Fayziyev A.Q

OLIY MATEMATIKA

BIR O‘ZGARUVCHILI FUNKSIYANING INTEGRAL HISOBI VA ULARNING TATBIQLARIGA DOIR MASALALAR TO‘PLAMI

USLUBIY QO‘LLANMA

Toshkent-2022

O‘ZBEKISTON RESPUBLIKASI OLIY VA O‘RTA MAXSUS TA’LIM VAZIRLIGI

ISLOM KARIMOV NOMIDAGI TOSHKENT DAVLAT TEXNIKA UNIVERSITETI

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FAYZIYEV A.Q. Oliy matematika. Bir o‘zgaruvchili funksiyaning integral hisobi va ularning tatbiqlariga doir masalalr to‘plami-Toshkent: ToshDTU, 2022, 78 b.

Ushbu uslubiy qo‘llanma bakalavriat ta’lim yo‘nalishlari talabalari uchun mo‘ljallangan bo‘lib, unda oliy matematika fanining bir o‘zgaruvchili funksiyaning integral hisobi va ularning tatbiqlari mavzulariga oid misol va masalalar keltirilgan.

Uslubiy qo‘llanma Islom Karimov nomidagi Toshkent davlat texnika universiteti ilmiy-uslubiy kengashi 2022 yil 27 aprel 8-sonli majlis qaroriga asosan chop etilgan.

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Kirish

Ko‘pchilik oliy o‘quv yurtlari talabalarining kasb egallashida o‘rganadigan dastlabki fanlaridan biri oliy matematika hisoblanadi.

Oliy matematikaning asosiy vazifasi shu fanning tushunchalari, tasdiqlari va boshqa matematik ma’lumotlar majmuasi bilan tanishtirishdangina iborat bo‘lmay, balki talabalarni mantiqiy fikrlashga, matematik usullarni amaliy masalalarni yechishda qanday qo‘llashni o‘rgatishdan iborat.

O‘zbekistonda kadrlar tayyorlash tizimini tubdan isloh qilish jarayonida talabalarni darslik hamda o‘quv qo‘llanmalar bilan ta’minlash muhim o‘rin egallaydi.

Mazkur uslubiy qo‘llanmada har bir bob (paragraf) zaruriy nazariy ma’lumotlarni keltirish bilan boshlanadi. Bunda muhim ta’riflar, teoremalar, formulalar keltiriladi. So‘ng bir nechta misol va masalalarning yechilish yo‘llari va batafsil yechimlari bayon etiladi. Bu keyingi misol va masalalarni mustaqil yechishga yordam beradi.

Har bir bob (paragraf) da mustaqil yechish uchun ko‘p sonda misol va masalalar keltirilgan. Ular tuzilishiga qarab (avval sodda yechiladigan masalalar, keyin o‘rtacha murakkablikka ega, pirovardida murakkabroq masalalar) joylashtirilgan.

Masalalar sonining ko‘pligi va ularni yuqorida aytilgan tartibda joylashtirilishi oliy matematikani turli hajmdagi dastur bo‘yicha o‘qitilishida ularga mos keladiganlarini tanlash imkonini beradi.

Har bir bobning so‘ngida shu bobda keltirilgan mavzularni mustahkamlash maqsadida savollar keltirilgan bo‘lib, ular talabalarning o‘zini o‘zi tekshirish imkoniyatini beradi.

1 bob
Aniqmas integral. Integralning sodda xossalari va integrallash usullari

1.1. Aniqmas integralning sodda xossalari. Integrallash usullari

1.1.1. “Boshlang‘ich funksiya” va “aniqmas integral” tushunchalari. Aniqmas integralning xossalari. $f(x)$ funksiya (a,b) da berilgan bo‘lib, shu intervalda differensiallanuvchi $F(x)$ funksiya uchun

$$F'(x) = f(x)$$

bo‘lsa, $F(x)$ funksiya (a,b) da $f(x)$ funksiyaning boshlang‘ich funksiyasi deyiladi.

$f(x)$ funksiya boshlang‘ich funksiyalarning umumiy ko‘rinishi (ifodasi)

$$F(x)+C, \quad C = const$$

$f(x)$ funksiyaning aniqmas integrali deyiladi va $\int f(x)dx$ kabi belgilanadi:

$$\int f(x)dx = F(x)+C$$

$f(x)$ funksiya (a,b) da uzluksiz bo‘lsa, uning aniqmas integrali mavjud bo‘ladi. Funksiyaning aniqmas integralini topish amali funksiyani integrallash deyiladi. U differensiyallash amaliga teskari amal bo‘ladi.

Integrallashning sodda xossalari:

1) $\left(\int f(x)dx\right)' = f(x)$, $\mathrm{d}\left(\int f(x)dx\right) = f(x)dx$,

$\int dF(x) = F(x)+C \quad (C = const)$

2) $\int kf(x)dx = k\int f(x)dx \qquad (k = const,\ k \neq 0).$

3) $\int\left(f(x)+g(x)\right)dx = \int f(x)dx + \int g(x)dx.$

Sodda funksiyalarning aniqmas integrallari jadvali:

1) $\int 1\cdot dx = \int dx = x+c$, chunki $(x+c)' = 1$.

2) $\int x^n dx = \dfrac{x^{n+1}}{n+1}+c \quad (n \neq -1)$, chunki $\left(\dfrac{x^{n+1}}{n+1}+c\right)' = x^n$.

3) $\int x^{-1}dx=\int\frac{dx}{x}=\ln|x|+c$, chunki

$x>0$ da $\int\frac{dx}{x}=\ln x+c$ va $(\ln x+c)'=x^{-1}$.

$x<0$ da $\int\frac{dx}{x}=\ln(-x)+c$ va $(\ln(-x)+c)'=\frac{1}{-x}\cdot(-1)=x^{-1}$..

4) $\int a^x\cdot dx=\frac{a^x}{\ln a}+c$, chunki $\left(\frac{a^x}{\ln a}+c\right)'=a^x$.

5) $\int e^x\cdot dx=e^x+c$, chunki $(e^x+c)'=e^x$.

6) $\int\sin xdx=-\cos x+c$, chunki $(-\cos x+c)'=\sin x$.

7) $\int\cos xdx=\sin x+c$, chunki $(\sin x+c)'=\cos x$.

8) $\int\frac{dx}{\sin^2 x}=-ctgx+c$, chunki $(-ctgx+c)'=\frac{1}{\sin^2 x}$.

9) $\int\frac{dx}{\cos^2 x}=tgx+c$, chunki $(tgx+c)'=\frac{1}{\cos^2 x}$.

10) $\int\frac{dx}{\sqrt{1-x^2}}=\arcsin x+c$, chunki $(\arcsin x+c)'=\frac{1}{\sqrt{1-x^2}}$.

11) $\int\frac{dx}{\sqrt{1-x^2}}=-\arccos x+c$, chunki $(-\arccos x+c)'=\frac{1}{\sqrt{1-x^2}}$.

12) $\int\frac{dx}{1+x^2}=arctgx+c$, chunki $(arctgx+c)'=\frac{1}{1+x^2}$.

13) $\int\frac{dx}{1+x^2}=-arcctgx+c$, chunki $(-arcctgx+c)'=\frac{1}{1+x^2}$.

14) $\int shxdx=chx+c$, chunki $(chx+c)'=shx$.

15) $\int chxdx=shx+c$, chunki $(shx+c)'=chx$.

1-misol. Agar $f'(x)=\sqrt{x}$ va $f(1)=2$ bo‘lsa, $f(x)$ funksiyani toping.

◀Integrallar jadvalidan foydalanib,

$$f(x)=\frac{2}{3}x^{\frac{3}{2}}+C$$

bo‘lishini topamiz. Ayni paytda,

$$f(1)=\frac{2}{3}+C=2$$

bo‘lganligidan $C=\frac{4}{3}$ bo‘lishi kelib chiqadi. Demak,

$$f(x)=\frac{2}{3}\left(x\sqrt{x}+2\right).$$ ▶

2-misol. Ma’lumki, m massali zarrachaning F kuch ta’sirida to‘g‘ri chiziq bo‘yicha harakati ushbu

$$m \cdot \frac{d^2 S(t)}{dt^2} = F$$

qonun bo‘yicha bo‘ladi (Nyutonning 3-qonuni). Agar zarrachaga hech qanday kuch ta’sir etmasa, ya’ni $F = 0$ bo‘lsa, harakat qanday bo‘ladi?

◀Yuqoridagi tenglikdan

$$m \cdot \frac{dv(t)}{dt} = 0$$

bo‘lishini topamiz, bunda $v(t)-$tezlik.

Bu tenglikdan

$$\frac{dv(t)}{dt} = 0$$

bo‘lib, undan

$$v(t) = const$$

bo‘lishi kelib chiqadi. Bu inersiya qonunidir: agar jismga hech qanday kuch ta’sir etmasa, u tinch holatda yoki to‘g‘ri chiziq bo‘yicha tekis harakatda bo‘ladi.▶

3-misol. Ushbu

$$\int \frac{x^2 - 3x + 5}{\sqrt{x}} dx$$

aniqmas integralni toping.

◀Bu integralni aniqmas integralning xossalari hamda integrallar jadvalidan foydalanib hisoblaymiz:

$$\int \frac{x^2 - 3x + 5}{\sqrt{x}} dx = \int \left(\frac{x^2}{\sqrt{x}} - \frac{3x}{\sqrt{x}} + \frac{5}{\sqrt{x}} \right) dx = \int \left(x^{\frac{3}{2}} - 3x^{\frac{1}{2}} + 5x^{-\frac{1}{2}} \right) dx =$$

$$= \int x^{\frac{3}{2}} dx - 3\int x^{\frac{1}{2}} dx + 5\int x^{-\frac{1}{2}} dx = \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} - 3\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + 5\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C =$$

$$= \frac{2}{5} x^{\frac{5}{2}} - 2x^{\frac{3}{2}} + 10x^{\frac{1}{2}} + C. ▶$$

1.1.2. Integrallash usullari. Integrallarni hisoblashda o‘zgaruvchini almashtirish hamda bo‘laklab integrallash usulidan keng foydalaniladi.

a) O‘zgaruvchini almashtirib integrallash usuli. Ushbu

$$\int f(x)dx$$

Integralda, $x=\varphi(t)$ almashtirishni bajaramiz, bunda t erkli o‘zgaruvchi, $\varphi(t)$ esa differensiallanuvchi funksiya. Natijada

$$\int f(x)dx=\int f(\varphi(t))\cdot\varphi'(t)dt \qquad (1)$$

bo‘ladi.

(1) formula integralda o‘zgaruvchini almashtirish formulasi deyiladi.

4-misol. Ushbu

$$\int\frac{dx}{e^x+e^{-x}}$$

integralni hisoblang.

◄Bu integralni hisoblash uchun

$$t=e^x$$

almashtirish bajaramiz. Unda $dt=e^xdx=t\cdot dx,\quad dx=\frac{1}{t}dt$ bo‘lib,

$$\int\frac{dx}{e^x+e^{-x}}=\int\frac{dt}{t\left(t+t^{-1}\right)}=\int\frac{dt}{t^2+1}$$

bo‘ladi. Jadvaldan foydalanib topamiz

$$\int\frac{dt}{t^2+1}=arctgt+C.$$

Demak,

$$\int\frac{dx}{e^x+e^{-x}}=arctge^x+C.$$ ►

5-misol. Ushbu

$$\int\frac{2x+1}{x^2+x+1}dx$$

integralni hisoblang.

◄Bu integralda $t=x^2+x+1$ deymiz. Unda

$$dt=d\left(x^2+x+1\right)=\left(x^2+x+1\right)'dx=(2x+1)dx$$

bo‘lib,

$$\int\frac{(2x+1)dx}{x^2+x+1}=\int\frac{dt}{t}=\ln|t|+c=\ln\left|x^2+x+1\right|+c$$

bo‘ladi. ►

Ko‘p hollarda o‘zgaruvchini almashtirish ifodasini yozish zaruriyati bo‘lmaydi. Masalan, ushbu

1) $d(x+a)=dx, \qquad (a=const)$

2) $d(ax)=adx,$ *яъни* $\quad dx=\frac{1}{a}d(ax) \qquad (a=const, a\neq 0)$

tengliklarni e’tiborga olish va uni tadbiq etish yetarli bo‘ladi.

6-misol. Ushbu

$$\int \sqrt{x+3}dx$$

integralni hisoblang.

◀Bu integral quyidagicha hisoblanadi:

$$\int \sqrt{x+3}dx=\int (x+3)^{\frac{1}{2}}d(x+3)=\frac{2}{3}(x+3)^{\frac{3}{2}}+C=\frac{2}{3}(x+3)\cdot\sqrt{x+3}+C.$$ ▶

Eslatma. Ravshanki, o‘zgaruvchi turli usullar bilan (turli funksiyalar bilan) almashtirilishi mumkin. Ular orasidan shundayini tanlash kerakki, natijada (1) tenglikning o‘ng tomonidagi integral sodda holga kelsin.

b) Bo‘laklab integrallash usuli. Aytaylik, $u=u(x)$ va $v=v(x)$ funksiyalar uzluksiz $u'(x)$ va $v'(x)$ hosilalarga ega bo‘lsin.

Ushbu

$$\int udv=uv-\int vdu \qquad (2)$$

formula **bo‘laklab integrallash formulasi** deyiladi. U $\int udv$ integralni hisoblashni $\int vdu$ integralni hisoblashga olib keladi.

Bo‘laklab integrallash formulasidan foydalanish uchun berilgan integral ostidagi ifodani $u(x)$ va $dv(x)$ lar ko‘paytmasi shunday yozib olinishi lozimki, bunda dv hamda $v(x)du$ lar oson hisoblanadigan bo‘lsin.

7-misol. Ushbu

$$\int x^n \ln xdx, \qquad (n\neq -1)$$

integralni hisoblang.

◀Bu integralni bo‘laklab integrallash formulasidan foydalanib hisoblaymiz.

Agar

$$u=\ln x, \qquad dv=x^n dx$$

deyilsa, unda

$$du=\frac{1}{x}dx, \qquad v=\int x^n dx=\frac{x^{n+1}}{n+1}$$

bo‘lib, (2) formulaga ko‘ra

$$\int x^n \ln x dx = \frac{x^{n+1}\cdot \ln x}{n+1} - \int \frac{x^{n+1}}{n+1}\cdot\frac{1}{x}dx$$

bo‘ladi. Ravshanki,

$$\int \frac{x^{n+1}}{n+1}\cdot\frac{1}{x}dx = \frac{1}{n+1}\cdot\int x^n dx = \frac{1}{n+1}\cdot\frac{x^{n+1}}{n+1} + C.$$

Demak,

$$\int x^n \ln x dx = \frac{x^{n+1}\cdot \ln x}{n+1} - \frac{x^{n+1}}{(n+1)^2} + C\,.$$ ►

8-misol. Ushbu

$$\int arctgx \cdot dx$$

integralni hisoblang.

◄Agar

$$u = arctgx, \qquad dv = dx$$

deyilsa, unda

$$du = \frac{1}{1+x^2}dx, \qquad v = x$$

bo‘lib, (2) formulaga ko‘ra

$$\int arctgx\cdot dx = xarctgx - \int \frac{xdx}{1+x^2}$$

bo‘ladi. Keyingi tenglikning o‘ng tomonidagi integral quyidagicha hisoblanadi

$$\int \frac{xdx}{1+x^2} = \frac{1}{2}\int \frac{d(1+x^2)}{1+x^2} = \frac{1}{2}\ln(1+x^2) + C$$

Demak,

$$\int arctgx\cdot dx = x\cdot arctgx - \frac{1}{2}\ln(1+x^2) + C\,.$$ ►

9-misol. Ushbu

$$J_n = \int \frac{dx}{(x^2+a^2)^n} \qquad (n = 1, 2, 3, \ldots)$$

integralni hisoblang.

◄Avvalo, $n=1$ bo‘lgan holni qaraymiz. Bu holda

$$J_1 = \int \frac{dx}{x^2+a^2} = \int \frac{dx}{a^2\left(1+\frac{x^2}{a^2}\right)} = \frac{1}{a}\int \frac{\frac{1}{a}dx}{1+\left(\frac{x}{a}\right)^2} = \frac{1}{a}\int \frac{d\left(\frac{x}{a}\right)}{1+\left(\frac{x}{a}\right)^2} = \frac{1}{a}arctg\frac{x}{a} + c$$

bo‘ladi.

Endi $J_n=\int\frac{dx}{\left(x^2+a^2\right)^n}$ da

$$u=\frac{1}{\left(x^2+a^2\right)^n},$$
$$dv=dx$$

deylik. U holda

$$du=\left(\frac{1}{\left(x^2+a^2\right)^n}\right)'dx=-\frac{2nx}{\left(x^2+a^2\right)^{n+1}}dx,$$
$$v=x$$

bo‘lib, (2) formulaga ko‘ra

$$J_n=\frac{x}{\left(x^2+a^2\right)^n}+2n\int\frac{x^2}{\left(x^2+a^2\right)^{n+1}}dx$$

bo‘ladi. Bu tenglikning o‘ng tomonidagi integralni quyidagicha yozib olamiz

$$\int\frac{x^2}{\left(x^2+a^2\right)^{n+1}}dx=\int\frac{\left(x^2+a^2\right)-a^2}{\left(x^2+a^2\right)^{n+1}}dx=\int\frac{x^2+a^2}{\left(x^2+a^2\right)^{n+1}}dx-$$
$$-\int\frac{a^2}{\left(x^2+a^2\right)^{n+1}}dx=\int\frac{1}{\left(x^2+a^2\right)^n}dx-a^2\int\frac{1}{\left(x^2+a^2\right)^{n+1}}dx=J_n-a^2\cdot J_{n+1}.$$

Natijada,

$$J_n=\frac{x}{\left(x^2+a^2\right)^n}+2n\cdot J_n-2na^2\cdot J_{n+1}$$

bo‘lib, undan

$$J_{n+1}=\frac{x}{2na^2\left(x^2+a^2\right)^n}+\frac{2n-1}{2na^2}\cdot J_n \tag{3}$$

bo‘lishi kelib chiqadi.

Yuqorida ko‘rdikki,

$$J_1=\frac{1}{a}arctg\frac{x}{a}+c.$$

(3) formulada $n=1$ deb

$$J_2=\int\frac{1}{\left(x^2+a^2\right)^2}dx=\frac{x}{2a^2\cdot\left(x^2+a^2\right)}+\frac{1}{2a^3}arctg\frac{x}{a}+c$$

bo‘lishini topamiz.

Shu tariqa (3) formula yordamida $n=3,4,\ldots$ bo‘lgan hollarda mos integrallar hisoblanadi.►

Odatda, (3) formula **rekurrent formula** deyiladi.

Aniqmas integrallar jadvali hamda integral xossalaridan foydalanib, quyidagi aniqmas integrallarni hisoblang.

1. a) $\int x^5 dx$, b) $\int \frac{x^6}{6} dx$, c) $\int \frac{2}{x^3} dx$.

2. a) $\int \sqrt{x} dx$, b) $\int \sqrt[5]{x} dx$, c) $\int \sqrt[6]{x^5} dx$.

3. a) $\int (x-1)(x+1)dx$, b) $\int (\sqrt{x}-x)(\sqrt{x}+x)dx$.

4. a) $\int (3x^2-3x+1)dx$, b) $\int (x^4-\frac{1}{3}x^2+x+1)dx$.

5. a) $\int (x^2+1)^2 dx$, b) $\int (x+1)(x-1)^2 dx$.

6. a) $\int \frac{dx}{\sqrt[10]{x}}$, b) $\int \frac{\sqrt{x}+\sqrt[3]{x}}{\sqrt[4]{x^3}} dx$.

7. a) $\int \frac{x-4}{\sqrt{x}-2} dx$, b) $\int \frac{x^3-1}{x^2+x+1} dx$.

8. a) $\int \frac{(x\sqrt{x}-3)^2}{x^3} dx$, b) $\int \frac{x^2}{x^2+1} dx$.

9. a) $\int \frac{x^3+x+2}{x^2+1} dx$ b) $\int \frac{dx}{x^2+2}$.

10. a) $\int 2^{x-1} dx$, b) $\int \frac{dx}{2^x}$.

11. a) $\int e^{-5x} dx$, b) $\int (e^x+e^{-x})^2 dx$.

12. a) $\int 3^x(3^x-1)dx$, b) $\int \frac{81^x-3^x}{9^x} dx$.

13. a) $\int 2^x e^x dx$, b) $\int (5^x-1)(e^x+1)dx$.

14. a) $\int (3^x+5^x)dx$, b) $\int \frac{3^x+5^{2x}}{7^x} dx$.

15. a) $\int (\frac{1}{2}\sin x+\frac{3}{5}\cos x)dx$, b) $\int \frac{dx}{\sin^2 x cox^2 x}$.

16. a) $\int tg^2 x dx$, b) $\int ctg^2 x dx$.

17. a) $\int \frac{dx}{\sin^2 x}$, b) $\int \frac{dx}{\cos^2 x}$.

18. a) $\int \frac{\cos 2x}{\cos x - \sin x} dx$, b) $\int \frac{\sin^2 x}{1+\cos x} dx$.

