu xildagi har qanday kesimda yarim o'qlari $a=4\sqrt{1+h^2/9}$ va $b=2\sqrt{1+h^2/9}$ lardan iborat ellips hosil bo'lar ekan.

Tenglamalari, $x=h$ va $y=h$ lar bilan berilgan tekisliklar orqali kesilganda, kesimlarda mos ravishda quyidagi giperbolalar hosil bo'ladi:

$$\begin{cases} \frac{x^2}{16}-\frac{z^2}{9}=1-\frac{h^2}{4}, \\ x=h \end{cases} \qquad \begin{cases} \frac{y^2}{16}-\frac{z^2}{9}=1-\frac{h^2}{4}, \\ y=h \end{cases}$$

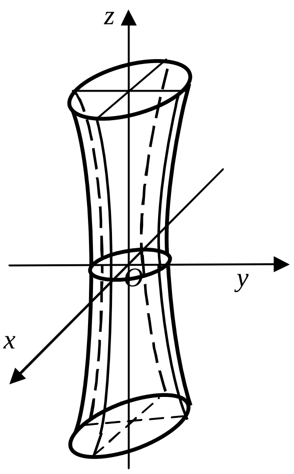
Xususan, agar $h=0$ bo'lganda biz bir pallali giperboloidning $z=0$ yoki $x=0$ yoki $y=0$ koordinata tekisliklar bilan kesilgandagi kesimlarni hosil qilamiz. Ularni odatda *bosh kesimlar* deb ataladi. (4.10-rasm) Bosh kesimlarning o'lchamlari quyidagichadir: $z=0$ tekislikda ellipsning yarim o'qlari $a=4$ va $b=2$; $x=0$ tekislikda giperbolaning haqiqiy yarim o'qi $b=2$, mavhum yarim o'qi esa, $c=3$; $y=0$ teislikda, giperbolaning haqiqiy yarim o'qi $a=4$ bo'lib, mavhum yarim o'qi $c=3$ dir. Koordinata tekisliklari, simmetriya tekisliklari bo'ladi. ◄

Muhandislik masalalarida ko'pincha, turli xildagi *aylanma sirtlar* deb ataluvchi sirtlar uchraydi, ya'ni, shunday sirtlarki, ular biror egri chiziqning, shu egri chiziq bilan bir tekislikda yotuvchi va sirtning *aylanish o'qi* deb ataladigan berilgan to'g'ri chiziq atrofida aylanishidan hosil bo'ladi.

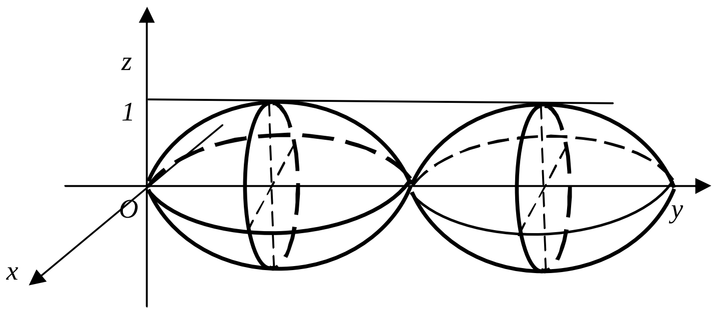
Agar egri chiziq Oyz tekislikda yotib, uning tenglamasi $F(y,z)=0$, $x=0$ bo'ladigan bo'lsa, uni Oz o'q atrofida aylantirishdan hosil bo'ladigan aylanma sirtning tenglamasi. $F\left(\pm\sqrt{x^2+y^2},z\right)=0$ kabi ko'rinishda bo'ladi; agar aylantirish Oy o'qi atrofida bajarsak, aylanma sirt tenglamasining ko'rinishi $F\left(y,\sqrt{x^2+z^2}\right)=0$ kabi yoziladi.

3- misol. Kanonik tenglamasi $\frac{y^2}{a^2}-\frac{z^2}{b^2}=1$ bo'lgan giperbolaning a) Oz o'qi atrofida; b) Oy o'qi atrofida aylanishidan hosil bo'lgan aylanma sirtlarning tenglamasi yozilsin.

► a) Yuqorida keltirilgan qoidalarga muvofiq, giperbola tenglamasidagi y ni $\pm\sqrt{x^2+y^2}$ bilan almashtiramiz: $\frac{x^2+y^2}{a^2}-\frac{z^2}{b^2}=1$ Mazkur tenglama, o'z navbatida bir pallali aylanma giperboloidning tenglamasidir. Uning gorizontal kesimlarida ellipslarning o'rnida aylanalar yotadi (2-misolga qaralsin).



4.10- rasm



4.11- rasm

b) Qaralayotgan giperbolani Oy o'qi atrofida aylantirishdan hosil bo'ladigan aylanma sirtning tenglamasini yozish uchun z ni $\pm\sqrt{x\ \ +y^2}$ bilan almashtiamiz u holda:

$$\frac{y^2}{a^2}-\frac{x^2+z^2}{b^2}=1 \text{ yoki } \frac{x^2}{b^2}-\frac{y^2}{a^2}+\frac{z^2}{b^2}=-1.$$

Bu tenglama, Oy o'qi bo'ylab cho'zinchoq bo'lgan ikki pallali giperboloidning tenglamasidir (1-misolga qaralsin). Bu sirtning $y=h>a$ tekisliklar bilan kesilishidan hosil bo'lgan kesimlar 1-misoldagidek ellipslar bo'lmasdan, balki, aylanalardir.◄

4-misol. Sinusoida yoyi $(z=\sin y,\ \ x=0(0\le y\le 2\pi))$ ning Oy o'qi atrofida aylanishdan hosil bo'lgan sirt tenglamasi yozilsin.

► Qoidaga muvofiq, mazkur aylanma sirt tenglamasi $z=\sin\left(\pm\sqrt{x^2+y^2}\right)$ yoki $z=\pm\sin\sqrt{x^2+y^2}$ kabi yoziladi (4.11-rasm).◄

4.2- AT

1. Parallel kesimlar usuli yordamida quyidagi sirtlarining shakllari tekshirilib, ularning chizmalari chizilsin.

a) $x^2+2y^2+4z^2=2;$ b) $2x^2-9y^2-z^2=36;$

c) $-2x^2+3y^2+4z^2=0;$ d) $2y^2+z^2=2x;$

e) $z^2-y^2=x;$ f) $2x^2+4z^2=4;$ j) $y^2-6z=0;$

2. Quyidagi sirtlarning ko'rinishi aniqlanib, chizmalari chizilsin.

a) $x^2+y^2+z^2-3x+5y-4z=0;$

b) $36x^2+16y^2-9z^2+18z=9;$

c) $x^2+y^2+z^2=2z;$

d) $5x^2+y^2+10x-6y-10z+14=0;$

e) $x^2+3z^2-8x+18z+34=0.$

3. Quyidagi tenglamalar ifodalaydigan sirtlar bilan chegaralangan jismlar tasvirlansin.

a) $x^2=z,\ \ z=0,\ \ 2x-y=0,\ \ x+y=9;$

b) $z^2=4-y,\ \ x^2+y^2=4y;$

c) $z=y^2,\ \ x^2+y^2=9;\ \ z=0,$

d) $z=y,\ \ z=0,\ \ y=\sqrt{4-x},\quad y=\frac{1}{2}(x-1).$

Mustaqil ish

Quyida keltirilgan tenglamalar orqali berilgan sirtlar bilan chegaralangan jismlar tasvirlansin.

1. $z=4-x^2,\ \ z=0,\ \ x^2+y^2=4;$

2. $z=2x^2+y^2,\ \ z=0,\ \ x=0,\ \ y=0,\ \ x+y=1;$

3. $x^2+y^2+z^2=9,\ \ z+1=x^2+y^2(z\geq -1);$

4. $x^2+y^2=z,\ \ x^2+y^2=4,\ \ z=0.$

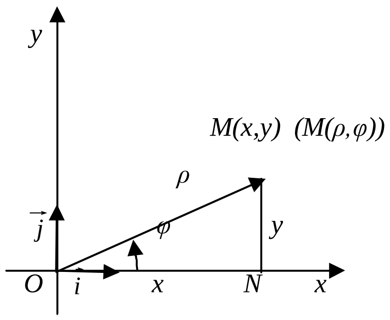
4.3 QUTB KOORDINATALARIDA VA PARAMETRIK TENGLAMALARI ORQALI BERILGAN CHIZIQLAR.

Nuqtaning qutb koordinatalari va chiziqning qutb koordinatalaridagi tenglamasi. Ma'lumki, tekislikdagi Oxy Dekart koordinatalari sistemasidagi har qanday *M* nuqta *x* va *u* sonlar orqali aniqlanar edi, ya'ni, $M(x,y)$ (4.12-rasm). Ushbu nuqtani masalan, $\rho=\left|\overrightarrow{OM}\right|$ masofa hamda *qutb o'qi* deb deb ataluvchi Ox o'qining $\overrightarrow{OM}$ radius vektorning soat strelkasiga teskari yo'nalishda hosil qilgan φ burchagi orqali ham aniqlash mumkin. Bu holda $M(\varphi,\rho)$ yozuvdan foydalanamiz. Bu erda, ρ masofani *qutb radiusi*, φ ni *qutb burchagi*, *O* nuqtani esa, qutb deb atalishi qabul qilingan. 4.12 rasmga ko'ra *M* nuqtaning *x* va *u* Dekart koordinatalari bilan ρ va φ qutb koordinatalari orasidagi bog'lanish quyidagicha ifodalanadi:

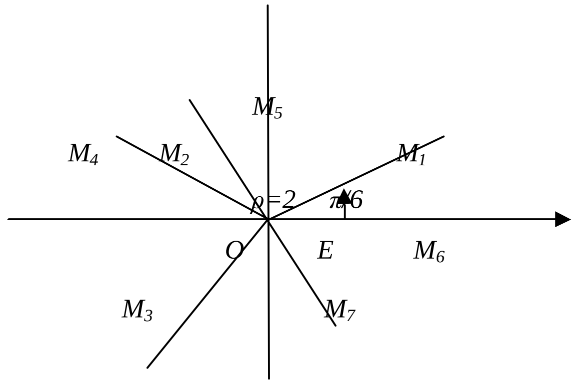
$$\left.\begin{array}{l} x=\rho\cos\varphi,\rho\geq 0 \\ y=\rho\sin\varphi, 0\leq\varphi<2\pi \end{array}\right\} \qquad (4.15)$$

yoki mazkur formuladan:

$$\rho=\sqrt{x^2+y^2},\qquad \cos\varphi=\frac{x}{\sqrt{x^2+y^2}},\qquad \sin\varphi=\frac{y}{\sqrt{x^2+y^2}}. \qquad (4.16)$$



4.12-rasm

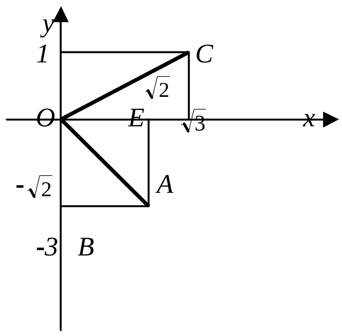


4.13-rasm

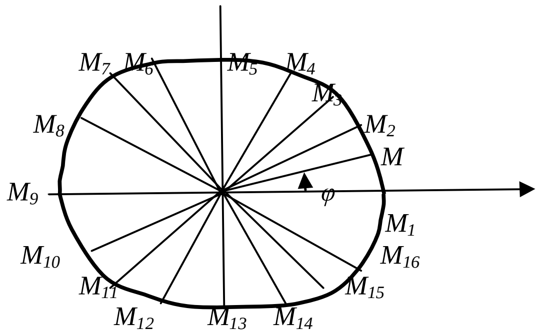
Yuqoridagi (4.15) va (4.16) formulalar orqali, u yoki bu chiziqning Dekart koordinatalari sistemasidagi tenglamasidan uning qutb koordinatalari sistemasidagi tenglamasini hosil qilish mumkin va aksincha.

1-misol. Qutb koordinatalari bilan berilgan quyidagi nuqtalarning o'rinlari topilsin: $M_1(2,\pi/6)$, $M_2(1{,}3\pi/4)$, $M_3(3{,}5\pi/4)$, $M_4(2{,}5\pi/6)$, $M_5(3/2,\pi/2)$, $M_6(4{,}0)$, $M_7(3,\pi/4)$.

► Avvalo, Ox o'qiga qutb o'qiga nisbatan φ burchak ostidagi nur o'tkazamiz, keyin esa, nur ustida O qutbdan boshlab ρ uzunlikdagi kesmani belgilaymiz. Natijada barcha nuqtalarning o'rinlarini aniqlaymiz. OE kesma uzunlik birligini aniqlaydi (4.13-rasm).◄



4.14-rasm.



4.15-rasm.

2-misol. _1-misolda berilgan $M_1, \ldots, M_7$ nuqtalarning Dekart koordinatalari aniqlansin.

►(4.15) formuladan foydalanib, $M_1(\sqrt{3},1)$, $M_2\left(-\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2}\right)$, $M_3-\left(\frac{3\sqrt{2}}{2},-\frac{3\sqrt{2}}{2}\right)$,

$M_4(-\sqrt{3},1)$ $M_5\left(0,\frac{3}{2}\right)$, $M_6(4{,}0)$ va $M_7\left(\frac{3\sqrt{2}}{2},-\frac{3\sqrt{2}}{2}\right)$ larni topamiz.◄

3-Misol. Dekart koordinatalari sistemasida $A(\sqrt{2},-\sqrt{2})$, $B(0,-3)$ va $C(\sqrt{3},1)$ nuqtalarining o'rinlarini topib, ularning qutb koordinatalari aniqlansin.

►(4.16) formulaga muvofiq, A nuqta uchun $\rho=2,\ tg\varphi=-1,\ \varphi=\frac{7\pi}{4}$ ni ya'ni,

$A\left(2,\frac{7\pi}{4}\right)$ ni hosil qilamiz; B nuqta uchun $\rho=3,\ \sin\varphi=-1,\ \varphi=\frac{3\pi}{2},\ B\left(3,\frac{3\pi}{2}\right)$ ni C nuqta uchun esa, $\rho=2,\ tg\varphi=\sqrt{3}/3,\ \varphi=\frac{\pi}{6},$ ya’ni, $c\left(2,\frac{\pi}{6}\right)$ larni hosil qilamiz.(4.14-rasm).◄

4-misol. $\left(x^2+y^2\right)^{3/2}=4\left(x^2-3y^2\right)$ tenglama bilan berilgan chiziqning qutb koordinatalardagi tenglamasi yozilsin.

► (4.15) formulaga binoan,

$$\rho^3=4\left(\rho^2\cos^2\varphi-3\rho^2\sin^2\varphi\right)$$

$\rho\neq 0$ deb hisoblab, $\rho=4\left(cos^2\varphi-\sin^2\varphi-2\sin^2\varphi\right)=4\left(\cos 2\varphi-1+\cos 2\varphi\right)$ yoki $\rho=4\left(2\cos 2\varphi-1\right)$ ni hosil qilamiz.◄

5-misol. $\rho^2=8\sin^2 2\varphi$ tenglama bilan berilgan egri chiziqning Dekart koordinatalariga nisbatan tenglama yozilsin.

► $\sin 2\varphi=2\sin\varphi\cos\varphi$ ekanligidan, $\rho^2=32\sin^2\varphi\cos^2\varphi$ da $\sin\varphi$ bilan $\cos\varphi$ larni (4.16) formuladagi qiymatlari bilan almashtiramiz:

$$\left(\sqrt{x^2+y^2}\right)^2=32\cdot\left(\frac{y}{\sqrt{x^2+y^2}}\right)^2\cdot\left(\frac{x}{\sqrt{x^2+y^2}}\right)^2$$

Bundan esa, $\left(x^2+y^2\right)^2=32x^2y^2$ ni aniqlaymiz.◄

6-Misol. $\rho=2+\cos^2\varphi$ tenglama bilan berilgan egri chiziqning shakli chizmada aks ettirilsin.

► φ_i burchaklarning qiymatlariga mos keluvchi $\rho_i\left(i=1{,}16\right)$ larning qiymatlarini aniqlab, quyidagi jadvalni tuzamiz;

φ_i	ρ_i	φ_i	ρ_i	φ_i	ρ_i	φ_i	ρ_i	φ_i	ρ_i	φ_i	ρ_i
0	3	$\frac{\pi}{3}$	$\frac{9}{4}$	$\frac{3}{4}\pi$	$\frac{5}{2}$	$\frac{7}{6}\pi$	$\frac{11}{4}$	$\frac{3}{2}\pi$	2	$\frac{11}{6}\pi$	$\frac{11}{4}$
$\frac{\pi}{6}$	$\frac{11}{4}$	$\frac{\pi}{2}$	2	$\frac{5}{6}\pi$	$\frac{11}{4}$	$\frac{5}{4}\pi$	$\frac{5}{2}$	$\frac{5}{3}\pi$	$\frac{9}{4}$	2π	3
$\frac{\pi}{4}$	$\frac{5}{2}$	$\frac{2\pi}{3}$	$\frac{9}{4}$	π	3	$\frac{4}{3}\pi$	$\frac{9}{4}$	$\frac{7}{4}\pi$	$\frac{5}{2}$		

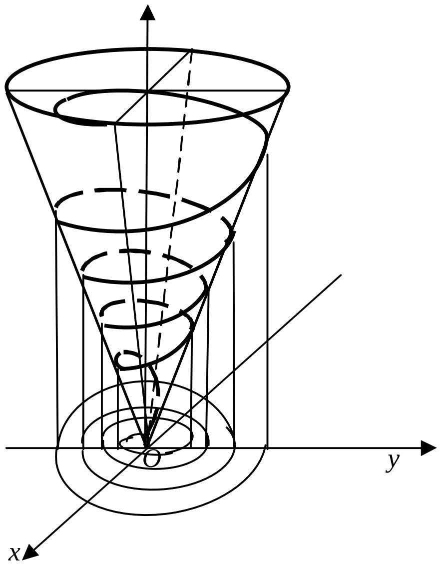
$M_i\left(\rho_i,\varphi_i\right)$nuqtalarning o’rinlarini topib (1-misolga qaralsin), ularni birlashtirib, qaralayotgan egri chiziqning yetarlicha aniq grafikni hosil qilamiz (4.15-rasm).◄

Chiziqning parametrik tenglamalari

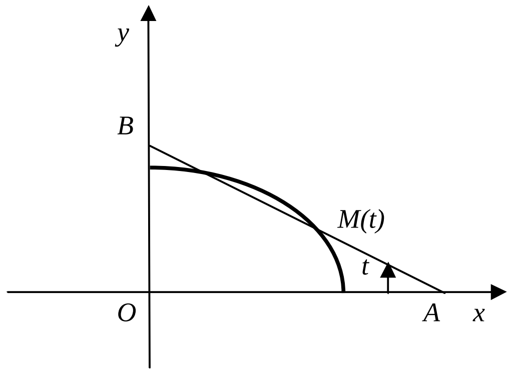
Quyidagi tenglamalar sistemasi

$$\left.\begin{aligned} x &= f_1(t), \\ y &= f_2(t), \\ z &= f_3(t), \end{aligned}\right\} t_1 \le t \le t_2 \qquad (4.17)$$

ni fazodagi chiziqning *parametrik tenglamalari* deb yuritiladi. Bu yerdagi $f_1(t)$ $f_2(t)$ va $f_3(t)$ lar t o'zgaruvchi parametrga bog'liq bo'lgan funksiyalardir. Xususan, $f_3(t) \equiv 0$ (yoki $f_1(t) \equiv 0$, yoki $f_2(t) \equiv 0$) bo'lganda, $z = 0$ tekislikdagi chiziqning parametrik tenglamalarini ($x = 0$ yoki $y = 0$) hosil qilamiz. Ta'kidlash lozimki, (4.17) tenglamalar, nafaqat chiziqlarning berilishini, balki $M(x, y, z)$ nuqtaning ushbu chiziq bo'yicha *"harakat qonuni"*ni ham ifodalaydi, ya'ni t parametrning har bir qiymatiga chiziqdagi nuqtaning ma'lum bir holati mos keladi.



4.16-rasm



4.17-rasm

7-misol. Quyidagi parametrik tenglamalar, ya'ni:

$$\left.\begin{aligned} x &= at\cos t, \\ y &= at\sin t, \quad a > 0, b > 0, t \ge 0 \\ z &= bt, \end{aligned}\right\}$$

lar qanday chiziqni tasvirlaydi?

► Bu tenglamalar, $z = 0$ tekislikdagi proeksiyasi *Arximed spirali* bo'lgan *ρ=aφ vint spirali* deb ataluvchi egri chiziqni ifoda etadi (4.16-rasm).◄

8-misol.

$$\left.\begin{aligned} x &= a\sin^2 t, \ a > 0 \\ y &= b\cos^2 t, \ b > 0, -\infty < t < \infty \\ z &= 0, \end{aligned}\right\}$$

parametrik tenglamalar qanday chiziq tenglamasi ekanligi aniqlansin.

►Bu tenglamalar, uchlari koordinata o'qlarining $A(a;0)$ va $B(0;b)$ nuqtalarida bo'lgan to'g'ri chiziq kesmasini ifodalaydi. t parametr $(-\infty;+\infty)$ oraliqda o'zgarganda, AV kesmaning $M(t)$ nuqtasi ushbu kesmada cheksiz davriy harakatlanadi.(4.17-rasm).◄

Agar (4.17) parametrik tenglamalardan t parametr yo'qotilsa, chiziqning

Dekart koordinatalardagi tenglamasi hosil bo'ladi. Fazodagi chiziq uchun har bir jufti tsilindirik sirtlarni ifodalaydigan ikkitadan tenglamalar sistemasi qaraladiki, ularning kesishishidan o'z navbatida chiziqning o'zi hosil bo'ladi. Masalan, 7-misolda qaralgan vint spirali deb ataluvchi egri chiziqni quyidagi bir juft silindrik sirtlarning tenglamalari sifatida qarash mumkin $\left(t=\frac{z}{b}\right)$:

$$x=\frac{a}{b}z\cos\frac{z}{b},\qquad y=\frac{a}{b}z\sin\frac{z}{b},\qquad z\geq 0.$$

Biror koordinata tekisligida yotuvchi chiziq tenglamasidagi t parametrni yo'qotish ham, Dekart koordinatalariga nisbatan bir juft tenglamalarga keltiriladiki, ulardan biri har doim mazkur chiziq yotadigan koordinata tekisligidan iborat bo'ladi. Masalan, 8-misoldagi AV kesmaning tenglamasini Dekart koordinatalaridagi bir juft tenglama bilan ifodalash mumkin:

$$\frac{x}{a}+\frac{y}{b}=1,\quad z=0\quad 0\leq x\leq a,\quad 0\leq y\leq b.$$

9-misol. Quyidagi parametrik tenglamalar

$$\left.\begin{aligned}&x=0\\&y=t-1/t+2,\\&z=t+1/t-3,\end{aligned}\right\}$$

bilan berilgan chiziq chizilsin.

► Ushbu chiziq Oyz koordinata tekisligida yotadi. Parametr t ga turli xil qiymatlar berib, chiziqda yotuvchi yetarli miqdordagi nuqtalar hosil qilinadiki, ularni birlashtirib chiziq hosil qilinadi. Mazkur chiziqni aniqroq o'rganish maqsadida parametrni yo'qotish usulidan foydalanamiz. Ikkinchi va uchinchi tenglamalardagi 2 bilan -3 ni tenglikning chap tomoniga o'tkazib, tengliklarni kvadratga oshiramiz, so'ngra, $(z+3)^2$ dan $(y-2)^2$ ni ayiramiz. U holda:

$$(z+3)^2-(y-2)^2=\left(t+\frac{1}{t}\right)^2-\left(t-\frac{1}{t}\right)^2=4,\qquad \frac{(z+3)^2}{4}-\frac{(y-2)^2}{4}=1.$$

Natijada, $x=0$ koordinata tekisligida teng tomonli giperbola $(a=b=2)$ hosil bo'ldi. Uning markazi $C(0;2;-3)$ nuqtadir (4.18-rasm).◄

4.3– AT

1. Qutb koordinatalarida berilgan chiziqlarni tasvirlab, ularning Dekart koordinatalaridagi tenglamalari yozilsin:

1) $\rho=5;$

2) $\varphi=\frac{\pi}{3};$

3) $\rho=a\varphi$ (*Arximed spirali*)

4) $\rho=6\cos\varphi;$

5) $\rho=10\sin\varphi;$

6) $\rho\cos\varphi=2;$

7) $\rho \sin\varphi = 1$

8) $\rho = \dfrac{4}{1-\cos\varphi}$ (*parabola)*

9) $\rho = a(1-\cos\varphi)$, (*кардоида*)

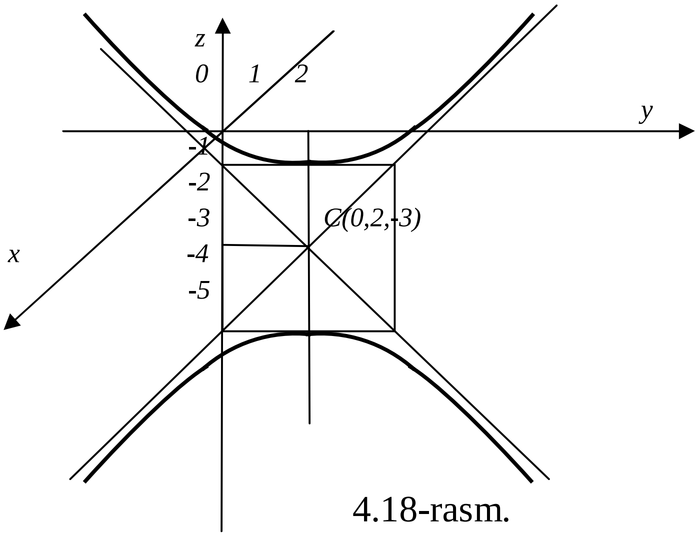
10) $\rho = 3/\varphi$ (*giperbolik spiral*)

11) $\rho = 2\varphi,\ \rho = (1/2)^{\varphi}$ (*logarifimik spiral*)

12) $\rho = a\sin 3\varphi$ (*uch yaproqli atirgul*);

13) $\rho = a\sin^2 2\varphi$ (*to'rt yaproqli atirgul*);

14) $\rho^2 = a^2\cos 2\varphi$; (*Bernulli lemniskatasi*).



4.18-rasm.

2. Quyidagi chiziqlarning qutb koordinatalaridagi tenglamalari yozilsin:

a) Qutb o'qidan perpendikulyar ravishda 3 birlik kesmani kesuvchi to'g'ri chiziq;

b) Qutb o'qiga parallel bo'lib, undan 5 birlik masofada joylashgan to'g'ri chiziqlar;

c) Qutbdan o'tib, markazi qutb o'qida joylashgan radiusi $R = 4$ bo'lgan aylana;

d) Qutb o'qiga qutbda urinma bo'lib o'tadigan $R = 3$ radiusli aylanalar. (*Javob:*a) $\rho\cos\varphi = 3$; b) $\rho\sin\varphi = \pm 5$; c) $\rho = 8\cos\varphi$; e) $\rho = \pm 6\sin\varphi.$).

AT–4.3

3. Parametrik tenglamalari bilan berilgan quyidagi chiziqlar chizilsin:

1) $x = 3t - 1,\ \ y = -2t + 5;$

2) $x = 3\cos t + 3,\ \ y = 3\sin t - 2;$

3) $x = 5 + 4\cos t,\ \ y = -1 + \sin t;$

4) $x = a(t - \sin t),\ \ y = a(1 - \cos t)$ (*tsikloida*);

5) $x = aCos^3 t,\ \ y = a\sin^3 t$ (*astroida*);

6) $x = aCost,\ \ y = a\sin t\ \ z = bt$ (*vint chiziqi*)

7) $x=\frac{a}{2}\left(t+\frac{1}{t}\right),\quad y=\frac{b}{2}\left(t-\frac{1}{t}\right),$

Mustaqil ish

1. Tekislikda o’zlarining parametrik tenglamalari bilan berilgan chiziqlarning parametrini yo’qotish usuli yordamida ularning Dekart koordinatalariga nisbatan $F(x;y)=0$ tenglamalari yozilib, har bir chiziqning turi hamda tekislikda qanday joylashganligi aniqlansin:

1) $x=\frac{a}{\cos t},\quad y=btgt$ (*giperbola*);

2) $x=2a\cos^2 t,\quad y=a\sin 2t$ (*aylana*);

3) $x=2a\sin 2t,\quad y=2a\sin^2 t$ (*aylana*);

4) $x=-2+3\sin 2t,\quad y=1+\cos 2t$ (*ellips*);

5) $x=4(1-t),\quad y=2\sqrt{t}$ ($y\geq 0$ *ni qanoatlantiruvchi parabolaning qismi*).

2. Quyidagi tenglamalar bilan berilgan chiziqlarning qutb koordinatalariga nisbatan tenglamalari yozilib, keyin ularning chizmalari chizilsin:

1) $x^2+y^2=5\left(\sqrt{x\;\;+y^2}-x\right);$

2) $x^4-y^4=\left(x^2+y^2\right)^3;$ 3) $\left(x^2+y^2\right)^2=y^2;$

4) $3x^2-y^2=\left(x^2+y^2\right)^{\frac{3}{2}};$ 5) $\left(x^2+y^2\right)^3=4x^2y^2.$

4.4 4- bo’limga individual uy topshiriqlari

4.1.–IUT

1. Quyidagi berilganlarga ko’ra, a) ellips; b) giperbola v) parabolalarning kanonik tenglamalari tuzilsin. Bu erda: *A*, *V* lar egri chiziqda yotuvchi nuqtalar; *G’*-fokus nuqta; *a*-katta (haqiqiy) yarim o’q; *b*-kichik (mavhum) yarim o’q; ε-ekstsentrisitet; $y=\pm kx$ –giperbola asimptotalarining tenglamalari; *D*-egri chiziqning direktrisasi; 2*s*- fokuslar orasidagi masofa.

1.1. *a) b= 15, F(-10, 0); b) a= 13, ε = 14/13; c) D:* $x=-4$.

1.2. *a) b= 2, F(4*$\sqrt{2}$*, 0); b) a= 7, ε =* $\sqrt{85}$*/7; c) D:* $x=5$.

1.3. *a) A(3, 0), B(2,*$\sqrt{5}$*/3); b)* $\kappa=\frac{3}{4},\ \varepsilon=\frac{5}{4}$*; c) D:* $y=-2$.

1.4. *a)* $\varepsilon=\frac{\sqrt{21}}{5}$*, A(-5, 0); b) A(*$\sqrt{80}$*, 3), B(4*$\sqrt{6}$ *, 3*$\sqrt{2}$*); c) D:* $y=1$.

1.5. *a) 2a=22,* $\varepsilon=\frac{\sqrt{57}}{11}$*; b)* $\kappa=\frac{2}{3}$*, 2c=*$10\sqrt{13}$*; c)* simmetriya o’qi *Ox* va *A(27,9).*

1.6. *a) b=*$\sqrt{15}$*,*$\varepsilon=\frac{\sqrt{10}}{25}$*; b)*$\kappa=\frac{3}{4}$*, 2a=16; c)Ox simmetriya o’qi va A(4,-8).*

1.7. *a)* *a = 4, F(3, 0) ;* *b) b = 2*$\sqrt{10}$ *, F(-11, 0); c) D:* $x=-2$.
1.8. *a) b= 4 , F(9, 0) ;* *b) a= 5, ε = 7/5;* *c) D:* $x=6$.
1.9. *a) A(0,* $\sqrt{3}$ *) , B(*$\sqrt{14/3}$ */1); b)* $\kappa=\frac{\sqrt{21}}{10}$, $\varepsilon=\frac{11}{10}$; *c) D:* $y=-4$.
1.10. *a)* $\varepsilon=\frac{7}{8}$, *A(8, 0); b) A(3, -*$\sqrt{3/5}$ *) , B(*$\sqrt{13/5}$ *, 6);* *c) D:* $y=4$.
1.11. *a) 2a=24,*$\varepsilon=\sqrt{22}/6$ *; b)*$\kappa=\sqrt{2/3}$ *, 2c=*10*; c)Ox* simmetriya o'qi va *A(-7,-7).*
1.12. *a) b = 2,* $\varepsilon=\frac{5\sqrt{29}}{29}$*; b)* $\kappa=\frac{12}{13}$*, 2a=26; c) Ox* simmetriya o'qi va *A(-5,15).*
1.13. *a)* *a = 6, F(-4 , 0) ;* *b) b = 3, F(7, 0); c) D:* $x=-7$.
1.14. *a) b= 7 , F(5, 0) ;* *b) a= 11, ε = 12/11;* *c) D:* $x=10$.
1.15. *a) A(*$\sqrt{17/3}$*, 1/3), B(*$\sqrt{21}$*/2, 1/2); b)* $\kappa=1/2$, $\varepsilon=\sqrt{5}/2$ *; c) D:* $y=-1$.
1.16. *a)* $\varepsilon=\frac{3}{5}$, *A(0, 8); b) A(*$\sqrt{6}$ *,0) , B(-2*$\sqrt{2}$ *, 1);* *c) D:* $y=9$.
1.17. *a) 2a=22,* $\varepsilon=10/11$*;b)*$\kappa=\sqrt{11}/5$ *, 2c=*12*; c)* Ox simmetriya o'qi va *A(-7,5).*
1.18. *a) b= 5,* $\varepsilon=\frac{12}{13}$*; b)* $\kappa=1/3$*, 2a= 6;* *c) Oy* simmetriya o'qi va *A(-9, 6).*
1.19. *a)* *a = 9, F(7, 0);* *b) b = 6, F(12, 0); c) D:* $x=-1/4$.
1.20. *. a) b= 5 , F(-10, 0);* *b) a= 9, ε = 4/3;* *c) D:* $x=12$.
1.21. *a) A(0, -2), B(*$\sqrt{15}$*/2, 1); b)* $\kappa=\frac{2\sqrt{10}}{9}$, $\varepsilon=11/9$*;* *c) D:* $y=5$.
1.22. *a)* $\varepsilon=\frac{2}{3}$, *A(-6, 0); b) A(*$\sqrt{8}$ *,0), B(*$\sqrt{20}$ */3 , 2);* *c) D:* $y=1$.
1.23. *a) 2a =50,* $\varepsilon=3/5$ *; b)* $\kappa=\sqrt{29}/14$*, 2c=*30 *;c)* simmetriya o'qi Oy va *A(4, 1).*
1.24. *a) b=2*$\sqrt{15}$*,* $\varepsilon=\frac{7}{8}$*; b)* $\kappa=5/6$*, 2a= 12; c) Oy simmetriya o'qi va A(-2,3*$\sqrt{2}$ *).*
1.25. *a)* *a= 13, F(-5 , 0) ;* *b) b= 44, F(-7, 0); c) D:* $x=-3/8$.
1.26. *a) b= 7 , F(13, 0) ;* *b) b = 4, F(-11, 0); c) D:* $x=13$.
1.27. *a) A(-3, 0) , B(1,*$\sqrt{40}$ */3); b)* $\kappa=\sqrt{2/3}$ *,* $\varepsilon=\sqrt{15}/3$ *;* *c) D:* $y=4$.
1.28. *a)* $\varepsilon=\frac{5}{6}$, *A(0, -*$\sqrt{11}$*); b) A(*$\sqrt{32/3}$ *,1), B(*$\sqrt{8}$ *, 0); c) D:* $y=-3$.
1.29. *a) 2a=30,*$\varepsilon=17/15$*;b)*$\kappa=\sqrt{17}/8$ *, 2c=*18*;c) Ox* simmetriya o'qi va *A(4, -10).*
1.30. *a) b=2*$\sqrt{2}$ *,*$\varepsilon=\frac{7}{9}$*; b)* $\kappa=\sqrt{2}/2$*, 2a=12; c) Oy* simmetriya o'qi va *A(-45, 15).*

2. Quyidagi berilganlarga binoan, markazi *A* nuqtada bo'lib, ko'rsatilgan nuqtalardan o'tuvchi aylana tenglamasi tuzilsin.

2.1. $12x^2-13y^2=156$ giperbolaning uchi, $A(0,-2)$
2.2. $4x^2-9y^2=36$ giperbolaning uchi, $A(0,4)$.
2.3. $24y^2-25x^2=600$ giperbolaning chap fokusi, $A(0,-3)$.

2.4. $O(0,0)$, A nuqta $y^2 = 3(x-4)$ parabolaning uchi.
2.5. $9x^2 + 25y^2 = 1$ ellipisning fokuslari, $A(0,6)$
2.6. $3x^2 - 4y^2 = 12$ giperbolaning fokuslari, $A(0,-3)$.
2.7. $3x^2 + 4y^2 = 12$ ellipisning fokuslari, "A" - uning yuqori uchidagi nuqta.
2.8. $x^2 - 16y^2 = 64$ giperbolaning fokuslari, $A(0,-2)$.
2.9. $4x^2 - 5y^2 = 80$ giperbolaning fokuslari, $A(0,-3)$.
2.10. A nuqta $y^2 = -\frac{(x+5)}{2}$ nuqtaning uchi va $O(0,0)$.
2.11. $33x^2 + 49y^2 = 1617$ ellipsning o'ng fokusi, $A(1,7)$.
2.12. $3x^2 - 5y^2 = 30$ giperbolaning chap fokusi, $A(0,6)$.
2.13. $16x^2 + 41y^2 = 656$ ellipsning fokuslari, A esa, uning uchi.
2.14. $2x^2 - 9y^2 = 18$ giperbolaning uchi, $A(0,4)$.
2.15. $5x^2 - 11y^2 = 55$ giperbolaning fokuslari, $A(0,5)$.
2.16. $A(0,-2)$ $B(1;4)$, A esa, $y^2 = \left(\frac{x-4}{3}\right)$ parabolaning uchi.
2.17. $3x^2 + 7y^2 = 21$ ellipsning chap fokusi, $A(-1,-3)$.
2.18. $5x^2 - 9y^2 = 45$ giperbolaning chap uchi, $A(0,-6)$.
2.19. $24x^2 - 25y^2 = 600$ ellipsning chap fokusi, A esa, uning yuqori uchi.
2.20. $3x^2 - 16y^2 = 48$ giperbolaning o'ng uchi, $A(1,3)$.
2.21. $7x^2 - 9y^2 = 63$ giperbolaning chap fokusi, $A(-1,-2)$.
2.22. $B(2,-5)$ $B(2;-5)$, A esa, $x^2 = -2(y+1)$ parabolaning uchi.
2.23. $x^2 + 4y^2 = 12$ ellipsning o'ng fokusi, $A(2,-7)$.
2.24. $40x^2 - 81y^2 = 3240$ giperbolaning o'ng uchi, $A(-2,5)$ $A(-2;5)$.
2.25. $x^2 + 10y^2 = 90$ ellipsning fokuslari, A esa,uning pastki uchi.
2.26. $3x^2 - 25y^2 = 75$ giperbolaning o'ng uchi, $A(-5,-2)$.
2.27. $4x^2 - 5y^2 = 20$ giperbolaning fokuslari, $A(0,-6)$.
2.28. $B(3,4)$, A esa, $y^2 = \left(\frac{x+7}{4}\right)$ parabolaning uchi.
2.29. $13x^2 + 49y^2 = 837$ ellipsning chap fokusi, $A(1,-8)$.
2.30. $57x^2 - 64y^2 = 3648$ giperbolaning o'ng fokuslari, $A(2,8)$.

3. Har bir M nuqtasi berilgan shartlarni qanoatlantiruvchi chiziq tenglamasi tuzilsin.

3.1. $A(1,3)$ nuqtagacha bo'lgan masofa, $x = -6$ to'g'ri chiziqqacha bo'lgan masofadan 2 marta kam.

3.2. $A(4,0)$nuqtagacha bo'lgan masofa, $x = -2$ to'g'ri chiziqqacha bo'lgan masofadan 2 marta kam.

3.3. $A(5,0)$nuqtagacha bo'lgan masofa, $y = -2$ to'g'ri chiziqqacha bo'lgan masofadan 3 marta kam.

3.4. M nuqtadan $A(2,3)$ gacha va $B(-1,2)$ gacha bo'lgan masofalarning nisbati $3/4$ ga teng.

3.5. M nuqtadan $A(4,0)$ va $B(-2,2)$ nuqtalargacha bo'lgan masofalar kvadratlarining yig'indisi 28 ga teng.

3.6. $x=8$ to'g'ri chiziqqacha bo'lgan masofa, $A(1,0)$ nuqtagacha bo'lgan masofadan 5 marta ortiq.

3.7. $B(-2,-1)$ nuqtagacha bo'lgan masofa, $A(4,1)$ nuqtagacha bo'lgan masofadan 4 marta kam.

3.8. $A(6,1)$ nuqtagacha bo'lgan masofa, $x=-5$ to'g'ri chiziqqacha bo'lgan masofadan 3 marta kam.

3.9. $A(0,-2)$ nuqtagacha bo'lgan masofa, $y=7$ to'g'ri chiziqqacha bo'lgan masofadan 5 marta kam.

3.10. M nuqtadan $A(0,-2)$ $A(-3;5)$ va $A(4,2)$ nuqtalargacha bo'lgan masofalarning nisbati $1/3$ ga teng.

3.11. M nuqtadan $A(-5,-1)$ va $B(3,2)$ nuqtalargacha bo'lgan masofalar kvadratlarining yig'indisi 40,5 ga teng.

3.12. $x=-5$ to'g'ri chiziqqacha bo'lgan masofa, $A(2,1)$ nuqtagacha bo'lgan masofadan 3 marta kam.

3.13. $B(5,1)$ nuqtagacha bo'lgan masofa, $A(-3,3)$ nuqtagacha bo'lgan masofadan 3 marta kam.

3.14. $A(-1,7)$ nuqtagacha bo'lgan masofa, $x=8$ to'g'ri chiziqqacha bo'lgan masofadan 2 marta kam.

3.15. $A(-1,2)$ $A(-1;2)$ nuqtagacha bo'lgan masofa $x=9$ to'g'ri chiziqqacha bo'lgan masofadan 4 marta ortiq.

3.16. M nuqtadan $A(2,-4)$ va $A(3,5)$ nuqtalargacha bo'lgan masofalarning nisbati $2/3$ ga teng.

3.17. M nuqtadan $A(-3,3)$ va $B(4,1)$ nuqtalargacha bo'lgan masofalar kvadratlarining yig'indisi 31 ga teng.

3.18. $x=3$ to'g'ri chiziqqacha bo'lgan masofa, $A(0,-5)$ nuqtagacha bo'lgan masofadan 2 marta ortiq.

3.19. $B(1,6)$ nuqtagacha bo'lgan masofadan, $A(4,-2)$ nuqtagacha bo'lgan masofa 2 marta kam.

3.20. $A(0,-2)$ $A(1;4)$ nuqtagacha bo'lgan masofa, $x=-7$ nuqtagacha bo'lgan masofadan 3 marta ortiq..

3.21. $A(2,3)$ nuqtagacha bo'lgan masofa, $x=14$ to'gri chiziqqacha bo'lgan masofadan 2 marta ortiq .

3.22. M nuqtadan $A(2-3)$ va $A(2,3)$ nuqtalargacha bo'lgan masofalarning nisbati $3/5$ ga teng.

3.23. M nuqtadan $A(-5,3)$ va $B(2,-4)$ nuqtalargacha bo'lgan masofalar kvadratlarining yig'indisi 65 ga teng.

3.24. $x=5$ to'g'ri chiziqqacha bo'lgan masofadan, $A(3,-4)$ nuqtagacha bo'lgan masofa 3 marta ortiq .

3.25. $A(5,7)$ nuqtagacha bo'lgan masofa, $B(-2,1)$ nuqtagacha bo'lgan masofadan 4 marta ortiq .

3.26. $x=2$ to'g'ri chiziqqacha bo'lgan masofa, $A(4,-3)$ nuqtagacha bo'lgan masofadan 5 marta ortiq .

3.27. $x=-7$ to'g'ri chiziqqacha bo'lgan masofa, $A(3,1)$ nuqtagacha bo'lgan masofadan 3 marta kam .

3.28. M nuqtadan $A(3,-5)$ va $B(4,1)$ nuqtalargacha bo'lgan masofalarning nisbati $\frac{1}{4}$ ga teng.

3.29. M nuqtadan $A(-1,2)$ va $B(3,-1)$ nuqtalargacha bo'lgan masofalar kvadratlarining yig'indisi 18,5 ga teng.

3.30. $x=-1$ to'g'ri chiziqqacha bo'lgan masofadan, $A(1,5)$ nuqtagacha bo'lgan masofa 4 marta kam .

4. Qutb koordinatalari sistemasidagi tenglamalari bilan berilgan egri chiziqlar chizilsin.

4.1. $\rho=2\sin 4\varphi$.

4.2. $\rho=2(1-\sin 2\varphi)$.

4.3. $\rho=2\sin 2\varphi$.

4.4. $\rho=3\sin 6\varphi$.

4.5. $\rho=2/(1+\cos\varphi)$.

4.6. $\rho=3(1+\sin\varphi)$.

4.7. $\rho=2(1-\cos\varphi)$.

4.8. $\rho=3(1-\cos 2\varphi)$.

4.9. $\rho=4\sin 3\varphi$.

4.10. $\rho=4\sin 4\varphi$.

4.11. $\rho=3(\cos\varphi+1)$.

4.12. $\rho=1/(2-\sin\varphi)$

4.13. $\rho=5(1-\sin 2\varphi)$

4.14. $\rho=3(2-\cos 2\varphi)$

4.15. $\rho=6\sin 4\varphi$.

4.16. $\rho=2\cos 6\varphi$.

4.17. $\rho=3/(1-\cos 2\varphi)$.

4.18. $\rho=2(1-\cos 3\varphi)$

4.19. $\rho=3(1-\cos 4\varphi)$.

4.20. $\rho=5(2-\sin\varphi)$.

4.21. $\rho=3\sin 4\varphi$.

4.22. $\rho=2\cos 4\varphi$.

4.23. $\rho=4(1+\cos 2\varphi)$.

4.24. $\rho=1/(2-\cos 2\varphi)$

4.25. $\rho=4(1-\sin\varphi)$.

4.26. $\rho=3(1+\cos 2\varphi)$.

4.27. $\rho=3\cos 2\varphi$.

4.28. $\rho=2\sin 3\varphi$.

4.29. $\rho=2(2-\cos\varphi)$

4.30. $\rho=2-\cos 2\varphi$.

5. Parametrik tenglamalari bilan berilgan egri chiziqlar chizilsin ($0\le t\le 2\pi$)

5.1. $\begin{cases} x=4\cos^3 t, \\ y=4\sin^3 t. \end{cases}$

5.2. $\begin{cases} x=2\cos^3 t, \\ y=2\sin^3 t. \end{cases}$

5.3. $\begin{cases} x=4\cos 2t, \\ y=3\sin 2t. \end{cases}$

5.4. $\begin{cases} x=2\sin t, \\ y=3(1-\cos t). \end{cases}$

5.5. $\begin{cases} x=4\cos t, \\ y=5\sin t. \end{cases}$

5.6. $\begin{cases} x=\cos^3 t, \\ y=4\sin^3 t. \end{cases}$

5.7. $\begin{cases} x=4\cos t, \\ y=5\sin t. \end{cases}$

5.8. $\begin{cases} x=5\cos^3 t, \\ y=5\sin^3 t. \end{cases}$

5.9. $\begin{cases} x = \cos 2t, \\ y = 3\sin 2t. \end{cases}$

5.10. $\begin{cases} x = 3\cos t, \\ y = 1 - \sin t. \end{cases}$

5.11. $\begin{cases} x = 2\cos t, \\ y = 4\sin t. \end{cases}$

5.12. $\begin{cases} x = 4\cos^3 t, \\ y = 5\sin^3 t. \end{cases}$

5.13. $\begin{cases} x = 2\cos t, \\ y = 5\sin t. \end{cases}$

5.14. $\begin{cases} x = 2\cos^3 t, \\ y = 2\sin^3 t. \end{cases}$

5.15. $\begin{cases} x = 3\cos 2t, \\ y = 2\sin 2t. \end{cases}$

5.16. $\begin{cases} x = 2\cos t, \\ y = 2(1 - \sin t). \end{cases}$

5.17. $\begin{cases} x = 5\cos t, \\ y = \sin t. \end{cases}$

5.18. $\begin{cases} x = 2\cos^3 t, \\ y = 5\sin^3 t. \end{cases}$

5.19. $\begin{cases} x = 4\cos 2t, \\ y = \sin 2t. \end{cases}$

5.20. $\begin{cases} x = 6\cos^3 t, \\ y = 6\sin^3 t. \end{cases}$

5.21. $\begin{cases} x = 4\cos 3t, \\ y = 2\sin 3t. \end{cases}$

5.22. $\begin{cases} x = \cos t, \\ y = 3(2 - \sin t). \end{cases}$

5.23. $\begin{cases} x = 9\cos t, \\ y = 5\sin t. \end{cases}$

5.24. $\begin{cases} x = 4\cos^3 t, \\ y = 2\sin^3 t. \end{cases}$

5.25. $\begin{cases} x = 3\cos 2t, \\ y = 3\sin 2t. \end{cases}$

5.26. $\begin{cases} x = 4\cos^3 t, \\ y = \sin^3 t. \end{cases}$

5.27. $\begin{cases} x = 5\cos 3t, \\ y = \sin 3t. \end{cases}$

5.28. $\begin{cases} x = 4\cos t, \\ y = 4(1 - \sin t). \end{cases}$

5.29. $\begin{cases} x = \cos t, \\ y = 3\sin t. \end{cases}$

5.30. $\begin{cases} x = 3\cos^3 t, \\ y = 4\sin^3 t. \end{cases}$

Namunaviy variantni yechish.

1.

a) ellipsning katta yarim o’qi 3 ga teng bo’lib fokus nuqtasi $F\left(\sqrt{5},0\right)$bo’lsa;

b) giperbolada mavhum yarim o’q 2 ga teng bo’lib, fokus nuqtasi $F\left(-\sqrt{13},0\right)$ bo’lsa;

c) parabolaning direktrisasi $x = -3$ bo’lsa, mos ravishda ularning kanonik tenglamalari tuzilsin.

►a) Shartga ko’ra, $a = 3$ va $c = \sqrt{5}$ bo’lganligi hamda $b^2 = a^2 - c^2$ dan, $b^2 = 9 - 5 = 4$. Demak, $\frac{x^2}{9} + \frac{y^2}{4} = 1;$

b) Shartga ko’ra, $b = 2$ va $c = \sqrt{13}$ bo’lib, $b^2 = c^2 - a^2$ekanligidan, $a^2 = 9$. Demak $\frac{x^2}{9} - \frac{y^2}{4} = 1;$

c) Parabolaning tenglamasi $y^2 = 2px$ hamda uning direktrisasi $\frac{p}{2} = -x$ ekanligidan va shartga ko’ra $x = -3$ yoki $p = 6$ dan, $y^2 = 12x$ kelib chiqadi

2. Markazi $x^2 + 4y^2 = 4$ ellipsning yuqori uchida bo’lib, ushbu ellipsning fokus

nuqtalaridan o’tuvchi aylananing tenglamasi tuzilsin.

► Ellipsning kanonik tenglamasi $\frac{x^2}{4}+\frac{y^2}{1}=1$ bo’lganligidan, $a=2$ va $b=1$, uning yuqori uchi $A(0,1)$ nuqtadadir. $c=\sqrt{a^2-b^2}=\sqrt{4-1}=\sqrt{3}$ bo’lganligi uchun fokus nuqtalari $F_1(-\sqrt{3},0)$ va $F_2(\sqrt{3},0)$ kabidir. Aylananing radiusini ikki nuqta orasidagi masofani topish formulasidan foydalanib aniqlaymiz:

$$R=|AF_1|=|AF_2|=\sqrt{(\pm\sqrt{3}-0)^2+(0-1)^2}=2.$$

U holda (4.2) ga ko’ra, $(x-0)^2+(y-1)^2=2^2$ yoki $x^2+(y-1)^2=4$. Bu esa tenglamasi tuzilishi lozim bo’lgan aylananing tenglamasidir.◄

3. M nuqtasi $A(3,2)$ nuqtadan $B(-1,0)$ nuqtagacha bo’lgan masofadan 3 marta katta bo’lgan chiziqning tenglamasi tuzilsin.

► Agar chiziqda yotuvchi ixtiyoriy nuqtani $M(x,y)$ deb olsak, u holda masalaning shartiga ko’ra, $|AM|=3|BM|$ tenglik o’rinlidir (4.19-rasmga qaralsin).

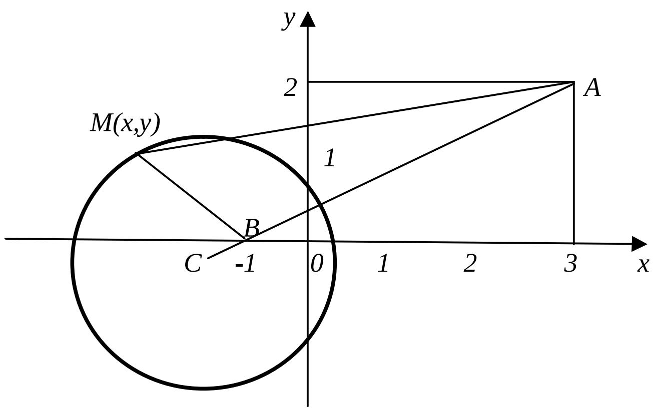
$|AM|=\sqrt{(x-3)^2+(y-2)^2}$ va $|BM|=\sqrt{(x+1)^2+y^2}$ bo’lganligi uchun

$$\sqrt{(x-3)^2+(y-2)^2}=3\sqrt{(x+1)^2+y^2}$$

ko’rinishida yoza olamiz. Bundan esa, $8x^2+24x+8y^2+4y-4=0$ ni hosil qilamiz. Ushbu tenglamada to’la kvadratlarni ajratib,

$$\left(x+\frac{3}{2}\right)^2+\left(y+\frac{1}{4}\right)^2=\frac{45}{16}$$

ni aniqlaymiz. Bu esa, markazi $C\left(-\frac{3}{2};-\frac{1}{2}\right)$ da bo’lib, radiusi $R=\frac{3\sqrt{5}}{4}$ dan iborat bo’lgan aylananing tenglamasidir.◄



4.19- rasm

4. Tenglamasi $\rho=4(1-\sin\varphi)$ bo’lgan kardioda chizilsin.

► Qutb burchagi $\varphi_i\left(i=\overline{1;16}\right)$ ga mos bo’lgan ρ_i larni aniqlab quyidagi jadvalni tuzamiz:

φ_i	ρ_i	φ_i	ρ_i	φ_i	ρ_i
0	4	$2\pi/3$	$\approx 0{,}6$	$5\pi/4$	$\approx 6{,}8$
$\frac{\pi}{6}$	2	$3\pi/4$	$\approx 1{,}2$	$4\pi/3$	$\approx 7{,}4$
$\frac{\pi}{4}$	$\approx 1{,}2$	$5\pi/6$	2	$3\pi/2$	8
$\frac{\pi}{3}$	$\approx 0{,}6$	π	4	$5\pi/3$	$\approx 7{,}4$
$\frac{\pi}{2}$	0	$7\pi/6$	6	$7\pi/4$	$\approx 6{,}8$
				$11\pi/6$	6

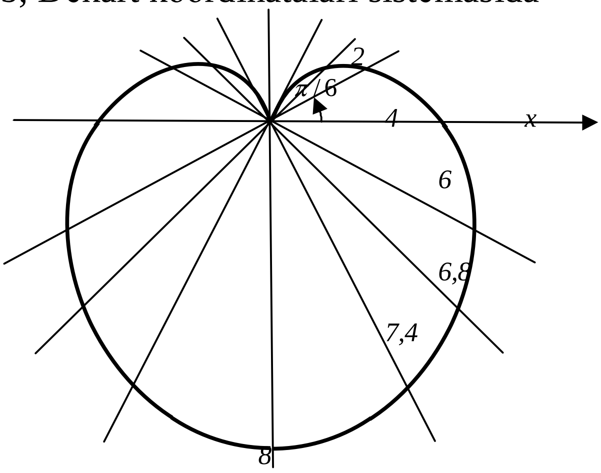
Aniqlangan $M_i(\rho_i, \varphi_i)$ nuqtalarning o'rinlarini qutb koordinatalar sistemasida topib, ularni (§4.3 dagi 1- misolga qaralsin) tekis chiziq bilan birlashtirsak, qaralayotgan kardiodaning shaklini hosil qilamiz.◄

5.

$$\left.\begin{aligned} x &= 1 + 3\cos t, \\ y &= 2 - 2\sin t \end{aligned}\right\} \qquad 0 \le t \le 2\pi$$

parametrik tenglamalar bilan berilgan egri chiziq chizilsin.

► Parametr t_i ning yetarlicha miqdordagi qiymatlariga mos bo'lgan x_i va y_i larning qiymatlarini aniqlab, Dekart koordinatalari sistemasida



4.20-rasm

$M_i(x_i, y_i)$ nuqtalarning o'rnini topamiz. So'ngra u nuqtalarni tekis chiziq bilan birlashtiramiz. Hosil bo'lgan egri chiziq, markazi *C(1,2)* nuqtada bo'lgan, yarim o'qlari *a*=3 va *b*=2 bo'lgan ellipsga juda o'xshashligini ko'rish mumkin. Uni qat'iy ravishda aniqlash uchun ***t*** parametrni yo'qotamiz, ya'ni:

$$\frac{x-1}{3} = \cos t, \quad \frac{y-2}{-2} = \sin t.$$

Bundan esa, $\frac{(x-1)^2}{9} + \frac{(y-2)^2}{4} = 1$ ni hosil qilamiz.◄

4.2IUT

1. Quyida keltirilgan tenglamalarga binoan, ularning har biri qanday sirtni ifoda etishi hamda shakllari chizilsin.

1.1. a) $4x^2 - y^2 - 16z^2 + 16 = 0;$ b) $x^2 + 4z = 0.$

1.2. a) $3x^2 + y^2 + 9z^2 - 9 = 0;$ b) $x^2 + 2y^2 - 2z = 0.$

1.3. a) $-5x^2 + 10y^2 - z^2 + 20 = 0;$ b) $y^2 + 4z^2 = 5x^2.$

1.4. a) $4x^2 - 8y^2 + z^2 + 24 = 0;$ b) $x^2 - y = -9z^2.$

1.5. a) $x^2 - 6y^2 + z^2 = 0;$ b) $7x^2 - 3y^2 - z^2 = 21.$

1.6. a) $z = 8 - x^2 - 4y^2;$ b) $4x^2 + 9y^2 + 36z^2 = 72.$

1.7. a) $4x^2 + 6y^2 - 24z^2 = 96;$ b) $y^2 + 8z^2 = 20x^2.$

1.8. a) $4x^2 - 5y^2 - 5z^2 + 40 = 0;$ b) $y = 5x^2 + 3z^2.$

1.9. a) $x^2 = 8(y^2 + z^2);$ b) $2x^2 + 3y^2 - z^2 = 18.$

1.10. a) $5z^2 + 2y^2 = 10x;$ b) $4z^2 - 3y^2 - 5x^2 + 60 = 0.$

1.11. a) $x^2 - 7y^2 - 14z^2 - 21 = 0;$ b) $2y = x^2 + 4z^2.$

1.12. a) $6x^2 - y^2 + 3z^2 - 12 = 0;$ b) $8y^2 + 2z^2 = x.$

1.13. a) $-16x^2 + y^2 + 4z^2 - 32 = 0;$ b) $6x^2 + y^2 - 3z^2 = 0.$

1.14. a) $5x^2 - y^2 - 15z^2 + 15 = 0;$ b) $x^2 + 3z = 0.$

1.15. a) $6x^2 + y^2 + 6z^2 - 18 = 0;$ b) $3x^2 + y^2 - 3z = 0.$

1.16. a) $-7x^2 + 14y^2 - z^2 + 21 = 0;$ b) $y^2 + 2z = 6x^2.$

1.17. a) $-3x^2 + 6y^2 - z^2 - 18 = 0;$ b) $x^2 - 2y = -z^2.$

1.18. a) $4x^2 - 6y^2 + 3z^2 = 0;$ b) $4x^2 - y^2 - 3z^2 = 12.$

1.19. a) $z = 4 - x^2 y^2;$ b) $3x^2 + 12y^2 + 4z^2 = 48.$

1.20. a) $4x^2 + 5y^2 - 10z^2 = 60;$ b) $7y^2 + z^2 = 14x^2.$

1.21. a) $9x^2 - 6y^2 - 6z^2 + 1 = 0;$ b) $15y = 10x^2 + 6y^2.$

1.22. a) $x^2 = 5(y^2 + z^2);$ b) $2x^2 + 3y^2 - z^2 = 36.$

1.23. a) $4x^2 + 3y^2 = 12x;$ b) $3x^2 - 4y^2 - 2z^2 + 12 = 0.$

1.24. a) $8x^2 - y^2 - 2z^2 - 32 = 0;$ b) $y - 4z^2 = 3x^2.$

1.25. a) $x^2 - 6y^2 + z^2 - 12 = 0;$ b) $x - 3z^2 = 9y^2.$

1.26. a) $2x^2 - 3y^2 - 5z^2 + 30 = 0;$ b) $2x^2 + 3z = 0.$

1.27. a) $7x^2 + 2y^2 + 6z^2 - 42 = 0;$ b) $2x^2 + 4y^2 - 5z = 0.$

1.28. a) $-4x^2 + 12y^2 - 3z^2 + 24 = 0;$ b) $2y^2 + 6z^2 = 3x.$

1.29. a) $3x^2 - 9y^2 + z^2 + 27 = 0;$ b) $z^2 - 2y = -4x^2.$

1.30. a) $27x^2 - 63y^2 + 21z^2 = 0;$ b) $3x^2 - 7y^2 - 2z^2 = 42.$

2. Quyida keltirilgan tenglamalarga binoan, ularni ifodalovchi chiziqning berilgan o'q atrofida aylanishidan hosil bo'lgan sirtlarning tenglamalari yozilib, ushbu sirtlarning ko'rinishi aniqlansin va shakli chizilsin.

