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**MATERIALLAR
QARSHILIGIDAN
MASALALAR
YECHISH**

O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA MAXSUS
TA'LIM VAZIRLIGI

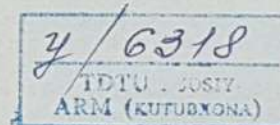
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*O'zbekiston Respublikasi Oliy va o'rta
maxsus ta'lim vazirligi tomonidan
oliy texnika ta'lim muassasalari talabalari uchun
o'quv qo'llanma sifatida tavsiya etilgan*

Kitobni quyida ko'rsatilgan
muddatdan kechiktirmay qaytaring



«O'zbekiston milliy ensiklopediyasi»
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X 30

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O'quv qo'llanmada "Materiallar qarshiligi" fanining asosiy mavzulariga oid masalalar yechimi hamda talabalar bajarishi lozim bo'lgan hisob-grafika ishlari variantlari keltirilgan. Bundan tashqari, oliy o'quv yurtlari talabalari o'rtasida "Materiallar qarshiligi" fanidan o'tkazib kelingan Respublika olimpiadalarida taqdim etilgan bir turkum masalalar yechimi ham keltirilgan.

O'quv qo'llanma oliy texnika ta'lim muassasalarining talabalari uchun mo'ljallangan. Undan muhandis-texnik xodimlar va o'rta maxsus kasb-hunar ta'lim tizimining o'qituvchilari ham foydalanishlari mumkin.

В учебном пособии приведены решения задач по основным разделам курса «Сопротивление материалов», а также приведены варианты расчетно-графических работ, выполняемых студентами. Кроме этого приводятся решения типовых задач, рекомендуемых для студентов высших учебных заведений, участвующих на Республиканских олимпиадах по дисциплине «Сопротивление материалов».

Предназначено для студентов высших технических учебных заведений и может быть использовано инженерно-техническими работниками и преподавателями средне-специальных учебных заведений.

In the scholar text book the decisions Of the tasks of the basic part - „Resistance of materials“, and also variants of accounting graphic work chart work done by students are given. Besides decisions for typical assignments for students of higher educational institutions who are take part in Republic olimpiad contests on the theme – “Resistance of materials”.

The book is intended for students of higher technical institutions, also used by engineers, teachers of secondary special educational establishment.

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SO'ZBOSHI

Kadrlar tayyorlash Milliy dasturining ikkinchi – sifat bosqichi talablari doirasida Oliy va o'rta maxsus ta'lim muassasalarida amaliy mashg'ulotlarga ajratilgan o'quv yuklamalari hajmini ko'paytirishga alohida e'tibor qaratilgan.

Hozirgi paytda "Materiallar qarshiligi" fanidan masalalar to'plami hamda misol va masalalar yechish bo'yicha Oliy texnika o'quv yurtlari talabalariga mo'ljallangan o'zbek tilidagi uslubiy ko'rsatmalar, qo'llanmalar va darsliklar kam. Shu bois mazkur o'quv qo'llanmada "Materiallar qarshiligi" fanida eng ko'p uchraydigan masalalarning yechimlari berilgan.

Ushbu o'quv qo'llanmasi qayta ishlangan va to'ldirilgan ikkinchi nashr bo'lib, unga muallifning ko'p yillar davomida Toshkent Davlat Texnika Universiteti, Toshkent Davlat aviatsiya hamda Toshkent avtomobil yo'llar institutlarida o'qigan ma'ruzalari, o'tkazgan amaliy mashg'ulotlari, yozgan uslubiy qo'llanmalari hamda foydalangan qo'lyozmalari asos qilib olingan.

O'quv qo'llanma oliy texnika ta'lim muassasalarida sanoatning mashinasozlik, samolyotsozlik, avtomobilsozlik, metallurgiya hamda transport va qurilish sohalari bo'yicha o'qiyotgan talabalarga mo'ljallangan. Undan mazkur ta'lim muassasalari magistrantlari va kasb-hunar kollejlarning mexanika fani o'qituvchilari, texniklari foydalanishlari mumkin.

“Materiallar qarshiligi” fanidan misol-masalalar yechish va hisob-grafika ishlarini bajarish bo'yicha rus va o'zbek tillarida chop etilgan ba'zi o'quv adabiyotlaridan muhandislik amaliyotida ko'p uchraydigan ayrim turdagi masalalar kitobga tanlab olindi. Muallif bunday yondashuvni eng avvalo, o'quv adabiyotlarning yangi avlodi hozircha taqchil bo'lgan davrda mavjud adabiyotlardan ham unumli foydalanishga talabalarni undashga asos bo'ladi, deb hisoblaydi.

XIV bobni yozishda Toshkent arxitektura va qurilish instituti professor-o'qituvchilarining turli yillarda “Materiallar qarshiligi” fanidan talabalar o'rtasida o'tkazilgan Respublika olimpiadalariga taklif qilgan masalalaridan foydalanilgan. Bu bobdan asosan professor-o'qituvchilar foydalanishini nazarda tutib, undagi masalalar tahlili qisqa holda berildi. Ba'zi boblarning oxirida talabalar uchun keltirilgan topshiriqlarni tuzishda Y. F. Vinokurov va boshqalarning o'quv qo'llanmalaridan [13] foydalanildi.

Muallif qo'lyozmani nashrga tayyorlashda bergan foydali maslahatlari uchun dotsentlar A. N. Nabiyeu, F. S. To'raxodjayev va A. Mo'yiddinovlarga samimiy minnatdorchilik bildiradi.

Qo'llanmaning mazmuni va sifatini boyitishga qaratilgan barcha tanqidiy fikr-mulohazalar uchun kitobxonlarga oldindan minnatdorchilik bildirgan holda, ularni quyidagi manzilga yuborishlarini iltimos qilamiz:

Materiallar qarshiligi fanidan qo'llaniladigan asosiy belgilashlar va ularning o'lchov birliklari:

A — ko'ndalang kesim yuzasi, m^2

F — kuch, kN ;

E — cho'zilish va siqilishdagi elastiklik moduli, MPa ;

G — siljishdagi elastiklik moduli, MPa ;

I — kesimning inersiya momenti, m^4 ;

I_x, I_y — o'qlarga nisbatan inersiya momenti, m^4 ;

I_{xy} — markazdan qochma inersiya momenti, m^4 ;

I_p — polyar inersiya momenti, m^4 ;

M — eguvchi moment, $kN \cdot m$;

N — bo'ylama kuch, kN ;

Q — ko'ndalang kuch, kN ;

S — kesimning statik momenti, m^3 ;

i — kesimning inersiya radiusi, m ;

m — to'plangan moment, juft kuch, $kN \cdot m$;

q — taralgan kuch intensivligi, kN/m ;

α — materialning temperatura ta'sirida chiziqli kengayish koeffitsiyenti, $1/grad$;

γ — solishtirma og'irlik, kN/m^3 ;

Δ, δ – chiziqli ko‘chish, mm;

ε – nisbiy deformatsiya;

θ – ko‘ndalang kesimning aylanish burchagi, rad;

λ – sterjenning egiluvchanligi;

μ – Puasson koeffitsiyenti;

σ – normal kuchlanish;

τ – urinma kuchlanish, MPa.

Karrali yoki ulushli birliklar

Qo‘shim- cha	Giga	Mega	Kilo	Gekto	Deka	Detsi	Santi	Milli	Mikro	Nano
Belgilash	G	M	k	g	Da	d	s	m	mk	n
Ko‘paytu vchi	10^9	10^6	10^3	10^2	10	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-9}

Mexanik qiymatlarning xalqaro o‘lchov birliklar sistemasidagi (SI) o‘lchov birliklari

Qiymatlar		O‘lchov birliklar (SI)		
Nomi	Belgilash	Nomi	Belgilash	Birliklar nisbati
Kuch	F, Q, N	Nyuton	N	$10N = 1kg \cdot kuch$
Kuchlanish	σ, τ	Paskal	Pa	$1Pa = 1N/m^2$
Elastiklik moduli	E, G	Paskal	Pa	$1MPa = 10kg \cdot kuch/sm^2$
Kuch momenti	M	Nyuton-metr	$N \cdot m$	$1N \cdot m = 0,1kg \cdot kuch \cdot M$
Taralgan kuch	q	Nyuton taqsim metr	N/m	$1N/m = 0,1kg \cdot kuch \cdot M$

I BOB

CHO'ZILISH VA SIQILISH

1.1- §. Asosiy mulohazalar

- Berilgan sterjenlarning ko'ndalang kesimlarida faqatgina bitta bo'ylama kuch N_z ta'sir ko'rsatsa, u holda cho'zilish yoki siqilish deformatsiyasi sodir bo'ladi.
- Kuch sterjenni cho'zsa, bo'ylama kuch ishorasi musbat, siqsa – manfiy olinadi.
- Sterjenning ixtiyoriy ko'ndalang kesimida hosil bo'ladigan normal kuchlanish quyidagiga teng bo'ladi:

$$\sigma = \frac{N}{A}. \quad (1.1)$$

Bu yerda, N – bo'ylama kuch; A – kesim yuzasi.

- Cho'zilish va siqilishda sterjen mustahkamlik sharti quyidagiga teng:

$$\sigma_{max} = \frac{N_{max}}{A} \leq [\sigma]. \quad (1.2)$$

Bu yerda, $[\sigma]$ – ruxsat etilgan normal kuchlanish.

- Cho'zilishda sterjen uzunligi ortadi, siqilishda esa kamayadi:

$$\Delta l = l - l_1.$$

Bu yerda, Δl – bo'ylama deformatsiya;

l – sterjenning dastlabki uzunligi;

l_1 – sterjenning deformatsiyadan keyingi uzunligi.

- Nisbiy bo'ylama deformatsiya quyidagiga teng:

$$\varepsilon = \Delta l / l. \quad (1.3)$$

- Cho'zilishda sterjen ko'ndalang kesimi kamayadi, siqilishda esa ko'payadi:

$$\Delta b = b_1 - b.$$

Bu yerda, Δb – absolyut qisqarish;

b – kesimning oldingi o'lchami;

b_1 – kesimning deformatsiyadan keyingi o'lchami.

- Nisbiy ko'ndalang deformatsiya quyidagiga teng:

$$\varepsilon_q = \frac{\Delta b}{b}. \quad (1.4)$$

- Cho'zilish va siqilishda Guk qonuni quyidagiga teng:

$$\sigma = E \cdot \varepsilon; \quad (1.5)$$

$$\Delta l = \frac{N \cdot l}{EA} \quad (1.6)$$

Bu yerda, E – materialning elastiklik moduli.

- Puasson koeffitsiyenti quyidagiga teng:

$$\mu = \frac{\varepsilon_q}{\varepsilon}. \quad (1.7)$$

1.2- §. Cho'zilish va siqilishdagi statik aniq masalalar

1-masala

Ko'ndalang kesim yuzasi $A=0,8 \cdot 10^4 \text{ m}^2$ va uzunligi $l=1 \text{ m}$ bo'lgan po'lat sterjenning $F = 10000 \text{ N}$ cho'zuvchi kuchdan hosil bo'lgan absolyut uzayishini aniqlang.

Yechish:

Sterjen materiali uchun $E=2 \cdot 10^3 \text{ N/m}^2$ ekanligini e'tiborga olib, (1.6) ifodadan absolyut uzayishni topamiz:

$$\Delta l = \frac{Fl}{EA} = \frac{10000 \cdot 1}{2 \cdot 10^3 \cdot 0,8 \cdot 10^4} = 0,625 \cdot 10^{-3} \text{ m}$$

2-masala

Ko'ndalang kesimi kvadrat shaklidagi po'lat brus tashqi kuch ta'siridan bo'ylama yo'nalishda $\Delta l = 3,2 \cdot 10^{-5}$ m ga uzayib, ko'ndalang yo'nalishda $\Delta h = 0,03 \cdot 10^{-5}$ m ga siqilgan. Agar brusning bo'ylama o'lchami (uzunligi) $l = 30 \cdot 10^{-2}$ m va ko'ndalang o'lchami $h = 1 \cdot 10^{-2}$ m bo'lsa, Puasson koeffitsiyenti nechaga teng?

Yechish

(1.7) ifodadan foydalanib, Puasson koeffitsiyentini aniqlaymiz:

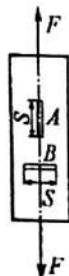
$$\mu = \frac{\Delta h}{\Delta l} \cdot \frac{l}{h} = \frac{0,03 \cdot 10^{-5}}{3,2 \cdot 10^{-5}} \cdot \frac{30 \cdot 10^{-2}}{1 \cdot 10^{-2}} = 0,28.$$

3-masala

Elastiklik moduli E_T va Puasson koeffitsiyenti μ_T larning tajribaviy qiymatlarini aniqlash uchun po'lat materialdan yasalgan tekis namuna cho'zilishiga sinalgan (1.1-shakl). Namunaning deformatsiyasi bo'ylama va ko'ndalang yo'nalishlarda o'rnatilgan A va B richagli tenzometrlar yordamida o'lchangan. Tenzometrlar bir xil bazali ($S_A = S_B = 20$ mm) bo'lib, ularning kattalashtirilishi mos ravishda $k_A = 950$ va $k_B = 1190$ ga teng.

Namunaning kesim yuzasi $A_0 = 1 \cdot 10^2$ mm² ga teng. Tajriba natijalari 1.1 – jadvalda keltirilgan.

1.1-jadval



1.1-shakl

Namunaning Yuklanishi F, kN	Tenzometrlarning ko'rsatishi, mm	
	nA	nB
2	4,5	36,0
12	14,5	32,5
22	24,0	30,0
32	34,5	25,5

Yechish:

Namunani cho'zuvchi kuch orttirilmasining o'rtacha qiymati

$$\Delta F = \frac{(12-2) + (22-12) + (32-22)}{3} = 10 \text{ kN ga teng.}$$

ΔF ga mos keluvchi tenzometr ko'rsatishi orttirilmasining o'rtacha qiymatlarini hisoblaymiz.

$$\Delta n_A = \frac{(14,5 - 4) + (24 - 14,5) + (34,5 - 24)}{3} = 10 \text{ mm;}$$

$$\Delta n_B = \frac{(32,5 - 36) + (30 - 32,5) + (25,5 - 30)}{3} = 3,5 \text{ mm.}$$

Absolyut uzayishning o'rtacha qiymati:

$$\Delta S_A = \frac{10}{950} = 1,05 \cdot 10^{-2} \text{ mm; } \Delta S_B = \frac{3,5}{1190} = 2,9 \cdot 10^{-3} \text{ mm.}$$

Shunday qilib,

$$E_T = \frac{\Delta F \cdot S_A}{A_0 \cdot \Delta S_A} = 1,9 \cdot 10^5 \text{ MPa; } \mu_T = \frac{\varepsilon_B}{\varepsilon_A} = \frac{\Delta S_B \cdot S_A}{\Delta S_A \cdot S_B} = 0,28.$$

4-masala

Uzunligi va diametri ma'lum bo'lgan pog'onali po'lat sterjen chizmada ko'rsatilgandek, $F_1=30$ kN, $F_2=65$ kN, $F_3=170$ kN bo'ylama kuchlar bilan yuklangan. Sterjenning xususiy og'irligini e'tiborga olmay, bo'ylama kuch va normal kuchlanish epyuralari qurilsin hamda sterjen uchining ko'chishi aniqlansin.

Bunda, $d=2$ sm; $l_1=20$ sm; $l_2=30$ sm; $l_3=20$ sm.

Yechish:

Sterjenni o'ng tomondan boshlab uchta oraliq (uchastka)ga ajratamiz. Kesish usulini qo'llab, avvalo har bir uchastka uchun bo'ylama kuch N va normal kuchlanish σ larni aniqlaymiz.

I oraliq: 1.2-shakl (b)da ko'rsatilgan sterjen bo'lagining muvozanat tenglamasini tuzamiz:

$$\sum Z_i = F_1 - N_1 = 0;$$

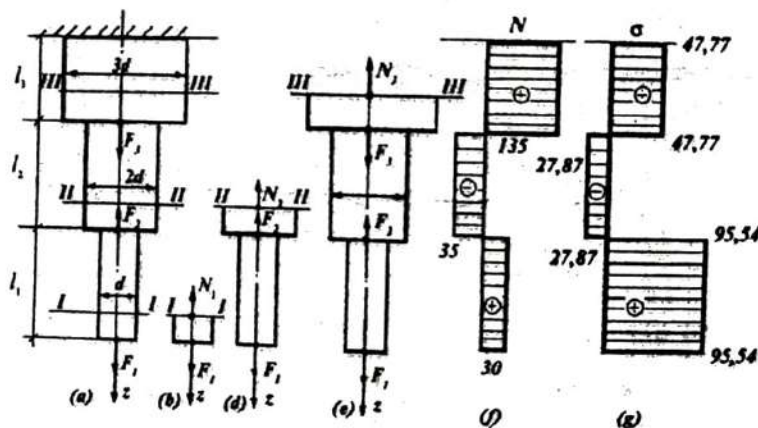
bundan, $N_1 = F_1$ yoki $N_1 = 30 \text{ kN}$.

(1.5) ifodaga ko'ra, normal kuchlanish

$$\sigma_1 = \frac{N_1}{A_1}$$

ga teng, bu yerda, $A_1 = \frac{\pi d^2}{4} \cong 3,14 \text{ sm}^2 = 3,14 \cdot 10^{-4} \text{ m}^2$.

$$\text{Demak, } \sigma_1 = \frac{30 \cdot 10^3}{3,14 \cdot 10^{-4}} = 95,54 \cdot 10^6 \frac{\text{N}}{\text{m}^2} = 95,54 \text{ MPa}.$$



1.2-shakl

II oraliq: 1.2-shakl (d) chizmada tasvirlangan bo'lakcha uchun:

$$\sum Z_1 = F_1 - F_2 - N_2 = 0.$$

Bundan, $N_2 = F_1 - F_2 = -35 \text{ kN}$;

$$\sigma_2 = \frac{N_2}{A_2} = \frac{-35 \cdot 10^3}{\frac{\pi (2d)^2}{4} \cdot 10^{-4}} = -27,87 \text{ MPa}.$$

III oraliq: 1.2-shakl (e) chizmadan foydalanib, quyidagilarni hisoblaymiz:

$$\sum Z_i = F_1 - F_2 + F_3 - N_3 = 0. \text{ Bundan, } N_3 = F_1 + F_3 - F_2 = 135 \text{ kN}.$$

$$\sigma_3 = \frac{N_3}{A_3} \quad \text{yoki} \quad \sigma_3 = \frac{135 \cdot 10^3}{\frac{\pi (3d)^2}{4} \cdot 10^{-4}} = 47,77 \text{ MPa}$$

1.2-shakl (f) va (g)larda, mos ravishda, bo'ylama kuch va normal kuchlanishlarning epyuralari ko'rsatilgan.

Sterjen uchidagi kesimning ko'chishini topishda (1.6) ifodadan foydalanamiz:

$$\Delta l = \frac{1}{E} \cdot \sum_{i=1}^3 \frac{N_i l_i}{A_i},$$

bu yerda, $E = 2 \cdot 10^{11} \text{ N/m}^2$ yoki

$$\Delta l = \frac{1}{E} \left(\frac{30 \cdot 10^3 \cdot 20 \cdot 10^{-2}}{3,14 \cdot 10^{-4}} - \frac{35 \cdot 10^3 \cdot 30 \cdot 10^{-2}}{12,56 \cdot 10^{-4}} + \frac{135 \cdot 10^3 \cdot 20 \cdot 10^{-2}}{28,56 \cdot 10^{-4}} \right) \approx 0,10 \cdot 10^{-3} \text{ m}$$

Demak, sterjen 0,101 mm ga uzayar ekan.

5-masala

Pastki uchi tayanchga qistirib mahkamlangan po'lat sterjenga $F = 80 \text{ kN}$ ga teng to'plangan kuch hamda $q(z) = ql/z$ ($q = 40 \text{ N/m}$ bo'lib, tayanch kesimga qo'yilgan) qonuniyat bo'yicha o'zgaruvchi tekis taralgan yuk ta'sir etmoqda (1.3-shakl, a). Sterjenning ko'ndalang kesim yuzasi $A = 15 \text{ sm}^2 = 15 \cdot 10^{-4} \text{ m}^2$ ga teng. Bo'ylama kuch va ko'ndalang kesimlar ko'chishining epyuralari qurilsin.

Yechish:

Sterjenni yuqori uchidan boshlab ikkita oraliqqa ajratamiz va kesish usulini tatbiq etamiz.

I oraliq: $0 \leq z_1 \leq 3 \text{ m}$

$$N(z_1) = - \int_0^{z_1} \frac{q}{l} z dz = - \frac{q z_1^2}{2l} = -4z_1^2,$$

bunda,

$$\begin{aligned} z_1 = 0, \quad N(0) &= 0; \\ z_1 = 2, \quad N(2) &= -16 \text{ kN}; \\ z_1 = 3, \quad N(3) &= -36 \text{ kN}. \end{aligned}$$

II oraliq: $3 \text{ m} \leq z_2 \leq 5 \text{ m}$

$$N(z_2) = F - \int_0^{z_2} \frac{q}{l} z dz = F - \frac{q z_2^2}{2l} \quad \text{yoki} \quad N(z_2) = 80 - 4z_2^2;$$

bunda,

$$\begin{aligned} z_2 = 3, \quad N(3) &= 80 - 36 = 44 \text{ kN}; \\ z_2 = 4,472, \quad N(4,472) &= 0; \\ z_2 = 5, \quad N(5) &= -20 \text{ kN}. \end{aligned}$$

1.3-shakl (e) da bo'ylama kuch epyurasi qurilgan.

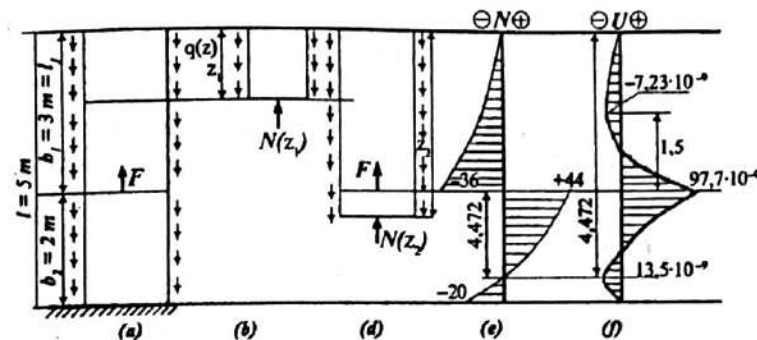
Tayanch kesimda ko'chish nolga teng. II oraliqda yotuvchi z kesimlarning ko'chishi quyidagicha aniqlanadi:

$$U(z_2) = \int_{z_2}^5 \frac{N(z) dz}{EA} = \int_{z_2}^5 \frac{(80 - 4z_2^2) dz}{EA} = \frac{1}{EA} \left(233,33 - 80z_2 + \frac{4}{3} z_2^3 \right),$$

bu yerda, $EA = 15 \cdot 10^{-4} \cdot 2 \cdot 10^{11} = 3 \cdot 10^8 \frac{\text{N}}{\text{m}^2} \cdot \text{m}^2 = 3 \cdot 10^8 \text{ N}$.

Eng katta ko'chishni topamiz:

$$\frac{dU(z_2)}{dz_2} = -80 + 4z_2^2 = 0, \quad \text{bundan, } z_2 \approx 4,472 \text{ m}.$$



1.3-shakl

Demak, $z_2 = 5 \text{ m}$ da $U(5) = 0;$

$$\begin{aligned} z_2 = 4,472 \text{ m da } U(4,472) &= \frac{4,0644}{EA} = -13,548 \cdot 10^{-9} \text{ m} \\ z_2 = 3 \text{ m da } U(3) &= \frac{29,33}{EA} = 97,77 \cdot 10^{-9}. \end{aligned}$$

I oraliq uchun ko'chishning ifodasini yozamiz:

$$U(z_1) = U(3) + \int_{z_1}^3 \frac{N(z) dz}{EA} = 97,77 \cdot 10^{-9} - \int_{z_1}^3 \frac{4z^2 dz}{EA} = -22,23 \cdot 10^{-9} + \frac{4}{3} \cdot \frac{z_1^3}{EA}.$$

$z_1 = 0$ bo'lsa, $U(0);$

$z_1 = 1,5 \text{ m}$ bo'lsa, $U(1,5) = -7,23 \cdot 10^{-9} \text{ m};$

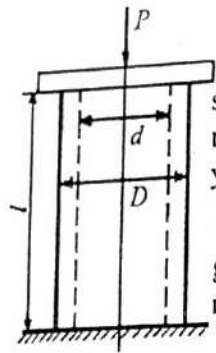
$z_1 = 3 \text{ m}$ bo'lsa, $U(3) = 97,77 \cdot 10^{-9} \text{ m}.$

Yuqorida hosil qilingan qiymatlar asosida ko'chish epyurasini quramiz (1.3-shakl, f).

6-masala

Ichki va tashqi diametrlarining nisbati mos ravishda: $d:D=4:5$ munosabatda bo'lgan $l=0,3 \text{ m}$ uzunlikdagi cho'yan ustun $F=150 \cdot 10^4 \text{ N}$ kuchni ko'tarib turadi (1.4-shakl). Ichki (d), tashqi (D) diametrlar hamda ustunning to'la qisqarishi aniqlansin. Cho'yaning siqilishdagi ruxsat etilgan kuchlanishi $\sigma_{adm} = 100 \text{ MPa}$, elastiklik moduli $E=0,8 \cdot 10^5 \text{ MPa}$ ma'lum deb hisoblansin.

Yechish:



1.4-shakl

Ustunning barcha ko'ndalang kesimlarida $N=F$ siquvchi bo'ylama kuchlar vujudga kelishi o'z-o'zidan ravshan. Ustunning ichi kavak bo'lganligi sababli, uning yuzasi

$$A = \frac{\pi}{4}(D^2 - d^2)$$

ga teng. Buni e'tiborga olib, quyidagini hosil qilish mumkin:

$$\frac{\pi}{4}(D^2 - d^2) \geq \frac{N}{\sigma_{adm}} \text{ yoki } \frac{\pi}{4}\left[D^2 - \left(\frac{4D}{5}\right)^2\right] \geq \frac{F}{\sigma_{adm}}$$

bundan,

$$D \geq \sqrt{\frac{100F}{2\pi\sigma_{adm}}} = \sqrt{\frac{100 \cdot 150 \cdot 10^4}{9 \cdot 3,14 \cdot 100 \cdot 10^6}} = 0,23 \text{ m.}$$

$D=25\text{sm}$ deb qabul qilamiz, u holda ustunning ichki diametri $d=4D/5=20 \text{ sm}$ ga teng bo'ladi. (1.6) ifodaga ko'ra,

$$\Delta l = \frac{Fl}{EA} = \frac{150 \cdot 10^4 \cdot 0,3}{0,8 \cdot 10^{11} \cdot \frac{\pi}{4}[(0,25)^2 - (0,2)^2]} \approx 0,318 \cdot 10^{-3} \text{ m.}$$

Demak, ustun $\Delta l=0,318 \text{ mm}$ ga qisqarar ekan.

7-masala

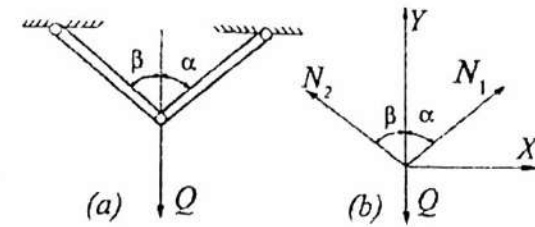
Sharnirlar vositasida ulangan po'lat sterjenlar ko'tara oladigan eng katta yuk aniqlansin (1.5-shakl). Sterjen diametric $d=2\text{sm}$, materialning cho'zilishdagi ruxsat etilgan kuchlanishi $\sigma_{adm} = 140 \text{ MPa}$, $\alpha = \beta = 60^\circ$.

Yechish:

Cho'zilish va siqilishdagi mustahkamlik shartidan har bir sterjenda paydo bo'luvchi bo'ylama kuchni aniqlaymiz:

$$N \geq A \cdot \sigma_{adm} = \frac{\pi}{4}(2 \cdot 10^{-2})^2 \cdot 140 \cdot 10^3 = 43,96 \text{ kN};$$

$$N_1 = N_2 = N.$$



1.5-shakl

Endi S tugunning muvozanatini tekshiramiz (1.5-shakl, b):

$$\sum Y_i = -Q + 2N \cdot \cos 60^\circ = 0.$$

Bundan, $Q = 2 \cdot 43,96 \cdot 0,5 = 43,96 \text{ kN}$.

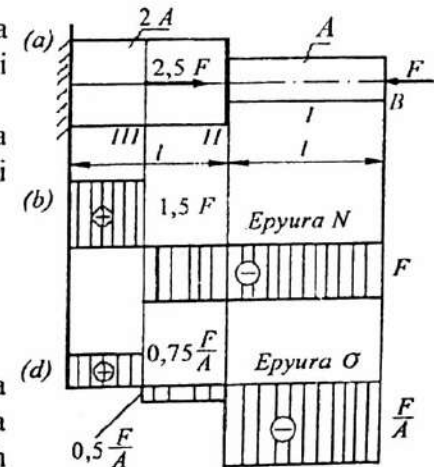
8-masala

Ikki pog'onali cho'yan brus ko'tara oladigan kuchning ruxsat etilgan qiymati aniqlansin (1.6-shakl, a).

Brus materialining cho'zilishdagi va siqilishdagi ruxsat etilgan kuchlanishlari mos ravishda $\sigma_{adm}^+ = 40 \text{ mPa}$ va $\sigma_{adm}^- = 10 \text{ mPa}$.

Yechish:

Kesish usulini qo'llab, brusning uchala uchastkasi (oralig'i) uchun ham bo'ylama kuch N (1.6-shakl, b) va normal kuchlanish σ (1.6-shakl, d)



1.6-shakl

va normal kuchlanish σ (1.6-shakl, d) larning epyuralarini ko'ramiz.

Normal kuchlanish epyurasidan ko'rinib turibdiki, eng katta cho'zuvchi kuchlanish III oraliqda va eng katta siquvchi kuchlanish I oraliqda paydo bo'lar ekan. Shuning uchun mustahkamlik shartlari quyidagicha yoziladi:

$$\frac{F}{A} \leq \sigma_{adm}^-; \quad 0,75 \frac{F}{A} \leq \sigma_{adm}^+.$$

Bulardan,
 $F^I = F \leq \sigma_{adm}^- \cdot A = 10^{-3} \cdot 120 = 120 \text{ kN}; \quad F^{II} = A \cdot \sigma_{adm}^-;$

$$F^{III} = \frac{A \cdot \sigma_{adm}^+}{0,75} = \frac{10^3 \cdot 40}{0,75} = 53,4 \text{ kN}.$$

Kuchning ruxsat etilgan qiymati $F_{adm} = F^{III} = 53,4 \text{ kN}$ ga teng.

9-masala

Bug' silindrlarning qopqog'i silindrga 8 ta bolt bilan biriktirilgan. Silindrlarning ichki diametri $D = 50 \cdot 10^{-2} \text{ m}$, boltlar uchun ruxsat etilgan kuchlanish $\sigma_{adm} = 50 \text{ MPa}$ va silindrdagi bug'ning bosimi $P = 6 \text{ atm}$ ($1 \text{ atm} = 9,81 \cdot 10^4 \text{ N/m}^2 = 9,81 \cdot 10^{-2} \text{ MPa}$).

Yechish:

Avval 8 ta boltning cho'zuvchi F kuchni topamiz:

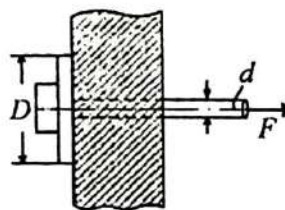
$$F = P \cdot \frac{\pi D^2}{4} = 6 \cdot 9,81 \cdot 10^4 \cdot \frac{3,14(50 \cdot 10^{-2})^2}{4} = 115,51 \text{ kN}.$$

Bu kuchni 8 ta bolt qabul qilganligi uchun hisob tenglamasi quyidagicha bo'ladi:

$$F \leq 8 \cdot \sigma_{adm} \cdot \frac{\pi d^2}{4}$$

bundan, $d = 1,92 \cdot 10^{-2} \text{ m}$.

10-masala



1.7-shakl

Diametri $d = 30 \cdot 10^{-3} \text{ m}$ bo'lgan tortqi F kuch bilan cho'zilganda, unda kichlanish paydo bo'ladi (1.7-shakl). Tortqiga kiygizilgan shayba-ni devorga ko'rsatadigan bosimi $q = 1,5 \text{ MPa}$ dan oshib ketmasligi uchun shayba diametri qancha bo'lishi kerak?

Yechish:

Oldin cho'zuvchi kuch F ni aniqlaymiz:

$$F = \sigma \cdot \frac{\pi d^2}{4} = 80 \cdot 10^3 \cdot \frac{3,14(30 \cdot 10^{-3})^2}{4} = 56,4 \text{ kN}.$$

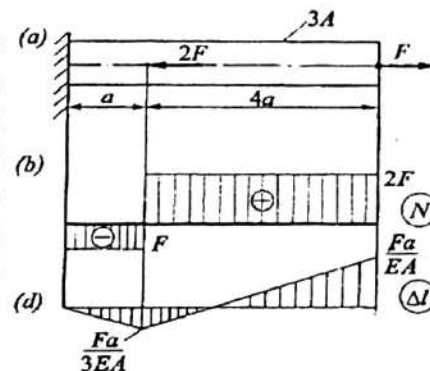
Mustahkamlik shartini $F \leq q \frac{\pi(D^2 - d^2)}{4}$ ko'rinishda yozib, undan shaybaning diametrini aniqlaymiz: $D = 4,7 \cdot 10^{-1} \text{ m} = 470 \text{ mm}$.

11-masala

Po'lat sterjen tashqi kuchlar bilan yuklangan (1.8-shakl, a). Tashqi kuchlarning bajargan ishi deformatsiyaning potentsial energiyasiga tengligini isbotlang.

Yechish:

Kesish usulini qo'llab, har bir oraliq uchun bo'ylama kuch N va ko'chish Δl larning epyuralarini ko'ramiz (1.8-shakl, d). (Epyuralarni mustaqil qurish o'quvchilarga havola qilinadi). Tashqi kuchlarning bajargan ishini hisoblashda $w_0 = F \cdot \Delta l / 2$ ifodadan foydalanamiz, shuning uchun



1.8-shakl

$$W = w_F + w_{2F} = \frac{1}{2} F \cdot \frac{5Fa}{3EA} + \frac{1}{2} (-2F) \cdot \frac{(-2F)a}{3EA} = \frac{9}{6} \cdot \frac{F^2 a}{EA}.$$

Deformatsiyaning potentsial energiyasini aniqlashda esa

$$U_0 = \frac{N^2 l}{2EA}$$

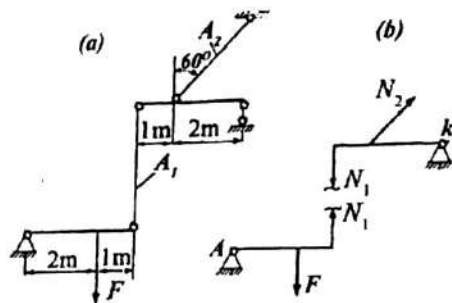
ifoda ishlatiladi. Demak,

$$U = U_I + U_{II} = \frac{F^2 5a}{6EA} + \frac{-(2F)^2 a}{6EA} = \frac{9}{6} \cdot \frac{F^2 a}{EA},$$

Shunga ko'ra, $W = U = \frac{3 F^2 a}{2 EA}$ ga teng ekan.

12-masala

1.9-shakl (a) da tasvirlangan 1-va 2-tortqilarning diametri aniqlansin. Tortqi materiali uchun ruxsat etilgan kuchlanish $\sigma_{adm} = 160$ MPa, tashqi kuch esa $F=900$ kN ga teng.



1.9-shakl

$$\sum M_{A_i} = F \cdot 2 - N_1 \cdot 3 = 0;$$

$$\sum M_{k_i} = N_2 \cdot 2 \cdot \cos 60^\circ - N_1 \cdot 3 = 0.$$

Bulardan, $N_1=600$ kN; $N_2=1800$ kN.

Mustahkamlik shartidan foydalanib, har bir tortqining zaruriy ko'ndalang kesim yuzasini hisoblaymiz:

$$A_1 \geq \frac{N_1}{\sigma_{adm}} = \frac{600 \cdot 10^3}{160 \cdot 10^6} = 3,75 \cdot 10^{-3} \text{ m}^2;$$

$$A_2 \geq \frac{N_2}{\sigma_{adm}} = \frac{1800 \cdot 10^3}{160 \cdot 10^6} = 11,25 \cdot 10^{-3} \text{ m}^2.$$

Endi tortqilarning kerakli diametrlarini aniqlaymiz:

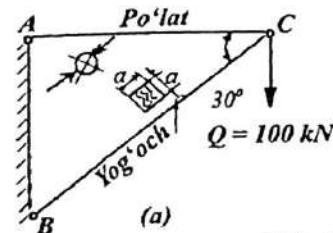
$$d_1 = \sqrt{\frac{4A_1}{\pi}} = 69,11 \cdot 10^{-3} \text{ m}; \quad d_2 = \sqrt{\frac{4A_2}{\pi}} = 119,7 \cdot 10^{-3} \text{ m}.$$

13-masala

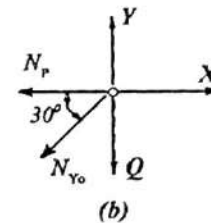
Kronshteyn ABC ga $Q=100$ kN yuk sharnir vositasida osilgan (1.10-shakl, a). Po'lat tortqi AC va yog'och tirgak BC larning zaruriy ko'ndalang kesim o'lchamlari aniqlansin. Po'lat materiali uchun ruxsat etilgan kuchlanish $\sigma_{adm}^P = 140$ MPa, yog'och uchun esa $\sigma_{adm}^{Yo} = 10$ MPa ga teng.

Yechish:

Tortqilarni fikran tekisliklar bilan kesib, ichki zo'riqish bo'ylama kuchlarni chizmada tasvirlaymiz (1.9-shakl, b). Statikaning muvozanat tenglamalarini tuzamiz:



1.10-shakl



Yechish:

Kesish usulidan foydalanib, C tugunining muvozanatini tekshiramiz (1.10-shakl, b):

$$\sum X_i = -N_p - N_{Yo} \cdot \cos 30^\circ = 0;$$

$$\sum Y_i = Q + N_{Yo} \cdot \cos 60^\circ = 0.$$

Bulardan, $N_{Yo} = -200$ kN (tirgak siqiladi); $N_p = 173,2$ kN (tortqi cho'ziladi). Tortqi kesimining zaruriy diametri:

$$d \geq \sqrt{\frac{4N_p}{\pi \sigma_{adm}^P}} = \sqrt{\frac{4 \cdot 173,2 \cdot 10^3}{3,14 \cdot 140 \cdot 10^6}} = 3,97 \cdot 10^{-2} \text{ m}.$$

Tirsak kesimini zaruriy o'lchami:

$$a \geq \sqrt{\frac{N_{Yo}}{\sigma_{adm}^{Yo}}} = \sqrt{\frac{200 \cdot 10^3}{10 \cdot 10^6}} = 14,14 \cdot 10^{-2} \text{ m}.$$

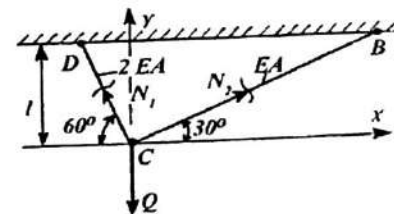
14-masala

1.11-shaklda tasvirlangan sterjenlar sistemasi C tugunining vertikal ko'chishi aniqlansin. Sterjenlarning bikirligi EA, osib qo'yilgan yuk Q va L masofa ma'lum deb hisoblansin.

Yechish:

Tashqi kuchning bajargan ishi quyidagiga teng:

$$W = \frac{1}{2} Q \cdot \lambda_c,$$



1.11-shakl

bu yerda, λ_c – C tugunning vertikal ko'chishi bo'lib, uning qiymati hozircha no'ma'lum.

Sterjenlar deformatsiyasining potensial energiyasi

$$U = \frac{N_1^2 l_{CB}}{2EA} + \frac{N_2^2 l_{CD}}{2 \cdot 2EA}$$

ko'rinishda ifodalanadi.

C tugun uchun muvozanat tenglamalarini yozamiz:

$$\sum X_i = N_2 \cos 30^\circ - N_1 \cos 60^\circ = 0;$$

$$\sum Y_i = N_1 \cos 30^\circ + N_2 \cos 60^\circ - Q = 0.$$

$$\text{Bulardan, } N_1 = \frac{\sqrt{3}}{2} Q; \quad N_2 = \frac{1}{2} Q.$$

Chizmadan $l_{CB} = 2l$, $l_{CD} = 2\sqrt{3}l$ ekanligi ma'lum.

Energiyaning saqlanish qonuniga asoslanib, quyidagi tenglamani (ifodani) yozib olamiz:

$$\frac{1}{2} Q \cdot \lambda_c = \frac{\left(\frac{\sqrt{3}}{2} Q\right)^2 \cdot 2l}{2EA} + \frac{\left(\frac{Q}{2}\right)^2 \cdot 2\sqrt{3}l}{4EA}.$$

$$\text{Bundan, } \lambda_c = 1,933 \frac{Ql}{EA}.$$

1.3-§. Sterjenning xususiy og'irligidan hosil bo'ladigan kuchlanish va deformatsiya

Sterjenning xususiy og'irligidan hosil bo'ladigan kuchlanish, agarda uning ko'ndalang kesimi o'zgarmas bo'lsa, uning materiali va uzunligiga bog'liq bo'ladi. Hosil bo'lgan umumiy kuchlanish esa tashqi kuchlar va xususiy og'irlikdan hosil bo'lgan kuchlanishlar yig'indisiga teng bo'ladi.

Sterjenning xususiy og'irligidan hosil bo'ladigan deformatsiya ham uning uzunligi bo'ylab o'zgaruvchan bo'ladi.

Uzunligi l ga teng bo'lgan sterjenning tashqi F kuch va xususiy og'irligidan hosil bo'ladigan absolyut uzayishi quyidagiga teng bo'ladi:

$$\Delta l = \frac{F \cdot l}{EA} + \frac{G \cdot l}{2EA}.$$

Bu yerda, $G = \gamma \cdot l \cdot A$.

15-masala

Pog'onali betondan qilingan kolonna ($\gamma = \frac{25 \text{ kN}}{\text{m}^3}$, $E = 27 \text{ GPa}$) F_1 , F_2 va o'zining xususiy og'irligi ta'sirida bo'lsin. Quyidagilarni aniqlash talab qilinadi:

- 1) kolonnaning mustahkamligini tekshirish;
- 2) har bir uchastkadagi kolonna ko'ndalang kesimining (A_1 , A_2 , A_3) mo'tadil qiymatini aniqlash;
- 3) I-I kesimning ko'chishini aniqlash.

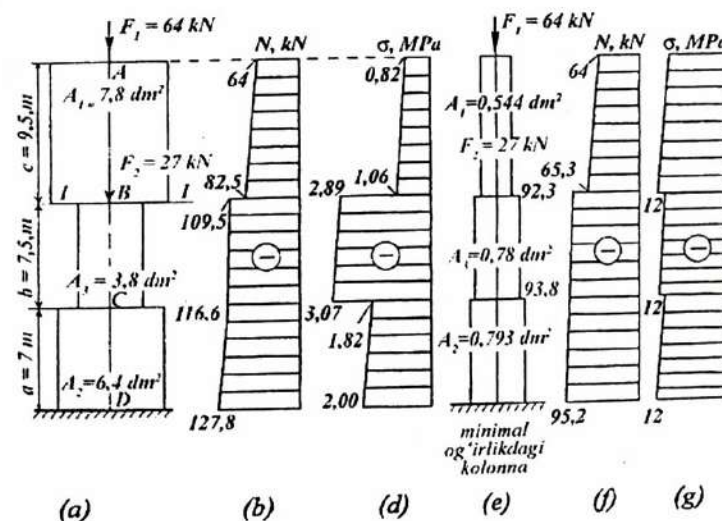
Yechish:

Kolonnaning har bir bo'lagi og'irligini aniqlaymiz ($G = g \cdot A \cdot l$):

$$G_1 = 25000 \cdot 7,8 \cdot 10^{-2} \cdot 9,5 = 18,5 \cdot 10^3 \text{ N} = 18,5 \text{ kN};$$

$$G_2 = 25000 \cdot 6,4 \cdot 10^{-2} \cdot 7 = 11,2 \text{ kN};$$

$$G_3 = 25000 \cdot 3,8 \cdot 10^{-2} \cdot 7,5 = 7,1 \text{ kN}.$$



1.12-shakl

Uchastkalardagi bo'ylama kuch qiymatlarini aniqlaymiz:

$$N_I = -F_1 \gamma \cdot A_1 \cdot z_1; \quad 0 \leq z_1 \leq c.$$

$$N_{I_{z_1=0}} = -F_1 - 64 \text{ kN}; \quad N_{I_{z_1=c}} = -64 - 18,5 - 82,5 \text{ kN}.$$

$$N_{II} = F_1 - F_2 - G_1 - \gamma \cdot A_3 \cdot z_2; \quad 0 \leq z_2 \leq b.$$

$$N_{II_{z_2=0}} = -109,5 \text{ kN}; \quad N_{II_{z_2=b}} = -64 - 27 - 18,5 - 7,1 = -116,6 \text{ kN}.$$

$$N_{III} = -F_1 - F_2 - G_1 - G_3 - \gamma A_2 \cdot z_3; \quad 0 \leq z_3 \leq a.$$

$$N_{III_{z_3=0}} = -116,6 \text{ kN}; \quad N_{III_{z_3=a}} = -116,6 - 11,2 - 127,8 \text{ kN}.$$

Topilgan qiymatlar bo'yicha bo'ylama kuch epyuralarini quramiz (1.12-shakl, b).