19. a) $\int \cos \pi x dx$, b) $\int \frac{dx}{\cos^2 5x}$.

20. a) $\int \frac{1+\cos 2x}{\cos x} dx$, b) $\int \frac{dx}{\sin^2(9x+1)}$.

21. a) $\int \frac{dx}{x^2+16}$, b) $\int \frac{dx}{3x^2+5}$.

22. a) $\int \frac{dx}{\sqrt{4-x^2}}$, b) $\int \frac{dx}{\sqrt{1-4x^2}}$.

23. a) $\int \frac{dx}{\sqrt{2-3x^2}}$, b) $\int \frac{dx}{\sqrt{25-4x^2}}$.

24. a) $\int \sin^2 \frac{x}{2} dx$, b) $\int \frac{1+\sqrt{1-x^2}}{\sqrt{1-x^2}} dx$.

25. a) $\int \sqrt{\frac{1+x^2}{1-x^4}} dx$, b) $\int \frac{1-\sin x}{x+\cos x} dx$.

O‘rniga qo‘yish (o‘zgaruvchilarini almaltirish) usulidan foydalanib quyidagi integrallarni hisoblang.

26. $\int \frac{dx}{2x+3}$. **27.** $\int x^2 e^{x^2} dx$.

28. $\int x \cdot \sqrt{x-1} dx$. **29.** $\int \frac{dx}{1+\sqrt{x}}$.

30. $\int \frac{dx}{x\sqrt{x^2-2}}$. **31.** $\int \frac{dx}{e^x+1}$.

32. $\int x(5x^2-3)^7 dx$. **33.** $\int \frac{e^x dx}{4+e^{2x}}$.

34. $\int \sin^3 x dx$.   **35.** $\int \frac{dx}{\sqrt{a^2 - x^2}}$.

36. $\int \frac{dx}{1+\sqrt{x+1}}$.   **37.** $\int \frac{dx}{x\sqrt{x^2-1}}$.

38. $\int \frac{x^2 dx}{(x^2+1)^2}$.   **39.** $\int \frac{dx}{\sin 2x}$

40. $\int \sqrt{a^2 - x^2} dx$.

Quyidagi integrallarni hisoblang.

41. $\int (x+1)^{100} dx$.   **42.** $\int x(1+x^2)^7 dx$.   **43.** $\int \frac{dx}{2^{7x+1}}$.

44. $\int (8x+5)^{10} dx$.   **45.** $\int \sqrt[4]{(2-3x)^3} dx$.   **46.** $\int x(2x+5)^{10} dx$.

47. $\int \frac{1+x}{1+\sqrt{x}} dx$.   **48.** $\int \frac{e^x dx}{(e^x-5)^3}$.   **49.** $\int (e^{2x}+5)^2 e^{2x} dx$.

50. $\int \frac{e^{5x} dx}{1-e^{10x}}$.   **51.** $\int \frac{\sqrt{tgx} dx}{4\cos^2 x}$.   **52.** $\int \frac{dx}{\sqrt[5]{tg^2 x} \cos^2 x}$.

53. $\int \frac{\sin^3 x dx}{\sqrt{\cos x}}$.   **54.** $\int \frac{\ln 2x dx}{x \ln 4x}$.   **55.** $\int \frac{x^3 dx}{\sqrt{2-x^2}}$.

56. $\int e^{arctg 3x} \cdot \frac{dx}{1+9x^2}$.   **57.** $\int \frac{e^{tgx}}{\cos^{2x}} dx$.   **58.** $\int \frac{\sin x}{\cos^5 x} dx$

59. $\int \frac{e^{\sqrt{x}} dx}{\sqrt{x}}$.   **60.** $\int \sin \frac{1}{x} \cdot \frac{dx}{x^2}$.   **61.** $\int \cos^5 2x dx$.

Bo‘laklab integrallash usulidan foydalanib quyidagi integrallarni hisoblang.

62. $\int \ln x dx \quad (u = \ln x, dv = dx)$.   **63.** $\int x \sin x dx \quad (u = x, \ dv = \sin x dx)$.

64. $\int x^2 e^x dx \quad (u = x^2, \ dv = e^x dx)$.   **65.** $\int x \ln x dx \quad (u = \ln x, \ dv = x dx)$.

66. $\int x^2 \cos x dx \quad (u = x^2, \ dv = \cos x dx)$.   **67.** $\int x arctgx dx \quad (u = arctgx, \ dv = x dx)$.

68. $\int x^2 \sin x dx \quad (u = x^2, \ dv = \sin x dx)$.

69. $\int \sqrt[3]{x^2} \ln x dx \ (u = \ln x, \quad dv = \sqrt[3]{x^2} dx)$.

70. $\int e^{3x} (x^2 - 6x + 2) dx \ \ (u = x^2 - 6x + 2, \quad dv = e^{3x} dx)$.

Quyidagi integrallarni hisoblang.

71. $\int (2x-3)\cos x dx$. **72.** $\int (1-4x)\sin 2x dx$. **73.** $\int 2x arctgx dx$.

74. $\int \frac{\ln x dx}{\sqrt{x}}$. **75.** $\int \frac{\ln x dx}{x^2}$. **76.** $\int \frac{x}{\sin^2 x} dx$.

77. $\int (2x+1) e^{2x} dx$. **78.** $\int \ln(1+x^2) dx$. **79.** $\int \frac{x}{e^x} dx$.

80. $\int \ln^2 x dx$. **81.** $\int (x+2)\cdot 3^x dx$. **82.** $\int \frac{\ln x dx}{\sqrt[5]{x}}$.

83. $\int \frac{x \cos x}{\sin^2 x}$. **84.** $\int \frac{\ln x dx}{(1+x)^2}$. **85.** $\int (\arcsin x)^2 dx$.

86. $\int \sin(\ln x) dx$. **87.** $\int e^{\frac{-x}{3}} x^3 dx$. **88.** $\int \sqrt{a^2 - x^2} dx$,

$(a \neq 0, \quad u = \sqrt{a^2 + x^2}), \quad dv = dx)$.

89. $\int e^{\alpha x} \cdot \sin \beta x dx \quad (u = e^{\alpha x}, \quad dv = \sin \beta x dx)$.

90. Ma’lumki, og‘irlik kuchi ta’siridagi jismning vertikal harakati ushbu

$$\frac{d^2 S(t)}{dt^2} = -g \quad \text{yoki} \quad \frac{dv(t)}{dt} = -g$$

qonun bo‘yicha bo‘ladi. Shu harakatning tezligini toping.

91. Og‘irlik kuchi ta’siridagi jismning vertikal harakatida jism vaqtning t momentida qanday balandlikda bo‘ladi?

92. Og‘irlik kuchi ta’siridagi jismning vertikal harakatida jism vaqtning qanday momentida yerga uriladi?

93. Qurol otilganda zarracha $12 \frac{км}{сек}$ tezlik bilan yuqoriga otilib chiqdi. Aytaylik, agar zarracha harakatni yer sathidan boshlagan desak, uning ko‘tarilish balandligi qanday bo‘ladi? Qanday vaqtdan so‘ng zarracha yerga uriladi?

94. 160*м* balandlikdan tosh tashlandi, boshlang‘ich tezligi $2\frac{м}{сек}$ bo‘lsa, qanday vaqtdan so‘ng u yerga yetadi? Qanday tezlik bilan?

95. Yerda turgan odam to‘pni yuqoriga otadi. Otilgan to‘p 18*м* balandlikga yetishi uchun yuqoriga uloqtirish qanday tezlikda bo‘lishi kerak?

96. Ushbu

$$\int e^x\left(1+\frac{e^{-x}}{x^2}\right)dx$$

integralni hisoblang.

A) $e^x+\frac{1}{x}+C,$ B) $e^x-\frac{1}{x}+C,$ C) $e^x-\frac{1}{x^2}+C,$ D) $e^x+\frac{1}{x^2}+C.$

97. Ushbu

$$\int\left(0,7\cdot x^{-0,1}+0,2\cdot(0,5)^x\right)dx$$

integralni hisoblang.

A) $\frac{1}{9}x^{0,9}-\frac{1}{2^x\cdot 5\ln 2}+C,$ B) $\frac{2}{9}x^{0,8}-\frac{1}{2^x\cdot 5\ln 2}+C,$

C) $\frac{7}{9}x^{0,9}+\frac{1}{2^x\cdot \ln 2}+C,$ D) $\frac{7}{9}x^{0,9}-\frac{1}{2^x\cdot 5\ln 2}+C.$

98. Ushbu $$\int\frac{\sqrt{x}}{x+16}dx$$

integralni hisoblang.

A) $2\sqrt{x}-8arctg\frac{\sqrt{x}}{4}+C,$ B) $\sqrt{x}-8arctg\frac{\sqrt{x}}{4}+C,$

C) $2\sqrt{x}+8arctg\frac{\sqrt{x}}{4}+C,$ D) $2\sqrt{x}-8arctg\frac{\sqrt{x}}{4}+C.$

99. Ushbu

$$\int\frac{x\sin x}{\cos^2 x}dx$$

integralni hisoblang.

A) $\dfrac{x}{\cos x}+\dfrac{1}{2}\ln\left|\dfrac{\sin x+1}{\sin x-1}\right|+C,$ B) $\dfrac{x}{\cos x}+\dfrac{1}{2}\ln\left|\dfrac{\sin x-1}{\sin x+1}\right|+C,$

C) $\dfrac{x}{\sin x}+\dfrac{1}{2}\ln\left|\dfrac{\sin x-1}{\sin x+1}\right|+C,$ D) $\dfrac{1}{\cos x}+\dfrac{1}{2}\ln\left|\dfrac{\sin x+1}{\sin x-1}\right|+C.$

100. Agar $f''(x)=3x^2,\ f'(0)=1,\ f(0)=-1$ bo‘lsa, $f(x)$ ni toping.

A) $f(x)=\dfrac{1}{3}x^3,$ B) $f(x)=\dfrac{1}{4}x^5+x-1,$ C) $f(x)=\dfrac{1}{4}x^4+x-1,$

D) $f(x)=\dfrac{1}{4}x^4-x+1.$

1.2. Ratsional funksiyalarni, ba’zi irratsional funksiyalarni hamda trigonometrik funksiyalarni integrallash.

1.2.1. Ratsional funksiyalarni integrallash.

Ma’lumki,

$$\frac{A}{x-a},\quad \frac{A}{(x-a)^n},\quad \frac{Bx+C}{x^2+px+q},\quad \frac{Bx+C}{\left(x^2+px+q\right)^n}$$

ko‘rinishdagi kasrlar **sodda kasrlar** deyiladi, bunda $a,\ p,\ q,\ A,\ B,\ C$ o‘zgarmas sonlar, $\dfrac{p^2}{4}-q<0,\quad n=2,3,4,...$. Ularning integrallari quyidagicha bo‘ladi:

1) $\displaystyle\int\frac{A}{x-a}dx=A\cdot\ln|x-a|+C;$

2) $\displaystyle\int\frac{A}{(x-a)^n}dx=\frac{A}{(1-n)(x-a)^{n-1}}+C,\qquad n=2,3,4,...;$

3) $\displaystyle\int\frac{Bx+C}{x^2+px+q}dx=\frac{B}{2}\ln\left(x^2+px+q\right)+\frac{2C-Bp}{\sqrt{4a-p^2}}\,arctg\,\frac{2x+p}{\sqrt{4a-p^2}}+C;$

4) $\displaystyle\int\frac{Bx+C}{\left(x^2+px+q\right)^n}dx=\frac{B}{2}\frac{1}{1-n}\frac{1}{\left(t^2+a^2\right)^{n-1}}+\left(c-\frac{Bp}{2}\right)\int\frac{dt}{\left(t^2+a^2\right)^n}$

bunda

$$x+\frac{p}{2}=t,\qquad q-\frac{p^2}{4}=a^2$$

bo‘lib,

$$J_n = \int \frac{dt}{\left(t^2 + a^2\right)^n}.$$

integral quyidagi formula

$$J_{n+1} = \frac{x}{2na^2\left(x^2 + a^2\right)^n} + \frac{2n-1}{2na^2} \cdot J_n$$

yordamida hisoblanadi.

Ravshanki, $J_1 = \int \frac{dx}{x^2 + a^2} = \frac{1}{a} arctg \frac{x}{a} + C$.

Ma’lumki, ratsional funksiya ikkita butun ratsional funksiyalar-ko‘phadlar nisbatidan iborat. Bu funksiyaning integrali quyidagicha topiladi:

1) agar kasr noto‘g‘ri bo‘lsa, uning butun qismini ajratib, butun ratsional funksiya (ko‘phad) ham to‘g‘ri kasr yig‘indisi sifatida yozib olinadi;

Ravshanki, butun rasional funksiyaning integrali oson hisoblanadi. Demak, noto‘g‘ri kasrni integrallash to‘g‘ri kasrni integrallashga keladi;

2) to‘g‘ri kasrni sodda kasrlar yig‘indisi sifatida yozib olinadi (qaralsin,[1]);

3)sodda kasrning integrallari hamda integral xossalaridan foydalanib, to‘g‘ri kasrlarning integrallari topiladi.

1-misol. Ushbu

$$\frac{2x-1}{x^2 - 5x + 6}$$

ratsional funksiyani sodda kasrlarga yoying.

◄Ma’lumki, $x^2 - 5x + 6 = (x-3)(x-2)$. Unda

$$\frac{2x-1}{x^2-5x+6} = \frac{A}{(x-3)} + \frac{B}{(x-2)} \tag{1}$$

bo‘ladi. Bu tenglikning ikki tomonini $x^2 - 5x + 6$ga ko‘paytirib.

$$2x - 1 = A(x-2) + B(x-3),$$

ya’ni

$$2x - 1 = (A+B)x - 2A - 3B$$

bo‘lishini topamiz. Keyingi tenglik ayniyat bo‘lgani uchun tenglikning o‘ng va chap tomonlaridagi x ning bir xil darajalari oldidagi koeffisietlar bir-biriga teng bo‘ladi.

Demak,

$$A+B=2,$$
$$2A+3B=1.$$

Bu sistemani yechib, $A=5,\quad B=-3$ bo‘lishini topamiz. Ularni (1) tenglikdagi A va B larning o‘rniga qo‘yamiz.

Shunday qilib,

$$\frac{2x-1}{x^2-5x+6}=\frac{5}{x-3}+\frac{3}{x-2}$$

bo‘ladi.►

2-misol. Ushbu

$$\frac{x^3+4x^2-2x+1}{x^4+x}$$

ratsional funksiyani sodda kasrlarga yoying.

◄Avvalo, kasr maxrajini ko‘paytuvchilarga ajratamiz:

$$x^4+x=x\left(x^3+1\right)=x(x+1)\left(x^2-x+1\right)$$

So‘ng berilgan ratsional funksiyani quyidagicha yozib olamiz:

$$\frac{x^3+4x^2-2x+1}{x(x+1)\left(x^2-x+1\right)}=\frac{A}{x}+\frac{B}{x+1}+\frac{Cx+D}{x^2-x+1}. \qquad (2)$$

Bu tenglikning ikki tomonini $x(x+1)\left(x^2-x+1\right)$ ga ko‘paytrib

$$x^3+4x^2-2x+1=A\left(x^3+1\right)+Bx\left(x^2-x+1\right)+(Cx+D)\left(x^2+x\right),$$

ya’ni

$$x^3+4x^2-2x+1=(A+B+C)x^3+(C+D-B)x^2+(B+D)x+A$$

bo‘lishini topamiz. Bu tenglikni ayniyat ekanligini e’tiborga olib, ushbu

$$A+B+C=1,$$
$$C+D-B=4,$$
$$B+D=-2,$$
$$A=1$$

sistemani hosil qilamiz. Keyingi sistemani yechib topamiz:

$$A=1,\quad B=-2,\quad C=2,\quad D=0$$

Demak, (2) formulaga ko‘ra

$$\frac{x^3+4x^2-2x+1}{x^4+x}=\frac{1}{x}-\frac{2}{x+1}+\frac{2x}{x^2-x+1}$$

bo‘ladi.►

3-misol. Ushbu

$$\int \frac{x^5-1}{x^3+x^2+x}dx$$

integralni hisoblang.

◄Integral ostidagi funksya noto‘g‘ri kasr. Bu kasrning suratini maxrajiga bo‘lib, uning butun qismini ajratamiz

$$\begin{array}{l|l}
x^5+0\cdot x^4+0\cdot x^3+0\cdot x^2-1 & x^3+x^2+x \\ \cline{2-2}
-\left(x^5+x^4+x^3\right) & x^2-x \\ \hline
-x^4-x^3+0\cdot x^2-1 & \\
-\left(-x^4-x^3-x^2\right) & \\ \hline
x^2-1 &
\end{array}$$

Demak,

$$\frac{x^5-1}{x^3+x^2+x}=x^2-x+\frac{x^2-1}{x^3+x^2+x}$$

bo‘lib, uning integrali

$$\int \frac{x^5-1}{x^3+x^2+x}dx=\int\left(x^2-x\right)dx+\int\frac{x^2-1}{x^3+x^2+x}dx=\frac{x^3}{3}-\frac{x^2}{2}+\int\frac{x^2-1}{x^3+x^2+x}dx \quad (3)$$

bo‘ladi.

Keyingi tenglikning o‘ng tomonidagi integralni hisoblash uchun integral ostidagi funksiyani (to‘g‘ri kasrni) sodda kasrlar orqali ifodasini topamiz.

Kasrning maxrajini ko‘paytuvchilarga

$$x^3+x^2+x=x\left(x^2+x+1\right)$$

ajratib, uni quyidagicha

$$\frac{x^2-1}{x\left(x^2+x+1\right)}=\frac{A}{x}+\frac{Bx+C}{x^2+x+1}$$

yozib olamiz. Bu tenglikning ikki tomonini $x\left(x^2+x+1\right)$ga ko‘paytirish bilan tenglik ushbu

$$x^2-1=A\left(x^2+x+1\right)+\left(Bx+C\right)\cdot x$$

ya’ni

$$x^2-1=\left(A+B\right)x^2+\left(A+C\right)x+A$$

ko‘rinishga keladi. Natijada,

$$\begin{cases} A+B=1 \\ A+C=0 \\ A=-1 \end{cases}$$

sistema hosil bo‘ladi. Uni yechib,

$$A=-1,\ B=2,\ C=1$$

bo‘lishini topamiz. Demak,

$$\frac{x^2-1}{x^3+x^2+x}=-\frac{1}{x}+\frac{2x+1}{x^2+x+1}.$$

Bu kasrning integrali quyidagicha hisoblanadi

$$\int\frac{x^2-1}{x^3+x^2+x}dx=-\int\frac{dx}{x}+\int\frac{2x+1}{x^2+x+1}dx=-\ln|x|+\int\frac{2x+1}{x^2+x+1}dx=$$
$$=\left[t=x^2+x+1,\ dt=(2x+1)dx\right]=-\ln|x|+\int\frac{dt}{t}=-\ln|x|+\ln|t|+C. \qquad (4)$$

(3) va (4) lardan

$$\int\frac{x^5-1}{x^3+x^2+x}dx=\frac{x^3}{3}-\frac{x^2}{2}+\ln\left|\frac{x^2+x+1}{x}\right|+C$$

bo‘lishi kelib chiqadi.►

1.2.2. Irratsional funksiyalarni integrallash. Ba’zi irratsional funksiyalarning integrallari mos almashtirishlar yordamida ratsional funksiyalarning integraliga keladi.

4-misol. Ushbu

$$J=\int\frac{\sqrt{x}}{1+\sqrt[4]{x}}dx$$

integralni hisoblang.

◄Bu integralda $x=t^4$ almashtirish bajarilsa, integrallash ratsional funksiyani integrallashga keldi. Ravshanki, $dx=4t^3dt$ bo‘lib,

$$\int\frac{\sqrt{x}}{1+\sqrt[4]{x}}dx=\int\frac{t^2}{1+t}\cdot 4t^3dt=4\int\frac{t^5}{1+t}dt$$

bo‘ladi. t^5ni $1+t$ ga bo‘lib topamiz:

$$\frac{t^5}{1+t}=t^4-t^3+t^2-t+1-\frac{1}{t+1}.$$

Natijada

$$\int\frac{t^5}{1+t}dt=4\int\left(t^4-t^3+t^2-t+1-\frac{1}{t+1}\right)dt=4\left(\frac{t^5}{5}-\frac{t^4}{4}+\frac{t^3}{3}-\frac{t^2}{2}+t-\ln|t+1|\right)+C$$

bo‘ladi, bunda $t=\sqrt[4]{x}$ ►

5-misol. Ushbu

$$\int \frac{x+\sqrt{1+x}}{\sqrt[3]{1+x}}dx$$

integralni hisoblang.