2.1. a) $y^2 = 2z, \quad Oz;$ b) $9y^2 + 4z^2 = 36. \ Oy.$

2.2. a) $4x^2 - 3y^2 = 12, \quad Ox;$ b) $x = 1, \quad y = 2, \ Oz.$

2.3. a) $x^2 = -3z, \quad Oz;$ b) $3x^2 + 5z^2 = 15. \ Ox.$

2.4. a) $3y^2 - 4z^2 = 12, \quad Oz;$ b) $y = 4, \quad z = 2, \ Ox.$

2.5. a) $x^2 = 3y, \quad Oy;$ b) $3x^2 + 4z^2 = 24, \ Oz.$

2.6. a) $2x^2 - 6y^2 = 12, \quad Ox;$ b) $y^2 = 4z, \ Oz.$

2.7. a) $x^2 + 3z^2 = 9, \quad Oz;$ b) $x = 4, \quad z = 6, \ Oy.$

2.8. a) $3x^2 - 5z^2 = 15, \quad Oz;$ b) $z = -1, \quad y = 3, \ Ox.$

2.9. a) $y^2 = 3z, \quad Oz;$ b) $2x^2 + 3z^2 = 6. \ Ox.$

2.10. a) $y^2 - 5x^2 = 5, \quad Oy;$ b) $y = 3, \quad z = 1, \ Ox.$

2.11. a) $x^2 = -4z, \quad Oz;$ b) $y^2 + 4z^2 = 4, \ Oy.$

2.12. a) $5x^2 - 6z^2 = 30, \quad Ox;$ b) $x = 3, \quad z = -2, Oy.$

2.13. a) $z^2 = 2y, \quad Oy;$ b) $2x^2 + 3z^2 = 6, \ Oz.$

2.14. a) $y^2 = -4z, \quad Oz;$ b) $3y^2 + z^2 = 6. \ Oy.$

2.15. a) $7x^2 - 5y^2 = 35, \quad Ox;$ b) $x = -1, \quad y = -3, Oz.$

2.16. a) $2x^2 = z, \quad Oz;$ b) $x^2 + 4z^2 = 4, \ Ox.$

2.17. a) $2y^2 - 5z = 10, \quad Oz;$ b) $y = 2, \quad z = 6, \ Ox.$

2.18. a) $x^2 = -5y, \quad Oy;$ b) $2x^2 + 3z = 6, \ Oz.$

2.19. a) $x^2 - 9y^2 = 9, \quad Ox;$ b) $3y^2 = z, \ Oz.$

2.20. a) $x^2 + 2z = 4, \quad Oz;$ b) $x = 3, \quad z = -1, \ Oy.$

2.21. a) $15x^2 - 3y^2 = 1, \quad Ox;$ b) $x = 3, \quad y = 4, \ Oz.$

2.22. a) $y^2 = 5z, \quad Oz;$ b) $3x^2 + 7y^2 = 21. \ Ox.$

2.23. a) $15y^2 - x^2 = 6, \quad Oy;$ b) $y = 5, \quad z = 2, \ Oy.$

2.24. a) $5z = -x^2, \quad Oz;$ b) $3y^2 + 18z^2 = 1, \ Oy.$

2.25. a) $3x^2 - 8y^2 = 288, \quad Ox;$ b) $x = 5, \quad z = -3, Oy.$

2.26. a) $2y^2 = 72, \quad Oz;$ b) $6y^2 + 5z^2 = 30, \ Oy.$

2.27. a) $5x^2 - 7y^2 = 35, \quad Ox;$ b) $x = 2, \quad y = -4, \ Oz.$

2.28. a) $3x^2 = -2z, \quad Oz;$ b) $8x^2 + 11z^2 = 88. \ Ox.$

2.29. a) $5y^2 - 8z^2 = 40, \quad Oz;$ b) $y = 3, \quad z = 1, Ox.$

2.30. a) $3x^2 = -4y, \quad Oz;$ b) $4x^2 + 3z^2 = 12, \ Oz.$

3. Quyida keltirilgan sirtlar bilan chegaralgan jism chizilsin.

3.1. a) $z = x^2 + y^2, \ z = 0, \ x = 1, \ y = 2, \ y = 0;$ b) $x^2 + y^2 = 2x, \ z = 0, \ z = x.$

3.2. a) $x^2 + y^2 = z^2, \ z = 0, \ y = 2x, \ y = 4x, \ x = 3; \ (z > 0);$ b) $x^2 + y^2 = 4y, \ z = 0, \ y + z = 5.$

3.3. a) $y^2 + 3z^2 = 6, \ 3x^2 - 25y^2 = 75, \ z \geq 0;$ b) $x = 4, \ y = 2, \ x + 2y + 3z = 12, \ x = 0, \ y = 0, \ z \geq 0.$

3.4. a) $z = 5y, \ x^2 + y^2 = 16, \ z = 0;$ b) $x + y + z = 5, \ 3x + y = 5 \ 2x + y = 5, \ y = 0, \ z = 0.$

3.5. a) $y = 3x \ y = 0, \ x = 2, \ z = xy, \ z = 0;$ b) $8(x^2 + y^2) = z^2, \ x^2 + y^2 = 1, \ 2x + y = 5, \ y \geq 0, \ z \geq 0.$

3.6. a) $y = x, \ y = 0, \ x = 1, \ z = x^2 + 5y^2, \ z = 0;$ b) $x^2 + y^2 + z^2 = 9, \ x^2 + y^2 \leq 1, \ x \geq 0.$

3.7. a) $y = x \ y = 0, \ x = 1, \ z = \sqrt{xy}, \ z = 0;$ b) $x^2 + y^2 + z^2 = 4, \ x^2 + y^2 = z^2, \ x \geq 0, \ z \geq 0.$

3.8. a) $y=2x\ y=0,\ x=2,\ z=xy,\ z=0;$ b) $x^2+y^2=z^2,\ x^2+y^2=1,\ y\geq 0,\ z\geq 0.$

3.9. a) $z=x^2+3y^2,\ z=0,\ y=x,\ y=0,\ x=1;$ b) $z=8(x^2+y^2)+3,\ z=16x+3.$

3.10. a) $y=4x\ y=0,\ x=1,\ z=\sqrt{xy},\ z=0;$ b) $z=3\sqrt{x^2+y^2},\ z=2-x^2-y^2.$

3.11. a) $y=x,\ y=0,\ x=1,\ z=3x^2+2y^2,\ z=0;$ b) $z=10(x^2+y^2)+1,\ z=1-20y.$

3.12. a) $y=x\ y=0,\ x=1,\ z=\sqrt{xy},\ z=0;$ b) $y=16\sqrt{2x},\ y=\sqrt{2x},\ z=0,\ x+z=2.$

3.13. a) $y=x\ y=0,\ x=2,\ z=0;$ b) $x+y=2,\ x=\sqrt{y}\ z=2x,\ z=0.$

3.14. a) $2z=x^2+y^2,\ z=0,\ x=2,\ y=3,\ x=0,\ y=0;$ b) $x^2+y^2=4x,\ z=0,\ z=x.$

3.15. a) $x^2+y^2=4z^2,\ z=0,\ y=x,\ y=8x,\ x=2,\ z>0;$ b) $x^2+y^2=8y,\ z=0,\ y+z=6.$

3.16. a) $y^2+4z^2=8, 16x^2-49y^2=784,\ z\geq 0;$ b) $x=1,\ y=3,\ x+5y+10z=20,\ x=0,\ y=0,\ z\geq 0.$

3.17. a) $z=3y^2,\ x^2+y^2=4,\ z=0;$ b) $x+2y+3z=6,\ 2x=y\ 2x+3y=6,\ y=0,\ z=0.$

3.18. a) $y=4x,\ y=0,\ x=1,\ z=xy,\ z=0;$ b) $4(x^2+y^2)=z^2,\ x^2+y^2=4,\ y\geq 0,\ z\geq 0.$

3.19. a) $y=2x,\ y=0,\ x=2,\ z=2x^2+y^2,\ z=0;$ b) $x^2+y^2+z^2=16,\ x^2+y^2\leq 4,\ x\geq 0.$

3.20. a) $y=4x\ y=0,\ x=4,\ z=\sqrt{xy},\ z=0;$ b) $x^2+y^2+z^2=9,\ x^2+z^2=y^2,\ x\geq 0,\ y\geq 0,\ z\geq 0.$

3.21. a) $y=3x\ y=0,\ x=3,\ z=xy,\ z=0;$ b) $4(x^2+y^2)=z^2,\ 4(x^2+y^2)=1,\ y\geq 0,\ z\geq 0.$

3.22. a) $z=16x^2+y^2,\ z=0,\ y=2x,\ y=0,\ x=1;$ b) $z-4=6(x^2+y^2),\ z=4x+1.$

3.23. a) $y=3x\ y=0,\ x=3,\ z=\sqrt{xy},\ z=0;$ b) $z=4\sqrt{x^2+y^2},\ z=5-x^2-y^2.$

3.24. a) $y=3x,\ y=0,\ x=2,\ z=x^2+y^2,\ z=0;$ b) $z-2=6(x^2+y^2),\ z=1-4y.$

3.25. a) $y=2x\ y=0,\ x=4,\ z=\sqrt{xy},\ z=0;$ b) $x+y=2,\ y=\sqrt{x},\ z=12y,\ z=0,\ x=0.$

3.26. a) $z=2x^2+3y^2,\ z=0,\ x=2,\ y=1,\ x=0,\ y=0;$ b) $x^2+y^2=6x,\ z=0,\ z=2x.$

3.27. a) $4(x^2+y^2)=z^2,\ z=0,\ y=x,\ y=4x,\ x=2,\ z>0;$ b) $x^2+y^2=4y,\ z=0,\ y+z=6.$

3.28. a) $2y^2+z^2=4,\ 3x^2-8y^2=48,\ z\geq 0;$ b) $x=1,\ y=3,\ x+2y+4z=24,\ x=0,\ y=0,\ z\geq 0.$

3.29. a) $z=3y^2,\ x^2+y^2=9,\ z=0;$ b) $x+y+z=8,\ x+2y=4,\ y=0,\ z=0.$

3.30. a) $y=5x\ y=0,\ x=3,\ z=0,\ z=0;$ b) $4(x^2+y^2)=z^2,\ x^2+y^2=4,\ y\geq 0,\ z\geq 0.$

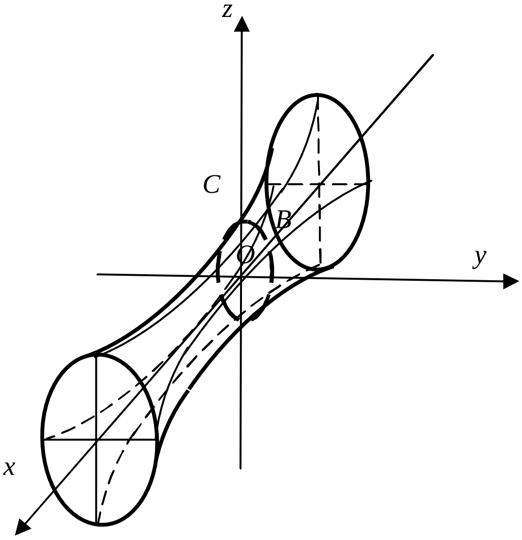
Namunaviy variantni yechish

1. Quyida berilgan tenglamalarga binoan, ularning har biri qanday sirt ekanligi (nomi) aniqlanib, ular chizilsin.

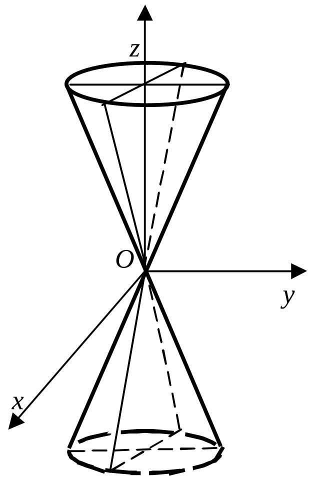
a) $-\dfrac{x^2}{6}+4y^2+\dfrac{1}{2}z^2-2=0$ b) $3x^2+\dfrac{y^2}{2}-\dfrac{z^2}{4}=0.$

► a) Tenglamani kanonik ko'rinishga keltiramiz: $-\dfrac{x^2}{12}+\dfrac{y^2}{1/2}-\dfrac{z^2}{4}=1.$ Bu esa, shakli 4.21-rasmda keltirilgan giperboloidning tenglamasi bo'lib, undagi ellipsning yarim o'qlari $OB=\dfrac{\sqrt{2}}{2}$ va *OC=2* lardir;

b) Tenglamaning kanonik shaklini yozsak, $\frac{x^2}{1}+\frac{y^2}{6}-\frac{z^2}{12}=0$ hosil bo’ladi. Bu esa, ikkinchi tartibli konus tenglamasidir. Uning shakli 4.22-rasmda tasvirlangan. Uning *z=const.* tekislik bilan kesilgandagi kesimlari ellipslardir.◄



4.21-rasm.



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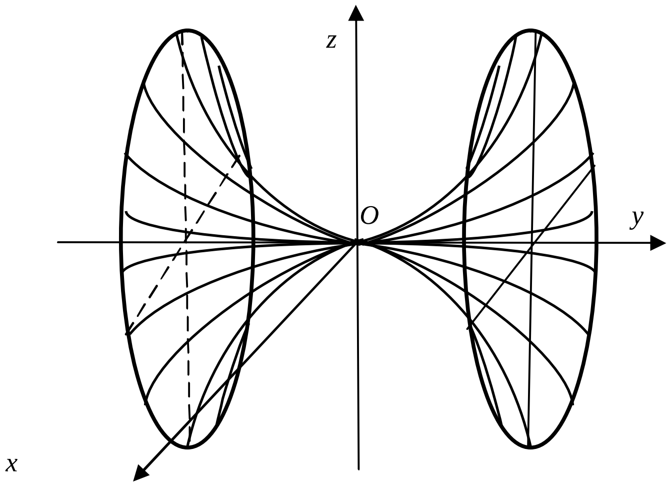

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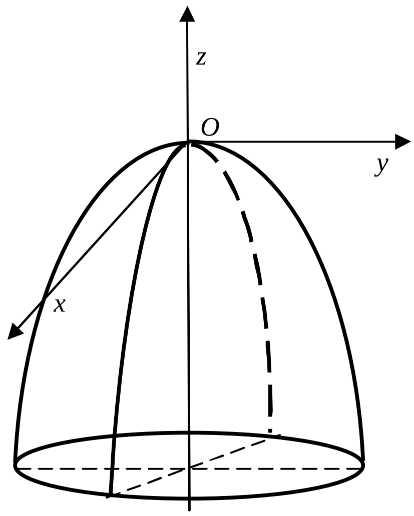


4.24-rasm

2. 1) $z=\frac{1}{2}y^2$ parabolaning a) *Oy* o’qi atrofida hamda, b) *Oz* o’q atrofida aylantirishdan hosil bo’lgan sirtlarning tenglamalari yozilsin;

2) $\frac{y^2}{64}+\frac{z^2}{4}=1$ ellipsning a) *Oz* va b) *Ou* o’qlar atrofida aylanishidan hosil bo’lgan sirtlarning tenglamalari yozilsin.

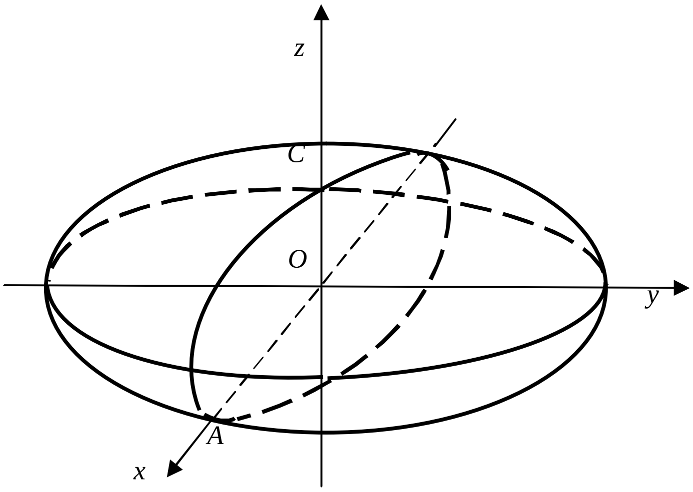
►1. Aylanma sirtlarning tenglamalarini hosil qilishning umumiy qoidasiga binoan (§ 4.2 ga qaralsin), yozamiz:

a) $\pm\sqrt{x^2+z^2}=-\frac{1}{2}y^2$, $4x^2-y^4+4z^2=0$. Bu esa, 4-tartibli algebraik sirtdir (4.23-rasmga qaralsin).

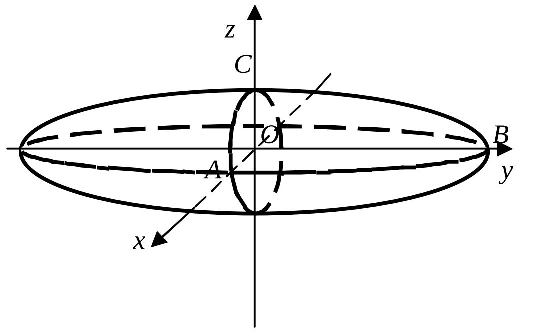
b) $z=-\frac{1}{2}\left(\pm\sqrt{x^2+y^2}\right)^2$, $z=-\frac{1}{2}\left(x^2+y^2\right)$.Bu esa, aylanma paraboloiddir (4.24-rasm).

2. a) Aylanma sirtni hosil qilish qoidasiga ko'ra $\frac{\left(\pm\sqrt{x^2+y^2}\right)^2}{64}+\frac{z^2}{4}=1$ yoki $\frac{x^2}{64}+\frac{y^2}{64}+\frac{z^2}{4}=1.$ Bu esa, *Oz* o'qi bo'ylab "siqilgan" aylanma ellipsoid (sferoid) bo'lib, uning bosh kesimining yarim o'qlari *OA=OC=2* va *OB*=8 (4.25-rasm) lardir.

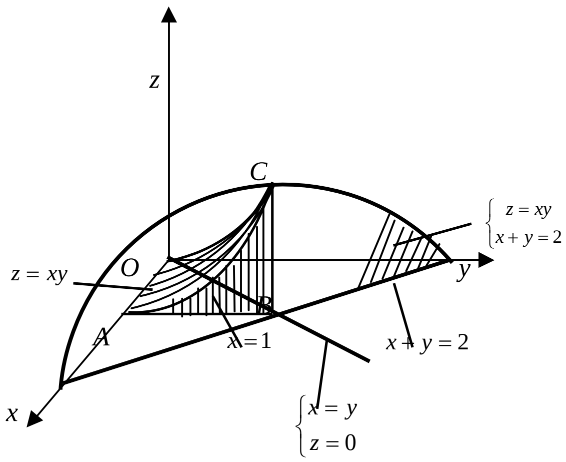
b) Yuqoridagi kabi $\frac{y^2}{64}+\frac{\left(\pm\sqrt{x^2+y^2}\right)^2}{4}=1.$ yoki $\frac{x^2}{4}+\frac{y^2}{64}+\frac{z^2}{4}=1.$ Bu esa, *Oy* o'qi bo'ylab chizilgan aylanma ellipsoid bo'lib, bu erda *OA*=*OC*=2, *OB*=8 (4.26-rasm) . ◄



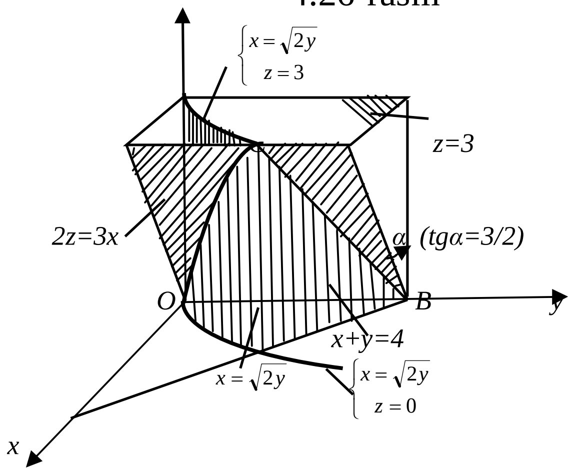
4.25-rasm.



4.26-rasm



4.27-rasm



4.28-rasm

3. Quyida keltirilgan tenglamalarni ifodalovchi sirtlar bilan chegaralangan jismlar chizilsin:

a) $y=x$, $x=1$, $z=0$, $z=xy$; b) $x+y=4$, $x=\sqrt{2y}$, $3x=2z$, $z=0$.

► a) Jismning chizmasi 4.27-rasmda keltirilgan: *OC* - $z=xu$ giperbolik paraboloid bilan $x=u$ tekislikning kesishishidan hosil bo'lgan parabolaning yoyi; *AS* yoy esa, $z=xu$ sirt bilan $x=1$ tekislikning kesishishidir; *A*(1;0;0), B(1;1;0) va C(1;1;1) lar esa, jismning xarakterli nuqtalaridir.

b) Jismning ko'rinishi 4.28-rasmda berilgan: *OS* parabolik tsilindrning $2z=3x$ tekislik bilan kesishishidan hosil bo'lgan parabola yoyi; *A*(2;2;0), B(0;4;0) va C(2;2;3) lar esa, jismga tegishli xarakterli nuqtalardir.◄

4.5. 4-bobga qo'shimcha masalalar

1. $x^2+4y^2=25$ ellipsning $A\left(\frac{7}{2},\frac{7}{4}\right)$ nuqtadan o'tuvchi hamda shu nuqtada teng ikkiga bo'linuvchi vatarining tenglamasi tuzilsin. (Javob: $x+2y-7=0$.)

2. Parabolaning optik xossasi deb ataluvchi quyidagi xossa isbotlansin, ya'ni: parabolaning fokus nuqtasidan chiqib, paraboladan qaytuvchi nur, uning o'qiga parallel bo'lgan to'gri chiziq bo'ylab yo'naladi.

3. $\frac{x^2}{3}-\frac{y^2}{4}=1$ giperbolaning *A(4,4)* nuqtadan o'tuvchi hamda shu nuqtada teng ikkiga bo'linuvchi vatarining tenglamasi tuzilsin. (Javob: $4x-3y-4=0$.)

4. $y^2=2px$ bo'lgan parabolaning ichida yotuvchi hamda uning uchiga urinib o'tuvchi eng katta aylananing radiusi topilsin. (Javob: $R=p$).

5. Asimptotalari $\sqrt{3}x \pm y=0$ bo'lib, $2x$-y-3=0 to'gri chiziqqa urinuvchi giperbolaning tenglamasi tuzilsin. (Javob: $\frac{x^2}{9}-\frac{y^2}{27}=1$.)

6. $y^2=-8x$ parabolaga shunday urinma tenglamasi tuzilsinki, urinmaning urinish nuqtasi bilan direktrisa orasidagi kesma *Oy* o'qi bilan teng ikkiga bo'linadigan bo'lsin. (Javob: $x+y-2=0$ yoki $x-y-2=0$.)

7. Giperbolaning asimptotalari hamda uning ixtiyoriy nuqtasiga o'tkazilgan urinmadan hosil bo'lgan har qanday uchburchaklar bir xil yuzaga ega ekanliklari isbotlanib, u yuza giperbolaning yarim o'qlari orqali ifodalansin. (Javob: *ab*.)

8. $y^2=16x$ parabolaning *A(1,5)* nuqtasiga o'tkazilgan urinmalarning tenglamalari tuzilsin hamda ushbu urinmalar bilan parabolaning direktrisasidan hosil bo'lgan uchburchakning yuzi hisoblansin. (Javob: $x-y+4=0$, $4x-y+1=0$, $S=37,5$.)

9. Qisqa intervalli tovush jo'natish manbai noma'lum *M* punktda. Tovush uchta kuzatish punktiga turli vaqtda etib bordi: *A* va S punktlarga *V* punktga nisbatan mos ravishda t_1 va t_2 muddatga kechikib keladi. Tovush tezligini 330 *m/s* deb hisoblab *M* punktning joylashgan o'rnini aniqlang. (Javob: *M* nuqta fokuslari *A* va *V* nuqtada bo'lgan $|AM|-|BM|=330t_1$ giperbolaning o'ng shoxida va fokuslari *V* va *S* nuqtada bo'lgan $|BM|-|CM|=-330t_2$ giperbolaning chap shoxida joylashgan.)

10. Osma ko’prikning zanjiri $y=px^2$ parabola shaklida bo’lib, ko’prikning uzunligi 50 metrga, zanjirning egilishi 5 metrga teng. Ko’prikning eng chetki nuqtasidagi egilish burchagi α ning qiymati aniqlansin. (Javob: $tg\alpha=0,4$, $\alpha \approx 21^0 5'$.)

11. Projektorning oynali sirti parabolaning o’z o’qi atrofida aylanishidan hosil bo’lgan. Oynaning diametri 80 *sm*, chuqurligi esa 20 *sm* ga teng. Nurlarni parallel dasta bilan qaytarish uchun, u parabola fokusida joylashishi lozim bo’lsa, yorug’lik manbaini parabola uchidan qanday masofada joylashtirish kerak. (Javob: 40 *sm*.)

12. *O* nuqta va undan $|OA| = a$ masofada joylashgan *l* to’gri chiziq berilgan. *O* nuqta atrofida *l* to’gri chiziqni o’zgaruvchi *R* nuqtada kesuvchi nur aylanmoqda. Bu nurda $|OP| \cdot |OM| = b^2$ shartni qanoatlantiruvchi *OM* kesma ajratilgan. Nurning aylanishida *M* nuqta chizadigan chiziq tenglamasini tuzing. Tenglamani qutb va dekart koordinatalarida yozing. (Javob: aylana: $\rho = \frac{b^2}{a}\cos\varphi, x^2 + y^2 = \frac{b^2}{a}x$.)

13. $x^2 + y^2 + z^2 = R^2$ sfera va $x^2 + y^2 - 2x = 0$ aylanma silindr kesishishidan hosil bo’lgan chiziq parametrik tenglamasini tuzing. Parametr sifatida chiziq ixtiyoriy *M* nuqtasi $\overrightarrow{OM}$ radius vektorining Oxy tekislikdagi proeksiyasi va Ox o’q musbat yo’nalishi bilan hosil qilgan φ burchak olinsin. (Javob: $x = R\cos^2\varphi, y = R\sin\varphi\cos\varphi, z = R\sin\varphi, 0 \le \varphi < 2\pi$.)

14. $x^2 +2y^2=2z$ va $x+2y+z =1$ kabi sirtlarning kesishishidan hosil bo’lgan chiziqning *Oxy* koordinata tekisligidagi proeksiyasining tenglamasi tuzilsin. (Javob: $x^2 +2y^2+2x+4y-2=0$.)

15. $x^2+2y^2-4z^2=-4$ giperboloid bilan $x+y+2z-2=0$ tekislikning kesishishi hosil bo’lgan kesimning markazi aniqlansin. (Javob: (*4,2,-2*).)

16. $x^2 +2y^2+4z^2=9$ ellipsoidni markazi *C(3,2,1)* nuqtada bo’lgan ellips bo’yicha kesadigan tekislikning tenglamasi tuzilsin.
(Javob: $3x+4y+4z-21=0$.)

17. Berilgan *M(1,1,1)* va *N(2,0,2)* nuqtalardan o’tuvchi hamda $x^2-y^2=z$ paraboloidni juft to’gri chiziqlar bo’yicha kesadigan tekislik tenglamasi tuzilsin. (Javob: $3x+y-2z-2=0$.)

18. Simmetriya tekisliklari koordinata tekisliklari bo’lib, *M*(3,1,1) nuqtani hamda $x^2+y^2-z^2=9$, $x-z=0$ aylanani o’z ichiga oladigan ellipsoid tenglamasi tuzilsin. (Javob: $3x^2 +4y^2+5z^2=36$.)

19. Tenglamasi $x^2+y^2+z^2-12x+4y-6z+24=0$, $2x+2y+z+1=0$ bo’lgan aylananing markazi hamda radiusi aniqlansin. (Javob: $\left(\frac{10}{3},\frac{14}{3},\frac{5}{3}\right)$, $R=3$.)

20. $x^2 +2y^2=4z+10$ paraboloid bilan $x^2 +y^2+z^2=6$ sferaning kesishish chizigi ikkita aylanadan iborat ekanligi isbotlanib, u aylanalarning kesishish nuqtalari hamda radiuslari aniqlansin.
(Javob: $M_1(\sqrt{2},0,-2)$, $M_2(-\sqrt{2},0,-2)$, $R=2$.)

5.1 SONLI TO'PLAMLAR. FUNKSIYANING TA'RIFI VA UNING BERILISH USULLARI

Ratsional **Q** va irratsional sonlar to'plami birgalikda haqiqiy sonlar to'plami **R** ni tashkil etadi. To'gri chiziqdagi nuqtalar to'plami bilan **R** to'plam orasida har doim o'zaro bir qiymatli moslik o'rnatish mumkin bo'ladi. Agar shunday moslik o'rnatilgan bo'lsa, to'gri chiziqni *son o'qi* deb yuritiladi. $a < x < b$ $(a \le x \le b)$ kabi shartni qanoatlantiradigan barcha x haqiqiy sonlar to'plamiga ochiq oraliq (kesma, segment) deb atalib (a; b) ([a, b]) kabi belgilanadi.

Biror a haqiqiy sonning *moduli (mutloq qiymati)* deb, shunday bir manfiy bo'lmagan $|a|$ songa aytiladi va u quyidagi shartlar bilan aniqlanadi: agar $a \ge 0$ bo'lsa , $|a| = a$ bo'lib, a<0 bo'lganda esa, $|a| = -a$ bo'ladi. Har qanday a va b haqiqiy sonlar uchun har doim $|a+b| \le |a| + |b|$ tengsizlik o'rinlidir.

Agarda har bir $x \in D$ elementga, biror aniq f qoidaga muvofiq, aniq bitta u element mos keladigan bo'lsa, u holda $u= f(x)$ funksiya berilgan deb ataladi. Bu erda x ni *erkli o'zgaruvchi* yoki *argument* deyiladi. D to'plamni *funksiyaning aniqlanish sohasi* deb atalib, $u= f(x)$ ning qabul qiladigan qiymatlar to'plamini esa *uning o'zgarish (qiymatlar) sohasi* deyiladi va E harfi bilan belgilanadi. Bundan buyon, D va E to'plamlarni sonli to'plamlar deb qabul (agar teskarisi aytilmagan bo'lsa) qilamiz. D va E lar sifatida , [a, b] kesmani yoki (a; b) ochiq oraliqni yoki [a, b) va (a; b] yarim ochiq oraliqlarni yoki son o'qining ayrim nuqtalarini yoki butun son o'qi $(-\infty;+\infty)$ larni olish mumkin.

Funksiyalar, odatda, jadval, grafik va analitik usullarda beriladi. Funksiyaning $u=f(x)$ analitik yozuvida, D va E lar ko'rsatilmasa, ular $f(x)$ funksiyaning hossalari bo'yicha aniqlanadi.

Misol. $y=lg(4\text{-}3x\text{-}x^2)$ funksiyaning aniqlanish va o'zgarish sohalari aniqlansin.

► Agar $4\text{-}3x\text{-}x^2>0$ shart bajarilsagina logarifmik funksiya aniqlangan bo'ladi. Bu esa, $(x+4)(x-1)<0$ tengsizlikka teng kuchlidir. Bundan esa, funksiyaning aniqlanish sohasi (-4;1) ochiq interval ekanligi kelib chiqadi. Shuningdek, D sohada $0 < 4 - 3x - x^2 \le \frac{7}{4}$ bo'lganligidan, funksiyaning qiymatlar sohasi E uchun $\left(-\infty; \lg\left(\frac{7}{4}\right)\right)$ ni aniqlaymiz.◄

Agarda, $y = f(x)$ funksiya, D sohani E sohaga o'zaro bir qiymatli akslantiradigan bo'lsa, u holda x ni u orqali bir qiymatli qilib ifodalash mumkin bo'ladi; $x = g(y)$. Bu funksiya $y = f(x)$ ga nisbatan teskari funksiya deb ataladi. $x = g(y)$ funksiya uchun E aniqlanish soha bo'lib, D esa, o'zgarish sohasi bo'ladi. Agar $g(f(x)) \equiv x$ va $f(g(y)) = y$ ekanligini nazarda tutadigan bo'lsak, u holda $y = f(x)$ bilan $x = g(y)$ lar *o'zaro teskari* funksiyalardir. Odatda, $x = g(y)$ funksiyada x va u larning o'rinlarini almashtirib uni

$y=g(x)$ kabi yoziladi. Masalan, $y=x^3$ bilan $y=\sqrt[3]{x}$ yoki $y=2^x$ bilan $y=\log_2 x$ yoki $y=\sin x$ bilan $y=\arcsin x$ kabi juftliklar mos ravishda o'zaro teskari funksiyalardir.

Agar $u=\varphi(x)$ funksiya D sohada aniqlangan bo'lib, G uning o'zgarish sohasi bo'ladigan bo'lsa, u holda $y=f(u)$ funksiya G sohada aniqlangan bo'lib, uni, ya'ni, $y=f[\varphi(x)]$ ni x argumentga nisbatan *murakkab funksiya* deb ataladi. Odatda, $y=f[\varphi(x)]$ ni $y=f(u)$ bilan $u=\varphi(x)$ funksiyalarning *kompozitsiyasi* deb yuritiladi. Umuman, murakkab funksiya 2 tadan ortiq funksiyalarning kompozitsiyalaridan ham iborat bo'lishi mumkin. Masalan, $y=\cos(x^2+1)$ kabi murakkab funksiya, *y=cosu* bilan (ikkita) funksiyalarning kompozitsiyalaridan iboratdir, $y=\lg(\sin 2^x)$ esa, uchta *y=lgu*, *u=sinv* va $v=2^x$ kabi funksiyalarning kompozitsiyasidan iboratdir. Yoki $y=\lg(\sin 2^{x^2})$ funksiya, *y=lgu*, *u=sinv*, $v=2^w$ va $w=x^2$ kabi to'rtta funksiyalarning kompozitsiyasi bo'ladi. Bu yerdagi, *u* va *w* lar *oraliq argumentlari* deb yuritiladi.

F(*x;y*)=0 ko'rinishdagi tenglama ham, umuman olganda *x* va *u* lar orasidagi funksional boglanishlarni ifoda etadi. Bu ko'rinshdagi tenglamani ta'rifga binoan, *u* ni *x* o'zgaruvchining *oshkormas funksiyasi* deb ataladi. Masalan, $y^3+x^3=8$ tenglama, *u* ning *x* ga nisbatan oshkormas funksiyasidir.

Biror $y=f(x)$ *funksiyaning grafigi* deb, *xOu* tekislikdagi shunday *M*(*x;y*) nuqtalar to'plamiga aytiladiki, u nuqtalarning koordinalari $y=f(x)$ funksional boglanishni qanoatlantiradi. O'zaro teskari bo'lgan $y=f(x)$ bilan $y=g(y)$ funksiyalarning grafiklari *u=x* bissektrisaga nisbatan simmetrik joylashadilar. Darajali, ko'rsatkichli, logarifmik, trigonometrik va teskari trigonometrik funksiyalar sinfini asosiy elementar funksiyalar deb yuritiladi.

5.1 AT

1. Quyida keltirilgan funksiyalarning aniqlanish sohalari topilsin:

a) $y=\sqrt{x^2-6x+5}$;

b) $y=arcCos\dfrac{2x}{x+1}$;

c) $y=\sqrt{25-x^2}+\lg(\sin x)$.

(Javob*:* a) (-∞:1]U[5;+∞); b) $[-1/3;1]$; c) [-5;-π)U(0;π).)

2. Quyida keltirilgan murakkab funksiyalarni asosiy elementar funksiyalardan iborat bo'lgan funksiyalarning kompozitsiyasi shaklida ifodalansin:

a) $y=2\sin\sqrt[3]{x}$;

b) $y=\sqrt[3]{\lg(\sin x^3)}$;

c) $y=tg\sqrt[5]{\lg x}$;

g) $y=arctg\sqrt[3]{2^{xy}}$.

3. Quyidagi funksiyalarning grafiklari chizilsin:

a) $y = \frac{(2x+3)}{(x-1)}$;

b) $y = |3x + 4 - x^2|$;

c) $y = -2\sin(2x+2)$;

g) $y = x\sin x$

Mustaqil ish.

1. 1. $y = \lg(2^{3x} - 4)$ funksiyaning aniqlanish sohasi topilsin: (Javob: $x > 2/3.$)

2. $y = \begin{cases} x, & agar \ \ x \le 0 \ \ bo'lsa \\ x^2, & agar \ \ x > 0 \ \ bo'lsa \end{cases}$ funksiyaga teskari funksiya topilsin. Berilgan va teskari funksiyalarning grafiklari chizilsin.

2. 1 $y = \lg(-x^2 - 5x + 6)$ funksiyaning aniqlanish sohasi topilsin. (Javob: $x \in (-6{,}1)$.)

2. $y = \begin{cases} x+1, & agar \ \ x < 0 \ \ bo'lsa \\ 2\sin x, & agar \ 0 \le x < \pi \ \ bo'lsa \\ x - \pi, & agar \ \ x \ge \pi \ \ bo'lsa \end{cases}$

funksiyaning grafigi yasalsin.

3. 1. $y = \frac{1}{\sqrt{x^2 + x}}$ funksiyaning aniqlanish sohasi topilsin. (Javob*: (-∞;-1) U (0,+∞).)*

2. $y = \begin{cases} -x, & agar \ \ x < 1 \ \ bo'lsa \\ x^2 - 1, & agar \ x \ge 1 \ \ bo'lsa \end{cases}$ funksiyaga teskari funksiya aniqlanib, har ikkalasining grafiklari yasalsin.

5.2. KETMA-KETLIKLAR VA FUNKSIYALARNING LIMITLARI. ENG SODDA ANIQMASLIKLARNI OCHISH.

Aytaylik, biror $\{x_n\}$ sonli ketma-ketlik qaralayotgan bo'lsin. Taэrifga binoan, agar har qanday ε>0 son uchun shunday bir N=$N(\varepsilon)$>0 butun son mavjud bo'lib, barcha n>N lar uchun $|x_n - A| < E$ tengsizlik o'rinli bo'ladigan bo'lsa, A soni $\{x_n\}$ *ketma-ketlikning limiti* deb atalib, uni $\lim\limits_{n\to\infty} x_n = A$ kabi yoziladi. Chekli limiti mavjud bo'lgan ketma-ketlikni *yaqinlashuvchi*, aks holda *uzoqlashuvchi* deyiladi.

1-misol. $\{x_n\} = \left\{\frac{2n+3}{n+1}\right\}$ ketma-ketlikning limiti A=2 ekanligi ko'rsatilsin.

► Har qanday $\varepsilon > 0$ son uchun shunday N=$N(\varepsilon)$>0 son mavjud bo'lib, barcha n>N lar uchun $|x_n - A| = \left|\frac{2n+3}{n+1} - 2\right| = \left|\frac{2n+3-2n-2}{n+1}\right| = \frac{1}{n+1} < \varepsilon$ ning bajarilishini ko'rsatamiz. $\frac{1}{n+1} < \varepsilon$ dan, $n > \frac{1}{\varepsilon} - 1$ ni hosil qilamiz. Demak, $N = \left[\frac{1}{\varepsilon} - 1\right] + 1$ (bu erda [α], α sonining butun qismidir). Bundan ko'rinmoqdaki, shunday N son mavjud ekanki, barcha $n>N$ lar uchun $|x_n - 2| < \varepsilon$ kabi shart bajariladi.◄

Faraz qilaylik, $y=f(x)$ funksiya biror x_0 nuqta atrofida aniqlangan bo'lsin. U holda, agar har qanday $\varepsilon>0$ son uchun shunday bir $\delta=\delta(E)>0$ son mavjud bo'lib, barcha $0<|x-x_0|<\delta$ lar uchun $|f(x)-A|<\varepsilon$ tengsizlik o'rinli bo'lsa, chekli A sonini $y=f(x)$ *funksiyaning* $x\to x_0$ *(*$x=x_0$ *nuqtada) dagi limiti* deb ataladi hamda $\lim\limits_{x\to x_0} f(x) = A$ yoziladi. Ayrim hollarda $x=x_0$ nuqtada funksiya aniqlanmagan ham bo'lishi mumkin. Agar har qanday $E>0$ son uchun $N=W(E)>0$ mavjud bo'lib, barcha $|x|>N$ lar uchun $|f(x)-A|<E$ bajariladigan bo'lsa, u holda $\lim\limits_{x\to\pm\infty} f(x) = A$ deb yoziladi. Agar $\lim\limits_{\substack{x\to x_0\\ x<x_0}} f(x)$ kabi (uni $\lim\limits_{x\to x_0-0} f(x)$ yoki $f(x_0-0)$ deb ham yozish mumkin) chekli limit mavjud bo'lsa, u limitni $f(x)$ funksiyaning x_0 nuqtadagi *chap bir tomonli limiti* deb ataladi. Shunga o'xshash $\lim\limits_{\substack{x\to x_0\\ x>x_0}} f(x)$ (yoki $\lim\limits_{x\to x_0+0} f(x)$ yoki $f(x_0+0)$) chekli limit mavjud bo'lsa, u holda uni $f(x)$ funksiyaning x_0 nuqtadagi *o'ng bir tomonli limiti* deb ataladi. Biror $f(x)$ funksiyaning belgilangan x_0 nuqtadagi chekli limiti mavjud bo'lishi uchun, uning shu nuqtadagi ham chap bir tomonli, ham o'ng bir tomonli chekli limitlari mavjud bo'lib, $f(x_0-0)=f(x_0+0)$ tenglikning bajarilishi zarur hamda yetarli shartdir.

Limitlar uchun quyidagi teoremalar o'rinlidir:

1-teorema. Agar $\lim\limits_{x\to x_0} f_i(x)$ $(i=\overline{1,n})$ chekli limit bo'lsa, u holda:

$$\lim_{x\to x_0}\sum_{i=1}^{n} f_i(x) = \sum_{i=1}^{n}\lim_{x\to x_0} f_i(x), \qquad \lim_{x\to x_0}\prod_{i=1}^{n} f_i(x) = \prod_{i=1}^{n}\lim_{x\to x_0} f_i(x) \text{ lar o'rinlidir.}$$

2-teorema. Agar $\lim\limits_{x\to x_0} f(x)$ va $\lim\limits_{x\to x_0}\varphi(x)\neq 0$ chekli limitlar mavjud bo'lsa, u holda: $\lim\limits_{x\to x_0}\dfrac{f(x)}{\varphi(x)} = \lim\limits_{x\to x_0} f(x)/\lim\limits_{x\to x_0}\varphi(x)$ tenglik o'rinlidir.

Yuqoridagi teoremalar $x_0=\pm\infty$ uchun ham o'z kuchida qolaveradi. Agar yuqoridagi teoremalarning shartlari bajarilmasa, u holda $\infty-\infty$, $\frac{\infty}{\infty}, \frac{0}{0}$ va boshqa xildagi aniqmasliklar paydo bo'ladiki, ularni sodda hollar uchun algebraik shakl almashtirishlar orqali aniq ko'rinishga keltiriladi.

2-misol. $\lim\limits_{x\to 2}\left(\dfrac{4}{x^2-4} - \dfrac{1}{x-2}\right)$ ni hisoblansin.

► Ushbu limitda $(\infty-\infty)$ turdagi aniqmaslik yuz bermoqda. Uni ochib chiqish maqsadida, umumiy maxrajga keltirilganda $\lim\limits_{x\to 2}\dfrac{2-x}{x^2-4}$ dan $\dfrac{0}{0}$ aniqmaslikga kelamiz. Agar $x-2\neq 0$ ga qisqartirsak, $\lim\limits_{x\to 2}\left(-\dfrac{1}{x+2}\right) = -\dfrac{1}{4}$ hosil bo'ladi.◄

3-misol . $\lim\limits_{x\to\pm\infty}\dfrac{2x^3 - x + 5}{x^3 + x^2 - 1}$ ni hisoblansin.

► Bu erda $\frac{\infty}{\infty}$ aniqmaslik ishtirok etmoqda. Kasrning surat va maxrajini x^3 ga bo'lib yuborsak, $\lim\limits_{x\to\pm\infty}\dfrac{2-\frac{1}{x^2}+\frac{5}{x^3}}{1+\frac{1}{x}-\frac{1}{x^3}}$ ni hosil qilamiz. Natijada, limitga o'tib 2 ni hosil kilamiz.◄

5.2– AT

1. $\{x_n\}=\left\{\dfrac{3n+5}{n-1}\right\}$ ketma-ketlikning limiti A=3 ekanligi isbotlansin.

Quyidagi limitlar hisoblansin.

2. $\lim\limits_{n\to\infty}\dfrac{3n^2+3n-5}{1-n^2}$ (Javob:-3.)

3. $\lim\limits_{n\to\pm\infty}\dfrac{2+4x^2+3x^3}{x^3-7x-10}$ (Javob:3.)

4. $\lim\limits_{n\to\pm\infty}\dfrac{7x^2+10x+20}{x^3-10x^2-1}$ (Javob:0.)

5. $\lim\limits_{n\to 2}\dfrac{x^3-3x^2+3}{x^2-3}$ (Javob:-1.)

6. $\lim\limits_{n\to 2}\dfrac{x^2-7x+10}{8-x^3}$ (Javob:1/4.)

7. $\lim\limits_{n\to 1}\dfrac{x^3-3x^2+2}{x^2-7x+6}$ (Javob:3/5.)

8. $\lim\limits_{n\to 2}\dfrac{\sqrt{x+7}-3}{\sqrt{x+2}-2}$ (Javob:2\3.)

9. $\lim\limits_{n\to\infty}\left(x\left(\sqrt{x^2+4}-x\right)\right)$ (Javob:2.)

10. $\lim\limits_{n\to 1}\left(\dfrac{1}{1-x}-\dfrac{3}{1-x^3}\right)$ (Javob:-1.)

Mustaqil ish

Quyidagi limitlar hisoblansin.

1. a) $\lim\limits_{n\to 1/2}\dfrac{8x^3-1}{6x^2-5x+1}$; b) $\lim\limits_{n\to 3}\dfrac{\sqrt{x+13}-4}{x^2-9}$. (Javob:a) 6; b) 1/148.)

2. a) $\lim\limits_{n\to 2}\dfrac{x^2-7x+10}{x^2-5x+6}$; b) $\lim\limits_{n\to 5}\dfrac{x^2-25}{\sqrt{x-1}-2}$. (Javob: a) 3; b) 40.)

3. a) $\lim\limits_{n\to 1}\dfrac{x^3-x^2+x-1}{x^2-4x+3}$; b) $\lim\limits_{n\to\infty}\left(x\left(\sqrt{x^2+5}-\sqrt{x^2+1}\right)\right)$ (Javob: a) -1; b) 2.)

5.3. AJOYIB LIMITLAR.

Quyidagi 1) $\lim\limits_{x\to 0}\frac{\sin x}{x}=1;$

2) $\lim\limits_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^x=\lim\limits_{\alpha\to 0}(1+\alpha)^{\frac{1}{\alpha}}=e\approx 2{,}71828;$

limitlar ajoyib limitlar deb atalib, ular limitlarni hisoblashda keng qo'llaniladi.

1-misol . $\lim\limits_{n\to 0}\frac{\sin 7x}{\sin 3x}$ hisoblansin.

► Agar $x\neq 0$ deb olsak, u holda

$$\lim_{n\to 0}\frac{\sin 7x}{\sin 3x}=\lim_{n\to 0}\frac{\frac{\sin 7x}{7x}\cdot 7x}{\frac{\sin 3x}{3x}\cdot 3x}=\lim_{n\to 0}\left(\frac{7x}{3x}\right)\cdot\lim_{n\to 0}\frac{\frac{\sin 7x}{7x}}{\frac{\sin 3x}{3x}}=\frac{7}{3}.$$ ◄

2-misol . $\lim\limits_{n\to\infty}\left(\frac{2x+1}{2x-1}\right)^{3x+1}$ ni hisoblansin.

► Quyidagicha shakl almashtiramiz:

$$\lim_{x\to\infty}\left(\frac{2x+1}{2x-1}\right)^{3x+1}=\lim_{x\to\infty}\left(\frac{2x-1+2}{2x-1}\right)^{3x+1}=\lim_{x\to\infty}\left(1+\frac{2}{2x-1}\right)^{3x+1};$$

Agar $\frac{2}{2x-1}=\frac{1}{y}$ deb belgilash kiritsak, $x=y-\frac{1}{2}$ dan $x\to\infty$ da $u\to\infty$.

$$\lim_{x\to\infty}\left(\frac{2}{2x-1}\right)^{3x+1}=\lim_{x\to\infty}\left(1+\frac{1}{y}\right)^{3x-\frac{1}{2}}=\lim_{y\to\infty}\left(\left(1+\frac{1}{y}\right)^{y}\right)^{3}\cdot\left(1+\frac{1}{y}\right)^{-\frac{1}{2}}=e^3$$

ni hosil qilamiz.◄

5.3–AT

Quyidagi limitlar hisoblansin.

1. $\lim\limits_{n\to 0}\frac{tg3x}{\sin 2x}.$ (Javob: 3/2)

2. $\lim\limits_{n\to 0}\frac{1-\cos 6x}{x\sin 3x}.$ (Javob: 6.)

3. $\lim\limits_{n\to 1}\frac{\sin(2(x-1))}{x^2-7x+6}.$ (Javob: -2/5.)

4. $\lim\limits_{n\to\pm\infty}\left(\frac{x+3}{2x-1}\right)^x.$ (Javob: 0 yoki ∞.)

5. $\lim\limits_{n\to\infty}\left(\frac{3x+2}{3x-1}\right)^{4x-1}.$ (Javob: e^4.)

6. $\lim\limits_{n\to\infty}((2x+1)(\ln(3x+1)-\ln(3x-2)))$ (Javob: 2.)

5.4 CHEKSIZ KICHIK FUNKSIYALARNI TAQQOSLASH. FUNKSIYANING UZLUKSIZLIGI.

Agar $\lim\limits_{x\to x_0}\alpha(x)=0$ bo'lsa, u holda $\alpha(x)$ ni $x\to x_0$ dagi *cheksiz kichik miqdor* deb ataladi. Ikkita $\alpha(x)$ va $\beta(x)$ ($x\to x_0$ dagi) cheksiz kichik miqdorlarni taqqoslash uchun ularning nisbatining limiti hisoblanadi:

$$\lim_{x\to x_0}\frac{\alpha(x)}{\beta(x)}=C \qquad (5.1)$$

Agar $C\neq0$ bo'lsa, $\alpha(x)$ bilan $\beta(x)$ larni *bir xil tartibli cheksiz kichik miqdorlar* deb yuritiladi; agar $C=0$ bo'lsa, $\alpha(x)$ni $\beta(x)$ ga nisbatan *yuqori tartibli cheksiz kichik miqdor* deb, $\beta(x)$ ni esa, $\alpha(x)$ ga nisbatan *quyi tartibli cheksiz kichik miqdorlar* deb ataladi.

Agar $\lim\limits_{x\to x_0}\frac{\alpha(x)}{(\beta(x))^k}=C$, $(0<|C|<\infty)$, bo'lsa, $\alpha(x)$ ni $x\to x_0$ da $\beta(x)$ ga nisbatan *k*-tartibli cheksiz kichik miqdor deyiladi.

Agar $\lim\limits_{x\to x_0}\frac{\alpha(x)}{\beta(x)}=1$ bo'lsa, $\alpha(x)$ bilan $\beta(x)$ funktsilarni $x\to x_0$ da *ekvivalent (teng kuchli) cheksiz miqdorlar* deb atalib, ularni $\alpha(x)\sim\beta(x)$ deb yoziladi. Masalan, $x\to0$ da $\sin ax\sim ax$, $tgx\sim x$, $\ln(1+x)\sim x$, $e^{ax}-1\sim ax$. Agar $x\to x_0$ da $\alpha(x)$ bilan $\beta(x)$ lar, hamda $\alpha^*(x)$ bilan $\beta^*(x)$ lar cheksiz kichik miqdorlar bo'lib, $\alpha(x)\sim\alpha^*(x)$ va $\beta(x)\sim\beta^*(x)$ bo'lsalar, u holda:

$$\lim_{x\to x_0}\frac{\alpha(x)}{\beta(x)}=\lim_{x\to x_0}\frac{\alpha^*(x)}{\beta(x)}=\lim_{x\to x_0}\frac{\alpha(x)}{\beta^*(x)}=\lim_{x\to x_0}\frac{\alpha^*(x)}{\beta^*(x)}$$

ekanligini isbotlash mumkin.

1-misol . $\lim\limits_{x\to x_0}\frac{\sin5x}{\ln(1+x)}$ ni hisoblansin.

► Agar $x\to0$ da $\sin5x\sim5x$ va $\ln(1+x)\sim x$ ekanligini inobatga olsak, $\lim\limits_{x\to x_0}\frac{\sin5x}{\ln(1+x)}=\frac{5x}{x}=5$ hosil bo'ladi.◄

Agar $u=f(x)$ funksiya biror x nuqtada va uning atrofida aniqlangan bo'lib, x nuqtada uning chekli limiti mavjud bo'lsa hamda u limit funksiyaning x_0 nuqtadagi qiymatiga teng bo'ladigan bo'lsa, yaэni:

$$\lim_{x\to x_0}f(x)=f(x_0) \qquad (5.2)$$

u holda $u=f(x)$ funksiyani $x=x_0$ *nuqtada uzluksiz* deb ataladi.

Agar $x=x_0+\Delta x$ olinsa, (5.2) uzluksizlik sharti quyidagi $\lim\limits_{\Delta x\to0}\Delta f(x_0)=\lim\limits_{\Delta x\to0}\left(f(x_0+\Delta x)-f(x_0)\right)=0$ shartga teng kuchlidir, yaэni, x_0 nuqtadagi argumentning cheksiz kichik orttirmasi Δx ga, funksiyaning ham $\Delta f(x_0)$ cheksiz kichik orttirmasi mos keladigan bo'lsagina, $u=f(x)$ funksiya $x=x_0$ nuqtada uzluksiz bo'ladi. Agar $u=f(x)$ funksiya biror sohaning barcha nuqtalarida uzluksiz bo'lsa, uni *shu sohada uzluksiz* deb ataladi.

2-misol . Har qanday $x\in R$ uchun $u=\sin5x$ funksiyaning uzluksizligi isbotlansin.

► Argumentning ixtiyoriy qiymatidagi Δx orttirma uchun funksiyaning orttirmasi

$$\Delta y=\sin(5x+\Delta x)-\sin 5x=2Cos\left(5x+\frac{5}{2}\Delta x\right)\cdot\sin\frac{5}{2}\Delta x$$

mos keladi. U holda:

$$\lim_{\Delta x\to 0}\Delta y=2\lim_{\Delta x\to 0}Cos\left(5x+\frac{5}{2}\Delta x\right)\lim_{\Delta x\to 0}\sin\frac{5}{2}\Delta x=0$$ ◄

Demak, $u=sin5x$ funksiya son o'qining barcha qiymatlarida uzluksiz ekan.

Agar x_0 nuqtadagi uzluksizlik shartlaridan hech bo'lmaganda bittasi bajarilmasa, x_0 ni *funksiyaning uzilish nuqtasi* deb ataladi. Xususan, agar $u=f(x)$ funksiyaning x_0 nuqtadagi chekli bir tomonli limitlari $f(x_0\text{-}0)$ va $f(x_0+0)$ mavjud bo'lib, $f(x_0\text{-}0)\neq f(x_0+0)$ bo'lsa, x_0 ni *1-tur uzilish nuqtasi* deyiladi. Agar x_0 ni nuqtadagi bir tomonli $f(x_0\text{-}0)$, $f(x_0+0)$ limitlardan hech bo'lmaganda birortasi mavjud bo'lmasa yoki cheksizlikka teng bo'lsa, x_0 ni *2-tur uzilish nuqtasi* deb ataladi. Agar $f(x_0\text{-}0)=f(x_0+0)$ bo'lib, $f(x)$ funksiya x_0 nuqtada aniqlanmagan bo'lsa, u holda x_0 ni funksiyaning *chetlantiriladigan uzilish nuqtasi* deyiladi. Masalan, $y=\frac{\sin x}{x}$ uchun, $x=0$ chetlantiriladigan uzilish nuqtasidir.

5.4– AT

1. $\lim_{x\to 2}\frac{\sin 3(x-2)}{x^2-3x+2}$ hisoblansin. (Javob: 3.)

2. $\lim_{x\to 0}\frac{e^{5x}-1}{\sin 10x}$ hisoblansin. (Javob: 1/2.)

3. $\lim_{x\to 0}\frac{\ln(1+7x)}{\sin 7x}$ hisoblansin. (Javob: 1.)

4. $y=\frac{7x^8}{x^4+1}$ $x\to x_0$ dagi cheksiz kichik funksiyaning x cheksiz kichik miqdorga nisbatan tartibi aniqlansin.

5. $y=\begin{cases}(x^2-9)/(x-3), & agar\ \ x\neq 3\ \ bo'lsa\\ A, & agar\ \ x=3\ \ bo'lsa\end{cases}$ funksiya A ning qanday qiymatida $x=3$ nuqtada uzluksiz bo'ladi?

6. $y=\frac{3x+3}{2x+4}$ funksiyaning uzluksizlik sohasi hamda uzilish nuqtalari aniqlansin.

7. Berilgan

$$f(x)=\begin{cases}x^2+1, & agar\quad x<0\ \ bo'lsa,\\ \sin x, & agar\quad 0\le x<\frac{\pi}{2}\ \ bo'lsa,\\ x-\frac{\pi}{2}+1, & agar\quad x\ge\frac{\pi}{2}\ \ bo'lsa,\end{cases}$$

funksiyaning uzilish nuqtalari aniqlanib, uning grafigi chizilsin.

8. $y=3^{\frac{1}{x+1}}+1$ funksiyani $x_1=1$ va $x_2=-1$ nuqtalarda uzluksizlikka tekshirilsin.

Mustaqil ish

1. 1. $\lim\limits_{x\to -1}\dfrac{\sin 3(x+1)}{x^2-4x-5}$ hisoblansin. (*Javob*: $-\dfrac{3}{4}$.)

2. $f(x)=(2x+4)/(3x+9)$ funksiyani x_1=-1 va x_2=2 nuqtalarda uzluksizlikka tekshirilsin hamda uning sxematik chizmasi yasalsin.

2. 1. $\lim\limits_{x\to 0}\dfrac{e^{\sin 7x}-1}{x^2+3x}$ hisoblansin. (Javob: 7/3.)

2. Berilgan

$$f(x)=\begin{cases} 1, & agar\ \ x<0\ \ bo'lsa, \\ \cos x, & agar\ \ 0\le x<\dfrac{\pi}{2}\ \ bo'lsa \\ 1-x, & agar\ \ x\ge\dfrac{\pi}{2}\ \ bo'lsaa, \end{cases}$$

funksiyani uzluksizlikka tekshirilib, uning grafigi chizilsin.

3. 1. $\lim\limits_{x\to 2}\dfrac{tg\left(x^2-3x+2\right)}{x^2-4}$ hisoblansin. (Javob: $\dfrac{1}{4}$.)

2. $f(x)=\dfrac{3x-2}{x+2}$ funksiyani x_1=0 va x_2 =-2 nuqtalarda uzluksizlikka tekshirilib, uning sxematik chizmasi yasalsin.

5.5. 5-bobga individual uy vazifalari.
5.1. IUT

Ko’rsatilgan limitlar hisoblansin.

1

1.1. $\lim\limits_{x\to 2}\dfrac{x^2-5x+6}{x^2-12x+20}$.

1.2. $\lim\limits_{x\to 0}\dfrac{x^3-x^2+2x}{x^2+x}$.

1.3. $\lim\limits_{x\to 3}\dfrac{6+x-x^2}{x^3-27}$.