Kesimlarda hosil bo'ladigan normal kuchlanishlar qiymatini $\sigma = N/A$ ifoda yordamida aniqlab uning epyurasini quramiz (1.12-shakl, d).

Bu epyuradan ko'rinib turibdiki, eng katta normal kuchlanish ikkinchi uchastkaning oxirgi kesimida hosil bo'lar ekan. Ya'ni,

$$\sigma_{\max} = 3,07 \text{ MPa} < [\sigma] = 12 \text{ MPa}.$$

Og'irligi eng kam miqdorga teng bo'lgan (mo'tadil) kolonnaning o'lchamlarini $\sigma_{\max} = [\sigma]$ dan foydalangan holda aniqlaymiz. Yechimni kolonnaning yuqori qismidan boshlaymiz.

$$\frac{N_{1\max}}{A_1} = \frac{F_1 + \gamma \cdot A_1 \cdot c}{A_1} \geq [\sigma].$$

$$\text{Bunda, } A_1 \geq \frac{F_1}{[\sigma] - \gamma \cdot c} = \frac{64000}{12 \cdot 10^6 - 25000 \cdot 9,5} = 5,44 \cdot 10^{-3} \text{ m}^2 = 0,544 \text{ dm}^2.$$

U holda, I uchastkaning mo'tadil og'irligi quyidagiga teng bo'ladi:

$$G_1 = 25000 \cdot 0,544 \cdot 10^{-2} \cdot 9,5 = 1,3 \text{ kN}.$$

Xuddi shuningdek,

$$A_3 = \frac{F_1 + F_2 + G_1}{R - \gamma \cdot b} = 0,781 \text{ dm}^2; \quad G_3 = 1,5 \text{ kN}.$$

$$A_2 = \frac{F_1 + F_2 + G_1 \cdot G_3}{R - \gamma \cdot a} = 0,793 \text{ dm}^2; \quad G_2 = 1,4 \text{ kN}.$$

I-I kesimning ko'chishi quyidagiga teng bo'ladi:

$$\Delta_{I-I} = \Delta_{Ia} + \Delta_{Ib}.$$

$$\text{Unda, } \Delta_{Ia} = \frac{(F_1 + F_2 + G_1 + G_3 + \frac{G_2}{2})a}{EA} \text{ qiymatlarni o'rniga qo'ysak,}$$

$$\Delta_{Ia} = 3,09 \text{ mm}.$$

$$\Delta_{Ib} = \frac{(F_1 + F_2 + G_1 + \frac{G_2}{2})b}{EA_3}, \quad \text{bu yerda, } \Delta_{Ib} = 3,31 \text{ mm}.$$

$$\text{U holda, } \Delta_{I-I} = 3,09 + 3,31 = 6,4 \text{ mm (pastga qarab ko'chadi).}$$

16-masala

F kuch hamda o'zining xususiy og'irligi ta'sirida bo'lgan po'lat sterjen uchun bo'ylama kuch epyurasi hamda I-I kesimning ko'chishi aniqlansin (1.13-shakl, a).

Berilgan: $a = 3 \text{ m}$; $b = 2 \text{ m}$; $c = 1,5 \text{ m}$; $A = 10 \text{ sm}^2$; $F = 1 \text{ kN}$; $E = 2 \cdot 10^5 \text{ MPa}$. Sterjenning solishtirma og'irligi: $\gamma = 7,8 \text{ kN/m}^3 = 7,8 \cdot 10^{-4} \text{ N/m}^3$.

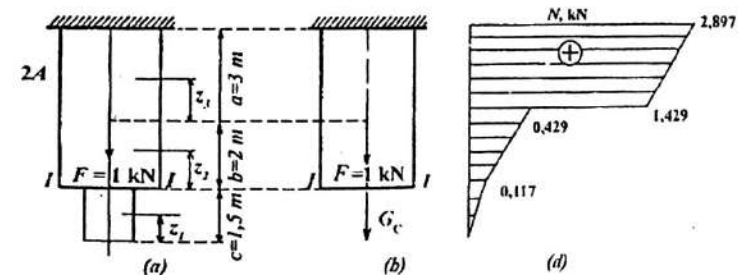
Yechish:

Uchastkalarining xususiy og'irligini aniqlaymiz:

$$G_c = \gamma \cdot A \cdot c = 7,8 \cdot 10^{-4} \cdot 10 \cdot 10^{-4} \cdot 1,5 = 117 \text{ N} = 0,117 \text{ kN};$$

$$G_b = \gamma \cdot 2A \cdot b = 7,8 \cdot 10^{-4} \cdot 2 \cdot 10 \cdot 10^{-4} \cdot 2 = 312 \text{ N} = 0,312 \text{ kN};$$

$$G_a = \gamma \cdot 2A \cdot a = 7,8 \cdot 10^{-4} \cdot 2 \cdot 10 \cdot 10^{-4} \cdot 3 = 468 \text{ N} = 0,468 \text{ kN}.$$



1.13-shakl

Uchastkalardagi bo'ylama kuch qiymatlarini aniqlaymiz:

$$\begin{aligned} N_I &= \gamma \cdot A \cdot z_1; \quad 0 \leq z_1 \leq c; \\ z_1 &= 0 \text{ da } N_I = 0; \quad z_1 = c \text{ da } N_I = 0,117 \text{ kN}; \\ N_{II} &= G_c + \gamma \cdot 2A \cdot z_2; \quad 0 \leq z_2 \leq b; \\ z_2 &= 0 \text{ da } N_{II} = 0,117 \text{ kN}; \quad z_2 = b \text{ da } N_{II} = 0,429 \text{ kN}; \\ N_{III} &= G_c + G_b + F + \gamma \cdot 2A \cdot z_3; \quad 0 \leq z_3 \leq a; \\ z_3 &= 0 \text{ da } N_{III} = 1,429 \text{ kN}; \quad z_3 = a \text{ da } N_{III} = 2,897 \text{ kN}. \end{aligned}$$

Topilgan qiymatlar bo'yicha bo'ylama kuch epyurasini quramiz (1.13-shakl, d).

I-I kesimning ko'chishini aniqlashda birinchi uchastkaning xususiy og'irligi G_c bo'lganda, uni to'plangan kuch sifatida ko'rsak, berilgan sterjenni 1.13-shakl (b) ko'rinishiga keltirish mumkin. Unda,

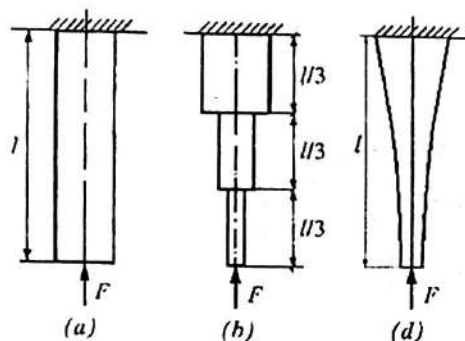
$$\begin{aligned} \Delta_{I-I} &= \Delta l_{a+b} = \frac{G_c(a+b)}{E \cdot 2A} + \frac{P \cdot a}{E \cdot 2A} + \frac{\gamma(a+b)^2}{2E} = \\ &= \frac{117 \cdot (3+2)}{2 \cdot 10^{11} \cdot 2 \cdot 10 \cdot 10^{-4}} + \frac{1 \cdot 10^3 \cdot 3}{2 \cdot 10^{11} \cdot 2 \cdot 10 \cdot 10^{-4}} + \frac{78 \cdot 10^3(3+2)^2}{2 \cdot 2 \cdot 10^{11}} = \\ &= (1,46 + 7,5 + 4,88) \cdot 10^{-6} \text{ m} = 13,84 \cdot 10^{-3} \text{ mm}. \end{aligned}$$

Ko'chish pastga qarab bo'ladi.

17-masala

1.14-shaklda ko'rsatilgan o'zgarmas kesimli, pog'onali va teng qarshilikli bruslarga $F=4\text{MN}$ (400t) markaziy siquvchi kuch qo'yilgan. Har bir brusning uzunligi $l=42 \text{ m}$ bo'lib, uning materiali uchun quyidagilar ma'lum: $\sigma_{adm}=12 \text{ MPa}$, $\gamma=22,0 \text{ kN/m}^3$.

Bruslarning og'irliklari solishtirilsin.



1.14-shakl

Yechish:

a) o'zgarmas kesimli brus.

Bu brusning yuzasini hisoblaymiz:

$$A_1 = \frac{F}{\sigma_{adm} - \gamma l} = \frac{4 \cdot 10^6}{1,2 \cdot 10^6 - 2,2 \cdot 10^4 \cdot 42} = 14,5 \text{ m}^2.$$

Brusning hajmi esa $V_1 = A_1 l = 603 \text{ m}^3$ ga teng. Shunga ko'ra uni og'irligi $G_1 = \gamma V_1 = 13398 \text{ kN} = 13398 \cdot 10^3 \text{ N} = 13,398 \text{ MN}$ bo'lar ekan.

b) pog'onali brus.

Navbat bilan pastki, o'rta va yuqori pog'onalarining ko'ndalang kesim yuzasini aniqlaymiz:

$$A_2^p = \frac{F}{\sigma_{adm} - \gamma \cdot \frac{l}{3}} = \frac{4 \cdot 10^6}{1,2 \cdot 10^6 - 14 \cdot 2,2 \cdot 10^4} = 4,48 \text{ m}^2;$$

$$A_2^{o'} = \frac{F}{\sigma_{adm} \left(1 - \frac{\gamma l}{3\sigma_{adm}}\right)^2} = \frac{4 \cdot 10^6}{1,2 \cdot 10^6 \left(1 - \frac{14 \cdot 2,2 \cdot 10^4}{1,2 \cdot 10^6}\right)^2} = 6,03 \text{ m}^2;$$

$$A_3^{yu} = \frac{F}{\sigma_{adm} \left(1 - \frac{\gamma l}{3\sigma_{adm}}\right)^2} = 8,12 \text{ m}^2.$$

Pog'onali brusning og'irligini hisoblaymiz:

$$G_2 = \gamma \sum_{i=1}^3 A_i l_i = \gamma (A_2^{yu} + A_2^{o'} + A_2^p) \cdot \frac{l}{3} = 5738040 \text{ N} = 5,74 \text{ MN}.$$

d) teng qarshilikli brus.

Teng qarshilikli brus ichidagi kesimning yuzasi quyidagiga teng.

$$A_3^0 = \frac{F}{\sigma_{adm}} = \frac{4 \cdot 10^6}{1,2 \cdot 10^6} = 3,33 \text{ m}^2.$$

Ixtiyoriy kesimning yuzasi: $A_3(z) = A_3^0 \cdot \frac{\gamma z}{l \sigma_{adm}}$

Brusning og'irligi:

$$G_3 = \gamma \int_0^1 A(z) dz = \gamma A_3^0 \int_0^1 \frac{\gamma z}{l^{\sigma_{adm}}} \cdot dz = F \left(\frac{\gamma l}{l^{\sigma_{adm}}} - 1 \right) = 4 \cdot 10^6 \left(\frac{2.2 \cdot 10^4 \cdot 4.2}{1.2 \cdot 10^6} - 1 \right) =$$

$$= 4,639 \cdot 10^6 \text{ N} = 4,639 \text{ MN}.$$

Bruslarning og'irliklarini o'zaro taqqoslaymiz:

$$G_1:G_2:G_3=13,358:5,74:4,639=2,89:1,24:1.$$

Demak, teng qarshilikli brusning og'irligi pog'onali brusning og'irligidan taxminan 20%, o'zgarmas kesimli brus og'irligidan esa 65% yengil ekan.

1.4- §. Cho'zilish va siqilishda statik noaniq masalalar

Cho'zilish va siqilish deformatsiyasiga oid masalalarni yechayotganda, sterjenlarning tayanchlarida hosil bo'luvchi reaksiya kuchlarini yoki ularning ko'ndalang kesimlaridagi zo'riqish kuchlarini statikaning muvozanat tenglamalari yordamida aniqlash mumkin bo'lmasa, bunday masalalarga cho'zilish va siqilishdagi statik noaniq masalalar deyiladi.

Bunday masalalarni yechish quyidagi tartibda amalga oshiriladi:

- bog'lanishlar reaksiya kuchlari bilan almashtiriladi;
- mumkin bo'lgan statika tenglamalari tuziladi;
- masalaning statik noaniqlik darajasi aniqlanadi: $S=m-n$, bu yerda, S - masalaning statik noaniqlik darajasi;
- m - noma'lum reaksiya kuchlari soni;
- n - tuzilishi mumkin bo'lgan statika tenglamalari soni;
- masalaning mahkamlanish shartidan foydalanib, ortiqcha deformatsiya tenglamasi tuziladi;
- statika tenglamalari bilan ortiqcha deformatsiya tenglamalari birgalikda yechilib, masala statik aniq masalaga aylantiriladi.

Harorat o'zgarishi natijasida sterjenlarda hosil bo'ladigan kuchlanishlarni aniqlashda ham yuqorida keltirilgan tartib saqlanib qoladi, lekin sterjenlar uzunligi o'zgarishini hisoblayotganda haroratning ta'siri e'tiborga olinadi. Harorat o'zgarishi natijasida sterjenlar uzunligining o'zgarishi quyidagi ifoda bilan aniqlanadi:

$$\Delta l = l \alpha \cdot \Delta t.$$

Bu yerda, l - sterjenning uzunligi;

α - sterjen materialining o'rtacha chiziqli kengayishi;

Δt - haroratning o'zgarish qiymati.

Ba'zi hollarda konstruksiya qismlarini tayyorlash jarayonida texnologik xatolarga yo'l qo'yilib, ularning ayrimlari keragidan uzunroq yoki qisqaroq qilib yasaladi. Agar konstruksiya statik aniq bo'lsa, yig'ilgan (montaj)dan keyingi sterjenlarida qo'shimcha zo'riqish kuchlari paydo bo'ladi. Lekin statik noaniq konstruksiyalarning sterjenlaridan birontasi loyihadagi o'lchamlari bo'yicha tayyorlanmasa, montajdan keyin mazkur sterjendagina emas, balki u bilan bog'langan boshqa sterjenlarda ham qo'shimcha zo'riqishlar hosil bo'ladi. Bunday masalalarni yechish uchun qo'shimcha deformatsiya tenglamalari tuzilayotganda sterjenlarni tayyorlashdagi xatoliklar natijasida hosil bo'ladigan deformatsiyalar ham e'tiborga olinadi.

18-masala

Chap uchi bilan mahkamlangan prizmatik po'lat sterjenning o'ng uchi B tayanchga $\delta = 2 \cdot 10^{-4} \text{ m}$ yetmaydi (1.15-shakl, a). Sterjen $F = 250 \text{ kN}$ kuch bilan yuklangan, ko'ndalang kesimlarda paydo bo'luvchi ko'chish epyuralari qurilsin.

Quyidagilar ma'lum deb hisoblansin: $E = 2 \cdot 10^5 \text{ MPa}$, $A = 20 \cdot 10^{-4} \text{ m}^2$, $l = 4 \text{ m}$.

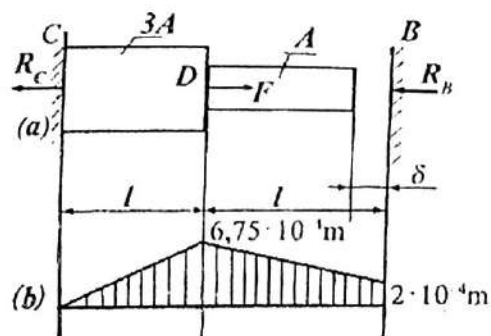
Yechish:

Faraz qilaylik, B tayanch olib tashlangan bo'lsin, u holda F kuch ta'siridan sterjenning o'ng uchi

$$\delta_b = \Delta l_F = \frac{F \cdot l}{E \cdot A} = 8,33 \cdot 10^{-4} \text{ m} \text{ ga ko'chadi.}$$

$\delta_b > \delta$ bo'lganligi sababli, δ oraliq yopilib, masala statik noaniq holatga o'tadi.

1.15-shakl (a) dan foydalanib, masalaning statik tomonini



1.15-shakl

tahlil qilamiz: $\sum Z = R_C - F - R_B = 0$.

Bitta muvozanat tenglamasi tarkibida ikkita (R_B, R_C) noma'lum ishtirok etmoqda: masala bir marta statik noaniq hisoblanadi.

Bu masalani yechish uchun yana bitta qo'shimcha tenglama zarur.

Kuchlar ta'sirining bir-biriga xalal bermaslik prinsipiga asoslanib, qo'shimcha deformatsiya tenglamasini tuzamiz:

$$\Delta l_F + \Delta l_{R_B} = \delta \text{ yoki } \frac{F \cdot l}{E \cdot 3A} - \frac{R_B \cdot l}{E \cdot A} - \frac{R_B \cdot l}{E \cdot 3A} - \delta = 0.$$

Bundan, $R_B = 47,5 \cdot 10^3 \text{ N} = 47,5 \text{ kN}$.

Demak, $R_C = R_B - F$ yoki $R_C = -202,5 \text{ kN}$.

Ko'chishlarni hisoblaymiz:

$$\delta_C = 0; \delta_D = \delta_C + \frac{202,5 \cdot 10^3 \cdot 4}{3 \cdot 2 \cdot 10^{11} \cdot 20 \cdot 10^{-4}} = 6,75 \cdot 10^{-4} \text{ m};$$

$$\delta_B = \delta_D + \frac{(-47,5 \cdot 10^3) \cdot 4}{2 \cdot 10^{11} \cdot 20 \cdot 10^{-4}} = 2 \cdot 10^{-4} \text{ m yoki } \delta_B = \delta.$$

Bu qiymatlar yordamida ko'chish epyurasi quriladi. (1.15-shakl, b).

19-masala

Ikki tomonidan mahkamlangan sterjenga F_1, F_2 kuchlar ta'sir etayotgan bo'lsin.

Berilgan: $A_1 = 2A_2$; $E_p = 2,1 \cdot 10^5 \text{ MPa}$; $[\sigma]_p = 210 \text{ MPa}$;

$E_{al} = 7 \cdot 10^4 \text{ MPa}$; $[\sigma]_{al} = 150 \text{ MPa}$; $a = 0,4 \text{ m}$; $b = 0,8 \text{ m}$;

$c = 1,4 \text{ m}$; (1.16-shakl, a).

Yechish:

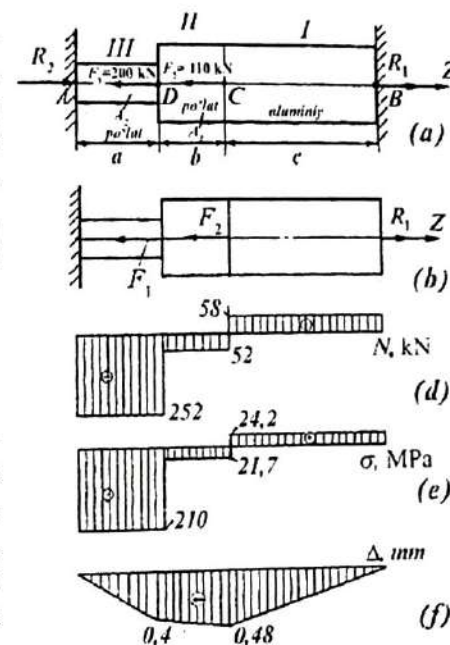
Mumkin bo'lgan statika tenglamalarini tuzamiz:

$$\sum Z = 0; R_1 + R_2 - F_1 - F_2 = 0.$$

Masalaning statik noaniqlik darajasini aniqlaymiz: $S = m - n = 2 - 1 = 1$. Demak, masala bir marta statik noaniq, masalani qo'shimcha deformatsiya tenglamasini yechish mumkin emas. Qo'shimcha tenglama quyidagi ko'rinishga ega bo'ladi:

$$\Delta l = \Delta l_a + \Delta l_b + \Delta l_c = 0. (a)$$

Sterjenning o'ng tomonidagi bog'lanishni tashlab yuborib, uning ta'sirini R_1 orqali ifodalaymiz (1.16-shakl, b). Shunda har bir uchastkada hosil bo'ladigan absolyut uzayishlar teng bo'ladi:



1.16-shakl

$$\Delta l_a = \frac{(-F_1 - F_2 + R_1) \cdot 0,4}{E_p \cdot A_2};$$

$$\Delta l_b = \frac{(-F_2 + R_1) \cdot 0,8}{E_p \cdot A_1};$$

$$\Delta l_c = \frac{R_1 \cdot 1,4}{E_{al} \cdot A_1}.$$

Bu qiymatlarni (a) ga qo'ysak:

$$\frac{(-F_1 - F_2 + R_1) \cdot 0,4}{E_p \cdot A_2} + \frac{(-F_2 + R_1) \cdot 0,8}{E_p \cdot A_1} + \frac{R_1 \cdot 1,4}{E_{al} \cdot A_1} = 0.$$

Bu yerda, $A_1 = 2A_2$; $E_p = 3E_{al}$ ekanligini e'tiborga olsak:

$$\frac{(-310 + R_1) \cdot 0,4}{3E_{al} \cdot A_2} + \frac{(-110 + R_1) \cdot 0,8}{3E_{al} \cdot 2A_2} + \frac{1,4 \cdot R_1}{E_{al} \cdot 2A_2} = 0.$$

Bu tenglamadan $R_1 = 58$ kN ga teng ekanligi aniqlanadi.

Shunday qilib, masala statik aniq masalaga aylandi. Bu masala uchun bo'ylama kuch epyurasi 1.16-shakl (d)da keltirilgan.

Sterjenning ko'ndalang kesim yuzasini aniqlaymiz:

$$A_2 \geq \frac{N_a}{[\sigma]_p} = \frac{252000}{210 \cdot 10^6} = 12 \cdot 10^{-4} \text{ m}^2. \text{ Unda, } A_1 = 24 \cdot 10^{-4} \text{ m}^2.$$

Har bir uchastka uchun $\sigma = N/A$ ifodadan kesimlarda bo'ladigan normal kuchlanish qiymatlarini aniqlaymiz va uning epyurasini ko'ramiz (1.16-shakl, e).

Sterjenning absolyut deformatsiyasi epyurasini ko'rish uchun uning C va D kesimlari deformatsiyasini aniqlash kifoyadir, chunki A va B kesimlarda Δ_A va Δ_B lar nolga teng.

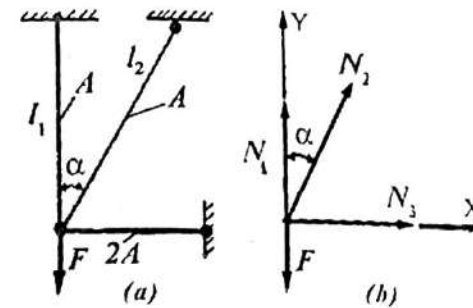
$$\Delta l_c = \frac{N_1 \cdot c}{E_{al} \cdot A_1} = \frac{58000 \cdot 1,4}{7 \cdot 10^{10} \cdot 24 \cdot 10^{-4}} = 0,48 \text{ mm (cho'zilayapti);}$$

$$\Delta l_a = \frac{N_3 \cdot a}{E_p \cdot A_2} = \frac{-252000 \cdot 1,4}{2,1 \cdot 10^{11} \cdot 12 \cdot 10^{-4}} = -0,4 \text{ mm (qisqarayapti).}$$

Topilgan qiymatlar bo'yicha Δ epyurasini quramiz (1.16-shakl, f).

20-masala

Energetik usul yordamida 1.17-shakl (a)da tasvirlangan sterjenlar tizimidagi ichki zo'riqishlar aniqlansin. Sterjenlar bir xil materialdan yasalgan bo'lib, quyidagi ma'lumotlar ma'lum: F ; l ; A ; $\alpha = 30^\circ$; $l_1 = l$; $l_2 = 1,548l$; $l_3 = 0,5774l$.



1.17-shakl

Yechish:

C tugunni fikran ajratib (1.17-shakl, b), uchta noma'lum bo'ylama kuchlar mavjudligiga ishonch hosil qilish mumkin. Ma'lumki, bir nuqta (tugun)da kesuvchi kuchlar tizimi ikkita muvozanat tenglamasini tuzishga imkon beradi:

$$\sum X_i = N_3 + N_2 \cdot \sin \alpha = 0; \quad \sum Y_i = N_1 + N_2 \cdot \cos \alpha - F = 0.$$

Demak, masala bir marta statik noaniq ekan.

Deformatsiyaning potensial energiyasini aniqlash ifodasini yozamiz:

$$U = \sum_{i=1}^3 \frac{N_i^2 l_i}{2EA_i} = \frac{N_1^2 l_1}{2EA} + \frac{N_2^2 l_2}{2EA} + \frac{N_3^2 l_3}{2E \cdot 2A}.$$

Endi deformatsiyaning potensial energiyasini funksiya $U = U(N_2)$ shaklida yozib olamiz:

$$U(N_2) = \frac{(F - N_2 \cos \alpha)^2 l}{2EA} + \frac{N_2^2 \cdot 1,1547l}{2EA} + \frac{(-N_2 \cdot \sin \alpha)^2 \cdot 0,5774l}{4EA}$$

$$\text{yoki } U(N_2) = \frac{l}{2EA} [(F - N_2 \cos \alpha)^2 + 1,1547N_2^2 + 0,0723N_2^2].$$

Deformatsiya potensial energiyasining minimallik shartiga ko'ra:

$$\frac{dU}{dN_2} = 0 \quad \text{yoki} \quad \frac{l}{2EA} [2(F - N_2 \cos \alpha)(-\cos \alpha) + 2,3094N_2 + 0,1446N_2] = 0;$$

bundan, $N_2 = 0,439 F$. Shu sababli, $N_1 = 0,6198 F$, $N_3 = -0,2195 F$. Demak, 1- va 2-sterjenlar cho'zilishga, 3-sterjen esa siqilishga qarshilik ko'rsatar ekan.

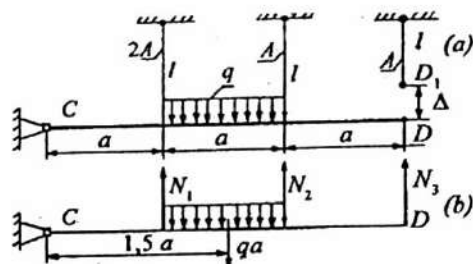
21-masala

Tekis taralgan q kuch bilan yuklangan absolyut bikir brus CD ni uchta sterjen yordamida shipga osish zarur (1.18-shakl, a). Ammo, 3-sterjen loyihadagi o'lchamga qaraganda Δ miqdorga qisqaroq qilib yasalgan. Sterjenlar bir xil materialdan yasalgan bo'lib, turlicha bikirliklarga ega. Montaj kuchlanishlari va tashqi kuchdan bo'lgan kuchlanishlar aniqlansin.

Yechish

Konstruksiyani yig'ish uchun D va D_1 nuqtalarga miqdor jihatdan teng, lekin qarama-qarshi yo'nalishlardagi bo'ylama kuchni qo'yish zarur (1.18-shakl, b).

CD brusning muvozanatini tekshiramiz:



1.18-shakl

$$\begin{aligned} \sum M_{C_1} &= 0, \\ qa \cdot 1,5a - N_1 \cdot a - N_2 \cdot 2a - N_3 \cdot 3a &= 0 \\ \text{yoki } N_1 + 2N_2 + 3N_3 - 1,5qa &= 0 \end{aligned}$$

Bitta tenglamada uchta (N_1 , N_2 , N_3) noma'lum bor: masala ikki marta statik noaniqdir.

Deformatsiyaning potensial energiyasini aniqlaymiz:

$$U = \frac{1}{2E} \sum_{i=1}^3 \frac{N_i^2 l_i}{A_i} \quad \text{yoki}$$

$$U = \frac{1}{2E} \left(\frac{N_1^2 l}{2A} + \frac{N_2^2 l}{A} + \frac{N_3^2 l}{A} \right) = \frac{l}{2EA} (0,5N_1^2 + N_2^2 + N_3^2).$$

Muvozanat tenglamasini nazarda tutib, deformatsiyaning potensial energiyasini

$$U(N_2, N_3) = \frac{1}{2EA} [0,5(-2N_2 - 3N_3 + 1,5qa)^2 + N_2^2 + N_3^2]$$

tarzida ifodalaymiz.

Oxirgi ifodadan navbat bilan N_2 va N_3 bo'yicha xususiy hosila olib, quyidagi algebraik tenglamalar tizimiga ega bo'lamiz:

$$\begin{cases} \frac{\partial u}{\partial N_2} = \frac{l}{2EA} = [(-2) \cdot (-2N_2 - 3N_3 + 1,5qa) + 2N_2] = \Delta \\ \frac{\partial u}{\partial N_3} = \frac{l}{2EA} [-3(2N_2 - 3N_3 + 1,5qa) + 2N_3] = \Delta \end{cases}$$

Bu tenglamalarni birgalikda yechib, quyidagi ichki zo'riqishlarni aniqlaymiz:

$$N_2 = 0,2qa - 0,4 \frac{EA\Delta}{l}; \quad N_3 = 0,3qa + 0,4 \frac{EA\Delta}{l}.$$

1-sterjendagi zo'riqish esa quyidagicha bo'ladi:

$$N_1 = 0,2qa - 0,4 \frac{EA\Delta}{l}.$$

Har bir sterjendagi normal kuchlanishlarni topamiz:

$$\sigma_1 = 0,1 \frac{qa}{A} - 0,2 \frac{EA\Delta}{l}; \quad \sigma_2 = 0,2 \frac{qa}{A} - 0,4 \frac{EA\Delta}{l}; \quad \sigma_3 = 0,3 \frac{qa}{A} + 0,4 \frac{EA\Delta}{l}.$$

Xususiy hollar:

a) $\Delta=0$ bo'lsin, ya'ni 3-sterjen ham loyiha bo'yicha tayyorlangan bo'lsin. Bu holda montaj kuchlanishlari nolga teng bo'lib, sterjenlarda faqatgina tashqi kuchlardan kuchlanishlar paydo bo'ladi:

$$\sigma'_1 = 0,1 \frac{qa}{A}; \quad \sigma'_2 = 0,2 \frac{qa}{A}; \quad \sigma'_3 = 0,3 \frac{qa}{A}.$$

b) $q=0$ bo'lsin. Bu holda sterjenlarda quyidagi montaj kuchlanishlari yuzaga keladi:

$$\sigma''_1 = -0,2 \frac{E\Delta}{l}; \quad \sigma''_2 = -0,4 \frac{E\Delta}{l}; \quad \sigma''_3 = 0,4 \frac{E\Delta}{l}.$$

22-masala

$F = 1,4 \text{ MN}$ kuch bilan yuklangan temir beton kolonnadagi (1.19-shakl) beton va temir armaturada bo'ladigan normal kuchlanish qiymati aniqlansin.

Berilgan: $A_{\text{arm}} = 10 \text{ sm}^2$; po'lat va beton elastiklik modullari orasidagi bog'lanish: $E_P: E_B = 15$.

Yechish:

F kuch ta'sirida beton va po'lat armaturada bo'ladigan reaksiya kuchlari har xil bo'ladi. Shuning uchun masala bir marta statik noaniqdir. Unda,

$$R_P + R_B = F. \quad (a)$$

Ularda bo'ladigan absolyut qisqarish deformatsiyasi quyidagiga teng bo'ladi:

$$\Delta l_B = \Delta l_P \quad \text{yoki} \quad \frac{R_B \cdot l}{E_B \cdot A_B} = \frac{R_P \cdot l}{E_P \cdot A_P},$$

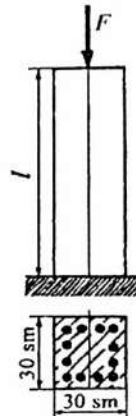
$$\text{unda} \quad R_P = R_B \cdot \frac{E_P \cdot A_P}{E_B \cdot A_B}.$$

Bu qiymatni (a) ga qo'yib, quyidagilarni hosil qilamiz:

$$R_B = \frac{F}{1 + \frac{E_P \cdot A_P}{E_B \cdot A_B}}$$

agarda, $A_B = 30 \cdot 30 \cdot 10 = 890 \text{ sm}^2$ bo'lsa, u holda:

$$R_B = \frac{1,4}{1 + 15 \cdot \frac{10}{890}} = 1,2 \text{ MN}.$$



1.19-shakl

Po'lat armaturada hosil bo'ladigan reaksiya kuchi $R_P = 1,4 - 1,2 = 0,2 \text{ MN}$ ga teng bo'ladi.

Unda kolonnaning ko'ndalang kesimida bo'ladigan normal kuchlanish (xususiyl og'irlikni hisobga olmaganda) quyidagiga teng bo'ladi:

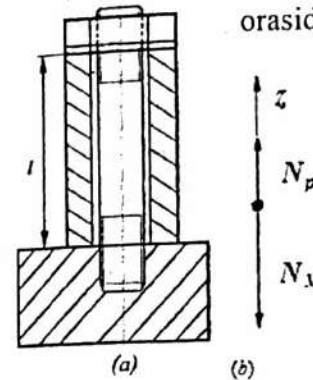
$$\sigma_P = \frac{R_P}{A_P} = \frac{-0,2 \cdot 10^6}{10 \cdot 10^{-4}} = -200 \text{ MPa}; \quad \sigma_B = \frac{R_B}{A_B} = \frac{-1,2 \cdot 10^6}{890 \cdot 10^{-4}} = -13,5 \text{ MPa}.$$

23-masala

Po'lat shpilka misdan yasalgan trubka (quvur)ning orasidan o'tkazilgan (1.20-shakl, a). Shpilka rezbasining qadami $t = 2,5 \text{ mm}$. Agar gaykani burab, $1/5$ ga aylantirilsa, shpilka va trubka kesimlarida qancha kuchlanish sodir bo'ladi?

Yechish:

Gayka buralganda shpilka cho'zilib, trubka siqiladi. Kesish usulini qo'llab, quyidagi muvozanat tenglamasiga ega bo'lamiz (1.20-shakl, b):



1.20-shakl

$$\sum Z = 0; \quad N_P - N_M = 0 \quad \text{yoki} \quad N_P = N_M = N,$$

bu yerda, N_P – shpilkani cho'zuvchi kuch;

N_M – trubkani (quvurni) siquvchi kuch. Demak, masala bir marta statik noaniq ekan.

Qo'shimcha $\Delta l \cdot M + \Delta l \cdot P = \frac{t}{5}$ tenglama ko'rinishda ifodalanadi.

Guk qonuniga asosan

$$\frac{Nl}{E_P A_P} + \frac{Nl}{E_M A_M} = \frac{t}{5} \cdot l.$$

$E_P = 2E_M = 2 \cdot 10^5 \text{ MPa}$ ekanligini e'tiborga olib, oxirgi ifodadan ichki zo'riqishni topamiz:

$$N = \frac{2tE_M A_M A_P}{5(A_M + 2A_P) \cdot l}$$

Shpilka va trubkaning kesim yuzalarini hisoblagandan so'ng, ifoda bo'yicha kuchlanishlarni osongina topish mumkin:

$$\sigma_p = 120,4 \text{ MPa}, \quad \sigma_M = -65 \text{ MPa}.$$

24-masala

Bir xil materialdan yasalgan va bir xil ko'ndalang kesimga ega sterjenlar 1.21-shakl (a)da ko'rsatilgandek mahkamlangan. 1-sterjen berilgan uzunligiga nisbatan Δ -qiymatga qisqaroq qilib yasalgan. Sterjenlarni yig'ish (montaj qilish) natijasida bo'ladigan boshlang'ich kuchlanishni aniqlash kerak. Berilgan: $\Delta=4\text{mm}$; $l=4\text{ m}$; $E=2 \cdot 10^5 \text{ MPa}$; $\alpha=45^\circ$.

Yechish:

Shakldan ko'rinib turibdiki, sterjenlar sistemasini yig'ishda uchta sterjenni ham cho'zishga to'g'ri keladi (shaklda punktir chiziqlar bilan ko'rsatilgan). C bog'lanishni kesib (1.21-shakl, b), unga ta'sir etayotgan kuchlardan muvozanat tenglamasini tuzamiz (α burchak o'zgarishini e'tiborga olmagan holda):

$$\sum X=0; \quad N_3 \cdot \sin \alpha - N_2 \sin \alpha = 0; \quad (a)$$

bu yerda, $N_2 = N_3$.

$$\sum Y=0; \quad N_1 - N_2 \cdot \cos \alpha - N_3 \cdot \cos \alpha = 0. \quad (b)$$

Demak, masala bir marta statik noaniqdir.

Qo'shimcha deformatsiya tenglamasi quyidagiga teng bo'ladi:

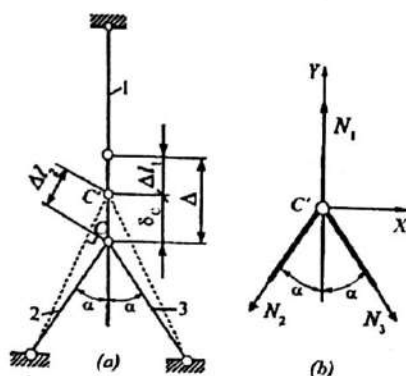
$$\Delta = \Delta l_1 + \delta_c, \quad \text{bu yerda,} \quad \delta_c = \frac{\Delta l_2}{\cos \alpha};$$

$$\text{unda } \Delta = \Delta l_1 + \frac{\Delta l_2}{\cos \alpha}.$$

Guk qonunini qo'llagan holda quyidagini hosil qilamiz:

$$\Delta = \frac{N_1 l}{EA} + \frac{N_2 l}{EA \cdot \cos \alpha}. \quad (d)$$

Bu yerda, 1-sterjen uzunligi (a), (b) va (d) tenglamalarni birgalikda yechib, sterjenlarda bo'ladigan zo'riqish kuchlarini aniqlaymiz:



1.21-shakl

$$N_1 = \frac{2\Delta EA \cdot \cos^2 \alpha}{(1 + 2\cos^2 \alpha)};$$

$$N_2 = N_3 = \frac{\Delta EA \cdot \cos^2 \alpha}{(1 + 2\cos^2 \alpha)}.$$

Sterjenlarda bo'ladigan normal kuchlanishlarni $\sigma = N/A$ ifoda yordamida aniqlaymiz:

$$\sigma_1 = \frac{2\Delta E \cos^2 \alpha}{(1 + 2\cos^2 \alpha)l} = \frac{2 \cdot 4 \cdot 10^{-4} \cdot 2 \cdot 10^5 \left(\frac{\sqrt{2}}{2}\right)^2}{\left[1 + 2\left(\frac{\sqrt{2}}{2}\right)^2\right] \cdot 4} = 100 \text{ MPa};$$

$$\sigma_2 = \sigma_3 = \frac{\Delta E \cos \alpha}{(1 + 2\cos^2 \alpha)l} = \frac{4 \cdot 10^{-4} \cdot 2 \cdot 10^5 \cdot \frac{\sqrt{2}}{2}}{\left[1 + 2\left(\frac{\sqrt{2}}{2}\right)^2\right] \cdot 4} = 70,7 \text{ MPa}.$$

25-masala

Po'lat materialdan yasalgan sterjenlar 1.22-shakl (a) da ko'rsatilgandek yig'ilgan. O'rtadagi sterjen Δl^0 sovutilganda, har bir sterjenda yuzaga keladigan ichki zo'riqishlarni aniqlang. Masalani yechishda energetik usul tavsiya qilinadi. Sterjenlarning moyillik koeffitsiyentlari $\beta_1 = \beta_3$ va β_2 lar ma'lum deb hisoblansin.

Yechish:

Agar o'rtacha sterjenning pastki uchi A tugunga ulanmaganda, u haroratning Δt^0 ga pasayishidan $\Delta l_2(t) = \alpha \cdot 2h \cdot \Delta t^0$ miqdorga qisqargan bo'lardi. Ammo yon tomonlardagi sterjenlarning qarshiligi tufayli o'rtadagi sterjen $\Delta l_2(t)$ dan kichik qiymatga qisqaradi. Umuman, o'rtadagi sterjenda cho'zuvchi, yon tomondagi sterjenda esa siquvchi bo'ylama kuchlar paydo bo'ladi (1.22-shakl).

A tugunning muvozanatini tekshirib, quyidagi ikkita tenglamaga ega bo'lamiz:

$$\sum X_1 = N_1 \sin \varphi - N_3 \sin \varphi = 0 \quad \text{yoki} \quad N_1 = N_3;$$

$$\sum Y_1 = N_2 - N_1 \cos \varphi - N_3 \cos \varphi = 0 \quad \text{yoki} \quad N_2 = 2N_1 \cos \varphi.$$

Demak, masala bir marta statik noaniq ekan.

Berilgan sistema uchun deformatsiyaning potensial energiyasini yozib olamiz:

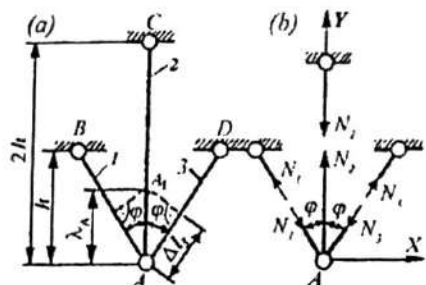
$$U = \frac{1}{2} \sum_{i=1}^3 N_i^2 \beta_i + 2l_2 \Delta t^0 N_2$$

yoki

$$U = \frac{1}{2} N_1^2 \beta_1 + \frac{1}{2} N_3^2 \beta_3 + \frac{1}{2} N_2^2 \beta_2 + \alpha \cdot 2h \Delta t^0 \cdot N_2.$$

Statika tenglamalarini nazarda tutib, quyidagiga ega bo'lamiz:

$$U(N_1) = \beta_1 N_1^2 + 2\cos^2 \beta \cdot \beta_2 N_1^2 + 4ah\Delta t^0 \cos \beta \cdot N_1.$$



1.22-shakl

$\frac{\partial u}{\partial N_1} = 0$ shartdan foydalanib, yon tomondagi sterjenlarda paydo bo'luvchi ichki zo'ri-qishlarni aniqlaymiz:

$$N_1 = N_3 = \frac{2ah\Delta t^0 \cos \varphi}{\beta_1 + 2\cos^2 \varphi \cdot \beta_2}.$$

O'rtacha sterjenda esa

$$N_2 = \frac{4ah\Delta t^0 \varphi}{\beta_1 + 2\cos^2 \varphi \cdot \beta_2}.$$

Ichki zoriqish yuzaga keladi.

26-masala

Ikkala uchi bilan mahkamlangan po'lat sterjen notekis qizdirilmoqda (1.23-shakl). Harorat $\Delta t(z) = \Delta t^0 \cdot z/l$ qonuniyat bo'yicha o'zgarmoqda. Sterjen kesimlarida haroratning o'zgarishi tufayli bo'lgan kuchlanish aniqlansin.

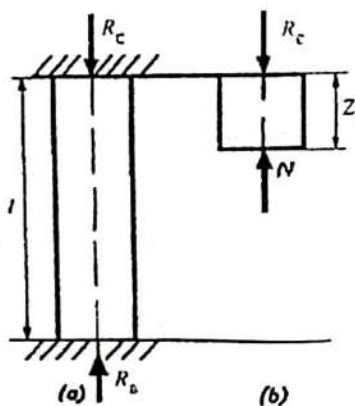
Yechish:

Haroratning o'zgarishi ikkala tayanchda ham reaksiya paydo qiladi:

$$\sum Z_1 = -R_c + R_B = 0.$$

Demak, masala bir marta statik noaniq ekan.

Kesish usulidan (1.23-shakl, b) foydalanib, $N = -R_c$ ekanligi aniqlanadi.



1.23-shakl

Unda:

$$U = \int_0^1 \frac{(-R_c)^2 dz}{2EA} + \alpha \int_0^1 (-R_c) \frac{\Delta t_0}{l} z dz = \frac{R_c^2 l}{2EA} - \frac{\alpha}{2} R_c \Delta t_0 l.$$

U dan R_c bo'yicha hosila olib, uni nolga tenglaymiz:

$$\frac{R_c l}{EA} - \frac{1}{2} \alpha \Delta t_0 l = 0;$$

bundan, $R_c = 0,5 \alpha EA \cdot \Delta t_0$.

B tayanchdagi reaksiya ham $0,5 \alpha EA \cdot \Delta t_0$ ga tengdir.

Demak, harorat kesimda $\sigma_1 = \frac{N}{A}$ yoki $\sigma_1 = 0,5 \alpha E \cdot \Delta t_0$ ga teng siquvchi kuchlanishni hosil qilar ekan.