◀Bu integralda

$$t=\sqrt[6]{1+x}$$

almashtirish bajaramiz. Unda

$$1+x=t^6,\ x=t^6-1,\ dx=6t^5dt$$

bo‘lib,

$$\frac{x+\sqrt{1+x}}{\sqrt[3]{1+x}}=\frac{t^6-1+t^3}{t^2}$$

bo‘ladi. Shularni e’tiborga olib topamiz:

$$\int \frac{x+\sqrt{1+x}}{\sqrt[3]{1+x}}dx=\int \frac{t^6-1+t^3}{t^2}6t^5dt=6\int\left(t^9+t^6-t^3\right)dt=6\left(\frac{t^{10}}{10}+\frac{t^7}{7}-\frac{t^4}{4}\right)+C=$$

$$=6\sqrt[3]{(1+x)^2}\left(\frac{1+x}{10}+\frac{\sqrt{1+x}}{7}-\frac{1}{4}\right)+C.$$ ▶

1.2.3. Trigonometrik funksiyalarni integrallash. Aytaylik, $f(x)$ funksiya o‘zgarmas son, $\sin x, \cos x$ funksiyalar orasida (ustida) qo‘shish, ayirish, ko‘paytirish va bo‘lish amallari bajarilishidan hosil bo‘lgan funksiya bo‘lsin. Uni $\sin x$ va $\cos x$ larning ratsional funksiyasi deyiladi va $R(\sin x, \cos x)$ kabi belgilanadi.

Bunday trigonometrik funksiyalarning integrallari $\int R(\sin x, \cos x)dx$ har doim ushbu

$$tg\frac{x}{2}=t$$

almashtirish natijasida ratsional funksiyalarning integraliga keladi. Bunda

$$x=2arctgt, \qquad dx=\frac{2}{1+t^2}dt,$$

$$\sin x=\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}}=\frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}}=\frac{2t}{1+t^2},$$

$$\cos x=\frac{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}}=\frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}=\frac{1-t^2}{1+t^2}$$

bo‘lishidan foydalaniladi.

6-misol. Ushbu

$$J=\int\frac{dx}{3+\sin x}$$

integralni hisoblang.

◀ Bu integralda

$$tg\frac{x}{2}=t$$

almashtirish bajaramiz. Unda

$$x=2arctgt,\qquad dx=\frac{2}{1+t^2}dt,$$

$$\sin x=\frac{2t}{1+t^2},\quad dx=\frac{2dt}{1+t^2}$$

bo‘lib,

$$\int\frac{dx}{3+\sin x}=\int\frac{1}{3+\frac{2t}{1+t^2}}\cdot\frac{2dt}{1+t^2}=2\int\frac{dt}{3t^2+2t+3}$$

bo‘ladi.

Keyingi integral quyidagicha hisoblanadi

$$2\int\frac{dt}{3t^2+2t+3}=\frac{2}{3}\int\frac{dt}{t^2+2\cdot\frac{1}{3}t+\frac{1}{9}-\frac{1}{9}+1}=\frac{2}{3}\int\frac{dt}{\left(t+\frac{1}{3}\right)^2+\frac{8}{9}}=\frac{2}{3}\int\frac{d\left(t+\frac{1}{3}\right)}{\left(t+\frac{1}{3}\right)^2+\left(\frac{2\sqrt{2}}{3}\right)^2}=$$

$$=\frac{2}{3}\cdot\left(\frac{2\sqrt{2}}{3}\right)^{-2}\cdot\int\frac{d\left(t+\frac{1}{3}\right)}{1+\left(\frac{t+\frac{1}{3}}{\frac{2\sqrt{2}}{3}}\right)^2}=\frac{2}{3}\cdot\left(\frac{2\sqrt{2}}{3}\right)^{-1}\cdot\int\frac{d\left(\frac{t+\frac{1}{3}}{\frac{2\sqrt{2}}{3}}\right)}{1+\left(\frac{t+\frac{1}{3}}{\frac{2\sqrt{2}}{3}}\right)^2}=\frac{\sqrt{2}}{2}arctg3\cdot\frac{t+\frac{1}{3}}{2\sqrt{2}}+C=$$

$$=\frac{\sqrt{2}}{2}arctg\frac{3tg\frac{x}{2}+1}{2\sqrt{2}}+C.$$

Demak,

$$J=\int\frac{dx}{3+\sin x}=\frac{\sqrt{2}}{2}arctg\frac{3tg\frac{x}{2}+1}{2\sqrt{2}}+C. \blacktriangleright$$

$\int R(\sin x,\cos x)dx$ integralning ba'zi xususiy hollari.

1) $\int\sin^m x\cos^n xdx,$ bunda m va n natural sonlar bo'lib, m yoki n toq son, masalan $n=2k+1$ $(k=0,1,2,...)$ bo'lsin. Bu holda

$$\int\sin^m x\cos^n xdx=\int\sin^m x\cdot\cos^{2k}x\cdot\cos xdx=$$
$$=[\sin x=t,\ \cos xdx=dt]=\int t^m\left(1-t^2\right)^k dt$$

bo'ladi.

7-misol. Ushbu

$$\int\sin^2 x\cos^5 xdx$$

integralni hisoblang.

◀Bu integral quyidagicha hisoblanadi

$$\int\sin^2 x\cos^5 xdx=\int\sin^2 x\left(1-\sin^2 x\right)^2\cos xdx=\int\sin^2 x\left(1-2\sin^2 x+\sin^4 x\right)^2 d(\sin x)=$$
$$=\int\left(\sin^2 x-2\sin^4 x+\sin^6 x\right)^2 d(\sin x)=\frac{\sin^3 x}{3}-\frac{2\sin^5 x}{5}+\frac{\sin^7 x}{7}+C. \blacktriangleright$$

8-misol. Ushbu

$$\int\cos^3 xdx$$

integralni hisoblang.

◀Bu integral quyidagicha hisoblanadi

$$\int\cos^3 xdx=\int\cos^2 x\cos xdx=\int\left(1-\sin^2 x\right)d(\sin x)=\sin x-\frac{\sin^3 x}{3}+C. \blacktriangleright$$

2) $\int\sin^m x\cos^n xdx,$ bunda m va n lar juft natural sonlar yoki 0.

Bunday integrallar trigonometriyaning

$$\cos^2\alpha=\frac{1+\cos 2\alpha}{2},\quad \sin^2\alpha=\frac{1-\cos 2\alpha}{2}$$

formulalarini qo'llash (takror qo'llash) bilan hisoblanadi.

3) $\int \sin mx \cos nx dx,\quad \int \cos mx \cos nx dx,\quad \int \sin mx \sin nx dx,$
bunda $m = const,\ n = const$.

Bunday integrallar trigonometriyaning

$$\sin\alpha\cdot\sin\beta = \frac{1}{2}\left[\cos(\alpha-\beta)-\cos(\alpha+\beta)\right],$$

$$\cos\alpha\cdot\cos\beta = \frac{1}{2}\left[\cos(\alpha-\beta)+\cos(\alpha+\beta)\right],$$

$$\sin\alpha\cdot\cos\beta = \frac{1}{2}\left[\sin(\alpha-\beta)+\sin(\alpha+\beta)\right],$$

$$\sin\alpha\cdot\cos\alpha = \frac{1}{2}\sin 2\alpha,$$

formulalarni qo‘llash bilan hisoblanadi.

9-misol. Ushbu

$$J = \int \sin^2 x \cos^4 x dx$$

integralni hisoblang.

◀Bu integralni hisoblashda yuqorida keltirilgan formulalardan foydalanamiz:

$$J = \int (\sin x \cos x)^2 \cos^2 x dx = \int \left(\frac{\sin 2x}{2}\right)^2 \cdot \frac{1+\cos 2x}{2} dx =$$

$$= \frac{1}{8}\int \sin^2 2x(1+\cos 2x)dx = \frac{1}{8}\int \sin^2 2x dx + \frac{1}{8}\int \sin^2 2x \cos 2x dx =$$

$$= \frac{1}{8}\int \frac{1-\cos 4x}{2}dx + \frac{1}{8}\cdot\frac{1}{2}\int \sin^2 2x d(\sin 2x) = \frac{1}{16}\int (1-\cos 4x)\cdot dx +$$

$$+\frac{1}{16}\int \sin^2 2x d(\sin 2x) = \frac{1}{16}x - \frac{\sin 4x}{64} + \frac{\sin^3 2x}{48} + c. \blacktriangleright$$

10-misol. Ushbu

$$\int \sin 3x \cdot \cos 5x dx$$

integralni hisoblang.

◀Bu integral quyidagicha hisoblanadi

$$\int \sin 3x \cdot \cos 5x dx = \frac{1}{2}\int \left[\sin 8x + \sin(-2x)\right]dx = \frac{1}{2}\int \sin 8x dx - \frac{1}{2}\int \sin 2x dx =$$

$$= \frac{1}{2}\cdot\frac{1}{8}\int \sin 8x d(8x) - \frac{1}{2}\cdot\frac{1}{2}\int \sin 2x d(2x) = \frac{-1}{16}\cos 8x +$$

$$+\frac{1}{4}\cos 2x + c = \frac{1}{4}\cos 2x - \frac{1}{16}\cos 8x + c. \blacktriangleright$$

Quyidagi ko'phadlarni ko'paytuvchilarga ajrating:

101. a) x^2-9, b) $1-x^2$, c) $9x^2-16$, d) x^2+x.

102. a) x^3-1, b) x^3+1 , c) x^3-8.

103. a) x^4-1, b) x^4-16 , c) x^4-9.

104. a) x^4+1 , b) x^4+x^2+1 c) x^2-4x+4.

105. a) x^3-2x^2+x , b) x^3-x^2+2x-2 .

106. a) x^3-5x^2+4x, b) x^4-5x^2+4.

107. x^3-3x^2+5x-3.

Quyidagi sodda kasrlarning integrallarini toping:

108. a) $\int \frac{3}{x+15}dx$, b) $\int \frac{5}{x-12}dx$, c) $\int \frac{dx}{\frac{1}{4}x-\frac{1}{2}}$.

109. a) $\int \frac{dx}{(x+2)^{10}}$, b) $\int \frac{dx}{(x-7)^{20}}$, c) $\int \frac{dx}{(2x-6)^{12}}$.

110. a) $\int \frac{xdx}{(x+1)^9}$, b) $\int \frac{xdx}{(x-2)^7}$, c) $\int \frac{xdx}{(3x+9)^{14}}$.

111. a) $\int \frac{dx}{3x^2+7}$, b) $\int \frac{dx}{5x^2-2}$.

112. $\int \frac{dx}{x^2+2x+5}$. **113.** $\int \frac{dx}{x^2+x+1}$. **114.** $\int \frac{dx}{x^2+6x+13}$.

115. $\int \frac{dx}{x^2+3x}$. **116.** $\int \frac{dx}{3x^2+5x+3}$. **117.** $\int \frac{xdx}{x^2+2x+5}$.

118. $\int \frac{xdx}{x^2+x+1}$. **119.** $\int \frac{x+1}{x^2-5x+7}dx$. **120.** $\int \frac{4x+1}{x^2+5x+8}dx$.

121. $\int \frac{3x+1}{x^2-2x+2}dx$.

Quyidagi ratsional funksiyalarning integrallarini hisoblang:

122. $\int \frac{x^2-5x+3}{x(x^2-1)}dx$. **123.** $\int \frac{dx}{x(x+1)^2}$. **124.** $\int \frac{xdx}{2x^4-2x^2+1}$.

125. $\int \frac{dx}{(x+1)(x^2+1)}$. **126.** $\int \frac{dx}{x^3+1}$. **127.** $\int \frac{xdx}{x^3-1}$.

128. $\int \frac{dx}{x^4+1}$. **129.** $\int \frac{x^2+1}{(x^2-1)(x^2-4)}dx$.

130. $\int \frac{xdx}{(x-1)^2(x^2+2x+2)}$. **131.** $\int \frac{dx}{x^4+x^2+1}$.

Quyidagi irratsional funksiyalarning integrallarini hisoblang:

132. $\int \frac{dx}{1+\sqrt{x}}$. **133.** $\int \frac{dx}{(1+\sqrt[3]{x})\sqrt{x}}$. **134.** $\int \frac{dx}{x\sqrt{x+1}}$.

135. $\int x\sqrt{3-x}dx$. **136.** $\int \sqrt[3]{2x-3}dx$. **137.** $\int \frac{x-1}{\sqrt{x+1}}dx$.

138. $\int \frac{dx}{\sqrt{x^2-2}}$. **139.** $\int \frac{1}{x^2}\sqrt{\frac{x+1}{x}}dx$. **140.** $\int \frac{x^2dx}{\sqrt{1-x^2}}$.

141. $\int \frac{dx}{\sqrt[4]{1+x^4}}$. **142.** $\int \frac{\sqrt{x}}{\sqrt[4]{x^3}+1}dx$. **143.** $\int \frac{x+1}{\sqrt[3]{3x+1}}dx$.

144. $\int \frac{x^3}{1+\sqrt[3]{x^4+1}}dx$. **145.** $\int \frac{dx}{\sqrt[6]{(1+x^6)^7}}$. **146.** $\int x^7\sqrt{1+x^4}dx$.

147. $\int \frac{\sqrt{x}}{(1+\sqrt[3]{x})^2}dx$. **148.** $\int \frac{dx}{\sqrt{x^2-6x+10}}$. **149.** $\int \frac{dx}{\sqrt{x^2+x}}$.

150. $\int \frac{dx}{\sqrt{x^2+4x+3}}$. **151.** $\int \frac{x^2}{\sqrt{x^2+2x+3}}dx$.

Quyidagi trigonometrik funksiyalarning integrallarini hisoblang:

152. $\int \cos^2 5xdx$. **153.** $\int \sin x \cdot \cos^3 xdx$. **154.** $\int \sin^3 xdx$.

155. $\int\frac{\cos^5 x}{\sin^3 x}dx$. **156.** $\int\frac{dx}{\cos^4 x}$. **157.** $\int\frac{\sin^3 x}{\cos^2 x+1}dx$.

158. $\int tg^6 xdx$. **159.** $\int\frac{1+tgx}{\sin^2 x}dx$. **160.** $\int\sin^4 x\cdot\cos^2 xdx$.

161. $\int\frac{dx}{3\sin x-4\cos x}$. **162.** $\int\frac{\cos^3 x}{4\sin^2 x-1}dx$. **163.** $\int\frac{dx}{1+2\cos^2 x}$.

164. $\int\frac{dx}{1+tgx}$. **165.** $\int\frac{dx}{1+\sin x+\cos x}$. **166.** $\int\cos\frac{x}{2}\cdot\cos\frac{x}{3}dx$.

1**67.** $\int\cos 2x\sin 4xdx$. **168.** $\int\sin 5x\cdot\sin 6xdx$. **169.** $\int\sin^3 x\cdot\sqrt{\cos x}dx$.

170. $\int\frac{dx}{7\cos^2 x+2\sin^2 x}$ **171.** $\int\frac{dx}{\sin x+tgx}$.

172. Ushbu

$$\int\frac{f'(x)}{f^2(x)}dx$$

integralni hisoblang.

A) $-\frac{1}{f(x)}+C$, B) $\frac{1}{f^2(x)}+C$, C) $f(x)+C$, D) $\frac{2}{f(x)}+C$.

173. Agar $\int f(x)dx=F(x)+C$ bo‘lsa, $\int xf'(x)dx$ nimaga teng bo‘ladi?

A) $f(x)+F(x)+C$, B) $xf(x)+F(x)+C$,

C) $2f(x)+F(x)+C$, D) $x^2 f(x)+F(x)+C$.

174. Ushbu

$$\int x\cdot 2^x dx$$

integralni hisoblang.

A) $\frac{2^x(x\ln 2-1)}{\ln 2}$, B) $\frac{2^x(\ln 2-1)}{\ln^2 2}$, C) $\frac{2^x(x\ln 2-1)}{\ln^2 2}$, D) $\frac{(x\ln 2+1)2^x}{\ln^2 2}$.

175. Ushbu

$$\int \frac{dx}{\sin x \cos x}$$

integralni hisoblang.

A) $\ln|\sin x| + C$, B) $\ln|\cos x| + C$, C) $\frac{1}{2}\ln|tgx| + C$, D) $\ln|tgx| + C$.

176. Ushbu

$$\int \sqrt[3]{5x-2}\,dx$$

integralni hisoblang.

A) $\frac{1}{20}\sqrt[3]{(5x-2)^4} + C$, B) $\frac{3}{20}\sqrt[3]{(5x-2)^4} + C$,

C) $\frac{7}{20}\sqrt[3]{(5x-2)^4} + C$, D) $\frac{9}{20}\sqrt[3]{(5x-2)^4} + C$.

Nazorat savollari

1.Funksiyaning boshlang'ich funksiyasiga ta'rif bering.
2. Aniqmas integralning sodda xossalarini keltring.
3.Aniqmas integralning o'zgaruvchilarni almashtirish usulini izoxlab bering.
4. Aniqmas integralni bo'laklab integrallash usulini izohlab bering.
5. Ratsional funksiyalarni integrallash usulini izohlab bering.
6. Irratsional funksiyalarni integrallash usulini izohlab bering.
7. Trigonom´etrik funksiyalarni integrallash usullarini izohlab bering.

2 bob

Aniq integral

2.1. Aniq integral va uning sodda xossalari. Aniq integralni hisoblash usullari

2.1.1. Aniq integral va uning sodda xossalari. Aytaylik, $y=f(x)$ funksiya $[a,b]$ segmentda berilgan va chegaralangan bo‘lsin. u kesmani

$$x_0, x_1, x_2, ..., x_{n-1}, x_n \ \left(x_0=a, \quad x_n=b, \quad x_0<x_1<...<x_n\right)$$

nuqtalar yordamida n ta

$$[x_0,x_1],[x_1,x_2],...,[x_k,x_{k+1}],...,[x_{n-1},x_n]$$

bo‘lakka ajratib, har bir bo‘lakda ixtiyoriy ravishda bittadan

$$\xi_0, \xi_1, ..., \xi_k, ..., \xi_{n-1} \qquad \left(\xi_k \in [x_k, x_{k+1}], k=0,1,...,n-1\right)$$

nuqtalarni olamiz. Bu nuqtalardagi funksiyalarning qiymatlari

$$f(\xi_0), f(\xi_1), ..., f(\xi_k), ..., f(\xi_{n-1})$$

ni mos bo‘lakchalar uzunliklari

$$\Delta x_0 = x_1 - x_0 \qquad (x_0=a),$$
$$\Delta x_1 = x_2 - x_1,$$
$$.........................$$
$$\Delta x_k = x_{k+1} - x_k,$$
$$.........................$$
$$\Delta x_{n-1} = x_n - x_{n-1} \qquad (x_n=b)$$

ga ko‘paytirib, quyidagi

$$\sigma = f(\xi_0)\cdot\Delta x_0 + f(\xi_1)\cdot\Delta x_1 + ... + f(\xi_k)\cdot\Delta x_k + \\ +...+ f(\xi_{n-1})\cdot\Delta x_{n-1} = \sum_{k=0}^{n-1} f(\xi_k)\cdot\Delta x_k \tag{1}$$

yig‘indini tuzamiz. (1) yig‘indi **integral yig‘indi** deyiladi.

Integral yig‘indi σ ning $\max\limits_k \Delta x_k \to 0$ dagi limiti mavjud bo‘lsa, $f(x)$ funksiya $[a,b]$ da integrallanuvchi, limitning qiymati esa $f(x)$ funksiyaning $[a,b]$ bo‘yicha **aniq integrali** deyiladi va $\int\limits_a^b f(x)dx$ kabi belgilanadi

$$\int_a^b f(x)dx = \lim_{\max_k \Delta x_k \to 0} \sum_{k=0}^{n-1} f(\xi_k)\cdot \Delta x_k .$$

Agar $f(x)$ funksiya $[a,b]$ segmentda

1) uzluksiz bo‘lsa, u $[a,b]$ da integrallanuvchi,

2) $[a,b]$ segmentning chekli sondagi nuqtalarida uzilishga ega va qolgan barcha nuqtalarda uzluksiz bo‘lsa, u $[a,b]$ da integrallanuvchi bo‘ladi.

Aniq integral quyidagi sodda xossalarga ega:

1) $\int_a^a f(x) = 0,$

2) $\int_a^b f(x)dx = -\int_b^a f(x)dx,$

3) $\int_a^b c\cdot f(x)dx = c\cdot\int_a^b f(x)dx, \quad c = const,$

4) $\int_a^b [f(x)+g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx,$

5) $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx, \quad (a<c<b).$

1-misol. Ushbu

$$\int_a^b c\cdot dx = c\cdot(b-a) \qquad\qquad (f(x) = c - const)$$

tenglikni isbotlang.

◄ $f(x)=c$ funksiyaning integral yig‘indisi

$$\sigma = f(\xi_0)\cdot\Delta x_0 + f(\xi_1)\cdot\Delta x_1 + f(\xi_2)\cdot\Delta x_2 + ... + f(\xi_k)\cdot\Delta x_k + ... + f(\xi_{n-1})\cdot\Delta x_{n-1} =$$
$$= c\cdot\Delta x_0 + c\cdot\Delta x_1 + c\cdot\Delta x_2 + ... + c\cdot\Delta x_k + ... + c\cdot\Delta x_{n-1} =$$
$$= c(\Delta x_0 + \Delta x_1 + \Delta x_2 + ... + \Delta x_k + ... + \Delta x_{n-1}) =$$
$$= c(x_1 - a + x_2 - x_1 + x_3 - x_2 + ... + x_{k+1} - x_k + ... + b - x_{n-1}) =$$
$$= c\cdot(b-a).$$

bo‘ladi.