1.4. $\lim\limits_{x\to 1}\dfrac{2x^2-x-1}{3x^2-x-2}$.

1.5. $\lim\limits_{x\to 2}\dfrac{2x^2-7x+4}{x^2-5x+6}$.

1.6. $\lim\limits_{x\to 3}\dfrac{12+x-x^2}{x^3-27}$.

1.7. $\lim\limits_{x\to 1/3}\dfrac{3x^2+2x-1}{27x^3-1}$.

1.8. $\lim\limits_{x\to -1}\dfrac{x^2-4x-5}{x^2-2x-3}$.

1.9. $\lim\limits_{x\to -1}\dfrac{3x^2+2x-1}{-x^2+x+2}$.

1.10. $\lim\limits_{x\to 3}\dfrac{3x^2-11x+6}{2x^2-5x-3}$.

1.11. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2+x-6}$.

1.12. $\lim\limits_{x\to -1}\dfrac{x^2-x-2}{x^3+1}$.

1.13. $\lim\limits_{x\to 4}\dfrac{x^2-16}{x^2+x-20}$.

1.14. $\lim\limits_{x\to -3}\dfrac{4x^2+11x-3}{x^2+2x-3}$.

1.15. $\lim\limits_{x\to 3}\dfrac{3x^2-7x-6}{2x^2-7x+3}$.

1.16. $\lim\limits_{x\to -2}\dfrac{4x^2+7x-2}{3x^2+8x+4}$.

1.17. $\lim\limits_{x\to -1}\frac{5x^2+4x-1}{3x^2+x-2}$.

1.18. $\lim\limits_{x\to -1}\frac{x^2-4x-5}{3x^2+2x-2}$.

1.19. $\lim\limits_{x\to -1}\frac{7x^2+4x-3}{2x^2+3x+1}$.

1.20. $\lim\limits_{x\to 4}\frac{3x^2-3x+2}{x^2-x-12}$.

1.21. $\lim\limits_{x\to 2}\frac{2x^2-9x+10}{x^2+3x-10}$.

1.22. $\lim\limits_{x\to 1}\frac{4x^2+x-5}{x^2-2x+1}$.

1.23. $\lim\limits_{x\to 2}\frac{-5x^2+11x-2}{3x^2-x-10}$.

1.24. $\lim\limits_{x\to 7}\frac{x^2-5x-14}{2x^2-9x-35}$.

1.25. $\lim\limits_{x\to 5}\frac{3x^2-6x-45}{2x^2-3x-35}$.

1.26. $\lim\limits_{x\to -3}\frac{4x^2+3x+15}{x^2-6x-27}$.

1.27. $\lim\limits_{x\to -5}\frac{x^2-2x-35}{2x^2+11x+5}$.

1.28. . $\lim\limits_{x\to -8}\frac{2x^2+15x-8}{3x^2+25x+8}$.

1.29. $\lim\limits_{x\to 4}\frac{3x^2-2x-40}{x^2-3x-4}$.

1.30. $\lim\limits_{x\to -3}\frac{2x^2+5x-3}{3x^2+10x+3}$.

2.

2.1. $\lim\limits_{x\to -3}\frac{2x^2+11x+15}{3x^2+5x-12}$.

2.2. $\lim\limits_{x\to 1}\frac{2x^2+5x-10}{x^3-1}$.

2.3. $\lim\limits_{x\to 1}\frac{x^3-3x+2}{x^2-4x+3}$.

2.4. $\lim\limits_{x\to 2}\frac{3x^2+2x+1}{x^3-8}$.

2.5. $\lim\limits_{x\to -1}\frac{x^4-x^2+x+1}{x^4+1}$.

2.6. $\lim\limits_{x\to 1}\frac{2x^2-3x-1}{x^4-1}$.

2.7. $\lim\limits_{x\to 2}\frac{x^2-x+3}{5x^2+3x-3}$.

2.8. $\lim\limits_{x\to -2}\frac{x^2+2x}{x^2+4x+4}$.

2.9. $\lim\limits_{x\to -1}\frac{x^2-1}{x^2+3x+2}$.

2.10. $\lim\limits_{x\to -4}\frac{2x^2+7x-4}{x^3+64}$.

2.11. $\lim\limits_{x\to -5}\frac{4x^2+19x-5}{2x^2+11x+5}$.

2.12. $\lim\limits_{x\to 1}\frac{x^3-x^2+x-1}{x^3+x-2}$.

2.13. $\lim\limits_{x\to 1}\frac{x^2-2x+1}{2x^2-7x+5}$.

2.14. $\lim\limits_{x\to 2}\frac{x^3-8}{2x^2-9x+10}$.

2.15. $\lim\limits_{x\to -2}\frac{9x^2+17x-2}{x^2+2x}$.

2.16. $\lim\limits_{x\to 1}\frac{x^3+x-2}{x^3-x^2-x+1}$.

2.17. $\lim\limits_{x\to 0}\frac{4x^3-2x^2+5x}{3x^2+7x}$.

2.18. $\lim\limits_{x\to 1}\frac{4x^4-5x^2+1}{x^2-1}$.

2.19. $\lim\limits_{x\to 3}\frac{3x^2+5x-1}{x^2-5x+6}$.

2.20. $\lim\limits_{x\to -5}\frac{x^2-x-30}{x^3+125}$.

2.21. $\lim\limits_{x\to 4}\frac{x^2+3x-28}{x^3-64}$.

2.22. $\lim\limits_{x\to 1/2}\frac{8x^3-1}{x^3-1/4}$.

2.23. $\lim\limits_{x\to 4}\frac{x^2+3x-28}{x^2-4x}$.

2.24. $\lim\limits_{x\to -2}\frac{3x^2+11x+10}{x^2-5x+14}$.

2.25. $\lim\limits_{x\to -2}\dfrac{x^2-4}{3x^2+x-10}$.

2.26. $\lim\limits_{x\to 0}\dfrac{3x^2+x}{4x^2-5x+1}$.

2.27. $\lim\limits_{x\to 6}\dfrac{2x^2-11x-6}{3x^2-20x+12}$.

2.28. $\lim\limits_{x\to -6}\dfrac{x^2+2x-24}{2x^3+15x+18}$.

2.29. $\lim\limits_{x\to 2}\dfrac{x^3-2x-4}{x^2-11x+18}$.

2.30. $\lim\limits_{x\to 4}\dfrac{x^3-64}{7x^2-27x-4}$.

3.

3.1. $\lim\limits_{x\to\infty}\dfrac{3x^3-5x^2+2}{2x^3+5x^2-x}$.

3.2. $\lim\limits_{x\to\infty}\dfrac{4x^3+7x}{2x^3-4x^2+5}$.

3.3. $\lim\limits_{x\to\infty}\dfrac{5x^4-3x^2+7}{x^4+2x^3+1}$.

3.4. $\lim\limits_{x\to\infty}\dfrac{7x^3-2x^2+4x}{2x^3+5}$.

3.5. $\lim\limits_{x\to\infty}\dfrac{x^3-4x^2+28x}{5x^3+3x^2+x-1}$.

3.6. $\lim\limits_{x\to\infty}\dfrac{3x^2+10x+3}{2x^2+5x-3}$.

3.7. $\lim\limits_{x\to\infty}\dfrac{-3x^4+x^2+x}{x^4+3x-2}$.

3.8. $\lim\limits_{x\to\infty}\dfrac{2x^2+7x+3}{5x^2-3x+4}$.

3.9. $\lim\limits_{x\to\infty}\dfrac{-x^2+3x+1}{3x^2+x-5}$.

3.10. $\lim\limits_{x\to\infty}\dfrac{x^3-3x^2+10}{7x^3+2x+1}$.

3.11. $\lim\limits_{x\to\infty}\dfrac{4x^2+5x-7}{2x^2-x+10}$.

3.12. $\lim\limits_{x\to\infty}\dfrac{3x^4+2x+1}{x^4-x^3+2x}$.

3.13. $\lim\limits_{x\to\infty}\dfrac{3x^2+2x+9}{2x^2-x+4}$.

3.14. $\lim\limits_{x\to\infty}\dfrac{3x^2+5x-7}{3x^2+x+1}$.

3.15. $\lim\limits_{x\to\infty}\dfrac{2x^3+7x-2}{3x^3-x-4}$.

3.16. $\lim\limits_{x\to 0}\dfrac{18x^2+5x}{8-3x-9x^2}$.

3.17. $\lim\limits_{x\to\infty}\dfrac{3x^4-6x^2+2}{x^4+4x-3}$.

3.18. $\lim\limits_{x\to\infty}\dfrac{8x^2+4x-5}{4x^2-3x+2}$.

3.19. $\lim\limits_{x\to\infty}\dfrac{8x^4-4x^2+3}{2x^4+1}$.

3.20. $\lim\limits_{x\to\infty}\dfrac{3x^2-4x+2}{6x^2+5x+1}$.

3.21. $\lim\limits_{x\to\infty}\dfrac{7x^3+4x}{x^3-3x+2}$.

3.22. $\lim\limits_{x\to\infty}\dfrac{1+4x-x^4}{x+3x^2+2x^4}$.

3.23. $\lim\limits_{x\to\infty}\dfrac{2x^3+7x^2-2}{6x^3-4x+3}$.

3.24. $\lim\limits_{x\to\infty}\dfrac{3x+14x^2}{1+2x+7x^2}$.

3.25. $\lim\limits_{x\to\infty}\dfrac{x-2x^2+5x^4}{2+3x^2+x^4}$.

3.26. $\lim\limits_{x\to\infty}\dfrac{3x^4-2x^2-7}{3x^4+3x+5}$.

3.27. $\lim\limits_{x\to\infty}\dfrac{4-5x^2-3x^5}{x^5+6x+8}$.

3.28. $\lim\limits_{x\to\infty}\dfrac{5x^3-7x^2+3}{2+2x-x^3}$.

3.29. $\lim\limits_{x\to\infty}\dfrac{4x^3-2x+1}{2x^3+3x^2+2}$.

3.30. $\lim\limits_{x\to\infty}\dfrac{5x^2-3x+1}{3x^2+x-5}$.

4.

4.1. $\lim\limits_{x\to-\infty}\dfrac{x^5-2x+4}{2x^4+3x^2+1}$.

4.2. $\lim\limits_{x\to\infty}\dfrac{3x^4+2x-5}{2x^2+x+7}$.

4.3. $\lim\limits_{x\to-\infty}\dfrac{3x^2+7x-4}{x^5+2x-1}$.

4.4. $\lim\limits_{x\to\infty}\dfrac{3x-x^6}{x^2-2x+5}$.

4.5. $\lim\limits_{x\to\infty}\dfrac{2x^3+7x-1}{3x^4+2x+5}$.

4.6. $\lim\limits_{x\to-\infty}\dfrac{2x^3+7x^2+4}{x^4+5x-1}$.

4.7. $\lim\limits_{x\to-\infty}\dfrac{3x^6-5x^2+2}{2x^3+4x-5}$.

4.8. $\lim\limits_{x\to\infty}\dfrac{x^7+5x^2-4x}{3x^2+11x-7}$.

4.9. $\lim\limits_{x\to-\infty}\dfrac{7x^2 5x+9}{1+4x-x^3}$.

4.10. $\lim\limits_{x\to\infty}\dfrac{3x^4+x^2-6}{2x^2+3x+1}$.

4.11. $\lim\limits_{x\to-\infty}\dfrac{2x^2+5x+7}{3x^4-2x^2+x}$.

4.12. $\lim\limits_{x\to\infty}\dfrac{3x^3+4x^2-7x}{2x^2+7x-3}$.

4.13. $\lim\limits_{x\to-1}\dfrac{5x^3-3x^2+7}{2x^4+3x^2+1}$.

4.14. $\lim\limits_{x\to\infty}\dfrac{5x^2-3x+1}{1+2x-x^4}$.

4.15. $\lim\limits_{x\to-\infty}\dfrac{2x^3+3x^2+5}{3x^2-4x+1}$.

4.16. $\lim\limits_{x\to\infty}\dfrac{6x^2-5x+2}{4x^3+2x-1}$.

4.17. $\lim\limits_{x\to-\infty}\dfrac{11x^3+3x}{2x^2-2x+1}$.

4.18. $\lim\limits_{x\to\infty}\dfrac{8x^2+3x+5}{4x^3-2x^2-1}$.

4.19. $\lim\limits_{x\to-\infty}\dfrac{6x^3+5x^2-3}{2x^2-x+7}$.

4.20. $\lim\limits_{x\to\infty}\dfrac{3x^2+4x-7}{x^4-2x^3+1}$.

4.21. $\lim\limits_{x\to-\infty}\dfrac{8x^5+4x^3+3}{2x^3+x-7}$.

4.22. $\lim\limits_{x\to\infty}\dfrac{2x^2-7x+1}{x^3+4x^2-3}$.

4.23. $\lim\limits_{x\to-\infty}\dfrac{5x^4-2x^2+3}{2x^2+3x-7}$.

4.24. $\lim\limits_{x\to\infty}\dfrac{8x^3+x^2-7}{2x^2-5x+3}$.

4.25. $\lim\limits_{x\to-\infty}\dfrac{3x^4+2x^2-8}{8x^3-4x+5}$.

4.26. $\lim\limits_{x\to\infty}\dfrac{3x^4+2x-4}{3x^2-4x+1}$.

4.27. $\lim\limits_{x\to-\infty}\dfrac{7x^3-2x+4}{2x^2+x-5}$.

4.28. $\lim\limits_{x\to\infty}\dfrac{4x^3+5x^2-3x}{3x^2+x+10}$.

4.29. $\lim\limits_{x\to-\infty}\dfrac{2x^2+10x-11}{3x^4-2x+5}$.

4.30. $\lim\limits_{x\to\infty}\dfrac{7x^3+3x-4}{2x^2-5x+1}$.

5.

5.1. $\lim\limits_{x\to\infty}\dfrac{2x^2+3x-5}{7x^3-2x^2+1}$.

5.2. $\lim\limits_{x\to-\infty}\dfrac{3x^2-7x+2}{x^4+2x-4}$.

5.3. $\lim\limits_{x\to\infty}\dfrac{7x^4-3x+4}{3x^2-2x+1}$.

5.4. $\lim\limits_{x\to-\infty}\dfrac{2x^2-x+7}{3x^4-5x^2+10}$.

5.5. $\lim\limits_{x\to-\infty}\dfrac{4x^3-2x^2+x}{3x^2-x}$.

5.6. $\lim\limits_{x\to\infty}\dfrac{3x^4-2x+1}{3x^2+2x-5}$.

5.7. $\lim\limits_{x\to\infty}\dfrac{2x^2-5x+2}{x^4+3x^2-9}$.

5.8. $\lim\limits_{x\to-\infty}\dfrac{5x^2-4x+2}{4x^3+2x-5}$.

5.9. $\lim\limits_{x\to\infty}\dfrac{2x^3+3x^2+2x}{x^2+7x+1}$.

5.10. $\lim\limits_{x\to-\infty}\dfrac{3x^2-7x+5}{4x^5-3x^3+2}$.

5.11. $\lim\limits_{x\to\infty}\dfrac{7x^5+6x^4-x^3}{2x^2+6x+1}$.

5.12. $\lim\limits_{x\to-\infty}\dfrac{4-3x-2x^2}{3x^4+5x}$.

5.13. $\lim\limits_{x\to-\infty}\dfrac{7-3x^4}{2x^3+3x^2-5}$.

5.14. $\lim\limits_{x\to\infty}\dfrac{8x^4+7x^3-3}{3x^2-5x+1}$.

5.15. $\lim\limits_{x\to-\infty}\dfrac{3x+7}{2-3x+4x^2}$.

5.16. $\lim\limits_{x\to-\infty}\dfrac{2x^3-3x+1}{7x+5}$.

5.17. $\lim\limits_{x\to\infty}\dfrac{10x-7}{3x^4+2x^3+1}$.

5.18. $\lim\limits_{x\to-\infty}\dfrac{5x^4-3x^2}{1+2x+3x^2}$.

5.19. $\lim\limits_{x\to\infty}\dfrac{5x+3}{x^3-4x^2-x}$.

5.20. $\lim\limits_{x\to-\infty}\dfrac{3x^4+5x}{2x^2-3x-7}$.

5.21. $\lim\limits_{x\to\infty}\dfrac{2x^2-5x+3}{3x^4-2x^2+x}$.

5.22. $\lim\limits_{x\to-\infty}\dfrac{2x^5-x^3}{4x^2+3x-6}$.

5.23. $\lim\limits_{x\to\infty}\dfrac{3x+1}{x^3-2x^2+x}$.

5.24. $\lim\limits_{x\to-\infty}\dfrac{2-x-3x^2}{x^3-16}$.

5.25. $\lim\limits_{x\to\infty}\dfrac{4x^2-10x+7}{2x^3-3x}$.

5.26. $\lim\limits_{x\to-\infty}\dfrac{2x^3-3x+1}{x^5+4x^3}$.

5.27. $\lim\limits_{x\to\infty}\dfrac{2x-13}{x^7-3x^5-4x}$.

5.28. $\lim\limits_{x\to-\infty}\dfrac{2x^2-3x+1}{x^2+2x^2+5}$.

5.29. $\lim\limits_{x\to\infty}\dfrac{x^3-81}{3x^2+4x+2}$.

5.30. $\lim\limits_{x\to-\infty}\dfrac{7x+4}{3x^3-5x+1}$.

6.

6.1. $\lim\limits_{x\to3}\dfrac{x^2+x-12}{\sqrt{x-2}-\sqrt{4-x}}$.

6.2. $\lim\limits_{x\to-4}\dfrac{\sqrt{x+12}-\sqrt{4-x}}{x^2+2x-8}$.

6.3. $\lim\limits_{x\to-3}\dfrac{\sqrt{x+10}-\sqrt{4-x}}{2x^2-x-21}$.

6.4. $\lim\limits_{x\to-2}\dfrac{\sqrt{2-x}-\sqrt{x+6}}{x^2-x-6}$.

6.5. $\lim\limits_{x\to1}\dfrac{\sqrt{3+2x}-\sqrt{x+4}}{3x^2-4x+1}$.

6.6. $\lim\limits_{x\to2}\dfrac{x^2-3x+2}{\sqrt{5-x}-\sqrt{x+1}}$.

6.7. $\lim\limits_{x\to-1}\dfrac{3x^2+4x+1}{\sqrt{x+3}-\sqrt{5+3x}}$.

6.8. $\lim\limits_{x\to4}\dfrac{2x^2-9x+4}{\sqrt{5-x}-\sqrt{x-3}}$.

6.9. $\lim\limits_{x\to5}\dfrac{\sqrt{2x+1}-\sqrt{x+6}}{2x^2-7x-15}$.

6.10. $\lim\limits_{x\to-5}\dfrac{\sqrt{3x+17}-\sqrt{2x+12}}{x^2+8x+15}$.

6.11. $\lim\limits_{x\to0}\dfrac{\sqrt{x^2+2}-\sqrt{2}}{\sqrt{x^2+1}-1}$.

6.12. $\lim\limits_{x\to0}\dfrac{\sqrt{7-x}-\sqrt{7+x}}{\sqrt{7x}}$.

6.13. $\lim\limits_{x\to0}\dfrac{3x}{\sqrt{1+x}-\sqrt{1-x}}$.

6.14. $\lim\limits_{x\to4}\dfrac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$.

6.15. $\lim\limits_{x\to-1}\dfrac{\sqrt{5+x}-2}{\sqrt{8-x}-3}$.

6.16. $\lim\limits_{x\to5}\dfrac{\sqrt{x+4}-3}{\sqrt{x-1}-2}$.

6.17. $\lim\limits_{x\to 7}\dfrac{\sqrt{x-3}-2}{\sqrt{x+2}-3}$.

6.18. $\lim\limits_{x\to 3}\dfrac{\sqrt{4x-3}-3}{x^2-9}$.

6.19. $\lim\limits_{x\to 3}\dfrac{\sqrt{5x+1}-4}{x^2+2x-15}$.

6.20. $\lim\limits_{x\to 0}\dfrac{2-\sqrt{x^2+4}}{3x^2}$.

6.21. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+4}-2}{\sqrt{x^2+16}-4}$.

6.22. $\lim\limits_{x\to 0}\dfrac{3x}{\sqrt{5-x}-\sqrt{5+x}}$.

6.23. $\lim\limits_{x\to 9}\dfrac{\sqrt{2x+7}-5}{3-\sqrt{x}}$.

6.24. $\lim\limits_{x\to 4}\dfrac{2-\sqrt{x}}{\sqrt{6x+1}-5}$.

6.25. $\lim\limits_{x\to 3}\dfrac{x^3-27}{\sqrt{3x}-x}$.

6.26. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+3x^2}-1}{x^3+x^2}$.

6.27. $\lim\limits_{x\to -4}\dfrac{\sqrt{x+20}-4}{x^3+64}$.

6.28. $\lim\limits_{x\to 1}\dfrac{3x^2-2}{\sqrt{8+x}-3}$.

6.29. $\lim\limits_{x\to 0}\dfrac{\sqrt{9+x}-3}{x^2+x}$.

6.30. $\lim\limits_{x\to 2}\dfrac{\sqrt{4x+1}-3}{x^3-8}$.

7.

7.1. $\lim\limits_{x\to\infty}\left(\dfrac{x+4}{x+8}\right)^{-3x}$

7.2. $\lim\limits_{x\to\infty}\left(\dfrac{x}{x+1}\right)^{2x-3}$.

7.3. $\lim\limits_{x\to\infty}\left(\dfrac{2x}{1+2x}\right)^{-4x}$

7.4. $\lim\limits_{x\to\infty}\left(\dfrac{x-1}{x}\right)^{2-3x}$.

7.5. $\lim\limits_{x\to\infty}\left(\dfrac{2x+5}{2x+1}\right)^{5x}$

7.6. $\lim\limits_{x\to\infty}\left(\dfrac{x+3}{x}\right)^{-5x}$.

7.7. $\lim\limits_{x\to\infty}\left(\dfrac{x+2}{x+1}\right)^{1+2x}$

7.8. . $\lim\limits_{x\to\infty}\left(\dfrac{x+3}{x-1}\right)^{x-4}$.

7.9. $\lim\limits_{x\to\infty}\left(\dfrac{2x}{2x-3}\right)^{3x}$

7.10. $\lim\limits_{x\to\infty}\left(\dfrac{x-7}{x}\right)^{2x+1}$.

7.11. $\lim\limits_{x\to\infty}\left(\dfrac{x-1}{x+4}\right)^{3x+2}$

7.12. $\lim\limits_{x\to\infty}\left(\dfrac{2x+1}{2x-1}\right)^{x+2}$.

7.13. $\lim\limits_{x\to\infty}\left(\dfrac{x-2}{x+1}\right)^{2x-3}$

7.14. $\lim\limits_{x\to\infty}\left(\dfrac{x}{x-3}\right)^{x-5}$.

7.15. $\lim\limits_{x\to\infty}\left(\dfrac{3x-4}{3x+2}\right)^{2x}$

7.16. $\lim\limits_{x\to\infty}\left(\dfrac{2x-1}{2x+4}\right)^{3x-1}$.

7.17. $\lim\limits_{x\to\infty}\left(\dfrac{2x-4}{2x}\right)^{-3x}$

7.18. $\lim\limits_{x\to\infty}\left(\dfrac{x+5}{x}\right)^{3x+4}$.

7.19. $\lim\limits_{x\to\infty}\left(\dfrac{x-7}{x+1}\right)^{4x-2}$

7.20. $\lim\limits_{x\to\infty}\left(\dfrac{x+2}{x}\right)^{3-2x}$.

7.21. $\lim\limits_{x\to\infty}\left(\dfrac{2-3x}{5-3x}\right)^{x}$

7.22. $\lim\limits_{x\to\infty}\left(\dfrac{1-x}{2-x}\right)^{3x}$.

7.23. $\lim\limits_{x\to\infty}\left(\frac{4x-1}{4x+1}\right)^{2x}$

7.24. $\lim\limits_{x\to\infty}\left(\frac{3x+4}{3x}\right)^{-2x}$.

7.25. $\lim\limits_{x\to\infty}\left(\frac{2x-1}{2x+4}\right)^{-x}$

7.26. $\lim\limits_{x\to\infty}\left(\frac{3x+4}{3x+5}\right)^{x+1}$.

7.27. $\lim\limits_{x\to\infty}\left(\frac{1+2x}{3+2x}\right)^{-x}$

7.28. $\lim\limits_{x\to\infty}\left(\frac{3x}{3x+2}\right)^{x-2}$.

7.29. $\lim\limits_{x\to\infty}\left(\frac{x}{x-1}\right)^{3-2x}$

7.30. $\lim\limits_{x\to\infty}\left(\frac{4-2x}{1-2x}\right)^{x+1}$.

8.

8.1. $\lim\limits_{x\to\infty}\left(\frac{2x+3}{5x+7}\right)^{x+1}$

8.2. $\lim\limits_{x\to\infty}\left(\frac{2x+1}{x-1}\right)^{x}$.

8.3. $\lim\limits_{x\to\infty}\left(\frac{x+1}{2x-1}\right)^{3x}$

8.4. $\lim\limits_{x\to-\infty}\left(\frac{2x-1}{4x+1}\right)^{3x-1}$.

8.5. $\lim\limits_{x\to\infty}\left(\frac{5x+8}{x-2}\right)^{x+4}$

8.6. $\lim\limits_{x\to-\infty}\left(\frac{x+1}{3x-1}\right)^{2x+1}$.

8.7. $\lim\limits_{x\to-\infty}\left(\frac{2x+1}{x-1}\right)^{4x}$

8.8. $\lim\limits_{x\to\infty}\left(\frac{x+1}{2x-1}\right)^{5x}$.

8.9. $\lim\limits_{x\to-\infty}\left(\frac{x+3}{2x-4}\right)^{x+2}$

8.10. $\lim\limits_{x\to-\infty}\left(\frac{2x+1}{3x-1}\right)^{x-1}$.

8.11. $\lim\limits_{x\to\infty}\left(\frac{5x-3}{x+4}\right)^{x+3}$

8.12. $\lim\limits_{x\to-\infty}\left(\frac{2x-3}{7x+4}\right)^{x}$.

8.13. $\lim\limits_{x\to-\infty}\left(\frac{x-5}{3x+4}\right)^{2x}$

8.14. $\lim\limits_{x\to\infty}\left(\frac{x+3}{4x-5}\right)^{2x}$.

8.15. $\lim\limits_{x\to-\infty}\left(\frac{x-2}{3x+1}\right)^{5x}$

8.16. $\lim\limits_{x\to-\infty}\left(\frac{3x-4}{x+6}\right)^{x-1}$.

8.17. $\lim\limits_{x\to\infty}\left(\frac{x-2}{3x+10}\right)^{3x}$

8.18. $\lim\limits_{x\to-\infty}\left(\frac{2x-3}{x+4}\right)^{6x+1}$.

8.19. $\lim\limits_{x\to-\infty}\left(\frac{x+3}{3x-1}\right)^{2x}$

8.20. $\lim\limits_{x\to\infty}\left(\frac{6x+5}{x-10}\right)^{5x}$.

8.21. $\lim\limits_{x\to-\infty}\left(\frac{3x+7}{x+4}\right)^{4x}$

8.22. $\lim\limits_{x\to\infty}\left(\frac{x-1}{4x+5}\right)^{3x}$.

8.23. $\lim\limits_{x\to-\infty}\left(\frac{5x-7}{x+6}\right)^{2x}$

8.24. $\lim\limits_{x\to\infty}\left(\frac{3-4x}{2-x}\right)^{6x}$.

8.25. $\lim\limits_{x\to\infty}\left(\frac{1-2x}{3-x}\right)^{-x}$

8.26. $\lim\limits_{x\to-\infty}\left(\frac{4+3x}{5+x}\right)^{7x}$.

8.27. $\lim\limits_{x\to-\infty}\left(\frac{3x-1}{2x+5}\right)^{3x}$

8.28. $\lim\limits_{x\to\infty}\left(\frac{1-x}{2-10x}\right)^{5x}$.

8.29. $\lim\limits_{x\to\infty}\left(\dfrac{3+x}{9x-4}\right)^{2x}$

8.30. $\lim\limits_{x\to-\infty}\left(\dfrac{x+5}{4x-2}\right)^{3x}$.

9.

9.1. $\lim\limits_{x\to 0}\dfrac{1-\cos 8x}{3x^2}$.

9.2. $\lim\limits_{x\to 0}\dfrac{\sin 3x-\sin x}{5x}$.

9.3. $\lim\limits_{x\to 0}\dfrac{\cos x-\cos 5x}{2x^2}$.

9.4. $\lim\limits_{x\to 0}\dfrac{tg3x}{2\sin x}$.

9.5. $\lim\limits_{x\to 0}\dfrac{tgx-\sin x}{3x^2}$.

9.6. $\lim\limits_{x\to 0}\dfrac{\arcsin 5x}{\sin 3x}$.

9.7. $\lim\limits_{x\to 1}(1-x)tg\dfrac{\pi x}{2}$.

9.8. $\lim\limits_{x\to\pi/2}\dfrac{1-\sin x}{\pi-2x}$.

9.9. $\lim\limits_{x\to 0}\dfrac{tg2x-\sin 2x}{x^2}$.

9.10. $\lim\limits_{x\to 0}\dfrac{1-\cos^2 x}{xtgx}$.

9.11. $\lim\limits_{x\to 0}\left(\dfrac{1}{tgx}-\dfrac{1}{\sin x}\right)$.

9.12. $\lim\limits_{x\to 0}\dfrac{\sin^2 3x-\sin^2 x}{x^2}$.

9.13. $\lim\limits_{x\to 0}\dfrac{\sin 7x+\sin 3x}{x\sin x}$.

9.14. $\lim\limits_{x\to 0}\dfrac{1-\cos 5x}{2x^2}$.

9.15. $\lim\limits_{x\to 0}\dfrac{\cos 2x-\cos 4x}{3x^2}$.

9.16. $\lim\limits_{x\to 0}\dfrac{arctg2x}{tg3x}$.

9.17. $\lim\limits_{x\to 0}\dfrac{tg3x-\sin 3x}{2x^2}$.

9.18. $\lim\limits_{x\to\pi/4}\dfrac{1-\sin 2x}{\pi-4x}$.

9.19. $\lim\limits_{x\to 0}\dfrac{\cos 4x-\cos^3 4x}{3x^2}$.

9.20. $\lim\limits_{x\to 0}\left(\dfrac{1}{\sin 2x}-\dfrac{1}{tg2x}\right)$.

9.21. $\lim\limits_{x\to 0}\dfrac{\cos^2 x-\cos^2 2x}{x^2}$.

9.22. $\lim\limits_{x\to 0}\dfrac{\arcsin 5x}{x^2-x}$.

9.23. $\lim\limits_{x\to 0}\dfrac{1-\cos^2 2x}{x\arcsin x}$.

9.24. $\lim\limits_{x\to 0}\dfrac{1-\cos 4x}{x\sin x}$.

9.25. $\lim\limits_{x\to 0}\dfrac{\cos 5x-\cos x}{4x^2}$.

9.26. $\lim\limits_{x\to 0}\dfrac{\sin 5x+\sin x}{\arcsin x}$.

9.27. $\lim\limits_{x\to\pi/2}\dfrac{1-\sin x}{(\pi/2-x)^2}$.

9.28. $\lim\limits_{x\to\pi/2}(\pi/2-x)tgx$.

9.29. $\lim\limits_{x\to 0}\dfrac{7x}{\sin x+\sin 7x}$.

9.30. $\lim\limits_{x\to 0}\dfrac{\cos x-\cos^3 x}{5x^2}$.

Namunaviy variantni yechish

Ko’rsatilgan limitlarni toping

1. $\lim\limits_{x\to -2}\dfrac{5x^2+13x+6}{3x^2+2x-8}$.

► $\lim\limits_{x\to -2}\dfrac{5x^2+13x+6}{3x^2+2x-8}=\lim\limits_{x\to -2}\dfrac{(x+2)(5x+3)}{(x+2)(3x-4)}=\lim\limits_{x\to -2}\dfrac{5x+3}{3x-4}=\dfrac{7}{10}=0{,}7$ ◄

2. $\lim\limits_{x\to 4}\dfrac{3x^2-10x-8}{4x^2+6x-64}$. ► $\lim\limits_{x\to 4}\dfrac{3x^2-10x-8}{4x^2+6x-64}=\dfrac{0}{24}=0$ ◄

3. $\lim\limits_{x\to\infty}\dfrac{7x^4+2x^3+5}{6x^4+3x^2-7x}$. ► $\lim\limits_{x\to\infty}\dfrac{7x^4+2x^3+5}{6x^4+3x^2-7x}=\lim\limits_{x\to\infty}\dfrac{x^4(7+2/x+5/x^4)}{x^4(6+3/x^2-7/x^3)}=\dfrac{7}{6}$ ◄

4. $\lim\limits_{x\to-\infty}\dfrac{10x-3}{2x^3+4x+3}$. ► $\lim\limits_{x\to-\infty}\dfrac{10x-3}{2x^3+4x+3}=\lim\limits_{x\to-\infty}\dfrac{x(10-3/x)}{x^3(2+4/x^2+3/x^3)}=$

$=\lim\limits_{x\to-\infty}\dfrac{10-3/x}{x^2(2+4/x^2+3/x^3)}=\dfrac{10}{\infty}=0.$ ◄

5. $\lim\limits_{x\to-\infty}\dfrac{2x^5+3x^3-4x}{3x^2-4x+2}$. ► $\lim\limits_{x\to-\infty}\dfrac{2x^5+3x^3-4x}{3x^2-4x+2}=\lim\limits_{x\to-\infty}\dfrac{x^5(2+3/x^2-4/x^4)}{x^2(3-4/x+2/x^2)}=$

$=\lim\limits_{x\to-\infty}\dfrac{x^3(2+3/x^2-4/x^4)}{3-4/x+2/x^2}=\dfrac{-\infty}{3}=-\infty.$ ◄

6. $\lim\limits_{x\to 4}\dfrac{\sqrt{21+x}-5}{x^3-64}$. ► $\lim\limits_{x\to 4}\dfrac{\sqrt{21+x}-5}{x^3-64}=\lim\limits_{x\to 4}\dfrac{(\sqrt{21+x}-5)(\sqrt{21+x}+5)}{(x^3-64)(\sqrt{21+x}+5)}=$

$=\lim\limits_{x\to 4}\dfrac{21+x-25}{(x-4)(x^2+4x+16)(\sqrt{21+x}+5)}=$

$=\lim\limits_{x\to 4}\dfrac{x-4}{(x^3-64)(\sqrt{21+x}+5)}=\lim\limits_{x\to 4}\dfrac{1}{(x^2-4x+16)(\sqrt{21+x+5})}=\dfrac{1}{480}.$ ◄

7. $\lim\limits_{x\to\infty}\left(\dfrac{2x}{2x-3}\right)^{2-5x}$. ► $\lim\limits_{x\to\infty}\left(\dfrac{2x}{2x-3}\right)^{2-5x}=\lim\limits_{x\to\infty}\left(1+\dfrac{2x}{2x-3}-1\right)^{2-5x}=$

$=\lim\limits_{x\to\infty}\left(1+\dfrac{2x-2x+3}{2x-3}\right)^{2-5x}=\lim\limits_{x\to\infty}\left(1+\dfrac{3}{2x-3}\right)^{2-5x}=$

$=\lim\limits_{x\to\infty}\left(\left(1+\dfrac{3}{2x-3}\right)^{(2x-3)/3}\right)^{3(2-5x)/(2x-3)}=\lim\limits_{x\to\infty}e^{3(2-5x)/(2x-3)}=e^{-15/2}.$ ◄

8. $\lim\limits_{x\to-\infty}\left(\dfrac{4x+3}{2x-5}\right)^{1+7x}$. ► $\lim\limits_{x\to-\infty}\left(\dfrac{4x+3}{2x-5}\right)^{1+7x}=\lim\limits_{x\to-\infty}2^{1+7x}=2^{-\infty}=0$ ◄

9. $\lim\limits_{x\to\pi}\dfrac{1-\sin(x/2)}{\pi^2-x^2}$. ► $\lim\limits_{x\to\pi}\dfrac{1-\sin(x/2)}{\pi^2-x^2}=\lim\limits_{x\to\pi}\dfrac{1-\cos(\pi/2-x/2)}{\pi^2-x^2}=$

$=\lim\limits_{x\to\pi}\dfrac{2\sin^2\big((\pi-x)/4\big)}{(\pi-x)(\pi+x)}=\lim\limits_{x\to\pi}\dfrac{2\sin\big((\pi-x)/4\big)\sin\big((\pi-x)/4\big)}{4\cdot\dfrac{\pi-x}{4}(\pi+x)}=$

$=\dfrac{1}{2}\lim\limits_{x\to\pi}\dfrac{\sin\big((\pi-x)/4\big)}{\pi+x}=\dfrac{1}{2}\dfrac{0}{2\pi}=0.$ ◄

5.2–IUT

1. Quyida keltirilgan misollardagi $f(x)$ bilan $\varphi(x)$ funksiyalarning $x\to 0$ da bir xil tartibdagi cheksiz kichik miqdorlar ekanligi isbotlansin.

1.1. $f(x)=tg2x,\quad \varphi(x)=\arcsin x.$

1.2. $f(x)=1-\cos x,\quad \varphi(x)=3x^2.$

1.3. $f(x)=arctg^2 3x, \quad \varphi(x)=4x^2.$

1.4. $f(x)=\sin 3x-\sin x, \quad \varphi(x)=5x.$

1.5. $f(x)=\cos 3x-\cos x, \quad \varphi(x)=7x^2.$

1.6. $f(x)=x^2-\cos 2x, \quad \varphi(x)=6x^2.$

1.7. $f(x)=\sqrt{1+x}-1, \quad \varphi(x)=2x.$

1.8. $f(x)=\sin x+\sin 5x, \quad \varphi(x)=2x.$

1.9. $f(x)=3x/(1-x), \quad \varphi(x)=x/(4+x).$

1.10. $f(x)=3x^2/(2+x), \quad \varphi(x)=7x^2.$

1.11. $f(x)=2x^3, \quad \varphi(x)=5x^3/(4-x).$

1.12. $f(x)=x^2/(5+x), \quad \varphi(x)=4x^2/(x-1).$

1.13. $f(x)=\sin 8x, \quad \varphi(x)=\arcsin 5x.$

1.14. $f(x)=\sin 3x+\sin x, \quad \varphi(x)=10x.$

1.15. $f(x)=\cos 7x-\cos x, \quad \varphi(x)=2x^2.$

1.16. $f(x)=1-\cos 2x, \quad \varphi(x)=8x^2.$

1.17. $f(x)=3\sin^2 4x, \quad \varphi(x)=x^2-x^4.$

1.18. $f(x)=tg(x^2+2x), \quad \varphi(x)=x^2+2x.$

1.19. $f(x)=\arcsin(x^2-x), \quad \varphi(x)=x^3-x.$

1.20. $f(x)=\sin 7x+\sin x, \quad \varphi(x)=4x.$

1.21. $f(x)=\sqrt{4+x}+2, \quad \varphi(x)=3x.$

1.22. $f(x)=\sin(x^2-2x), \quad \varphi(x)=x^4-8x.$

1.23. $f(x)=2x/(3-x), \quad \varphi(x)=2x-x^2.$

1.24. $f(x)=x^2/(7+x), \quad \varphi(x)=3x^3-x^2.$

1.25. $f(x)=\sin(x^2+5x), \quad \varphi(x)=x^3-25x.$

1.26. $f(x)=\cos x-\cos^3 x, \quad \varphi(x)=6x^2.$

1.27. $f(x)=\arcsin 2x, \quad \varphi(x)=8x.$

1.28. $f(x)=1-\cos 4x, \quad \varphi(x)=x\sin 2x.$

1.29. $f(x)=\sqrt{9-x}-3, \quad \varphi(x)=2x.$

1.30. $f(x)=\cos 3x-\cos 5x, \quad \varphi(x)=x^2.$

2. Ekvivalent cheksiz kichik miqdorlardan foydalanib quyidagi limitlar hisoblansin.

2.1. $\lim\limits_{x\to 0}\dfrac{\ln(1+3x^2)}{x^3-5x^2}.$

2.2. $\lim\limits_{x\to 0}\dfrac{\arcsin 5x}{tg3x}.$

2.3. $\lim\limits_{x\to 0}\dfrac{\sin 7x}{tg2x}.$

2.4. $\lim\limits_{x\to 0}\dfrac{e^{3x}-1}{x^3+27x}.$

2.5. $\lim\limits_{x\to 0}\dfrac{arctg6x}{2x^2-3x}.$

2.6. $\lim\limits_{x\to 0}\dfrac{\arcsin 3x}{2x}.$

2.7. $\lim\limits_{x\to 0}\dfrac{\sin 5x}{arctg2x}.$

2.8. $\lim\limits_{x\to 0}\dfrac{\ln(1+3x)}{\sin 2x}.$

2.9. $\lim\limits_{x\to 0}\dfrac{e^{2x}-1)}{tg3x}.$

2.10. $\lim\limits_{x\to 3}\dfrac{\sin(x-3)}{x^2-5x+6}.$

2.11. $\lim\limits_{x\to 0}\dfrac{\cos 3x-\cos x}{2x^2}$.

2.12. $\lim\limits_{x\to 0}\dfrac{1-\cos 6x}{4x^2}$.

2.13. $\lim\limits_{x\to 0}\dfrac{arctg 3x}{\ln(1+2x)}$.

2.14. . $\lim\limits_{x\to 0}\dfrac{\arcsin 4x}{tg 5x}$.

2.15. $\lim\limits_{x\to 0}\dfrac{e^{5x}-1}{\sin 2x}$.

2.16. $\lim\limits_{x\to -2}\dfrac{tg(x+2)}{x^2-4}$.

2.17. $\lim\limits_{x\to -2}\dfrac{\sin(x+2)}{x^3+8}$.

2.18. $\lim\limits_{x\to 0}\dfrac{\arcsin 2x}{tg 4x}$.

2.19. $\lim\limits_{x\to 4}\dfrac{x^3-64}{tg(x-4)}$.

2.20. $\lim\limits_{x\to 0}\dfrac{\cos 2x-\cos 4x}{3x^2}$.

2.21. $\lim\limits_{x\to 0}\dfrac{\ln(1+4x^3)}{2x^3}$.

2.22. $\lim\limits_{x\to 0}\dfrac{arctg 5x}{tg 2x}$.

2.23. $\lim\limits_{x\to 0}\dfrac{\sin 3x}{\ln(1+2x)}$.

2.24. $\lim\limits_{x\to 0}\dfrac{\arcsin 8x}{tg 4x}$.

2.25. $\lim\limits_{x\to 0}\dfrac{e^{5x}-1}{tg 2x}$.

2.26. $\lim\limits_{x\to 0}\dfrac{\ln(1+4x)}{\sin 2x}$.

2.27. $\lim\limits_{x\to 3}\dfrac{\sin(x-3)}{x^3-27}$.

2.28. $\lim\limits_{x\to -5}\dfrac{tg(x+5)}{x^2-25}$.

2.29. $\lim\limits_{x\to 0}\dfrac{1-\cos 8x}{2x^2}$.

2.30. $\lim\limits_{x\to 0}\dfrac{\ln(1+5x)}{\sin 3x}$.

3. Quyida keltirilgan funksiyalar uzluksizlikka tekshirilib, ularning grafigi chizilsin.

3.1. $f(x)=\begin{cases} x+4, & x<-1, \\ x^2+2 & -1\le x<1, \\ 2x, & x\ge 1. \end{cases}$

3.2. $f(x)=\begin{cases} x+1, & x\le 0, \\ (x+1)^2 & 0<x\le 2, \\ -x+4, & x>2. \end{cases}$

3.3. $f(x)=\begin{cases} x+2, & x\le -1, \\ x^2+1 & -1<x\le 1, \\ -x+3, & x>1. \end{cases}$

3.4. $f(x)=\begin{cases} -x, & x\le 0, \\ -(x-1)^2 & 0<x<2, \\ x-3, & x\ge 2. \end{cases}$

3.5. $f(x)=\begin{cases} -2(x+1), & x\le -1, \\ (x+1)^3 & -1<x<0, \\ x, & x\ge 0. \end{cases}$

3.6. $f(x)=\begin{cases} -x, & x\le 0, \\ x^2 & 0<x\le 2, \\ x+1, & x>2. \end{cases}$

3.7. $f(x)=\begin{cases} x^2+1, & x\le 1, \\ 2x & 1<x\le 3, \\ x+2, & x>3. \end{cases}$

3.8. $f(x)=\begin{cases} x-3, & x<0, \\ x+1 & 0\le x\le 4, \\ 3+x, & x>4. \end{cases}$

3.9. $f(x)=\begin{cases} \sqrt{1-x}, & x\le 0, \\ 0 & 0<x\le 2, \\ x-2, & x>2. \end{cases}$

3.10. $f(x)=\begin{cases} 2x^2, & x\le 0, \\ x & 0<x\le 1, \\ 2+x, & x>1. \end{cases}$

3.11. $f(x)=\begin{cases}\sin x, & x<0,\\ x, & 0\le x\le 2,\\ 0, & x>2,\end{cases}$

3.12. $f(x)=\begin{cases}\cos x, & x\le \pi/2,\\ 0, & \pi/2<x<\pi,\\ 2, & x\ge \pi,\end{cases}$

3.13. $f(x)=\begin{cases}x-1, & x\le 0,\\ x^2 & 0<x<2,\\ 2x, & x\ge 2.\end{cases}$

3.14. $f(x)=\begin{cases}x+1, & x<0,\\ x^2-1, & 0\le x<1,\\ -x, & x\ge 1.\end{cases}$

3.15. $f(x)=\begin{cases}-x, & x<0,\\ x^2+1, & 0\le x<2\\ x+1, & x\ge 2.\end{cases}$

3.16. $f(x)=\begin{cases}x+3, & x\le 0,\\ 1, & 0<x\le 2,\\ x^2-2, & x>2.\end{cases}$

3.17. $f(x)=\begin{cases}x-1, & x<0,\\ \sin x, & 0\le x<\pi,\\ 3, & x\ge \pi.\end{cases}$

3.18. $f(x)=\begin{cases}-x+1, & x<-1,\\ x^2+1, & -1\le x\le 2,\\ 2x, & x>2.\end{cases}$

3.19. $f(x)=\begin{cases}1, & x\le 0,\\ 2^x, & 0<x\le 2,\\ x+3, & x>2.\end{cases}$

3.20. $f(x)=\begin{cases}-x+2, & x\le -2,\\ x^3, & -2<x\le 1,\\ 2, & x>1.\end{cases}$

3.21. $f(x)=\begin{cases}3x+4, & x\le -1,\\ x^2-1, & -1<x<2,\\ x, & x\ge 2.\end{cases}$

3.22. $f(x)=\begin{cases}x, & x\le 1,\\ (x-2)^2 & 1<x<3,\\ -x+6, & x\ge 3.\end{cases}$

3.23. $f(x)=\begin{cases}x-1, & x<1,\\ x^2+2, & 1\le x\le 2,\\ -2x, & x>2.\end{cases}$

3.24. $f(x)=\begin{cases}x^3, & x<-1,\\ x-1, & -1\le x\le 3,\\ -x+5, & x>3.\end{cases}$

3.25. $f(x)=\begin{cases}x, & x<-2,\\ -x+1, & -2\le x\le 1,\\ x^2-1, & x>1.\end{cases}$

3.26. $f(x)=\begin{cases}x+3, & x\le 0,\\ -x^2+4, & 0<x<2,\\ x-2, & x\ge 2.\end{cases}$

3.27. $f(x)=\begin{cases}0, & x\le -1,\\ x^2-1, & -1<x\le 2,\\ 2x, & x>2.\end{cases}$

3.28. $f(x)=\begin{cases}-1, & x<0,\\ \cos x & 0\le x\le \pi,\\ 1-x, & x>\pi.\end{cases}$

3.29. $f(x)=\begin{cases}2, & x<-1,\\ 1-x, & -1\le x\le 1,\\ \ln x, & x>1.\end{cases}$

3.30. $f(x)=\begin{cases}-x, & x\le 0,\\ x^3 & 0<x\le 2,\\ x+4, & x>2.\end{cases}$

4. Berilgan funksiyalarni ko’rsatilgan nuqtalarda uzluksizlikka tekshiring.

4.1. $f(x)=2^{1/(x-3)}+1;\quad x_1=3,\quad x_2=4.$

4.2. $f(x)=5^{1/(x-3)}-1;\quad x_1=3,\quad x_2=4.$

4.3. $f(x)=(x+7)/(x-2);\quad x_1=2,\quad x_2=3.$

4.4. $f(x)=(x-5)/(x+3);\quad x_1=-2,\quad x_2=-3.$

4.5. $f(x)=4^{1/(3-x)}+2;\quad x_1=2,\quad x_2=3.$

4.6. $f(x)=9^{1/(2-x)}$; $x_1=0$, $x_2=2$.
4.7. $f(x)=2^{1/(x-5)}+1$; $x_1=4$, $x_2=5$.
4.8. $f(x)=5^{1/(x-4)}-2$; $x_1=3$, $x_2=4$.
4.9. $f(x)=6^{1/(x-3)}+3$; $x_1=3$, $x_2=4$.
4.10. $f(x)=7^{1/(5-x)}+1$; $x_1=4$, $x_2=5$.
4.11. $f(x)=(x-3)(x+4)$; $x_1=-5$, $x_2=-4$.
4.12. $f(x)=(x+5)/(x-2)$; $x_1=3$, $x_2=2$.
4.13. $f(x)=5^{2/(x-3)}$; $x_1=3$, $x_2=4$.
4.14. $f(x)=4^{2/(x-1)}-3$; $x_1=1$, $x_2=2$.
4.15. $f(x)=2^{5/(1-x)}-1$; $x_1=0$. $x_2=1$.
4.16. $f(x)=8^{4/(x-2)}-1$; $x_1=2$, $x_2=3$.
4.17. $f(x)=5^{4/(3-x)}+1$; $x_1=2$, $x_2=3$.
4.18. $f(x)=3x/(x-4)$; $x_1=4$, $x_2=5$.
4.19. $f(x)=2x/(x^2-1)$; $x_1=1$, $x_2=2$.
4.20. $f(x)=2^{3/(x+2)}+1$; $x_1=-2$, $x_2=-1$.
4.21. $f(x)=4^{3/(x-2)}+2$; $x_1=2$, $x_2=3$.
4.22. $f(x)=3^{2/(x+1)}-2$; $x_1=-1$, $x_2=0$.
4.23. $f(x)=5^{3/(x+4)}+1$; $x_1=-5$ $x_2=-4$.
4.24. $f(x)=(x-4)/(x+2)$; $x_1=-2$, $x_2=-1$.
4.25. $f(x)=(x-4)/(x+3)$; $x_1=-3$ $x_2=-2$.
4.26. $f(x)=(x+5)/(x-3)$; $x_1=3$, $x_2=4$.
4.27. $f(x)=3^{4/(1-x)}+1$; $x_1=1$, $x_2=2$.
4.28. $f(x)=4x/(x+5)$; $x_1=-5$, $x_2=-4$.
4.29. $f(x)=6^{2/(4-x)}$; $x_1=3$, $x_2=4$.
4.30. $f(x)=(x+1)/(x-2)$; $x_1=2$, $x_2=3$.

Namunaviy variantni yechish.

1. $f(x)=\cos 2x-\cos^3 2x$ va $\varphi(x)=3x^2-5x^3$ funksiyalar $x\to 0$ da bir xil tartibdagi cheksiz kichik ekanligini isbotlang.

► Quyidagi hisoblashlarni bajaramiz.

$$\lim_{x\to 0}\frac{f(x)}{\varphi(x)}=\lim_{x\to 0}\frac{\cos 2x-\cos^3 2x}{3x^2-5x^3}=$$

$$=\lim_{x\to 0}\frac{\cos 2x(1-\cos^2 2x)}{x^2(3-5x)}=\lim_{x\to 0}\left(2\cdot\frac{\cos 2x\cdot\sin^2 2x}{x^2(3-5x)}\right)=$$

$$=\lim_{x\to 0}\frac{8\cos 2x\cdot\sin 2x\cdot\sin 2x}{2x2x(3-5x)}=8/3.$$

$f(x)$ va $\varphi(x)$ funksiyalar nisbatlarining limiti noldan farqli chekli songa teng bo'lganligi uchun, ta'rifga asosan (5.1 formulani qarang) bu funksiyalar bir xil tartibli cheksiz kichiklar bo'ladi. ◄

2. O'zaro ekvivalent cheksiz kichik miqdorlardan foydalanib limitni toping.

► Ekvivalent cheksiz kichik miqdorlardan foydalansak

$$\lim_{x\to 0}\frac{\arcsin 8x}{\ln(1+4x)}=\lim_{x\to 0}\frac{8x}{4x}=2. \blacktriangleleft$$

3. Berilgan funksiyani uzluksizlikka tekshiring va uning grafigini chizing.

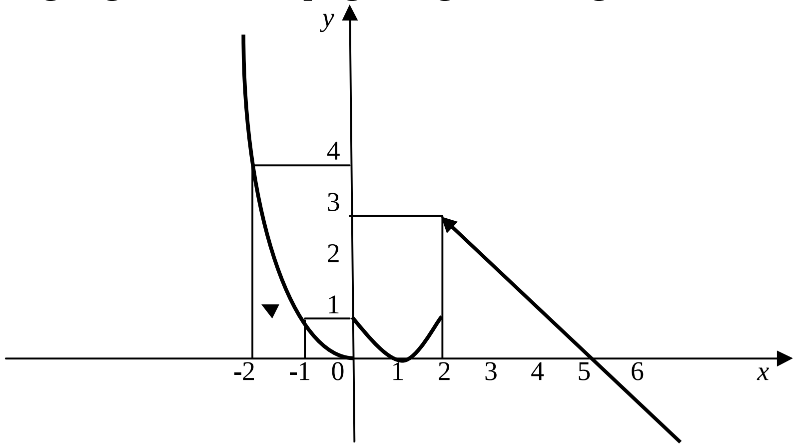
$$f(x)=\begin{cases} x^2, & -\infty < x \le 0, \\ (x-1)^2, & 0 < x \le 2, \\ 5-x, & 2 < x < +\infty. \end{cases}$$

► $f(x)$ funksiya $(-\infty;0)$, $(0;2)$ $(2;+\infty)$ intervallarda aniqlangan va uzluksiz bo'lib, bu intervallarda u elementar funksiyalar bilan berilgan. O'z navbatida, uzilish faqat $x_1=0$ va $x_2=2$ nuqtalardagina bo'lishi mumkin.

$x_1=0$ nuqta uchun $\lim\limits_{x\to 0-0} f(x)=\lim\limits_{x\to 0-0} x^2=0,$ $\lim\limits_{x\to 0+0} f(x)=\lim\limits_{x\to 0+0}(x-1)^2=1,$

$f(0)=x^2|_{x=0}=0,$ larga ega bo'lamiz, ya'ni $f(x)$ funksiya $x_1=0$ nuqtada 1- tur uzilishga ega. $x_2=2$ nuqta uchun esa $\lim\limits_{x\to 2-0} f(x)=\lim\limits_{x\to 2-0}(x-1)^2=1,$

$\lim\limits_{x\to 2+0} f(x)=\lim\limits_{x\to 2+0}(5-x)=3,$ $f(2)=(x-1)^2|_{x=2}=1,$ ya'ni $x_2=2$ nuqtada ham funksiya 1-tur uzilishga ega bo'ladi.Topilganlarga asosan grafikni chizamiz(5.1-Rasm).



5.1 rasm ◄

4. $f(x)=8^{1/(x-3)}+1$ funksiyani $x_1=3$ va $x_2=4$ nuqtalarda uzluksizlikka tekshiring.

► $x_1=3$ nuqta uchun quyidagilarni hosil qilamiz:

$$\lim_{x\to 3-0} f(x)=\lim_{x\to 3-0}(8^{1/(x-3)}+1)=8^{-\infty}+1=1,$$

$$\lim_{x\to 3+0} f(x)=\lim_{x\to 3+0}(8^{1/(x-3)}+1)=8^{\infty}+1=\infty,$$

Ya'ni, $x_1=3$ nuqtada funksiya cheksiz katta uziladi ($x_1=3$- ikkinchi tur uzilish nuqtasi).

$x_2 = 4$ nuqta uchun

$$\lim_{x\to 4-0} f(x) = \lim_{x\to 4-0}(8^{1/(x-3)}+1) = 9,$$
$$\lim_{x\to 4+0} f(x) = \lim_{x\to 4+0}(8^{1/(x-3)}+1) = 9,$$
$$f(4) = 8^{1/(4-3)}+1 = 9.$$

demak, bu nuqtada funksiya uzluksiz. ◄

5.6. 5- bo'limga qo'shimcha mashqlar

Ko'rsatilgan limitlarni toping.

1. $\lim\limits_{n\to\infty}\dfrac{(2+n)^{100}-n^{100}-200n^{99}}{n^{98}-10n^2+1}$. (*Javob*: 19800.)

2. $\lim\limits_{n\to\infty}\dfrac{\lg(n^2+2n\cos n+1)}{1+\lg(n+1)}$. (Javob: 2.)

3. $\lim\limits_{n\to\infty}\dfrac{\ln(n^2-n+1)}{\ln(n^{10}+n+1)}$. (Javob: $\dfrac{1}{5}$.)

4. $\lim\limits_{n\to\infty}\dfrac{a^n-a^{-n}}{a^n+a^{-n}}(a\neq 0)$. (Javob: $\begin{cases} 1, & |a|>1, \\ 0, & |a|=1, \\ -1, & |a|<1. \end{cases}$)

5. $\lim\limits_{n\to\infty}\dfrac{1^2+2^2+3^3+...+n^2}{n^3}$. (Javob: $\dfrac{1}{3}$.)

6. $\lim\limits_{x\to 1}\dfrac{x^{101}-101x+100}{x^2-2x+1}$. (Javob:5050.)

7. $\lim\limits_{x\to 0}\dfrac{\sqrt[2]{2x^2+10x+1}-\sqrt[7]{2x^2+10x+1}}{x}$. (Javob $\dfrac{4}{7}$.)

8. $\lim\limits_{x\to 0}\left(\sqrt[n]{(1+x^2)(2+x^2)...(n+x^2)}-x^2\right)(n\in N)$. (Javob: $\dfrac{n+1}{2}$.)

9. $\lim\limits_{x\to\infty}(\sqrt{1+x}-x)^{1/x}$. (Javob: $\dfrac{1}{\sqrt{e}}$.)

10. $\lim\limits_{x\to\pi}\dfrac{\sin x}{\pi^2-x^2}$. (Javob : $\dfrac{1}{2\pi}$.)

11. $\lim\limits_{x\to\infty}\dfrac{(ax+1)^n}{x^n+b}$, bu erda $n\in Z, n\neq 0$; a,b –o'zgarmas sonlar.

$$\left(\text{javob}: \begin{cases} a^n, & agar \quad n>0 \\ 0, & agar \quad n<o, b\neq 0 \\ a^n, & agar \quad n<0, a\neq 0, b=0 \\ \infty, & agar \quad n<0, a=b=0 \end{cases}\right)$$

12. $\lim\limits_{x\to\infty}\left(\dfrac{1}{3}-\dfrac{1}{9}+...+(-1)^{n-1}\dfrac{1}{3^n}\right)$. (Javob: ¼.)

Berilgan funksiyalarning uzilish nuqtalarini toping, uzilish nuqtalarini tavsiflang va bu funksiyalar grafigini chizing.

13. $y = 1/\lg|x|$. (Javob: $x_1 = 0$- tuzatiladigan uzilish nuqtasi, $x_{2,3} = \pm 1$ ikkinchi tur uzilish nuqtalari.)

14. $y = x\sin(\pi/x)$ (Javob*:* $x = 0$- tuzatiladigan uzilish nuqtasi.)

15. $y = 1/(1+3^{1/x})$. (Javob*:* $x = 0$- birinchi tur uzilish nuqtasi.)

16. $y = 1/(1+2^{tgx})$. (Javob*:* $x = \frac{\pi}{2}(2k+1),\ k \in Z$ birinchi tur uzilish nuqtalari.)

17. $y = (1+1/x)^x$. (Javob*:* $x = -1$- ikkinchi tur uzilish nuqtasi, $x = 0$ tuzatiladigan uzilish nuqtasi.)

18. $y = 1/(1-e^{1-x})$. (Javob*:* $x = 1$- ikkinchi tur uzilish nuqtasi.)