27-masala

Uzunligi $l = 12$ m, qalinligi $h = 40 \cdot 10^{-2}$ m bo'lgan temir-beton ko'prigi plitasi C tayanchga $\Delta = 0,3 \cdot 10^{-2}$ m oraliq bilan o'rnatilgan (1.24-shakl). Temir-betonning chiziqli kengayish koeffitsiyenti $\alpha = 0,00001$, elastiklik moduli esa $E = 2 \cdot 10^4$ MPa. $\Delta t^0 = 30^\circ \text{C}$ ga oshirilganda plitada hosil bo'luvchi kuchlanish hisoblansin.

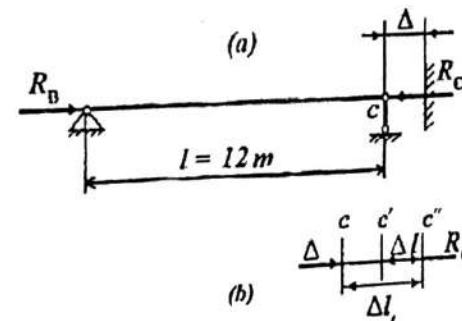
Yechish:

Agar B tayanch xalaqit bermaganda, oshuvidan Δ plita oraliqni yopib:

$$\Delta l_1 = \alpha l \cdot \Delta t = 0,00001 \cdot 12 \cdot 30 = 0,36 \cdot 10^{-2} \text{ m}$$

miqdorga erkin uzaygan bo'lar edi.

Bundan chiqdi, tayanchlarda bir xil, ammo yo'nalishlari qarama-qarshi bolgan R_B va R_c reaksiyalar, kesimlarda esa $N = R_B = -R_c$ zo'riqishlar vujudga kelar ekan. Bu reaksiyalarni yoki zo'riqishlarni aniqlash uchun muvozanat tenglamasining o'zi yetarli emas, chunki masala statik noaniq. Shu sababli yana bitta qo'shimcha tenglama tuzish zarur.



1.24-shakl

Masalaning mohiyatidan kelib chiqib deformatsiya tenglamasini tuzamiz:

$$\Delta t = \Delta \quad \text{yoki} \quad \Delta l_1 - \Delta l_{R_c} - \Delta = 0.$$

Guk qonunini e'tiborga olib, oxirgi ifodadan zo'riqishni aniqlaymiz:

$$N = \frac{EA}{l} (\Delta - \alpha l \Delta t^0).$$

Demak, plitaning eni $b=1\text{m}$ uzunlikdagi qismiga

$$N = \frac{2 \cdot 10^{10} \cdot 1 \cdot 40 - 10^{-2}}{12} (0,3 \cdot 10^{-2} - 0,36 \cdot 10^{-2}) = -400 \text{ kN}$$

Ichki zo'riqish to'g'ri kelar ekan.

$$\text{Plitadagi kuchlanishni hisoblaymiz: } \sigma = \frac{N}{A} = -1,0 \text{ MPa}.$$

28-masala

1.25-shaklda ko'rsatilgan sterjen bikir tekislikka chap uchi bilan mustahkamlangan, sterjenning o'ng uchi ikkinchi bikir tekislikka $\Delta=0,03 \text{ mm}$ ga yetmaydi.

Sterjenni 30^0 ga qizdirilsa (egilish deformatsiyasi e'tiborga olinmay), uning qismlaridagi kuchlanishlar aniqlansin.

Berilgan:

$$E_M = 1 \cdot 10^6 \text{ kg/sm}^2; E_P = 2 \cdot 10^6 \text{ kg/sm}^2; \alpha_p = 165 \cdot 10^{-7}; \\ \alpha_M = 125 \cdot 10^{-7}; a = 1,5 \text{ m}; b = 1 \text{ m}; A = 100 \text{ sm}^2.$$

Yechish:

1. Temperatura ta'siridan hosil bo'ladigan sterjenning absolyut uzayishini hisoblaymiz:

$$\Delta l_t = \alpha_p \cdot a \cdot \Delta t + \alpha_M \cdot b \cdot \Delta t = 165 \cdot 10^{-7} \cdot 1,5 \cdot 30^0 + \\ + 125 \cdot 10^{-7} \cdot 30 \cdot 1 = 0,112 \text{ sm} \quad (a)$$

$\Delta > \Delta$ demak masala statik noaniq.

2. Reaksiya kuchlarini R_A va R_B orqali ifodalasak, unda statika tenglamasi quyidagiga teng bo'ladi: $\sum Z=0$; $R_A - R_B = 0$.

Bu yerda:

$$R_A = R_B = R. \quad (b)$$

3. Qo'shimcha (deformatsiya) tenglamani tuzamiz:

$$\Delta l_{R_B} = \Delta l_t - \Delta. \quad (d)$$

Bu yerda:

$$\Delta l_{R_B} = \frac{R \cdot a}{E_M \cdot A_M} + \frac{R \cdot b}{E_P \cdot A_P} = R \left(\frac{150}{1 \cdot 10^6 \cdot 200} + \frac{100}{2 \cdot 10^6 \cdot 100} \right) = \\ = 1,25 R \cdot 10^{-6} \text{ sm}. \quad (e)$$



1.25-shakl

(a), (b), (e) larni (d) ga qo'yib, quyidagini hosil qilamiz: $1,25R \cdot 10^{-6} = 0,112 - 0,003 = 0,109$.

Bunda:

$$R = \frac{0,109 \cdot 10^6}{1,25} = 87000 \text{ kg} = 87 \text{ t}.$$

4. Sterjenlardagi kuchlanishlarni aniqlaymiz:

$$\sigma_M = -\frac{R}{A_M} = -\frac{87000}{200} = -435 \text{ kg/sm}^2;$$

$$\sigma_P = -\frac{R}{A_P} = -\frac{87000}{100} = -870 \text{ kg/sm}^2.$$

1.5-§. Cho'zilish va siqilishga doir topshiriqlar

1-topshiriq

Markaziy bo'lgan cho'zilish va siqilishdagi statik aniq masalalar.

Berilgan: Po'latdan yasalgan pog'onali sterjenga bo'ylama F_i kuchlar ta'sir etmoqda. Ruxsat etilgan normal kuchlanish $[\sigma]=210\text{MPa}$.

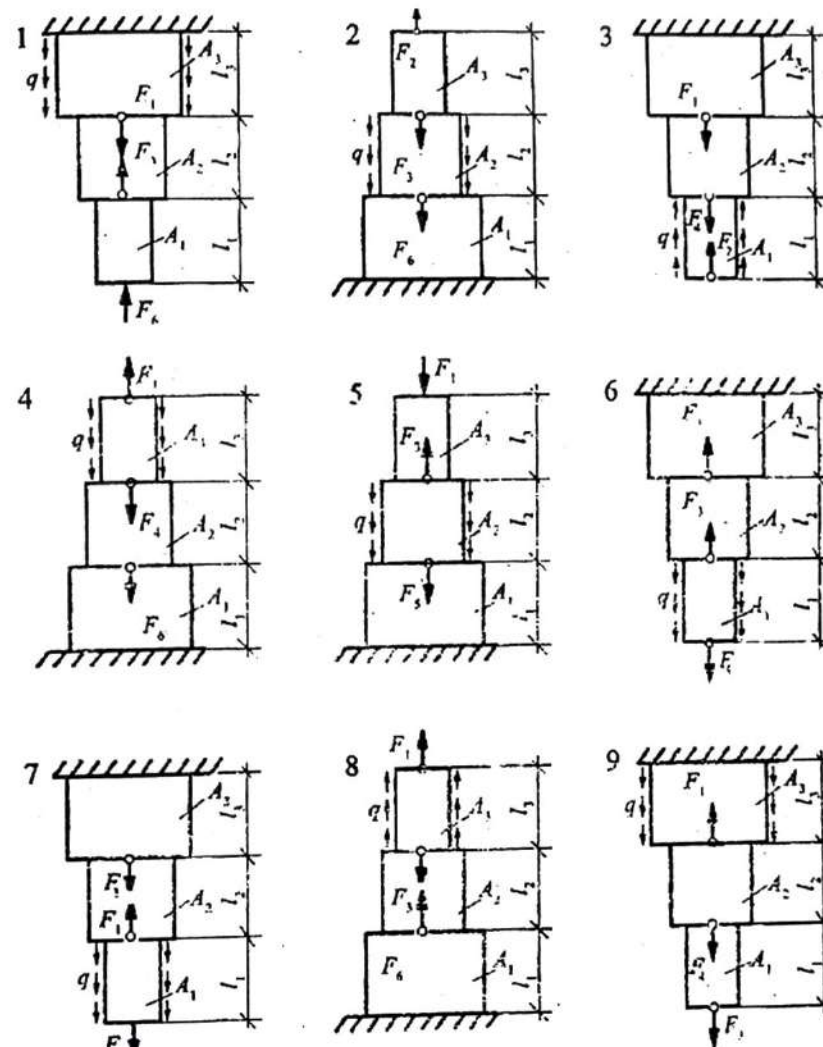
Topish kerak:

- 1) bo'ylama kuch, normal kuchlanish va deformatsiya epyuralari qurilsin;
- 2) sterjenning mustahkamligi tekshirilsin.

Kerakli ma'lumotlar 1.2-jadvalda, hisoblash sxemalari esa 1.26-shaklda keltirilgan.

1.2-jadval

№	Uchastka uzunligi, sm			Ko'ndalang kesim yuzasi, sm^2			Tashqi kuchlar, kN						Taralgan kuch, kN/sm
	l_1	l_2	l_3	A_1	A_2	A_3	F_1	F_2	F_3	F_4	F_5	F_6	q
1	40	80	50	8	4	6	60	180	160	140	100	80	0,30
2	50	46	70	10	4	4	120	80	200	160	120	60	0,40
3	80	40	30	14	4	8	80	140	160	60	60	80	0,20
4	42	60	80	12	8	6	100	140	100	120	40	60	0,35
5	52	42	62	12	16	8	60	120	160	80	100	40	0,25
6	78	50	60	8	4	16	120	80	140	100	60	120	0,45
7	30	80	42	10	12	6	80	100	120	80	60	80	0,50
8	42	62	50	6	12	4	120	140	100	60	80	60	0,50
9	60	30	48	10	4	8	140	80	60	100	120	40	0,65
10	70	50	690	6	8	4	100	120	100	140	40	80	0,70



1.26-shakl

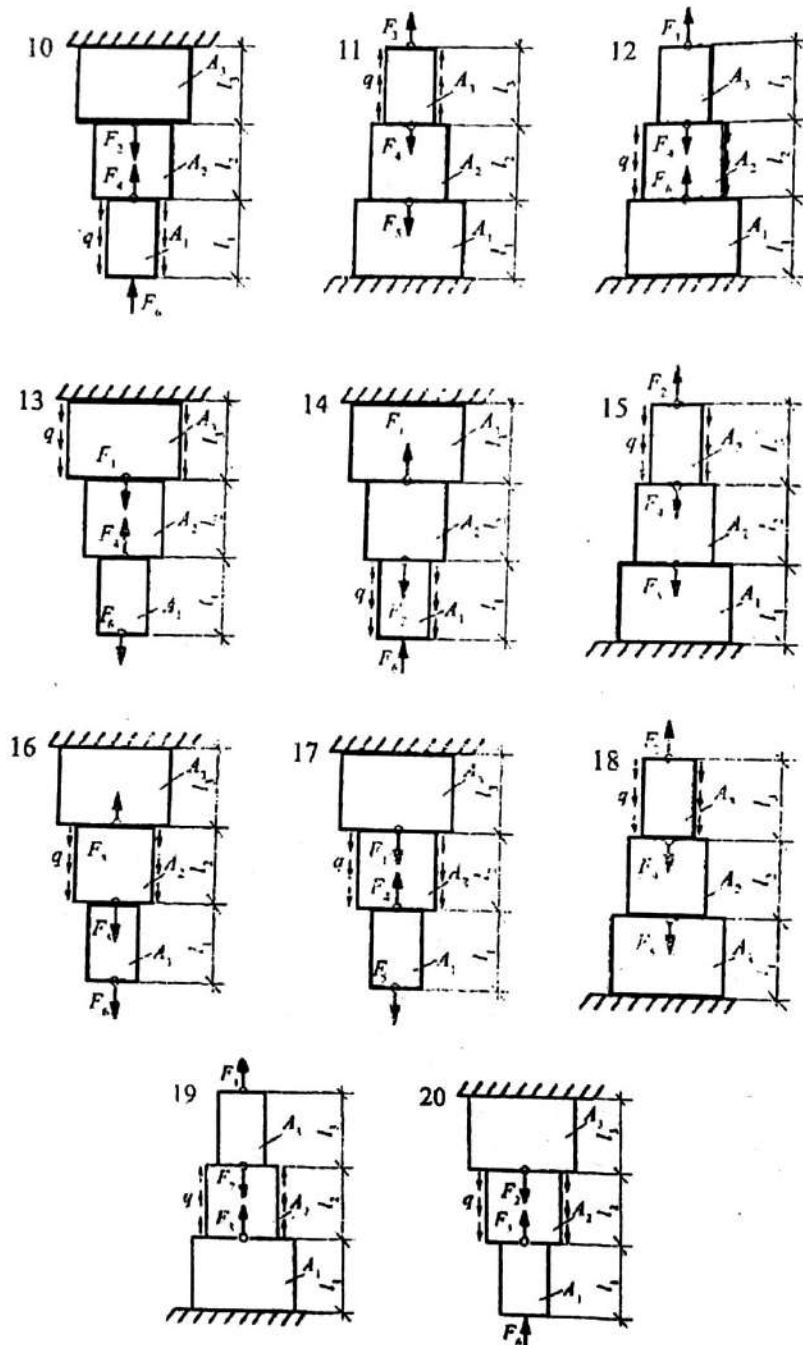
2-topshiriq

Markaziy bo'lgan cho'zilish va siqilishdagi statik noaniq masalalar.

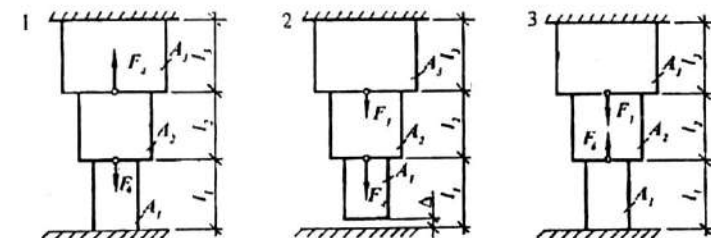
Berilgan: Po'latdan yasalgan pog'onali sterjenga bo'ylama F kuchlar ta'sir etmoqda. Ruxsat etilgan normal kuchlanish $[\sigma]=210$ MPa. Oraliq masofa $\Delta=0,1$ mm.

Topish kerak:

- 1) Tayanch reaksiya kuchlari aniqlansin;
 - 2) bo'ylama kuch, normal kuchlanish va deformatsiya epyuralari qurilsin;
 - 3) sterjenning mustahkamligi tekshirilsin.
- Kerakli ma'lumotlar 1.2-jadvalda, hisoblash sxemalari 1.27-shaklda keltirilgan.

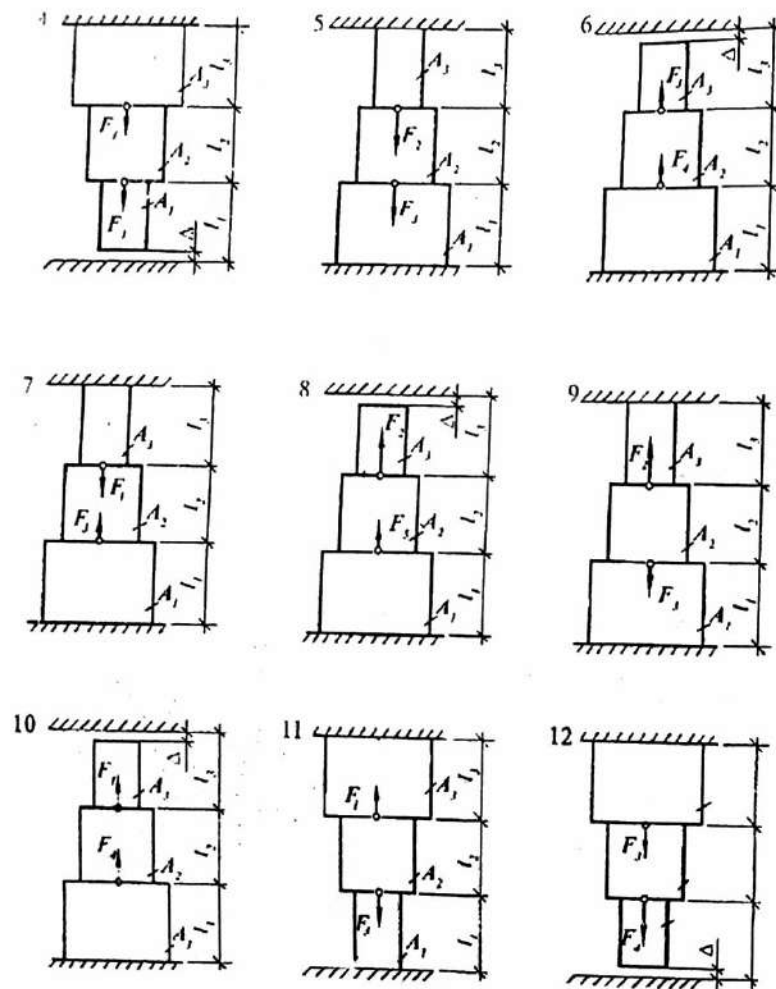


1.26-shakl (davomi)

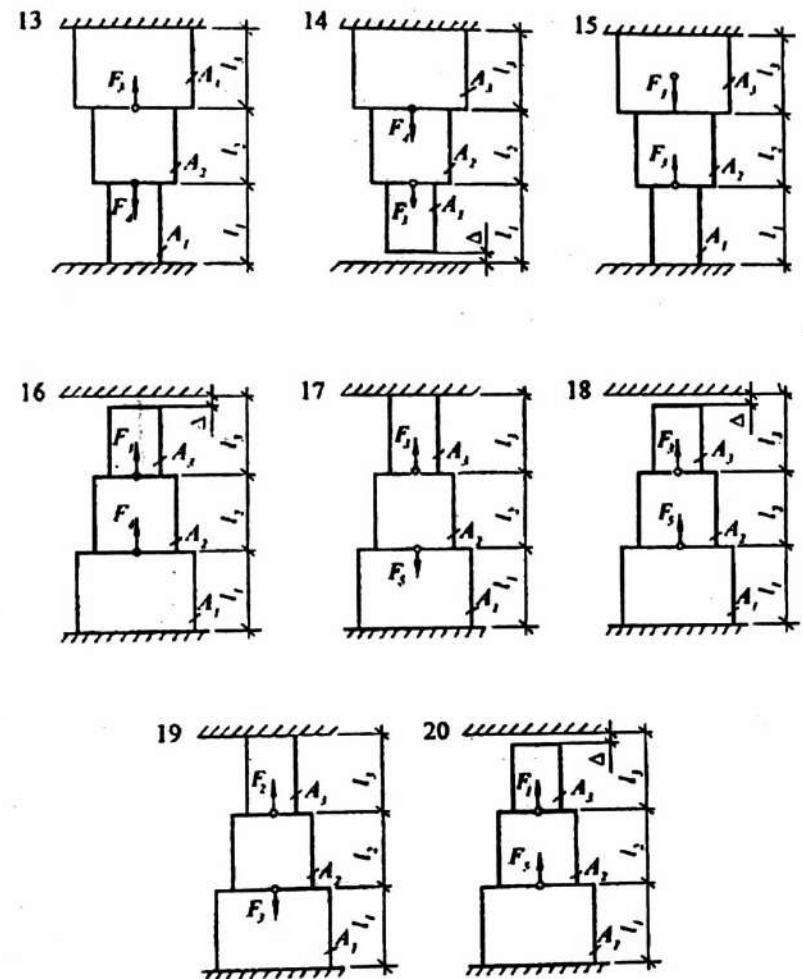


1.27-shakl

1.27-shakl (davomi)



1.27-shakl (davomi)



3-topshiriq

Absolyut qattiq balka po'lat tortqilar yordamida mahkamlangan. Unga to'plangan kuch F_1 va taralgan kuch q lar ta'sir etmoqda.

Tortqi uchun ruxsat etilgan normal kuchlanish $[\sigma]=210\text{MPa}$ ga teng.

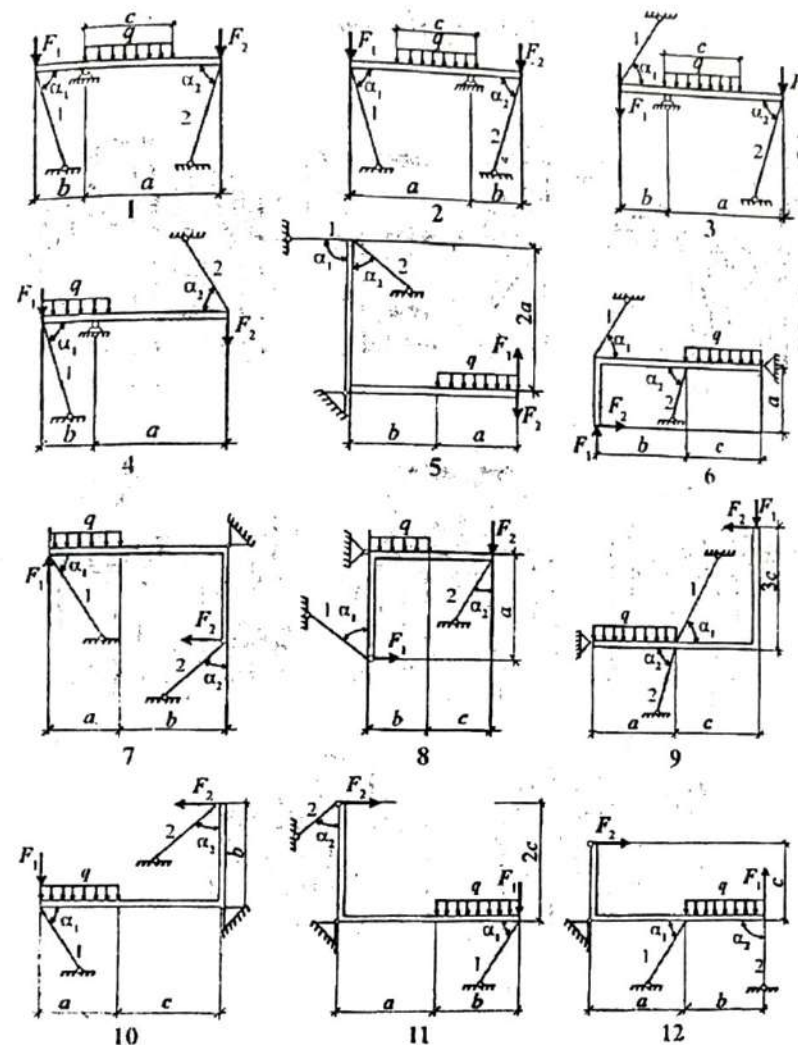
Topish kerak:

- 1) tortqilardagi bo'ylama kuch va kuchlanishlar topilsin;
- 2) tortqilardagi eng katta zo'riqish kuchi bo'yicha mustahkamlik shartiga binoan, eng katta ta'sir kuchi aniqlansin.

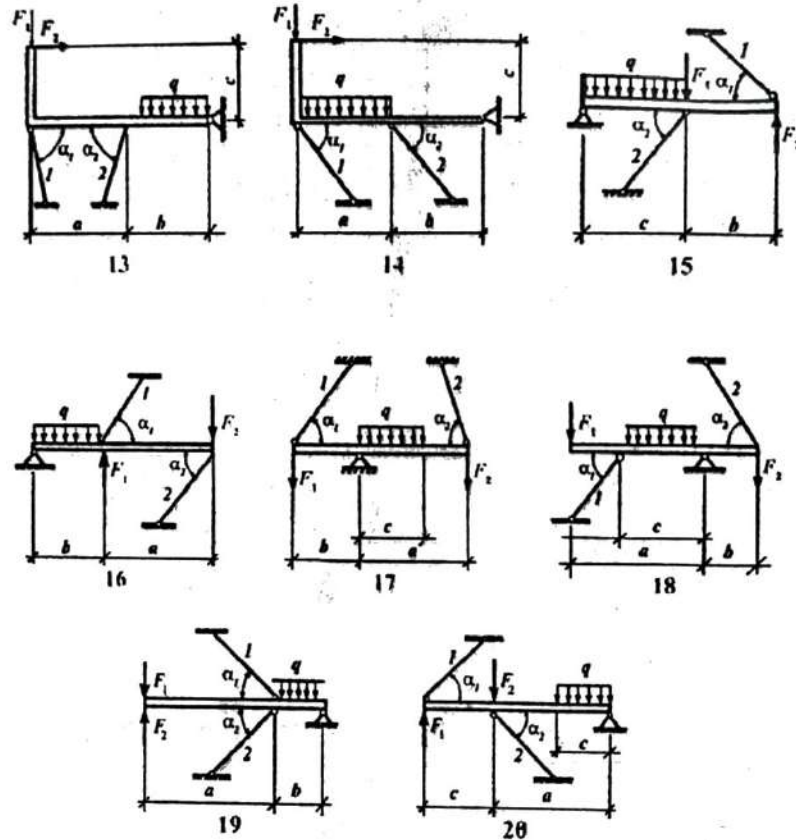
Kerakli ma'lumotlar 1.3-jadvalda, hisoblash sxemalari 1.28-shakl-da keltirilgan.

1.3-jadval

№	Tashqi kuchlar			Uchastka uzunligi, m					Ko'ndalang kesim yuzasi, sm^2		Burchak, grad	
	KN	KN	KN/m									
	F_1	F_2	q	a	b	c	l_1	l_2	A_1	A_2	α_1	α_2
1	-	50	-	5.4	2.6	0.8	1.0	1.2	6	12	90	45
2	30	-	-	4.8	2.8	1.2	1.4	1.0	8	10	45	90
	12	-	-	4.6	2.4	1.4	1.0	1.4	6	8	90	30
4	-	-	10	4.4	2.8	0.6	1.2	1.4	6	6	60	90
5	-	25	-	4.8	2.6	1.4	1.4	1.6	10	8	90	120
6	-	20	-	4.6	2.6	1.0	1.2	1.4	6	10	135	90
7	50	-	-	5.2	2.4	0.8	1.4	1.0	8	12	90	135
8	-	-	12	4.8	3.2	1.4	1.6	1.2	6	6	60	90
9	-	-	10	4.8	2.6	1.2	1.0	1.2	8	8	90	45
10	-	40	-	4.6	2.4	1.0	1.2	1.0	1.2	8	45	90



1.28-shakl



II BOB

NUQTADAGI KUCHLANISH VA DEFORMATSIYALANISH HOLATLARI

2.1-§. Asosiy mulohazalar

➤ Nuqtadagi kuchlanish holati deb, mazkur nuqta orqali o'tkaziladigan barcha yuzachalarda paydo bo'ladigan normal va urinma kuchlanishlarning to'plamiga aytiladi.

➤ Urinma kuchlanishlar ta'sir ko'rsatmaydigan yuzalarga bosh yuzalar deyiladi; bu yuzadagi normal kuchlanishlarga esa bosh kuchlanishlar deyiladi. Ular σ_1 ; σ_2 ; σ_3 orqali belgilanib, quyidagi munosabatda bo'ladi: $\sigma_1 > \sigma_2 > \sigma_3$.

➤ Kuchlanish holatlari chiziqli, tekis va fazoviy bo'ladi.

➤ Chiziqli kuchlanish holatida qiya kesimlardagi kuchlanishlar quyidagilarga teng:

$$\sigma_\alpha = \cos^2 \alpha \quad (2.1)$$

$$\tau_\alpha = \frac{\sigma_1}{2} \sin 2\alpha \quad (2.2)$$

➤ Tekis kuchlanish holatida qiya kesimlarda hosil bo'ladigan kuchlanishlar quyidagilarga teng:

$$\sigma_\alpha = \sigma_x \cdot \sin^2 \alpha + \sigma_y \cdot \cos^2 \alpha - \tau_{xy} \cdot \sin 2\alpha \quad (2.3)$$

$$\tau_\alpha = \frac{(\sigma_y - \sigma_x)}{2} \sin 2\alpha + \tau_{xy} \cdot \cos 2\alpha. \quad (2.4)$$

➤ Istalgan ikkita o'zaro perpendikulyar yuzalardagi urinma kuchlanishlar miqdor jihatdan bir-biriga teng, lekin yo'nalishlari qarama-qarshi bo'ladi:

$$\tau_{xy} = -\tau_{yx}$$

➤ Tekis kuchlanish holatida normal va urinma kuchlanishlarning ekstremal qiymatlari quyidagilarga teng:

$$\sigma_{\max/\min} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}; \quad (2.5)$$

$$\tau_{\max/\min} = \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2}. \quad (2.6)$$

➤ Bosh yuzalarning holati quyidagicha aniqlanadi:

$$\operatorname{tg} 2\alpha_0 = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad (2.7)$$

➤ Fazoviy kuchlanish holatida umumlashgan Guk qonuni quyidagicha ifodalanadi:

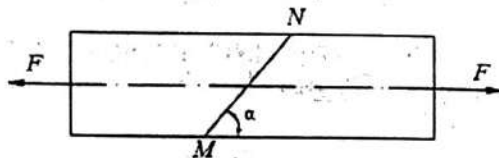
$$\begin{cases} \varepsilon_1 = \frac{1}{E} [\sigma_1 - \mu(\sigma_2 + \sigma_3)] \\ \varepsilon_2 = \frac{1}{E} [\sigma_2 - \mu(\sigma_1 + \sigma_3)] \\ \varepsilon_3 = \frac{1}{E} [\sigma_3 - \mu(\sigma_1 + \sigma_2)] \end{cases} \quad (2.8)$$

bu yerda, $\varepsilon_1; \varepsilon_2; \varepsilon_3$ – nisbiy deformatsiyalar.

2.2-§. Nuqtadagi kuchlanish va deformatsiyalanish holatlariga doir masalalar

29-masala

O'zaro qarama-qarshi yo'nalgan ikkita kuch bilan doiraviy sterjenning MN, qiya kesimida hosil bo'ladigan kuchlanishlarni hisoblang (2.1-shakl). Qaysi og'ma yuzalarda urinma kuchlanishlar ekstremal qiymatga ega bo'ladi? Quyidagilar ma'lum: $F = 300$ kN, $d = 8 \cdot 10^{-2}$ m, $\alpha = 30^\circ$.



2.1-shakl

Yechish:

Dastlab sterjen o'qiga nisbatan tik o'tkazilgan kesmadagi kuchlanishni aniqlaymiz:

$$\sigma_0 = \frac{4F}{\pi d^2} = \frac{4 \cdot 300}{3,14(8 \cdot 10^{-2})^2} = 5,97 \cdot 10^4 \frac{\text{kN}}{\text{m}^2} = 59,7 \text{ MPa}.$$

(2.1) va (2.2) formulalar yordamida qiya kesimlardagi kuchlanishlar hisoblanadi:

$$\sigma_\alpha = \sigma_0 \cos^2 \alpha = 4,48 \cdot 10^4 \frac{\text{kN}}{\text{m}^2} = 44,8 \text{ MPa};$$

$$\tau_\alpha = 0,5\sigma_0 \sin 2\alpha = 2,58 \cdot 10^4 \frac{\text{kN}}{\text{m}^2} = 25,8 \text{ MPa}.$$

Oxirgi formuladan ko'rinib turibdiki, $\sin 2\alpha + 1$ yoki 1 larni qabul qilgandagina, urinma kuchlanish, mos ravishda, o'zining eng katta yoki eng kichik qiymatlariga erishadi, xolos. Shunga ko'ra, sterjen o'qiga 45° va 135° burchaklarda qiyalangan yuzalarda urinma kuchlanishlar ekstremal qiymatlarga erishar ekan:

$$\tau_{\alpha=45}(\max) = 0,5\sigma_0 = 2,985 \cdot 10^4 \text{ kN/m}^2$$

$$\tau_{\alpha=135}(\min) = -0,5\sigma_0 = -2,985 \cdot 10^4 \text{ kN/m}^2$$

30-masala



2.2-shakl

O'zaro tik joylashgan va og'ma yuzachalardagi normal kuchlanishlarni $\sigma_\alpha = \sigma_0/3 = 30$ MPa ga teng bo'lgan, ko'ndalang kesimi kvadrat ($a = 16 \cdot 10^{-2}$ m) namunani cho'zuvchi kuchning qiymati topilsin (2.2-shakl). Namunaning bo'ylama o'qi bilan og'ma yuza normali orasidagi burchaklar aniqlansin.

Yechish:

Ko'ndalang kesimdagi normal kuchlanish σ_0 ni og'ma yuzachalardagi kuchlanishlar orqali ifodalaymiz:

$$\sigma_\alpha = \sigma_0 \cos^2 \alpha; \quad \sigma_\beta = \sigma_0 \cos^2 \beta = \sigma_0 \sin^2 \alpha,$$

bu yerda, $\beta = -(90 - \alpha)$.

Hadlab qo'shamiz: $\sigma_\alpha + \sigma_\beta = \sigma_0$ Bundan, $\sigma_0 = 120 \text{ MPa}$.

Namunani cho'zuvchi kuch:

$$F = \sigma_0 \cdot 2\alpha^2 = 2 \cdot 120 \cdot 10^3 (16 \cdot 10^{-2})^2 = 384 \text{ kN}.$$

Yuqoridagi ifodalardan:

$$\operatorname{tg}^2 \alpha = \frac{\sigma_\beta}{\sigma_\alpha} = 3; \alpha = 60^\circ; \beta = -(90^\circ - \alpha) = -30^\circ$$

31-masala

2.3-shakl (a) da tasvirlangan tekis kuchlanish holati uchun normal va urinma kuchlanishlarning ekstremal qiymatlari hamda ular ta'sir etuvchi yuzachalarning holatlarini aniqlash talab etiladi. Kuchlanishlar MPa larda berilgan.

Yechish:

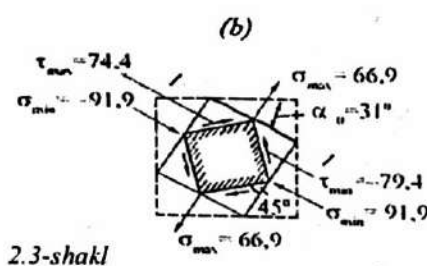
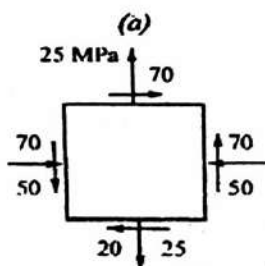
Quyidagi formula bo'yicha ekstremal normal kuchlanishlar aniqlanadi:

$$\sigma_{\max/\min} = \frac{\sigma_x + \sigma_y}{2} \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = 0,5 [(-50 + 25) \pm \sqrt{(-50 + 25)^2 + 4 \cdot (70)^2}] = -12,5 \pm 79,4.$$

Shunday qilib, $\sigma_{\max} = 66,9 \text{ MPa}$; $\sigma_{\min} = -91,9 \text{ MPa}$.

(11.6) formula yordamida ekstremal urinma kuchlanishlar topiladi:

$$\tau_{\max/\min} = \pm \frac{1}{2} \sqrt{(\sigma_x - \sigma_y)^2 + 4\tau_{xy}^2} = \pm 0,5 \sqrt{(-50 + 25)^2 + 4 \cdot (70)^2} = \pm 79,4.$$



2.3-shakl

Demak, $\tau_{\max} = 79,4 \text{ MPa}$; $\tau_{\min} = -79,4 \text{ MPa}$.

Bosh yuzachalarning holatini aniqlashda (2.7) formula ishlatiladi:

$\operatorname{tg} 2\alpha_0 = 2 \cdot 70 / (-50 - 25) = -1,866 \Rightarrow 2\alpha_0 = \operatorname{arctg}(-1,866) \approx -62^\circ \Rightarrow \alpha_0 = -31^\circ$.
 $\sigma_{\max}$ bosh kuchlanish ta'sir etadigan bosh yuzaning holatini chizmada tasvirlash uchun gorizontaal yuzani (chunki bu yuzadagi normal kuchlanishning algebraik qiymati vertikal yuzadagi normal kuchlanishdan katta) α_0 burchakka buramiz; α_0 ning yo'nalishini tanlashda shunga e'tibor berish kerakki, buralayotgan yuzachadagi (bizning misolimizda gorizontaal yuzacha) urinma kuchlanish element yuzining markaziga nisbatan aylantirishga intilsin. Shu qoidaga amal qilib, $\sigma_{\max}$ ning yo'nalishi doimo τ_{xy} va τ_{yx} larning yo'nalish (strelka) lari uchrashadigan koordinata o'qlarining ikkita choragi orqali o'tishiga ishonch hosil qilish mumkin (2.3-shakl, b).

2.3-shakl (b) da $\tau_{\max/\min}$ lar ta'sir etadigan siljish yuzalari ham ko'rsatilgan.

32-masala

Berilgan kuchlanish tenzoriga ko'ra kuchlanish holati aniqlansin (kuchlanishlar MPa larda berilgan).

$$\text{a) } \begin{vmatrix} 50 & 50 & 50 \\ 50 & 50 & 50 \\ 50 & 50 & 50 \end{vmatrix} \quad \text{b) } \begin{vmatrix} 0 & 40 & 60 \\ 40 & 0 & 50 \\ 60 & 50 & 0 \end{vmatrix}$$

Yechish:

Kuchlanish tenzorining invariantlarini hisoblaymiz:

$$\begin{aligned} \text{a) } S_1 &= 50 + 50 + 50 = 150 \text{ MPa}; \\ S_2 &= 50 \cdot 50 + 50 \cdot 50 + 50 \cdot 50 - 50^2 - 50^2 - 50^2 = 0; \\ S_3 &= 50 \cdot 50 \cdot 50 - 50 \cdot 50^2 - 50 \cdot 50^2 - 50 \cdot 50^2 + 2 \cdot 50 \cdot 50 \cdot 50 = 0, \end{aligned}$$

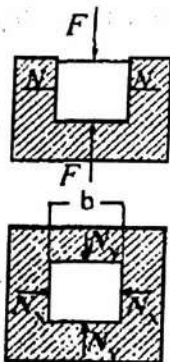
Demak, $S_1 = 150 \text{ MPa}$; $S_2 = S_3 = 0$ chiziqli kuchlanish holati.

$$\begin{aligned} \text{b) } S_1 &= 0; \\ S_2 &= -40^2 - 60^2 - 50^2 = -7700 \text{ MPa} \neq 0; \\ S_3 &= 2 \cdot 40 \cdot 60 = 24 \cdot 10^4 \text{ MPa} \neq 0. \end{aligned}$$

Bu hajmiy kuchlanish holatiga to'g'ri keladi, chunki $\theta = 0$ ga teng.

33-masala

Mis materialidan yasalgan elementar kubik «deformatsiyalanmaydigan» detal o'yig'iga tirqishsiz erkin o'rnatilgan bo'lib, $F = 400$ kN kuch bilan markaziy siqilgan (2.4-shakl). Kubik hajmining nisbiy o'zgarishi hamda deformatsiyaning solishtirma potensial energiyasi aniqlansin. Quyidagilar ma'lum deb hisoblansin: $E = 1 \cdot 10^5$ MPa, $\mu = 0,34$, $b = 10 \cdot 10^{-2}$ m.



2.4-shakl

Yechish:

Detal yetarlicha bikir bo'lganligi tufayli, quyidagi ikkita ko'ndalang nisbiy deformatsiyalar nolga teng bo'ladi:

$$\left. \begin{aligned} \varepsilon_x &= -\frac{\sigma_x}{E} + \mu \frac{\sigma_y}{E} + \mu \frac{\sigma_z}{E} = 0 \\ \varepsilon_y &= -\frac{\sigma_y}{E} + \mu \frac{\sigma_x}{E} + \mu \frac{\sigma_z}{E} = 0 \end{aligned} \right\} \quad (a)$$

bu yerda, $\sigma_x = \frac{N_x}{b^2}$; $\sigma_y = \frac{N_y}{b^2}$; $\sigma_z = \frac{F}{b^2}$. (b)

Deformatsiya tenglamasidan $\sigma_x = \sigma_y = \frac{\mu}{(1-\mu)} \cdot \sigma_z$ ekanligi kelib chiqadi. (b) ifodani e'tiborga olib, kubik devorlariga ta'sir etuvchi reaksiyalarni hisoblaymiz:

$$N_x = N_y = \frac{\mu}{(1-\mu)} \cdot F = 206,1 \text{ kN}.$$

Demak, kubikdagi bosh kuchlanishlar quyidagicha bo'lar ekan:

$$\sigma_1 = \sigma_x = -\frac{N_x}{b^2} = -20,61 \text{ MPa};$$

$$\sigma_2 = \sigma_y = -\frac{N_y}{b^2} = -20,61 \text{ MPa};$$

$$\sigma_3 = \sigma_z = -\frac{F}{b^2} = -40 \text{ MPa}.$$

Kubik hajmining nisbiy o'zgarishini hisoblaymiz:

$$\theta = \frac{1-2\mu}{E} (\sigma_1 + \sigma_2 + \sigma_3) = -25,99 \cdot 10^{-3}.$$

Hajmning o'zgarishidan hosil bo'lgan solishtirma potensial energiya quyidagiga teng bo'ladi:

$$a_h = \frac{1-2\mu}{6E} (\sigma_1 + \sigma_2 + \sigma_3)^2 = 351,82 \cdot 10^{-3} \cdot 10^3 \frac{\text{kN} \cdot \text{m}}{\text{m}^3} = 3,5182 \frac{\text{kN} \cdot \text{m}}{\text{m}^3}.$$

Endi shaklning o'zgarishidan hosil bo'lgan solishtirma potensial energiyani aniqlaymiz:

$$\begin{aligned} a_{sh} &= \frac{1+\mu}{6E} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] = \\ &= \frac{1+0,34}{6 \cdot 1 \cdot 10^5} [(-20,61 + 40)^2 + (-40 + 20,61)^2] = 1,6793 \frac{\text{kN} \cdot \text{m}}{\text{m}^3}. \end{aligned}$$

Shunday qilib, kubikning to'la solishtirma potensial energiyasi $a = a_h + a_{sh} = 5,1915 \text{ kN m/m}^3$ ga teng ekan.

34-masala

Berilgan nuqtadagi tekis kuchlanganlik holati uchun analitik va grafik usullarda quyidagilar aniqlansin (2.5-shakl, a):

- 1) ixtiyoriy yuzachadagi ($\alpha = 40^\circ$) kuchlanish;
- 2) $\sigma_{\max}$ va $\sigma_{\min}$ bosh kuchlanishlar;
- 3) bosh yuzalarning holati;
- 4) urinma kuchlanishlarning ekstremal qiymatlari.

Yechish:

I. Analitik usul

Ixtiyoriy yuzachadagi kuchlanishlarni ($\alpha = 40^\circ$) quyidagi formula yordamida aniqlaymiz (2.5-shakl, b):

$$\begin{aligned}\sigma_{\alpha} &= \sigma_x \cdot \cos^2 \alpha + \sigma_y \cdot \sin^2 \alpha - \tau_y \cdot \sin 2\alpha = \\ &= 40 \cos^2 40^\circ + (-20) \cdot \sin^2 40^\circ - (-30) \cdot \sin 80^\circ = 44,8 \text{ MPa};\end{aligned}$$

$$\tau_{\alpha} = \frac{\sigma_x - \sigma_y}{2} \sin 2\alpha + \tau_y \cdot \cos 2\alpha = \frac{40 - (-20)}{2} \sin 80^\circ = 24,3 \text{ MPa}.$$

Bosh kuchlanishlar quyidagiga teng bo'ladi:

$$\begin{aligned}\sigma_{\max/\min} &= \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_y^2} = \\ &= \frac{40 + (-20)}{2} \pm \sqrt{\left[\frac{40 - (-20)}{2}\right]^2 + (-30)^2} = 10 \pm 42,4 \text{ MPa}.\end{aligned}$$

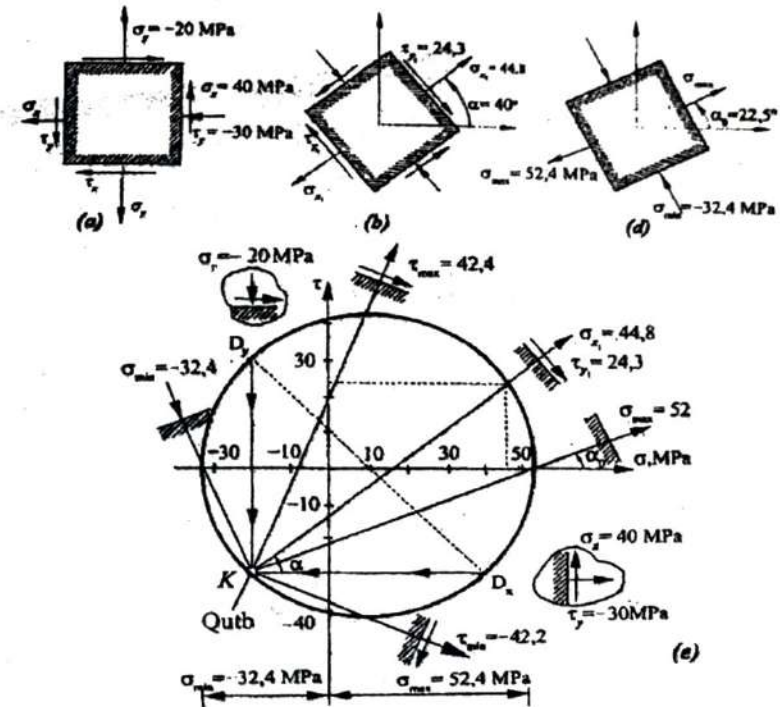
$$\sigma_{\max} = 52,4 \text{ MPa}; \quad \sigma_{\min} = -32,4 \text{ MPa}.$$

Endi bosh yuzalarning holatini aniqlaymiz (2.5-shakl, d):

$$\operatorname{tg} \alpha_0 = -\frac{\tau_y}{\sigma_{\max} - \tau_y} = -\frac{(-30)}{52,4 - (-20)} = 0,414; \quad \alpha_0 = 22,5^\circ.$$

Ekstremal urinma kuchlanishlar esa quyidagiga teng bo'ladi:

$$\tau_{\max/\min} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_y^2} = \pm 42,4 \text{ MPa}.$$



2.5-shakl

II. Grafik usul (Mor aylanasi yordamida)

1) Masshtab asosida X o'qi yo'nalishi bo'ylab σ va Y o'qi yo'nalishi bo'ylab τ qiymatlarini qo'yamiz (2.5-shakl, e).