Undan

$$\lim_{\max \Delta x_k \to 0} \sigma = \lim_{\max \Delta x_k \to 0} c\cdot(b-a) = c\cdot(b-a)$$

bo‘lishi kelib chiqadi. Ta'rifga binoan

$$\int_a^b c\cdot dx = c\cdot(b-a)$$

bo‘ladi.►

2.1.2. Aniq integralni hisoblash usullari.

1) Aniq integrallarni Nyuton-Leybnis formulasi yordamida hisoblash. Aytaylik, $f(x)$ funksiya $[a,b]$ da uzluksiz bo‘lib, $F(x)$ esa uning boshlang‘ich funksiyasi bo‘lsin ($F'(x)=f(x)$) Unda

$$\int_a^b f(x)dx = F(b)-F(a) = F(x)\Big|_a^b \tag{2}$$

bo‘ladi. (2) formula Nyuton-Leybnits formulasi deyiladi. Uning yordamida aniq integrallar hisoblanadi.

2-misol. Ushbu.

$$\int_0^4 \left(1+e^x\right)dx$$

integralni hisoblang.

◄Bu integralni aniq integral xossalari hamda Nyuton-Leybnis formulasidan foydalanib topamiz:

$$\int_0^4 \left(1+e^x\right)dx = \int_0^4 dx + \int_0^4 e^x dx = x\Big|_0^4 + e^x\Big|_0^4 = 4+e^4-e^0 = 3+e^4. ►$$

2) Aniq integrallarni o‘zgaruvchini almashtirish usuli bilan hisoblash.

Agar $f(x)$ funksiya $[a,b]$ da uzluksiz, $x=\varphi(t)$ funksiya esa $[\alpha,\beta]$ da uzluksiz, uzluksiz $\varphi'(t)$ hosilaga ega va $\varphi(\alpha)=a,\quad \varphi(\beta)=b$ bo‘lsa,

$$\int_a^b f(x)dx = \int_\alpha^\beta f(\varphi(t))\cdot\varphi'(t)dt \tag{3}$$

bo‘ladi. (3) formula aniq integrallarda **o‘zgaruvchini almashtirish formulasi** deyiladi.

3-misol. Ushbu.

$$\int_0^1 x\sqrt{1+x^2}\,dx$$

integralni hisoblang.

◀Berilgan integralda

$$\sqrt{1+x^2}=t \quad \text{ya'ni} \quad x=\sqrt{t^2-1}$$

deylik. Unda

$$x=0 \ \text{ da } \ t=1$$
$$x=1 \ \text{ da } \ t=\sqrt{2}$$

bo‘lib,

$$dx=\left(\sqrt{t^2-1}\right)'\cdot dt=\frac{t}{\sqrt{t^2-1}}dt$$

bo‘ladi. Natijada, berilgan integral quyidagi ko‘rinishga keladi

$$\int_0^1 x\sqrt{1+x^2}dx=\int_1^{\sqrt{2}}\sqrt{t^2-1}\cdot t\cdot\frac{t}{\sqrt{t^2-1}}dt=\int_1^{\sqrt{2}}t^2dt$$

Nyuton-Leybnis formulasiga ko‘ra

$$\int_1^{\sqrt{2}}t^2dt=\frac{t^3}{3}\Bigg|_1^{\sqrt{2}}=\frac{2\sqrt{2}-1}{3}$$

bo‘lib,

$$\int_0^1 x\sqrt{1+x^2}dx=\frac{2\sqrt{2}-1}{3}$$

bo‘ladi.▶

3) Aniq integrallarni bo‘laklab integrallash usuli yordamida hisoblash.

Agar $f(x)$ va $g(x)$ funksiyalar $[a,b]$ segmentda uzluksiz hamda uzluksiz $f'(x)$ va $g'(x)$ hosilalarga ega bo‘lsa,

$$\int_a^b f'(x)\cdot g(x)dx=f(x)\cdot g(x)\Bigg|_a^b-\int_a^b f(x)\cdot g'(x)dx \qquad (4)$$

bo‘ladi. (4) formula aniq integrallarda **bo‘laklab integrallash formulasi** deyilib, uni quyidagicha

$$\int_a^b f(x)dg(x)=f(x)\cdot g(x)\Bigg|_a^b-\int_a^b g(x)df(x)$$

ham yozish mumkin.

4-misol. Ushbu

$$\int_1^2 x\log_2 xdx$$

integralni hisoblang.

◀Bu integralni bo‘laklab integrallash usulidan foydalanib hisoblaymiz. Berilgan integral ostidagi $x\log_2 xdx$ ifodada

$$u=\log_2 x,\quad dv=xdx$$

deymiz. Unda

$$du=\frac{1}{x\ln 2}dx,\quad v==\frac{x^2}{2}$$

bo‘lib, bo‘laklab integrallash formulasiga ko‘ra

$$\int_1^2 x\log_2 xdx=\frac{1}{2}x^2\log_2 x\Big|_1^2-\int_1^2\frac{xdx}{2\ln 2}.$$

bo‘ladi. Ravshanki,

$$\frac{1}{2}x^2\log_2 x\Big|_1^2=\frac{1}{2}\cdot 4\log_2 2-\frac{1}{2}\cdot 1\log_2 1=2,$$

$$\int_1^2\frac{xdx}{2\ln 2}=\frac{1}{2\ln 2}\cdot\frac{x^2}{2}\Big|_1^2=\frac{1}{2\ln 2}\left(\frac{4}{2}-\frac{1}{2}\right)=\frac{3}{4\ln 2}.$$

Demak,

$$\int_1^2 x\log_2 xdx=2-\frac{3}{4\ln 2}.$$ ▶

Nyuton-Leybnis formulasidan foydalanib quyidagi integrallarni toping:

177. a) $\int_1^4 x^2dx$, b) $\int_0^{36}\sqrt{x}dx$.

178. a) $\int_1^4 \frac{1}{x^2}dx$, b) $\int_{\frac{1}{4}}^{\frac{1}{3}}\frac{2}{x^4}dx$.

179. a) $\int_1^8 \frac{1}{\sqrt[3]{x}}dx$, b) $\int_1^3\frac{|x|}{x}dx$.

180. a) $\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\cos xdx$, b) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\frac{1}{\sin^2 x}dx$.

181. a) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{1}{\cos^2 x}dx$, b) $\int_0^{\frac{\pi}{2}}\sin xdx$.

182. a) $\int_1^5\frac{1}{3x-2}dx$, b) $\int_2^3\frac{x}{x^2+1}dx$.

183. a) $\int_{-1}^0 e^{-2x}dx$, b) $\int_0^4 e^{\frac{x}{4}}dx$.

184. a) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}}\cos 2xdx$, b) $\int_0^{\frac{\pi}{2}}\sin 4xdx$.

185. a) $\int\limits_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{1}{\sin^2 \frac{x}{2}} dx$, b) $\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{4}} tgx dx$. **186.** a) $\int\limits_{5}^{5\sqrt{3}} \frac{1}{25+x^2} dx$, b) $\int\limits_{0}^{1} \frac{1}{(2x+1)^3} dx$.

Integralning xossalari hamda Nyuton – Leybnis formulasidan foydalanib quyidagi integrallarni hisoblang:

187. $\int\limits_{-1}^{2} (3x^2+4x-1)dx$. **188.** $\int\limits_{1}^{4} (2x-\frac{9}{2}\sqrt{x}-\frac{8}{x^2})dx$.

189. $\int\limits_{0}^{1} \frac{x-1}{x+1} dx$. **200.** $\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{4}} ctg^2 x dx$.

201. $\int\limits_{0}^{\frac{\pi}{4}} (e^{-x}-tg^2 x)dx$. **202.** $\int\limits_{1}^{2} \frac{e^x}{e^x-1} dx$.

203. $\int\limits_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{1}{\cos^2 x \cdot \sin^2 x} dx$. **204.** $\int\limits_{0}^{1} \frac{x^3}{x^8+1} dx$.

205. $\int\limits_{1}^{e} \frac{\sin(\ln x)}{x} dx$. **206.** $\int\limits_{0}^{2} |1-x| dx$

O‘zgaruvchilarini almashtirish usulidan foydalanib, quyidagi aniq integrallarni hisoblang:

207. a) $\int\limits_{-2}^{5} \sqrt[3]{5x+2} dx$, b) $\int\limits_{1}^{4} \frac{1}{(1+2x)^2} dx$.

208. a) $\int\limits_{0}^{1} (x^3-1)^2 \cdot x^2 dx$, b) $\int\limits_{-1}^{0} \frac{x^2}{1-4x^3} dx$.

209. a) $\int\limits_{-0,5}^{0,5} \frac{3^x}{1+9^x} dx$, b) $\int\limits_{\sqrt{\frac{\pi}{3}}}^{\frac{\sqrt{\pi}}{2}} \frac{8x}{\sin^2 x^2} dx$.

210. a) $\int_1^3 \frac{\sqrt{x}}{x+1}dx$, b) $\int_3^8 \frac{x}{\sqrt{1+x}}dx$.

211. a) $\int_1^{e^3} \frac{1}{x\sqrt{1+\ln x}}dx$, b) $\int_0^{\frac{\pi}{4}} \frac{1}{(\sin x+\cos x)^2}dx$.

212. $\int_0^1 \frac{x^2}{(x+1)^4}dx$ $(t=x+1)$.

213. $\int_0^{\ln 2} \sqrt{e^x-1}dx$ $(t=\sqrt{e^x-1})$.

214. $\int_{\sqrt{3}}^{\sqrt{7}} \frac{x^3}{\sqrt[3]{(x^2+1)^2}}dx$ $(t=x^2+1)$.

215. $\int_1^e \frac{\sqrt[4]{1+\ln x}}{x}dx$ $(t=1+\ln x)$.

216. $\int_{-3}^{3} x^2\sqrt{9-x^2}dx$ $(x=3\cos t)$.

217. $\int_0^1 x\sqrt{1+x^2}dx$ $(t=\sqrt{1+x^2})$.

218. $\int_0^1 (1-\sqrt[3]{x^2})^{\frac{3}{2}}dx$ $(x=\sin^3 t)$.

219. $\int_0^a x^2\sqrt{\frac{a-x}{a+x}}dx$ $(x=a\cos t)$.

220. $\int_a^{2a} \frac{dx}{\sqrt{x^2+a^2}}$ $(x=atgt)$.

221. $\int_0^{2a} \sqrt{2ax-x^2}dx$ $(x=2a\sin^2 t)$.

222. $\int_{\ln 2}^{\ln 3} \frac{dx}{e^x-e^{-x}}$ $(t=e^x)$.

223. $\int_0^2 \sqrt{4-x^2}dx$ $\left(x=2\sin t\right)$.

224. $\int_0^1 \frac{dx}{(1+x)\sqrt{x}}$ $\left(x=t^2\right)$.

225. $\int_0^3 \frac{2xdx}{\sqrt{16+x^2}}$.

226. $\int_{2\sqrt{2}}^{4} x\sqrt{x^2-7}dx$.

227. $\int_{\frac{\pi}{2}}^{\pi} \frac{\sin xdx}{(1-\cos x)^2}$.

228. $\int_0^{\frac{\pi}{3}} \frac{\sin xdx}{\cos^4 x}$.

Bo‘laklab integrallash usulidan foydalanib, quyidagi aniq integrallarni hisoblang:

229. $\int_0^{\pi} x\cos xdx$.

230. $\int_0^{\pi} x\sin xdx$.

231. $\int_0^5 xe^x dx$.

232. $\int_1^e \ln xdx$.

233. $\int_0^{2\pi} x\sin 2xdx$.

234. $\int_0^1 (x-1)e^{-x}dx$.

235. $\int_1^{3\sqrt{e}} x^2 \ln x dx$.   **236.** $\int_0^1 arctgxdx$.   **237.** $\int_1^{e^4} \sqrt{x} \ln x dx$.

238. $\int_{-1}^0 \arccos dx$.   **239.** $\int_0^1 xarctgxdx$.   **240.** $\int_0^2 (3-2x)e^{-3x} dx$.

241. $\int_0^1 x^3 arctgxdx$.   **242.** $\int_0^{\pi} e^x \sin 2x dx$.   **243.** $\int_0^1 x \ln(1+x^2) dx$.

244. $\int_0^{\pi} x^3 \sin x dx$.

Quyidagi aniq integrallarni hisoblang:

245. $\int_9^{16} \frac{3^{\sqrt{x}}-1}{2\sqrt{x}} dx$.   **246.** $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\cos^2 x}{\sin^6 x} dx$.   **247.** $\int_{-5}^{-1} \frac{dx}{x^2+6x+13}$.

248. $\int_1^2 \frac{dx}{\sqrt{3+2x-x^2}}$.   **249.** $\int_0^{\frac{\pi}{4}} \frac{2+\sqrt{tgx}}{\cos^2 x} dx$.   **250.** $\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \sin 7x \cos x dx$.

251. $\int_1^2 \frac{\sqrt{x}dx}{x+3}$.   **252.** $\int_{\ln 2}^{2\ln 2} \sqrt{e^x-1} dx$.

253. Ushbu $\int_0^{\frac{\pi}{2}} \cos^2\left(\frac{\pi}{6}-x\right) dx$ integralni hisoblang.

A) $\pi - \frac{\sqrt{3}+1}{2}$,   B) $\frac{1}{4}\left(\pi + \frac{\sqrt{3}}{2}\right)$,   C) $\frac{1}{4}\left(\pi + \frac{\sqrt{3}+1}{2}\right)$,   D) $\frac{1}{4}\left(\pi - \frac{\sqrt{3}+1}{2}\right)$.

254. Ushbu

$$\int_0^1 \frac{4arctgx - x}{1+x^2} dx$$

integralni hisoblang.

A) $\frac{\pi^2}{8}$,   B) $\frac{\pi^2}{8} - \frac{1}{2}\ln 2$,   C) $\frac{\pi}{4} + \frac{1}{2}\ln 2$,   D) $\frac{\pi^2}{8} + \frac{1}{2}\ln 2$.

255. Ushbu

$$\int_0^{\frac{\pi}{4}} tg^3 x dx$$

integralni hisoblang.

A) $\frac{1-\ln 2}{2}$, B) $\frac{1+\ln 2}{2}$, C) $\frac{1-\ln 2}{4}$, D) $\frac{1+\ln 2}{4}$.

256. Ushbu

$$\int_0^1 \sqrt{1+x}dx$$

integralni hisoblang.

A) $\frac{1}{3}\left(\sqrt{8}-1\right)$, B) $\frac{2}{3}\left(\sqrt{8}+1\right)$, C) $\frac{2}{3}\left(\sqrt{8}-1\right)$, D) $\frac{3}{4}\left(\sqrt{8}-1\right)$.

257. Agar

$$a=\int_0^1 4^{-x}dx, \quad b=\int_0^1 5^{-x}dx$$

bo‘lsa,

$$1)\,a=b,\ \ 2)\,a>b,\ \ 3)\,a<b,\ \ 4)\,a\ge b.$$

munosabatlardan qaysi biri o‘rinli?

A) 2, B) 1, C) 4, D) 3.

2.2. Aniq integrallarni taqribiy hisoblash

Ko‘pincha $\int_a^b f(x)dx$ integral mavjud bo‘lib, uni hisoblash ancha qiyin bo‘ladi. Bunday holda integralni taqribiy hisoblashga to‘g‘ri keladi.

2.2.1. To‘g‘ri to‘rtburchaklar formulasi.

$$\int_a^b f(x)dx \approx \frac{b-a}{n}\sum_{k=1}^{n-1} f(x_k) \qquad (1)$$

taqribiy formula o‘rinli, bunda

$$x_k = a + k \cdot \frac{b-a}{n} \qquad (k = 0,1,2,...,n-1)$$

(1) taqribiy formulaning xatoligi

$$R_1 = \frac{(b-a)^3}{24n^2} f'(c) \quad (c \in (a,b))$$

bo‘ladi.

2.2.2. Trapetsiyalar formulasi.

$$\int_a^b f(x)dx \approx \frac{b-a}{n} \sum_{k=1}^{n-1} \frac{f(x_k) + f(x_{k+1})}{2} \qquad (2)$$

taqribiy formula o‘rinli. Uning xatoligi

bo‘ladi.

2.2.3. Parabolalar (Simpson) formulasi.

( ) —— 1 ( ) ( ) ( ) (3)

taqribiy formula o‘rinli bo‘lib, uning xatoligi

bo‘ladi.

1- misol. Ushbu

integralni taqribiy hisoblang.

◀ segmentni 10 ta teng bo‘lakka bo‘lamiz. Bo‘linish nuqtalari qiymati bilan taqqoslab, trapetsiyalar formulasi yordamida topilgan integralning qiymati aniqroq ekanligini ko‘ramiz.►

Quyidagi aniq integrallarni taqribiy hisoblang:

258. $\int_1^7 \frac{e^x}{x}dx$ ($[1,7]$ oraliq $n=6$ ta teng bo‘linib, Simpson formulasidan foydalanilsin).

259. $\int_0^1 \sqrt{1+x^2}dx$ ($[0,1]$ oraliq $n=10$ ta teng bo‘lakka bo‘linib, trapetsiya formulasidan foydalanilsin).

260. $\int_0^1 e^{-x^2}dx$ ($[0,1]$ oraliq $n=5$ ta teng bo‘lakka bo‘linib, to‘g‘ri to‘rtburchak formulasidan foydalanilsin).

261. $\int_{\frac{\pi}{2}}^{\frac{13\pi}{10}} \frac{\cos x}{x}dx$ ($\left[\frac{\pi}{2}, \frac{13\pi}{10}\right]$ oraliq $n=8$ ta teng bo‘lakka bo‘linib, Simpson formulasidan foydalanilsin).

262. Integrallash oralig‘ini $n=10$ ta teng bo‘lakka bo‘lib, Simpson formulasidan foydalanib, quyidagi integrallarni taqribiy hisoblang:

263. $\int_2^3 \frac{dx}{\ln x}$ **264.** $\int_0^{\frac{\pi}{4}} \sin(x^2)dx$

Nazorat savollari.

1. Aniq integralga ta'rif bering.
2. Aniq integralning xossalarini keltring.
3. Nyuton-Leybnis formulasini izohlab bering.
4. Aniq integrallarni o‘zgaruvchini almashtirish usuli bilan hisoblashni izohlab bering.
5. Aniq integrallarni bo‘laklab, integrallash usuli yordamida hisoblashni izohlab bering.
6. Aniq integrallarni taqribiy hisoblashni izohlab bering.
7. To‘g‘ri to‘rtburchak formulasi yordamida hisoblashni izohlab bering.
8. Trapetsiyalar formulasi yordamida hisoblashni izohlab bering.
9. Parabolalar (Simpson) formulasi yordamida hisoblashni izohlab bering.

3 bob

Aniq integralning ba'zi- bir tatbiqlari. Xosmas integrallar

3.1. Aniq integralning geometrik masalalarni yechishga tatbiqlari

Aniq integral yordamida ko‘pgina masalalar yechiladi. Jumladan, tekis shaklning yuzini yoy uzunligini, aylanma sirtning yuzini, statik momentlar hamda og‘irlik markazlarini topish masalalari hal etiladi.

3.1.1. Tekis shaklning yuzini hisoblash. $f(x)$ funksiya $[a,b]$ da uzluksiz bo‘lib, ixtiyoriy $x\in[a,b]$ da $f(x)\geq 0$ bo‘lsin.

Yuqoridan $f(x)$ funksiya grafigi, yon tomonlardan $x=a$, $x=b$ vertikal chiziqlar hamda pastdan abssissa o‘qi bilan chegeralangan (S) shakl – egri chiziqli trapetsiyaning yuzi

$$S=\int_a^b f(x)dx \tag{1}$$

bo‘ladi.

Aytaylik, $f_1(x)$ va $f_2(x)$ funksiyalar $[a,b]$ da uzluksiz bo‘lib, $x\in[a,b]$ da

$$0\leq f_1(x)\leq f_2(x)$$

bo‘lsin.

Yuqorida $f_2(x)$ funksiyaning grafigi, pastdan $f_1(x)$ funksiya grafigi, yon tomonlardan $x=a$, $x=b$ vertikal chiziqlar bilan chegaralangan shakilning yuzi

$$S=\int_a^b [f_2(x)-f_1(x)]dx \tag{2}$$

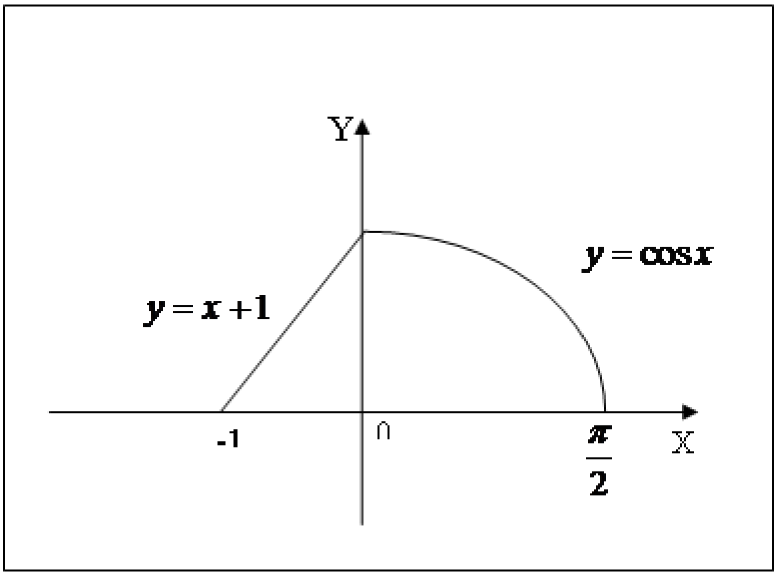
bo‘ladi.