19. Chetlari mahkamlangan *a* radiusli doiraviy plastina, markazga qo'yilgan *P* kuch ta'sirida turibdi. Plastina markazidan *x* masofadagi egilish quyidagi formula bilan ifodalanadi: $y = Pkx^2 \ln\frac{x}{a} + P\frac{k}{2}(a^2 - x^2),$ bu yerda *k* – plastina tuzilishi va materialining mustahkamlik xarakteristikalari bilan bogliq koeffitsient. Plastina markazidagi egilishni toping. (Javob: $Pka^2/2.$)

20. Tekis taqsimlangan *q* kuchlanish va siquvchi *N* kuch ta'siridagi sharnirli- tayanchli to'sin egilmoqda. To'sin o'rtasidagi egilish quyidagi formula bilan aniqlanadi:

$$f = \frac{ql^4}{EIu^4}\left(\frac{1}{\cos(u/2)} - 1 - \frac{u^2}{8}\right),$$

bu yerda, $u = l\sqrt{\frac{N}{EI}}$; EI - to'sin qattiqligi, *l*- to'sin uzunligi.

Quyidagilarni ko'rsating:

a) $u \to 0$ $(EI \to \infty)$ da to'sin egilmasligini, ya'ni $f \to 0$;

b) $u \to \pi$ $(N \to \pi^2 EI/l)$ da $f \to \infty$, ya'ni shunday kritik kuch borki, bunda to'sin "sinadi", bu holat matematik jihatdan uning cheksiz egilishiga mos keladi.

6. BIR O'ZGARUVCHILI FUNKSIYANING DIFFERENSIAL HISOBI VA UNING TATBIQLARI.

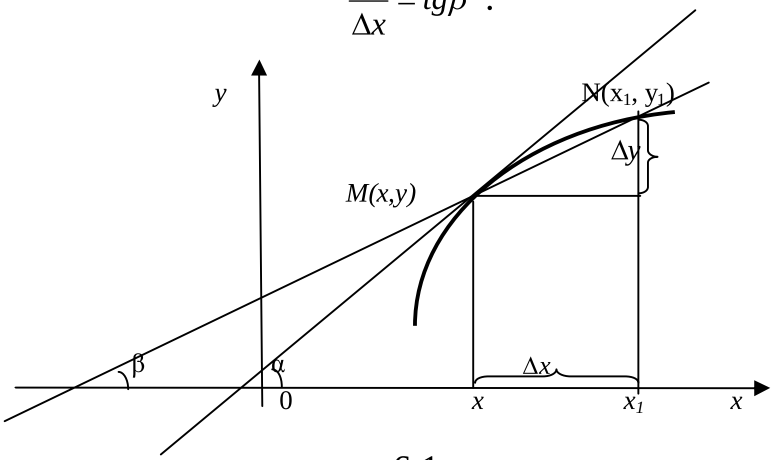
6.1. HOSILA, UNING GEOMETRIK VA FIZIK MA'NOSI. DIFFERENSIALLASH QOIDALARI VA FORMULALARI.

Eslatib o'tamizki, $y = f(x)$ funksiyaning *orttirmasi* deb

$$\Delta y = f(x + \Delta x) - f(x)$$

ayirmaga aytiladi, bu yerda Δx argument x ning orttirmasi. 6.1 rasmdan ko'rinib turibdiki

$$\frac{\Delta y}{\Delta x} = tg\beta \quad . \qquad (6.1)$$



6.1. rasm

Funksiya orttirmasi Δy ning, argument orttirmasi Δx ga nisbatining Δx ixtiyoriy tarzda nolga intilgandagi limitiga, $y = f(x)$ *funksiyaning x nuqtadagi hosilasi* deyiladi va quyidagilarning biri orqali belgilanadi: $y', f'(x), \frac{dy}{dx}$. Shunday qilib, ta'rifga ko'ra

$$y' = f'(x) = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad . \qquad (6.2)$$

Agar yuqoridagi (6.2) formulada limit mavjud bo'lsa, $f(x)$ funksiya x *nuqtada differensiallanuvchi,* hosila olish amalini esa d*ifferensiallash* deyiladi.

(6.1) tenglik va hosila ta'rifidan, hosilaning x nuqtadagi qiymati $f(x)$ funksiya grafigiga $M(x,y)$ nuqtada o'tkazilgan urinmaning Ox o'qi musbat yo'nalishi bilan hosil qilgan burchagi tangensiga tengligi kelib chiqadi.

Osongina ko'rsatish mumkinki, fizik jihatdan qaraganda, $y' = f'(x)$ hosilaning qiymati x argumentga nisbatan funksiya o'zgarishining tezligiga teng.

Agar C- o'zgarmas son bo'lsa va $u = u(x), v = v(x)$ differensiallanuvchi funksiyalar bo'lsa, u holda quyidagi differensiallash formulalari o'rinli:

1) $C' = 0$;
2) $x' = 1$;

3) $(u \pm v)' = u' \pm v'$;

4) $(Cu)' = Cu'$;

5) $(uv)' = u'v + uv'$;

6) $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}, \quad v \neq 0$;

7) $\left(\frac{C}{v}\right)' = -\frac{Cv'}{v^2}, \quad v \neq 0$;

8) agar $y = f(u)$, $u = \varphi(x)$, ya'ni $y = f(\varphi(x))$ murakkab funksiya differensiallanuvchi funksiyalardan tuzilgan bo'lsa, u holda

$$y_x' = y_u' u_x' \text{ yoki } \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}.$$

9) agar $y = f(x)$ funksiyaning, $x = g(y)$ va $\frac{dg}{dy} = g'(y) \neq 0$ Differensiallanuvchi teskari funksiyasi mavjud bo'lsa, u holda

$$f'(x) = \frac{1}{g'(y)}.$$

Hosilaning ta'rifi va differensiallash qoidalariga asosan *asosiy elementar funksiyalarning hosilalari* jadvalini tuzish mumkin:

1) $(u^\alpha)' = \alpha u^{\alpha-1} u' \; (\alpha \in \mathbf{R})$;

2) $(a^u)' = a^u \cdot \ln a \cdot u'$;

3) $(e^u)' = e^u \cdot u'$;

4) $(\log_a u)' = \frac{u'}{u \cdot \ln a}$;

5) $(\ln u)' = \frac{u'}{u}$;

6) $(\sin u)' = \cos u \cdot u'$;

7) $(\cos u)' = -\sin u \cdot u'$;

8) $(tgu)' = \frac{u'}{\cos^2 u}$;

9) $(ctgu)' = -\frac{u'}{\sin^2 u}$;

10) $(\arcsin u)' = \frac{u'}{\sqrt{1-u^2}}$;

11) $(\arccos u)' = -\frac{u'}{\sqrt{1-u^2}}$;

12) $(arctgu)' = \frac{u'}{1+u^2}$;

13) $(arcctgu)' = -\frac{u'}{1+u^2}$;

14) $(shu)' = chu \cdot u'$;

15) $(chu)' = shu \cdot u'$;

16) $(thu)' = \frac{u'}{ch^2 u}$;

17) $(cthu)' = -\frac{u'}{sh^2 u}$.

$y = f(x)$ egri chiziqqa $M_0(x_0, f(x_0))$ nuqtada o'tkazilgan *urinma tenglamasi*

$$y - f(x_0) = f'(x_0)(x - x_0) \quad (f'(x_0) \neq 0).$$

$y = f(x)$ egri chiziqqa $M_0(x_0, f(x_0))$ nuqtada o'tkazilgan *normal tenglamasi*

$$y - f(x_0) = -\frac{1}{f'(x_0)}(x - x_0) \quad (f'(x_0) \neq 0).$$

$f'(x_0) = 0$ da normal tenglamasi $x = x_0$ ko'rinishda bo'ladi.

Ikki egri chiziqning, ular kesishish nuqtasida hosil qilgan burchagi deb, shu nuqtada ularga o'tkazilgan urinmalar orasidagi burchakka aytiladi.

1-misol. Hosila ta'rifidan foydalanib $y=\frac{2x}{3x+1}$ funksiyaning hosilasini toping.

► Ta'rifga asosan ixtiyoriy Δx da:

$$\Delta y=\frac{2(x+\Delta x)}{3(x+\Delta x)+1}-\frac{2x}{3x+1}=\frac{6x^2+6x\Delta x+2x+2\Delta x-6x^2-6x\Delta x-2x}{(3(x+\Delta x)+1)(3x+1)}=\frac{2\Delta x}{(3(x+\Delta x)+1)(3x+1)}$$

$\frac{\Delta y}{\Delta x}=\frac{2}{(3(x+\Delta x)+1)(3x+1)}$ ekanligidan $y'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{2}{(3(x+\Delta x)+1)(3x+1)}=\frac{2}{(3x+1)^2}$ kelib chiqadi.◄

2-misol. $y=|x|$ funksiyaning $x=0$ nuqtadagi hosilasi qiymatini toping.

► Argumentning $x=0$ nuqtadagi ixtiyoriy Δx orttirmasi uchun funksiya orttirmasi

$$\Delta y=|\Delta x|=\begin{cases}-\Delta x, агар.\Delta x<0\\ \Delta x, агар.\Delta x>0\end{cases}.$$

Hosilaning ta'rifiga ko'ra esa

$$y'=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\begin{cases}-1, агар.\Delta x<0\\ 1, агар.\Delta x>0\end{cases}.$$

Bu esa $y=|x|$ funksiya $x=0$ nuqtada hosilaga ega emasligini ko'rsatadi. Shuni ta'kidlash lozimki, funksiya $x=0$ nuqtada uzluksiz, chunki

$$\lim_{\Delta x\to 0}|\Delta y|=\lim_{\Delta x\to 0}|\Delta x|=0$$ ◄

Shunday qilib, funksiya biror nuqtada uzluksiz ekanligidan, uning Differensiallanuvchi ekanligi kelib chiqmaydi. Osongina ko'rsatish mumkinki, funksiyaning Differensiallanuvchi bo'lgan barcha nuqtalarida uzluksiz bo'ladi.

6.1-AT

1. Hosila ta'rifidan foydalanib $y=3x^3-2x^2+3x-1$ funksiyaning hosilasini toping.

2. $y=\sqrt[3]{x}$ funksiyani $x=0$ nuqtada uzluksiz va Differensiallanuvchi bo'lishini tekshiring.

3. Quyidagi funksiyalarning hosilalarini toping:

a) $y=5x^4-3\sqrt[7]{x^3}+\frac{7}{x^5}+4$; b) $y=x^3\sin x$;

c) $y=(x^4+1)/(x^4-1)$; dg) $y=(x^5+3x-1)^4$;

e) $y=\sqrt[3]{\left(\frac{x^3+1}{x^3-1}\right)^2}$.

4. $y=x^3+2x-2$ egri chiziq grafigiga $x_0=1$ abssissali nuqtada o'tkazilgan urinma va normal tenglamalarini tuzing. (Javob: $y-5x+4=0,\ 5y+x-6=0$.)

5. $y=x^2$ va $x^2+2y^2=3$ tenglamalar bilan berilgan chiziqlarning, qanday burchak ostida kesishishini toping. (Javob: $90^\circ, 90^\circ$.)

Mustaqil ish

1. 1. Quyidagi funksiyalarning hosilalarini toping:

a) $y=3x^3+5\sqrt[3]{x^5}-\frac{4}{x^3}$; b) $y=x^3\sin x\cdot\ln x$; v) $y=\sqrt{\frac{x^3+1}{x^3-1}}$;

2. $y=\ln(x^2-4x+4)$ egri chiziq grafigiga $x_0=1$ nuqtada o'tkazilgan urinma va normal tenglamalarini yozing. (Javob: $2x+y-2=0$; $x-2y-1=0$.)

2. 1. Hosila ta'rifidan foydalanib, $y=\frac{3x-1}{2x+3}$ funksiyaning hosilasini toping. (Javob: $y'={17}/{(2x+5)^2}$.)

2. Quyidagi funksiyalarning hosilalarini toping:

a) $y=7\sqrt[7]{x^5}-\frac{2}{x^4}+7x^6$; b) $y=(x^9+1)\cos 5x$; v) $y=\left(\frac{x^4+1}{x^4-1}\right)^3$;

3. 1. Quyidagi funksiyalarning hosilalarini toping:

a) $y=4\sqrt{x}+\frac{4}{\sqrt{x}}+3x^2$; b) $y=x^3 tgx\cdot e^{2x}$; v) $y=\frac{\sin^2 x}{x^3+1}$;

2. Moddiy nuqtaning *t* sekundda bosib o'tgan yo'li $s=\frac{t^4}{4}-\frac{t^3}{3}+2t+1$ bo'lsin (*s*-metrda). Bu nuqtaning $t=0;1;2$ sekundlardagi oniy tezligini toping. (Javob: 2 *m/s;* 2 *m/s;* 6 *m/s*.)

6.2- AT

Differensiallash qoidalari va formulalaridan foydalanib, berilgan funksiyalarning hosilalarini toping.

1. a) $y=x^3\sin 3x$; b) $y=e^x tg4x$;
v) $y=\sqrt[3]{x^4+\sin^4 x}$; g) $y=xctg^2 7x$;
d) $y=2^{-\cos^4 5x}$; e) $y=e^{arctg\sqrt{x}}$.

2. a) $y=\left(2^{x^4}-tg^4x\right)^3$; b) $y=\ln^5\left(x-2^{-x}\right)$;
v) $y=\sin\left(tg\sqrt{x}\right)$; g) $y=x\sin^2 x\cdot 2^{x^2}$;
d) $y=2^{\frac{x}{\ln x}}$; e) $y=arctg\sqrt{1+x^2}$.

3. a) $y=e^{-\sqrt{x^2+2x+2}}$; b) $y=sh^3x^2$;
v) $y=\left(2^{tg3x}+tg3x\right)^2$; g) $y=3^{tg^2 5x}$.

Mustaqil ish

Quyidagi funksiyalarning hosilalarini toping.

1. a) $y=x\sin^3 x$; b) $y=\sqrt{\frac{\cos^2 x+1}{\sin 2x+1}}$;

v) $y=\left(2^{\cos 3x}+\sin 3x\right)^3$; g) $y=x\cos^2 x\cdot e^{x^2}$.

2. a) $y=x^3e^{tg3x}$ y; b) $y=\left(\sin^3 x+\cos^3 2x\right)^2$;

v) $y=\ln(x^4-\sin^3 x)$; g) $y=x\sin 7x\cdot tg^2 x$.

3. a) $y=x\cdot ctg^2 5x$; b) $y=\left(x^3+tg^3 2x\right)^2$;

v) $y=\sin(x^5-tg^2 x)$; g) $y=x^3\cos 3x\cdot e^{-x^2}$.

6.2 LOGARIFMIK DIFFERENSIALLASH

$y=f(x)$ funksiyaning *logarifmik hosilasi* deb, bu funksiyaning logarifmidan olingan hosilaga aytiladi, ya'ni

$$\left(\ln f(x)\right)'=\frac{f'(x)}{f(x)}$$

Funksiyani logarifmlash va Differensiallashning ketma- ket qo'llanilishi *logarifmik Differensiallash* deb ataladi. Ba'zi hollarda avval funksiyani logarifmlash, uning hosilasini topishni osonlashtiradi. Masalan, $y=u^v$, (bu yyerda $u=u(x), v=v(x)$) funksiyaning hosilasini topishda, avval logarifmlash quyidagi formulaga olib keladi:

$$y'=u^v\ln u\cdot v'+vu^{v-1}\cdot u'.$$

1-misol. $y=(\sin 2x)^{x^3}$ funksiyaning hosilasini toping.

► Berilgan funksiyani logarifmlasak,

$$\ln y=x^3\ln\sin 2x.$$

Tenglikning ikkala tomonini *x* bo'yicha Differensiallab

$$(\ln y)'=(x^3)'\cdot\ln\sin 2x+x^3(\ln\sin 2x)'$$

ni hosil qilamiz. Bu yyerdan

$$\frac{y'}{y}=3x^2\cdot\ln\sin 2x+x^3\frac{1}{\sin 2x}2\cos 2x$$

yoki

$$y'=y(3x^2\cdot\ln\sin 2x+2x^3ctg2x),$$

$$y'=\left(\sin 2x\right)^{x^3}(3x^2\cdot\ln\sin 2x+2x^3ctg2x). \blacktriangleleft$$

Agar *y* va *x* o'zgaruvchilar orasidagi bog'lanish oshkormas ko'rinishda bo'lib, $F(x,y)=0$ tenglama bilan berilgan bo'lsa, u holda $y'=y'_x$ hosilani topish uchun eng sodda hollarda *y* ni *x* ning funksiyasi deb qarab $F(x,y)=0$ tenglamaning ikkala tomonini Differensiallab, hosil bo'lgan chiziqli tenglamadan y' ni topish mumkin.

2-misol. Agar $x^3+y^3-3xy=0$ bo'lsa, y' toping.

► *y* ni *x* ning funksiyasi deb hisoblab, berilgan tenglamaning ikkala tomonini Differensiallaymiz:

$$3x^2+3y^2y'-3y-3xy'=0.$$

Bu yerdan

$$y'=\frac{3x^2-3y}{3x-3y^2}. \blacktriangleleft$$

6.3-AT

1. Berilgan funksiyalarning hosilasini toping:

a) $y = 3^{x^2} - tg^4 2x$;   b) $y = x^3 th^3 x$;

v) $y = \lg^4(x^5 - \sin^5 2x)$;   g) $y = arctg\sqrt{1+e^{-x}}$.

2. Quyidagi funksiyalarning hosilasini toping:

a) $y = (\sin 3x)^{\cos 5x}$;   b) $y = (x^3+1)^{tg 2x}$.

3. Quyidagi oshkormas tenglamalari bilan berilgan funksiyalarning hosilalarini toping:

a) $e^{xy} - x^3 - y^3 = 3$;   b) $xy - arctg\dfrac{x}{y} = 3$;   v) $\sqrt[3]{x} + \sqrt[3]{y} = a$.

(Javob: a) $y' = \dfrac{3x^2 - e^{xy}y}{-3y^2 + e^{xy}x}$; b) $y' = -\dfrac{x^2y + y^3 - x}{x^3 + xy^2 + y}$; v) $y' = -\sqrt[3]{\left(\dfrac{y}{x}\right)^2}$.)

Mustaqil ish

Berilgan funksiyalarning hosilalarini toping.

1. a) $y = x^3\ln^2(\sin^2 x - tg^2 x)$;   b) $y = (tg 3x)^{x^4}$;

v) $e^{x^2y^2} - x^4 + y^4 = 5$ (Javob: $y' = -\dfrac{e^{x^2y^2}\cdot 2xy^2 - 4x^3}{4y^3 + e^{x^2y^2}\cdot 2x^2y}$.)

2. a) $y = ctg^2 3x \cdot e^{-\cos^2 3x}$;   b) $y = (1+x^4)^{tg 7x}$;

v) $y^2 + x^2 - \sin(x^2y^2) = 5$. (Javob: $y' = \dfrac{2xy^2\cdot\cos(x^2y^2) - 2x}{2y - 2yx^2\cos(x^2y^2)}$.)

3. a) $y = e^{-x^2} arctg\sqrt{x^3 - 1}$;   b) $y = (ctg 5x)^{x^3-1}$;

v) $2^x + 2^y = 2^{x+y}$.   (Javob: $y' = -\dfrac{2^x - 2^{x+y}}{2^y - 2^{x+y}}$.)

6.3. YUQORI TARTIBLI HOSILALAR

Biror u=f(x) funksiyaning ikkinchi tartibli hosilasi deb, uning birinchi tartibli hosilasidan olingan hosilaga aytiladi, ya’ni $(y')'$ va uni y'' va $f''(x)$ yoki $\dfrac{d^2y}{dx^2}$ kabi belgilarning biri orqali yoziladi. Agar moddiy nuqtaning to’g’ri chiziqli harakati $S=S(t)$ funksiya bilan berilgan bo’lsa, $S'=S'(t)$ va $S''= S''(t)$ lar mos ravishda uning tezligi va tezlanishini ifodalaydi.

Agar u funksiyaning x argumentga bo’lgan bog’lanishi $x=x(t)$, $u=u(t)$ parametrik tenglamalar bilan berilgan bo’lsa, u holda:

$$\frac{dy}{dx} = \frac{y'(t)}{x'(t)}, \qquad \frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{y'(t)}{x'(t)}\right)\cdot\frac{1}{x'(t)}, \qquad (6.3)$$

bu yerda shtrix t bo’yicha hosilani bildiradi.

u=*f*(*x*) *funksiyaning n-tartibli hosilasi* deb uning (*n-1*)- tartibli hosilasidan olingan hosilaga aytiladi. Uni belgilash uchun quyidagi belgilarning biri ishlatiladi: $y^{(n)}$, $f^{(n)}(x)$, $\frac{d^n y}{dx^n}$. Demak, $y^{(n)}=(y^{(n-1)})'$ ekan.

1-misol. $y=\ln\left(x+\sqrt{x^2+a^2}\right)$ funksiyaning 2- tartibli hosilasi hisoblansin.

►

$$y'=\frac{\left(x+\sqrt{x^2+a^2}\right)'}{x+\sqrt{x^2+a^2}}=\frac{1}{x+\sqrt{x^2+a^2}}\cdot\left(1+\frac{x}{\sqrt{x^2+a^2}}\right)=\frac{\sqrt{x^2+a^2}+x}{\left(x+\sqrt{x^2+a^2}\right)\cdot\sqrt{x^2+a^2}}=\frac{1}{\sqrt{x^2+a^2}},$$

$$y''=-\frac{1}{2}\left(x^2+a^2\right)^{\frac{-3}{2}}\cdot 2x=-\frac{x}{\sqrt{\left(x^2+a^2\right)^3}}.$$ ◄

2-misol. *y*=$(2x-1)^4$ funksiya uchun *y*'(1) va *y*'(-1) hamda *y*''(1), *y*''(-1) hisoblansin.

►*y*'=$8(2x-1)^3$ bo'lganligidan *y*'(1)=8, *y*'(-1)=-216, *y*''=$48(2x-1)^2$ dan esa, *y*''=(+1)=48 va *y*''=(-1)=432. ◄

3-misol. $y=\sin x$ funksiyaning *n*- tartibli hosilasi hisoblansin.

$$► \; y'=\cos x=\sin\left(x+\frac{\pi}{2}\right),$$

$$y''=\cos\left(x+\frac{\pi}{2}\right)=\sin\left(x+2\cdot\frac{\pi}{2}\right),$$

$$y'''=\cos\left(x+2\cdot\frac{\pi}{2}\right)=\sin\left(x+3\cdot\frac{\pi}{2}\right),$$

....

$$y^{(n)}=\cos\left(x+(n-1)\frac{\pi}{2}\right)=\sin\left(x+n\frac{\pi}{2}\right).$$ ◄

4- misol *x*=*lnt* va *y*=t^3+2t+1 parametrik tenglamalar bilan berilgan funksiyaning 2- tartibli hosilasi hisoblansin.

► (6.3) formulaga binoan, $\frac{dy}{dx}=\frac{3t^2+2}{1/t}=3t^3+2t,\quad \frac{d^2y}{dx^2}=\frac{9t^2+2}{1/t}=9t^3+2t.$ ◄

6.4- AT

1. *y*=$(1+4x^2)$*arctg*2*x* funksiyaning 2-tartibli hosilasi hisoblansin.

2. *y*=x^3-5x^2+7x-2 funksiyaning ixtiyoriy tartibli hosilasining *x*=2 nuqtadagi qiymati hisoblansin.

3. Agar nuqtaning *Ox* o'qi bo'ylab harakat tenglamasi $x=100-5t-0{,}001t^3$ (*x* metrlarda va *t* esa sekundlarda o'lchanadi) bo'lsa, uning *v* tezligi bilan *w* tezlanishining $t_0=0$, $t_1=1$, $t_2=10$ sekundlardagi qiymatlari aniqlansin. (Javob: *v*=5; 4,997; 4,7 m/s, *w*=0; -0,006; -0,06m/sek^2)

4. Quyidagi tenglamalari bilan berilgan funksiyalarning ikkinchi tartibli hosilalari hisoblansin:

a) $\begin{cases} y = t^3 + t^2 - 1, \\ x = t^2 + t + 1; \end{cases}$ b) $\begin{cases} y = 2\sin^3 t, \\ x = 2\cos^3 t; \end{cases}$

5. x^4-xy+y^4-1=0 tenglama bilan berilgan *u* funksiyaning *M*(0;1) nuqtadagi 2-tartibli hosilasining qiymati hisoblansin. (Javob: -1/16)

6. $x = \frac{1+t}{t^3}, \quad y = \frac{3}{2t^2} + \frac{1}{2t}$ tenglama bilan berilgan egri chiziqning M_0(2;2) nuqtasidagi urinma va normalining tenglamasi yozilsin. (Javob: 7*x*-10*y*+6=0, 10*x*+7*y*-34=0.)

7. $u=C_1e^{2x}+C_2\ e^{3x}$ funksiya C_1 va C_2 o'zgarmaslarning har qanday qiymatlarida ham y''-5y'+6y=0 tenglamani qanoatlantirishligi ko'rsatilsin.

Mustaqil ish

1. 1) $y=(x^2+1)\ln(1+x^2)$ funksiyaning 2- tartibli hosilasi topilsin;

2) $y=t^3+t$, $x=t^2-2t$ berilgan funksiyaning 2- tartibli hosilasi topilsin;

3) $e^y+y-x=0$ bo'lsa, y'' ning M_0(1;0) nuqtadagi qiymati hisoblansin; (Javob: -1/8)

2. 1) $y=e^{-3x}(cos2x+sin2x)$ funksiyaning 2- tartibli hosilasi topilsin;

2) $y=t^3+t^2+1$, $x=1/t$ berilgan funksiyaning 2- tartibli hosilasi topilsin;

3) $x^3+y^3-xy=1$ bo'lsa, y'' ning M_0(1;1) nuqtadagi qiymati aniqlansin; (Javob: -7.)

3. 1) $y = \sqrt{1-4x^2} \cdot \arcsin 2x$ funksiyaning 2- tartibli hosilasi topilsin;

2) $y=(2t+1)cost$, $x=lnt$ tenglamalari bilan berilgan funksiyaning 2- tartibli hosilasi hisoblansin;

3) $x^2+2y^2-xy+x+y=4$ tenglama bilan berilgan *u* funksiyaning 2-tartibli hosilasining M_1(1;1) nuqtadagi qiymati hisoblansin; (Javob: -1)

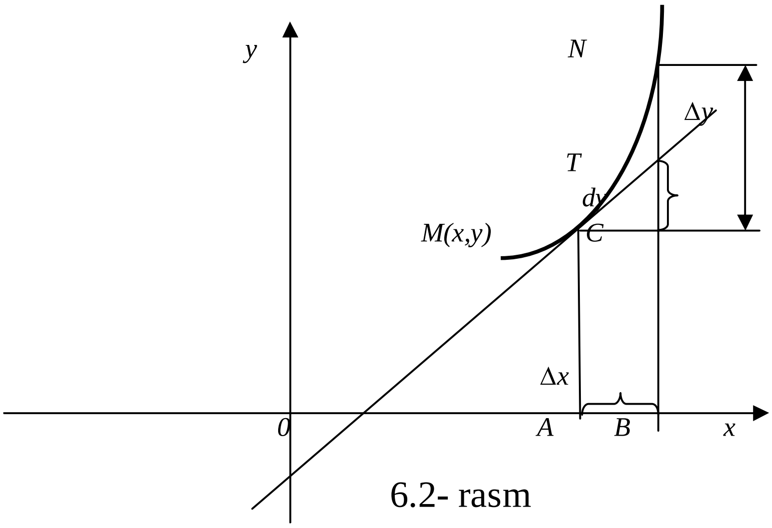
6.4. BIRINCHI VA YUQORI TARTIBLI DIFFERENSIALLAR HAMDA ULARNING TATBIQLARI.

Biror $y=f(x)$ *funksiyaning ixtiyoriy x nuqtadagi 1-tartibli differensiali* deb, uning shu nuqtadagi orttirmasining, $\Delta x=dx$ orttirmaga nisbatan chiziqli bo'lgan qismiga aytiladi. Umuman, funksiyaning differensiali *du* ni hisoblash uchun uning *x* nuqtadagi hosilasini *dx* ga ko'paytiriladi. $dy=y'dx=f'(x)dx$. Shu boisdan, $y' = \frac{dy}{dx}$ tenglik har doim o'rinlidir. 6.2- rasmdan ko'rinib turibdiki, agar *MN*, $u=f(x)$ funksiya grafiginining yoyi bo'lib, *MT* esa, uning *M*(*x*,*y*) nuqtasiga o'tkazilgan urinmasi hamda $AB=\Delta x=dx$ kabi bo'ladigan bo'lsa, u holda C$T=dy$ va C$N=\Delta y$ kabi bo'ladi. Funksiyaning *dy* Differensiali o'zining Δy orttirmasidan, Δx ga nisbatan yuqori tartibdagi cheksiz kichik miqdorgagina farq qiladi.

Differensialning ta'rifidan hamda hosilani hisoblash qoidalariga ko'ra, $u=u(x)$ va $v=v(x)$ funksiyalar uchun quyidagilarni yozish mumkin:

1) $dC=0$ (C=const.);

2) $dx=\Delta x$ (bu yerda x erkli o'zgaruvchi);
3) $d(u\pm v)=du\pm dv$;
4) $d(uv)=udv+vdu$;
5) $d(Cu)=Cdu$;
6) $d(u/v)=(vdu-udv)/v^2$ $(v\neq 0)$;
7) $df(u)=f_u'(u)\,u'dx=f'(u)du$.



6.2- rasm

1-misol. $y=sin^5 3x$ ning Differensiali hisoblansin.

► Funksiyaning hosilasi $y'=5sin^4 3x\cdot sos3x\cdot 3$ bo'lganligidan, quyidagini hosil qilamiz $dy=15sin^4 3x\cdot sos3xdx$.◄

y=f(x) funksiyaning n-tartibli Differensiali deb, uning (n-1)- tartibli Differensialidan hisoblangan Differensialiga aytiladi, ya'ni, $d^n y=d(d^{n-1}y)$. Ta'rifga binoan $d^2y=y''dx^2$, $d^3y=y'''dx^3,\ldots,d^ny=y^{(n)}dx^n$ bo'ladi. $(dx^n=(dx)^n)$.

Agar $y=f(u)$ va $u=\varphi(x)$ bo'lsa, u holda: $d^2y=y''(du)^2+y'\,d^2u$, bu yerda, Differensiallash u o'zgaruvchiga nisbatan bajariladi.

2-misol.$y=ln(1+x^2)$ funksiya uchun d^2y hisoblansin.

► $y'=\dfrac{2x}{1+x^2}$ va $y''=\dfrac{2(1-x^2)}{(1+x^2)^2}$ bo'lganligidan, $d^2y=\dfrac{2(1-x^2)}{(1+x^2)^2}dx^2$.◄

Yuqorida ta'kidlaganidek, $\Delta y\approx dy$ yoki $f(x+\Delta x)-f(x)\approx f'(x)dx$ deb yozish mumkin, bundan esa, $f(x+\Delta x)\approx f(x)+f'(x)dx$ ni hosil qilamiz.

Ushbu formula, ko'pincha, $f(x)$ funksiyaning qiymatini taqribiy hisoblash uchun qo'llaniladi (bu yerdagi Δx argument x ning kichik orttirmasidir).

3-misol. Agar kubning hajmi 27 birlikdan 27,1m^3 gacha ortgan bo'lsa, uning tomonlarining orttirmasi hisoblansin.

►Agar kubning hajmini x deb belgilasak, u holda uning tomonlari $y=\sqrt[3]{x}$ ga teng bo'ladi. Masalaning shartiga ko'ra, x=27, Δx =0,1 bo'lganligi uchun kub tomonlarining orttirmasi $\Delta y\approx dy=y'(x)\cdot\Delta x=\dfrac{1}{3\sqrt[3]{27^2}}\cdot 0{,}1=\dfrac{0{,}1}{27}\approx 0{,}0037$ ga teng bo'ladi.◄

4-misol. $sin31^\circ$ taqriban hisoblansin.

► Agar $x=\frac{\pi}{6}$ desak, u holda, $\Delta x = 1^\circ \cdot \frac{\pi}{180^\circ} = 0.017.$

$Sin31^\circ \approx \sin\frac{\pi}{6} + \cos\frac{\pi}{6} \cdot 0{,}017 = 0{,}5 + 0{,}017 \cdot \frac{\sqrt{3}}{2} \approx 0{,}515.$ ◄

Agar funksiya argumentining mutloq ε_x xatoligi aniq bo'lsa, uning ε_y mutloq xatoligini funksiyaning Differensiali yordamida aniqlash mumkin bo'ladi. Amaliy masalalarda, argumentning qiymatlarini o'lchashlar yordamida aniqlanib, uning mutloq xatoligini aniq deb hisoblanadi.

Aytaylik, $y=f(x)$ funksiya argumenti x ning biror qiymatiga ko'ra, funksiyaning qiymatini hisoblash lozim bo'lsin. Bu yerda, x ning aniq qiymati oldindan noma'lum ekanligini, lekin uning x_0 taqribiy qiymati, mutloq xatoligi ε_x bilan berilgan bo'lsin deb faraz qilamiz:

$$x=x_0+dx, \quad |dx| \le \varepsilon_x$$

U holda:

$$|f(x) - f(x_0)| \approx |f'(x_0)| \cdot |dx| < |f'(x_0)| \cdot \varepsilon_x$$

Demak, bundan ko'rinmoqdaki, $\varepsilon_y = |f'(x_0)| \cdot \varepsilon_x$. Funksiyaning nisbiy xatoligi quyidagi formula orqali ifodalanadi:

$$\delta_y = \frac{\varepsilon_x}{|f'(x_0)|} = \left|\frac{f'(x_0)}{f(x_0)}\right| \cdot \varepsilon_x = \left|(\ln f(x_0))'\right| \cdot \varepsilon_x$$

Masalan, 4-misolda ε_x=0,017 deb olinsa, u holda,

$$\varepsilon_y = \left|\cos\frac{\pi}{6}\right| \cdot 0{,}017 = 0{,}015, \quad \delta_y = \frac{0{,}015}{0{,}5} \cdot 100\% = 3\%.$$

6.5- AT

1. $y=x^3-2x^2+2$ funksiya bilan $x_0=1$ nuqta berilgan. Erkli o'zgaruvchi x ning har qanday Δx orttirmasi uchun funksiya orttirmasining bosh qismi ajratilib, funksiya orttirmasi bilan Differensiali orasidagi ayirmaning berilgan nuqtadagi mutloq qiymati aniqlansin. Agar: a) Δx=0,1; b) Δx=0,01 bo'lsa; ushbu ayirmani funksiya Differensialining mutloq qiymati bilan taqqoslansin. (Javob: a) $\varepsilon = |\Delta y - dy| = 0{,}011$, $\varepsilon / |dy| \cdot 100\% = 11\%$; b) ε=0,000101, $\varepsilon / |dy| \cdot 100\%$=1,01%)

2. Quyidagi funksiyalarning 1- tartibli Differensiallari hisoblansin:

a) $y = xtg^3 x$; b) $y = \sqrt{arctgx} + (ar\sin x)^2$ v) $y = \ln(x + \sqrt{4 + x^2})$

3. $y - e^{-x^2}$ funksiyaning ikkinchi tartibli Differensiali hisoblansin.

4. a) $y=sin^2 2x$; b) $y = \frac{\ln x}{x}$ funksiyalarning uchinchi tartibli Differensiallari hisoblansin.

5. $y=x^3-4x^2+5x+3$ funksiyaning x=1,03 dagi taqribiy qiymatini vyerguldan keyin ikki xonali son aniqlik bilan hisoblansin. (Javob: 5,00)

6. $\sqrt[4]{17}$ ning vyerguldan keyin ikki xonali son aniqlikdagi taqribiy qiymati aniqlansin. (Javob: 2,03)

Mustaqil ish

1. 1) $y=x^3 lnx$ funksiyaning birinchi, ikkinchi va uchinchi tartibli Differensiallari hisoblansin;

2) $y=\sqrt[3]{\frac{1-x}{1+x}}$ funksiyaning x=0,1 dagi taqribiy qiymatini 0,01 aniqlikda hisoblansin. (Javob: 1,03)

2. 1) $y=(x^2+1)arctgx$ funksiyaning birinchi va ikkinchi tartibli Differensiallari hisoblansin;

2) $y=\sqrt{x^2-7x+10}$ funksiyaning x=0,98 dagi taqribiy qiymatini 0,01 aniqlikda hisoblansin. (Javob: 2,09)

3. 1) $y=e^{-3x}sos2x$ funksiya uchun d^2y bilan d^3y lar hisoblansin;

2) $y=\sqrt[3]{x^2-5x+12}$ funksiyaning x=1,3 qiymatdagi taqribiy qiymati 0,01 aniqlik bilan hisoblansin. (Javob*:* 2,08)

6.5. O'rta qiymat haqidagi teoremalar. Lopital-Bernulli qoidalari.

1-teorema (Roll). *Agar u=f(x) funksiya,* [*a;b*] *kesmada uzluksiz bo'lib, kesmaning ichidagi barcha nuqtalarda Differensiallanuvchi hamda f(a)=f(b) bo'lsa, u holda hech bo'lmaganda shunday bir x=s nuqta (a<s<b) mavjudki, har doim uning uchun f '(c)=0 shart o'rinli bo'ladi*.

2- teorema (Lagranj). *Agar u=f(x) funksiya,* [*a;b*] *kesmada uzluksiz bo'lib, kesmaning ichidagi barcha nuqtalarda Differensiallanuvchi bo'lsa, u holda hech bo'lmaganda shunday bir x=s nuqta (a<s<b) mavjud bo'ladiki, har doim uning uchun f(b)-f(a)=f'(c)(b-a) formula o'rinli bo'ladi*.

Ushbu formulani *chekli orttirmalar haqidagi Lagranj formulasi* deb yuritiladi.

3- teorema (Koshi). *Agar u=f(x) bilan u=φ(x) funksiyalar,* [*a; b*] *kesmada uzluksiz hamda shu kesma ichida Differensiallanuvchi (φ'(x)≠0) bo'lsalar, u holda hech bo'lmaganda shunday bir x=s nuqta mavjud bo'ladiki (a<s<v), har doim quyidagi tenglik o'rinli bo'ladi:*

$$\frac{f(b)-f(a)}{\phi(b)-\phi(a)}=\frac{f'(c)}{\phi'(c)},\ \varphi'(c)\neq 0$$

Lopital qoidasi ($\frac{0}{0}$ ***va*** $\frac{\infty}{\infty}$ ***kabi aniqmasliklarni ochish).*** *Agar u=f(x) bilan u=φ(x) funksiyalar, biror* $x=x_0$ *nuqta atrofida Koshi teoremasining shartlarini qanoatlantirib,* $x\to x_0$ *da 0 ga yoki ±∞ ga intiladigan bo'lsalar hamda* $\lim\limits_{x\to x_0}\frac{f'(x)}{\varphi'(x)}$ *chekli limit mavjud bo'lsa, u holda* $\lim\limits_{x\to x_0}\frac{f(x)}{\varphi(x)}$ *chekli limit mavjud bo'ladi va bu limitlar o'zaro teng bo'ladilar, ya'ni:* $\lim\limits_{x\to x_0}\frac{f(x)}{\varphi(x)}=\lim\limits_{x\to x_0}\frac{f'(x)}{\varphi'(x)}$.

Ushbu qoida $x_0=\pm\infty$ bo'lganda ham o'rinli bo'lib qolavyeradi.

Agar $\frac{f'(x)}{\varphi'(x)}$ nisbat uchun qaralayotgan nuqtada yana yuqoridagi aniqmasliklar yuz byeradigan bo'lsa, u holda, agar *f'(x)* va *φ'(x)* lar uchun *f(x)* bilan *φ(x)* larga nisbatan yuqorida keltirilgan shartlar o'rinli bo'ladigan bo'lsa, ikkinchi tartibli hosilalarning nisbati uchun ham limitga o'tish mumkin va hokazo. Ta'kidlash lozimki, garchi funksiyalar nisbatining limiti mavjud bo'lsada, ammo hosilalar nisbatining hech qanday limiti mavjud bo'lmasligi ham mumkin.

1-misol. $\lim_{x\to\infty}\frac{x+\sin x}{x+\cos x}$ ni hisoblansin.

► $\lim_{x\to\infty}\frac{x+\sin x}{x+\cos x}=\lim_{x\to\infty}\frac{1+\frac{\sin x}{x}}{1+\frac{\cos x}{x}}=1.$

Ammo, $\lim_{x\to\infty}\frac{(x+\sin x)'}{(x+\cos x)'}=\lim_{x\to\infty}\frac{1+\cos x}{1-\sin x}$ ning hech qanaqa limiti mavjud emas, chunki, x→∞ da, kasrning surat hamda maxraji [0;2] kesmadagi har qanday qiymatini qabul qilish mumkin, hosilalarning nisbati esa, ixtiyoriy manfiy bo'lmagan qiymatlarni qabul qiladi. Demak, bu holatda Lopital qoidasini qo'llash mumkin emas.◄

2-misol. $\lim_{x\to 0}\frac{e^{3x}-1}{\sin 5x}$ ni hisoblansin.

►Bu yerda Lopital qoidasining barcha shartlari bajarilmoqda. Shuning uchun,

$$\lim_{x\to 0}\frac{e^{3x}-1}{\sin 5x}=\lim_{x\to 0}\frac{3e^{3x}}{5\cos 5x}=\frac{3}{5}.◄$$

Agar 0·∞ yoki kabi aniqmaslikni ochish lozim bo'lsa, uni yoki $\frac{0}{0}$ yoki $\frac{\infty}{\infty}$ ga keltirib, so'ngra Lopital qoidasi qo'llaniladi.

3-misol. $\lim_{x\to\infty}x^3e^{-x}$ ni hisoblang.

►Bu yerda 0·∞ kabi aniqmaslikni ochishga to'g'ri keladi, uni quyidagicha o'zgartiramiz:

$$\lim_{x\to\infty}x^3e^{-x}=\lim_{x\to\infty}\frac{x^3}{e^x}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{3x^2}{e^x}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{6x}{e^x}=\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{6}{e^x}=\frac{6}{e^\infty}=0.◄$$

Agar $\lim_{x\to x_0}f_1(x)=\infty$ va $\lim_{x\to x_0}f_2(x)=\infty$ bo'lsa, u holda $f_1(x)$-$f_2(x)$ ning limiti ∞-∞ aniqmaslikka olib keladi. Ammo, $f_1(x)-f_2(x)=f_1(x)\left(1-\frac{f_2(x)}{f_1(x)}\right)$ deb yozsak, hamda $\lim_{x\to x_0}\frac{f_2(x)}{f_1(x)}=1$ bo'lsa, 0·∞ kabi aniqmaslikka kelamiz.

Endi $f(x)^{\varphi(x)}$ kabi funksiyani qaraymiz.

1. Agar $\lim_{x\to x_0}f(x)=0$, $\lim_{x\to x_0}\varphi(x)=0$ bo'lsa, 0^0 aniqmaslik hosil bo'ladi.

2. Agar $\lim_{x\to x_0}f(x)=1$, $\lim_{x\to x_0}\varphi(x)=0$ bo'lsa, 1^∞ aniqmaslikka ega bo'lamiz.

3. Agar $\lim\limits_{x\to x_0} f(x)=\infty$, $\lim\limits_{x\to x_0}\varphi(x)=0$ bo'lsa, ∞^0 aniqmaslik hosil bo'ladi.

Bu xildagi aniqmasliklarni ochib chiqish uchun logarifmlash usuli deb ataluvchi usuldan foydalanamiz, ya'ni faraz qilaylik, $\lim\limits_{x\to x_0} f(x)^{\varphi(x)}=A$ bo'lsin. Logarifimik funksiyaning uzluksiz ekanligidan, $\lim\limits_{x\to x_0}\ln y=\ln\left(\lim\limits_{x\to x_0} y\right)$ ni yozish mumkin. U holda, $\ln A=\lim\limits_{x\to x_0}(\varphi(x)\cdot\ln f(x))$. Natijada, yuqoridagi aniqmasliklarning barchasi $0\cdot\infty$ ga keltiriladi.

4-misol. $\lim\limits_{x\to 0}\left(e^x+x\right)^{\frac{1}{x}}$ ni hisoblansin.

► Hisoblanishi lozim bo'lgan limitni *A* deb belgilaymiz. U holda:

$\ln A=\lim\limits_{x\to 0}\left(\frac{1}{x}\cdot\ln(e^x+x)\right)=\lim\limits_{x\to 0}\frac{\ln(e^x+x)}{x}=\lim\limits_{x\to 0}\frac{e^x+1}{e^x+x}=2$. Demak $A=e^2$.◄

6.6-AT

1. $f(x)=x-x^3$ funksiyaning [-1;0] va [0;1] kesmalarda Roll teoremasining shartlarini qanoatlantirishi ko'rsatilsin va unga mos *x*=*s* ning qiymati aniqlansin. (Javob: $C=\pm\frac{1}{\sqrt{3}}$.)

2. $y=x^2$ parabolaning *A*(1;1) va *V*(3;9) nuqtalar orasidagi yoyida shunday bir nuqta topilsinki, unga o'tkazilgan urinma *AV* vatarga parallel bo'lsin, (Javob: (2;4) .)

3. Limitlar hisoblansin:

a) $\lim\limits_{x\to 1}\frac{x^3-7x^2+4x+2}{x^3-5x+4}$; b) $\lim\limits_{x\to 0}\frac{x\cos x-\sin x}{x^3}$; v) $\lim\limits_{x\to 0}\frac{e^{7x}-1}{tg3x}$;

g) $\lim\limits_{x\to 1}\left(\frac{x}{x-1}-\frac{1}{\ln x}\right)$; d) $\lim\limits_{x\to a}\left(2-\frac{x}{a}\right)^{\frac{tg\pi x}{2a}}$;

e) $\lim\limits_{x\to\frac{\pi}{2}}(tgx)^{2x-\pi}$; j) $\lim\limits_{x\to\infty}\left(\frac{2}{x}+1\right)^{x}$.

(Javob: a) 7/2; b) -1/3; v) 7/3; g) ½; d) $e^{2/\pi}$; e) 1; j) e^2.)

Mustaqil ish

Ko'rsatilgan limitlar hisoblansin:

1. 1. a) $\lim\limits_{x\to 0}\frac{1-\cos 7x}{x\sin 7x}$; b) $\lim\limits_{x\to 0}(\cos 2x)^{1/x^2}$ (Javob: a) 7/2; b) e^{-2}.)

2. 2. a) $\lim\limits_{x\to 2}\frac{ctg(\pi x/4)}{x-2}$ b) $\lim\limits_{x\to 0}\left(\frac{1}{x}\right)^{\sin x}$ (Javob: a) $-\frac{\pi}{4}$; b) 1.)

3. 3. a) $\lim\limits_{x\to\infty}\left(x\sin\frac{3}{x}\right)$ b) $\lim\limits_{x\to 1}x^{1/(1-x)}$. (Javob: a) 3; b) e^{-1}.)

6.6. HOSILANING FUNKSIYA VA UNING GRAFIGINI TEKSHIRISHGA QO'LLANILISHI

Differensial hisobning muhim amaliy masalalaridan biri funksiya o'zgarishini tekshirishning umumiy usullarini ishlab chiqishdir.

Biror X oraliqda aniqlangan $y=f(x)$ funksiya argument x ning shu oraliqdagi $x_1<x_2$ shartni qanoatlantiruvchi qiymatlari uchun $f(x_1)<f(x_2)$ (yoki $f(x_1)>f(x_2)$) tengsizlikni qanoatlantirsa, u holda, funksiyani shu oraliqda o'suvchi (yoki kamayuvchi) deb ataladi.

Funksiyaning o'sishi (*kamayishi*) belgilarini sanab o'tamiz:

1. Agar $[a;b]$ kesmada Differensiallanuvchi bo'lgan $y=f(x)$ funksiya o'suvchi (yoki kamayuvchi) bo'lsa, u holda kesmaning barcha nuqtalarida funksiyaning hosilasi manfiy (yoki musbat) bo'la olmaydi, yani, $f'(x)\geq 0$ (yoki $f'(x)\leq 0$) dir.

2. Agar $[a;b]$ kesmada uzluksiz hamda kesmaning ichida Differensiallanuvchi bo'lgan funksiya musbat (yoki manfiy) hosilaga ega bo'lsa, u holda funksiya shu kesmada o'suvchi (yoki kamayuvchi) bo'ladi.

Shuningdek, agar $y=f(x)$ funksiya biror X oraliqning ixtiyoriy $x_1<x_2$ qiymatlarida $f(x_1)\leq f(x_2)$ (yoki $f(x_1)\geq f(x_2)$) kabi shartni qanoatlantirsa, uni shu oraliqda *kamaymaydigan* (yoki *o'smaydigan*) funksiya deyiladi.

Funksiyaning kamaymaydigan yoki o'smaydigan oraliqlarini uning *monotonlik oraliqlari* deb yuritiladi. Funksiyaning aniqlanish sohasidagi monotonligining xarakteri, faqatgina uning birinchi tartibli hosilasining ishorasi o'zgaradigan nuqtalardagina o'zgarishi mumkin. Funksiyaning birinchi tartibli hosilasi nolga yoki uzilishga ega bo'ladigan nuqtalar, uning *kritik nuqtalari* deb ataladi.

1-misol. $y=2x^2-lnx$ funksiyaning kritik nuqtalari va monotonlik oraliqlari aniqlansin.

► Funksiya $x>0$ qiymatlarda aniqlangan. Uning hosilasi, $y'=4x-\frac{1}{x}=\frac{4x^2-1}{x}$ bo'lib, $x_0=\frac{1}{2}$ da $y'=0$ bo'lganligi uchun kritik nuqta orqali funksiyaning aniqlanish sohasini $\left(0;\frac{1}{2}\right)$ va $\left(\frac{1}{2};+\infty\right)$ larga ajratamiz. Birinchi oraliqda $y'<0$ bo'lib, ikkinchi oraliqda esa, $y'>0$. Demak, funksiya $(0;0,5)$ da kamayuvchi bo'lib, $(0,5;+\infty)$ da esa o'suvchi bo'lar ekan.◄

Tarifga binoan, $y=f(x)$ funksiya uchun har qanday yetarlicha kichik $|\Delta x|\neq 0$ larda $f(x_1+\Delta x)<f(x_1)$ kabi tengsizlik o'rinli bo'lsa, u holda x_1 nuqtani funksiyaning *lokal maksimumi* deb ataladi. Aksincha, har qanday yetarlicha kichik $|\Delta x|\neq 0$ lar uchun $f(x_2+\Delta x)>f(x_2)$ shart bajarilsa, x_2 ni funksiyaning *lokal minimumi* deb ataladi. Maksimum va minimum nuqtalar birgalikda funksiyaning *ekstremum nuqtalari* deyilib, funksiyaning maksimumi bilan minimumi birgalikda uning *ekstremal qiymatlari* deb ataladi.

1-teorema (***lokal ekstremum mavjudligining zaruriy belgisi***). *Agar* $x=x_0$ *nuqtada* $y=f(x)$ *funksiya ekstremumga yerishadigan bo'lsa, u holda, yoki* $f'(x_0)=0$ *bo'ladi, yoki* $f'(x_0)$ *mavjud bo'lmaydi.*

Differensiallanuvchi funksiyaning ekstremum nuqtalarida uning grafigiga o'tkazilgan *urinmalar* har doim *Ox* o'qiga parallel bo'ladi.

2-misol. $y=(x+1)^3$ funksiyani ekstremumga tekshirilsin.

► Funksiyaning hosilasi $u'=3(x+1)^2$ bo'lib, $x=-1$ da $u'=0$ bo'ladi. Ammo, $x=-1$ da funksiya ekstremumga ega emas, chunki, $x>-1$ da $(x+1)^3>0$ bo'lib, $x<-1$ da esa, $(x+1)^3<0$ dir, hamda $x=-1$ da $(x+1)^3=0$. Demak, funksiya hosilasining nolga aylanishi, uning ekstremumi mavjudligini taminlamas ekan.◄

3-misol. $u=|x|$ funksiyani ekstremumga tekshirilsin.

► $y(0)=0$ bo'lganligidan, hamda $x\neq 0$ da $y=|x|>0$ ligi uchun $x=0$, minimum nuqtadir. Lekin, §6.1 ning 2-misolida funksiyaning $x=0$ nuqtada hosilasi mavjud emasligi ko'rsatilgan edi.◄

Yuqoridagi misollardan ko'rinadiki, har qanday kritik nuqtalarda ham funksiya ekstremumga ega bo'lavyermas ekan. Ammo, biror nuqtada funksiya ekstremumga yerishadigan bo'lsa, u nuqta har doim ham funksiyaning kritik nuqtasi bo'ladi.

Funksiyaning ekstremumini topish uchun quyidagicha ish yuritiladi: barcha kritik nuqtalar aniqlangandan so'ng, ularning har birida alohida-alohida funksiyaning ekstremumi mavjudligi yoki umuman ekstremum mavjud emasligi tekshiriladi.

2-teorema (***lokal ekstremum mavjudlining birinchi yetarli sharti***) *Aytaylik,* $u=f(x)$ *funksiya, biror* $x=x_0$ *kritik nuqta yotadigan oraliqda uzluksiz bo'lib, u oraliqning barcha nuqtalarida Differensiallanuvchi* (x_0 *nuqtada Differensiallanuvchi bo'lmasligi ham mumkin) bo'lsin. Agar* $x<x_0$ *da* $f'(x)>0$, $x>x_0$ *da esa,* $f'(x)<0$ *bo'ladigan bo'lsa, funksiya* $x=x_0$ *da maksimumga yerishadi. Aksincha, yani,* $x<x_0$ *da* $f'(x)<0$ *va* $x>x_0$ *da* $f'(x)>0$ *bo'lsa, funksiya* $x=x_0$ *nuqtada minimumga yerishadi.*

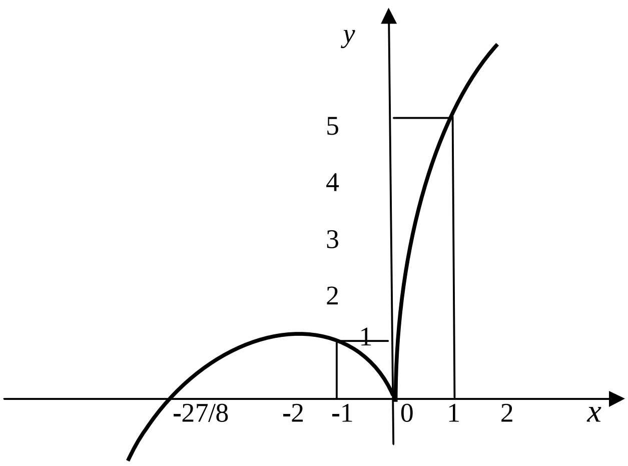
Teorema shartida keltirilgan tengsizliklar, $x=x_0$ kritik nuqtaning yetarlicha kichik atrofida bajarilishi lozimligini yana bir bor takidlaymiz. Birinchi tartibli hosila yordamida funksiyani ekstremumga tekshirish sxemasi jadval ko'rinishida yozilishi mumkin (6.1-jadvalga qaralsin).

4-misol. $y=2x+3\sqrt[3]{x^2}$ funksiyani ekstremumga tekshirilsin.

► Qaralayotgan funksiya barcha $x\in R$lar uchun aniqlangan hamda uzluksiz. $y'=2+\frac{2}{\sqrt[3]{x}}=\frac{2}{\sqrt[3]{x}}\left(\sqrt[3]{x}+1\right)$ ekanligi uchun, kritik nuqtalar $x_1=-1$ bilan $x_2=0$ lardir, chunki ularning birinchisida $y'=0$ bo'lib, ikkinchisida esa, y' uzilishga ega. Bu nuqtalar funksiyaning aniqlanish sohasini $(-\infty;-1)$, $(-1;0)$ va $(0;+\infty)$ kabi oraliqlarga ajratadi. Bu oraliqlarda hosila o'z ishorasini saqlaydi. $(\infty; -1)$ da, $y'>0$, ya'ni, funksiya o'suvchi; $(-1;0)$ da esa, $u'<0$ va funksiya kamayuvchi; $(0;+\infty)$ da, $y'>0$ va funksiya o'suvchidir. Demak, $x_1=-1$ kritik nuqta funksiyaning lokal maksimum nuqtasi bo’lib, $y_{max}= y(-1)=1$; $x_2=0$ nuqta esa, lokal minimum nuqta bo’lib, $y_{min}=y(0)=0$. (6.3-rasmga qaralsin). ◄

6.1- jadval

$f'(x)$ ning x_0 kritik nuqta atrofidagi ishoralari			Kritik nuqtaning xaraktyeri
$x<x_0$	$x=x_0$	$x>x_0$	
+	$f'(x_0)=0$ yoki mavjud emas	-	Maksimum nuqtasi
-	— ׀ ׀ —	+	Minimum nuqtasi
+	— ׀ ׀ —	+	Ekstremum mavjud emas (f-ya o’suvchi)
-	— ׀ ׀ —	-	Ekstremum mavjud emas (f-ya kamayuvchi)



6.3-rasm.

3-teorema ***(lokal ekstremum mavjudligining ikkinchi yetarli sharti)****. Aytaylik, $u=f(x)$ funksiya, biror $x_0 \in X$ oraliqda ikki marta Differensiallanuvchi bo’lib, $f'(x_0)=0$ bo’lsin. U holda, agar, $f''(x_0)<0$ shart bajarilsa, funksiya $x=x_0$ nuqtada lokal maksimumga erishadi, aksincha, $f''(x_0)>0$ bo’lsa, lokal minimumga erishadi*.

Agarda, $f''(x_0)=0$ bo’lsa, $x=x_0$ nuqtada ekstremum mavjud bo’lmasligi ham mumkin.

5- misol. $u=x^2e^{-x}$ funksiyani ikkinchi tartibli hosila yordamida ekstremumga tekshirilsin.

► Birinchi hamda ikkinchi tartibli hosilalarni hisoblaymiz.

$$y'=2xe^{-x}-x^2e^{-x}=(2x-x^2)\,e^{-x},\quad y''=(2-2x)\,e^{-x}-(2x-x^2)\,e^{-x}=(x^2-4x+2)\,e^{-x},$$

$y'=0$ dan, $x_1=0$ va $x_2=2$ kritik nuqtalarni aniqlaymiz. Ikkinchi tartibli hosilaning kritik nuqtalaridagi qiymatlari, $y''(0)=2>0$ va $y''(2)=-2e^{-2}<0$ bo’lganliklaridan, $x_1=0$ nuqtada funksiya minimumga, $x_2=2$ nuqtada esa, maksimumga ega bo’ladi; $y_{min}=y(0)=0$, $y_{max}=y(2)=4e^{-2}$.◄

Qoidaga binoan, [a;b] kesmada uzluksiz bo'lgan $y=f(x)$ funksiya o'zining eng kichik m va eng katta M qiymatlariga yoki (a;b) ochiq oraliqda yotuvchi kritik nuqtalarda yoki kesmaning chegaralarida erishishi mumkin.

6-misol. $y=x^3-3x+3$ funksiyaning [-2;3] kesmadagi eng kichik va eng katta qiymatlari topilsin.

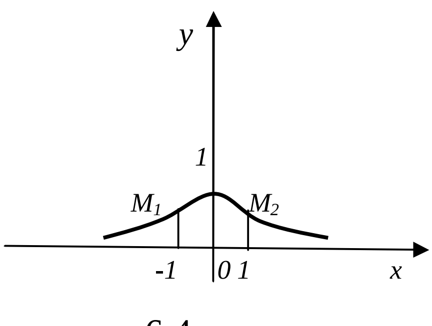
► Funksiyaning hosilasi $y'=3x^2-3$ ekanligidan, $x_1=-1$ bilan $x_2=1$ lar funksiyaning kritik nuqtalaridir va ular (-2;3) oraliqda yotadi. Shuning uchun: *y(-1)=5, y(1)=1, y(-2)=1* va y*(3)=21* larni aniqlaymiz. Ushbu aniqlangan qiymatlarni o'zaro taqqoslab, funksiyaning eng kichik qiymati $m=1$ ga va eng katta qiymati $M=21$ ga teng ekanliklarini topamiz.◄

Aytaylik, egri chiziq (a;b) oraliqda o'zining $u=f(x)$ funksiya bilan ifodalanadigan tenglamasi bilan berilgan bo'lsin. U holda,egri chiziqning barcha nuqtalari uning har qanday nuqtasiga o'tkazilgan urinmadan pastda joylashgan bo'lsa, uni (a;b) oraliqda *qavariq* deb aytiladi. Aksincha bo'lsa, ya'ni, yuqorida joylashsa, uni (a;b) oraliqda *botiq* deyiladi. Egri chiziqning biror $M_0(x_0;\ f(x_0))$ nuqtasi, uning qavariqlik qismini botiqlik qismidan ajratgan bo'lsa, uni egri chiziqning *burilish nuqtasi* deb yuritiladi. Bu holda M_0 nuqtada o'tkazilgan urinma mavjud deb faraz qilinadi.