2) $D_x(\sigma_x; \tau_y)$ va $D_y(\sigma_y; \tau_x)$ nuqtalarni berilgan koordinatalari bo'yicha 2.5-shakl. (e) ga qo'yamiz. Bu yerda, $\sigma_x = 40$, $\tau_y = -30$; $\sigma_y = -20$ va $\tau_x = -30$ ga tengdir.

3) D_x va D_y nuqtalardan o'tgan to'g'ri chiziqlarning gorizont σ o'qi bilan kesishidan hosil bo'lgan nuqta Mor doirasining markazidir.

4) D_x nuqtadan gorizont hamda D_y nuqtadan vertikal o'qlarga parallel chiziqlar o'tkazamiz. Bunda ularning kesishgan nuqtasi doiraning qutbini (K nuqta) hosil qiladi.

5) K nuqtadan (qutbdan) bizni qiziqtirayotgan tekislikka o'tkazilgan normalga parallel to'g'ri chiziq o'tkazib, Mor doirasi yordamida kerakli qiymatlarni aniqlaymiz (2.5-shakl, e).

2.3-§. Nuqtadagi kuchlanish va deformatsiyalanish holatlari bo'yicha topshiriq

4-topshiriq

Nuqtadagi tekis kuchlanlik holati.

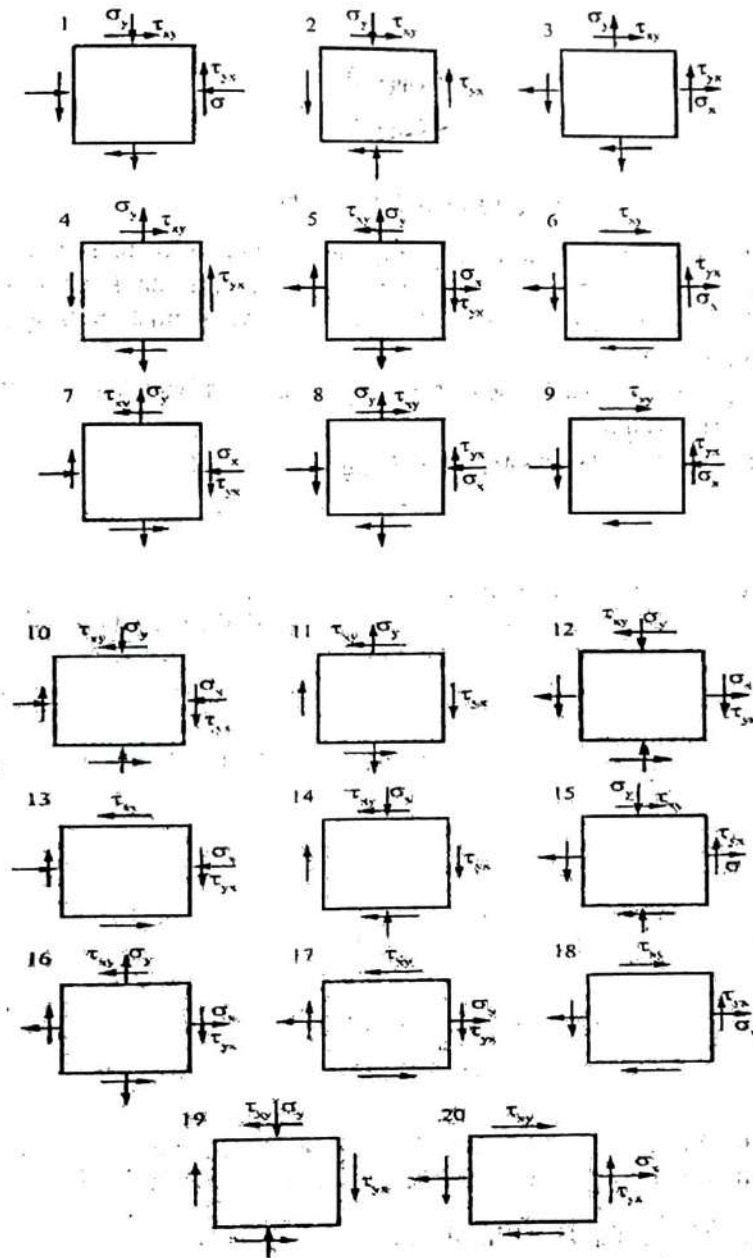
Berilgan: elastik deformatsiyalanuvchi jismning (2.6-shakl) xavfli nuqtasidan ajratib olingan to'g'ri to'rtburchakning qirralariga normal va urinma kuchlanishlar ta'sir etmoqda. Jism po'latdan yasalgan bo'lib, ruxsat etilgan normal kuchlanish $[\sigma] = 210 \text{ MPa}$ ga teng.

Topish kerak:

- 1) bosh kuchlanishlarning qiymati va yo'nalishi analitik usulda hamda Mor aylanasi yordamida aniqlansin;
 - 2) shaklda bosh yuzalarning holati va bosh kuchlanishlarning yo'nalishi ko'rsatilsin;
 - 3) eng katta urinma kuchlanishning qiymati aniqlansin;
 - 4) mustahkamlik nazariyalaridan birini qo'llab, jism materialining shu nuqtadagi mustahkamligi tekshirilsin;
 - 5) X, Y, Z o'qlari bo'yicha elementning nisbiy chiziqli deformatsiyasi topilsin;
 - 6) elementning nisbiy hajmiy deformatsiyasi topilsin.
- Hisoblash uchun qiymatlar 2.1-jadvalda, chizmalar esa 2.6-shaklda keltirilgan.

2.1-jadval

No	σ_x, MPa	σ_y, MPa	τ_{xy}, MPa
1	40	130	40
2	70	70	80
3	30	120	40
4	80	50	70
5	90	40	60
6	40	140	40
7	50	120	60
8	110	30	70
9	60	40	60
10	90	40	60



2.6-shakl

III-BOB SILJISH

3.1-§. Asosiy mulohazalar

- Elastik sterjendan ma'lum qiyalikdagi tekisliklar yordamida ajratib olingan elementar kubning tomonlariga faqat urinma kuchlanishlar ta'sir qilsa, u holda kubning bunday tekis kuchlanish holatiga sof siljish deyiladi.
- Faqat urinma kuchlanishlar paydo bo'ladigan yuzalar sof siljish yuzalari deyiladi.
- Siljishdagi Guk qonuni quyidagiga teng:

$$\tau = G \cdot \gamma, \quad (3.1)$$

bu yerda, τ – urinma kuchlanish;

G – siljishdagi elastiklik moduli va u quyidagicha ifodalanadi:

$$G = \frac{E}{2(1 + \mu)}, \quad (3.2)$$

γ – nisbiy siljish burchagi va u quyidagicha ifodalanadi:

$$\gamma = \frac{\delta}{a}, \quad (3.3)$$

bu yerda, δ – absolyut siljish;

a – elementning balandligi.

3.2-§. Siljishga doir masalalar

35- masala

Cho'zuvchi F kuch bilan yuklangan bolt qalpog'ining diametri D va balandligi h ni aniqlang (3.1-shakl). Quyidagi joiz kuchlanishlar ma'lum:

$\delta_{adm} = 140$ MPa (cho'zilishdagi);

$\tau_{adm} = 140$ MPa (kesilishdagi);

$\sigma'_{con} = 250$ MPa (ezilishdagi);

$d_0 = 3,4 \cdot 10^{-2}$ m; $d = 3,2 \cdot 10^{-2}$ m.

Material cho'zilish, kesilish va ezilishlarga bir xil qarshilik ko'rsatishi inobatga olingan holda, hisoblash olib borilsin.

Yechish

Chizmadan ko'rinib turibdiki, boltning sterjen qismi cho'zilishga, bosh qismi kesilishga, devorga tegib turgan tayanch yuzasi esa ezilishga qarshilik ko'rsatadi.

a) sterjenning cho'zilishdagi mustahkamlik shartidan joiz kuchni aniqlaymiz:

$$F_{adm} = \sigma_{adm} \frac{\pi d^2}{4} = 140 \cdot 10^6 \cdot \frac{3,14 (3,2 \cdot 10^{-2})^2}{4} = 112,6 \text{ kN}.$$

b) ezilishdagi mustahkamlik shartidan tayanch yuzaning diametrini aniqlaymiz:

$$\frac{F}{A_{ex}} \leq \sigma'_{con},$$

bu yerda, $A_{ex} = \frac{\pi (D^2 - d_0^2)}{4}$; $F = F_{adm}$ (masalaning

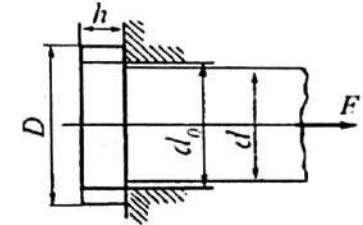
shartiga ko'ra).

$$\text{Bundan, } D \geq \sqrt{\frac{4 F_{adm}}{\pi \sigma'_{con}} + d_0^2} = \sqrt{\frac{4 \cdot 112,6}{3,14 \cdot 250 \cdot 10^6} + (3,4 \cdot 10^{-2})^2} = 4,17 \cdot 10^{-2} \text{ m},$$

$D = 4,2 \cdot 10^{-2}$ m deb qabul qilamiz.

c) kesilishga mustahkamlik shartini yozamiz: $\frac{F}{A_{kes}} \leq \tau_{adm},$

bu yerda, $A_{kes} = \pi d_0 h$; $F = F_{adm}$ (masalaning shartiga asosan).



3.1-shakl

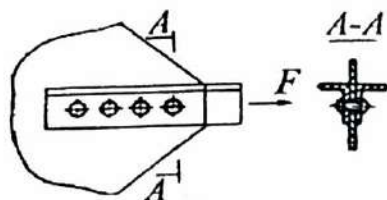
Demak,

$$h \geq \frac{F_{adm}}{\pi d_0 \tau_{adm}} = \frac{112,6}{3,14 \cdot 3,4 \cdot 10^{-2} \cdot 100 \cdot 10^6} = 1,055 \cdot 10^{-2} \text{ m};$$

$h = 1,1 \cdot 10^{-2} \text{ m}$ deb qabul qilinadi.

36- masala

Ikkita $90 \times 56 \times 8$ li burchaklikdan iborat ferma raskosini $\delta = 1,2 \cdot 10^{-2} \text{ m}$ qalinlikdagi kosinkaga biriktirish uchun diametri $d = 2,3 \cdot 10^{-2} \text{ m}$ li parchin mixdan nechta zarur (3.2-shakl). Raskosdagi cho'zuvchi kuch $F = 300 \text{ kN}$ bo'lib, uning materiali uchun $\sigma_{con} = 280 \text{ MPa}$, $\tau_{adm} = 100 \text{ MPa}$ ga teng.



3.2-shakl

Yechish:

Birikmada ikki kesilishli parchin mix ishlatilganligi sababli, uning kesilmaslik sharti

$$\tau_{kes} = \frac{F}{2n \cdot \frac{\pi d^2}{4}} \leq \tau_{adm} \quad \text{ko'rinishda yoziladi.}$$

$$\text{Bunda,} \quad n \geq \frac{N}{2 \cdot \frac{\pi d^2}{4} \cdot \tau_{adm}} = \frac{300 \cdot 10^3}{2 \cdot \frac{3,14(2,3 \cdot 10^{-2})^2}{4} \cdot 100} = 3,6.$$

Ezilmaslik sharti esa quyidagicha:

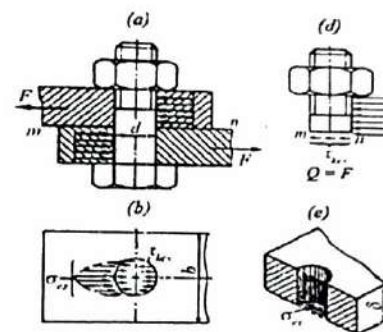
$$\sigma_{ez} = \frac{F}{n \cdot \delta \cdot d} \leq \sigma_{con}.$$

$$\text{Bunda,} \quad n \geq \frac{F}{\delta d \sigma_{con}} = \frac{300 \cdot 10^3}{1,2 \cdot 2,3 \cdot 10^{-2} \cdot 280} = 3,9.$$

Demak, birikmaga $n=4$ ta parchin mix yetarli ekan.

37- masala

3.3-shaklda tasvirlangan birikmadagi boltning diametrini aniqlash talab etiladi. List (polosa) ning qalinligi $\delta = 2 \cdot 10^{-2} \text{ m}$ bo'lib, uni cho'zuvchi kuch $F = 200 \text{ kN}$ ga teng. Bolt materiali uchun joiz kuchlanishlar ma'lum: $[\sigma]_{ez} = 200 \text{ MPa}$, $[\tau]_{kes} = 80 \text{ MPa}$.



3.3-shakl

Yechish:

Bolt bir kesilishli bo'lganligi sababli kesilishdagi mustahkamlik shartini

$$\frac{F}{\frac{\pi d^2}{4}} \leq [\tau]_{kes}$$

ko'rinishida yozib, undan boltning diametrini topamiz:

$$d \geq \sqrt{\frac{4F}{\pi [\tau]_{kes}}} = \sqrt{\frac{4 \cdot 200 \cdot 10^3}{3,14 \cdot 80 \cdot 10^6}} = 5,6 \cdot 10^{-2} \text{ m} \approx 6 \cdot 10^{-2} \text{ m}.$$

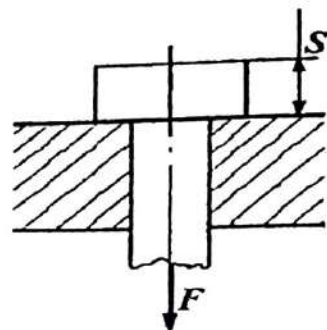
Ezilishdagi mustahkamlik $\frac{F}{\delta d} \leq [\sigma]_{ez}$ shartidan diametrni aniqlaymiz (3.3-shakl, e):

$$d \geq \frac{F}{\delta [\sigma]_{ez}} = \frac{200 \cdot 10^3}{2 \cdot 10^{-2} \cdot 200 \cdot 10^6} = 5 \cdot 10^{-2} \text{ m}.$$

Shunday qilib, $d = 6 \cdot 10^{-2} \text{ m}$ qabul qilindi.

38- masala

Cho'zuvchi kuch $F=40\text{kN}$ bo'lganda tortqining qalpog'ida hosil bo'luvchi kesuvchi kuchlanishning qiymatini aniqlang. Boltning diametri $d=2\cdot 10^{-2}\text{ m}$, qalpog'ning qalinligi esa $S=2,4\cdot 10^{-2}\text{ m}$ ga teng (3.4-shakl).



3.4-shakl

Yechish:

Shartli kesilish yuzasini hisoblaymiz:

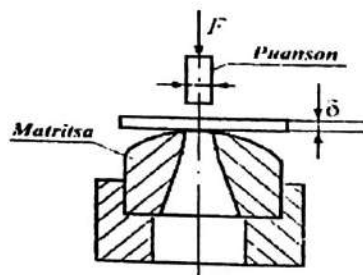
$$A_{ks} = \pi ds = 3,14 \cdot 2,4 \cdot 10^{-2} = 15,072 \cdot 10^{-4} \text{ m}^2.$$

Tortqining qalpog'idagi ezuvchi kuchlanish quyidagiga teng:

$$\sigma_{ks} = \frac{F}{A_{ks}} = \frac{40}{15,072 \cdot 10^{-4}} = 0,26 \cdot 10^7 \frac{\text{kN}}{\text{m}^2} = 0,26 \text{ MPa}.$$

39- masala

Qalinligi $\delta=1,4\cdot 10^{-2}\text{ m}$ bo'lgan po'lat listda (3.5-shakl) $d=1,4\cdot 10^{-2}\text{ m}$ li teshik o'yish uchun puansonga qanday F kuch qo'yilishi kerak? List materialining kesilishiga qarshilik qiluvchi mustahkamlik chegarasi $\tau_b=360\text{ MPa}$ ga teng.



3.5-shakl

Yechish:

Listga teshik o'yishda puanson kesgan yuza:

$$A_{ks} = \pi d \delta = 3,14 \cdot 1,4 \cdot 10^{-2} \cdot 1,2 \cdot 10^{-2} = 5,28 \cdot 10^{-4} \text{ m}^2.$$

Teshik o'yish uchun zarur bo'lgan kuch esa quyidagicha topiladi:

$$F = \tau_b \cdot A_{ks} = 360 \cdot 10^6 \cdot 5,28 \cdot 10^{-4} = 190,08 \text{ kN}.$$

3.3-§. Siljish deformatsiyasi bo'yicha topshiriq

5-topshiriq

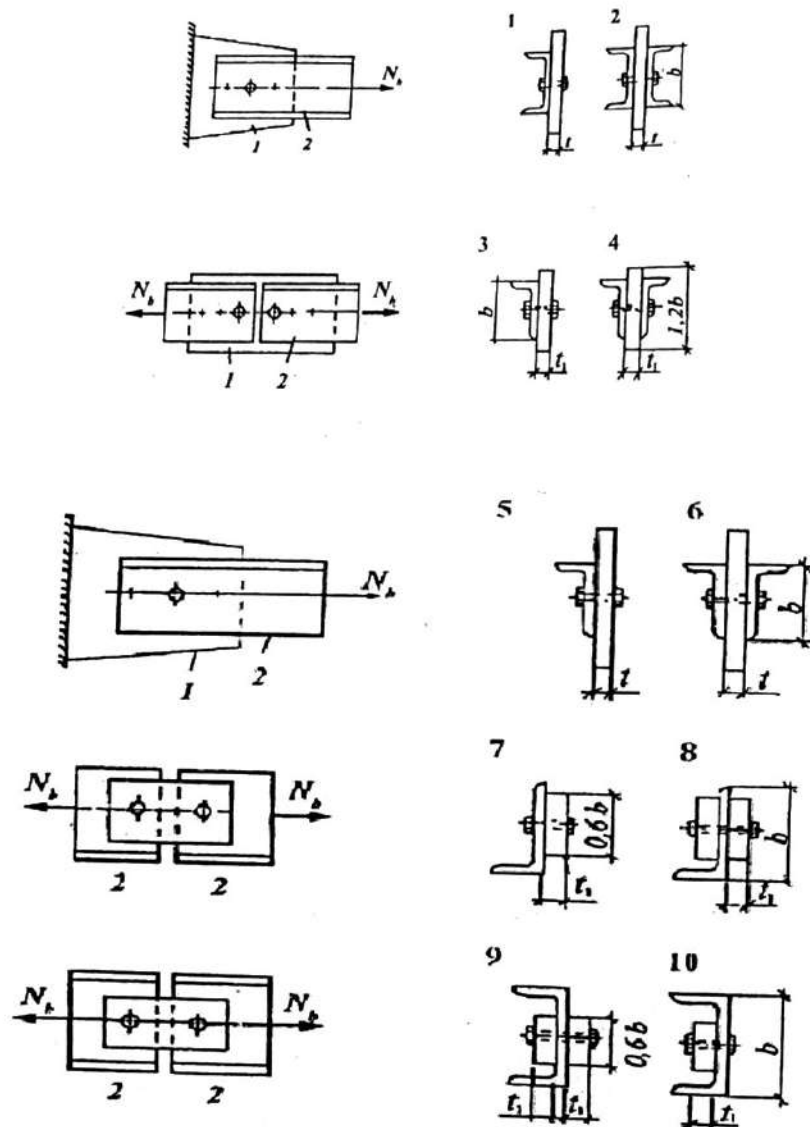
To'g'ri to'rtburchak va prokat profillardan tashkil topgan (3.6-shakl) kesimning (simmetrik) quyidagi qiymatlari aniqlansin:

- 1) kesimning og'irlik markazi;
- 2) bosh markaziy inersiya momentlari;
- 3) kesim masshtabda chizilib, kerakli o'qlar va o'lchamlar ko'rsatilsin.

Hisoblash uchun qiymatlar 3.1-jadvalda, kesim chizmalari 3.6-shaklda keltirilgan

3.1-jadval

№	O'lchashlar				Nomer	
	List, sm	Burchaklik, mm			Qo'shtavr (ГОСТ-72)	Shveller (ГОСТ-72)
	h	b	Teng yonli (ГОСТ 8509-72)	Teng yonsiz (ГОСТ 8510-72)		
1	16	1.8	80x80x8	-	18a	22
2	16	2.2	80x80x7	-	18	20a
3	16	1.4	80x80x6	-	16	22
4	18	2.0	-	100x63x6	20	22a
5	18	2.4	-	110x70x7	18a	22
6	18	1.6	-	90x56x6	18	22a
7	20	2.2	110x70x7	-	20a	24
8	20	2.4	-	110x70x7	18a	22
9	20	1.8	100x100x8	-	18a	20
10	22	2.4	-	125x80x12	22	24a



3.6-shakl

IV BOB

TEKIS SHAKLLARNING GEOMETRIK TAVSIFNOMALARI

4.1-§. Asosiy mulohazalar

Ko'rib o'tganimizdek, markaziy cho'zilish yoki siqilish deformatsiyasida brusning ko'ndalang kesim yuzi mustahkamlik va bikirlikni tavsiflovchi asosiy miqdor ekan.

Ammo buralish, egilish, murakkab deformatsiya, bo'ylama egilish (siqilgan sterjenlarning ustuvorligi) va dinamik yuklarning ta'sirlarini o'rganish jarayonida ko'ndalang kesim yuzi tekis shakllarning geometrik tavsifnomasi bo'la olmaydi.

Shuning uchun bu deformatsiya turlarini o'rganishda tekis shakllarning quyidagi geometrik tavsifnomalarini ham bilish muhimdir:

- tekis shakllarning o'qqa nisbatan statik momentlari;
- tekis shakllarning inersiya momentlari;
- tekis shakllarning markazdan qochirma inersiya momentlari;
- tekis shakllarning qarshilik momentlari.

4.2-§. Tekis shakllarning inersiya momentlari

1. Tekis shakllarning o'qqa nisbatan statik momentlari

Kesimning X va Y o'qlariga nisbatan statik momentlari (4.1-shakl) quyidagiga teng:

$$S_x = \int_{(A)} y \cdot dA, \quad (4.1)$$

$$S_y = \int_{(A)} x \cdot dA. \quad (4.2)$$

Bunda tekis shakllarning og'irlik markazi koordinatalari quyidagilarga teng bo'ladi:

$$x_c = \frac{S_y}{A}, \quad (4.3)$$

$$y_c = \frac{S_x}{A} \quad (4.4)$$

2. Tekis shakllarning inersiya momentlari

Ixtiyoriy tekis shaklning o'qli yoki ekvatorial inersiya momenti deb, miqdor jihatdan quyidagi integralga teng bo'lgan geometrik tavsifnomaga aytiladi:

a) X o'qiga nisbatan

$$J_x = \int_{(A)} y^2 \cdot dA; \quad (4.5)$$

b) Y o'qiga nisbatan

$$J_y = \int_{(A)} x^2 \cdot dA. \quad (4.6)$$

Tekis shaklning qutb inersiya momenti deb, quyidagi integral bilan aniqlanuvchi geometrik tavsifnomaga aytiladi:

$$J_\rho = \int_{(A)} \rho^2 \cdot dA; \quad (4.7)$$

bunda, ρ – elementar, dA – yuzachadan qutb nuqtasi – 0 gacha bo'lgan masofa.

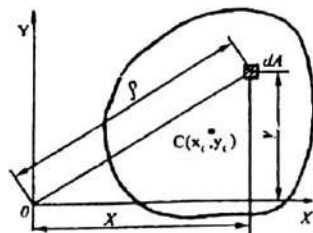
Tekis shakllarning o'qli (ekvatorial) va qutb inersiya momentlari faqat musbat kattaliklardir.

3. Tekis shakllarning markazdan qochirma inersiya momentlari

Tekis shaklning markazdan qochirma inersiya momenti deb quyidagi integralga teng bo'lgan geometrik tavsifnomaga aytiladi:

$$J_{xy} = \int_{(A)} xy dA; \quad (4.8)$$

Bittasi yoki ikkalasi ham tekis shaklning simmetriya o'qlari hisoblanuvchi o'qlarga nisbatan markazdan qochirma inersiya momentlari nolga teng bo'ladi. Bundan tashqari, XY ko'paytma musbat



4.1-shakl

yoki manfiy qiymatlarga ham ega bo'lishi mumkin.

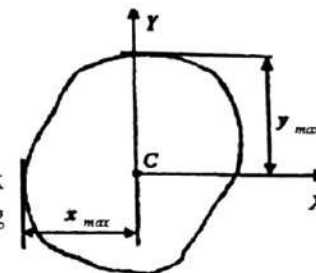
4. Tekis shakllarning qarshilik momentlari

Tekis shaklning o'qli qarshilik momenti deb biror o'qqa nisbatan olingan inersiya momentining shu o'qdan mazkur shaklda joylashgan eng uzoqdagi nuqtachaga bo'lgan masofaga nisbati bilan o'lchanadigan kattalikka aytiladi (4.2-shakl):

$$X \text{ o'qiga nisbatan } W_x = \frac{S_x}{y_{\max}};$$

$$Y \text{ o'qiga nisbatan } W_y = \frac{S_y}{x_{\max}};$$

Tekis shaklning qutb qarshilik momenti deb qutb inersiya momentining qutb nuqtasidan mazkur shaklda



4.2-shakl

$$W_\rho = \frac{S_\rho}{\rho_{\max}};$$

5. Oddiy tekis shakllarning inersiya momentlarini hisoblash

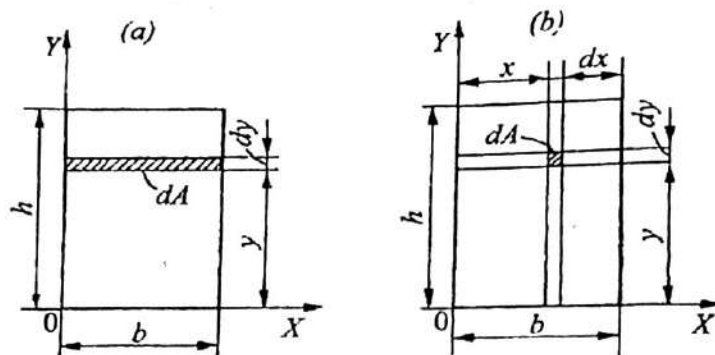
a) to'g'ri to'rtburchak. Asosi b va balandligi h bo'lgan to'g'ri to'rtburchakning asosidan o'tuvchi X o'qqa nisbatan inersiya momentini hisoblaymiz (4.3-shakl). Buning uchun X o'qidan ixtiyoriy Y masofada yuzasi $dA = bdy$ ga teng bo'lgan cheksiz yupqa qatlam ajratib olamiz. Inersiya momentining ta'rifiga asosan:

$$J_x = \int_{(A)} y^2 \cdot dA = \int_{(A)} y^2 \cdot bdy.$$

Oxirgi ifodani integrallashda uning 0 dan h gacha o'zgarishini e'tiborga olamiz:

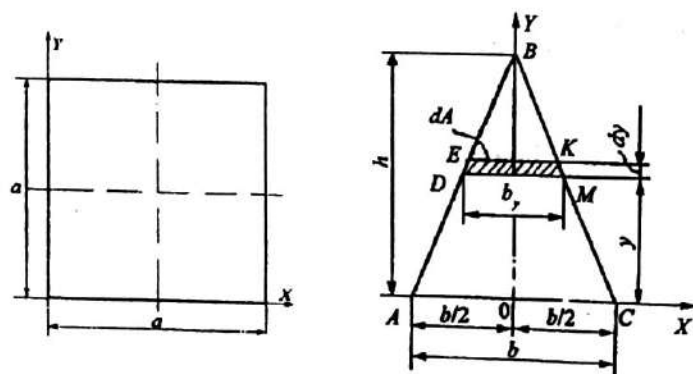
$$J_x = \int_0^h y b dy = \frac{bh^3}{3}$$

Xuddi shu tartibda vertikal Y o'qqa nisbatan inersiya momentini aniqlab, uning $J_y = \frac{bh^2}{3}$ ekanligiga ishonch hosil qilish mumkin.



4.3-shakl

b) kvadrat. (4.5) va (4.6) formulalarga asosan, tomonlari $b=h=a$ bo'lgan kvadrat uchun o'qli inersiya momentlari quyidagicha bo'ladi (4.4-shakl):



4.4-shakl

4.5-shakl

c) uchburchak. Asosi b va balandligi h ga teng bo'lgan ixtiyoriy uchburchakning asosidan o'tuvchi X o'qqa nisbatan inersiya

momentini hisoblaymiz (4.5-shakl). Uchburchakning asosidan ixtiyoriy y masofada qalinligi dy bo'lgan cheksiz yupqa DEKM trapetsiya ajratib olamiz. Agar trapetsiyaning yuzasini to'g'ri to'rtburchakning yuzasiga taxminan teng deb olsak, u holda, $dA \approx b_y dy$ bo'ladi.

ABC va DBM uchburchaklarning o'xshashligidan:

$$\frac{b_y}{b} = \frac{h-y}{h} \quad \text{yoki} \quad b_y = \frac{b}{h}(h-y)$$

munosabatni yozib olib, quyidagi formulani hosil qilamiz:

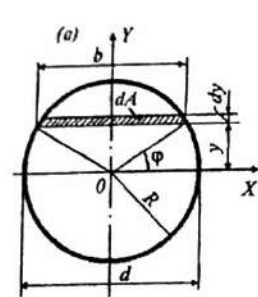
$$J_x = \int_{(A)} y^2 \cdot dA = \int_0^h y^2 \cdot \frac{b}{h}(h-y) dy = \frac{bh^3}{12}.$$

e) doira. Dastlab doiraning qutb inersiya momentini aniqlaymiz: buning uchun doira markazidan ixtiyoriy masofada yuzasi $dA = 2\rho dc$ bo'lgan cheksiz yupqa doira ajratib olamiz (4.6-shakl).

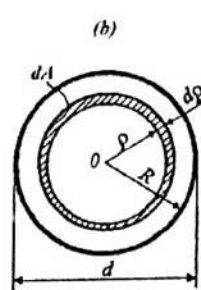
$$\text{U holda, } J_p = 2\pi \int_0^R \rho^2 d\rho = \frac{\pi R^4}{2} = \frac{\pi D^4}{32} \text{ bo'ladi.}$$

(4.7) formuladan foydalanib, doiraning ekvatorial inersiya momentlarini aniqlaymiz. Doira OX va OY o'qlarga nisbatan simmetrik shakl bo'lganligi uchun uning ekvatorial inersiya momentlari o'zaro teng bo'ladi:

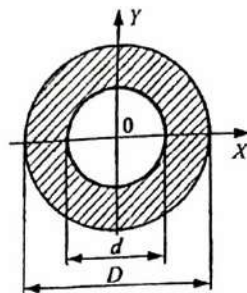
$$J_x = J_y = 0,5 J_p = \frac{\pi D^4}{64}.$$



4.6-shakl



4.7-shakl



f) halqa. 4.7-shaklda tasvirlangan halqa uchun inersiya momenti tashqi va ichki doiralarning qutb inersiya momentlari ayirmasiga teng bo'ladi:

$$J_p = \frac{\pi D^4}{32} - \frac{\pi d^4}{32} = \frac{\pi D^4}{32} (1 - c^4);$$

bu yerda, $c = \frac{d}{D}$.

Halqaning ekvatorial inersiya momentlari quyidagicha topiladi:

$$J_x = J_y = \frac{\pi D^4}{64} (1 - c^4).$$

g) murakkab tekis shakllarning inersiya momentlari. Murakkab tekis shakllarning inersiya momentlarini hisoblash aniq integralning quyidagi xossasiga asoslangan:

$$\iint_{(A)} y^2 dA = \sum_{i=1}^n \left[\left(\iint_{(A_i)} y^2 dA \right) = \int_{A_1} y^2 dA + \int_{A_2} y^2 dA + \dots + \int_{A_n} y^2 dA \right];$$

bundan, $A = A_1 + A_2 + A_3 + \dots + A_n$.

Bu integrallarning har biri tegishli qismning birorta x o'qiga nisbatan inersiya momentini ifodalaydi:

$$J_x = J_x^1 + J_x^2 + \dots + J_x^n.$$

Oxirgi ifoda murakkab tekis shaklning inersiya momenti uni tashkil etuvchi alohida qismlar inersiya momentlarining yig'indisiga teng, deyishga asos bo'ladi.

Bu qoida yordamida murakkab tekis shakllarning markazdan qochirma inersiya momentlarini ham aniqlash mumkin.

Murakkab ko'rinishdagi tekis shakllarning inersiya momentlarini osonroq aniqlash maqsadida uni, albatta, inersiya momentlari oldindan ma'lum bo'lgan bir necha oddiy: to'g'ri to'rtburchak, uchburchak, doira va shu kabi tekis shakllarga ajratish ma'qul.

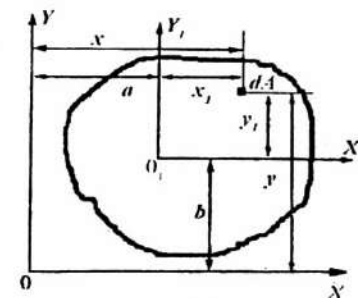
h) o'qlarni parallel ko'chirganda inersiya momentlarining o'zgarishi. Tekis kesim yuzaning markaziy o'qlariga parallel bo'lgan ixtiyoriy o'qlarga nisbatan inersiya momentlari quyidagicha aniqlanadi (4.8-shakl):

$$J_x = J_{x_c} + Fb^2; \quad (IV.9)$$

$$J_x = J_{x_1} + Fb^2; \quad J_y = J_{y_1} + Fa^2;$$

$$J_{xy} = J_{x_1 y_1} + Fab.$$

Bu yerda, X_1 va Y_1 – markaziy o'qlar.



4.8-shakl

7. O'qlar burilganda inersiya momentlarining o'zgarishi

$$J_{x_1} = J_x \cos^2 \alpha + J_y \sin^2 \alpha - J_{xy} \sin 2\alpha;$$

$$J_{y_1} = J_x \sin^2 \alpha + J_y \cos^2 \alpha + J_{xy} \sin 2\alpha;$$

$$J_{x_1 y_1} = \frac{1}{2} (J_x - J_y) \sin 2\alpha + J_{xy} \cos 2\alpha.$$

Quyidagi formuladan bosh inersiya o'qlarining vaziyati aniqlanadi:

$$\operatorname{tg} 2\alpha_0 = \frac{-2J_{xy}}{J_x - J_y}.$$

Bosh inersiya momentlari quyidagicha topiladi:

$$J_{\max/\min} = 0,5[(J_x + J_y) \pm \sqrt{(J_x - J_y)^2 + 4J_{xy}^2}].$$

E'tibor bering!

➤ Agarda kesim ikkita shakldan iborat bo'lsa, uning og'irlik markazi shu shakllar og'irlik markazini birlashtiruvchi to'g'ri chiziq ustida yotadi.

➤ Koordinata o'qlaridan biri simmetriya o'qi bo'lsa, unda Y va unga perpendikulyar yo'nalgan o'q bosh o'qlar bo'ladi, J_{xy} esa nolga teng bo'ladi.

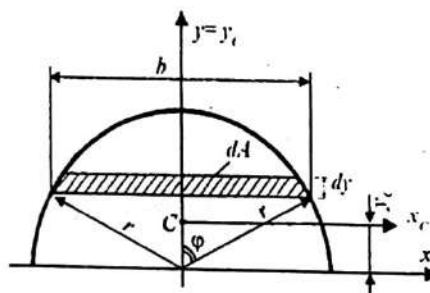
1-ilovada eng ko'p uchraydigan kesim yuzalarining geometrik tavsifnomalari berilgan.

40- masala

4.9-shaklda tasvirlangan yarim doira uchun quyidagilarni aniqlash talab etiladi:

- og'irlik markazi koordinatalari;
- bosh markaziy inersiya o'qlarining holati;
- bosh markaziy inersiya momentlarining qiymati:

$$W_x = \frac{S_x}{y_{\max}}.$$



4.9-shakl

Yechish:

a) Berilgan tekis shakl vertikal o'qqa nisbatan simmetrik joylashganligi uchun, uning og'irlik markazi shu o'q ustida yotishi tabiiy. Shuning uchun og'irlik markazi koordinatasi hisoblangan y_C masofani aniqlash kifoya.

Shtrixlangan qatlamning yuzachasi $dA \approx b \cdot dy = 2r \sin \varphi \cdot dy$ chizmadan $y = r \cos \varphi$ ga teng. Yangi o'zgaruvchini kiritamiz:

$$dy = -r \sin \varphi \cdot d\varphi.$$

U holda, kesimning x o'qiga nisbatan statik momenti quyidagiga teng bo'ladi:

$$\begin{aligned} S_x &= \int_{(A)} y dA = \int_{y=0}^{y=r} y \cdot 2r \cdot \sin \varphi \cdot dy = \int_{\varphi=\frac{\pi}{2}}^{\varphi=0} r \cdot \cos \varphi \cdot 2r \cdot \sin \varphi (-r \cdot \sin \varphi) d\varphi = \\ &= -2r^3 \int_{\varphi=\frac{\pi}{2}}^{\varphi=0} \sin^2 \varphi \cdot \cos \varphi d\varphi = \frac{2}{3} r^3. \end{aligned}$$

$$\text{b) Demak, } y_C = \frac{S_x}{A} = \frac{4r}{3\pi}.$$

Markaziy bosh inersiya o'qlaridan biri y_C simmetriya o'qi, ikkinchisi esa unga tik joylashgan x_C gorizontal o'qidir. Boshqacha qilib aytganda, tekis shakl bitta simmetriya o'qiga ega bo'lganligi uchun $J_{x_C y_C} = 0$ bo'ladi; bundan chiqdi, $\operatorname{tg} \alpha_0 = 0$ yoki $\alpha_0 = 0$

d) (4.3) formulaga binoan:

$$J_y = J_{y_C} = 0,5 \frac{\pi d^4}{64} = \frac{\pi r^4}{8} \approx 0,393 \cdot r^4;$$

$$J_{x_C} = J_x - y_C^2 A = \frac{\pi r^4}{8} - \left(\frac{4r}{3\pi}\right)^2 \cdot \frac{1}{2} \pi r^2 \approx 0,11r^4.$$

Buni quyidagicha ifodalash ma'qul: tekis shaklning og'irlik markazidan o'tuvchi x_C va y_C o'qlar bosh markaziy inersiya o'qlari bo'la oladi, chunki tekis shakl bitta simmetriya o'qiga ega bo'lganligi uchun $J_{x_C y_C} = 0$; bundan chiqdi, $\operatorname{tg} \alpha_0 = 0$ yoki $\alpha_0 = 0$.

41- masala

4.10-shaklda tasvirlangan kesim yuzaning X, Y o'qlariga nisbatan o'qli va markazdan qochma inersiya momentlari topilsin.

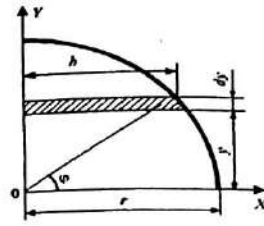
Yechish:

Chizmadan $dA \approx b \cdot dy = \sqrt{r^2 - y^2} dy$ ekanligi ma'lum. Yangi o'zgaruvchi kiritamiz:

$y = r \cdot \sin \varphi; \quad dy = r \cdot \cos \varphi d\varphi.$
Integral chegarasini 0 dan $\pi/2$ gacha olib, quyidagilarni hisoblaymiz:

$$J_x = \int_{(A)} y^2 dA \quad \text{yoki}$$

$$J_x = \int_0^{\pi/2} r^2 \sin^2 \varphi \sqrt{r^2 - r^2 \sin^2 \varphi} \cdot r \cos \varphi d\varphi = \frac{\pi \cdot r^4}{16}.$$



4.10-shakl

Xuddi shunga o'xshash:

$$J_y = \frac{\pi \cdot r^4}{16}.$$

Shtrixlangan qatlam uchun dJ_{xy} ni topishda quyidagi formuladan foydalanamiz:

$$dJ_{xy} = \frac{b}{2} \cdot y dA = \frac{r^4}{2} \cdot \cos^3 \varphi \sin \varphi d\varphi.$$

$$\text{Bundan, } J_{xy} = \frac{r^4}{2} \cdot \int_0^{\pi/2} \cos^3 \varphi \sin \varphi d\varphi = \frac{r^4}{8} = \frac{d^4}{128}.$$

42- masala

To'g'ri burchakli uchburchakning katetlariga parallel bo'lgan markaziy o'qlarga nisbatan markazdan qochma inersiya momenti topilsin (4.11-shakl).

Yechish:

Berilgan tekis shakl uchun quyidagilar ma'lum:

$$J_x = \frac{bh^3}{36}; \quad J_y = \frac{hb^3}{36}; \quad J_z = \frac{lc^3}{36},$$

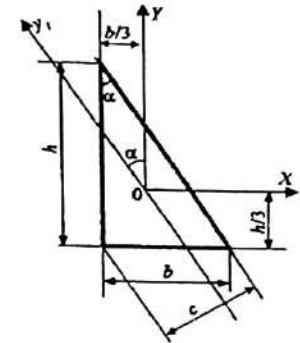
$$\text{bu yerda, } c = \frac{bh}{l}, \text{ chunki } A = \frac{bh}{2} = \frac{cl}{2};$$

u holda, $J_{x_1} = J_x \cos^2 \alpha + J_y \sin^2 \alpha + J_{xy} \cos 2\alpha;$

$$\text{Bu yerda, } \cos \alpha = \frac{b}{l}; \quad \sin \alpha = \frac{h}{l}.$$

Tegishli ixchamlashtirishdan so'ng

$$J_{xy} = -\frac{b^2 h^2}{72} \quad \text{ekanligi kelib chiqadi.}$$



4.11-shakl

43-masala

4.12-shaklda tasvirlangan 90x90x9 mm li teng yoqli burchaklik (GOST 8509-72) ning markaziy o'qlarga nisbatan markazdan qochma inersiya momentlari hisoblansin. Sortament jadvalidan quyidagi ma'lumotlar olingan: $J_{max} = J_{x_0} = 186 \text{ sm}^4, J_{min} = J_{y_0} = 48,6 \text{ sm}^4.$

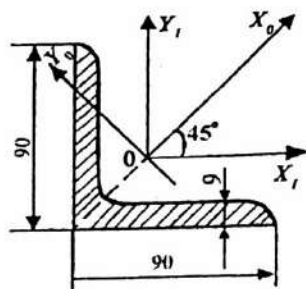
Yechish:

Quyidagi formula yordamida burchaklikning X_1 va Y_1 o'qlariga nisbatan markazdan qochma inersiya momentlarini aniqlaymiz:

$$J_{x_1 y_1} = \frac{I}{2} (J_{x_0} - J_{y_0}) \sin 2\alpha_0 + J_{x_0 y_0} \cdot \cos 2\alpha_0.$$

X_0 shaklning simmetriya o'qi hamda X_0 va Y_0 lar markaziy bosh o'qlar bo'lganligi uchun $J_{x_0 y_0} = 0$ ga teng. $X = 45^\circ$, chunki markazdan qochma inersiya momenti aniqlanishi kerak bo'lgan X va Y o'qlar markaziy bosh o'qlarga nisbatan soat strelkasi harakati bo'yicha burilgan.

$$\text{Shunday qilib, } J_{x_1 y_1} = \frac{186 - 48,6}{2} \cdot (-1) = -68,7 \text{ sm}^4$$



4.12-shakl

44-masala

To'g'ri burchakli to'rtburchak ($20 \times 2,75$ sm) va burchaklikdan № 12,5/8 (125×8012) iborat kesim berilgan (4.13-shakl). Kesimning bosh inersiya momenti (I_x ; I_y) va bosh o'qlarning yo'nalishi aniqlansin.

