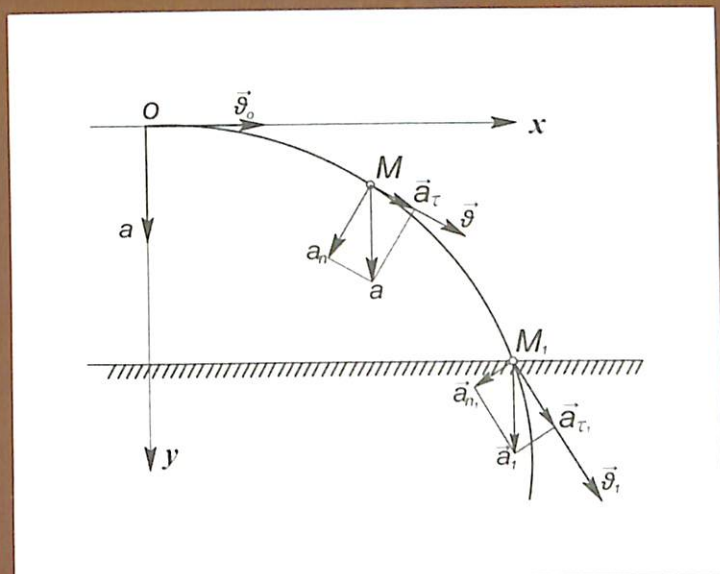


K.KENJAYEV

NAZARIY MEXANIKA MISOL VA MASALALARDA

2-qism

KINEMATIKA



Toshkent - 2020

Kitob quyidagi ko'rsatilgan
muddatda topshirilishi shart

Oldingi foydalanishlar
miqdori

26407

va masalalarda

O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS
TA'LIM VAZIRLIGI

TOSHKENT ARHITEKTURA QURILISH INSTITUTI

K. KENJAYEV

NAZARIY MEXANIKA
MISOL VA MASALALARDA

II-qism

KINEMATIKA

“Shafoat Nur Fayz”
Toshkent 2020

47-§	Ko'chirma harakat ilgarilanma harakat bo'lmagan holda nuqtaning absolyut tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar.	155
48-§	Ko'chirma harakat ilgarilanma harakat bo'lmagan hol uchun nuqtaning absolyut tezlanishini aniqlashga doir masalalar.	156
49-§	Talabalarga mustaqil o'rganish uchun tavsiya etiladigan muammo.	170
50-§	Talabalar tomonidan mustaqil bajariladigan masalalar variantlari (keys, hisob chizma ishlari).	173
IV-bob	Qattiq jismning tekislikka parallel harakati.	184
51-§	Tekis shakl harakatini qutb bilan birgalikda oniy ilgarilanma va qutb atrofida oniy aylanma harakatlarga ajratish	184
52-§	Qattiq jismning tekislikka parallel harakati tenglamalari	185
53-§	Tekis shaklning burchak tezligi va burchak tezlanishi.	186
54-§	Tekis shaklning harakat tenglamalari, tekis shakl nuqtasining harakat tenglamalari, tekis shakl burchak tezligi va burchak tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar.	187
55-§	Tekis shaklning harakat tenglamalari, burchak tezligi va burchak tezlanishini aniqlashga doir masalalar.	189
56-§	Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar.	194
57-§	Tekis shakl nuqtasining tezligini qutb usulida aniqlash.	196
58-§	Tekis shakl ikki nuqtasi tezliklarining proeksiyalariga oid teorema.	198
59-§	Tezliklarning oniy markazi.	199

MUNDARIJA

SO'Z BOSHI.	8
I-bob Nuqta kinematikasi.	9
1-§ Kinematikaning asosiy tushunchalari.	9
2-§ Moddiy nuqta harakatining berilish usullari.	10
3-§ Nuqtaning tezligi	13
4-§ Nuqtaning tezlanishi	17
5-§ Nuqta harakatining hususiy hollari.	23
6-§ Nuqta harakatining tenglamalari va traektoriyasini yechish uchun uslubiy ko'rsatmalar.	26
7-§ Nuqta harakatining tenglamalari, traektoriyasini aniqlashga doir masalalar	28
8-§ Mustaqil o'rganish uchun talabalarga tavsiya etiladigan masalalar.	35
9-§ Nuqtaning tezligini aniqlashga oir masalalarni yechish uchun uslubiy ko'rsatmalar	36
10-§ Nuqtaning tezligini aniqlashga doir masalalar	38
11-§ Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar	43
12-§ Nuqtaning tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar	45
13-§ Nuqtaning tezlanishini aniqlashga doir masalalar.	47
14-§ Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar.	55
15-§ Talabalar mustaqil o'rganishi uchun keyslar.	61
II-bob Qattiq jismning ilgarilanma va qo'zg'almas o'q atrofida aylanma harakati.	64
16-§ Qattiq jismning ilgarilanma harakati	64
17-§ Qattiq jismning ilgarilanma harakatiga doir masalalarni yechish uchun uslubiy ko'rsatmalar.	67
18-§ Qattiq jismning ilgarilanma harakatiga doir masalalar.	68



SO'Z BOSHI

Nazariy mexanikaning kinematika bo'limida moddiy nuqtaning harakati harakatni yuzaga keltiruvchi sabablarga bog'lanmagan holda o'rganiladi. Shuning uchun odatda kinematika "harakat geometriyasi" ham deyiladi.

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Qurilish yo'nalishlarida ta'lim oluvchi talabalar uchun turli xil mashinalarni tashkil etuvchi mexanizmlarning kinematik va dinamik xususiyatlarni o'rganish, harakat qonunlarini keltirib chiqarish muhim vazifa hisoblanadi. Buning uchun talabalar kursning nazariy asoslarini chuqur o'rganishi va amaliy masalalarni yechish malakalariga ega bo'lishlari lozim.

O'quv qo'llanmani tuzishda: Engineering mechanics statics. J.L. Meriam, L.G. Kraige 2007, Statics and Dynamics. R.C.Hibbeler 2013, Theoretical mechanics. Vasile Szolga 2010, Engineering mechanics. R.S. Khurmi 2011, xorijiy adabiyotlardan foydalanildi.

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34-§	Nuqtaning nisbiy va absolyut harakatlarida uning traektoriyasi va harakat tenglamalarini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar.....	118
35-§	Nuqta absolyut harakatining tenglamalari va traektoriyasini aniqlashga doir masalalar.....	120
36-§	Nuqta nisbiy harakatining tenglamalari va traektoriyasini aniqlashga doir masalalar.....	123
37-§	Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar.....	126
38-§	Nuqtaning nisbiy, ko'chirma va absolyut tezligini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar.....	128
39-§	Murakkab harakatda nuqtaning nisbiy, ko'chirma va absolyut tezligini aniqlashga doir masalalar.....	128
40-§	Murakkab harakatda nuqtaning nisbiy, ko'chirma va absolyut tezligini aniqlashga doir mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar.....	134
41-§	Murakkab harakatda ko'chirma harakat ilgarinma harakat bo'lgan hol uchun nuqtaning absolyut tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar.....	137
42-§	Ko'chirma harakat ilgarilanma harakatdan iborat bo'lganda nuqtaning absolyut tezlanishini aniqlashga doir masalalar.....	139
43-§	Talabalarga mustaqil yechish uchun tavsiya etiladigan muammolar.....	145
44-§	Koriolis tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar.....	148
45-§	Murakkab harakatda nuqtaning Koriolis tezlanishini aniqlashga doir masalalar.....	149
46-§	Talabalarga mustaqil yechish uchun tavsiya etiladigan muammolar.....	152

Harakati vektor usulida berilgan nuqtaning tezlanishi.

Nuqtaning harakati vektor usulida

$$\vec{r} = \vec{r}(t) \quad (1.18)$$

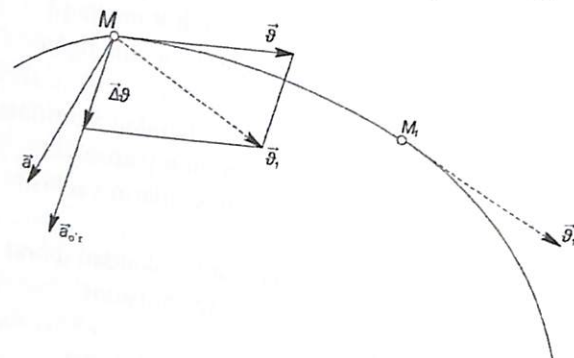
tenglama bilan berilganda, uning tezligi

$$\vec{v} = \frac{d\vec{r}}{dt} \quad (1.19)$$

bo'lishini e'tiborga olsak, nuqtaning tezlanish vektori uning tezlik vektoridan vaqt bo'yicha olingan birinchi tartibli hosilaga yoki radius vektoridan vaqt bo'yicha olingan ikkinchi tartibli hosilaga teng bo'ladi:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} \quad (1.20)$$

Nuqta bir tekislikda yotuvchi traektoriya bo'ylab harakatlansa, tezlanish vektori, o'rtacha tezlanish $\vec{a}_{ur}$ kabi, traektoriya tekisligida yotadi hamda traektoriyaning botiq tomoniga yo'naladi.



1.11 b-rasm

Agar nuqtaning traektoriyasi bir tekislikda yotmaydigan egri chiziqdan iborat bo'lsa, tezlanish vektori egrilik tekisligida yotadi va traektoriyaning botiq tomoniga yo'naladi (1.11a,b-rasm).

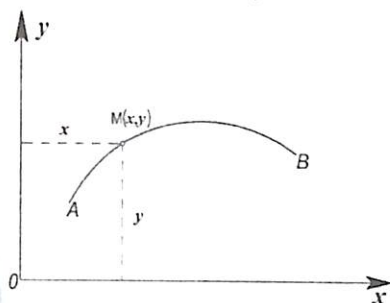
60-§	Bazi hollarda tezliklarning oniy markazini aniqlash.....	201
61-§	Tekis shakl nuqtalarining tezliklarini tezliklarning oniy markazida foydalanib aniqlash.....	204
62-§	Tekislikka parallel harakatdagi jism nuqtalarining tezliklarini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar.....	205
63-§	Tekislikka parallel harakatdagi jism nuqtalarining tezliklarini aniqlashga doir masalalar.....	206
64-§	Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar.....	214
65-§	Tekis shakl nuqtasining tezlanishi.....	216
66-§	Tezlanishlarning oniy markazi va undan foydalanib tekis shakl nuqtalarining tezlanishlarini aniqlash.....	219
67-§	Tekislikka parallel harakatda bo'lgan jism nuqtalarining tezlanishlarini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar....	221
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69-§	Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar.....	243
70-§	Mustaqil yechish uchun talabalarga tavsiya etiladigan masalalar.....	245
	FOYDALANILGAN ADABIYOTLAR.....	257

$$\begin{cases} x = x(t), \\ y = y(t), \\ z = z(t). \end{cases} \quad (1.2)$$

Agar nuqta koordinatalari bilan vaqt orasidagi munosabatlar berilgan bo'lsa, nuqtaning istalgan paytdagi holatini aniqlash mumkin bo'ladi. Shu sababli (1.2) tenglamalar nuqta harakatining Dekart koordinatalaridagi kinematik tenglamalarini ifodalaydi.

(1.2) tenglamalar nuqta traektoriyasining parametrik tenglamalarini ham ifodalaydi. Bunda parametr sifatida t vaqt olingan.

(1.2) tenglamalardan t vaqtni yo'qotib, nuqtaning koordinatalar formasidagi traektoriya tenglamasi aniqlanadi.



1.3-rasm

1 nuqtaning O koordinatalar boshiga nisbatan radius-vektorini

koordinata o'qlarining birlik yo'naltiruvchi vektorlarini $\vec{i}, \vec{j}, \vec{k}$ belgilasak (1.2- rasm), harakatning vektor va Dekart koordinata orqali aniqlash usullari orasidagi bog'lanishni ifodalovchi quyidagi tenglama o'rinli bo'ladi:

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}. \quad (1.3)$$

Agar nuqta xy tekisligida harakatlansa (1.3 - rasm), nuqtaning ikdagi harakat tenglamalari quyidagi ko'rinishda bo'ladi:

$$\begin{cases} x = x(t), \\ y = y(t). \end{cases} \quad (1.4)$$

nuqta to'g'ri chiziqli harakatda bo'lsa (1.4-rasm), harakat traektoriyasi bo'ylab x o'qini yo'naltiramiz. Bu holda nuqtaning to'g'ri harakat tenglamasi quyidagi ko'rinishda yoziladi

$$X = x(t) \quad (1.5)$$

SO'Z BOSHI

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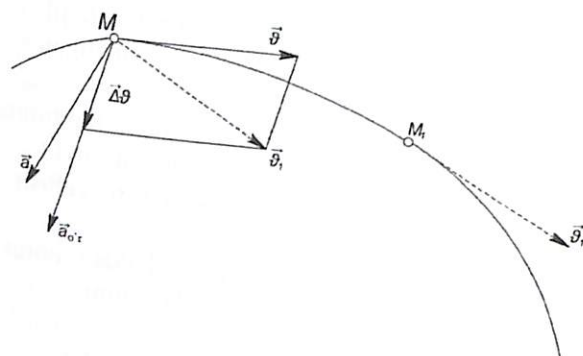
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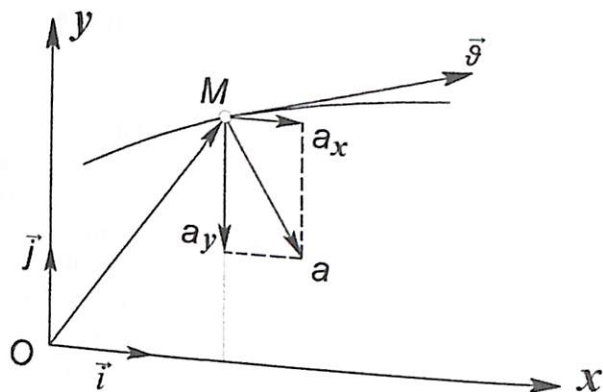
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Agar nuqta Oxyz tekisligida harakatlansa (1.13-rasm), $a_z = \ddot{z} = \ddot{z} = 0$ bo'lib, tezlanish miqdori va yo'nalishi quyidagi formulalar bilan aniqlanadi:

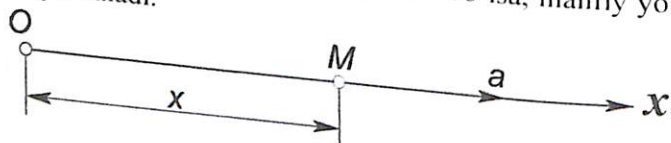
$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\ddot{x}^2 + \ddot{y}^2}; \quad \cos(\vec{a}, \vec{i}) = \frac{a_x}{a}, \quad \cos(\vec{a}, \vec{j}) = \frac{a_y}{a}. \quad (1.25)$$



1.13-rasm

Agar nuqta Ox o'qi bo'ylab to'g'ri chiziqli harakat qilsa (1.14-rasm), tezlanish moduli

$a = |a_x| = |\ddot{x}|$ formula bilan aniqlanadi. Agar $\ddot{x} > 0$ bo'lsa, tezlanish vektori $\vec{a}$ Ox o'qining musbat yo'nalishi bo'yicha, $\ddot{x} < 0$ bo'lsa, manfiy yo'nalishi bo'yicha yo'naladi.



1.14-rasm.

7-§ Nuqta harakatining tenglamalari, traektoriyasini aniqlashga doir masalalar

1. Masala

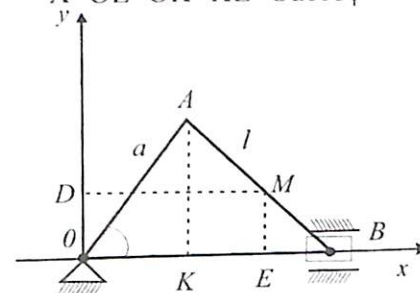
Krivoship-shatun mehanizmda OA krivoship doimiy ω burchak tezligi bilan aylanadi va $OA = l$, $l = a$. shatun o'rtasidagi M nuqtaning harakat tenglamasi va trayektoriya tenglamasini aniqlang. Shuningdek, B polzunning harakat tenglamasini toping. Harakat boshlanishida B polzun o'ngdagi eng chetki holatda bo'lsin. Koordinata o'qlari shakilda ko'rsatilgan bo'lsin.

Yechish.

M nuqtadan koordinata o'qlariga MD va ME perpendikularlar tushiramiz.

1.19 rasmdan (1.58)

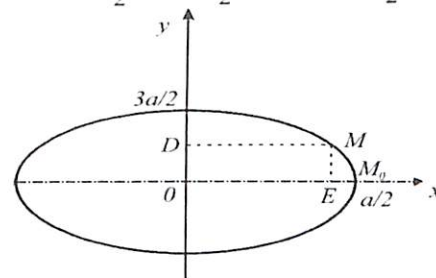
$$X = OE = OK + KE = Oa \cos \varphi + AM \cos \psi \quad (1.58)$$



1.19-rasm

$l = a$ bo'lgani uchun $\varphi = \psi$ bo'ladi, u holda

$$x = a \cos \varphi + \frac{a}{2} \cos \varphi = \frac{3a}{2} \cos \varphi; \quad y = \frac{a}{2} \sin \varphi \quad (1.59)$$



1.20-rasm

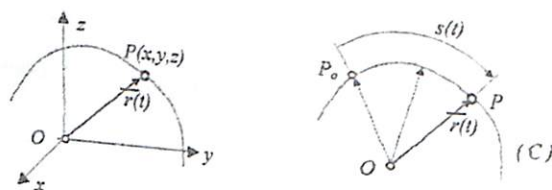
2-§. Moddiy nuqta harakatining berilish usullari

Kinematikada nuqtaning harakati vektor, koordinatalar va tabiiy usulda beriladi.

1. Vektor usuli.

Harakatdagi M nuqtaning Oxyz sanoq sistemasiga nisbatan holati O markazdan o'tkazilgan $\vec{r}$ radius – vektor bilan aniqlanadi (15.1-rasm). M nuqta harakatlanganda vaqt o'tishi bilan uning radius – vektori $\vec{r}$ miqdor va yo'nalish jihatdan o'zgaradi, ya'ni skalyar argument t ning vektorli funksiyasidan iborat bo'ladi:

$$\vec{r} = \vec{r}(t). \quad (1.1)$$



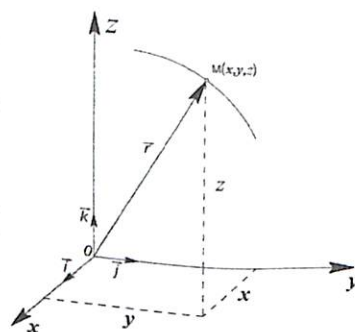
1.1-rasm

Agar $\vec{r}(t)$ funksiyasi ma'lum bo'lsa, nuqtaning fazodagi holati vaqtning har bir payti uchun aniq bo'ladi. Shu sababli (1.1) tenglama nuqta harakatining vektor ko'rinishdagi kinematik tenglamasi deyiladi. Ko'riladigan masalalarda $\vec{r}(t)$ funksiya bir qiymatli, uzluksiz va kamida ikkinchi tartibli hosilaga ega bo'lishi lozim.

2. Koordinatalar usuli.

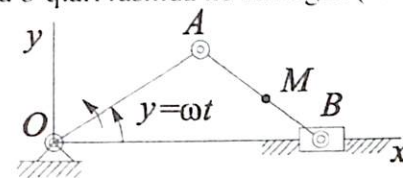
M nuqta Oxyz sanoq sistemasiga nisbatan harakatlanayotgan bo'lsin. Nuqtaning holatini uning uchta x, y, z Dekart koordinatalari orqali aniqlash mumkin (1.2-rasm).

Nuqta harakatlanganda uning koordinatalari vaqt o'tishi bilan o'zgaradi, ya'ni ular t vaqtning funksiyasidan iborat bo'ladi:



1.2-rasm

topilsin; harakat boshlanganida B polzun o'ngdagi eng chetki holatda bo'lgan; koordinata o'qlari rasmda ko'rsatilgan (1.21-rasm)



1.21-rasm

Yechish. Koordinata boshi sifatida O nuqtani tanlab, Ox o'qini gorizontal, Oy o'qini vertikal holda o'tkazamiz. Mehanizmning berilgan holati uchun M nuqtaning koordinatalarini aniqlaymiz:

$$x = OA \cos \varphi + \frac{OA}{2} \cos \varphi = \frac{3}{2} OA \cos \varphi = 1.5 OA \cos \omega t$$

$$y = \frac{OA}{2} \sin \varphi = \frac{OA}{2} \sin \omega t$$

Demak, berilgan mehanizm M nuqtasining harakat qonuni quyidagicha ko'rinishda bo'lar ekan:

$$\begin{cases} x_m = 120 \cos 10t \\ y_m = 40 \sin 10t \end{cases}$$

Mehanizm "M" nuqtasining traektoriyasini aniqlash uchun traektoriya tenglamasini tuzamiz. Buning uchun harakat qonunini ifodalovchi tenglamalardan parametr t ni qisqartiramiz.

$$\left(\frac{x}{120} \right)^2 = \cos^2 10t,$$

$$\left(\frac{y}{40} \right)^2 = \sin^2 10t.$$

Yozilgan tenglamalardan

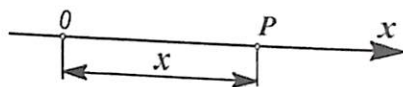
$$\frac{x^2}{120^2} + \frac{y^2}{40^2} = 1$$

Hosil bo'lgan tenglama ellips tenglamasi hisoblanadi.

"B" nuqtaning harakat tenglamasini aniqlaymiz:

$$x = 2OA \cos \omega t = 160 \cos 10t$$

3. masala. Avtomobil to'g'ri chiziqli yo'lda o'zgarmas 20 m/s tezlik bilan harakatlanadi; uning $R=1$ m radiusli gardishida yotuvchi



1.4-rasm.

3. Tabiiy usul.

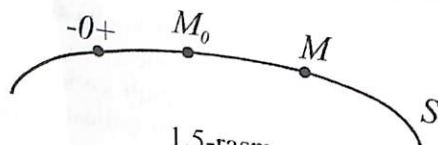
Harakatlanayotgan nuqtaning trayektoriyasi oldindan ma'lum bo'lsa, nuqta harakatini tabiiy usulda aniqlash qulay. Nuqtaning trayektoriyasi to'g'ri chiziqdan yoki egri chiziqdan iborat bo'ladi. Trayektoriyada qo'zg'almas O nuqtani olib, bu nuqtaga nisbatan yoy koordinatasini o'tkazamiz (1.5-rasm). Harakatlanayotgan M nuqtaning trayektoriyadagi holatini O nuqtadan trayektoriya bo'yicha $OM=S$ yoy koordinatasi bilan aniqlaymiz. O nuqtadan bir tomonga qo'yilgan masofani musbat, ikkinchi tomonga qo'yilgan masofani manfiy deb hisoblaymiz. Vaqtning o'tishi bilan harakatlanayotgan nuqtadan qo'zg'almas O nuqtagacha bo'lgan OM masofa o'zgaradi, ya'ni koordinatasi vaqtning funksiyasidan iborat:

$$S=f(t) \quad (1.6)$$

Bu munosabatga **nuqtaning tabiiy usuldagi harakat tenglamasi** yoki **harakat qonuni** deyiladi.

Agar $f(t)$ funksiya ma'lum bo'lsa, u holda t vaqtning har bir payti uchun OM ni aniqlab, O nuqtadan trayektoriya bo'yicha qo'ya-miz. Natijada M nuqtaning berilgan t paytdagi holati aniqlanadi. Shunday qilib, uqtaning harakatini tabiiy usulda aniqlash uchun uning trayektoriyasida O qo'zg'almas nuqta (hisoblash boshi) va yoy koordinatasining hisoblash yo'nalishi hamda $S=f(t)$ harakat tenglamasi bo'lishi kerak. Nuqtaning S yoy koordinatasi bilan trayektoriya ustidan o'tgan OM yo'li doimo bir xil bo'lavermaydi.

Agar M nuqtaning harakati O qo'zg'almas nuqtadan boshlanib $\Delta t = t - t_0$ vaqt oralig'ida doimo musbat yo'nalishi bo'yicha bo'lsa, t vaqtda nuqtaning yoy koordinatasi bilan Δt vaqt oralig'ida o'tilgan yo'l o'zaro teng.



1.5-rasm

I –BOB.

1-§. Kinematikaning asosiy tushunchalari

Nazariy mexanikaning kinematika bo'limida moddiy nuqta va absolyut qattiq jismning harakati shu harakatni vujudga keltirgan sabablarga bog'lanmagan holda faqat geometrik nuqtai nazardan o'rganiladi.

Harakat tushunchasi harakatlanuvchi moddiy nuqta (yoki absolyut qattiq jism), vaqt va fazo tushunchalari bilan chambarchas bog'liqdir.

Ko'chish va harakat tushunchalari nazariy mexanikaning asosiy tushunchalari hisoblanadi. *Moddiy nuqtaning ma'lum vaqt ichida fazoda biror sanoq sistemasiga nisbatan bir holatdan boshqa holatga ixtiyoriy ravishda o'tishi ko'chish deyiladi.*

Nuqtaning boshlang'ich holatdan oxirgi holatga aniq bir usulda vaqtga bog'liq holda o'tishi esa harakat deyiladi.

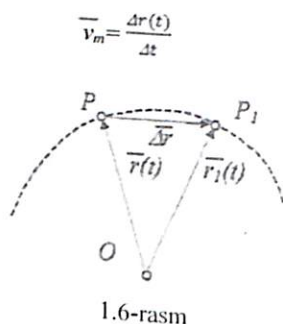
Klassik mexanikada fazo uch o'lchovli, absolyut qo'zg'almas Evklid fazosi deb qaraladi va undagi barcha o'lchamlar Evklid geometriyasi asosida olib boriladi.

Vaqt ob'yektiv borliqda ro'y beruvchi hodisalarning qancha davom etishini ifodalaydi va u absolyut deb qaraladi. Vaqt barcha sanoq sistemalarida bir xil o'tadi va bir sistemaning ikkinchi sistemaga nisbatan harakatiga bog'liq bo'lmaydi. SI sistemasida sekund vaqt birligi hisoblanadi.

Harakatlanayotgan moddiy nuqtaning fazoda biror sanoq sistemasiga nisbatan holati bilan vaqt orasidagi bog'lanishni ifodalovchi tenglama nuqtaning harakat qonunini ifodalaydi. Agar moddiy nuqtaning biror sanoq sistemasiga nisbatan harakat qonuni berilgan bo'lsa, uning trayektoriyasi, tezligi va tezlanishini aniqlash mumkin bo'ladi. Trayektoriya deb – moddiy nuqta yoki absolyut qattiq jismning harakatlanishi tufayli tekislik yoki fazoda qoldirgan iziga aytiladi.

Kinematikaning asosiy vazifasi moddiy nuqta va absolyut qattiq jismning harakat qonunlarini o'rganishdan iborat.

Tezlik vektori nuqta traektoriyasiga harakat yo'nalishi bo'yicha o'tkazilgan urinma bo'ylab yo'naladi (1.6-rasm).



1.6-rasm

2. Harakati koordinatalar usulida berilgan nuqtaning tezligi.

Agar nuqtaning harakati koordinatalar usulida

$$\begin{cases} x = x(t), \\ y = y(t), \\ z = z(t), \end{cases} \quad (1.8)$$

tenglamalar bilan berilgan bo'lsa, nuqta tezligining biror qo'zg'almas Dekart koordinata o'qidagi proeksiyasi mos koordinatasidan vaqt bo'yicha olingan birinchi tartibli hosilaga teng bo'ladi. Shuning uchun:

$$\vartheta_x = \frac{dx}{dt}, \quad \vartheta_y = \frac{dy}{dt}, \quad \vartheta_z = \frac{dz}{dt}. \quad (1.9)$$

Agar tezlikning koordinataga o'qlaridagi proeksiyalari ma'lum bo'lsa, uning moduli

$$\vartheta = \sqrt{\vartheta_x^2 + \vartheta_y^2 + \vartheta_z^2} \quad (1.10)$$

formula bilan, yo'nalishi esa

$$\cos(\vec{\vartheta} \sim \vec{i}) = \frac{\vartheta_x}{\vartheta}, \quad \cos(\vec{\vartheta} \sim \vec{j}) = \frac{\vartheta_y}{\vartheta}, \quad \cos(\vec{\vartheta} \sim \vec{k}) = \frac{\vartheta_z}{\vartheta}. \quad (1.11)$$

formulalar yordamida aniqlanadi. Bunda $\vec{i}, \vec{j}, \vec{k}$ lar Dekart koordinata o'qlarining birlik vektorlari (1.7-rasm).

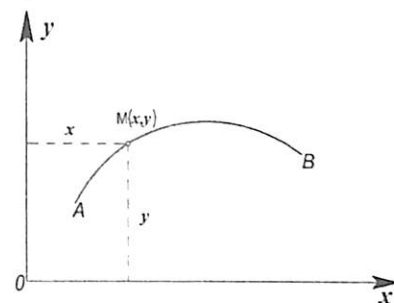
Agar nuqta tekislikda harakatlansa, uning harakati

$$\begin{cases} x = x(t), \\ y = y(t), \\ z = z(t). \end{cases} \quad (1.2)$$

Agar nuqta koordinatalari bilan vaqt orasidagi munosabatlar berilgan bo'lsa, nuqtaning istalgan paytdagi holatini aniqlash mumkin bo'ladi. Shu sababli (1.2) tenglamalar nuqta harakatining Dekart koordinatalaridagi kinematik tenglamalarini ifodalaydi.

(1.2) tenglamalar nuqta traektoriyasining parametrik tenglamalarini ham ifodalaydi. Bunda parametr sifatida t vaqt olingan.

(1.2) tenglamalardan t vaqtni yo'qotib, nuqtaning koordinatalar formasidagi traektoriya tenglamasi aniqlanadi.



1.3-rasm

M nuqtaning O koordinatalar boshiga nisbatan radius-vektorini

$\vec{r}$, koordinata o'qlarining birlik yo'naltiruvchi vektorlarini $\vec{i}, \vec{j}, \vec{k}$ bilan belgilasak (1.2-rasm), harakatning vektor va Dekart koordinatalari orqali aniqlash usullari orasidagi bog'lanishni ifodalovchi quyidagi tenglama o'rinli bo'ladi:

$$\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}. \quad (1.3)$$

Agar nuqta xy tekisligida harakatlansa (1.3-rasm), nuqtaning tekislikdagi harakat tenglamalari quyidagi ko'rinishda bo'ladi:

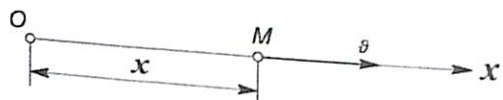
$$\begin{cases} x = x(t), \\ y = y(t). \end{cases} \quad (1.4)$$

Nuqta to'g'ri chiziqli harakatda bo'lsa (1.4-rasm), harakat traektoriyasi bo'ylab x o'qini yo'naltiramiz. Bu holda nuqtaning to'g'ri chiziqli harakat tenglamasi quyidagi ko'rinishda yoziladi

$$X = x(t) \quad (1.5)$$

Bunday holda nuqta tezligining moduli tezlik vektorining koordinata o'qidagi proeksiyasining absolyut qiymatiga teng bo'ladi (1.9-rasm)

$$\vartheta = |\vartheta_x| = \left| \frac{dx}{dt} \right| \quad (1.15)$$



1.9-rasm

3. Harakati tabiiy usulda ifodalangan nuqtaning tezligi.

Agar nuqta berilgan traektoriya bo'ylab $s=s(t)$ qonun asosida harakatlansa, tezlik vektori quyidagi formula orqali ifodalanadi:

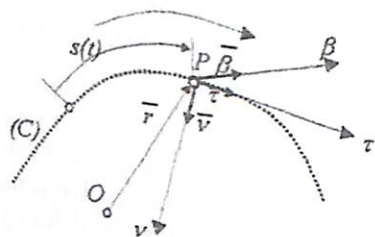
$$\vec{v} = \frac{ds}{dt} \vec{\tau}^0 \quad (1.16)$$

(1.16) da $\frac{ds}{dt}$ hosila $\dot{s}$ tezlikning urinmadagi proeksiyasi ϑ ni ifodalaydi va tezlikning algebraik qiymati deyiladi.

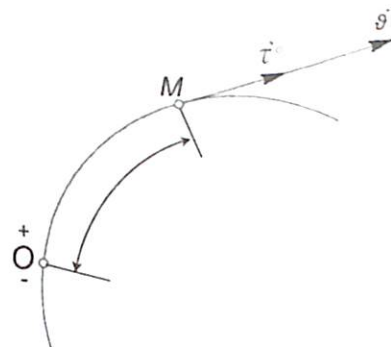
ϑ ning absolyut qiymati tezlikning moduliga teng bo'ladi:

$$\vartheta = |\vartheta_\tau| = \left| \frac{ds}{dt} \right| \quad (1.17)$$

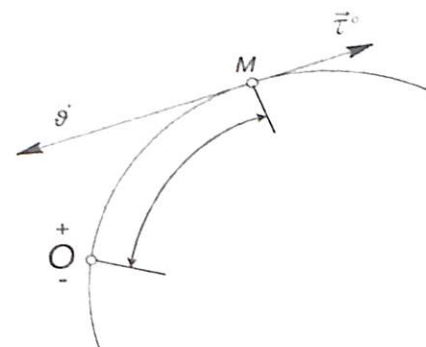
Bunda $\frac{ds}{dt} > 0$ bo'lsa, yoy koordinatasi s orta boradi va nuqta tezligi $\dot{s}$ ning yo'nalishi $\vec{\tau}^0$ bilan ustma-ust tushadi. Agar $\frac{ds}{dt} < 0$ bo'lsa, yoy koordinatasi s kamaya boradi va $\dot{s}$ tezlik vektori $\vec{\tau}^0$ ga qarama-qarshi yo'naladi (1.10a,b,c-rasmlar).



1.10a-rasm



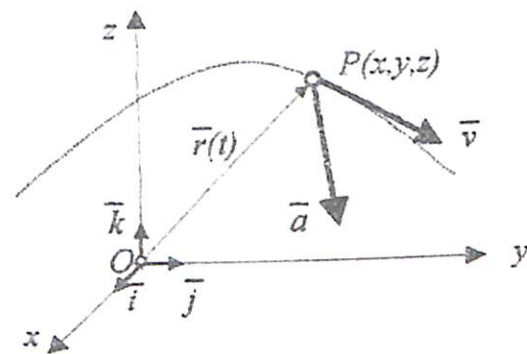
1.10b-rasm



1.10c-rasm

4-§ Nuqtaning tezlanishi

Harakatdagi nuqta tezligining vaqt o'tishi bilan miqdor va yo'nalish jihatidan o'zgarishini ifodalovchi vektor kattalik tezlanish deyiladi.



1.11a-rasm

Tezlanish vektori urinma tezlanish $\vec{a}_\tau$ va normal tezlanish $\vec{a}_n$ larning geometrik yig'indisiga teng bo'ladi:

$$\vec{a} = \vec{a}_\tau + \vec{a}_n \quad (1.34)$$

Bu tezlanishlar o'zaro perpendikulyar yo'nalganidan, to'la tezlanish moduli

$$a = \sqrt{a_\tau^2 + a_n^2} \quad (1.35)$$

yoki

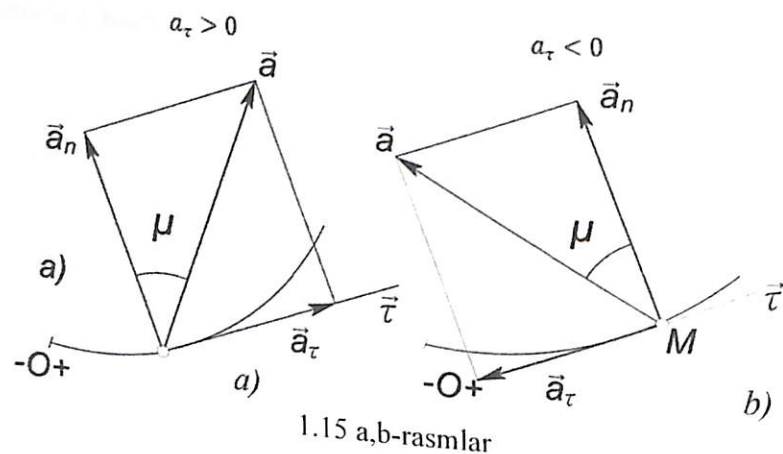
$$a = \sqrt{\left(\frac{d\vartheta}{dt}\right)^2 + \left(\frac{v^2}{\rho}\right)^2} \quad (1.36)$$

formula bilan, yo'nalishi esa

$$\operatorname{tg} \mu = \frac{|a_\tau|}{a_n} \quad (1.37)$$

formula bilan aniqlanadi (1.15a,b- rasmlar).

Bunda $\vec{a}_n$ har doim traektoriyaning botiq tomoniga yo'naladi ($a_n > 0$), $\vec{a}_\tau$ proeksiyaning ishorasiga bog'liq holda $M\tau$ o'qning musbat yoki manfiy tomoniga qarab yo'naladi (1.15a,b-rasmlar).



2. Harakati koordinatalar usulida berilgan nuqtaning tezlanishi.

Nuqtaning harakati koordinatalar usulida berilganda nuqta tezligining koordinata o'qlaridagi proeksiyalari

$$\vartheta_x = \frac{dx}{dt}, \quad \vartheta_y = \frac{dy}{dt}, \quad \vartheta_z = \frac{dz}{dt} \quad (1.21)$$

formulalar yordamida aniqlangan edi.

Nuqta tezlanishining biror o'qdagi proeksiyasi nuqta tezligining mazkur o'qdagi proeksiyasidan vaqt bo'yicha olingan birinchi tartibli hosilaga yoki radius vektoridan vaqt bo'yicha olingan ikkinchi tartibli hosilaga teng bo'ladi.

Shuning uchun:

$$a_x = \frac{d\vartheta_x}{dt} = \frac{d^2x}{dt^2}, \quad a_y = \frac{d\vartheta_y}{dt} = \frac{d^2y}{dt^2}, \quad a_z = \frac{d\vartheta_z}{dt} = \frac{d^2z}{dt^2}. \quad (1.22)$$

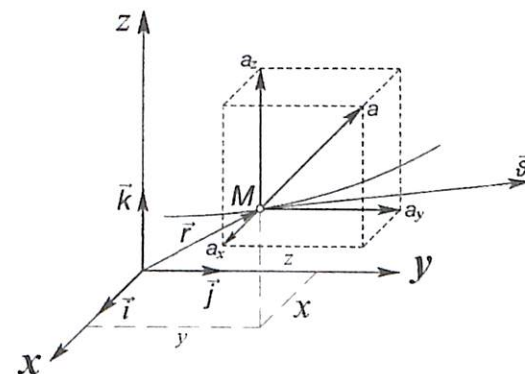
Tezlanishning koordinata o'qlaridagi proeksiyalari ma'lum bo'lsa, uning moduli

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2} = \sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2} \quad (1.23)$$

formula bilan, yo'nalishi esa,

$$\cos(\vec{a} \sim \vec{i}) = \frac{a_x}{a}, \quad \cos(\vec{a} \sim \vec{j}) = \frac{a_y}{a}, \quad \cos(\vec{a} \sim \vec{k}) = \frac{a_z}{a} \quad (1.24)$$

formulalar yordamida aniqlanadi. Bunda $\vec{i}, \vec{j}, \vec{k}$ lar koordinata o'qlarining birlik vektorlari (1.12-rasm).



1.12-rasm

Bunda nuqtaning normal tezlanishi $\vec{a}_n$ doimo egri chiziqning botiq tomoniga yo'nalgan bosh normal bo'ylab yo'naladi. $\vartheta = \text{const}$ bo'lgani uchun, bu tezlanish nuqtaning tezligi vaqt o'tishi bilan faqat yo'nalishini o'zgartirishidan hosil bo'ladi. Shu sababli, normal tezlanish nuqta tezligining yo'nalish jihatdan o'zgarishini ifodalaydi.

Agar $\vartheta = \frac{ds}{dt}$ ekanligini e'tiborga olsak, ($\vartheta = \vartheta_0$)

$$ds = \vartheta dt$$

Bu tenglikni mos chegaralar bo'yicha integrallasak

$$\int_{s_0}^s ds = \int_0^t \vartheta_0 dt$$

yoki

$$s = s_0 + \vartheta_0 t$$

tenglama hosil bo'ladi.

Agar

$$s_0 = 0 \text{ bo'lsa, } s = \vartheta_0 t$$

(1.44)

(1.44) tenglama nuqtaning egri chiziqli tekis harakati tenglamasi deyiladi.

3. Egri chiziqli tekis o'zgaruvchan harakat.

Agar nuqtaning harakati davomida doimo $a_\tau = \text{const}$ bo'lsa, bunday harakat tekis o'zgaruvchan harakat deyiladi.

Agar $t=0$ da $s=s_0$ va $\vartheta=\vartheta_0$ bo'lsa,

$$a_\tau = \frac{d\vartheta}{dt} = \frac{d^2 s}{dt^2}$$

(1.45)

tenglamadan

$$d\vartheta = a_\tau ds \quad (1.46)$$

tenglik hosil bo'ladi. $a_\tau = \text{const}$ ekanligini e'tiborga olib, (1.46) tenglikni mos chegaralar bo'yicha integrallasak

$$\int_{\vartheta_0}^{\vartheta} d\vartheta = \int_0^t a_\tau ds$$

yoki

$$\vartheta = \vartheta_0 + a_\tau t$$

(1.47)

3. Harakati tabiiy usulda berilgan nuqtaning tezlanishi.

Nuqtaning harakati tabiiy usulda berilganda uning tezligi quyidagicha ifodalanar edi:

$$\vec{\vartheta} = \frac{ds}{dt} \vec{\tau}^0 = \vartheta \vec{\tau}^0. \quad (1.27)$$

Tezlanish vektori, tezlik vektoridan vaqt bo'yicha olingan birinchi tartibli hosilaga teng bo'ladi:

$$\vec{a} = \frac{d\vec{\vartheta}}{dt} = \frac{d\vartheta}{dt} \vec{\tau}^0 + \vartheta \frac{d\vec{\tau}^0}{dt} = \frac{d\vartheta}{dt} \vec{\tau}^0 + \vartheta \frac{d\vec{\tau}^0}{ds} \frac{ds}{dt} \quad (1.28)$$

Analitik geometriyadan ma'lumki

$$\frac{d\vec{\tau}^0}{ds} = \frac{1}{\rho} \vec{n}^0 \quad (1.29)$$

bunda ρ – traektoriyaning egrilik radiusi, $\vec{n}^0$ – traektoriyaga o'tkazilgan bosh normal birlik vektori.

Bularni e'tiborga olsak,

$$\vec{a} = \frac{d\vartheta}{dt} \vec{\tau}^0 + \frac{\vartheta^2}{\rho} \vec{n}^0 \quad (1.30)$$

Bu ifodada $\frac{d\vartheta}{dt} \vec{\tau}^0$ vektor kattalik traektoriyaga M nuqtada o'tkazilgan urinma bo'ylab yo'naladi va *urinma tezlanish* deyiladi:

$$\vec{a}_\tau = \frac{d\vartheta}{dt} \vec{\tau}^0. \quad (1.31)$$

$\frac{\vartheta^2}{\rho} \vec{n}^0$ vektor kattalik traektoriyaga M nuqtada o'tkazilgan bosh normal bo'ylab yo'naladi va *normal tezlanish* deyiladi:

$$\vec{a}_n = \frac{\vartheta^2}{\rho} \vec{n}^0. \quad (1.32)$$

Urinmaning birlik vektori $\vec{\tau}^0$ va bosh normalning birlik vektori $\vec{n}^0$ traektoriyaning M nuqtasiga o'tkazilgan egrilik tekisligida yotganligi tufayli, tezlanish vektori ham mazkur egrilik tekisligida yotadi. Shu sababli tezlanishning binormaldagi tashkil etuvchisi nolga teng bo'ladi.

Tezlanishning tabiiy koordinata o'qlaridagi proeksiyalari quyidagicha aniqlanadi:

$$a_\tau = \frac{d\vartheta}{dt} = \frac{d^2 s}{dt^2}, \quad a_n = \frac{\vartheta^2}{\rho}. \quad (1.33)$$

Nuqtaning maksimal uchish uzoqligi (qancha uzoqqa borishi) traektoriya tenglamasidan $y=0$ bo'lgan holatda (harakatlanayotgan jism yerga tushganda) aniqlanadi:

$$tg\alpha \cdot x - \frac{gx^2}{2v_0^2 \cos^2 \alpha} = 0 \quad (1.74)$$

Bu tenglamadan x ning ikki qiymati

$$x_1 = 0, \quad x_2 = \frac{v_0^2 \sin 2\alpha}{g} \quad (1.75)$$

aniq bo'ladi. Bunda x_1 nuqtaning boshlang'ich holatini, x_2 esa, nuqtaning gorizontal yo'nalishda uchish uzoqligini ifodalaydi. Binobarin, nuqtaning maksimal uchish uzoqligi quyidagiga teng bo'lar ekan:

$$x_2 = s_{max} = \frac{v_0^2 \sin 2\alpha}{g}$$

4-masala. Nuqta harakatining berilgan tenglamalariga qarab uning traektoriya tenglamasi topilsin; shuningdek, masofani nuqtaning boshlang'ich holatidan hisoblab, nuqtaning traektoriya bo'ylab harakatlanish qonuni ko'rsatilsin.

$$x=3\sin t, \quad y=3\cos t$$

Yechish. Nuqta traektoriyasini aniqlash uchun harakatning berilgan tenglamalaridan vaqt t ni qisqartiramiz:

$$\left. \begin{aligned} \sin t &= \frac{x}{3} \\ \cos t &= \frac{y}{3} \end{aligned} \right\}$$

Tenglamalarning har ikki tomonlarini kvadratga ko'tarib qo'shsak, quyidagi ko'rinishdagi tenglamaga ega bo'lamiz:

$$\frac{x^2}{9} + \frac{y^2}{9} = 1 \text{ yoki } x^2 + y^2 = 9$$

Mazkur tenglama, radiusi $R=3$ bo'lgan aylana tenglamasini ifodalaydi. Nuqtaning tezligi quyidagicha aniqlanadi:

$$v_x = \frac{dx}{dt} = 3\cos t, \quad v_y = -3\sin t, \quad v = \sqrt{v_x^2 + v_y^2} = 3$$

Nuqtaning tezligini bilgan holda traektoriya bo'ylab harakatlanish qonunini aniqlaymiz:

5-§ Nuqta harakatining xususiy hollari

Nuqtaning tezlanishi tabiiy koordinata o'qlaridagi tashkil etuvchilari orqali quyidagicha yoziladi:

$$\vec{a} = \frac{d\vartheta}{dt} \vec{\tau}^0 + \frac{\vartheta^2}{\rho} \vec{n}^0 \quad (1.38)$$

Nuqtaning tezlanishiga qarab harakat turlarini aniqlash mumkin.

1. To'g'ri chiziqli tekis harakat.

Nuqtaning traektoriyasi to'g'ri chiziqdan iborat bo'lsa, $\rho = \infty$ bo'ladi.

Bunday holda

$$a_n = \frac{\vartheta^2}{\rho} = 0 \quad (1.39)$$

bo'lib, nuqtaning tezlanishi faqat urinma tezlanishdan iborat bo'ladi:

$$a = a_\tau = \frac{d\vartheta_\tau}{dt} \quad (1.40)$$

Bunday holda nuqtaning tezligi faqat miqdor jihatdan o'zgaradi ($\rho = \infty$). Shuning uchun ham urinma tezlanish tezlikning son qiymati jihatdan o'zgarishini ifodalaydi.

Nuqtaning harakati davomida doimo $\vec{a}_\tau = 0$, $\vec{a}_n = 0$ ya'ni $\vec{a} = 0$ bo'lsa, $\frac{d\vartheta_\tau}{dt} = 0$ bo'lib, $\vartheta = |\vartheta_\tau| = \text{const}$ bo'ladi.

$\frac{\vartheta^2}{\rho} = 0$ bo'lganidan $\rho = \infty$ ekanligi kelib chiqadi.

Bunday holda nuqta to'g'ri chiziqli tekis harakatda bo'ladi.

2. Egri chiziqli tekis harakat.

Nuqta egri chiziqli tekis harakatda bo'ladi, agarda tezlikning son qiymati harakat davomida doimo o'zgarmas holda saqlansa:

$$\vartheta = \text{const}.$$

Bunday holda

$$a_\tau = \frac{d\vartheta}{dt} = 0 \quad (1.41)$$

bo'lib, nuqtaning tezlanishi faqat normal tezlanishdan iborat bo'ladi:

$$a = a_n = \frac{\vartheta^2}{\rho} \quad (1.42)$$

nuqtaning tezligi nolga teng bo'ladi? Harakat boshlangan paytdan $t=3s$ vaqt davomida bosib o'tgan yo'l aniqlansin. (1.23-rasm)



1.23-rasm

9-§ Nuqtaning tezligini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Nuqtaning tezligi deb berilgan sanoq sistemasida har qanday vaqt onida nuqta harakatining qanchalik ildamligi va yo'nalishini ifodalovchi vektor kattalikka aytiladi:

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x \vec{i} + v_y \vec{j} + v_z \vec{k} \quad (1.76)$$

Bunda $\vec{i}, \vec{j}, \vec{k}$ lar koordinata o'qlari birlik vektorlari.

Tezlik vektorining Dekart o'qlaridagi proyeksiyalari quyidagicha aniqlanadi:

$$v_x = \frac{dx}{dt} = x, \quad v_y = \frac{dy}{dt} = y, \quad v_z = \frac{dz}{dt} = z,$$

Tezlik moduli

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2} \quad (1.77)$$

formula asosida, uning yo'nalishi esa

$$\cos(\vec{v} \wedge \vec{i}) = \frac{v_x}{v}, \quad \cos(\vec{v} \wedge \vec{j}) = \frac{v_y}{v}, \quad \cos(\vec{v} \wedge \vec{k}) = \frac{v_z}{v} \quad (1.78)$$

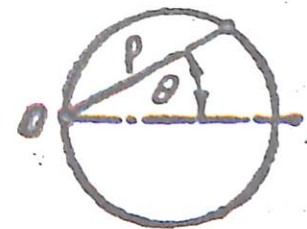
fo'rmular asosida aniqlanadi.

Ko'pincha masalalarda harakatdagi nuqtaning ma'lum vaqt oralig'idagi o'rtacha sur'atini aniqlash talab etiladi:

$$v_{\text{ort}} = \frac{\Delta S}{\Delta t}$$

Ba'zi hollarda harakatdagi nuqtaning "average speed" - o'rtacha tezligini topish ham ma'lum qiziqish uyg'otadi.

$$v_{\text{ort}} = \frac{S_T}{\Delta t}$$



1.33-rasm.

12-§ Nuqtaning tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Nuqtaning tezlanishi deb nuqta tezligining vaqt o'tishi bilan miqdor va yo'nalish jixatdan o'zgarishini ifodalovchi vektor kattalikka aytiladi:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} = a_x \vec{i} + a_y \vec{j} + a_z \vec{k} \quad (1)$$

Bu ifodada

$$a_x = \frac{dv_x}{dt} = x'', \quad a_y = \frac{dv_y}{dt} = y'', \quad a_z = \frac{dv_z}{dt} = z''$$

Tezlanishning koordinata o'qlaridagi pyeksiyalari mumkin bo'lsa tezlanish moduli quyidagicha aniqlanadi:

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

Tezlanish vektorining yo'nalishi esa uning yo'naltiruvchi qoidalari orqali aniqlanadi.

$$\cos(\vec{a} \wedge \vec{i}) = \frac{a_x}{a}, \quad \cos(\vec{a} \wedge \vec{j}) = \frac{a_y}{a}, \quad \cos(\vec{a} \wedge \vec{k}) = \frac{a_z}{a}$$

Ba'zan, masalalar yechishda nuqtaning ma'lum vaqt oralig'ida gi o'rtacha tezlanishini aniqlash talab etiladi.

$$a_{o'r} = \frac{\Delta v}{\Delta t}$$

Bunda $\Delta v = v' - v$ nuqtaning tezlanishini Δt vaqt oraligida o'zgarishi.

Nuqtaning harakati tabiiy usulda berilganda uning tezlanishi

$$\vec{a} = \vec{a}_\tau + \vec{a}_n = \frac{dv}{dt} \vec{i}_0 + \frac{v^2}{\rho} \vec{n}_0 \quad (2)$$

6-§ Nuqta harakatining tenglamalari va traektoriyasini aniqlashga doir masalalarni yechish uchun uslubiy korsatmalar

Nuqta kinematikasida nuqtaning harakat tenglamalari berilgan bo'lib, uning traektoriyasi, tezligi, tezlanishi kabi kinematik kattaliklarni aniqlash talab etiladi.

Bunday holda nuqta kinematikasi masalalarini quyidagi tartibda yechish maqsadga muvofiq bo'ladi:

1. Koordinatalar sistemasi tanlab olinadi;
2. Tanlangan koordinatalar sistemasida nuqtaning harakat tenglamalari tuziladi;
3. Nuqtaning harakat tenglamalarini bilgan holda, traektoriya tenglamasi tuziladi. Buning uchun harakat tenglamalaridan t vaqt yo'qotiladi;

4. Nuqtaning harakat tenglamalarini bilgan holda, tezlikning o'qlardagi proektsiyalarini, ular orqali esa, tezlikning miqdori va yo'nalishi aniqlanadi;

5. Tezlikning o'qlardagi proektsiyalarini bilgan holda, tezlanishning o'qlardagi proektsiyalarini, ular orqali esa, tezlanishning miqdori va yo'nalishi aniqlanadi.

Agar masalada nuqtaning traektoriyasi berilgan bo'lsa, tezlanishning miqdori va yo'nalishi aniqlanadi.

Agar masalada nuqtaning traektoriyasi berilgan bo'lsa, tezlanishning miqdori va yo'nalishi aniqlanadi.

1. Tezlanishning urinma o'qdagi proektsiyasi aniqlanadi:

$$a_t = \frac{v_x a_x + v_y a_y}{v} \quad (1.51)$$

2. Tezlanishning bosh normaldagi proektsiyasi aniqlanadi:

$$a_n = \frac{v_x a_y - v_y a_x}{v} \quad (1.52)$$

Ular orqali nuqtaning to'la tezlanishi topiladi:

$$a = \sqrt{a_t^2 + a_n^2} \quad (1.53)$$

3. Traektoriyaning egrilik radiusi:

$$\rho = \frac{v^2}{a_n} \quad (1.54)$$

formula yordamida aniqlanadi.

6. Nuqtaning biror vaqt uchun ma'lum bo'lgan koordinatalaridan foydalanib, integrallash o'zgarmaslari aniqlanadi.

7. Hosil bo'lgan nuqtaning harakat tenglamalaridan vaqtni yo'qotib (qisqartirib), koordinatalar formasidagi trayektoriya tenglamasi tuziladi

13-§ Nuqtaning tezlanishini aniqlashga doir masalalar

1-Masala.

Samolyotdan $h=320\text{m}$ balandlikdan tashlangan yuk

$$x=60t, \quad y=5t^2 \quad (3)$$

tenglamalarga asosan harakatlanadi, bunda x, y lar metrlarda, t – sekundlarda o'lchanadi.

Yukning traektoriyasi, samolyotdan tashlash va yerga tushish nuqtalari orasidagi gorizonttal masofa, yerga tushish paytidagi tezligi va tezlanishi, tushish nuqtasida traektoriyaning egrilik radiusi aniqlansin (1.34a-rasm).

Yechimi.

Yukning traektoriyasini aniqlaymiz. Buning uchun harakat tenglamalarining biridan t vaqtni topib ikkinchi tenglamaga qo'yamiz:

$$t = \frac{x}{60}; \quad y = 5\left(\frac{x}{60}\right)^2 = \frac{1}{720}x^2.$$

Natijada

$$y = \frac{1}{720}x^2 \quad (4)$$

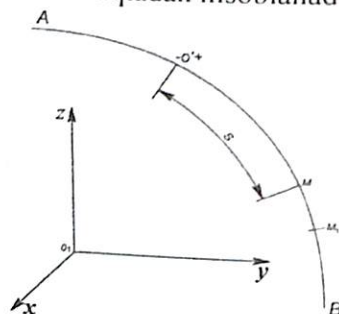
ko'rinishdagi parabola tenglamasi hosil bo'ladi. Demak, yukning traektoriyasi y o'qiga simmetrik, uchi koordinata boshida bo'lgan parabola ekan (1.34a-rasm).

Yukning samolyotdan tashlash va yerga tushish nuqtalari orasidagi gorizonttal masofani aniqlaymiz. Yukning M_1 tushish nuqtasidagi $y_1=h$, $x_1=l$ koordinatalarni aniqlash uchun yukning harakat tenglamalaridan foydalanamiz (1.34b-rasm):

$$y = 5t^2, \quad t_1 = \sqrt{\frac{y_1}{5}} = \sqrt{\frac{h}{5}} = 8\text{s}. \quad (5)$$

Shuning uchun

Bu ifodada S - egri chiziqli koordinata bo'lib, traektoriya bo'ylab tanlab olingan biror O nuqtadan hisoblanadi (1.17-rasm).



1.17-rasm

Bunda nuqtaning traektoriyasi to'g'ri chiziq bo'lishi ham mumkin (1.18-rasm).



1.18-rasm

Nuqta harakatining tenglamalari va traektoriyasini aniqlashga doir masalalar quyidagi tartibda yechiladi:

1. Qo'zg'almas o'qlar sistemalari (to'g'ri burchakli, qutb va h. k), ularning boshi (qo'yilish nuqtalari) tanlab olinadi;
2. Masala shartiga ko'ra tanlab olingan koordinatalar sistemasi uchun nuqtaning harakat tenglamalari tuziladi;
3. Tuzilgan harakat tenglamalariga ko'ra istalgan vaqt oni uchun nuqtaning o'rni, harakatining yo'nalishi, traektoriyasi aniqlanadi.