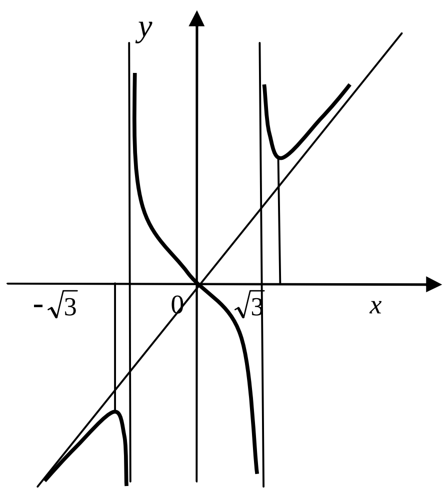
4-teorema (***funksiya grafigining qavariqligi (botiqligi) ning yetarli sharti***) *Agar (a;b) oraliqning barcha nuqtalarida y=f(x) funksiyaning ikkinchi tartibli hosilasi manfiy (musbat) bo'lsa, ya'ni,* $f''(x)<0$ ($f''(x)>0$) *bo'lsa, u holda y=f(x) egri chiziq (a;b) oraliqda har doim qavariq (botiq) dir.*

Egri chiziqning burilish nuqtasining chap va o'ng tomonida u'' hosilaning ishoralari har xil bo'lganligi bois, bu nuqtada $y''=0$ yoki $y''=\infty$ dir.

5-teorema (***burilish nuqtasi mavjudligining yetarli sharti***). *Agar* $x=x_0$ *nuqtada* $f''(x_0)=0$ *yoki* $f''(x_0)$ *mavjud bo'lmasa, hamda* $f''(x)$ *hosila* $x=x_0$ *nuqtadan o'tganda ishorasini o'zgartiradigan bo'lsa, u holda abstsissasi* $x=x_0$ *bo'lgan nuqta y=f(x) egri chiziqning burilish nuqtasidir*.



6.4- rasm



6.5-rasm.

7-misol. Tenglamasi $y = e^{-\frac{x^2}{2}}$ funksiya bo'lgan egri chiziq (*Gauss egri chizig'i*) ning qavariqlik va botiqlik oraliqlari hamda burilishi nuqtalari topilsin. ► Funksiyaning birinchi va ikkinchi tartibli hosilalarini hisoblaymiz: $y' = -xe^{-\frac{x^2}{2}}$, $y'' = (x^2 - 1)e^{-\frac{x^2}{2}}$. Har qanday $x \in R$ uchun *u*' hamda *u*" lar har doim mavjud. *y*"=0 dan x_1=-1 va x_2=1 larni topamiz. Agar *x*<-1 bo'lsa, *y*">0 va *x*>-1 bo'lganda *u*"<0 ekanligini inobatga olsak, $M_1(-1;e^{-1/2})$ ning burilish nuqtasi ekanligini aniqlaymiz. Demak, (-∞;-1) oraliqda egri chiziq botiq bo'lib, (-1;1) oraliqda esa, qavariqdir. Shuningdek, $M_2(-1;e^{-1/2})$ nuqta ham burilish nuqta bo'lib, egri chiziq (-1;1) oraliqda qavariq bo'ladi, (1; +∞) oraliqda esa, botiqdir. Ushbu funksiyaning sxematik grafigi 6.4- rasmda keltirilgan.◄

Tenglamasi *y*=*f*(*x*) funksiya bo'lgan *egri chiziqning M* nuqtasidan biror *L* to'g'ri chiziqqacha bo'lgan masofa, *M* nuqta egri chiziq bo'ylab cheksizlikka intilganda 0 ga intiladigan bo'lsa, u *L* to'g'ri chiziqni egri chiziqning *asimptotasi* deb ataladi. Ta'rifdan faqat cheksiz *egri chiziqlar* (chegaralanmagan egri chiziq) uchun asimptotalar mavjud bo'lishi mumkin ekanligi kelib chiqadi.

7- misoldagi *Gauss egri chizig'i*, *y*=0 asimptotaga egadir (6.4-rasmga qaralsin).

Agar shunday $x=x_i$ ($i=\overline{1,n}$) sonlar mavjud bo'lib, $\lim_{x \to x_i} f(x) = \pm\infty$ bo'lsa, ya'ni, *y*=*f*(*x*) funksiya cheksiz uzilishga ega bo'lsa, u holda $x=x_i$ to'g'ri chiziqlarni *y*=*f*(*x*) egri chiziqning *vertikal asimptotalari* deb yuritiladi.

Agar $k = \lim_{x \to \pm\infty} \frac{f(x)}{x}$ va $b = \lim_{x \to \pm\infty} (f(x) - kx)$ kabi chekli limitlar mavjud bo'lsa, u holda *y*=*kx*+*b* to'g'ri chiziqlar *y*=*f*(*x*) egri chiziqning *og'ma asimptotalari* deb ataladi. Xususan, *k*=0 bo'lsa, *y*=*b* ni *gorizontal asimptota* deyiladi.

8-Misol . $y = \frac{x^3}{x^2 - 1}$ egri chiziqning asimptotalari topilsin.

► $\lim_{x \to \pm 1} \frac{x^3}{x^2 - 1} = \pm\infty$ bo'lganligidan, egri chiziq ikkita *x*=±1 vertikal asimptotalarga ega. Og'ma asimptotalarni qidiramiz.

$$k = \lim_{x \to \pm\infty} \frac{y}{x} = \lim_{x \to \pm\infty} \frac{x^2}{x^2 - 1} = 1 \qquad b = \lim_{x \to \pm\infty} (f(x) - kx) = \lim_{x \to \pm\infty} \left(\frac{x^3}{x^2 - 1} - 1 \right) = \lim_{x \to \pm\infty} \frac{x}{x^2 - 1} = 0$$

Demak, qaralayotgan egri chiziqning bitta *u*=*x* og'ma asimptotasi mavjud ekan.(6.5-rasm).◄

6.7- AT

1. $y=x^4-2x^2-5$ funksiyaning monotonlik oraliqlari topilsin. (Javob: funksiya (-∞;-1) bilan (0;1) da kamayuvchi, (-1;0) bilan (1;+∞) da esa, o'suvchi).

2. $y = \frac{x}{x^2 - 6x - 16}$ funksiyaning monotonlik oraliqlari topilsin. (Javob: funksiya (-∞;-2), (-2;8) va (8;+∞) oraliqda kamayuvchi).

3. $y=\sqrt[3]{\left(x^2-6x+5\right)^2}$ funksiyani ekstremumga tekshirilsin. (Javob: x=1 bilan x=5 da y_{min}=0, x=3 da esa, $y_{\max}=2\sqrt[3]{2}$).

4. $y=x-ln(1+x)$ funksiyani ekstremumga tekshirilsin. (Javob: $u_{min}(0)=0$.)

5. $y=x\cdot ln^2x$ funksiyani ekstremumga tekshirilsin. (Javob: $x=e^{-2}$ da $y_{max}=4/e^2$; $y_{min}(1)=0$.)

6. $y=2x^3+3x^2-12x+1$ funksiyaning [-1;5] kesmadagi eng kichik va eng katta qiymatlari topilsin. (Javob: x=1 da m=-6 va x=5 da M=266).

7. $y=ln(1+x^2)$ funksiya grafigining qavariqlik, botiqlik oraliqlari hamda burilish nuqtalari aniqlansin. (Javob: M_1 (1;ln2), M_2 (-1;$ln2$).)

8. $y=\dfrac{x^2}{\sqrt{x^2-1}}$ funksiya grafigining asimptotalari topilsin. (Javob: $x=\pm1$, $y=\pm x$.)

Mustaqil ish

1. 1) $y=\sqrt[3]{\left(x^2-1\right)^2}$ funksiyani ekstremumga tekshirilsin.

2) $y=\dfrac{x^3}{2(x+1)^2}$ funksiya grafigining asimptotalari topilsin. (Javob:

1) $y_{min}(\pm1)=0$; $y_{max}(0)=1$; 2) x=-1, $y=\frac{1}{2}x+1$.)

2. 1) $y=arctgx-x$ funksiya grafigining qavariqlik, botiqlik oraliqlari hamda burilish nuqtalari topilsin.

2) $y=x+3\sqrt[3]{x}$ funksiyaning [-1;1] kesmadagi eng kichik va eng katta qiymatlari aniqlansin. (Javob: 1) O(0;0), (-∞;0) oraliqda qavariq (0;+∞) oraliqda esa, botiq; 2) m= -4; M=4.)

3. 1) $y=x^3-3x^2-9x+7$ funksiyaning monotonlik oraliqlari hamda ekstremum nuqtalari aniqlansin.

3) $x>0$ bo'lsa, $x>ln(1+x)$ tengsizlikning to'g'riligi isbotlansin (Javob: 1) $y_{max}(-1)=12$, $y_{min}(3)=-20$.)

6.7. FUNKSIYANI TO'LA TEKSHIRISH VA UNING GRAFIGINI YASASH SXEMASI.

Funksiyani to'la tekshirish va uning grafigini yasash uchun quyidagi sxemani tavsiya etish mumkin:

1) funksiyaning aniqlanish sohasini ko'rsatish;

2) funksiyaning uzilish nuqtalari, uning grafigining koordinata o'qlari bilan kesishish nuqtalari hamda vertikal asimptotalari (agar ular mavjud bo'lsalar)ni aniqlash;

3) funksiyaning juft yoki toqligi, davriyligini aniqlash;

4) funksiyani monotonlikka va ekstremumga tekshirish;

5) qavariqlik, botiqlik oraliqlari hamda burilish nuqtalarini aniqlash;

6) funksiya grafigining asimptotalarini topish;

7) boshqa kerakli qo'shimcha hisoblashlarni bajarish;

8) funksiyaning grafigini yasash.

Misol $y=\sqrt[3]{(x+3)x^2}$ funksiyani to'la tekshirib uning grafigi yasalsin.

►1. Funksiya barcha $x \in R$ lar uchun aniqlangan.

2. Funksiyaning uzilish nuqtalari mavjud emas, grafik *Ox* o'qini *x*=-3 va *x*=0 nuqtalarda kesib, koordinata boshidan o'tadi.

3. Funksiya juft ham, toq ham, davriy ham emas.

4. Hosilani hisoblaymiz: $f'(x)=\dfrac{x+2}{\sqrt[3]{x(x+3)^2}}$.

Hosila, x_1=-2 da 0 ga teng va x_2=-3 bilan x_3=0 nuqtalarda mavjud emas. Ushbu nuqtalar, funksiyaning aniqlanish sohasini (-∞:-3), (-3;-2), (-2;0) va (0;+∞) kabi oraliqlarga ajratadi. Ulardan, (-∞:-3), (-3;-2) va (0;+∞) oraliqlarda $f'(x)>0$ bo'lib, (-2;0) oraliqda esa, $f'(x)<0$ dir. Bundan ko'rinmoqdaki, funksiya (-∞:-2) va (0;+∞) oraliqlarda o'suvchi bo'lib, (-2;0) oraliqda esa, kamayuvchidir. Shuningdek x_1=-2 maksimum nuqtasi bo'lib, $y_{\max}=\sqrt[3]{4}$ hamda x_3=0 esa, minimum nuqtasidir va $y_{min}=y(0)=0$; x_2 =-3 nuqtada funksiya ekstremumga ega emas.

5. Ikkinchi tartibli hosilani topamiz:

$$f''(x)=-\frac{2}{\sqrt[3]{x^4(x+3)^5}},$$

bu hosila argument *x* ning hech bir chekli qiymatida 0 ga teng bo'la olmaydi. Shu sababli, ikkinchi tartibli hosila mavjud bo'lmaydigan nuqtalargina ya'ni, x_2=-3 bilan x_3=0 nuqtalargina egri chiziq burilish nuqtalarining absissalari bo'lishi mumkin. Ushbu nuqtalar orqali, funksiyaning aniqlanish sohasini (-∞:-3), (-3;0) va (0;+∞) kabi bo'laklarga ajratib, ularning har birida *f*(*x*) ning ishoralarini aniqlaymiz: (-∞:-3) oraliqda *f''*(*x*)>0 bo'lganligi uchun u oraliqda egri chiziq botiq bo'lib, (-3;0) bilan (0;+∞) oraliqlarda *f''*(*x*)<0 bo'lganligidan, egri chiziq u oraliqlarda qavariqdir. x_2=-3 nuqtaning atrofida *f''*(*x*) ning ishoralari turli xil bo'lganligi uchun *M*(-3;0) nuqta, egri chiziqning burilish nuqtasidir. Ammo, x_3=0 nuqta atrofida *f''*(*x*) ning ishorasi bir xil bo'lganligi sababli, u burilish nuqtasi bo'la olmaydi.

6. Qaralayotgan funksiya cheksiz uzilish nuqtalariga ega bo'lmaganligi bois, vertikal asimptotalari yo'q. Og'ma asimptotalar mavjudligini tekshiramiz:

$$k=\lim_{x\to\pm\infty}\frac{y}{x}=\lim_{x\to\pm\infty}\frac{\sqrt[3]{(x+3)x^2}}{x}=\lim_{x\to\pm\infty}\sqrt[3]{1+\frac{3}{x}}=1,$$

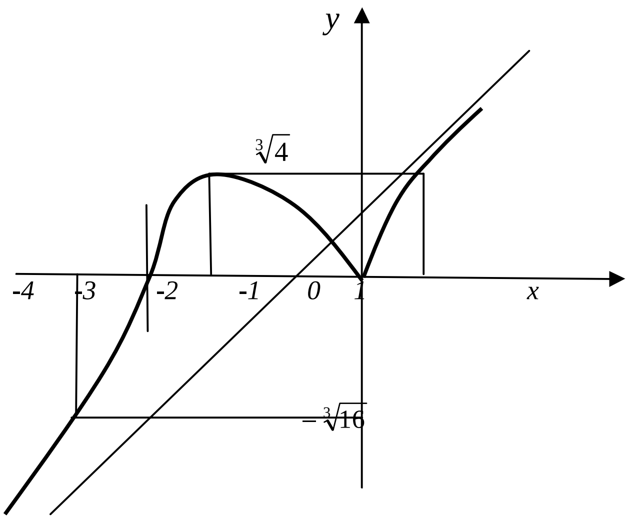
$$b=\lim_{x\to\pm\infty}(y-kx)=\lim_{x\to\pm\infty}\left(\sqrt[3]{(x+3)x^2}-x\right)=\lim_{x\to\pm\infty}\frac{\left(\sqrt[3]{(x+3)x^2}-x\right)\left(\sqrt[3]{(x+3)^2x^4}+x\sqrt[3]{(x+3)x^2}+x^2\right)}{\left(\sqrt[3]{(x+3)^2x^4}+x\sqrt[3]{(x+3)x^2}+x^2\right)}=$$

$$=\lim_{x\to\pm\infty}\frac{(x+3)x^2-x^3}{\left(\sqrt[3]{(x+3)^2x^4}+x\sqrt[3]{(x+3)x^2}+x^2\right)}=\lim_{x\to\pm\infty}\frac{3x^2}{\left(\sqrt[3]{(x+3)^2x^4}+x\sqrt[3]{(x+3)x^2}+x^2\right)}=$$

$$=\lim_{x\to\pm\infty}\frac{3}{\left(\sqrt[3]{(1+\frac{3}{x})^2}+\sqrt[3]{(1+\frac{3}{x})}+1\right)}=1,$$

Demak, $y=x+1$ to'g'ri chiziq og'ma asimptota bo'lar ekan.

7. Funksiyaning grafigini chizishdan avval, grafik Ox o'qini $x_2=-3$ va $x_3=0$ nuqtalarda qanday α burchak ostida kesib o'tishligini aniqlash maqsadga muvofiqdir. Bu nuqtalarda $u'=tg\alpha=\infty$ ligi uchun $\alpha=\frac{\pi}{2}$ ga teng bo'ladi. Agar $x_3=0$ nuqta funksiyaning minimum nuqtasi bo'lganligi sababli, funksiyaning grafigi bu nuqta atrofida Ox o'qidan yuqorida joylashgan bo'ladi, hamda $x_3=0$ nuqta funksiya grafigining *"qaytishi" nuqtasidir*.



6.6-rasm.

8. Tekshirishlar natijasiga ko'ra, qaralayotgan funksiyaning grafigini yasaymiz. (6.6-rasm).◄

6.8- AT

Quyida berilgan funksiyalarni to'la tekshirib, ularning grafiklari yasalsin.

1. $y=x^3-3x^2$ (Javob: $y_{max}(0)=0$; $y_{min}(2)=-4$; burilish nuqtasi $M_1(-1,2)$.)

2. 2. $y=x^2+\frac{2}{x}$ (Javob: $y_{min}(1)=3$; $M_1(-\sqrt[3]{2};0)$ burilish nuqtasi: $x=0$-asimptota.)

3. $y=x^3/(3-x^2)$ (Javob: $x=\pm\sqrt{3}$ uzilish nuqtalari: $y_{min}(-3)=4,5$; $y_{max}(3)=-4,5$; $M_1(0,0)$ burilish nuqtasi: $x=\pm\sqrt{3}$ va $y=-x$ lar asimptotalar.)

Mustaqil ish

Quyida keltirilgan funksiyalar to'la tekshirilib, ularning grafiklari yasalsin.

1. $y=ln(x^2+2x+2)$. (Javob: $y_{min}(-1)=0$; $M_1(-2;\ ln2)$ va $M_2(0;\ ln2)$ lar burilish nuqtalari.)

2. $y=\frac{2x-1}{(x-1)^2}$ (Javob: $y_{min}(0)=-1$; $M_1\left(-\frac{1}{2};-\frac{8}{9}\right)$ burilish nuqtasi: $x=1$ va $y=0$ lar asimptotalar.)

3. $y=-ln(x^2-4x+5)$. (Javob: $y_{max}(2)=0$; $M_1(1;\ ln2)$ bilan $M_2(3;\ ln2)$ lar burilish nuqtalari.)

6.8. EKSTREMUMGA AMALIY MASALALAR

1-misol. Agar usti ochiq tsilindrik bakni tayyorlash uchun $S=27\pi \approx 84{,}42\ m^2$ matyerial ishlatiladigan bo'lsa, uning maksimal sig'imiga ega bo'lishi uchun asos radiusi R bilan balandligi N qanday o'lchamlarda bo'lishi kerak bo'ladi?

► Bakning sig'imi $V=\pi R^2H$ va uni tayyorlash uchun yuzasi $S=\pi R^2+2\pi RH$ bo'lgan matyerial kerak bo'ladi. Oxirgi tenglikdan bakning balandligi $H=\dfrac{S-\pi R^2}{2\pi R}$ ekanligidan, bakning sig'imi $V=\pi R^2\dfrac{S-\pi R^2}{2\pi R}=\dfrac{SR-\pi R^3}{2}=V(R)$ bo'ladi.

Bu ifodadan $V(R)$ maksimal sig'im bo'lishi kerak bo'ladigan R ning qiymatini aniqlaymiz (6.5 –qarang): $V'=\dfrac{1}{2}(S-3\pi R^2)$ dan, $V'=0$ bo'lishi uchun $S-3\pi R^2=0$ va $R=\sqrt{\dfrac{S}{3\pi}}=\sqrt{\dfrac{27\pi}{3\pi}}=3м$. $V''=-3\pi R<0$ ekanligini inobatga olsak, topilgan $R=3$ qiymatida bakning sig'imi maksimal darajada bo'ladi. U hoda, bakning balandligi $H=\dfrac{S-\pi R^2}{2\pi R}=\sqrt{\dfrac{S}{3\pi}}=3м$ ekan.◄

2-misol. Sug'orish kanalining kesimi teng yonli trapetsiya shaklida bo'lib, uning yon tomonlari kichik asosiga teng bo'lsa, yon tomonlarining kichik asos tekisligi bilan tashkil etgan α burchagi qanday bo'lganda kanalning kesimi eng katta yuzaga ega bo'ladi (6.7-rasm)?

► Kanal kesimining yuzini, yon tomonlari bilan kichik asos tomoni a deb belgilagan holda quyidagicha aniqlaymiz:

$$S=\frac{|AB|+|DC|}{2}\cdot|CE|=\frac{2a+2a\cos 2\alpha}{2}\cdot a\sin\alpha=a^2\left(\sin\alpha+\frac{1}{2}\sin 2\alpha\right)$$

Hosil bo'lgan yuzani α ga bog'liq funksiya sifatida ekstremumga tekshiramiz:

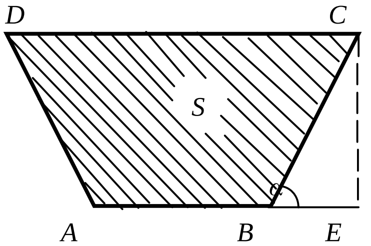
$$S'=a^2(cos\alpha+cos2\alpha),$$

Kritik nuqtalarni aniqlash uchun

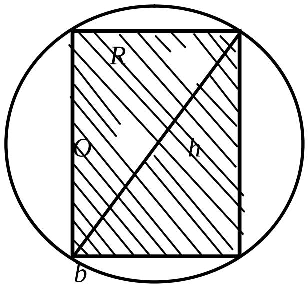
$$S'=0,\ \text{yoki}\ cos\alpha+cos2\alpha=2\cos\frac{3\alpha}{2}\cdot\cos\frac{\alpha}{2}=0$$

ni hosil qilamiz. Agar $0<\alpha<\dfrac{\pi}{2}$ ekanligini inobatga olsak, $Cos\dfrac{\alpha}{2}\neq 0$ va $Cos\dfrac{3\alpha}{2}=0$ dan $\dfrac{3\alpha}{2}=\dfrac{\pi}{2}$ va $\alpha=\dfrac{\pi}{3}$ ni aniqlaymiz. Bu esa, qaralayotgan funksiya uchun kritik nuqta bo'ladi. Ana shu $\alpha=\dfrac{\pi}{3}$ nuqtada S funksiya, $\left[0;\dfrac{\pi}{2}\right]$ kesmada o'zining eng katta qiymatiga yerishishligini isbotlaymiz. Haqiqatan ham,

$S''=a^2(-\sin\alpha-2\sin 2\alpha)$, $S''\left(\frac{\pi}{3}\right)=-\frac{3\sqrt{3}}{2}a^2<0$. Shuning uchun, $\alpha=\frac{\pi}{3}$ nuqtada lokal maksimumga ega bo'lamiz: $S\left(\frac{\pi}{3}\right)=S_{\max}=\frac{3\sqrt{3}}{4}a^2$. Agar $S(0)=0$ va $S\left(\frac{\pi}{2}\right)=a^2<S_{\max}$ ekanligini inobatga olsak, S funksiyaning $\left[0;\frac{\pi}{2}\right]$ kesmadagi eng katta qiymati $\frac{3\sqrt{3}}{4}a^2$ ekanligi hosil bo'ladi.◄



6.7-rasm.



6.8-rasm.

3-misol. Agar to'g'ri to'rtburchakli ko'ndalang kesimdagi brusning mustahkamligi uning eni b ga va balandligi h ning kvadratiga proportsional bo'lsa, radiusi $R=2\sqrt{3}$ bo'lgan g'o'ladan kesib olingan brusning o'lchamlari qanday bo'lganda uning mustahkamligi eng katta bo'ladi (6.8-rasm)

► Brusning mustahkamligi, $N=kh^2b$ (k -proportsionallik koeffitsiyenti bo'lib, $k>0$) formula bilan aniqlanadi. 6.8- rasmdan ko'rinidaki, $h^2+b^2=4R^2$ kabidir. U holda, $N=k(4R^2-b^2)b$ ni hosil qilamiz va uning ekstremumini topamiz:

$$N'(b)=k(4R^2-3b^2),$$

agar $N'=0$ bo'lsa, $b=\frac{2R}{\sqrt{3}}=\frac{2\cdot 2\sqrt{3}}{\sqrt{3}}=4\partial m.$ U holda: $h=\sqrt{4R^2-b^2}=\sqrt{2}b=4\sqrt{2}\,dm$.

Endi, $N''=-64kb<0$ ekanligidan, ayta olamizki, brusning maksimal mustahkamligi $b=4$ *dm* va $h=4\sqrt{2}$ *dm* kabi qiymatlarda yuz beradi.◄

6.9- AT

1. Sig'imi $V=16\pi\approx 50$ m^3 dan iborat bo'lgan yopiq silindrik bak yasashda eng kam miqdordagi matyerial sarf bo'lishi uchun uning o'lchamlari (R radius va N balandlik) qanday bo'lishi kerak? (Javob: $R=2m$, $N=4m$.)

2. Radiusi R bo'lgan sharga eng katta hajmli konusni ichki chizish uchun uning balandligi qanday bo'lishi kerak? (Javob: $H=\frac{4R}{3}$.)

3. Tenglamasi $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ bo'lgan ellipsga, eng katta yuzali to'g'ri to'rtburchakni ichki chizish uchun uning tomonlari qanday bo'lishi kerak? (Javob: $a\sqrt{2}$ va $b\sqrt{2}$.)

4. Doiradan, markaziy burchagi α bo'lgan sektor kesib olinib undan konus sirt hosil qilingan bo'lsa, α ning qanday qiymatida hosil bo'lgan konusning hajmi eng katta bo'ladi? (Javob: $\alpha = 2\pi\sqrt{\frac{2}{3}} \approx 293^\circ 56'$.)

Mustaqil ish

1. Berilgan M(1;4) nuqtadan shunday bir to'g'ri chiziq o'tkazilsinki, uning koordinata o'qlaridan kesgan musbat kesmalar uzunliklarining yig'indisi eng kichik bo'lsin. Ushbu to'g'ri chiziq tenglamasi yozilsin. (Javob: $\frac{x}{3}+\frac{y}{6}=1$.)

2. Radiusi R bo'lgan sharga eng katta hajmli ichki chizilgan silindrning balandligi N topilsin (Javob: $H = \frac{2R}{\sqrt{3}}$)

3. Yasovchisi 20 sm bo'lgan konus shaklidagi voronka yasash lozim bo'lsin. Uning hajmi eng katta bo'lish uchun voronkaning balandligi qanday bo'lishi kerak? (Javob: $\frac{20\sqrt{3}}{3}$.)

6.9. YOY UZUNLIGINING DIFFERENSIALI VA EGRI CHIZIQNING EGRILIGI.

1. Ta'rifga ko'ra, $y=f(x)$ funksiya berilgan *egri chiziq yoyining ds Differensiali*

$$ds = \sqrt{1+\left(f'(x)\right)^2}\,dx$$

formula orqali ifodalanadi.

Agar $x=\varphi$(e) funksiya berilgan *egri chiziq yoyining ds Differensiali*

$$ds = \sqrt{1+\left(\varphi'(y)\right)^2}\,dy$$

formula orqali ifodalanadi.

Agar egri chiziq $x=\varphi(t)$ va $y=\psi(t)$ kabi parametrik tenglamalar bilan beriladigan bo'lsa, u holda $ds = \sqrt{\left(\varphi'(t)\right)^2+\left(\psi'(t)\right)^2}\,dt$.

Agar qutb koordinatalari sistemasida egri chiziq $\rho=\rho(\varphi)$ tenglama bilan berilsa, uning yoyining Differensiali uchun $ds = \sqrt{\rho^2+\left(\rho'\right)^2}\,d\varphi$ formula o'rinli bo'ladi.

1-misol. $x=a(1-sint)$, $y=a(1-cost)$ $(a>0)$ sikloida yoyi uzunligining Differensiali topilsin.

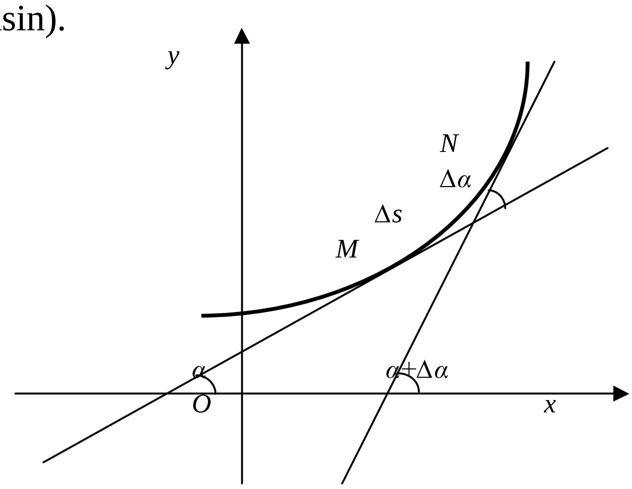
► $x'=a(1-cost)$, $y'=a\ sint$ bo'lganligidan,

$$ds = \sqrt{a^2(1-\cos t)^2+a^2\sin^2 t}\,dt = a\sqrt{2(1-\cos t)}\,dt = a\sqrt{4\sin^2\frac{t}{2}}\,dt = 2a\sin\frac{t}{2}\,dt\ .◄$$

2. *Ixtiyoriy egri chiziqning biror M nuqtasidagi egriligi K* deb, egri chiziqning M va N nuqtalariga o'tkazilgan urinmalarning musbat yo'nalishlari orasidagi burchak modulining $MN=\Delta s$ yoy uzunligiga bo'lgan nisbatining N nuqta M ga intilgandagi limitiga aytiladi, ya'ni:

$$k=\lim_{\Delta s\to 0}\left|\frac{\Delta\alpha}{\Delta s}\right|=\left|\frac{d\alpha}{ds}\right|$$

Bu yerda α M nuqtadagi urinmaning Ox o'qiga og'ish burchagi.(6.9-rasmga qaralsin).



6.9- rasm

R egrilik radiusi deb, egri chiziq egriligi K ga teskari qiymatga aytiladi, ya'ni: $R=\frac{1}{K}$. Masalan, aylana uchun $K=\frac{1}{R}$ (R - aylana radiusi) bo'lib, to'g'ri chiziq uchun K=0. Umuman qaralganda, ixtiyoriy egri chiziqning egriligi o'zgarmas miqdor emas.

Agar egri chiziq $y=f(x)$ tenglama bilan berilgan bo'lsa, uning har qanday nuqtasidagi egriligi quyidagi formula bilan aniqlanadi:

$$K=\frac{|y''|}{\left(1+(y')^2\right)^{\frac{3}{2}}}.$$

Xususan, egri chiziq $x=\varphi(t)$, $y=\psi(t)$ prametrik tenglamalar yoki $\rho=\rho(\varphi)$ kabi (qutb koordinatalari sistemasida) tenglama bilan qaraladigan bo'lsa, ularning egriliklari mos ravishda quyidagi formulalar bilan hisoblanadi:

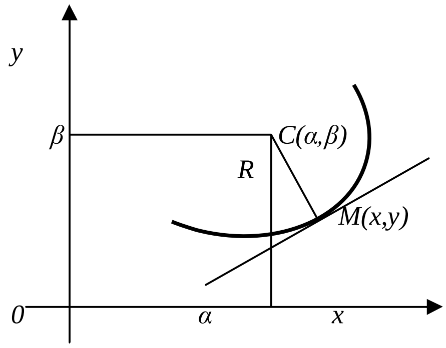
$$K=\frac{|y''x'-x''y'|}{\left((x')^2+(y')^2\right)^{\frac{3}{2}}}$$ (hosilalar t ga nisbatan olinadi);

$$K=\frac{\left|\rho^2+2(\rho')^2-\rho\rho''\right|}{\left(\rho^2+(\rho')^2\right)^{\frac{3}{2}}}$$ (hosilalar φ ga nisbatan olingan).

2-misol. $y=x^2$ kabi egri chiziqning $M(1;1)$ nuqtadagi egriligi hamda egrilik radiusi topilsin.

► $y'=2x$ va $y'(1)=2$ hamda $y''=2$ bo'lganligidan, $K=\frac{|y''|}{\left(1+(y')^2\right)^{\frac{3}{2}}}=\frac{2}{5\sqrt{5}}$ va $R=\frac{1}{K}=\frac{5\sqrt{5}}{2}$. ◄

3. Biror egri chiziqning $M(x;u)$ nuqtasida uning botiqlik tomoniga qarab unga normal o'tkazamiz va u normalda egri chiziqning M nuqtadagi egrilik radiusi R ga teng bo'lgan $|MS|$ kesma olamiz. (6.10-rasm)



6.10- rasm

Ta'rifga binoan, S nuqtani *egri chiziqning M nuqtadagi egrilik markazi* deb atalib, markazi S nuqtada bo'lib radiusi R bo'lgan doira (aylana)ni esa, *egri chiziqning M nuqtadagi egrilik doira (aylana)si* deb ataladi.

Egrilik markazining α va β koordinatalarini quyidagi formulalar orqali aniqlanadi:

$$\alpha = x - y' \cdot \frac{1+(y')^2}{y''}, \qquad \beta = y + \frac{1+(y')^2}{y''}. \qquad (6.4)$$

$y=f(x)$ egri chiziq egriligining barcha markazlar to'plamini *evolyuta* deb ataladi. Har qanday aylananing evolyutasi uning markazidir, to'g'ri chiziq esa, evolyutaga ega emas. Yuqorida keltirilgan (6.4) formulalar evolyutaning x o'zgaruvchi parametrga nisbatan parametrik tenglamalaridir.

3-misol. $y=x^2-6x+10$ egri chiziqning $M_0(3;1)$ nuqtadagi egrilik aylanasining tenglamasi yozilsin.

► $y'=2x-6$ va $y''=2$ bo'lganligidan, $y'(3)=0$ va $y''=2$. Shuning uchun $M_0(3;1)$ nuqtadagi egrilik $K=2$ bo'lib, egrilik radiusi $R=1/2$ ga tengdir. Egrilik markazining koordinatalari (6.4-formula), $\alpha=3$ va $\beta=3/2$ ga teng bo'lib, egrilik aylananing tenglamasi esa, $(x-3)^2+\left(y-\frac{3}{2}\right)^2=\frac{1}{4}$ kabi yoziladi.◄

4-misol. $y^2=2px$ parabola evolyutasining tenglamasi yozilsin.

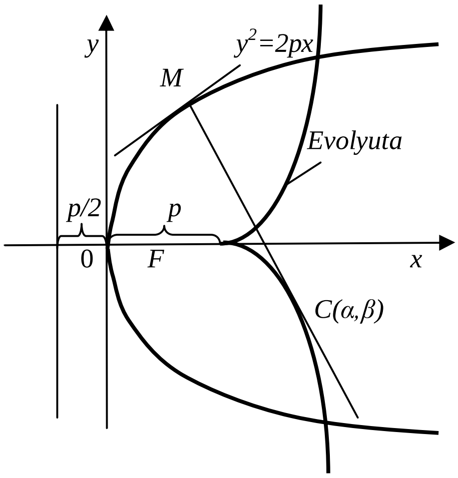
► Ixtiyoriy nuqtada y' va y'' larni hisoblaymiz:

$$2yy'=2p \quad \text{yoki} \quad y'=\frac{p}{y}, \quad y''=-\frac{p}{y^2}\cdot y'=-\frac{p^2}{y^3}.$$

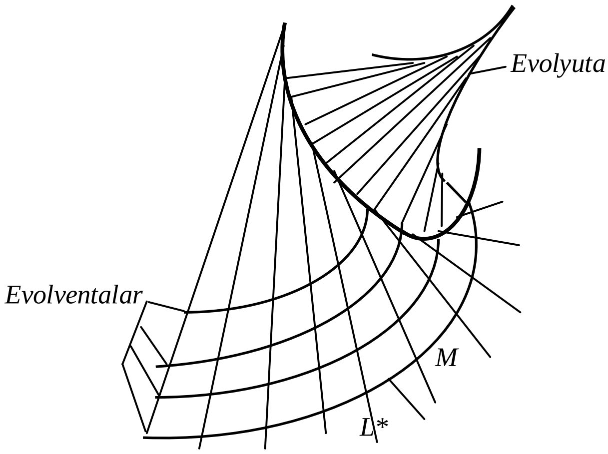
U holda, 6.4 formuladan:

$$\alpha = x-\frac{p}{y}\cdot\frac{1+\frac{p^2}{y^2}}{-\frac{p^2}{y^2}}=3x+p, \qquad \beta = y+\frac{p}{y}\cdot\frac{1+\frac{p^2}{y^2}}{-\frac{p^2}{y3}}=-\frac{y^3}{p^2}=-\frac{(2x)^{3/2}}{\sqrt{p}},$$

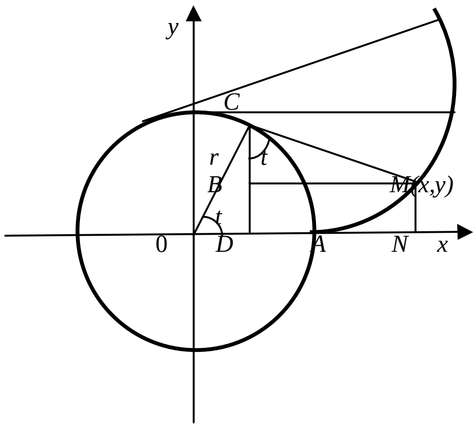
Ushbu tenglamalardan x parametrni yo'qotib, $\beta^2 = \frac{8}{27p}(\alpha\text{-}\rho)^3$ ni hosil qilamiz. Bu esa, yarim kubik parabolaning tenglamasidir (6.11-rasm).◄



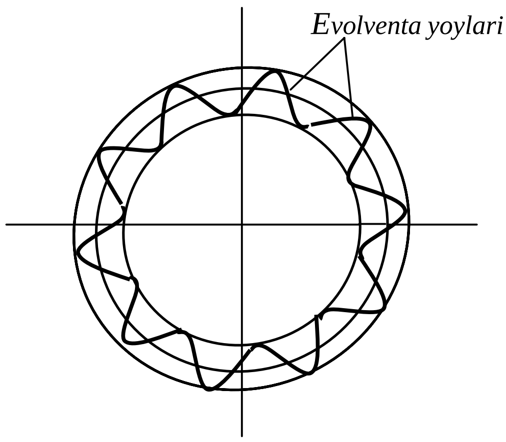
6.11-rasm



6.12-rasm



6.13-rasm.



6.14-rasm

Berilgan biror L egri chiziq urinmasining belgilangan M nuqtasi egri chiziq bo'ylab sirpanmasdan yumalab harakatlanishi natijasida hosil bo'lgan L^* chiziqni L *egri chiziqning evolventa (yoyilmasi)* deb yuritiladi (6.12-rasm). Berilgan L egri chiziqning cheksiz ko'p miqdordagi evolventalari va bittagina evolyutasi mavjuddir.

5-misol. Tenglamasi $x^2+y^2=r^2$ bo'lgan aylananing $A(r;0)$ nuqtadan chiqadigan evolventalarning parametrik tenglamalari tuzilsin.

►6.13-rasmda ko'rsatilgan usul yordamida t parametr kiritib hamda AS yoy uzunligining $|MS|=rt$ ga teng ekanligini inobatga olib, evolventaning ixtiyoriy $M(x;y)$ nuqtasining koordinatalarini aniqlaymiz.

$x=|ON|=|OD|+|DN|=rcost+rtsint=r(cost+tsint)$;

$y=|DB|=|DC|-|BC|=rsint-rtcost=r(sint-tcost)$.◄

Ta'kidlash lozimki, silindrik sisternalarning tishlari, ko'p hollarda aylana evolventasi bo'yicha chiziladi (6.14-rasm), chunki shu holatda tishlarning eng ko'p shovqinsiz hamda tekis ulanishlari yuz beradi.

6.10- AT

1. Agar egri chiziq o'zining $x=atsint$, $y=atcost$ kabi parametrik tenglamalari bilan berilgan bo'lsa, uning yoy uzunligining Differensiali topilsin. (Javob: $ds = 3a\ cost\ sint\ dt$.)

2. 2. $y=\sqrt{x^3}$ egri chiziq yoyining differensiali topilsin.

3. $\rho=a(1+cos\varphi)$ egri chiziq yoyining differensiali topilsin. (Javob: $ds = a\cos\frac{\varphi}{2}d\varphi$.)

4. $x^2+xy+y^2=3$ egri chiziqning $A(1;1)$ nuqtadagi egriligi va egrilik radiusi topilsin. (Javob: $K=\frac{1}{(3\sqrt{2})}$, $R=3\sqrt{2}$)

5. Parametrik tenglamalari $x=3t^2$, $y=3t-t^3$ kabi bo'lgan egri chiziqning $V(3;2)$ nuqtadagi egriligi va egrilik radiusi topilsin. (Javob: $K=1/6$, $R=6$.)

6. $y=\frac{1}{x}$ egri chiziqning $A(1;1)$ nuqtadagi egrilik markazi hamda egrilik aylanasining tenglamasi yozilsin. (Javob: $(x-2)^2+(y-2)^2=2$.)

Mustaqil ish

1. 1) $y=tgx$ egri chiziq uzunligining Differensiali topilsin;

2) $y^2=x^3$ egri chiziqning $M(4;8)$ nuqtadagi egrilik va egrilik radiusi topilsin. (Javob: $K=\frac{3}{40}$, $R=\frac{40}{3}$.)

2. 1) $x=acos^3t$, $y=asin^3t$ egri chiziq yoyi uzunligining Differensiali topilsin;

2) $y=x^2-2x$ egri chiziqning $M_0(2;0)$ nuqtadagi egrilik markazining koordinatalari topilib, egrilik aylanasining tenglamasi yozilsin. (Javob: $(x+3)^2+\left(y-\frac{5}{2}\right)^2=\frac{125}{4}$.)

3. 1) $\rho=a(1+sin\varphi)$ egri chiziq yoyi uzunligining Differensiali topilsin;

2) $y=lnx$ egri chiziqning $M_1(1;0)$ nuqtadagi egrilik markazi aniqlanib, egrilik aylanasining tenglamasi yozilsin. (Javob: $(x-3)^2+(y+2)^2=8$.)

6.10. 6- BO'LIMGA INDIVIDUAL UY TOPSHIRIQLARI

6.1-IUT

Berilgan funksiyalarni differensiyallang.

1.

1.1. $y = 2x^5 - \frac{4}{x^3} + \frac{1}{x} + 3\sqrt{x}.$

1.2. $y = \frac{3}{x} + \sqrt[5]{x^2} - 4x^3 + \frac{2}{x^4}.$

1.3. $y = 3x^4 + \sqrt[3]{x^5} - \frac{2}{x} - \frac{4}{x^2}.$

1.4. $y = 7\sqrt{x} - \frac{2}{x^5} - 3x^3 + \frac{4}{x}.$

1.5. $y = 7x + \frac{5}{x^2} - \sqrt[7]{x^4} + \frac{6}{x}.$

1.6. $y = 5x^2 - \sqrt[3]{x^4} + \frac{4}{x^3} - \frac{5}{x}.$

1.7. $y = 3x^5 - \frac{3}{x} - \sqrt{x^3} + \frac{10}{x^5}.$

1.8. $y = \sqrt[3]{x^7} + \frac{3}{x} - 4x^6 + \frac{4}{x^5}.$

1.9. $y = 8x^2 + \sqrt[3]{x^4} - \frac{4}{x} - \frac{2}{x^3}.$

1.10. $y = 4x^6 + \frac{5}{x} - \sqrt[3]{x^7} - \frac{7}{x^4}.$

1.11. $y = 2\sqrt{x^3} - \frac{7}{x} + 3x^2 - \frac{2}{x^5}.$

1.12. $y = 4x^3 - \frac{3}{x} - \sqrt[5]{x^2} + \frac{6}{x^2}.$

1.13. $y = 5x^3 - \frac{8}{x^2} + 4\sqrt{x} + \frac{1}{x}.$

1.14. $y = \frac{9}{x^3} + \sqrt[3]{x^4} - \frac{2}{x} + 5x^4.$

1.15. $y = \frac{4}{x^5} - \frac{9}{x} + \sqrt[5]{x^2} - 7x^3.$

1.16. $y = \frac{8}{x^3} + \frac{3}{x} - 4\sqrt{x^3} + 2x^7.$

1.17. $y = 5x^2 - \frac{4}{x} - \sqrt[3]{x^7} - 2x^6.$

1.18. $y = 10x^2 + 3\sqrt{x^5} - \frac{4}{x} - \frac{5}{x^4}.$

1.19. $y = \sqrt{x^5} - \frac{3}{x} + \frac{4}{x^3} - 3x^2.$

1.20. $y = 9x^3 + \frac{5}{x} - \frac{7}{x^4} + \sqrt[3]{x^7}.$

1.21. $y = 3\sqrt{x} + \frac{4}{x^5} + \sqrt[3]{x^2} - \frac{7}{x}.$

1.22. $y = \sqrt{x^3} + \frac{2}{x} - \frac{4}{x^5} - 5x^3.$

1.23. $y = 7x^2 + \frac{3}{x} - \sqrt[5]{x^4} + \frac{8}{x^3}.$

1.24. $y = 8x^3 - \frac{4}{x} - \frac{7}{x^4} + \sqrt[7]{x^2}.$

1.25. $y = 8x - \frac{5}{x^4} + \frac{1}{x} - \sqrt[5]{x^4}.$

1.26. $y = \sqrt[4]{x^3} - \frac{5}{x} + \frac{4}{x^5} + 3x.$

1.27. $y = 4x^3 + \frac{3}{x} - \sqrt[3]{x^5} - \frac{2}{x^4}.$

1.28. $y = 4x^5 - \frac{5}{x} - \sqrt{x^3} + \frac{2}{x^3}.$

1.29. $y = \frac{7}{x} + \frac{4}{x^3} - \sqrt[5]{x^3} - 2x^6.$

1.30. $y = \frac{6}{x^4} - \frac{3}{x} + 3x^3 - \sqrt{x^7}.$

2.

2.1. $y = \sqrt[3]{3x^4 + 2x - 5} + \frac{4}{(x-2)^5}.$

2.2. $y = \sqrt[3]{(x-3)^4} - \frac{3}{2x^3 - 3x + 1}.$

2.3. $y = \sqrt{(x-4)^5} + \frac{5}{(2x^2 + 4x - 1)^2}.$

2.4. $y = \sqrt[5]{7x^2 - 3x + 5} - \frac{5}{(x-1)^3}.$

2.5. $y = \sqrt[4]{3x^2 - x + 5} - \frac{34}{(x-5)^4}.$

2.6. $y = \sqrt{3x^4 - 2x^3 + x} - \frac{4}{(x+2)^3}.$

2.7. $y = \sqrt[3]{(x-7)^5} + \frac{5}{4x^2 + 3x - 5}.$

2.8. $y = \sqrt[5]{(x+4)^6} - \frac{2}{2x^2 - 3x + 7}.$

2.9. $y = \frac{3}{(x-4)^7} - \sqrt{5x^2 - 4x + 3}.$

2.10. $y = \sqrt[3]{4x^2 - 3x - 4} - \frac{2}{(x-3)^5}.$

2.11. $y=\frac{7}{(x-1)^3}+\sqrt{8x-3+x^2}.$

2.12. $y=\sqrt[5]{3x^2+4x-5}+\frac{4}{(x-4)^4}.$

2.13. $y=\sqrt[3]{5x^4-2x-1}+\frac{8}{(x-5)^2}.$

2.14. $y=\frac{3}{(x+2)^5}-\sqrt[7]{5x-7x^2-3}.$

2.15. $y=\sqrt[4]{(x-1)^5}-\frac{4}{7x^2-3x+2}.$

2.16. $y=\sqrt[5]{(x-2)^6}-\frac{3}{7x^3-x^2-4}.$

2.17. $y=\frac{3}{(x+4)^2}-\sqrt[3]{4+3x-x^4}.$

2.18. $y=\frac{2}{(x-1)^3}-\frac{8}{6x^2+3x-7}.$

2.19. $y=\sqrt{1+5x-2x^2}+\frac{3}{(x-3)^4}.$

2.20. $y=\sqrt[3]{5+4x-x^2}-\frac{5}{(x+1)^3}.$

2.21. $y=\sqrt[4]{5x^2-4x+1}-\frac{7}{(x-5)^2}.$

2.22. $y=\sqrt[5]{3-7x+x^2}-\frac{4}{(x-7)^5}.$

2.23. $y=\sqrt{(x-3)^7}+\frac{9}{7x^2-5x-8}.$

2.24. $y=\sqrt[3]{(x-8)^4}-\frac{2}{1+3x-4x^2}.$

2.25. $y=\frac{3}{4x-3x^2+1}-\sqrt{(x+1)^5}.$

2.26. $y=\frac{3}{x-4}+\sqrt[6]{(2x^2-3x+1)^5}$

2.27. $y=\frac{4}{(x-7)^3}-\sqrt[3]{(3x^2-x+1)^4}.$

2.28. $y=\sqrt{(x-4)^7}-\frac{10}{(3x^2-5x+1)}.$

2.29. $y=\frac{7}{(x+2)^5}-\sqrt{8-5x+2x^2}.$

2.30. $y=\sqrt[3]{(x-1)^5}+\frac{5}{2x^2-4x+7}.$

3.

3.1. $y=\sin^3 2x\cdot\cos 8x^5.$

3.2. $y=\cos^5 3x\cdot tg(4x+1)^3.$

3.3. $y=tg^4 x\cdot\arcsin 4x^5.$

3.4. $y=\arcsin^3 2x\cdot ctg 7x^4.$

3.5. $y=ctg 3x\ \cdot\arccos 3x^2.$

3.6. $y=\arccos^2 4x\cdot\ln(x-3).$

3.7. $y=\ln^5 x\cdot arctg 7x^4.$

3.8. $y=arctg^3 4x\cdot 3^{\sin x}.$

3.9. $y=2^{\cos x}\cdot arcctg 5x^3.$

3.10. $y=4^{-x}\cdot\ln^5(x+2).$

3.11. $y=3^{tgx}\cdot\arcsin 7x^4.$

3.12. $y=5^{x^2}\cdot\arccos 2x^5.$

3.13. $y=\sin^4 3x\cdot arctg 2x^3.$

3.14. $y=\cos^3 4x\cdot arcctg\sqrt{x}.$

3.15. $y=tg^3 2x\cdot\arcsin x^5.$

3.16. $y=ctg^7 x\cdot\arccos 2x^3.$

3.17. $y=e^{-\sin x} tg 7x^6$

3.18. $y=e^{\cos x} ctg 8x^3.$

3.19. $y=\cos^5 x\cdot\arccos 4x.$

3.20. $y=\sin^3 7x\cdot arcctg 5x^2.$

3.21. $y=\sin^2 3x\cdot arcctg 3x^5.$

3.22. $y=\cos\sqrt[5]{x}\cdot arctg x^4.$

3.23. $y=tg^6 2x\cdot\cos 7x^2.$

3.24. $y=ctg^3 4x\cdot\arcsin\sqrt{x}.$

3.25. $y=ctg\frac{1}{x}\cdot\arccos x^4.$

3.26. $y=tg\sqrt{x}\cdot arcctg 3x^5.$

3.27. $y=tg^3 2x\cdot\arccos 2x^3.$

3.28. $y=2^{tgx} arctg^5 3x.$

3.29. $y=\sin^5 3x\cdot arctg\sqrt{x}.$

3.30. $y=\cos^4 3x\cdot\arcsin 3x^2.$

4.

4.1. $y = arcctg^2 5x \cdot \ln(x-4).$

4.2. $y = arctg^3 2x \cdot \ln(x+5).$

4.3. $y = \arccos^4 x \cdot \ln(x^2+x-1).$

4.4. $y = \sqrt{\arccos 2x} \cdot 3^{-x}.$

4.5. $y = tg^4 3x \cdot arctg 7x^2.$

4.6. $y = 5^{-x^2} \arcsin 3x^3.$

4.7. $y = arctg^5 x \cdot \log_2(x-3).$

4.8. $y = \log_3(x+5) \cdot \arccos 3x.$

4.9. $y = e^{-x} \cdot \arcsin^2 5x.$

4.10. $y = \log_4(x-1) \cdot \arcsin^4 x.$

4.11. $y = (x-4)^5 \cdot arcctg 3x^2.$

4.12. $y = ctg^3 4x \cdot arctg 2x^3.$

4.13. $y = e^{-\cos x} \cdot arctg 7x^5.$

4.14. $y = (x+1)\arccos 3x^4.$

4.15. $y = 2^{\sin x} arcctg x^4.$

4.16. $y = 3^{-x^3} arctg 2x^5.$

4.17. $y = 3^{\cos x} \arcsin^2 3x.$

4.18. $y = \ln(x-10) \cdot arccox^2 4x.$

4.19. $y = \lg(x-2) \cdot \arcsin^5 x.$

4.20. $y = \log_3(x+1) \cdot arctg^5 7x.$

4.21. $y = \ln(x+9) \cdot arcctg^3 2x.$

4.22. $y = \lg(x+2) \cdot \arcsin^2 3x.$

4.23. $y = 4^{-\sin x} arctg 3x.$

4.24. $y = 2^{\cos x} arcctg^3 x.$

4.25. $y = \lg(x-3) \cdot \arcsin^2 5x.$

4.26. $y = \log_2(x+3) \cdot \arccos^2 x.$

4.27. $y = 2^{-x} arctg^3 4x.$

4.28. $y = \ln(x-4) \cdot arcctg^4 3x.$

4.29. $y = \lg(x+3) \cdot arcctg^2 5x.$

4.30. $y = \log_5(x+1) \cdot arctg^2 x^3.$

5.

5.1. $y = tg^4 3x \cdot \arcsin 2x^3.$

5.2. $y = (x-2)^4 \arcsin 5x^4.$

5.3. $y = 2^{-x^3} arctg\, 7x^4.$

5.4. $y = (x+6)^5 arcctg 3x^5.$

5.5. $y = 3^{\cos x} \ln(x^2-3x+7).$

5.6. $y = \log_2(x-7) \cdot arctg \sqrt{x}.$

5.7. $y = \arccos^3 5x \cdot tgx^4.$

5.8. $y = (x-5)^7 arcctg 7x^3.$

5.9. $y = \arccos x^2 \cdot ctg 7x^3.$

5.10. $y = 5^{-x^2} \arccos 5x^4.$

5.11. $y = arctg^4 x \cdot \cos 7x^4.$

5.12. $y = 4(x-7)^6 \arcsin 3x^5.$

5.13. $y = (x+5)^4 \arccos^3 5x.$

5.14. $y = 2^{-\sin x} \arcsin^3 2x.$

5.15. $y = (x+2)^7 \arccos \sqrt{x}.$

5.16. $y = (x-7)^5 \arcsin 7x^4.$

5.17. $y = \ln(x-3) \cdot \arccos 3x^4.$

5.18. $y = \log_2(x-4) \cdot arctg^3 4x.$

5.19. $y = (x-4)^4 arcctg^2 7x.$

5.20. $y = \sqrt[3]{x-3} \arccos^4 2x.$

5.21. $y = \sqrt[3]{x-4} \arcsin^4 5x.$

5.22. $y = (x-3)^5 \arccos 3x^6.$

5.23. $y = \sqrt{(x+3)^5} \arcsin 2x^3.$

5.24. $y = \sqrt[3]{(x+1)^2} \arccos 3x.$

5.25. $y = tg^3 x \cdot arcctg 3x.$

5.26. $y = \sqrt{(x-2)^3} arctg(7x-1).$

5.27. $y = \sqrt[5]{(x+4)^2} \arcsin 7x^2.$

5.28. $y = \arcsin^3 4x \cdot ctg 3x.$

5.29. $y = e^{-\cos x} \arcsin 2x.$

5.30. $y = \sqrt{(x+5)^3} \arccos^4 x.$

6.

6.1. $y = (x-3)^4 \arccos 5x^3.$

6.2. $y = (3x-4)^4 \arccos 3x^2.$

6.3. $y = sh^3 4x \cdot \arccos \sqrt{x}.$

6.4. $y = th^2 \sqrt{x} \cdot arcctg 3x^2.$

6.5. $y = cth^3 5x \cdot \arcsin 3x^2.$

6.6. $y = ch\frac{1}{x} \cdot arctg(7x+2).$

6.7. $y = ch^3 4x \cdot \arccos 4x^2.$

6.8. $y = sh^3 3x \cdot arcctg 5x^2.$

6.9. $y = th^5 3x \cdot \arcsin\sqrt{x}.$

6.10. $y = cth^2(x+1) \cdot \arccos\frac{1}{x}.$

6.11. $y = sh^4 2x \cdot \arccos x^2.$

6.12. $y = ch^3(3x+2) \cdot arctg 3x.$

6.13.6 $y = th^3 4x \cdot arcctg 3x^4.$

6.14. $y = cth^4 7x \cdot \arcsin\sqrt{x}.$

6.15. $y = sh^3 2x \cdot \arcsin 7x^2.$

6.16. $y = th^5 4x \cdot \arccos 3x^4.$

6.17. $y = ch^2 5x \cdot arctg\sqrt{x}.$

6.18. $y = cth^4 2x \cdot arctgx^3.$

6.19. $y = sh^4 5x \cdot \arccos 3x^2.$

6.20. $y = ch^3 9x \cdot arctg(5x-1).$

6.21. $y = th^4 x \cdot arcctg\frac{1}{x}.$

6.22. $y = cth^3 4x \cdot \arcsin(3x+1).$

6.23. $y = ch^2 5x \cdot arctgx^4.$

6.24. $y = th^4 7x \cdot \arccos x^3.$

6.25. $y = cth 4x^5 * \arccos 2x$.

6.26. $y = cth 3x * \arcsin^4 2x.$

6.27. $y = th^5 3x * arctg\sqrt{x}.$

6.28. $y = sh^4 3x * \arccos 5x^4$.

6.29. $y = cth^2 4x * \arcsin x^3.$

6.30. $y = th^3 5x * arctg(2x-5).$

7.

7.1. $y = \frac{e^{\arccos^3 x}}{\sqrt{x+5}}.$

7.2. $y = \frac{(x-4)^2}{e^{arcctgx}}.$

7.3. $y = \frac{e^{-x^3}}{\sqrt{x^2+5x-1}}.$

7.4. $y = \frac{e^{-ctg 5x}}{(3x^2-4x+2)}.$

7.5. $y = \frac{\sqrt{7x^3-5x+2}}{e^{\cos x}}.$

7.6. $y = \frac{e^{tg 3x}}{\sqrt{3x^2-x+4}}.$

7.7. $y = \frac{e^{\sin x}}{(x-5)^7}.$

7.8. $y = \frac{\sqrt[3]{2x^2-3x+1}}{e^{-x}}.$

7.9.7.9 $y = \frac{\sqrt{x^3+4x-5}}{e^{x^3}}$

7.10. $y = \frac{e^{ctg 5x}}{(x+4)^3}.$

7.11. $y = \frac{\sqrt{3+2x-x^2}}{e^x}.$

7.12. $y = \frac{e^{3x}}{\sqrt{3x^2-4x-7}}.$

7.13. $y = \frac{e^{-\sin 2x}}{(x+5)^4}.$

7.14. $y = \frac{e^{\cos 5x}}{\sqrt{x^2-5x-2}}.$

7.15. $y = \frac{(2x+5)^3}{e^{tgx}}$.

7.16. $y = \frac{e^{-tg 3x}}{4x^2-3x+5}.$

7.17. $y = \frac{e^{-\sin 4x}}{(2x-5)^6}.$

7.18. $y = \frac{3x^2-5x+10}{e^{-x^4}}.$

7.19. $y = \frac{e^{-x}}{(2x^2-x+4)^2}.$

7.20. $y = \frac{e^{4x}}{(3x+5)^3}.$

7.21. $y = \frac{e^{ctg 5x}}{(3x-5)^4}.$

7.22. $y = \frac{(2x-3)^7}{e^{-2x}}.$

7.23. $y = \frac{(3x+1)^4}{e^{4x}}.$

7.24. $y = \frac{5x^2+4x-2}{e^{-x}}$

7.25. $y = \frac{\sqrt{5x^2-x+1}}{e^{3x}}.$

7.26. $y = \frac{e^{-x^2}}{(2x-5)^7}.$

7.27. $y=\frac{e^{\cos 3x}}{(2x+4)^5}$.

7.28. $y=\frac{e^{\sin 5x}}{(3x-2)^2}$.

7.29. $y=\frac{\sqrt{x^2-3x-7}}{e^{-x^3}}$.

7.30. $y=\frac{e^{-tgx}}{4x^2+7x-5}$.

8.

8.1. $y=\frac{\log_5(3x-7)}{ctg7x^3}$.

8.2. $y=\frac{\ln(5x-3)}{4tg3x^4}$.

8.3. $y=\frac{\ln(7x+2)}{5\cos 42x}$.

8.4. $y=\frac{\sin^3 5x}{\ln(2x-3)}$.

8.5. $y=\frac{\cos^2 3x}{\lg(3x-4)}$.

8.6. $y=\frac{tg^3 2x}{\lg(5x+1)}$.

8.7. $y=\frac{\log_3(4x+5)}{2ctg\sqrt{x}}$.

8.8. $y=\frac{\ln(7x-3)}{3tg^2 4x}$.

8.9. $y=\frac{\lg(11x+3)}{\cos^2 5x}$.

8.10. $y=\frac{ctg^2 5x}{\ln(7x-2)}$.

8.11. $y=\frac{tg^2(x-2)}{\lg(x+3)}$.

8.12. $y=\frac{\sin^3(5x+1)}{\lg(3x-2)}$.

8.13. $y=\frac{\cos^4(7x-1)}{\lg(x+5)}$.

8.14. $y=\frac{\sin^3(4x+3)}{\ln(7x+1)}$.

8.15. $y=\frac{ctg^3(2x-3)}{\log_3(x+2)}$.

8.16. $y=\frac{\lg^3 x}{\sin 5x^2}$.