Yechish:

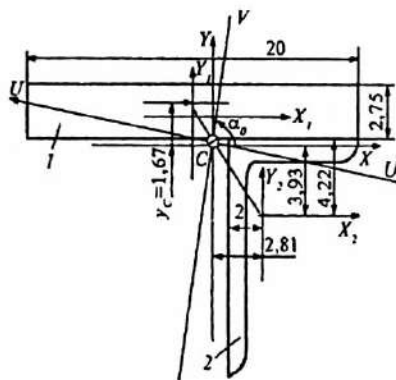
Sortament jadvalidan burchaklikning geometrik xarakteristikalarini aniqlaymiz:

$$A_1 = 23,36 \text{ sm}^2; X_0 = 2 \text{ sm}$$

$$Y_0 = 4,22 \text{ sm}; I_{x_1} = 364,79 \text{ sm}^4;$$

$$I_{y_1} = 116,84 \text{ sm}^4; J_{x_1 y_1} = 118 \text{ sm}^4.$$

X_1 va Y_1 o'qlarga nisbatan butun kesimning og'irlik markazini aniqlaymiz:



4.13-shakl

$$X_c = \frac{\sum A_i \cdot x_i}{\sum A_i} = \frac{A_1 \cdot x_1 + A_2 \cdot x_2}{A_1 + A_2};$$

$$Y_c = \frac{\sum A_i \cdot y_i}{\sum A_i} = \frac{A_1 \cdot y_1 + A_2 \cdot y_2}{A_1 + A_2};$$

Bu formulalarda X_1 va Y_1 lar nolga teng, chunki X_1 va Y_1 o'qlar to'g'ri burchakli to'rtburchakning og'irlik markazidan o'tadi. Unda:

$$X_C = \frac{23,36 \cdot \left(\frac{20}{2} - 8 + 2 \right)}{78,36} = 1,19 \text{ sm};$$

$$Y_C = \frac{-23,36 \cdot \left(\frac{2,75}{2} + 4,22 \right)}{78,36} = -1,67 \text{ sm}.$$

Topilgan qiymatlarni shaklga qo'yib butun kesim uchun og'irlik markazni C nuqta orqali ifodalaymiz. C nuqtadan X va Y koordinata o'qlarini o'tkazamiz va ularga nisbatan inersiya momentlarini aniqlaymiz:

$$I_x = \frac{20 \cdot (2,75)^3}{12} + 1,67^2 \cdot 55 + 364,79 + 3,93^2 \cdot 23,36 = 913 \text{ sm}^4;$$

$$I_y = \frac{2,75 \cdot 20^3}{12} + 1,19^2 \cdot 55 + 116,84 + 2,81^2 \cdot 23,36 = 2213 \text{ sm}^4;$$

$$I_{xy} = 0 + (-1,19) \cdot 1,67 \cdot 55 + 118 + 2,81 \cdot (-3,93) \cdot 23,36 = -249 \text{ sm}^4.$$

Bosh inersiya momentlarini aniqlaymiz:

$$I_u = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2} \right)^2 + I_{xy}^2} = \frac{913 + 2213}{2} \pm \sqrt{\left(\frac{913 - 2213}{2} \right)^2 + (-249)^2} = 1563 \pm 696 \text{ sm}^4.$$

Bu yerdan, $I_u = 2259 \text{ sm}^4$; $I_v = 867 \text{ sm}^4$.

Bosh o'qlarning yo'nalishini quyidagi formula yordamida aniqlaymiz:

$$\operatorname{tg} \alpha_0 = -\frac{I_{xy}}{I_v - I_y} = -\frac{(-249)}{2259 - 2213} = 5,41;$$

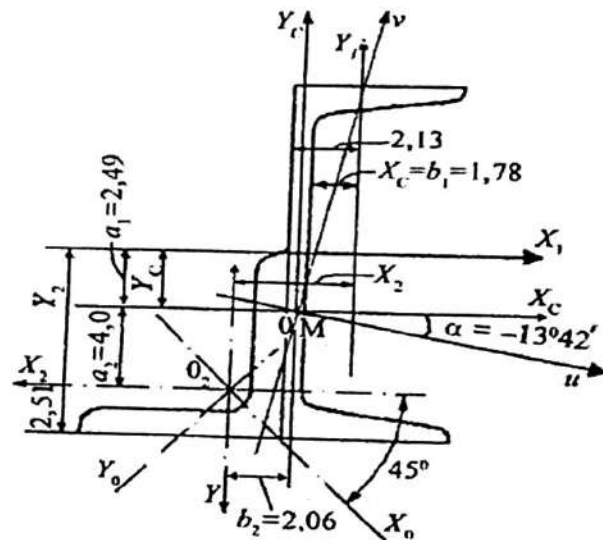
$$\alpha_0 = 79,5^\circ$$

α_0 burchak X o'qining musbat yo'nalishidan boshlab hisoblanadi. $\alpha_0 > 0$ bo'lganda, burchak shaklda soat strelkasi harakatiga teskari yo'nalishda, $\alpha_0 < 0$ bo'lganda esa, soat strelkasi harakati yo'nalishi bo'yicha olinadi.

45-masala

4.14-shaklda ko'rsatilgan 18a nomerli shvellar hamda 90x90x8 o'lchamli teng yonli burchaklikdan iborat kesim uchun quyidagilar topilsin:

- 1) og'irlik markazi;
- 2) og'irlik markazidan o'tuvchi ixtiyoriy o'qlar sistemasi (X_C , Y_C) ga nisbatan inersiya momentlari va markazdan qochirma inersiya momenti;
- 3) markaziy bosh inersiya o'qlarining yo'nalishi;
- 4) bosh inersiya momentlari.



4.14-shakl

Yechish:

ГОСТ 8240-56 dan shvellerga tegishli, ГОСТ 8509-57 dan teng yonli burchaklikka tegishli qiymatlarni ko'chirib yozamiz:

18a nomerli shvellar o'lchamlari 90x90x8 o'lchamli burchaklik

$$h_1 = 180 \text{ mm}$$

$$b_1 = 74 \text{ mm}$$

$$d_1 = 5,1 \text{ mm}$$

$$A_1 = 22,2 \text{ sm}^2$$

$$J_{x1} = 1190 \text{ sm}^4$$

$$J_{y1} = 132 \text{ sm}^4$$

$$Z_0 = 2,13 \text{ sm}$$

$$b_2 = 74 \text{ mm}$$

$$d_2 = 8 \text{ mm}$$

$$A_2 = 13,9 \text{ sm}^2$$

$$J_{x2} = J_{y2} = 106 \text{ sm}^4$$

$$J_{x0} = J_{y0} = 168 \text{ sm}^4$$

$$J_{y0} = J_{x0} = 43,8 \text{ sm}^4$$

$$Z_0 = 2,51 \text{ sm}$$

1. Yordamchi o'qlarga nisbatan (yordamchi o'qlar sifatida shvellarning X_1 va Y_1 o'qlari tanlangan) murakkab shaklning og'irlik markazini topamiz:

$$Y_C = \frac{\sum S_{x_i}}{\sum A} = \frac{A_1 \cdot Y_1 + A_2 \cdot Y_2}{A_1 + A_2} = \frac{0 + 13,9 \cdot [-(9 - 2,51)]}{22,2 + 14} = -2,49 \text{ sm},$$

$$X_C = \frac{\sum S_{y_i}}{\sum A} = \frac{A_1 \cdot X_1 + A_2 \cdot X_2}{A_1 + A_2} = \frac{0 + 13,9 \cdot [-(2,13 + 2,51)]}{22,2 + 14} = -1,78 \text{ sm}.$$

Qabul qilingan masshtabda X_C va Y_C larni 4.14-shaklga qo'yib chiqamiz va murakkab shaklning og'irlik markazi O ni topamiz. U O_1 va O_2 nuqtalarini birlashtiruvchi to'g'ri chiziq ustida yotadi.

2. Topilgan markaziy o'qlarga nisbatan murakkab shaklning inersiya momentlari hamda markazdan qochirma inersiya momentini formulaga ko'ra hisoblaymiz:

$$J_{x_C} = J_{x1} + A_1 \cdot a_1^2 + J_{x2} + A_2 \cdot a_2^2 = 1190 + 22,2 \cdot 2,49^2 + 106 + 13,9 \cdot 4^2 = 1656,5 \text{ sm}^4;$$

$$J_{y_C} = J_{y1} + A_1 \cdot b_1^2 + J_{y2} + A_2 \cdot b_2^2 = 132 + 22,2 \cdot 1,78^2 + 106 + 13,9 \cdot 2,86^2 = 422,3 \text{ sm}^4.$$

Ko'p hollarda teng yonli burchaklikning markazdan qochirma inersiya momentini topishda xatolikka yo'l qo'yiladi. Murakkab shaklning X_C , Y_C o'qlariga nisbatan markazdan qochirma inersiya momenti J_{x_C} ni topish uchun, avvalo, burchaklikning X_C va Y_C ga parallel bo'lgan markaziy o'qlari X_2 , Y_2 ga nisbatan markazdan qochirma inersiya momentini topish zarur.

Burchakning X_2, Y_2 o'qlarga nisbatan markazdan qochirma inersiya momenti quyidagicha topiladi:

$$J_{X_2Y_2} = \frac{J_{X_0} - J_{Y_0}}{2} \sin 2\alpha + J_{X_0Y_0} \cos 2\alpha.$$

X_0, Y_0 burchaklikning bosh inersiya o'qlari bo'lganidan, X_0, Y_0 o'qlari bilan X_2, Y_2 o'qlari orasidagi burchak $\alpha = 45^\circ$.

$$J_{X_2Y_2} = \frac{168 - 43,6}{2} \sin(2 \cdot 45^\circ) = 62,2 \text{ sm}^4.$$

$$J_{X_cY_c} = J_{X_1Y_1} + A_1 \cdot a_1 \cdot b_1 + J_{X_2Y_2} + A_2 \cdot a_2 \cdot b_2 =$$

$$= 0 + 22,2 \cdot (+1,78) \cdot (+2,49) + 62,2 + 13,9 \cdot (-2,86) \cdot (-4) = 319,7 \text{ sm}^4.$$

Murakkab shakl bosh inersiya o'qlarining markaziy o'qlarga nisbatan og'ish burchagi α ni quyidagicha topamiz:

$$\operatorname{tg} 2\alpha = \frac{-2J_{X_cY_c}}{J_{X_c} - J_{Y_c}} = -\frac{2 \cdot 319,7}{1656,5 - 422,3} = -0,518;$$

$$2\alpha = -27^\circ 24', \alpha = -13^\circ 42'.$$

Bosh inersiya momentlarini topamiz:

$$J_u = J_{X_c} \cos^2 \alpha + J_{Y_c} \sin^2 \alpha - J_{X_cY_c} \sin 2\alpha =$$

$$= 1656,5 \cdot (\cos(-13^\circ 42'))^2 + 422,3 \cdot (\sin(-13^\circ 42'))^2 - 319,7 \cdot (\sin(-27^\circ 24')) =$$

$$= 1735,7 \text{ sm}^4.$$

$$J_v = J_{X_c} \sin^2 \alpha + J_{Y_c} \cos^2 \alpha - J_{X_cY_c} \sin 2\alpha =$$

$$= 1656,5 \cdot (\sin(-13^\circ 42'))^2 + 422,3 \cdot (\cos(-13^\circ 42'))^2 - 319,7 \cdot (\sin(-27^\circ 24')) =$$

$$= 345,7 \text{ sm}^4.$$

Tekshirish:

$$J_{X_c} + J_{Y_c} = 1656,5 + 422,3 = 2078,8 \text{ sm}^4;$$

$$J_u + J_v = 1735,7 + 345,7 = 2081,4 \text{ sm}^4$$

Farqi $(J_u + J_v) - (J_{X_c} + J_{Y_c}) = 2,6 \text{ sm}$ Chiqqan xato ruxsat etiladigan chegarada.

$$J_{uv} = \frac{J_{X_c} - J_{Y_c}}{2} \sin 2\alpha + J_{X_cY_c} \cos 2\alpha =$$

$$= \frac{1656,5 - 422,3}{2} (-0,4602) + 319,7 \cdot 0,8879 = -284 + 284 = 0$$

Bosh inersiya momentlarini quyidagi formuladan hisoblasa ham bo'ladi:

$$J_{\frac{u}{v}} = J_{\frac{max}{min}} = \frac{J_{X_c} + J_{Y_c}}{2} \pm \frac{1}{2} \sqrt{(J_{X_c} - J_{Y_c})^2 + 4J_{X_cY_c}^2} =$$

$$= \frac{1656,5 + 422,3}{2} \pm \frac{1}{2} \sqrt{(1656,5 - 422,3)^2 + 4 \cdot 319,7^2} = 1039,4 \pm 695.$$

$$J_u = J_{max} = 1734,4 \text{ sm}^2; J_v = J_{min} = 344,4 \text{ sm}^2; J_u + J_v = 2078,8 \text{ sm}^2.$$

Bu yerda, $(J_{X_c} + J_{Y_c})$ bilan $(J_u + J_v)$ o'rtasida xatolik yo'q.

4.3-§. Tekis shakllarning geometrik tavsifnomalari bo'yicha topshiriqlar

6- topshiriq

To'g'ri to'rtburchak va prokat profilardan tashkil topgan kesimning (simmetrik) quyidagi qiymatlari aniqlansin:

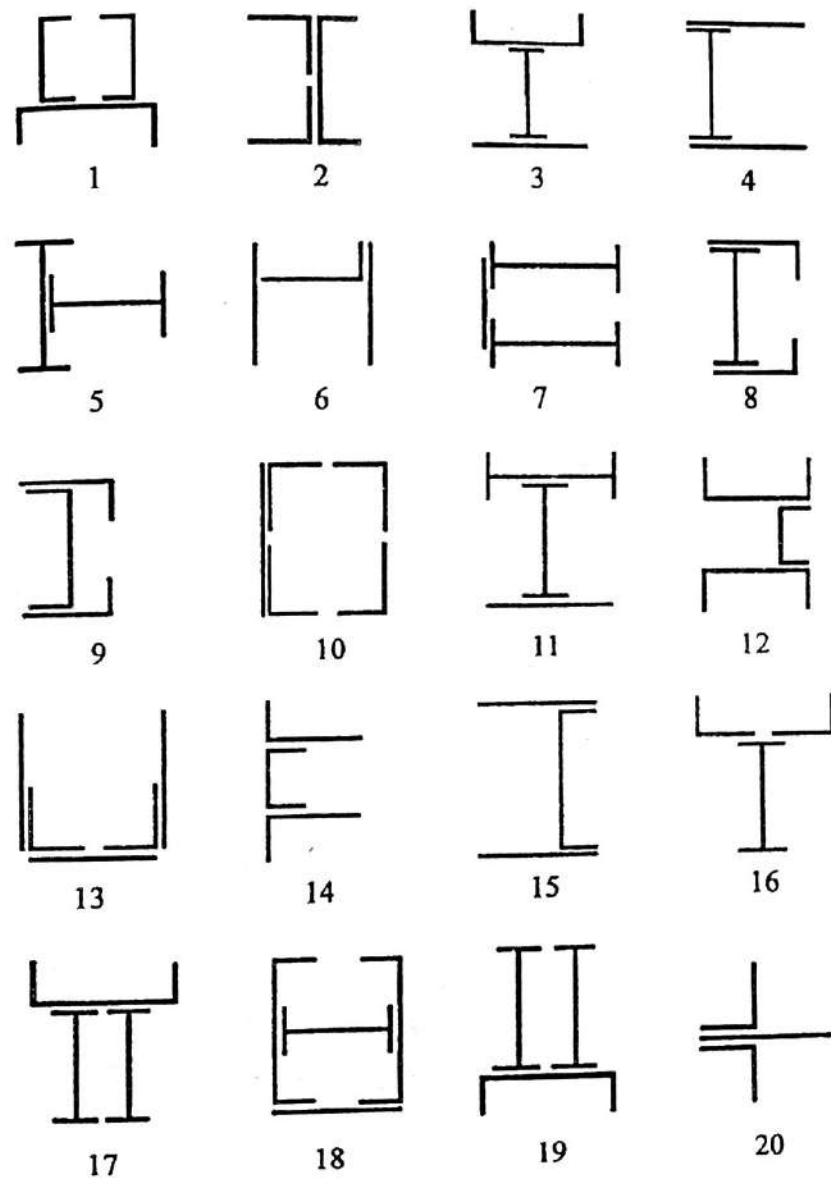
- 1) kesimning og'irlik markazi aniqlansin;
- 2) bosh markaziy inersiya momentlari aniqlansin;

3) kesim masshtabda chizilib, kerakli o'qlar va o'lchamlar ko'rsatilsin.

Hisoblash uchun qiymatlar 4.1-jadvalda, kesimlar 4.15-shaklda keltirilgan

4.1-jadval

№	O'lchashlar				Nomer	
	List, sm		Burchaklik, mm		Qo'shtavr (ГОСТ-72)	Shveller (ГОСТ-72)
	h	b	Teng yonli (ГОСТ 8509-72)	Teng yonsiz (ГОСТ 8510-72)		
1	16	1.8	80x80x8	—	18a	22
2	16	2.2	80x80x7	—	18	20a
3	16	1.4	80x80x6	—	16	22
4	18	2.0	—	100x63x6	20	22a
5	18	2.4	—	110x70x7	18a	22
6	18	1.6	—	90x56x6	18	22a
7	20	2.2	110x70x7	—	20a	24
8	20	2.4	—	110x70x7	18a	22
9	20	1.8	100x100x8	—	18a	20
10	22	2.4	—	125x80x12	22	24a

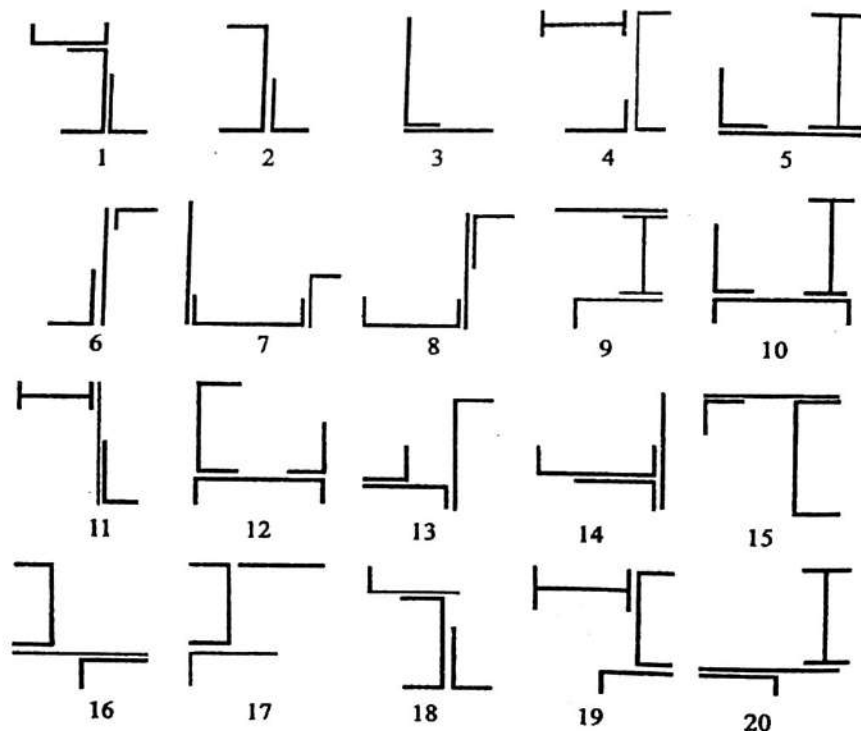


4.15- shakl

7-topshiriq

To'g'ri to'rtburchak va prokat profillardan tashkil topgan kesimning (nosimmetrik) quyidagi qiymatlari aniqlansin:

- 1) kesimning og'irlik markazi;
- 2) markaziy o'qlarga nisbatan kesimning inersiya hamda markazdan qochirma inersiya momentlari;
- 3) bosh markaziy inersiya o'qlarining yo'nalishi;
- 4) bosh markaziy inersiya momentlarining qiymatlari;
- 5) kesim masshtabda chizilib, kerakli o'qlar va o'lchamlar ko'rsatilsin. Hisoblash uchun qiymatlar 4.1-jadvalda, kesimlar 4.16-shaklda keltirilgan.



4.16-shakl

V BOB BURALISH

5.1- §. Asosiy mulohazalar

- Agarda kuchlanish holatidagi brusning ko'ndalang kesimlarida ichki kuchlardan faqat burovchi moment mavjud bo'lib, qolganlari nolga teng bo'lsa, u holda buralish deformatsiyasi sodir bo'ladi.
- Buralishga qarshilik ko'rsatuvchi doiraviy kesimli brusga val deyiladi. Valning og'irlik markazidan ixtiyoriy ρ masofada yotuvchi nuqtalarida hosil bo'ladigan urinma kuchlanish quyidagiga teng bo'ladi:

$$\tau_{\rho} = \frac{T_e}{J_{\rho}} \cdot \rho, \quad (5.1)$$

bu yerda, T_e – burovchi moment; J_{ρ} – polyar inersiya momenti.

- Eng katta urinma kuchlanish val ko'ndalang kesimining eng chekka nuqtalarida paydo bo'ladi:

$$\tau_{max} = \frac{T_e}{W_{\rho}}, \quad (5.2)$$

bu yerda, W_{ρ} – polyar qarshilik momenti.

- Buralish burchagi quyidagi Guk qonuni bo'yicha aniqlanadi:

$$\varphi = \frac{T_e \cdot l}{G \cdot J_{\rho}}, \quad (5.3)$$

bu yerda, l – val uzunligi; $G \cdot J_{\rho}$ – valning buralishdagi bikirligi; φ – radianda o'lchanadi.

- Nisbiy buralish burchagi quyidagiga teng:

$$\theta = \frac{\varphi}{l} = \frac{T_e}{G \cdot J_{\rho}}, \quad (5.4)$$

- Valni loyihalashdagi hisoblar ikki xil shart bo'yicha amalga oshiriladi:

– mustahkamlik sharti bo'yicha:

$$\tau_{max} = \frac{T_e^{max}}{W_{\rho}} \leq \tau_{adm}, \quad (5.5)$$

– bikirlik sharti bo'yicha

$$\theta_{max} = \frac{T_e^{max}}{GJ_p} \leq \theta_{adm} \quad (5.6)$$

5.2- §. Buralishga doir masalalar

46-masala

Aylanishlar soni $n=300$ ayl/min, quvvati $P = 450$ o.k ga teng bo'lgan yaxlit valning diametri aniqlansin. Aylanish burchagi val uzunligining 2 m ga 1° dan oshmasligi kerak.

$$[\tau] = 40 \text{ MPa}, G = 8 \cdot 10^4 \text{ MPa}.$$

Yechish:

Burovchi moment qiymatini aniqlaymiz.

$$T_e = 7160 = 7160 \frac{P}{n} = 7160 \frac{450}{300} = 10740 \text{ N} \cdot \text{M}.$$

Valning diametrini aniqlaymiz:

$$d \geq \sqrt[3]{\frac{16T_e}{\pi[\tau]}} = \sqrt[3]{\frac{16 \cdot 10740}{\pi \cdot 40 \cdot 10^6}} = 0.111 \text{ m}.$$

Valning diametri bikirlik sharti bo'yicha quyidagiga teng:

$$d \geq \sqrt[4]{\frac{32T_e}{\pi[\varphi_0] \cdot G}} = \sqrt[4]{\frac{32 \cdot 10740}{\pi \cdot \frac{1}{2.57.2} \cdot 8 \cdot 10^{10}}} = 0.112 \text{ m}.$$

Valning diametrini $d = 0.112$ m deb qabul qilamiz.

47- masala

Ikkita bir xil materialdan yasalgan, bir xil uzunlikdagi o'zaro teng burovchi moment uzatuvchi val berilgan. Ulardan biri yaxlit, ikkinchisi esa g'ovak bo'lib, uning g'ovakli koeffitsiyenti $\alpha = 0.8$ ga teng. Yaxlit val, g'ovakli valga nisbatan necha marta og'ir.

Yechish

Bir xil materialdan yasalgan, bir xil mustahkamlikka ega vallar deb burovchi momentning biror qiymatiga ega katta urinma hosil bo'ladigan vallar aytiladi.

Unda:

$$\frac{T_e}{W_S^{yaxl}} = \frac{T_e}{W_S^{g'ov}},$$

$$\text{bu yerdan } W_S^{yaxl} = W_S^{g'ov}, \text{ unda } \frac{\pi D^{yaxl^3}}{16} = \frac{\pi D^{g'ov^3}}{16} (1 - \alpha^2).$$

Vallar massasining nisbati ular ko'ndalang kesimlarining nisbatiga tengligidan:

$$m^{yaxl} : m^{g'ov} = F^{yaxl} : F^{g'ov} = \frac{\pi D^{yaxl^3}}{16} (1 - \alpha^2)$$

Bu tenglamaga diametrlar nisbatini keltirib qo'yib quyidagini hosil qilamiz.

$$\frac{m^{yaxl}}{m^{g'ov}} = \frac{(1 - \alpha^2)^{\frac{2}{3}}}{(1 - \alpha^2)} = 1.955;$$

Demak bir xil mustahkamlikda g'ovakli val yaxlit valga nisbatan ikki barobar yengil ekan.

48- masala

Uzunliklari o'zaro teng va bir xil materialdan yasalgan ikkita val bir xil mustahkamlikka ega. Ulardan biri yaxlit bo'lib, diametri d ga, ikkinchisining ichi g'ovak bo'lib ichki va tashqi diametrlarining nisbati $\alpha = d_2 : d_1 = 0.8$ ga teng. Ikkinchi val birinchisiga qaraganda qanchaga yengil?

Yechish

Masalaning shartiga ko'ra, ikkala valning kesimida paydo bo'luvchi eng katta urinma kuchlanishlar bir xil qiymatga ega:

$$\tau_{max} = \frac{16 \cdot T_e^{max}}{\pi d^3} = \frac{16 \cdot T_e^{max}}{\pi d_1^3 (1 - \alpha^4)};$$

$$d_1 = \frac{1}{\sqrt[3]{1 - \alpha^4}} \cdot d = 1,19d; \quad d_2 = \frac{\alpha}{\sqrt[3]{1 - \alpha^4}} \cdot d = 0,954d.$$

Yaxlit val og'irligining ichi g'ovak val og'irligiga nisbatini aniqlaymiz:

$$\frac{\gamma l A_v}{\gamma l A_k} = \frac{\gamma l \frac{\pi d^2}{4}}{\gamma l \frac{\pi}{4} (d_1^2 - d_2^2)} = 1,98.$$

Shunday qilib, ichi g'ovak val yaxlit valdan taxminan 2 marta yengil ekan.

49- masala

Agar ichki va tashqi diametrlarining nisbati $a = d_2/d_1 = 0,7$ bo'lgan g'ovak val bilan diametri d ga teng yaxlit vallarning ko'ndalang kesim yuzalari bir xil bo'lsa, qaysi val chidamli bo'ladi? Ikkala valning bikirligi solishtirilsin.

Yechish

Masalaning shartidan $A = \frac{\pi d^2}{4} = \frac{\pi d_1^2}{4} (1 - \alpha^2)$ munosabatni yozib olamiz.

Bundan, $d = \sqrt{1 - \alpha^2} d_1 = 0,714d_1$ kelib chiqadi.

Ikkala valda ham eng katta urinma kuchlanish kesimdagi eng chekka nuqtalarda paydo bo'ladi:

$$\tau_{ya}^{max} = \frac{16 \cdot T_e^{max}}{\pi d^3};$$

$$\tau_{g'}^{max} = \frac{16 \cdot T_e^{max}}{\pi d_1^3 (1 - \alpha^4)}.$$

Bundan chiqdi, g'ovak val yaxlit valga qaraganda

$$\frac{d_1^3 (1 - \alpha^4)}{d^3} = 2,088.$$

marta mustahkamroq bo'lar ekan.

Vallarning bikirliklari quyidagi nisbatda bo'ladi:

$$\frac{d_1^4 (1 - \alpha^4)}{d^4} = 2,92$$

Demak, yaxlit valning bikirligi g'ovak valning bikirligi dan taxminan 3 marta katta ekan.

50- masala

Uzunligi $l = 1,5$ m bo'lgan yaxlit valga dvigateldan quvvat oladigan bitta yetakchi va stanoklarga aylanma harakat beradigan uchta yetaklanuvchi shkiqlar o'rnatilgan

(5.1-shakl, a). Yetaklanuvchi shkiqlarning quvvati mos ravishda 25,15 va 12 ot kuchiga teng. Val po'latdan yasalgan bo'lib, minutiga 200 marta aylanadi.

Ruxsat etilgan urinma kuchlanish $\tau_{adm} = 35 \text{ MPa}$, ruxsat etilgan nisbiy buralish burchagi $\theta_{adm}^0 = 0,25 \text{ grad/m}$, siljishdagi elastiklik moduli $G = 8 \cdot 10^4 25 \text{ MPa}$ va hajmi og'irligi $\gamma_{adm} = 7,85 \cdot 10^4 \text{ N/m}^3$ lar ma'lum.

Yetakchi shkiqlarning o'rnini qaysi yetaklanuvchi shkiqlar bilan almashtirilganda, valni tayyorlash uchun eng kam material sarf qilinadi?

Podshipniklardagi ishqalanish kuchlari e'tiborga olinmasin.

Yechish

Masalani yechish ikki bosqichda olib boriladi.

1. Dastlab, shkiqlarning o'rnini almashtirmasdan, valni yasash uchun qancha material sarflanishini hisoblash lozim. Buning uchun quyidagi ishlar ketma-ket bajariladi:

a) burovchi momentlarning epyurasini qurish:

b) valni mustahkamlik va bikirlikka tekshirib, zarur diametrini aniqlash.

Valni va unga o'rnatilgan shkiqlarni chizish oson bo'lishi uchun ularning fazoviy ko'rinishini tekislikda tasvirlaymiz (5.1-shakl, b).

Yetaklanuvchi shkivlarga qo'yilgan burovchi momentlar quyidagi formula yordamida aniqlanadi:

$$T_e = 7028,8 \cdot \frac{N_{ok}}{n},$$

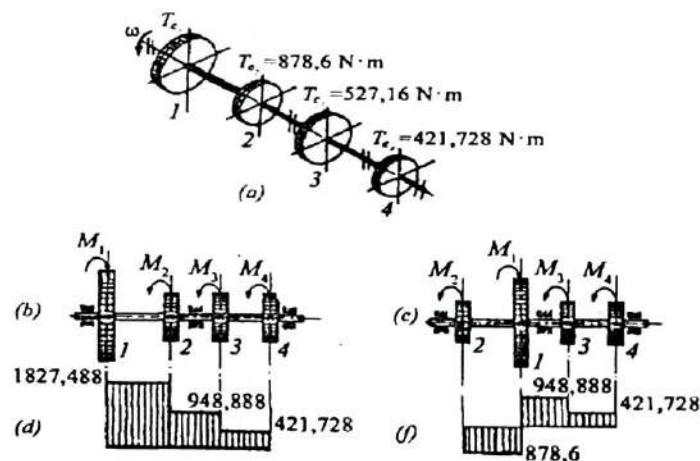
bu yerda, N_{ok} – quvvat, n – aylanishlar soni.

U holda,

$$T_{e_2} = 7028,8 \cdot \frac{25}{200} = 878,6 \text{ N} \cdot \text{m}; \quad T_{e_2} = 7028,8 \cdot \frac{15}{200} = 527,16 \text{ N} \cdot \text{m};$$

$$T_{e_4} = 7028,8 \cdot \frac{12}{200} = 421,728 \text{ N} \cdot \text{m}.$$

Yetakchi shkiv uzatayotgan burovchi moment esa $T_{e_1} = T_{e_2} + T_{e_3} + T_{e_4} = 1827,488 \text{ N} \cdot \text{m}$ ga teng bo'ladi.



5.1-shakl

Kesish usulini tatbiq etib, valga shkivlar oralig'ida ta'sir etuvchi burovchi momentlarni aniqlaymiz:

I–I kesimda: $T_e^I = \sum_{i=1}^n M_{zi} = T_{e_1} = 1827,488 \text{ N} \cdot \text{m};$

II–II kesimda: $T_e^{II} = T_{e_1} - T_{e_2} = 948,888 \text{ N} \cdot \text{m};$

III–III kesimda: $T_e^{III} = T_{e_1} - T_{e_2} - T_{e_3} = 421,728 \text{ N} \cdot \text{m}.$

5.1-shakl (a). Bu qiymatlar yordamida burovchi moment epyurasi quriladi (5.1-shakl, d): epyuradan ko'rinib turibdiki, $T_e^{\max} = 1827,988 \text{ N} \cdot \text{m}$ ga teng ekan.

Valning mustahkamlik shartini qanoatlantiruvchi zaruriy diametrni aniqlaymiz:

$$d_\tau = 1,72 \cdot \sqrt[4]{\frac{T_e^{\max}}{\tau_{adm}}} = 1,72 \cdot \sqrt[4]{\frac{1827,488}{35 \cdot 10^6}} = 1,72 \cdot 3,7377 \cdot 10^{-2} = 6,4288 \cdot 10^{-2} \text{ m}.$$

Valning bikirlik shartini qanoatlantiruvchi zaruriy diametr esa quyidagicha aniqlanadi:

$$d_\theta = 4,92 \cdot \sqrt[4]{\frac{T_e^{\max}}{G\theta'_{adm}}} = 4,92 \cdot \sqrt[4]{\frac{1827,488}{8 \cdot 10^{10} \cdot 0,25}} = 4,92 \cdot 1,7386 \cdot 10^{-2} = 8,554 \cdot 10^{-2} \text{ m}.$$

Nihoyat, valning diametri uchun yaxlitlab olingan $d_0 = 9 \cdot 10^{-2} \text{ m}$ qiymat qabul qilinadi.

Bundan chiqdi, birinchi holatda valni tayyorlash uchun

$$Q^I = \gamma \cdot l \cdot \frac{\pi d_0^2}{4} = 7,85 \cdot 10^4 \cdot 1,5 \cdot \frac{3,14}{4} (9 \cdot 10^{-2})^2 = 748,71 \text{ N}$$

og'irlikdagi material sarflanar ekan.

Yuqoridagi hisoblashlardan ko'rinib turibdiki, materialni tejash uchun burovchi momentning eng katta qiymati $T_e^{\max}$ ni mumkin qadar kamaytirish zarur ekan. Bunga erishish uchun yetakchi shkivni valga ratsional tartibda o'rnatish kifoya: yetakchi shkiv yetaklanuvchi shkivlar orasiga shunday o'rnatilishi kerakki, iloji boricha yetakchi shkivdan ikkala tomonga uzatilayotgan quvvatlarning yig'indisi teng bo'lsin.

1- va 2-shkivlarning o'rinarini almashtirib (5.1-shakl, e), yana oldingi tartibda hisoblashlarni davom ettiramiz (o'quvchilarning mustaqil bajarishi tavsiya etiladi).

Burovchi momentlarning epyurasi qurilgach (5.1-shakl, f), valning zaruriy diametrlarini aniqlaymiz:

$$d_{\tau} = 1,72 \cdot \sqrt[3]{\frac{948,888}{35 \cdot 10^6}} = 5,167 \cdot 10^{-2} \text{ m};$$

$$d_{\theta} = 4,92 \cdot \sqrt[4]{\frac{948,888}{8 \cdot 10^{10,0,25}}} = 7,26 \cdot 10^{-2} \text{ m}.$$

Yaxlitlangan $d_0 = 8 \cdot 10^{-2} \text{ m}$ o'ldam qabul qilinadi.

Demak, po'lat materialdan uzunligi $l = 1,5 \text{ m}$ va diametri $d_0 = 8 \cdot 10^{-2} \text{ m}$ bo'lgan valni yasash uchun

$$Q'' = 7,85 \cdot 10^4 \cdot 1,5 \frac{3,14(8 \cdot 10^{-2})^2}{4} = 591,576 \text{ N}$$

material zarur ekan.

Tejalgan materialni foizlarda hisoblaymiz:

$$\Delta Q = \frac{(748,71 - 591,576)}{748,71} \cdot 100\% = 21\%.$$

51- masala

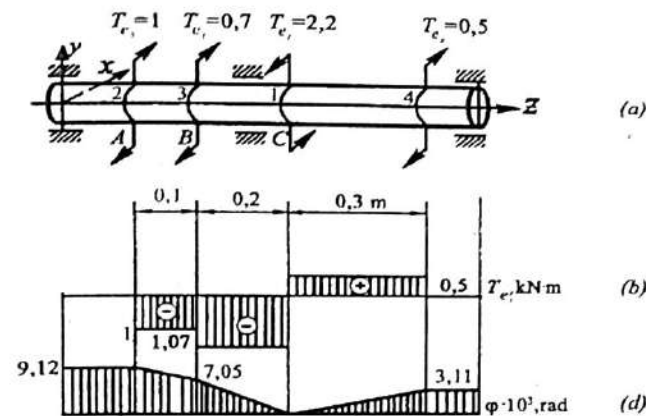
O'zgaras burchak tezlik bilan aylanayotgan 1-shkiv qolgan 3 ta shkivni aylanma harakatga keltiradi (5.2-shakl). Bu shkivlarda hosil bo'lgan momentlar quyidagilarga teng: $T_{e2} = 1 \text{ kN} \cdot \text{m}$; $T_{e3} = 0,7 \text{ kN} \cdot \text{m}$; $T_{e4} = 0,5 \text{ kN} \cdot \text{m}$.

Ko'ndalang kesimi yaxlit hamda g'ovak ko'rinishdagi ($d:D = 0,8$) vallarning diametrlari aniqlansin ($[\tau]_{ort} = 80 \text{ MPa}$). Burovchi moment hamda buralish burchaklari epyuralari ko'rilsin. Bikirlik shartlari quyidagi tengliklar bo'yicha tekshirilsin: $[\theta] = 3 \text{ grad/m}$; $G = 8 \cdot 10^4 \text{ MPa}$.

1-shkivdagi buruvchi moment qiymatini aniqlaymiz:

$$\sum M_z = 0; T_{e1} = T_{e2} + T_{e3} + T_{e4} = 2,2 \text{ kN} \cdot \text{m}.$$

Valni uchta uchastkaga bo'lib, ulardagi eguvchi momentlarni aniqlaymiz (oldingi masaladagidek) va uning epyurasini quramiz (5.2-shakl, b). Demak, eng katta buruvchi moment



5.2-shakl

$T_e^{\max} = 1,7 \text{ kN} \cdot \text{m}$ ga teng. Buralishdagi mustahkamlik shartiga binoan, val ko'ndalang kesimining qutb inersiya momentini aniqlaymiz:

$$W_p \geq \frac{T_e^{\max}}{[\tau]_{ort}} = \frac{1700}{80 \cdot 10^6} = 21,25 \cdot 10^{-6} \text{ m}^3 = 21,25 \text{ sm}^3.$$

Aylanma ko'rinishdagi kesimning qarshilik momenti $W_p = \pi D^3/16$ ekanligini e'tiborga olib, ko'ndalang kesim yaxlit bo'lgan valning diametrini aniqlaymiz:

$$D \geq \sqrt[3]{\frac{16W_p}{\pi}} = \sqrt[3]{\frac{16 \cdot 21,25}{3,14}} = 4,77 \text{ sm}.$$

Kesimi g'ovak ko'rinishidagi val uchun:

$$W_p = \frac{\pi D^3}{16} \left[1 - \left(\frac{d}{D} \right)^4 \right] = \frac{3,14 D^3}{16} (1 - 0,8^4) = 0,116 D^3.$$

Shunga ko'ra, uning ichki va tashqi diametrlari:

$$D \geq \sqrt[3]{\frac{W_p}{0,116}} = \sqrt[3]{\frac{21,25}{0,116}} = 5,68 \text{ sm};$$

$$d = 0,8 \cdot 5,68 = 4,54 \text{ sm}$$

ga teng bo'ladi.

Yaxlit va g'ovak vallarning ko'ndalang kesim yuzalarini aniqlaymiz:

$$A_{yaxlit} = \frac{\pi D^2}{4} = \frac{3,14 \cdot 4,77^2}{4} = 17,9 \text{ sm}^2;$$

$$A_{g'ovak} = \frac{\pi D^2}{4} \left[1 - \left(\frac{d}{D} \right)^2 \right] = \frac{3,14 \cdot 5,68^2}{4} (1 - 0,8^2) = 9,12 \text{ sm}^2.$$

Bu qiymatlarni solishtirish shuni ko'rsatadiki, g'ovak valni qo'llaganda unga sarf bo'ladigan metall miqdori yaxlit valga nisbatan ikki baravar kam bo'lar ekan.

Keyingi hisoblarni g'ovak val uchun amalga oshiramiz.

Val ko'ndalang kesimi yuzalarining buralish burchaklarini 1-shkiv mahkamlangan kesim yuzasini qo'zg'almas deb hisoblab, shu kesimga nisbatan aniqlaymiz:

$$\varphi_{CD} = \frac{T_e^{CD} \cdot l_{CD}}{G \cdot I_p} = \frac{500 \cdot 0,3}{80 \cdot 10^{10} \cdot 60,3 \cdot 10^{-8}} = 3,11 \cdot 10^{-3} \text{ rad}.$$

Bu yerda,

$$I_p = \frac{\pi D^4}{32} \left[1 - \left(\frac{d}{D} \right)^4 \right] = \frac{3,14 \cdot 5,68^4}{32} [1 - 0,8^4] = 6,03 \text{ sm}^4;$$

$$\varphi_{CB} = \frac{T_e^{BC} \cdot l_{BC}}{G \cdot I_p} = \frac{1700 \cdot 0,2}{80 \cdot 10^{10} \cdot 60,3 \cdot 10^{-8}} = 7,05 \cdot 10^{-3} \text{ rad};$$

$$\varphi_{BA} = \frac{T_e^{AB} \cdot l_{AB}}{G \cdot I_p} = \frac{100 \cdot 0,1}{80 \cdot 10^{10} \cdot 60,3 \cdot 10^{-8}} = 2,07 \cdot 10^{-3} \text{ rad};$$

$$\varphi_{CA} = \varphi_{CB} + \varphi_{BA} = 7,05 \cdot 10^{-3} + 2,07 \cdot 10^{-3} = 9,12 \cdot 10^{-3} \text{ rad}.$$

Valning bikirlik shartini tekshiramiz. Unda nisbiy buralish burchaklari: CD uchastkada $\theta_{CD} = \varphi_{CD} / l_{CD} = 3,11 \cdot 10^{-3} / 0,3 = 0,0104 \text{ rad/m}$; CB uchastkada $\theta_{CB} = \varphi_{CB} / l_{CB} = 7,05 \cdot 10^{-3} / 0,2 = 0,0353 \text{ rad/m}$; BA uchastkada $\theta_{BA} = \varphi_{BA} / l_{BA} = 2,07 \cdot 10^{-3} / 0,1 = 0,0207 \text{ rad/m}$; ga teng bo'ladi.

Topilgan qiymatlardan ko'rinib turibdiki, eng katta nisbiy buralish burchagi quyidagiga teng:

$$\theta_{max} = \theta_{CB} = 0,0353 < [\theta] = 3:57,3 = 0,0524 \text{ rad/m}.$$

Bu yerda, $l_{rad} = 57,3 \text{ grad}$.

Demak, valning bikirligi qanoatlantiradigan darajada ekan.

52- masala

Chap uchi bilan qistirib mahkamlangan, uchiga esa qo'zg'almas qilib biriktirilgan bikir travers o'rnatilgan brusni tashqi moment burmoqda (5.3-shakl, a). Traversning aylanishini maxsus tayanch cheklaydi.

Berilgan brus uchun burovchi moment epyurasini qurish talab etladi.

Yechish

Brusning o'ng uchini tayanchdan ozod qilib, B kesimning qanchaga aylanishini hisoblaymiz (5.3- shakl, b):

$$\varphi_{BT_e} = \frac{T_e \cdot 2a}{G \frac{\pi d^4}{32} (1 - \alpha^4)} = \frac{T_e \cdot 2a}{G \frac{\pi d^4}{32} [1 - (0,7)^4]} = 26,8 \frac{T_e \cdot a}{G d^4}.$$

Bundan chiqdi, masala statik noaniq ekan, chunki brus T_e moment bilan yuklanganda oraliq yopilib, travers va tayanch orasidagi qiymati hozircha noma'lum bo'lgan T_{ex} juft kuch vujudga keladi (5.3- shakl, b).