(1.47) egri chiziqli tekis o'zgaruvchan harakatdagi nuqtaning tezligini ifodalaydi. Agar

$$v = \frac{ds}{dt}$$

ekanligini e'tiborga olsak, (1.47) tenglama quyidagicha yoziladi:

$$\frac{ds}{dt} = v_0 + a_t t \quad (1.48)$$

Bu tenglamaning har ikkala tomoni mos chegaralar bo'yicha integrallansa, tekis o'zgaruvchan harakat tenglamasi hosil bo'ladi:

$$s = s_0 + v_0 t + \frac{a_t t^2}{2} \quad (1.43)$$

To'g'ri chizikli tekis o'zgaruvchan harakat tezligi va harakat tenglamasi quyidagi ko'rinishda bo'ladi:

$$\dot{x} = v_0 + a_x t. \quad (1.49)$$

$$x = x_0 + v_0 t + \frac{a_x t^2}{2} \quad (1.50)$$

Takrorlash uchun savollar

1. Nuqtaning tezligi deb qanday kattalikka aytiladi?
2. Nuqtaning tezligi vektor usulida qanday aniqlanadi?
3. Nuqtaning tezligi koordinatalar usulida qanday aniqlanadi?
4. Nuqtaning tezligi tabiiy usulda qanday aniqlanadi?
5. Nuqtaning tezlanishi vector usulida qanday aniqlanadi?
6. Nuqtaning tezlanishi koordinatalar usulida qanday aniqlanadi?
7. Nuqtaning tezlanishi. Tabiiy usulda qanday aniqlanadi?
8. Nuqtaning urinma tezlanishi qanday harakatda yuzaga keladi?
9. Nuqtaning normal tezlanishi qanday harakatda yuzaga keladi?
10. Nuqtaning tezlanishi doimo nolga teng bo'lsa, u qanday harakatda bo'ladi?
11. Tekis o'zgaruvchan harakatni ta'riflang.
12. Egri chiziqli biror M nuqtadagi egriligi qanday aniqlanadi.
13. Egrilik radiusini ta'riflang.

Bizda $\varphi = \psi \cdot t$ u holda (1.59) tenglama quyidagicha yoziladi:

$$\left. \begin{aligned} x &= \frac{3a}{2} \cos \omega t, \\ y &= \frac{a}{2} \sin \omega t. \end{aligned} \right\} \quad (1.60)$$

(1.60) tenglamalar sistemasi M nuqtaning harakat tenglamalari bo'ladi. Bu tenglamalardan vaqt t ni yo'qotsak, trayektoriya tenglamalarini topamiz. Sinus va kosinus funksiyalarning argumentlari bir xil bolsa, vaqt t ni yo'qotish uchun (1.60) tenglamalarni quyidagi ko'rinishda yozamiz:

$$\left. \begin{aligned} \cos \omega t &= \frac{2}{3} \cdot \frac{x}{a}, \\ \sin \omega t &= \frac{2}{a} \cdot y. \end{aligned} \right\} \quad (1.61)$$

(1.61) tenglamalarning ikkala tomonini kvadratga ko'taramiz:

$$\left. \begin{aligned} \cos^2 \omega t &= \frac{4}{9} \cdot \frac{x^2}{a^2}, \\ \sin^2 \omega t &= \frac{4}{a^2} \cdot y^2. \end{aligned} \right\} \quad (1.62)$$

O'zaro qo'ship quyidagini hosil qilamiz:

$$\frac{4x^2}{9a^2} + \frac{4y^2}{a^2} = 1 \text{ yoki } \frac{x^2}{\frac{9a^2}{4}} + \frac{y^2}{\frac{a^2}{4}} = 1 \quad (1.63)$$

(1.63) tenglama M nuqtaning trayektoriya tenglamasi. Trayektoriya yarim o'qlari $\frac{3a}{2}$ va $\frac{a}{2}$ ga teng bolgan va ellipsdan iborat (1.20-rasm)

Endi B polzuning harakat tenglamasini topamiz.
1.19-rasmdan:

$$x_B = OB = a \cos \varphi + l \cos \varphi + a \cos \varphi = 2a \cos \varphi \text{ yoki } x_B = 2a \cos \varphi \quad (1.64)$$

(1.64) tenglama B polzuning harakat tenglamasini ifodalaydi.
2. masala.

OA krivoship $\varphi = 10$ rad/s doimiy burchak tezlik bilan aniqlanadi. Uzunlik OA=AB=80 sm. Shatun o'rtasidagi M nuqtaning harakat tenglamasi va trayektoriyasi, shuningdek B polzuning harakat tenglamasi

Harakatdagi nuqtaning fazoda qoldirgan izi uning traektoriyasi deyiladi. Nuqtaning traektoriyasi tekislikda yoki fazoda yotuvchi chiziq bo'lishi mumkin. Nuqtaning harakati uning harakat qonuni orqali ifodalanadi. Nuqtaning harakat qonuni (tenglamasi) uning tekislikda yoki fazodagi o'rni va vaqt oralig'idagi bog'lanishni ifodalaydi. Nuqtaning harakati vektor usulida berilganida ixtiyoriy vaqt onidagi o'rni kordinatalar boshidan harakatdagi nuqtaga o'tkazilgan $\vec{r}$ radius vektor orqali aniqlanadi (1.16-rasm).

$$\vec{r} = \vec{r}(t) \quad (1.55)$$

Nuqtaning harakati koordinatalar usulida berilganda uning ixtiyoriy vaqt oralig'idagi o'rni:

$$\left. \begin{aligned} \text{a) fazoda } x &= f_1(t), y = f_2(t), z = f_3(t) \\ \text{b) tekislikda } x &= f_1(t), y = f_2(t) \\ \text{c) nuqta to'g'ri chiziqli harakatda bo'lganda } x &= f(t) \end{aligned} \right\} \quad (1.56)$$

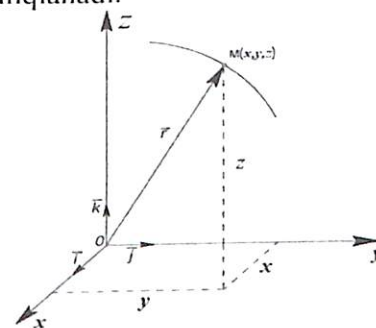
koordinatalari orqali aniqlanadi.

Nuqtaning harakati qutb, slindrik va sferik koordinatalarda ham beriladi.

Agar nuqta harakatining traektoriyasi oldindan ma'lum bo'lsa, uning harakatini tabiiy usulda berish qulay bo'ladi. Bunday holda nuqtaning traektoriyadagi o'rni

$$S = f(t) \quad (1.57)$$

tenglama orqali aniqlanadi.



1.16-rasm

nuqtaning harakat tenglamasi va traektoriyasi aniqlansin. G'ildirakni sirpanmasdan g'ildiraydi deb hisoblansin; koordinata boshini Ox o'q sifatida olingan yo'lining harakat boshlanadigan nuqtasida olinsin.

Yechish. Avtomobil g'ildiragi gardishidagi nuqtaning harakat tenglamasi quyidagi ko'rinishda yoziladi.

$$x = R\varphi - d \sin \varphi$$

$$y = R - d \cos \varphi$$

Masalada $R=1$ $\varphi=\omega t$. Shuning uchun

$$\omega = \frac{v}{r} = \frac{20}{1} 20 \frac{1}{s}$$

Natijada avtomobil g'ildiragi gardishidagi nuqtaning harakat tenglamasi quyidagi ko'rinishda yoziladi:

$$x=20t-\sin 20t \quad y=1-\cos 20t$$

4. masala. Nuqtaning harakati

$$x=\vartheta_0 t \cos \alpha \quad (1.65)$$

$$y = \vartheta_0 t \sin \alpha - \frac{gt^2}{2} \quad (1.66)$$

tenglamalar bilan berilgan. Bundagi ϑ_0 va g – lar doimiy miqdorlar.

Nuqtaning traektoriyasi, maksimal ko'tarilish balandligi va bunday holatda gorizonttal yo'nalishda s siljishi, hamda qancha uzoqqa borishi aniqlansin (1.22-rasm).

Yechimi.

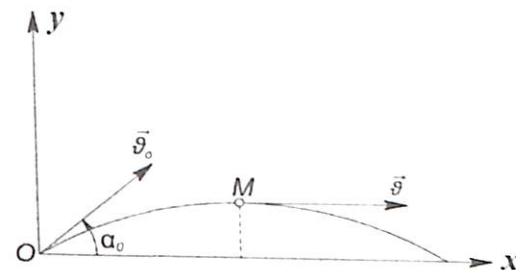
Traektoriyaning tenglamasini aniqlash uchun nuqtaning harakat tenglamalarining biridan t vaqtini topib, ikkinchi tenglamaga qo'yamiz:

$$t = \frac{x}{\vartheta_0 \cos \alpha} \quad (1.67)$$

$$y = t g \alpha \cdot x - \frac{g}{2 \vartheta_0^2 \cos^2 \alpha} x^2 \quad (1.68)$$

(1.68) ifoda parabola tenglamasidir.

Nuqtaning traektoriyasi mazkur parabolaning $x \geq 0$ shartni qanoatlantiruvchi qismidan iborat (1.22-rasm).



1.22-rasm

Nuqta eng yuqori holatga ko'tarilguncha o'tgan vaqt va maksimal ko'tarilish balandligini aniqlash uchun tezlikning koordinata o'qlaridagi proeksiyalarini aniqlaymiz:

$$\vartheta_x = \dot{x} = \vartheta_0 \cos \alpha$$

$$\vartheta_y = \dot{y} = \vartheta_0 \sin \alpha - gt \quad (1.69)$$

Nuqta maksimal balandlikni egallaganda uning tezligi x o'qiga parallel bo'ladi. Shu sababli

$$\vartheta_y = 0$$

yoki

$$\vartheta_0 \sin \alpha - gt_1 = 0 \quad (1.70)$$

bo'ladi, bunda t_1 nuqta eng yuqori holatga ko'tarilguncha o'tgan vaqt. (1.70) dan

$$t_1 = \frac{\vartheta_0 \sin \alpha}{g} \quad (1.71)$$

Vaqt t_1 ning qiymatini (1.66)ga qo'yib, nuqtaning maksimal ko'tarilish balandligini aniqlaymiz:

$$h = y_{\max} = \frac{\vartheta_0^2 \sin^2 \alpha}{g} - \frac{g \vartheta_0^2 \sin^2 \alpha}{2g^2} = \frac{\vartheta_0^2 \sin^2 \alpha}{2g} \quad (1.72)$$

Nuqta maksimal balandlikka ko'tarilganda boshlang'ich holatidan gorizonttal yo'nalishda s siljishini aniqlash uchun vaqt t ning qiymatini (1.65)ga qo'yamiz:

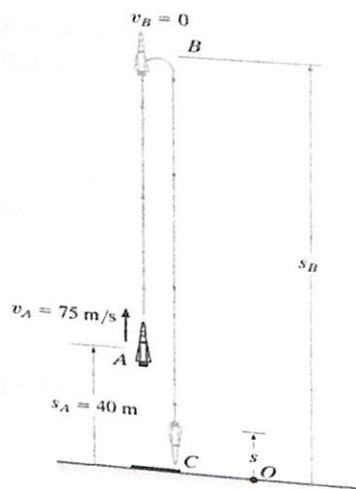
$$s_1 = x_1 = \vartheta_0 \cos \alpha \cdot \frac{\vartheta_0 \sin \alpha}{g} = \frac{\vartheta_0^2 \sin 2\alpha}{2g} \quad (1.73)$$

10-§ Nuqtaning tezligini aniqlashga doir masalalar.

1. Masala.

Sinov paytida raketaning dvigateli u yerdan 40 m balandlikka ko'tarilganda ishdan chiqqan. U paytda raketa tezligi 75 m/s bo'lgan. Raketaning maksimal ko'tarilish balandligi va u qaytib yerga tushganda qanday tezlikka ega bo'lishi aniqlansin. Erkin tushish tezlanishi $a_c = 9.81 \text{ m/s}^2$, u vertical payti yo'nalgan.

Havo qarshiligi e'tiborga olinmasin.



(1.25- rasm)

Yechilishi:

Koordinata boshi sifatida yer sirtidagi O nuqtani tanlab, koordinata o'qini raketa harakati tomon vertical va yuqoriga yo'naltiramiz. Raketaning maksimal ko'tarilish balandligini aniqlaymiz. Raketa maksimal balandlik B nuqtaga yetganda uning tezligi quyidagicha ifodalanadi:

$$v_B^2 = v_A^2 + 2a_c(s_B - s_A)$$

$$v = \frac{ds}{dt}; \quad 3 = \frac{ds}{dt} \quad \text{bundan} \quad \int_0^s ds = \int_0^t 3dt$$

Natijada nuqtaning traektoriya bo'ylab harakatlanish qonuni uchun $S=3t$ tenglamaga ega bo'lamiz.

8-§ . Mustaqil o'rgamish uchun talabalarga tavsiya etiladigan masalalar

Masala -1. Nuqtaning koordinata usulida berilgan harakat tenglamasiga ko'ra uning traektoriya tenglamasi topilsin va rasmda harakat yo'nalishi ko'rsatilsin.

$$X=3t-5 \quad y=4-2t$$

Masala -2. Nuqtaning koordinata usulida berilgan harakat tenglamasiga ko'ra uning traektoriya tenglamasi topilsin va rasmda harakat yo'nalishi ko'rsatilsin.

$$X=5\sin 10t, \quad y=3\cos 10t$$

Masala -3. Nuqta harakatining berilgan tenglamalariga qarab uning traektoriya tenglamasi topilsin; shuningdek, masofani nuqtaning boshlang'ich holatidan hisoblab, nuqtaning traektoriya bo'ylab harakatlanish qonuni ko'rsatilsin.

$$X=3t^2 \quad y=4t^2$$

Masala -4. Nuqta harakatining berilgan tenglamalariga qarab uning traektoriya tenglamasi topilsin; shuningdek, masofani nuqtaning boshlang'ich holatidan hisoblab, nuqtaning traektoriya bo'ylab harakatlanish qonuni ko'rsatilsin.

$$X=a\cos^2 t \quad y=a\sin^2 t$$

Masala -5. Nuqtaning harakati $x=2a\cos^2 \frac{kt}{2}$, $y=as\text{inkt}$ tenglamalar bilan berilgan, bundagi a va k musbat o'zgarmaslar. Masofani nuqtaning boshlang'ich holatidan hisoblab, harakat traektoriyasi va traektoriya bo'ylab harakat qonuni aniqlansin.

Masala -6. Moddiy nuqtaning harakati $S=(2t^2-8t+6)\text{m}$ tenglama orqali berilgan (t -sekundlarda o'lchanadi). Qanday vaqt momentida

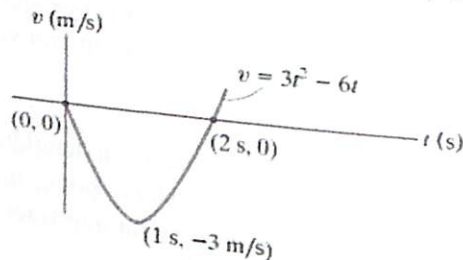
2. Nuqtaning berilgan trayektoriyadagi o'rnini aniqlash usuli

$$\begin{aligned} \dot{s} &= ds/dt \\ ds &= \dot{s} dt = (3t^2 - 6t) dt \\ \int_0^s ds &= \int_0^t (3t^2 - 6t) dt \end{aligned}$$

Tenglamani integrallasak va harakatning boshlang'ich shartlaridan foydalansak, nuqtaning istalgan vaqt momentida trayektoriyadagi o'rnini aniqlash uchun quyidagi tenglama (munbosabatga) ega bo'lamiz.

$$S = (t^3 - 3t^2) \text{ m}$$

Avtomobilning $t = 3.5 \text{ s}$ vaqt onidagi trayektoriyada egallagan o'rnini aniqlash uchun harakat grafigini tuzamiz (1.27-rasm).



1.27-rasm

Harakat grafigidan ko'rinib turibdiki, $0 < t < 2 \text{ s}$ vaqt oralig'ida avtomobil tezligi manfiy ishoraga ega bo'lar ekan va avtomobil O nuqtadan chap tomonga harakatlanar ekan.

$t > 2 \text{ s}$ dan boshlab, avtomobil tezligi musbat ishoraga ega bo'lib, u o'ng tomonga harakatlanar ekan. Avtomobil tezligi grafigida $t = 0$, $t = 2$ s va $t = 3.5 \text{ s}$ vaqt onlari uchun tezliklari ko'rsatilgan.

Avtomobil mazhur vaqt oralig'ida trayektoriyadagi o'rnini aniqlash uchun.

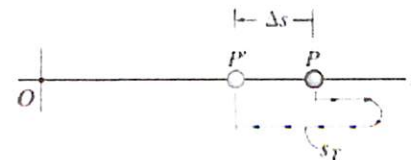
$$S = (t^3 - 3t^2)$$

Munosabatdan foydalanilamiz:

- $T = 0 \quad S = 0$
- $t = 2$
- $t = 3.5 \text{ s}$

O'rtacha tezlik har doim musbat kattalik hisoblanadi.

O'rtacha sur'at va o'rtacha tezlik quyidagi rasmdan yaqqol ko'rinadi:



1.24-rasm

Agar nuqtaning harakati tabiiy usulda berilgan bo'lsa, uning tezligi quyidagicha aniqlanadi:

$$\vec{v} = \frac{ds}{dt} \vec{r} = v_r \vec{r} \quad (1.79)$$

Bunda $\vec{r}$ - urinmaning birlik vektori, u yoy koordinatasi S ning o'sishi tomon yo'naladi.

Tezlik moduli quyidagi formula yordamida aniqlanadi:

$$v = \frac{ds}{dt} = s'$$

bunda: $v_r > 0$ bo'lsa, nuqta yoy koordinatasining o'sish tomoniga harakatlanadi.

$v_r < 0$ bo'lsa, nuqta yoy koordinatasining kamayishi tomoniga harakatlanadi.

Nuqta kinematikasida nuqtaning tezligini aniqlashga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

- Koordinata o'qlari sistemasini tanlab olinadi.
- Tanlab olingan koordinata o'qlari sistemasida nuqta harakatining tenglamalari tuziladi.
- Nuqta harakatining tenglamalariga ko'ra tezlik vektorining o'qlaridagi proyeksiyalari aniqlanadi.
- Nuqtaning tezligini o'qlaridagi proyeksiyalarga ko'ra uning miqdori va yo'nalishi aniqlanadi.

Poezdning harakat tenglamasini yozish uchun yoy koordinatasining sanoq boshini tanlashimiz kerak. Bunday nuqta sifatida poezdning boshlang'ich holatini olamiz va poezdning harakat yo'nalishini musbat yunalish deb qabul qilamiz. Bu holda $S_0=0$

Nuqtaning tekis sekinlanuvchan harakatida uning harakat tenglamasi va tezligi quyidagi formulalar asosida ifodalanadi:

$$s = v_0 t - \frac{a_\tau t^2}{2}, \quad (11)$$

$$v = v_0 - a_\tau t, \quad (12)$$

bunda a_τ – urinma tezlanish moduli.

Masala shartidan harakatdagi M nuqtaning yoy oxiridagi yoy koordinatasi $S=560$ m, boshlang'ich tezligi $v_0 = 36 \frac{\text{km}}{\text{s}} = 10 \text{ m/s}$, boshlang'ich tezlanish $a_0=0.125 \text{ m/s}^2$, hamda traektoriyaning egrilik radiusi $R=100 \text{ m}$ berilgan.

M nuqtaning yoy boshidagi normal tezlanishini quyidagi formula asosida aniqlaymiz:

$$a_{no} = \frac{v_0^2}{R} = \frac{100}{1000} = 0,1 \text{ m/s}^2.$$

M nuqtaning yoy boshidagi to'la tezlanishini bilgan holda, uning yoy boshidagi urinma tezlanishini aniqlaymiz:

$$a_0^2 = a_{no}^2 + a_\tau^2; \quad a_\tau = \sqrt{a_0^2 - a_{no}^2} = \sqrt{0,125^2 - 0,1^2} = 0,075 \text{ m/s}^2.$$

Nuqtaning harakati tekis sekinlanuvchan bo'lganligi uchun

$$a_\tau = \text{const.}$$

(11) va (12) tenglamalarga aniqlangan kattaliklarning qiymatlarini qo'yamiz:

$$560 = 10t - 0,075t^2, \quad (13)$$

$$v = 10 - 0,075 t. \quad (14)$$

Bu tenglamalardan harakatlanish vaqti t – aniqlanadi:

$$0,075t^2 - 20t + 1120 = 0$$

$$t = \frac{10 \pm \sqrt{100 - 1120 \cdot 0,075}}{0,075} = \frac{10 \pm 4}{0,075} \text{ s.}$$

Harakatlanish vaqti uchun kichik ildiz qiymatini tanlaymiz:

Raketaning maksimal balandlikdagi tezligi v_B bo'ladi. Shuning uchun

$$0 = (75 \text{ m/s})^2 + 2(-9,81 \text{ m/s}^2)(S_B - 40 \text{ m})$$

Bu ifodadan $S_B = 327 \text{ m}$,

Raketa C nuqtaga tushganda uning tezligi quyidagiga teng bo'ladi:

$$v_C^2 = v_B^2 + 2a_c(S_C - S_B) = 0 + 2(-9,81 \frac{\text{m}}{\text{s}^2})(0 - 327)$$

Bu ifodadan

$$v_C = -80,1 \text{ m/s}$$

$\vec{v}_C$ ning (-) ishorasi arametr pastga yo'nalganligidan darak beradi.

Raketaning yerga tushgandagi uning AC uchaskadagi harakatini o'rganishdan ham aniqlanadi.

$$v_C^2 = v_A^2 + 2a_c(S_C - S_B) = (75 \frac{\text{m}}{\text{s}})^2 + 2(-9,81 \frac{\text{m}}{\text{s}^2})(0 - 40);$$

Bundan

$$v_C = -80,1 \frac{\text{m}}{\text{s}}, \quad |v_C| = 80,1 \frac{\text{m}}{\text{s}}.$$

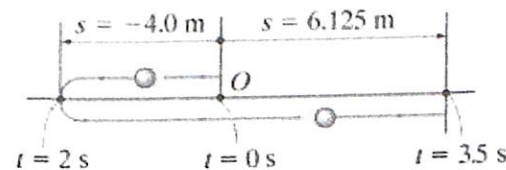
2.masala. Moddiy nuqta yo'lining qismida qismida

$v = (3t^2 - 6t) \text{ m/s}$ tezlik bilan harakatlanmoqda, bunda t sekundlarda o'lchanadi.

Agar, dastlab nuqta O holatda bo'lsa, 3,5s. Davomida nuqta bosib o'tgan masofa va shu vaqt orasidagi o'rtacha sur'at va o'rtacha tezlik aniqlansin.

Yechish. 1. Koordinata o'qini nuqtaning to'g'ri chiziqli harakati trayektoriyasi bo'ylab o'ng tomon yo'naltiramiz.

Koordinata boshi sifatida nuqtaning boshlang'ich ($t=0$) holatini tanlaymiz (1.26-rasm).



1.26-rasm

$$\sin^2\left(\frac{\pi t}{3}\right) + \cos^2\left(\frac{\pi t}{3}\right) = 1. \quad (20)$$

Masalada:

$$\sin^2\left(\frac{\pi t}{3}\right) = \frac{y}{4}, \quad \cos^2\left(\frac{\pi t}{3}\right) = \frac{x-2}{4}, \quad (21)$$

(20) ni e'tiborga olsak

$$\frac{y}{4} + \frac{x-2}{4} = 1, \text{ yoki } y = 6 - x.$$

Nuqtaning traektoriyasi to'g'ri chiziqdan iborat ekan.

Harakat tenglamalaridan foydalanib, nuqtaning $t=1/2$ ekunddagi koordinatalarini topamiz va shaklda ko'rsatamiz (1-36 rasm):

$$x = 4\cos^2\frac{\pi}{6} + 2 = 4\left(\frac{\sqrt{3}}{2}\right)^2 + 2 = 5 \text{ sm},$$

$$y = 4\sin^2\frac{\pi}{6} = 4 \cdot 0,25 = 1 \text{ sm} \quad (22)$$

Demak, $t=1/2$ sekundda nuqtaning koordinatalari $x=5, y=1$ bo'lar ekan.

Nuqtaning tezligini uning koordinata o'qlaridagi proeksiyalari orqali aniqlaymiz:

$$v = \sqrt{v_x^2 + v_y^2}.$$

Buning uchun nuqta harakat tenglamalaridan vaqt bo'yicha birinchi tartibli hosila olamiz:

$$v_x = \dot{x} = -\frac{8\pi}{3} \cos\left(\frac{\pi t}{3}\right) \sin\left(\frac{\pi t}{3}\right) = -\frac{4\pi}{3} \sin\left(\frac{2\pi t}{3}\right) \quad (23)$$

$$v_y = \dot{y} = \frac{8\pi}{3} \sin\left(\frac{\pi t}{3}\right) \cos\left(\frac{\pi t}{3}\right) = \frac{4\pi}{3} \sin\left(\frac{2\pi t}{3}\right). \quad (24)$$

$t=1/2$ s da,

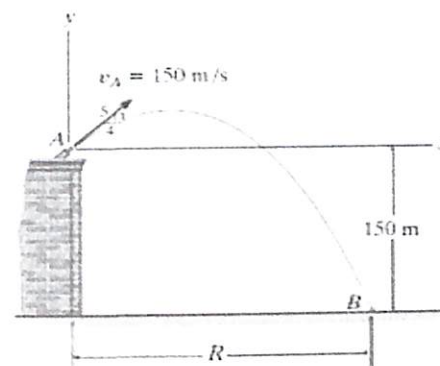
$$v_x = -\frac{4\pi^2}{3} \sin\left(\frac{2\pi}{6}\right) = -\frac{4\pi}{3} \cdot \frac{\sqrt{3}}{2} = -3,6 \text{ m/s},$$

$$v_y = \frac{4\pi^2}{3} \sin\left(\frac{2\pi}{6}\right) = \frac{4\pi}{3} \cdot \frac{\sqrt{3}}{2} = 3,6 \text{ sm/s}.$$

Binobarin,

$$v = \sqrt{v_x^2 + v_y^2} = 5,1 \text{ m/s}.$$

Tezliklar uchun masshtab tanlab, ularni shaklda ko'rsatamiz (1.36 rasm).



1.47 – rasm.

15-§ Talabalar mustaqil o'rganishi uchun keyslar

Nuqta harakatining berilgan tenglamalariga ko'ra uning tezligi va tezlanishini aniqlash.

M nuqtaning berilgan harakat tenglamalariga ko'ra traektoriya-sining ko'rinishi aniqlansin va $t=t_1(s)$ vaqt oni uchun nuqtaning traektoriyadagi o'rni, uning tezligi, to'la, urinma va normal tezlanish-lari, hamda traektoriyaning egrilik radiusi topilsin.

Topshiriqni yechish uchun zarur bo'lgan ma'lumotlar quyidagi jadvalda keltirilgan.

Variantlar raqamlari	Harakat tenglamalari		$t_1,$ s
	$x = x(t), \text{ sm}$	$y = y(t), \text{ sm}$	
1.	$x = 3t$	$y = 4t^2 + 1$	$\frac{1}{2}$
2.	$x = 7\sin^2\left(\frac{\pi t}{6}\right) - 5$	$y = -7\cos^2\left(\frac{\pi t}{6}\right)$	1
3.	$x = 1 + 3\cos\left(\frac{\pi t^2}{3}\right)$	$y = 3\sin\left(\frac{\pi t^2}{3}\right) + 3$	1
4.	$x = -5t^2 - 4$	$y = 3t$	1

$$(2) \text{ dan } t = \frac{v_0 \sin \alpha_0}{g} \pm \sqrt{\frac{v_0^2 \sin^2 \alpha_0}{2g} - \frac{2y}{g}} \quad (4)$$

(3) va (4)larning o'ng tomonlarini tenglashtirsak, quyidagi tenglamaga ega bo'lamiz:

$$\frac{x}{v_0 \cos \alpha_0} = \frac{v_0 \sin \alpha_0}{g} \pm \sqrt{\frac{v_0^2 \sin^2 \alpha_0}{2g} - \frac{2y}{g}}$$

$$\left(\frac{x}{v_0 \cos \alpha_0} - \frac{v_0 \sin \alpha_0}{g} \right)^2 = \left(\frac{v_0^2 \sin^2 \alpha_0}{2g} - \frac{2y}{g} \right)$$

4-masala. Nuqtaning harakati.

$$x=2t, \quad y=t^2 \quad (1)$$

tenglamalar bilan berilgan (t – sekundlarda, x va y – santimetrlarda o'lchanadi).

$t=1$ vaqt uchun tezlik va tezlanishning qiymati topilsin va shaklda ko'rsatilsin.

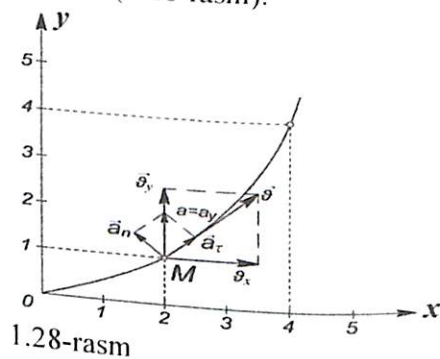
Yechish. Traektoriya tenglamasini tuzish uchun harakat tenglamalarining biridan vaqt t ni aniqlab, ikkinchisiga qo'yamiz:

$$t = \frac{x}{2}, \quad y = \frac{x^2}{4}. \quad (2)$$

Bu tenglama parabola tenglamasi. Binobarin, nuqtaning traektoriyasi paraboladan iborat ekan.

Traektoriyani chizish uchun (2) tenglamada x ga qiymatlar berib, unga mos y ning qiymatlarini topamiz (1.28-rasm).

x	0	2	4
y	0	1	4



22.	$x = 7 \sin\left(\frac{\pi t^2}{6}\right) + 3$	$y = 2 - 7 \cos\left(\frac{\pi t^2}{6}\right)$	1
23.	$x = 3t^2 - t + 1$	$y = 5t^2 - \frac{5}{3}t - 2$	1
24.	$x = 3t^2 + 2$	$y = -4t$	$\frac{1}{2}$
25.	$x = 2 \sin\left(\frac{\pi t}{3}\right)$	$y = -3 \cos\left(\frac{\pi t}{3}\right) + 4$	1
26.	$x = 4t + 4$	$y = -\frac{4}{(t+1)}$	2
27.	$x = -2t^2 + 3$	$y = -5t$	$\frac{1}{2}$
28.	$x = 4 \cos^2\left(\frac{\pi t}{3}\right) + 2$	$y = 4 \sin^2\left(\frac{\pi t}{3}\right)$	1
29.	$x = -\cos\left(\frac{\pi t^2}{3}\right) + 3$	$y = \sin\left(\frac{\pi t^2}{3}\right) - 1$	1
30.	$x = -3 - 9 \sin\left(\frac{\pi t^2}{6}\right)$	$y = -9 \cos\left(\frac{\pi t^2}{6}\right) + 5$	1

Muammo -2. Shar vertikal holda yuqoriga Yerdan 15 m/s tezlik bilan harakatlana boshlagan. Shar Yerga qancha vaqt o'tgach qaytip tushadi? (1.30-rasm).

Muammo -3. Harakatdagi nuqtaning traektoriyadagi o'ni $S=(2t^2-8t+6) \text{ m}$ masofa orqali aniqlanadi. Harakat boshlangandan qanday vaqt o'tgach nuqta tezligi 0 ga teng bo'ladi? Nuqta $t=3 \text{ s}$ vaqt davomida qanday masofani bosib o'tadi? (1.31-rasm)



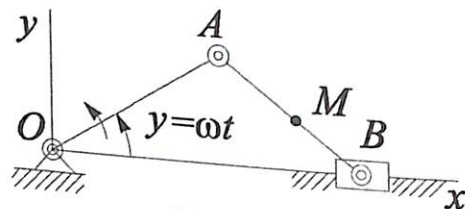
1.30-rasm



1.31-rasm

Muammo -4. Nuqta to'g'ri chiziq bo'ylab $a=(12t-3t^2) \text{ m/s}^2$ tezlashish bilan harakatlanmoqda, bunda t sekundlarda o'lanadi. Nuqtaning tezligi va to'g'ri chiziqdagi holati (o'ni) vaqt funksiyasi sifatida aniqlansin. $t=0$ da $v=0$, $S=15 \text{ m}$ bo'lgan.

Muammo -5. OA krivoship ω o'zgarmas burchak tezlik bilan aylanadi. Krivoship n-polzunli mexanizm shatunining o'rtasidagi M nuqtaning tezligi va polzunning tezligi vaqt funksiyasi sifatida topilsin; $OA=AB=a$ (1.32-rasm)



1.32-rasm.

Muammo -6. Elektrovozning tezligi $v_0=72 \text{ km/soat}$; g'ildiragining radiusi $R=1 \text{ m}$; g'ildirak to'g'ri chiziqli temir izda sirpanmasdan g'ildirab boradi. G'ildirak gardishidagi M nuqtaning radiusi v_0 tezlik yo'nalishi bilan $\frac{\pi}{2}+\alpha$ burchak hosil qilgan paytda shu nuqta v tezligining miqdori va yo'nalishi aniqlansin (1.33-rasm).

Avtomobilning $t=3.5 \text{ s}$ vaqt davomida bosib o'tgan masofa quyidagicha aniqlanadi:

$$S_r=4.0+4.0+6.125=14.125=14.12 \text{ m}$$

Avtomobil $t=0$ dan $t=3.5 \text{ s}$ vaqt oralig'ida ko'chishi quyidagiga teng

$$\Delta S = S|_{t=3.5s} - S|_{t=0} = 6.125 \text{ m} - 0 = 6.125 \text{ m}.$$

Buni e'tiborga olsak, shu vaqt orasidagi o'rtacha sur'at (tezlikni o'zgarish jadalligi) quyidagiga teng bo'ladi:

$$v = \frac{\Delta S}{\Delta t} = \frac{6.125 \text{ m}}{3.5 \text{ s} - 0} = 1.75 \text{ m/s}.$$

O'rtacha tezlik esa

$$v = \frac{S_r}{\Delta t} = \frac{14.125 \text{ m}}{3.5 \text{ s} - 0} = 4.04 \text{ m/s}$$

3. masala. Nuqta harakati.

$$x = v_0 t \cos \alpha_0$$

$$y = v_0 t \sin \alpha_0 - \frac{1}{2} g t^2$$

Tenglamalar bilan berilgan; Ox o'q gorizontal, Oy o'q parametr bo'yicha yuqoriga yo'nalgan, v_0 , g va $\alpha_0 < \frac{\pi}{2}$ doimiy miqdorlar.

1. Nuqta trayektoriyasi.
2. Uning yuqori holatining koordinatalari
3. Nuqta Ox o'qda bo'lgan paytdagi tezligining koordinata o'qlaridagi proyeksiyalari topilsin.

Yechilishi:

1) Nuqtaning trayektoriyasini aniqlaymiz.

Masala shartiga nuqta trayektoriyasining parametrik tenglamalari berilgan

Koordinatalar formasidagi trayektoriya tenglamasini tuzish uchun

berilgan tenglamalardan parameter "t" ni qisqartiramiz:

$$x = v_0 t \cos \alpha_0 \quad (1)$$

$$y = v_0 t \sin \alpha_0 - \frac{1}{2} g t^2 \quad (2)$$

$$(1) \text{ dan } t = \frac{x}{v_0 \cos \alpha_0}; \quad (3)$$

formula asosida aniqlanadi.

Bu ifodada $\vec{a}_\tau$ va $\vec{a}_n$ lar nuqtaning urinma va normal tezlanishlarini ifodalaydi.

Bunday holda tezlanish moduli

$$a = \sqrt{a_\tau^2 + a_n^2}$$

formula asosida hisoblanadi.

Tezlanishning yo'nalishi esa quyidagi formulada aniqlanadi.

$$\operatorname{tg} \mu = \frac{|a_\tau|}{a_n}$$

Nuqta kinematikasida nuqtaning tezlanishini aniqlashga doir masalalarni quyidagi tartibda yechish tavsiya etiladi.

1. Koordinata o'qlari sistemasi tanlab olinadi.
2. Tanlab olingan koordinata o'qlari sistemasida nuqta harakatining tenglamalari tuziladi.
3. Nuqta harakatining tenglamalariga ko'ra tezlanish vektorining o'qlardagi proyeksiyalari aniqlanadi.
4. Nuqtaning tezlanishini o'qlardagi proyeksiyalariga ko'ra uning miqdori va yo'nalishi aniqlanadi.

Agar moddiy nuqtaning tezlanishi mavxum bo'lsa, u orqali nuqta harakatining tenglamalari va trayektoriyasini aniqlash mumkin.

Nuqta harakatining tezlanishi orqali uning harakati tenglamalarini va trayektoriyasini aniqlashda quyidagi amallarni bajarish tavsiya etiladi:

1. Koordinata o'qlari sistemasi tanlab olinadi.
2. Tezlanishning tanlab olingan o'qlardagi proyeksiyalari aniqlanadi.
3. Hosil bo'lgan tenglamani integrallab, nuqta tezligining o'qlardagi proyeksiyalari aniqlanadi.
4. Nuqta tezligining ma'lum vaqt oni uchun mumkin bo'lgan qiymatlaridan foydalanib hosil bo'lgan ifodalarda ishtirok etuvchi integrallash o'zgaraslarini aniqlanadi.
5. Hosil bo'lgan tezlikning o'qlardagi proyeksiyalari bo'lmish ifodalarni integrallab, nuqtaning harakat tenglamalari aniqlanadi.

t sekundda nuqtaning traektoriyada o'rnini topamiz. Buning uchun berilgan harakat tenglamalaridagi t ning o'rniga uning qiymatini qo'yib, nuqtaning koordinatalarini topamiz.

$$t=1\text{s. da} \quad x=2\text{sm}, \quad y=1\text{sm}$$

Demak, $t=1$ sekundda nuqtaning koordinatalari (2,1) bo'lar ekan

Nuqtaning tezligini koordinata o'qlaridagi proeksiyalari orqali aniqlaymiz:

$$v_x = \dot{x} = 2 \text{ sm/s}, \quad (v_x = \text{const}), \quad v_y = \dot{y} = 2t \text{ sm/s}. \quad (3)$$

$$t=1\text{s. da}, \quad v_x=2\text{m/s} \quad v_y=2*1=2\text{m/s}$$

Natijada

$$v = \sqrt{v_x^2 + v_y^2} = 2\sqrt{2} \text{ sm/s}. \quad (4)$$

tezlik uchun masshtabni 1 smda 2 m/s deb tanlaymiz va chizmada ko'rsatamiz (4- rasm).

11-§ Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

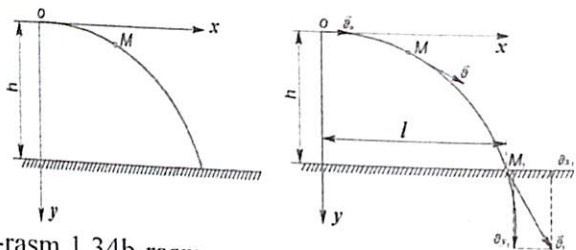
Muammo -1. Moddiy nuqta to'g'ri chiziq bo'ylab $v=(4t-3t^2)$ m/s tezlik bilan harakatlanmoqda, bu ifodada t sekundlarda o'lchanadi. Agar $t=0$ da $S=0$ bo'lsa, nuqtaning $t=4\text{s}$ da traektoriyadagi o'rni aniqlansin (1.29-rasm).



1.29-rasm

$$l = x_1 = 60t = 60 \cdot 8 = 480 \text{ m}.$$

Demak, yukning samolyotdan tashlash va yerga tushish nuqtalari orasidagi gorizontal masofa 480 m ekan.



1.34a-rasm 1.34b-rasm

Yukning tushish nuqtasidagi tezligi va tezlanishini aniqlaymiz. Yukning tezligi uning koordinata o'qlaridagi proeksiyalari orqali aniqlanadi:

$$v_1 = \sqrt{v_{x1}^2 + v_{y1}^2} = \sqrt{60^2 + (10t)^2}. \quad (6)$$

Yuk yerga tushganda $t_1=8\text{s}$ shuning uchun

$$v_1 = \sqrt{3600 + 6400} = 100 \text{ m/s}.$$

Yukning tezlanishi ham uning tezligi kabi aniqlanadi (1v-rasm).

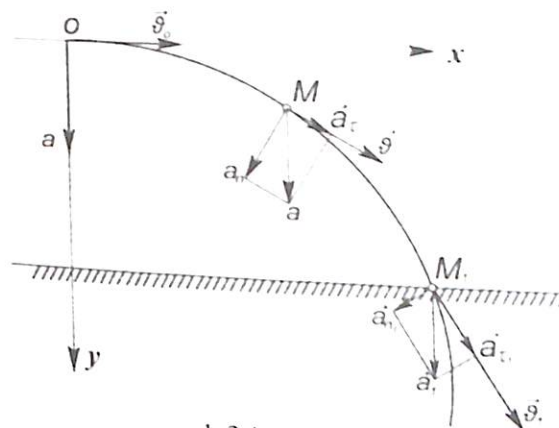
$$a_1 = \sqrt{a_{x1}^2 + a_{y1}^2},$$

$$a_{x1} = \frac{dv_{x1}}{dt} = 0, \quad a_{y1} = \frac{dv_{y1}}{dt} = 10.$$

Shuning uchun

$$a_1 = \sqrt{a_{x1}^2 + a_{y1}^2} = 10 \text{ m/s}^2. \quad (7)$$

Yukning yerga tushish nuqtasida traektoriyaning egrilik radiusini aniqlash uchun uning *urinma* va *normal* tezlanishini aniqlaymiz (1.34v-rasm).



1.34v-rasm

Yukning *urinma* tezlanishini quyidagi formula yordamida aniqlaymiz:

$$a_t = \left| \frac{dv}{dt} \right| = \frac{v_x a_x + v_y a_y}{v} = 8 \text{ m/s}^2. \quad (8)$$

Yukning *normal* tezlanishi quyidagicha aniqlanadi:

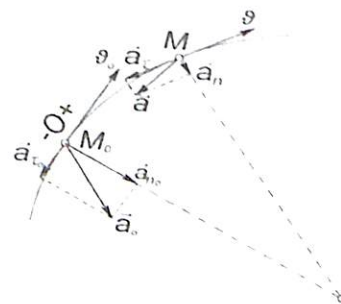
$$a^2 = a_t^2 + a_n^2, \quad a_n = \sqrt{a^2 - a_t^2} = \sqrt{100 - 64} = 6 \text{ m/s}^2. \quad (9)$$

Traektoriyaning yuk tushgan a_0 nuqtasining egrilik radiusi

$$\rho = \frac{v^2}{a_n} = \frac{100^2}{6} = 1667 \text{ m}. \quad (10)$$

2-Masala.

Poezd radiusi $R=1 \text{ km}$ bo'lgan aylana yoyi bo'ylab tekis sekinlanuvchan harakat qiladi va $S=1 \text{ m}$ yo'l bosadi. Uning boshlang'ich tezligi $v_0 = 36 \frac{\text{km}}{\text{soat}}$, boshlang'ich tezlanishi esa $a_0=0.125 \text{ m/s}^2$. Poezdning yoy oxiridagi tezligi va tezlanishi aniqlansin (1.35-rasm).



1.35-rasm

Yechimi.

Poezd nuqtalaridan birining, masalan, og'irlik markazining harakatini o'rganamiz.

Hisoblash natijalarini quyidagi jadvalda joylashtiramiz:

Nuqta koordinatalari (sm)		Nuqta tezligi (sm/s)			Nuqta tezlanishi (sm/s ²)					Egrilik radiusi (sm)
x	y	v_x	v_y	v	a_x	a_y	a	a_τ	a_n	ρ
5	1	-3,6	3,6	5,1	-4,4	4,4	6,2	6,2	0	∞

Aniqlangan kattaliklar 1.36-rasmda ko'rsatilgan.

4-masala.

Avtomobil yo'lining to'g'ri chiziqli uchastkasida ma'lum qisqa vaqt harakatlanib $v=(3t^2+2t)$ m/s tezlikka ega bo'ladi (ifodada t sekundda o'lchanadi), harakat boshlangan vaqtda $t_1=3$ s o'tgach avtomobilning bisib o'tgan yo'li va tezlanishi aniqlansin. $t=0$ da $S=0$ bo'lgan.

Yechimi.

1. Koordinata o'qini avtomobil harakati tomon yo'naltiramiz.
2. Avtomobilning $t=3$ s da bosib o'tgan yo'lini aniqlaymiz. Koordinata boshi sifatida avtomobilning boshlang'ich holatini tanlaymiz. Nuqtaning trayektoriyadagi o'rni $v = \frac{ds}{dt}$ formuladan aniqlanadi. $t=0$ da $S=0$ bo'lgan. $v = \frac{ds}{dt} = (3t^2 + 2t)$

$$\int_0^3 ds = \int_0^3 (3t^2 + 2t) dt$$

$$S = t^3 + t^2$$

Harakat boshlangandan $t=3$ s vaqt o'tgach avtomobil bosib o'tgan yo'l quyidagiga teng bo'ladi.

$$S = (3)^3 + (3)^2 = 36 \text{ m}$$

3. Avtomobilning $t=3$ s dagi tezlanishini aniqlaymiz.

$$a = \frac{dv}{dt} = \frac{d(3t^2 + 2t)}{dt} = 6t + 2$$

Harakat boshlangandan $t=3$ s vaqt o'tgach

$$a = 6(3) + 2 = 20 \text{ m/s}^2$$

$$t = \frac{6}{0.075} = 80 \text{ s}, \quad (15)$$

chunki katta ildiz qiymati (187s) nuqtaning to'xtashi uchun ($v=0$) ketgan vaqtdan katta.

$$(t_{\text{to'xtash}} = \frac{v_0}{a} = \frac{10}{0.075} = 133 \text{ sek.}). \quad (16)$$

(1.12) tenglamadan nuqtaning yoy oxiridagi tezligini aniqlaymiz:

$$v = v_0 - at = 10 - 0.075 \cdot 80 = 4 \text{ m/s}. \quad (17)$$

Nuqtaning yoy oxiridagi normal tezlanishi quyidagiga teng bo'ladi:

$$a_n = \frac{v^2}{R} = \frac{4^2}{1000} = 0,016 \text{ m/s}^2. \quad (18)$$

Nuqtaning yoy oxiridagi to'la tezlanishi quyidagi formuladan aniqlanadi:

$$a = \sqrt{a_\tau^2 + a_n^2} = \sqrt{(0,075)^2 + (0,016)^2} = 0,0767 \text{ m/s}^2. \quad (19)$$

Aylana yoyi bo'ylab tekis sekinlanuvchan harakatda nuqtaning urinma tezlanishining moduli o'zgarmaydi. to'la tezlanish moduli esa, normal tezlanish modulining kamayishi tufayli kamayadi.

Aniqlangan tezlik va tezlanishlar 1.36-rasmda tasvirlangan.

3-Masala.

M nuqtaning berilgan harakat tenglamalariga ko'ra traektoriyasining ko'rinishi aniqlansin va $t=t_1$ vaqt oni uchun nuqtaning traektdagi o'rni, uning tezligi, to'la, urinma va normal tezlanishlari, hamda traektoriyaning egrilik radiusi topilsin.

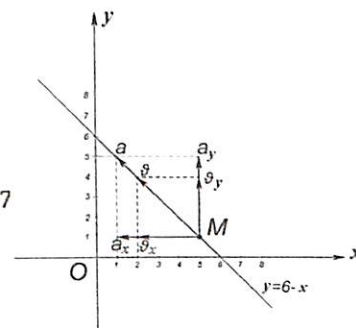
$$x = 4 \cos^2\left(\frac{\pi t}{3}\right) + 2 \text{ sm},$$

$$y = 4 \sin^2\left(\frac{\pi t}{3}\right) \text{ sm}.$$

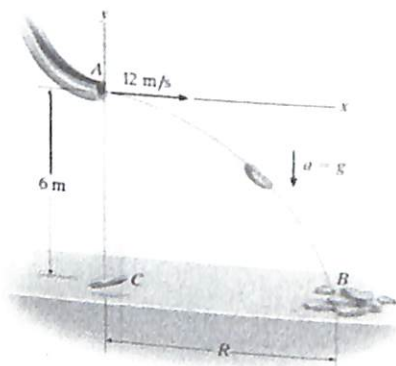
$$t = 1/2 \text{ s}.$$

Yechimi.

Nuqtaning traektoriyasini aniqlaymiz. Traektoriya tenglamasini tuzish uchun harakat tenglamalaridan t vaqtini yo'qotamiz. Buning uchun, berilgan masalada quyidagi ayniyatdan foydalanamiz:

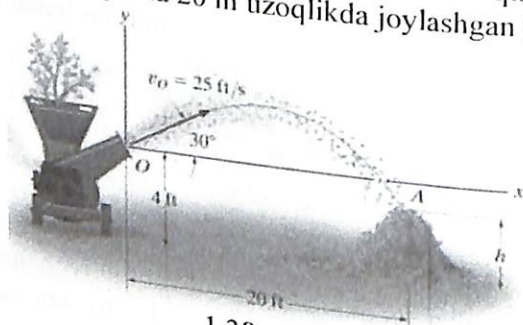


5-muammo. Qop nishablikda harakatlanib, A nuqtaga $v_A = 12 \text{ m/s}$ gorizontaal tezlikka ega bo'ladi. Agar A nuqtaning poldan balandligi 6 m bo'lsa, qopning B nuqtaga tushishi uchun ketadigan vaqt va B nuqtaning C nuqtadan qanday uzoqlikda yotishi aniqlansin (1.37-rasm).



1.37 - rasm

6-muammo. Yog'och qirqadigan mashinaning a nuqtasidan yog'och qirindisi $v_0 = 25 \text{ m/s}$ tezlik bilan otilib chiqadi. Agar qirindining otilib chiqish tezligi gorizontaal bilan 90° burchak tashkil qilsa, uning A tushish nuqtasining yerdan qanday h balandlikda bo'lishi aniqlansin. A nuqta qirindining otilib chiqishi nuqtasidan yer bo'ylab hisoblanganda 20 m uzoqlikda joylashgan (1.38-rasm).



1.38 -rasm

Nuqtaning tezlanishini uning koordinata o'qlaridagi proeksiyalari orqali aniqlaymiz:

$$a = \sqrt{a_x^2 + a_y^2}. \quad (25)$$

Buning uchun ϑ_x , ϑ_y lardan vaqt bo'yicha birinchi tartibli hosila olamiz:

$$a_x = \vartheta'_x = \ddot{x} = -\frac{8\pi^2}{9} \cos\left(\frac{2\pi t}{3}\right),$$

$$a_y = \frac{8\pi^2}{9} \cos\left(\frac{2\pi t}{3}\right). \quad (26)$$

$t = 1/2 \text{ s}$ da,

$$a_x = -\frac{8\pi^2}{9} \cos\left(\frac{2\pi}{6}\right) = -\frac{8\pi^2}{9} \cdot 0,5 = -4,4 \text{ sm/s}^2.$$

$$a_y = \frac{8\pi^2}{9} \cos\left(\frac{2\pi}{6}\right) = \frac{8\pi^2}{9} \cdot 0,5 = 4,4 \text{ sm/s}^2.$$

Binobarin,

$$a = \sqrt{a_x^2 + a_y^2} = 6,2 \text{ m/s}^2.$$

Nuqtaning urinma tezlanishi quyidagiga teng bo'ladi:

$$a_\tau = \left| \frac{d\vartheta}{dt} \right| = \frac{\vartheta_x a_x + \vartheta_y a_y}{\vartheta} = 6,2 \text{ sm/s}^2. \quad (27)$$

Nuqtaning normal tezlanishi quyidagicha aniqlanadi:

$$a^2 = a_\tau^2 + a_n^2; a_n = \sqrt{a^2 - a_\tau^2} = 0. \quad (28)$$

Tezlanishlar uchun masshtab tanlab, ularni shaklda ko'rsatamiz (1.37-rasm).

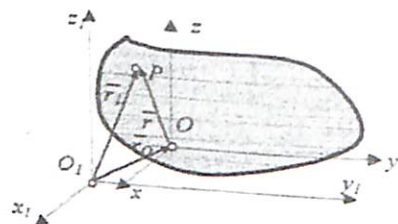
Traektoriyaning egrilik radiusi quyidagi formula asosida aniqlanadi:

$$a_n = \frac{v^2}{\rho}; \quad \rho = \frac{v^2}{a_n} = \infty. \quad (29)$$

Masalada, nuqtaning traektoriyasi to'g'ri chiziq bo'lganligi uchun, egrilik radiusi ∞ ga teng.

esa – jismning ilgarilanma harakat tezlanishi deyiladi. Ilgarilanma harakat tezlik vektori $\vec{v}$ va tezlanish vektori $\vec{a}$ larni jismning ixtiyoriy nuqtasiga qo'yilgan holda ko'rsatish mumkin. Bu hol qattiq jismning faqat ilgarilanma harakatida o'rinli bo'ladi. Boshqa harakatlarda jismning turli nuqtalari turlicha tezlik va turlicha tezlanishga ega bo'ladi.

Qattiq jismning ilgarilanma harakatida jism O nuqtasining tezligi $\vec{v}_0 \neq 0$ bo'lib, burchak tezlik $\vec{\omega}_0 = 0$ bo'ladi (2.3-rasm).



2.3-rasm

Rasmdan

$$\vec{r}_1 = \vec{r}_0 + \vec{r}$$

Ilgarilanma harakat ta'rifidan,

$$\vec{r} = \text{const.}$$

Shuning uchun, $\frac{d\vec{r}}{dt} = 0$.

$$\frac{d\vec{r}}{dt} = 0; \frac{d\vec{r}_1}{dt} = \frac{d\vec{r}_0}{dt} \text{ bo'ladi.}$$

Bundan,

$$\vec{v}_p = \vec{v}_0$$

natija kelib chiqadi.

Yuqoridan qattiq jismning ilgarilanma harakatida uning barcha nuqtalarining tezlanishini ham bir xil miqdor va yo'nalishga ega bo'lishi ma'lum bo'ladi:

$$\vec{a}_p = \vec{a}_0$$

2.4a-rasmda tramvayning ilgarilanma harakatida uning A va B nuqtalarining trayektoriyasi ko'rsatilgan.

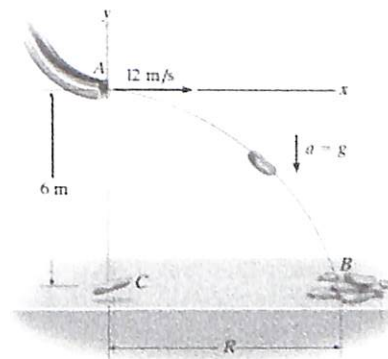
14-§ Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

1-muammo.

Moddiy nuqta to'g'ri chiziqli bo'ylab $a = (10 - 0.25)m/s^2$ tezlanish bilan harakatlanadi (S-metrlarda o'lchanadi). Nuqta S = 10 m masofani bosib o'tgach qanday tezlikka ega bo'ladi. Nuqtaning boshlang'ich tezligi $v_0 = 5 m/s$ ga teng.

2-muammo.

Moddiy nuqta to'g'ri chiziqli bo'ylab $v = (20 - 0.05s^2) m/s$ tezlik bilan harakatlanmoqda (S-metrlarda o'lchanadi). Nuqta S = 15 m masofani bosib o'tgach qanday tezlanishga ega bo'ladi?



3-muammo.

Moddiy nuqtaning trayektoriyadagi holati to'g'ri chiziqli harakatida

$$S = (1.5t^3 - 13.5t^2 + 22.5t)$$

tenglama orqali aniqlanadi. Moddiy nuqtaning $t = 6 s$ bo'lganda trayektoriyadagi holati va 6 s vaqt intervalida bosib o'tgan yo'li aniqlansin.

4-muammo.

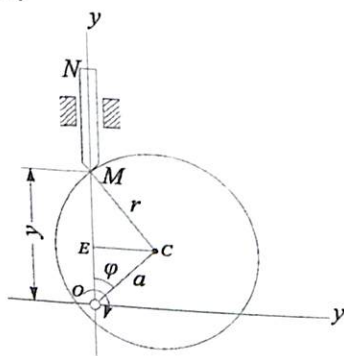
Moto to'g'ri chiziqli harakatida uning trayektoriyadagi holati rasmda ko'rsatilgan grafik asosida aniqlanadi.

$0 \leq t \leq 30 s$ vaqt intervali uchun $v-t$ va $a-t$ garafiklari tuzilsin.

18-§ Qattiq jismning ilgarilanma harakatiga doir masalalar

1-masala. Diametri $d=2r$ bo'lgan eksentrik O nuqata atrofida o'tuvchi o'q aylanadi; bunda φ burchak $\varphi = \frac{\pi}{2}t$ qonunga muvofiq o'zgaradi. Eksentrik geometrik markazi bo'lgan C va O nuqtalar orasidagi masofa $OC = a = \frac{r}{3}$. Vertikal yo'nalishda harakatlanuvchi MN sterjen M nuqtasining to'g'ri chiziqli harakat tenglamasi tuzilsin (2.3- rasm) hamda $t_1=1s$ vaqt oni uchun mazkur nuqtaning tezligi va tezlanishi aniqlansin.

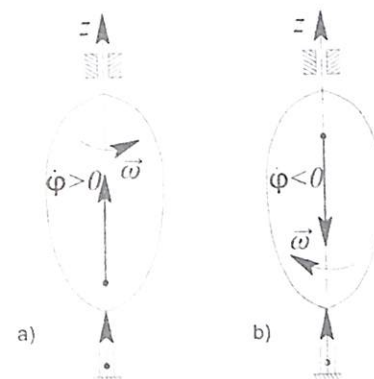
Yechimi. Masala shartiga ko'ra MN sterjen O nuqtadan o'tuvchi vertikal chiziq bo'ylab to'g'richiziqli harakatda bo'ladi, yani MN sterjen ilgarilanma harakat sodir etadi. Shuning uchun sterjenning M nuqtasi ham O nuqtasidan o'tuvchi to'g'ri chiziq bo'ylab harakatlanadi.