8.17. $y=\frac{\ln^2(x+1)}{\cos 3x^4}$.

8.18. $y=\frac{\log_2(7x-5)}{tg\sqrt{x}}$.

8.19. $y=\frac{\log_3(4x-2)}{ctg 2x}$.

8.20. $y=\frac{\ln^3(x-5)}{tg(\frac{1}{x})}$.

8.21. $y=\frac{\lg(x+2)}{\sin 2x^5}$.

8.22. $y=\frac{tg^3 7x}{\ln(3x+2)}$.

8.23. $y=\frac{ctg\sqrt{x-2}}{\lg(3x+5)}$.

8.24. $y=\frac{tg(3x-5)}{\ln^2(x+3)}$.

8.25. $y=\frac{\cos^2 x}{\lg(x^2-2x+1)}$.

8.26. $y=\frac{\log_2(3x+7)}{tg3x}$.

8.27. $y=\frac{\ln^3 x}{ctg(x-3)}$.

8.28. $y=\frac{tg^4 5x}{\ln(x+7)}$.

8.29. $y=\frac{\log_3(x+4)}{\cos^5 x}$.

8.30. $y=\frac{tg^4 3x}{\lg(x^2-x+4)}$.

9.

9.1. $y=\frac{arcctg^4 5x}{sh\sqrt{x}}$.

9.2. $y=\frac{arctg^3 2x}{ch(\frac{1}{x})}$.

9.3. $y=\dfrac{ar\cos 3x^4}{th^2x}.$

9.4. $y=\dfrac{\arcsin 5x^3}{ch\sqrt{x}}.$

9.5. $y=\dfrac{cth^3(x+1)}{\arccos 2x}.$

9.6. $y=\dfrac{th3x^5}{arctg^2 3x}.$

9.7. $y=\dfrac{\arccos^7 2x}{thx^5}.$

9.8. $y=\dfrac{\arcsin^3 4x}{sh(3x+1)}.$

9.9. $y=\dfrac{th^4(2x+5)}{\arccos 3x}.$

9.10. $y=\dfrac{\sqrt[3]{arctg 2x}}{sh^2x}.$

9.11. $y=\dfrac{\arcsin^2 4x}{th(5x-3)}.$

9.12. $y=\dfrac{ch^2(4x+2)}{arctgx^3}.$

9.13. $y=\dfrac{\arcsin 4x^5}{th^3x}.$

9.14. $y=\dfrac{arctg^3(2x+1)}{ch\sqrt{x}}.$

9.15. $y=\dfrac{\arccos 4x^3}{sh^4x}.$

9.16. $y=\dfrac{cth^2(x-2)}{\arccos 3x}.$

9.17. $y=\dfrac{th^3(2x+2)}{\arcsin 5x}.$

9.18. $y=\dfrac{cth^2(3x-1)}{\arccos x^2}.$

9.19. $y=\dfrac{sh^5x}{\arccos 4x}.$

9.20. $y=\dfrac{\sqrt{ch^3x}}{arctg 5x}.$

9.21. $y=\dfrac{th^2(x+3)}{arctg\sqrt{x}}.$

9.22. $y=\dfrac{\arcsin^2 3x}{ch(x-5)}.$

9.23. $y=\dfrac{arcctg^3x}{sh(2x-5)}.$

9.24. $y=\dfrac{\arccos^3 5x}{th(x-2)}.$

9.25. $y=\dfrac{\sqrt{\arccos 3x}}{sh^2x}.$

9.26. $y=\dfrac{\arcsin^2 3x}{\sqrt{thx}}.$

9.27. $y=\dfrac{arctg^2 5x}{\sqrt[3]{cthx}}.$

9.28. $y=\dfrac{arctg^2 5x}{th(x+3)}.$

9.29. $y=\dfrac{\sqrt{sh^3x}}{arctg 5x}.$

9.30. $y=\dfrac{\sqrt[5]{ch3x}}{arctg(x+2)}.$

10.

10.1. $y=\dfrac{9arctg(x+7)}{(x-1)^2}.$

10.2. $y=\dfrac{8arctg(2x+3)}{(x+1)^3}.$

10.3. $y=\dfrac{7\arccos(4x-1)}{(x+2)^4}.$

10.4. $y=\dfrac{6\arcsin(x+5)}{(x-2)^5}.$

10.5. $y=\dfrac{3arcctg(2x-5)}{(x+1)^4}.$

10.6. $y=\dfrac{2arctg(3x+2)}{(x-3)^2}.$

10.7. $y=\dfrac{4\arccos 3x}{(x+2)^5}.$

10.8. $y=\dfrac{\arcsin(3x+8)}{(x-7)^3}.$

10.9. $y=\dfrac{7arctg(4x+1)}{(x-4)^2}.$

10.10. $y=\dfrac{3\arcsin(2x-7)}{(x+2)^4}.$

10.11. $y=\frac{2\lg(4x+5)}{(x+6)^4}$.

10.12. $y=\frac{5\ln(5x+7)}{(x-7)^2}$.

10.13. $y=\frac{4\log_3(3x+1)}{(x+1)^2}$.

10.14. $y=\frac{7\log_4(2x-5)}{(x-1)^5}$.

10.15. $y=\frac{\ln(7x+2)}{(x-6)^4}$.

10.16. $y=\frac{4\lg(3x+7)}{(x+1)^7}$.

10.17. $y=\frac{5\log_2(x^2+1)}{(x-3)^4}$.

10.18. $y=\frac{6\log_3(2x+9)}{(x+4)^2}$.

10.19. $y=\frac{3\log_2(5x-4)}{(x-3)^5}$.

10.20. $y=\frac{7\log_5(x^2+x)}{(x+3)^3}$.

10.21. $y=\frac{\log_7(2x^2+5)}{(x-4)^2}$.

10.22. $y=\frac{2\ln(3x-10)}{(x+5)^7}$.

10.23. $y=\frac{8\lg(4x+5)}{(x-1)^5}$.

10.24. $y=\frac{2\log_3(4x-7)}{(x+3)^4}$.

10.25. $y=\frac{3\log_4(2x+9)}{(x-7)^2}$.

10.26. $y=\frac{\lg(x^2+2x)}{(x+8)^4}$.

10.27. $y=\frac{3\ln(x^2+5)}{(x-7)^3}$.

10.28. $y=\frac{4\log_2(3x-5)}{(x-2)^2}$.

10.29. $y=\frac{2\ln(2x^2+3)}{(x-7)^4}$.

10.30. $y=\frac{4\lg(3x+7)}{(x-5)^3}$.

11.

11.1. $y=\sqrt{\frac{2x+1}{2x-1}}\log_2(x-3x^2)$.

11.2. $y=\sqrt[3]{\frac{2x-5}{2x+3}}\lg(4x+7)$.

11.3. $y=\sqrt[4]{\frac{x+3}{x-3}}\ln(5x^2-2x+1)$.

11.4. $y=\sqrt[5]{\frac{x+1}{x-1}}\log_3(x^2+x+4)$.

11.5. $y=\sqrt[6]{\frac{7x-4}{7x+4}}\log_5(3x^2+2x)$.

11.6. $y=\sqrt[7]{\frac{2x-3}{2x+1}}\lg(7x-10)$.

11.7. $y=\sqrt[8]{\frac{5x+1}{5x-1}}\ln(3x-x^2)$.

11.8. $y=\sqrt[9]{\frac{x+3}{x-3}}\log_5(2x-3)$.

11.9. $y=\sqrt{\frac{6x+5}{6x-5}}\lg(4x+7)$.

11.10. $y=\sqrt[3]{\frac{4x-1}{4x+1}}\ln(2x^3-3)$.

11.11. $y=\sqrt[4]{\frac{x+6}{x-6}}\sin(3x^2+1)$.

11.12. $y=\sqrt[5]{\frac{x-7}{x+7}}\cos(2x^3+x)$.

11.13. $y=\sqrt[6]{\frac{x-9}{x+9}}tg(3x^2-4x+1)$.

11.14. $y=\sqrt[7]{\frac{x-4}{x+4}}ctg(2x+5)$.

11.15. $y=\sqrt[8]{\frac{x-2}{x+2}}\sin(4x^2-7x+2)$.

11.16. $y=\sqrt[9]{\frac{x-3}{x+3}}\cos(x^2-3x+2)$.

11.17. $y=\sqrt{\frac{3x-2}{3x+2}}tg(2x^2-9)$.

11.18. $y=\sqrt{\frac{2x+3}{2x-3}}ctg(3x^2+5)$.

11.19. $y=\sqrt[4]{\frac{x+5}{x-5}}\sin(3x^2-x+4).$

11.20. $y=\sqrt[5]{\frac{x-6}{x+6}}\cos(7x+2).$

11.21. $y=\sqrt[6]{\frac{x-7}{x+7}}\arcsin(2x+3).$

11.22. $y=\sqrt[7]{\frac{x-8}{x+8}}\arccos(3x-5).$

11.23. $y=\sqrt[8]{\frac{x-4}{x+4}}arctg(5x+1).$

11.24. $y=\sqrt[9]{\frac{x-1}{x+1}}arcctg(7x+2).$

11.25. $y=\sqrt{\frac{7x-4}{7x+4}}\arcsin(x^2+1).$

11.26. $y=\sqrt[3]{\frac{8x-3}{8x+3}}\arccos(x^2-5).$

11.27. $y=\sqrt[4]{\frac{2x-5}{2x+5}}arctg(3x+2).$

11.28. $y=\sqrt[5]{\frac{3x-4}{3x+4}}arcctg(2x+5)$.

11.29. $y=\sqrt[6]{\frac{x^2-1}{x^2+1}}\arcsin 2x.$

11.30. $y=\sqrt[7]{\frac{x^2+3}{x^2-3}}\arccos 4x.$

12.

12.1. $y=(cth3x)^{\arcsin x}.$

12.2. $y=(\cos(x+2))^{\ln x}.$

12.3. $y=(\sin 3x)^{\arccos x}.$

12.4. $y=(th5x)^{\arcsin(x+1)}.$

12.5. $y=(sh(x+2))^{\arcsin 2x}.$

12.6. $y=(\cos 5x)^{arctg\sqrt{x}}.$

12.7. $y=(\sqrt{3x+2})^{arcctg 3x}.$

12.8. $y=(\ln(x+3))^{\sin\sqrt{x}}.$

12.9. $y=(\log_2(x+4))^{ctg 7x}.$

12.10. $y=(sh3x)^{arctg(x+2)}.$

12.11. $y=(ch3x)^{ctg\frac{1}{x}}.$

12.12. $y=(\arcsin 5x)^{tg\sqrt{x}}.$

12.13. $y=(\arccos 5x)^{\ln x}.$

12.14. $y=(arctg 2x)^{\sin x}.$

12.15. $y=(\ln(x+7))^{ctg 2x}.$

12.16. $y=(ctg(7x+4))^{\sqrt{x+3}}.$

12.17. $y=(th\sqrt{x+1})^{arctg 2x}.$

12.18. $y=(cth\frac{1}{x})^{\arcsin 7x}.$

12.19. $y=(\cos(x+5))^{\arcsin 3x}.$

12.20. $y=(\sqrt{x+5})^{\arccos 3x}.$

12.21. $y=(\sin 4x)^{arctg\frac{1}{x}}.$

12.22. $y=(tg 3x^4)^{\sqrt{x+3}}.$

12.23. $y=(ctg 2x^3)^{\sin\sqrt{x}}.$

12.24. $y=(tg 7x^5)^{\sqrt{x+2}}.$

12.25. $y=(\arccos x)^{\sqrt{\cos x}}.$

12.26. $y=(ctg 7x)^{sh(x+3)}.$

12.27. $y=(sh5x)^{arctg(x+2)}.$

12.28. $y=(arctgx)^{th(3x+1)}.$

12.29. $y=(cth\sqrt{x})^{\sin(x+3)}.$

12.30. $y=(sh3x)^{arcctg 2x}.$

13.

13.1. $y=(\arccos(x+2))^{tg 3x}.$

13.2. $y=(\arcsin 2x)^{ctg(x+1)}.$

13.3. $y=(arctg(x+7))^{\cos 2x}.$

13.4. $y=(arcctg(x-3))^{\sin 4x}.$

13.5. $y=(ctg(3x-2))^{\arcsin 3x}.$

13.6. $y=(tg(4x-3))^{\arccos 2x}.$

13.7. $y=(\cos(2x-5))^{arctg 5x}.$

13.8. $y=(\sin(7x+4))^{arcctgx}.$

13.9. $y=(\arcsin 2x)^{\ln(x+3)}.$

13.10. $y=(\arccos 3x)^{\lg(5x-1)}.$

13.11. $y=(arctg 5x)^{\log_2(x+4)}.$

13.12. $y=(arctg 7x)^{\lg(x+1)}.$

13.13. $y=(\log_4(2x+3))^{\arcsin x}.$

13.14. $y=(\log_5(3x+2))^{\arccos x}.$

13.15. $y=(\lg(7x-5))^{arctg\,2x}$.

13.16. $y=(\ln(5x-4))^{arcctgx}$.

13.17. $y=(\log_2(6x+5))^{\arcsin 2x}$.

13.18. $y=(\lg(4x-3))^{\arccos 4x}$.

13.19. $y=(\ln(7x-3))^{arctg\,5x}$.

13.20. $y=(\log_5(2x+5))^{arctgx}$.

13.21. $y=(\sin(8x-7))^{cth(x+3)}$.

13.22. $y=(\cos(3x+8))^{th(x-7)}$.

13.23. $y=(tg(9x+2))^{ch(2x-1)}$.

13.24. $y=(ctg(7x+5))^{sh3x}$.

13.25. $y=(sh(3x-7))^{\cos(x+4)}$.

13.26. $y=(ch(2x-3))^{tg(x+5)}$.

13.27. $y=(th(7x-5))^{\sin(x+2)}$.

13.28. $y=(ch(3x+2))^{\cos(x+4)}$.

13.29. $y=(\ln(7x+4))^{tgx}$.

13.30. $y=(\lg(8x+3))^{tg5x}$.

14.

14.1. $y=\dfrac{\sqrt{x+7}(x-3)^4}{(x+2)^5}$.

14.2. $y=\dfrac{(x-3)^5(x+2)^3}{\sqrt{(x-1)^3}}$.

14.3. $y=\dfrac{(x-2)^3\sqrt{(x+1)^5}}{(x-4)^2}$.

14.4. $y=\dfrac{(x+3)\sqrt[5]{(x-2)^2}}{(x+1)^7}$.

14.5. $y=\dfrac{(x+2)^7(x-3)^3}{\sqrt{(x+1)^5}}$.

14.6. $y=\dfrac{(x-1)^4(x+2)^5}{\sqrt[3]{(x-4)^2}}$.

14.7. $y=\dfrac{(x-3)^2\sqrt{x+4}}{(x+2)^7}$.

14.8. $y=\dfrac{(x-7)^{10}\sqrt{3x-1}}{(x+3)^5}$.

14.9. $y=\dfrac{(x+1)^8(x-3)^2}{\sqrt{(x+2)^5}}$.

14.10. $y=\dfrac{(x+2)(x-7)^4}{\sqrt[3]{(x-1)^4}}$.

14.11. $y=\dfrac{\sqrt[5]{(x+4)^3}}{(x-1)^2(x+3)^5}$.

14.12. $y=\dfrac{\sqrt[3]{(x-1)^7}}{(x+1)^5(x-5)^3}$.

14.13. $y=\dfrac{\sqrt{(x+2)^3}(x-1)^4}{(x+2)^7}$.

14.14. $y=\dfrac{\sqrt[3]{(x-2)^5}(x+3)^2}{(x-7)^3}$.

14.15. $y=\dfrac{\sqrt[4]{x-8}(x+2)^6}{(x-1)^5}$.

14.16. $y=\dfrac{\sqrt[5]{x+1}(x-3)^7}{(x+8)^3}$.

14.17. $y=\dfrac{\sqrt[7]{(x-2)^4}}{(x+1)^2(x-6)^5}$.

14.18. $y=\dfrac{\sqrt[5]{(x+1)^2}}{(x-3)^4(x-4)^3}$.

14.19. $y=\dfrac{\sqrt{x^2+2x-3}}{(x+3)^7(x-4)^2}$.

14.20. $y=\dfrac{\sqrt[3]{(x-2)^4}}{(x-5)(x+1)^7}$.

14.21. $y=\dfrac{(x+4)^3(x-2)^3}{\sqrt[5]{(x-2)^2}}$.

14.22. $y=\dfrac{(x-1)^6(x+2)^3}{\sqrt[5]{(x+3)^2}}$.

14.23. $y=\dfrac{(x-1)^4(x-7)^2}{\sqrt[3]{(x+2)^5}}$.

14.24. $y=\dfrac{(x+7)^2(x-3)^5}{\sqrt{x^2+3x-1}}$.

14.25. $y=\dfrac{\sqrt[3]{x-3}(x+7)^5}{(x-4)^2}$.

14.26. $y=\dfrac{\sqrt{x+10}(x-8)^3}{(x-1)^5}$.

14.27. $y=\dfrac{\sqrt[5]{(x-2)^3}(x-1)}{(x+3)^4}$.

14.28. $y=\dfrac{\sqrt[4]{(x+1)^3}(x-2)^5}{(x-3)^2}$.

14.29. $y=\dfrac{\sqrt[6]{(x-1)^5}}{(x+2)^4(x-5)^7}.$

14.30. $y=\dfrac{\sqrt[5]{(x+2)^3}}{(x-1)^4(x-3)^5}.$

Namunaviy variantni yechish

Berilgan funksiyalarni differensiyalang

1. $y=9x^5-\dfrac{4}{x^3}+\sqrt[3]{x^7}-3x+4.$

► $y'=9\cdot 5x^4-4(-3)x^{-4}+\dfrac{7}{3}x^{\frac{4}{3}}-3=45x^4+\dfrac{12}{x^4}+\dfrac{7}{3}\sqrt[3]{x^4}-3.$ ◄

2. $y=\sqrt[4]{(2x^2-3x+1)^3}-\dfrac{6}{(x+1)^3}.$

► $y'=\dfrac{3}{4}(2x^2-3x+1)^{-\frac{1}{4}}(4x-3)-6(-3)(x+1)^{-4}=\dfrac{3}{4}\dfrac{4x-3}{\sqrt[4]{2x^2-3x+1}}+\dfrac{18}{(x+1)^4}.$ ◄

3. $y=tg^5(x+2)\cdot\arccos 3x^2.$

► $y'=5tg^4(x+2)\cdot\dfrac{1}{\cos^2(x+2)}\arccos 3x^2+tg^5(x+2)\cdot\left(\dfrac{1}{\sqrt{1-9x^4}}\right)\cdot 6x=$

.

$=\dfrac{5tg^4(x+2)\cdot\arccos 3x^2}{\cos^2(x+2)}-\dfrac{tg^5(x+2)\cdot 6x}{\sqrt{1-9x^4}}$ ◄

4. $y=\arcsin^5 4x\cdot\log_2(x-5).$

► $y'=5\arcsin^4 4x\cdot\dfrac{1}{\sqrt{1-16x^2}}\cdot 4\log_2(x-5)+\arcsin^5 4x\cdot\dfrac{1}{(x-5)\ln 2}=$

$=\dfrac{20\arcsin^4 4x\cdot\log_2(x-5)}{\sqrt{1-16x^2}}+\dfrac{\arcsin^5 4x}{(x-5)\ln 2}.$ ◄

5. $y=3^{-x^4}ctg\,7x^3.$

► $y'=3^{-x^4}\ln 3\cdot(-4x^3)ctg7x^3+3^{-x^4}(\dfrac{1}{-\sin^2 7x^3})\cdot 21x^2=-4\ln 3\cdot 3^{-x^4}ctg7x^3-\dfrac{21x^3\cdot 3^{-x^4}}{\sin^2 7x^3}.$ ◄

6. $y=cth^2 3x\cdot arctg\sqrt{x}.$

► $y'=2cth3x\cdot\left(-\dfrac{1}{sh^2 3x}\right)\cdot 3arctg\sqrt{x}+cth^2 3x\times\dfrac{1}{1+x}\cdot\dfrac{1}{2\sqrt{x}}=$

$=-\dfrac{6cth3x\cdot arctg\sqrt{x}}{sh^2 3x}+\dfrac{cth^2 3x}{(1+x)2\sqrt{x}}.$ ◄

7. $y=\sqrt{3x^2-7x+\dfrac{5}{e^{-x^4}}.}$

► $y'=(\sqrt{3x^2-7x+\dfrac{5}{e^{-x^4}}.})'=\dfrac{(6x-7)e^{x^4}}{2\sqrt{3x^2-7x+5}}+\sqrt{3x^2-7x+5}\cdot e^{x^4}\cdot 4x^3=$

$$=\frac{(6x-7)e^{x^4}}{2\sqrt{3x^2-7x+5}}+4x^3e^{x^4}\sqrt{3x^2-7x+5}. ◀$$

8 $y=\frac{(\lg(x^2-3x+5))}{arcctg^2 5x}.$

$$▶\ y'=\left(\frac{2x-3}{(x^2-3x+5)\ln 10}arcctg^2 5x-\lg(x^2-3x+5)\times 2\cdot arcctg5x\cdot\left(-\frac{1}{1+25x^2}\right)\cdot 5\right)*$$

$$*\,arcctg^4 5x=\left(\frac{(2x-3)arcctg^2 5x}{(x^2-3x+5)\ln 10}+\frac{10\lg(x^2-3x+5)\cdot arcctg5x}{1+25x^2}\right)\cdot arcctg^{-4}5x. ◀$$

9. $y=\sqrt{\frac{\arcsin 3x}{sh^2 x}}.$

$$▶\ y'=\frac{\frac{1}{2\sqrt{\arcsin 3x}}\frac{1}{\sqrt{1-9x^2}}\cdot 3sh^2x-2shxchx\sqrt{\arcsin 3x}}{sh^4 x}=.$$

$$=\frac{\frac{3sh^2x}{2\sqrt{\arcsin 3x}\sqrt{1-9x^2}}-ch2x\sqrt{\arcsin 3x}}{sh^4x} ◀$$

10. $y=\frac{(3\ln(x^2-5))}{(x+3)^7}.$

$$▶\ y'=3\frac{\frac{1}{x^2-5}\cdot 2x(x+3)^7-7(x+3)^6\ln(x^2-5)}{(x+3)^{14}}=$$

$$=3\frac{\cdot\frac{2x(x+3)}{x^2-5}-7\cdot\ln(x^2-5)}{(x+3)^8} ◀$$

11. $y=\sqrt[7]{\frac{(x+5)}{(x-5)}}ctg(3x-4).$

$$▶\ y'=\frac{1}{7}\left(\frac{x+5}{x-5}\right)^{-\frac{6}{7}}\frac{x-5-(x+5)}{(x-5)^2}ctg(3x-4)-\frac{1}{\sin^2(3x-4)}\cdot 3\sqrt[7]{\frac{x+5}{x-5}}=$$

$$=-\frac{10}{7}\frac{ctg(3x-4)}{\sqrt[7]{\frac{(x+5)^6}{(x-5)^6}}}-\frac{3}{\sin^2(3x-4)}\sqrt[7]{\frac{x+5}{x-5}}. ◀$$

12. $y=(th\sqrt{x+2})^{\ln(3x+2)}.$

▶Berilgan funksiyani logarifmlaymiz:

$\ln y=\ln(3x+2)\ln(th\sqrt{x+2}).$ Bu tenglikni differensiyalasak

$$\frac{1}{y}y'=\frac{3}{3x+2}\ln(th\sqrt{x+2})+\ln(3x+2)\cdot\frac{1}{th\sqrt{x+2}ch^2\sqrt{x+2}}\frac{1}{2\sqrt{x+2}}.$$

Oxirgi tenglikdan y' ni topamiz

$$y'=(th\sqrt{x+2})^{\ln(3x+2)}\left(\frac{3\ln(th\sqrt{x+2})}{3x+2}+\frac{\ln(3x+2)}{2\sqrt{x+2}sh\sqrt{x+2}ch\sqrt{x+2}}\right). ◀$$

13. $y=(\sin 7x)^{arctg\,(3x-5)}$.

► Berilgan funksiyani logarifmlaymiz: $\ln y = arctg(3x-5)\cdot \ln(\sin 7x)$

Bu tenglikni differensiyalasak,

$$\frac{1}{y}y'=\frac{3}{1+(3x-5)^2}\ln(\sin 7x)+7\frac{\cos 7x}{\sin 7x}arctg(3x-5)$$

Bu yerdan

$$y'=(\sin 7x)^{arctg(3x-5)}\cdot\left(\frac{3\cdot\ln(\sin 7x)}{1+(3x-5)^2}+7ctg7x\cdot arctg(3x-5)\right)$$ ◄

14. $y=\sqrt[7]{(x+5)^6}\,/\left((x-1)^2(x+3)^5\right)$.

► Logarifmik differensiyalash usulini qo'llab ketma- ket topamiz:

$$\ln y=\frac{6}{7}\ln(x+5)-2\ln(x-1)-5\ln(x+3)$$

$$\frac{1}{y'}y=\frac{6}{7}\frac{1}{x+5}-2\frac{1}{x-1}-5\frac{1}{x+3}$$

$$y'=\frac{\sqrt[7]{(x+5)^6}}{(x-1)^2(x+3)^5}\left(\frac{6}{7(x+5)}-\frac{2}{x-1}-\frac{5}{x+3}\right)$$ ◄

6.2 -IUT

1. . y' va y'' larni toping:

1.1. $y^2 = 8x.$

1.2. $x^2/5 + y^2/7 = 1.$

1.3. $y = x + arctgy.$

1.4. $x^2/5 + y^2/3 = 1.$

1.5. $y^2 = 25x - 4$.

1.6. $arcctgy = 4x + 5y.$

1.7. $y^2 - x = \cos y.$

1.8. $3x + \sin y = 5y.$

1.9. $tgy = 3x + 5y$.

1.10. $xy = ctgy.$

1.11. $y = e^y + 4x.$

1.12. $\ln y - y/x = 7.$

1.13. $y^2 + x^2 = \sin y.$

1.14. $e^y = 4x - 7y.$

1.15. $4\sin^2(x+y) = x.$

1.16. $\sin y = 7x + 3y.$

1.17. $tgy = 4y - 5x$.

1.18. $y = 7x - ctgy.$

1.19. $xy - 6 = \cos y.$

1.20. $3y = 7 + xy^3.$

1.21. $y^2 = x + \ln(y/x).$

1.22. $xy^2 - y^3 = 4x - 5.$

1.23. $x^2y^2 + x = 5y.$

1.24. $x^4 + x^2y^2 + y = 4.$

1.25. $\sin y = xy^2 + 5$.

1.26. $x^3 + y^3 = 5x.$

1.27. $\sqrt{x} + \sqrt{y} = \sqrt{7}.$

1.28. $y^2 = (x-y)/(x+y).$

1.29. $\sin^2(3x + y^2) = 5.$

1.30. $ctg^2(x+y) = 5x.$

2. y' va y'' larni toping:

2.1. $\begin{cases} x = (2t+3)\cos t \\ y = 3t^3 \end{cases}$

2.2. $\begin{cases} x = 2\cos^2 t \\ y = 3\sin^2 t \end{cases}$

2.3. $\begin{cases} x = 6\cos^3 t \\ y = 2\sin^3 t \end{cases}$

2.4. $\begin{cases} x = 1/(t+2) \\ y = (t/(t+2))^2 \end{cases}$

2.5. $\begin{cases} x = e^{-2t} \\ y = e^{4t} \end{cases}$

2.6. $\begin{cases} x = \sqrt{t} \\ y = \sqrt[5]{t} \end{cases}$

2.7. $\begin{cases} x = 2t/(1+t^3) \\ y = t^2/(1+t^2) \end{cases}$

2.8. $\begin{cases} x = \sqrt{t^2 - 1} \\ y = (t+1)/\sqrt{t^2 - 1} \end{cases}$

2.9. $\begin{cases} x = 4t + 2t^2 \\ y = 5t^3 - 3t^2 \end{cases}$

2.10. $\begin{cases} x = (\ln t)/t \\ y = t\ln t \end{cases}$

2.11. $\begin{cases} x = e^t \cos t \\ y = e^t \sin t \end{cases}$

2.12. $\begin{cases} x = t^4 \\ y = \ln t \end{cases}$

2.13. $\begin{cases} x = 5\cos t \\ y = 4\sin t \end{cases}$

2.14. $\begin{cases} x = 5\cos^2 t \\ y = 3\sin^2 t \end{cases}$

2.15. $\begin{cases} y = arctgt \\ y = \ln(1+t^2) \end{cases}$

2.16. $\begin{cases} x = \arcsin t \\ y = \sqrt{1-t^2} \end{cases}$

2.17. $\begin{cases} x = 3(t - \sin t) \\ y = 3(1 - \cos t) \end{cases}$

2.18. $\begin{cases} x = 3(\sin t - t\cos t) \\ y = 3(\cos t + t\sin t) \end{cases}$

2.19. $\begin{cases} x = \sin 2t \\ y = \cos^2 t \end{cases}$

2.20. $\begin{cases} x = e^{3t} \\ y = e^{-3t} \end{cases}$

2.21. $\begin{cases} x = (\ln t)/t \\ y = t^2 \ln t \end{cases}$

2.22. $\begin{cases} x = \arccos t \\ y = \sqrt{1-t^2} \end{cases}$

2.23. $\begin{cases} x = 1/(t+1) \\ y = (t/(t+1))^2 \end{cases}$

2.24. $\begin{cases} x = 5\sin 3t \\ y = 3\cos^3 t \end{cases}$

2.25. $\begin{cases} x = e^{-3t} \\ y = e^{8t} \end{cases}$

2.26. $\begin{cases} x = \sqrt[3]{(t-1)^2} \\ y = \sqrt{t-1} \end{cases}$

2.27. $\begin{cases} x = \ln^2 t \\ y = t + \ln t \end{cases}$

2.28. $\begin{cases} x = te^t \\ y = t/e^t \end{cases}$

2.29. $\begin{cases} x = 6t^2 - 4 \\ y = 3t^5 \end{cases}$

2.30. $\begin{cases} x = \arcsin t \\ y = \ln t \end{cases}$

3. Berilgan funksiya y va x_0 argument uchun $y'''(x_0)$ hosilani hisoblang.

3.1. $y = \sin^2 x,\ x_0 = \pi/2.$

3.2. $y = \mathrm{arctg}\, x,\ x_0 = 1.$

3.3. $y = \ln(2 + x^2),\ x_0 = 0.$

3.4. $y = e^x \cos x,\ x_0 = 0.$

3.5. $y = e^x \sin 2x,\ x_0 = 0.$

3.6. $y = e^{-x} \cos x,\ x_0 = 0.$

3.7. $y = \sin 2x,\ x_0 = \pi.$

3.8. $y = (2x + 1)^5,\ x_0 = 1.$

3.9. $y = \ln(1 + x),\ x_0 = 2.$

3.10. $y = \frac{1}{2} x^2 e^x,\ x_0 = 0.$

3.11. $y = \arcsin x,\ x_0 = 0.$

3.12. $y = (5x - 4)^5,\ x_0 = 2.$

3.13. $y = x\sin x,\ x_0 = \pi/2.$

3.14. $y = x^2 \ln x,\ x_0 = 1/3.$

3.15. $y = x\sin 2x,\ x_0 = -\pi/4.$

3.16. $y = x\cos 2x,\ x_0 = \pi/12.$

3.17. $y = x^4 \ln x,\ x_0 = 1.$

3.18. $y = x + \mathrm{arctg} x,\ x_0 = 1.$

3.19. $y = \cos^2 x,\ x_0 = \pi/4.$

3.20. $y = \ln(x^2 - 4),\ x_0 = 3.$

3.21. $y = x^2 \cos x,\ x_0 = \pi/2.$

3.22. $y = x \arccos x,\ x_0 = \sqrt{3}/2.$

3.23. $y = (x+1)\ln(x+1),\ x_0 = -1/2.$

3.24. $y = \ln^3 x,\ x_0 = 1.$

3.25. $y = 2^{x^2},\ x_0 = 1.$

3.26. $y = (4x\ \ 3)^5,\ x_0 = 1.$

3.27. $y = x\,\mathrm{arcctg}\, x,\ x_0 = 2.$

3.28. $y = (7x - 4)^6,\ x_0 = 1.$

3.29. $y = x\sin 2x,\ x_0 = \pi/4.$

3.30. $y = \sin(x^3 + \pi),\ x_0 = \sqrt[3]{\pi}.$

4. Quyidagi funksiyalarning *n* – tartibli hosilasini toping.

4.1. $y = \ln x.$

4.2. $y = 1/x.$

4.3. $y = 2^x.$

4.4. $y = \cos x.$

4.5. $y = \sin x.$

4.6. $y = 1/(x + 5).$

4.7. $y = e^{-2x}.$

4.8. $y = \ln(3 + x).$

4.9. $y = \sqrt{x}$.

4.10. $y = xe^{3x}$.

4.11. $y = 1(x - 3)$.

4.12. $y = \ln(5 - x^x)$.

4.13. $y = e^x$.

4.14. *$y = 1/(x - 7)$*

4.15. $y = 5^x$.

4.16. $y = e^{-3x}$.

4.17. $y = \ln(4 + x)$.

4.18. $y = 1/(x\ \ 6)$.

4.19. $y = 10^x$.

4.20. $y = 7^x$.

4.21. $y = \cos 3x$.

4.22. $y = \ln(3x - 5)$.

4.23. $y = \dfrac{x}{(x+5)}$.

4.24. $y = \ln\dfrac{1}{4-x}$.

4.25. $y = \sqrt{x + 7}$.

4.26. $y = xe^x$.

4.27. $y = \dfrac{4}{x+3}$.

4.28. $y = \dfrac{1+x}{\sqrt{x}}$.

4.29. $y = \dfrac{1}{1+x}$.

4.30. $y = \ln(5x-1)$.

5.1. Ushbu $y=x^2 -7x+3$ egri chiziq uchun absissasi $x=1$ bo’lgan nuqtada urinma tenglamasini yozing.

5.2. Ushbu $y=x^2-16x+7$ egri chiziq uchun absissasi $x=1$ bo’lgan nuqtada normal tenglamasini yozing.

5.3. Ushbu $y = \sqrt{x - 4}$ egri chiziq uchun absissasi $x = 8$ bo’lgan nuqtada urinma tenglamasini yozing.

5.4. Ushbu $y = \sqrt{x + 4}$ egri chiziq uchun absissasi $x = -3$ bo’lgan nuqtada urinma tenglamasini yozing.

5.5. Ushbu $y=x^3-2x^2+4x-7$ egri chiziq uchun (2,1) nuqtada urinma tenglamasini yozing.

5.6. Ushbu $u=x^3-5x^2+7x-2$ egri chiziq uchun (1,1) nuqtada urinma tenglamasini yozing.

5.7. Ushbu $x^2-y^2+xu-11=0$ egri chiziqqa (3,2) nuqtadagi urinmaning burchak koeffitsiyentini toping?

5.8. Qaysi nuqtada $y^2=4x^3$ egri chiziqning urinmasi $x+3y-1=0$ to’g’ri chiziqqa perpendikulyar bo’ladi?

5.9. Ushbu $y=x^2-6x+2$ egri chiziqqa absissasi $x=2$ bo’lgan nuqtada urinma tenglamasini yozing.

5.10. Ushbu $y=x^2/4-x+5$ egri chiziqqa absissasi $x=4$ bo’lgan nuqtada urinma tenglamasini yozing.

5.11. Ushbu $y=x^2/4-27x+60$ egri chiziqqa absissasi $x=2$ bo’lgan nuqtada urinma tenglamasini yozing.

5.12. Ushbu $y = -\dfrac{x^2}{2} + 7x - \dfrac{15}{2}$ egri chiziqqa absissasi $x=3$ bo’lgan nuqtada urinma tenglamasini yozing.

5.13. Ushbu $y = 3tg2x+1$ egri chiziqqa absissasi $x=\pi/2$ bo’lgan nuqtada urinma tenglamasini yozing.

5.14. Ushbu $y=4tg3x$ egri chiziqqa absissasi $x=\pi/9$ bo'lgan nuqtada urinma tenglamasini yozing.

5.15. Ushbu $y=6tg5x$ egri chiziqqa absissasi $x=\pi/20$ bo'lgan nuqtada normalning tenglamasini yozing.

5.16. Ushbu $y=4sin6x$ egri chiziqqa absissasi $x=\pi/18$ bo'lgan nuqtada urinmaning tenglamasini yozing.

5.17. Ushbu $y=sin2x$ egri chiziqqa urinma qaysi nuqtada Ox o'ki bilan $\pi/4$ burchak hosil qiladi.

5.18. Ushbu $y=2x^3-1$ egri chiziqqa urinma qaysi nuqtada Ox o'qi bilan $\pi/3$ burchak hosil qiladi.

5.19. Ushbu $y=x^3/3-x^2/2–7x+9$ egri chiziqqa urinma qaysi nuqtada Ox o'qi bilan $-\pi/4$ burchak hosil qiladi.

5.20. Ushbu $y=x^3/3–5x^2/2+7x+4$ egri chiziqqa urinma qaysi nuqtada Ox o'qi bilan $\pi/4$ burchak hosil qiladi.

5.21. Ushbu $y=x^3/3–9x^2/2+20x-7$ egri chiziqqa urinma qaysi nuqtada Ox o'qiga parallel bo'ladi.

5.22. Ushbu $y = x^4/4 –7$ egri chiziqqa urinma qaysi nuqtada $y = 8x - 4$ o'qiga parallel bo'ladi.

5.23. Ushbu $y=-3x^2+4x+7$ egri chiziqqa urinma qaysi nuqtada $x-20y+5=0$ o'qiga perpendikulyar bo'ladi.

5.24. Ushbu $u=3x^2-4x+6$ egri chiziqqa urinma qaysi nuqtada $8x–y–5=0$ o'qiga parallel bo'ladi.

5.25. Ushbu $y=5x^2-4x+1$ egri chiziqqa urinma qaysi nuqtada $x+6y+15=0$ o'qiga perpendikulyar bo'ladi.

5.26. Ushbu $y=3x^2-5x-11$ egri chiziqqa urinma qaysi nuqtada $x–y+10=0$ o'qiga parallel bo'ladi.

5.27. Ushbu $y=-x^2+7x+16$ egri chiziqqa urinma qaysi nuqtada $y=3x+4$ o'qiga parallel b o'ladi.

5.28. Ushbu $y=4x^2-10x+13$ egri chizikka urinma qaysi nuqtada $y=6x-7$ o'qiga parallel b o'ladi.

5.29. Ushbu $y=7x^2-5x+4$ egri chiziqqa urinma qaysi nuqtada $23y+x–1=0$ o'qiga perpendikulyar bo'ladi.

5.30. Ushbu $y=x^2/4–7x+5$ egri chiziqqa urinma qaysi nuqtada $y=2x+5$ o'qiga parallel bo'ladi.

6. Quyidagi masalalarni yeching

6.1. Jismning harakatlanish traektoriyasi $12u= x^3$ kubik parabola bo'yicha harakat qiladi. Parabolaning qaysi nuqtalarida absissa va ordinataning o'sish tezligi bir xil.(Javob: (2;2/3); (-2;-2/3.)

6.2. Material nuqta quyidagi $S=3t^2/4–3t+7$ qonun bilan harakatlanadi. Qachon uning tezligi 2 m/s ga teng bo'ladi.(Javob: 10/3 s.)

6.3. Ikkita material nuqta $x=4t^2-7$ va $x=3t^2-4t+38$ qonun bilan Ox o'qida harakatlanadi. Bu nuqtalar uchrashgandan so'ng bir biridan qanday tezlik bilan uzoqlashadi. (Javob: *40 m/s* va *26 m/s*.)

6.4. Material nuqta *xu=12* giperbola bo'yicha harakat qiladi. Uning absissasi *1 m/s* tezlik bilan tekis o'sadi. Material nuqta (6,2) joydan o'tayotganda uning ordinatasi o'zgarishi tezligini toping.(Javob: - 1/3 *m/s*.)

6.5. Quyidagi $u^2=4x$ parabolaning ordinatasi qanday nuqtada absissadan 2 marta tez o'sadi.(Javob: ¼,1.)

6.6. Material nuqta $S = t^4 - 3t^2 + 2t - 4$ qonun bilan harakatlanadi. Ushbu nuqtaning *t=2c* paytdagi tezligini aniqlang.(Javob: *22m/s*.)

6.7. Material nuqta $S = 3t^4 - t^3 + 4t^2 + 6$ qonun bilan harakatlanadi. Ushbu nuqtaning *t=2s* paytidagi tezligini aniqlang.(Javob: *100m/s*.)

6.8. Material nuqta *S=4cos(t/4+π/4)+6* qonun bilan harakatlanadi. Ushbu nuqtaning *t=π* s paytdagi tezligini aniqlang.(Javob: *-1m/s.)*

6.9. Material nuqta *S=4sin(t/3+π/6)-8* qonun bilan harakatlanadi. Ushbu nuqtaning *t=π/2* c paytidagi tezligini aniqlang.(Javob: *2/3 m/s*.)

6.10. Material nuqta *S=-3cos(t/4+π/12)+10* qonuni bilan harakatlanadi. Ushbu nuqtaning *t=π/3* paytdagi tezligining aniqlang. (Javob: *3/8 m/s*.)

6.11. Material nuqta $S=5/3t^3-½t^2+7$ qonun bilan harakatlanadi. Qanday vaqtda ushbu nuqtaning tezligi *t=42 m/s* ga teng bo'ladi.(Javob: *3s*.)

6.12. Material nuqta $S=4t^3-2t+11$ qonun bilan harakatlanadi. Qanday vaqtda bu nuqtaning tezligi *190 m/s* ga teng bo'ladi.(Javob: *4s*.)

6.13. Material nuqtaning harakatlanish qonuni $S=5/3t^3-2t+7$ bo'lsin. Ushbu nuqtaning *t=4s* paytidagi tezligini aniqlang.

6.14. Material nuqta $S = 2t^5 - 6t^3 - 58$ qonun bilan harakatlansin. Ushbu nuqtaning *t=2c* paytdagi tezligini aniqlang.

6.15. Ikkita material nuqta *Ox* o'qi bo'yicha $x=3t^2-8$ va $x=2t^2+5t+6$ qonun bo'yicha harakatlanadi. Ular uchrashgandan so'ng bir-biridan qanday tezlikda uzoqlashadi. (Javob: *42 m/s*, *33 m/s*.)

6.16. Ikkita material nuqta *Ox* o'qi bo'yicha $x=5t^2-t+6$ va $x=4t^2+18$ qonun bo'yicha harakatlanadi. Ular uchrashgandan so'ng bir-biridan qanday tezlikda uzoqlashadi. (Javob: *39 m/s*, *32 m/s*)

6.17. Ikkita material nuqta *Ox* o'qi bo'yicha $x=4/3t^3-7t+16$ va $x=t^3+2t^2+5t-8$ qonun bo'yicha harakatlanadi. Qanday paytda ularning tezligi teng bo'ladi. (Javob: *6s*.)

6.18. Material nuqta $s=1/3t^3-2t^2-11t+275$ qonun bo'yicha harakatlanadi. Qaysi paytda uning tezligi *10 m/s* ga teng bo'ladi (Javob: *7s*.)

6.19. Material nuqta *xy=20* giperbola bo'yicha harakatlanadi. Agar uning abstsisassi tezlik bilan o'sadi, uning ordinatasi (4,5) holatdan o'tganda ordinatasi qanday tezlikda o'zgaradi. (Javob: - *1,25 m/s*.)

6.20. Ushbu $y^2=8x$ parabolaning ordinatasi qanday nuqtada absissasidan 2 marta tez o'sadi. (javob: (*1/2, 2*) .)

6.21. Ikkita material nuqta *Ox* o'qida $x=5t^2+2t+16$ va $x=4t^2+3t+18$ qonun bo'yicha harakatlanadi. Ular uchrashgandan so'ng bir-biridan qanday tezlikda uzoqlashadi. (Javob: *42 m/s*, *35 m/s*.)

6.22. Ushbu $y^2=16x$ parabolaning qanday nuqtasida ordinatasi, absissasidan 4 marta tez o'sadi. (Javob: ¼, *2*.)

6.23. Ushbu $x^2=9y$ parabolaning qaysi nuqtasida absissasi ordinatasidan 2 marta tez o'sadi. (Javob: *9/4, 9/16*.)

6.24. Ushbu $x^2=10u$ parabolaning qanday nuqtasida absissasi ordinatasidan 5 marta tez o'sadi (Javob : *1,01*.)

6.25. Ikkita material nuqta *Ox* o'qi bo'yicha $x=2t^3-2t^2+6t-7$ va $x=\frac{5}{3}t^3-t^2+14t+4$ qonun bo'yicha harakatlanadi. Qanday paytda ularning tezligi teng bo'ladi (*Javob*: *4s.*)

6.26. Material nuqta to'g'ri chiziqda $S=\frac{1}{3}t^3-\frac{1}{2}t^2-30t+18$ qonuni bo'yicha harakatlansin. Qanday paytda nuqtaning tezligi nolga teng bo'ladi. (Javob: *6s*.)

6.27. Jism *Ox* o'qida $x=\frac{1}{3}t^3-\frac{7}{2}t^2+10t-16$ qonun bo'yicha harakat qiladi. Jismning tezlik va tezlanishini aniqlang. Jism qaysi vaqtlarda harakat yo'nalishini o'zgartiradi. (Javob: *2s, 5s*.)

6.28. Kimyoviy reaktsiya natijasida olinadigan moddaning massasi va vaqt o'rtasidagi bog'lanish $x=7\left(1-e^{-4t}\right)$ tenglama orqali aniqlanadi. Kimyoviy reaksiya boshlanganda uning tezligi qanday bo'lishini aniqlang. (Javob: *28 kg/s*.)

6.29. Material nuqta to'g'ri chiziqli harakat qiladi va uning tenglamasi $V^2=6x$, bu erda V – tezlik, x – bosib o'tilgan masofa bo'lsin. Tezlik *6 m/s* bo'lganda nuqtaning tezlanishini aniqlang. (Javob: *½ m/s²*.)

6.30. Nuqtaning harakatlanish qonuni $S=3t+t^3$ bo'lsin, uning $t=2c$ bo'lgandagi tezligini aniqlang. (Javob: *15 m/s*.)

Namunaviy variantni yechish

1. Agar $x^3y-y^2=6x$ bo'lsa, y' va y'' ni toping.

►Yuqoridagi ifodadan hosila olamiz

$3x^2y+x^3y'-2yy'=6$, bu erdan $y'=(6-3x^2y)/(x^2-2y)$.

Ushbu $3x^2y+x^3y'-2yy'=6$ tenglikdan hosila olamiz:

$$6xy+3x^2y'+3x^2y'+x^3y''-2y'^2-2yy''=0.$$

Bundan $y''(x^3-2y)=2y'^2-6x^2y'-6xy$

$$y''=2\frac{(6-3x^2y)^2}{(x^2-2y)^3}-6x^2\frac{6-3x^2y}{(x^2-2y)^2}-\frac{6xy}{x^2-2y}\quad .◄$$

2. Agar $\begin{cases} x=3t^4-t^2 \\ y=t^3-5 \end{cases}$, bo'lsa y' va y'' ni hisoblang.

Ma'lumki $\left.\begin{matrix} x'=12t^3-2t \\ y'=3t^2 \end{matrix}\right\}$ va $\left.\begin{matrix} x''=36t^2-2 \\ y''=6t \end{matrix}\right\}$ u holda

$$y_x' = \frac{y_t'}{x_t'} = \frac{3t^2}{(12t^3-2t)} = \frac{3t}{(12t^2-2)}$$

$$y_x'' = \frac{y_t''x_t' - x_t''y_t'}{x_t'^3} = \frac{6t(12t^2-2t)-(36t^2-2)3t^2}{(12t^3-2t)^3} =$$

$$\frac{72t^4-12t^2-108t^4+6t^2}{(12t^3-2t)^3} = -\frac{3(6t^2+1)}{4t(6t^2-1)^3}$$

◄

Agar $y = \frac{1}{8} - \frac{1}{4}\cos^2 x$ bo'lsa $y'''(\pi/4)$ ni toping.

► Ketma-ket hisoblaymiz:

$$y' = \frac{1}{2}\cos x \sin x = \frac{1}{4}\sin 2x,$$

$$y'' = \frac{1}{2}\cos 2x, \quad y''' = -\sin 2x,$$

$$y'''(\pi/4) = -\sin(\pi/2) = -1. ◄$$

3. Quyidagi $y = xe^x$ funksiya uchun *n*-tartibli hosilaning formulasini yozing.

►Ketma-ket xosila olamiz

$y' = e^x + xe^x$, $y'' = e^x + e^x + xe^x = 2e^x + xe^x$, $y''' = 2e^x + e^x + xe^x = 3e^x + xe^x$

Yuqorida y', y'' va y''' uchun olingan ifoddalarni tiqqoslab quyidagiga ega bo'lamiz $y^{(n)} = ne^x + xe^x$ ◄

4. Ushbu $y=x^2-9x-4$ egri chiziqqa abstsisasi $x=-1$ bo'lgan nuqtada urinmaning tenglamasini yozing

►Urinmaning ordinatasi $y'=2x-9$. Urinish nuqtasida $y'(-1)=-11$

Demak,urinmaning tenglamasi (-11 burchak koeffitsiyenti bilan (-1,6) nuqtada)

$y-6=-11(x+1)$, $u=-11x-5$ ◄

Ikkita material nuqta *Ox* o'qi bo'yicha $x_1 = \frac{t^3}{3} - 4$ va $x_2 = \frac{7}{2}t^2 - 12t + 3$ qonunga ko'ra harakatlanadi. Qanday paytda ularning tezligi bir xil bo'ladi?

►Ikkala nuqtaning ham tezligini topamiz $\dot{x}_1 = t^2$ va $\dot{x}_2 = 7t - 12$. Masala sharti bo'yicha $\dot{x}_1 = \dot{x}_2$, u holda $t^2=7t-12$, $t^2-7t+12=0$, $t_1=3c$, $t_2=4c$◄

6.3.–IUT

Quyidagi limitlarni Lopital qoidasi yordamida xisoblang

1.

1.1. $\lim\limits_{x\to\infty}\frac{\ln(x+5)}{\sqrt[4]{x+3}}$.

1.2. $\lim\limits_{x\to 1}\frac{a^{\ln x}-x}{x-1}$.

1.3. $\lim\limits_{x\to 0}\frac{tgx-x}{x-\sin x}$.

1.4. $\lim\limits_{x\to 1}\frac{1-4\sin^2(\pi x/6)}{1-x^2}$.

1.5. $\lim\limits_{x\to a}\arcsin\frac{x-a}{a}ctg(x-a)$.

1.6. $\lim\limits_{x\to\infty}(\pi-2arctgx)\ln x$.

1.7. $\lim\limits_{x\to\infty}(a^{1/x}-1)x$.

1.8. $\lim\limits_{x\to 1}(\frac{1}{\ln x}-\frac{x}{\ln x})$.

1.9. $\lim\limits_{x\to 0}\frac{1-\cos x^2}{x^2-\sin x^2}$.

1.10. $\lim\limits_{x\to 0}\frac{tgx-x}{2\sin x+x}$.

1.11. $\lim\limits_{x\to\infty}\frac{e^{1/x^2}-1}{2arctgx^2-\pi}$.

1.12. $\lim\limits_{x\to 1}\frac{x^3-2x^2-x+2}{x^3-7x+6}$.

1.13. $\lim\limits_{x\to 0}\frac{x\cos x-\sin x}{x^2}$.

1.14. $\lim\limits_{x\to\infty}\frac{e^x}{x^5}$.

1.15. $\lim\limits_{x\to 1}\frac{1-x}{1-\sin(\pi x/2)}$.

1.16. $\lim\limits_{x\to\infty}\frac{\ln x}{\sqrt[3]{x}}$.

1.17. $\lim\limits_{x\to 0}\frac{chx-1}{1-\cos x}$.

1.18. $\lim\limits_{x\to 0}\frac{\pi/x}{ctg(\pi x/2)}$.

1.19. $\lim\limits_{x\to\pi/4}\frac{1/\cos^2 x-2tgx}{1+\cos 4x}$.

1.20. $\lim\limits_{x\to 0}\frac{\ln(\sin mx)}{\ln(\sin x)}$.

1.21. $\lim\limits_{x\to\pi/2}\frac{tgx}{tg5x}$.

1.22. $\lim\limits_{x\to 0}(1-\cos x)ctgx$.

1.23. $\lim\limits_{x\to 1}(1-x)tg(\pi x/2)$.

1.24. $\lim\limits_{x\to\infty}x\sin(3/x)$.

1.25. $\lim\limits_{x\to -1}\frac{\sqrt[3]{1+2x}+1}{\sqrt{2+x}+x}$.

1.26. $\lim\limits_{x\to 0}\frac{x\cos x-\sin x}{x^3}$.

1.27. $\lim\limits_{x\to 1}\frac{1-x}{1-\sin(\pi x/2)}$.

1.28. $\lim\limits_{x\to 0}\frac{tgx-\sin x}{4x-\sin x}$.

1.29. $\lim\limits_{x\to\pi/2}\frac{tg3x}{tg5x}$.

1.30. $\lim\limits_{x\to\pi/4}\frac{\sec^2 x-2tgx}{1+\cos 4x}$.

2.

2.1. $\lim\limits_{x\to 0}\frac{1-\cos 8x}{tg^2 2x}$

2.2. $\lim\limits_{x\to\infty}x^4\sin(a/x)$.

2.3. $\lim\limits_{x\to 1}\ln x\ln(x-1)$.

2.4. $\lim\limits_{x\to 3}(\frac{1}{x-3}-\frac{5}{x^2-x-6})$.

2.5. $\lim\limits_{x\to 1}(\frac{1}{2(1-\sqrt{x}}-\frac{1}{3(1-\sqrt[3]{x}})$.

2.6. $\lim\limits_{x\to 0}\frac{e^{ax}-e^{bx}}{\sin x}$

2.7. $\lim\limits_{x\to\pi/2}(\frac{x}{ctgx}-\frac{\pi}{2\cos x})$

2.8. $\lim\limits_{x\to\pi}(\pi-x)tg(x/2)$

2.9. $\lim\limits_{x\to 0}\frac{x-arctgx}{x^3}$

2.10. $\lim\limits_{x\to\pi/(2a)}\frac{1-\sin ax}{(2ax-\pi)^2}$.

2.11. $\lim\limits_{x\to\pi/(2a)}\frac{1-\sin ax}{(2ax-\pi)^2}$.

2.12. $\lim\limits_{x\to 0}\frac{e^{2x}-1}{\ln(1+2x)}$.

2.13. $\lim\limits_{x\to 0}\frac{a^x-1}{c^x-1}$.

2.14. $\lim\limits_{x\to 1}\frac{\ln x}{1-x^3}$.

2.15. $\lim\limits_{x\to 1}\frac{\ln x}{ctgx}$.

2.16. $\lim\limits_{x\to 0}\frac{1-\cos ax}{1-\cos bx}$.

2.17. $\lim\limits_{x\to a}\frac{x-a}{x^n-a^n}$.

2.18. $\lim\limits_{x\to 0}\frac{e^x-1}{\sin 2x}$.

2.19. $\lim\limits_{x\to 0}(x\ln x)$.

2.20. $\lim\limits_{x\to 0}(\frac{1}{x\sin x}-\frac{1}{x^2})$.

2.21. $\lim\limits_{x\to 0}(1-e^{2x})ctgx$.

2.22. $\lim\limits_{x\to 0}\dfrac{a^x-b^x}{x\sqrt{1-x^2}}$.

2.23. $\lim\limits_{x\to 0}\dfrac{e^{x^3}-1-x^3}{\sin^2 2x}$.

2.24. $\lim\limits_{x\to 0}\dfrac{e^{a\sqrt{x}}-1}{\sqrt{\sin bx}}$.

2.25. $\lim\limits_{x\to 0}\dfrac{\ln(1+x^{2)}}{\cos 3x-e^{-x}}$.

2.26. $\lim\limits_{x\to\infty}\dfrac{e^x}{x^5}$.

2.27. $\lim\limits_{x\to+\infty}\dfrac{\ln(x+7)}{\sqrt[7]{x-3}}$.

2.28. $\lim\limits_{x\to 0}\dfrac{\pi / x}{ctg(5x/2)}$.

2.29. $\lim\limits_{x\to 0}(1-\cos 2x)ctg 4x$.

2.30. $\lim\limits_{x\to\infty}(x^2\sin b/x)$.

3.

3.1. $\lim\limits_{x\to 0}\dfrac{\arcsin 4x}{5-5e^{-3x}}$.

3.2. $\lim\limits_{x\to 0}\dfrac{\ln\cos x}{x}$.

3.3. $\lim\limits_{x\to 0}\dfrac{e^{x^2}-1}{\cos x-1}$.

3.4. $\lim\limits_{x\to 0}\dfrac{e^x-x^2/2-x-1}{\cos x-x^2/2-1}$.

3.5. $\lim\limits_{x\to 0}\dfrac{e^{tgx}-1}{tgx-x}$.

3.6. $\lim\limits_{x\to 1}\dfrac{\ln(1-x)+tg(\pi x/2)}{ctg\pi x}$.

3.7. $\lim\limits_{x\to a}\dfrac{\cos x\ln(x-a)}{\ln(e^x-e^a)}$.

3.8. $\lim\limits_{x\to 1}\dfrac{1}{\cos(\pi x/2)\ln(1-x)}$.

3.9. $\lim\limits_{x\to 0}\dfrac{\cos(e^{x^2}-1)}{\cos x-1}$.

3.10. $\lim\limits_{x\to 0}\dfrac{e^{\alpha x}-\cos\alpha x}{e^{\beta x}-\cos\beta x}$.

3.11. $\lim\limits_{x\to a}\dfrac{x^m-a^m}{x^n-a^n}$.

3.12. $\lim\limits_{x\to\infty}x\sin\dfrac{a}{6x}$.

3.13. $\lim\limits_{x\to 0}\dfrac{3tg4x-12tgx}{3\sin 4x-12\sin x}$.

3.14. $\lim\limits_{x\to\pi/4}\dfrac{\sqrt{tgx}-1}{2\sin^2 x-1}$.

3.15. $\lim\limits_{x\to 0}\dfrac{x(e^x+1)-2(e^x-1)}{x^3}$.

3.16. $\lim\limits_{x\to 0}\dfrac{\arcsin 2x-2\arcsin x}{x^3}$.

3.17. $\lim\limits_{x\to 0}\dfrac{a^x-a^{\sin x}}{x^3}$.

3.18. $\lim\limits_{x\to\pi/4}(tgz)^{tg2x}$.

3.19. $\lim\limits_{x\to 0}\dfrac{\ln(\cos ax)}{\ln(\cos bx)}$.

3.20. $\lim\limits_{x\to\pi/4}\dfrac{\sqrt[3]{tgx}-1}{2\sin^2 x-1}$.

3.21. $\lim\limits_{x\to 0}\left(\dfrac{1}{x}-\dfrac{1}{e^x-1}\right)$.

3.22. $\lim\limits_{x\to\infty}x^2e^{-0,01x}$.

3.23. $\lim\limits_{x\to 0}\dfrac{\ln(1+xe^x)}{\ln(x+\sqrt{1+x^2})}$.

3.24. $\lim\limits_{x\to 1}(1-x)^{\log 2x}$.

3.25. $\lim\limits_{x\to\infty}\dfrac{e^{4/x^2}-1}{2arctgx^2-\pi}$.

3.26. $\lim\limits_{x\to 1/2}\ln 2x\cdot\ln(2x-1)$.

3.27. $\lim\limits_{x\to 1/3}(\dfrac{x}{3x-1}-\dfrac{1}{\ln 3x})$.

3.28. $\lim\limits_{x\to 0}\arcsin x\cdot tgx$.

3.29. $\lim\limits_{x\to\infty}(x^3e^{-x})$.

3.30. $\lim\limits_{x\to 1}(x-1)^{x-1}$.

4.

4.1. $\lim\limits_{x\to 0}(1-\sin 2x)^{ctgx}$.

4.2. $\lim\limits_{x\to 0}(\ln(1/x))^{x}$.

4.3. $\lim\limits_{x\to 0}(\cos x)^{ctgx}$.

4.4. $\lim\limits_{x\to 0} x^{x}$.

4.5. $\lim\limits_{x\to\infty}(\ln 2x)^{1/\ln x}$.

4.6. $\lim\limits_{x\to 0}(1+\sin^2 x)^{1/tg^2 x}$.

4.7. $\lim\limits_{x\to 1}(1-x)^{\ln x}$.

4.8. $\lim\limits_{x\to 0}(\ln(x+e))^{1/x}$.

4.9. $\lim\limits_{x\to 0}(\sin x)^{tgx}$.

4.10. $\lim\limits_{x\to\infty}\sqrt[x]{x}$.

4.11. $\lim\limits_{x\to 0} x^{\sin x}$.