Muvozanat tenglamasini tuzamiz:

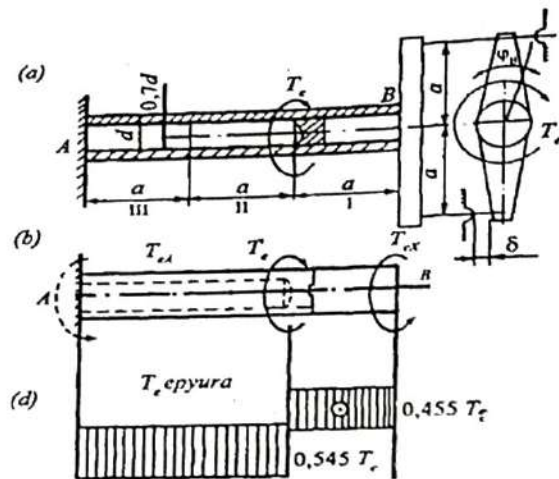
$$-T_{eA} + T_e - T_{eX} = 0 \text{ yoki } T_{eA} + T_{eX} = T_e.$$

Kuchlar ta'sirining bir-biriga xalal bermaslik prinsipiga tayanib, qo'shimcha tenglama, ya'ni ko'chish tenglamasini tuzamiz:

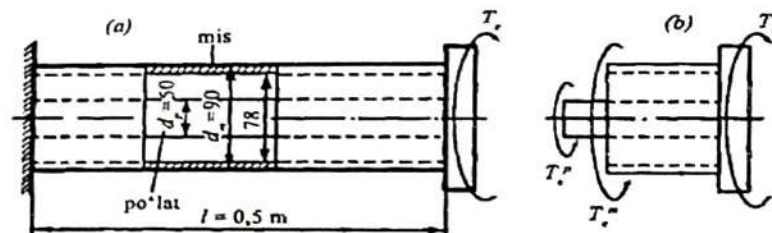
$$\frac{T_e \cdot 2a}{G \frac{\pi d^4}{32} (1 - \alpha^4)} - \frac{T_{eX} \cdot 2a}{G \frac{\pi d^4}{32} (1 - \alpha^4)} - \frac{T_{eX} \cdot 2a}{G \frac{\pi d^4}{32}} = \frac{\delta}{a}.$$

Tegishli soddalashtirishdan so'ng, $T_{eX} = 0,455 \cdot T_e$ ekanligi kelib chiqadi. Demak, qistirib mahkamlangan tayanchda, $T_{eA} = T_e - T_{eX} = 0,545 T_e$ ga teng bo'lgan burovchi moment hosil bo'lar ekan.

Kesish usulini qo'llab, burovchi moment epyurasini quramiz (5.3- shakl, d).



5.3-shakl



5.4-shakl

53- masala

Po'lat valik va mis trubka chap uchi bilan devorga qistirib mahkamlangan, o'ng uchiga esa bikir disk biriktirilgan (5.4-shakl, a). Diskka T_e tashqi moment qo'yilgan.

T_e momentning ruxsat etilgan qiymatini aniqlash talab etiladi.

Quyidagi ma'lumotlar berilgan:

po'lat uchun: $\tau_{adm}^p = 10 \text{ MPa}$; $G_p = 8 \cdot 10^4 \text{ MPa}$;

mis uchun: $\tau_{adm}^m = 50 \text{ MPa}$; $G_m = 4 \cdot 10^4 \text{ MPa}$.

Yechish:

Kesish usulini tatbiq etib (5.4- shakl, b) quyidagi muvozanat tenglamasini tuzamiz:

$$T_e^p + T_e^m - T_e = 0,$$

bu yerda, T_e^p va T_e^m — tegishli po'lat valik va mis trubkalar qabul qiladigan burovchi momentlar.

Masala bir marta statik noaniq hisoblanadi. Shuning uchun qo'shimcha tenglama tuzish zarur.

Valik va trubkaning o'ng uchlariidagi kesimning buralish burchaklari teng, chunki disk ularga bikir qilib mahkamlangan. Bu shartni quyidagicha yozamiz:

$$\varphi_p = \varphi_m \text{ yoki } \frac{T_e^p l}{G_p J_p^p} = \frac{T_e^m l}{G_m J_p^m}.$$

Bu formulaga son qiymatlarini qo'yib, $T_e^p = 0,302 T_e$; $T_e^m = 0,698 T_e$ ekanligiga ishonch hosil qilamiz. Valikni mustahkamlikka tekshiramiz:

$$\tau_{max}^p = \frac{0,302 \cdot T_e}{W_{pp}} \leq \tau_{adm}^p.$$

Bundan,

$$T_{adm}^p = T_e = \frac{1}{0,302} \cdot W_{pp} \tau_{max}^p = \frac{\frac{\pi}{16} \cdot 50^3 \cdot 100}{0,302} = 8 \text{ kN} \cdot \text{m};$$

trubkaning mustahkamlik shartidan esa,

$$T_{adm}^m = \frac{1}{0,698} \cdot W_{pm} \tau_{adm}^m = \frac{\frac{\pi}{16} \cdot 90^3 \left[1 - \left(\frac{78}{90} \right)^4 \right] \cdot 50}{0,698} = 4,52 \text{ kN} \cdot \text{m};$$

kelib chiqadi.

Bundan chiqdi, tashqi momentning ruxsat etilgan miqdori bo'lib, mis trubkaning mustahkamligi bilan aniqlanadi. Agar konstruksiya ruxsat etilgan momentga teng tashqi moment bilan yuklansa, u holda po'lat valik to'liq yuklanmaydi. Balki u ruxsat etilganiga nisbatan $\frac{8-4,52}{8} \cdot 100\% = 43,5\%$ ga kam yuklanadi. Ammo bundan po'lat valikning dia-

metrini kichiklashtirish mumkin ekan, degan xulosa kelib chiqmaydi. Chunki diametr o'zgarsa, tashqi moment qayta taqsimlanadi.

5.3- §. Buralishda valni mustahkamlikka va bikirlikka hisoblash bo'yicha topshiriqlar

8- topshiriq

Berilgan: ko'ndalang kesimi aylanadan iborat bo'lgan po'lat val (5.5-shakl) tashqi burovchi momentlar bilan yuklangan. Val uchun ruxsat etilgan kuchlanish $[\tau] = 130 \text{ MPa}$ ga teng.

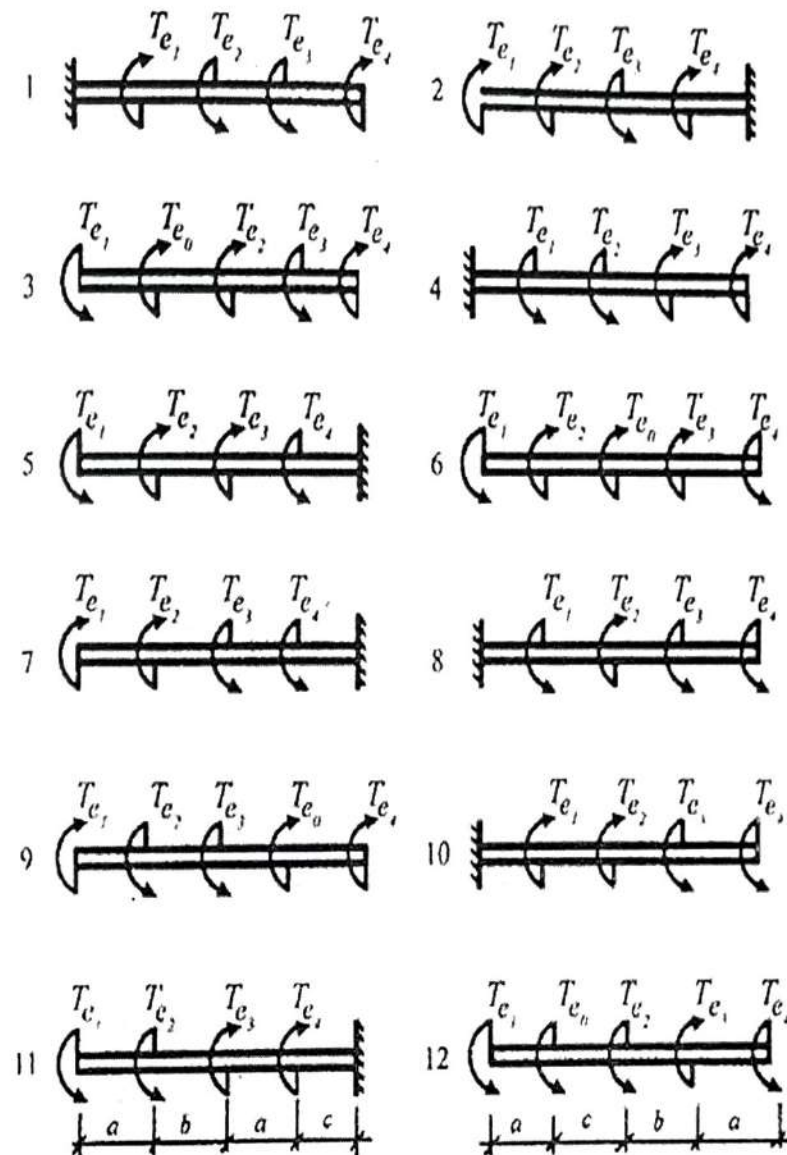
Topish kerak:

- 1) berilgan val uchun burovchi momentlar epyurasi qurilsin;
- 2) valning diametri mustahkamlik va bikirlik shartiga binoan tanlansin;
- 3) valning aylanish burchagi epyurasi qurilsin;
- 4) eng katta nisbiy buralish burchagi aniqlansin.

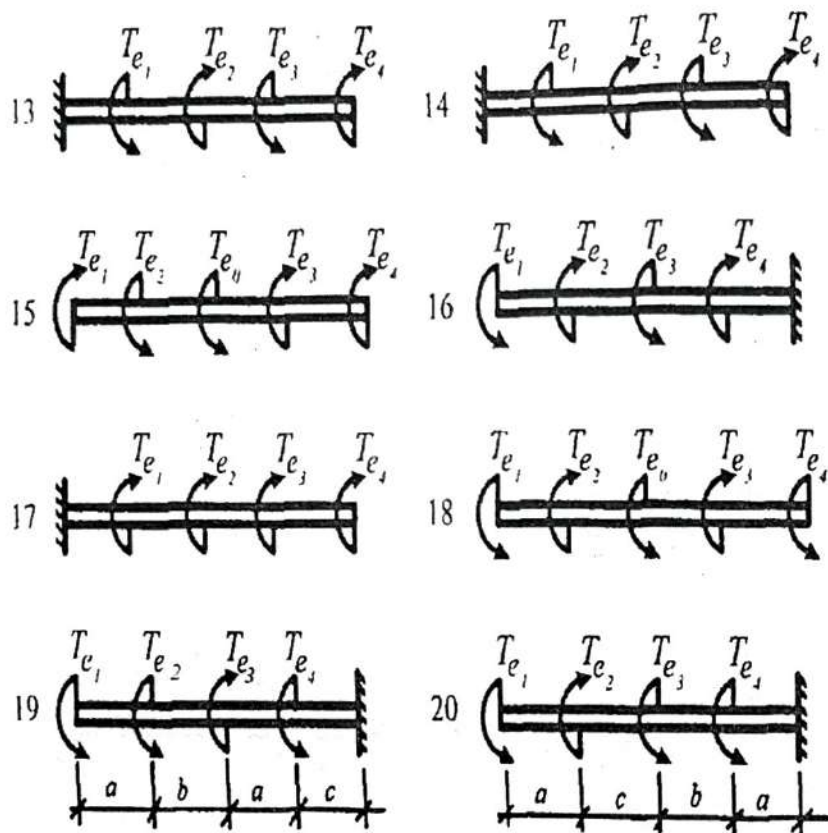
Kerakli ma'lumotlar 5.1-jadvalda, hisoblash sxemalari esa 5.5-shaklda keltirilgan.

5.1-jadval

№	Uchastkalar uzunligi			Moment, kN·m				θ_{adm}° grad/m
	a	b	c	T_{e1}	T_{e2}	T_{e3}	T_{e4}	q
1	1,0	0,8	1,7	28	14	23	6	3,0
2	1,3	0,8	1,4	5	22	10	26	3,0
3	1,1	1,7	0,4	10	17	9	28	2,0
4	1,1	1,9	0,6	7	24	11	21	1,4
5	1,3	0,8	1,4	5	22	10	26	3,0
6	1,2	1,0	0,9	33	8	25	8	1,5
7	1,2	1,1	0,8	14	10	24	12	2,4
8	1,4	0,5	1,1	27	12	22	6	2,6
9	1,3	0,8	1,1	5	18	8	25	1,0
10	1,3	0,6	1,5	19	11	10	22	1,8



5.5-shakl



VI BOB

TO'G'RI EGILISHDA BALKA VA RAMALAR UCHUN
ICHKI KUCH FAKTORLARI EPYURALARINI QURISH

6.1-§. Epyura qurish qoidalari

Bruslarni mustahkamlik va bikirlikka hisoblashda ularning qaysi kesimlarida ichki kuchlar ekstremal (eng katta yoki eng kichik) qiymatlarga erishishini, aniqrog'i, ichki kuchlarning brus uzunligi bo'yicha o'zgarish qonuniyatini bilish muhimdir. Odatda, bu qonuniyatni analitik bog'lanishlar va ular yordamida quriladigan epyuralar orqali ifodalash mumkin.

Ichki kuchlarning brus uzunligi bo'yicha o'zgarish qonunini ko'rsatuvchi grafik yoki diagramma mazkur ichki kuchlarning epyuralari yoki, qisqacha, epyura deyiladi.

Demak, epyuralarni to'g'ri qurish muhim ahamiyatga ega ekan, chunki ular yordamida brusning xavfli kesimi tanlanadi hamda ichki kuchlarning hisobiy qiymatlari aniqlanadi.

Har bir ichki kuchning o'ziga aynan bitta tenglama mos keladi.

Epyuralarni qurishda quyidagi umumiy tartiblardan foydalanish tavsiya etiladi:

1) statikaning muvozanat tenglamalari yordamida tayanch reaksiya kuchlari aniqlanadi;

2) brusni tegishli «oraliq»larga ajratib, ma'lum tartibda *I*, *II*, *III*,... raqamlar bilan belgilanadi; oraliqning chegaralari quyidagilardan iborat bo'lishi mumkin (6.1-shakl):

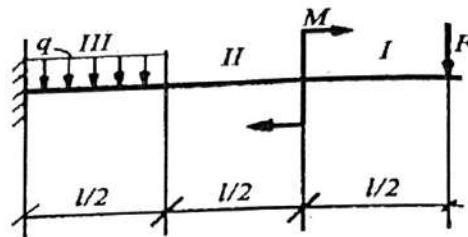
- a) to'plangan kuchlar va juft kuchlar qo'yilgan kesimlar;
- b) bitta qonuniyat bo'yicha o'zgaruvchi taqsimlangan kuchlarning boshlanish va oxirgi kesimlari;
- d) bruslarning siniq joylari;

3) kesish usulidan foydalanib, har bir oraliq uchun ichki kuchlarning analitik ifodalari yoziladi; bunda ichki kuchlarning ishoralariga alohida e'tibor berish zarur;

4) analitik ifodalar tarkibidagi o'zgaruvchi «Z»ga tegishli qiymatlar berib, har bir oraliqning o'ziga xos kesimlaridagi ichki kuchlarning miqdorlari - ordinatalari hisoblanadi;

5) ordinatalar aniq masshtab bilan brus o'qiga parallel qilib o'tkazilgan nol chizig'iga tik (perpendikulyar) qilib joylashtiriladi va epyura chiziladi; odatda, epyura musbat qiymatlar nol chizig'ining yuqori, manfiy qiymatlar esa pastki qismiga joylashtiriladi;

6) epyura nol chizig'iga tik bo'lgan chiziqlar bilan shtrixlanadi.



6.1-shakl

Egilgan to'sinlarning barcha kesimlarida ko'ndalang kuch Q va eguvchi moment M paydo bo'ladi. Shu sababli, to'sinning tayanchlari, tayanch reaksiyalarini aniqlash hamda turli xil to'sinlar uchun Q va M larning epyuralarini qurish tartiblarini mukammalroq bilish zarur.

Dastlab, egilishdagi ichki kuchlar uchun quyidagi qoidalarini bayon qilamiz:

– to'sindan ajratib olingan bo'lakchaga ta'sir etayotgan barcha kuchlar (tashqi kuchlar va tayanch reaksiyalari)ning to'sin o'qiga tik yo'nalgan o'qqa nisbatan olingan proyeksiyalar yig'indisi ko'ndalang kuch deyiladi hamda ushbu ko'rinishda aniqlanadi;

$$Q_Y(z) = \pm \sum Y_i \quad (a)$$

– to'sindan ajratib olingan bo'lakchaga ta'sir etayotgan barcha kuchlar (tashqi yuklar va tayanch reaksiyalari) dan qirqilgan kesim markaziga nisbatan olingan momentlarning algebraik yig'indisi eguvchi moment deyiladi hamda

$$M(z) = \pm \sum M_{0i} \quad (b)$$

ko'rinishdagi formula yordamida aniqlanadi.

Q va M lar uchun ishoralar qoidasi quyidagicha:

– agar tashqi kuchlar (tashqi yuklar va tayanch reaksiyalari) vektori to'sindan ajratib olingan bo'lakchani qirqilgan kesim markaziga nisbatan soat strelkasi harakati yo'nalishi bo'yicha aylantirsa,

ko'ndalang kuch musbat va aksincha, soat strelkasi harakati yo'nalishiga teskari aylantirsa, manfiy ishorali qilib olinadi (6.2-shakl);

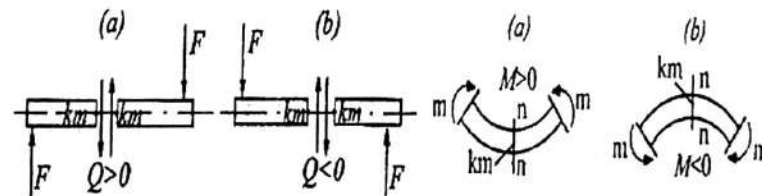
– agar kuchlar (tashqi yuklar va tayanch reaksiyalari) vektori to'sindan ajratib olingan bo'lakchani qirqilgan kesim markaziga nisbatan olingan momentlar to'sindan ajratib olingan bo'lakchani yuqori tolalarini siqsa, u holda eguvchi moment musbat va, aksincha, yuqori tolalarini cho'zsa, manfiy ishorali qilib tanlanadi (6.3-shakl).

Eguvchi moment, kesuvchi kuch va yoyilgan kuch intensivligi orasida quyidagi differensial va integral munosabatlar mavjuddir:

$$\frac{dM_x}{dz} = Q_y; \quad (6.1)$$

$$\frac{dM_x}{dz^2} = \frac{dQ_y(z)}{dz} = -q. \quad (6.2)$$

Ya'ni:



6.2-shakl

eguvchi momentdan Z abssissa bo'yicha olingan hosila tekshirilayotgan kesimdagi ko'ndalang kuchga tengdir;

eguvchi momentdan Z abssissa bo'yicha olingan ikkinchi hosila yoyilgan yuk intensivligining teskari ishora bilan olingan qiymatiga tengdir.

Yuqorida keltirilgan differensial bog'lanishlardan, birinchidan, ko'ndalang kuch Q va eguvchi moment M larning epyuralarini qurishga, ikkinchidan esa qurilgan epyuralarning to'g'ri yoki noto'g'riligini tekshirishga imkon beruvchi quyidagi muhim xulosalar kelib chiqadi:

- yoyilgan yuk intensivligi bo'tmagan uchastkalarda Q ning epyurasi to'sin o'qiga parallel yo'nalgan to'g'ri chiziq, M_x ning epyurasi esa to'sin o'qiga og'ma yo'nalgan to'g'ri chiziq bilan chegaralangan bo'ladi;

- yoyilgan yuk intensivligi ta'sir etayotgan uchastkalarda Q ning epyurasi to'sin o'qiga og'ma to'g'ri chiziq, M ning epyurasi esa kvadratik parabola yoyi bilan chegaralangan bo'ladi;

- kesuvchi kuch nolga teng bo'lgan kesimda eguvchi moment ekstremal qiymatga erishadi;

- $Q > 0$ bo'lgan uchastkalarda chapdan o'ngga tomon M ning ordinatasi orta boshlaydi, ya'ni eguvchi momentning musbat qiymati oshadi, manfiy qiymati esa kamayadi;

- aksincha, $Q_x < 0$ bo'lgan uchastkalarda esa M ning ordinatasi kamaya boshlaydi;

- to'plangan kuch qo'yilgan kesimlarda Q ning epyurasi shu kuch miqdori qadar sakraydi, M ning epyurasini chegaralovchi chiziq esa o'z yo'nalishini o'zgartiradi;

- juft kuch qo'yilgan kesimlarda Q ning epyurasida hech qanday o'zgarish bo'lmaydi, M ning epyurasida esa shu juft kuch miqdori qadar sakrash sodir bo'ladi;

- chetki shamirli tayanchlarda kesuvchi kuch tayanch reaksiyalariga, eguvchi moment esa nolga teng bo'ladi (agar shu kesimlarga juft kuch qo'yilmagan bo'lsa);

- to'sin (konsol)ning erkin uchiga juft kuch qo'yilmagan bo'lsa, eguvchi moment shu kesimda nolga teng bo'ladi; agar konsol uchiga to'plangan kuch ham qo'yilmagan bo'lsa, shu kesimda kesuvchi kuch ham nolga teng bo'ladi;

- qistirib mahkamlangan tayanchda kesuvchi kuch shu tayanchning reaksiya kuchiga, eguvchi moment esa reaktiv momentiga teng bo'ladi.

6.2-§. To'sinlarning tayanch reaksiyalarini aniqlash

Umumiy holda tekislikdagi statik aniq tizim (to'sin, rama, arka, egri brus) larning tayanch reaksiyalari quyidagi uchta statikaning muvozanat tenglamalaridan topiladi:

$$\begin{aligned} \sum Z_i &= 0 & \sum Z_i &= 0 \\ \sum Y_i &= 0 & \sum M_{A_i} &= 0; \\ \sum M_{A_i} &= 0; & \sum M_{B_i} &= 0. \end{aligned} \quad (6.3)$$

Bu yerda, A va B harflari tayanchlarga tegishli nuqtani ifodalaydi.

Biz faqat to'sinning geometrik o'qiga tik yo'nalgan tashqi kuchlar ta'siri ostidagi egilishni o'rganish bilan chegaralanamiz, xolos. Shu sababli, bunday to'sinlarning tayanch reaksiyalarini aniqlash uchun $\sum Z_i = 0$ tenglamani tuzishning zaruriyati qolmaydi, ya'ni tayanchning gorizontal tuzuvchisi $N = 0$ bo'ladi.

Konsolning tayanch reaksiyalarini aniqlash uchun esa quyidagi ikkita tenglamani tuzish tavsiya etiladi:

$$\sum Y_i = 0; \quad \sum M_{A_i} = 0,$$

Parallel kuchlar sistemasi ta'siri ostidagi ikki tayanchli oddiy va konsol uchli to'sinlarning tayanch reaksiyalarini aniqlash uchun esa A va B tayanchlarga nisbatan hamma kuchlardan olingan momentlar tenglamasidan foydalanish zarur:

$$\sum M_A(P_i) = 0; \quad \sum M_B(P_i) = 0.$$

keyin esa tayanch reaksiyalarning qiymatlari

$$\sum Y_i = 0,$$

tenglama yordamida tekshirib ko'riladi.

54- masala

Ikki uchi bilan shamirli qo'zg'almas va qo'zg'aluvchi tayanchlarga tiralgan oddiy to'singa 6.4-shaklda ko'rsatilgandek, to'plangan va juft

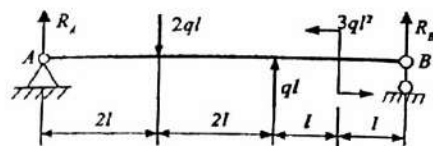
kuchlar ta'sir etmoqda. Mazkur to'sinning tayanch reaksiyalarini aniqlash talab etiladi.

Yechish:

To'sinning chap tayanchini R_A va o'ng tayanchini esa R_B reaksiyalar bilan almashtiramiz. Barcha kuchlardan A va B tayanchlarga nisbatan momentlar olib, berilgan masala uchun quyidagi ifodani tuzamiz:

$$\begin{aligned}\sum M_A(P_i) &= 0 \text{ yoki } 2ql \cdot 2l - ql \cdot (2l + 2l) - 3ql^2 - R_B \cdot 6l = 0; \\ \sum M_B(P_i) &= 0 \text{ yoki } R_A \cdot 6l - 2ql \cdot 4l + ql \cdot 2l - 3ql^2 = 0.\end{aligned}$$

Oxirgi tenglamalardan noma'lum reaksiyalarni aniqlaymiz:
 $R_A = 3ql/2$; $R_B = -ql/2$.



6.4-shakl

Demak, R_B tayanch reaksiyasining haqiqiy yo'nalishi Y o'qiga teskari yo'nalgan ekan.

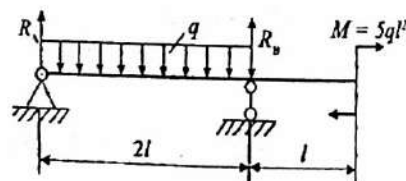
Quyidagi tenglamani tuzib, R_A va R_B larning qiymatlari to'g'ri topilganligiga ishonch hosil qilamiz:

$$R_A - 2ql + ql + R_B = 0 \text{ yoki } \frac{3ql}{2} - 2ql + ql - \frac{ql}{2} = 0; \\ 0 = 0.$$

55- masala

6.5-shaklda tasvirlangan konsol uchli oddiy to'sin juft kuch va tekis taralgan yoyilgan kuch ta'sirida bo'lsin. To'sinning tayanch reaksiyalarini aniqlaymiz.

Quyidagilar ma'lum: q , M , l .



6.5-shakl

Yechish:

To'sinning tayanchlarini R_A va R_B reaksiyalar bilan almashtiramiz. Shuni eslatib o'tish lozimki, yoyilgan kuchlardan biror nuqtaga moment olish uchun, dastlab, ularning teng ta'sir etuvchisi aniqlanadi.

Statikaning muvozanat tenglamalarini tuzamiz:

$$\sum M_A = 0; \quad q \cdot 2l \cdot \frac{2l}{2} - R_B \cdot 2l + M = 0, \text{ bunda, } R_B = \frac{7ql}{2};$$

$$\sum M_{B_i} = 0; \quad R_A \cdot 2l - q \cdot 2l \cdot \frac{2l}{2} + M = 0, \text{ bunda, } R_A = -\frac{3ql}{2}.$$

Tekshiramiz:

$$\sum Y_i = 0; \quad R_A - q \cdot 2l + R_B = \frac{-3ql}{2} - 2ql + \frac{7ql}{2} = 0.$$

Oxirgi tenglama tayanch reaksiyalarining to'g'ri aniqlanganligini tasdiqlaydi.

56- masala

6.6-shaklda tasvirlangan ramaning tayanch reaksiyalari topilsin. Quyidagilar ma'lum: $F = 5 \cdot 10^4 \text{ N}$; $q = 2 \cdot 10^4 \text{ N/m}$; $l = 1,2 \text{ m}$.

Yechish:

Ramaning sharnirli qo'zg'almas tayanchini H_A va R_A , sharnirlar qo'zg'aluvchi tayanchini esa R_B reaksiyalar bilan almashtiramiz.

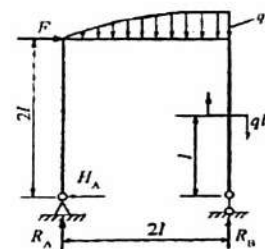
Muvozanat tenglamalarini tuzamiz:

$$\sum Z_i = 0; \quad F - H_A = 0; \quad H_A = 5 \cdot 10^4 \text{ N};$$

$$\begin{aligned}\sum M_{A_i} &= 0; \\ F \cdot 2l + \frac{2q \cdot 2l}{3} \cdot \frac{5 \cdot 2l}{8} + M - R_B \cdot 2l &= 0;\end{aligned}$$

$$R_B = 8,2 \cdot 10^4 \text{ kN}$$

$$\begin{aligned}\sum M_{B_i} &= 0; \quad R_A \cdot 2l + F \cdot 2l - \frac{2q \cdot 2l}{3} \cdot \frac{3 \cdot 2l}{8} + M = 0; \\ R_A &= -5 \cdot 10^4 \text{ kN}.\end{aligned}$$



6.6-shakl

Tekshirish:

$$\sum Y_i = 0; \quad R_A + R_B - \frac{2q \cdot 2l}{3} = 0; \quad -5 \cdot 10^4 + 8,2 \cdot 10^4 - 3,2 \cdot 10^4 = 0.$$

Demak, tayanch reaksiyalari to'g'ri topilgan.

57- masala

Egri brusga to'plangan va yoyilgan kuch ta'sir etmoqda (6.7-shakl); A va B tayanchlardagi reaksiyalar aniqlansin.

Quyidagilar ma'lum:

$$\alpha = 45^\circ; \quad q = l \cdot 10^4 \frac{N}{m}; \quad F = 2 \cdot 10^4 N; \quad l = 1 m.$$

Yechish:

Tayanchlarni tegishli nuqtalar bilan almashtirib, muvozanat tenglamalarini tuzamiz:

$$\sum Z_i = 0; \quad F \cdot \cos \alpha - H_A = 0;$$

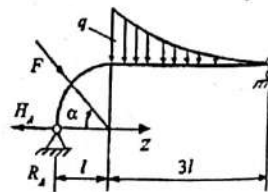
$$\sum M_{Ai} = 0, \quad F \cdot \sin \alpha (l - l \cos \alpha) + \frac{q \cdot 3l}{3} \cdot \left(\frac{3l}{4} + l \right) - R_B \cdot 4l + F \cdot \cos \alpha \cdot l \sin \alpha = 0;$$

$$\sum M_{Bi} = 0, \quad R_A \cdot 4l + H_A \cdot l - F \cdot \cos \alpha (l - l \sin \alpha) - F \cdot \cos \alpha (3l + l \cos \alpha) - \frac{q \cdot 3l}{3} \cdot \frac{3 \cdot 3l}{4} = 0;$$

Demak, $H_A = 1,414 \cdot 10^4 N$; $R_A = 1,6205 \cdot 10^4 N$ va $R_B = 0,79 \cdot 10^4 N$ ga teng ekan, bu qiymatlarning to'g'riligini tekshirish uchun $\sum Y_i = 0$ tenglamani tuzamiz:

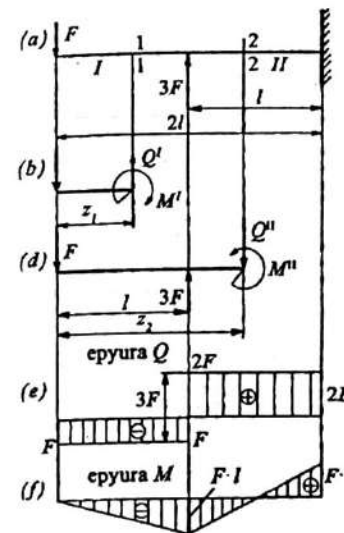
$$R_A - F \cdot \sin \alpha - \frac{q \cdot 3l}{3} + R_B = 0 \text{ yoki} \\ 2,41 \cdot 10^4 - 2,41 \cdot 10^4 = 0.$$

Demak, reaksiya kuchlari to'g'ri topilgan.



6.7-shakl

6.3-§. Tashqi kuchlar bilan yuklatilgan turli xil to'sinlar uchun ko'ndalang kuch va eguvchi moment epyuralarini qurish



6.8-shakl

Konsoldan ajratib olingan bo'lakcha uchun ichki kuchlarning analitik ifodalarini tuzamiz:

I oraliq: $0 \leq z_1 \leq l$

$$Q^I(z_1) = \pm \sum_{i=1}^1 Y_i = -F = \text{const};$$

$$M^I(z_1) = \pm \sum_{i=1}^1 M_{0i} = -F \cdot z_1.$$

II oraliq: $l \leq z_2 \leq 2l$

$$Q^{II}(z_2) = \pm \sum_{i=1}^2 Y_i = -F + 3F = 2F = \text{const};$$

$$M^{II}(z_2) = \pm \sum_{i=1}^2 M_{0i} = -F \cdot z_2 + 3F(z_2 - l).$$

58- masala

Ikki oraliqdan iborat konsol uchun ko'ndalang kuch va eguvchi moment epyuralarini quring (6.8-shakl, a).

Yechish:

Kesish usuliga muvofiq har bir oraliqni z masofada fikran qirib, konsoldan ajratilgan bo'lakchani alohida chizib olamiz (6.8-shakl, b, d).

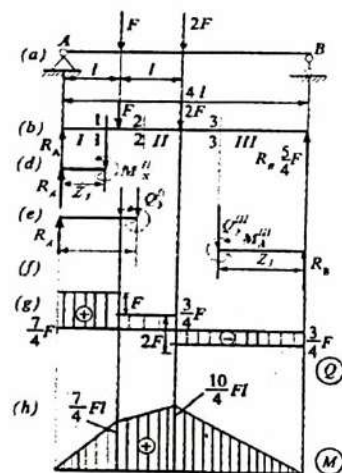
Kesim og'irlik markaziga navbat bilan $Q^I(z_1)$, $Q^{II}(z_2)$ ko'ndalang kuchlarni va $M^I(z_1)$, $M^{II}(z_2)$ eguvchi momentlarni qo'yamiz.

Hosil bo'lgan ifodalardan ko'rinib turibdiki, ikkala oraliqda ham ko'ndalang kuchlar $Q'(z_1) = \text{const}$;
 $Q''(z_2) = \text{const}$; eguvchi moment $M'(z_1)$, $M''(z_2)$ lar chiziqli qonuniyat bilan o'zgarar ekan.

Har bir oraliq uchun quyidagi jadvalni tuzamiz.

I oraliq			II oraliq		
z_1	Q	M	z_2	Q	M
0	$-F$	0	0	$2F$	$-F l$
l	$-F$	$-F l$	$2l$	$2F$	$F l$

Jadvaldagi qiymatlardan foydalanib, ichki kuchlarning miqdorlari – ordinatalarini aniq masshtab bilan konsol o'qiga parallel o'tkazilgan nol chizig'iga perpendikulyar qilib joylashtiramiz. Natijada, har bir oraliq uchun ko'ndalang kuch $Q(z)$ va eguvchi moment $M(z_1)$ larning epyuralari paydo bo'ladi (6.8-shakl, e, f).



6.9-shakl

To'sinning muvozanatini tekshiramiz:

59- masala

Oddiy to'singa F va $2F$ kuchlar ta'sir ko'rsatmoqda (6.9-shakl, a). Ichki zo'riqishlarning epyuralarini qurish talab etiladi.

Yechish:

Dastlab A (qo'zg'almas sharnirli) va B (qo'zg'aluvchan sharnirli) tayanchlarda paydo bo'luvchi reaksiyalarni aniqlash zarur (6.9-shakl, b). A tayanchdagi gorizonttal tashkil etuvchi $H_A = 0$ ga teng.

$$\sum_{i=0}^n M_A(F_i) = 0;$$

$$Fl + 2F \cdot 2l - R_B \cdot 4l = 0;$$

$$\sum_{i=0}^n M_B(F_i) = 0;$$

$$-2F \cdot 2l - F \cdot 3l + R_A \cdot 4l = 0;$$

muvozanat tenglamalarini tayanch reaksiyalariga nisbatan yechsak,
 $R_A = 7F/4$ va $R_B = 5F/4$ lar kelib chiqadi. Bu qiymatlarning to'g'rili-giga ishonch hosil qilish uchun

$$\sum_{i=1}^n Y_i = 0;$$

tenglamani tuzamiz:

$$R_A - F - 2F + R_B = 0 \quad \text{yoki} \quad \frac{7F}{4} - F - 2F + \frac{5F}{4} = 0.$$

Oxirgi ifodaning chap tomoni ham nolga teng. Demak, tayanch reaksiyalari to'g'ri topilgan. Kesish usulidan foydalanib, uchala oraliq uchun ham to'sindan «bo'lakcha» ajratib olamiz va kesim og'irlik markaziga ko'ndalang kuch va eguvchi momentlarni joylashtiramiz (6.9-shakl, d, e, f).

Har bir bo'lakcha uchun quyidagi analitik ifodalarni tuzamiz:

I oraliq: $0 \leq z_1 \leq l$

$$Q'(z_1) = \pm \sum_{i=1}^1 Y_i = R_A = \frac{7F}{4} = \text{const};$$

$$M'(z_1) = \pm \sum_{i=1}^1 M_{0i} = R_A \cdot z_1 = \frac{7F \cdot z_1}{4}$$

II oraliq: $l \leq z_2 \leq 2l$

$$Q''(z_2) = \pm \sum_{i=1}^2 Y_i = R_A - F = \frac{3F}{4} = \text{const};$$

$$M''(z_2) = \pm \sum_{i=1}^2 M_{0i} = R_A \cdot z_2 - F(z_2 - l).$$

III oraliq: (o'ng tomon) $l \leq z_3 \leq 2l$

$$Q'''(z_3) = \pm \sum_{i=1}^1 Y_i = -R_B = -\frac{5F}{4} = \text{const};$$

$$M'''(z_3) = \pm \sum_{i=1}^2 M_{0i} = R_B \cdot z_3 = \frac{5F}{4} \cdot z_3.$$

Quyidagi jadvalni tuzamiz:

I oraliq			II oraliq			III oraliq		
z_1	Q	M	z_2	Q	M	z_3	Q	M
0	$\frac{7Fl}{4}$	0	0	$\frac{3F}{4}$	$\frac{7Fl}{4}$	0	$-\frac{5F}{4}$	0
l	$\frac{7Fl}{4}$	$\frac{7Fl}{4}$	$2l$	$\frac{3F}{4}$	$\frac{7Fl}{4}$	l	$-\frac{5F}{4}$	$\frac{10Fl}{4}$

Jadvaldagi qiymatlar asosida qat'iy masshtab bilan ko'ndalang kuch va eguvchi moment epyuralarini quramiz (6.9-shakl, g, h).

60- masala

Konsol uchli to'singa F va $M=3Fl$ tashqi kuch ta'sir etmoqda (6.10-shakl, a). Ushbu to'sin uchun Q va M epyuralarini quring.

Yechish:

6.10-shakl (b) dan foydalanib, A va B tayanchlardagi reaksiya kuchlarini aniqlaymiz.

$$\sum_{i=0}^n M_A(F_i) = 0; \quad Fl + M - R_B \cdot 3l = 0;$$

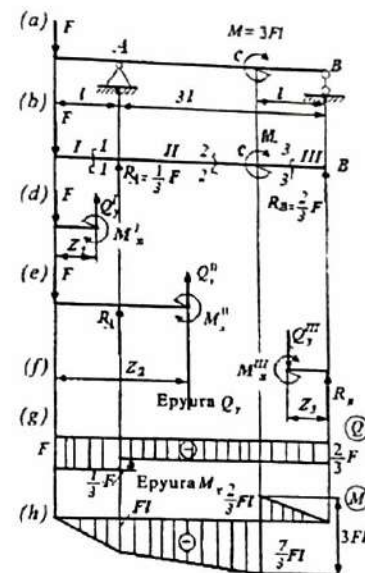
$$\sum_{i=0}^n M_B(F_i) = 0;$$

$$-F \cdot 4l + M + R_A \cdot 3l = 0$$

muvozanat tenglamalaridagi tayanch reaksiyalariga nisbatan yechsak,

$$R_A = \frac{F}{3} \quad \text{va} \quad R_B = \frac{2F}{3}$$

lar kelib chiqadi.



6.10-shakl

Endi R_A va R_B larning qiymatlarini tekshirib ko'ramiz:

$$\sum_{i=0}^n Y_i = 0;$$

$$R_A - F + R_B = 0; \quad \text{yoki} \quad \frac{F}{3} - F + \frac{2F}{3} = 0.$$

Oxirgi tenglama chap tomonining nolga tengligi reaksiya kuchlarining to'g'ri aniqlanganligini tasdiqlaydi.

Oldingi masalalardagi kabi har bir oraliqni navbat bilan fikran qirqamiz (6.10-shakl, d, e, f) hamda ichki zo'riqishlarning analitik ifodalarini tuzamiz:

I oraliq: $0 \leq z_1 \leq l$

$$Q_y^I(z_1) = \pm \sum_{i=1}^1 Y_i = -F = \text{const}; \quad M_x^I(z_1) = \pm \sum_{i=1}^1 M_{0i} = F \cdot z_1.$$

II oraliq: $l \leq z_2 \leq 4l$

$$Q_y^{II}(z_2) = \pm \sum_{i=1}^2 Y_i = R_A - F = \frac{F}{3} - F = -\frac{2}{3}F = \text{const};$$

$$M_x^{II}(z_2) = \pm \sum_{i=1}^2 M_{0i} = R_A(z_2 - l) - F \cdot z_2 = \frac{F \cdot (z_2 - l)}{3} - F \cdot z_2.$$

III oraliq: (o'ng tomon) $0 \leq z_3 \leq l$

$$Q_y^{III}(z_3) = \pm \sum_{i=1}^1 Y_i = -R_B = -\frac{2}{3}F = \text{const};$$

$$M_x^{III}(z_3) = \pm \sum_{i=1}^1 M_{0i} = R_B \cdot z_3 = \frac{2}{3}F \cdot z_3.$$

6.10-shakl (g), (h)larda olingan qiymatlar bo'yicha qurilgan ko'ndalang kuch Q va eguvchi moment M larning epyuralari keltirilgan.

61- masala

Qo'zg'almas va qo'zg'aluvchan tayanchlarda yotuvchi oddiy to'singa $q = \text{const}$ yoyilgan kuch intensivligi ta'sir ko'rsatmoqda (6.11-shakl, a), Q va M epyuralari qurilsin.

Yechish:

Har galgidek, masalani yechishni tayanch reaksiyalarni aniqlashdan boshlaymiz (6.11-shakl, b). Buning uchun statikaning muvozanat tenglamalarini tuzish kifoya:

$$\sum_{i=0}^n M_A(F_i) = 0;$$

$$ql \cdot \frac{l}{2} - R_B \cdot 2l = 0;$$

$$\sum_{i=0}^n M_B(F_i) = 0;$$

$$-ql \cdot \left(\frac{l}{2} + l\right) + R_A \cdot 2l = 0.$$

Bulardan, $R_A = \frac{3}{4}ql$ va $R_B = \frac{ql}{4}$ ekanligi ma'lum.

Tayanch reaksiyalarning qiymatlarini tekshirib ko'rish maqsadida

$$\sum_{i=1}^n Y_i = 0; \quad R_A - ql + R_B = 0$$

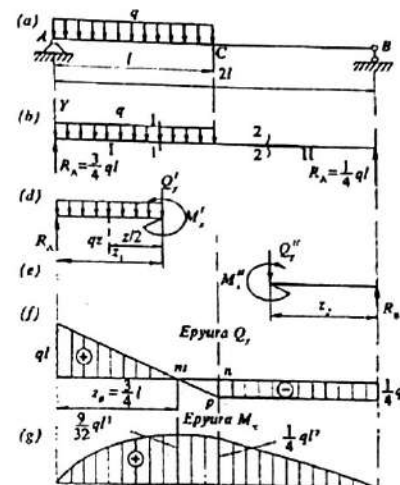
tenglamani tuzamiz.

$$\text{Demak, } \frac{3}{4}ql - ql + \frac{1}{4}ql = 0 \text{ yoki } 0 = 0.$$

Kesish usulini qo'llab, 6.11-shakl (d, e) chizmalarni hosil qilamiz. Natijada, Q va M larning analitik ifodalari quyidagi ko'rinishda yoziladi:

I oraliq: $0 \leq z_1 \leq l$

$$Q_y^I(z_1) = \pm \sum_{i=1}^2 Y_i = R_A - qz_1 = \frac{3}{4}ql - qz_1;$$



6.11-shakl

$$M_x^I(z_1) = \pm \sum_{i=1}^2 M_{0i} = R_A \cdot z_1 - qz_1 \cdot \frac{z_1}{2} = \frac{3ql \cdot z_1}{4} - \frac{qz_1^2}{2},$$

(bu yerda, $z_1/2$ – kuch yelkasi, ya'ni teng ta'sir etuvchi qz_1 qo'yilgan nuqtadan I-I kesimning og'irlik markazigacha bo'lgan masofa).

II oraliq: (o'ng tomon) $0 \leq z_2 \leq l$

$$Q_y^{II}(z_2) = \pm \sum_{i=1}^1 Y_i = -R_B = -\frac{ql}{4} = \text{const};$$

$$M_x^{II}(z_2) = \pm \sum_{i=1}^1 M_{0i} = R_B \cdot z_2 = \frac{qlz_2}{4}$$

ifodaning birinchisi yordamida ko'ndalang kuch epyurasi quriladi:

$$z_1 = 0 \text{ da } Q_y^I(0) = \frac{3}{4}ql;$$

$$z_2 = l \text{ esa } Q_y^{II}(0) = -\frac{ql}{4} \text{ bo'ladi}$$

Demak, I oraliqning boshlanishida ko'ndalang kuch musbat, oxirida esa manfiy qiymatga ega bo'lar ekan. Boshqacha aytganda, $0 \leq z_1 \leq l$ oraliqda ko'ndalang kuch nolga teng bo'lgan kesim mavjud bo'ladi. Buni aniqlash uchun shu oraliqda ko'ndalang kuch tenglamasini nolga tenglab, $z_1 = z_0$ ni aniqlaymiz: $Q_y^I(z_1) = 0$ yoki $\frac{3ql}{4} - qz_0 = 0$.

Bundan, $z_0 = \frac{3l}{4}$ ekanligi kelib chiqadi.