2.3- rasm

Mazkur to'g'ri chiziq bo'ylab Oy koordinata o'qini o'tqazamiz; koordinata boshi sifatida O nuqta olinadi; Ox o'qi gorizontal holda yo'naltiriladi. Rasmdan $OM=y$; Bu masofa vaqtga bog'liq holda o'zgaradi. Bu holni aniqlash uchun OM masofani φ burchak orqali ifodalash lozim. Buning uchun C nuqtadan mos uchburcphakning OM tomoniga CE balandlikni o'takazamiz. Natijada $OM=OE+EM$ Lekin $OE=a \cos \varphi$, $EC=a \sin \varphi$.

$$EM = \sqrt{(MC)^2 - (EC)^2} = \sqrt{r^2 - a^2 \sin^2 \varphi},$$



2.14-rasm

Bu holda $\frac{d\varphi}{dt} = \omega_0 = \text{const}$, shuning uchun

$$d\varphi = \omega_0 dt \quad (2.9)$$

Vaqt 0 dan t gacha o'zgarganda aylanish burchagi φ_0 dan φ gacha o'zgarishini e'tiborga olib, (2.9) ni integrallasak

$$\varphi = \varphi_0 + \omega_0 t \text{ bo'ladi.} \quad (2.10)$$

(2.10) ifoda jismning tekis aylanma harakat tenglamasini ifodalaydi.

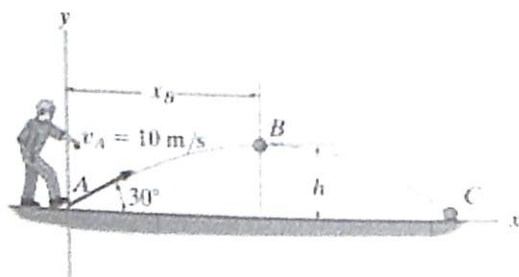
Texnikada tekis aylanma harakatda burchak tezlik ko'pincha bir minutdagi aylanishlar soni bilan o'lchanadi.

Jism bir marta to'la aylanganda $\varphi=2\pi$ bo'ladi. Agar jism bir minutda n marta aylansa, tekis aylanma harakatning burchak tezligi quyidagiga teng bo'ladi:

$$\omega = \frac{2\pi n}{60} = \frac{\pi n}{30} \text{ rad/s.} \quad (2.11)$$

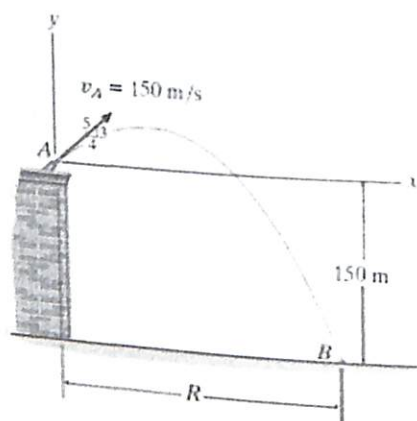
22-§ Qo'zg'almas o'q atrofida aylanma harakatda bo'lgan jismning burchak tezlanishi. Tekis o'zgaruvchan aylanma harakat

Burchak tezlanishi aylanma harakatda bo'lgan jism burchak tezligining o'zgarishini ifodalovchi kattalik bo'lib, u burchak tezligidan vaqt bo'yicha olingan birinchi tartibli hosilaga yoki aylanish



1.41 - rasm

10-muammo. Koptok A nuqtada $v_A = 10 \text{ m/s}$ tezlik bilan tepiladi. Koptokning uchish uzoqligini va Yerga tushgandagi tezligi aniqlansin (1.42 – rasm).



1.42- rasm

11-muammo. Basketbol to'pini A nuqtadan gorizont bilan $\alpha = 30^\circ$ burchak hosil qiluvchi v_A tezlik bilan otilib, Yerdan 3m balandlikda turuvchi basketbol setkasiga tushadi. Basketbol to'pining otilish tezligi v_A aniqlansin (1.43 – rasm).

Bunday holda

$$\frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2} = \varepsilon = \varepsilon_0 = \text{const.} \quad (2.14)$$

Vaqt 0 dan t gacha o'zgarganda, burchak tezlik ω dan ω_0 gacha o'zgarishini e'tiborga olib, (2.14) ni integrallasak,

$$\omega = \omega_0 + \varepsilon t \quad (2.15)$$

bo'ladi. (2.15) tenglik yordamida tekis o'zgaruvchan aylanma harakat burchak tezligi aniqlanadi.

Tekis o'zgaruvchan aylanma harakat tenglamasini ifodalash uchun (2.15) ni quyidagicha yozamiz:

$$\frac{d\varphi}{dt} = \omega_0 + \varepsilon t. \quad (2.16)$$

Bundan,

$$d\varphi = (\omega_0 + \varepsilon t) dt. \quad (2.17)$$

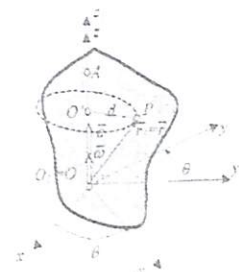
(2.10) ni e'tiborga olib, (2.17) ni integrallasak,

$$\varphi = \varphi_0 + \omega_0 t + \frac{\varepsilon t^2}{2} \quad (2.18)$$

ko'rinishdagi tekis o'zgaruvchan aylanma harakat tenglamasi kelib chiqadi.

Qattiq jismning qo'zg'almas o'q atrofida aylanma harakatini quyidagi rasm orqali soddaroq holda tushuntirish mumkin (2.16-rasm)

Rasmda θ burchak orqali jismning OZ aylanish o'qi atrofidagi aylanma harakatida burilish burchagi ko'rsatilgan (OXYZ – koordinata o'qlari sistemi qo'zg'aluvchan sistema, u jism bilan bog'langan).

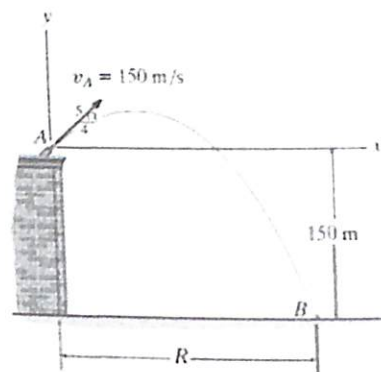


2.16-rasm

Chizmada $\vec{r}_i$ vektor orqali jism ixtiyoriy nuqtasining radius vektori ko'rsatilgan. Shuning uchun, $\vec{r}_i = \vec{r}$ bo'lib, jismning aylanma harakat burchak tezligi va burchak tezlanishlari quyidagicha aniqlanadi:

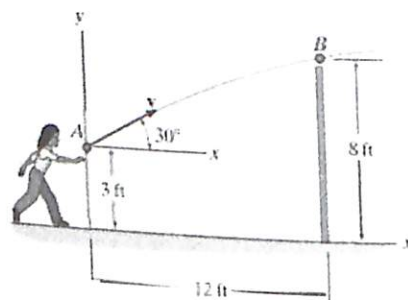
$$\omega = \omega_z = \theta'$$

$$\varepsilon = \frac{d\omega}{dt} = \frac{d\omega_z}{dt} = \theta''$$



1.45 – rasm. Рашм йўқ.

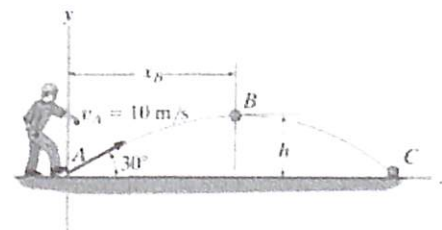
14-muammo. To'p A nuqtadan otiladi. U Yerdan 8m, otilish nuqtasidan 12m masofada joylashgan B nuqtaga tushishi uchun, qanday tezlikda otiladi? (1.46- rasm).



1.46 – rasm

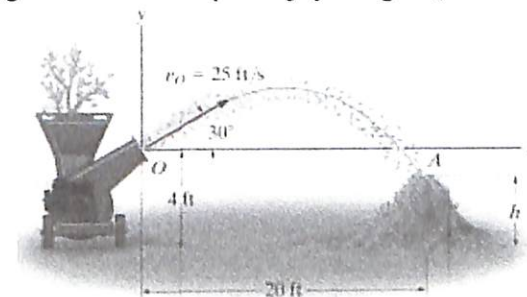
15-muammo. Reaktiv snaryad A nuqtadan $v_A=250\text{m/s}$ tezlik bilan otiladi. Agar A nuqta Yerdan 150m balandlikda joylashgan bo'lsa, snaryadning uchish uzoqligi aniqlansin (1.47 – rasm).

7-muammo. Qop nishablikda harakatlanib, A nuqtaga $v_A=12\text{m/s}$ gorizontal tezlikka ega bo'ladi. Agar A nuqtaning poldan balandligi 6m bo'lsa, qopning B nuqtaga tushishi uchun ketadigan vaqt va B nuqtaning C nuqtadan qanday uzoqlikda yotishi aniqlansin (1.39- rasm).



1.39- rasm

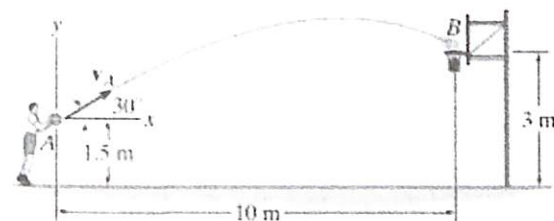
8-muammo. Yog'och qirqadigan mashinaning a nuqtasidan yog'och qirindisi $v_0=25\text{m/s}$ tezlik bilan otilib chiqadi. Agar qirindining otilib chiqish tezligi gorizontal bilan 90° burchak tashkil qilsa, uning A tushish nuqtasining yerdan qanday h balandlikda bo'lishi aniqlansin. A nuqta qirindining otilib chiqishi nuqtasidan yer bo'y-lab hisoblanganda 20 m uzoqlikda joylashgan (1.40- rasm).



1.40 - rasm

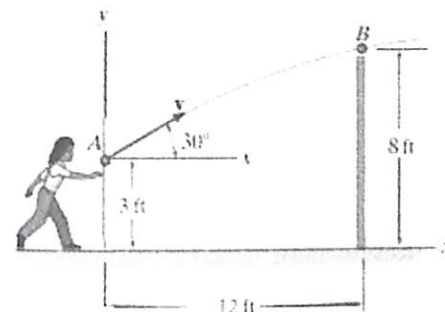
9-muammo. Koptok A nuqtadan $v_A=10\text{m/s}$ tezlik bilan tepiladi. Koptokning maksimal ko'talish balandligini aniqlang (1.41 – rasm).

5.	$x = 2 - 3t - 6t^2$	$y = 3 - \frac{3}{2}t - 3t^2$	0
6.	$x = 6 \sin\left(\frac{\pi t^2}{6}\right) - 2$	$y = 6 \cos\left(\frac{\pi t^2}{6}\right) + 3$	1
7.	$x = 7t^2 - 3$	$y = 5t$	$\frac{1}{4}$
8.	$x = 3 - 3t^2 + 1$	$y = 4 - 5t^2 + \frac{5}{3}t$	1
9.	$x = -4 \cos\left(\frac{\pi t}{3}\right) - 1$	$y = -4 \sin\left(\frac{\pi t}{3}\right)$	1
10.	$x = -6t$	$y = -2t^2 - 4$	1
11.	$x = 8 \cos^2\left(\frac{\pi t}{6}\right) + 2$	$y = -8 \sin^2\left(\frac{\pi t}{6}\right) - 7$	1
12.	$x = -4t^2 + 1$	$y = -3t$	1
13.	$x = 5t^2 + \frac{5}{3}t - 3$	$y = 3t^2 + t + 3$	1
14.	$x = 2 \cos\left(\frac{\pi t^2}{3}\right) - 2$	$y = -2 \sin\left(\frac{\pi t^2}{3}\right) + 3$	1
15.	$x = 4 \cos\left(\frac{\pi t}{3}\right)$	$y = -3 \sin\left(\frac{\pi t}{3}\right)$	1
16.	$x = -2t - 2$	$y = -\frac{2}{(t+1)}$	2
17.	$x = 5 \cos\left(\frac{\pi t^2}{3}\right)$	$y = -5 \sin\left(\frac{\pi t^2}{3}\right)$	1
18.	$x = 5 \sin^2\left(\frac{\pi t}{6}\right)$	$y = -5 \cos^2\left(\frac{\pi t}{6}\right) - 3$	1
19.	$x = -4t^2 + 1$	$y = -3t$	$\frac{1}{2}$
20.	$x = -4 \cos\left(\frac{\pi t}{3}\right)$	$y = -2 \sin\left(\frac{\pi t}{3}\right) - 3$	1
21.	$x = -\frac{3}{(t+2)}$	$y = 3t + 6$	2



1.43 – rasm

12-muammo. To'p A nuqtadan otiladi. U Yerdan 8m, otilish nuqtasidan 12m masofada joylashgan B nuqtaga tushish uchun, qanday θ_A tezlik bilan otiladi? (1.44 – rasm).



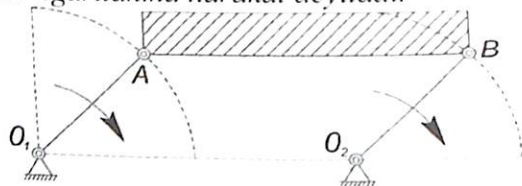
1.44 - rasm

13-muammo. Reaktiv snaryad A nuqtadan $\theta_A=250\text{m/s}$ tezlik bilan otiladi. Agar A nuqta Yerdan 150m balandlikda joylashgan bo'lsa, snaryadning uchish uzoqligi aniqlansin (1.45 – rasm).

II-BOB. QATTIQ JISMNING ILGARILANMA VA QO'ZG'ALMAS O'Q ATROFIDA AYLANMA HARAKATI

16-§ Qattiq jismning ilgarilanma harakati

Jismda olingan har qanday kesma jism harakati davomida doimo o'zining boshlang'ich holatiga parallel qolsa, jismning bunday harakati ilgarilanma harakat deyiladi.



2.1-rasm

Jismning ilgarilanma harakatini uning to'g'ri chiziqli harakati bilan aralashtirib bo'lmaydi. Ilgarilanma harakatdagi jism nuqtasi-ning traektoriyasi egri chiziqdan iborat bo'lishi ham mumkin. Masalan, 2.1-rasmda ko'rsatilgan AB sparnikning harakati davomida O_1A va O_2A kripovshiplar O_1, O_2 nuqtalardan o'tuvchi o'qlar atrofida aylanadi, AB sparnik esa hamma vaqt o'z-o'ziga parallel qoladi, ya'ni ilgarilanma harakatda bo'ladi.

Ilgarilanma harakatda bo'lgan qattiq jismning kinematik xarakteristiklari quyidagi teoremda o'z ifodalarini topgan:

Teorema. Ilgarilanma harakatdagi jismning hamma nuqtalari bir xil traektoriya chizadi va har onda bir xil tezlik, hamda bir xil tezlanishga ega bo'ladi.

Teoremani isbotlash uchun jismning berilgan Oxyz qo'zg'almas koordinatalar sistemasiga nisbatan ilgarilanma harakatini o'rganamiz (2.2-rasm). Jismda ixtiyoriy A va B nuqtalarni olib, ularning radius vektorlarini $\vec{r}_A$ va $\vec{r}_B$ bilan belgilaymiz. Rasmdan:

$$\vec{r}_B = \vec{r}_A + \vec{AB} \quad (2.1)$$

Jism harakatlanganda $\vec{r}_A, \vec{r}_B$ o'zgaradi, ammo AB kesmaning uzunligi va yo'nalishi o'zgarmaydi.

B nuqtaning tezligini aniqlash uchun (2.1) dan vaqt t bo'yicha hosila olamiz:

$$(2.2)$$

bunda, $\frac{d\vec{AB}}{dt} = 0$ bo'lgani uchun,

$$(2.3)$$

yoki $\vec{v}_B = \vec{v}_A$ bo'ladi.

Bu tenglik ilgarilanma harakatdagi jism barcha nuqtalarining harondagi tezliklari bir xil bo'lishini ifodalaydi.

Agar (2.3) dan vaqt t bo'yicha hosila olsak:

$$(2.4)$$

yoki $\vec{a}_B = \vec{a}_A$ bo'ladi.

(2.4) tenglik ilgarilanma harakatdagi jism barcha nuqtalarining harondagi tezlanishlari bir xil bo'lishini ifodalaydi.

Shunday qilib teorema isbotlandi.

Isbotlangan teoremdan jismning ilgarilanma harakati uning biror nuqtasining harakati bilan aniqlanishi mumkinligi ma'lum bo'ladi. Odatda bunday nuqta sifatida jismning og'irlik markazi C nuqta olinadi.

Olingan nuqtaning harakat tenglamalarini koordinata usulida quyidagicha yozish mumkin:

$$x_c = f_1(t), \quad y_c = f_2(t), \quad z_c = f_3(t) \quad (2.5)$$

(2.5) tenglama C nuqtaning harakat tenglamasi bo'lib, jismning ilgarilanma harakat tenglamasini ham ifodalaydi.

Jismning ilgarilanma harakatida hamma nuqtalari uchun bir xil bo'lgan tezlik jismning ilgarilanma harakat tezligi deyiladi, tezlanish

M nuqtaning tezligi $t_1=1$ s da quyidagi qiymatlarga ega bo'ladi:

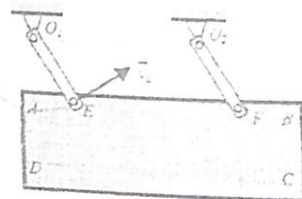
$$v_1 = \frac{\pi}{2} a \quad \text{sm/s},$$

$$a_1 = \frac{\pi^2}{8\sqrt{2}} a \quad \text{sm/s}.$$

$\vartheta_1 > 0$ va $a_1 > 0$ bo'lganligi uchun, M nuqtaning tezligi va tezlanishi O nuqtadan vertikal holda yuqoriga yo'nalgan bo'ladi, yani M nuqta tezlanuvchan harakatda bo'ladi.

2-masala. O'lchamlari $l_{AB}=3$ va $l_{BC}=30$ sm bo'lgan to'g'ri burchakli plastina O_1 va O_2 nuqtalardagi sharnirlarga sterjenlar yordamida biriktirilgan plastinka E nuqtasining tezligi $v_E = 0.6$ m/s. Boshlang'ich paytda sterjenlar vertikal xolatda bo'lgan plastina ABC va D nuqtalarining $t=t_1=0.5$ s dagi tezliklari va tezlanishlari aniqlansin (2.4- rasm).

$$O_1O_2 = EF = 2l, \quad O_1E = O_2F = l.$$



2.4- rasm

Yechimi.

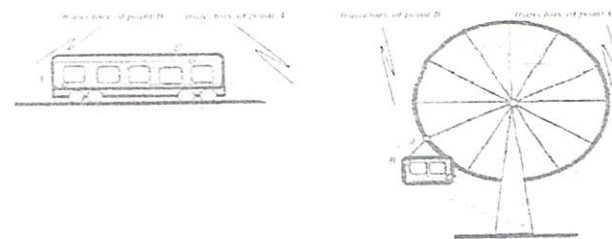
Masalaga ABCD plastina ilgarilanma harakat sodir etiladi, chunki y O_1E va O_2E strjenlarning aylanma harakatlari natijasida har doim o'zining boshlang'ich holatiga parallel qolgan holda harakatlanadi. Boshlang'ich paytda sterjenlar vertikal holatda bo'lgan. Plastinkaning $t=t_1$ vaqt momentidagi vaziyati E nuqtaning burchak tezligi orqali quydagicha aniqlanadi:

$$\omega_E = \frac{v_E}{l} = \frac{0.6}{0.3} = 2 \text{ rad/s}.$$

Demak, O_1E va O_2E sterjenlar tekis aylanma harakatda bo'ladi. Sterjenlarning boshlang'ich holatiga nisbatan (vertikal holati) og'ish burchagi quydagicha topiladi:

$$\theta = \frac{\omega_E}{l}.$$

2.4b-rasmda charxpalak A,B,C nuqtalarining trayektoriyasi ko'rsatilgan.



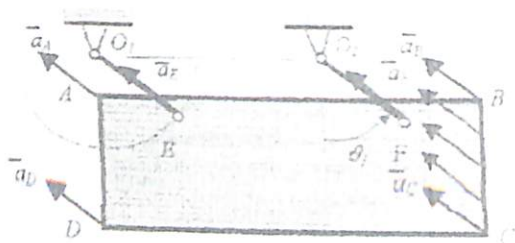
2.4 a, b-rasmlar

17-§ Qattiq jismning ilgarilanma harakatiga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Qattiq jismning ilgarilanma harakatiga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1. Koordinatalar sistemasi tanlab olinadi, bunda koordinata o'qlaridan birini jismning ilgarilanma harakati yo'nalishida o'tkazish maqsadga muvofiq bo'ladi.
2. Masala shartidan ilgarilanma harakatda bo'ladigan jism tanlab olinadi.
3. Tanlab olingan koordinata o'qlari sistemasida jismning ilgarilanma harakati tenglamasi tuziladi.
4. Jismning ilgarilanma harakat tenglamalariga ko'ra tezlik vektorining o'qlardagi proeksiyalari aniqlanadi.
5. Jism ilgarilanma harakat tezligining o'qlardagi proeksiyalariga ko'ra uning miqdori va yo'nalishi aniqlanadi.
6. Jismning ilgarilanma harakat tenglamalari yoki jism tezligining o'qlardagi proeksiyalariga ko'ra uning tezlanishini o'qlardagi proeksiyalari aniqlanadi.
7. Jism tezlanishining o'qlardagi proeksiyalariga ko'ra uning miqdori va yo'nalishi aniqlanadi.

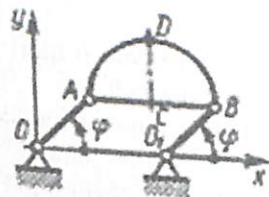
Plastina nuqtalari tezlanishlarining taqsimoti (2.6-rasm)da ko'rsatilgan.



2.6-rasm

19-§ Mustaqil o'rganish uchun talabalarga tavsiya etiladigan masalalar

1-masala. Uzunliklari $OA=OB=0,16\text{m}$ bo'lgan ikki krivoship-larning harakat qonuni $\varphi=\pi t$ bo'lib, yarim aylana shaklidagi ABD jismini ilgarilanma harakatga keltiradi. Agar $AB=0,25\text{m}$ bo'lsa, $t=2\text{s}$ da jismning D nuqtasi trayektoriyasining egrilik radiusini toping (2.7-rasm).



2.7- rasm

2-masala. 1 g'ilof ichida 2 polzun harakat qiladi. Agar polzun-ning ilgarilanma harakat qonuni $x_A=0,1 \cos t$, $y_A=0,1$, $z_A=0$ bo'lsa $t=\pi$ (sek) paytda B nuqtaning tezligini aniqlang. Bunda masofa $AB=0,3\text{m}$ (2.8- rasm).

Shuning uchun

$$y = OM = a \cos \varphi + \sqrt{r^2 - a^2 \sin^2 \varphi}$$

Hosil bo'lgan ifodaga φ burchak qiymatini qo'yib, $r=3a$ ekanligini e'tiborga olsak, M nuqtaning harakat tenglamasi uchun quyidagi ifoda kelib chiqadi:

$$y = a \left(\cos \frac{\pi}{2} t + \sqrt{9 - \sin^2 \frac{\pi}{2} t} \right).$$

M nuqtaning tezligini quyidagicha aniqlaymiz:

$$\begin{aligned} v = \frac{dy}{dt} &= \frac{d}{dt} a \left(\cos \frac{\pi}{2} t + \sqrt{9 + \sin^2 \frac{\pi}{2} t} \right) = -\frac{\pi}{2} a \sin \frac{\pi}{2} t \cdot \left(1 + \frac{\cos \frac{\pi}{2} t}{\sqrt{9 - \sin^2 \frac{\pi}{2} t}} \right) \\ &= -\frac{\pi a \sin \frac{\pi}{2} t}{2 \sqrt{9 - \sin^2 \frac{\pi}{2} t}} \cdot \left(\cos \frac{\pi}{2} t + \sqrt{9 - \sin^2 \frac{\pi}{2} t} \right), \end{aligned}$$

yoki

$$v = -\frac{\pi}{2} \cdot \frac{\sin \frac{\pi}{2} t}{\sqrt{9 - \sin^2 \frac{\pi}{2} t}} \cdot y$$

M nuqtaning tezlanishini aniqlaymiz:

$$a = \frac{dv}{dt} = \frac{d^2 y}{dt^2} = -\frac{\pi}{2} \left[y \frac{d}{dt} \left(\frac{\sin \frac{\pi}{2} t}{\sqrt{9 - \sin^2 \frac{\pi}{2} t}} \right) + \frac{\sin \frac{\pi}{2} t}{\sqrt{9 - \sin^2 \frac{\pi}{2} t}} \frac{dy}{dt} \right]$$

Agar $\frac{dy}{dt} = v$ va $\frac{d}{dt} \left(\frac{\sin \frac{\pi}{2} t}{\sqrt{9 - \sin^2 \frac{\pi}{2} t}} \right) = \frac{9 \pi \cos \frac{\pi}{2} t}{2(9 - \sin^2 \frac{\pi}{2} t)^{\frac{3}{2}}}$,

ekanligini e'tiborga olsak,

$$a = -\frac{\pi}{2} \left[y \frac{9 \cos \frac{\pi}{2} t}{2(9 - \sin^2 \frac{\pi}{2} t)^{\frac{3}{2}}} + v \frac{\sin \frac{\pi}{2} t}{\sqrt{9 - \sin^2 \frac{\pi}{2} t}} \right]$$

Shunday qilib,

$$a = -\frac{\pi^2}{4} y \frac{9 \cos \frac{\pi}{2} t - \sin^2 \frac{\pi}{2} t \sqrt{9 - \sin^2 \frac{\pi}{2} t}}{(9 - \sin^2 \frac{\pi}{2} t)^{\frac{3}{2}}}$$

$$a_\tau = \frac{d\theta_\tau}{dt} = \frac{d}{dt}(R\omega) = R\varepsilon, \quad (2.22)$$

$$a_n = \frac{\theta^2}{\rho} = \frac{R^2\omega^2}{R} = \omega^2 R \quad (2.23)$$

bo'ladi.

Urinma tezlanish $\vec{a}_\tau$ aylanma harakat tezlanuvchan bo'lganda, traektoriyaga o'tkazilgan urinma bo'ylab harakat yo'nalishida, sekinlanuvchan aylanma harakatda esa, unga teskari yo'naladi. Normal tezlanish $\vec{a}_n$ doimo bosh normal bo'yicha aylanish o'qi tomon yo'naladi (2.19a-rasm). Ba'zan $\vec{a}_\tau$ aylanma tezlanish, $\vec{a}_n$ esa markazga intilma tezlanish deb ham yuritiladi.

M nuqta tezlanishining moduli:

$$a = \sqrt{a_\tau^2 + a_n^2} = R\sqrt{\varepsilon^2 + \omega^4} \quad (2.24)$$

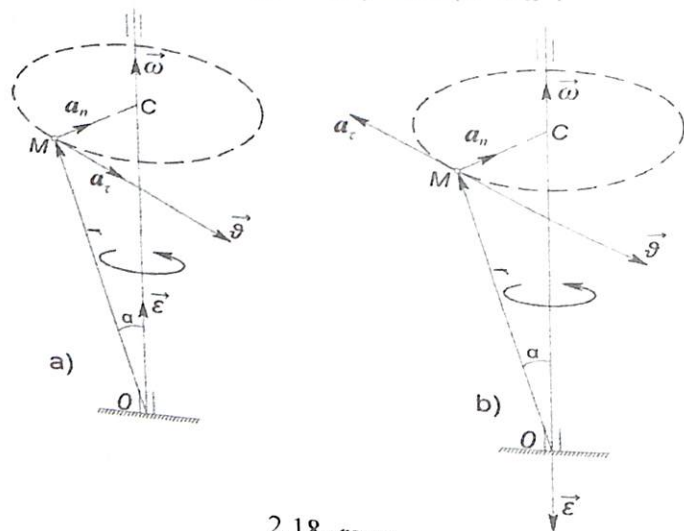
formula orqali aniqlanadi.

Aylanma harakatdagi qattiq jism ixtiyoriy M nuqtasining tezlanishini quyidagicha aniqlash ham mumkin (2.19b-rasm)

$$\vec{a}_p = \varepsilon \vec{k} * \vec{r} + \omega \vec{k} * (\omega \vec{k} * \vec{r}) = (-\varepsilon y - \omega^2 x)\vec{i} + (\varepsilon x - \omega^2 y)\vec{j}$$

Tezlanish moduli esa quyidagicha aniqlanadi:

$$a_p = \sqrt{(x^2 + y^2)(\varepsilon^2 + \omega^4)} = d * \sqrt{\varepsilon^2 + \omega^4}.$$



2.18- rasm

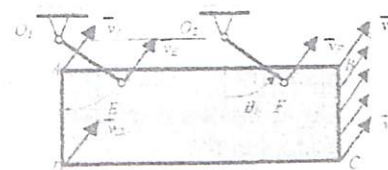
Bunday $t=t_1$ vaqt momenti uchun sterjenlarning boshlang'ich holatidan og'ish burchagi quydagiga teng boladi:

$$\theta = \theta(t_1) = 2 \cdot 0.5 = 1 \text{ rad} = 57^\circ 32'.$$

Demak, $t=t_1$ vaqt momentiga boshlang'ich holati vertikal bo'lgan sterjenlar $\theta=57^\circ 32'$ burchakga burilar ekan.

Plastinka ilgarilanma harakatda bo'lishi tufayli, uning barcha nuqtalarining tezligi E nuqtaning tezligiga teng boladi va $\vec{v}_E$ biron bir xil yo'nalishda bo'ladi:

$$v_A = v_B = v_C = v_D = v_E = 0.6 \text{ m/s}.$$



2.5-rasm

Plastinka nuqtalari tezliklarining taqsimoti (2.5rasm)da ko'rsatilgan.

Plastina E nuqtasining tezlanishini aniqlaymiz. E nuqtaning tezlanishi urinma va normal tashkil etuvchilardan iborat bo'ladi;

$$a_n = a_{Er} + a_{En}$$

Bunda

$$a_{Er} = g_E,$$

$$a_{En} = \frac{v_E^2}{0.3} = \frac{(0.6)^2}{0.3} = 1.2 \text{ m/s}^2.$$

Demak, E nuqtaning tezlanishi y tekis aylanma harakatda bo'lishi sababli, normal markazga intilma tezlanishdan iborat bo'lar ekan:

$$a_n = a_{En} = 1.2 \text{ m/s}^2$$

Bu tezlanish E nuqta chizadigan aylanma radiusi bo'ylab, aylanma markazi tomon yo'naladi. Plastina ilgarilanma harakatda bo'lishi tufayli, uning barcha nuqtalarining tezlanishlari ham o'zaro teng bo'ladi va $\vec{a}_E$ yo'nalishi bilan bir xil bo'ladi:

$$a_A = a_B = a_C = a_D$$

Takrorlash uchun savollar

1. Qattiq jismning qanday harakatiga ilgariylanma harakat deyiladi va bu harakatning asosiy xususiyatlari?
2. Qattiq jismning qanday harakatiga qo'zg'almas o'q atrofida gi aylanma harakat deyiladi?
3. Aylanma harakat tenglamasi.
4. Aylanma harakat qilayotgan qattiq jismning burchak tezlik va burchak tezlanish modullari qanday formula bilan aniqlanadi?
5. Qo'zg'almas o'q atrofida gi aylanma harakat qilayotgan qattiq jism burchak tezlik va burchak tezlanish vektorlari qanday yo'nalgan bo'ladi?
6. Aylanma harakat qilayotgan nuqtaning chiziqli tezligi qanday formula orqali ifodalanadi?
7. Aylanma harakat qilayotgan nuqtaning chiziqli tezlanishi qanday formula orqali ifodalanadi?
8. Eyler formulasi qanday ko'rinishda bo'ladi?
9. Aylanma harakat qilayotgan nuqtaning tezlik vektori qanday ifodalanadi?
10. Aylanma harakat qilayotgan nuqtaning tezlanish vektori qanday ifodalanadi?

25-§ Qattiq jismning ilgariylanma va qo'zg'almas o'q atrofida aylanma harakatiga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Qattiq jismning qo'zg'almas o'q atrofida aylanishiga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1. Koordinatalar sistemasi tanlab olinadi, bunda koordinata o'qlaridan birini (Z o'qini) aylanish o'qi bo'ylab yo'naltirish maqsadga muvofiq bo'ladi.
2. Qattiq jismning aylanma harakati tenglamasi tuziladi.
3. Qattiq jismning aylanish burchagidan vaqt bo'yicha birinchi tartibli hosila hisoblab, burchak tezlikning aylanish o'qidagi proeksiyasi aniqlanadi.

$$a_n = \omega^2 R = 400t^2 \cdot 4000t^2 \text{ sm/s}^2.$$

Shuning uchun,

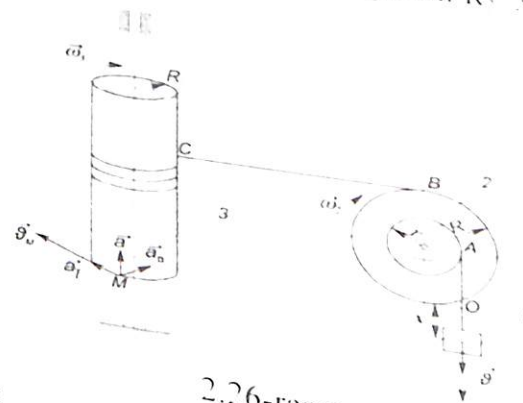
$$a = \sqrt{(200)^2 + (4000t^2)^2} = 200\sqrt{1 + (4000t^4)} \text{ sm/s}^2.$$

27-§ Jismlarning ilgariylanma va aylanma harakatlarini mexanizmlarda qo'llanilishiga doir masalalar

5-masala.

1-jismning harakat tenglamasi $x = 10 + 30t^2$ ga ko'ra, uning vertikal o'q bo'ylab, $S = 0.3 \text{ m}$ yo'lni o'tgan vaqt momentida, 3-jism M nuqtasining tezligi, urinma, normal va to'la tezlanishi topilsin (2.25-rasm).

Shkiv va tsilindr o'lchamlari quyidagicha: $R = 30 \text{ cm}$, $r = 25 \text{ cm}$, $R_1 = 30 \text{ cm}$



2.26-rasm

Yechimi:

Birinchi jismning

Bundan,

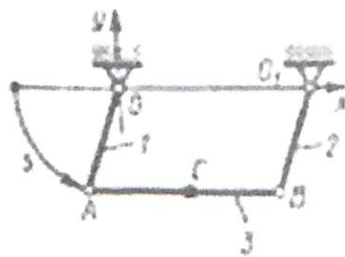
$$\tau = \sqrt{s/30} = \sqrt{s/30} = 1 \text{ s}.$$

Birinchi jism tezligini aniqlash uchun uning harakat tenglamasidan vaqt bo'yicha birinchi tartibli hosila hisoblaymiz:

$$v_1 = \frac{dx}{dt} = (10 + 30t^2)' = 60t.$$

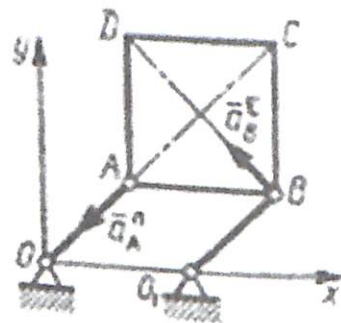
Agar birinchi jism osilgan, hamda 2- va 3- jismlarni biriktiruvchi tasmlarni cho'zilmaydi deb olsak, $\vec{v}_A = \vec{v}_1$ bo'ladi. U vaqtda

5-masala. Bir xil uzunlikdagi $OA=OB=0.2\text{m}$ 1 va 2 krivoshiplarga o'rnatilgan 3 sterjen Oxy tekisligida ilgarilanma harakat qiladi. Uning A nuqtasining harakat qonuni $s=0.2\pi t$ bo'lsa, $t=0$ paytdagi sterjen o'rtasidagi C nuqtaning tezlanishini aniqlang. Bunda masofa $AB=0.36\text{m}$ (2.11 – rasm).

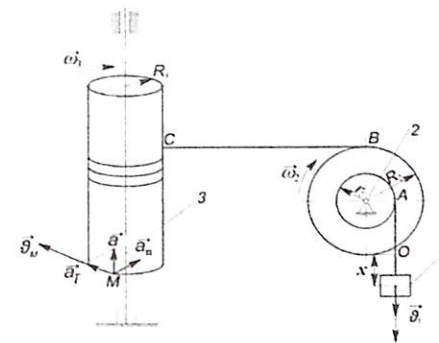


2.11- rasm

6-masala. ABCD kvadrat plastina Oxy tekisligida ilgarilanma harakat qiladi. Agar uning A nuqtasi $a_A^n = 4\text{m/s}^2$ normal tezlanishga va B nuqtasi $a_B^t = 3\text{m/s}^2$ urinma tezlanishga ega bo'lsa, C nuqtasining tezlanishini toping (2.12 – rasm).

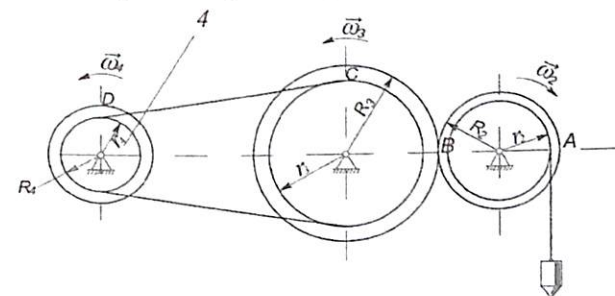


2.12 -rasm



2.27-rasm

6-masala. 1-jismning harakat tenglamasi $x=5+8t^2$ ga ko'ra, uning vertikal o'q bo'ylab, $s=0.32\text{m}$ yo'lni bosib o'tgan vaqt momentida, 4-jism M nuqtasining tezligi, urinma, normal va to'la tezlanishlari topilsin. 2,3,4 Jismlar radiuslari $r_2=16\text{sm}$, $R_2=20\text{sm}$, $r_3=20\text{sm}$, $R_3=25\text{sm}$ teng (2.28-rasm).



2.28-rasm

Yechimi:

1-jismning $s=0.32\text{m}$ yo'lni bosib o'tish vaqti τ ni aniqlaymiz:

$$s = x_{t=\tau} - x_{t=0} = 5 + 8\tau^2 - 5 = 8\tau^2$$

Bundan,

$$\tau = \sqrt{s/8} = \sqrt{32/8} = 2\text{ s.}$$

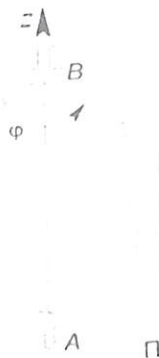
Birinchi jism tezligini topamiz. Buning uchun uning harakat tenglamasidan vaqt bo'yicha birinchi tartibli hosila hisoblaymiz:

$$v_1 = \frac{dx}{dt} = 16t.$$

21-§ Qo'zg'almas o'q atrofida aylanma harakatda bo'lgan jismning burchak tezligi. Tekis aylanma harakat.

Burchak tezlik aylanma harakatda bo'lgan jism aylanish burchagining o'zgarishini ifodalovchi kattalik bo'lib, u aylanish burchagidan vaqt bo'yicha olingan birinchi tartibli hosilaga teng:

$$\omega = \varphi' = \frac{d\varphi}{dt} \quad (2.7)$$



2.13-rasm

Burchak tezlik φ burchakning o'zgarish qonuniga mos ravishda musbat yoki manfiy qiymatga ega bo'lishi mumkin.

Agar $\varphi' = \frac{d\varphi}{dt} > 0$ bo'lsa, aylanma harakat aylanish o'qining musbat yo'nalishidan qaraganda soat milining aylanishiga teskari yo'nalishda yuz beradi; $\varphi' = \frac{d\varphi}{dt} < 0$ bo'lsa, jism soat milining aylanish yo'nalishida aylanma harakatda bo'ladi.

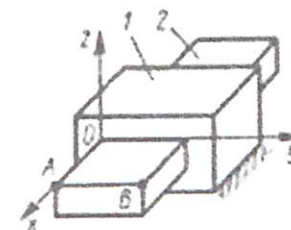
Burchak tezlik vektori aylanish o'qi bo'ylab yo'naladi va uning musbat yo'nalishidan qaraganda, aylanish soat mili harakatiga teskari yo'nalishda ko'rinadi (2.13-rasm). Burchak tezlik vektori aylanish o'qining ixtiyoriy nuqtasiga qo'yiladi. Shuning uchun ham u erkin vektor hisoblanadi. Burchak tezlik vektorining moduli

$$\omega = \left| \frac{d\varphi}{dt} \right| \quad (2.8)$$

formula yordamida aniqlanadi.

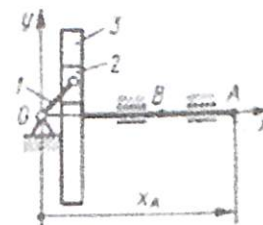
Burchak tezlik SI birliklar sistemasida rad/s yoki 1/s da o'lchanadi.

Jism harakati davomida $\omega = \omega_0 = \text{const}$ bo'lsa, u tekis aylanma harakatda bo'ladi.



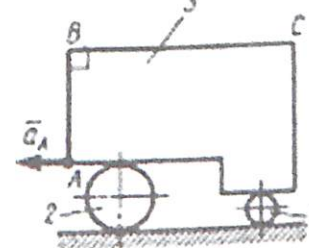
2.8 - rasm

3-masala. Krivoship 1 va polzun 2 yordamida ilgarilanma harakatga keluvchi 3 kulisali mexanizm $x_A = 0,4 - 0,1 \sin t^2$ qonun asosida siljisa, $t=2s$ dagi B nuqtaning tezligini aniqlang (2.9 - rasm).



2.9 - rasm.

4-masala. Ikki 1 va 2 silindirik o'qlarga o'rnatilgan 3 jism ilgarilanma harakat qiladi. Agar masofalar $BC=2AB=1m$ bo'lib, jismning A nuqtasi $2m/s^2$ tezlanishga ega bo'lsa, C nuqtasining tezlanishini hisoblang (2.10 - rasm).



2.10 - rasm

o'qi atrofida aylanish burchagidan vaqt bo'yicha olingan ikkinchi tartibli hosilaga teng bo'ladi:

$$\varepsilon = \frac{d\varphi}{dt} = \frac{d^2\varphi}{dt^2} \quad (2.12)$$

Burchak tezlanish ham, burchak tezlik kabi vektor kattalik hisoblanadi.

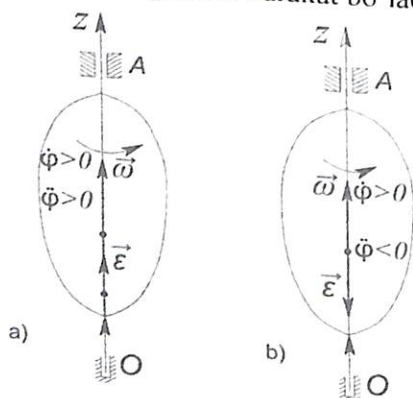
Agar $\frac{d\varphi}{dt}$ va $\frac{d^2\varphi}{dt^2}$ bir xil ishorali bo'lsa, ya'ni aylanma harakat tezlanuvchan bo'lsa, burchak tezlik va burchak tezlanish vektorlari aylanish o'qi bo'ylab bir tomonga (2.15a-rasm), turli ishorali bo'lsa, ya'ni aylanma harakat sekinlanuvchan bo'lsa, qarama-qarshi tomonlarga yo'naladi (2.15b-rasm). Burchak tezlanish vektorining moduli

$$\varepsilon = \left| \frac{d^2\varphi}{dt^2} \right| = \left| \frac{d\varphi}{dt} \right| \quad (2.13)$$

formula yordamida aniqlanadi. Burchak tezlanish SI birlilar sistemasida rad/s^2 yoki $1/\text{s}^2$ larda o'lchanadi.

Agar aylanma harakat davomida $\frac{d\varphi}{dt} > 0$ bo'lsa, φ orta boradi va bunday harakat **tezlanuvchan aylanma harakat** deyiladi; $\frac{d\varphi}{dt} < 0$ bo'lsa, ω kamaya boradi va bunday harakat **sekinlanuvchan aylanma harakat** deyiladi.

Agar aylanma harakat davomida $\varepsilon = \varepsilon_0 = \text{const}$ bo'lsa, jismning harakati tekis o'zgaruvchan aylanma harakat bo'ladi.



2.15a,b-rasm

20-§ Qattiq jismning qo'zg'almas o'q atrofida aylanma harakati

Qattiq jismning harakatida ikki nuqtasi doimo qo'zg'almasdan qolsa, uning bunday harakati qo'zg'almas o'q atrofida aylanma harakat, qo'zg'almas nuqtalardan o'tuvchi o'q esa aylanish o'qi deyiladi.

Qattiq jismning qo'zg'almas o'q atrofida aylanma harakatida uning aylanish o'qida yotuvchi barcha nuqtalari qo'zg'almas bo'ladi. Aylanish o'qida yotmaydigan boshqa barcha nuqtalar aylanish o'qiga perpendikulyar tekisliklarda yotuvchi, markazi aylanish o'qida bo'lgan aylanalar bo'ylab harakatlanadi.

Qattiq jismning aylanma harakatini o'rganish uchun aylanish o'qi orqali o'tuvchi qo'zg'almas Π_0 va jismga mahkam biriktirilgan, u bilan birga harakatlanadigan Π tekisliklarni o'tkazamiz.

Jism aylanish o'qi Az atrofida harakatlanganda Π tekislik Π_0 tekislikka nisbatan φ burchakka buriladi. Bu burchak aylanish burchagi deyiladi va Π tekislik jism bilan mahkam biriktirilganligidan jismning holati φ burchak bilan aniqlanadi.

Jism Az o'q atrofida aylanganda uning aylanish burchagi φ vaqtning uzluksiz, bir qiymatli funksiyasi sifatida o'zgaradi:

$$\varphi = f(t) \quad (2.6)$$

Bu tenglama jismning qo'zg'almas o'q atrofida aylanma harakatining kinematik tenglamasi deyiladi. Aylanish burchagi radianlarda o'lchanadi.

Qo'zg'almas o'q atrofida aylanma harakatda bo'lgan jismning asosiy kinematik xarakteristikalarini uning burchak tezligi va burchak tezlanishi hisoblanadi.

Qo'zg'almas o'q atrofida aylanma harakatda bo'lgan jismning asosiy kinematik xarakteristikalarini uning burchak tezligi va burchak tezlanishi hisoblanadi.

ni aniqlaymiz. Bunda ham integrallash o'zgarmaslari boshlang'ich kattaliklar orqali aniqlanadi.

3. Burchak tezlikning aylanish o'qidagi proyeksiyasi ifodasidan foydalanib, jism nuqtalarining tezligini va normal tezlanishini aniqlaymiz.

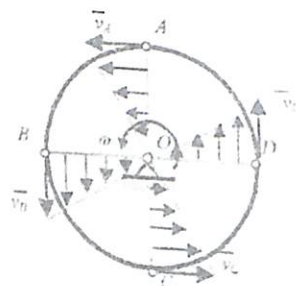
4. Burchak tezlanishning aylanish o'qidagi proyeksiyasi ifodasidan foydalanib, jism nuqtalarining urinma tezlanishlarini aniqlaymiz.

5. nuqtalarining normal va urinma tezlanishlarini bilgan holda uning to'la tezlanishi aniqlanadi.

26-§ Qattiq jismning qo'zg'almas o'q atrofida aylanma harakatiga doir masalalar

1-masala.

Radius $R=40\text{sm}$ bo'lgan disk qo'zg'almas O nuqta atrofida qo'zg'almas $\omega=5$ radius tezlik bilan aylanadi. (2.20-rasm) Disk gorizontal va vertikal deametrleri uchlaridagi nuqtalarning tezligi va tezlanishi aniqlansin va mazkur diametrlar nuqtalari tezliklarining taqsimoti ko'rsatilsin.



2.20-rasm

Yechish: Disk qo'zg'almas O nuqtadan o'tuvchi o'q atrofida o'zgarmas burchak tezlik bilan aylanma harakatda bo'lishi sababli, disk gorizontal va vertikal diametrlari uchlaridagi nuqtalarning tezliklari quydagicha aniqlanadi:

$$v_A = OA \cdot \omega = 40 \cdot 0.5 = 20 \text{ sm/s};$$

$$v_C = OC \cdot \omega = 40 \cdot 0.5 = 20 \text{ sm/s};$$

$$v_B = OB \cdot \omega = 40 \cdot 0.5 = 20 \text{ sm/s};$$

$$v_D = OD \cdot \omega = 40 \cdot 0.5 = 20 \text{ sm/s};$$

M nuqta tezlanishining yo'nalishi bosh normal bilan $\vec{a}$ tezlanish vektorini orasidagi μ burchak orqali aniqlanadi (2.18- rasm):

$$\tan \mu = \frac{|\vec{a}_n|}{a_n} = \frac{\varepsilon}{\omega^2}. \quad (2.25)$$

Aylanma harakatdagi jismning barcha nuqtalari uchun ω va ε lar bir xil bo'lganidan, jism nuqtalarining tezlanishi aylanish o'qidan mazkur nuqtalargacha bo'lgan masofalarga proporsional ravishda o'zgaradi. Berilgan onda jismning barcha nuqtalari uchun μ burchak ham bir xil bo'ladi.

Urinma va normal tezlanishlarning vektorli ifodalarini aniqlash uchun (2.20) dan vaqt bo'yicha hosila olamiz:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}. \quad (2.26)$$

Bunda ,

$$\frac{d\vec{\omega}}{dt} = \vec{\varepsilon}, \quad \frac{d\vec{r}}{dt} = \vec{v}. \quad (2.27)$$

Shuning uchun,

$$\vec{a} = \vec{\varepsilon} \times \vec{r} + \vec{\omega} \times \vec{v}. \quad (2.28)$$

Bu formulada,

$$\vec{\varepsilon} \times \vec{r} = \vec{a}_n \quad (2.29)$$

urinma tezlanish vektorini ifodalaydi.

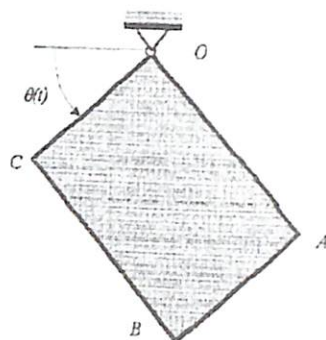
Ko'rinib turibdiki, qo'zg'almas o'q atrofida aylanma harakatdagi jism ixtiyoriy nuqtasining urinma tezlanishi jismning burchak tezlanishi vektorini bilan mazkur nuqtaning aylanish o'qidagi ixtiyoriy O nuqtaga nisbatan radius – vektorining vektorli ko'paytmasiga teng bo'lar ekan.

(2.28) formulada

$$\vec{a}_n = \vec{\omega} \times \vec{v} \quad (2.30)$$

normal tezlanish vektorini ifodalaydi.

Demak, qo'zg'almas o'q atrofida aylanma harakatdagi jism ixtiyoriy nuqtasining normal yoki markazga intilma tezlanishi jismning burchak tezlik vektorini bilan mazkur nuqta chiziqli tezligining vektorli ko'paytmasiga teng bo'lar ekan



2.22-rasm

Yechimi. 1. Plastinaning $t_1=1$ s vaqt onida egallagan o'rnini aniqlaymiz.

Bundan

$$\theta_1 = \theta(t_1) = \sin(1 \text{ rad}) = 984 \text{ rad}$$

$$\theta_1 = 48^\circ 23'$$

Plastina nuqtalarining tezliklarini aniqlash uchun dastlab uning burchak tezligini aniqlaymiz:

$$\omega_1 = \frac{d\theta_1}{dt} = \cos t_1 = 0,54 \frac{1}{s}$$

Burchak tezlik burilish tezlik o'sish tomoni qarab yo'naladi (ularning ishoralari bir xil).

Plastina A nuqtasining tezligini aniqlaymiz:

$$w_B = w_1 \cdot OB = w_1 \cdot \sqrt{(OA)^2 + (AB)^2} = 0,54 \cdot 50 = 27 \frac{\text{sm}}{s}$$

$\vec{v}_B$ tezlik vektori aylanish markazi O nuqtadan o'tkazilgan OB radiusga perpendikulyar holda ω_1 tomon yo'naladi

Plastina C nuqtasining tezligi quyidagiga teng bo'ladi:

$$v_C = \omega_1 \cdot OC = 0,54 \cdot 30 = 16,2 \frac{\text{sm}}{s}$$

$\vec{v}_C$ tezlik vektori aylanish markazi O nuqtadan o'tkazilgan OC radiusga perpendikulyar holda ω_1 tomon yo'naladi.

4. Qattiq jismning aylanish burchagidan vaqt bo'yicha ikkinchi tartibli hosila hisoblab, burchak tezlanishning aylanish o'qidagi proektsiyasi aniqlanadi.

5. Aylanma harakat burchak tezligini bilgan holda, jism nuqtasining chiziqli tezligi va normal tezlanishi aniqlanadi.

6. Aylanma harakat burchak tezlanishini bilgan holda, jism nuqtasining urinma tezlanishi aniqlanadi.

7. Aniqlangan normal va urinma tezlanishlar orqali jism nuqtasining to'la tezlanishi aniqlanadi.

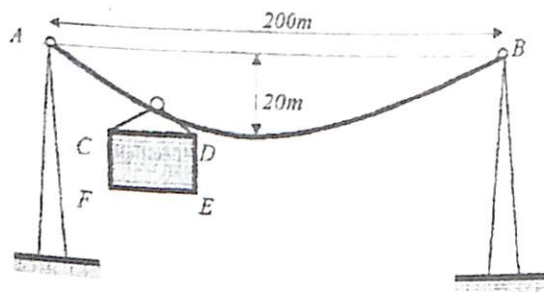
Jismda olingan har qanday kesma jism harakati davomida doimo o'zining boshlang'ich holatiga parrallel qolsa, jismning bunday harakati ilgariylanma harakati deyiladi. Ilgarilanma harakatda jismning barcha nuqtalari bir xil traektoriya bo'ylab harakatlanadi va har yerda bir xil tezlik, hamda bir xil tezlanishga ega bo'ladi. Binobarn jismning ilgariylanma harakati uning biror nuqtasining harakati bilan aniqlanishi mumkin. Nuqta kinematikasida uning kinematik xarakteristikalarini aniqlash bilan tanishgan edik. Nuqtaning harakat tenglamalari trayektoriyasi, tezligi, tezlanishi va h.k larni aniqlash yo'llari bilan batafsil tanishdik. Shuning uchun mazkur amaliy mashg'ulotda qattiq jismning ilgariylanma harakatiga doir masalalar o'rganilmaydi.

Qattiq jismning harakatida ikki nuqtasi doimo qo'zg'almasdan qolsa, uning bunday harakati qo'zg'almas o'q atrofidagi aylanma harakat, qo'zg'almas nuqtalardan o'tuvchi o'q esa aylanish o'qi deyiladi. Agar masalada qattiq jismning burchak tezlanishi yoki burchak tezligi berilgan bo'lib, aylanma harakat tenglamasini qattiq jism nuqtalarining tezligi va teshlanishini aniqlash talab etilsa, masalani quyidagi tartibda yechish maqsadga muvofiq bo'ladi.

1. Qattiq jism burchak tezlanishining aylanish o'qidagi proyeksiyasini ifodalovchi tenglamani integrallab, burchak tezlikning aylanish o'qidagi proyeksiyasini aniqlaymiz. Bundan integrallash doimiylari — o'zgarmaslari boshlang'ich kattaliklar orqali aniqlanadi.

2. Burchak tezlikning aylanish o'qidagi proyeksiyasini ifodalovchi tenglamani integrallab, jismning aylanma harakat tenglamasi-

Muammo -2. Lift kabinasi parabola qismi bo'ylab A nuqtadan B nuqtaga tortilgan arqonga bog'langan holda harakatlanmoqda. Kabina harakati A nuqtadan gorizontol $v_{hor}=1\text{m/s}$ o'zgarmas tezlik bilan boshlangan. Kabina AB arqon o'rtasida bo'lgan paytda $t_1=3\text{s}$ vaqt momenti uchun uning $CDFE$ nuqtalarining tezligi va tezlanishi aniqlansin. (2.31-rasm)



2.31-rasm

Muammo -3. Kvadrat plastina chizma tekisligida o'zgarmas $\varepsilon=1\text{rad/s}^2$ burchak tezlanish bilan aylanma harakat qilmoqda. Agar $t=0$ vaqt onida plastina OA tomoni gorizontol holatni egallagan bo'lib, B nuqtaning tezligi $v_{B0}=1$ ga teng bo'lsa, kvadrat plastina uchlarining tezligi va tezlanishi aniqlansin va $t=1\text{s}$ vaqt oni uchun kvadrat plastina dioganallarida yotuvchi nuqtalar tezliklarining taqsimoti ko'rsatilsin. Kvadrat plastina tomoni $l=0,5\text{m}$ ga teng.

Muammo -4. Jism qo'zg'almas o'q atrofida $\varepsilon=5\text{rad/s}^2$ burchak tezlanish bilan aylanadi. Boshlang'ich paytda, $t_0=0$ da, jismning burchak tezligi $\omega_0=0$ bo'lsa, $t=2\text{s}$ da uning aylanish o'qidan $r=0,2\text{m}$ masofadagi nuqtasining tezligini aniqlang.