4.12. $\lim\limits_{x\to 1}(1-x)^{\cos(\pi x/2)}$.

4.13. $\lim\limits_{x\to 0}(1+x^2)^{1/x}$.

4.14. $\lim\limits_{x\to 1} x^{1/(x-1)}$.

4.15. $\lim\limits_{x\to 1}(tg\frac{\pi x}{4})^{tg(\pi x/2)}$.

4.16. $\lim\limits_{x\to 1}(ctg\frac{\pi x}{4})^{tg(\pi x/2)}$.

4.17. $\lim\limits_{x\to 0}(\frac{1}{x})^{tgx}$.

4.18. $\lim\limits_{x\to\infty}(\frac{x-4}{x+3})^{3x}$.

4.19. $\lim\limits_{x\to 0}(ctgx)^{\sin x}$.

4.20. $\lim\limits_{x\to\infty}(\ln x)^{1/x}$.

4.21. $\lim\limits_{x\to\infty} x^{6/(1+2\ln x)}$.

4.22. $\lim\limits_{x\to\infty}(1-e^{x})^{1/x}$.

4.23. $\lim\limits_{x\to\infty}(x-1)^{1/\ln(2(x-1))}$.

4.24. $\lim\limits_{x\to\infty}(\cos\frac{m}{x})^{x}$.

4.25. $\lim\limits_{x\to 0}(ctg\, 2x)^{1/\ln x}$.

4.26. $\lim\limits_{x\to 5}(\frac{1}{x-5}-\frac{5}{x^2-x-20})$.

4.27. $\lim\limits_{x\to\infty} x^2\sin\frac{a}{x}$.

4.28. $\lim\limits_{x\to 1}(\frac{1}{2(1-\sqrt{x})}-\frac{1}{3(1-\sqrt[3]{x})})$.

4.29. $\lim\limits_{x\to 1}(1-x)^{\cos(\pi x/2)}$.

4.30. $\lim\limits_{x\to 0}(ctgx)^{\sin x}$.

5.

5.1. $\lim\limits_{x\to\infty} x(\ln(2+x)-\ln(x+1))$.

5.2. $\lim\limits_{x\to\infty}(\cos\frac{m}{x}+\lambda\sin\frac{m}{x})^{x}$.

5.3. $\lim\limits_{x\to\infty}(x+2^{x})^{1/x}$.

5.4. $\lim\limits_{x\to 0}(1+3tg^2 x)^{ctg^2 x}$.

5.5. $\lim\limits_{x\to\infty}(\cos(m/\sqrt{x}))^{x}$.

5.6. $\lim\limits_{x\to 0}(\cos 2x)^{3/x^2}$.

5.7. $\lim\limits_{x\to 0}(\ln ctgx)^{tgx}$.

5.8. $\lim\limits_{x\to a}(2-x/a)^{tg(\pi x/(2a))}$.

5.9. $\lim\limits_{x\to 0}(\frac{5}{2+\sqrt{9+x}})^{1/\sin x}$.

5.10. $\lim\limits_{x\to\infty}(1+3/x)^{x}$.

5.11. $\lim\limits_{x\to 0}(e^{x}+x)^{1/x}$.

5.12. $\lim\limits_{x\to\pi/2}(tgx)^{2x-\pi}$.

5.13. $\lim\limits_{x\to\infty}(\frac{2}{\pi}arctgx)^{x}$.

5.14. $\lim\limits_{x\to 0}(\frac{\cos x}{\cos 2x})^{1/x^2}$.

5.15. $\lim\limits_{x\to\infty}(\frac{1+tgx}{1+\sin x})$.

5.16. $\lim\limits_{x\to\infty}(\cos(1/x)+\sin(1/x))^{x}$.

5.17. $\lim\limits_{x\to 1}(x-1)^{e^{1/(\pi-1)}}$.

5.18. $\lim\limits_{x\to 0}(\frac{tgx}{x})^{1/x^2}$.

5.19. $\lim\limits_{x\to\infty}(\frac{x^2+1}{x^2-2})^{x^2}$.

5.20. $\lim\limits_{x\to 0}\sqrt[x]{1-2x}$.

5.21. $\lim\limits_{x\to\infty}(\sin\frac{2}{x}+\cos\frac{2}{x})^{x}$.

5.22. $\lim_{x\to 1}(1+\sin\pi x)^{ctg\pi x}$.

5.23. $\lim_{x\to\infty}\left(\frac{x+a}{x-a}\right)^{x}$.

5.24. $\lim_{x\to\infty} x^{1/x}$.

5.25. $\lim_{x\to 0} x^{3/(4+\ln 4)}$.

5.26. $\lim_{x\to 0} x^{\sin x}$.

5.27. $\lim_{x\to 0}\left(\frac{1}{x}\right)^{tgx}$.

5.28. $\lim_{x\to 1}\frac{x^3-2x^2-x+2}{x^3-7x+6}$.

5.29. $\lim_{x\to 0}\sqrt[x]{1-2x}$.

5.30. $\lim_{x\to\infty}(\cos(1/x)+\sin(1/x))^{x}$.

6. Differensial yordamida quyidagi miqdorlarni taqriban hisoblang va nisbiy xatoni verguldan keyin 2 ta raqamigacha aniqlikda hisoblang.

6.1. $\sqrt[5]{34}$.

6.2. $\sqrt[3]{26,19}$.

6.3. $\sqrt[4]{16,64}$.

6.4. $\sqrt{8,76}$.

6.5. $\sqrt[5]{31}$.

6.6. $\sqrt[3]{70}$.

6.7. $(2,01)^3+(2,01)^2$.

6.8. $\sqrt[3]{65}$.

6.9. $2,9/\sqrt{(2,9)^2+16}$.

6.10. $\sqrt{\frac{4-3,02}{1+3,02}}$.

6.11. $\sqrt[4]{15,8}$.

6.12. $\sqrt[3]{10}$.

6.13. $\sqrt[5]{200}$.

6.14. $(3,03)^5$.

6.15. $\sqrt{\frac{(2,037)^2-3}{(2,037)^2+5}}$.

6.16. $\sqrt[7]{130}$.

6.17. $\sqrt[3]{27.5}$.

6.18. $\sqrt{17}$.

6.19. . $\sqrt{640}$.

6.20. $\sqrt{1,2}$.

6.21. $\sqrt[10]{1025}$.

6.22. $(3,02)^4+(3,02)^2$.

6.23. $(5,07)^3$.

6.24. $(4,01)^{1,5}$.

6.25. $\sqrt[3]{1,02}$.

6.26. $\cos 151°$.

6.27. $arctg 1,05$.

6.28. $\cos 61°$.

6.29. $tg 44°$.

6.30. $arctg 0,98$.

7.

7.1. $\arcsin 0,6$.

7.2. $arctg 0,95$.

7.3. $e^{0,2}$.

7.4. $\lg 11$.

7.5. $\arcsin 0,54$.

7.6. $\cos 59°$.

7.7. $e^{2,01}$.

7.8. $\ln tg 46°$.

7.9. $arctg\sqrt{1,02}$.

7.10. $arctg\sqrt{0,97}$.

7.11. $arctg 1,01$.

7.12. $\ln(e^2+0,2)$.

7.13. $arctg 1.03$.

7.14. $\ln tg 47°15'$.

7.15. $\lg 9,5$.

7.16. $arctg\sqrt{3,1}$.

7.17. $2^{2,1}$.

7.18. $4^{1,2}$.

7.19. $tg 59°$.

7.20. $\log_2 1,9$.

7.21. $arctg\sqrt{3,2}$.

7.22. $ctg 29°$.

7.23. $\sin 93°$.

7.24. $\lg 1,5$.

7.25. $\sin 29°$.

7.26. $\lg 101$.

7.27. $\sin 101°$.

7.28. $\lg 0,9$.

7.29. $e^{0,25}$.

7.30. $\sqrt{15}$.

Namunaviy variantni yechish

Quyidagi limitlari Lopital qoidasi yordamida hisoblang

1. $\lim_{x\to\infty}\frac{\ln(x^2+1)}{\sqrt[5]{3x-1}}$.

Limit ostidagi ifodaning surati va maxraji $x \to \infty$ ga intilganda cheksizga intilgani uchun $\frac{\infty}{\infty}$* ko'rinishdagi noaniqlikka ega bo'lamiz. Demak Lopital qoidasini qo'llash mumkin.

►Quydagiga ega bulamiz

$$\lim_{x\to\infty}\frac{\ln(x^2+1)}{\sqrt[5]{3x-1}}=\frac{\infty}{\infty}=\lim_{x\to\infty}\frac{2x/(x^2+1)}{3/\sqrt[5]{(3x-1)^4}}=\frac{2}{3}\lim_{x\to\infty}\frac{x\sqrt[5]{(3x-1)^4}}{x^2+1}=\frac{\infty}{\infty}=$$

$$\frac{2}{3}\lim_{x\to\infty}\frac{\sqrt[5]{(3x-1)^4}+x\cdot\frac{4}{5}(3x-1)^{-\frac{1}{5}}\cdot 3}{2x}=$$

$$=\frac{2}{3}\lim_{x\to\infty}\frac{15x-5+12x}{10x\sqrt[5]{(3x-1)}}=\frac{1}{15}\lim_{x\to\infty}\frac{27x-5}{x\cdot\sqrt[5]{(3x-1)}}=\frac{1}{15}\lim_{x\to\infty}\frac{27-5/x}{\sqrt[5]{(3x-1)}}=0$$

◄

2. $\lim_{x\to\pi/2}\frac{1-\sin x}{tg^2 2x}$.

►Argument $x\to\pi/2$ ga intilganda $\frac{0}{0}$ ko'rinishdagi noaniqlikka ega bo'lamiz. Bu yerda Lopital qoidasini qo'llaymiz:

$$\lim_{x\to\pi/2}\frac{1-\sin x}{tg^2 2x}=\lim_{x\to\pi/2}\frac{-\cos x}{2tg2x\frac{2}{\cos^2 2x}}=$$

$$\lim_{x\to\pi/2}\frac{-\cos^3 2x\cos x}{4\sin 2x}=\frac{1}{4}\lim_{x\to\pi/2}(-\cos^3 2x)$$

$$\lim_{x\to\pi/2}\frac{\cos x}{2\sin x\cos x}=\frac{1}{4}\lim_{x\to\pi/2}\frac{1}{2\sin x}=\frac{1}{8}$$

◄

3. $\lim_{x\to 0}\frac{arctg4x}{e^{5x}-1}$.

►Ushbu limitda $\frac{0}{0}$, ko'rinishdagi noaniklikka ega bo'lamiz va uni Lopital qoidasi yordamida hisoblaymiz

$$\lim_{x\to 0}\frac{arctg4x}{e^{5x}-1}=\lim_{x\to 0}(\frac{4/(1+16x^2)}{5e^{5x}})=\frac{4}{5}$$ ◄

4. $\lim_{x\to 0}(\frac{1}{2-\sqrt{4+x^2}}-\frac{3}{\sqrt{16+x}-4})$.

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*Limitlarni hisoblaganda quyidagi simvolik yozuvlarga kelishib olamiz. Agar *u(x)* va *v(x)* funksiyalar $x\to x_0$ ga yoki $\to x\to\pm\infty$ ga intilganda mos ravishda «0» va «0», yoki «∞» va «∞», yoki «1» va «∞» va h.k. bo'lsa quydagi ko'rinishda yozamiz: $\lim\frac{u}{v}=\frac{0}{0}$, yoki $\lim\frac{u}{v}=\frac{\infty}{\infty}$, yoki $\lim(u\cdot v)=0\cdot\infty$, yoki $\lim u^v=1^\infty$ va hokazo.

►Bu misolda $\infty-\infty$ ko'rinishdagi aniqmaslikka ega bo'lamiz va uni $\frac{0}{0}$ ko'rinishga keltiramiz

$$\lim_{x\to 0}\left(\frac{1}{2-\sqrt{4+x^2}}-\frac{3}{\sqrt{16+x}-4}\right)=\infty-\infty=\lim_{x\to 0}\frac{\sqrt{16+x}-4-6+3\sqrt{4+x^2}}{(2-\sqrt{4+x^2})(\sqrt{16+x}-4)}=\frac{0}{0}$$

$$=\lim_{x\to 0}\frac{1/(2\sqrt{16+x})+3x/\sqrt{4+x^2}}{-\frac{x}{\sqrt{4+x^2}}(\sqrt{16+x}-4)+\frac{1}{2\sqrt{16+x}}(2-\sqrt{4+x^2})}=\frac{\frac{1}{8}}{0}=\infty$$

◄

5. $\lim_{x\to 0}\left(\frac{x^2+3x-4}{x^2-x-3}\right)^x$.

►Ushbu limitda 1^∞ ko'rinishdagi aniqmaslikka ega bo'lamiz. Quydagi belgilashni kiritamiz $y=\left(\frac{x^2+3x-4}{x^2-x-3}\right)^x$, u holda $\ln y = x\ln\left(\frac{x^2+3x-4}{x^2-x-3}\right)$.

$$\lim_{x\to 0}\ln y=\lim_{x\to 0}\frac{\ln\left(\frac{x^2+3x-4}{x^2-x-3}\right)}{1/x}=\frac{0}{0}=\lim_{x\to 0}\frac{\frac{x^2-x-3}{x^2+3x-4}\cdot\frac{(2x+3)(x^2-x-3)-(2x-1)(x^2+3x-4)}{(x^2-x-3)^2}}{-1/x^2}=$$

$$\lim_{x\to 0}(-x^2(2x^3-2x^2-6x+3x^2-3x-9-2x^3-6x^2+8x+x^2+3x-4))/((x^2+3x-4)(x^2-x-3))^-$$

$$=\lim_{x\to 0}\frac{-x^2(-4x^2+2x-13)}{(x^2+3x-4)(x^2-x-3)}=4$$

Demak $\ln\lim_{x\to 0}\left(\frac{x^2+3x-4}{x^2-x-3}\right)^x=4$, u holda $\lim_{x\to 0}\left(\frac{x^2+3x-4}{x^2-x-3}\right)^x=e^4$.◄

Diferensial yordamida quyidagi miqdorlarni taqriban hisoblang va nisbiy xatoni verguldan keyin 2 ta songacha aniqlikda hisoblang.

► $\sqrt[3]{84}$. Ushbu miqdorni quyidagi ko'rinishda yozamiz $\sqrt[3]{84}=\sqrt[3]{4^3+20}$ va $y=\sqrt[3]{x}$ funksiyani kiritamiz, bu yerda $x = x_0 + \Delta x$ $x_0=64$; $\Delta \tilde{o}=20$. Takribiy hisoblash formulasiga ko'ra $y(x_0+\Delta x)=y(x_0)+y'(x_0)\Delta x$, $y(x_0)=\sqrt[3]{64}=4$, $y'=\frac{1}{3\sqrt[3]{x^2}}$, $y'(64)=\frac{1}{48}$.

Hisoblaymiz $\sqrt[3]{84}=4+20/48=4{,}42$ nisbiy xato $b=\frac{4{,}42-4{,}3}{4{,}42}100\%=2{,}7\%$.◄

6. $arctg0{,}98$.

►Yuqoridagi sxemadan foydalanamiz

$y=arctgx, x_0=1, \Delta x=0{,}98-1=-0{,}02, y(x_0)=arctg1=\pi/4, y'$

$(1)=0{,}5, arctg0{,}98\approx\pi/4-0{,}5\cdot 0{,}02=0{,}77$

$\delta=\left|\frac{0{,}77-0{,}78}{0{,}77}\right|100\%=13\%$.◄

IUT-6.4

1. Quyidagi masalarni eching

1.1. O'tov konus shaklida bo'lib uning xajmi "*V*" ga teng. O'tovga eng kam material ketishi uchun uning balandligini asosining radiusiga nisbati nechaga teng bo'lishi kerak ? (javob: $\sqrt{2}$.)

1.2. Asosi *a* ga, asosidagi burchagi α ga teng bo'lgan teng yonli uchburchakka parallelogramm shunday ichki chizilganki, uning bir tomoni uchburchakning asosida, ikinchi tomoni uchburchakning yon tomonida yotadi. Parallelogrammning yuzi eng katta bo'lishi uchun uning tomonlari qanday bo'lishi kerak. (Javob: *a/2, a/4cos*α *.)*

1.3. Silindrning hajmi *V* bo'lib uning to'la sirti eng kichik bo'lishi uchun uning radiusi *R* va balandligi *H* orasidagi munosobatni toping. (Javob: *H=2R*)

1.4. Konussimon varonkaning yasovchisi 20 sm. ga teng. Voronkaning hajmi eng kichik bo'lishi uchun, uning balandligi qanday bo'lishi kerak? (Javob: $20\sqrt{3}/3 см$.)

1.5. Teng yonli uchburchakning perimetri *2p* ga teng. Uchburchakning asosi atrofida aylanishidan xosil bo'lgan jismning hajmi eng katta bo'lishda uchun uning asosi qanday bo'lishi kerak? (Javob: *p/2*.)

1.6. *R* – radiusli sharga ichki chizilgan konusning hajmi eng katta bo'lishi uchun balandligi qanday bo'lishi kerak? (Javob: *4R/3*.)

1.7. Doiraviy sektor ko'rinishida bo'lgan gulzorni o'rash uzunligi *l* ga teng bo'lgan sim ishlatildi. Gulzorning yuzi eng katta bo'lishi uchun doiraning radiusi qanday bo'lishi kerak? (Javob*: l/4m*.)

1.8. Radiusi *à* ga teng bo'lgan yarim doiraga ichki chizilgan to'g'ri burchakli uchburchak yuzining eng katta qiymatini toping? (Javob: a^2.)

1.9. Uzunligi 20 m bo'lgan g'o'la kesik konus shaklida bo'lib, uning asoslarini diametrlari 2 m va 1 m. O'qi ushbu g'o'laning o'qi bilan ustma ust tushuvchi, ko'ndalang kesimi kvadratdan iborat qilib yasalgan to'sinning hajmi eng katta bo'lishi uchun, uning o'lchamlari qanday bo'lishi kerak? (Javob: to'sining uzunligi *40/3 m*, ko'ndlang kesimi tomonining uzunligi $2\sqrt{2}/3\,m$.)

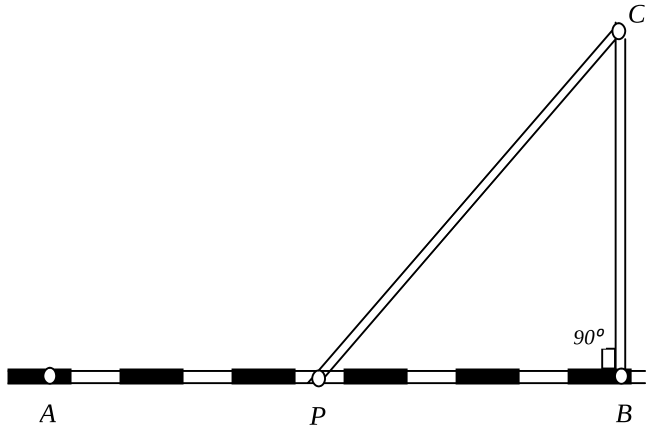
1.10. Kema qirg'oqdan *9 km* masofada langar tashlagan. Qirg'oqning kemaga eng yaqin nuqtasidan *15 km* masofada joylashgan lagerga chopar jo'natilgan. Choparning qayiqdagi tezligi *4 km/s*, piyoda yurganda *5 km/s*. U qirgoqqa lagerdan qanday masofada chiqsa yo'lga eng kam vaqt ketadi? (Javob: lagerdan *3 km* masofada.)

1.11. Tunukaning eni *a* ga teng to'g'ri to'rtburchak ko'rinishida, u silindrik tarnov sifatida, kesimnning shakli segment bo'lib egilgan. Tarnovning sig'imi eng katta bo'lishi uchun segmetning yoyiga tayangan markaziy burchak qanday bo'lishi kerak? (Javob: $\varphi=\pi$.)

1.12. Diametri *d* ga teng bo'lgan dumaloq shaklidagi g'o'ladan, ko'ndalang kesimi to'g'ri to'rtburchak bo'lgan to'sin yasalgan . To'sin gorizontal joylashgan va unga ta'sir etuvchi tashqi kuchlar tekis taqsimlangan bo'lsa, uning eng kam egilishi uchun ko'ndalang kesimining eni *b* va balandligi *h* qanday bo'lishi kerak?

(Egilishning miqdori kubining ko'ndalang kesimning eni *b* va balandangi *h* ko'paytmasiga teskari proportsianal). (Javob: $b=d/2$, $h=\frac{d\sqrt{3}}{2}$.)

1.13. Temir yo'lda yuk tashish narxi *1 km (AB)* uchun k_1 so'm, avtomobilda esa *(RS)* - k_2 so'm. ($k_1 < k_2$). Qanday *R* nuqtadan shosse qurishni boshlash kerakki, *A* punktdan S punktgacha yuk tashish eng arzon bo'lsin? Bu erda $|AB| = a$, $|BC| = b$ (6.15. rasm). (Javob: A nuqtadan $à - \frac{k_1 b}{\sqrt{k_2^2 - k_1^2}}$ masofada .)



6.15. rasm

1.14. Yo'lovchi daryoning qarama - qarshi qirg'oqlarida joylashgan *A* punktdan *V* punktga borishi kerak. Agar yo'lovchining qirg'oq bo'ylab tezligi suvdagidan k marta katta bo'lsa, u *V* punktga eng kam vaqtda borishi uchun daryoni qanday burchak ostida kesib o'tishi kerak? Daryoning eni *h*, *A* va *V* punktlar orasidagi masofa *d* ga teng . (Javob: $max(\arccos(1/k), arctg(h/a))$.)

1.15. To'g'ri chiziqli *AB* kesmada *A* nuqtada quvvati *P* va *B* nuqtada quvvati *q* bo'lgan yoruglik manbai joylashgan. Agar $|AB| = a$ va yoritilish darajasi yorug'lik manbaidan bo'lgan masofaning kvadratiga teskari proporsional bo'lsa, shu kesmada eng kam yoritilayotgan *M* punktni toping, (Javob: *A* nuqtadan $\frac{a\sqrt[3]{P}}{\sqrt[3]{P} + \sqrt[3]{q}}$ masofada.)

1.16. Chiroq radiusi *r* ga teng bo'lgan stolning markazi ustida osilib turibdi. Chiroq qanday balandlikda bo'lganda, stolning chetidagi predmetning yoritilish darajasi eng kechik bo'ladi? (yoritilish, yorig'lik nuri tushish burchagining kosinusiga to'g'ri, yoruglik manbagacha bo'lgan masofaning kvadratiga teskari proportsional). (Javob: $r/\sqrt{2}$.)

1.17. Berilgan konusga ichki chizilgan silindrlardan yon sirti eng katta bo'lganini toping. Konusning balandligi *H*, asosining radiusi *R*.(Javob: silindr asosining radiusi $R/2$, balandligi *N/2.)*

1.18. Doiraviy qog'ozdan sektor kesib olingan, qolgan qismidan konussimon idish (voronka) yasalgan. Idishning hajmi maksimal bo'lishi uchun sektorning burchagi qanday bo'lishi kerak? (Javob: $2\pi\sqrt{2/3}$.)

1.19. Yon sirti S ga teng bo'lgan hamma konuslardan hajmi eng katta konusni toping. (Javob: konus asosining radiusi $\sqrt{\frac{S}{\pi\sqrt{3}}}$, balandligi $\sqrt{\frac{S(3\pi-1)}{\pi\sqrt{3}}}$.)

1.20. B punkt temir yo'ldan 60 *km* masofada joylashgan. Temir yo'ldagi *A* punktdan B punktga eng yaqin bo'lgan C punktgacha bo'lgan masofa 285 km. *C* nuqtadan qanday masofada stantsiya qurish kerakki, undan B punktga tamon qurilgan shosse yordamida *A* dan B ga borish uchun eng kam vakt sarflansin. Temir yo'ldagi tezlik *52 km*, shossedagi *20m/s*. (Javob: *25 km*.)

1.21. Eni *a* ga teng bo'lgan kanal to'g'ri burchak ostida eni *b* ga teng kanalga quyiladi. Shu kanallar orqali oqizish mumkin bo'lgan g'o'lalarning eng uzunini toping (Javob: ($a^{\frac{2}{3}}+b^{\frac{2}{3}})^{\frac{3}{2}}$ m.)

1.22. Radiusi *R* ga teng bo'lgan sharga tashqi chizilgan konusning hajmi eng kichik bo'lgan konusning balandligini toping. (Javob: *8R*.)

1.23. Teng yonli trapetsiyaning yon tomoni *b* kichik asosi *a* ga teng bo'lsa, uning asosidagi burchaklari qanday bo'lganida, uning yuzi eng katta bo'ladi. (Javob: $\cos\varphi=(\sqrt{a^2+b^2}-a)/(4b)$).)

1.24. Quyidagi $y=3\sqrt{x}$ egri chiziq va *x=4, y=0* to'g'ri chiziqlar bilan chegaralangan figuradan yuzasi eng katta bo'lgan to'rburchakni yasang. (Javob: $S=9{,}22$).

1.25. Radiusi *R* ga teng bo'lgan aylanaga ichki chizilgan teng yonli uchburchakni, uning uchidan asosiga parallel qilib o'tkazilgan to'g'ri chiziq atrofida aylantirildi. Hosil bo'lgan jismning hajmi eng katta bo'lishi uchun, uchburchakning balandligi qanday bo'lishi kerak. (Javob: $5R/3$).

1.26. Usti ochiq silindrik ko'rinishdagi idish (bak) tayyorlash uchun ostiga *1 kvmi* P_1 sumlik, yoniga esa *1 kvmi* P_2 sumlik sumlik material ishlatiladi. Agar idishning xajmi *V* ga teng bo'lsa idish asosining radiusi uning balandliga nisbatan qanday bo'lganda eng kam material ketadi. (Javob: P_2/P_1) .

1.27. Balandligi *N* bo'lgan vertikal devorli idish yopishqoq bo'lmagan suyuqlik bilan to'ldirilib tekis joyda turibdi. Agar Toricheli qonuni bo'yicha oqib chiqayotgan suyuqlikning tezligi $\sqrt{2gx}$ ga teng bo'lib, bu erda *x* tirqishdan suyuqlikning sirtigacha bo'lgan masofa, g – erkin tezlanish bo'lsa oqib chiqayotgan suyuqlik tirqishdan eng uzoq masofaga tushishi uchun, u qayerda joylashi kerak. (Javob: *N* balandnikning o'rtasida.)

1.28. Deraza to'g'ri to'rtburchak shaklida bo'lib uning bir tomoni yarim doira shaklida, perimetrii *15 m* ga teng. Yarim doirining radiusi qanday bo'lganda u eng ko'p yorug'lik o'tkazadi. (javob: *2,1 m*.)

1.29. Kitobning har bir betidagi bosma matn yuzasi *S* ga teng, yuqori va pastdagi matnsiz masofalar *a* ga teng, o'ng va chap tomondagi masofalar *b* . Matn eni va bo'yining qanday nisbatida kitob betining yuzasi eng kichik bo'ladi? (Javob: *b/a*.)

1.30. Dumaloq g'o'laning diametri *d* ga teng, undan ko'ndalang kesimi to'g'ri to'rtburchak bo'lgan to'sin yasalgan. To'sinning egilishga qarshiligi eng katta bo'lishi uchun uning ko'ndalang kesimining o'lchamlari qandiy bo'lishi kerak? To'sinning egilishga qarshiligi *Q*, uning eni *x* va balandligi *u* ning kvadratiga proportsional, $Q \approx kxy^2$, *k*= const. (Javob: $x = \frac{d\sqrt{3}}{3}, y = \frac{d\sqrt{6}}{3}$.)

2. Quyidagi funksiyalarni to'la tekshiring va grafigini yasang.

2.1. $y = \frac{x^2 - 2x + 2}{x - 1}$.

2.2. $y = \frac{x+1}{(x-1)^2}$.

2.3. $y = e^{1/(5+x)}$.

2.4. $y = x/(9 - x)$.

2.5. $y = \frac{4x - x^2 - 4}{x}$.

2.6. $y = \frac{x^2}{4x^2 - 1}$.

2.7. $y = \frac{\ln x}{\sqrt{x}}$.

2.8. $y = x + \frac{\ln x}{x}$.

2.9. $y = x - \ln(1 + x^2)$.

2.10. $y = \frac{x^3}{x^2 - x + 1}$.

2.11. $y = x^2 - 2\ln x$.

2.12. $y = x^3 e^{-x^2/2}$.

2.13. $y = \frac{x^2 - x - 1}{x^2 - 2x}$.

2.14. $y = \frac{(x-2)^2}{x+1}$.

2.15. $y = -\ln\frac{1+x}{1-x}$.

2.16. $y = \ln(x^2 + 1)$.

2.17. $y = \frac{x^2 + 6}{x^2 + 1}$.

2.18. $y = x\ln x$.

2.19. $y = (x - 1)e^{3x+1}$.

2.20. $y = \frac{x^2 - 3x + 2}{x + 1}$.

2.21. $y = \frac{2x - 1}{(x-1)^2}$.

2.22. $y = \frac{x^5}{x^4 - 1}$.

2.23. $y = (x^3 + 4)/x^2$.

2.24. $y = \frac{1}{3}\sqrt[3]{x^2}(x - 5)$.

2.25. $y = x^3/(x^4 - 1)$.

2.26. $y = (e^{2x} + 1)/e^x$.

2.27. $y = x^2 + 1/x^2$.

2.28. $y = (5x^4 + 3)/x$.

2.29. $y = \frac{4 - 2x}{1 - x^2}$.

2.30. $y = \frac{5x}{4 - x^2}$.

3. Quyidagi funksiyalarni to'la tekshiring va grafigini yasang.

3.1. $y = e^{2x - x^2}$.

3.2. $y = x + \ln(x^2 - 4)$.

3.3. $y = \frac{2(x+1)^2}{x - 2}$.

3.4. $y = x\ln^2 x$.

3.5. $y = (4e^{x^2} - 1)/e^{x^2}$.

3.6. $y = x^2 e^{-x^2/2}$.

3.7. $y = xe^{1/x}$.

3.8. $y = \frac{2 + x}{(x+1)^2}$.

3.9. $y = \frac{(1-x)^3}{(x-2)^2}$.

3.10. $y = xe^x$.

3.11. $y = x^2 e^{1/x}$.

3.12. $y = x^2/(x + 2)^2$.

3.13. $y = (x + 2)e^{1-x}$.

3.14. $y=\frac{\ln x}{x}$.

3.15. $y=(\frac{x-2}{x+1})^2$.

3.16. $y=\frac{x^3}{9-x^3}$.

3.17. $y=(x+1)e^{2x}$.

3.18. $y=4x/(4+x^2)$.

3.19. $y=x^4/(x^3-1)$.

3.20. $y=\ln(x^2-2x+6)$.

3.21. $y=\ln(1-1/x^2)$.

3.22. $y=x^3e^{x+1}$.

3.23. $y=x-\ln(1+x^2)$.

3.24. $y=1-\ln^3 x$.

3.25. $y=(x-1)e^{4x+2}$.

3.26. $y=\frac{2x^2+2+4x}{2-x}$.

3.27. $y=-x\ln^2 x$.

3.28. $y=x^2-2\ln x$.

3.29. $y=e^{1/(2-x)}$.

3.30. $y=\ln(4-x^2)$.

4. Funksiyaning $[a;b]$ kesmadagi eng katta va eng kichik qiymatini toping.

4.1. $y=\ln(x^2-2x+2),[0;3]$.

4.2. $y=3x/(x^2+1),[0;5]$.

4.3. $y=(2x-1)/(x-1)^2,[-1/2;0]$.

4.4. $y=(x+2)e^{1-x},[-2;2]$.

4.5. $y=\ln(x^2-2x+4),[-1;3/2]$.

4.6. $y=x^3/(x^2-x+1),[-1;1]$.

4.7. $y=((x+1)/x)^3,[1;2]$.

4.8. $y=\sqrt{x-x^3},[-2;2]$.

4.9. $y=4-e^{-x^2},[0;1]$.

4.10. $y=(x^3+4)/x^2,[1;2]$.

4.11. $y=xe^x,[-2;0]$.

4.12. $y=(x-2)e^x,[-2;1]$.

4.13. $y=(x-1)e^{-x},[0;3]$.

4.14. $y=x/(9-x^2),[-2;2]$.

4.15. $y=(1+\ln x)/x,[1/e;e]$.

4.16. $y=e^{4x-x^2},[1;3]$.

4.17. $y=(x^5-8)/x^4,[-3;-1]$.

4.18. $y=\frac{e^{2x}+1}{e^x},[-1;2]$.

4.19. $y=x\ln x,[1/e^2;1]$.

4.20. $y=x^3e^{x+1},[-4;0]$.

4.21. $y=x^2-2x+2/(x-1),[-1;3]$.

4.22. $y=(x+1)\sqrt[3]{x^2},[-4/5;3]$.

4.23. $y=e^{6x-x^2},[-3;3]$.

4.24. $y=(\ln x)/x,[1;4]$.

4.25. $y=3x^4-16x^3+2,[-3;1]$.

4.26. $y=x^5-5x^4+5x^3+1,[-1;2]$.

4.27. $y=(3-x)e^{-x},[0;5]$.

4.28. $y=\sqrt{3}/2+\cos x,[0;\pi/2]$.

4.29. $y=108x-x^4,\left[-1;4\right]$.

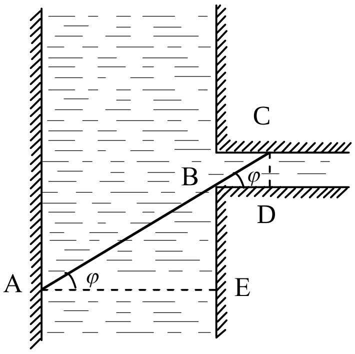
4.30. $y=x^4/4-6x^3+7,[16;20]$.

Namunaviy variantni yechish.

1. Eni 32 m bo’lgan kanaldan to’g’ri burchak ostida eni 4 m bo’lgan kanal ajralib chiqqan. Shu kanallar orqali oqizish mumkin bo’lgan g’o’lalarning eng uzunini toping. (G’o’lalarning qalinligi hisobga olinmaydi)

►G’o’laning uzunligini l deb belgilaymiz.

U xolda $l=|AC|=|AB|+|BC|$, $|AB|=\frac{|AE|}{\cos\varphi}=\frac{32}{\cos\varphi}$, $|BC|=\frac{|CD|}{\sin\varphi}=\frac{4}{\sin\varphi}$; $l=\frac{32}{\cos\varphi}+\frac{4}{\sin\varphi}$.



(6.16- rasm).

Bu funksiyaning ekstremumini qidiramiz: $l' = \frac{dl}{d\phi} = \frac{32}{\cos^2\phi}\sin\phi - \frac{4}{\sin^2\phi}\cos\phi = \frac{32\sin^3\phi - 4\cos^3\phi}{\sin^2\phi\cos^2\phi}$

Agar $l' = 0$, $32\sin^3\varphi - 4\cos^3\varphi = 0.$ Trigonometrik funksiya $\cos\varphi \neq 0$ bo'lgani uchun oxirgi tenglamadan $tg^3\varphi = 1/8$, $tg\varphi = 1/2$, $\sin\varphi = 1/\sqrt{5}$, $\cos\varphi = 2/\sqrt{5}$, $\varphi \approx 26°34'$.Bu qiymat atrofida hosila l' ning ishorasi suratning ishorasi bilan aniqlanadi, ya'ni $u(\varphi) = 32\sin^3\varphi - 4\cos^3\varphi$

6.16- rasm

Quydagiga ega bo'lamiz:

$u(\phi)\big|_{\phi=26°} \approx 32\cdot 0,438^3 - 4\cdot 0,899^3 \approx 2,696 - 2,904 < 0$.

$u(\phi)\big|_{\phi=27°} \approx 32\cdot 0,454^3 - 4\cdot 0,891^3 \approx 2,994 - 2,829 > 0$.

Demak $\varphi = 20°34'$, $|AC|$ masofa eng kichik bo'ladi va oqizilayotgan g'o'lalarning uzunligi $l_{\max}$, bundan uzun bo'la olmaydi . Shunday qilib quyidagiga ega bo'lamiz $l_{\max} = 20\sqrt{5} \approx 44,72\,\text{m}$.

2. Funksiyani to'liq tekishiring va grafigini yasang. $y = (x+3)^2 / x - 4$.

►Funksiyani § 6.7. dagi asosiy sxemaga asosan tekshiramiz.

1. Funksiyaning aniqlanish sohasi quyidagi to'plamni hosil qiladi

$$x \in (-\infty;4) \cup (4;+\infty).$$

2. Grafikning ordinatalari agar *x>4* bo'lsa, *y>0* va agar *x<4* bo'lsa *y<0.*

3. Bu funksiya grafigining koordinata o'qlari bilan kesishish nuqtalari *(0;-9/4)* va *(-3;0)* .

4. Ko'rinib turibdiki *x=4* – veritikal asimptotasi

$$\lim_{x\to 4-0} y = \lim_{x\to 4-0} \frac{(x+3)^2}{x-4} = -\infty,\ \lim_{x\to 4+0} \frac{(x+3)^2}{x-4} = +\infty .$$

Og'ma asimptotalarni topamiz: $k = \lim_{x\to\pm\infty} \frac{f(x)}{x} = \lim_{x\to\pm\infty} \frac{(x+3)^2}{x(x-4)} = 1$.

$$b = \lim_{x\to\pm\infty} (f(x) - kx) = \lim_{x\to\pm\infty} (\frac{(x+3)^2}{x-4} - x) = \lim_{x\to\pm\infty} \frac{10x+9}{x-4} = 10 .$$

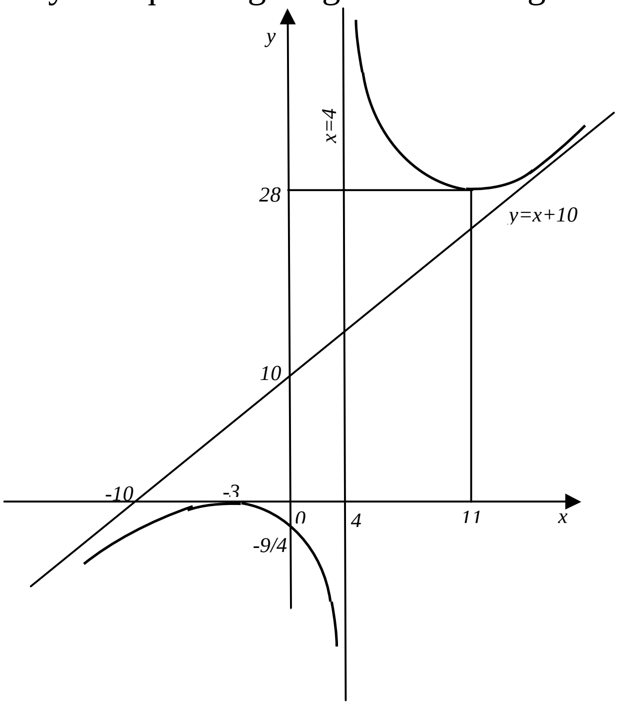
Shunday qilib yagona u=x+10 og'ma asimtota mavjud.

5. Funksiyani o'sish va kamayish oraliqlari va ekstremumlirini aniqlaymiz: $y' = \frac{2(x+3)(x-4) - (x+3)^2}{(x-4)^2} = \frac{2x^2 - 2x - 24 - x^2 - 6x - 9}{(x-4)^2} = \frac{x^2 - 8x - 33}{(x-4)^2}$.

Hosila $y' = 0$ dan $x^2 - 8x - 33 = 0$, bundan $x_1 = 11, x_2 = -3$ kelib chiqadi . Quyidagi $(-\infty;-3)$ oraliqda $y' > 0$ va funksiya bu oraliqda o'suvchi , (-3;4) oraliqda $y' < 0$ funksiya kamayuvchi. Shuning uchun x=-3 da funksiya *y(-3)=0* maksimumga ega bo'ladi. Ushbu (4,11) oraliqda $y' < 0$ va funksiya kamayuvchi, $(11;\infty)$ oraliqda $y' > 0$ va funksiya bu oraliqda o'suvchi. Funksiya x=11 nuqtada *u(11)=28* lokal minimumga ega bo'ladi.

6. Funksiya grafigini botiqlik va qavariqlikka tekshiramiz va burilish nuqtalarini topamiz. Buning uchun ikkinchi tartibli hosilani topamiz $y''=\frac{(2x-8)(x-4)^2-(x^2-8x-33)2(x-4)}{(x-4)^4}=\frac{2x^2-8x-8x+32-2x^2+16x+66}{(x-4)^3}=\frac{98}{(x-4)^3}$.

Ko'rinib turubdiki $x<4$ bo'lsa $y''<0$, x>4 bo'lsa $y''>0$, demak, (-∞;4) oraliqda grafik qavariq, (4;+∞) oraliqda funksiya botiq. Ma'lumki x=4 nuqtada funksiya aniqlanmaganligi uchun uning burilish nuqtasi yo'q.



6.17- rasm

7. Funksiyaning grafigi 6.17 rasmda berilgan.

3. $y=xe^{\frac{-x^2}{2}}$ funksiyani to'liq tenshiring va grafigini chizing.

Funksiyani to'liq tekshirishning umumiy sxemasidan foydalanamiz.

1. Funksiyaning aniqlanish sohasi (-∞;+∞).

2. Argument x=0 bo'lganda u=0 bo'lgani uchun funksiya koordinata boshidan o'tadi.

3. Funksiya (0;+∞) oraliqda musbat, (-∞;0) oraliqda manfiy qiymat qabul qiladi.

4. Vertikal asimtotalar yo'q. Og'ma asimptotalarni topamiz:

$k=\lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{1}{e^{\frac{x^2}{2}}}=0$; $b=\lim_{x\to\pm\infty}(f(x)-kx)=\lim_{x\to\pm\infty}\frac{x}{e^{\frac{x^2}{2}}}=\lim_{x\to\pm\infty}\frac{1}{e^{\frac{x^2}{2}}}=0$.

Demak gorizontal asimptota *y=0.*

5. Funksiya $y(-x)=-xe^{\frac{x^2}{2}}=-y(x)$ shartni qanoatlantirgani uchun toq va uning grafigi koordinata boshiga nisbatan simmetrik bo'ladi.

6. Funksiyaning monotonligini tekshiramiz

$$y'=\frac{e^{\frac{x^2}{2}}-x\cdot xe^{\frac{x^2}{2}}}{e^{x^2}}=\frac{e^{\frac{x2}{2}}(1-x^2)}{e^{x^2}}.$$

Agar $y'=0$ bo'lsa $1-x^2=0,$ bundan $x_1=-1, x_2=1.$ Bu nuqtalar sonlar o'qini 3 ta oraliqqa ajratadi: $(-\infty;-1)$ oraliqda $y'>0$ va funksiya o'suvchi, $(-1;1)$ oraliqda $y'<0$ va funksiya bu oraliqda kamayuvchi $(1;+\infty)$ oraliqda $y'>0$ va funksiya bu oraliqda o'suvchi.

Argumentning x=-1 qiymatida funksiya minimumga ega $y(-1)=\frac{-1}{e^{\frac{1}{2}}}\approx -0,6$, x=1 nuqtada $y(1)=\frac{1}{e^{\frac{1}{2}}}\approx 0,6$.

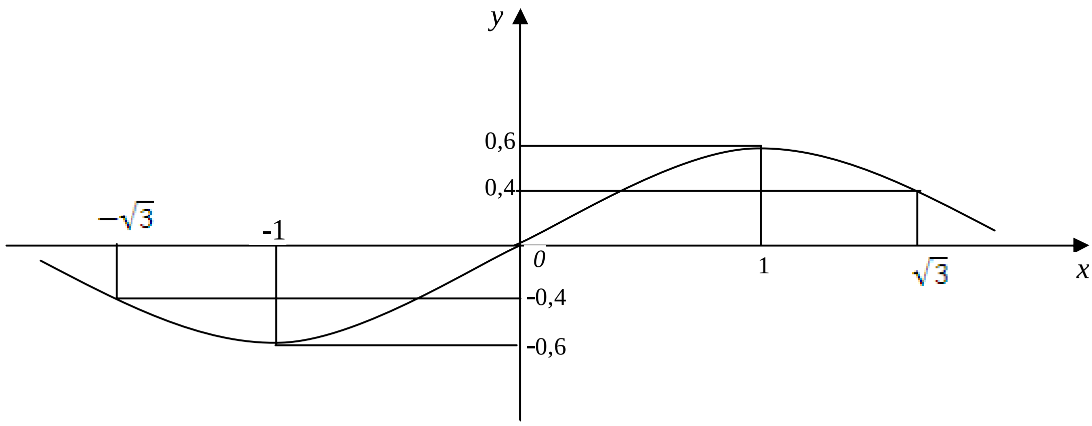
7. Funksiyaning ikkinchi tartibli hosilasini hisoblaymiz :

$$y'=\frac{1-x^2}{e^{\frac{x^2}{2}}}; y''=\frac{-2xe^{\frac{x^2}{2}}-(1-x^2)xe^{\frac{x^2}{2}}}{e^{x^2}}=\frac{xe^{\frac{x^2}{2}}(-2-1+x^2)}{e^{x^2}}=\frac{x(x^2-3)}{e^{\frac{x^2}{2}}}.$$

Agar $y''=0$ bo'lsa $x(x^2-3)=0$, bundan $x_1=0, x_2=-\sqrt{3}, x_3=\sqrt{3}$.

Quyidagi $(-\infty;-\sqrt{3})$ oraliqda $y''<0$ grafik qavariq; $(-\sqrt{3};0)$ oraliqda $y''>0$ grafik botiq; $(0;\sqrt{3})$ oraliqda $y''<0$ grafik qavariq; $(\sqrt{3};+\infty)$ oraliqda $y''>0$ grafik botiq bo'ladi.

Ushbu $x=\pm\sqrt{3}, x=0$ nuqtalarda ikkinchi tartibli hosila ishorasini o'zgartiradi va bu nuqtalar burilish nuqtalari bo'ladi va ularning ordinatalarini hisoblaymiz: $y(\pm\sqrt{3})=\pm\sqrt{3}/e^{3/2}\approx\pm 0,4; y(0)=0$.



6.18-rasm

8. Ushbu ma'lumotlarga tayanib funksiyaning grafigini chizamiz (rasm 6.18) .

4. Quyidagi $y=2\sin x+\cos 2x$ funksiyaning $[0;\frac{\pi}{2}]$ kesmadagi eng katta va eng kichik qiymatini toping.

► Kritik nuqtalarni topamiz: $y'=2\cos x-2\sin 2x$. Agar $y'=0$ bo'lsa $2\cos x-4\sin x\cos x=0,\quad 2\cos x(1-2\sin x)=0$,agar $\cos x=0,$ bo'lsa $x=\frac{\pi}{2}+2k\pi,$ agar $\sin x=\frac{1}{2}$ bo'lsa $x=(-1)^n\frac{\pi}{6}+\pi n, k_1 n\in f$.

Hamma topilgan kritik nuqtalardan faqat $x=\frac{\pi}{6}, x=\frac{\pi}{2}$ nuqtalar berilgan $[0;\frac{\pi}{2}]$ kesmaga tegishli bo'ladi. Funksiyaning $x=0,\quad x=\frac{\pi}{6},\quad x=\frac{\pi}{2}$ nuqtalardagi qiymatini hisoblaymiz:

$$y(0)=1,\quad y(\frac{\pi}{6})=2\sin\frac{\pi}{6}+\cos\frac{\pi}{3}=1+\frac{1}{2}=1{,}5,\quad y(\frac{\pi}{2})=2\sin\frac{\pi}{2}+\cos\pi=2-1=1.$$

Demak, funksiya $[0;\frac{\pi}{2}]$ kesmada eng katta qiymatini $x=\frac{\pi}{6}$ nuqtada, eng kichik qiymatini x=0 va $x=\frac{\pi}{2}$ nuqtada qabul qiladi: $y(\frac{\pi}{6})=1{,}5\quad y(0)=y(\frac{\pi}{2})=1$ ◄

6.11 6-BO'LIMGA DOIR QO'SHIMCHA MASALALAR

1. Quyidagi funksiyalarning grafigi qaysi nuqtada va qanday burchak ostida kesishadi: $a)\, f(x)=x^3,\quad y(x)=\frac{1}{x^2};\quad$ *b)* $f(x)=x^2-4x+4,\quad y(x)=-x^2+6x-4$ (Javob: *a)* $(1{,}2),\ \varphi=\frac{\pi}{4};\quad b)\ (1{,}1)\ \ (4{,}4),\ \varphi=arctg\frac{6}{7}.$)

2. Dekart va qutb koordinatalar sistemasida qutb burchagi $\varphi=\frac{\pi}{6}$ bo'lgan nuqtada $\rho=a(1+\cos\varphi)$ kardoidaga normalning tenglamasi tuzilsin (Javob: $x-y-(1+2\sqrt{3})a/4=0,\quad \rho=\frac{1+2\sqrt{3}a}{4(\cos\varphi-\sin\varphi)}$.)

3. Massasi 1,5 kg bo'lgan jism $S(t)=t^2+t+1$(S- metr,t-sekund) qonun bilan to'g'ri chiziqli harakat qilmoqda. Jismning 5 sek harakatidan so'ng kinetik energiyasi topilsin (Javob: 90,75 Dj.)

4. Material nuqta $\rho=a\varphi$ Arximed spirali bo'yicha harakat qilmoqda, radiusning aylana burchak tezligi o'zgarmas va $\frac{\pi}{30}$ rad/s teng. Agar *a=10 m* bo'lsa qutb radiusi ρ ning uzayish tezligini toping. (Javob: $\frac{\pi}{3}м/c$.)

5. 1 kg suvni $0°$ dan t^0C gacha isitish uchun ketadigan issiqlik miqdori Q Dj ushbu formala yordamida aniqlanadi : $Q=t+2\cdot10^{-5}\cdot t^2+3\cdot10^{-7}t^3$. Suvning $t=100°C$ bo'lgandagi issiqlik sig'imi topilsin (Javob: 1,013 Dj/(kg·grad) .)

6. Tosh boshlangich tezlik bilan gorizontga nisbatin α burchak ostida otilgan. Havoning qarshiligini hisobga olmaganda, α ning qanday qiymatida tosh eng uzoq masofaga borib tushadi (Javob: $\frac{\pi}{4}$.)

7. Galvanik elementning ichki qarshiligi *R OM* ga teng. Qanday tashqi qarshiligda, bu elementdan tashqi zanjir bo'yicha olingan tokning quvvati eng katta bo'ladi (Javob*: R OM.*)

8. Quyidagi funksiyalarni to'la tekshiring va grafiklarini chizing:

a) $x = t^3 + 2t^2 + t, \quad y = -3t^3 + 3t - 2$ b) $x = (t-1)^2(t-2), \quad y = (t-1)^2(t-3)$ c) $x^3 - y^3 = 1$

d) $y^2(2-x) = x^3$.

9. Quyidagi limitlarni toping

a) $\lim\limits_{x\to\frac{\pi}{4}}(tgx)^{tg2x}$; b) $\lim\limits_{x\to a}(2-\frac{x}{a})^{tg(\pi x/(2a))}$ c) $\lim\limits_{x\to 1}(\frac{1}{2(1-\sqrt{x})} - \frac{1}{3(1-\sqrt[3]{x})})$ d) $\lim\limits_{x\to 0} x^{\sin x}$

(Javob: *a)* e^{-1}; *b)* e^{π} *; c)* 1/12; *d) 1.*)

10. Funksiyani Makloren formulasiga ko'ra yoyilishidan foydalanib, limitni hisoblang $\lim\limits_{x\to 0}\frac{\cos x - e^{\frac{-x^2}{2}}}{x^4}$ (Javob: -1/12.)

11. Botqoqlikni quritish uchun ko'ndalang kesimi teng yonli trapetsiya bo'lgan kanal qurish kerak. Kanal shunday qurilishida, agar ko'ndalang kesimning yuzi «S», kanalning chuqurligi «h» bo'lsa, suvning ishqalanishi eng kichik bo'lishi uchun, qirg'oqning nishablik burchagi «α» qanday bo'lishi kerak? (Javob: $\alpha = \frac{\pi}{6}$.)

12. Kanalning ko'ndalang kesimi to'g'ri to'rtburchak bo'lib, yarim aylana bilan tugaydi. Kesimning perimetrii 45 *m*. Yarim aylananing radiusi qanday bo'lganda ko'ndalang kesimning yuzi eng katta bo'ladi ? (Javob: $\frac{45}{45+\pi}$ *m*.)

13. Suv qalin devordagi tirqishdan oqib chiqadi. Suvning sekunddagi sarfi quyidagi formula orqali aniqlanadi $Q = cy\sqrt{h-y}$, bu erda *s*-o'zgarmas, *u*-tirqishning diametri, *h*-uning eng pastki nuqtasining chuqurligi. Tirqishning diametri qanday bo'lganda suvning sarfi eng katta bo'ladi. (Javob: *2/3 h.*)

14. Binoning balandligi «*H*» eni «*a*» ga teng. Ushbu binoni plita bilan qoplash uchun balandligi «*h*» bo'lgan kran, strelasi va bino orasidagi masofa «*m*» dan kichik bo'lmagan holda, binoga parallel harakatlanadi. Kran plitani shunday uzatishi kerakki, uning ilmog'i binonig o'rtasida bo'lsin. Qurilish kranining strelasining uzunligi toping va masalani umumiy holda eching, bu yerda *N=125m, m=15m, a=10m, h=116m*. (Javob: *23,3m*.)

15. Ko'ndalang kesimining radiusi «*r*» bo'lgan trubadan suv oqadi. Ma'lumki, suvning oqish tezligi gidravlika radiusi deb nomlanuvchi - *R* ga to'g'ri proporsional.. *R=S/P,* bu yerda *S* – suv oqimi ko'ndalang kesimining yuzi, *R* – trubaning ko'ndalang kesimbiing suv osti perimetrii. Trubani suv bilan to'ldirishning qanday markaziy burchagida suvning oqish tezligi maksimal bo'ladi? (Javob: $258°$.)

16. Tekis siqilgan g'o'laning uzunligi «l» bo'lib uning egilish vaqtinig maksimum nuqtasi g'o'laning markazida bo'lishini isbot qiling.(M nuqtadagi egilish vaqtni quyidagi formula bilan aniqlanadi $M = \frac{1}{2}lx - \frac{1}{2}wx^2$, *w*- solishtirma yuklama, *x* - nuqtadan g'o'lagacha bo'lgan masofa.)

17. Bir jinsli sterjen *AV*, *A* nuqtaning atrofida aylanishi mumkin va bu nuqtadan *S* masofada og'irligi *Q* ga teng bo'lgan yukni, sterjenning ohiri *V* punktiga quyilgan vertikal «*R*» kuch yordamida muvozanitda ushlab turadi. Sterjenning bir santimetrning og'irligi *q* ga teng bo'lsa sterjenning qanday uzunligida, vertikal «*R*» kuch eng kichik bo'ladi? (Javob: $|AB| = \sqrt{\frac{2SQ}{q}}, P = \sqrt{2SgQ}$.)

18. Agar sferaning radiusini o'lchaganida nisbiy xato 1% bo'lsa, sferaning sirti yuzini hisoblaganda nisbiy xatoni butun sonlarga yaxlitlab hisoblang. (Javob: 2%)

19. Agar o'tkazgichning qarshiligi 1% ga oshishi, tok kuchi necha foizga oshishini butun songacha yaxlitlab hisoblang (Javob: 2%.)

20. Uzunligi *l=20 cm* bo'lgan mayatnikni tebranish davri *T - 0,05* sekundga oshishi uchun uning uzunligini necha sm oshirish kerak ($T = 2\pi\sqrt{l/q}$).(Javob: *2,23sm.*)

21. Quyidagi chiziqlarning ixtiyoriy nuqtadagi egrilik markazlarning (Evolyuta larning parametrik tenglamalarini) koordinatalari topilsin? a) Giperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ b) Astroida $x^{2/3} + y^{2/3} = a^{2/3}$. (Javob: a) $\xi = (a^2 + b^2)x^3/a^4$, $\eta = -(a^2 + b^2)y^3/b^4$; b) $\xi = x + 3x^{\frac{1}{3}}y^{\frac{2}{3}}$, $\eta = y + 3x^{2/3}y^{1/3}$.)

22. Ushbu chiziqning egrilik radiusining eng katta qiymati hisoblansin $\rho = a\sin^3\frac{\varphi}{3}$. (Javob: $\frac{3}{4}a$).

23. Ushbu $y = e^x$ chiziqning (0,1) nuqtadagi egrilik aylanasiniig tenglamasi aniqlansin. (Javob: $(x+2)^2 + (y-3)^2 = 8$).

ILOVA

«Vektor algebrasi» dan nazorat ishi (2 soat)

1.

Ushbu *K* va *L* nuqtalar *ABCD* parallelogramning *BC* va *CD* tomonlarining o'rtalari. Agar $\overline{AK}=\overline{a}$ va $\overline{AL}=\overline{b}$ bo'lsa quyidagi vektorlarni $\overline{a}$ va $\overline{b}$ orqali ifodalang.

1.1. $\overline{BC},\overline{CD}$.
1.2. $\overline{AC},\overline{AB}$.
1.3. $\overline{BD},\overline{BL}$.
1.4. $\overline{KD},\overline{KL}$.
1.5. $\overline{BK},\overline{DL}$.
1.6. $\overline{CK},\overline{BA}$.
1.7. $\overline{DA},\overline{DB}$.
1.8. $\overline{LB},\overline{LC}$.
1.9. $\overline{CA},\overline{KB}$.
1.10. $\overline{DB},\overline{DA}$.

2.Tomonlari *2* ga teng bo'lgan muntazam *ABCDEF* olti burchak, A uchidan $\overline{AB}$ vektor yo'nalishi bo'yicha birlik vektor $\overline{m}$ va $\overline{AF}$ vektor yo'nalishi bo'yicha $\overline{n}$ vektor chiqarilgan. Quyidagi vektorlarni $\overline{m}$ va $\overline{n}$ orqali ifodalang.

1.11. $\overline{AD},\overline{EC}$.
1.12. $\overline{BD},\overline{DF}$.
1.13. $\overline{AE},\overline{DF}$.
1.14. $\overline{AC},\overline{BE}$.
1.15. $\overline{BC},\overline{BD}$.
1.16. $\overline{FB},\overline{AE}$.
1.17. $\overline{AD},\overline{CF}$.
1.18. $\overline{DA},\overline{FC}$.
1.19. $\overline{AC},\overline{BD}$.
1.20. $\overline{CE},\overline{FB}$.

OABC tetraedr berilgan. Agar $\overline{OA}=\overline{a}$, $\overline{OB}=\overline{b}$, $\overline{OC}=\overline{c}$ bo'lsa, ushbu vektorlar orqali quyidagi vektorlarni aniqlang (*M,R,R* nuqtalar mos ravishda *OA, OB* va *OC* qirralarnig o'rtalari *N,Q,S* nuqtalar qarama-qarshi qirralarning o'rtalari)

1.21. $\overline{MN},\overline{MC}$.
1.22. $\overline{PQ},\overline{PA}$.
1.23. $\overline{RS},\overline{RB}$.
1.24. $\overline{NM},\overline{NO}$.
1.25. $\overline{QP},\overline{OQ}$.
1.26. $\overline{SR},\overline{OS}$.
1.27. .$\overline{MP},\overline{CS}$.
1.28. $\overline{NP},\overline{CM}$.
1.29. $\overline{NQ},\overline{BR}$.
1.30. $\overline{RN},\overline{MB}$.

2.

Ushbu $\overline{a}$ va $\overline{b}$ vektorlarda yasalgan uchburchakning yuzini toping.

2.1. $\vec{a}=-2j+3k,\vec{b}=3i-2j$.
2.2. $\vec{a}=2i-3j+k,\vec{b}=i+2j-4k$.
2.3. $\vec{a}=5i-2j-k,\vec{b}=-2i+j-7k$.
2.4. $\vec{a}=6i-4j+k,\vec{b}=2i+3j-4k$.
2.5. $\vec{a}=7i-4j+2k,\vec{b}=i+3k-4k$.
2.6. $\vec{a}=i+2j-3k,\vec{b}=3j-k$.
2.7. $\vec{a}=4i-j+6k,\vec{b}=2j-3k$.
2.8. $\vec{a}=-3i+6j-2k,\vec{b}=i+2j+4k$.
2.9. $\vec{a}=3i+7j-2k,\vec{b}=i-j+5k$.
2.10. $\vec{a}=i+6j-2k,\vec{b}=5i+4j$.

Parallelogramm $\bar{a}$ va $\bar{b}$ vektorlarga qurilgan. Uning $\bar{a}$ vektor bilan ustma ust tushuvchi tomoniga tushirilgan balandligini toping.

2.11. $\vec{a} = 5i + 7j - 3k, \vec{b} = -i + 2j + 4k$.

2.12. $\vec{a} = -4i - 9j + 2k, \vec{b} = i - 4j + k$.