1-misol. Ushbu

$$y=x+1,\qquad y=\cos x$$

chiziqlar hamda OX o‘qi bilan chegaralangan shaklning yuzini toping.

◄Bu shakl 1-chizmada tasvirlangan



1-chizma

Ravshanki,

$$f(x) = \begin{cases} x+1, & агар \quad -1 \le x \le 0, \\ \cos x, & агар \quad 0 \le x \le \dfrac{\pi}{2} \end{cases}$$

bo‘lib, u $\left[-1, \frac{\pi}{2}\right]$ oraliqda uzluksiz. Unda qaralayotgan shaklning yuzini (1) formulaga ko‘ra

$$S = \int_{-1}^{\frac{\pi}{2}} f(x)dx$$

bo‘ladi.

Endi aniq integralni hisoblaymiz

$$S = \int_{-1}^{\frac{\pi}{2}} f(x)dx = \int_{-1}^{0} (x+1)dx + \int_{0}^{\frac{\pi}{2}} \cos x dx = \frac{(x+1)^2}{2}\Bigg|_{-1}^{0} + \sin x \Bigg|_{0}^{\frac{\pi}{2}} = \frac{1}{2} + 1 = \frac{3}{2}.$$

Demak, izlanayotgan shaklning yuzini

$$S = \frac{3}{2}$$

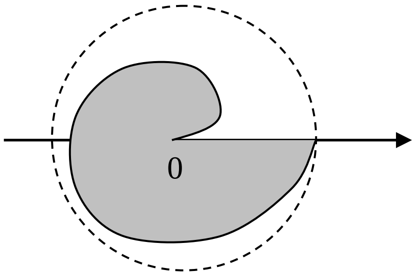
bo‘ladi.►

2-misol. Qutb o‘qi hamda ushbu

$$r = k\varphi$$

tenglama bilan berilgan Arximed spiralining birinchi aylanishdan hosil bo‘lgan chiziq bilan chegaralangan shaklning yuzini toping.

◀Bu shakl 2-chizmada tasvirlangan.



2-chizma.

Ravshanki, qaralayotgan hol uchun

$$0 \le \varphi \le 2\pi$$

bo‘ladi.

Bunday shaklning yuzini

$$S = \frac{1}{2}\int_{\alpha}^{\beta} r^2 d\varphi \tag{3}$$

formula yordamida topiladi. (3) formulaga ko‘ra

$$S = \frac{1}{2}\int_{0}^{2\pi} k^2\varphi^2 d\varphi = \frac{k^2}{2}\int_{0}^{2\pi} \varphi^2 d\varphi = \frac{k^2}{2}\left(\frac{\varphi^3}{3}\right)\Bigg|_{0}^{2\pi} = \frac{4}{3}k^2\pi^3 . \blacktriangleright$$

3.1.2. Yoy uzunligini hisoblash. $f(x)$ funksiya $[a,b]$ segmentda uzluksiz va uzluksiz $f'(x)$ hosilaga ega bo‘lib, uning grafigi $\overset{\cup}{AB}$ yoyni (egri chiziqni) tasvirlansin. Bu $\overset{\cup}{AB}$ yoyning uzunligi

$$l = \int_{a}^{b} \sqrt{1 + f'^2(x)}dx \tag{4}$$

bo‘ladi.

Aytaylik, $\overset{\cup}{AB}$ egri chiziq ushbu

$$\begin{cases} x = \varphi(t) \\ y = \psi(t) \end{cases}, \quad (\alpha \le t \le \beta)$$

tenglamalar sistemasi bilan (parametrik ko‘rinishda) berilgan bo‘lib, bunda $\varphi(t)$ va $\psi(t)$ funksiyalar $[\alpha,\beta]$ da uzluksiz $\varphi'(t)$ va $\psi'(t)$ hosilalarga ega bo‘lsin. Bu yoyning uzunligi

$$l=\int_{\alpha}^{\beta}\sqrt{\varphi'^2(t)+\psi'^2(t)}dt \tag{5}$$

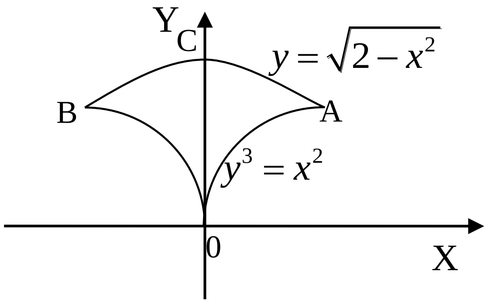
bo‘ladi

3-misol. Ushbu

$$y^3=x^2,\qquad y=\sqrt{2-x^2}$$

chiziqlar bilan chegaralangan shaklning perimetrini toping.

◀ Bu shakl 3-chizmada tasvirlangan.



3-chizma

Egri chiziq tenglamalarini sistema qilib, so‘ng uni echib, bu egri chiziqlarning kesishish nuqtalari $A=A(1,1)$, $B=B(-1,1)$ bo‘lishini topamiz.

Demak, izlanayotgan perimetr $\overset{\cup}{OA}$, $\overset{\cup}{AB}$, $\overset{\cup}{OB}$ yoylar uzunliklarinig yig‘indisiga teng bo‘ladi. $\overset{\cup}{OA}$ va $\overset{\cup}{OB}$ yoylarning uzunliklari bir-biriga teng bo‘lgani uchun perimetr

$$l=l_{\overset{\cup}{AB}}+2l_{\overset{\cup}{OA}}$$

bo‘ladi.

(4) formuladan foydalanib topamiz

$$l_{\overset{\cup}{AB}}=\int_{-1}^{1}\sqrt{1+\left(\sqrt{2-x^2}\right)'^2}dx=\int_{-1}^{1}\sqrt{1+\frac{x^2}{2-x^2}}dx=\sqrt{2}\int_{-1}^{1}\frac{dx}{\sqrt{2-x^2}}=\sqrt{2}\arcsin\frac{x}{\sqrt{2}}\Bigg|_{-1}^{1}=\frac{\pi}{2}\sqrt{2},$$

$$l_{\overset{\cup}{OA}}=\int_{0}^{1}\sqrt{1+\left(\frac{dx}{dy}\right)^2}dy=\int_{0}^{1}\sqrt{1+\frac{9}{4}y}dy=\frac{4}{9}\int_{0}^{1}\sqrt{1+\frac{9}{4}y}d\left(1+\frac{9}{4}y\right)=$$

$$=\frac{8}{27}\left(1+\frac{9}{4}y\right)^{\frac{3}{2}}\Bigg|_0^1=\frac{8}{27}\left(\frac{13\sqrt{13}}{8}-1\right)$$

Demak, perimetr

$$l=2\frac{13\sqrt{13}}{27}+\frac{\pi}{2}\sqrt{2}$$

bo‘ladi.►

4-misol. Ushbu

$$\begin{cases} x=e^t\sin t, \\ y=e^t\cos t, \qquad \left(0\le t\le \dfrac{\pi}{2}\right) \end{cases}$$

tenglamalar sistemasi bilan berilgan egri chiziqning uzunligini toping.

◄Bu egri chiziq uzunligini (5) formuladan foydalanib topamiz:

$$l=\int_0^{\frac{\pi}{2}}\sqrt{\left(e^t\sin t\right)'^2+\left(e^t\cos t\right)'^2}dt=\int_0^{\frac{\pi}{2}}\sqrt{e^{2t}\left[\left(\sin t+\cos t\right)^2-\left(\cos t-\sin t\right)^2\right]}dt=$$

$$=\int_0^{\frac{\pi}{2}}\sqrt{2}e^t dt=\sqrt{2}e^t\Bigg|_0^{\frac{\pi}{2}}=\sqrt{2}\left(e^{\frac{\pi}{2}}-1\right)$$

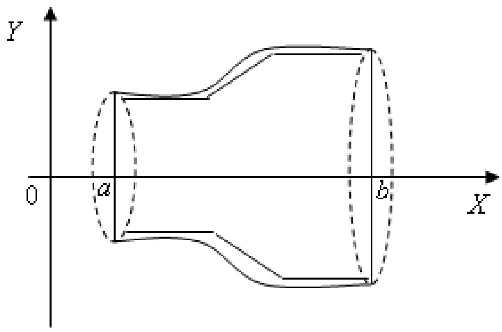
Demak,

$$l=\sqrt{2}\left(e^{\frac{\pi}{2}}-1\right).►$$

3.1.3. Aylanma sirtning yuzinini hisoblash. Aytaylik, $f(x)$ funksiya $[a,b]$ da uzluksiz bo‘lib, $\forall x\in[a,b]$ da $f(x)\ge 0$ bo‘lsin. Bu funksiya grafigining

$$\left(a, f(a)\right), \quad \left(b, f(b)\right)$$

nuqtalari orasidagi bo‘lagi $\breve{AB}$ yoyni OX o‘qi atrofida aylantirishdan hosil bo‘lgan sirtning - aylanma sirt yuzini (4-chizma)



4-chizma

$$S = 2\pi \int_a^b f(x) \cdot \sqrt{1 + f'^2(x)}dx \qquad (6)$$

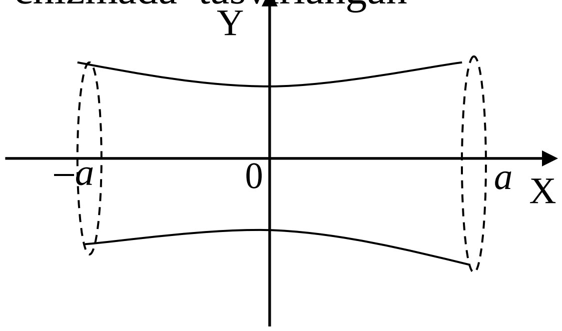
bo‘ladi.

5-misol. Ushbu

$$f(x) = \frac{a}{2}(e^{\frac{x}{a}} + e^{\frac{-x}{a}}), \quad a > 0, \ 0 \le x \le a$$

egri chiziqning (zanjir chizig‘inining) OX o‘qining $x_1 = -a, \quad x_2 = a \quad (a > 0)$ qismi atrofida aylantirishdan hosil bo‘lgan aylanma sirtning yuzini toping.

◀Bu sirt 5-chizmada tasvirlangan



5-chizma

Funksiyaning hosilasini hisoblaymiz

$$f'(x) = \frac{a}{2}\left(e^{\frac{x}{a}} \cdot \frac{1}{a} - e^{-\frac{x}{a}} \cdot \frac{1}{a} \right) = \frac{1}{2}(e^{\frac{x}{a}} - e^{\frac{-x}{a}}).$$

Qaralayotgan aylanma sirt OY koordinata o‘qiga nisbatan simmetrik bo‘lishini etiborga olib, (6) formuladan foydalanib, izlanayotgan aylanma sirtning yuzinini toping

$$S=4\pi\int_0^a \frac{a}{2}(e^{\frac{x}{a}}+e^{-\frac{x}{a}})\sqrt{1+\frac{1}{4}(e^{\frac{x}{a}}-e^{-\frac{x}{a}})^2}\,dx=\pi a\int_0^a (e^{\frac{x}{a}}+e^{-\frac{x}{a}})^2dx=$$

$$=\frac{\pi a^2}{2}\left[e^{\frac{2x}{a}}+2-e^{-\frac{2x}{a}}\right]_0^a=\frac{\pi a^2}{2}\left[e^2-e^{-2}+2a\right].\ ▶$$

3.1.4. Aylanma jismning hajmi. Aytaylik, $f(x)$ funksiya $[a,b]$ segmentda uzluksiz bo‘lib, u shu segmentda $f(x)\geq 0$ bo‘lsin.

Yuqoridan $f(x)$ funksiya grafigi, yon tomonlaridan $x=a$ va $x=b$ vertikal chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning OX va OY o‘qlari atrofida aylantirishdan hosil bo‘lgan jismning hajmi

$$V_x=\pi\int_a^b f^2(x)dx, \tag{7}$$

$$V_y=2\pi\int_a^b xf(x)dx, \tag{8}$$

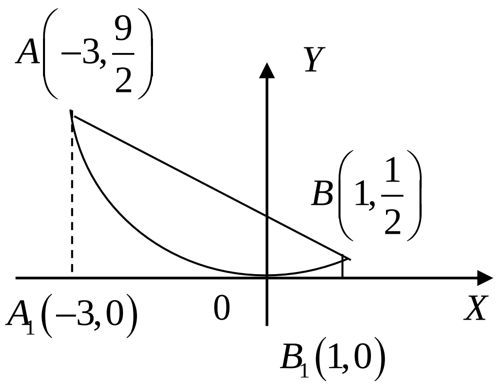
bo‘ladi.

6-misol. Ushbu

$$2y=x^2,\quad 2x+2y-3=0$$

chiziqlar bilan chegaralangan shaklni OX o‘qi atrofida aylantirishdan hosil bo‘lgan aylanma jismning hajmini toping.

◀Bu chiziqlar bilan chegarlangan shakl 1- chizmada tasvirlangan.



6- chizma

Bu shaklni OX o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmini

$$A_1ABB_1 \text{ va } A_1AOBB_1$$

egri chiziqli trapetsiyalarni OX o‘qi atrofida aylantirishdan hosil bo‘lgan jismlar hajmlari V_1 va V_2 larning ayirmasi deb qarash mumkin. Unda (7) formulaga ko‘ra

$$V_1 = \pi\int_a^b f^2(x)dx = \pi\int_{-3}^{1}(-1{,}5-x)^2 dx = \pi\int_{-3}^{1}(x-1{,}5)^2 d(x-1{,}5) = \frac{\pi(x-1{,}5)^3}{3}\Bigg|_{-3}^{1} = \frac{91}{3}\pi,$$

$$V_2 = \pi\int_{-3}^{1}\left(\frac{1}{2}x^2\right)^2 dx = \frac{\pi}{4}\int_{-3}^{1}x^4dx = \frac{\pi}{4}\cdot\frac{x^5}{5}\Bigg|_{-3}^{1} = \frac{\pi}{20}(1+243) = \frac{61}{\pi}$$

bo‘ladi. Demak, izlanayotgan hajm

$$V = V_1 - V_2 = \frac{91}{3}\pi - \frac{61}{5}\pi = 18\frac{2}{15}\pi$$

bo‘ladi. ►

3.1.5. Statik momentlar va og‘irlik markazlari koordinatalarini hisoblash.

Aytaylik, $\breve{AB}$ egri chiziq ($\breve{AB}$ yoyi)

$$y = f(x), \qquad a \le x \le b,$$

tenglama bilan aniqlangan bo‘lsin, bunda $f(x)$ funksiya $[a,b]$ da uzluksiz $f'(x)$ hosilaga ega. Bu egri chiziqning OX va OY koordinata o‘qlariga nisbatan statik momentlari ushbu

$$I_x = \int_a^b f(x)\cdot\sqrt{1+f'^2(x)}\,dx, \quad I_y = \int_a^b x\cdot\sqrt{1+f'^2(x)}\,dx, \tag{9}$$

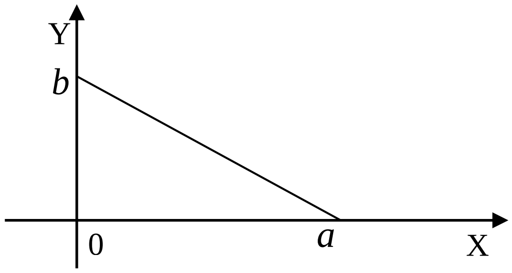
formulalar bilan topiladi.

7-misol. Ushbu

$$\frac{x}{a}+\frac{y}{b}=1, \quad x=0, \quad y=0$$

chiziqlar bilan chegaralangan uchburchakning OX va OY koordinata o‘qlariga nisbatan statik momentlarini toping.

◀Bu uchburchak 6-chizmada tasvirlangan



7-chizma

Ravshanki, $y = b\left(1 - \frac{x}{a}\right)$. Endi (9) formuladan foydalanib toping

$$I_x = \frac{b^2}{2}\int_0^a \left(1 - \frac{x}{a}\right)^2 dx = \frac{ab^2}{6}, \quad I_y = b\int_0^a x \cdot \left(1 - \frac{x}{a}\right) dx = \frac{a^2 b}{6}. ▶$$

Massali $\breve{AB}$ egri chiziqning og‘irlik markazi $C = C(x*, y*)$ ning koordinatalari quyidagi formulalar yordamida topiladi

$$x* = \frac{\int_a^b x \cdot \sqrt{1 + f'^2(x)}\, dx}{\int_a^b \sqrt{1 + f'^2(x)}\, dx}, \qquad y* = \frac{\int_a^b f(x) \cdot \sqrt{1 + f'^2(x)}\, dx}{\int_a^b \sqrt{1 + f'^2(x)}\, dx} \qquad (10)$$

8-misol. Ushbu

$$x^2 + y^2 = a^2, \quad y \geq 0$$

yarim aylana yoyning og‘irlik markazini toping.

◀Ravshanki,

$$y = \sqrt{a^2 - x^2}, \qquad y' = \frac{-x}{\sqrt{a^2 - x^2}}$$

bo‘ladi. Shularni e’tiborga olib toping:

$$\int_{-a}^{a} x\cdot\frac{a}{\sqrt{a^2-x^2}}dx=0,\qquad \int_{-a}^{a}\sqrt{a^2-x^2}\cdot\frac{a}{\sqrt{a^2-x^2}}dx=2a,$$

$$\int_{-a}^{a}\frac{a}{\sqrt{a^2-x^2}}dx=a\pi.$$

Demak, (10) formulaga ko‘ra og‘irlik markazi $C=C(x*,y*)$ koordinatalari

$$x^*=0,\qquad y^*=\frac{2}{\pi}a$$

bo‘ladi : $C=\left(0,\dfrac{2a}{\pi}\right)$.►

Quyidagi tekis shaklning yuzini toping:

265. Ushbu $x=0$, $x=2$, $y=0$, $y=x+3$ to‘g‘ri chiziqlar bilan chegaralangan tekis shaklning yuzini toping.

266. Ushbu $y=0$, $y=x^2$, $x=1$, $x=3$ chiziqlar bilan chegaralangan shaklning yuzini toping.

267. Ushbu $y=0$, $y=\dfrac{x^2}{2}$, $x=1$, $x=3$ chiziqlar bilan chegaralangan shaklning yuzini toping.

268. Ushbu $y=0$, $y=e^{-2x}$, $x=1$, $x=-\dfrac{1}{2}$ chiziqlar bilan chegaralangan shaklning yuzini toping.

269. Ushbu $y=0$, $y=\ln x$, $x=1$, $x=e$ chiziqlar bilan chegaralangan shaklning yuzini toping.

270. Ushbu $y=x$, $y=2-x^2$, chiziqlar bilan chegaralangan shaklning yuzini toping.

271. Ushbu $y^2 = x$, $y = x^2$, chiziqlar(parabolalar) bilan chegaralangan shaklning yuzini toping.

272. Ushbu $y = 0$, $y = 6x - 3x^2$, $x = 3$ chiziqlar bilan chegaralangan shaklning yuzini toping.

273. $x^2 + y^2 \le 36$ aylana hamda $x = 3, x = 3\sqrt{3}$ to‘g‘ri chiziqlar bilan chegaralangan shaklning yuzini toping.

274. Ushbu $y = 0$, $y = 4 - x$, $y = (x+1)^2$ chiziqlar bilan chegaralangan shaklning yuzini toping.

275. Ushbu $x = 0{,}1$, $x = 0$, $y = 0$, $y = |\ln x|$ chiziqlar bilan chegaralangan shaklning yuzini toping.

276. Ushbu $x = 0$, $y = 2$, $y = 2^x$ chiziqlar bilan chegaralangan shaklning yuzini toping.

277. $y = \dfrac{3}{x}$ giperboladan $x + y - 4 = 0$ to‘g‘ri chiziq ajrashgan qismining yuzini toping.

278. $y = \dfrac{x^2}{2}$, $y = \dfrac{1}{1+x^2}$ chiziqlar bilan chegaralangan shaklning yuzini toping.

279. Ushbu $\begin{cases} x = a\cos t, \\ y = b\sin t \end{cases}$ ellipsning yuzini toping.

280. Ushbu $\begin{cases} x = a\left(2\cos t - \cos 2t\right), \\ y = a\left(2\sin t - \sin 2t\right) \end{cases}$ chiziqlar bilan (kardioda bilan) chegaralangan shaklning yuzini toping.

281. Ushbu $r = a\sin 2\varphi$ chiziqlar bilan chegaralangan shaklning yuzini toping.

282. Ushbu $\begin{cases} x = a\cos^3 t, \\ y = a\sin^3 t \end{cases}$ $(0 \le t \le 2\pi)$ chiziq (astroida) bilan chegaralangan shakl yuzini toping.