Muammo -5. Qo'zg'almas o'q atrofida aylanayotgan jismning aylanish o'qidan $r=0,2\text{m}$ masofadagi nuqtasining tezligi $v=4t^2$ qonun bo'yicha o'zgarsa, $t=2\text{s}$ dagi jismning burchak tezlanishini toping.

Muammo -6. Jismning burchak tezligi $\omega=1+t$ qonun bo'yicha o'zgarsa, $t=1\text{s}$ paytda uning aylanish o'qidan $r=0,2\text{m}$ masofadagi nuqtasining tezlanishini toping.

Mazkur tezliklar nuqtalar radiuslariga perpendikulyar holda ω yo'nalishi tomon yo'naladi:

$$\vec{v}_A \perp \vec{OA}; \vec{v}_B \perp \vec{OB}; \vec{v}_C \perp \vec{OC}; \vec{v}_D \perp \vec{OD}.$$

Diskning garizontal va vertikal diametridagi nuqtalar tezliklarining taqsimoti 1-rasmida ko'rsatilgan.

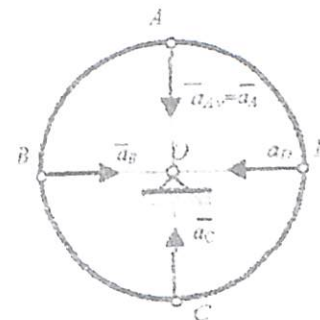
Diskning garizontal va vertikal deametrleri uchlaridagi nuqtalar tezlanishlarining normal va urinma tashkil etuvchilarini aniqlaymiz.

A nuqtaning tezlanishi:

$$\vec{a}_{At} = \vec{OA} * \varepsilon = \vec{OA} * \omega = 0$$

$$\vec{a}_{An} = \vec{OA} * \omega^2 = 40 * (0.5)^2 = 10\text{m/s}^2$$

Nuqta normal tezlanishi uchun radiusi bo'ylab, disk markazi tomon yo'naladi.



2.21-rasm

Disk boshqa B, C, D nuqtalarning tezlanishlari ham miqdor jihatdan A nuqtasining tezlanishiga teng bo'lib, nuqtalar radiuslari bo'ylab, disk markazi tomon yo'naladi (2.21-rasm).

2-masala. Ko'rsatilgan $OABS$ plastina chizma tekisligida O nuqta atrofida aylanadi. Agar plastinaning aylanma harakat tenglamasi

$$\theta(t) = \sin t (\text{rad})$$

bo'lib, $OA=40\text{sm}$, $AB=30\text{sm}$ bo'lsa, plastina A, B va C nuqtalarining tezligi, tezlanishi aniqlansin. Plastina OA va AB tomonlari nuqtalarining $t_1=1\text{s}$ vaqt onidagi tezliklarining taqsimoti ham ko'rsatilsin (2.22-rasm).

29-§ Talabalar mustaqil bajarishi uchun ko'p variantli keyslar (hisob chizma ishlari uchun)

Ilgarilama va aylanma harakatlarda qattiq jism nuqtalarining tezliklari va tezlanishlarini aniqlash.
1-yukning harakati;

$$x = c_2 t^2 + c_1 t + c_0$$

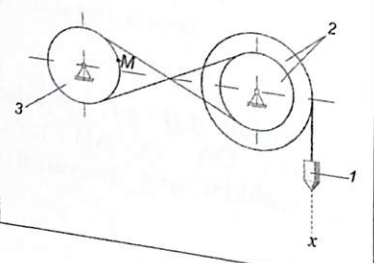
tenglama bilan tavsiflanishi kerak. Bu yerda t -vaqt, s ; s_0 ; s_1 ; s_2 - doimiylar.

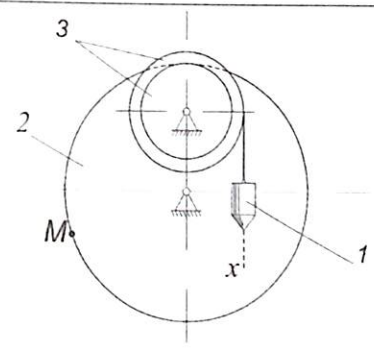
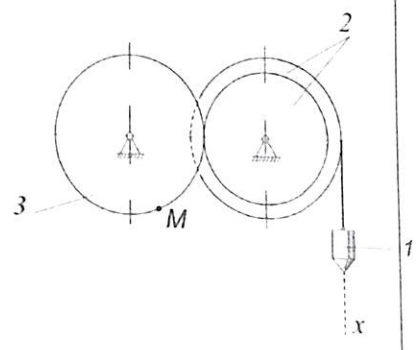
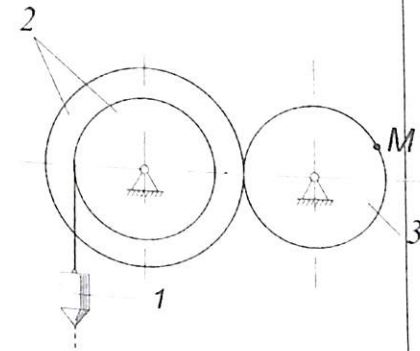
Vaqtning boshlang'ich onida ($t=0$) yukning koordinatasi x_0 , tezligi esa v_0 bo'lishi kerak.

Undan tashqari $t=t_2$ vaqt onida yukning koordinatasi x_2 ga teng bo'lishi lozim.

s_0 , s_1 , s_2 koeffitsientlar shunday aniqlansinki, bunda yuk 1-ning talab qilgan harakati amalga oshsin. Shuningdek, $t=t_1$ vaqt onida yukning hamda mexanizm g'ildiraklaridan birining M nuqtasining tezligi va tezlanishi aniqlansin.

Mexanizmlarning sxemalari, hisoblash uchun kerakli ma'lumotlar jadvalda keltirilgan.

Variant raqam-lari	Mexanizmlarning sxemalari	Radiuslar, sm	1 yukning koordinatalari va tezliklari	Hisob uchun vaqt onlari
1.	2.	3.	4.	5.
1.		$R_2 = 20 \text{ sm}$ $r_2 = 15 \text{ sm}$ $R_3 = 15 \text{ sm}$	$x_0 = 4 \text{ sm}$ $v_0 = 6 \text{ sm/s}$ $x_2 = 220 \text{ sm}$	$t_2 = 4 \text{ s}$ $t_1 = 3 \text{ s}$

26.		$R_2 = 80 \text{ sm}$ $R_3 = 45 \text{ sm}$ $r_3 = 30 \text{ sm}$	$x_0 = 3 \text{ sm}$ $v_0 = 15 \text{ sm/s}$ $x_2 = 102 \text{ sm}$	$t_2 = 3 \text{ s}$ $t_1 = 2 \text{ s}$
27.		$R_2 = 58 \text{ sm}$ $r_2 = 45 \text{ sm}$ $R_3 = 60 \text{ sm}$	$x_0 = 4 \text{ sm}$ $v_0 = 4 \text{ sm/s}$ $x_2 = 172 \text{ sm}$	$t_2 = 4 \text{ s}$ $t_1 = 3 \text{ s}$
28.		$R_2 = 120 \text{ sm}$ $r_2 = 72 \text{ sm}$ $R_3 = 90 \text{ sm}$	$x_0 = 8 \text{ sm}$ $v_0 = 6 \text{ sm/s}$ $x_2 = 40 \text{ sm}$	$t_2 = 4 \text{ s}$ $t_1 = 2 \text{ s}$

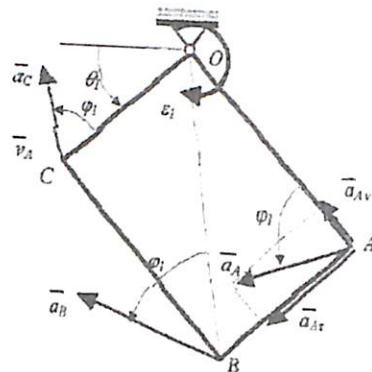
Plastina A nuqtaning urinma tezlanishi quyidagiga teng bo'ladi:

$$a_{A\tau} = OA \cdot \varepsilon_1 = 40 \cdot 0.841 = 33.64 \text{ sm/s}^2$$

Plastina A nuqtasining normal tezlanishi esa quyidagiga teng bo'ladi:

$$a_{An} = OA \cdot \omega_1^2 = 40 \cdot (0.54)^2 = 11.64 \text{ sm/s}^2$$

$\vec{a}_{A\tau}$ vektor OA radiusga perpendikulyar holda ε_1 tomon $\vec{a}_{An}$ vektor esa A nuqtadan OA radius bo'ylab aylanish markazi tomon yo'naladi.



2.24-rasm

A nuqta tezlanishining moduli quyidagiga teng:

$$a_A = \sqrt{(a_{A\tau})^2 + (a_{An})^2} = \sqrt{(33.64)^2 + (11.64)^2} = 35.59 \text{ sm/s}^2$$

$\vec{a}_A$ ning yo'nalishi quyidagi formula asosida aniqlanadi:

$$\operatorname{tg} \varphi_1 = \frac{a_{A\tau}}{a_{An}} = \frac{33.64}{11.64} = -2.89$$

$$\varphi_1 = \arctg 2.89 = 70^\circ 91'$$

Plastina B nuqtasi tezlanishining miqdori va yo'nalishi quyidagicha topiladi:

$$a_B = OB \cdot \sqrt{\varepsilon_1^2 + \omega_1^4} = 50 \sqrt{(0.84)^2 + (0.54)^2} = 50 \cdot 0.89 = 44.5 \text{ sm/s}^2$$

$$\operatorname{tg} \varphi_1 = \frac{\varepsilon_1}{\omega_1^2} = \frac{0.841}{(0.54)^2} = 2.89$$

Plastina C nuqtasi tezlanishi miqdorini va yo'nalishi quyidagicha aniqlanadi:

III-BOB. NUQTANING MURAKKAB HARA-KATI

30-§ Nuqtaning nisbiy, ko'chirma va absolyut harakatlari



3.1-rasm

Nuqta bir vaqtning o'zida ikki yoki undan ortiq harakatda ishtirok etsa, bunday harakat murakkab harakat deyiladi.

Nuqtaning murakkab harakatini o'rganish uchun qo'zg'almas O_1 , x_1, y_1, z_1 va unga nisbatan ixtiyoriy ravishda harakatlanadigan oxyz koordinatalar sistemasini tanlab olamiz (3.1-rasm).

M nuqtaning qo'zg'aluvchi Oxyz koordinatalar sistemasiga nisbatan harakati nisbiy harakat deyiladi.

Nuqtaning bunday harakatdagi tezlik va tezlanishi mos ravishda nisbiy tezlik va nisbiy tezlanish deyiladi hamda $\vec{v}_n$ va $\vec{a}_n$ bilan belgilanadi.

M nuqtaning qo'zg'aluvchi koordinatalar sistemasi bilan birgalikda qo'zg'almas koordinatalar sistemasiga nisbatan harakati ko'chirma harakat deyiladi. Qo'zg'aluvchi koordinatalar sistemasining berilgan onda M nuqta bilan ustma-ust tushuvchi nuqtasining tezligi va tezlanishi ko'chirma tezlik va ko'chirma tezlanish deyiladi hamda $\vec{v}_k$ va $\vec{a}_k$ bilan belgilanadi.

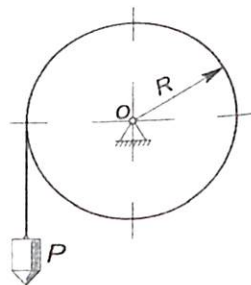
M nuqtaning qo'zg'almas koordinatalar sistemasiga nisbatan harakati absolyut harakat deyiladi. Nuqtaning absolyut harakati o'z navbatida nisbiy va ko'chirma harakatlardan tashkil topgani tufayli nuqtaning absolyut harakatini murakkab deb atash mumkin. Absolyut harakatdagi nuqtaning tezlik va tezlanishi mos ravishda absolyut tezlik va absolyut tezlanish deyiladi hamda $\vec{v}_a$ va $\vec{a}_a$ bilan belgilanadi.

$$a_t = \varepsilon R = 5 \cdot 2 = 10 \text{ m/s}^2,$$

$$a_n = \vartheta_1^2 / R = 11250 \text{ m/s}^2$$

3-masala.

Radiusi $R=10$ sm bo'lgan val unga ipda osilgan P tosh bilan aylantiriladi. Toshning harakati $x=100t$ tenglama bilan ifodalanadi, bunda x – toshdan qo'zg'almas OO_1 gorizontalgacha bo'lgan, santimetrlar hisobida ifodalgan masofa, t vaqt (sekundlar hisobida). t paytida valning burchak tezligi va burchak tezlanishi, shuningdek, val sirtidagi M nuqtaning tezligi va to'la tezlanishi aniqlansin (2.25-rasm).



2.25-rasm

Yechimi.

Toshning tezligi uning harakat tenglamasidan vaqt bo'yicha olingan birinchi tartibli hosilaga teng:

$$\vartheta = x = (100t)' = 200.$$

Tosh osilgan ipni cho'zilmaydi deb faraz qilsak, val O_1 nuqtasining chiziqli tezligi tosh tezligiga teng bo'ladi ($\vartheta = \vartheta_0$). Shuning uchun valning burchak tezligini quyidagicha aniqlash mumkin:

$$\vartheta = \vartheta_0 = \omega \cdot R.$$

Bundan,

$$\omega = \frac{\vartheta_0}{R} = v/R = 20t \text{ rad/s}.$$

Valning burchak tezlanishi uning burchak tezligidan vaqt bo'yicha hisoblangan birinchi tartibli hosilaga teng:

$$\varepsilon = \frac{d\omega}{dt} = (20t)' = 20 \text{ rad/s}^2.$$

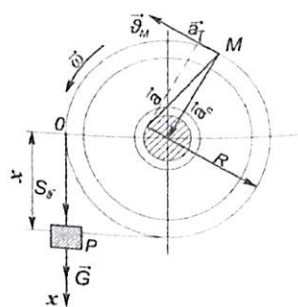
ω va ε lar yo'nalishlari $\vec{\vartheta}$ yo'nalishi orqali aniqlanadi, $\vec{\vartheta}$ esa toshning harakati tomon yo'naladi (2.25-rasm).

Val sirtidagi nuqtaning to'la tezlanishi quyidagicha aniqlanadi:

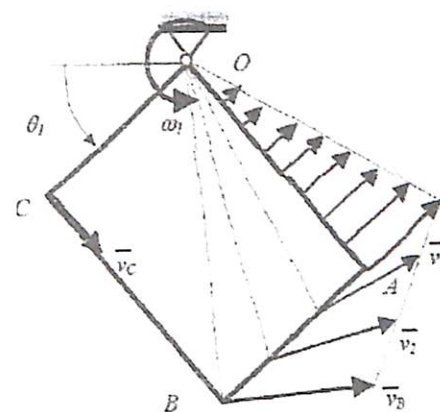
$$a = \sqrt{a_t^2 + a_n^2}.$$

Bunda,

$$a_t = \varepsilon \cdot R = 20 \cdot 10 = 200 \text{ sm/s}^2,$$



Plastina OA tomoni nuqtalari tezliklarining taqsimoti 2.23-rasmda ko'rsatilgan



2.23-rasm.

Plastina OA tomoni nuqtalarining aylanish nuqtasiga bo'lgan masofalarga to'g'ri proporsional holda o'sib boradi.

Plastina AB tomoni nuqtalari tezliklarining taqsimoti ham 2.23-rasmda ko'rsatilgan.

Plastina nuqtalari tezliklarining miqdorlari nuqtalardan aylanish markazigacha bo'lgan masofalarga to'g'ri proporsional bo'ladi. Mazkur vektorlar uchlari $\vec{v}_B$ va $\vec{v}_A$ vektorlar uchlari bir kesishtiruvchi to'g'ri chiziqli kesmasida yotadi.

2.24-rasmda Plastina AB tomoni 1 va 2 nuqtalari tezliklarining vektorlari ham ko'rsatilgan.

Plastina nuqtalarining tezlanishlarini aniqlash uchun plastinaning aylanma harakat burchak tezlanishini aniqlaymiz:

$$\varepsilon_1 = \frac{d\omega_1}{dt} = -\sin t_1 = -0.841 \frac{1}{s}$$

Burchak tezlanishining "manfiy" ishorasi plastinaning aylanma harakati tekis sekinlashuvlar ekanligidan dalolat beradi. ε_1 va O_1 yo'nalishlari qarama qarshi bo'ladi.

2- jismning burchak tezligi

$$\omega_2 = \frac{\vartheta_A}{r_2} = \frac{\vartheta_2}{r_2} = \frac{60t}{20} = 3t \text{ bo'ladi.}$$

Bu jism B nuqtasining tezligi:

$$\vartheta_B = \omega_2 \cdot R_2 = 3t \cdot 30 = 90$$

BC tasmani ham cho'zilmaydi, deb olsak,

$$\vartheta_B = \vartheta_C = 90t \text{ bo'ladi.}$$

Bu holatda 3-jismning burchak tezligi quyidagi formula bilan aniqlanadi:

$$\omega_3 = \frac{\vartheta_C}{R_3} = \frac{90t}{30} = 3t \frac{1}{s}$$

3-jismning burchak tezlanishi esa uning burchak tezligidan vaqt bo'yicha hisoblangan birinchi tartibli hosilaga teng:

$$\varepsilon_3 = \frac{d\omega_3}{dt} = (3t)' = 3 \frac{1}{s^2}$$

M va C nuqtalar t silindrning sirtida, ya'ni aylanish o'qidan bir xil masofada yotganligi uchun, ularning tezliklari, urinma, normal va to'la tezlanishlari o'zaro teng bo'ladi. Shuning uchun:

$$\vartheta_M = \vartheta_C = \omega_3 \cdot R_3 = 3t \cdot 30 = 90t,$$

$$a_t = \varepsilon_3 R_3 = 3 \cdot 30 = 90$$

$$\vartheta_n = \omega_3^2 \cdot R_3 = 9t^2 \cdot 30 = 270t^2,$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{90^2 + (270t^2)^2} = \sqrt{8100 + 72900t^4}.$$

$$t = \tau = 1 \text{ sekunda:}$$

$$\vartheta_M = 90 \cdot 1 = 90 \text{ sm/s,}$$

$$a_t = 90 \text{ sm/s}^2,$$

$$a_n = 270 \cdot 1 = 270 \text{ sm/s}$$

$$a = \sqrt{8100 + 72900} = 284,6 \text{ sm/s}^2.$$

M nuqtaning tezligi, urinma, normal va to'la tezlanishi (2.27-rasmda ko'rsatilgan).

$$a_C = OC \cdot \sqrt{\varepsilon_1^2 + \omega_1^4} = 30 \cdot 0,89 = 26,7 \text{ sm/s}^2$$

$\vec{a}_C$ vektorining plastina OC tomoni bilan hosil qilgan burchagi ham φ ga teng bo'ladi.

Plastina B va C nuqtalarning tezlanishlari ham 2.24-rasmda ko'rsatilgan.

3-Masala.

Radiusi R m bo'lgan maxovik tinch holatdan boshlab tekis tezlanish bilan aylanadi, gardishida yotuvchi nuqtalar $t=10s$ dan keyin $\vartheta=100m/s$ chiziqli tezlikka ega bo'ladi. G'ildirak gardishida yotgan nuqtaning $t_1=15$ bo'lgan vaqtdagi tezligi, urinma va normal tezlanishlari topilsin.

Yechimi.

Maxovik tinch holatdan boshlab, tekis tezlanish bilan aylanadi. Shuning uchun, $\omega_0=0$. $t=10s$ vaqt oni uchun maxovikning burchak tezligini aniqlaymiz:

$$\vartheta = \omega \cdot R$$

Bundan,

$$\omega = \frac{\vartheta}{R} = \frac{100m/s}{2m} = 50 \frac{rad}{s}.$$

Maxovikning shu ondagi burchak tezlanishi esa quyidagicha aniqlanadi:

$$\omega = \varepsilon t$$

Bundan,

$$\varepsilon = \frac{\omega}{t} = \frac{50 \text{ rad/s}}{10 \text{ s}} = 5 \frac{rad}{s^2} = const$$

Maxovik gardishida yotgan nuqtaning $t_1 = 15s$ dagi tezligi quyidagiga teng bo'ladi:

$$\vartheta_1 = \omega_1 \cdot R$$

Bunda,

$$\omega_1 = \varepsilon t_1 \text{ rad/s.}$$

Shuning uchun,

$$\vartheta_1 = 150 \text{ m/s.}$$

Maxovik gardishida yotgan nuqtaning urinma va normal tezlanishlarini aniqlaymiz:

Yuk osilgan arqonni choʻzilmaydi deb hisoblasak, 2 jismning burchak tezligi quyidagicha aniqlanadi:

$$\omega_2 = \frac{\vartheta_A}{r_2} = \frac{\vartheta_1}{r_2} = \frac{16t}{16} = t;$$

bunda, $\vartheta_a = \vartheta_1$ ekanligi eʼtiborga olindi.

2-jism B nuqtasining tezligi esa $\vartheta_B = \omega_2 \cdot R$, boʻladi.

B nuqtani 2- va 3- jismlar uchun umumiy deb olib, 3 jismning burchak tezligini topamiz:

$$\omega_3 = \frac{\vartheta_B}{R_3} = \frac{20t}{25} = 0,8t.$$

3- jism c nuqtasining tezligi esa quyidagiga teng boʻladi:

$$\vartheta_c = \omega_3 \cdot r_3 = 0,8t \cdot 20 = 16t.$$

Agar 3- va 4- jismlarni biriktiruvchi tasmani choʻzilmaydi deb hisoblasak,

$$\vartheta_c = \vartheta_D$$

boʻladi.

Shuning uchun 4- jismning burchak tezligi

$$\omega_4 = \frac{\vartheta_d}{r_4} = \frac{16t}{8} = 2t \text{ boʻladi.}$$

4-jismning burchak tezlanishini aniqlash uchun burchak tezligidan vaqt boʻyicha birinchi tartibli hosila hisoblaymiz:

$$\varepsilon_4 = \frac{d\omega_4}{dt} = (2t)' = 2.$$

4-jism M nuqtasining tezligi, urinma, normal va toʻla tezlanishlari quyidagicha aniqlanadi:

$$\vartheta_M = \omega_4 \cdot R_4 = 2t \cdot 12 = 24t,$$

$$a_t = \varepsilon_4 \cdot R_4 = 2 \cdot 12 = 24,$$

$$a_n = \omega_4^2 \cdot R_4 = 4t^2 \cdot 12 = 48t^2,$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{576 + 2304t^2}.$$

$t = \tau = 2$ sekundda:

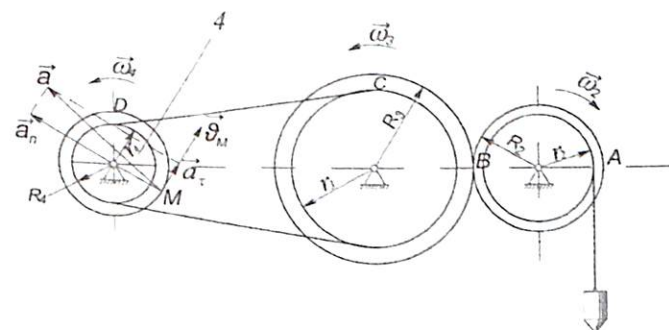
$$\vartheta_m = 24 \cdot 2 = 48 \text{ sm/s}^2,$$

$$a_t = 24 \text{ sm/s}^2,$$

$$a_n = 48 \cdot 4 = 192 \text{ sm/s}^2,$$

$$a = \sqrt{576 + 36864} = 193,4 \text{ sm/s}^2.$$

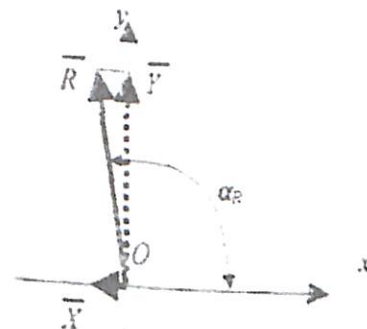
Tezlik va tezlanishlar uchun masshtab tanlab, ularni chizmada koʻrsatamiz: M nuqtaning tezlik vektori nuqtadan traektoriyaga oʻtkazilgan urinma boʻylab, toʻla tezlanish vektori esa urinma va normal tezlanishlarga qurilgan parallelogramning diagonalini boʻylab yoʻnaladi (2.29-rasm).



2.29-rasm

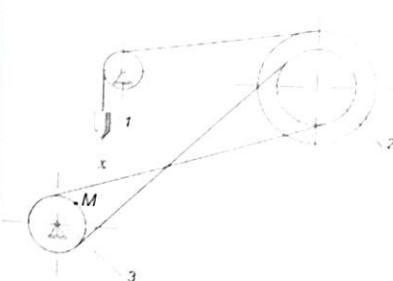
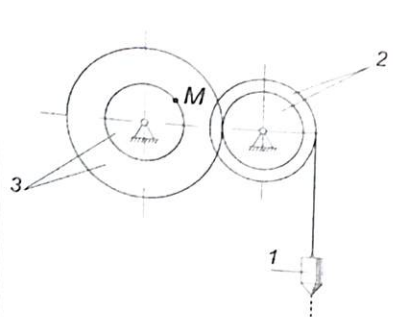
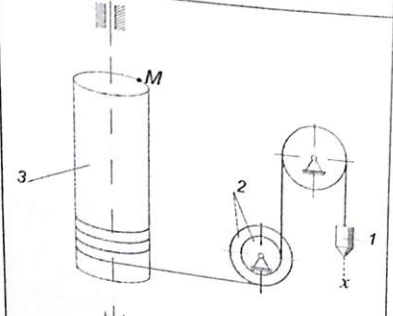
28-§ Mustaqil oʻrganish uchun talabalarga tavsiya etiladigan muammolar

Muammo-1. Toʻgʻri burchakli prizma gorizont tekislikda harakatlanmoqda (2.30-rasm).

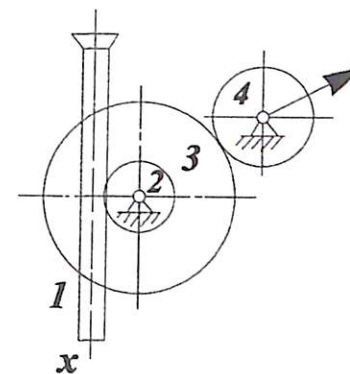


2.30-rasm

Agar A nuqta oʻzgarmas $a_A = 230 \text{ sm/s}^2$ tezlanishga ega boʻlsa, $t_1 = 1 \text{ s}$ vaqt momenti uchun prizma A, B, C nuqtalarining tezligi va tezlanishi aniqlansin.

5.		$R_2 = 30 \text{ sm}$ $r_2 = 20 \text{ sm}$ $R_3 = 15 \text{ sm}$	$x_0 = 10 \text{ sm}$ $\dot{\varphi}_0 = 8 \text{ sm/s}$ $x_2 = 277 \text{ sm}$	$t_2 = 3 \text{ s}$ $t_1 = 1 \text{ s}$
6.		$R_2 = 25 \text{ sm}$ $r_2 = 15 \text{ sm}$ $R_3 = 35 \text{ sm}$ $r_3 = 15 \text{ sm}$	$x_0 = 6 \text{ sm}$ $\dot{\varphi}_0 = 5 \text{ sm/s}$ $x_2 = 356 \text{ sm}$	$t_2 = 5 \text{ s}$ $t_1 = 2 \text{ s}$
7.		$R_2 = 50 \text{ sm}$ $r_2 = 25 \text{ sm}$ $R_3 = 40 \text{ sm}$	$x_0 = 7 \text{ sm}$ $\dot{\varphi}_0 = 6 \text{ sm/s}$ $x_2 = 103 \text{ sm}$	$t_2 = 2 \text{ s}$ $t_1 = 1 \text{ s}$

Muammo -7. Strelkani indikator mexanizmida harakat o'lchov shtiftining 1 reykasidan 2 shesternyaga uzatiladi; 2 shesterniyaning o'qiga 3 tishli g'ildirak o'rnatilgan, 3 g'ildirak esa strelka biriktirilgan 4 shesternya bilan tishlashadi. Agar shtiftning harakati $x = asin kt$ tenglama bilan berilgan bo'lsa va tishli g'ildiraklarning radiuslari tegishli r_2 , r_3 va r_4 bo'lsa, strelkaning burchak tezligi aniqlansin (2.32 – rasm).

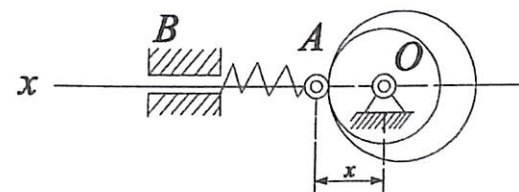


2.32 – rasm

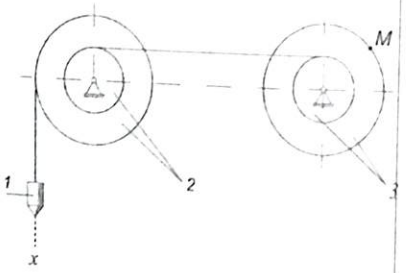
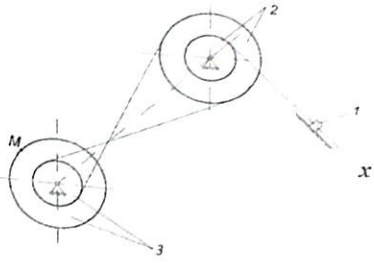
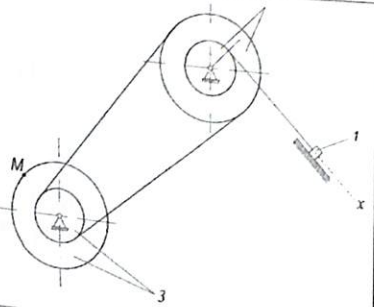
Muammo -8. Kulak O o'q atrofida tekis aylanib, AB sterjenni teng o'lchovchi ilgarilamma-qaytma harakatga keltiradi. Kulakning bir marta to'liq aylanish vaqti 8 c, sterjenning shu vaqt ichidagi harakati tenglamasi:

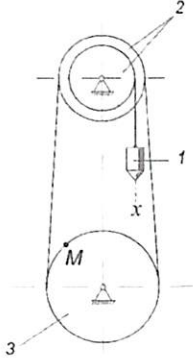
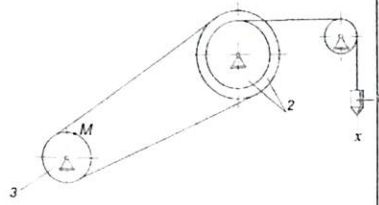
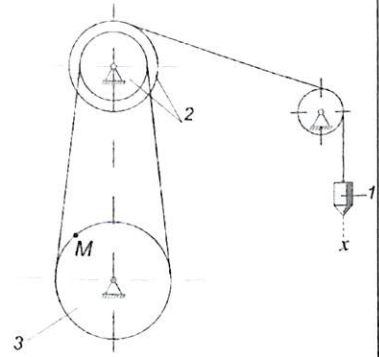
$$x = \begin{cases} 30 + 5t, & 0 \leq t \leq 4, \\ 70 - 5t, & 4 \leq t \leq 8 \end{cases}$$

ko'rinishga ega (x -santimetrlar, t -sekundlar hisobida). Kulak konturining tenglamasi topilsin va sterjen harakatining grafigi chizilsin (2.33 – rasm).



2.33 - rasm

11.		$R_2 = 40 \text{ sm}$ $r_2 = 20 \text{ sm}$ $R_3 = 40 \text{ sm}$ $r_3 = 20 \text{ sm}$	$x_0 = 6 \text{ sm}$ $\vartheta_0 = 14 \text{ sm/s}$ $x_2 = 862 \text{ sm}$	$t_2 = 4s$ $t_1 = 2s$
12.		$R_2 = 50 \text{ sm}$ $r_2 = 20 \text{ sm}$ $R_3 = 50 \text{ sm}$ $r_3 = 20 \text{ sm}$	$x_0 = 5 \text{ sm}$ $\vartheta_0 = 10 \text{ sm/s}$ $x_2 = 193 \text{ sm}$	$t_2 = 2s$ $t_1 = 1s$
13.		$R_2 = 50 \text{ sm}$ $r_2 = 25 \text{ sm}$ $R_3 = 50 \text{ sm}$ $r_3 = 20 \text{ sm}$	$x_0 = 8 \text{ sm}$ $\vartheta_0 = 5 \text{ sm/s}$ $x_2 = 347 \text{ sm}$	$t_2 = 3s$ $t_1 = 2s$

2.		$R_2 = 20 \text{ sm}$ $r_2 = 15 \text{ sm}$ $R_3 = 25 \text{ sm}$	$x_0 = 8 \text{ sm}$ $\vartheta_0 = 4 \text{ sm/s}$ $x_2 = 44 \text{ sm}$	$t_2 = 2s$ $t_1 = 1s$
3.		$R_2 = 25 \text{ sm}$ $r_2 = 20 \text{ sm}$ $R_3 = 15 \text{ sm}$	$x_0 = 3 \text{ sm}$ $\vartheta_0 = 12 \text{ m/s}$ $x_2 = 211 \text{ sm}$	$t_2 = 4s$ $t_1 = 1s$
4.		$R_2 = 20 \text{ sm}$ $r_2 = 15 \text{ sm}$ $R_3 = 25 \text{ sm}$	$x_0 = 5 \text{ sm}$ $\vartheta_0 = 10 \text{ sm/s}$ $x_2 = 505 \text{ sm}$	$t_2 = 5s$ $t_1 = 3s$

Absolyut tezlik moduli va yo'nalishi quyidagi formulalar asosida aniqlanadi:

$$\begin{aligned} \vartheta_a &= \sqrt{\vartheta_{ax}^2 + \vartheta_{ay}^2} \\ \cos(\vec{\vartheta}_a \wedge x) &= \frac{\vartheta_{ax}}{\vartheta_a}, \quad \cos(\vec{\vartheta}_a \wedge y) = \frac{\vartheta_{ay}}{\vartheta_a} \end{aligned} \quad (3.7)$$

Shuni ta'kidlash lozimki, nuqtaning nisbiy tezligini aniqlash uchun ko'chirma harakat xayolan to'xtatiladi.

Ko'chirma harakat tezligini aniqlash uchun nisbiy harakat xayolan to'xtatiladi va berilgan onda qo'zg'aluvchan sanoq sistemasining M nuqta bilan ustma - ust tushuvchi nuqtasining tezligi aniqlanadi.

32-§ Koriolis tezlanishi

Nisbiy tezlikning yo'nalishini ko'chirma harakatda, ko'chirma tezling miqdori va yo'nalishini nisbiy harakatda o'zgarishlarini xarakterlovchi kattalik Koriolis tezlanishi deyiladi.

Koriolis tezlanishi murakkab harakatdagi nuqtaning ko'chirma harakat burchak tezligi vektori bilan nisbiy harakat tezligi vektorining vektir ko'paytmasining ikkilanganiga teng:

$$\vec{a}_c = 2(\vec{\omega}_k \times \vec{\vartheta}_n) \quad (3.8)$$

Agar $\vec{\omega}_k$ bilan $\vec{\vartheta}_n$ orasidagi burchakni α bilan belgilasak, Koriolis tezlanishining moduli

$$a_c = 2\omega_k \vartheta_n \sin \alpha \quad (3.9)$$

formuladan aniqlanadi.

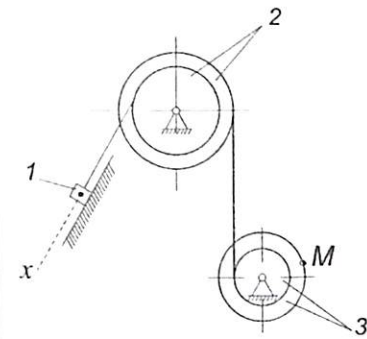
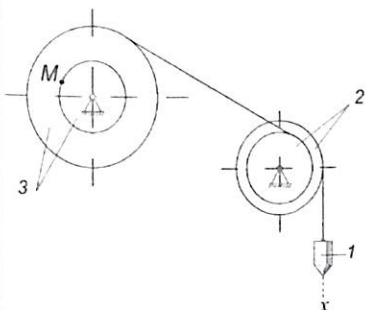
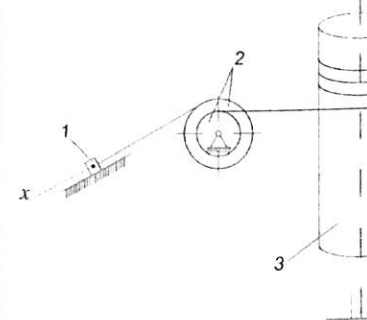
Koriolis tezlanishining yo'nalishini aniqlash uchun nuqtaning nisbiy tezligini ko'chirma harakat aylanish o'qiga perpendikulyar tekislikka proeksiyalab, bu proeksiyani mazkur tekislikda ko'chirma harakat aylanishi yo'nalishida 90° burchakka burish kerak (3.3-rasm).

Bu usul Jukovskiy qoidasi deyiladi.

Agar $\vec{\omega}_k \perp \vec{\vartheta}_n$ bo'lsa (3.4-rasm), $\sin \alpha = 1$ bo'ladi.

U holda

$$a_c = 2\omega_k \cdot \vartheta_n. \quad (3.10)$$

8.		$R_2 = 50 \text{ sm}$ $r_2 = 40 \text{ sm}$ $R_3 = 40 \text{ sm}$ $r_3 = 20 \text{ sm}$	$x_0 = 5 \text{ sm}$ $\vartheta_0 = 9 \text{ sm/s}$ $x_2 = 194 \text{ sm}$	$t_2 = 3 \text{ s}$ $t_1 = 2 \text{ s}$
9.		$R_2 = 40 \text{ sm}$ $r_2 = 20 \text{ sm}$ $R_3 = 50 \text{ sm}$ $r_3 = 25 \text{ sm}$	$x_0 = 9 \text{ sm}$ $\vartheta_0 = 8 \text{ sm/s}$ $x_2 = 105 \text{ sm}$	$t_2 = 4 \text{ s}$ $t_1 = 2 \text{ s}$
10.		$R_2 = 60 \text{ sm}$ $r_2 = 25 \text{ sm}$ $R_3 = 70 \text{ sm}$	$x_0 = 8 \text{ sm}$ $\vartheta_0 = 4 \text{ sm/s}$ $x_2 = 119 \text{ sm}$	$t_2 = 3 \text{ s}$ $t_1 = 2 \text{ s}$

Nuqtaning ko'chirma tezlanishini aniqlash uchun nisbiy harakat xayolan to'xtatiladi va berilgan onda qo'zg'aluvchan sanoq sistemasining M nuqta bilan ustma – ust tushuvchi nuqtasinnig tezlanishi aniqlanadi.

Agar qo'zg'aluvchan koordinatalar sistemasining nuqtalari egri chiziqli harakatda bo'lsa, hamda nuqtaning nisbiy harakat traektoriyasi egri chiziqdan iborat bo'lsa, ko'chirma va nisbiy tezlanishlarni normal va urinma tezlanishlarning geometrik yig'indisidan iborat deb qarash mumkin. U vaqtda (3.11) quyidagicha yoziladi:

$$\vec{a}_a = \vec{a}_n^n + \vec{a}_k^n + \vec{a}_k^\tau + \vec{a}_c^\tau + \vec{a}_c \quad (3.12)$$

bunda,

a_n^n ; a_k^n – nisbiy va ko'chirma harakatlardagi normal tezlanishlar.

$\vec{a}_k^\tau$, $\vec{a}_c^\tau$ – nisbiy va ko'chirma harakatlardagi urinma tezlanishlar.

M nuqtaning murakkab harakatida absolyut tezlanishning modulini aniqlash uchun proektsiyalash usulidan foydalanish ham mumkin. Buning uchun koordinata o'qlari o'tkaziladi va (18.9) tenglik koordinata o'qlariga proektsiyalanadi:

$$a_{ax} = a_{nx}^n + a_{nx}^\tau + a_{kx}^n + a_{kx}^\tau + a_{cx} \quad (3.13)$$

$$a_{ay} = a_{ny}^n + a_{ny}^\tau + a_{ky}^n + a_{ky}^\tau + a_{cy}$$

$$a_{az} = a_{nz}^n + a_{nz}^\tau + a_{kz}^n + a_{kz}^\tau + a_{cz}$$

Absolyut tezlanish moduli va yo'nalishi quyidagi formulalar asosida aniqlanadi:

$$a_a = \sqrt{a_{ax}^2 + a_{ay}^2 + a_{az}^2} \quad (3.14)$$

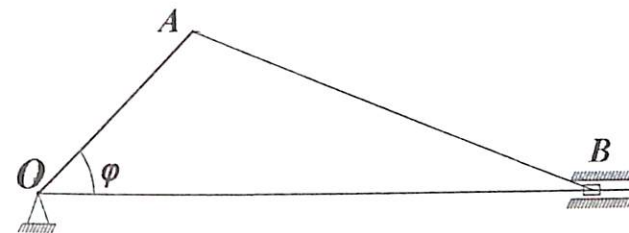
$$\cos(\vec{a}_a \wedge \vec{x}) = \frac{a_{ax}}{a}, \quad \cos(\vec{a}_a \wedge \vec{y}) = \frac{a_{ay}}{a},$$

$$\cos(\vec{a}_a \wedge \vec{z}) = \frac{a_{az}}{a}.$$

Ko'chirma harakati ilgarilanma harakatdan iborat bo'lgan nuqtaning absolyut tezlanishi uning nisbiy va ko'chirma tezlanishlarning geometrik yig'indisiga teng:

$$\vec{a}_a = \vec{a}_n + \vec{a}_k \quad (3.15)$$

Shatun uzunligi $AB=l$, B polzun O nuqtadan o'tuvchi gorizontaal chiziqli bo'ylab harakatlanadi. B polzunni moddiy nuqta sifatida qarab, uning nisbiy harakati tenglamasi tuzilsin (3.12 – rasm).

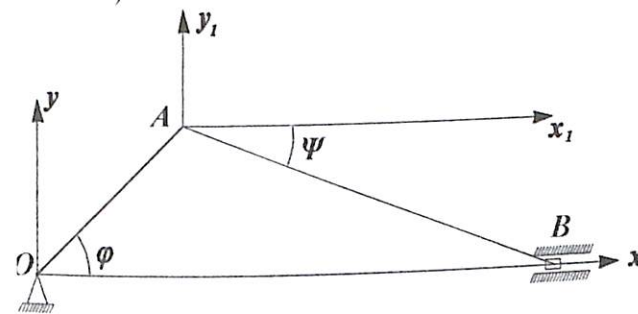


3.12 - rasm

Yechimi. AB shatun murakkab harakatini ikki soda harakatlarga ajratamiz:

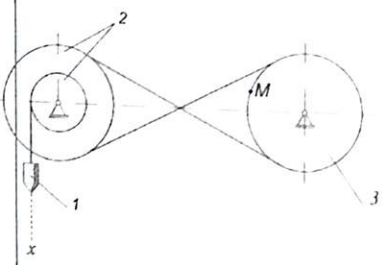
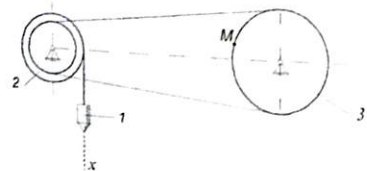
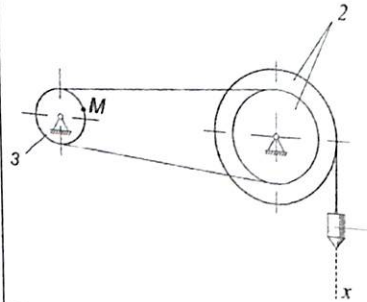
- O nuqta atrofida ω_0 burchak tezlik bilan yuz beruvchi ko'chirma harakat
- A nuqta atrofida notekis yuz beruvchi aylanma harakat – nisbiy harakat.

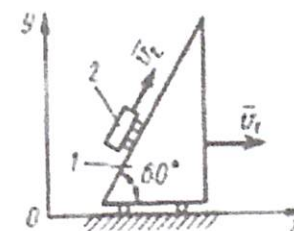
Qo'zg'almas sanoq sistemasida Yer bilan bog'langan O nuqtadan o'tuvchi Oxy koordinata o'qlari sistemasini tanlaymiz. Qo'zg'aluvchan sanoq sistemasida Krivoship va shatun birlashadigan A nuqtadan o'tuvchi Ax_1y_1 koordinata o'qlari sistemasini oladi (3.13 –rasm).



3.13 - rasm

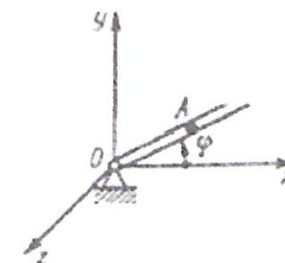
AB shatunning mazkur sanoq sistemasidagi holati
 $\psi = \angle x_1AB = \angle ABO$

17.		$R_2 = 30 \text{ sm}$ $r_2 = 20 \text{ sm}$ $R_3 = 30 \text{ sm}$	$x_0 = 6 \text{ sm}$ $\vartheta_0 = 3 \text{ sm/s}$ $x_2 = 80 \text{ sm}$	$t_2 = 2 \text{ s}$ $t_1 = 1 \text{ s}$
18.		$R_2 = 40 \text{ sm}$ $r_2 = 25 \text{ sm}$ $R_3 = 50 \text{ sm}$	$x_0 = 7 \text{ sm}$ $\vartheta_0 = 0 \text{ sm/s}$ $x_2 = 557 \text{ sm}$	$t_2 = 5 \text{ s}$ $t_1 = 2 \text{ s}$
19.		$R_2 = 40 \text{ sm}$ $r_2 = 30 \text{ sm}$ $R_3 = 20 \text{ sm}$	$x_0 = 5 \text{ sm}$ $\vartheta_0 = 10 \text{ sm/s}$ $x_2 = 179 \text{ sm}$	$t_2 = 3 \text{ s}$ $t_1 = 2 \text{ s}$



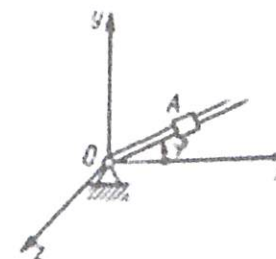
3.14 -rasm

3-muammo. Oz o'qi atrofida $\varphi=4t$ qonun bo'yicha aylanayotgan naycha ichidagi A sharcha $OA=5t^2$ tenglama asosida harakat qilsa, $t=0,25\text{s}$ paytdagi A nuqtaning x_A koordinatasini toping (3.15 - rasm)

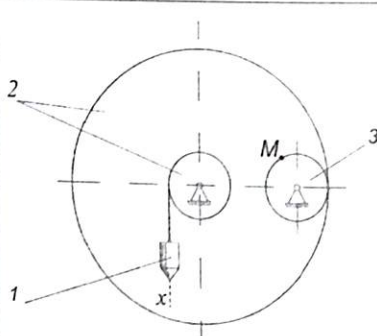
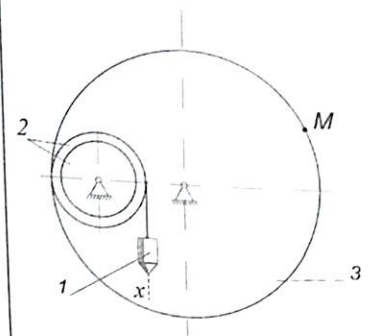
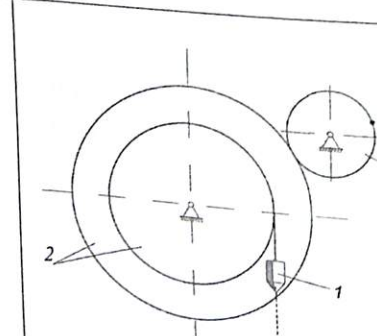


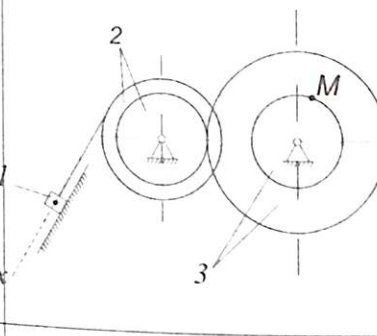
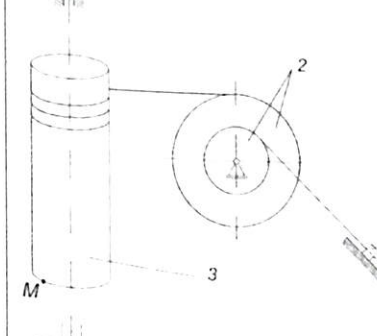
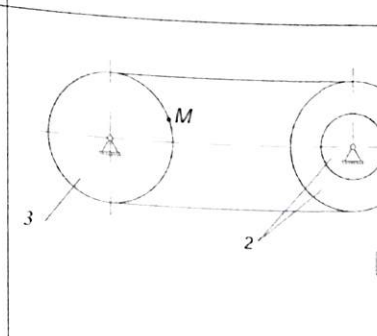
3.15 - rasm

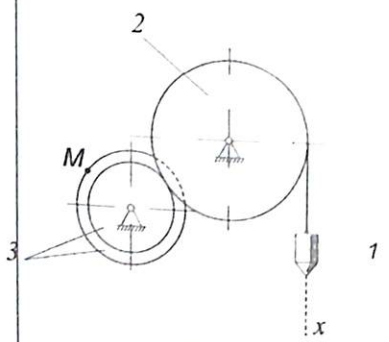
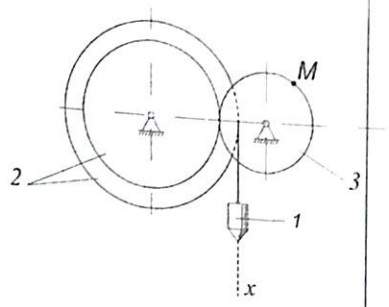
4-muammo. Oz o'qi atrofida $\varphi=2t$ qonun bo'yicha aylanayotgan sterjen bo'ylab A polzun $OA=3t^2$ tenglama asosida harakat qilsa, polzunning o'lchamlarini hisobga olmay, $t=0,5\text{s}$ paytdagi uning y_A koordinatasini hisoblang (3.16-rasm).

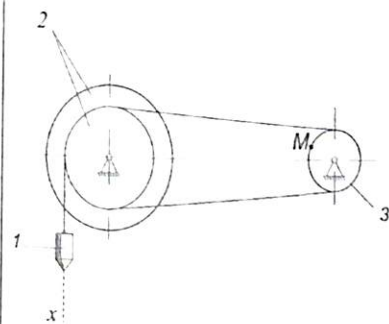
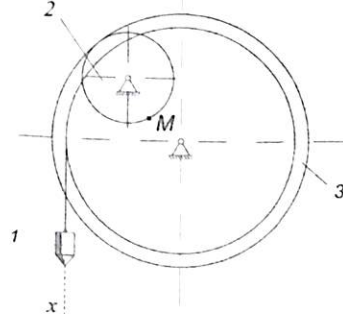
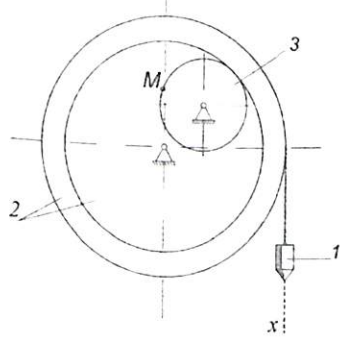


3.16-rasm

23.		$R_2 = 40 \text{ sm}$ $r_2 = 15 \text{ sm}$ $R_3 = 15 \text{ sm}$	$x_0 = 6 \text{ sm}$ $\vartheta_0 = 2 \text{ sm/s}$ $x_2 = 111 \text{ sm}$	$t_2 = 3 \text{ s}$ $t_1 = 2 \text{ s}$
24.		$R_2 = 60 \text{ sm}$ $r_2 = 45 \text{ sm}$ $R_3 = 130 \text{ sm}$	$x_0 = 8 \text{ sm}$ $\vartheta_0 = 5 \text{ sm/s}$ $x_2 = 124 \text{ sm}$	$t_2 = 4 \text{ s}$ $t_1 = 3 \text{ s}$
25.		$R_2 = 120 \text{ sm}$ $r_2 = 72 \text{ sm}$ $R_3 = 36 \text{ sm}$	$x_0 = 7 \text{ sm}$ $\vartheta_0 = 16 \text{ sm/s}$ $x_2 = 215 \text{ sm}$	$t_2 = 4 \text{ s}$ $t_1 = 2 \text{ s}$

14.		$R_2 = 30 \text{ sm}$ $r_2 = 22 \text{ sm}$ $R_3 = 60 \text{ sm}$ $r_3 = 30 \text{ sm}$	$x_0 = 4 \text{ sm}$ $\vartheta_0 = 6 \text{ sm/s}$ $x_2 = 32 \text{ sm}$	$t_2 = 2 \text{ s}$ $t_1 = 1 \text{ s}$
15.		$R_2 = 40 \text{ sm}$ $r_2 = 20 \text{ sm}$ $R_3 = 25 \text{ sm}$	$x_0 = 10 \text{ sm}$ $\vartheta_0 = 7 \text{ sm/s}$ $x_2 = 128 \text{ sm}$	$t_2 = 2 \text{ s}$ $t_1 = 1 \text{ s}$
16.		$R_2 = 30 \text{ sm}$ $r_2 = 20 \text{ sm}$ $R_3 = 30 \text{ sm}$	$x_0 = 5 \text{ sm}$ $\vartheta_0 = 2 \text{ sm/s}$ $x_2 = 189 \text{ sm}$	$t_2 = 4 \text{ s}$ $t_1 = 2 \text{ s}$

29.		$R_2 = 100 \text{ sm}$ $r_2 = 75 \text{ sm}$ $R_3 = 60 \text{ sm}$	$x_0 = 5 \text{ sm}$ $v_0 = 10 \text{ sm/s}$ $x_2 = 41 \text{ sm}$	$t_2 = 2s$ $t_1 = 1s$
30.		$R_2 = 60 \text{ sm}$ $r_2 = 45 \text{ sm}$ $R_3 = 36 \text{ sm}$	$x_0 = 2 \text{ sm}$ $v_0 = 12 \text{ sm/s}$ $x_2 = 173 \text{ sm}$	$t_2 = 3s$ $t_1 = 2s$

20.		$R_2 = 50 \text{ sm}$ $r_2 = 30 \text{ sm}$ $R_3 = 25 \text{ sm}$	$x_0 = 9 \text{ sm}$ $v_0 = 8 \text{ sm/s}$ $x_2 = 65 \text{ sm}$	$t_2 = 2s$ $t_1 = 1s$
21.		$R_2 = 30 \text{ sm}$ $R_3 = 80 \text{ sm}$ $r_3 = 70 \text{ sm}$	$x_0 = 5 \text{ sm}$ $v_0 = 3 \text{ sm/s}$ $x_2 = 129 \text{ sm}$	$t_2 = 4s$ $t_1 = 3s$
22.		$R_2 = 40 \text{ sm}$ $r_2 = 30 \text{ sm}$ $R_3 = 15 \text{ sm}$	$x_0 = 10 \text{ sm}$ $v_0 = 7 \text{ sm/s}$ $x_2 = 48 \text{ sm}$	$t_2 = 2s$ $t_1 = 1s$

Nuqtaning nisbiy va ko'chirma harakatini bilgan holda uning absolyut harakatini, binobarin, absolyut harakat tezligi va tezlanishini aniqlash nuqta murakkab harakati kinematikasining asosiy masalasi hisoblanadi.

31-§ Murakkab harakatdagi nuqtaning tezliklarini qo'shish haqidagi teorema

Faraz qilaylik M nuqta qo'zg'almas O_1, x_1, y_1, z_1 koordinatalar sistemasiga nisbatan murakkab harakatda bo'lsin (3.2-rasm). [4]

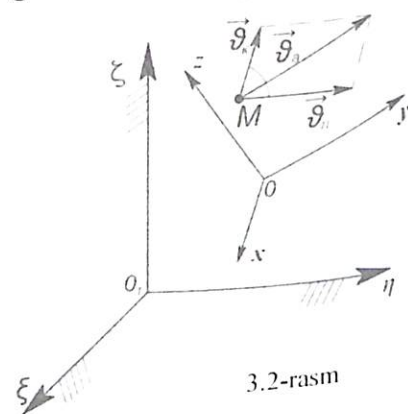
Nuqtaning qo'zg'almas va qo'zg'aluvchan koordinatalar sistemasiga nisbatan holatini aniqlovchi radius - vektorlarni $\vec{r}_1$ va $\vec{r}_0$ deb, qo'zg'aluvchan sistemani qo'zg'almas sistemaga nisbatan holatini aniqlovchi radius - vektorni $\vec{r}$ deb belgilasak, 18.1-rasmdan:

$$\vec{r}_1 = \vec{r}_0 + \vec{r} \quad (3.1)$$

Nuqtaning tezligi uning holatini aniqlovchi radius - vektordan vaqt bo'yicha hisoblangan birinchi tartibli hosilaga teng:

$$\vec{v} = \frac{d\vec{r}_1}{dt} = \frac{d\vec{r}_0}{dt} + \frac{d\vec{r}}{dt} \quad (3.2)$$

$\vec{r}$ - vector M nuqtaning ko'zg'almas koordinatalar sistemasiga nisbatan holatini aniqlovchi radius - vektor bo'lgani uchun $\frac{d\vec{r}}{dt}$ hosila nuqtaning absolyut tezligi $\vec{v}_a$ ni, $\vec{r}_0$ - vector M nuqtaning qo'zg'aluvchi koordinatalar sistemasiga nisbatan holatini aniqlovchi radius - vektor bo'lgani uchun $\frac{d\vec{r}_0}{dt}$ hosila nuqtaning nisbiy harakat tezligi $\vec{v}_n$ ni, $\vec{r}_0$ - vektor qo'zg'aluvchi koordinatalar sistemasining qo'zg'almas sistemaga nisbatan holatini aniqlovchi radius - vektor bo'lgani uchun, $\frac{d\vec{r}_0}{dt}$ hosila nuqtaning ko'chirma harakat tezligi $\vec{v}_k$ ni ifodalaydi. Shuning uchun (3.2) dan:



3.2-rasm

$$\vec{v}_a = \vec{v}_n + \vec{v}_k, \vec{v}_k = \vec{v}_0 + \vec{\omega} * \vec{r} \quad (3.3)$$

Agar ekvivalent e'tiborga olsak, nuqtaning absolyut tezlanishi quyidagicha aniqlanadi:

$$\vec{a}_a = \vec{a}_0 + \vec{\omega} * \vec{r} + \frac{d\vec{r}}{dt}$$

Binobarin, murakkab harakatdagi nuqtaning absolyut tezligi nisbiy va ko'chirma harakat tezliklarining geometrik yig'indisiga teng. (3.3) tenglama murakkab harakatdagi nuqtaning tezliklarini qo'shish haqidagi teoremani ifodalaydi.