$M_x^I(z_1)$ eguvchi momentning tenglamasi ikkinchi tartibli egri chiziq tenglamasidan iborat; shu sababli epyura qurish uchun eng kamida yana bitta nuqtadagi eguvchi moment qiymatini bilish shart. $z_0 = 3l/4$ bo'lgan «o'ziga xos» kesimda eguvchi momentning qiymatini topamiz:

$$M_0 = \frac{3ql}{4} \cdot z_0 - \frac{q}{2} \cdot z_0^2 = \frac{3ql}{4} \cdot \frac{3l}{4} - \frac{q}{2} \cdot \left(\frac{3l}{4}\right)^2 = \frac{9ql^2}{32}$$

Eslatma: materiallar qarshiligi to'la kursida ko'ndalang kuch nolga teng bo'lgan kesimda eguvchi moment ekstremal (max yoki min) qiymatga erishishi isbotlangan. Shu bois, biz tekshirayotgan hol uchun:

$$z_0 = \frac{3l}{4} \text{ da } M_0 = \frac{9ql^2}{32} \text{ bo'ladi.}$$

Odatdagidek, har bir oraliq uchun jadval tuzamiz:

I oraliq			II oraliq		
z_1	Q	M	z_2	Q	M
0	$\frac{3ql}{4}$	0	0	$-\frac{ql}{4}$	0
$\frac{3l}{4}$	0	$\frac{9ql^2}{32}$			
l	$-\frac{ql}{4}$	$\frac{ql^2}{4}$	l	$-\frac{ql}{4}$	$\frac{ql^2}{4}$

Jadvaldagi qiymatlar bo'yicha ma'lum masshtabda ko'ndalang kuch Q va eguvchi moment M larning epyuralarini quramiz (6.11-shakl, f, g)

62-masala

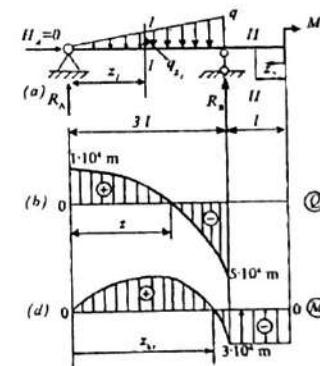
Konsol uchli to'singa juft kuch va Q yoyilgan kuch ta'sir ko'rsatmoqda (6.12-shakl, a). Shu to'sin uchun ko'ndalang kuch va eguvchi momentning epyurasi qurilsin. Quyidagilar ma'lum: $q=4 \cdot 10^4$ N/m; $M=3 \cdot 10^4$ N·m; $l=1$ m.

Yechish:

To'sinning tayanchlarini tayanch reaksiyalari bilan almash-tiramiz: R_A va R_B larni aniqlash uchun esa statikaning muvozanat tenglamalarini tuzamiz:

$$\sum M_A = 0, \\ q \cdot \frac{3l}{2} \cdot 2 \cdot \frac{3l}{3} + M - R_B \cdot 3l = 0,$$

bundan, $R_B = 5 \cdot 10^4$ N;



6.12-shakl

$$\sum M_B = 0, R_A \cdot 3l - q \cdot \frac{3l}{2} \cdot \frac{3l}{3} + M = 0, \text{ bundan,}$$

$$R_A = 1 \cdot 10^4 \text{ N};$$

Tekshirish:

$$\sum Y_i = 0, R_A - q \cdot \frac{3l}{2} + R_B = 0;$$

yoki

$$1 \cdot 10^4 - 1,5 \cdot 4 \cdot 10^4 \cdot 1 + 5 \cdot 10^4 = 0, 0 = 0.$$

Oxirgi tenglama tayanch reaksiyalarining to'g'ri topilganligi tasdiqlaydi.

To'sin o'qi bo'yicha Z ni yo'naltirib, oraliqni mos ravishda I va II raqamlari bilan belgilaymiz. Yana kesim usuliga murojaat etib, har bir oraliqni tegishli I-I va II-II tekisliklari bilan fikran qirqamiz.

Qirqimning ixtiyoriy tomonidan bo'lakchalar ajratib olib, Q va M larning tenglamalarini tuzamiz:

I oraliq: (chap tomon uchun) $0 \leq z_1 \leq 3l$

$$Q(z_1) = R_A - q(z_1) \cdot \frac{z_1}{2}; \quad M(z_1) = R_A \cdot z_1 - q(z_1) \cdot \frac{z_1}{2} \cdot \frac{z_1}{3},$$

bu yerda, $q(z_1)$ – ixtiyoriy z_1 uzunlikka to'g'ri keluvchi yoyilgan kuch intensivligi bo'lib, quyidagi munosabatdan topiladi:

$$\frac{q(z_1)}{q} = \frac{z_1}{3l} \text{ yoki } q(z_1) = \frac{q \cdot z_1}{3l};$$

Shunday qilib, I oraliqda ichki zo'riqishlar quyidagi formulalar yordamida ifodalanadi (chiziqli bo'lmagan bog'lanish):

$$Q_y(z_1) = R_A - \frac{q \cdot z_1^2}{6l}; \quad M(z_1) = R_A \cdot z_1 - \frac{q \cdot z_1^3}{18l};$$

$$z_1 = 0 \text{ bo'lsa, } Q_y(0) = 1 \cdot 10^4 \text{ N, } M(0) = 0;$$

$$z_1 = 3m \text{ bo'lsa, } Q_y(3) = -5 \cdot 10^4 \text{ N, } M(3) = -3 \cdot 10^4 \text{ N} \cdot m.$$

Bu oraliqda ichki zo'riqishlar to'sin o'qi bo'yicha chiziqli bo'lmagan qonuniyatga ko'ra o'zgaruvchanligi sababli, ularning epyurasini qurishda 1-jadvaldan foydalanamiz (oldingi masalaga qarang). Unda:

$$Q'(z_1) = 0, \quad R_A - \frac{q \cdot z_1^2}{6l} = 0$$

Bundan kritik nuqtani topamiz: $z = 0$.

Demak, I oraliqning boshida Q ning epyurasiga urunma o'tkazish lozim.

	$0 \leq z_1 \leq 3l$
$Q'(z)$	–
$Q(z)$	(kamayadi)

Ikkinchi tartibli hosila $Q''(z_1) = -\frac{q}{3l} < 0$ bo'lganligi uchun epyurasi $[0, 3l]$ oraliqda qavariq bo'ladi; bu epyuraning abssissa o'qi bilan kesishuv nuqtasi $M^1(z_1) = 0$ tenglamadan topiladi:

$$z_{kr1} = \left[\sqrt{\frac{6R_A \cdot l}{q}} \right] = \sqrt{\frac{3}{2}} m$$

tenglik o'rinli bo'ladi.

$$Q'(z_1) = R_A - \frac{q \cdot z_1^2}{6l} = 0 \text{ tenglamadan kritik nuqtani topamiz:}$$

$$z_{kr1} = \sqrt{\frac{6}{4}} m = \sqrt{\frac{3}{2}} m.$$

	$0 < z_1 < z_{kr}$	z_{kr}	$z_{kr} < z_1 < 3l$
$M'(z)$	+	0	–
$M(z)$	↗	max	↘

Tekshirilayotgan oraliqda ikkinchi tartibli hosila $M''(z) = -\frac{q \cdot z_1}{3l} < 0$ bo'lganligi uchun M epyurasi qavariq bo'ladi. Eguvchi moment epyurasi I oraliqda Z o'qini ikki joyda: koordinata boshida va $M(z_1) = 0$ tenglamadan aniqlanadigan

$$z_{kr2} = \sqrt{\frac{18R_A \cdot l}{q}} = 2,12 \text{ m}$$

masofada kesib o'tadi,

II oraliq: (o'ng tomon uchun) $0 \leq z_2 \leq l$

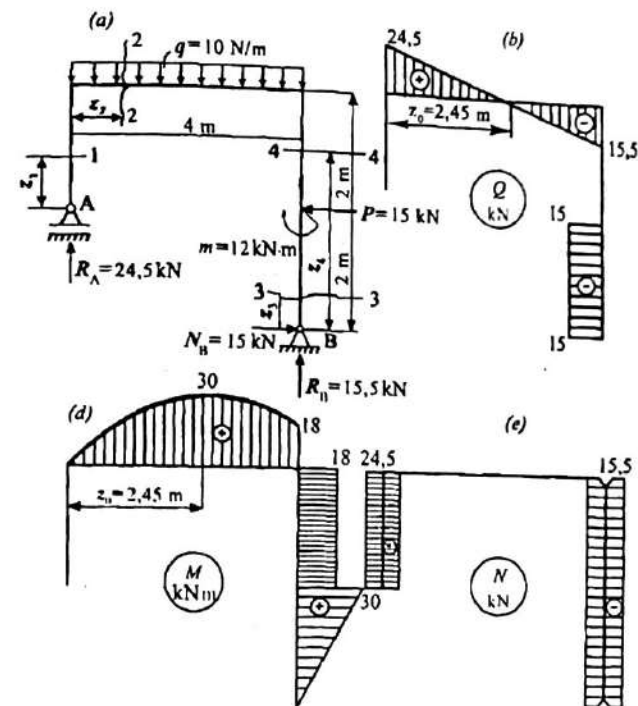
$$Q(z_2) = 0; \quad M(z_2) = -M = \text{const.}$$

6.12-shakl (b, d) da Q va M epyuralari tasvirlangan.

6.4- §. Statik aniq rama, siniq va egri balkalar uchun ichki kuch faktorlari epyuralarini qurish

63-masala

Quyida berilgan statik aniq rama uchun kesuvchi kuch (Q) eguvchi moment (M) va bo'ylama kuch (N) epyuralari qurilsin. Qiymatlar 6. 13-shaklda berilgan.



6. 13-shakl

Yechish:

1. Tavan ch reaksiya kuchlarini aniqlaymiz:

$$\sum X = 0; \quad H_B - P = 0 \Rightarrow H_B = P = 15 \text{ kN}$$

$$\sum M_A = 10 \cdot 4 \cdot 2 - 15 \cdot 2 + 12 - 4R_B = 0 \Rightarrow R_B = 15,5 \text{ kN};$$

$$\sum M_B = 4R_A - 10 \cdot 4 \cdot 2 - 15 \cdot 2 + 12 = 0 \Rightarrow R_A = 24,5 \text{ kN}.$$

$$\text{Tekshirish: } \sum Y = 0; \quad 24,5 + 15,5 - 10 \cdot 4 = 0.$$

Demak, tayanch reaksiya kuchlari to'g'ri topilgan.

Ramani to'rtta uchastkaga bo'lib, kesish usulini qo'llab, uchastkalardagi Q , M va N qiymatlarini aniqlaymiz.

2. Kesuvchi kuch qiymatlarini aniqlaymiz:

I uchastka:

$$0 \leq z_1 \leq 2 \text{ m} \quad Q = 0;$$

II uchastka: $0 \leq z_2 \leq 4m$ $Q = R_A - q \cdot z_2$;
 $z_2 = 0$ bo'lsa, $Q = 24,5 \text{ kN}$;
 $z_2 = 4m$ bo'lsa, $Q = 24,5 - 10 \cdot 4 = -15,5 \text{ kN}$;

III uchastka: $0 \leq z_3 \leq 2m$ $Q = -H_B = -15 \text{ kN}$;
IV uchastka: $2m \leq z_4 \leq 4m$ $Q = -H_B + P = -15 + 15 = 0$.

Aniqlangan qiymatlar bo'yicha Q epyurasini quramiz (6.13-shakl, b).

3. Eguvchi moment qiymatlarini aniqlaymiz:

I uchastka: $0 \leq z_1 \leq 2m$ $M = 0$;

II uchastka: $0 \leq z_2 \leq 4m$ $M = R_A \cdot z_2 - q \cdot z_2^2/2$;
 $z_2 = 0$ bo'lsa, $M = 0$;
 $z_2 = 4m$ bo'lsa, $M = 24,5 - 10 \cdot 4^2/2 = 18 \text{ N} \cdot m$.

M ning ekstremal qiymatlarini aniqlaymiz:

$$\frac{dM}{dz} = R_A - q \cdot z_2; 24,5 - 10 \cdot z_2 = 0;$$

$$z_0 = z_2 = 2,45 \text{ m bo'lsa, } M_{max} = 24,5 \cdot 2,45 - 10 \cdot 2,45^2/2 = 31,2 \text{ kN} \cdot m.$$

III uchastka: $0 \leq z_3 \leq 2m$ $M = H_B \cdot z_3$;

$z_3 = 0$ bo'lsa, $M = 0$;

$z_3 = 2m$ bo'lsa, $M = 15 \cdot 2 = 30 \text{ kN} \cdot m$.

IV uchastka: (sof egilish, chunki $Q_4 = 0$)

$$M = \text{const} = H_B \cdot 2m = 30 - 12 = 18 \text{ kN} \cdot m.$$

Aniqlangan qiymatlar bo'yicha M epyurasini quramiz (6.13-shakl, d).

4. Bo'ylama kuch qiymatlarini aniqlaymiz:

I uchastka: $0 \leq z_1 \leq 2m$ $N = -R_A = -24,5 \text{ kN}$;

II uchastka: $0 \leq z_2 \leq 4m$ $N = 0$;

III uchastka: $0 \leq z_3 \leq 2m$ $N = -R_B = -15,5 \text{ kN}$;

IV uchastka: $2m \leq z_4 \leq 4m$ $N = -R_B = -15,5$.

Aniqlangan qiymatlar bo'yicha N epyurasini quramiz (6.13-shakl, e).

64- masala

Berilgan siniq balka uchun M , Q va N epyuralari aniqlansin.
 Qiymatlar 6.14-shaklda berilgan.

Yechish:

1. Tavanch reaksiya kuchlarini aniqlaymiz:

$$\sum X = 0; H_A - 10 \cdot 2 = 0 \Rightarrow H_A = 20 \text{ kN};$$

$$\sum Y = 0; R_A - 15 = 0 \Rightarrow R_A = 15 \text{ kN};$$

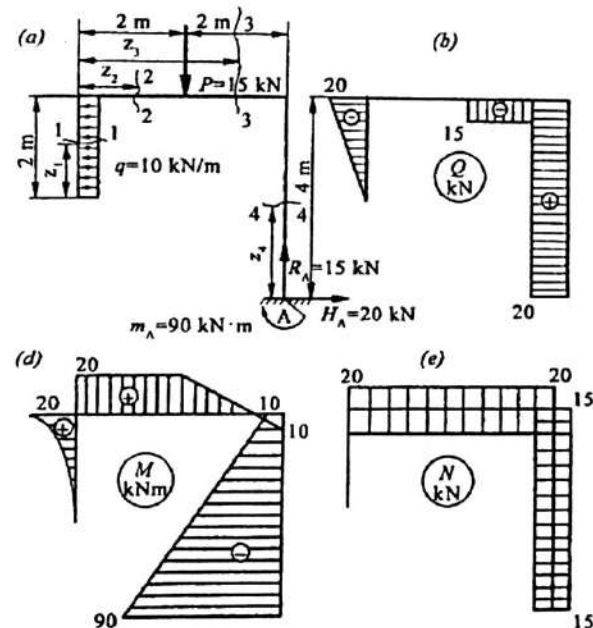
$$\sum M_A = 0; 10 \cdot 2 \cdot 3 + 15 \cdot 2 - m_A = 0 \Rightarrow m_A = 90 \text{ kN} \cdot m;$$

2. Kesuvchi kuch qiymatlarini aniqlaymiz:

I uchastka: $0 \leq z_1 \leq 2m$ $Q = q \cdot z_1$;

$z_1 = 0$ bo'lsa, $Q = 0$;

$z_1 = 2m$ bo'lsa, $Q = 10 \cdot 2 = 20 \text{ kN}$.



6.14-shakl

II uchastka: $0 \leq z_2 \leq 2m$ $Q = 0$;

III uchastka: $2m \leq z_3 \leq 4m$ $Q = -P = -15 \text{ kN}$;

IV uchastka: $0 \leq z_4 \leq 4$ $Q = -H_A = -20 \text{ kN}$.

Aniqlangan qiymatlar bo'yicha Q epyurasini quramiz (6.14-shakl, b).

3) Eguvchi moment qiymatlarini aniqlaymiz:

I uchastka: $0 \leq z_1 \leq 2m$ $M = q \cdot z_1^2 / 2$;

$z_1 = 0$ bo'lsa, $M = 0$;

$z_1 = 2m$ bo'lsa, $Q = 10 \cdot 2^2 / 2 = 20 \text{ kN} \cdot m$.

II uchastka: $0 \leq z_2 \leq 2m$ $M = \text{const} = 10 \cdot 2 = 20 \text{ kN} \cdot m$;

III uchastka: $2m \leq z_3 \leq 4m$ $M = q \cdot 2 \cdot 1 - P(z_3 - 2)$;

$z_3 = 2m$, $M = 20 \text{ kN} \cdot m$;

$z_3 = 4m$, $M = 20 - 15(4 - 2) = -10 \text{ kN} \cdot m$;

IV uchastka $0 \leq z_4 \leq 4m$ $M = -m_A + H_A \cdot z_4$;

$z_4 = 0$, $M = -m_A = -90 \text{ kN} \cdot m$;

$z_4 = 4m$, $M = -90 + 20 \cdot 4 = -10 \text{ kN} \cdot m$.

Aniqlangan qiymatlar bo'yicha M epyurasini quramiz (6.14-shakl, d).

4. Bo'ylama kuch qiymatlarini aniqlaymiz:

I uchastka: $0 \leq z_1 \leq 2m$ $N = 0$;

II uchastka: $0 \leq z_2 \leq 2m$ $N = 20 \text{ kN}$;

III uchastka: $0 \leq z_3 \leq 4m$ $N = 20 \text{ kN}$;

IV uchastka: $2m \leq z_4 \leq 4m$ $N = -R_A = -15 \text{ kN}$.

Aniqlangan qiymatlar bo'yicha N epyurasini quramiz (6.14-shakl, e).

65- masala

Berilgan siniq balka uchun M , Q va N epyuralari qurilsin. Qiymatlar 6.15-shaklda berilgan.

Yechish:

1. Tayanch reaksiya kuchlarini aniqlaymiz:

$$\sum X = 0; H_A = 0;$$

$$\sum Y = 0; R_A - 10 \cdot 2 - 15 = 0 \Rightarrow R_A = 35 \text{ kN};$$

$$\sum M_A = 0; M_A - 10 \cdot 2 \cdot 1 - 15 \cdot 2 - 12 = 0 \Rightarrow M_A = 62 \text{ kN} \cdot m;$$

2. Kesuvchi kuch qiymatlarini aniqlaymiz:

I uchastka: $0 \leq z_1 \leq 2m$;

$z_1 = 0$ bo'lsa, $Q = 35 \text{ kN}$;

$z_1 = 2m$ bo'lsa, $Q = 35 - 10 \cdot 2 = 15 \text{ kN}$.

II uchastka: $2m \leq z_2 \leq 4m$; $Q = 15 \text{ kN}$;

III uchastka: $0 \leq z_3 \leq 2m$; $Q = -P = -15 \text{ kN}$;

IV uchastka: $0 \leq z_4 \leq 2m$; $Q = 0$

Aniqlangan qiymatlar bo'yicha Q epyurasini quramiz (6.15-shakl, b).

3. Eguvchi moment qiymatlarini aniqlaymiz:

I uchastka: $0 \leq z_1 \leq 2m$; $M = -M_A + R_A \cdot z_1 - q \cdot z_1^2 / 2$;

$z_1 = 0$ bo'lsa, $M = -M_A = -62 \text{ kN} \cdot m$;

$z_1 = 2m$ bo'lsa, $M = -62 + 35 \cdot 2 - 10 \cdot \frac{2^2}{2} = -62 + 70 - 20 = -12 \text{ kN} \cdot m$.

II uchastka: $2 \leq z_2 \leq 4m$ $M = -M_A + R_A \cdot z_2 - q \cdot 2(z_2 - 1)$;

$z_2 = 2m$ bo'lsa, $M = -62 + 35 \cdot 2 - 10 \cdot 2 \cdot 1 = -12 \text{ kN} \cdot m$;

$z_2 = 4m$ bo'lsa, $M = -62 + 35 \cdot 2 - 10 \cdot 2 \cdot 3 = 18 \text{ kN} \cdot m$

III uchastka: $0 \leq z_3 \leq 2m$; $M = P \cdot z_3$

$z_3 = 0$ bo'lsa, $M = 0$;

$z_3 = 2m$ bo'lsa, $M = 15 \cdot 2 = 30 \text{ kN} \cdot m$

IV uchastka: $0 \leq z_4 \leq 2m$ $M = P \cdot z_4$

$z_4 = 0$ bo'lsa, $M = 0$;

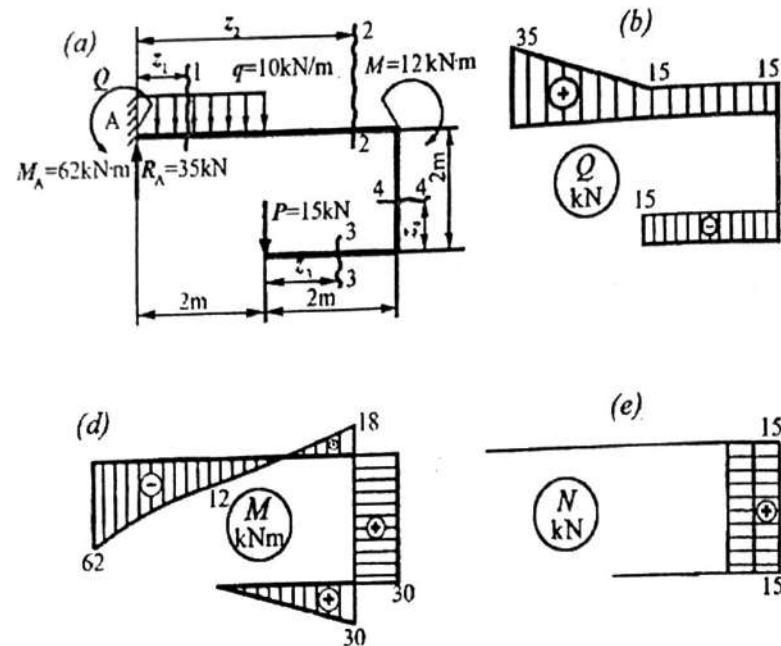
$z_4 = 2m$ bo'lsa, $M = 15 \cdot 2 = 30 \text{ kN} \cdot m$

Aniqlangan qiymatlar bo'yicha M epyurasini quramiz (6.15-shakl, d).

4. Bo'ylama kuch qiymatlarini aniqlaymiz:

I uchastka $0 \leq z_1 \leq 2m$; $N = 0$

II uchastka: $2m \leq z_2 \leq 4m$; $N = 0$



6.15-shakl

III uchastka: $0 \leq z_3 \leq 2\text{m}$; $N=0$;

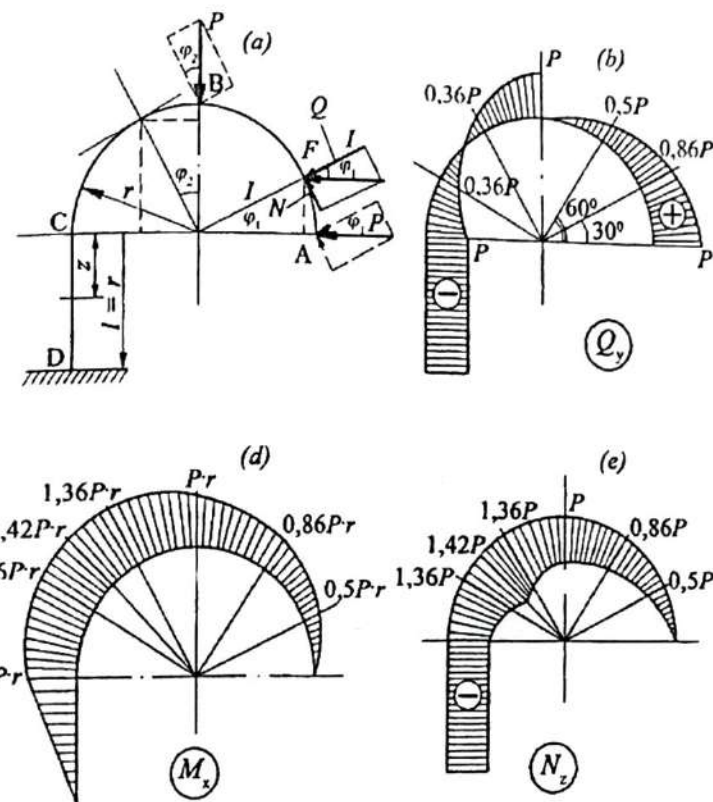
IV uchastka: $0 \leq z_4 \leq 2\text{m}$; $N=15\text{kN}$.

Aniqlangan qiymatlar bo'yicha N epyurasini quramiz (6.15-shakl, e).

Yuqorida keltirilgan masalani siniq balkalardagi tayanch reaksiya kuchlarini aniqlamasdan yechish ham mumkin. Unda balka uchastkalari erkin uchidan boshlab fikran kesilib, har gal mahkamlangan tomon tashlab yuboriladi.

66- masala

Berilgan egri sterjen uchun M , Q va N epyuralari qurilsin. Qiymatlar 6.16-shaklda berilgan.



6.16-shakl

Yechish:

Bu masalani tayanch reaksiya kuchlarini aniqlamasdan, erkin uchidan boshlab yechamiz. Brus uchta uchastkadan iborat bo'lib, ular egri chiziqli AB va BC hamda to'g'ri chiziqli CD lardir.

Bu uchastkalarni alohida ko'rib o'tamiz:

1. AB uchastka: $0 \leq \varphi_1 \leq \pi/2$

AB uchastkadan fikran F nuqta orqali I-I kesim o'tkazamiz. R kuchni F nuqtaga ko'chiramiz. Unda birinchi uchastkada hosil bo'lgan ichki kuch faktorlari teng bo'ladi:

$$Q^I(\varphi_1) = P \cdot \cos \varphi_1; M_z^I(\varphi_1) = P \cdot r \cdot \sin \varphi_1; N_z^I(\varphi_1) = -P \cdot \sin \varphi_1$$

Yuqoridagi tenglamalarga φ_1 ning qiymatlarini qo'yib, quyidagi jadvalni tuzamiz:

	φ_1	0	$\pi/6$	$\pi/3$	$\pi/2$
1	$Q^I(\varphi_1) = P \cdot \cos \varphi_1$	P	$0,86 P$	$-0,5 P$	0
2	$M_z^I(\varphi_1) = P \cdot r \cdot \sin \varphi_1$	0	$0,5 Pr$	$0,86 Pr$	Pr
3	$N_z^I(\varphi_1) = -P \cdot \sin \varphi_1$	0	$-0,5 P$	$0,86 P$	$-P$

2. BC uchastka $\frac{\pi}{2} \geq \varphi_2 \geq 0$.

Ikkinchi uchastkada esa tenglamalar quyidagi ko'rinishda bo'ladi:

$$\begin{aligned} Q^{II}(\varphi_2) &= -P \cdot \sin \varphi_2 + P \cdot \cos \varphi_2; \\ M_z^{II}(\varphi_2) &= P \cdot r \cdot \cos \varphi_2 + P \cdot \sin \varphi_2; \\ N_z^{II}(\varphi_2) &= -P \cdot \cos \varphi_2 - P \cdot \sin \varphi_2; \end{aligned}$$

Bu tenglamalarga ham φ_2 ning qiymatlarini qo'yib, quyidagi jadvalni tuzamiz:

	φ_2	0	$\pi/6$	$\pi/3$	$\pi/2$
1	$Q^{II}(\varphi_2) = -P \cdot \sin \varphi_2 + P \cdot \cos \varphi_2$	P	$0,36 P$	$-0,36 P$	$-P$
2	$M_z^{II}(\varphi_2) = P \cdot r \cdot \cos \varphi_2 + P \cdot \sin \varphi_2$	Pr	$1,36 Pr$	$1,36 Pr$	Pr
3	$N_z^{II}(\varphi_2) = -P \cdot \cos \varphi_2 - P \cdot \sin \varphi_2$	$-P$	$-1,36 P$	$1,36 P$	$-P$

2. CD uchastka: $0 \leq z_3 \leq l$

Uchinchi uchastkada hosil bo'lgan ichki kuch faktorlari quyidagiga teng bo'ladi:

$$\begin{aligned} Q^{III}(z) &= -P; & N^{III}(z) &= -P; \\ M_z^{III}(z) &= -P \cdot z + P \cdot 2; & M_z^{III}(0) &= P \cdot r; & M_z^{III}(l) &= 0 \end{aligned}$$

Topilgan qiymatlar bo'yicha Q (6.16-shakl, b), M (6.16-shakl, d) va N (6.16-shakl, e) epyuralarini quramiz.

6.5-§. To'g'ri egilishda balka va ramalar uchun ichki kuch faktorlari epyuralarini qurish bo'yicha topshiriqlar

9-topshiriq

To'g'ri egilishda kesuvchi kuch va eguvchi moment epyuralarini qurish. To'sinlar ko'ndalang kesimlarini tanlash.

Berilgan: Keltirilgan shakllarda tashqi tomondan yuklangan to'sinlar berilgan.

Topish kerak:

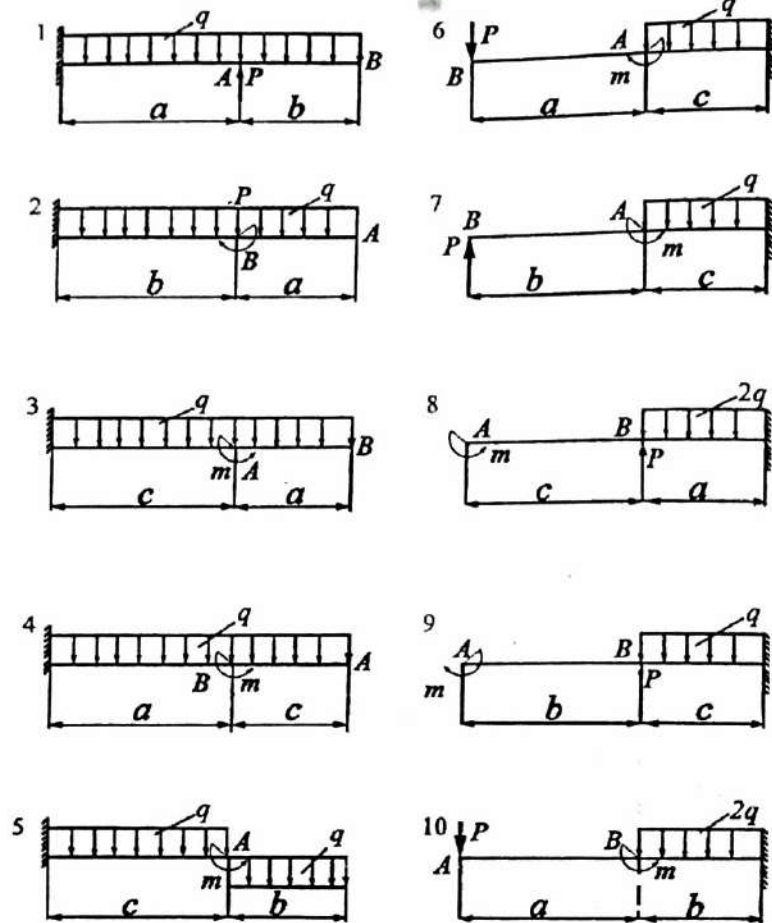
1) berilgan to'sinlar uchun kesuvchi kuch va eguvchi moment epyuralari qurilsin;

2) mustahkamlik shartiga binoan, to'sinning ko'ndalang kesimi tanlansin (kesim ko'rinishi – o'qituvchi ko'rsatmasi bo'yicha).

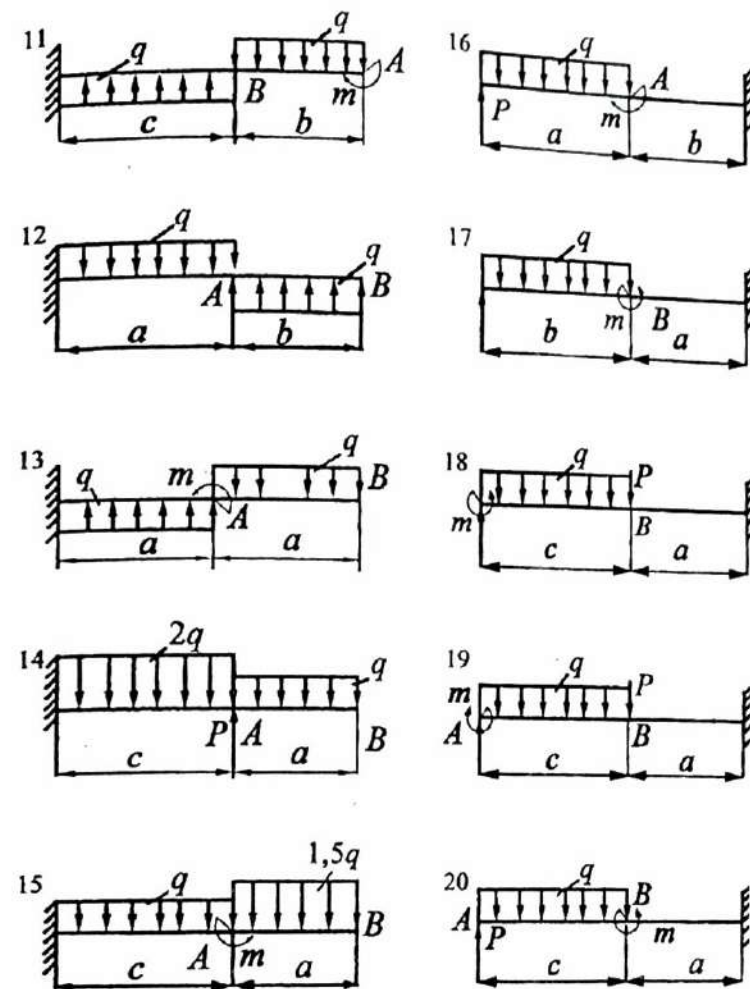
Kerakli ma'lumotlar 6.1-jadvalda, hisoblash sxemalari esa 6.17-shaklda berilgan.

6.1-jadval

N_2	α	b	C	P	M	$q, \text{kV/m}$	l, m
1	1	$2l$	$3l$	ql	$2ql^2$	2	1
2	$2l$	$3l$	$4l$	$2ql$	$3ql^2$	3	1
3	$3l$	$4l$	l	$3ql$	$4ql^2$	4	1
4	$4l$	l	$2l$	$4ql$	$5ql^2$	5	1
5	l	$2l$	$3l$	$5ql$	$6ql^2$	6	1
6	$2l$	$3l$	$4l$	$6ql$	$7ql^2$	2	1
7	$3l$	$4l$	l	$7ql$	$8ql^2$	3	1
8	$4l$	l	$2l$	$8ql$	ql^2	4	1
9	l	$2l$	$3l$	$9ql$	$2ql^2$	5	1
10	$2l$	$3l$	$4l$	ql	$3ql^2$	6	1



6.17-shakl



6.17 shakl (davomi)

10- topshiriq

To'g'ri egilishda kesuvchi kuch va eguvchi moment epyuralarini qurish. To'sinlar ko'ndalang kesimlarini tanlash.

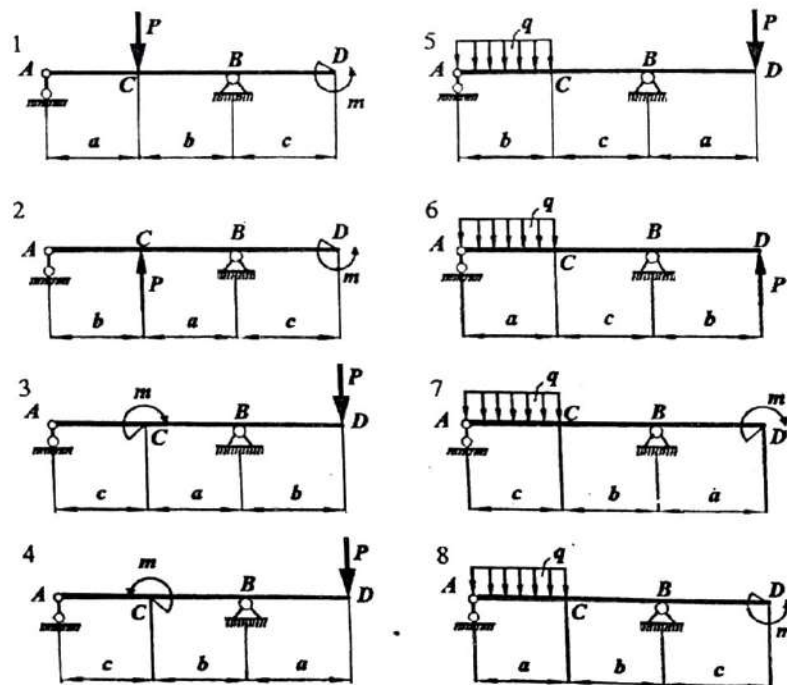
Berilgan: Keltirilgan shakllarda (6.18-shakl) tashqi tomondan yuklangan to'sinlar berilgan.

Topish kerak:

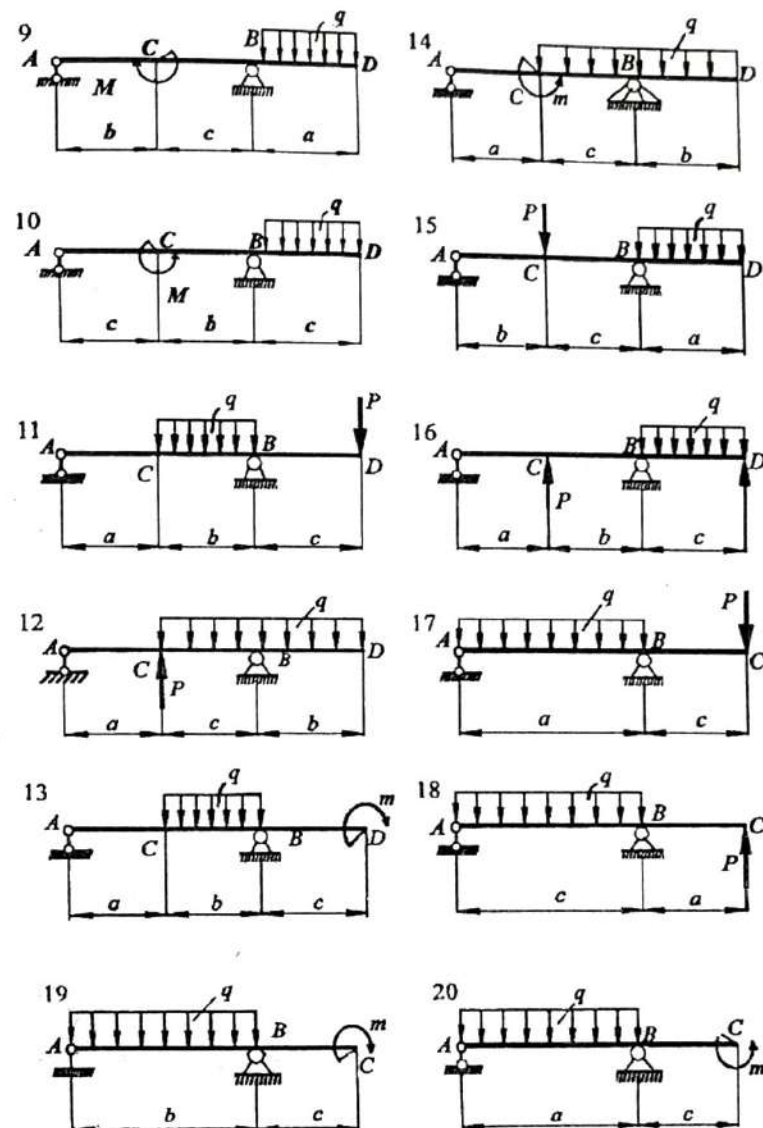
1) berilgan to'sinlar uchun kesuvchi kuch va eguvchi moment epyuralari qurilsin;

2) mustahkamlik shartiga binoan to'sinning ko'ndalang kesimi tanlansin (kesim ko'rinishi-o'qituvchi ko'rsatmasi bo'yicha).

Kerakli ma'lumotlar 6.1-jadvalda, hisoblash sxemalari esa 6.18-shaklda berilgan.



6.18-shakl



6.18-shakl (davomi)

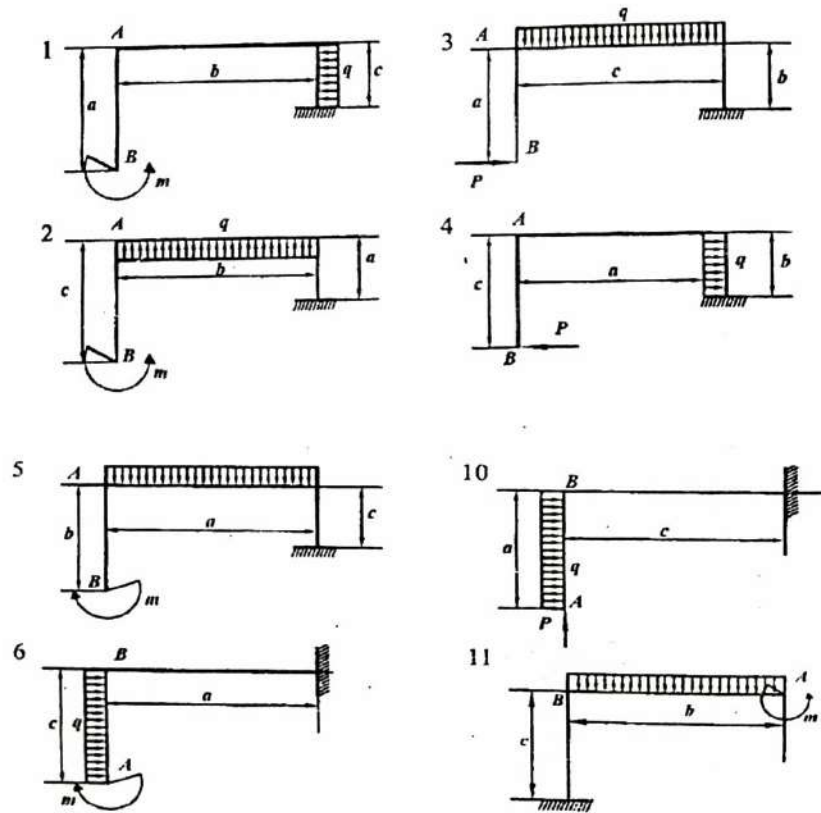
11- topshiriq

Siniq sterjenlar uchun ichki kuch faktorlari epyuralarini qurish.

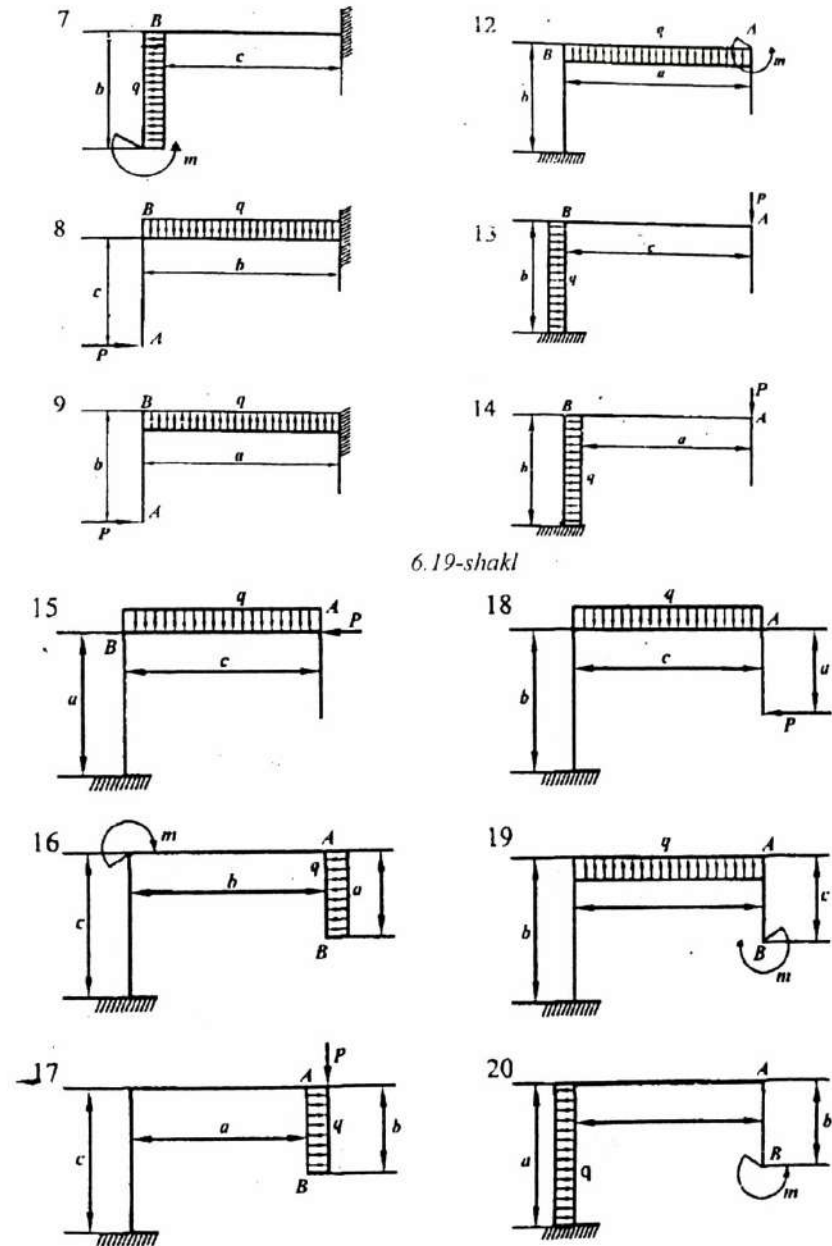
Topish kerak:

- 1) berilgan siniq sterjen uchun (6.19-shakl) bo'ylama kuch, kesuvchi kuch va eguvchi moment epyuralari qurilsin;
- 2) mustahkamlik shartiga binoan, sterjenning ko'ndalang kesimi tanlansin (kesim ko'rinishi o'qituvchi ko'rsatmasi bo'yicha).