2.13. $\vec{a} = 3i - 2j + 6k, \vec{b} = 5j - 4k$.

2.14. . $\vec{a} = 4i - 6j - k, \vec{b} = i - 2j + 5k$.

2.15. $\vec{a} = 4i - 3j + k, \vec{b} = 2i - 6j + 3k$.

2.16. $\vec{a} = 5i + 2j + 3k, \vec{b} = 5i + 2k$.

2.17. $\vec{a} = 4i + j + k, \vec{b} = 2i + j - k$.

2.18. $\vec{a} = 3i - 2j + 4k, \vec{b} = i + 3j - k$.

2.19. $\vec{a} = -3i + 5j + 2k, \vec{b} = 2i - 3j + 6k$.

2.20. $\vec{a} = 11i - 5j + 4k, \vec{b} = 2i - j$.

Agar $|\vec{a}| = k, |\vec{b}| = l, \vec{a} \cdot \vec{b} = p$ bo'lsa $|\vec{a} \times \vec{b}|$ ni toping

2.21. $k = \sqrt{29}, l = \sqrt{61}, p = 36$.

2.22. $k = \sqrt{74}, l = \sqrt{20}, p = 20$.

2.23. $k = \sqrt{45}, l = \sqrt{14}, p = 5$.

2.24. $k = \sqrt{33}, l = \sqrt{59}, p = 25$.

2.25. $k = \sqrt{46}, l = \sqrt{38}, p = -24$.

2.26. $k = \sqrt{30}, l = \sqrt{29}, p = -28$.

2.27. $k = \sqrt{50}, l = \sqrt{14}, p = -23$.

2.28. $k = \sqrt{45}, l = \sqrt{21}, p = 20$.

2.29. $k = \sqrt{53}, l = \sqrt{30}, p = 12$.

2.30. $k = \sqrt{98}, l = \sqrt{21}, p = 10$.

3.

$\bar{c}$ - vektorning $\bar{d}$ - vektor yo’nalishi bo’yicha proektsiyasini toping

3.1. $\vec{c} = (-2,0,1), \vec{d} = (1,2,-3)$.

3.2. $\vec{c} = (4,-5,1), \vec{d} = (3,2,-4)$.

3.3. $\vec{c} = (2,-8,1), \vec{d} = (-3,-1,2)$.

3.4. $\vec{c} = (-4,5,2), \vec{d} = (3,4,-6)$.

3.5. $\vec{c} = (9,5,-4), \vec{d} = (3,2,6)$.

3.6. $\vec{c} = (3,-4,11), \vec{d} = (-2,5,3)$.

3.7. $\vec{c} = (3,7,-5), \vec{d} = (1,4,-9)$.

3.8. $\vec{c} = (3,-6,5), \vec{d} = (1,4,4)$.

3.9. $\vec{c} = (-7,-5,1). \vec{d} = (3,4,-2)$.

3.10. $\vec{c} = (5,4,-1), \vec{d} = (2,-4,6)$.

x– vektor, *a* – vektorga kolleniar bo’lib *Oz* o’qi bilan o’tkir burchak tashkil qiladi.Agar $|\vec{x}| = t$ bo’lsa, *x* – vektorning koordinatalarini toping.

3.11. $\vec{a} = (4,-7,1), \vec{t} = \sqrt{264}$.

3.12. $\vec{a} = (5,-3,-1), \vec{t} = \sqrt{315}$.

3.13. $\vec{a} = (4,5,-6), \vec{t} = \sqrt{308}$.

3.14. $\vec{a} = (3,-5,7), \vec{t} = \sqrt{1328}$.

3.15. $\vec{a} = (4,-2,2), \vec{t} = 10\sqrt{6}$.

3.16. $\vec{a} = (5,6,-7), \vec{t} = 3\sqrt{110}$.

3.17. $\vec{a} = (5,-3,9), \vec{t} = 2\sqrt{115}$.

3.18. $\vec{a} = (5,-3,1), \vec{t} = 5\sqrt{35}$.

3.19. $\vec{a} = (7,-4,2), \vec{t} = 4\sqrt{69}$.

3.20. $\vec{a} = (3,-1,7), \vec{t} = 6\sqrt{59}$.

Agar ***x*** – vektor ***a*** va ***b*** vektorlarga perpendikulyar bo'lib Oy o'qi bilan o'tmas burchak tashkil qilsa va $|\vec{x}| = p$ shart bajarilsa, uning koordinatalarini toping

3.21. $\vec{a} = (4,2,-2), \vec{b} = (5,1,-3), \vec{p} = \sqrt{15}$.

3.22. $\vec{a} = (7,5,2), \vec{b} = (0,4,3), \vec{p} = \sqrt{26}$.

3.23. $\vec{a} = (4,3,1), \vec{b} = (3,4,8), \vec{p} = \sqrt{42}$.

3.24. $\vec{a} = (2,0,2), \vec{b} = (4,-6,0), \vec{p} = \sqrt{22}$.

3.25. $\vec{a} = (3,4,-1), \vec{b} = (4,6,-4), \vec{p} = \sqrt{42}$.

3.26. $\vec{a} = (4,6,5), \vec{b} = (-4,2,7), \vec{p} = \sqrt{17}$.

3.27. $\vec{a} = (-2,7,10), \vec{b} = (0,3,4), \vec{p} = \sqrt{26}$.

3.28. $\vec{a} = (-1,9,2), \vec{b} = (14,-1,-3), \vec{p} = \sqrt{27}$.

3.29. $\vec{a} = (4,5,8), \vec{b} = (5,2,-7), \vec{p} = \sqrt{26}$.

3.30. $\vec{a} = (12,3,-2), \vec{b} = (11,7,-1), \vec{p} = \sqrt{56}$.

4.

Quyidagi $\bar{a}$ va $\bar{b}$ vektorlar orasidagi burchakni toping

4.1. $|\vec{a}| = 1, |\vec{b}| = 2, (\vec{a} - \vec{b})^2 + (\vec{a} + 2\vec{b})^2 = 20.$

4.2. $|\vec{a}| = 2, |\vec{b}| = 3, (2\vec{a} - 3\vec{b})^2 - (\vec{a} + 4\vec{b})^2 = 69.$

4.3. $|\vec{a}| = 4, |\vec{b}| = 1, (3\vec{a} + 2\vec{b})^2 + (\vec{a} - 5\vec{b})^2 = 189.$

4.4. $|\vec{a}| = 3, |\vec{b}| = 5, (\vec{a} - 3\vec{b})^2 + (2\vec{a} + 4\vec{b})^2 = 595.$

4.5. $|\vec{a}| = 5, |\vec{b}| = 4, (4\vec{a} + \vec{b})^2 - (3\vec{a} - 2\vec{b})^2 = 77.$

4.6. $|\vec{a}| = 4, |\vec{b}| = 3, (2\vec{a} - 5\vec{b})^2 - (\vec{a} + 2\vec{b})^2 = 93.$

4.7. $|\vec{a}| = 6, |\vec{b}| = 1, (\vec{a} - 8\vec{b})^2 - (2\vec{a} + 3\vec{b})^2 = 31.$

4.8. $|\vec{a}| = 5, |\vec{b}| = 4, (3\vec{a} - \vec{b})^2 - (\vec{a} + 6\vec{b})^2 = 0$

4.9. $|\vec{a}| = 7, |\vec{b}| = 2, (\vec{a} + 4\vec{b})^2 + (3\vec{a} - 7\vec{b})^2 = 274.$

4.10. $|\vec{a}| = 3, |\vec{b}| = 6, (5\vec{a} - 2\vec{b})^2 - (\vec{a} + 3\vec{b})^2 = 270.$

Agar $\bar{a}$ va $\bar{b}$ vektorlar o'zaro perpendikulyar bo'lib $|\bar{m}| = |\bar{n}| = 1$ bo'lsa ***m*** va ***n*** vektorlar orasidagi burchakni toping.

4.11. $\vec{a} = 5\vec{m} - 4\vec{n}, \vec{b} = \vec{m} + 2\vec{n}$.

4.12. $\vec{a} = 3\vec{m} + 2\vec{n}, \vec{b} = \vec{m} - \vec{n}$.

4.13. $\vec{a} = \vec{m} + \vec{n}, \vec{b} = 2\vec{m} - \vec{n}$.

4.14. $\vec{a} = \vec{m} + 2\vec{n}, \vec{b} = 5\vec{m} - 4\vec{n}$.

4.15. $\vec{a} = \vec{m} - 2\vec{n}, \vec{b} = 5\vec{m} + 4\vec{n}$.

4.16. $\vec{a} = 3\vec{m} - 2\vec{n}, \vec{b} = \vec{m} + 4\vec{n}$.

4.17. $\vec{a} = 2\vec{m} - 3\vec{n}, \vec{b} = \vec{m} - \vec{n}$.

4.18. $\vec{a} = 2\vec{m} + \vec{n}, \vec{b} = \vec{m} - \vec{n}$.

4.19. $\vec{a} = 2\vec{m} + 4\vec{n}, \vec{b} = \vec{m} - \vec{n}$.

4.20. $\vec{a} = 3\vec{m} - 4\vec{n}, \vec{b} = \vec{m} + \vec{n}$.

Qanday $\bar{a}$ va $\bar{b}$ vektorlar uchun quyidagi shartlar bajariladi

4.21. $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$.

4.22. $|\vec{a} + \vec{b}| = |\vec{a}| - |\vec{b}|$.

4.23. $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$.

4.24. $|\vec{a} - \vec{b}| = |\vec{a}| + |\vec{b}|$.

4.25. $|\vec{a}| + |\vec{b}| = 0$.

4.26. $\vec{a} / |\vec{a}| = \vec{b} / |\vec{b}|$.

4.27. $(\vec{a} + \vec{b})^2 = |\vec{a}|^2 + |\vec{b}|^2$.

4.28. $\vec{a} = |\vec{a}| \vec{b}$.

4.29. $(\vec{a} + \vec{b}) \times (\vec{a} + \vec{b}) = 2\vec{a} \times \vec{b}$.

4.30. $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2$.

5.

O'zgarmas α ning qanday qiymatida $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlar komplanar bo'ladi.

5.1. $\vec{a} = (3,-1,4), \vec{b} = (2, \alpha, -5), \vec{c} = (1,0,2)$.

5.2. $\vec{a} = (4,-2, \alpha), \vec{b} = (-5,1,3), \vec{c} = (2,4,-3)$.

5.3. $\vec{a} = (3,-1,4), \vec{b} = (1,-4,0), \vec{c} = (\alpha,3,2)$.

5.4. $\vec{a} = (\alpha,2,-5),\ \vec{b} = (3,1,1),\ \vec{c} = (4,-1,0)$.

5.5. $\vec{a} = (-1,5,-7), \vec{b} = (4,2, \alpha), \vec{c} = (3,5,1)$.

5.6. $\vec{a} = (2,1,-1), \vec{b} = (4,-2,1), \vec{c} = (\alpha,-3,-2)$.

5.7. $\vec{a} = (4,-5,3), \vec{b} = (2, \alpha, -1), \vec{c} = (1,5,6)$.

5.8. $\vec{a} = (3,-2,1), \vec{b} = (1,-5,2), \vec{c} = (\alpha,4,-1)$.

5.9. $\vec{a} = (2,-3,5), \vec{b} = (1,-4, \alpha), \vec{c} = (2,1,-3)$.

5.10. $\vec{a} = (1,1, \alpha), \vec{b} = (-3,3,1), \vec{c} = (2,3,-3)$.

Ushbu $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlarga qurilgan piramidaning hajmini toping.

5.11. $\vec{a}=(5,2,0), \vec{b}=(2,5,0), \vec{c}=(1,2,4)$.

5.12. $\vec{a}=(-12,2,-4), \vec{b}=(-4,2,3), \vec{c}=(-3,4,-3)$.

5.13. $\vec{a}=(0,1,-1), \vec{b}=(1,0,-1), \vec{c}=(3,2,0)$.

5.14. $\vec{a}=(-5,6,-8), \vec{b}=(-2,-3,1), \vec{c}=(-3,1,1)$.

5.15. $\vec{a}=(4,4,-6), \vec{b}=(1,3,1), \vec{c}=(0,-2,0)$.

5.16. $\vec{a}=(1,2,-1), \vec{b}=(0,2,2), \vec{c}=(-1,1,-2)$.

5.17. $\vec{a}=(-1,3,3), \vec{b}=(0,4,2), \vec{c}=(3,3,-4)$.

5.18. $\vec{a}=(-3,6,2), \vec{b}=(-4,-1,-5), \vec{c}=(1,0,5)$.

5.19. $\vec{a}=(3,-2,1), \vec{b}=(1,4,0), \vec{c}=(5,2,3)$.

5.20. $\vec{a}=(-3,0,-2), \vec{b}=(-1,-1,3), \vec{c}=(-4,-1,0)$.

Agar $\vec{a}$ va $\vec{b}$ vektorlarga qurilgan parallelogramm $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlarga qurilgan paralelopiped asosi bo'lsa, shu paralelopiped balandligini toping.

5.21. $\vec{a}=(2,3,-1), \vec{b}=(-2,4,5), \vec{c}=(3,-1,4)$.

5.22. $\vec{a}=(3,6,-8), \vec{b}=(-2,4,-6), \vec{c}=(5,2,-1)$.

5.23. $\vec{a}=(-4,5,-4), \vec{b}=(-4,0,2), \vec{c}=(-3,3,-5)$.

5.24. $\vec{a}=(-1,-2,5), \vec{b}=(-4,-2,5), \vec{c}=(1,-3,-2)$.

5.25. $\vec{a}=(2,-1,1), \vec{b}=(-3,0,4), \vec{c}=(0,4,3)$.

5.26. $\vec{a}=(-2,5,5), \vec{b}=(-2,1,-1), \vec{c}=(-5,1,5)$.

5.27. $\vec{a}=(-2,3,0), \vec{b}=(-2,0,6), \vec{c}=(0,3,-2)$.

5.28. $\vec{a}=(4,-6,4), \vec{b}=(4,-1,2), \vec{c}=(3,2,7)$.

5.29. $\vec{a}=(-12,2,-4), \vec{b}=(-4,2,3), \vec{c}=(-3,4,-3)$.

5.30 $\vec{a}=(5,2,0), \vec{b}=(2,5,0), \vec{c}=(1,2,4)$.

2. «Limitlar»dan nazorat ishi

Limitlarni toping

1

1.1. $\lim\limits_{x\to 1}\dfrac{3x^2-2x-1}{x^2+4x+1}$.

1.2. $\lim\limits_{x\to 2}\dfrac{x^2+x+1}{x^2-x-2}$.

1.3. $\lim\limits_{x\to 3}\dfrac{x^2-4x+3}{2x^2-5x+1}$.

1.4. $\lim\limits_{x\to 4}\dfrac{x^2-x-2}{x^2-5x-4}$.

1.5. $\lim\limits_{x\to 5}\dfrac{x^2-25}{x^2-4x+5}$.

1.6. $\lim\limits_{x\to -1}\dfrac{x^2+3x+1}{2x^2-3x-5}$.

1.7. $\lim\limits_{x\to -3}\dfrac{2x^2+5x+1}{x^2+2x-3}$.

1.8. $\lim\limits_{x\to -2}\dfrac{x^2+x-2}{2x^2-x+1}$.

1.9. $\lim\limits_{x\to -4}\dfrac{x^2-16}{x^2+5x+2}$.

1.10. $\lim\limits_{x\to 3}\dfrac{x^2+x-3}{x^2-4}$.

1.11. $\lim\limits_{x\to 1}\dfrac{2x^2+x-3}{x^2+x-2}$.

1.12. $\lim\limits_{x\to 2}\dfrac{2x^2-3x-2}{x^2-3x+2}$.

1.13. $\lim\limits_{x\to 3}\dfrac{3x^2-10x+3}{x^2-2x-3}$.

1.14. $\lim\limits_{x\to 5}\dfrac{3x^2-14x+5}{x^2-6x+5}$.

1.15. $\lim\limits_{x\to 7}\dfrac{2x^2-13x-7}{x^2-9x+14}$.

1.16. $\lim\limits_{x\to 3}\dfrac{3m^2-5m-3}{m^2-5m+6}$.

1.17. $\lim\limits_{x\to 5}\dfrac{2x^2-11x+5}{x^2-7x+10}$.

1.18. $\lim\limits_{x\to 6}\dfrac{2x^2-9x-18}{x^2-7x+6}$.

1.19. $\lim\limits_{x\to 7}\dfrac{3x^2-17x-28}{x^2-9x+14}$.

1.20. $\lim\limits_{x\to 3}\dfrac{3x^2-8x-3}{x^2-x-6}$.

1.21. $\lim\limits_{x\to -2}\dfrac{x^2-x-6}{2x^2+x-6}$.

1.22. $\lim\limits_{x\to 2}\dfrac{x^2-x-2}{x^2+x-6}$.

1.23. $\lim\limits_{x\to -1}\dfrac{3x^2+x-2}{3x^2+4x+1}$.

1.24. $\lim\limits_{t\to -1}\dfrac{2t^2-5t-7}{3t^2+t-2}$.

1.25. $\lim\limits_{x\to -5}\dfrac{x^2+2x-15}{2x^2+7x-15}$.

1.26. $\lim\limits_{x\to -4}\dfrac{2x^2+9x+4}{x^2-x-20}$.

1.27. $\lim\limits_{x\to 1}\dfrac{3x^2-5x+2}{x^2-4x+3}$.

1.28. $\lim\limits_{x\to -2}\dfrac{x^2+3x+2}{2x^2+5x+2}$.

1.29. $\lim\limits_{x\to 4}\dfrac{x^2-x-12}{x^2-2x-8}$.

1.30. $\lim\limits_{x\to -3}\dfrac{x^2-x-12}{x^2+5x+6}$.

2.

2.1. $\lim\limits_{x\to 2}\dfrac{2x^2+3x-2}{3x^2+2x-8}$.

2.2. $\lim\limits_{x\to 1}\dfrac{3x^2-5x+2}{2x^2-x-1}$.

2.3. $\lim\limits_{x\to 2}\dfrac{10x-3x^2-8}{3x^2-8x+4}$.

2.4. $\lim\limits_{x\to 1}\dfrac{2x^2-x-3}{x^2-3x-4}$.

2.5. $\lim\limits_{x\to 3}\dfrac{7x-x^2-12}{2x^2-11x+15}$.

2.6. $\lim\limits_{x\to -3}\dfrac{3-8x-3x^2}{x^2+x-6}$.

2.7. $\lim\limits_{x\to 5}\dfrac{2x^2-17x+35}{x^2-x-20}$.

2.8. $\lim\limits_{x\to 1}\dfrac{2x^2-x-1}{4-3x^2-x}$.

2.9. $\lim\limits_{x\to -1}\dfrac{2x^2-16x+1}{3x^2+5x-2}$.

2.10. $\lim\limits_{x\to -1}\dfrac{3x^2+2x-1}{x^2-1}$.

2.11. $\lim\limits_{x\to 1}\dfrac{2x^2+x-3}{3x^2+x+2}$.

2.12. $\lim\limits_{x\to 1}\dfrac{4x^2+2x-3}{x^2+x-2}$.

2.13. $\lim\limits_{x\to 3}\dfrac{3x^2+10x+5}{x^2-2x-3}$.

2.14. $\lim\limits_{x\to 5}\dfrac{3x^2-14x-5}{2x^2+6x+5}$.

2.15. $\lim\limits_{x\to -2}\dfrac{3x^2-6x+2}{3x^2+5x+6}$.

2.16. $\lim\limits_{x\to -2}\dfrac{4x^2+9x+2}{x^2-3x-10}$.

2.17. $\lim\limits_{x\to -1}\dfrac{5x^2+4x-1}{x^2-6x-7}$.

2.18. $\lim\limits_{x\to -3}\dfrac{x^2+x-6}{2x^2+3x-7}$.

2.19. $\lim\limits_{x\to 2}\dfrac{x^2-5x+6}{3x^2-4x-3}$.

2.20. $\lim\limits_{x\to 5}\dfrac{x^2-6x+5}{2x^2-7x-18}$.

2.21. $\lim\limits_{x\to 4}\dfrac{x^2-x-12}{x^2-4x+3}$.

2.22. $\lim\limits_{x\to 2}\dfrac{4x^2-7x-2}{x^2-7x+10}$.

2.23. $\lim\limits_{x\to -3}\dfrac{x^2-x-12}{x^2+16}$.

2.24. $\lim\limits_{x\to 1}\dfrac{3x^2+x+4}{2x^2+x-3}$.

2.25. $\lim\limits_{x\to 1}\dfrac{2x^2-3x+2}{4x-3x^2-1}$.

2.26. $\lim\limits_{x\to}\dfrac{3x^2+x-2}{x^2-2x+3}$.

2.27. $\lim\limits_{x\to 2}\dfrac{2x^2+3x-14}{3x^2-7x+2}$.

2.28. $\lim\limits_{x\to 2}\dfrac{3x^2-10x+8}{2x^2-3x-2}$.

2.29. $\lim\limits_{x\to -1}\dfrac{3x^2-2x-5}{x^2+5x+4}$.

2.30. $\lim\limits_{x\to 3}\dfrac{2x^2-3x-9}{3x^2-5x-10}$.

3.

3.1. $\lim\limits_{x\to 2}\dfrac{\sqrt{3x-2}-2}{x^2-4}$.

3.2. $\lim\limits_{x\to -3}\dfrac{\sqrt{x+4}-1}{\sqrt{3-2x}-3}$.

3.3. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+4}-2}{\sqrt{9-x^2}-3}$.

3.4. $\lim\limits_{z\to -2}\dfrac{\sqrt{z+6}-2}{z^2-4}$.

3.5. $\lim\limits_{x\to 3}\dfrac{\sqrt{x+13}-4}{x^2-9}$.

3.6. $\lim\limits_{x\to 4}\dfrac{\sqrt{x}-2}{\sqrt{2x+1}-3}$.

3.7. $\lim\limits_{x\to 0}\dfrac{\sqrt{9-x}-3}{\sqrt{x+4}-2}$.

3.8. $\lim\limits_{x\to 3}\dfrac{\sqrt{2x+3}-3}{2-\sqrt{x+1}}$.

3.9. $\lim\limits_{n\to 0}\dfrac{\sqrt{n^2+9}-3}{\sqrt{4-n^2}-2}$.

3.10. $\lim\limits_{m\to 4}\dfrac{5-\sqrt{m^2+9}}{\sqrt{2m+1}-3}$.

3.11. $\lim\limits_{x\to 3}\dfrac{\sqrt{5x+1}-4}{x^2-9}$.

3.12. $\lim\limits_{x\to 0}\dfrac{\sqrt{4+3x}-\sqrt{4-3x}}{7x}$.

3.13. $\lim\limits_{x\to 2}\dfrac{\sqrt{5x-1}-3}{x^2-2x}$.

3.14. $\lim\limits_{x\to 1}\dfrac{\sqrt{5x+4}-3}{\sqrt{2x-1}-1}$.

3.15. $\lim\limits_{x\to 2}\dfrac{x^3-8}{\sqrt{4x+1}-3}$.

3.16. $\lim\limits_{b\to 5}\dfrac{\sqrt{b-1}-2}{\sqrt{2b-1}-3}$.

3.17. $\lim\limits_{x\to 0}\dfrac{\sqrt{9+x}-3}{x^2+3}$.

3.18. $\lim\limits_{a\to 2}\dfrac{\sqrt{3a+10}-4}{a^2-4}$.

3.19. $\lim\limits_{m\to 3}\dfrac{9-m^2}{\sqrt{4m-3}-3}$.

3.20. $\lim\limits_{x\to 7}\dfrac{x^2-49}{\sqrt{2x+11}-5}$.

3.21. $\lim\limits_{z\to 1}\dfrac{\sqrt{1+3z^2}-2}{z^2-z}$.

3.22. $\lim\limits_{x\to 3}\dfrac{\sqrt{2x-2}-2}{\sqrt{x+1}-2}$.

3.23. $\lim\limits_{t\to 5}\dfrac{\sqrt{1+3t}-\sqrt{2t+6}}{t^2-5t}$.

3.24. $\lim\limits_{x\to 2}\dfrac{x^3-8}{\sqrt{2x}-2}$.

3.25. $\lim\limits_{m\to 4}\dfrac{\sqrt{6m+1}-5}{\sqrt{m}-2}$.

3.26. $\lim\limits_{z\to 3}\dfrac{\sqrt{z-1}-\sqrt{2}}{\sqrt{2z+3}-3}$.

3.27. $\lim\limits_{x\to 9}\dfrac{\sqrt{x}-3}{\sqrt{2x-2}-4}$.

3.28. $\lim\limits_{x\to 0}\dfrac{\sqrt{x^2+9}-3}{\sqrt{x^2+25}-5}$.

3.29. $\lim\limits_{n\to 0}\dfrac{\sqrt{3-n}-\sqrt{3+n}}{5n}$.

3.30. $\lim\limits_{a\to 4}\dfrac{a-4}{\sqrt{5a+5}-5}$.

4.

4.1. $\lim\limits_{a\to\infty}\dfrac{2a^3-a+1}{a^2+2a-5}$.

4.2. $\lim\limits_{x\to\infty}\dfrac{2x^2-3x+1}{x^2+x-4}$.

4.3. $\lim\limits_{z\to\infty}\dfrac{2z^3+3z-1}{2z^3+z^2-4}$.

4.4. $\lim\limits_{n\to\infty}\dfrac{2n^2-3n+1}{n^2+2n-3}$.

4.5. $\lim\limits_{m\to\infty}\dfrac{3m^3+2m-5}{m^4+5m^2-1}$.

4.6. $\lim\limits_{z\to\infty}\dfrac{2z^2+z-3}{z^2+3z+1}$.

4.7. $\lim\limits_{a\to\infty}\dfrac{3a^2-4a+1}{a^3+3a-4}$.

4.8. $\lim\limits_{m\to\infty}\dfrac{m^3-8m+1}{3m^3-m+4}$.

4.9. $\lim\limits_{n\to\infty}\dfrac{3n^2-4n+1}{2n^2+n-3}$.

4.10. $\lim\limits_{z\to\infty}\dfrac{2-3z-z^2}{2z^3+z-1}$.

4.11. $\lim\limits_{n\to\infty}\dfrac{3n^2-4n^5+1}{2n^5+3n^3-n}$.

4.12. $\lim\limits_{a\to\infty}\dfrac{4a^3+3a^2-1}{2a^3-3a+1}$.

4.13. $\lim\limits_{y\to-\infty}\dfrac{y^6-3y^2-2}{2y^6+4y+5}$.

4.14. $\lim\limits_{z\to\infty}\dfrac{6z^5-3z^2+1}{3z^5-2z+3}$.

4.15. $\lim\limits_{x\to-\infty}\dfrac{x^3-4x^2+5}{3x^4+2x^2-x}$.

4.16. $\lim\limits_{a\to\infty}\dfrac{a^4-3a^2+2}{5a^4-3a-2}$.

4.17. $\lim\limits_{b\to-\infty}\dfrac{9b^5-4b^3+2}{3b^4-2b+3}$.

4.18. $\lim\limits_{x\to\infty}\dfrac{3x^2-4x+1}{x^3-2x^2-1}$.

4.19. $\lim\limits_{n\to\infty}\dfrac{4n^5-3n^2+1}{2n^5-2n+3}$.

4.20. $\lim\limits_{z\to\infty}\dfrac{9z^3-4z^2+1}{6z^3+3z+2}$.

4.21. $\lim\limits_{x\to-\infty}\dfrac{3x^5-x^2+x}{x^4+2x+5}$.

4.22. $\lim\limits_{n\to-\infty}\dfrac{6n^3-2n+7}{3n^3-5n+2}$.

4.23. $\lim\limits_{n\to-\infty}\dfrac{7n^4-2n^3+2}{n^4+2n}$.

4.24. $\lim\limits_{a\to\infty}\dfrac{3a^7+6a-5}{4a^7+2a^3-3}$.

4.25. $\lim\limits_{x\to\infty}\dfrac{8x^5-3x^2+9}{2x^5+2x+5}$.

4.26. $\lim\limits_{n\to\infty}\dfrac{n^4-5n+2}{2n^4+3n^2-n}$.

4.27. $\lim\limits_{x\to-\infty}\dfrac{6x^4-4x^3+8}{2x^3-3x^2+1}$.

4.28. $\lim\limits_{a\to-\infty}\dfrac{3a^4-4a^2+5}{6a^4+2a^3-1}$.

4.29. $\lim\limits_{n\to\infty}\dfrac{2+3n^2-n^5}{2n+n^2-3n^5}$.

4.30. $\lim\limits_{z\to\infty}\dfrac{2z^3+7z-4}{6z^3-3z^2+2}$.

5.

5.1. $\lim\limits_{\alpha\to\infty}\sin 3\alpha\cdot ctg\,2\alpha$.

5.2. $\lim\limits_{\varphi\to\infty}\dfrac{1-\cos 4\varphi}{\sin^2 3\varphi}$.

5.3. $\lim\limits_{\beta\to\infty}\dfrac{\arcsin 6\beta}{2\beta}$.

5.4. $\lim\limits_{\varphi\to0}\dfrac{\varphi\sin 2\varphi}{tg^2 3\varphi}$.

5.5. $\lim\limits_{\alpha\to0}\dfrac{arctg5\alpha}{3\alpha}$.

5.6. $\lim\limits_{x\to0}\dfrac{x\sqrt{1-\cos 4x}}{\sin^2 3x}$.

5.7. $\lim\limits_{x\to0}\dfrac{\sin^2 6x}{x\,tg\,2x}$.

5.8. $\lim\limits_{y\to0}\dfrac{\sin 5y}{\arcsin 2y}$.

5.9. $\lim\limits_{\varphi\to0}\dfrac{\arcsin^2\varphi}{3\varphi\sin\varphi}$.

5.10. $\lim\limits_{\beta\to0}\dfrac{tg^2 3\beta}{1-\cos 4\beta}$.

5.11. $\lim\limits_{x\to0}\dfrac{tg^2 4x}{\sin^2 3x}$.

5.12. $\lim\limits_{\alpha\to0}\dfrac{1-\cos 4\alpha}{\alpha\sin 3\alpha}$.

5.13. $\lim\limits_{x\to0}\dfrac{1-\cos 4x}{tg^2 5x}$.

5.14. $\lim\limits_{x\to0}\dfrac{arctg3x}{4x}$.

5.15. $\lim\limits_{x\to 0}\frac{\cos x-\cos^3 x}{4x^2}$.

5.16. $\lim\limits_{x\to 0}\frac{\cos x-\cos 5x}{3x^2}$.

5.17. $\lim\limits_{x\to 0}\frac{1-\cos 5x}{x tg 2x}$.

5.18. $\lim\limits_{x\to 0}\sin 5x\cdot ctg 3x$.

5.19. $\lim\limits_{z\to 0}\frac{5z^2}{\sin 3z\cdot tg 2z}$.

5.20. $\lim\limits_{\alpha\to 0}\sin 8\alpha\cdot ctg\,\alpha$.

5.21. $\lim\limits_{x\to 0}\frac{5x}{arcctg 3x}$.

5.22. $\lim\limits_{x\to 0}tg^2 3x\cdot ctg^2 2x$.

5.23. $\lim\limits_{\alpha\to 0}\frac{1-\cos 8\alpha}{1-\cos 2\alpha}$.

5.24. $\lim\limits_{x\to 0}3x ctg 7x$.

5.25. $\lim\limits_{\alpha\to 0}\frac{\alpha\sin 3\alpha}{\cos\alpha-\cos^3\alpha}$.

5.26. $\lim\limits_{x\to 0}\frac{x\sin 2x}{1-\cos 4x}$.

5.27. $\lim\limits_{\alpha\to 0}\frac{\arcsin^2 3\alpha}{2\alpha\sin 5\alpha}$.

5.28. $\lim\limits_{x\to 0}\frac{x tg 2x}{1-\cos 3x}$.

5.29. $\lim\limits_{x\to 0}\sin^2 3x\cdot ctg^2 5x$.

5.30. $\lim\limits_{\varphi\to 0}\frac{\sin^2 3\varphi}{arctg^2 2\varphi}$.

6.

6.1. $\lim\limits_{x\to -\infty}(1+\frac{3}{2x-1})^{4x}$.

6.2. $\lim\limits_{x\to\infty}(1-\frac{2}{x+4})^{2x-5}$.

6.3. $\lim\limits_{x\to\infty}(\frac{2x-3}{2x+1})^{3x-4}$.

6.4. $\lim\limits_{x\to -\infty}(\frac{3x+2}{3x+5})^{4-x}$.

6.5. $\lim\limits_{y\to\infty}(1-\frac{4}{3y-1})^{y+2}$.

6.6. $\lim\limits_{x\to\infty}(\frac{3x-1}{3x+5})^{2x-4}$.

6.7. $\lim\limits_{x\to\infty}(\frac{2x-3}{2x+1})^{5-2x}$.

6.8. $\lim\limits_{x\to\infty}(\frac{3x-1}{3x+2})^{2x-4}$.

6.9. $\lim\limits_{x\to\infty}(1-\frac{2}{3x-1})^{1-4x}$.

6.10. $\lim\limits_{x\to\infty}(1+\frac{2}{3x-4})^{1-6x}$.

6.11. $\lim\limits_{x\to -\infty}(\frac{2+x}{2-x})^{3x+1}$.

6.12. $\lim\limits_{x\to\infty}\ln(\frac{x+2}{x+3})^{2x+3}$.

6.13. $\lim\limits_{x\to -\infty}(\frac{2x+1}{2x-5})^{x-1}$.

6.14. $\lim\limits_{x\to\infty}\ln(\frac{2-4x}{1-4x})^{x+3}$.

6.15. $\lim\limits_{x\to -\infty}(\frac{2x+3}{2x-2})^{3x}$.

6.16. $\lim\limits_{x\to\infty}\ln(\frac{2x-3}{2x-1})^{x}$.

6.17. $\lim\limits_{x\to 2}(2x-3)^{x^2/(x-2)}$.

6.18. $\lim\limits_{x\to -\infty}(\frac{4x+5}{4x-1})^{x+3}$.

6.19. $\lim\limits_{x\to 1}(3x-2)^{5x/(x-1)}$.

6.20. $\lim\limits_{x\to 3}(3x-8)^{(x+1)/(x-3)}$.

6.21. $\lim\limits_{x\to\infty}\ln(\frac{2x+3}{2x-1})^{x}$.

6.22. $\lim\limits_{x\to\infty}(\frac{3x-2}{3x+4})^{x-1}$.

6.23. $\lim\limits_{n\to\infty}\ln(\frac{n+1}{n+2})^{n+3}$.

6.24. $\lim\limits_{x\to\infty}(\frac{2x+4}{2x-4})^{x-3}$.

6.25. $\lim\limits_{x\to\infty}(\frac{5x+1}{5x-1})^{x-4}$.

6.26. $\lim\limits_{x\to 1}(3x-2)^{x/(x^2-1)}$.

6.27. $\lim\limits_{t\to\infty}\ln(\frac{4+3t}{1+3t})^{t-2}$.

6.28. $\lim\limits_{x\to 2}(5-2x)^{x^2/(x-2)}$.

6.29. $\lim\limits_{x\to\infty}\ln(\frac{x+3}{x-4})^{x}$.

6.30. $\lim\limits_{x\to 1}(7-6x)^{x(3x-3)}$.

3. «Hosilalar va ularning tadbiqlari» dan nazorat ishi (2 soat).

1. Birinchi tartibli hosilani y' hisoblang

1.1. $y=(\frac{2}{27x}-\frac{1}{9x^2})\sqrt{3x+x^2}$.

1.2. $y=x\sqrt[3]{\frac{1+x}{1-x}}$.

1.3. $y=\sqrt{\frac{x+\sqrt{x}}{x-\sqrt{x}}}$.

1.4. $y=\sqrt[3]{\frac{x+3}{3x-5}}$.

1.5. $y=\frac{\sqrt{1+3x^2}}{2+3x^2}$.

1.6. $y=\frac{\sqrt{1+\cos^3 x}}{1+\sin 3x}$.

1.7. $y=\left(1+\sqrt{\frac{1+x}{1-x}}\right)^3$.

1.8. $y=\frac{x}{(x+1)^2(x^2+1)^3}$.

1.9. $y=\sqrt[5]{x+x\sqrt[3]{x}}$.

1.10. $y=\sqrt[3]{\frac{1+\sin 3x}{3+2\sin 3x}}$.

1.11. $y=\sqrt[3]{x+\sqrt{x}}$.

1.12. $y=\frac{3}{\sqrt[3]{x^3+3x+1}}-2\sqrt{6x+5}$.

1.13. $y=\frac{x}{\sqrt[3]{1+x^3}}$.

1.14. $y=x\sqrt[3]{\frac{1+x^2}{1-x^2}}$.

1.15. $y=\sqrt{x+\sqrt[3]{x}}$.

1.16. $y=\sqrt{x^2+1}+\sqrt[3]{x^3+1}$.

1.17. $y=\sqrt{\frac{x^2+\sqrt{x}}{x^3-\sqrt{x}}}$.

1.18. $y=5\sqrt{x^2+\sqrt{x}+1/x}$.

1.19. $y=1+\sqrt{\frac{1+x}{x-1}}$.

1.20. $y=\sqrt[3]{\frac{x^2+1}{3x-2}}$.

1.21. $y=\sqrt[4]{x^2+3x}-\sqrt[5]{(6x-1)^2}$.

1.22. $y=\frac{2x}{\sqrt{1+x}}-4\sqrt{1+x}$.

1.23. $y=\sqrt[3]{\frac{1+x^3}{1-x^3}}$.

1.24. $y=\sqrt{x+\sqrt{x}}$.

1.25. $y=\sqrt[5]{3x^2+1}+\sqrt[3]{x^3-4}$.

1.26. $y=x\sqrt{1+x^2}$.

1.27. $y=5\sqrt[5]{4x+3}-\frac{2}{\sqrt{x^3+x+1}}$.

1.28. $y=3\sqrt[3]{x^5+5x^4-5/x}$.

1.29. $y=\frac{1}{x+\sqrt{1+x^2}}$.

1.30. $y=x+\sqrt[5]{\frac{1+x^5}{1-x^5}}$.

2. Birinchi tartibli– y' hosilani hisoblang

2.1. $y=3^{arctg^2(4x+1)}$.

2.2. $s=\ln\frac{5+\sqrt{25-t^2}}{t}$.

2.3. $z=y^{\arcsin((2y+1)/3)}$.

2.4. $y=(1+ctg^2 3x)e^{-x}$.

2.5. $y=e^{\varphi^2}\cos^3(2\varphi+3)$.

2.6. $y=e^{-\sqrt{x}}/(1+e^{2x})$.

2.7. $y=e^{-1/\cos x}$.

2.8. $y=\sqrt[3]{(1+\sin^3 2x)^2}$.

2.9. $y=3^{x\cos^3 x}$.

2.10. $y=e^{x/\sqrt{3}}arctg^2 x$.

2.11. $y=\dfrac{1+\sin 2x}{1-\sin 2x}$.

2.12. $y=\cos 2x\cdot\sin^2 x$.

2.13. $y=\sin^3 5x\cdot\sin^5 3x$.

2.14. $Q=e^{\cos^2 3\varphi}$.

2.15. $y=e^{tgx}\cos x$.

2.16. $y=\arcsin(tgx)$.

2.17. $y=e^{\cos x}\sin^2 x$.

2.18. $y=\ln\ln\sqrt{\dfrac{1-\cos x}{1+\cos x}}$.

2.19. $z=(\sin y)/(1+tgy)$.

2.20. $s=e^t/\cos t$.

2.21. $y=(1+e^x)/(1-e^x)$.

2.22. $y=\sin^2 3x$.

2.23. $y=\sqrt{1+\ln^2 x}$.

2.24. $y=\dfrac{4\ln x}{1-\ln x}$.

2.25. $y=\dfrac{1}{3}tg^3x-ctgx+x$.

2.26. $y=\ln\sqrt{\dfrac{1+tgx}{1-tgx}}-x$.

2.27. $y=\ln\sqrt{\dfrac{1-\sin x}{1+\cos x}}$.

2.28. $y=\ln(e^x+\sqrt{1+e^{2x}})$.

2.29. $y=\dfrac{\ln x}{\sqrt{x^2+1}}$.

2.30. $y=tg^2(x^3+1)$.

3. Birinchi tartibli y' hosilani argumentning yoki parametrning berilgan qiymatlarida hisoblang

3.1. $f(x)=(1-2x)/(1+\sqrt[3]{2x}), x=4$.

3.2. $f(x)=\sqrt{x+2\sqrt{x}},\quad x=1$.

3.3. $f(x)=xe^{x/a}, x=0$.

3.4. $f(t)=\ln(1+a^{-2t}), t=0$.

3.5. $f(t)=\sqrt{a^2+b^2-2ab\cos t}, t=\pi/2$.

3.6. $f(x)=x/(2x-1), x=-2$.

3.7. $f(x)=\sqrt[3]{x^2}, x=-8$.

3.8. $f(x)=(\sqrt{x}-1)^2/x, x=0{,}01$.

3.9. $f(x)=x^2-1/(2x^2), x=\pm 2$.

3.10. $f(x)=x^3/3-x^2+x, x=-1$.

3.11. $f(x)=e^{-x}\cos 3x, x=0$.

3.12. $f(x)=\ln(1+x)+\arcsin(x/2), x=1$.

3.13. $f(x)=tg^3(\pi x/6), x=2$.

3.14. $2y=1+xy^3, x=1; y=1$.

3.15. $y=(x+y)^3-27(x-y), x=2; y=1$.

3.16. $ye^y=e^{x+1}, x=0; y=1$.

3.17. $y^2=x+\ln(y/x), x=1; y=1$.

3.18. $x=t\ln t, y=(\ln t)/t, t=1$.

3.19. $x=a(t-\sin t), y=a(1-\cos t), t=\pi/2$.

3.20. $x=e^t\cos t, y=e^t\sin t, t=\pi/4$.

3.21. $y(x)=(1+x^3)(5-1/x^2), x=1; x=0$.

3.22. $s(t)=3/(5-t)+t^2/5, t=0; t=2$.

3.23. $\varphi(z)=z(1+\sqrt{z^3}), z=0$.

3.24. $\rho(\varphi)=\varphi/(1-\varphi^2), \varphi=2$.

3.25. $\varphi(z)=(a-z)/(1+z), z=1$.

3.26. $s(t)=3/(5-t)+t^2/5, t=0; t=2$.

3.27. $y=e^{\sqrt{\ln x}}, x=e$.

3.28. $y=\sqrt[3]{tg(x/2)}, x=\pi/2$.

3.29. $f(x)=(x^2+x+1)(x^2-x+1), x=0; x=1$.

3.30. $f(x)=1/(x+2)+3/(x^2+1), x=0; x=1$.

4. Ikkinchi tartibli y'' – hosilani hisoblang

4.1. $y=\dfrac{x-1}{x+1}e^{-x}$.

4.2. $y=arctg(x^2)$.

4.3. $y=x^2\ln x$.

4.4. $y=\sqrt{a^2-x^2}/x$.

4.5. $y=\ln ctg4x$.

4.6. $y=\sqrt[3]{(1-x)^2}$.

4.7. $y=2^{ctg3x}$.

4.8. $y=xe^{1/x}$.

4.9. $y=xe^{-x}$.

4.10. $y=\ln\ln x$.

4.11. $y=x\sqrt{1+x^2}$.

4.12. $y=x/\sqrt{1-x^2}$.

4.13. $y=(\ln x)/x$.

4.14. $y=x^2\ln x$.

4.15. $y=x^3e^{5x}$.

4.16. $y=(1+x^2)tgx$.

4.17. $y=e^x\cos^4 x$.

4.18. $y=e^{-x}\cos x$.

4.19. . $y=\sqrt{x}e^x$.

4.20. $y=xe^{-x^3}$.

4.21. $y=arctg\dfrac{2x}{1-x^2}$.

4.22. $y=x^3\ln x$.

4.23. $y=xe^{\sin x}$.

4.24. $y=\ln tg(\dfrac{\pi}{4}+\dfrac{x}{2})$.

4.25. $y=xarctgx$.

4.26. $y=x/(x^2-1)$.

4.27. $y=x-arctgx$.

4.28. $y=\sin x-\dfrac{1}{3}\cos^3 x$.

4.29. $y=arctg\sqrt{x}$.

4.30. $y=\ln(x+\sqrt{x})$.

5. .Ikkinchi tartibli d^2y/dx^2 hosilani hisoblang

5.1. $\begin{cases} x=t+\ln\cos t \\ y=t-\ln\sin t \end{cases}$.

5.2. $\begin{cases} x=2t-\sin 2t \\ y=\sin^3 t \end{cases}$.

5.3. $\begin{cases} x=t+\dfrac{1}{2}\sin t \\ y=\cos^3 t \end{cases}$.

5.4. $\begin{cases} x=t^5+2t \\ y=t^3-8t-1 \end{cases}$.

5.5. $\begin{cases} x=t^3/3+t^2/2+t \\ y=t^2/2+1/t \end{cases}$.

5.6. $\begin{cases} x=\arcsin(t^2-1) \\ y=\arccos 2t \end{cases}$.

5.7. $\begin{cases} x=t^2+t+1 \\ y=t^3+t \end{cases}$.

5.8. $\begin{cases} x=ctgt \\ y=1/\cos^2 t \end{cases}$.

5.9. $\begin{cases} x=(2-t)/(2+t^2) \\ y=t^2/(2+t^2) \end{cases}$.

5.10. $\begin{cases} x=2\cos^3 2t \\ y=\sin^3 2t \end{cases}$.

5.11. $\begin{cases} x=2t-t^2 \\ y=3t-t^3 \end{cases}$.

5.12. $\begin{cases} x=3\cos t \\ y=4\sin^2 t \end{cases}$.

5.13. $\begin{cases} x=2\cos^3 t \\ y=4\sin^3 t \end{cases}$.

5.14. $\begin{cases} x=\cos t+t\sin t \\ y=\sin t-t\cos t \end{cases}$.

5.15. $\begin{cases} x=2\cos t-\cos 2t \\ y=2\sin t-\sin 2t \end{cases}$.

5.16. $\begin{cases} x=2t^2+t \\ y=\ln t \end{cases}$.

5.17. $\begin{cases} x=3t-t^3 \\ y=3t^2 \end{cases}$.

5.18. $\begin{cases} x=2t-t^3 \\ y=2t^2 \end{cases}$.

5.19. $\begin{cases} x=ctgt \\ y=1/\cos^2 t \end{cases}$.

5.20. $\begin{cases} x=\ln t \\ y=(t+1/t)/2 \end{cases}$.

5.21. $\begin{cases} x=t^2 \\ y=t^3/3-t \end{cases}$.

5.22. $\begin{cases} x=t-\sin t \\ y=1-\cos t \end{cases}$.

5.23. $\begin{cases} x=\sin(t/2) \\ y=\cos t \end{cases}$.

5.24. $\begin{cases} x=\cos at \\ y=\sin at \end{cases}$.

5.25. $\begin{cases} x=e^{2t} \\ y=\cos t \end{cases}$.

5.26. $\begin{cases} x=\cos(t/2) \\ y=t-\sin t \end{cases}$.

5.27. $\begin{cases} x=tgt+ctgt \\ y=2\ln ctgt \end{cases}$.

5.28. $\begin{cases} x=t^2+1 \\ y=e^{t^2} \end{cases}$.

5.29. $\begin{cases} x=3\cos^2 t \\ y=2\sin^3 t \end{cases}$.

5.30. $\begin{cases} x=t\cos t \\ y=at\sin t \end{cases}$.

6. Quyidagi masalalarni eching

6.1. Sinusoida $y=sinx$, $y=\frac{1}{2}$ to'g'ri chiziqni qaysi burchak ostida kesib o'tadi?

6.2. Giperbolalar $xy=8$ va $x^2-y^2=12$ to'g'ri burchak ostida kesishishini ko'rsating

6.3. Egri chiziqlar $x^2+y^2=8$ va $y^2=2x$ qanday burchak ostida kesishadi?

6.4. Giperbola $y=1/x$ va parabola $y=\sqrt{x}$ qanday burchak ostida kesishadi?

6.5. Quyidagi $y=x^2$ parabolada absissalari $x_1=1$ va $x_2=3$ bo'lgan nuqtalar olinib, shu nuqtalar orqali kesuvchi o'tkazilgan. Parabolaning qanday nuqtasidagi urinma kesuvchiga parallel bo'ladi?

6.6. Osma ko'pirkning arqonni, parabola ko'rinishida bo'lib vertikal ustunlarga maxkamlangan. Arqonning eng past nuqtasi osilgan nuqtasidan 40m pastda. Agar vertikal ustunlar orasidagi masofa 200m bo'lsa, arqon va ustunlar orasidagi burchakni toping

6.7. Parametr *a* ning qanday qiymatida $y=(ax+x^3)/4$ egri chiziq *Ox* o'qni $45°$ burchak ostida kesadi?

6.8. Quyidagi $y = x - x^3$ egri chiziq va $y = 5x$ to'g'ri chiziq orasidagi burchakni toping

6.9. Ushbu chiziqlar orasidagi kesishish burchagini toping: $y = 1 + \sin x$ va $y = 1$.

6.10. Quyidagi $y = \sqrt{2}\sin x$ va $y = \sqrt{2}\cos x$ chiziqlar orasidagi kesishish burchakni toping.

6.11. Egri chiziqlar $y = x^3$, $y = \frac{1}{x^2}$ lar orasidagi kesishish burchagini toping.

6.12. YArim kubik parabolaga $x = t^2$, $y = t^3, t = 2$ nuqtada o'tkazilgan urinmaning va normalning tenglamasi tuzilsin.

6.13. Quyidagi egri $x^2 + y^2 = 5, y^2 = 4x$ chiziqlar kesishish burchagini toping.

6.14. Ushbu $y = (x-1)/1 + x^2$ egri chiziq *Ox* o'qini qanday burchak ostida kesadi?

6.15. Berilgan $f(x) = x^3 - x - 1$ va $\varphi(x) = 3x^2 - 4x + 1$ egri chiziqlar urinmalari parrallel bo'ladigan nuqtalarni toping.

6.16. Quyidagi $x^2 + y^2 + 4x - 2y - 3 = 0$ egri chiziqni uning *Oy* o'qi bilan kesishish nuqtasidagi urinma va normal tenglamasi tuzilsin.

6.17. Berilgan $y = 4x - x^3$ egri chiziqning *Ox* o'qi bilan kesishigan nuqtalaridan o'tkazilgan urinma va normal tenglamasi tuzilsin.

6.18. Quyidagi $xy = 4$ giperbolaga absissasi $x_1 = 1, x_2 = -4$ nuqtalarda o'tkazilgan urinmalar orasdagi burchak topilsin.

6.19. Ushbu $y = x^2 + 5x + 3$ parabolaga absissalari $x = -2, x = 3$ bo'lgan nuqtalarda kesuvchi o'tkazilgan. Parrabolaning qaysi nuqtasidagi urinma, kesuvchiga paralel bo'ladi?

6.20. Berilgan $4x^3 - 3xy^2 + 6x^2 - 5xy - 3y^2 + 9x + 14 = 0$ egri chiziqni (-2,3) nuqtada urinma va normalning tenglamasi tuzilsin.

6.21. $x = a\cos^3 t, y = a\sin^3 t$ astroidaga $t = \frac{\pi}{4}$ nuqtada o'tkazilgan normalning tenglamasi tuzilsin.

6.22. Quyidagi $y = \ln(2x+1)$ egri chiziqning birinchi va uchunchi chorak koordinata burchagi bissektrisasiga perpendikulyar bo'lgan normal tenglamasi tuzilsin.

6.23. $y = x^2 - 4x + 5$ parabolaning uchidan uning Ou o'qi bilan kesishish nuqtasidan o'tkazgan urinmagacha bo'lgan masofani toping.

6.24. Agar $y = x^2 + bx + c$ parabola *x=2* nuqtada *y=x* to'g'ri chiziqqa urinsa, *b,c* parametrlani toping.

6.25. Ushbu $y = (x+9)/(x+5)$ egri chiziqqa koordinata boshidan o'tuvchi urinma o'tkazilsin. Urinmaning tenglamasi tuzilsin.

6.26. Parabolalar $y = (x-2)^2$ va $y = -4 + 6x - x^2$ kesishish burchagi topilsin.

6.27. Ellips $\frac{x^2}{4} + y^2 = 1$ va $4y = 4 - 5x^2$ parabolalar kesishish burchagi topilsin.

6.28. $y = arctg(x/2)$ egri chiziqning $x - 2 = 0$ to'g'ri chiziq bilan kesishish nuqtalaridan o'tkazilgan urinmalarning tenglamasi tuzilsin.

6.29. Ushbu $4x^2 + y^2 = 80$ egri chiziqning $x + y - 6 = 0$ to'g'ri chiziqqa paralel bo'lgan urinmasining tenglamasi tuzilsin.

6.30. *a* parametr ning qanday qiymatida $y = ax^3$ parabola $y = \ln x$ egri chiziqqa urinadi. ?

7. Quyidagi masalalarni yeching

7.1. Material nuqta $S = t^3 - 3t^2 + 3t + 5$ qonun bilan to'g'ri chiziq bo'yicha harakat qiladi. Parametr «t» ning qanday qiymatida tezlik nolga teng bo'ladi?

7.2. Ikkita nuqta $S_1 = t^2 - 3t$ va $S_2 = t^3 - 5t^2 + 17t - 4$ qonun bilan to'g'ri chiziq bo'yicha harakat qilmoqda. Qanday paytda ularning tezligi teng bo'ladi?

7.3. Yuqori otilgan jism $S = -\frac{1}{3}t^3 + \frac{17}{2}t^2 + 60t - 49$ qonun bilan harakatlanadi. Qanday vaqtda jismning tezligi 0 ga teng bo'ladi. Jismning ko'tarilgan eng yuqori balandligi topisin.

7.4. To'g'ri chiziqli harakat qilayotgan jismning tezigi $V = 3t + t^2$ formula bilan aniqlanadi. Jism harakat boshlangandan 4 *sek* keyin qanday tezlanishga ega bo'ladi?

7.5. Massasi 100 kg bo'lgan jism to'g'ri chiziq bo'yicha $S = 2t^2 + 3t + 1$ qonun bilan harakatlanadi. Jismning harakat boshlangandan 5 *sek* keyin $\frac{mV^2}{2}$ kinetik energiyasi aniqlansin.

7.6. Jism yuqoriga *a* m/s boshlang'ich tezlik bilan vertikal otilgan. Qanday vaqtda yer sirtidan qanday masofada jism eng yuqori nuqtaga erishadi.

7.7. Sol qirg'oqqa 50 m/min tezlik bilan havozaga o'ralayotgan arqon yordamida tortiladi. Agar soldan qirg'oqgacha bo'lgan masofa 25 metr bo'lgan vaqtda, qirg'oqdagi havoza suvdan $6\sqrt{6}$ balanlikda joylashgan bo'lsa, uning harakat tezligini toping.

7.8. Zaryad o'tkazgichdan $t = 0$ vaqtdan o'tgan boshlab $Q = t^3 - 9t^2 + 15t + 1$ formula bilan aniqlanadi. Qaysi paytlarda o'tkazgichda tok kuchi nolga teng bo'ladi

7.9. Massasi *6 t* bo'lgan jism to'g'ri chiziq bo'yicha $S = -1 + \ln(t+1) + (t+1)^3$ qonun bilan harakat qiladi. Harakat boshlangandan 1*sek* keyin jismning $\frac{m\text{V}^2}{2}$ kinetik energiyasi hisoblansin

7.10. Yo'lning vaqt bo'yicha to'g'ri chiziqli harakatda quyidagi $S = \frac{1}{5}t^5 + \frac{2}{\pi}\sin\frac{\pi}{8}t$ tenglama bilan berilgan. Jismning harakat boshlangandan 2*sek* keyin tezligi topilsin.

7.11. Reaktsiya natijisida olinadigan «*x*» moddaning miqdori vaqt orqali $x = 7(1 - e^{-3t})$ aniqlanadi. Tajriba boshlangandan 2*sek* keyin reaksiya tezligi aniqlansin (*t=0*).

7.12. G'ildarak aylanganda, burilish burchagi vaqtning kubiga proporsional. G'ildirakning birinchi 2 ta to'la aylanishiga 4 *sek* vaqt ketgan bo'lsa, harakat boshlangandan 16 *sek* keyin g'ildirakning *w* burchak tezligi aniqlansin.

7.13. Jism to'g'ri chiziq bo'yicha Ox o'qida $x = \frac{t^3}{3} - 2t^2 + 3t$ qonun bilan harakatlanmoqda. Harakatning tezligi va tezlanishi aniqlansin. Qanday paytda jism harakat yo'nalishini o'zgartiradi?

7.14. Nuqta $y = x(8 - x)$ parabola bo'yicha shunday harakat qiladi uning absissa vaqt bo'yicha $x = t\sqrt{t}$ qonun bilan o'zgaradi. Ordinataning M(1,7) nuqtadagi o'zgarish tezligini toping.

7.15. Nuqta $y = 10/x$ giperbola bo'yicha harakatlanayotganda uning absissasi 1m/s tezlik bilan tekis o'sadi. Uning ordinatasi (5,2) nuqtadan o'tganda qanday tezlik bilan o'zgaradi?

7.16. Nuqtaning Ox – o'qi bo'yicha harakat qilish qonuni $S = 5t - t^2$. Nuqtaning, $t_1 = 0,\ t_2 = 1\ cek$ paytdagi tezligi va tezlanishi topilsin.

7.17. Nuqta formula $y = \sqrt{6x}$ parabola bo'yicha harakat qiladi. Uning absissasi 10 *sm/s* tezlik bilan o'zgaradi. Agar *x=6* bo'lsa, shu nuqtadagi ordinataning o'zgarish tezligi topilsin.

7.18. Nuqtaning to'g'ri chiziq bo'yicha harakat qonuni $S = 5t - 4/t^2 + 3$ formula bilan berilgan. Harakat boshlangandan *1sek* keyin tezlik va tezlanish topilsin.

7.19. Nuqta *1* chorakda $y = \sqrt[3]{x}$ egri chiziq bo'yicha harakatlanadi. Absissaning o'zgarish tezligi, ordinataning o'zgarish tezligidan 12 marta katta bo'lgan vaqtdagi punktning koordanataliri topilsin.

7.20. Nuqta $S = 4t^3 + 2t^2 - 5$ (*sm*) qonun bilan harakatlanadi. Nuqtaning 2*sek* dan keyingi harakat tezligi va tezlanishi topilsin.

7.21. Sharning radiusi 5 sm/s tezlik tekis o'sadi. Radius 50 sm bo'lganda sharning sirti va xajmi qanday tezlikda o'zgaradi.

7.22. O'tkazilgan elektr zaryadining o'tishi $t = 0$ vaqtdan boshlab $Q = 2t^2 + 10t + 9$ formula bilan aniqlanadi. Tokning $t = 15$ sekundagi kuchi aniqlansin

7.23. $16x^2 + 9y^2 = 400$ tenglama bilan berilgan ellipsning qanday nuqtasida ordinata kamayish tezligi, absissaning o'sish tezligi bir hil bo'ladi?

7.24. Kvadrat tomonining o'sish tezligi *5 m/s*. Kvadratning tomoni *50 m* bo'lganda, kvadrat perametri va yuzasining o'zgarish tezligini toping.

7.25. G'ildirak aylanganda, uning burilish burchagi vaqtning kvadratiga proportsional. Birinchi aylanishni g'ildirak 8 *sek* bajargan bo'lsa, 32 sek harakat boshlangandan so'ng, g'ildirakning tezligi topilsin.

7.26. Jismning bosadigan yo'li $65S = \frac{t^3}{8} + 3t^2 + t$ formula bilan aniqlanadi, uning $t = 10$ *sek* dagi tezlik va tezlanishi aniqlansin.

7.27. Aylanuvchi g'ildirak tormoz bilan to'xtatilayotganda, $t$sek vaqt ichida $\varphi = a + bt - ct^2$ burchakka buriladi, bu yerda *a,b,s* o'zgarmas musbat sonlar. G'ildirakning aylanish burchak tezligi, tezlanishi topilsin. Qachon g'ildirak to'xtaydi?

7.28. Nuqta to'g'ri chiziq bo'yicha harakatlanadi, $V^2 = 2bx$, V - nuqtaning tezligi, *x* – bosilgan yo'l, *b* – o'zgarmas son. Nuqta harkatining tezlanishi topilsin.

7.29. Harakat paytida maxovik $\varphi = \frac{t^3}{10}$ qonun bilan aylanadi. Harakat boshlangandan necha *sek* keyin uning tezligi 60 π *rad/s* ga teng bo'ladi? Bunday paytda jismning burchak tezlanishi nechaga teng bo'ladi?

7.30. Nuqta $S = 60t - 5t^3$ qonun bilan to’g’ri chiziq bo’yicha harakat qiladi. Harakat boshlangandan qancha vaqtdan keyin nuqta to’xtaydi? Shu vaqt ichida nuqtaning o’tgan yo’li masofasi toping.

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