Absolyut tezlik vektori nisbiy va ko'chirma harakat tezliklariga qurilgan parallelogramning diagonalini bo'ylab yo'nalgan bo'lib, moduli quyidagi formula asosida aniqlanadi (3.2-rasm):

$$v = \sqrt{v_n^2 + v_k^2 + 2v_nv_k \cos \alpha} \quad (3.4)$$

Bunda:

a) Agar $\alpha = 90^\circ$, ya'ni $\vec{v}_n \perp \vec{v}_k$ bo'lsa, absolyut tezlik moduli

$$v_a = \sqrt{v_n^2 + v_k^2}$$

formula yordamida hisoblanadi.

b) Agar $\alpha = 0^\circ$ bo'lsa, ya'ni $\vec{v}_n$ va $\vec{v}_k$ bir to'g'ri chiziq bo'ylab bir tomonga yo'nalsa, absolyut tezlik moduli

$$v_a = \sqrt{v_n^2 + v_k^2 + 2v_nv_k} = (v_n + v_k)$$

formula orqali aniqlanadi.

v) Agar $\alpha = 180^\circ$ bo'lsa, ya'ni $\vec{v}_n$ bilan $\vec{v}_k$ bir to'g'ri chiziq bo'ylab qarama - qarshi tomonga yo'nalsa, absolyut tezlik moduli

$$v_a = \sqrt{v_n^2 + v_k^2 - 2v_nv_k} = (v_n - v_k) \quad (3.5)$$

formuladan aniqlanadi.

Absolyut tezlik modulini proektsiyalash usuli yordamida ham aniqlash mumkin. Buning uchun koordinata o'qlari o'tqaziladi va (3.3) tenglik koordinata o'qlariga proektsiyalanadi:

$$\begin{aligned} v_{ax} &= v_{nx} + v_{kx}, \\ v_{ay} &= v_{ny} + v_{ky}. \end{aligned} \quad (3.6)$$

Takrorlash uchun savollar

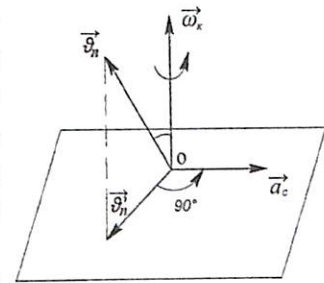
1. Nuqtaning qanday harakatga nisbiy harakat deyiladi?
2. Nuqtaning qanday harakatiga ko'chirma harakat deyiladi?
3. Nuqtaning qanday harakatiga absolyut harakat deyiladi?
4. Nuqtaning nisbiy tezligi qanday topiladi?
5. Nuqtaning ko'chirma tezligi qanday topiladi?
6. Nuqtaning absolyut tezligi qanday topiladi?
7. Tezliklarni qo'shish haqidagi teoremani aytib bering.
8. Absolyut tezlik modulini topish formulasini yozib bering.
9. Nuqtaning nisbiy tezlanishi qanday aniqlanadi?
10. Nuqtaning ko'chirma tezlanishi qanday aniqlanadi?
11. Nuqtaning absolyut tezlanishi qanday aniqlanadi?
12. Koriolis tezlanishining yuzaga kelish shartlarini aytib bering.
13. Koriolis tezlanishi qanday yo'naladi?
14. Koriolis tezlanishining moduli qanday ifodalanadi?
15. Koriolis tezlanishining nolga teng bo'lishi shartlarini aytib bering.

34-§ Nuqtaning nisbiy va absolyut harakatlarida uning traektoriyasi va harakat tenglamalarini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Mazkur mavzuga doir masalalarni ikki asosiy turga ajratish mumkin.

- 1) Nuqtaning nisbiy va ko'chirma harakatlarini bilgan holda absolyut harakat traektoriyasining tenglamalarini aniqlash.
 - 2) Nuqtaning absolyut va ko'chirma harakatlarini bilgan holda nisbiy harakat traektoriyasining tenglamalarini aniqlash.
- Birinci turdagi masalalarni yechishda nuqtaning nisbiy va ko'chirma harakatlarini qo'shish lozim.

Koriolis tezlanishining yo'nalishini ($\vec{\omega}_k \times \vec{v}_n$) vektor ko'paytma qoidasiga muvofiq aniqlash ham mumkin. Qoidaga ko'ra, Koriolis tezlanishi $\vec{\omega}_k$ va $\vec{v}_n$ vektorlar joylashgan tekislikka perpendikulyar holda shunday tomonga qarab yo'nalgan bo'ladiki, u tomondan qaraganda $\vec{\omega}_k$ vektorni $\vec{v}_n$ vektor bilan kichik burchak orqali ustma-ust tushirish uchun qilinadigan aylanma harakat soat mili harakatiga qarama-qarshi yo'nalishda ro'y beradi (3.4- rasm). Koriolis tezlanishi quyidagi hollarda nolga teng bo'ladi:



3.4-rasm

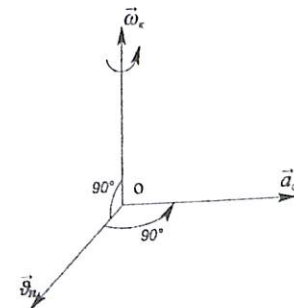
- a) agar ω_k ya'ni ko'chirma harakat ilgarilanma harakatdan iborat bo'lsa;
- b) agar $v_n=1$ ya'ni nisbiy harakat tezligi biror onda nolga teng bo'lsa;
- v) agar $\alpha=0$ yoki $\alpha=180^\circ$, ya'ni nisbiy harakat ko'chirma harakat aylanish o'qiga parallel ravishda sodir bo'lsa, yoki berilgan onda nisbiy harakat tezligi mazkur o'qqa parallel bo'lsa.

33-§. Murakkab harakatdagi nuqtaning tezlanishlarini qo'shish haqidagi Koriolis teoremasi

Murakkab harakatdagi nuqtaning tezlanishi nisbiy, ko'chirma va Koriolis tezlanishlarining geometrik yig'indisiga teng:

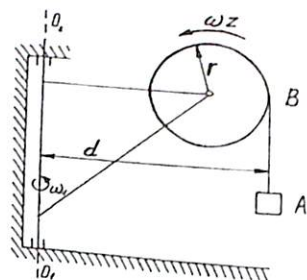
$$\vec{a}_a = \vec{a}_n + \vec{a}_k + \vec{a}_c \quad (3.11)$$

(3.11) tenglik ko'chirma harakati ilgarilanma harakat bo'lmagan holda nuqtaning tezlanishlarini qo'shish haqidagi Koriolis teoremasini ifodalaydi. Nuqtaning nisbiy tezlanishini aniqlash uchun ko'chirma harakat xayolan to'xtatiladi va nisbiy tezlanish aniqlanadi.



35-§ Nuqta absolyut harakatining tenglamalari va traektoriyasini aniqlashga doir masalalar

1-masala. Aylanuvchi kranning O_1O_2 o'q atrofida ω_1 o'zgarmas burchak tezlik bilan aylanishida A yuk B barabanga o'ralgan kanat yordamida yuqoriga ko'tariladi. r radiusli B baraban ω_2 o'zgarmas burchak tezlik bilan aylanadi. Agar kranning qulochi d ga teng bo'lsa, yukning absolyut harakati traektoriyasi aniqlansin (3.5 - rasm).



3.5- rasm

Yechimi: Aylanuvchi kranning O_1O_2 o'q atrofida ω_1 o'zgarmas burchak tezlik bilan aylanishi ko'chirma harakat hisoblanadi. A nuqtaning B barabanga o'ralgan kanat yordamida yuqoriga ko'tarilishi nisbiy harakat hisoblanadi.

Aylanuvchi kran asosi bilan bog'langan $O_1x_1y_1z_1$ koordinata o'qlari sistemasi qo'zg'almas sanoq sistemasini tashkil etadi. Aylanuvchi kran bilan bog'lashgan va u bilan birga aylanuvchi O_2xyz koordinata o'qlari sistemasi qo'zg'aluvchan sanoq sistemasini tashkil etadi (3.6 - rasm).

Bunda x_1 o'q O_1O_2 o'q va yukning boshlang'ich holatidan o'tadi, z_1 o'q esa kran aylanish o'qi bo'ylab yo'naladi.

A yukning holati uning absolyut harakatida quyidagi koordinatalar orqali aniqlanadi (3.7 - rasm)

$$\begin{cases} x_1 = d \cos \omega_1 t \\ y_1 = d \sin \omega_1 t \\ z_1 = r \omega_2 t \end{cases}$$

Bunday xolda absolyut tezlanish nisbiy tezlanish $\vec{a}_n$ va ko'chirma tezlanish $\vec{a}_k$ larga qurilgan parallelogramning diagonali bilan ifodalanadi. Absolyut tezlanish moduli quyidagicha hisoblanadi:

$$a_a = \sqrt{a_n^2 + a_k^2 + 2a_n a_k \cos(\vec{a}_n \wedge \vec{a}_k)} \quad (3.16)$$

Nuqtaning absolyut tezlanishini (3.1-rasm) quyidagi usul bilan ham aniqlash mumkin:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d\vec{r}_0}{dt} + \frac{d\vec{r}}{dt} = \vec{v}_0 + (\vec{\omega} \times \vec{r}) + \frac{d\vec{r}}{dt}$$

Nuqtaning absolyut tezlanishi absolyut tezlikdan vaqt bo'yicha hisoblashgan birinchi tartibli hosilaga teng:

$$\vec{a}_a = \frac{dv_a}{dt} = \frac{dv_0}{dt} + \left(\frac{d\vec{\omega}}{dt} \times \vec{r} \right) + \vec{\omega} \times \frac{d\vec{r}}{dt} + \frac{d^2\vec{r}}{dt^2}$$

Agar

$$\frac{dv_a}{dt} = a_a; \quad \frac{dv_0}{dt} = \vec{a}_0; \quad \frac{d\vec{\omega}}{dt} = \vec{\epsilon};$$

$$\frac{d\vec{r}}{dt} = \frac{d\vec{r}}{dt} + \vec{\omega} \times \vec{r}; \quad \frac{d}{dt} \left(\frac{d^2\vec{r}}{dt^2} \right) = \frac{d^2\vec{r}}{dt^2} + \vec{\omega} \times \frac{d\vec{r}}{dt}$$

ekanligini e'tiborga olsak

$$\begin{aligned} \vec{a}_a &= \vec{a}_0 + \vec{\epsilon} \times \vec{r} + \vec{\omega} \times \left(\frac{d\vec{r}}{dt} + \vec{\omega} \times \vec{r} \right) + \frac{d^2\vec{r}}{dt^2} + \vec{\omega} \times \frac{d\vec{r}}{dt} \\ &= \vec{a}_0 + \vec{\epsilon} \times \vec{r} + \vec{\omega} * (\vec{\omega} \times \vec{r}) + \frac{d^2\vec{r}}{dt^2} + 2\vec{\omega} \times \frac{d\vec{r}}{dt} \end{aligned}$$

Bu ifodada

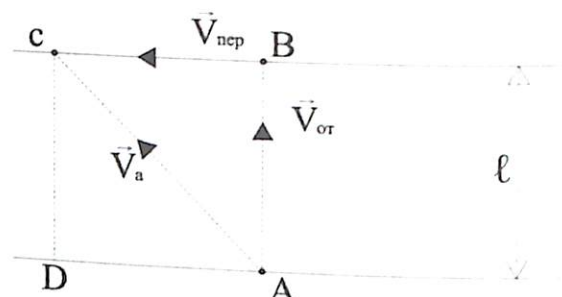
$\vec{a}_0 + \vec{\epsilon} \times \vec{r} + \vec{\omega} * (\vec{\omega} \times \vec{r}) = \vec{a}_k$ - nuqtaning ko'chirma tezlanishi.

$\frac{d^2\vec{r}}{dt^2} = \vec{a}_n$ - nuqtaning nisbiy tezlanishi

$2\vec{\omega} \times \frac{d\vec{r}}{dt} = 2(\vec{\omega}_k \times \vec{v}_n) = \vec{a}_c$ - Koraolis tezlanishi.

Yozilganlarni e'tiborga olsak, murakkab harakatdagi nuqtaning absolyut tezlanishini ifodalovchi (18.8) tenglamasi hosil bo'ladi.

$$\vec{a}_a = \vec{a}_k + \vec{a}_n + \vec{a}_c$$



3.19-rasm

Bunday harakat $t_1 = 10$ minutda amalga oshadi.

Masalada qayiqning daryo oqimiga nisbatan harakatidagi tezligi nisbiy tezlik hisoblanadi va u $\vec{v}_n$ vektor orqali belgilanadi. Daryoning qayiq turgan nuqtasining qirg'oqqa nisbatan oqimi tezligi hisoblanadi va u $\vec{v}_k$ vektor orqali belgilanadi. 3.19- rasmdan

$$V = V_{nep} = \frac{AD}{t_1} = \frac{120}{10} = 12 \text{ m/min.}$$

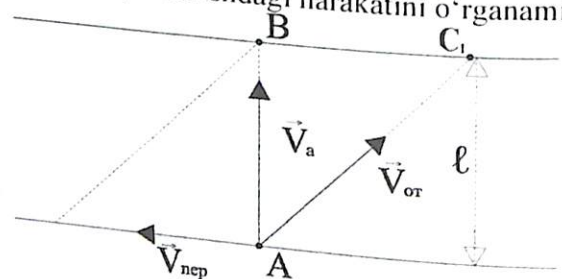
Bunda daryo kengligi quyidagi formula asosida aniqlanadi:

$$l = \vec{v}_n \cdot t_1,$$

bundan,

$$\vec{v}_n = \frac{l}{t_1} \quad (1)$$

b) Qayiqning AB tog'ri chiziqqa ma'lum burchak ostida daryo oqimiga qarshi yo'nalishdagi harakatini o'rganamiz.



3.20- rasm

Bunda qayiq qarama - qarshi qirg'oq $t_2 = 12,5$ minut vaqt o'tgach yetadi 3.20- rasmdan:

Ikkinchi turdagi masalalarni yechishda nuqtaning berilgan absolyut harakatini masala shartida ma'lum bo'lgan ko'chirma va no-ma'lum nisbiy harakatlarga ajratish talab etiladi.

Masalalarni yechishda, dastavval qo'zg'almas va qo'g'aluvchan sanoq sistemalari tanlab olinadi va qo'zg'aluvchan sanoq sistemalari bog'langan jism harakati yani ko'chirma harakat o'rganiladi.

Natijada nuqtaning absolyut va nisbiy harakatlarining xususiyatlarini oson aniqlash imkoni tug'iladi.

Birinchi turdagi masalalarni quyidagi tartibda yechish maqsadga muvofiq bo'ladi:

1) Masala shartidan ma'lum bo'lgan nuqta absolyut harakati ko'chirma va nisbiy harakatlarga ajratiladi.

2) Shartli ravishda qo'zg'almas deb qabul qilingan absolyut va harakatdagi jism bilan bog'langan nisbiy sanoq sistemalari tanlab olinadi.

3) Nuqta nisbiy harakatining tenglamalari tuziladi.

4) Nuqta absolyut harakatining parametric ko'rinishdagi tenglamalari tuziladi.

Ikkinchi turga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1) Nuqtaning masala shartidan ma'lum bo'lgan absolyut harakati ko'chirma va nisbiy harakatlarga ajratiladi.

2) Shartli ravishda qo'zg'almas deb qabul qilingan absolyut va harakatdagi jism bilan bog'langan nisbiy sanoq sistemalari tanlab olindi.

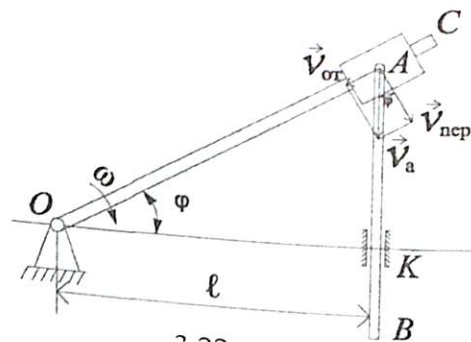
3) Nuqta absolyut harakatining tenglamalari tuziladi.

4) Nuqta nisbiy harakatining parametrik formadagi tenglamalari tuziladi.

5) Nisbiy harakatning parametrik tenglamalaridan parametrvagtni qisqartirib, koordinatalar ko'rinishidagi nisbiy harakat tenglamalari tuziladi.

Yechilishi. Masalada A polzun uchun, OC krivoshipning chizma tekisligiga perpendikulyar holda O nuqtadan o'tuvchi o'q atrofidagi tebranishi, ko'chirma harakat hisoblanadi. Polzunning OC krivoshipga nisbatan harakati esa nisbiy harakat deb qaraladi.

OC krivoshipning qaralayotga vaqt momentida A polzun bilan ustma ust tushuvchi nuqtaning tezligi A polzun uchun ko'chirma tezlik hisoblanadi.



3.22-rasm

Shuning uchun

$$\vartheta_k = \omega * OA$$

$\vec{\vartheta}_k$ vektor OC krivoshipga perpendikulyar holda krivoshipning A nuqtasidan uning aylanishi tomon yo'naladi. (3.22- rasm) dan

Shuning uchun

$$OA = l / \cos \varphi$$

$$V_{nep} = \frac{\omega l}{\cos \varphi} \cdot (1)$$

A polzunning nisbiy tezligi $\vec{\vartheta}_n$ OC krivoship bo'ylab yo'naladi. Mexanizmning A nuqtasida tezliklar parallelogramni chizamiz

Demak, $\vartheta_k = 7 \text{ m/s}$

$\vec{v}_k$ vektor, $\vec{x}'_k$ kattalik oldidagi ishora musbat bo'lganligi uchun, x ning o'sish tomoniga, ya'ni aravachaning harakat yo'nalishi tomon yo'naladi (3.31a-rasm).

M nuqtaning absolyut tezligi uning nisbiy va ko'chirma harakat tezliklaridan ko'rilgan parallelogramning diagonalini orqali ifodalanadi.

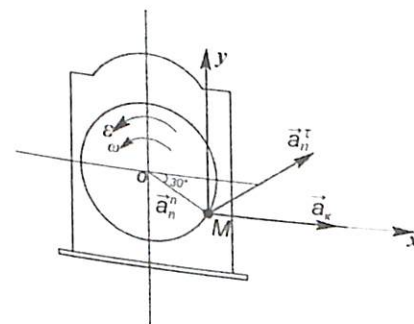
Uning moduli:

$$\vartheta_m = \sqrt{\vartheta_n^2 + \vartheta_k^2 + 2\vartheta_n\vartheta_k \cos 60^\circ} = 7,21 \text{ sm/s.}$$

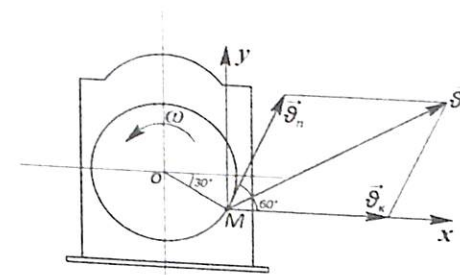
M nuqtaning absolyut tezlanishini nuqtaning murakkab harakatida tezlanishlarni qo'shish teoremasidan aniqlaymiz. Ko'chirma harakat ilgari tanilgan harakat bo'lganligi uchun

$$\vec{a}_M = \vec{a}_n + \vec{a}_k. \quad (1.4)$$

yoki, yoyilgan ko'rinishda



3.31a-rasm



3.31b-rasm

$$\vec{a}_M = \vec{a}_n^r + \vec{a}_n^n + \vec{a}_k. \quad (1.5)$$

Nisbiy urinma tezlanishning moduli:

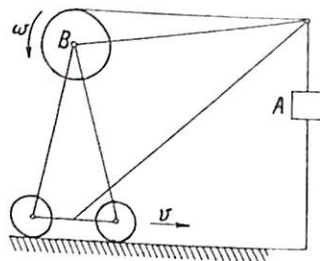
$$a_n^r = R \varepsilon_n. \quad (1.6)$$

bu yerda $\varepsilon_n = |\ddot{\varphi}_n|$ – rotor burchaktezlanishining moduli.

$$\ddot{\varphi}_n = \frac{d^2 \varphi_n}{dt^2} = 2 \frac{\text{rad}}{s^2}, \quad \varepsilon_n = \frac{2 \text{ rad}}{s^2}$$

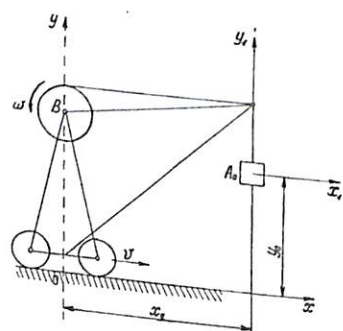
$\vec{\varepsilon}_n$ va $\vec{\omega}_n$ larning ishoralari bir xil. Demak, $\vec{a}_n^r$ va $\vec{\vartheta}_n$ vektorlar bir xil yo'nalishga ega bo'ladi (3.31a,b-rasmlar). (3.22)ga asosan

siljiydi. $r=0,5 \text{ m}$ radiusli B barabanga o'raglan kanat vositasida A yuk ushlab turiladi. B baraban ishga tushirilishida $\omega=2\pi \text{ rad/s}$ burchak tezlik bilan aylanadi. Kran gorizontaal yo'nalishda $v=0,5 \text{ m/s}$ doimiy tezlik bilan siljiydi. Agar yukning boshlang'ich koordinatalari $x_0=10 \text{ m}$, $y_0=6 \text{ m}$ bo'lsa, uning absolyut traektoriyasi aniqlansin (3.8- rasm).



3.8- rasm

Yechimi: Kranning gorizontaal yo'nalishda $v=0,5 \text{ m/s}$ doimiy tezlik bilan siljishi ko'chirma harakat deyiladi. A nuqtaning vertikal yo'nalishda siljishi nisbiy harakat sifatida qaraladi. Yer bilan bog'langan Oxy koordinata o'qlari sistemasi shartli ravishda qo'zg'almas sanoq sistemasi sifatida qaraladi. Gorizontaal yo'nalishda $v=0,5 \text{ m/s}$ doimiy tezlik bilan siljuvchi kran bilan bog'langan Aox_1y_1 koordinata o'qlari sistemasi qo'zg'luvchan sanoq sistemasini tashkil etadi, bunda qo'zg'luvchan sanoq sistemasining boshi A yukning boshlang'ich holati bilan ustma ust tushadi (3.9 – rasm).



3.9- rasm

Masalada:

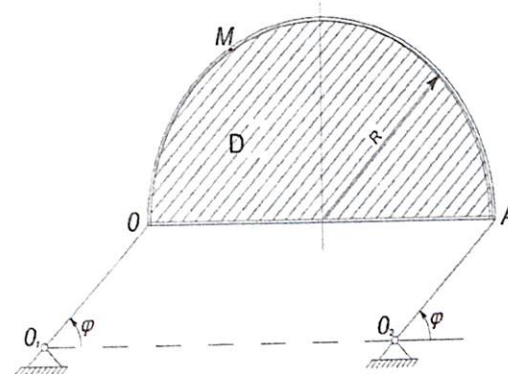
$$OM=s_n=6\pi t^2$$

$$\varphi = \frac{\pi t^3}{6} (\text{rad}),$$

$$t_1=1 \text{ s}$$

$$R=18 \text{ sm}$$

$$o_1o=o_2A=20 \text{ sm}.$$



3.32a-rasm

Yechimi: Masalada to'rt zvenolikning O_1O va O_2A sterjenlari O_1 va O_2 sharnirlar atrofida aylanadi, OA sterjen esa ilgari lanma harakatda bo'ladi. Yarim doira ham OA sterjenga mahkamlanganligi tufayli ilgari lanma harakatda bo'ladi. M nuqta uchun D yarim doira-ning harakati ko'chirma harakat hisoblanadi. Shuning uchun masalada ko'chirma harakat ilgari lanma harakat bo'ladi. M nuqtaning D jismga nisbatan harakati esa nisbiy harakat hisoblanadi.

Berilgan vaqt momentida M nuqtaning D jismdagi o'rni $\alpha = \frac{s_n}{R}$ burchak orqali aniqlanadi:

$$t_1 = 1 \text{ s. da}$$

$$\alpha = \frac{6\pi t_1^2}{18} = \frac{\pi}{3} = 60^\circ.$$

D jismning tekislikdagi holati φ burchak orqali aniqlanadi:

$$t_1 = 1 \text{ s. da}$$

$$\varphi = \frac{\pi t_1^3}{6} = \frac{\pi}{6} = 30^\circ.$$

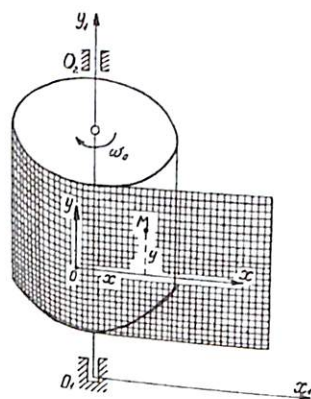
Nuqtaning murakkab harakatida tezliklarni qo'shish haqidagi teorema asosan M nuqtaning absolyut tezligi uning nisbiy va ko'chirma harakat tezliklarining geometrik yig'indisiga teng bo'ladi:

$$\vec{v}_a = \vec{v}_n + \vec{v}_k.$$

Nisbiy tezlikning miqdorini aniqlaymiz:

$$v_n = s_n' = (6\pi t) = 12\pi t$$

Yechimi. Yozib oluvchi moslama barabanning ω_0 burchak tezlik bilan aylanishi ko'chirma harakat hisoblanadi. O'ziyozar apparat perosining harakati nisbiy harakat sifatida qaraladi. Yer bilan bog'langan va shartli ravishda qo'zg'almas deb qabul qilingan $O_1x_1y_1$ koordinata o'qlari sistemi qo'zg'almas – absolyut sanoq sistemasini tashkil etadi. Aylanuvchi baraban bilan bog'langan Oxy koordinata o'qlari sistemi qo'zg'aluvchan sanoq sistemi sifatida qaraladi (3.11 – rasm).



3.11 – rasm.

Faraz qilaylik t vaqt onida o'zi yozar apparat perosi M holatda bo'lsin. Pero M holatining koordinatalari quyidagicha aniqlanadi:

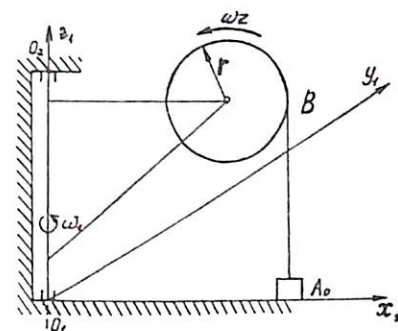
$$x = vt = \omega_0 r t$$

$$y = a \sin \omega_1 t$$

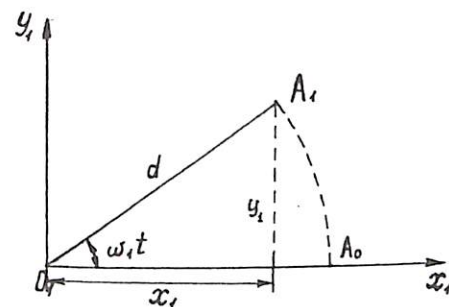
Yozilgan tenglamalar o'ziyozar apparat perosi nisbiy harakatining parametrik tenglamalarini ifodalaydi. Pero nisbiy harakatining koordinatalar ko'rinishidagi tenglamasini yozish uchun yuqoridagi tenglamalardan parametr – vaqtni qisqartiramiz. Natijada pero nisbiy harakatining quyidagi ko'rinishdagi tenglamasiga ega bo'lamiz:

$$t = \frac{x}{\omega_0 r}; \quad y = a \sin \frac{\omega_1 x}{\omega_0 r}.$$

2-masala. Krivoship shatunli mexanizmda uzunligi $OA = r$ bo'lgan privoship O nuqtadan chizma tekisligiga perpendikulyar holda o'tuvchi o'q atrofida o'zgaras ω_0 burchak tezlik bilan aylanadi, bunda $\varphi = \omega_0 t$.



3.6 – rasm



3.7 – rasm

3.8

Hosil bo'lgan tenglamalarni A yukning absolyut harakat traektoriyasining parametrik ko'rinishidagi tenglamalari sifatida qarash mumkin.

Koordinatalar fermasidagi traektoriya tenglamasini tuzish uchun yuqoridagi tenglamalardan parametr-vaqtni qisqartiramiz. Natijada A yuk absolyut harakati traektoriyasining tenglamalari hosil bo'ladi:

$$x_1 = d \cos \frac{\omega_1 x}{\omega_0 r}, \quad y_1 = d \sin \frac{\omega_1 x}{\omega_0 r}.$$

2-masala. Yukni ko'tarish va kranni siljitish mexanizmlarining ishlarini birlashtirishda A yuk gorizonta va vertikal yo'nalishlarda

Burchak orqali aniqlanadi. Ψ burchak qiymati sinuslar teoremasi orqali aniqlanadi:

$$\frac{r}{\sin \Psi} = \frac{l}{\sin \varphi},$$

bundan

$$\sin \Psi = \frac{r \sin \varphi}{l} = \frac{r \sin \omega_0 t}{l};$$

yoki

$$\Psi = \arcsin \left(\frac{r}{l} \sin \omega_0 t \right)$$

Hosil bo'lgan tenglama AB shatun nisbiy harakatining tenglamasini ifodalaydi.

B polzun nisbiy harakati tenglamalarini tuzish uchun uning nisbiy koordinatalari x_1, y_1 larni yuqorida aniqlangan Ψ burchak qiymati orqali ifodalashimiz lozim.

$$x_1 = l \cos \Psi = l \sqrt{1 - \frac{r^2}{l^2} \sin^2 \omega_0 t}$$

$$y_1 = l \sin \Psi = -r \sin \varphi = -r \sin \omega_0 t \text{ (m).}$$

37-§. Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

1-muammo. Platforma gorizontaal yo'l bo'ylab 1 m/s tezlik bilan tekis harakatlanadi. Platforma ichidagi moddiy nuqta ham shu yo'nalish bo'yicha unga nisbatan $s=0,5t$ qonun asosida siljisa, boshlang'ich paytda $t=0$ va $x=0$ deb, $t=4$ s paytdagi nuqtaning x koordinatasini hisoblang.

2-muammo. 1 jism o'zgarmas $\vartheta_1=2$ m/s tezlik bilan gorizontaal tekislik bo'ylab harakat qiladi. Uning ustida esa 2 jism o'zgarmas $\vartheta_2=2$ m/s tezlik bilan yuqoriga ko'tarilmoqda. Agar boshlang'ich paytda, $t=0$ s. da $x_2=0$ bo'lsa, $t=0,5$ s paytdagi 2 jismning x_2 koordinatasini aniqlang. 2 jism moddiy nuqta deb qaralsin (3.14-rasm).

A yukning qo'zg'almas – absolyut sanoq sistemasidagi holati quyidagi koordinatalar orqali aniqlanadi:

$$\begin{cases} x = x_0 + Vt, \\ y = y_0 + \omega \cdot rt. \end{cases}$$

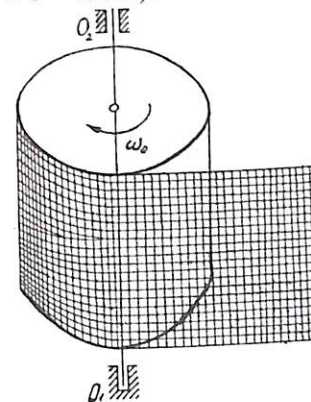
Hosil bo'lgan tenglamalar sistemasini A yuk absolyut harakati traektoriyasining parametrik tenglamalari sifatida qarash mumkin. A yuk absolyut harakati traektoriyasining koordinatalar formasidagi tenglamalarini tuzish uchun yuqoridagi tenglamalardan parametr – vaqtni qisqartiramiz. Natijada A yuk absolyut harakati traektoriyasining tenglamalari hosil bo'ladi:

$$t = \frac{x-x_0}{V};$$

$$y = y_0 + \omega r \left(\frac{x-x_0}{V} \right) = 6 + 2\pi \cdot 0,5 \left(\frac{x-10}{0,5} \right) = 6,28x - 56,8 \text{ m.}$$

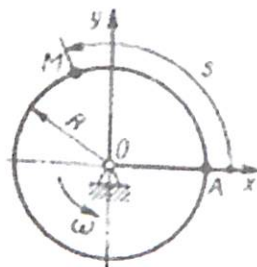
36-§ Nuqta nisbiy harakatining tenglamalari va traektoriyasini aniqlashga doir masalalar

1-masala. Yozib oluvchi moslamaning barabani ω_0 burchak tezlik bilan bir tekis aylanadi. Barabanning radiusi r . O'ziyozar, vertikal yo'nalishda $y=asin\omega_1 t$ qonun bilan harakatlanuvchi detal bilan birlashtirilgan. Qozg'oz lentada pero yozib olgan egri chiziqlarning tenglamasi topilsin (3.10 – rasm).



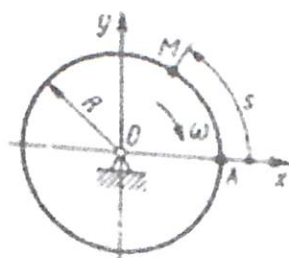
3.10 - rasm

5-muammo. Radiusi $R=0,5m$ bo'lgan disk o'zgarmas burchak tezlik $\omega=2rad/s$ bilan aylanadi. Diskning gardishi esa $s=2t^2$ tenglama asosida M nuqta Ox o'qida bo'lgan bo'lsa, $t=0,5s$ paytdagi nuqtaning yoy koordinatasi s ni aniqlang. (3.17-rasm)



3.17 - rasm

6-muammo. Radiusi $R=0,5m$ bo'lgan disk o'zgarmas burchak tezlik $\omega=2rad/s$ bilan aylanadi. M nuqta esa diskning gardishi bo'ylab $s=2t^2$ qonun asosida harakat qiladi. Agar boshlang'ich paytda M nuqta Ox o'qida bo'lgan bo'lsa, $t=1s$ paytda nuqtaning yoy koordinatasi s ni aniqlang (3.18 - rasm).



3.18-rasm

38-§. Nuqtaning nisbiy, ko'chirma va absalyut tezligini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Nuqtaning murakkab harakatida absalyut tezligini aniqlashga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

- 1) Masala shartiga ko'ra nuqtaning nisbiy, ko'chirma va absalyut harakatlari aniqlanadi.
- 2) Qo'zg'almas va qo'zg'aluvchan sanoq sistemalari tanlab olinadi.
- 3) Ko'chirma harakat xayolan to'xtatiladi va nuqtaning nisbiy harakat tezligi aniqlanadi.
- 4) Nisbiy harakat xayolan to'xtatiladi va nuqtaning ko'chirma tezligi aniqlanadi.
- 5) Murakkab harakatda nuqtaning tezliklarini qo'shish haqidagi teoremdan foydalanib, nuqtaning absalyut tezligi aniqlanadi.

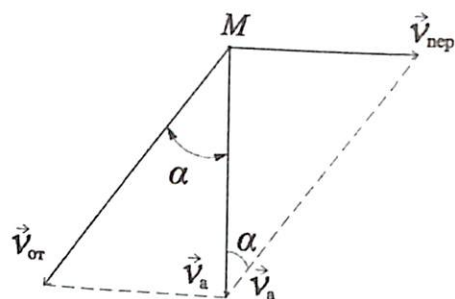
39-§. Murakkab harakatda nuqtaning nisbiy, ko'chirma va absalyut tezligini aniqlashga doir masalalar

1-masala. Daryo qirg'oqlari parallel; qayiq A nuqtadan chiqib, qirg'oqlarga tik kurs oldi va jo'naganidan 10 minut keyin narigi qirg'oqqa borib yetdi. Bunda u, A nuqtadan daryoning oqimi bo'ylab hisoblaganda 120 m pastdagi C nuqtaga keldi. A nuqtadan chiqib, qirg'oqqa tik bo'lgan AB to'g'ri chiziqqa nisbatan qandaydir burchak ostida va oqimga qarshi kurs olishi kerak; bu holda qayiq narigi qirg'oqqa, 12,5 minutda yetadi. Daryo kengligi l , qayiqning suvga nisbatan nisbiy tezligi u va daryo oqimining tezligi v aniqlansin.

Yechimi. Masalada qayiqning daryo oqimiga nisbatan harakati nisbiy harakat deyiladi. Daryoning qirg'oqqa nisbatan harakati (qirg'oq qo'zg'almas sanaladi) ko'chirma harakat sifatida qaraladi.

Qayiqning qirg'oqqa nisbatan harakati absalyut harakat hisoblanadi.

a) Qayiqning A nuqtadan qirg'oqqa perpendikulyar yo'nalishdagi harakatini o'rganamiz. Bunda qayiq qarama – qarshi qirg'oqning C nuqtasiga borib yetadi. (3.19- rasm).



3.24-rasm

Chizilgan paralelogramdan

$$\frac{v_k}{v_a} = \tan 40^\circ.$$

Agar

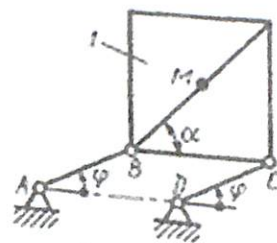
$$v_k = 72 \frac{\text{km}}{\text{s}} = 20 \text{ m/s}$$

Ekanligini etiborga olsak

$$v_a = \frac{v_k}{\tan 40^\circ} = \frac{20 \text{ m/s}}{0,839} = 23,8 \text{ m/s}.$$

40-§. Murakkab harakatda nuqtaning nisbiy, ko'chirma va absolyut tezligini aniqlashga doir mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

1-muammo. O'zaro teng krivoshiplar $AB=CD=0,5\text{m}$, $\varphi=0,25\pi$ qonun bo'yicha aylanadi. Krivoshiplarga o'rnatilgan kvadrat plastina diagonali bo'ylab harakatlanayotgan M nuqtaning tenglamasi $BM=0,1 t^2$ bo'lsa, $t=1\text{s}$ paytdagi M nuqtaning absolyut tezligini aniqlang (3.25- rasm).



3.25- rasm

$$l = 9a \cdot t_2.$$

Bundan

$$\vartheta_a = \frac{l}{t_2}. \quad (2)$$

ΔABC_1 dan

$$\vartheta_n^2 = \vartheta_a^2 + \vartheta_k^2. \quad (3)$$

(1) va (2) ifodalarni (3) ifodaga qo'ysak:

$$\frac{l^2}{t_1^2} = \frac{l^2}{t_2^2} + \vartheta_k^2,$$

yoki

$$\frac{l^2}{10^2} = \frac{l^2}{(12,5)^2} + 144$$

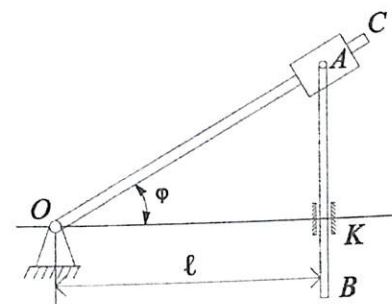
Hosil bo'lgan ifodadan daryo kengligi aniqlanadi:

$$l = \sqrt{\frac{144}{0,0036}} = 200 \text{ m}.$$

Bunday holda qayiqning daryo oqimiga nisbatan tezligi quyidagicha aniqlanadi:

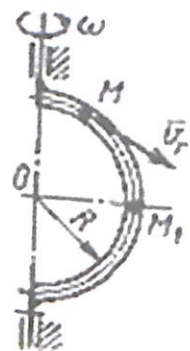
$$\vartheta_n = \frac{l}{t_1} = \frac{200}{10} = 20 \text{ m/s}.$$

2-masala. Kulisali mexanizmda OC krivoshipning rasm tekisligiga perpendikulyar bo'lgan O o'q atrofida tebranishi natijasida, A polzun OC krivoship bo'ylab surilib, vertikal k yo'naltiruvchilarda harakatlanuvchi AB sterjenni harakatga keltiradi. Masofa $OK=l$ A polzunning OC krivoshipga nisbatan harakatidagi tezligi krivoshipning burchak tezligi ω va aylanish burchagi φ funksiyasi sifatida aniqlansin.



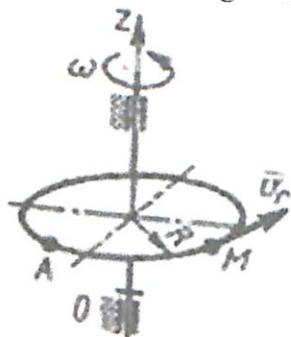
3.21 - rasm

sharcha o'zgarmas nisbiy tezlik $v_r=3\text{m/s}$ bilan harakatlansa, M shar-
chaning M_1 holatga kelgan paytdagi absolyut tezligini aniqlang (3.28
- rasm).



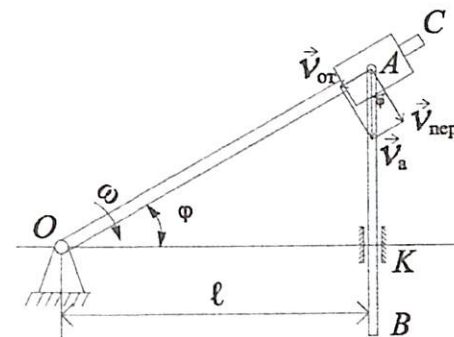
3.28 - rasm

5-muammo. Radiusi $R=1\text{m}$ bo'lgan disk Oz o'qi atrofida
 $\varphi=4\sin 3t$ qonun bilan aylanadi. M nuqta esa diskning gardishi bo'y-
lab $AM=0,66\sin 6t+4$ tenglama bo'yicha harakatlanadi. Vaqtning
 $t=0,35\text{s}$ paytda M nuqtaning absolyut tezligini toping (3.29- rasm).



3.29 - rasm

$OA=0,1\text{m}$ bo'lgan 1 krivoshipning O o'q atrofida $\omega_1=5\text{rad/s}$ burchak
tezlik bilan aylanadi. Shaklda ko'rsatilgan holat uchun 2 polzunning
3 kulisaga nisbatan tezligini aniqlang (3.30 - rasm).



3.23 - rasm

Tezliklar parallelogramidan

$$V_{or} = V_{nep} \cdot \operatorname{tg} \varphi \quad (2)$$

(1) ni (2) ga qo'ysak, A polzunning nisbiy tezligi uchun
quyidagi ifodaga ega bo'lamiz.

$$(2) \quad V_{or} = \frac{\omega l}{\cos \varphi} \cdot \operatorname{tg} \varphi.$$

3-masala. Gorizontol yo'lda 72 km/soat tezlik bilan borayotgan
avtomobildagi passajir kabinaning yon oynasiga tushgan yomg'ir
tomchisining vertikalga nisbatan 40° ga teng burchakka og'gan tra-
ektoriyasini kuzatadi. Vertikal tushayotgan yomg'ir tomchisining
absolyut tezligini aniqlansin. Tomchi bilan oyna orasidagi
ishqalanish hisobga olinmasin.

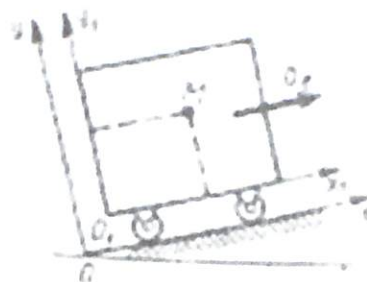
Yechilishi: Avtomobilning gorizontol yo'ldagi harakati pasajir
uchun ko'chirma harakat hisoblanadi.

Yomg'ir tomchisining avtomobil oynasida vertikalga nisbatan
 40° burchakka og'gan traektoriyasi nisbiy harakatni ifodalaydi. Ver-
tikal tushayotgan yomg'ir tomchisining harakati absolyut harakat hi-
soblanadi.

Murkkab harakatda tezliklarni qo'shish teoremasiga asosan
yomg'ir tomchisining absolyut tezligi uning nisbiy va ko'chirma
tezliklarining geometrik yig'indisiga teng bo'ladi:

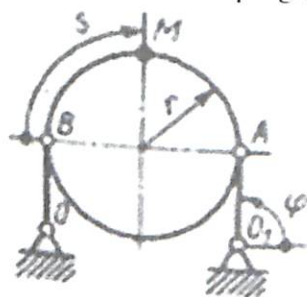
$$\vec{v}_a = \vec{v}_n + \vec{v}_k$$

Ko'chirma, nisbiy va absolyut tezliklarning yo'nalishini bilgan
holda tezliklar parallelogramini chizamiz (3.24-rasm).



3.33-rasm

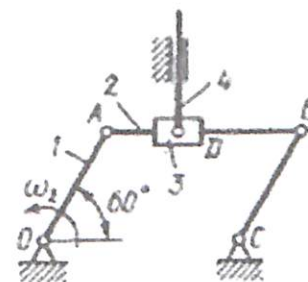
2-muammo. O_1A zveno $\varphi=2t$ qonun bilan aylanib, radiusi $r=0,5m$ li diskni harakatga keltiradi. Diskning gardishi bo'ylab esa M nuqta $s=2rt$ tenglama asosida aylanadi. Nuqtaning $t=0,25\pi$ paytdagi absolyut tezlanishi miqdorini toping (3.34-rasm).



3.34-rasm

3-muammo. Uzunligi $OA=0,1m$ bo'lgan sterjen $\omega=4rad/s$ burchak tezlik va $\varepsilon=0,4rad/s^2$ burchak tezlanish bilan aylanib, $OABO_1$ sharnirli paralelogramni harakatga keltiradi. M nuqta AB sterjen bo'ylab $a=0,4m/s^2$ tezlanish bilan harakat qiladi. M nuqtaning absolyut tezlanish modulini $\varphi=0,5$ holat uchun aniqlang. (3.35-rasm).

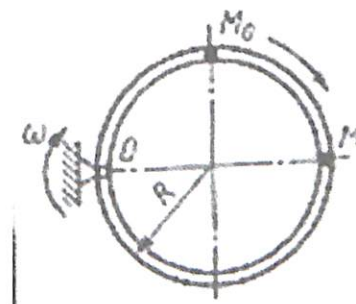
2-muammo. $OABC$ sharnirli paralelogramning 2 shatuni bo'ylab 3 haqasimon polzun (vtulka) harakat qiladi. O'z o'rnida 3 polzun 4 sterjenni harakatga keltiradi. Mexanizmning berilgan holati uchun 1 krivoship A nuqtasining tezligini $2m/s$ deb olib, 4 sterjenning tezligini toping (3.26 – rasm).



3.26 – rasm

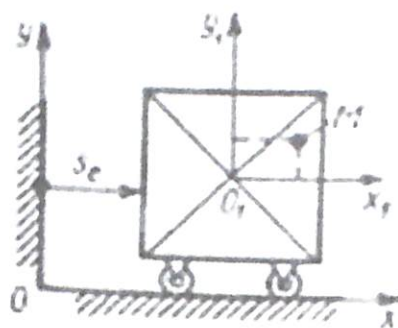
3.27

3-muammo. Radiusi $R=0,1m$ bo'lgan halqa shakl tekisligida O nuqta atrofida o'zgarmas $\omega=4rad/s$ burchak tezlik bilan aylanadi. Halqadagi M shar esa $M_0M=0,1t$ qonun bo'yicha nisbiy harakat qilsa, ko'rsatilgan holat uchun M sharning absalyut tezligini toping (3.27 – rasm).



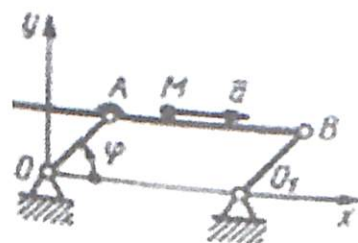
3.27 – rasm

4-muammo. Radiusi $R=1m$ bo'lgan yarim doira shaklidagi naycha $\omega=3rad/s$ burchak tezlik bilan aylanadi. Naycha ichidagi M



3.37-rasm

6-muammo. Uzunligi $OA=2\text{m}$ bo'lgan sterjen $\varphi=t$ qonun bilan aylanib, $OABO_1$ sharnirli parallelogramni harakatga keltiradi. M nuqta AB sterjen bo'ylab esa $a=\cos t$ tezlanish bilan harakat qilsa, M nuqtaning $t=\pi$ paytdagi absolyut tezlanish miqdorini toping (3.38-rasm).



3.38-rasm

44-§. Koriolis tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Kariolis tezlanishini aniqlashga doir masalalarni quyidagi tartibda yechish tavsiya etiladi.
1. Masala shartiga ko'ra murakkab harakatdagi nuqtaning nisbiy harakati tenglamasi aniqlanadi.

M nuqtaning absolyut tezligini nuqtaning murakkab harakatida tezliklarni qo'shish haqidagi teorema asosan nisbiy va ko'chirma tezliklarning geometrik yig'indisi kabi topamiz:

$$\vec{v}_M = \vec{v}_n + \vec{v}_k \quad (1)$$

Nisbiy tezlikning moduli:

$$v_n = |\vec{v}_n| \quad (2)$$

bu yerda,

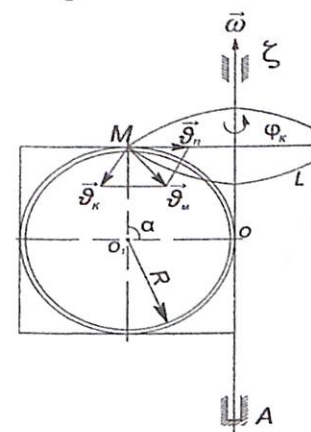
$$\vec{v}_n = \frac{ds_n}{dt} = -\frac{40\pi^2}{3} \sin \frac{\pi t}{3}.$$

$$t=1 \text{ sekundda, } \vec{v}_n = \frac{-40 \cdot (3.14)^2}{3} \cdot 0.86 = -113.06 \text{ sm/s.}$$

$$v_n = 113.06 \text{ sm/s}$$

$\vec{v}_n$ kattalikning oldidagi manfiy ishora M nuqtaning nisbiy tezligi s_n ning kamayish tomoniga qarab aylanaga urinma holda yo'nalishini bildiradi (6-rasm).

Ko'chirma tezlikning moduli:



3.49-rasm

$$v_k = R_k \omega_k \quad (3)$$

bu yerda R_k to'g'ri burchakli ramkaning qaralayotgan vaqt onida M nuqta bilan ustma - ust tushuvchi nuqtasi tomonidan A ξ o'q atrofida chizadigan L aylanasi radiusi, $R_k=R=40\text{sm}$

Mavzuga doir masalalarni ikki usulda yechish tavsiya etiladi: geometrik va analitik usullar.

Masalalarni geometrik usulda yechishda tanlangan masshtabda tezlanishlar parallelogram yoki ko'p burchagi chiziladi.

Masalalarni analitik usulda yechishda proeksiyalar metodidan foydalanish tavsiya etiladi. Buning uchun koordinata o'qlari o'tkaziladi va (2) tenglamani chap va o'ng tomonlari tanlab olingan koordinata o'qlariga proeksiyalanadi:

$$(a_a)_x = (a_n^{mi})_x + (a_n^{ayl})_x + (a_k^n)_x + (a_k^t)_x,$$

$$(a_a)_y = (a_n^{mi})_y + (a_n^{ayl})_y + (a_k^n)_y + (a_k^t)_y,$$

$$(a_a)_z = (a_n^{mi})_z + (a_n^{ayl})_z + (a_k^n)_z + (a_k^t)_z.$$

Bunda absolyut tezlanishning moduli

$$a_a = \sqrt{(a_a)_x^2 + (a_a)_y^2 + (a_a)_z^2}$$

formula yordamida, yo'nalishi esa

$$\cos(\vec{a}_a \wedge x) = \frac{(a_a)_x}{a_a}$$

$$\cos(\vec{a}_a \wedge y) = \frac{(a_a)_y}{a_a}$$

$$\cos(\vec{a}_a \wedge z) = \frac{(a_a)_z}{a_a}.$$

formulalar asosida aniqlanadi.

Mavzuga doir masalalarni quyidagi tartibda yechish maqsadga muvofiq bo'ladi.

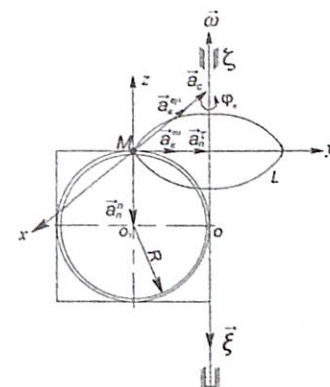
1) Masala shartidan nuqtaning nisbiy, ko'chirma va absolyut harakatlari aniqlab olinadi;

2) Qo'zg'almas va qo'zg'aluvchan koordinata o'qlari sistemasi tanlab olinadi;

3) Ko'chirma harakat hayolan to'xtatilib, nuqtaning nisbiy tezligi va nisbiy tezlanishi aniqlab olinadi;

4) Nisbiy harakat xayolan to'xtatilib, nuqtaning ko'chirma harakat tezligi va tezlanishi aniqlab olinadi;

5) Masalani geometrik usulda yechishda tezlanishlar parallelogram yoki ko'p burchagi chiziladi va ulardan noma'lum tezlanish aniqlanadi.



3.50-rasm

$$a_n^t = 21,91 \text{ sm/s}^2.$$

$\vec{a}_n^t$ ning manfiy ishorasi $\vec{a}_n^t$ vektorining S_n ning kamayish tomoniga qarab yo'nalganligini ko'rsatadi. $\vec{a}_n^t$ va $\vec{v}_n$ ishoralari bir xil. Demak, $\vec{a}_n^t$ va $\vec{v}_n$ vektorlari bir xil yo'nalishga ega. Nisbiy normal tezlanish:

$$a_n^n = \frac{v_n^2}{R} = 319,56 \text{ sm/s}^2.$$

a_n^n vektor O_1 nuqtaga qarab yo'nalgan. Ko'chirma aylanma tezlanishning moduli

$$a_k^{ayl} = R_k \varepsilon_k; \quad (7)$$

bu yerda $\varepsilon_k = |\vec{\varepsilon}_k|$ - to'g'ri burchakli ramkaning burchak tezlanishining moduli

$$\varepsilon_k = \frac{d^2 \varphi_k}{dt^2} = -3t. \quad (8)$$

sekundda,

$$\varepsilon_k = -3 \text{ rad/s}^2, \quad \varepsilon_k = 3 \text{ rad/s}^2.$$

ε_k va $\vec{\omega}_k$ larning ishoralari har xil. Demak, to'g'ri burchakli ramkaning aylanishi sekinlanuvchan, $\vec{\omega}_k$ va $\vec{\varepsilon}_k$ vektorlarning yo'nalishlari qarama - qarshi bo'ladi.

(7) ga asosan,

$$a_k^{ayl} = 40 \cdot 3 = 120 \text{ sm/s}^2.$$

$$a_n^r = 0,2 \cdot 2 = 0,4 \text{ sm/s}^2.$$

Nisbiy normal tezlanishning moduli:

$$a_n^n = R\omega_n^2 = 0,2 \cdot 4 = 0,8 \text{ sm/s}^2.$$

$\vec{a}_n$ vektor rotor M nuqtasining nisbiy harakatda chizgan aylanashining markazi O nuqta tomon yo'naladi (3.31b-rasm).

M nuqtaning ko'chirma tezlanishi qaralayotgan vaqt momenti-da motor korpusi – aravachaning M nuqtasi bilan ustma – ust tushuvchi nuqtasining tezlanishiga teng bo'ladi:

$$a_k = |\ddot{x}_k| = |6t|.$$

t=1 sekunda

$$\ddot{x}_k = 6 \text{ sm/s}^2, \quad x'' = 6 \text{ sm/s}^2.$$

Demak, $a_k = 6 \text{ sm/s}^2$

$\ddot{x}'$ va $\ddot{x}''$ kattaliklarning ishoralari bir xil bo'lganligi uchun $\ddot{\theta}_k$ va $\vec{a}_k$ vektorlarning yo'nalishlari ustma – ust tushadi (3.31a,b-rasmlar).

M nuqta absolyut tezlanishining modulini proektsiyalash usuli yordamida topamiz:

$$a_{Mx} = a_k - a_n^n \cos 30^\circ + a_n^r \cos 60^\circ = 5,52 \text{ sm/s}^2,$$

$$a_{My} = a_n^n \cos 60^\circ + a_n^r \cos 30^\circ = 0,74 \text{ sm/s}^2,$$

$$a_M = \sqrt{a_{Mx}^2 + a_{My}^2} = 5,6 \text{ sm/s}^2.$$

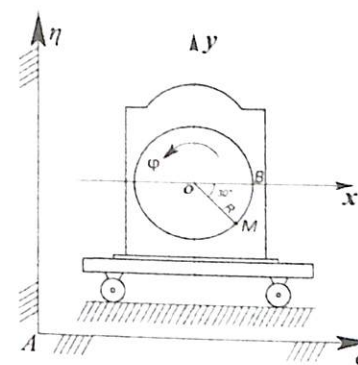
Hisob natijalari jadvalda ko'rsatilgan.