Quyidagi egri chiziqlarning uzunligini toping:

283. Ushbu $y = \frac{2}{3}x^{\frac{3}{2}}$ egri chiziq (yarim kubik parabola)ning abssissa $x = 0$, $x = 8$ bo‘lgan nuqtalar orasidagi qismining uzunligini toping.

284. Ushbu $y = \frac{x^2}{4} - \frac{1}{2}\ln x$ egri chiziqning abssissalari $x = 1$, $x = e$ bo‘lgan nuqtalari orasidagi qismining uzunligini toping.

285. Ushbu $y^2 = (x = 1)^3$ egri chiziqning abssissalari $x = 4$ to‘g‘ri chiziqdagi ajrashgan qismi uzunligini toping.

286. Ushbu $y = 2\left(e^{\frac{x}{4}} + e^{-\frac{x}{4}}\right)$ egri chiziqning (zanjir chizig‘ining) abssissadagi $x = 0$, $x = 4$ bo‘lgan nuqtalari orasidagi qismining uzunligi toping.

287. Ushbu $y = \ln\sin x$ egri chiziqning abssissalari $x = \frac{\pi}{3}$, $x = \frac{2\pi}{3}$ bo‘lgan nuqtadagi orasidagi qismi uzunligini toping.

288. Ushbu $\begin{cases} x = e^t \sin t, \\ y = e^t \cos t \end{cases}$ $(0 \le t \le \frac{\pi}{4})$ egri chiziqning uzunligi toping.

289. Ushbu $\begin{cases} x = a(\cos t + t\sin t) \\ y = a(\sin t - t\cos t) \end{cases}$ $(0 \le t \le 2\pi)$ egri chiziqning uzunligi toping.

290. Ushbu $r = a(1 + \cos\varphi)$ egri chiziqning (kardiodaning) uzunligini toping.

Quyidagi aylanma sirtlarning hajmini toping.

291. OY o‘qi, $y=\frac{1}{2}x^2$ va $y=2\sqrt{2}$ chiziqlar bilan chegaralangan egri chiziqli trapetsiyaning OY o‘qi atrofiga aylantirishdan hosil bo‘lgan jismning hajmi toping.

292. $y=3x^2$ egri chiziqning (parabolani) $x=0$ va $x=2$ nuqtalar orasidagi qismini OX o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmi toping.

293. Ushbu $y=\sin x$ $(0\le x\le\pi)$ egri chiziqning OX o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmi toping.

294. Asoslarining radiuslari r va R, balandligi, h bo‘lgan kesik konusning OX o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmi toping.

295. Radiusi r ga teng bo‘lgan shar hajmi toping.

Quyidagi shaklning ko‘rsatilgan koordinatalar o‘qi atrofida aylantirishdan hosil bo‘lgan jismning hajmini toping

296. $y^2=2px$, $x=a$, OX o‘qi atrofida .

297. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, OY o‘qi atrofida .

298. $2y=x^2$, $2x+2y-3=0$, OX o‘qi atrofida.

299. $x=a\cos^3 t$, $y=a\sin^3 t$, OX o‘qi atrofida.

3.2. Aniq integralning mexanika, fizika, texnika va boshqa soha masalalarini yechishga tatbiqlari.

3.2.1. Jismning bosib o‘tgan yo‘li. Aytaylik, jism $v=v(t)$ tezlik bilan $[t_1,t_2]$vaqt oralig‘ida harakatlansin. Uning bosib o‘tgan yo‘li

$$S=\int_{t_1}^{t_2} v(t)\,dt \qquad (1)$$

bo‘ladi.

1-misol. Avtobus $1 \frac{м}{сек^2}$ tezlanish bilan harakatlana boshlagan bo‘lsa, u harakat boshlaganida 12 sekund o‘tganda qancha yo‘lni bosib o‘tgan?

◀Ravshanki, avtobusning tezligi

$$v(t)=t\,\frac{м}{c} \qquad \text{(chunki, } v'(t)=1\text{)}$$

bo‘ladi. Unda avtobusning $t_1=0, t_2=12\,сек$ vaqt oralig‘ida bosib o‘tgan yo‘li (1) formulaga ko‘ra

$$S=\int_0^{12} t\,dt=\left.\frac{t^2}{2}\right|_0^{12}=72\,м$$

bo‘ladi. ▶

3.2.2. O‘zgaruvchi kuchning bajargan ishi.

Aytaylik, jism $F=F(x)$ kuch ta'sirida OX o‘qi bo‘ylab $[a,b]$oraliqda harakatlansin. Uning bajargan ishi

$$A=\int_a^b F(x)\,dx \qquad (2)$$

bo‘ladi.

2-misol. Vintsimon prujinaning bir uchi mustahkamlangan, ikkinchi uchiga ega $F=F(x)$ kuch ta'sir etib, prujina qisilgan. Agar prujinaning qisilishi unga ta'sir etayotgan $F(x)$ kuchga proporsional bo‘lsa, prujinani a birlikka qisish uchun $F(x)$ kuchning bajargan ishini toping.

◀Agar $F(x)$ kuch ta'sirida prujinaning qisilish miqdorini x deyilsa, u holda

$$F(x)=k\cdot x$$

bo‘ladi, bunda $k-$proporsionallik koeffitsiyenti.

(2) formuladan foydalanib, bajarilgan ishni topamiz:

$$A=\int_0^a kx\,dx=k\cdot\left.\frac{x^2}{2}\right|_0^a=\frac{ka^2}{2}. ▶$$

3.2.3. Na'munani qizdirishga sarflanadigan issiqlik miqdori

$Q(t)$-qandaydir moddani t temperaturagacha qizdirish uchun sarflanadigan issiqlik miqdori.

$\Delta Q = Q(t+\Delta t)$ - $Q(t)$ - moddani t dan $t+\Delta t$ temperaturagacha qizdirsh uchun kerak bo‘ladigan issiqlik miqdori.

$\dfrac{\Delta Q}{\Delta t}$ - ning qiymati o‘rtacha issiqlik sig‘imi deyiladi.

$\lim\limits_{\Delta t \to 0} \dfrac{\Delta Q}{\Delta t} = \dfrac{dQ}{dt}$ - esa solishtirma issiqlik sig‘imi bo‘lib, $C_p(t)$ bilan belgilanadi.

$$\frac{dQ}{dt} = C_p(t) \text{ yoki } dQ = C_p(t)dt$$

3-misol. Agar temirning $0^\circ C$ dan $200^\circ C$ gacha bo‘lgan sohada issiqlik sig‘imi

$$C_p(t) = 0.1053 + 0.00142t$$

formula bilan ifodalansa, a kg temirni 20° dan $100^\circ C$ gacha qizdirish sarflanadigan issiqlik miqdorini aniqlang.

◀ $dQ = C_p(t)dt$ tenglamadan 1 kg temirni $20^\circ C$ dan $100^\circ C$ gacha qizdirish uchun sarflanadigan issiqlik miqdorini topamiz:

$$Q = \int_{20}^{100} (0.1053 + 0.000142t)dt = 0.1053t + 0.000071t^2 \Big|_{20}^{100} = 9.106$$

a kg temir uchun sarflangan issiqlik miqdori 9.106 a kkal.▶

Vodorodning yonish temperaturasi

4-misol. Agar vodorodning normal sharoitdagi yonish issiqligi 57,8 kkal$\cdot$mol^{-1} Vodorod kislorod aralashmasining solishtirma issiqlik sig‘imi C_p (kal/g.grad) ning temperaturaga bog‘liqligi.

$$C_p(d) = 0.375 + 5 \cdot 10^{-5}t$$

ligi aniq bo‘lsa, H_2 va O_2 ning stexiometrik aralashmasini yonishga erishadigan maksimal temperaturani hisoblang.

◀H_2 va kislorod aralashmasini yonishidan hosil bo‘lgan suv bug‘larining temperaturasi, reaksiyadan ajralgan issiqlikning hammasi mahsulotni qizdirishga sarflanganda maksimal bo‘ladi.

Suv bug‘larining temperaturasini dt ga o‘zgartirish uchun sarflanadigan issiqlik miqdori dQ larni quyidagicha ifodalaymiz

$$dQ = C_p(t)dt.$$

t_0 (boshlang‘ich temperatura normal sharoit $25^\circ C$) dan t gacha oraliqda integrallaymiz

$$Q = \int_{t_0}^{t} C_0(t)dt$$

bunda: $Q = 57{,}8 \cdot 10^3 / 18$ - aralashma yondirilganda ajraladigan issiqlik miqdori. Bundan

$$\frac{57{,}8 \cdot 10^3}{18} = 0{,}375(t - 25) + \frac{5 \cdot 10^{-5}}{2}\left(t^2 - 25^2\right)$$

yoki

$$45 \cdot 10^{-5} t^2 + 6{,}714t - 37968 = 0$$

Buning yechimini diskriminantdan topamiz. Tenglamaning yechimi yonish issiqligida erishishi mumkin bo‘lgan maksimal temperaturani aniqlaydi.

Amalda H_2 ning yonish issiqligi biz hisoblaganimizdan kamroq, chunki ajralgan issiqlikning ma'lum qismi gazlarning kengayishi bo‘yicha ishi sarflanadi.►

Buger-Lambert-Ber qonuni

Eritma qatlami orqali o‘tadigan yorug‘lik bog‘lamining intensivligi I_0, yutuvchi moddaning konsentratsiyasi c bo‘lsin.

Umumiy yorug‘lik oqimi yutilishi natijasida intensivligi kamayadi. Δx qalinlikdagi qatlam orqali o‘tayotgan yorug‘lik intensivligining kamayishi ΔI- bu qatlamning qalinligi, yutuvchi moddaning konsentratsiyasini va yorug‘lik intensivliklari ko‘paytmasiga proporsional bo‘ladi.

$$\Delta I = \alpha c \cdot \Delta x \cdot I$$

bu yerda $\alpha > 0$ - yutilish koeffitsiyenti.

Kyuvetaga kiradigan, yorug‘lik oqimi intensivligi I_0 undan chiqadigan yorug‘lik intensivligi oqimi I orasidagi bog‘liqlikdan optik yutilish xossasi yuzaga chiqadi, ya'ni

$$\Delta I ::= I(x + \Delta x) - I(x) < 0.$$

Bunda $\Delta I = \alpha c \cdot \Delta x \cdot I$ formuladan olingan $\alpha > 0$ ligidan quyidagi ifoda olinadi.

$$dI = -\alpha \cdot I \cdot c \cdot dx.$$

Agar eritma na'munasi gomogen va konsentratsiyasi c ni x ga bog‘liq emas deb hisoblasak ,oxirgi tenglamani quyidagicha yozish mumkin:

$$d(\ln I) = -\alpha c dx.$$

Bu tenglamani integrallasak bunda kyuveta qalinligi ℓ ga teng,

$$\int_{I_0}^{I(\ell)} d(\ln(I)) = -\int_0^{\ell} \alpha c dx$$

va

$$\ln I - \ln I_0 = -\alpha c\ell \quad \text{yoki} \quad \frac{I}{I_0} = e^{-\alpha c\ell}$$

dan

$$I(\ell) = I_0 \cdot \exp(-ac\ell) \qquad (*)$$

ni hosil qilamiz. Bu ifoda Buger-Lombert Ber qonunining analitik ifodasidir.

5-misol. Buger-Lamber-Ber qonuni ifodasini hisobga olib, optik zichlik $D = \ln\frac{I_0}{I}$ ni toping.

◀(*) tenglamani logarifmlasak

$$\ln I(\ell) = \ln I_0 - \alpha c\ell$$

yoki

$$\ln\frac{I_0}{I} = \alpha c\ell.$$

$D = \ln\frac{I_0}{I}$ (optik yutilish xarakteristkasi) ning qiymatini, quyidagicha oddiy ko‘rinishda ifodalash mumkin:

$$D = \varepsilon c\ell.$$

Bu yerda $\varepsilon = \alpha / 2{,}303$ ektinshor koeffitsiyenti.▶

Shakar inversiyasi

Shakarning inversiyasi – gidrolizlanish jarayoni bo‘lib, eritmaning qutblangan nur yozuvsini burishi orqali oson kuzatish mumkin. Δt oralig‘ida inversiyalangan shakarning miqdori eritmadagi miqdoriga proporsional bo‘ladi.

a - shakarning eritmadagi miqdori. Shuningdek t vaqt o‘tgandan momentdagi inversiyalangan miqdorini x deb belgilasak. Δt vaqt oralig‘ida Δx - shakarning inversiyalangan miqdori

$$\Delta x = k(a - x)t$$

bu yerda k - inversiyalanish jarayonining konstantasi.

Ushbu

$$dx = k(a + x)t \qquad (3)$$

tenglama shakarning differensial inversiallanish qonunini ifodalaydi.

6-misol. Inversiallanish jarayonining konstantasi k va shakarning dastlabki eritmadagi maqdor a berilgan bo‘lsin. t vaqt o‘tgandan keyin shakarning eritmadagi miqdorini toping.

◀(3) tenglamani quyidagi ko‘rinishda yozib olamiz:

$$\frac{dx}{a - x} = kdt \ .$$

Agar t 0 dan T gacha, x esa 0 dan x_1 gacha o‘zgarganligi hisobga olsak, hamda $t = 0$ da $x(0) = 0$ bo‘lishini hisobga olib integrallaymiz:

$$\int_0^{x_1} \frac{dx}{a - x} = k\int_0^T dt \, .$$

Natija quyidagicha bo‘ladi

$$-\ln(a - x)\Big|_0^{x_1} = kT,$$

$$\ln a - \ln(a - x) = kt \, .$$

Demak,

$$a - x_1 = a \cdot e^{-kt} \, .$$ ▶

Populyatsiya miqdori.

Vaqt o‘tishi bilan Populyatsiya miqdori o‘zgarib turadi. Agar Populyatsiya uchun sharoit yetarlicha yaxshi bo‘lsa, u holda tug‘ilish soni o‘lishga nisbatan ko‘p bo‘ladi. Populyatsiya tezligini $v = v(t)$ (birlik t vaqt ichida) bilan belgilaymiz. Eski Populyatsiya yashash joyida $v(t)$ tezlik kamayadi va asta-sekin nolga yaqinlashadi. Lekin Populyatsiyayosh bo‘lsa, o‘zaro bir-biri bilan bo‘lgan munosabatlar hali o‘rnatilmagan bo‘lsa yoki mavjud bo‘lgan bazi tashqi ta'sirlar bunga ta'sir qilsa, misol uchun insonning aralashuvi, u holda $v(t)$ tezlik sezilarli darajada ko‘payib yoki kamayib to‘radi.

Agar Populyatsiya tezligi $v(t)$ ma'lum bo‘lsa, u holda biz t_0 dan T vaqtgacha bo‘lgan oraliqda Populyatsiya miqdorining o‘sishini topa olamiz. Haqiqatdan ham t vaqt ichida $v(t)$ ta'rifidan, bu funksiya N(t) Populyatsiya miqdoridan olingan hosilaga teng va bundan kelib chiqadiki N(t) Populyatsiya miqdori $v(t)$ funksiyaning boshlang‘ich funksiyasi. shuning uchun

$$N(T) - N(t_0) = \int_{t_0}^{T} v(t)dt \qquad (4)$$

Ma'lumki juda yaxshi sharoitda Populyatsiya tezligining o‘sishi $v(t) = ae^{kt}$ bo‘ladi. Populyatsiya bu holda keksaymaydi. Bu holda (1) formuladan foydalanib, quyidagiga ega bo‘lamiz

$$N(T) = N(t_0) + a\int_{t_0}^{T} e^{kT} dt = N(t_0) + \frac{a}{k} e^{kt} \Big|_{t_0}^{T} = N(t_0) + \frac{a}{k}\left(e^{kT} - e^{kt_0}\right) \qquad (5)$$

Bu formula orqali ba'zi zamburug‘larning (penitsillin ajratib chiqaradigan) miqdorini hisoblab chiqarish mumkin.

Populyatsiya biomassasi.

Shunday biomassalarni qaraymizki hayoti davomida massasi sezilarli o‘zgaradi va shu Populyatsiyaumumiy biomassasini hisoblaymiz.

◄ Aytaylik, τ qaysidir birlik vaqtning o‘sishini aniqlasin, N(τ) o‘sishi τ ga teng populyatsiyaning maxsus miqdori. R(τ)-maxsus τ o‘sishidagi o‘rtacha massa, M(τ)- 0 dan τ o‘sishgacha bo‘lgan biomassa.

Shuni aniqlaymizki, N(τ)R(τ) ko‘paytma τ o‘sishdagi biomassaga teng. Ayirmani qaraylik,

$$M(\tau + \Delta\tau) - M(\tau)$$

Yuqoridagi ayirma τ dan $\tau + \Delta\tau$ o‘sishdagi biomassa, quyidagini qanoatlantiradi

$$N(\tau)P(\tau)\Delta\tau \le M(\tau + \Delta\tau) - M(\tau) \le$$

$$N\left(\check{\tau}\right)P\left(\check{\tau}\right)\Delta\tau \le M(\tau + \Delta\tau) - M(\tau) \le N\left(\hat{\tau}\right)P\left(\hat{\tau}\right)\Delta\tau$$

(6)

bu yerda $N\left(\overset{\vee}{\tau}\right)P\left(\overset{\vee}{\tau}\right)$-yetarlicha kichik, $N\left(\overset{\wedge}{\tau}\right)P\left(\overset{\wedge}{\tau}\right)$-yetarlicha katta miqdor ($[\tau,\tau+\Delta\tau]$ oraliqda $N\left(\overset{\wedge}{\tau}\right)P\left(\overset{\wedge}{\tau}\right)$funksiyaning). $\Delta\tau>0$ ni hisobga olib (6) tengsizlikdan quyidagiga kelamiz.

$$N\left(\overset{\vee}{\tau}\right)P\left(\overset{\vee}{\tau}\right)\le\frac{M(\tau+\Delta\tau)-M(\tau)}{\Delta\tau}\le N\left(\overset{\wedge}{\tau}\right)P\left(\overset{\wedge}{\tau}\right) \qquad (7),$$

N(τ)R(τ) funksiyalar uzluksizligidan (yani N(τ) va R(τ) larning uzluksiz bo‘lgani uchun) quyidagiga kelamiz.

$$\lim_{\Delta\tau\to 0}\left[N\left(\overset{\vee}{\tau}\right)P\left(\overset{\vee}{\tau}\right)\right]=\lim_{\Delta\tau\to 0}\left[N\left(\overset{\wedge}{\tau}\right)P\left(\overset{\wedge}{\tau}\right)\right]=N(\tau)P(\tau),$$

bundan $\lim\limits_{\Delta\tau\to 0}\frac{M(\tau+\Delta\tau)-M(\tau)}{\Delta\tau}=N(\tau)P(\tau)$ yoki $\frac{dM(\tau)}{d\tau}=N(\tau)P(\tau)$ kelib chiqadiki, M(τ) biomassa N(τ)R(τ) larning boshlang‘ich funksiyasi. Demak

$$M(T)-M(0)=\int_0^{\pi}N(\tau)P(\tau)d\tau,$$

bu yerda T-berilgan biomassaning maksimal yoshi. M(0) aniqki nolga teng, u holda

$$M(T)=\int_0^{\pi}N(\tau)P(\tau)d\tau$$

bu ifoda orqali populyatsiyaning umumiy biomassasini hisoblaymiz.►

Quyidagi masalalarni yeching.

300. Ushbu

$$y=\frac{1}{3}(3-x)\cdot\sqrt{x},\quad 0\le x\le 3$$

egri chiziqning OX o‘qi atrofidan aylantirishda hosil bo‘lgan sirtning yuzini toping.

301. Ushbu

$$x^2+y^2=R^2,\quad a<x<R,\quad (0<a<R)$$

aylana yoyining OX o‘qi atrofidan aylantirishda hosil bo‘lgan sirtning yuzini toping.

302. Ushbu

$$y^2 = 2x, \quad a \le x \le 4$$

parabola yoyining OX o‘qi atrofidan aylantirishda hosil bo‘lgan sirtning yuzini toping.

303. Radiusi R ga teng bo‘lgan shar sirtini toping.

304. Ushbu

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

ellipsning OX o‘qi atrofidan aylantirishda hosil bo‘lgan sirtning yuzini toping.

305. Ushbu

$$y = \frac{1}{3}x^2, \quad -1 \le x \le 1$$

egri chiziqning OX o‘qi atrofida aylantirishdan hosil bo‘lgan sirtning yuzini toping.

306. Ushbu

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = R^{\frac{2}{3}}$$

egri chiziqni (astroidani) OX o‘qi atrofida aylantirishdan hosil bo‘lgan sirtning yuzini toping.

307. Agar egri chiziq ushbu

$$\begin{cases} x = x(t), \\ y = y(t) \end{cases} \quad ,\alpha \le t \le \beta$$

tenglamalar sistemasi yordamida parametrik ko‘rinishda berilgan bo‘lsa, uni OX o‘qi atrofida aylantirishdan hosil bo‘lgan sirtning yuzini

$$S = 2\pi \int_{\alpha}^{\beta} y(t)\sqrt{x'(t)^2 + \left(y'(t)\right)^2}\,dt$$

bo‘lishini isbotlang.

308. Ushbu

$$\begin{matrix} x = \dfrac{t^3}{3}, & \\ & , \quad 0 \le t \le 2\sqrt{2} \\ y = 4 - \dfrac{t^2}{2} & \end{matrix}$$

egri chiziqni OX o‘qi atrofida aylantirishdan hosil bo‘lgan sirtning yuzini toping.