ω_n , rad/s.	Tezlik, sm/s.			ε , rad/s ² .	Tezlanish, sm/s ² .					
	$\dot{\theta}_n$	$\dot{\theta}_k$	$\dot{\theta}_M$		a_n^r	a_n^n	a_k	a_{Mx}	a_{My}	a_M
2	0,4	7	7,21	2	0,4	0,8	6	5,52	0,74	5,6

2-masala. M nuqta D jismga nisbatan $OM = S_n = 6\pi t^2$ tenglama bo'yicha harakatlanadi. D jism $O_1 O A O_2$ sharnirli to'rt zvenolikga mahkamlangan. To'rt zvenolikning $O_1 O$ va $O_2 A$ sterjenlari O_1 va O_2 nuqtalar atrofida $\varphi = \frac{\pi}{6}$ qonunga muvofiq aylanadi. M nuqtaning $t=1$ vaqt onidagi absolyut tezligi va absolyut tezlanishi aniqlansin (3.32a-rasm).

6) Masalani analitik usulda yechishda proektsiyalar usulidan foydalanish tavsiya etiladi yani absolyut tezlanishning o'qlardagi proektsiyalari aniqlanadi

7) Absolyut tezlanishning o'qlardagi proektsiyalariga ko'ra uning moduli va yo'nalishi topiladi.



3.31-rasm

42-§. Ko'chirma harakat ilgari lanma harakatdan iborat bo'lganda nuqtaning absolyut tezlanishini aniqlashga doir masalalar

1-masala.

O'ng tomonga gorizontaal yo'nalishda $x_k = t^3 + 4t$ m. qonunga muvofiq harakat qiluvchi aravachaga elektr motori o'rnatilgan. Uning rotori harakatga keltirish vaqtida $\varphi_n = t^2$ tenglamaga muvofiq aylanadi, bunda φ_n burchak radianlarda o'lchanadi. Rotor gardishidagi M nuqtaning $t=1$ bo'lgandagi absolyut tezligi va absolyut tezlanishi aniqlansin. Rotoring radiusi 0,2m.ga teng. Shu paytda M nuqta rasmda ko'rsatilgan holda turadi (3.31-rasm).

Masalada:

$$x_k = t^3 + 4t \text{ m.}$$

Yechimi:

$$\Phi_n = t^2 \text{ rad, } R = 0,2 \text{ m, } t = 1 \text{ s}$$

$$t_1 = 1 \text{ s. da}$$

$$\vartheta_n = 12 \cdot \pi \cdot 1 = 37.68 \text{ sm/s}$$

Ilgarilanma harakatdagi jismning barcha nuqtalari bir xil traektoriya bo'ylab harakatlanadi va har onda miqdor va yo'nalishlari bir xil bo'lgan tezlik va tezlanishga ega bo'ladi. Shuning uchun M nuqtaning ko'chirma tezligi O nuqtaning ko'chirma tezligiga teng bo'ladi:

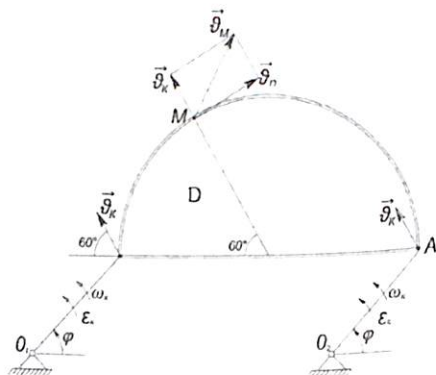
$$\vartheta_k = \vartheta_0 = \omega \cdot 0_1 0_1$$

Bunda:

$$\omega_k = \varphi' = \left(\frac{\pi t^3}{6} \right)' = \frac{\pi t^2}{2};$$

$$t_1 = 1 \text{ s da}$$

$$\omega_k 1.57 \text{ rad/s}$$



3.32b-rasm

Binobarin,

$$\vartheta_k = \omega \cdot 0_1 0 = 1.57 \cdot 20 = 3.14 \text{ rad/s}$$

$\vartheta_n = \vartheta_k$ vektorlar o'zaro perpendikulyar yo'nalgan (3.32b-rasm). Shuning uchun M nuqta absolyut tezligining miqdori quyidagicha aniqlanadi:

$$\vartheta_M = \sqrt{\vartheta_n^2 + \vartheta_k^2} = \sqrt{(37.68)^2 + (3.14)^2} = 49.05 \text{ sm/s}.$$

M nuqtaning absolyut tezlanishini aniqlaymiz. Ko'chirma harakat ilgarilanma harakat bo'lganligi uchun kariolis tezlanishi $\vec{a}_c = 2(\vec{\omega}_k \times \vec{\vartheta}_n) = 0$

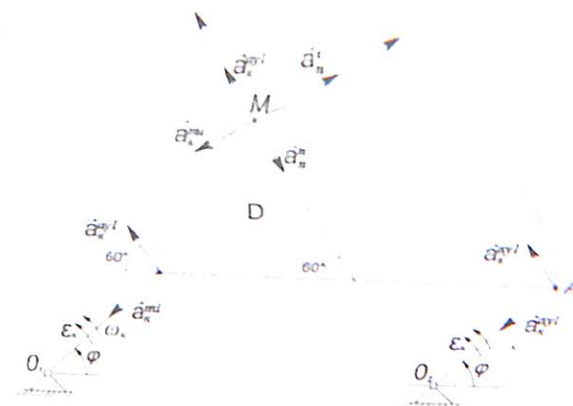
Shuning uchun

$$\vec{a}_2 = \vec{a}_n + \vec{a}_\tau = \vec{a}_n^i + \vec{a}_n^k + \vec{a}_\tau^k$$

M nuqta nisbiy tezlanishlarining miqdorlari:

$$a_n^i = \frac{\vartheta_n^2}{R} = \frac{(37.68)^2}{18} = 78.88 \text{ sm/s}^2.$$

$$a_n^k = \frac{d\vartheta_n}{dt} = 12 = 37.68 \text{ sm/s}^2.$$



3.32v-rasm

M nuqtaning ko'chirma tezlanishlarining miqdorlari:

$$a_k^{mi} = \omega^2 \cdot O_1 O = (1.57)^2 \cdot 20 = 49.30 \text{ sm/s}^2;$$

$$a_k^{ayl} = \varepsilon \cdot O_1 O.$$

Bunda:

$$\varepsilon = \varepsilon_k = \frac{d\omega_k}{dt}.$$

$$t = 1 \text{ s. da}, \varepsilon_k = \frac{d\omega_k}{dt} = \pi = 3.14 \text{ rad/s}^2.$$

Shuning uchun,

$$a_k^{ayl} = 3.14 \cdot 20 = 62.8 \text{ sm/s}^2.$$

M nuqtaning nisbiy va ko'chirma tezlanishlari 3.32v-rasmga ko'rsatilgan. M nuqtaning absolyut tezlanishining miqdorini proektsiyalash usulidan foydalanib aniqlaymiz.

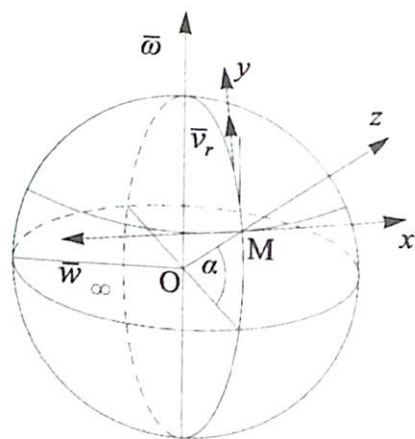
$$(a_M)_x = a_n^i - a_k^{mi} = 78.88 - 49.30 = -11.62,$$

$$(a_M)_y = a_n^k - a_k^{ayl} = -78.88 + 62.8 = -16.08,$$

$$a_M = \sqrt{(a_M)_x^2 + (a_M)_y^2} = 19.84 \text{ sm/s}^2$$

43-§. Talabalarga mustaqil yechish uchun tavsiya etiladigan muammolar

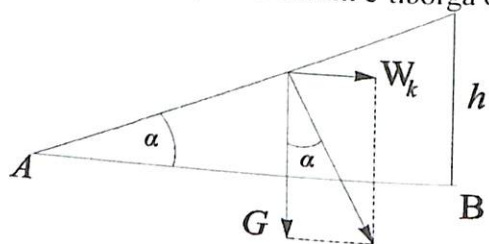
1-muammo. Arava qiya tekislikda $a_c = 2 \text{ m/s}^2$ tezlanish bilan harakat qiladi. Aravadagi M nuqta esa shakil tekisligida $x_1 = 3t^2$ va $y_1 = 4t^2$ tenglamalar bo'yicha harakatlanadi. Nuqtaning absolyut tezlanishin toping (3.33-rasm).



3.39-rasm

$$a_c = 2 \cdot 0.000073 \cdot 1.5 \cdot 0.87 = 1.89 \cdot 10^{-4} \text{ m/s}^2.$$

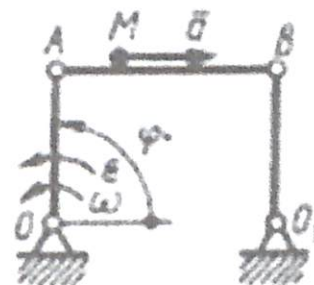
Yerning shimoliy yarim sharida, Yer aylanishi tufayli, Yer sirtida harakatlanayotgan har qanday jism o'ng tomonga og'adi. Binobarin, suv daryoning o'ng qirg'og'ida baland bo'ladi. Daryo suvining o'ng qirg'og'i qancha baland bo'lishini aniqlash uchun suv sathi, koriolis tezlanishiga teng va unga qarama-qarshi yo'nalgan vektor bilan og'irlik kuchining tezlanishi $\vec{g}$ vektoring yig'indisiga teng bo'lgan vektor yo'nalishiga perpendikulyar bo'lishini e'tiborga olamiz.



3.40-rasm

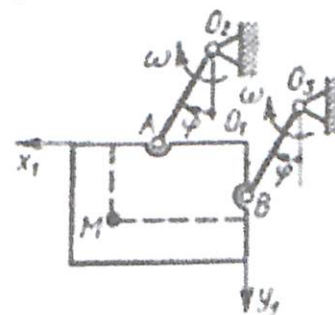
3.40-rasmdan $h = \tan \alpha$ Ikkinchi tomondan $\tan \alpha = \frac{a_k}{g}$.
Shuning uchun

$$h = 500 \cdot \tan \alpha = 500 \cdot \frac{1.89 \cdot 10^{-4}}{9.81} = 0.0096 \text{ m}.$$



3.35-rasm

4-muammo. To'rtburchak shakildagi plastina uzunliklari $AO_2 = BO_3 = 1 \text{ m}$ bo'lgan krivoshiplar yordamida harakatga keltiriladi. M nuqta esa plastina bo'ylab $x_1 = 0.2t^3$ va $y_1 = 0.3t^2$ tenglama asosida harakatlanadi. Agar krivoshiplar o'zgarmas $\omega = 2\pi$ burchak tezlik bilan aylansa, $t = 1 \text{ s}$ paytda $\varphi = 30^\circ$ holat uchun M nuqtaning absolyut tezlanishini hisoblang (3.36-rasm).



3.36-rasm

5-muammo. Arava gorizontaal yo'lda $s_c = 0.5t^3$ qonun bilan harakatlanadi. Aravadagi M nuqta esa vertikal shakil tekisligida $x_1 = 0.3t$ va $y_1 = 0.1t^2$ tenglamalar asosida harakat qiladi. $t = 1 \text{ s}$ paytdagi nuqtaning absolyut tezlanishini toping (3.37-rasm).

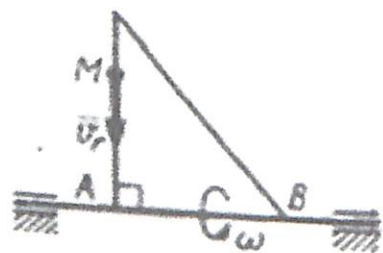
3-masala. Shimoliy kenglik paralleli bo'ylab o'tkazilgan temir yo'lda teplovoz g'arqdan sharqqa qarab $\vartheta_r=20\text{m/s}$ tezlik bilan harakat qiladi. teplovozning koriolis tezlanishi ω_c topilsin.

Yechimi: Masala shartiga ko'ra shimoliy kenglik paralleli bo'ylab, o'tkazilgan temir yo'lda teplovoz g'arbdan sharqqa qarab $\vartheta_n=20\text{m/s}$ tezlik bilan harakat qiladi. Yerning aylanishi teplovoz uchun ko'chirma harakat hisoblanadi. Uning burchak tezlik vektori $\vec{\omega}_k$ yerning aylanish o'qi bo'ylab yuqoriga yo'nalgan. Shuning uchun $\vec{\omega}_k \perp \vec{\vartheta}_n$. Natijada teplovozning koriolis tezlanishi quyidagiga teng bo'ladi:

$$a_c = 2 \cdot \omega_k \cdot \vartheta_n = 2 \cdot 0,000073 \cdot 20 = 2,91 \cdot 10^{-3} \text{ m/s}$$

46-§. Talabalarga mustaqil yechish uchun tavsiya etiladigan muammolar

1-muammo. Uchburchak shaklidagi jism AB tomoni atrofida $\omega=8\text{rad/s}$ burchak tezlik bilan aylanadi. M nuqta esa uchburchakning AB ga perpendikulyar tomoni bo'ylab $v=4\text{m/s}$ nisbiy tezlik bilan harakat qiladi. M nuqtaning Koriolis tezlanishini toping (3.42- rasm).



3.42- rasm

2-muammo. Yarim doira shaklidagi jism $\omega=4\text{rad/s}$ burchak tezlik bilan aylanadi. M nuqta esa uning yoyi bo'ylab $\vec{\vartheta}_r$ tezlik bilan harakat qilsa, M nuqtaning Koriolis tezlanishini toping (3.43- rasm).

2. Nuqtaning nisbiy harakat tenglamasiga ko'ra uning nisbiy tezligining miqdori va yo'nalishi aniqlanadi.

3. Masala shartiga ko'ra ko'chirma harakat tenglamasi aniqlanadi.

4. Ko'chirma harakat tenglamasiga ko'ra nuqta ko'chirma harakatining burchak tezligining miqdori va yo'nalish aniqlanadi.

5. Nuqtaning aniqlangan nisbiy tezligi va ko'chirma harakati burchak tezligining miqdori va yo'nalishiga asosan uning Koriolis tezlanishi aniqlanadi.

45-§. Murakkab harakatda nuqtaning Koriolis tezlanishini aniqlashga doir masalalar

1-masala. Eni 500 m bo'lgan daryo janubdan shimolga qarab 1,5 m/s tezlik bilan oqadi. 60° shimoliy kenglikda suv zarrasining ω_c koriolis tezlanishi aniqlansin. Keyin, suv daryoning qaysi qirg'og'ida ekanligi va qancha baland ekanligi aniqlansin; suv sathi, koriolis tezlanishiga teng va unga qarama-qarshi yo'nalgan vektor bilan og'irlik kuchining tezlanishi g vektorning yig'indisiga teng bo'lgan vektor yo'nalishiga perpendikulyar.

Yechilishi: Daryo suv zarrasining koriolis tezlanishi quyidagi formula asosida aniqlanadi:

$$\vec{a}_c = 2(\vec{\omega}_k \times \vec{v}_n).$$

Uning moduli esa

$$a_c = 2\omega_k \vartheta_n \sin(\vec{\omega}_k \wedge \vec{\vartheta}_n)$$

formula yordamida hisoblanadi.

Agar

$$\vartheta_n = \frac{1,5 \text{ m}}{\text{s}}, \quad \vartheta_k = \frac{2\pi}{24 \cdot 60 \cdot 60} = 0,000073 \frac{1}{\text{s}}; \quad \sin(\vec{\omega}_k \wedge \vec{\vartheta}_n) = \sin 60^\circ = 0,87$$

Ekanligini e'tiborga olsak, daryo suv zarrasining Koriolis tezlanishi quyidagi miqdorga teng bo'ladi.

$$a_n^n = \frac{u^2}{r}; \quad a_k^{mi} = \omega^2 \cdot r; \quad a_c = 2\omega_k \vartheta_n \cdot \sin(\vec{\omega}_k \vec{\vartheta}_n) = 0:$$

Shuning uchun

$$a_1 = a_k^{mi} - a_n^n = \omega^2 r - \frac{u^2}{r};$$

2-nuqtada suyuqlik zarralarining absolyut tezlanishining miqdorini aniqlaymiz (3.51b-rasm).

$$a_n^n = \frac{u^2}{r}; \quad a_k^{mi} = \omega^2 \cdot 2r.$$

$$a_c = 2 \cdot \omega_k \vartheta_n \sin(\vec{\omega}_k \vec{\vartheta}_n) = 2 \cdot \omega_k \cdot \vartheta_n = 2\omega \cdot u.$$

Bularni e'tiborga olsak,

$$\begin{aligned} \sqrt{(a_c)^2 + (-a_n^n)^2 + (-a_k^{mi})^2} &= \sqrt{4\omega^2 u^2 + \frac{u^4}{r^2} + 4\omega^4 r^2} = \\ &= \sqrt{\frac{4\omega^2 u^2 r^2 + u^4 + 4\omega^4 r^4}{r^2}} = \frac{1}{r} \sqrt{(u^2 + 2\omega^2 r^2)^2} = \\ &= \frac{1}{r} (u^2 + 2\omega^2 r^2) = \frac{u^2}{r} + 2\omega^2 r. \end{aligned}$$

3-nuqtada suyuqlik zarralarining absolyut tezlanishining miqdori quyidagiga teng bo'ladi (3.51b-rasm).

$$a_n^n = \frac{u^2}{r}; \quad a_k^{mi} = \omega^2 \cdot 3r.$$

Shuning uchun $a_c = 2 \cdot \omega_k \vartheta_n \sin(\vec{\omega}_k \vec{\vartheta}_n) = 0$.

$$a_3 = a_n^n + a_k^{mi} = \frac{u^2}{r} + 3\omega^2 r.$$

4-nuqtada suyuqlik zarralarining absolyut tezlanishining miqdorini aniqlaymiz. (3.51b-rasm).

$$a_n^n = \frac{u^2}{r}; \quad a_k^{mi} = \omega^2 \cdot 2r.$$

$$a_c = 2 \cdot \omega_k \vartheta_n \sin(\vec{\omega}_k \vec{\vartheta}_n) = 2\omega u.$$

Shuning uchun

$$\begin{aligned} a &= \sqrt{(-a_c)^2 + (-a_n^n)^2 + (-a_k^{mi})^2} \\ &= \sqrt{(-2\omega u)^2 + \left(\frac{u^2}{r}\right)^2 + (2\omega^2 r)^2} = \frac{u^2}{r} + 2\omega^2 r. \end{aligned}$$

2-masala. Meridian bo'yicha harakatlanuvchi elektrovoz ekvatorni kesib o'tayotgan paytda uning g'ildirigidagi M_1, M_2, M_3 va M_4 nuqtalarning koriolis tezlanishlari aniqlansin. Elektrovoz g'ildiragi markazining tezligi $v_0 = 40$ m/s.

Yechimi. Elektrovoz g'ildiragi Yerning meridian bo'ylab harakatlanib ekvatorni kesib o'tadi. Elektrovoz g'ildiragining shu holatida M_1 nuqta g'ildirak nuqtalarining Yer sirtidagi nisbiy harakati tezliklarining oniy markazi bo'ladi (3.41-rasm).

Rasimdan $M_1 M_2 = M_1 M_4 = r\sqrt{2}$ bunda r - elektrovoz g'ildiragining radiusi.

Elektrovoz g'ildiragining oniy burchak tezligini aniqlaymiz:

$$\omega = \frac{\vartheta_0}{r} = \frac{\vartheta_2}{r\sqrt{2}} = \frac{\vartheta_4}{r\sqrt{2}}$$

bundan $\vartheta_2 - \vartheta_4 = \vartheta_0 \sqrt{2}$

ϑ_2 va ϑ_4 - lar g'ildirak 2va 4 nuqtalarining nisbiy tezliklari Yerning aylanishi elektrovoz uchun ko'chirma harakat hisoblanadi.

Uning burchak tezligi

$$\omega_k = \frac{2\pi}{24 \cdot 60 \cdot 60} = 0,000073 \frac{1}{c}.$$

Ma'lumki, elektrovoz g'ildiragi nuqtalarining koriolis tezlanishi quyidagi formula asosida aniqlanadi.

$$a_c = 2 \cdot \omega_k \cdot \vartheta_n \sin \alpha.$$

Bu ifodada $\alpha - \vec{\omega}_k$ va $\vec{\vartheta}_n$ vektorlar orasidagi burchak.

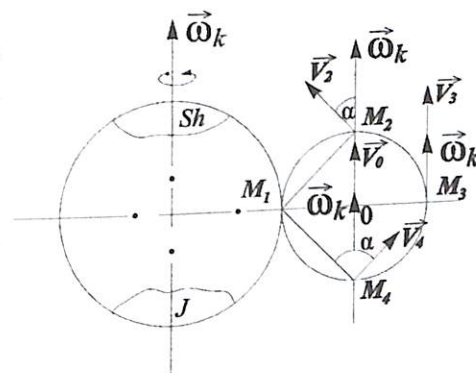
Elektrovoz g'ildiragi nuqtalarning koriolis tezlanishlarini aniqlaymiz:

$$a_{c1} = 2 \cdot \omega_k \cdot \vartheta_1 \sin \alpha = 0 \text{ chunki } \vartheta_1 = 0$$

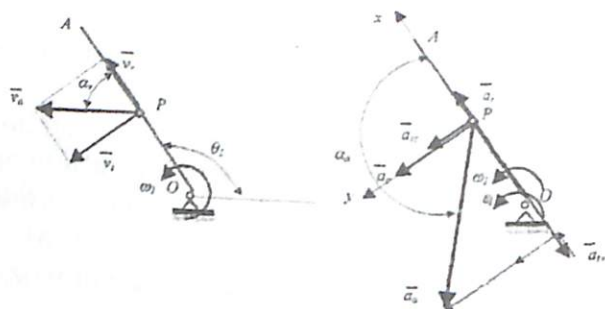
$$a_{c2} = 2 \cdot \omega_k \cdot \vartheta_2 \sin 45 = 2 \cdot 0,000073 \cdot 40 \cdot \sqrt{2} \cdot 0,71 = 5,81 \cdot 10^{-3} \text{ m/s}^2.$$

$$a_{c3} = 2 \cdot \omega_k \cdot \vartheta_3 \sin 0 = 0 \text{ chunki } \sin 0 = 0$$

$$a_{c4} = a_{c2} = 5,81 \cdot 10^{-3} \text{ m/s}^2$$



Natijada



3.53-rasm

Polzunning nisbiy tezlik va nisbiy tezlanishini aniqlash uchun sterjenning aylanma harakatini-ko'chirma harakatini hayolan to'xtatamiz. Natijada P polzunning nisbiy tezligi va nisbiy tezlanishi uchun quyidagi kattaliklarga ega bo'lamiz:

$$v_n = \frac{dS(t_1)}{dt} = 25t_1 = \frac{50 \text{ sm}}{s}$$

$$a_n = \frac{d^2S(t_1)}{dt^2} = \frac{dv_n}{dt} = \frac{25 \text{ sm}}{s^2}$$

P polzun ko'chirma harakat tezligi va tezlanishini aniqlash uchun P polzunning nisbiy harakatini hayolan to'xtatamiz. Natijada P polzun ko'chirma tezligi va ko'chirma tezlanishlari uchun quyidagi ifodalarga ega bo'lamiz:

$$v_k = OP(t_1) * w(t_1) = \frac{l}{2} * \frac{\pi t_1}{3} = 104,7 \frac{\text{cm}}{s}$$

$\vec{v}_k$ vektor OP kesmaga perpendikulyar holda $w(t_1)$ tomon yo'naladi.

P polzunning ko'chirma tezlanishi uning urinma va normal tezlanishlaridan tashkil topadi:

$$\vec{a}_k = \vec{a}_{k\tau} + \vec{a}_{kn}$$

Ko'chirma urinma tezlanishi:

$$a_{k\tau} = OP(t_1) * \varepsilon(t_1) = OP(t_1) * \frac{dw(t_1)}{dt} = \frac{1}{2} * \frac{\pi}{3} = 52,34 \frac{\text{sm}}{s^2}$$

Ko'chirma normal tezlanishi:

$$a_{kn} = OP(t_1)w^2(t_1) = \frac{1}{2} \frac{\pi^2 t_1^2}{9} = 219,1 \frac{\text{sm}}{s^2}$$

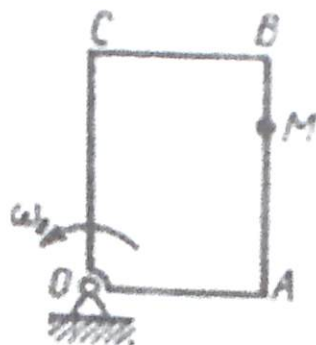
50-§. Talabalar tomonidan mustaqil bajariladigan masalalar variantlari (keys, hisob chizma ishlari) Tekis mexanizmning kinematika tahlili

Mexanizmning berilgan holati uchun B va C nuqtalarning tezliklari va tezlanishlari hamda shu nuqtalar tegishli bo'lgan zveno-ning burchak tezligi va burchak tezlanishi topilsin.

Mexanizmlarning sxemalari va hisoblash uchun kerakli ma'lumotlar quyidagi jadvalda keltirilgan.

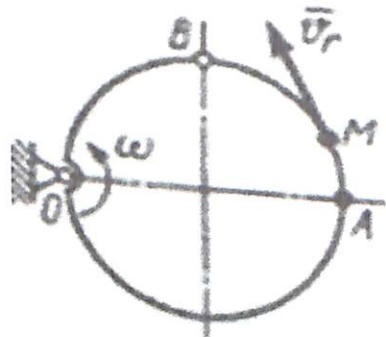
Variant raqam-lari	Mexanizmlarning sxemalari	Hisoblash uchun kerak ma'lumotlar
1.		$OA=60 \text{ sm}$ $r=20 \text{ sm}$ $\omega_{OA}=2 \text{ rad/s}$ $\varepsilon_{OA}=4 \text{ rad/s}^2$

5-muammo. To'rtburchak shaklidagi plastina shakl tekisligida O nuqta atrofida aylanadi. M nuqta AB qirrasi bo'ylab $AM=3\sin(\pi/3)t$ qonun bo'yicha harakatlanadi. Agar $t=2\text{sek}$ da M nuqtaning Koriolis tezlanishi $4\pi(\text{m/s})$ bo'lsa, plastinaning ω_B ko'chirma burchak tezligini toping (3.46- rasm).



3.46- rasm

6-muammo. Shakl tekisligida $\omega=2\text{rad/s}$ burchak tezlik bilan aylanuvchi diskning gardishi bo'ylab M nuqta $\vartheta=0.2\text{m/s}$ nisbiy tezlik bilan harakat qiladi. M nuqta A holatdan B holatga o'tgan bo'lsa, uning Koriolis tezlanishining miqdori o'zgaradimi? (3.47- rasm).



3.47- rasm

5.		$AB=30\text{ sm}$ $AC=10\text{ sm}$ $\vec{v}_A=10\text{ m/s}$ $a_A=15\text{ m/s}^2$
6.		$OA=30\text{ sm}$ $AB=60\text{ sm}$ $AC=20\text{ sm}$ $\omega_{OA}=2\text{ rad/s}$ $\epsilon_{OA}=6\text{ rad/s}^2$
7.		$OA=40\text{ sm}$ $AB=60\text{ sm}$ $AC=40\text{ sm}$ $\omega_{OA}=3\text{ rad/s}$ $\epsilon_{OA}=8\text{ rad/s}^2$

ω_k - to'g'ri burchakli ramka burchak tezligining moduli:

$$\omega_k = |\tilde{\omega}_k|, \tilde{\omega}_k = \frac{d\varphi_k}{dt} = 3 - 1,5t^2.$$

$t=1$ sekundda

$$\tilde{\omega}_k = 1,5 \text{ rad/s}, \omega_k = 1,5 \text{ rad/s}$$

$\tilde{\omega}_k$ kattalikning musbat ishorasi to'g'ri burchakli ramkaning $A\xi O\eta$ atrofidagi aylanishi φ_k burchakning o'sish tomoniga ro'y berishini ko'rsatadi. Shuning uchun $\tilde{\omega}_k$ Ko'chirma tezlikning moduli (3) formula bo'yicha hisoblanadi:

$$\vartheta_k = 40 \cdot 1,5 = 60 \text{ sm/s}.$$

$\tilde{\vartheta}_k$ vektor L aylanaga urinma bo'ylab, to'g'ri burchakli ramkaning aylanish tomoniga qarab yo'nalgan. $\tilde{\vartheta}_k$ va $\tilde{\vartheta}_n$ vektorlar o'zaro perpendikulyar bo'lgani uchun M nuqta absolyut tezligining moduli:

$$\vartheta_M = \sqrt{\tilde{\vartheta}_n^2 + \tilde{\vartheta}_k^2} = 128 \text{ sm/s}.$$

M nuqtaning absolyut tezlanishini nuqtaning murakkab harakatida tezlanishlarni qo'shish teoremasidan aniqlaymiz. Masalada, ko'chirma harakat ilgarilanma bo'lmagan murakkab harakat bo'lganligi uchun, absolyut tezlanish nisbiy, ko'chirma va Koriolis tezlanishlarining geometrik yig'indisiga teng:

$$\tilde{a}_M = \tilde{a}_n + \tilde{a}_k + \tilde{a}_s, (4)$$

yoki, yoyilgan ko'rinishda

$$\tilde{a}_M = \tilde{a}_n^r + \tilde{a}_n^n + \tilde{a}_k^{ayl} + \tilde{a}_k^{mi} + \tilde{a}_c. (5)$$

Nisbiy urinma tezlanishning moduli

bu yerda,

$$a_n^r = |\tilde{a}_n^r| \quad (6)$$

$t = 1$
sekundda,

$$\tilde{a}_n^r = \frac{d^2 s_n}{dt^2} = -\frac{40\pi^2}{9} \cos \frac{\pi t}{3}.$$

$$\tilde{a}_n^r = -21,91 \text{ sm/s}^2,$$

47-§. Ko'chirma harakat ilgarilanma harakat bo'lmagan holda nuqtaning absolyut tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Nuqtaning murakkab harakatida uning absolyut tezlanishini aniqlashga doir masalalarni yechishda ko'chirma harakatning ko'rinishi muhim ahamiyat kasb etadi.

Agar nuqtaning murakkab harakatida ko'chirma harakat ilgarilanma harakat bo'lmasa, yani qo'zg'aluvchi koordinatalar sistemasi-ning berilgan ondagi burchak tezligi ω_k ma'lum bo'lsa, nuqtaning absolyut tezlanishi uning nisbiy, ko'chirma va Koriolis tezlanishlarining geometrik yig'indisidan iborat bo'ladi:

$$\tilde{a}_a = \tilde{a}_n + \tilde{a}_k + 2(\tilde{\omega}_k \times \hat{\vartheta}_n) = \tilde{a}_n + \tilde{a}_k + \tilde{a}_c.$$

Murakkab harakatda ko'chirma harakat ilgarilanma harakat bo'lmagan holda nuqtaning absolyut tezlanishini aniqlashda quyidagi tartibga rioya etish tavsiya etiladi:

1. Masala shartiga ko'ra nuqtaning nisbiy, ko'chirma va absolyut harakatlari aniqlab olinadi
2. Qo'zg'almas va qo'zg'aluvchan sanoq sistemalari tanlab olinadi.
3. Ko'chirma harakat hayolan to'xtatilib, nuqtaning nisbiy tezlanishi aniqlab olinadi.
4. Nisbiy harakat hayolan to'xtatilib, nuqtaning ko'chirma tezlanishi aniqlab olinadi.
5. Nuqtaning nisbiy tezligi va ko'chirma harakatning burchak tezliklarini bilgan holda nuqtaning Koriolis tezlanishi aniqlanadi.
6. Koriolis teoremasiga asosan nuqtaning absolyut tezlanishi aniqlanadi.

$$\vec{v}_1 \rightarrow \vec{v}_k \quad \vec{a}_{1\tau} = \vec{a}_{k\tau} \varepsilon_1 \rightarrow \varepsilon_k$$

4-masala. M nuqta D jismga (doiraga) nisbatan $OM = s_n = 20 \sin \frac{\pi t}{6}$ qonun bo'yicha harakat qiladi. D jism (doira) chizma tekisligiga perpendikulyar o'q atrofida $\varphi_k = 2t - 0.5t^2$ qonun bo'yicha aylanadi. M nuqtaning $t=1$ sekunddagi absolyut tezligi va absolyut tezlanishi topilsin (3.54-rasm).

Masalada: $OM = s_n = 20 \sin \frac{\pi t}{6}$ sm., $\varphi_k = 2t - 0.5t^2$ rad. $t=1$ s, $a=20$ sm

Yechimi:

Chizmada ko'rsatilgan $A\xi\eta$ o'qlar sistemasi qo'zg'almas sanoq sistemasini, doira bilan bog'langan va u bilan birga aylanuvchi O_1xy sanoq sistemasi quzg'aluvchan sanoq sistemasini tashkil etadi.

M nuqtaning qo'zg'aluvchan O_1xy sanoq sistemasiga nisbatan harakati nisbiy, D jismning qo'zg'aluvchan

O_1xy sanoq sistemasi bilan birgalikda qo'zg'almas $A\xi\eta$ sanoq sistemasiga nisbatan harakati M nuqta uchun ko'chirma va M nuqtaning bevosita qo'zg'almas sanoq sistemasiga nisbatan harakati murakkab harakat hisoblanadi.

Nuqtaning $t=1$ sekunddagi holatini uning to'g'ri chiziq bo'ylab harakat qonunidan foydalanib topamiz va 3.54-rasm chizmada ko'rsatamiz (3.54a-rasm).

$t=1$ sekunda $OM = s_n = 20 \sin \frac{\pi t}{6} = 10$ sm.

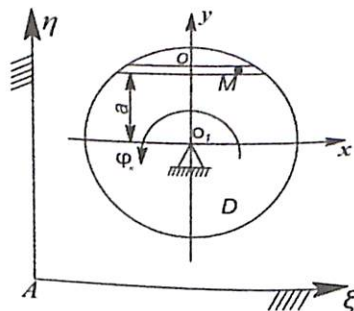
M nuqtaning absolyut tezligini nuqtaning murakkab harakatida tezliklarni qo'shish haqidagi teoreмага asosan, nisbiy va ko'chirma tezliklarning geometrik yig'indisi kabi topamiz:

$$\vec{v}_M = \vec{v}_n + \vec{v}_k. \quad (1.1)$$

Nisbiy tezlikning moduli

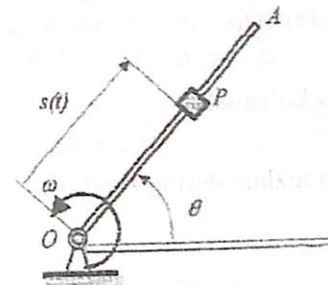
$$v_n = |\vec{v}_n|; \quad (1.2)$$

bu yerda,



3-masala. Uzunligi $l=1$ m bo'lgan sterjen chizma tomoniga perpendikulyar holda O nuqtadan o'tuvchi o'q atrofida $w = \frac{\pi t}{3} \frac{1}{s}$ burchak tezlik bilan aylanmoqda. Shu vaqtning o'zida sterjen bo'ylab P polzun $S(t)=OP=12.5t^2$ qonunga muvofiq harakatlanadi.

Sterjen harakati gorizontol holatdan boshlanadi deb faraz qilib, P polzuning, u sterjenning yarmida bo'lgan holatida, absolyut tezligi va absolyut tezlanishi aniqlansin.



3.52-rasm

Yechimi: Sterjenning O nuqtadan o'tuvchi o'q atrofida aylanma harakati P polzun uchun ko'chirma harakat, P polzuning sterjen bo'ylab O nuqtadan sterjenga nisbatan harakati nisbiy harakat hisoblanadi.

Agar t_1 vaqt onida polzun sterjenning yarmida bo'ladi deb faraz qilsak,

$$S(t_1) = 1/2; 12.5t^2 = 50 \text{ bo'ladi.}$$

Bundan $t_1 = 2$ s

Shu vaqt onida sterjenning boshlang'ich holati (gorizontol holat) bilan tashkil etgan burchak quyidagicha aniqlanadi:

$$w = \frac{d\theta}{dt}; d\theta = w dt$$

$$\theta(t) = \int w dt + C$$

yoki

$$\theta(t) = \frac{\pi t^2}{6} + C$$

Bu ifoda C-integrallash doimiysi.

Agar boshlang'ich paytda $\theta=0$ ekanligini e'tiborga olsak $C=0$ bo'ladi.

$$\begin{aligned} \vartheta_{Mx} &= -10.9 \text{ sm/s;} \\ \vartheta_{My} &= 10.06 \text{ sm/s} \\ \vartheta_M &= \sqrt{\vartheta_{Mx}^2 + \vartheta_{My}^2} = 14.83 \text{ sm/s.} \end{aligned}$$

M nuqtaning absolyut tezlanishi nuqtaning murakkab harakati-da tezlanishlarni qo'shish teoremasidan aniqlanadi. Masalada, ko'chirma harakat ilgari nima bo'lmagan murakkab harakat bo'lganligi uchun, absolyut tezlanish nisbiy, ko'chirma va Koriolis tezlanishlarining geometrik yig'indisiga teng:

$$\vec{a}_M = \vec{a}_n + \vec{a}_k + \vec{a}_c, \quad (1.6)$$

yoki, yoyilgan ko'rinishda:

$$\vec{a}_M = \vec{a}_n^r + \vec{a}_n^t + \vec{a}_k^{ayl} + \vec{a}_k^{mi} + \vec{a}_c \quad (1.7)$$

Nisbiy urinma tezlanishning moduli

$$a_n^r = |\vec{a}_n^r| \quad (1.8)$$

bu yerda,

$$\vec{a}_n^r = \frac{d^2 S_n}{dt^2} = -\frac{20\pi^2}{36} \sin \frac{\pi t}{6}.$$

sekundda

$$\vec{a}_n^r = -2.74 \text{ sm/s}^2, \quad a_n^r = 2.74 \text{ sm/s}^2.$$

$\vec{a}_n^r$ ning manfiy ishorasi $\vec{a}_n^r$ vektorining S_n ning kamayish tomoniga qarab yo'nalganligini ko'rsatadi. $\vec{a}_n^r$ va $\vec{v}_n$ larning ishoralari har xil. Demak, $\vec{a}_n^r$ va $\vec{v}_n$ vektorlar qarama-qarshi tomonlarga yo'nalgan bo'ladi.

M nuqtaning nisbiy harakatdagi normal tezlanish

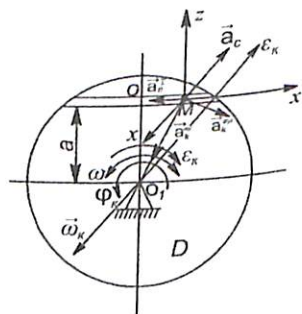
$$a_n^t = \frac{v_n^2}{\rho} = 0,$$

chunki nisbiy harakat traektoriyasi to'g'ri chiziqli ($\rho = \infty$).

Ko'chirma aylanma tezlanishning moduli

$$a_k^{ayl} = R_k \varepsilon_k;$$

bu yerda: $\varepsilon_k = |\vec{\varepsilon}_k|$ – doira burchak tezlanishi-ning moduli



3.54b-rasm

P polzunung Koriolisma tezlanishini aniqlaymiz:

$$\begin{aligned} \vec{w}_{1k} &= w_1(t_1) * \vec{k} = 2.09 \vec{k} \\ \vec{v}_n &= v_n \vec{i} = 50 \vec{i} \end{aligned}$$

Bulardan foydalansak, P polzun Koriolis tezlanishini quyidagiga teng bo'ladi:

$$\vec{a}_s = 2\vec{w}_{1k}(t_1) * \vec{v}_n(t_1) = 2 * 2.09 \vec{k} * 50 \vec{i} = 209.4 \vec{j}$$

Yuqorida hisoblangan kattaliklarnin gqiymatlarini e'tiborga olsak, P polzunung absolyut tezligi quyidagiga teng bo'ladi:

$$v_a = \sqrt{v_n^2 + v_k^2} = \sqrt{(104.7)^2 + (261.74)^2} = 325.82 \frac{\text{sm}}{\text{s}^2}$$

Absolyut tezlikning yo'nalishini quyidagicha topiladi:

$$\text{tg } \alpha_\lambda = \frac{v_k}{v_n} = \frac{104.7}{50} = 2.09$$

$$\alpha_\lambda = \arctg \alpha_\lambda = 64.47'$$

P polzunung absolyut tezlanishini aniqlaymiz:

$$\vec{a}_n = \vec{a}_k + \vec{a}_n + \vec{a}_s$$

Absolyut tezlanishi moduli va yo'nalishini aniqlash uchun proyeksiya usulidan foydalanilamiz:

$$a_{ax} = a_n - a_{ki} = 25 - 219.1 = -194.1 \frac{\text{sm}}{\text{s}^2}$$

$$a_{ay} = a_{kt} + a_s = 52.34 + 209.4 = 261.74 \text{ sm/s}^2$$

Natijada P polzun absolyut tezlanishi quyidagiga teng bo'ladi:

$$a_a = \sqrt{a_{ax}^2 + a_{ay}^2} = \sqrt{(-194.1)^2 + (261.74)^2} = 325.82 \frac{\text{sm}}{\text{s}^2}$$

Absolyut tezlanish yo'nalishi esa uning Px o'qi bilan hosil qilgan burchak orqali aniqlanadi:

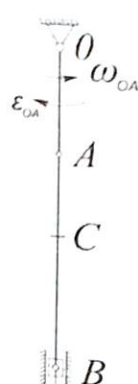
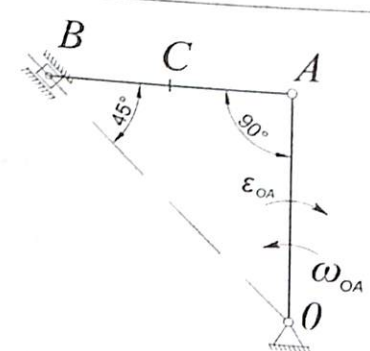
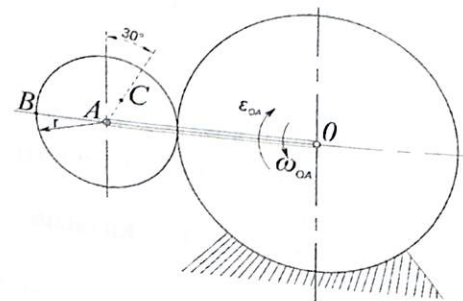
$$\cos \alpha_\lambda = \frac{Q_x}{Q_a} = \frac{-194.1}{325.82} = -0.595$$

$$\alpha_\lambda = \arccos \alpha_\lambda = 180^\circ - 53^\circ 44' = 126^\circ 56'$$

Absolyut tezlik vektori, absolyut tezlanish vektori va ularning yo'nalishlari 3.53-rasmda ko'rsatilgan.

Masalani yechishda quyidagi belgilashlar kiritiladi:

$$\vec{v}_r \rightarrow \vec{v}_n \quad \vec{a}_r = \vec{a}_n \quad w_1 \rightarrow w_k$$

14.		$OA=20 \text{ sm}$ $AB=40 \text{ sm}$ $AC=15 \text{ sm}$ $\omega_{OA}=4 \text{ rad/s}$ $\varepsilon_{OA}=6 \text{ rad/s}^2$
15.		$OA=40 \text{ sm}$ $AC=20 \text{ sm}$ $\omega_{OA}=4 \text{ rad/s}$ $\varepsilon_{OA}=8 \text{ rad/s}^2$
16.		$OA=50 \text{ sm}$ $r=20 \text{ sm}$ $AC=10 \text{ sm}$ $\omega_{OA}=1 \text{ rad/s}$ $\varepsilon_{OA}=8 \text{ rad/s}^2$

$$\tilde{\vartheta}_n = \frac{dS_n}{dt} = \frac{20\pi}{6} \cos \frac{\pi t}{6}.$$

$t=1$ sekundda,

$$\tilde{\vartheta}_n = 9 \text{ sm/s}, \quad \vartheta_n = 9 \text{ sm/s}.$$

$\tilde{\omega}_k$ oldidagi musbat ishora $\tilde{\vartheta}_n$ vektorining S_n ning o'sish tomoniga qarab yo'nalganligini ko'rsatadi.

Ko'chirma tezlikning moduli

$$\vartheta_k = R_k \omega_k \quad (1.3)$$

bu yerda R_k – doiraning berilgan onda M nuqta bilan ustma – ust tushuvchi nuqtasi tomonidan chiziladigan L aylananing radiusi, ω_k – doira burchak tezligining moduli.

$$R_k = OM = \sqrt{a^2 - (OM)^2} = 22,36 \text{ sm}. \quad (1.4)$$

$$\omega_k = |\tilde{\varphi}_k|, \quad \tilde{\omega}_k = \frac{d\varphi_k}{dt} = 2 - t. \quad (1.5)$$

$t=1$ sekundda,

$$\tilde{\omega}_k = 1 \frac{\text{rad}}{\text{s}}, \quad \omega_k = 1 \text{ rad/s}.$$

(L aylana rasmda ko'rsatilmagan).

$\tilde{\varphi}_k$ kattalikning oldidagi musbat ishora doiraning O_1 nuqta atrofida aylanishi φ_k burchakning o'sish tomoniga qarab ro'y berishini ko'rsatadi. Shuning uchun $\tilde{\omega}_k$ vektor chizma tekisligiga perpendikulyar holda O_1 nuqtadan o'tkazilgan aylanish o'qi bo'ylab tepaga qarab yo'nalgan (3.7a-rasm).

Ko'chirma tezlikning moduli (1.3)3.35 formula bo'yicha topiladi:

$$\vartheta_k = 22.36 \text{ sm/s}$$

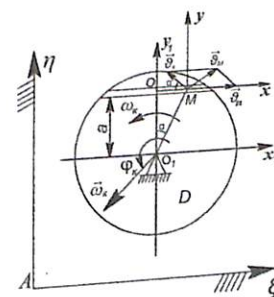
ϑ_k vektor L aylanaga urinma bo'ylab, doiraning aylanish tomoniga qarab yo'nalgan.

M nuqtaning absolyut tezligi uning nisbiy va ko'chirma harakat tezliklaridan qurilgan parallelogramning diagonalini orqali ifodalanadi. Uning modulini proektsiyalash usuli orqali aniqlaymiz:

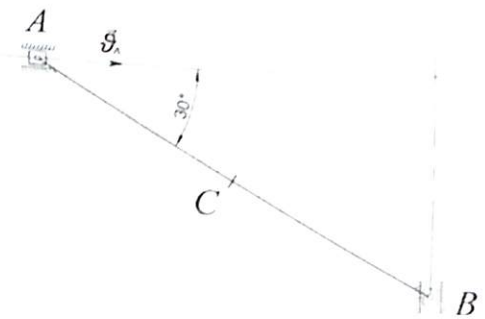
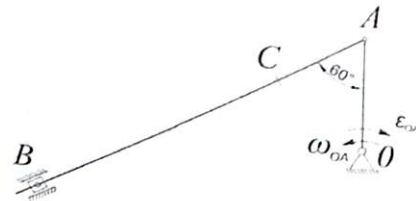
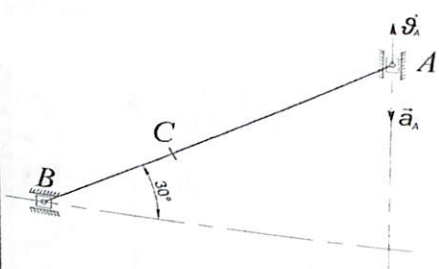
$$\vartheta_{Mx} = \vartheta_n - \vartheta_k \cos \alpha; \quad \cos \alpha = 0.89.$$

$$\vartheta_y = \vartheta_k \sin \alpha; \quad \sin \alpha = 0.45$$

Shuning uchun,



3.54a-rasm

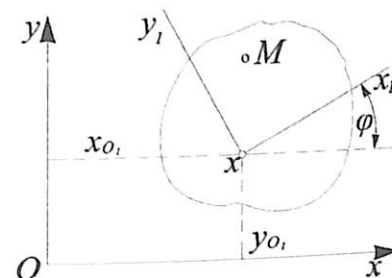
20.		$AB=40 \text{ sm}$ $AC=20 \text{ sm}$ $\vec{v}_A=10 \text{ sm/s}$ $a_A=0$
21.		$OA=25 \text{ sm}$ $AB=80 \text{ sm}$ $AC=20 \text{ sm}$ $\omega_{OA}=2 \text{ rad/s}$ $\epsilon_{OA}=2 \text{ rad/s}^2$
22.		$AB=50 \text{ sm}$ $AC=30 \text{ sm}$ $\vec{v}_A=20 \text{ sm/s}$ $\vec{a}_A=10 \text{ sm/s}^2$

5. Tekis shakl burilish burchagidan birinchi tartibli hosila olinib, burchak tezlik aniqlanadi.

6. Tekis shakl burchak tezligidan birinchi tartibli xosila hisoblanib, burchak tezlanish aniqlanadi.

55-§. Tekis shakilning harakat tenglamalari, burchak tezligi va burchak tezlanishini aniqlashga doir masalalar

1-masala. Krivoship - polzunli mexanizmida krivoshipning aylanish markazi B polzun traektoriyasidan a masofa uzoqlikda joylashgan. Krivoship O nuqta atrofida $\Psi=kt$ qonunga muvofiq aylanadi, bunda k - doimiy koeffitsient. Krivoship uzunligi $OA=r$, shatun uzunligi $AB=l$ ekanligini e'tiborga olib (4.4-rasm),



4.4-rasm

AB shatun harakat tenglamasini aniqlang.

Yechilishi. Qo'zg'almas sanoq sistemasi sifatida markazi O nuqtada bo'lgan, Ox o'qi gorizontol holda o'ng tomon, Oy o'qi vertikal yuqoriga yo'nalgan o'qlar sistemasini tanlaymiz. Qo'zg'aluvchan sanoq sistemasi sifatida markazi A nuqtada bo'lgan, Ax1 o'qi AB shatun bo'ylab, Ay1 o'qi unga perpendikulyar holda yo'nalgan o'qlar sistemasi olinadi.

Shatun A nuqtasi qutb sifatida tanlaymiz. Qutbning harakat tenglamasi quyidagi ko'rinishda yoziladi:

$$x_A = OA \cos \psi = r \cos kt$$

$$y_A = OA \sin \psi = r \sin kt$$

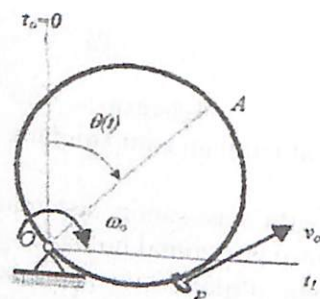
Hisoblash natijalari quyidagi jadvalda keltirilgan.

ω_k , rad/s	Tezlik, sm/s			ε_k , rad/s ²	Tezlanish, sm/s ²								
	ϑ_k	ϑ_n	ϑ_M		a_k^{mi}	a_k^{ayl}	a_n^r	a_n^n	a_c	$a_{M,1}$	$a_{M,2}$	$a_{M,3}$	a_M
1	22,36	9	14,83	-1	22,36	22,36	2,74	0	18	18	7,1	30,5	36,12

49-§. Talabalarga mustaqil o'rganish uchun tavsiya etiladigan muammolar

Muammo-1.

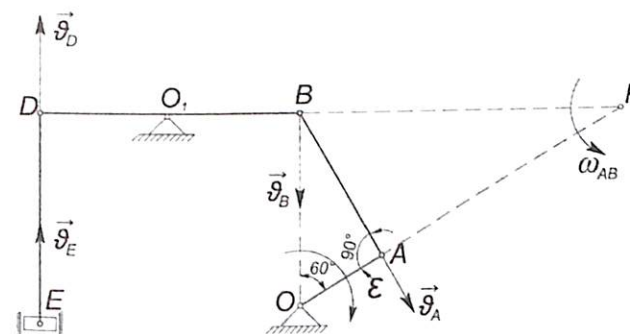
Radiusi $R=30\text{sm}$ bo'lgan doira chizma tekisligiga O nuqtadan o'tuvchi o'q atrofida $\omega_0=1\text{rad/s}$ o'zgarmas tezlik bilan aynalmoqda. Shu vaqtning o'zida P nuqta O nuqtadan $v_0=25\text{sm/s}$ tezlik bilan doira gardishi bo'ylab harakatlana boshlaydi. Boshlang'ich paytda vertical holatda bo'lgan doira diametrik gorizontal holatini egallaganda P nuqtaning absolyut tezligi va absolyut tezlanishi aniqlanadi (3.55-rasm).



3.55-rasm

Muammo-2.

Disk uning diametridan o'tuvchi gorizontal to'g'ri chiziq bo'ylab $OA=at^2$ qonungamuvofiq ilgariylanma harakatlanmoqda. Shu



4.5b-rasm

Shuning uchun:

$$\omega_{BD} = \frac{\vartheta_B}{O_1B} = \frac{\vartheta_D}{O_1D}.$$

Lekin $O_1B = O_1D$ bo'lganligi uchun, $\vartheta_B = \vartheta_D$.

Rasmda VD krivoship gorizontal holatda bo'lganligi uchun

$$\dot{\vartheta}_D \parallel \dot{\vartheta}_B; \vartheta_D = \vartheta_B.$$

Binobarin, $\vartheta_D = \vartheta_B$.

Demak, E nuqtaning tezligini aniqlash uchun B nuqtaning tezligini aniqlash yetarli ekan.

B nuqtaning tezligini aniqlash uchun berilgan mexanizm AB qismining harakatini o'rganamiz. AB qism nuqtalari tezliklarining oniy markazi $\vec{\vartheta}_A$ va $\vec{\vartheta}_B$ vektorlarga o'tkazilgan perpendikulyarlarning kesishish nuqtasi P hisoblanadi (4.5b-rasm).

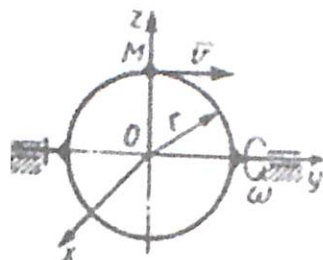
Shuning uchun:

$$\omega_{AB} = \frac{\vartheta_A}{AP} = \frac{\vartheta_B}{BP}.$$

Bundan,

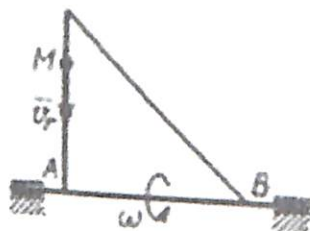
$$\vartheta_B = \frac{\vartheta_A}{AP} \cdot BP.$$

Agar,



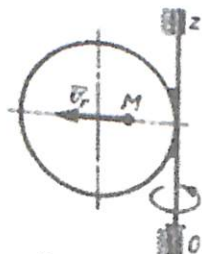
3.58- rasm

Muammo-5. Uchburchak shaklidagi jism AB tomoni atrofida ω burchak tezlik bilan aylanadi. M nuqta esa uning tomoni bo'ylab $v_r = 3t^2$ nisbiy tezlik bilan harakatlanadi. Nuqtaning $t=2s$ paytdagi nisbiy tezlanishini aniqlang (3.59- rasm).



3.59- rasm

Muammo-6. Disk Oz o'qi atrofida aylanadi. M nuqta esa $v_r = 4t^3$ nisbiy tezlik bilan diskning diametric bo'ylab harakatlanadi. Nuqtaning $t=1s$ paytdagi nisbiy tezlanishini toping (3.60- rasm).



3.60- rasm

$$\tilde{\varepsilon}_k = \frac{d^2 \varphi_k}{dt^2} = -1 \text{ rad/s}^2, \varepsilon_k = 1 \text{ rad/s}^2.$$

$\tilde{\varepsilon}_k$ va $\tilde{\omega}_k$ larning ishoralari har xil. Demak, doira sekinlanuvchan aylanma harakatda bo'lar ekan. $\tilde{\omega}_k$ va $\tilde{\varepsilon}_k$ vektorlar qarama – qarshi tomonlarga yo'naladi (3.54a,b-rasmlar).

(1.9) ga asosan,

$$a_k^{ayl} = 22,36 \text{ sm/s}^2.$$

$\tilde{a}_k^{ayl}$ vektor $\tilde{v}_k$ vektorga qarama-qarshi holda yo'nalgan.

M nuqtaning ko'chirma harakat markazga intilma tezlanishining moduli:

$$a_k^{mi} = R_k \omega_k^2 = 22,36 \text{ sm/s}^2 \quad (1.10)$$

$\tilde{a}_k^{mi}$ vektor L aylananing markaziga (O_1 nuqtaga) yo'nalgan.

Koriolis tezlanishi:

$$\tilde{a}_c = 2(\tilde{\omega}_k \times \tilde{v}_n) \quad (1.11)$$

Koriolis tezlanishining moduli

$$a_c = 2\omega_k v_n \sin(\tilde{\omega}_k \times \tilde{v}_n).$$

masalada,

$$\sin(\tilde{\omega}_k \times \tilde{v}_n) = \sin 90^\circ = 1.$$

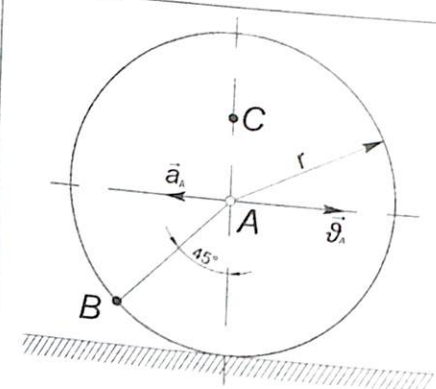
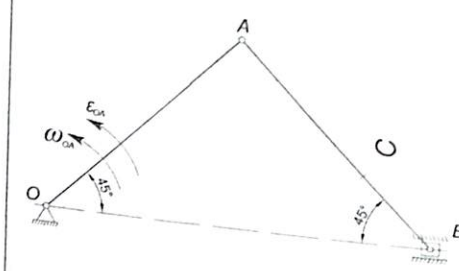
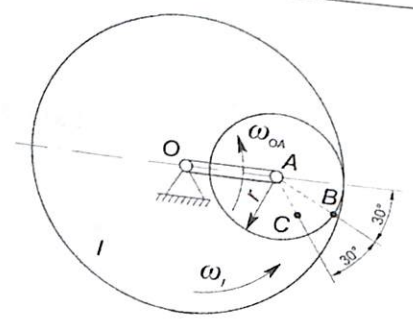
ω_k va v_n larning yuqorida topilgan qiymatlarini hisobga olsak,

$$a_c = 18 \text{ sm/s}^2$$

$\tilde{a}_c$ vektor ($\tilde{\omega}_k \times \tilde{v}_n$) vektor ko'paytma qoidasiga muvofiq yo'naladi (3.54b-rasm).

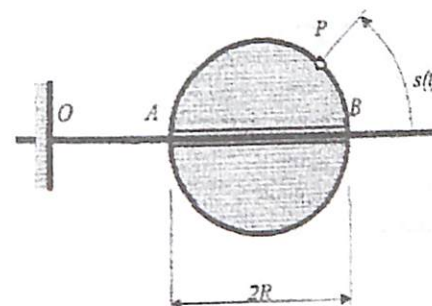
M nuqtaning absolyut tezlanishining modulini proeksiyalash usuli yordamida aniqlaymiz:

$$\begin{aligned} a_{Mx} &= -a_c = -18 \text{ sm/s}^2 \\ a_{My} &= -a_n^r - a_k^{mi} \sin \alpha + a_k^{ayl} \cos \alpha = 7,1 \text{ sm/s}^2; \\ a_{Mz} &= -a_k^{mi} \cos \alpha - a_k^{ayl} \sin \alpha = 30,5 \text{ sm/s}^2; \\ a_M &= \sqrt{a_{Mx}^2 + a_{My}^2 + a_{Mz}^2} = 36,12 \text{ sm/s}^2. \end{aligned}$$

2.		$r=45 \text{ sm}$ $AS=15 \text{ sm}$ $\vec{\omega}_A=100 \text{ sm/s}$ $a_A=50 \text{ sm/s}^2$
3.		$OA=20 \text{ sm}$ $AB=20 \text{ sm}$ $AC=10 \text{ sm}$ $\omega_{OA}=2 \text{ rad/s}$ $\epsilon_{OA}=6 \text{ rad/s}^2$
4.		$OA=30 \text{ sm}$ $r=20 \text{ sm}$ $AC=15 \text{ sm}$ $\omega_{OA}=1 \text{ rad/s}$ $\omega_I=2,5 \text{ rad/s}$ $\epsilon_{OA}=0 \text{ rad/s}^2$

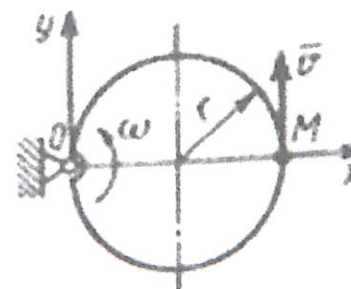
vaqtning o'zida disk gardishi bo'ylab P nuqta $BP = S(t) = \frac{R\pi t^2}{12}$ qonuniga muvofiq harakatlanadi.

Agar $a=10 \text{ sm/s}^2$, $R=25 \text{ sm}$ bo'lsa, $t=2 \text{ s}$ vaqt oni uchun P nuqtaning absolyut tezligi va absolyut tezlanishi aniqlansin (3.56- rasm).



3.56-rasm

Muammo-3. Radiusi $r=0,5 \text{ m}$ bo'lgan halqa shakl tekisligida o'zgarmas $\omega=4 \text{ rad/s}$ burchak tezlik bilan aylanadi. M nuqta esa halqa bo'ylab o'zgarmas $v=2 \text{ m/s}$ tezlik bilan harakat qiladi. Ko'rsatilgan holat uchun M nuqtaning absolyut tezlanishini toping (3.57- rasm).



3.57- rasm

Muammo-4. Oy o'qi atrofida o'zgarmas $\omega=4 \text{ rad/s}$ burchak tezlik bilan aylanayotgan halqa bo'ylab M nuqta o'zgarmas $v=2 \text{ m/s}$ tezlik bilan harakat qiladi. Agar halqaning radiusi $r=0,5 \text{ m}$ bo'lsa, ko'rsatilgan holat uchun M nuqtaning absolyut tezlanishini toping (3.58- rasm).

26.		$OA = 25 \text{ sm}$ $AC = 20 \text{ sm}$ $\omega_{OA} = 1 \text{ rad/s}^2$ $\epsilon_{OA} = 1 \text{ rad/s}^2$
27.		$OA = 40 \text{ sm}$ $AC = 50 \text{ sm}$ $\omega_{OA} = 4 \text{ rad/s}$ $\epsilon_{OA} = 8 \text{ rad/s}^2$
28.		$r = 50 \text{ sm}$ $\vec{\omega}_A = 50 \text{ sm/s}$ $a_A = 100 \text{ sm/s}^2$

17.		$OA = 60 \text{ sm}$ $r = 25 \text{ sm}$ $\omega_{OA} = 3 \text{ rad/s}$ $AC = 10 \text{ sm}$ $\omega_I = 12 \text{ rad/s}$
18.		$OA = 30 \text{ sm}$ $AB = 60 \text{ sm}$ $AC = 40 \text{ sm}$ $\omega_{OA} = 2 \text{ rad/s}$ $\epsilon_{OA} = 4 \text{ rad/s}^2$
19.		$AB = 40 \text{ sm}$ $AC = 25 \text{ sm}$ $\vec{\omega}_A = 20 \text{ sm/s}$ $a_A = 20 \text{ sm/s}^2$

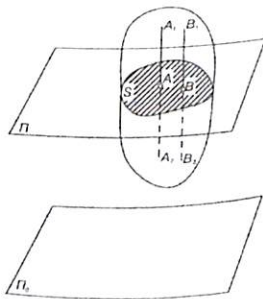
IV BOB. QATTIQ JISMNING TEKISLIKKA PARALLEL HARAKATI

Agar jismning barcha nuqtalari berilgan qo'zg'almas tekislikka parallel tekisliklarda harakatlansa, uning bunday harakati tekislikka parallel harakat deyiladi.

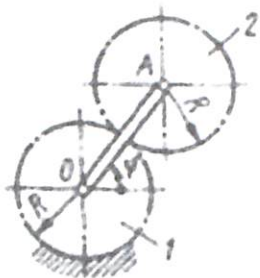
Jismning tekislikka parallel harakatiga misol tariqasida to'g'ri chiziqli relisda g'ildirakning dumalashini, bir tekislikda harakatlanuvchi mashina va mexanizm qismlarining harakatini va hokazolarini keltirish mumkin.

51-§. Tekis shakl harakatini qutb bilan birgalikda oniy ilgarilanma va qutb atrofida oniy aylanma harakatlarga ajratish

Jismning tekislikka parallel harakatini o'rganish uchun uni qo'zg'almas π_0 tekislikka parallel bo'lgan π tekislik bilan fikran kesamiz. Kesish natijasida hosil bo'lgan kesimni S bilan belgilab, uni *tekis shakl* deb ataymiz. Jismning tekislikka parallel harakati ta'rifi-ga ko'ra, jismning harakati davomida bu tekis shakl doimo qo'zg'al-mas π tekislikka parallel bo'lgan π tekislikda harakatlanadi. Tekislikka parallel harakatdagi jismda π tekislikka perpendikulyar qilib olingan A_1A_2 kesma o'ziga parallel holda ko'chadi, ya'ni, kesma ilgarilanma harakatda bo'ladi. Shu sababli jismning bu kesmada yotgan barcha nuqtalarining harakatini o'rganish o'rniga, ularidan birining, masalan, S tekis shakl A nuqtasining harakatini o'r-ganish yetarli bo'ladi. π tekislikka perpen-dikulyar B_1B_2 kesmaning harakatini o'rganishda ham xuddi shunday xulosaga kelish mumkin. Shunday qilib, qattiq jismning tekislikka paral-lel harakatini o'rganish uchun π_0 (4.1 – rasm) qo'zg'almas tekislikka parallel bo'lgan tekis shaklning π tekislikdagi harakatini o'rganish kifoya bo'lar ekan. Tekis shakl harakatlanadigan π tekislik tekis shaklning harakat tekisligi deyiladi (4.1-rasm).

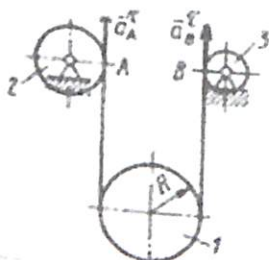


23.		$AB=30 \text{ m}$ $AC=15 \text{ m}$ $\vec{v}_A=40 \text{ m/s}$ $\vec{a}_A=20 \text{ m/s}^2$
24.		$OA=35 \text{ m}$ $AB=75 \text{ m}$ $AC=60 \text{ m}$ $\omega_{OA}=4 \text{ rad/s}$ $\epsilon_{OA}=10 \text{ rad/s}^2$
25.		$OA=60 \text{ m}$ $r=15 \text{ m}$ $AC=6 \text{ m}$ $\omega_{OA}=1 \text{ rad/s}$ $\omega_1=1 \text{ rad/s}$ $\epsilon_{OA}=0$



4.12- rasm

6-muammo. Qo'zg'almas 2 va 3 bloklarning A va B nuqtalari $a_A^t = 5m/s^2$ va $a_B^t = 10m/s^2$ tangensial tezlanishga ega bo'lsa, radiusi $R=0,5m$ li 1 qo'zg'luvchan blokning burchak tezlanishini toping (4.13- rasm).



4.13- rasm

57-§. Tekis shakl nuqtasining tezligini qutb usulida aniqlash

Tekis shakl nuqtalarining tezliklari orasidagi bog'lanish quyidagi teorema yordamida ifodalanadi.

Teorema. Ekis shakl ixtiyoriy P nuqtasining tezligi qutb sifatida olingan O nuqtaning tezligi bilan mazkur nuqtaning qutb atrofida aylanma bo'ylab harakatidagi chiziqli tezlikning geometrik yig'indisidan iborat bo'ladi (4.14- rasm).

62-§. Tekislikka parallel harakatdagi jism nuqtalarining tezliklarini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Umuman, tekis shakl nuqtalarining tezliklarini quyidagi 3 usulda aniqlash mumkin:

1. Analitik usul.
2. Grafik usul.
3. Grafoanalitik usul.

Mazkur o'quv qo'llanmada tekis shakl nuqtalarining tezliklarini aniqlashni grafoanalitik usuli bilan tanishamiz.

Grafoanalitik usulning o'zi ham ikki yo'ldan iborat.

a) Tekis shakl nuqtalarining tezliklarini qutb usulida aniqlash.

Bu usulda tekis shakl nuqtalarining tezliklari quyidagicha aniqlanadi.

1. Tezligi ma'lum yoki masala shartiga ko'ra aniqlanishi mumkin bo'lgan tekis shakl nuqtasi qutb sifatida tanlanadi.

2. Tekis shaklda tezligining yo'nalishi ma'lum bo'lgan boshqa nuqta aniqlanadi.

3. Bu nuqtaning tezligi tekis shakl nuqtalarining tezliklari haqidagi teorema asosida hisoblanadi.

4. Tekis shaklning shu vaqt onidagi burchak tezligi aniqlanadi.

5. Tekis shakl burchak tezligini bilgan holda, yuqorida bayon etilgan tekis shakl nuqtalarining tezliklari haqidagi teoremadan, tekis shaklning so'ralgan nuqtasining tezligi aniqlanadi.

b) Tekis shakl nuqtalarining tezliklarini tezliklarning oniy markazi orqali aniqlash.

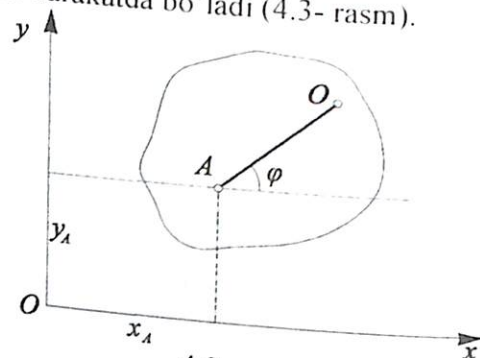
Bu usulda tekis shakl nuqtalarining tezliklari quyidagicha aniqlanadi:

1. Tekis shakl nuqtalari tezliklarining oniy markazi aniqlanadi.

2. Tekis shaklning tezligi ma'lum bo'lgan nuqtasining oniy radiusi aniqlanadi va tezlik modulini oniy radiusga bo'lib, tekis shaklning burchak tezligi topiladi.

ravishda harakatlanib boradi va tekis shakl yoki tekis shakl taluqli bo'lgan qattiq jism ilgarilanma harakatda bo'ladi.

Agar qattiq jismning harakati davomida A nuqta (qutb) koordinatalari x_A, y_A lar o'zgarmas holda qolib, φ burchak o'zgarsa, u holda jismning A nuqtasi qo'zg'almasdan qoladi va tekis shakl A nuqta atrofida aylanma harakatda bo'ladi, yani qattiq jism A nuqtadan o'tuvchi va shakl tekisligiga perpendikulyar bo'lgan o'q atrofida aylanma harakatda bo'ladi (4.3- rasm).



4.3- rasm

53-§. Tekis shakilning burchak tezligi va burchak tezlanishi

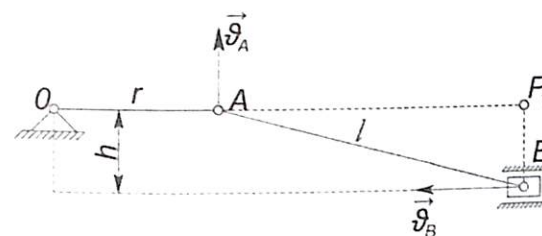
Tekis shakl qutb atrofida aylanganda uning barcha nuqtalari har onda bir xil burchak tezlik va bir xil burchak tezlanishga ega bo'ladi:

Tekis shakilning aylanish burchagidan vaqt bo'yicha olingan hosila tekis shaklning burchak tezligi deyiladi.

$$\omega = \frac{d\varphi}{dt}$$

Tekis shaklning burchak tezligidan vaqt bo'yicha olingan birinchi tartibli hosila yoki tekis shakl aylanish burchagidan vaqt bo'yicha olingan tartibli hosila tekis shaklning burchak tezlanishi deyiladi.

Tekis shaklning burchak tezligi va burchak tezlanishi qutbning tanlab olinishiga bog'liq bo'lmaydi, chunki tekis shaklning qutb at-



4.25b-rasm

Krivoship A nuqtasining tezligi:

$$v_A = \omega \cdot OA = 1,5 \cdot 40 = 60 \text{ sm/s}; \quad \vec{v}_A \perp \vec{OA}$$

B polzun tezligini aniqlash uchun, AB shatun nuqtalari tezliklarining oniy markazini aniqlaymiz. Buning uchun $\vec{v}_A$ va $\vec{v}_B$ yo'nalishlariga perpendikulyar chiziqlar o'tkazib, ularning kesishish nuqtasi P_1 ni topamiz. P_1 nuqta AB shatun nuqtalari tezliklarining oniy markazini ifodalaydi. P_1 nuqta qutb sifatida qabul qilinsa, krivoship A nuqtasining tezligi quyidagicha yoziladi:

$$v_A = \omega_{AB} \cdot AP_1$$

Bundan,

$$\omega_{AB} = \frac{v_A}{AP_1}$$

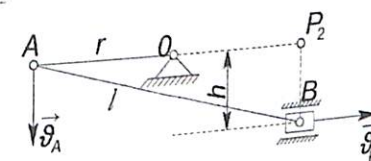
Bunday holda,

$$v_B = \omega_{AB} \cdot BP_1$$

Agar $BP_1 = h$ ekanligini e'tiborga olsak:

$$v_B = \frac{v_A}{AP_1} \cdot h = \frac{60}{\sqrt{l^2 - n^2}} \cdot h = 6,03 \text{ sm/s}$$

2. Xuddi shunday mulohazalar asosida, krivoshipning ikkinchi gorizontal holatida B polzunning tezligini aniqlaymiz (4.25v-rasm).



4.25v-rasm

4.25v-rasmdan ko'rinib turibdiki, krivoshipning ikkinchi gorizontal holatida ham B polzun tezligining miqdori l gorizontal holatidagi tezligiga teng bo'lar ekan:

Qutb A nuqtaning aylanma harakatini ifodalovchi tenglamani tuzish uchun AB kesmani Oy o'qiga proeksiyalaymiz.

Agar Ax_1 va Ox o'qlar orasidagi burchakni φ orqali belgilasak 4.4-rasmdan

$$AB \sin \varphi = OA \psi + a.$$

Agar $AB=l$, $OA=r$, $\psi=kt$ ekanligini e'tiborga olsak,

$$\sin \varphi = \frac{r}{l} \sin kt + \frac{a}{l}.$$

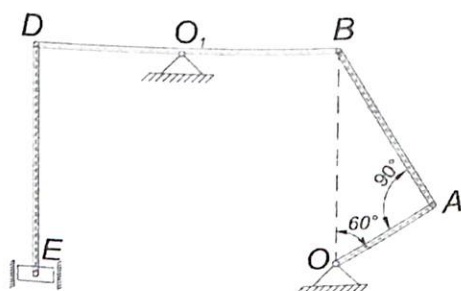
Bu ifodadan AB polzun harakat tenglamalaridan uchinchi quyidagi ko'rinishda yozilishi ma'lum bo'ladi:

$$\varphi = \arcsin \left(\frac{r}{l} \sin kt + \frac{a}{l} \right).$$

Shunday qilib, AB polzuning quyidagi ko'rinishdagi harakat tenglamalariga esa bo'lamiz.

2-masala. OA

krivoship 2 rad/s burchak tezlik bilan bir tekis aylanadi. Agar $OA=20\text{sm}$, $O_1B=O_1D$ bo'lsa, rasmda ko'rsatilgan holat uchun nasosning uzatmali mexanizmi E porshenining tezligi aniqlansin (4.5a-rasm).



4.5a-rasm

Yechimi: A nuqtaning tezligi $\vec{v}_A$ OA krivoshipga perpendikulyar holda ω_{OA} tomon yo'naladi ($\vec{v}_A \perp OA$).

BD krivoship O_1 nuqta atrofida aylanishi tufayli D nuqtaning tezligi $\vec{v}_D$, O_1D krivoshipga perpendikulyar holda, B nuqtaning tezligi $\vec{v}_B$ esa O_1B kesmaga perpendikulyar holda ω_{BD} tomon yo'naladi:

$$\vec{v}_D \perp O_1D, \vec{v}_B \perp O_1B$$

rofida aylanish burchagi qutbni tanlashga bog'liq bo'lmasligini yuqorida aytib o'tgan edik.

$$\varphi = \frac{d\varphi}{dt}, \quad \varepsilon = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2}. \quad (4.4)$$

Burchak tezlik $\vec{\omega}$ va burchak tezlanish $\vec{\varepsilon}$ vektorlari tekis shakl tekisligiga A qutb orqali perpendikulyar xolda o'tgan o'qda yotadi. Agar tekis shaklning qutb atrofida aylanma harakati tezlanuvchan bo'lsa, $\vec{\omega}$ va $\vec{\varepsilon}$ lar bir tomonga, sekinlanuvchan bo'lsa, qarama-qarshi tomonga yo'naladi.

Takrorlash uchun savollar

1. Qattiq jismning tekislikka parallel harakatini tariflang.
2. Qanday tekislik shaklning harakat tekisligi deyiladi?
3. Qanday nuqta qutb sifatida tanlanadi?
4. Tekis shaklning harakat tekisligidagi har qanday ko'chishi qanday harakatlardan tashkil topadi?
5. Tekis shaklning tekislikka parallel harakati tenglamalarini yozing.
6. Agar tekis shaklning harakat tekisligida $\varphi=\text{const}$ bo'lsa tekis shakl qanday harakatda bo'ladi?
7. Tekis shaklning burchak tezligini ta'riflang.
8. Tekis shaklning burchak tezlanishini ta'riflang.
9. Tekis shakl qutb atrofida tezlanuvchan harakatda bo'lish shartini ta'riflang.
10. Tekis shakl qutb atrofida sekinlashuvchan harakatda bo'lish shartini ta'riflang.

54-§. Tekis shaklning harakat tenglamalari, tekis shakl nuqtasining harakat tenglamalari, tekis shakl burchak tezligi va burchak tezlanishini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Tekis shaklning harakat tekisligida har qanday ko'chishi qutb bilan birgalikdagi ilgarilanma harakati hamda qutbdan harakat tekisligiga perpendikulyar ravishda o'tuvchi o'q atrofida aylanma harakatidan tashkil topadi.

58-§. Tekis shakl ikki nuqtasi tezliklarining proeksiyalariga oid teorema

Tekis shakl nuqtalarining tezliklari orasidagi bog'lanish quyidagi ikkita teorema orqali aniqlanadi:

1-Teorema. Tekis shakl ixtiyoriy nuqtasining tezligi qutb tezligi bilan, nuqtaning qutb atrofidagi aylana bo'ylab harakatidagi chiziqli tezligining geometrik yig'indisiga teng bo'ladi.

Isboti: Tekis shakl harakatini qo'zg'almas Oxy koordinatalar sistemasiga nisbatan o'rganamiz. Agar A nuqta deb qutb sifatida olinsa, A va B nuqtalar radius vektorlari quyidagicha bog'lanadi:

$$\vec{r}_B = \vec{r}_A + \vec{AB}. \quad (4.6)$$

Tezlik tarifiga ko'ra:

$$\frac{d\vec{r}_B}{dt} = \frac{d\vec{r}_A}{dt} + \frac{d\vec{AB}}{dt} = \vec{v}_A + \vec{v}_{BA} = \vec{\omega} \times \vec{AB}. \quad (4.7)$$

Bunda,

$$\frac{d\vec{r}_A}{dt} = \vec{v}_A, \quad \frac{d\vec{AB}}{dt} = \vec{v}_{BA} = \vec{\omega} \times \vec{AB}. \quad (4.8)$$

Shuning uchun,

$$\vec{v}_B = \vec{v}_A + \vec{v}_{BA} = \vec{v}_A + \vec{\omega} \times \vec{AB}. \quad (4.9)$$

Tekis shakl biror nuqtasining tezligi va tekis shakl aylanna harakatining burchak tezligi berilganda, tekis shakl boshqa biror nuqtasining tezligini (4.9) formula vositasida aniqlash, uni qutb usulida aniqlash deyiladi (4.15b- rasm).

Teorema. Tekis shaklning ikkita nuqtasi tezliklarining shu nuqtalardan o'tuvchi o'qdagi proeksiyalari o'zaro teng bo'ladi.

$$(\vec{v}_B)_x = (\vec{v}_A)_x + (\vec{v}_{BA})_x \quad (4.10)$$

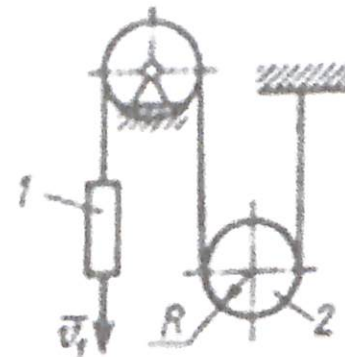
Teoremaga ko'ra (4.10) ni har ikki tomonini A_x o'qiga proeksiyalaymiz:

$$(\vec{v}_B)_x = (\vec{v}_A)_x + (\vec{v}_{BA})_x \quad (4.11)$$

Lekin, $(\vec{v}_{BA})_x = 0$, chunki $\vec{v}_{BA} \perp A_x$.

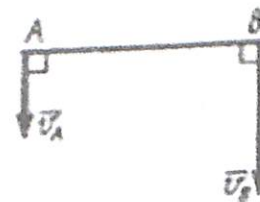
Shuning uchun 4.16 - rasmdan:

$$(\vec{v}_B)_x = (\vec{v}_A)_x \quad (4.12)$$



4.10-rasm

4-muammo. Uzunligi 80 sm bo'lgan AB sterjen shakl tekisligida harakat qilib, A va B nuqtalari $v_A=0.2\text{m/s}$ va $v_B=0.6\text{m/s}$ tezlikka ega bo'lsa, sterjenning burchak tezligini aniqlang (4.11- rasm).



4.11-rasm

5-muammo. OA krivoship $0.5t^2$ qonun bo'yicha aylansa, 2 g'ildirakning burchak tezlanishini aniqlang (4.12- rasm).

katning burchak tezligi ω berilgan bo'lsin. P nuqtaning tezligini topamiz:

$$\vec{v}_P = \vec{v}_O + \vec{v}_{PO}. \quad (4.13)$$

Bunda,

$$v_{PO} = \omega \cdot OP, \quad OP = \frac{v_O}{\omega}$$

bo'lgani uchun,

$$v_{PO} = \omega \cdot \frac{v_O}{\omega} = v_O, \quad \vec{v}_{PO} = -\vec{v}_O.$$

U holda, (4.13) dan $\vec{v}_P = 0$ bo'ladi. Demak, P nuqta tekis shakl tezliklarining oniy markazi ekan.

Berilgan onda tekis shakl nuqtalari tezliklarining oniy markazini qutb deb olsak, (4.9) formulaga asosan, tekis shakl A, B, C nuqtalarining tezliklari quyidagicha aniqlanadi:

$$\vec{v}_A = \vec{v}_P + \vec{v}_{AP}; \quad \vec{v}_B = \vec{v}_P + \vec{v}_{BP}; \quad \vec{v}_C = \vec{v}_P + \vec{v}_{CP}. \quad (4.14)$$

Lekin $\vec{v}_P = 0$.

Shuning uchun,

$$\vec{v}_A = \vec{v}_{AP}; \quad \vec{v}_B = \vec{v}_{BP}; \quad \vec{v}_C = \vec{v}_{CP}. \quad (4.15)$$

Bunda,

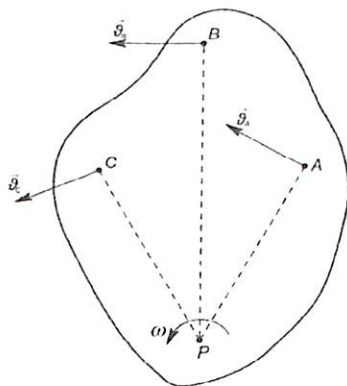
$$v_A = \omega \cdot AP, \quad v_B = \omega \cdot BP, \quad v_C = \omega \cdot CP. \quad (4.16)$$

va

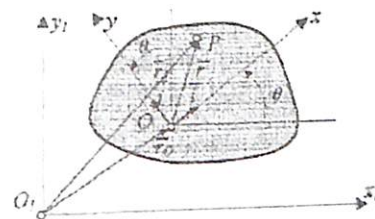
$$\vec{v}_A \perp \overline{AP}, \quad \vec{v}_B \perp \overline{BP}, \quad \vec{v}_C \perp \overline{CP}.$$

Demak, biror onda tezliklarining oniy markazi ma'lum bo'lgan tekis shakl nuqtalarining tezliklarini, oniy markaz atrofida aylanma harakatdagi nuqtalarining tezliklari kabi topish mumkin ekan.

Agar tezliklar oniy markazi tekis shakl konturidan tashqarida yotsa, tezliklar oniy markazi uchun tekis shaklga birlashtirilgan tekislikning nuqtasi olinadi. (4.16) dan tekis shakl



4-18rasm



4.14-rasm

Isboti. Agar tekis shaklning o'z tekisligidagi Ox va O_1x_1 o'qlar orasidagi burchakni θ orqali belgilasak, qo'zg'aluvchan Oxy o'qlar sistemasining birlik vektorlari quyidagicha ifodalanadi:

$$\vec{i} = \cos \theta \vec{i}_1 + \sin \theta \vec{j}_1$$

$$\vec{j} = \sin \theta \vec{i}_1 + \cos \theta \vec{j}_1$$

Tekis shaklning qutb atrofida aylanma harakati qutbdan shakl tekisligiga perpendikulyar bo'lgan Oz o'q atrofida yuzaga kelishini e'tiborga olsak, aylanma harakatning burchak tezligini quyidagicha aniqlash mumkin:

$$\vec{\omega} = \omega \vec{k} = \dot{\theta} \vec{k} = \dot{\theta} \vec{k}_1.$$

Bunday holda qutb sifatida olingan O nuqtaning tezligi quyidagi formula asosida yoziladi:

$$\vec{v}_O = v_{Ox_1} \cdot \vec{i}_1 + v_{Oy_1} \cdot \vec{j}_1 = v_{Ox} \cdot \vec{i} + v_{Oy} \cdot \vec{j}.$$

Natijada rekis shaklda olingan ixtiyoriy P nuqtaning tezligi teorema asosan quyidagicha ifodalanadi:

$$\begin{aligned} \vec{v}_P &= \vec{v}_O + \vec{\omega} \times \vec{r} = v_{Ox} \cdot \vec{i} + v_{Oy} \cdot \vec{j} + \omega \vec{k} \times (x \cdot \vec{i} + y \cdot \vec{j}) = \\ &= \vec{i}(v_{Ox} - \omega y) + \vec{j}(v_{Oy} - \omega x) \quad (4.5) \end{aligned}$$

Bu ifodadan P nuqta tezligining qo'zg'aluvchan O_x va O_y o'qlardagi proeksiyalari quyidagi ifodalar orqali aniqlanishi ma'lum bo'ladi:

$$v_{Px} = v_{Ox} + \omega y,$$

$$v_{Py} = v_{Oy} + \omega x.$$

Tekis shakl nuqtasining tezligini (4.5) formula asosida aniqlash qutb usulida aniqlash deyiladi.

Demak, E nuqtaning tezligini aniqlash uchun B nuqtaning tezligini aniqlash yetarli ekan.

B nuqtaning tezligini aniqlash uchun berilgan mexanizm AB qismining harakatini o'rganamiz. AB qism nuqtalari tezliklarining oniy markazi $\vec{v}_A$ va $\vec{v}_B$ vektorlarga o'tkazilgan perpendikulyarlarining kesishish nuqtasi P hisoblanadi (4.26b-rasm). Shuning uchun:

$$\omega_{AB} = \frac{v_A}{AP} = \frac{v_B}{BP}$$

Bundan,

$$v_B = \frac{v_A}{AP} \cdot BP$$

Agar,

$$v_A = \omega_{OA} \cdot OA = 2 \cdot 20 = 40 \text{ sm/s};$$

$$OB = \frac{AO}{\cos 60} = 2 \cdot AO = 40 \text{ sm};$$

$$OP = \frac{OB}{\cos 60} = 2 \cdot OB = 80 \text{ sm};$$

$$AP = OP - OA = 80 - 20 = 60 \text{ sm};$$

$$BP = OB \tan 60^\circ = 40\sqrt{3} = 69,3 \text{ sm}$$

ekanligini e'tiborga olsak,

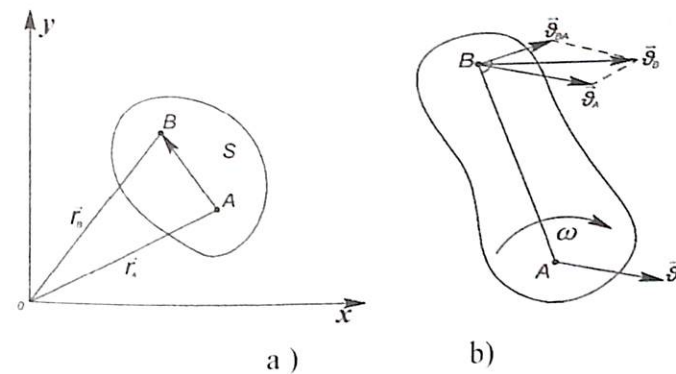
$$v_B = \frac{v_A}{AP} \cdot BP = \frac{40}{60} \cdot 69,3 = 46,2 \text{ sm/s}.$$

Demak,

$$v_E = v_B = 46,2 \text{ sm/s}$$

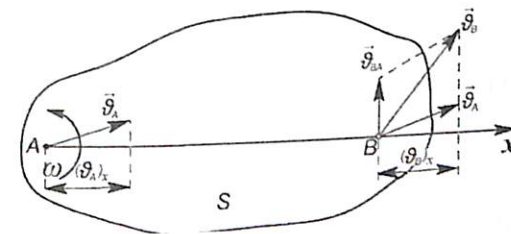
3-masala. Sharnirli $OABO_1$ parallelogramning AB sterjeni o'r-tasida D nuqtaga K polzunni ilgarilanma-qaytma harakatga keltiruv-chi DE sterjen sharnir yordamida birlashtirilgan. Agar $OA=OB=2DE=20$ sm bo'lsa, mexanizmning rasmda tasvirlangan holati uchun K polzunning tezligi va DE sterjenning burchak tezligi aniqlansin; OA zvenoning berilgan paytdagi burchak tezligi 1 rad/s .

Yechilishi: Chizmada sharnirli parallelogramning A va B nuqtalari tezliklarining yo'nalishlarini ko'rsatamiz (4.28- rasm):



4.15-rasm

Mazkur teoremani isbotlashda 1-teoremadan foydalandik.



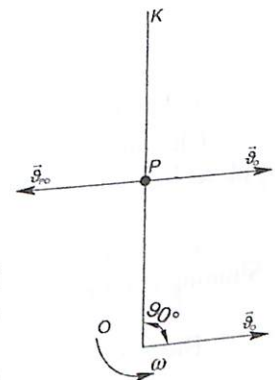
4.16-rasm

59-§. Tezliklarning oniy markazi

Tekis shaklning berilgan onda tezligi nolga teng bo'lgan nuqtasi tezliklar oniy markazi yoki aylanish oniy markazi deyiladi.

Agar tekis shaklning burchak tezligi noldan farqli bo'lsa, albatta tezliklar oniy markazi mavjud bo'ladi (4.17-rasm).

Tekis shakl biror O nuqtasining tezligi $\vec{v}_O$ va shu O nuqta atrofidagi aylanma hara-



4.17-rasm

Bunday holda DE shatun burchak tezligi quyidagicha aniqlandi:

$$\omega_{DE} = \frac{V_D}{DP} = \frac{V_E}{EP}.$$

Bunda

$$V_D = V_A = \omega_{OA} = 1 \cdot 20 = 20 \text{ rad/s}$$

$$DP = DE \tan 30^\circ = \frac{10\sqrt{3}}{3} = 5,8 \text{ sm.}$$

$$EP = \frac{DE}{\cos 30^\circ} = \frac{10 \cdot 2 \cdot 10}{\frac{\sqrt{3}}{2} \sqrt{3}} = 11,5 \text{ sm.}$$

Aniqlangan kattaliklarni hisobga olsak

$$\omega_{ED} = \frac{V_D}{DP} = \frac{20}{5,8} = 3,5 \frac{1}{s}.$$

4-masala. 1 qo'zg'aluvchi va 2 qo'zg'almas bloklar cho'zil-maydigan ip bilan bog'langan. Ipning uchiga biriktirilgan K yuk $x=2t^2$ m qonun bilan vertikal bo'ylab pastga tushadi. $t=1$ s bo'lgan paytda rasmda tasvirlangan holat uchun harakatlanuvchi blok gardishida yotuvchi C, D, B va E nuqtalarning tezliklari topilsin: qo'zg'aluvchi 1 blok radiusi 0,2 m ga teng, $CD \perp BE$. Shuningdek, 1 blokning burchak tezligini ham toping.

Yechilishi: K yuk tezligini aniqlaymiz

$$v_K = \frac{Dx_K}{dt} = (2t^2)' = 4t,$$

$$t_1 = 1 \text{ s. da}$$

$$v_K = 4 \cdot 1 = 4 \text{ m/s.}$$

$\vec{v}_K$ K nuqtaga qo'yilgan bo'lib, vertikal pastga yo'nalgan.

Chizmadan harakatlanuvchan 1- blok D nuqtasining tezligi K yuk tezligiga teng bo'lishi ma'lum:

$$v_D = v_K = 4 \text{ m/s}$$

1 - blok uchun C nuqta tezliklar oniy markazini ifodalaydi. Shuning uchun

$$v_C = 0$$

Bunday holda 1 - blok burchak tezligi quyidagicha aniqlanadi:

67-§. Tekislikka parallel harakatda bo'lgan jism nuqtalarining tezlanishlarini aniqlashga doir masalalarni yechish uchun uslubiy ko'rsatmalar

Tekislikka parallel harakatda bo'lgan jism nuqtalarining tezlanishlarini aniqlashga doir masalalarni quyidagi tartibda yechish tavsiya etiladi:

1. Tekis shakl nuqtalari tezliklarining oniy markazini aniqlash usullaridan foydalanib, berilgan masalada, tekis shakl nuqtalari tezliklarining oniy markazi aniqlanadi.

2. Tekis shaklning burchak tezligini bilgan holda, tekis shakl ikkinchi nuqtasining birinchi nuqta atrofidagi aylanma harakatidagi markazga intilma tezlanishi topiladi.

3. Ikkinchi nuqtaga uning tezlanishini tashkil etuvchi tezlanishlar vektorlari qo'yiladi. Agar birinchi nuqta A, ikkinchi nuqta B bo'lsa:

$$\vec{a}_B = \vec{a} + \vec{a}_{BA} = \vec{a}_A^t + \vec{a}_A^n + \vec{a}_{BA}^{ayl} + \vec{a}_{BA}^{mi}.$$

4. Koordinata o'qlarini o'tkazib, yuqoridagi vektor tenglikning har ikki tomoni koordinata o'qlariga proeksiyalanadi.

5. Hosil bo'lgan proeksiyalar tenglamalaridan noma'lum $\vec{a}_{BA}^{ayl}$ va $\vec{a}_B$ lar aniqlanadi.

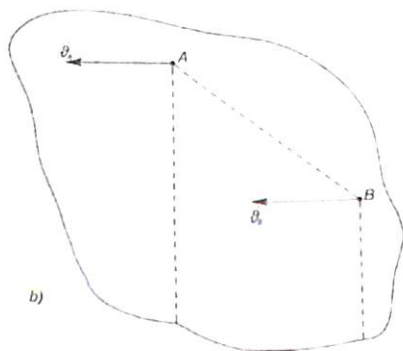
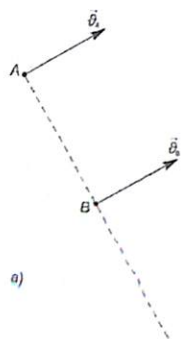
6. Proeksiyalar tenglamalaridan topilgan $\vec{a}_{BA}^{ayl}$ tezlanish modulini bilgan holda, tekis shakl burchak tezlanishi aniqlanadi:

$$\vec{a}_{BA}^{ayl} = \varepsilon \cdot AB;$$

$$\varepsilon = \frac{\vec{a}_{BA}^{ayl}}{AB}.$$

7. Tekis shakl burchak tezligi va burchak tezlanishini bilgan holda, tekis shakl nuqtalarining tezlanishlari haqidagi teorema yordamida, so'ralgan ixtiyoriy nuqtaning tezlanishi aniqlanadi.

Izoh: Tekis shaklda, $\vec{a}_B$ va $\vec{a}_{BA}^{ayl}$ larning modullarini, B nuqtada tanlangan masshtabda chizilgan, tomonlari tashkil etuvchi tezlanishlar, yopuvchi tomoni esa nuqtaning tezlanishi bo'lgan ko'p burchakdan, grafik usulda aniqlash mumkin.

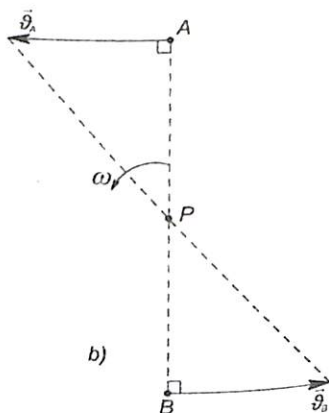
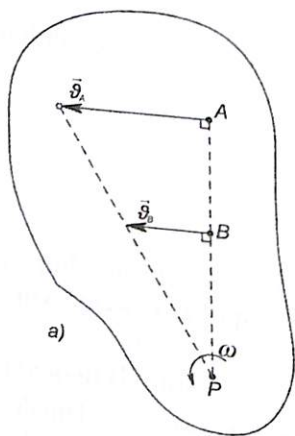


4.20a,b-rasmlar

2) Tekis shakl A va B nuqtalarining tezliklari parallel va AB ga perpendikulyar bo'lsa, tezliklarning oniy markazini aniqlash uchun tezliklar moduli ham ma'lum bo'lishi kerak. (4.17) ga ko'ra:

$$\frac{v_B}{v_A} = \frac{BP}{AP} \quad (4.19)$$

Shuning uchun ham, A va B nuqtalar tezliklarining uchi oniy markaz orqali o'tuvchi chiziqli yotadi. Shu chiziqli AB chiziqli bilan kesishgan nuqtasi tezliklar oniy markazini ifodalaydi



4.21a,b-rasmlar

yoki

$$\frac{v_A}{P_A} = \frac{v_B}{P_B} = \frac{v_C}{P_C}$$

Masala shartiga ko'ra:

$$PA = r = 30 \text{ sm}; \quad PB = \sqrt{r^2 + r^2 - 2r^2 \cos 45^\circ} = 22,8 \text{ sm}.$$

$$PC = r + AC = 30 + 10 = 40 \text{ sm}.$$

Shuning uchun,

$$v_B = \frac{v_A \cdot PB}{PA} = 38,1 \text{ sm/s};$$

$$v_C = \frac{v_A \cdot PC}{PA} = 66,7 \text{ sm/s}.$$

G'ildirak nuqtalarining tezliklarini g'ildirakning burchak tezligi orqali ham topish mumkin:

$$v_A = \omega \cdot PA$$

Bundan,

$$\omega_1 = \frac{v_A}{PA} = \frac{50}{30} = 1,67 \text{ rad/s}.$$

Burchak tezlikning yo'nalishi $\vec{v}_A$ yo'nalishi orqali aniqlanadi (4.39b-rasm).

Bunday holda g'ildirak B va S nuqtalarining tezligi quyidagilarga teng bo'ladi:

$$v_B = \omega \cdot PB = 1,67 \cdot 22,8 = 38,1 \text{ sm/s}.$$

$$v_C = \omega \cdot PC = 1,67 \cdot 40 = 66,7 \text{ sm/s}.$$

II. G'ildirak nuqtalarining tezlanishlari va g'ildirak burchak tezlanishini aniqlash.

Masala shartida A nuqtaning tezlanishi a_A berilgan.

61-§. Tekis shakl nuqtalarining tezliklarini tezliklarning oniy markazida foydalanib aniqlash

Berilgan onda tekis shakl nuqtalari tezliklarining oniy markazini qutb deb olsak, (4.9) formulaga asosan, tekis shakl A,B,C nuqtalarining tezliklari quyidagicha aniqlanadi:

$$\vec{v}_A = \vec{v}_P + \vec{v}_{AP}; \quad \vec{v}_B = \vec{v}_P + \vec{v}_{BP}; \quad \vec{v}_C = \vec{v}_P + \vec{v}_{CP} \quad (4.20)$$

Lekin $\vec{v}_P = 0$.

Shuning uchun,

$$\vec{v}_A = \vec{v}_{AP}; \quad \vec{v}_B = \vec{v}_{BP}; \quad \vec{v}_C = \vec{v}_{CP}. \quad (4.21)$$

Bunda,

$$v_A = \omega \cdot AP, \quad v_B = \omega \cdot BP, \quad v_C = \omega \cdot CP \quad (4.22)$$

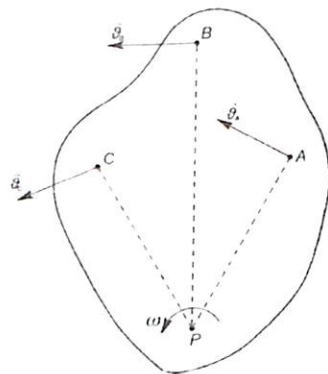
va $\vec{v}_A \perp AP, \quad \vec{v}_B \perp BP, \quad \vec{v}_C \perp CP$.

Demak, biror onda tezliklarining oniy markazi ma'lum bo'lgan tekis shakl nuqtalarining tezliklarini, oniy markaz atrofida aylanma harakatdagi nuqtalarining tezliklari kabi topish mumkin ekan.

Agar tezliklar oniy markazi tekis shakl konturidan tashqarida yotsa, tezliklar oniy markazi uchun tekis shaklga biriktirilgan tekislikning nuqtasi olinadi. (4.22) dan tekis shakl nuqtalarining ayni paytdagi tezliklari orasidagi quyidagi munosabatni aniqlash mumkin:

$$\frac{v_A}{AP} = \frac{v_B}{BP} = \frac{v_C}{CP} \quad (4.23)$$

Demak, tekis shakl nuqtalarining tezliklari, shu nuqtalardan tezliklar oniy markazigacha bo'lgan masofalarga to'g'ri proporsional bo'lar ekan (4.6-rasm).



4-24rasm

nuqtalarining ayni paytdagi tezliklari orasidagi quyidagi munosabatni aniqlash mumkin:

$$\frac{v_A}{AP} = \frac{v_B}{BP} = \frac{v_C}{CP}. \quad (4.17)$$

Demak, tekis shakl nuqtalarining tezliklari, shu nuqtalardan tezliklar oniy markazigacha bo'lgan masofalarga to'g'ri proporsional bo'lar ekan (4.6-rasm).

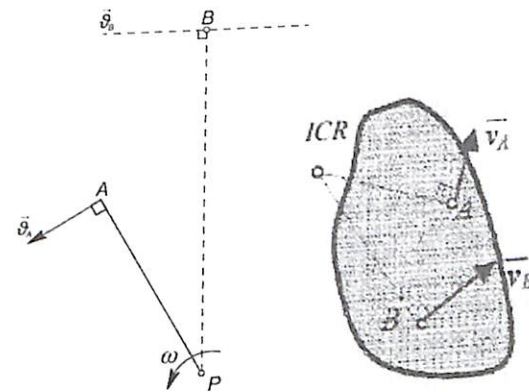
60-§. Bazi hollarda tezliklarning oniy markazini aniqlash

1) Tekis shakl biror A nuqtasining tezligi $\vec{v}_A$ va B nuqtasining tezligini yo'nalishi ma'lum bo'lsin. Bunday holda tekis shakl nuqtalari tezliklarining oniy markazi A va B nuqtalar tezliklariga o'tkazilgan perpendikulyarlarning kesishgan nuqtasida bo'ladi (4.19-rasm).

A nuqta tezligining moduli ma'lum bo'lgani uchun (4.16) dan tekis shaklning burchak tezligini aniqlaymiz:

$$\omega = \frac{v_A}{AP}. \quad (4.18)$$

AP masofa chizmadan aniqlanadi.



U paytda B nuqtaning tezligi quyidagicha teng bo'ladi:

$$v_B = \omega \cdot BP$$

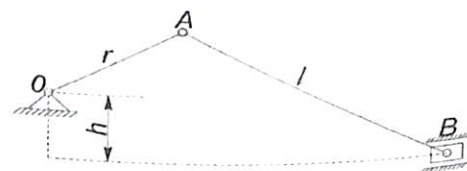
3. Tekis shaklning burchak tezligini bilgan holda, so'ralgan nuqtaning tezligi aniqlanadi.

Takrorlash uchun savollar

1. Tekis shakl nuqtalarining tezliklari orasidagi bog'lanishni ta'riflang.
2. Tekis shakl nuqtasining tezligini qutb usulida aniqlash deb qanday usulga aytiladi?
3. Tekis shakl ikki nuqtasi tezliklarining proeksiyalariga oid teoremani ta'riflang.
4. Tezliklar oniy markazi deb qanday nuqtaga aytiladi?
5. Tekis shakl nuqtalari tezliklarining oniy marazini aniqlash hollarini ko'rsating.
6. Agar tekis shakl A va B nuqtalarining tezliklari teng va parallel yo'nalgan bo'lsa, tezliklar oniy markazi qayerda joylashadi?

63-§. Tekislikka parallel harakatdagi jism nuqtalarining tezliklarini aniqlashga doir masalalar

1-masala. O val atrofi-da $\omega = 1,5 \text{ rad/s}$ burchak tezlik bilan aylanuvchi krivoshipning ikkita gorizontaal va ikkita vertikal holatida, markaziy



4.25a-rasm

bo'lmagan krivoship mexanizmi B polzuni tezligining qancha bo'lishi topilsin; $OA = 40 \text{ sm}$, $AB = 200 \text{ sm}$ (4.25a-rasm).

Yechimi:

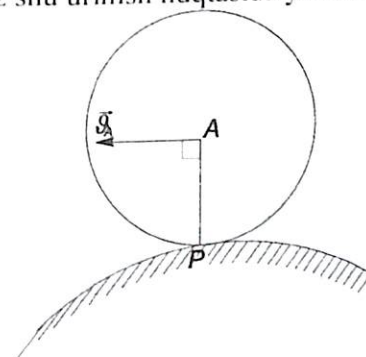
1. Krivoshipning birinchi gorizontaal holatida B polzunning tezligini qancha bo'lishini aniqlaymiz (4.25b-rasm).

Agar tekis shakl A va B nuqtalarining tezliklari o'zaro teng va parallel yo'nalgan bo'lsa, u holda tezliklar oniy markazi cheksizlikda bo'ladi ($AP = \infty$).

Tekis shakl burchak tezligi bunday holda nolga teng bo'lib, u berilgan onda ilgarilanma harakatda bo'ladi (4.21a,b-rasmlar):

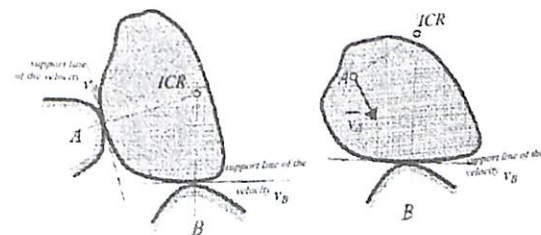
$$\omega = \frac{v_A}{AP} = \frac{v_A}{\infty} = 0.$$

3) Tekis shakl konturi biror qo'zg'almas chiziq ustida sirpanmasdan dumalasa, tekis shakl konturining qo'zg'almas chiziqqa tegib turgan nuqtasining tezligi nolga teng bo'ladi. Shuning uchun oniy markaz shu urinish nuqtasida yotadi (4.22-rasm).



4.22-rasm

4) Tekis shakl konturi A va B qo'zg'almas b) rasm yoki B qo'zg'almas c) rasm chizma ustida sirpanmasdan dumalasa (4.23-rasm).

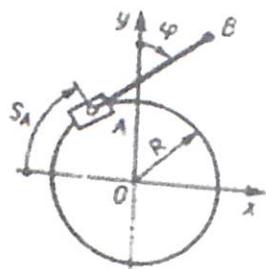


4.23 -rasm

Shakl tezliklarining oniy markazi A va B nuqtalar tezliklariga o'tkazilgan perpendikulyarlarning kesishgan nuqtasida bo'ladi.

64-§. Mustaqil o'rganish uchun talabalarga
tavsiya etiladigan muammolar

1-muammo. AB sterjenning A nuqtasi radiusi $R=1\text{m}$ bo'lgan aylana bo'ylab $S_A=1,05t$ qonun bo'yicha harakat qiladi. Bir vaqtning o'zida sterjen $\varphi=t$ qonun bilan aylanadi. Agar sterjenning uzunligi $AB=1\text{m}$ bo'lsa, $t_1=1\text{s}$ paytda uning B nuqtasi tezligining Oy o'qiga proyeksiyasini aniqlang (4.30 –rasm).



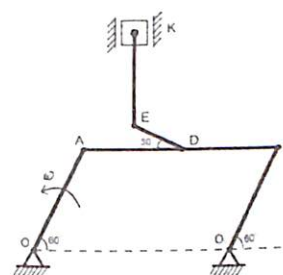
4.30 –rasm

2-muammo. Sharnirli parallelogram OABD ga CE shatun biriktirilgan bo'lib, uning uchida E polzun harakatlanadi. Agar A nuqtaning tezligi $0,4\text{m/s}$ va parallelogramning o'lchamlari $OA=BD=20\text{sm}$, $BC=BD/2$ bo'lsa, E polzunning tezligini toping (4.31 –rasm).



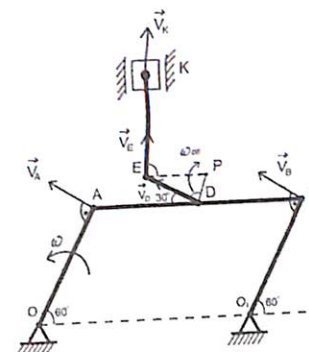
4.31 – rasm

3-muammo. Uzunligi $0,2\text{m}$ bo'lgan OA krivoship $\omega=8\text{rad/s}$ burchak tezlik bilan tekis aylanadi. AB shatunning C nuqtasiga CD shatun biriktirilgan. Berilgan holat, $\alpha=20^\circ$ uchun D polzunning tezligini aniqlang (4.32- rasm).



4.27 – rasm

$$\vec{V}_A \perp OA;$$



4.28 – rasm

$$\vec{V}_A \perp O_1B.$$

Sharnirli parallelogramda

$$OA=O_1B$$

Shuning uchun sharnirli parallelogramning harakati davomida OA va O_1B krivoshiplarning burilish burchaklari o'zaro teng bo'ladi. Keltirilgan mulohazalar sharnirli parallelogramda AB serjen tekislikka parallel harakatda bo'lishini e'tirof etadi.

Bu hol $\theta_A=\theta_B$ bo'lishini taqozo etadi.

D nuqta ham AB sterjingga taluqli. Shuning uchun

$$\vec{v}_D = \vec{v}_A = \vec{v}_B;$$

$$\vec{v}_D \parallel \vec{v}_A \parallel \vec{v}_B.$$

Agar $\angle AED=30^\circ$ ekanliginni e'tiborga olsak, D nuqtaning tezligi $\vec{v}_D$ DE shatun bo'ylab yo'nalishi ma'lum bo'ladi.

Mexanizmda K polzun vertikal yo'nalishda harakatlanadi. K va E nuqtalar KE sterjingga taaluqli. KE sterjin vertikal yo'nalishda ilgariylanma-qaytalanma harakatda bo'lishi tufayli K nuqtaning tezligi E nuqta tezligiga teng bo'ladi.

$$\vec{v}_K = \vec{v}_E.$$

KE sterjin E nuqtasining tezligini aniqlash uchun DE shatunning harakatini o'rganamiz. DE shatun shakl tekisligida harakatlanadi. DE shatun nuqtalari tezliklarining vektorlariga perpendikulyar chizmalar tushiramiz. Ularning kesishish nuqtasi tezliklarning oniy markazini ifodalaydi (4.28-rasm):

Yechish.

1. Nuqtalarning tezliklarini va zvenoning burchak tezligini aniqlash.

Mexanizmning berilgan harakatida OA krivoship A panjasi tezligining modulini hisoblaymiz:

$$v_A = \omega_{OA} \cdot OA = 2 \cdot 80 \text{ sm/s} \quad (1.1)$$

A nuqtaning tezligi $\vec{v}_A$ OA krivoshipga perpendikulyar holda ω_{OA} yo'nalishi bo'yicha yo'naladi.

B polzunning tezligi gorizontol holda B nuqtadan A nuqta tomon yo'nalgan. AB shatun nuqtalari tezliklarining oniy markazi P_{AB} A va B nuqtalardan, ularning tezliklariga o'tkazilgan perpendikulyarlarning kesishgan nuqtasida yotadi.

AB shatun burchak tezligi quyidagi formuladan topiladi

$$\omega_{AB} = \omega_{AB} = \omega_{P_{AB}},$$

bundan,

$$\omega_{AB} = \frac{v_A}{AP_{AB}} \quad (1.2)$$

ω_{AB} ning yo'nalishi $\vec{v}_A$ vektor yo'nalishi orqali aniqlanadi.

Shatun B va C nuqtalari tezliklarining modullari quyidagi formulalardan aniqlanadi:

$$v_B = \omega_{AB} \cdot BP_{AB}, v_C = \omega_{AB} \cdot CP_{AB} \quad (1.3)$$

$AP_{AB}, BP_{AB}, CP_{AB}$ masofalar chizmadagi ABP_{AB} va ACP_{AB} uchburchaklardan topiladi (4.40a-rasm).

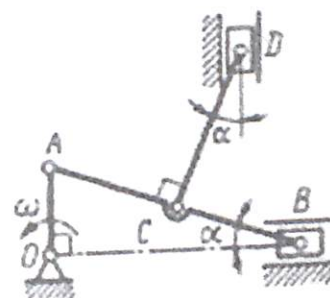
$$AP_{AB} = \frac{AB}{\cos 60^\circ} = 160 \text{ sm}, BP_{AB} = AP_{AB} \cdot \sin 60^\circ = 137,6 \text{ sm};$$

$$CP_{AB} = \sqrt{(BC)^2 + (BP_{AB})^2} = \sqrt{2500 + 18933,8} = 146,4 \text{ sm}.$$

Yuqoridagilarni e'tiborga olsak:

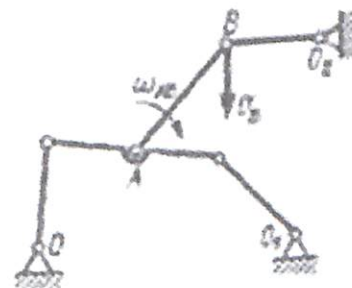
$$\omega_{AB} = 0,5 \text{ rad/s}; v_B = 68,8 \text{ sm/s}; v_C = 73,2 \text{ sm/s}$$

$\vec{v}_C$ vektor CP_{AB} kesmaga perpendikulyar holda, ω_{AB} yo'nalishi tomon yo'nalgan (4.40a-rasm).



4.32 – rasm

4-muammo. Berilgan paytda B nuqtaning tezligi 20m/s va AB zvenoning burchak tezligi 10rad/s bo'lsa, B nuqtadan AB zvenoning tezliklar oniy markazigacha bo'lgan masofani aniqlang (4.33- rasm).