309. Ushbu

$$\begin{cases} x = R(t - \sin t), \\ & 0 \le t \le 2\pi \\ y = R(1 - \cos t) \end{cases}$$

egri chiziqni (sikloidani) OX o‘qi atrofida aylantirishdan hosil bo‘lgan sirtning yuzini toping.

310. Ushbu

$$\sqrt{x} + \sqrt{y} = \sqrt{a}$$

parabola va OX, OY o‘qlari bilan chegaralangan bir jinsli shaklning (plastinkaning) og‘irlik markazining koordinatalarini toping.

311. Ushbu

$$x^2 + y^2 = a^2$$

bir jinsli doiraning parabola va OX o‘qining yuqorisida joylashgan qismining og‘irlik markazini toping.

312. Ushbu

$$y = x^2, \ y = \sqrt{x}$$

chiziqlar bilan chegaralangan bir jinsli shaklning og‘irlik markazini toping.

313. Ushbu

$$ax = y^2, \ ay = x^2 \quad (x > 0)$$

parabolalar bilan chegaralangan bir jinsli shaklning (plastinkaning) og‘irlik markazini toping.

314. Ushbu

$$\begin{cases} x = a(t - \sin t), \\ & 0 \le t \le 2\pi \\ y = a(1 - \cos t) \end{cases}$$

chiziq (sikloida) bilan chegaralangan shaklning og‘irlik markazini toping.

315. Ushbu

$$\frac{x}{a} + \frac{y}{b} = 1, \quad x = 0, \ y = 0$$

chiziqlar bilan chegaralangan uchburchak shaklning OX va OY koordinata o‘qlariga nisbatan statik momentlarini toping.

316. Ushbu

$$x = a(t - \sin t), \qquad 0 \le t \le 2\pi$$
$$y = a(1 - \cos t)$$

egri chiziq (sikloida) bilan chegaralangan shaklning (plastinkaning) OX o‘qiga nisbatan statik momentlarini toping.

3**17.** Ushbu

$$x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$$

egri chiziqning (astroidaning) OX va OY koordinata o‘qlariga nisbatan statik momentlarini toping.

3.3. Xosmas integrallar.

3.3.1. Chegaralari cheksiz (cheksiz oraliq bo‘yicha) integrallar. Aytaylik, $f(x)$ funksiya $[a,+\infty)$ oraliqda (cheksiz oraliqda) uzluksiz bo‘lsin. Ravshanki, bu funksiya ixtiyoriy $[a,t]$ oraliq ($(a < t < +\infty)$chekli oraliq) bo‘yicha integralanuvchi bo‘ladi.

Ushbu

$$\lim_{t\to+\infty}\int_a^t f(x)dx \tag{1}$$

limit mavjud bo‘lsa, bu limit $f(x)$ funksiyaning $[a,+\infty)$ oraliq bo‘yich (chegarasi cheksiz) xosmas integrali deyiladi va

$$\int_a^{+\infty} f(x)dx$$

kabi belgilanadi

$$\int_a^{+\infty} f(x)dx = \lim_{t\to+\infty}\int_a^t f(x)dx. \tag{2}$$

Agar (1) limit chekli bo‘lsa, $\int_a^{+\infty} f(x)dx$ xosmas integral yaqinlashuvchi aks holda esa xosmas integral uzoqlashuvchi deyiladi.

Shunga o‘xshash $f(x)$ funksiya $(-\infty,a]$ oraliqda uzluksiz bo‘lganda

$$\int_{-\infty}^{a} f(x)dx = \lim_{t\to-\infty}\int_{t}^{a} f(x)dx \quad ,$$

xosmas integral, $f(x)$ funksiya $(-\infty,+\infty)$ da uzluksiz bo‘lganda

$$\int_{-\infty}^{+\infty} f(x)dx = \int_{-\infty}^{a} f(x)dx + \int_{a}^{+\infty} f(x)dx = \lim_{t\to-\infty}\int_{t}^{a} f(x)dx + \lim_{t\to+\infty}\int_{a}^{t} f(x)dx$$

xosmas integrallar ta’riflanadi.

1-misol. Ushbu

$$\int_{a}^{+\infty}\frac{dx}{x^{\alpha}} \qquad (a>0)$$

xosmas integralni yaqinlashuvchilikka tekshiring.

◀Aytaylik, $\alpha\neq 1$ bo‘lsin. Bu holda

$$\int_{a}^{+\infty}\frac{dx}{x^{\alpha}} = \lim_{t\to+\infty}\int_{a}^{t} x^{-\alpha}dx = \lim_{t\to+\infty}\frac{x^{-\alpha+1}}{-\alpha+1}\bigg|_{a}^{t} = \lim_{t\to+\infty}\frac{1}{1-\alpha}\left(\frac{1}{t^{\alpha-1}}-\frac{1}{a^{\alpha-1}}\right)$$

bo‘ladi, $\alpha>1$ bo‘lganda

$$\int_{a}^{+\infty}\frac{dx}{x^{\alpha}} = \frac{1}{\alpha-1}\cdot\frac{1}{a^{\alpha-1}}$$

bo‘ladi, $\alpha<1$ bo‘lganda

$$\int_{a}^{+\infty}\frac{dx}{x^{\alpha}} = +\infty$$

bo‘ladi. Demak, berilgan integral $\alpha>1$ da yaqinlashuvchi, $\alpha<1$ bo‘lganda uzoqlashuvchi.

Aytaylik, $\alpha=1$ bo‘lsin. Bu holda

$$\int_{a}^{+\infty}\frac{dx}{x} = \lim_{t\to+\infty}\int_{a}^{t}\frac{dx}{x} = \lim_{t\to+\infty}(\ln t-\ln a) = +\infty$$

bo‘lib, berilgan integral uzoqlashuvchi bo‘ladi.▶

3.3.2. Yaqinlashuvchi xosmas integrallarning xossalari.

Xosmas integrallar yaqinlashuvchiligini aniqlashda quyidagi tasdiqlardan foydalaniladi:

1) agar $[a,+\infty)$ da $f(x)$ va $g(x)$ funksiyalar uzluksiz bo‘lib,

$$0\le f(x)\le g(x)$$

bo‘lsa, u holda $\int_a^{+\infty} g(x)dx$ integralning yaqinlashuvchi bo‘lishidan

$\int_a^{+\infty} f(x)dx$

integralning yaqinlashuvchi bo‘lishi kelib chiqadi.

Endi $[a,+\infty)$ oraliqda uzluksiz bo‘lgan ixtiyoriy funksiyani qaraymiz. Bu funksiya yordamida tuzilgan

$$|f(x)|$$

funksiya $[a,+\infty)$da manfiy bo‘lmaydi: $|f(x)|\geq 0$

2) agar $\int_a^{+\infty} |f(x)|dx$ integral yaqinlashuvchi bo‘lsa, u holda

$\int_a^{+\infty} f(x)dx$

xosmas integral ham yaqinlashuvchi bo‘ladi.

Bu holda $\int_a^{+\infty} f(x)dx$ xosmas integral absolyut yaqinlashuvchi integral deyiladi.

2-misol. Ushbu

$$\int_0^{+\infty} e^{-x^2}dx$$

xosmas integralni yaqinlashuvchilikka tekshiring..

◄Avvalo , berilgan xosmas integralni quyidagicha yozib olamiz

$$\int_0^{+\infty} e^{-x^2}dx = \int_0^{1} e^{-x^2}dx + \int_1^{+\infty} e^{-x^2}dx.$$

Ravshanki, $\int_0^{1} e^{-x^2}dx$ yaqinlashuvchi. Demak $\int_1^{+\infty} e^{-x^2}dx$ ni yaqinlashuvchi bo‘lishini ko‘rsatish yetarli. Bu integral yaqinlashuvchi bo‘ladi, chunki

$x\geq 1$ bo‘lganda

$$e^{-x^2}\leq e^{-x} \tag{3}$$

bo‘lib,

$$\int_1^{+\infty} e^{-x}dx = \lim_{t\to+\infty}\int_1^{t} e^{-x}dx = \lim_{t\to+\infty}\left(-e^{-t}+e^{-1}\right) = e^{-1}.$$

bo‘lgani uchun $\int_{1}^{+\infty} e^{-x}dx$ integral yaqinlashuvchi bo‘ladi. Keltrilgan tasdiqqa ko‘ra

$$\int_{0}^{+\infty} e^{-x^2}dx$$

integral yaqinlashuvchi bo‘ladi.▶

3.3.3. Xosmas integrallarni hisoblash.

Aytaylik, $f(x)$ funksiya $[a,+\infty)$ oraliqda uzluksiz bo‘lib, uning xosmas integrali

$$\int_{a}^{+\infty} f(x)dx \tag{4}$$

yaqinlashuvchi bo‘lsin.

Ma’lumki, bu holda $f(x)$ funksiya $[a,+\infty)$ oraliqda boshlang‘ich funksiyaga ega bo‘ladi. Uni $F(x)$ bilan belgilaylik, $(F'(x) = f(x))$.

Agar

$$\lim_{t\to+\infty} F(t) = F(+\infty)$$

deyilsa, unda

$$\int_{a}^{+\infty} f(x)dx = F(+\infty) - F(a) = F(x)\Big|_{a}^{+\infty}. \tag{5}$$

bo‘ladi.

Xosmas integrallar (5) formula bilan shungdek o‘zgaruvchilarni almashtrish hamda bo‘laklab itegrallash usullari yordamida hisoblanadi.

3-misol. Ushbu

$$\int_{0}^{2} \frac{x^3dx}{\sqrt{4-x^2}}$$

xosmas integralni hisoblang.

◀Berilgan integralda $x = 2\sin t$ almashtrish bajaramiz. Unda

$dx = 2\cos dt,$ $x = 0$ da $t = 0$, $x = 2$ da $t = \frac{\pi}{2}$

bo‘lib,

$$\int_0^2 \frac{x^3dx}{\sqrt{4-x^2}} = 8\int_0^{\frac{\pi}{2}} \frac{\sin^3 t\cos t}{\cos t}dt = 8\int_0^{\frac{\pi}{2}} \sin^3 dt$$

bo‘ladi. Keyingi integralni hisoblaymiz

$$\int_0^{\frac{\pi}{2}} \sin^3 dt = -\int_0^{\frac{\pi}{2}} \left(1-\cos^2 t\right)d\left(\cos t\right) = -\left(\cos t - \frac{1}{3}\cos^3 t\right)\Bigg|_0^{\frac{\pi}{2}} = \frac{2}{3}.$$

Demak,

$$\int_0^2 \frac{x^3dx}{\sqrt{4-x^2}} = \frac{2}{3}. ►$$

4-misol. Ushbu

$$\int_0^{+\infty} \frac{dx}{a^2+x^2}, \qquad (a>0)$$

xosmas integral yaqinlashuvchilikka tekshiring va qiymatini hisoblang.

◄Bu xosmas integralni , ta’rifga ko‘ra, yaqinlashuvchilikka tekshirib, qiymatini hisoblaymiz

$$\int_0^t \frac{dx}{a^2+x^2} = \frac{1}{a}\int_0^t \frac{d\left(\frac{x}{a}\right)}{1+\left(\frac{x}{a}\right)^2} = \frac{1}{a}arctg\frac{x}{a}\Bigg|_0^t = \frac{1}{a}arctg\frac{t}{a} - \frac{1}{a}arctg0 = \frac{1}{a}arctg\frac{t}{a}$$

bo‘lib,

$$\lim_{t\to+\infty}\int_0^t \frac{dx}{a^2+x^2} = \lim_{t\to+\infty}\frac{1}{a}arctg\frac{t}{a} = \frac{\pi}{2a}$$

bo‘ladi. Demak, berilgan xosmas integral yaqinlashuvchi va

$$\int_0^{+\infty} \frac{dx}{a^2+x^2} = \frac{\pi}{2a}$$

ga teng.►

3.3.4. Chegaralanmagan funksiyaning xosmas integrallari.

Faraz qilaylik, $f(x)$ funksiya $[a,b)$ da uzluksiz bo‘lsin. Bu funksiya $x\to b-0$ da cheksizga intilsin $\lim\limits_{x\to b-0} f(x)=\infty$.

Demak, $f(x)$ funksiya $[a,b)$ da chegaralanmagan (aniqrog‘i, $f(x)$ funksiya b nuqta atrofida chegaralanmagan).

Ushbu

$$\lim_{t\to b-0}\int_a^t f(x)dx$$

limit mavjud bo‘lsa, bu limit chegaralanmagan $f(x)$ funksiyaning $[a,b)$ bo‘yicha xosmas integrali deyiladi va

$$\int_a^b f(x)dx = \lim_{t\to b-0}\int_a^t f(x)dx \qquad (6)$$

kabi belgilanadi.

Agar (6) limit mavjud va chekli bo‘lsa, $\int_a^b f(x)dx$ xosmas integral yaqinlashuvchi deyiladi.

Agar (6) limit cheksiz yoki mavjud bo‘lmasa, $\int_a^b f(x)dx$ xosmas integral uzoqlashuvchi deyiladi.

Aytaylik, $f(x)$ funksiya $(a,b]$ da uzluksiz bo‘lib, $x\to a+0$ da cheksizga intilsin. Ravshanki, bu funksiyaning $[t,b]$ oraliq $(a<t<b)$ bo‘yicha integrali mavjud bo‘ladi. Bu holda chegaralanmagan funksiyaning xosmas integrali quyidagicha ta’riflanadi

$$\int_a^b f(x)dx = \lim_{t\to a+0}\int_t^b f(x)dx. \qquad (7)$$

Agar (7) limiti mavjud va chekli bo‘lsa, $\int_a^b f(x)dx$ xosmas integral yaqinlashuvchi deyiladi, cheksiz yoki mavjud bo‘lmasa, $\int_a^b f(x)dx$ xosmas integral uzoqlashuvchi deyiladi.

Shunga o‘xshash (a,b) hol uchun

$$\int_a^b f(x)dx = \lim_{\substack{t\to a+0\\ u\to b-0}}\int_t^u f(x)dx$$

bo‘ladi.

Masalan,

$$J_1=\int_a^b \frac{dx}{(x-a)^{\alpha}} \quad , \quad (\alpha>0)$$

chegaralanmagan funksiyaning xosmas integrali $\alpha<1$ da yaqinlashuvchi, $\alpha\ge 1$ da uzoqlashuvchi bo‘ladi, chunki

$$\lim_{t\to a+0}\int_t^b \frac{dx}{(x-a)^{\alpha}} = \lim_{t\to a+0}\int_t^b (x-a)^{-\alpha}d(x-a) = \lim_{t\to a+0}\frac{(x-a)^{-\alpha+1}}{-\alpha+1}\Bigg|_t^b =$$

$$= \frac{1}{1-\alpha}\lim_{t\to a+0}[(b-a)^{1-\alpha}-(t-a)^{1-\alpha}],$$

bo‘lib, $\alpha<1$ bo‘lganda

$$\lim_{t\to a+0}\int_t^b \frac{dx}{(x-a)^{\alpha}} = \frac{1}{(1-\alpha\cdot)(b-a)^{\alpha-1}},$$

berilgan xosmas integral yaqinlashuvchi, $\alpha>1$ bo‘lganda

$$\lim_{t\to a+0}\int_t^b \frac{dx}{(x-a)^{\alpha}} = +\infty \quad ,$$

berilgan xosmas integral uzoqlashuvchi bo‘ladi.

Aytaylik, $\alpha=1$ bo‘lsin. Bu holda

$$\lim_{t\to a+0}\int_t^b \frac{dx}{x-a} = \lim_{t\to a+0}(\ln(x-a))\Bigg|_t^b = \lim_{t\to a+0}\left[\ln(b-a)-\ln(t-a)\right] = +\infty$$

bo‘ladi. Demak, berilgan xosmas integral uzoqlashuvchi.

Shunday qilib,

$$J_1 = \int_a^b \frac{dx}{(x-a)^{\alpha}}$$

xosmas integral $0<\alpha<1$ bo‘lganda yaqinlashuvchi, $\alpha\geq 1$ bo‘lganda uzoqlashuvchi bo‘ladi.

Quyidagi chegaralari cheksiz xosmas integrallarni hisoblang:

318 . a) $\int_0^{+\infty}\frac{dx}{1+x^2}$, b) $\int_0^{+\infty} e^{-x}dx$. **319**. a) $\int_1^{+\infty}\frac{1}{x^4dx}$, b) $\int_0^{+\infty}\frac{1}{x^2+9}dx$.

320. a) $\int_0^{+\infty} e^{-5x}dx$, b) $\int_{-\infty}^{+\infty}\frac{dx}{1+x^2}$. **321.** a) $\int_{-\infty}^{0} xe^{x}dx$, b) $\int_0^{+\infty} x^2e^{-\frac{x}{2}}dx$.

322. a) $\int_1^{+\infty}\frac{dx}{x+x^2}$, b) $\int_1^{+\infty}\frac{arctgx}{1+x^2}dx$. **323.** a) $\int_{-\infty}^{+\infty}\frac{dx}{x^2+2x+5}$, b) $\int_0^{+\infty} xe^{-x^2}dx$.

324. a) $\int_{-\infty}^{+\infty}\frac{dx}{x^2+4x+9}$, b) $\int_2^{+\infty}\frac{dx}{(x^2-1)^2}$.

Quyidagi chegaralari heksiz xosmas integrallarni yaqinlashuvchilikka tekshiring:

325. $\int_0^{+\infty}\sqrt{x}\cdot e^{-x}dx$. **326.** $\int_0^{+\infty} x\cos x dx$. **327.** $\int_2^{+\infty}\frac{1}{x^2\ln x}dx$.

328. $\int_2^{+\infty}\frac{1}{x\ln x}dx$. **329.** $\int_e^{+\infty}\frac{1}{x\ln^2 x}dx$. **330.** $\int_0^{+\infty} x\sin x dx$.

331. $\int_{1}^{+\infty}\frac{1}{\sqrt{x}}dx$. **332.** $\int_{-\infty}^{+\infty}\frac{1}{x^2+2x+2}dx$. **333.** $\int_{0}^{+\infty}\frac{\cos x}{1+x^2}dx$.

334. $\int_{1}^{+\infty}\frac{1}{\sqrt{x^3+1}}dx$.

Quyidagi chegaralanmagan funksiyalarning xosmas integrallari hisoblang:

335. a) $\int_{0}^{1}\frac{1}{\sqrt{x}}dx$, b) $\int_{0}^{1}\frac{1}{\sqrt{1-x^2}}dx$. **336**. a) $\int_{-1}^{1}\frac{1}{\sqrt[3]{x^2}}dx$, b) $\int_{0}^{1}\frac{1}{\sqrt{x\sqrt{x}}}dx$.

337. a) $\int_{0}^{e}\frac{1}{x\ln^2 x}dx$, b) $\int_{-1}^{1}\frac{3x^2+2}{\sqrt[3]{x^2}}dx$. **338.** a) $\int_{0}^{2}\frac{1}{\sqrt[3]{(x-1)^2}}dx$, b) $\int_{0}^{1}\frac{1}{\sqrt{x(1-x)}}dx$.

Quyidagi chegaralanmagan funksiyalarning xosmas integrallari yaqinlashuvchilikka tekshiring:

339. $\int_{0}^{a}\frac{1}{\sqrt{a^2-x^2}}dx$ $(a>0)$, **340.** $\int_{0}^{\frac{\pi}{2}}\frac{1}{1-\cos x}dx$.

341. $\int_{0}^{\frac{1}{2}}\frac{1}{x\ln x}dx$. **342.** $\int_{0}^{\pi}\frac{1-\cos x}{\sqrt{x}}dx$.

343. $\int_{0}^{1}\frac{1}{e^x-\cos x}dx$. **344.** $\int_{-\frac{1}{2}}^{1}\frac{1}{\sqrt{1-x^2}}dx$.

Nazorat savollari.

1. Tekis shakl yuzini hisoblashni izohlab bering.
2. Yoy uzunligini hisoblashga misol keltring.
3. Aylanma sirtning yuzinini hisoblashni izohlab bering.
4. Statik momentni hisoblashga misol keltiring.
5. Og‘irlik markazini hisoblashni izohlab bering.
6. Aylanma sirtning hajmini hisoblashga misol keltring.
7. Jismning bosib o‘tgan yo‘lini hisoblashga misol keltring.
8. O‘zgaruvchi kuchning bajargan ishini hisoblashni izohlab bering.
9. Suyuqlik bosimining kuchini hisoblashni izohlab bering.
10. Chegaralari cheksiz (cheksiz oraliq bo‘yicha) integrallarni izohlab bering.
11. Yaqinlashuvchi xosmas integralni izohlab bering.
12. Chegaralanmagan funksiyaning xosmas integrallarini izohlab bering.