4.33- rasm

5-muammo. Mexanizmning OA krivoshipi tekis aylanma harakat qilib, OB yo'nalishga $\varphi = 90^\circ$ burchak tashqil qilgan paytda B pozundan AB zveno tezliklari oniy markazigacha bo'lgan masofani toping (4.34- rasm).

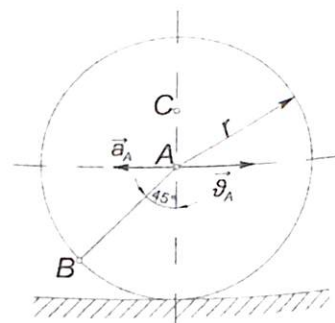
68-§. Tekislikka parallel harakatda bo'lgan jism nuqtalarining tezlanishlarini aniqlashga doir masalalar

1-masala. 4-masala. Radiusi $r_1 = 30 \text{ sm}$, bo'lgan g'ildirak yo'lning to'g'ri chiziqli gorizonttal uchastkasida sirg'anmay dumalaydi. Bu paytda g'ildirak markazining tezligi $\vartheta_A = 50 \text{ m/s}$, tezlanishi $a_A = 30 \text{ m/s}^2$, $AS = 10 \text{ sm}$.

G'ildirak B va C nuqtalarining tezligi va tezlanishi aniqlansin (4.39a-rasm).

Yechimi: 1. Nuqtalarning tezliklarini va g'ildirak burchak tezligini aniqlash.

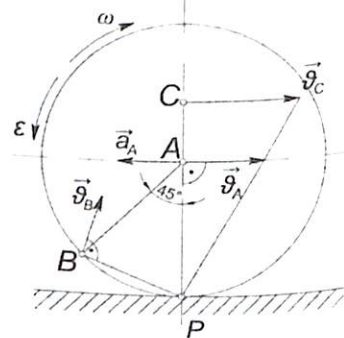
Masala shartida g'ildirak markazi A nuqtaning tezligi ϑ_A berilgan.



4.39a-rasm

G'ildirakning qo'zg'almas chiziqqa tegib turgan nuqtasining tezligi nolga teng bo'lishi sababli, g'ildirak nuqtalari tezliklarining oniy markaz shu urinish nuqtasida yotadi (4.39b-rasm).

Berilgan onda g'ildirak nuqtalari tezliklarining oniy markazi P nuqtani qutb deb olsak, g'ildirak nuqtalarining shu ondagi tezliklarini, oniy markaz atrofida aylanma harakatdagi jism nuqtalarining tezliklari kabi topish mumkin bo'ladi:



4.39b-rasm

$$\vartheta_A = \omega \cdot PA$$

$$\vartheta_B = \omega \cdot PB$$

$$\vartheta_C = \omega \cdot PC$$

66-§. Tezlanishlarning oniy markazi va undan foydalanib tekis shakl nuqtalarining tezlanishlarini aniqlash

Tekis shaklning berilgan ondagi tezlanishi nolga teng bo'lgan nuqtasi (yoki tekis shaklga bog'langan va u bilan birga harakatlanuvchi tekislikning nuqtasi) tezlanishlarning oniy markazi deyiladi.

Tezlanishlarning oniy markazini aniqlash uchun tekis shakl biror A nuqtasining tezlanishi $\vec{a}_A$ va tekis shaklning burchak tezligi ω hamda burchak tezlanishi ε berilgan bo'lishi kerak.

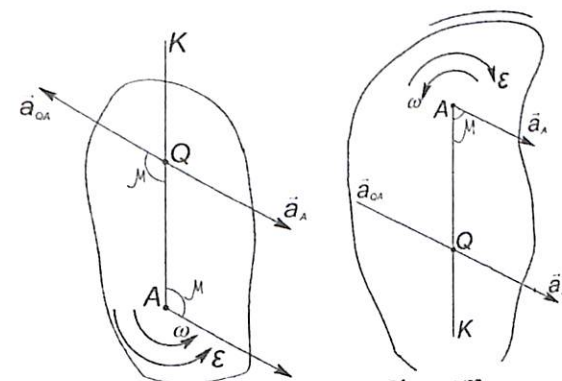
Dastlab,

$$tg \mu = \frac{|\varepsilon|}{\omega^2}$$

formula orqali μ burchak topiladi. So'ngra, tezlanuvchan aylanma harakatda $\vec{a}_A$ vektorga harakat yo'nalishida, sekinlanuvchan aylanma harakatda, harakat yo'nalishiga teskari yo'nalishda μ burchak ostida to'g'ri chiziq o'tkazib, A nuqtadan

$$AQ = \frac{a_A}{\sqrt{\varepsilon^2 + \omega^2}} \quad (4.30)$$

masofa uzoqlikda yotuvchi Q nuqtani aniqlaymiz (4.12a,b-rasmlar). Bu nuqta tekis shakl nuqtalari tezlanishlarining oniy markazini ifodalaydi.



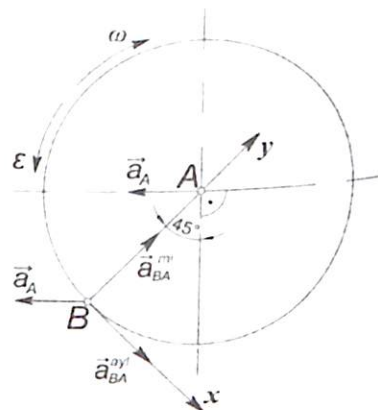
4.38a-rasm 4.38b-rasm

Tekis shakl nuqtalarining tezlanishlari haqidagi teorema asosan:

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}^{mi} + \vec{a}_{BA}^{ayl}$$

Bunda A nuqta qutb sifatida qabul qilindi.

G'ildirakning A qutb atrofida aylanma harakatida B nuqtasining markazga intilma tezlanishi:



4.39v-rasm

$$a_{BA}^{mi} = \omega_g^2 \cdot BA = 83.7 \text{ sm/s}^2.$$

$\vec{a}_{BA}^{mi}$ vektor B nuqtadan A nuqtaga qarab yo'naladi.

G'ildirakning A qutb atrofida aylanma harakatida B nuqtasining aylanma tezlanishi:

$$a_{BA}^{ayl} = \varepsilon_g \cdot BA$$

G'ildirakning burchak tezlanishini aniqlaymiz:

$$\varepsilon_g = \frac{d\omega_g}{dt} = \frac{d}{dt} \left(\frac{v_A}{PA} \right) = \frac{1}{PA} \frac{dv_A}{dt} = \frac{a_A}{PA} = 1 \text{ rad/s}^2.$$

Shuning uchun,

$$a_{BA}^{ayl} = \varepsilon_g \cdot BA = 30 \text{ sm/s}^2.$$

$\vec{a}_{BA}^{ayl}$ vektor g'ildirakning B nuqtasiga ε_g yo'nalishida o'tkazilgan urinma bo'ylab yo'naladi (4.39v-rasm).

B nuqtaning tezlanishini proeksiyalash yo'li bilan aniqlaymiz:

$$(a_B)_x = a_{BA}^{ayl} - a_A \cos 45^\circ = 30 - 30 \cdot 0.71 = 8.7 \text{ sm/s}^2;$$

$$(a_B)_y = a_{BA}^{mi} - a_A \cos 45^\circ = 83.7 - 30 \cdot 0.71 = 62.4 \text{ sm/s}^2;$$

$$a_B = \sqrt{(a_B)_x^2 + (a_B)_y^2} = \sqrt{75.69 + 3893.76} = 63 \text{ sm/s}^2.$$

G'ildirak C nuqtasining tezlanishini aniqlaymiz.

Tekis shakl nuqtalarining tezlanishlari haqidagi teorema asosan:

$$\vec{a}_C = \vec{a}_A + \vec{a}_{CA},$$

yoki

$$\vec{a}_C = \vec{a}_A + \vec{a}_{CA}^{mi} + \vec{a}_{CA}^{ayl}.$$

G'ildirakning A qutb atrofida aylanma harakatida C nuqtasining markazga intilma tezlanishi:

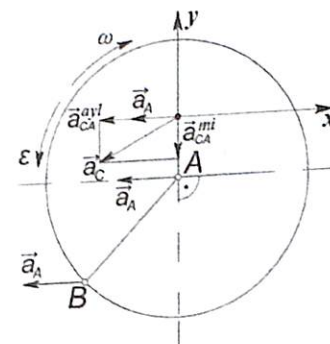
$$a_{CA}^{mi} = \omega_g^2 \cdot CA = 16.7 \text{ sm/s}^2$$

G'ildirakning A qutb atrofida aylanma harakatida S nuqtasining aylanma tezlanishi:

$$a_{CA}^{ayl} = \varepsilon_g \cdot CA = 10 \text{ sm/s}^2.$$

$\vec{a}_{CA}^{mi}$, $\vec{a}_{CA}^{ayl}$ vektorlar 4.39g-rasmda ko'rsatilgan.

C nuqtaning tezlanishini ham proeksiyalash yo'li bilan aniqlaymiz:



4.39g-rasm

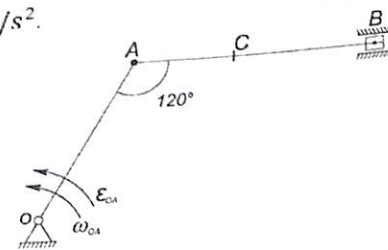
$$(a_C)_x = -a_A - a_{CA}^{ayl} = -30 - 10 = -40 \text{ sm/s}^2,$$

$$(a_C)_y = -a_{CA}^{mi} = -16.7 \text{ sm/s}^2.$$

$$a_C = \sqrt{(a_C)_x^2 + (a_C)_y^2} = 43.34 \text{ sm/s}^2.$$

2-masala. Mexanizmning berilgan holati uchun A, B, C nuqtalarining tezliklari va tezlanishlari hamda shu nuqtalar tegishli bo'lgan zvenoning burchak tezligi va burchak tezlanishi topilsin (4.40-rasm)

Masalada: $OA=40\text{sm}$
 $AB=80\text{sm}$, $AC=30\text{sm}$
 $\omega_{OA}=2\text{rad/s}$, $\varepsilon_{OA}=6\text{rad/s}^2$



4.40-rasm

asosan B nuqtadan boshlab, tanlangan masshtabda ketma – ket $\vec{a}_A^{ayl}$, $\vec{a}_A^{mi}$ va $\vec{a}_{BA}^{mi}$ vektorlarni qo'yamiz (4.40v-rasm). $\vec{a}_A^{ayl}$ vektorning oxiri orqali AB shatunga perpendikulyar holda o'tkazilgan to'g'ri chiziqni, $\vec{a}_B$ tezlanishning yo'nalish chizig'i bilan kesishguncha davom ettiramiz.

Mazkur to'g'ri chiziq uzunligi tanlangan masshtabda $\vec{a}_{BA}^{ayl}$ ning modulini ifodalaydi. $\vec{a}_B$ vektori tezlanishlar ko'p burchagining yopuvchi tomoni kabi aniqlanadi. Shuning uchun ko'p-burchakning yopuvchi tomonining uzunligi tanlangan masshtabda $\vec{a}_B$ modulini ifodalaydi (4.40v-rasm).

C nuqtaning tezlanishini aniqlaymiz:

$$\vec{a}_C = \vec{a}_A + \vec{a}_{CA} = \vec{a}_A^{ayl} + \vec{a}_A^{mi} + \vec{a}_{CA}^{ayl} + \vec{a}_{CA}^{mi} \quad (1.11)$$

AB shatunning A nuqta atrofida aylanma harakatida C nuqtaning aylanma va markazga intilma tezlanishlari quyidagilarga teng bo'ladi:

$$a_{CA}^{ayl} = \varepsilon_{AB} \cdot AC = 6,6 \text{ sm/s}^2,$$

$$a_{CA}^{mi} = \omega_{AB}^2 \cdot AC = 7,5 \text{ sm/s}^2.$$

$\vec{a}_{CA}^{mi}$ vektor C nuqtadan A nuqta tomon yo'naladi. $\vec{a}_{CA}^{ayl}$ vektor esa, $\vec{a}_{CA}^{mi}$ vektorga perpendikulyar holda, ε_{AB} burchak tezlanishining yo'nalishi tomon yo'naladi.

$\vec{a}_C$ ning modulini proektsiyalash usuli bilan aniqlaymiz.

Buning uchun (4.38) ni x va y o'qlarga proektsiyalaymiz (4.40b-rasm):

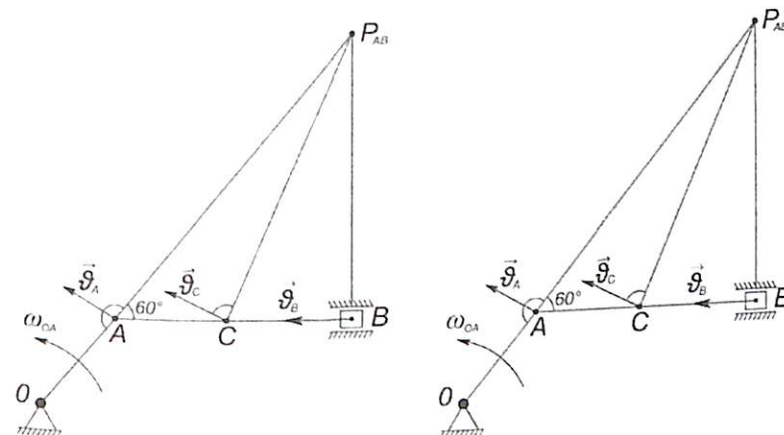
$$(a_C)_x = +a_A^{ayl} \cos 60^\circ - a_A^{mi} \cos 30^\circ + a_{CA}^{ayl} = -11 \text{ sm/s}^2;$$

$$(a_C)_y = a_A^{ayl} \cos 30^\circ + a_A^{mi} \cos 60^\circ + a_{CA}^{mi} = 293,9 \text{ sm/s}^2.$$

Natijada

$$a_C = \sqrt{(a_C)_x^2 + (a_C)_y^2} = 294,1 \text{ sm/s}^2.$$

3-masala. Radius $r=20$ sm bo'lgan tishli g'ildirak radiusi $R=40$ sm bo'lgan qo'zg'almas tishli g'ildirakning O o'qi atrofida aylanuvchi OA krivoship bilan harakatga keltiriladi; krivoship shu paytda $\omega=2 \text{ rad/s}$ burchak tezligiga ega bo'lib, $\varepsilon=2 \text{ rad/s}^2$ burchak tezlanish bilan aylanadi.



4.40a-rasm

Bajarilgan hisoblashlarning to'g'riligiga ishonch hosil qilish uchun, B nuqtaning tezligini, tekis shakl ikki nuqtasi tezliklarining bu nuqtalardan o'tuvchi o'qdagi proektsiyalarining o'zaro tengligi haqidagi teoremdan foydalanib aniqlaymiz.

Y o'qini shatun bo'ylab B nuqtadan A nuqtaga qarab yo'naltiramiz. Teoremaga asosan:

$$v_A \cos(\vec{v}_A \wedge y) = v_B \cos(\vec{v}_B \wedge y) \quad (1.4)$$

4.40a-rasmdan:

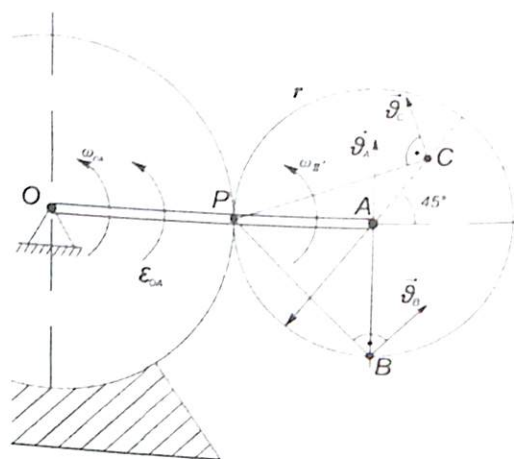
$$v_A \cos 30^\circ = v_B$$

Demak, $v_B=68.8 \text{ sm/s}$, hisoblashlar to'g'ri bajarilgan.

C nuqtaning avval topilgan tezligi v_C ham shu teorema yordamida tekshirilishi mumkin.

2. Nuqtalarning tezlanishlari va zvenoning burchak tezlanishini aniqlash.

A nuqta O nuqta atrofida aylana bo'ylab harakatlanishi tufayli uning tezlanishi aylanma va markazga intilma tezlanishlardan tashkil topadi (4.40b-rasm).



4.41a-rasm

Qo'zg'aluvchi g'ildirak B va C nuqtalari tezliklarining modullari quyidagi formulalardan aniqlanadi:

$$\vartheta_B = \vartheta_B \cdot BP \quad (1.3)$$

$$\vartheta_C = \omega_B \cdot CP \quad (1.4)$$

Chizmadan

$$BP = r\sqrt{2} = 28,2 \text{ sm}$$

$$CP = \sqrt{r^2 + (AC)^2 + 2 \cdot r \cdot AC \cos 45^\circ} = 28 \text{ sm}$$

Yuqoridagilarni e'tiborga olsak:

$$\vartheta_B = 169,2 \text{ sm/s}, \vartheta_C = 168 \text{ sm/s}$$

$\vec{\vartheta}_B$ vektor BP kesmaga, $\vec{\vartheta}_C$ vektor CP kesmaga perpendikulyar holda ω_B yo'nalishi tomon yo'naladi (4.41a-rasm).

2. Nuqtalarning tezlanishlari va qo'zg'aluvchi g'ildirak burchak tezlanishini aniqlash.

A nuqta O nuqta atrofida aylanma harakatda bo'lishi tufayli, uning tezlanishi aylanma va markazga intilma tezlanishlardan tashkil topadi (4.41b-rasm):

$$\vec{a}_A = \vec{a}_A^{ayl} + \vec{a}_A^{mi} \quad (1.5)$$

Bunda:

$$a_A^{ayl} = \varepsilon_{OA} \cdot OA = 120 \text{ sm/s}^2, a_A^{mi} = \omega_{OA}^2 \cdot OA = 240 \text{ sm/s}^2$$

A nuqta tezlanishining moduli quyidagiga teng bo'ladi:

AB shatunning A qutb atrofidagi aylanma harakatida B nuqtaning markazga intilma tezlanishi quyidagiga teng bo'ladi:

$$a_{BA}^{mi} = \omega_{AB}^2 \cdot AB = 20 \text{ m/s}^2 \quad (1.7)$$

$\vec{a}_{BA}^{mi}$ vektor B nuqtadan A nuqta tomon yo'naladi.

B nuqtaning tezlanishi $\vec{a}_B$ va B nuqtaning A qutb atrofidagi aylanma harakatidagi aylanma tezlanishi $\vec{a}_{BA}^{ayl}$ larning faqat yo'nalish chiziqlari ma'lum: $\vec{a}_B$ gorizontal, $\vec{a}_{BA}^{ayl}$ esa AB shatunga perpendikulyar yo'nalgan. Ularning ko'rsatilgan yo'nalish chiziqlari bo'ylab qaysi tomonlarga yo'nalishlarini ixtiyoriy tanlab olamiz (4.40b-rasm).

Bu tezlanishlarning modullarini (1.6) vektor tenglikning koordinata o'qlariga proektsiyalari tenglamalaridan aniqlaymiz. Javobning ishorasiga qarab, vektorning haqiqiy yo'nalishini, hisoblashda qabul qilinganiga mos kelishi yoki kelmasligi aniqlanadi. x va y o'qlarining yo'nalishlarini 4.40b-rasmda ko'rsatilgandek o'tkazib, quyidagilarni hosil qilamiz:

$$0 = -a_A^{mi} \cos 30^\circ + a_A^{ayl} \cos 60^\circ + a_{BA}^{ayl}; \quad (1.8)$$

$$a_B = a_{BA}^{mi} + a_A^{mi} \cos 60^\circ + a_A^{ayl} \cos 30^\circ. \quad (1.9)$$

(1.8) dan:

$$a_{BA}^{ayl} = a_A^{mi} \cos 30^\circ - a_A^{ayl} \cos 60^\circ = 17,6 \text{ sm/s}^2$$

(1.9) dan

$$a_B = 306,4 \text{ sm/s}^2$$

Javoblarning ishoralari musbat. Shuning uchun $\vec{a}_{BA}^{ayl}$ va $\vec{a}_B$ vektorlarning haqiqiy yo'nalishlari, hisoblashda qabul qilingan yo'nalishlarga mos kelar ekan.

AB shatunning burchak tezlanishini quyidagi formuladan topamiz:

$$a_{BA}^{ayl} = \varepsilon_{AB} \cdot AB$$

Bundan,

$$\varepsilon_{AB} = \frac{a_{BA}^{ayl}}{AB} = 0,22 \text{ rad/s}^2 \quad (1.10)$$

$\vec{a}_B$ va $\vec{a}_{BA}^{ayl}$ larni grafik usulda B nuqtada tezlanishlar ko'p burchagini chizish orqali ham aniqlash mumkin. Buning uchun (1.6) ga

Burchak tezlanish yo'nalishi burchak tezlik yo'nalishi bilan ustma-ust tushadi, chunki ularning ishoralari bir xil.

B nuqtaning tezlanishini aniqlaymiz.

Birinchi yo'l.

Tekis shakl nuqtalarini tezlanishlari haqidagi teorema ko'ra, qutb sifatida AB sterjen nuqtalari tezlanishlarning oniy markazi θ nuqta olinsa

$$a_B = QB \sqrt{\varepsilon_1^2 + \omega_1^4} = 0,5 \cdot 3 = 1,5 \frac{m}{s^2}.$$

$\vec{a}_B$ ning yo'nalishi φ_1 burchak orqali aniqlanadi:

$$\tan \varphi_1 = \frac{\varepsilon_1}{\omega_1^2} = 0,337; \quad \varphi_1 = 18,66^\circ$$

Ikkinchi yo'l:

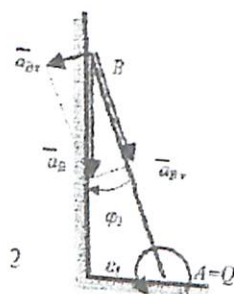
$$\vec{a}_B = \vec{a}_B^r + \vec{a}_B^n.$$

Bunda:

$$a_B^r = AB \cdot \varepsilon_1 = 0,5 \cdot 0,96 = 0,48 \text{ m/s}^2$$

$$a_B^n = AB \cdot \omega_1^2 = 0,5 \cdot (1,688)^2 = 1,424 \text{ m/s}^2.$$

Mazkur tezlanishlarning yo'nalishlari 4.46- rasmda ko'rsatilgan.



4.46- rasm

Natijada B nuqta tezlanishi quyidagiga teng bo'ladi:

$$a_B = \sqrt{(a_B^r)^2 + (a_B^n)^2} = \sqrt{(0,48)^2 + (1,424)^2} = 1,5 \text{ m/s}^2.$$

$\vec{a}_B$ ning yo'nalishini aniqlaymiz:

$$\operatorname{tg} \varphi_1 = \frac{a_B^r}{a_B^n} = \frac{0,48}{1,424} = 0,337;$$

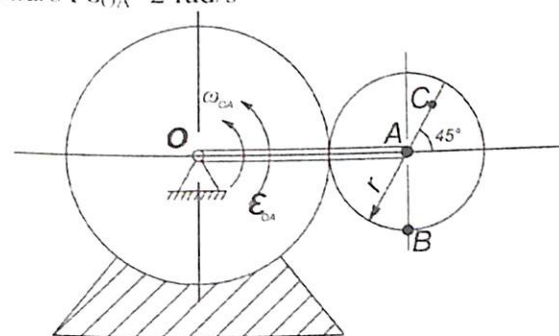
$$\varphi_1 = 18,66^\circ$$

Krivoship A nuqtasining va qo'zg'aluvchi g'ildirakning B va C nuqtalarining tezliklari va tezlanishlari hamda qo'zg'aluvchi g'ildirakning burchak tezligi va burchak tezlanishi aniqlansin (4.41-rasm).

Masalada:

$R=40$ sm, $r=20$ sm, $AC=10$ sm.,

$\omega_{OA}=2$ rad/s, $\varepsilon_{OA}=2$ rad/s



4.41-rasm

Yechish.

1. Nuqtalarning tezliklarini va ko'zg'aluvchi g'ildirak burchak tezligini aniqlash.

Mexanizmning berilgan holatida OA krivoship A panjasi tezligining modulini aniqlaymiz:

$$v_A = \omega_{OA} \cdot OA = 2 \cdot 0,1 = 0,2 \text{ m/s. (1.1)}$$

A nuqtaning tezligi $\vec{v}_A$ OA krivoshipga perpendikulyar holda ω_{OA} yo'nalishi tomon yo'naldi.

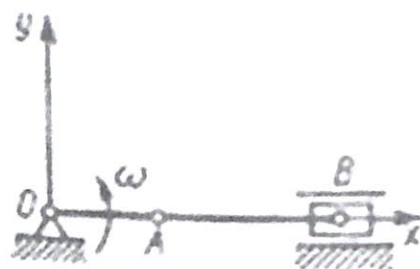
Qo'zg'aluvchi g'ildirak nuqtalari tezliklarining oniy markazi P nuqta bo'lganligi uchun (4.41a-rasmga qarang).

$$v_A = \omega_g \cdot r.$$

Bundan,

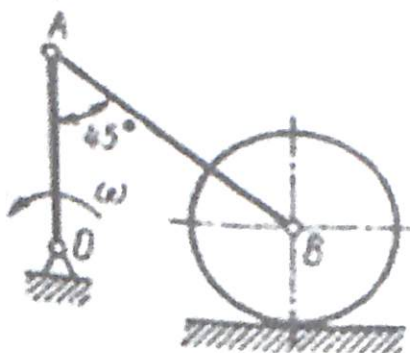
$$\omega_g = \frac{v_A}{r} = 6 \text{ rad/s} \quad (1.2)$$

Qo'zg'aluvchi g'ildirak burchak tezligi ω_g ning yo'nalishi $\vec{v}_A$ vektor yo'nalishi orqali aniqlanadi (4.41a-rasm).



4.48– rasm

4-muammo. Krivoship –shatunli mexanizmning o'lchamlari $OA=0,3\text{m}$ va $AB=0,45\text{m}$ bo'lib, OA krivoship o'zgarmas burchak tezlik $\omega=10\text{rad/s}$ bilan aylanadi. AB shatunning burchak tezlanishini hisoblang (4.49- rasm).



4.49 –rasm

5-muammo. Sharnirli parallelogrammning OA krivoshipi o'zgarmas burchak tezlik $\omega=0,4\text{rad/s}$ bilan aylanadi. Agar mexanizmning o'lchamlari $OA=20\text{sm}$, $CD=30\text{sm}$ bo'lsa, ko'rsatilgan holat uchun CD shatunning burchak tezlanishini toping (4.50-rasm).

29		$OA=40\text{ sm}$ $r=20\text{ sm}$ $AC=10\text{ sm}$ $\omega_{OA}=3\text{ rad/s}$ $\epsilon_{OA}=2\text{ rad/s}^2$
30		$OA=40\text{ sm}$ $r=15\text{ sm}$ $AC=8\text{ sm}$ $\omega_{OA}=1\text{ rad/s}$ $\epsilon_{OA}=1\text{ rad/s}^2$

Eslatma. ω_{OA} , ϵ_{OA} – OA krivoship mexanizmning berilgan vaziyatidagi burchak tezligi va burchak tezlanishi; ω_1 – g' ildirakning burchak tezligi (doimiy); v_A –va a_A – A nuqtaning tezligi va tezlanishi. G' ildiraklar sirpanishsiz aylanadi.

$\vec{a}_{BA}^{ayl}$ vektor $\vec{a}_{BA}^{mi}$ vektorga perpendikulyar holda ε_g yo'nalishi tomon yo'naladi, uning moduli quyidagi formuladan topiladi:

$$a_{BA}^{ayl} = \varepsilon_g \cdot AB = 360 \text{ sm/s}^2. \quad (1.9)$$

$\vec{a}_B$ vektor modulini aniqlash uchun $\vec{a}_A^{ayl}$, $\vec{a}_A^{mi}$, $\vec{a}_{BA}^{ayl}$, $\vec{a}_{BA}^{mi}$ vektorlarni B nuqtaga qo'yamiz va proektsiyalash usulidan foydalanamiz. x o'qini B nuqtadan gorizontal, y o'qini esa vertikal yo'naltiramiz. 1.6 ni x va y o'qlariga proektsiyalasak:

$$(a_B)_x = -a_A^{mi} + a_{BA}^{ayl} = 120 \text{ sm/s}^2;$$

$$(a_B)_y = a_A^{ayl} + a_{BA}^{mi} = 840 \text{ sm/s}^2;$$

Natijada

$$a_B = \sqrt{(a_B)_x^2 + (a_B)_y^2} = 848,5 \text{ sm/s}^2.$$

C nuqtaning tezlanishi B nuqtaning tezlanishi kabi topiladi:

$$\vec{a}_C = \vec{a}_A + \vec{a}_{CA}. \quad (1.10)$$

yoki

$$\vec{a}_C = \vec{a}_A^{ayl} + \vec{a}_A^{mi} + \vec{a}_{CA}^{ayl} + \vec{a}_{CA}^{mi}. \quad (1.11)$$

Bunda,

$$a_{CA}^{ayl} = \varepsilon_g \cdot AC = 60 \text{ sm/s}^2,$$

$$a_{CA}^{mi} = \omega_g^2 \cdot AC = 360 \text{ sm/s}^2.$$

$\vec{a}_C$ vektor modulini ham proektsiyalash usulidan foydalanib aniqlaymiz. Buning uchun C nuqtaga $\vec{a}_A^{ayl}$, $\vec{a}_A^{mi}$, $\vec{a}_{CA}^{ayl}$, $\vec{a}_{CA}^{mi}$ vektorlarni qo'yamiz. x o'qini C nuqtadan gorizontal, y o'qini esa vertikal yo'naltiramiz.

(1.11) ni x va y o'qlariga proektsiyalasak:

$$(a_C)_x^2 = -a_A^{mi} - a_{CA}^{mi} \cos 45^\circ - a_{CA}^{ayl} \cos 45^\circ,$$

$$(a_C)_y^2 = a_A^{ayl} - a_{CA}^{mi} \cos 45^\circ + a_{CA}^{ayl} \cos 45^\circ.$$

Natijada

$$a_C = \sqrt{(a_C)_x^2 + (a_C)_y^2} = 546,2 \text{ sm/s}^2. \quad (1.12)$$

4-masala. Uzunligi 60 sm bo'lgan OA krivoship chizma tekisligiga perpendikulyar bo'lgan qo'zg'almas Ox o'q atrofida

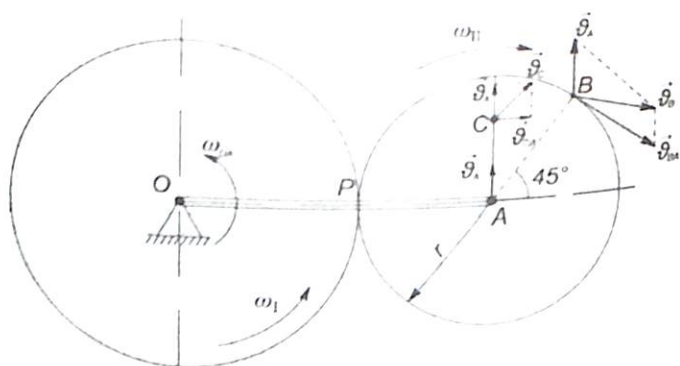
K. KENJAYEV

NAZARIY MEXANIKA MISOL VA MASALALARDA

II-qism

KINEMATIKA

Muharrir:	— I.T. Nishanbayeva
Musahhih:	— M.X. Mustafayeva
Rassom:	— J.R. Azimov
Kompyuterda sahifalovchi:	— J.R. Azimov



4.42a-rasm

A nuqtaning tezligi $\vec{v}_A$ OA krivoshipga perpendikulyar holda, ω_{OA} yo'nalishi tomon yo'naladi.

II-g'ildirak B va C nuqtalarining tezliklarini aniqlash uchun, tekis shakl nuqtalarining tezliklari haqidagi teoremdan foydalanamiz. Buning uchun, qutb sifatida tezligi $\vec{v}_A$ ma'lum bo'lgan A nuqtani tanlaymiz. Teorema asosan, B nuqtaning tezligi quyidagicha aniqlanadi:

$$\vec{v}_B = \vec{v}_A + \vec{v}_{BA}.$$

Bunda:

$$v_{BA} = \omega_{II} \cdot BA = 160 \text{ sm/s},$$

Shuning uchun,

$$v_B = \sqrt{v_A^2 + v_{BA}^2 + 2v_A v_{BA} \cos 45^\circ} = 288 \text{ sm/s}.$$

II-g'ildirak C nuqtasining tezligi ham B nuqtaning tezligi kabi topiladi:

$$\vec{v}_C = \vec{v}_A + \vec{v}_{CA}, \quad v_{CA} = \omega_{II} \cdot CA = 80 \text{ sm/s}.$$

$$v_C = \sqrt{v_A^2 + v_{CA}^2} = 252,8 \text{ sm/s}$$

$\vec{v}_B$ va $\vec{v}_C$ vektorlar $\vec{v}_A$ va $\vec{v}_{BA}$, hamda $\vec{v}_A$ va $\vec{v}_{CA}$ vektorlardan qurilgan parallelogrammlar diagonallari orqali ifodalanadi (4.42a-rasm).

B) II-g'ildirak A, B, C nuqtalarining tezlanishlarini aniqlash.

$$a_A = \sqrt{(a_A^{ayl})^2 + (a_A^{mi})^2} = 268,3 \text{ sm/s}^2.$$

$\vec{a}_A^{mi}$ vektor A nuqtadan O nuqta tomon yo'naladi, $\vec{a}_A^{ayl}$ vektor $\vec{a}_A^{mi}$ vektorga perpendikulyar holda, ϵ_{OA} yo'nalishi tomon yo'naladi (4.41b-rasm).

Qo'zg'aluvchi g'ildirak B va C nuqtalarining tezlanishlarini, tekis shakl nuqtalarining tezlanishlari haqidagi teoremdan foydalanib aniqlaymiz:

$$\vec{a}_B = \vec{a}_A + \vec{a}_{BA}$$

yoki

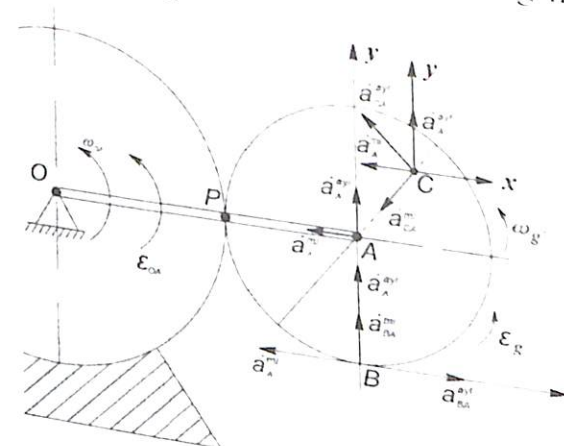
$$\vec{a}_B = \vec{a}_A^{ayl} + \vec{a}_A^{mi} + \vec{a}_{BA}^{ayl} + \vec{a}_{BA}^{mi} \quad (1.6)$$

Bunda tezlanishi $\vec{a}_A$ ma'lum bo'lgan A nuqta *qutb* deb olinadi.

$\vec{a}_{BA}^{mi}$ vektor B nuqtadan A nuqta tomon yo'naladi, uning moduli quyidagi formuladan topiladi:

$$a_{BA}^{mi} = \omega_g^2 \cdot BA = 720 \text{ sm/s}^2. \quad (1.7)$$

$\vec{a}_{BA}^{ayl}$ vektorni aniqlash uchun qo'zg'aluvchi g'ildirak burchak tezlanishini aniqlash lozim:



4.41.b-rasm

$$\epsilon_g = \frac{d\omega_g}{dt} = \frac{1}{r} \frac{dv_A}{dt} = \frac{a_A^{ayl}}{r} = 6 \text{ rad/s}^2.$$

ϵ_g ishorasi ω_g ishorasi bilan bir xil bo'lganligi uchun, ular bir xil yo'nalishga ega bo'ladilar. (1.8)

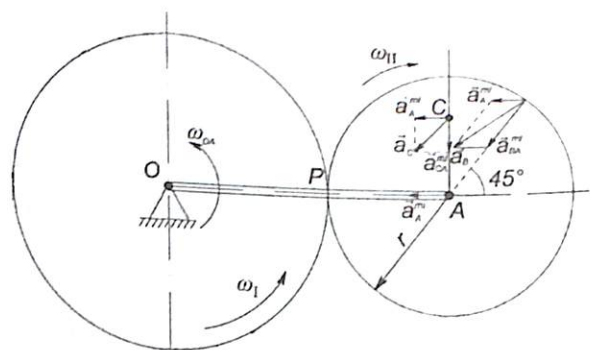
$$a_{CA}^{mi} = \omega_{II}^2 \cdot CA = 640 \text{ sm/s}^2.$$

$\vec{a}_{CA}^{mi}$ vektor C nuqtadan A nuqta tomon yo'naladi.

$\vec{a}_C$ vektor $\vec{a}_A^{mi}$ va $\vec{a}_{CA}^{mi}$ vektorlardan qurilgan parallelogramm diagonali orqali ifodalanadi (4.42b-rasm).

Uning moduli:

$$a_C = \sqrt{(a_A^{mi})^2 + (a_{CA}^{mi})^2} = 1153 \text{ sm/s}^2 \quad (1.11)$$

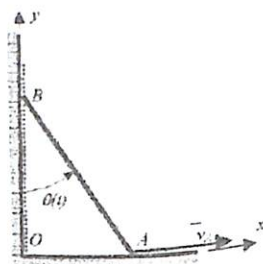


4.42b-rasm

5-masala. Uzunligi $l_{AB}=50\text{sm}$ bo'lgan sterjen A nuqtada gorizontaal polga, B nuqtada esa vertikal devorga tayanadi. Sterjenning shakl tekisligidagi harakatida A nuqta gorizontaal polga $\vartheta_A = \vartheta_0 = 0.8\text{m/s}$ tezlik harakatlanadi, ϑ_0 – A nuqtaning boshlang'ich tezligi. sterjenning B nuqtasi vertikal devor bo'ylab harakatlanadi (4.43-rasm).

a) Sterjen nuqtalari tezliklarining $t_1=0.2\text{s}$ vaqt onidagi taqsimoti aniqlansin.

b) Sterjen A va B nuqtalarining vaqt onidagi tezlanishi topilsin.

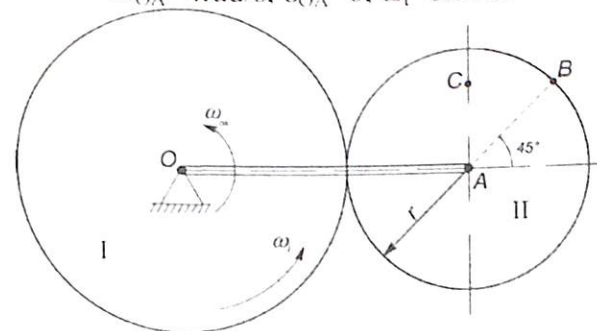


4.43 - rasm

$\omega_{OA}=4\text{rad/s}$ burchak tezlik bilan aylanadi. Xuddi shu Ox o'qqa I – g'ildirak o'tqazilgan, krivoship A nuqtasiga esa radiusi $r=20\text{ sm}$ bo'lgan, I g'ildirakka tashqari tomonidan ilashgan, II – g'ildirak o'rnatilgan. I-g'ildirakning burchak tezligi $\omega_I=15\text{ rad/s}$. $AC=10\text{sm}$ (4.42-rasm). Ikkinchi g'ildirak A,B,C – nuqtalarining tezliklari va tezlanishlari aniqlansin, hamda ikkinchi g'ildirakning burchak tezligi topilsin.

Masalada: $OA=60\text{sm}$, $r=20\text{sm}$, $AC=10\text{sm}$.

$\omega_{OA}=4\text{rad/s}$, $\varepsilon_{OA}=0$, $\omega_I=6\text{rad/s}$.



4.42-rasm

Yechimi.

A) II-g'ildirak burchak tezligi va A,B,C – nuqtalarining tezliklarini aniqlash.

I- va II – g'ildiraklar tashqari tomonidan ilashganligi uchun, II – g'ildirakning burchak tezligi ω_{II} ni Villis formulasidan aniqlaymiz (4.42a-rasm):

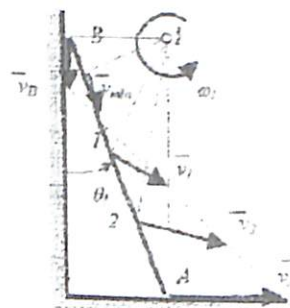
$$\frac{\omega_I - \omega_{CA}}{\omega_{II} - \omega_{OA}} = -\frac{r}{OA - r} \quad (1.1)$$

bundan $\omega_{II} = 8\text{ rad/s}$.

ω_I va ω_{II} larning yo'nalishlari chizmada ko'rsatilgan (4.19a-rasm).

Mexanizmning berilgan holatida krivoship A panjasi tezligining modulini aniqlaymiz:

$$\vartheta_A = \omega_{OA} \cdot OA = 4 \cdot 60 = 240 \text{ sm/s}. \quad (1.2)$$



4.44 – rasm

Rasmdan:

$$YA = AB \cos \theta.$$

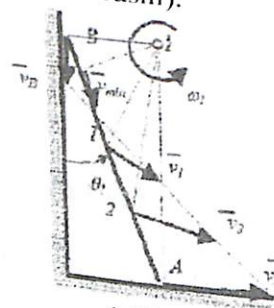
AB stejen oniy aylanish burchak tezligi quyidagi formula yordamida topiladi

$$\omega_1 = \frac{v_A}{YA} = \frac{0,8}{0,5 \cdot 0,947} = 1,688 \frac{\text{rad}}{\text{s}}.$$

U paytda B nuqtaning tezligi quyidagi formula orqali aniqlanadi:

$$V_B = \omega_1 \cdot YB = \omega_1 \cdot AB \sin \theta = 1,688 \cdot 0,5 \cdot 0,32 = 0,27 \text{ m/s}$$

B nuqtaning tezligi $\vec{v}_B$ YB kesmaga perpendikulyar holda ω_1 tomon yo'naladi. AB stejen nuqtalari tezliklarining $t_1 = 0,2 \text{ s}$ vaqt onidagi taqsimotini aniqlash uchun $\vec{v}_B$ va $\vec{v}_A$ vektorlar uchlarini birlashtiramiz. AB stejen nuqtalari tezliklari vektorlari mazkur nuqtalardan tezliklar oniy markaziga o'tkazilgan kesmalarga perpendikulyar holda yo'naladi, ularning uchlari $\vec{v}_A$ va $\vec{v}_B$ vektorlar uchlarini birlashtiruvchi chiziqda yotadi (4.45- rasm).



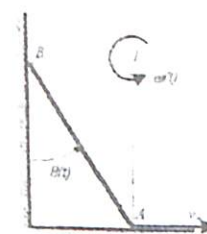
4.45- rasm

b) stejen A nuqtasining $t_1 = 0,2 \text{ s}$ vaqt onidagi tezlanishini aniqlashda, masal shartiga ko'ra, A nuqta o'zgarmas $v_A = 0,8 \text{ m/s}$ tezlik bilan harakatlanishini e'tiborga olish lozim. Binobarin,

$$a_A = \frac{dv_A}{dt} = 0.$$

A nuqta tezlanishi qaralayotgan vaqt onida nolga teng ekanligi sababli mazkur nuqta AB stejen nuqtalari tezlanishlarining oniy markazini ifodalaydi. Shuning stejen boshqa nuqtalarini tezlanishlari mazkur nuqtalarning tezlanishlar oniy markazi atrofidagi aylanma harakat tezlanishi kabi aniqlanadi (4.46- rasm).

Tezlanishlar oniy markazi Q orqali belgilanadi $A = Q$.



4.46- rasm

Ma'lumki

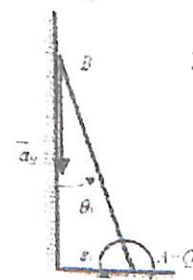
$$v_A = \omega(t) \cdot YA$$

Shuning uchun

$$\omega(t) = \frac{v_A}{YA} = \frac{v_A}{l \cos \theta(t)}$$

Sterjin burchak tezlanishi uning burchak tezligidan vaqt bo'yicha hisoblangan birinchi tartibli hosilaga teng (4.47- rasm).

$$\epsilon_1 = \epsilon(t_1) = \frac{d\omega(t)}{dt} = \frac{0,8 \cdot \theta(t_1) \cdot \sin \theta_1}{0,5 \cos^2 \theta_1} = \frac{0,8 \cdot \omega_1 \cdot \sin \theta_1}{0,5 \cos^2 \theta_1} = 0,96 \text{ rad/s}^2$$



4.47- rasm

Mexanizmlarning sxemalari va hisoblash uchun kerakli ma'lumotlar quyidagi jadvalda keltirilgan.

Variant raqamlari	Mexanizmlarning sxemalari	Hisoblash uchun kerakli ma'lumotlar
1.		$OA=60 \text{ sm}$ $r=20 \text{ sm}$ $\omega_{OA}=2 \text{ rad/s}$ $\epsilon_{OA}=4 \text{ rad/s}^2$
2.		$r=45 \text{ sm}$ $AS=15 \text{ sm}$ $\vec{\omega}_A=100 \text{ sm/s}$ $a_A=50 \text{ sm/s}^2$
3.		$OA=20 \text{ sm}$ $AB=20 \text{ sm}$ $AC=10 \text{ sm}$ $\omega_{OA}=2 \text{ rad/s}$ $\epsilon_{OA}=6 \text{ rad/s}^2$
4.		$OA=30 \text{ sm}$ $r=20 \text{ sm}$ $AC=15 \text{ sm}$ $\omega_{OA}=1 \text{ rad/s}$ $\omega_I=2.5 \text{ rad/s}$ $\epsilon_{OA}=0 \text{ rad/s}^2$

Ko'rinib turibdiki, har ikkala yo'lda ham B nuqtaning tezlanishi bir xil miqdor va yo'nalishga ega bo'ldi.

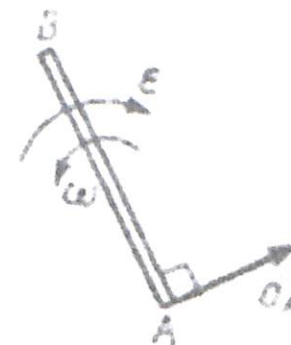
69-§. Mustaqil o'rganish uchun talabalarga tavsiya etiladigan muammolar

1-muammo. Uzunligi $AB=1.5 \text{ m}$ bo'lgan sterjen shakl tekisligida harakatlanadi.

Sterjen A nuqtasining tezligi gorizontol holda yo'nalgan bo'lib $\vec{v}_A=0.5 \text{ m/s}$ ga teng ($\vec{v}_A = \text{const}$).

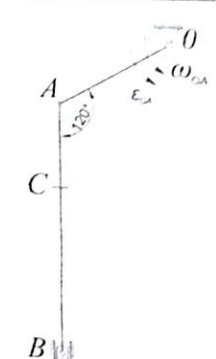
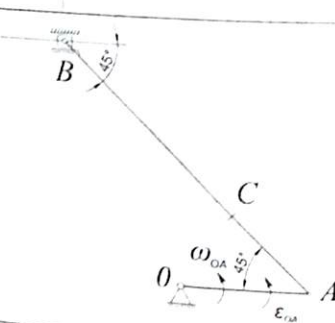
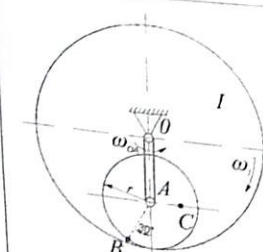
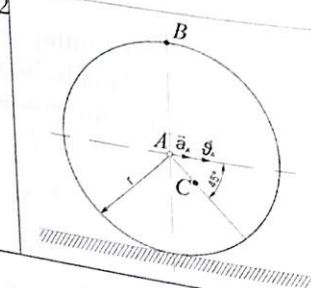
t_1 = vaqt oni uchun sterjen B nuqtasining tezligi va tezlanishi aniqlansin $AC=1 \text{ m}$.

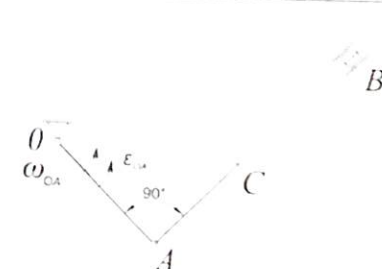
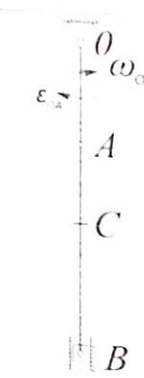
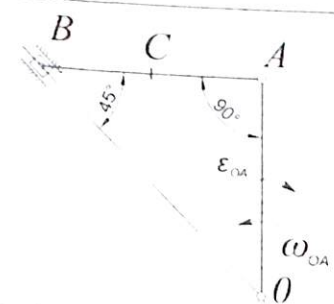
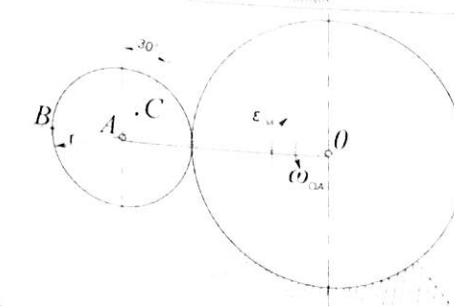
2-muammo. Uzunligi $AB=1 \text{ m}$ bo'lgan sterjen tekislik bo'ylab harakatlanadi. Agar uning burchak tezligi $\omega=2 \text{ rad/s}$, burchak tezlanishi $\epsilon=2 \text{ rad/s}^2$ va A nuqtasining tezlanishi $a_A=1 \text{ m/s}^2$ bo'lsa, B nuqtasining tezlanishini hisoblag (4.47 -rasm).



4.47- rasm

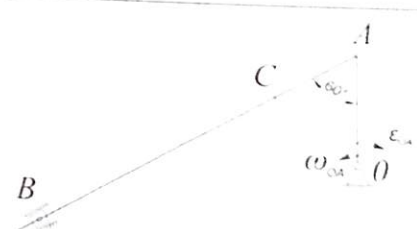
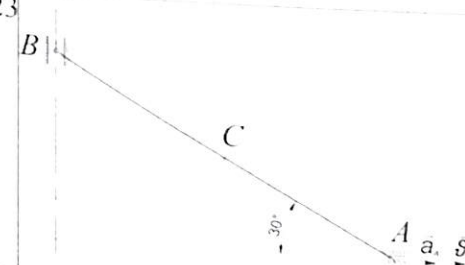
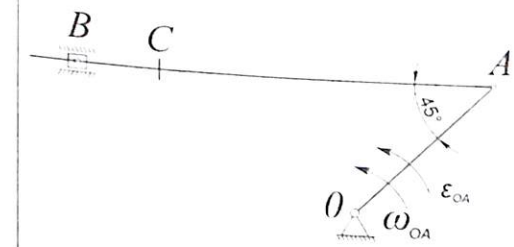
3-muammo. Krivoship -polzunli mexanizmning OA krivoshipi o'zgarmas $\omega=10 \text{ rad/s}$ burchak tezlik bilan aylanadi. Shaklda ko'rsatilgan holat uchun AB shatunning burchak tezlanishini toping (4.48-rasm).

9.		$OA=30 \text{ sm}$ $AB=40 \text{ sm}$ $AC=15 \text{ sm}$ $\omega_{OA}=3 \text{ rad/s}$ $\epsilon_{OA}=3 \text{ rad/s}^2$
10.		$OA=30 \text{ sm}$ $AB=80 \text{ sm}$ $AC=25 \text{ sm}$ $\omega_{OA}=1 \text{ rad/s}$ $\epsilon_{OA}=2 \text{ rad/s}^2$
11.		$OA=20 \text{ sm}$ $r=15 \text{ sm}$ $AC=10 \text{ sm}$ $\omega_{OA}=7.0 \text{ rad/s}$ $\omega_I=1.2 \text{ rad/s}$ $\epsilon_{OA}=0$
12.		$r=20 \text{ sm}$ $AC=10 \text{ sm}$ $\ddot{\theta}_A=60 \text{ sm/s}^2$ $a_A=30 \text{ sm/s}^2$

13.		$OA=30 \text{ sm}$ $AB=60 \text{ sm}$ $AC=25 \text{ sm}$ $\omega_{OA}=1 \text{ rad/s}$ $\epsilon_{OA}=1 \text{ rad/s}^2$
14.		$OA=20 \text{ sm}$ $AB=40 \text{ sm}$ $AC=15 \text{ sm}$ $\omega_{OA}=4 \text{ rad/s}$ $\epsilon_{OA}=6 \text{ rad/s}^2$
15.		$OA=40 \text{ sm}$ $AC=20 \text{ sm}$ $\omega_{OA}=4 \text{ rad/s}$ $\epsilon_{OA}=8 \text{ rad/s}^2$
16.		$OA=50 \text{ sm}$ $r=20 \text{ sm}$ $AC=10 \text{ sm}$ $\omega_{OA}=1 \text{ rad/s}$ $\epsilon_{OA}=8 \text{ rad/s}^2$

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21		$OA=25 \text{ sm}$ $AB=80 \text{ sm}$ $AC=20 \text{ sm}$ $\omega_{OA}=2 \text{ rad/s}$ $\epsilon_{OA}=2 \text{ rad/s}^2$
22		$AB=50 \text{ sm}$ $AC=30 \text{ sm}$ $\vec{v}_A=20 \text{ sm/s}$ $\vec{a}_A=10 \text{ sm/s}^2$
23		$AB=30 \text{ sm}$ $AC=15 \text{ sm}$ $\vec{v}_A=40 \text{ sm/s}$ $\vec{a}_A=20 \text{ sm/s}^2$
24		$OA=35 \text{ sm}$ $AB=75 \text{ m}$ $AC=60 \text{ sm}$ $\omega_{OA}=4 \text{ rad/s}$ $\epsilon_{OA}=10 \text{ rad/s}^2$

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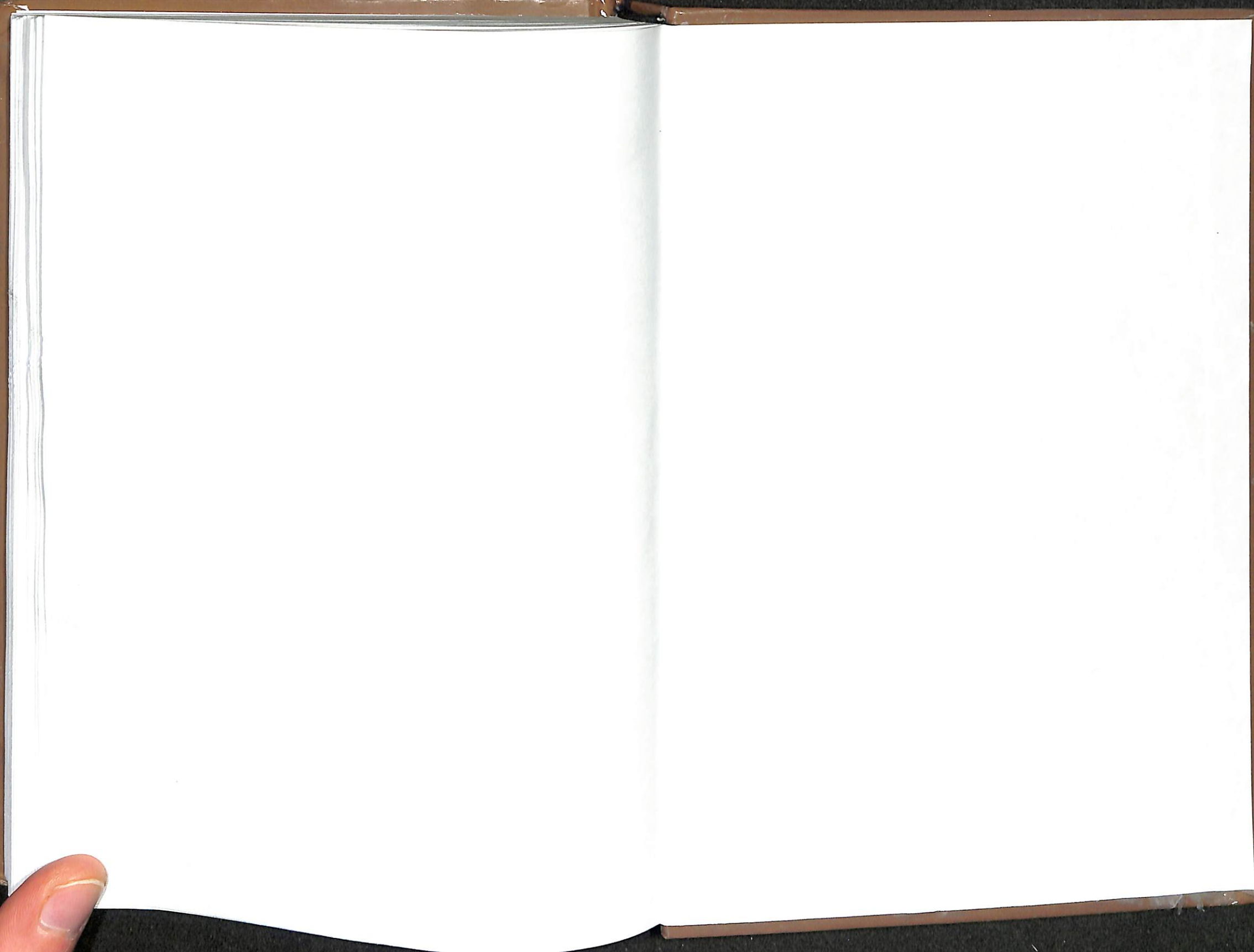
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