JAVOBLAR

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26. Ko‘rsatma: $(2x+3=t)$. $\frac{1}{2}\ln|2x+3|+C$. **27.** Ko‘rsatma: $\left(x^2=t\right)$. $\frac{1}{3}e^{x^2}+C$. **28.** Ko‘rsatma: $(\sqrt{x-1}=t)$. $\frac{2}{5}(x-1)^{\frac{5}{2}}+\frac{2}{3}(x-1)^{\frac{3}{2}}+C$. **29.** Ko‘rsatma: $(\sqrt{x}=t)$. $2\sqrt{x}-2\ln(1+\sqrt{x})+C$. **30.** Ko‘rsatma: ($x=\frac{1}{t}$). $\frac{1}{\sqrt{2}}\arccos\frac{\sqrt{2}}{x}+C$. **31.** Ko‘rsatma: ($x=-\ln t$). $-\ln(1+e^{-x})+C$. **32.** Ko‘rsatma: ($5x^2-3=t$). $\frac{1}{80}(5x^2-3)^8+C$. **33.** Ko‘rsatma: ($e^x=t$). $\frac{1}{2}arctg\frac{e^x}{2}+C$. **34.** Ko‘rsatma: ($\cos x=t$). $-\cos x+\frac{1}{3}\cos^3 x+C$. **35.** Ko‘rsatma: ($\frac{x}{a}=t$) . $\arcsin\frac{x}{a}+C$. **36.** Ko‘rsatma: ($x+1=t^2$). $2[\sqrt{x+1}-\ln(1+\sqrt{x+1})]+C$. **37** Ko‘rsatma: ($x=\frac{1}{t}$) . $-\arcsin\frac{1}{x}+C$. **38.** Ko‘rsatma: ($x=tgt$). $\frac{1}{2}(arctgx-\frac{x}{x^2+1})+C$. **39.** Ko‘rsatma: ($tgx=t$). $\frac{1}{2}\ln|tgx|+C$. **40.** Ko‘rsatma: ($x=a\sin t$). $\frac{a}{2}\arcsin\frac{x}{a}+\frac{x}{2}\sqrt{a^2-x}+C$.

41. $\frac{(x+1)^{101}}{101}+C$. **42.** $\frac{(1+x^2)^8}{16}+C$. **44.** $\frac{1}{88}(8x+5)^{11}+C$. **45.** $-\frac{4}{21}\sqrt[4]{(2-3x)^7}+C$. **46.** $\frac{1}{4}[\frac{(2x+5)^{12}}{12}-\frac{5(2x+5)^{11}}{11}]+C$. **47.** $2(\frac{\sqrt{x^3}}{3}-\frac{x}{2}+2\sqrt{x}-2\ln|1+\sqrt{x}|)+C$. **48.** $-\frac{1}{2(e^x-5)^2}+C$. **49.** $\frac{1}{8}(e^{2x}+5)^4+C$. **50.** $\frac{1}{10}\ln|\frac{1+e^{5x}}{1-e^{5x}}|+C$. **51.** $\frac{1}{6}\sqrt{tg^3x}+C$. **52.** $\frac{5}{3}\sqrt[5]{tg^3x}+C$. **53.** $\frac{2}{5}(\cos^2 x-5)\sqrt{\cos x}+C$. **54.** $\ln x-\ln 2|\ln x+2\ln 2|+C$. **55.** $-\frac{x^2}{3}\sqrt{2-x^2}-\frac{4}{3}\sqrt{2-x^2}+C$. **56.** $\frac{1}{3}e^{arctg3x}+C$. **57.** $e^{tgx}+C$. **58.** $\frac{1}{4\cos^4 x}+C$. **59.** $2e^{\sqrt{x}}+C$. **60.** $\cos\frac{1}{x}+C$. **61.** $\frac{1}{2}\sin 2x-\frac{1}{3}\sin^3 2x+\frac{1}{10}\sin^5 2x+C$. **62.** $x\ln x-x+C$. **63.** $-x\cos x+\sin x+C$. **64.** $e^x(x^2 2x+2)+C$. **65.** $\frac{x^2}{2}\ln x-\frac{x^2}{4}+C$. **66.** $x^2\sin x+2x\cos x-2\sin x+C$. **67.** $\frac{x^2}{2}arctgx-\frac{x}{2}+\frac{1}{2}arctgx+C$. **68.** $-x^2\cos x+2x\sin x+2\cos x+C$. **69.** $\frac{3}{5}\sqrt[3]{x^5}\ln x-\frac{9}{25}\sqrt[3]{x^5}+C$. **70.** $e^{3x}(\frac{x^2}{3}-\frac{20}{9}x+\frac{38}{27})+C$. **71.** $(2x-3)\sin x+2\cos x+C$. **72.** $(2x-0{,}5)\cos 2x-\sin 2x+C$. **73.** $(x^2+1)arctgx-x+C$. **74.** $2\sqrt{x}\ln x-4\sqrt{x}+C$. **75.** $-\frac{\ln x}{x}-\frac{1}{x}+C$. **76.** $-xctgx+\ln|\sin x|+C$. **77.** $xe^{2x}+C$. **78.** $x\ln(1+x^2)-2x+2arctgx+C$. **79.**

$-\frac{x+1}{e^x}+C$. **80.** $x\ln^2 x-2x\ln x+2x+C$. **81.** $\frac{3^x}{\ln^2 x}[(x+2)\ln 3-1]+C$. **82.** $\frac{5}{16}\sqrt[5]{x^4}(4\ln x-5)+C$. **83.** $-\frac{x}{\sin x}+\ln|tg\frac{x}{2}|+C$. **84.** $\frac{x}{1+x}\ln x-\ln(1+x)+C$. **85.** $x(\arcsin)^2+2\sqrt{1-x^2}arctgx-2x+C$. **86.** $\frac{x}{2}[\sin(\ln x)-\cos(\ln x)]+C$. **87.** $-3e^{-\frac{x}{3}}(x^3+9x^2+54x+162)$. **88.** $\frac{1}{2}x\sqrt{a^2+x^2}+\frac{a^2}{2}\ln(x+\sqrt{a^2+x^2}+C$. **89.** $\frac{e^{\alpha x}}{\alpha^2+\beta^2}(\alpha\sin\beta x-\beta\cos\beta x)+C$. **90.** $v(t)=-gt+v_0,\ v_0-$boshlang‘ich tezligi. **91.** $S(t)=-\frac{1}{2}gt^2+v_0t+S_0,\ S_0=S(0)$. **92.** $t=\frac{2v_0}{g},\ v_0-$boshlang‘ich tezlik. **95.** $v_0=6\sqrt{g}\approx 18{,}8\frac{м}{сек^2}$. **112.** $\frac{1}{2}arctg\frac{x+1}{2}+C$. **113.** $\frac{2}{\sqrt{3}}arctg\frac{2x+1}{\sqrt{3}}+C$. **114.** $\frac{1}{2}arctg\frac{x+3}{2}+C$. **115.** $\frac{1}{3}\ln\left|\frac{x}{x+3}\right|+C$. **116.** $\frac{2}{\sqrt{\pi}}arctg\frac{6x+3}{\sqrt{\pi}}+C$. **117.** $\frac{1}{2}\ln\left|x^2+2x+5\right|-\frac{1}{2}arctg\frac{x+1}{2}+C$. **118.** $\frac{1}{2}\ln\left|x^2+x+1\right|-\frac{1}{\sqrt{3}}arctg\frac{2x+1}{\sqrt{3}}+C$. **122.** $-3\ln|x|+\frac{9}{2}\ln|x+1|-\frac{1}{2}\ln|x-1|+C$. **123.** $\frac{1}{x+1}+\ln\left|\frac{x}{x+1}\right|+C$. **124.** $\frac{1}{2}arctg(2x^2-1)+C$.

125. $\frac{1}{2}\ln|x+1|-\frac{1}{4}\ln(x^2+1)+\frac{1}{2}arctgx+C$. **126.** $\frac{1}{6}\ln\frac{(x+1)^2}{x^2-x+1}+\frac{1}{\sqrt{3}}arctg\frac{2x-1}{\sqrt{3}}+C$.

127. $\frac{1}{6}\ln\frac{(x-1)^2}{x^2+x+1}+\frac{1}{\sqrt{3}}arctg\frac{2x+1}{\sqrt{3}}+C$. **128.** $\frac{1}{4\sqrt{2}}\ln\frac{x^2+x\sqrt{2}+1}{x^2-x\sqrt{2}+1}+\frac{\sqrt{2}}{4}arctg\frac{x\sqrt{2}}{1-x^2}+C$.

129. $\frac{1}{3}\ln\frac{x+1}{x-1}+\frac{5}{12}\ln\frac{x-2}{x+2}+C$. **130.** $-\frac{1}{5(x-1)}+\frac{1}{10}\ln\frac{(x-1)^2}{x^2+2x+2}-\frac{8}{25}arctg(x+1)+C$. **131.** $\frac{1}{4}\ln\frac{x^2+x+1}{x^2-x+1}+\frac{1}{2\sqrt{3}}arctg\frac{x^2-1}{x\sqrt{3}}+C$. **132.** $2(\sqrt{x}-\ln(1+\sqrt{x})+C)$. **133.** $6\sqrt[6]{x}-6arctg\sqrt[6]{x}+C$. **134.** $\ln\left|\frac{\sqrt{x+1}-1}{\sqrt{x+1}+1}\right|+C$. **135.** $0{,}4(x^2-x-6)\sqrt{3-x}+C$. **136.** $\frac{3}{8}\sqrt[3]{(2x-3)^4}+C$. **137.** $\frac{2}{3}\sqrt{(x+1)^3}-4\sqrt{x+1}+C$. **138.** $\ln\left|x+\sqrt{x^2-2}\right|+C$. **139.** $-\frac{2}{3}\sqrt{(\frac{x+1}{x})^3}+C$. **140.** $-\frac{x\sqrt{1-x}}{2}+\frac{1}{2}\arcsin x+C$. **141.** $\frac{1}{4}\ln\frac{\sqrt[4]{x^{-4}+1}+1}{\sqrt[4]{x^{-4}+1}-1}-\frac{1}{2}arctg\sqrt[4]{x^{-4}+1}+C$. **142.** $\frac{4}{3}(\sqrt[4]{x^3}-\ln\left|\sqrt[4]{x^3}+1\right|+C)$. **143.** $\frac{x+2}{5}\sqrt[3]{(3x+2)^2}+C$. **145.** $\frac{x}{\sqrt[6]{1+x^6}}+C$. **146.** $\frac{1}{30}(3x^4-2)(1+x^4)\cdot\sqrt{1+x^4}+C$. **147.** $(x=t^6)$. $\frac{6}{5}\sqrt[6]{x^5}-4\sqrt{x}+18\sqrt[6]{x}-21arctg\sqrt[6]{x}+3\frac{\sqrt[6]{x}}{\sqrt[3]{x}+1}+C$. **148.** $\ln\left|x-3+\sqrt{x^2-6x+10}\right|+C$.

149. $\ln\left|x+\frac{1}{2}+\sqrt{x^2+x}\right|+C$. **150.** $\ln\left|x+2+\sqrt{x^2+4x+3}\right|+C$. **151.** $\frac{x-3}{2}\sqrt{x^2+2x+3}+C$.

152. $\frac{x}{2}+\frac{1}{20}\sin 10x+C$. **153.** $-\frac{1}{4}\cos^4 x+C$. **154.** $-\cos x+\frac{1}{3}\cos^3 x+C$. **155.** $\frac{\sin^2 x}{2}-\frac{1}{2\sin^2 x}-2\ln|\sin x|+C$. **156.** $tgx+\frac{1}{3}tg^3x+C$. **157.** $\cos x-2arctg(\cos x)+C$.

158. $\frac{1}{5}tg^5x-\frac{1}{3}tg^3x+tgx-x+C$. **159.** $\frac{1}{2}tgx+\frac{1}{2}\ln|tgx|+C$.

160. $\frac{x}{16}-\frac{\sin 4x}{64}-\frac{\sin^2 2x}{48}+C$. **161.** $\frac{1}{5}\ln\left|\frac{tg\frac{x}{2}-\frac{1}{2}}{tg\frac{x}{2}+2}\right|+C$. **162.** $-\frac{1}{4}\sin x+\frac{3}{16}\ln\left|\frac{2\sin x-1}{2\sin x+1}\right|+C$. **163.** $\frac{1}{\sqrt{3}}arctg(\frac{tgx}{\sqrt{3}})+C$. **164.** $\frac{1}{2}x+\frac{1}{2}\ln|\sin x+\cos x|+C$.

165. $\ln\left|1+tg\frac{x}{2}\right|+C$. **166.** $\frac{3}{5}\sin\frac{5x}{6}+3\sin\frac{x}{6}+C$. **167.** $-\frac{1}{12}\cos 6x-\frac{1}{4}\cos 2x+C$. **168.** $\frac{1}{2}\sin x-\frac{1}{22}\sin 11x+C$. **169.** $2(\frac{1}{7}\cos^2 x-\frac{1}{3})\cos x\cdot\sqrt{\cos x}+C$. **170.** $\frac{1}{\sqrt{114}}arctg(\sqrt{\frac{2}{7}}tgx)+C$.

171. $-\frac{1}{4}tg^2\frac{x}{2}+\frac{1}{2}\ln\left|tg\frac{x}{2}\right|+C$.

2- bob

177. a) 21, **b)** 144). **178. a)** $\frac{3}{4}$, **b)** $24\frac{2}{3}$. **179. a)** $4{,}5$, **b)** 2). **180. a)** $\frac{1}{2}$, **b)** 1). **181. a)** $\frac{2\sqrt{3}}{3}$, **b)** 1). **182. a)** $\frac{1}{3}\ln 13$, **b)** $\frac{1}{2}\ln 2$). **183. a)** $\frac{e^2-1}{2}$, **b)** $4(e-1)$. **184. a)** $-\frac{1}{2}$, **b)** 0). **185. a)** $2(\sqrt{3}-1)$, **b)** $\frac{1}{2}(\ln 3-\ln 2)$. **186. a)** $\frac{\pi}{60}$, **b)** $\frac{2}{9}$). **187.** 12. **188.** -12. **189.** $1-2\ln 2$. **190.** $\sqrt{3}-1-\frac{\pi}{12}$. **191.** $\frac{\pi}{4}-e^{-\frac{\pi}{4}}$. **192.** $\ln(e-1)$. **193.** $\frac{4}{3}\sqrt{3}$. **194.** $\frac{\pi}{6}$. **195.** $1-\cos 1$. **196.** 1. **197. a)** $9{,}75$, **b)** $\frac{1}{9}$.

198. a) $\frac{1}{9}$, **b)** $\frac{1}{12}\ln 5$. **199. a)** $\frac{\pi}{6\ln 3}$, **b)** $4(\sqrt{3}-1)$. **200. a)** $2\sqrt{3}-2-\frac{\pi}{6}$, **b)** $\frac{32}{3}$.

201. a) 2, **b)** $\frac{1}{2}$. **202.** $\frac{1}{24}$. **203.** $\frac{4-\pi}{2}$. **204.** 3. **205.** $0{,}8(2\sqrt[4]{2}-1)$. **206.** $\frac{81\pi}{8}$. **207.** $\frac{2\sqrt{2}-1}{3}$. **208.** $\frac{3\pi}{32}$. **209.** $(\frac{\pi}{4}-\frac{2}{3})a^3$. **210.** $\ln\frac{2+\sqrt{5}}{1+\sqrt{2}}$. **211.** $\frac{\pi a^2}{2}$.

212. $\frac{1}{2}\ln\frac{3}{2}$. **213.** π. **214.** $\frac{\pi}{2}$. **215.** 2. **216.** $8\frac{2}{3}$. **217.** $\frac{1}{2}$. **218.** $2\frac{1}{3}$.

219. -2. **220.** π. **221.** $4e^5+1$. **222.** 1. **223.** $-\pi$. **224.** $-e^{-1}$. **225.** $\frac{1}{9}$.

226. $\pi-1$. **227.** $\frac{4}{9}\left(5e^5+1\right)$. **228.** $\pi-1$. **229.** $\frac{\pi-2}{4}$. **230.** $\frac{5e^{-6}+7}{9}$. **231.**

$\frac{1}{6}$. **232.** $-\frac{2(e^{\pi}-1)}{\ln(e)^2+4}$. **233.** $\ln 2-\frac{1}{2}$. **234.** $\pi^3-6\pi$. **235.** $\frac{2}{35}$. **236.** $\frac{8}{15}$. **237.** $\frac{\pi}{4}$. **238.** $\frac{\pi}{6}$. **239.** $\frac{8}{3}$. **240.** 0. **241.** $\frac{1}{3}\left(12-\pi\sqrt{3}\right)$. **242.** $\frac{1}{6}\left(12\sqrt{3}-12-\pi\right)$. **248.** $\approx 189{,}403$. **249.** $\approx 1{,}148$. **250.** $\approx 0{,}748$. **251.** -0.77. **253.** ≈ 1.118. **254.** $\approx 0{,}157$.

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255. 8. **256.** $8\frac{2}{3}$. **257.** $4\frac{1}{3}$. **258.** $\frac{e^3-1}{2e^2}$. **259.** 1. **260.** $\frac{9}{2}$. **261.** $\frac{1}{3}$. **262.** 8. **263.** 6π. **264.** $10\frac{2}{3}$. **265.** $9{,}9-8{,}1\lg e$. **266.** $2-\frac{1}{\ln 2}$. **267.** $4-\ln 3$. **268.** $\frac{3\pi-2}{6}$. **269.** πab. **270.** $a^2\left(\frac{\pi}{3}+\frac{\sqrt{3}}{2}\right)$. **271.** $\frac{\pi a^2}{4}$. **272.** $\frac{3}{8}\pi a^3$. **273.** $17\frac{1}{3}$. **274.** $\frac{1}{4}\left(e^2+1\right)\approx 2{,}1$. **275.** $\frac{670}{27}$. **276.** $2\left(e-\frac{1}{e}\right)$. **277.** $\ln 3$. **278.** $\sqrt{2}\left(e^{\frac{x}{2}}-1\right)$. **279.** $6a$. **280.** $16a$. **281.** 8π. **282.** $57{,}6\pi$. **283.** $\frac{1}{2}\pi^2$. **284.** $\frac{\pi h}{3}\left(r^2+rR+R^2\right)$. **285** Ko'rsatma; sharni $y=\sqrt{r^2-x^2}$ $(-r\le x\le r)$ yarim doirani OX o'qi atrofida aylantirishdan hosil bo'lgan jism deb qaraladi. $\frac{4}{3}\pi r^3$. **286.** πpa^2. **287.** $\frac{4}{3}\pi a^2 b$. **288.** $18\frac{2}{15}\pi$. **289.** $\frac{32}{105}\pi a^3$. **290.** 3π. **291.** $2\pi R(R-a)$. **292.** $\frac{52}{3}\pi$. **293.** $4\pi R^2$. **295.** $\frac{2}{9}\pi\left(2\sqrt{2}-1\right)$. **296.** $\frac{12}{5}\pi R^2$. **298.** $\frac{133}{5}\pi$. **299.** $\frac{64}{3}\pi R^2$. **300.** $x_c=\frac{a}{5},\ y_c=\frac{a}{5}$. **301.** $0,\ y_c=\frac{4a}{3\pi}$. **302.** $x_c=\frac{9}{20},\ y_c=\frac{9}{20}$. **303.** $x_c=\frac{9}{20}a,\ y_c=\frac{9}{20}a$. **304.** $x_c=\pi a,\ y_c=\frac{5}{6}a$. **305.** $M_x=\frac{ab^2}{6},\ M_y=\frac{a^2b}{6}$. **306.** $\frac{5}{2}\pi a^3$. **307.** $M_x=\frac{3}{5}a,\quad M_y=\frac{3}{5}a$. **308.** **a)** $\frac{\pi}{2}$, **b)** 1). **309.** **a)** $\frac{1}{3}$, **b)** $\frac{\pi}{6}$). **310.** **a)** $\frac{1}{5}$, **b)** π). **311.** **a)** -1, **b)** 16). **312.** **a)** $\frac{1}{2}\ln 2$, **b)** $\frac{\pi^2}{8}$. **313.** **a)** π, **b)** $\frac{1}{2}$). **314.** **a)** $\frac{\pi}{\sqrt{5}}$, **b)** $\frac{1}{3}+\frac{1}{4}\ln 3$. **315.** Yaqinlashuvchi. **316.** Uzoqlashuvchi. **317.** Yaqinlashuvchi. **318.** Uzoqlashuvchi. **319**. Yaqinlashuvchi. **320.** Uzoqlashuvchi. **321**. Uzoqlashuvchi. **322**. Yaqinlashuvchi. **323**. Yaqinlashuvchi. **324**. Yaqinlashuvchi. **325.** **a)** 2, **b)** $\frac{\pi}{2}$. **326.** **a)** 6, **b)** 4. **327.** **a)** $2\sqrt{\ln 2}$, **b)** $14\frac{4}{7}$. **328.** **a)** 6, **b)** π. **329.** Yaqinlashuvchi. **330**. Uzoqlashuvchi. **331.**

Uzoqlashuvchi. **332.** Yaqinlashuvchi. **333.** Uzoqlashuvchi**.** **334.** Yaqinlashuvchi**.**

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MUNDARIJA

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OLIY MATEMATIKA

BIR O‘ZGARUVCHILI FUNKSIYANING INTEGRAL HISOBI VA ULARNING TATBIQLARIGA DOIR MASALALAR TO‘PLAMI
USLUBIY QO‘LLANMA

Muharrir:Sidikova K.A.