Kerakli ma'lumotlar 6.1-jadvalda, hisoblash sxemalar 6.19-shaklda keltirilgan.



6.19-shakl



6.19-shakl

12- topshiriq

Egri sterjenlar uchun ichki kuch faktorlari epyuralarini qurish.

Berilgan: Keltirilgan shakllarda (6.20-shakl) tashqi tomondan yuklangan to'rsinlar berilgan.

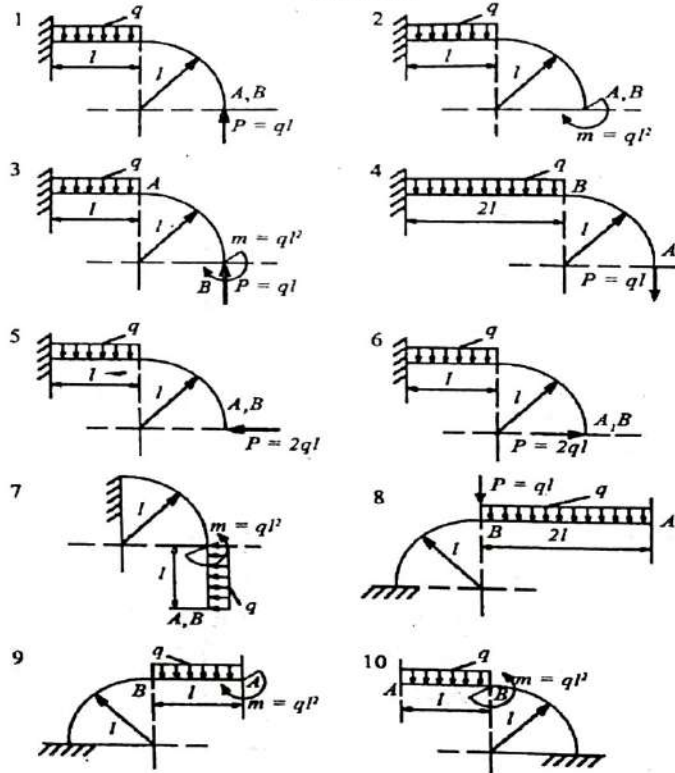
Topish kerak:

1) berilgan egri sterjen uchun bo'ylama kuch, kesuvchi kuch va eguvchi moment epyuralari qurilsin;

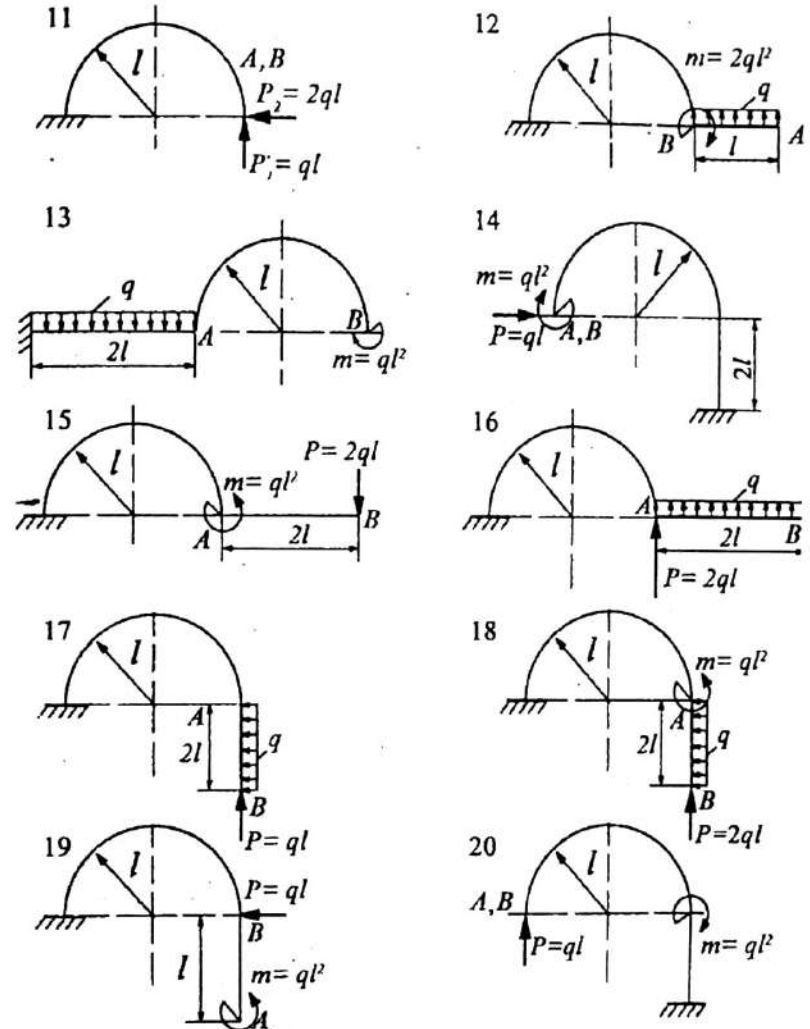
2) mustahkamlik shartiga binoan, egri sterjenning ko'ndalang kesimi tanlansin (kesim ko'rinishi-o'qituvchi ko'rsatmasi bo'yicha).

Kerakli ma'lumotlar 6.1-jadvalda, hisoblash sxematari 6.20-shaklda keltirilgan.

6.20-shakl



6.20-shakl davomi



13- topshiriq

Statik aniq ramalar uchun ichki kuch faktorlari epyuralarini qurish.

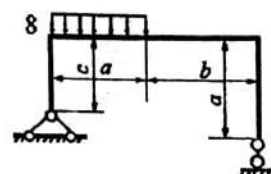
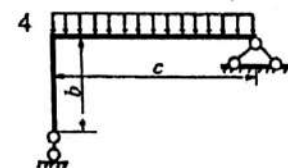
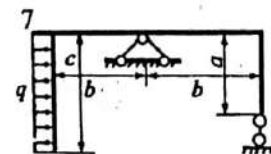
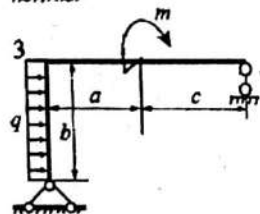
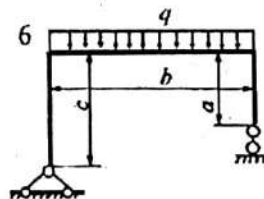
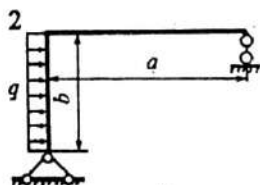
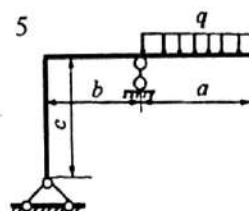
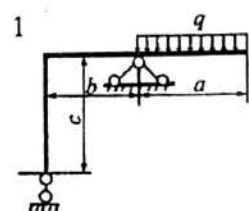
Berilgan: Keltirilgan shakllarda (6.21-shakl) tashqi tomondan yuklangan statik aniq rama berilgan.

Topish kerak:

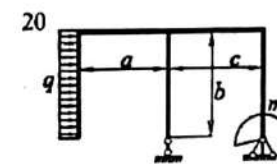
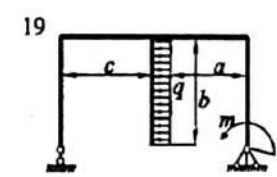
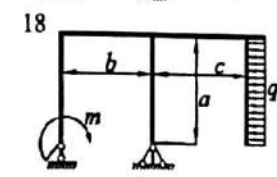
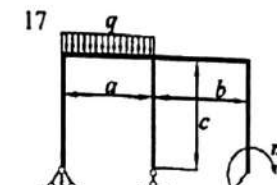
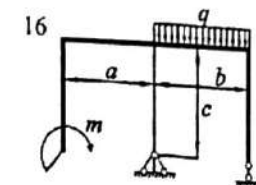
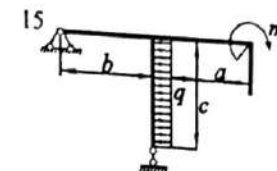
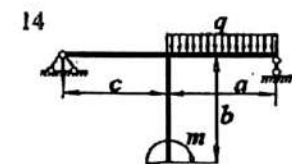
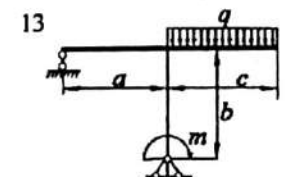
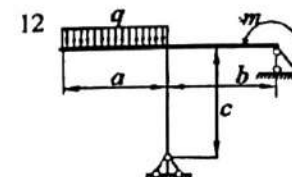
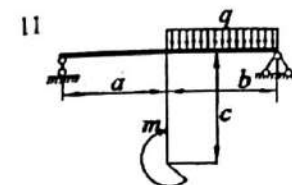
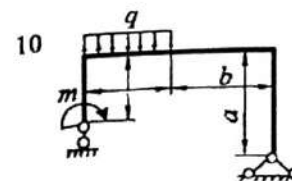
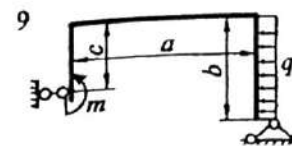
1) berilgan egri sterjen uchun bo'ylama kuch, kesuvchi kuch va eguvchi moment epyuralari qurilsin;

2) mustahkamlik shartiga binoan, egri sterjenning ko'ndalang kesimi tanlansin (kesim ko'rinishi-o'qituvchi ko'rsatmasi bo'yicha).

Kerakli ma'lumotlar 6.1-jadvalda, hisoblash sxemalari 6.21-shaklda keltirilgan.



6.21-shakl



6.21-shakl (davomi)

VII BOB

EGILISHDA TO'SINLARNI MUSTAHKAMLIK SHARTI BO'YICHA HISBLASH

7.1-§. Asosiy mulohazalar

– Ko'ndalang egilishga qarshilik ko'rsatuvchi to'sinlarning ixtiyoriy kesimlarida normal va urinma kuchlanishlar hosil bo'ladi.

– Normal kuchlanishlari eng katta qiymatga erishgan nuqtalar xavfli nuqtalar hisoblanib, ular yuzaning eng chekka tolalarida joylashgan bo'ladi. Normal kuchlanish bo'yicha mustahkamlik sharti quyidagiga teng:

$$\sigma_{max} = \frac{M_{max}}{W_x} \leq \sigma_{adm}, \quad (7.1)$$

bu yerda, W_x – kesim qarshilik momenti;

σ_{adm} – ruxsat etilgan normal kuchlanish.

– Urinma kuchlanishlari eng katta qiymatga erishgan nuqtalar ham xavfli nuqtalar hisoblanib, ular neytral qatlamda joylashadi. Urinma kuchlanish bo'yicha mustahkamlik sharti quyidagiga teng:

$$\tau_{max} = \frac{Q_{max} \cdot S_{max}}{b \cdot J_x} \leq \tau_{adm}, \quad (7.2)$$

bu yerda, Q_{max} – eng katta kesuvchi kuch;

S_{max} – kesimning eng katta statik momenti;

b – kesim eni;

J_x – inersiya momenti;

τ_{adm} – ruxsat etilgan urinma kuchlanish.

– Normal va urinma kuchlanishlari birgalikda yetarli darajada katta qiymatga ega bo'lgan nuqtalar ham xavfli nuqtalar deb hisoblanadi. Bunday holatda bosh kuchlanishlar aniqlanib, mustahkamlik sharti yoziladi.

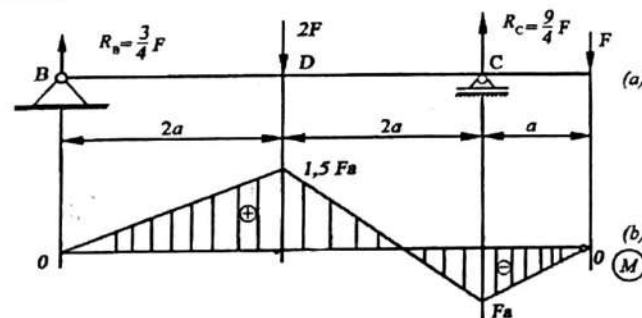
– Quyidagi vaziyatlarda σ va τ lar birgalikda yetarli darajada katta qiymatga ega bo'ladi:

- eguvchi moment va ko'ndalang kuchlar birgalikda o'zlarining eng katta yoki unga yaqinroq qiymatlariga erishgan kesimlarida;
- ko'ndalang kesim o'zgargan joylarda.

7.2-§. Egilishda to'sinlarni mustahkamlik sharti bo'yicha hisoblashga doir masalalar

67-masala

Uzunligi $l = 5a$ bo'lgan qo'shtavrli to'sinning sortament jadvalidagi raqami 24 (ГОСТ 8239-72) bo'lib, to'plangan kuchlar bilan yuklangan (88-shakl, a) normal kuchlanish $\sigma = 120$ MPa dan oshmasligi uchun F kuchning eng katta qiymati qancha bo'ladi? Hisoblashda $a = 0,9$ m deb qabul qilinsin.



7.1-shakl

Yechish:

N24 profil uchun $W_x = 289 \text{ sm}^3$ to'sinning mustahkamlik shartidan.

$$M_{max} = \sigma_{max} \cdot W_z = 120 \cdot 10^6 \cdot 289 = 34,68 \text{ kN} \cdot \text{m} \quad (a)$$

bo'ladi.

Eguvchi moment epyurasidan ko'rinib turibdiki (7.1-shakl, b) to'sinning D kesimt xavfli kesim bo'lib, unga eng katta eguvchi moment

$$M_{max} = 1,5Fa \quad (b)$$

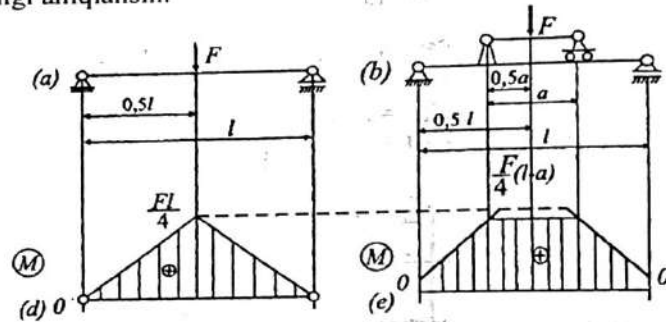
mos keladi.

(a) va (b) larni solishtirib, kuchning eng katta qiymatini aniqlaymiz:

$$1,5Fa = 34,68; F = 25,69 \text{ kN}.$$

68-masala

Asosiy to'sinning o'rtasiga shunday F kuch qo'yilganki, uning ta'sirida xavfli kesimda vujudga keladigan eng katta normal kuchlanish to'sin materiali uchun ruxsat etilgan normal kuchlanish σ_{adm} dan 20 % oshib ketadi (7.2-shakl, *a*). Ortiqcha kuchlanishni yo'qotish maqsadida yordamchi to'sindan foydalanilgan (7.2-shakl, *b*). Yordamchi to'sinning uzunligi aniqlansin.



7.2-shakl

Yechish:

Ikkala hol uchun ham eguvchi moment epyuralarini qurib, xavfli kesimlardagi eng katta eguvchi momentlarni aniqlaymiz:

1-hol. Yordamchi to'sin bo'lmaganda (7.2-shakl, *d*):

$$M_{\max} = \frac{Fl}{4}$$

2-hol. Yordamchi to'sin bo'lganda (7.2-shakl, *e*):

$$M_{\max} = \frac{Fl}{4} - \frac{Fa}{4}$$

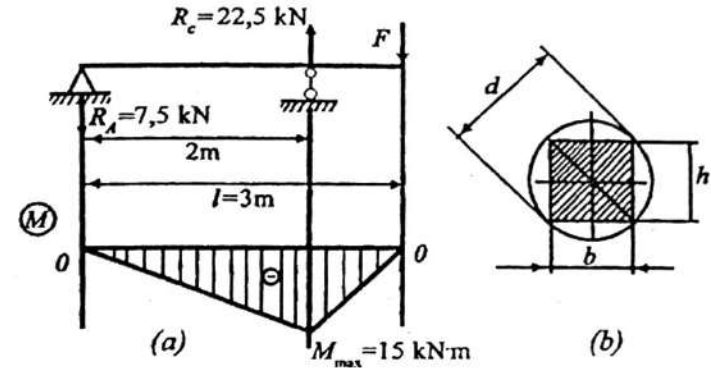
ortiqla kuchlanishni yo'qotish uchun $\frac{Fl}{4}$ ni 20 % kamaytirish kifoya.

$$0,2 \cdot \frac{Fl}{4} - \frac{Fa}{4} = 0,$$

bundan, $a = 0,2l$ bo'ladi.

69-masala

7.3-shaklda tasvirlangan to'sinning ko'ndalang kesimi $b \times h$ o'lchamli to'g'ri to'rtburchakli bo'lib, qarag'ay yog'ochidan yasalgan.



7.3-shakl

To'sinning uzunligi $l = 3$ m, unga qo'yilgan kuch esa $F = 15$ kN, ruxsat etilgan kuchlanish $\sigma_{adm} = 10$ MPa. Ko'ndalang kesimi doiraviy brusdan, ya'ni xuddi shunday o'lchamdagi to'sin arralash uchun brusning minimal diametri qancha bo'lishi kerak? $b : h$ ning qanday qiymatida to'sinning qarshilik momenti eng katta bo'ladi?

Yechish:

Ta'sir reaksiyalarini aniqlab, eguvchi moment epyurasini quramiz (7.3-shakl, *b*). Xavfli kesimda $M_{\max} = 15$ kN·m ta'sir etayotganligi uchun qarshilik momenti

$$W_x = \frac{M_{\max}}{\sigma_{adm}} = \frac{15 \cdot 10^3}{10 \cdot 10^6} = 15 \cdot 10^{-4} \text{ m}^3$$

bo'ladi.

Ikkinchi tomondan, ko'ndalang kesimning qarshilik momentini kesim yuza o'lchamlari orqali quyidagicha yozish mumkin:

$$W_x = \frac{bh^2}{6} = \frac{b(d^2 - b^2)}{6}$$

Qarshilik momenti eng katta qiymatga erishishi uchun

$$\frac{dW_x}{db} = \frac{1}{6}(d^2 - 3b^2) = 0$$

bo'lishi kerak.
Bundan,

$$b = \frac{d}{\sqrt{3}}; \quad h = \sqrt{\frac{2}{3}} \cdot d; \quad \frac{b}{h} = \frac{\sqrt{2}}{2}$$

kelib chiqadi. Demak, $W_{max} = \frac{d^3}{9\sqrt{3}}$.

Oxirgi ifodani $15 \cdot 10^{-4} \text{ m}^3$ ga tenglab, d ni topamiz: $d = 28,6 \cdot 10^{-2} \text{ m}$.

70-masala

Uzunligi $l=5a$ bo'lgan konsol uchun quyidagi ikki xil ko'rinishdagi kesim yuza tanlansin (7.4-shakl, a):

- 1) qo'shtavrli kesim yuza;
- 2) yonma-yon qo'yilgan ikkita shvellardan iborat kesim yuza.

Hisoblashlarda $a=0,8 \text{ m}$, $F=25 \text{ kN}$ va $\sigma_{adm}=160 \text{ MPa}$ ga teng deb olinsin.

Yechish:

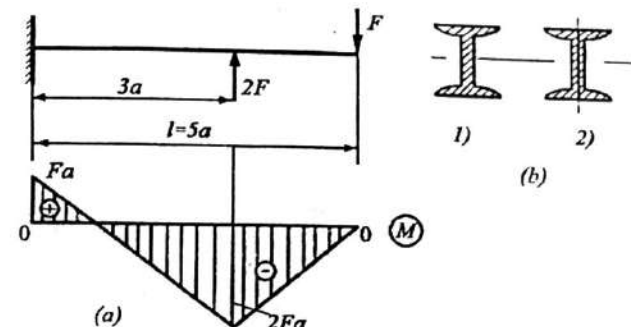
Kesish usulidan foydalanib, eguvchi moment epyurasini quramiz (7.4-shakl, b). Hisob tenglamasidan qarshilik momentini aniqlaymiz:

$$W_x \geq \frac{M_{max}}{\sigma_{adm}} = \frac{2Fa}{\sigma_{adm}} = \frac{2 \cdot 25 \cdot 10^3 \cdot 0,8}{160 \cdot 10^6} = 0,25 \cdot 10^{-3} \text{ m}^3 = 250 \text{ sm}^3$$

1) Qarshilik momentining bu qiymati bo'yicha ГОСТ 8239-72 dan profil raqami 22^a bo'lgan qo'shtavrni tanlaymiz; bu qo'shtavr qarshilik momenti $W_{xi} = 254 \text{ sm}^3$; massasi $q_I=25,8 \text{ kg/mga}$ tengdir.

2) Bitta shveller uchun $W_x^1 = 0,5 W_x = 125 \text{ sm}^3$ bo'lgani uchun ГОСТ 8240-72 dan profil raqami 18 bo'lgan shveller tanlanadi; bu shveller uchun: $W_x=121 \text{ sm}^3$; $q_2=16,3 \text{ kg/m}$.

Shunday qilib qo'shtavrli to'sin kesimi yonma-yon qo'yilgan ikkita shvellardan iborat to'sindan 20,8 % yengil ekan.



7.4-shakl

71-masala

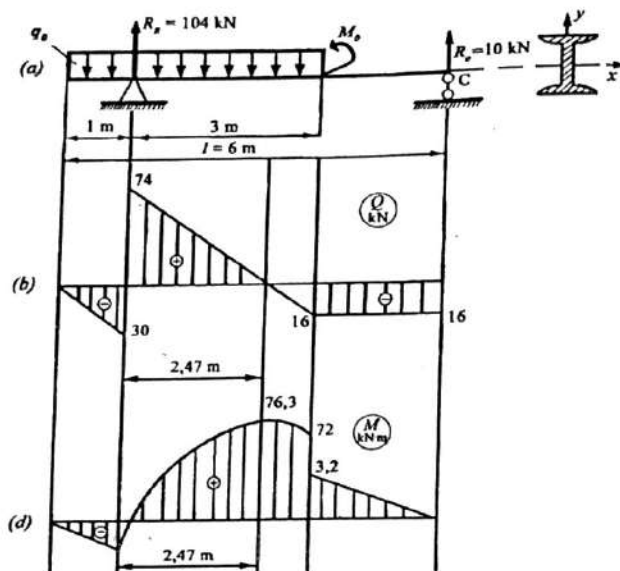
Uzunligi $l = 6 \text{ m}$ bo'lgan qo'shtavrli to'sin juft kuch va tekis taralgan kuchlar bilan yuklangan (7.5-shakl, a). To'sinning mustahkamligi normal va urinma kuchlanishlar bo'yicha tekshirilsin.

Hisoblashlarda $q_0=30 \text{ kN/m}$, $M_0=40 \text{ kNm}$, $\sigma_{adm}=160 \text{ MPa}$, $\tau_{adm}=100 \text{ MPa}$ ga teng deb olinsin.

Yechish:

Tayanch reaksiyalarini aniqlab, ko'ndalang kuch va eguvchi moment epyuralarini quramiz (7.5-shakl, b). Qo'zg'almas tayanchdan 2,47 m o'ngda joylashgan xavfli kesim uchun mustahkamlik shartini yozamiz:

$$\sigma_{max} = \frac{M_{max}}{W_x} \leq \sigma_{adm}$$



7.5-shakl

Bundan, zaruriy qarshilik momenti:

$$W_x \geq \frac{M_{max}}{\sigma_{adm}} = \frac{76,3 \cdot 10^3}{160 \cdot 10^6} = 0,477 \cdot 10^{-3} m^3 = 477 sm^3$$

Qarshilik momentining bu qiymati bo'yicha sortament jadvalidan profil raqami 30 bo'lgan qo'shtavr tanlaymiz; bu qo'shtavr uchun: $W_{xI} = 472 sm^3$; $I_{xI} = 7080 sm^4$; $b = 0,65 sm$ (qo'shtavr devorining qalinligi); $S_{max} 268 sm^3$.

Normal kuchlanish bo'yicha mustahkamlikka tekshiramiz:

$$\sigma_{max} = \frac{M_{max}}{W_{xI}} = \frac{76,3 \cdot 10^3}{472 \cdot 10^{-6}} = 161,65 \cdot 10^6 \frac{N}{m^2} = 161,65 MPa$$

bu kuchlanish xavfli emas, chunki u σ_{adm} dan atigi 1,03 % gina katta. Urinma kuchlanish bo'yicha mustahkamlikka tekshiramiz:

$$\tau_{max} = \frac{Q_{max} S_{max}}{b \cdot I_{xI}} \leq \tau_{adm},$$

bu yerda, $Q_{max} = 74 kN$ bo'lib, ko'ndalang kuch epyurasidan olinadi. Shu tufayli

$$\tau_{max} = \frac{74 \cdot 10^3 \cdot 268 \cdot 10^{-6}}{0,65 \cdot 10^{-2} \cdot 7080 \cdot 10^{-8}} = 43,094 \cdot 10^6 \frac{N}{m^2} = 43,09 MPa,$$

bu kuchlanish, m . dan 56,91 % kichik.

Shunday qilib, to'sinning kesim yuza o'lchamlari normal va urinma kuchlanishlar bo'yicha mustahkamlik shartini qanoatlantiradi.

72-masala

Uzunligi $l = 4 m$ bo'lgan to'sin tekis taralgan kuchlar bilan yuklangan (7.6-shakl, a). Chizmada tasvirlangan yetti xil ko'rinishdagi kesim yuzalardan qaysi biri ratsional kesim yuza bo'la oladi?

Hisoblashlarda $q_0 = 50 kN/m$; $l = 4 m$; $160 MPa$ ga teng deb olinsin.

Yechish:

Tayanch reaksiyalarini topib, eguvchi moment epyurasini quramiz (7.6-shakl, b). Xavfli kesim tayanchlar o'rtasida bo'lib, unga

$$M_{max} = \frac{q_0 l^2}{16} = 50,0 kN \cdot m \text{ moment ta'sir etadi.}$$

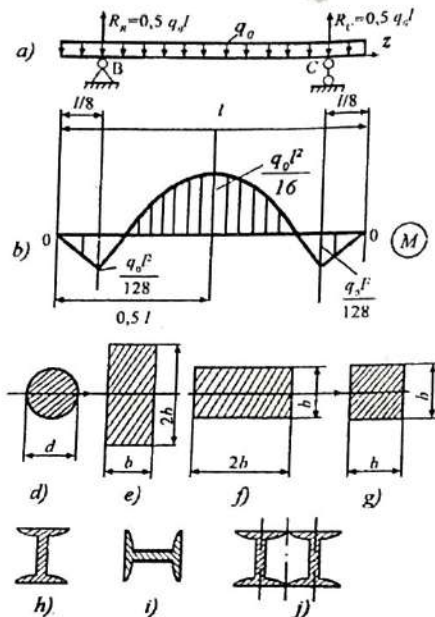
Hisob tenglamasidan qarshilik momentining zaruriy qiymatini aniqlaymiz:

$$W_x \geq \frac{M_{max}}{\sigma_{adm}} = \frac{50 \cdot 10^3}{160 \cdot 10^6} = 0,3125 \cdot 10^{-3} m^3 = 312,5 sm^3.$$

Endi har bir kesim yuza uchun navbat bilan tegishli hisoblashlarni bajaramiz.

1. Doiraviy kesim yuza (7.6-shakl, d):

$$W = \frac{\pi d^3}{32} \geq 312,5 sm^3, \quad d \geq \sqrt[3]{\frac{32 \cdot 312,5}{3,14}} = 14,71 sm,$$



7.6-shakl

qabul qilamiz: $d_0 = 15$ sm, u holda:

$$W_0 = \frac{\pi d_0^3}{32} = 331,17 \text{ sm}^3; \quad A = \frac{\pi d_0}{4} = 176,625 \text{ sm}^2;$$

$$\delta_\sigma = \frac{W_x - W_0}{W_0} \cdot 100\% = -5,46\%;$$

$$\eta = \sqrt[3]{\frac{A^3}{W^2}} = \sqrt[3]{\frac{(176,625)^3}{(331,17)^2}} = 3,6898.$$

2. To'g'ri to'rtburchakli kesim yuza (7.6-shakl, e):

$$W = \frac{b \cdot (2b)^2}{6} = \frac{2}{3} b^3 \geq 312,5 \text{ sm}^3; \quad b \geq \sqrt[3]{468,75} = 7,7679 \text{ sm}$$

qabul qilamiz: $b_0 = 8$ sm, u holda: $W_0 = \frac{2b_0^3}{3} = 341,33 \text{ sm}^3;$

$$A = 2b_0^2 = 128 \text{ sm}^2;$$

$$\delta_\sigma = \frac{W_x - W_0}{W_0} \cdot 100\% = -8,45\%;$$

$$\eta = \sqrt[3]{\frac{(128)^3}{(341,33)^2}} = 2,62.$$

3. To'g'ri to'rtburchakli kesim yuza (7.6-shakl, f):

$$W = \frac{2b \cdot b^2}{6} = \frac{b^3}{3} \geq 312,5 \text{ sm}^3; \quad b = 9,787 \text{ sm}$$

Qabul qilamiz: $b_0 = 10$ sm, u holda: $W_0 = \frac{b_0^3}{3} = 333,333 \text{ sm}^3;$

$$A = 2b_0^2 = 200 \text{ sm}^2;$$

$$\delta_\sigma = \frac{W_x - W_0}{W_0} \cdot 100\% = \frac{312,5 - 333,333}{333,33} \cdot 100\% = -6,25\%;$$

$$\eta = \sqrt[3]{\frac{A^3}{W_0^2}} = \sqrt[3]{\frac{200^3}{333,33^2}} \approx 41,16$$

4. Kvadrat kesim yuza (7.6-shakl, g):

$$W = \frac{b \cdot 2b^2}{6} = \frac{b^3}{3} \geq 312,5 \text{ sm}^3; \quad b = 12,33 \text{ sm}$$

qabul qilamiz: $b_0 = 13$ sm, u holda: $W_0 = \frac{b_0^3}{3} = 366,166 \text{ sm}^3;$

$$A = 2b_0^2 = 169 \text{ sm}^2$$

$$\delta_\sigma = \frac{W_x - W_0}{W_0} \cdot 100\% = \frac{312,5 - 366,166}{366,166} \cdot 100\% = -14,66\%;$$

$$\eta = \sqrt[3]{\frac{A^3}{W_0^2}} = \sqrt[3]{\frac{169^3}{366,166^2}} = 3,3.$$

5. Qo'shtavrli kesim yuza (7.6-shakl, h):

ГОСТ 8239-72 dan tanlaymiz: №24^a; $W_1 = 317 \text{ sm}^3; A = 37,5 \text{ sm}^2;$

$$\delta_\sigma = \frac{W_x - W_1}{W_1} \cdot 100\% = -1,42\%; \quad \eta = \sqrt[3]{\frac{A_1^3}{W_1^2}} = \sqrt[3]{\frac{37,5^3}{317^2}} = 0,8066$$

6. Qo'shtavrli kesim yuza (7.6-shakl, i):

ГОСТ 8239-56 dan tanlaymiz: №70^a; $W_H = 309 \text{ sm}^3; A = 202 \text{ sm}^2;$

$$\delta_\sigma = \frac{W_x - W_H}{W_H} \cdot 100\% = \frac{325,5 - 309}{309} \cdot 100\% = 1,32\%;$$

$$\eta_H = \sqrt[3]{\frac{A_H^3}{W_H^2}} = \sqrt[3]{\frac{202^3}{309^2}} = \sqrt[3]{86,325} = 4,42$$

7. Yonma-yon joylashgan ikkita qo'shtavrli kesim yuza (7.6-shakl, j):

ГОСТ 8239-72 dan tanlaymiz:
№18^a, $W_1^0 = 159 \text{ cm}^3$; $A_1^0 = 25,4 \text{ cm}^2$; $W_1 = 2 \cdot 159 = 318 \text{ cm}^3$;
 $A_1 = 50,8 \text{ cm}^2$,

$$\delta_\sigma = \frac{W_x - W_t}{W_t} = \frac{312,5 - 2 \cdot 159}{2 \cdot 159} \cdot 100\% = -1,73\%; \quad \eta_H = \sqrt[3]{\frac{(2 \cdot 25,4)^3}{(2 \cdot 159)^2}} = 1,09$$

Hisoblangan qiymatlardan quyidagi xulosaga kelamiz: kesim shaklining ratsional koeffitsenti eng kichik, ya'ni $\eta = 0,8066$ bo'lgan qo'shtavrli to'sin boshqalariga nisbatan ancha tejamli ekan.

73- masala

Uzunligi $l = 3a$ kesimi bo'lgan halqasimon to'sin juft kuch va tekis taralgan kuchlar bilan yuklangan (7.7-shakl, a). To'singa qo'yilgan tashqi kuchning ruxsat etilgan qiymatini aniqlang.

Hisoblashda $\sigma_{adm} = 160 \text{ MPa}$, $\alpha = d:D = 0,7$, $a = 1,1 \text{ m}$, $D = 25 \cdot 10^{-2} \text{ m}$ ga teng deb olinsin.

Yechish:

Tayanch reaksiyalarini topib, eguvchi moment epyurasini quramiz (7.7-shakl, b). Xavfli kesim uchun mustahkamlik shartini yozamiz:

$$\sigma_{max} = \frac{M_{max}}{W_x} \leq \sigma_{adm},$$

bu yerda, $M_{max} = \frac{3qa^2}{4}$ bo'lib, eguvchi moment epyurasidan olinadi;

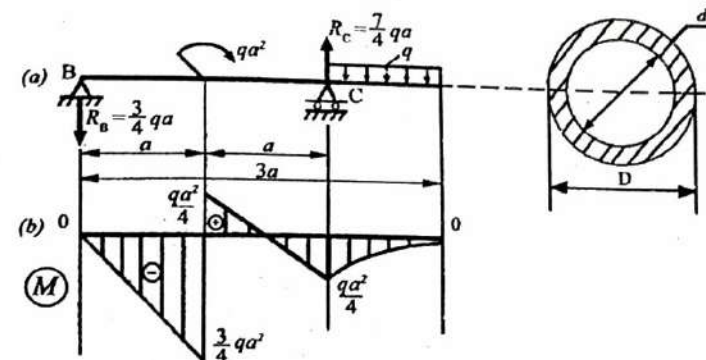
$$W_x = \frac{\pi D^3 \cdot (1 - d^4)}{32} - \text{qarshilik momenti.}$$

Demak, tashqi kuchning ruxsat etilgan qiymati:

$$q_{adm} = q \leq \frac{4 \cdot \pi D^3 (1 - d^4) \sigma_{adm}}{3 \cdot 32 \cdot a^2}$$

yoki

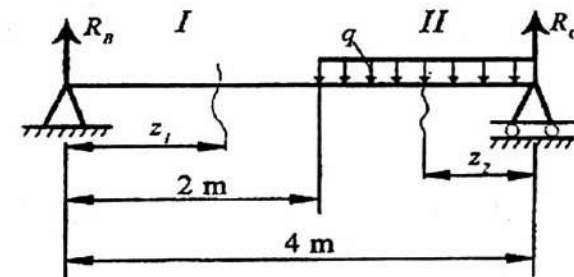
$$q_{adm} = \frac{4 \cdot 3,14 \cdot (25 \cdot 10^{-2})^3 (1 - 0,2401) \cdot 160 \cdot 10^6}{96 \cdot 1,21} = 205,4 \text{ kN/m ga teng ekan.}$$



7.7-shakl

74-masala

Profil raqami 60 bo'lgan qo'shtavrli po'lat to'sin $q = 400 \text{ kN/m}$ tekis taralgan kuchlar bilan yuklangan (7.8-shakl). Mazkur to'sin uchun deformatsiyaning potensial energiyasini aniqlash talab etiladi. Qo'shtavr uchun ko'ndalang kesim yuzining shakliga bog'liq bo'lgan koeffitsiyent $K = 1,9$ deb olinsin.



7.8-shakl

Yechish:

Dastlab, tayanch reaksiyalarini topamiz: $R_B = 200 \text{ kN}$, $R_C = 600 \text{ kN}$. Keyin esa to'sinning har bir oralig'i uchun ichki zo'riqlashlarning ifodalarini tuzamiz:

$$0 \leq z_1 \leq 2; Q(z_1) = R_B; M(z_1) = R_B z_1;$$

$$0 \leq z_2 \leq 2; Q(z_2) = qz_2 - R_C; M(z_2) = R_C \cdot z_2 - 0,5q \cdot z_2$$

Egilishdagi potensial energiyani aniqlaymiz:

$$U_M = \sum \int_{(e)} \frac{M^2(z) dz}{2EI_x} = \frac{1}{2EI_x} \left[\int_0^2 (200z_1)^2 dz_1 + \int_0^2 (600z_2 - 200z_2^2)^2 dz_2 \right] = 1180J.$$

Siljishdagi potensial energiyani aniqlaymiz:

$$U_Q = \sum \int_{(1)} \frac{KQ^2(z) dz}{2GF} = \frac{1,9}{2GF} \left[\int_0^2 (200)^2 dz_1 + \int_0^2 (400z_2 - 600)^2 dz_2 \right] = 229J.$$

Hisoblashlardan ko'rinib turibdiki, siljishdagi potensial energiya egilishdagi potensial energiyaning 19,4 % ni tashkil qilar ekan. Shunday qilib, potensial energiya:

$$U_M + U_Q = 1409J.$$

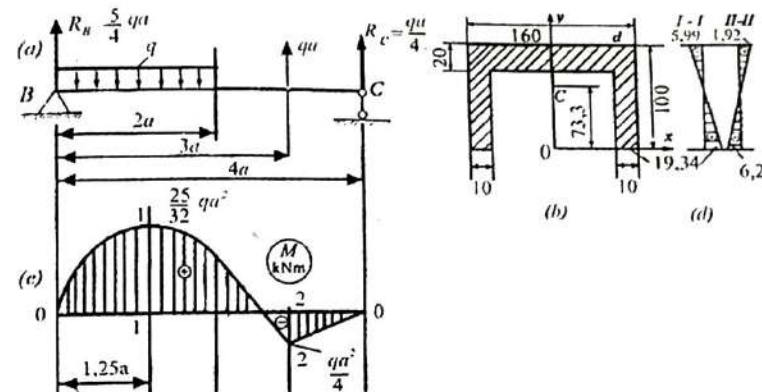
75- masala

To'plangan kuch va tekis taralgan kuchlar bilan yuklangan oddiy to'sin "II" shaklli ko'ndalang kesimga ega (7.9-shakl, a). To'sin cho'yandan yasalgan bo'lib, tayanchlar orasidagi masofa $l = 4a$ ga teng.

To'sin mustahkamlikka tekshirilsin. Hisoblashlarda

$$\sigma_{adm}^+ = 35MPa; \sigma_{adm}^- = 110 MPa;$$

$a = 0,5$ m; $q = 5$ kN/m ga teng deb olinsin. Kesimning o'lchamlari shaklda mm hisobida berilgan (7.9-shakl, b).



7.9-shakl

Yechish:

Tayanch reaksiyalari aniqlangach, eguvchi moment epyurasini quramiz (7.9-shakl, d). Epyuradan xavfli kesimlardagi eguvchi momentlarni aniqlaymiz:

I—I kesimda (bu yerda neytral o'qdan yuqoridagi tolalar siqiladi):

$$M_{max} = \frac{25qa^2}{32} = 0,9765 \text{ kNm}.$$

II—II kesimda (bu yerda, neytral o'qdan yuqoridagi tolalar cho'ziladi):

$$M_{max} = \frac{qa^2}{4} = 0,3125 \text{ kNm}.$$

Kesim yuza vertikal o'qqa nisbatan simmetrik bo'lgani uchun, Y_c neytral o'qning holatini aniqlaydi: $Y_c = \frac{10 \cdot 16 \cdot 5 - 8 \cdot 14 \cdot 4}{10 \cdot 16 - 8 \cdot 14} = 7,33 \text{ sm} = 73,3 \text{ mm}.$

Inersiya momentini hisoblaymiz:

$$I_{xc} = \frac{16 \cdot 10^3}{12} + 2,33^2 \cdot 16 \cdot 10 - \frac{14 \cdot 8^3}{12} - 3,33^2 \cdot 14,8 = 370 \text{ sm}^4$$

Kesimlardagi cho'zuvchi va siquvchi kuchlanishlarni aniqlaymiz.

I—I kesim uchun eng katta cho'zuvchi kuchlanishni aniqlaymiz:

$$\sigma_A = \frac{M_{max}}{I_{xc}} \cdot y_A = \frac{25qa^2}{32 \cdot 370 \cdot 10^{-8}} \cdot 7,33 \cdot 10^{-2} = 19,4 \cdot 10^6 \frac{N}{m^2} = 19,34 \text{ MPa}$$

bo'lib σ_{adm} dan 44,7 % kichikdir.

Eng katta siquvchi kuchlanishni aniqlaymiz:

$$\sigma_B = -\frac{M_{max}}{I_{xc}} \cdot y_B = -\frac{25qa^2 \cdot 2,27 \cdot 10^{-2}}{32 \cdot 370 \cdot 10^{-8}} = -5,99 \cdot 10^6 \text{ N/m}^2 = -5,99 \text{ MPa.}$$

II-II kesim uchun eng katta cho'zuvchi kuchlanish quyidagiga teng:

$$\sigma_B = \frac{M_{max}}{I_{xc}} \cdot y_B = \frac{0,3125 \cdot 10^3 \cdot 2,27 \cdot 10^{-2}}{370 \cdot 10^{-8}} = 1,92 \cdot 10^6 \text{ N/m}^2 = 1,92 \text{ MPa.}$$

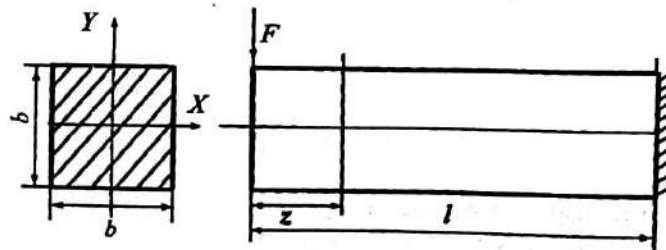
Eng katta siquvchi kuchlanish esa quyidagiga teng:

$$\sigma_A = -\frac{M_{max}}{I_{xc}} \cdot y_A = -\frac{0,3125 \cdot 7,33 \cdot 10^3 \cdot 10^{-2}}{370 \cdot 10^{-8}} = -6,2 \cdot 10^6 \text{ N/m}^2 = -6,2 \text{ MPa.}$$

7.9-shakl (e) da keltirilgan kuchlanish epyuralaridan ko'rinib turibdiki, to'sinning mustahkamligi ta'minlangan.

76-masala

Kesimi kvadrat shaklidagi ($b = 5 \cdot 10^{-2} \text{ m}$) konsolning uchiga F kuch qo'yilgan (7.10-shakl). Konsolning shunday ko'ndalang kesimi topilsinki, unda eng katta normal kuchlanish eng katta urinma kuchlanishdan 3 marta katta bo'lsin.



7.10-shakl

Yechish:

Ma'lumki, eng katta normal kuchlanish kesimning chekka nuqtasida bo'lib,

$$\sigma_{max} = \frac{M_{zmax}}{W_x} = \frac{F \cdot z}{\frac{b^3}{6}} = \frac{6F \cdot z}{b^3}$$

formuladan topiladi; urinma kuchlanishning eng katta qiymati esa xolis qatlamda yotuvchi nuqtalarda bo'ladi:

$$\tau_{max} = \frac{3}{2} \cdot \frac{F}{b^2}$$

Masalaning shartiga ko'ra:

$$\frac{6Fz}{b^3} = 3 \cdot \frac{3}{2} \cdot \frac{F}{b^2}$$

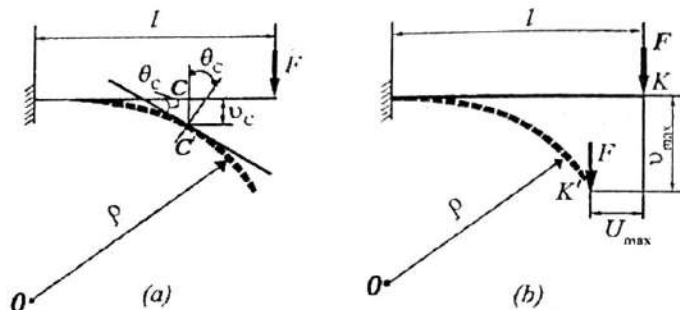
bo'lishi kerak, bunda, $z = \frac{3}{4}b$ ekanligi kelib chiqadi.

VIII BOB

EGILISHDA KO'CHISHLARNI ANIQLASH

8.1-§. Asosiy mulohazalar

Egilishda to'sinning mustahkamligini ta'minlash bilan birga uning bikirligini ham ta'minlash zarurdir. To'sinlarni bikirlikka hisoblash jarayonlarida ularning tashqi kuchlar natijasida olgan deformatsiyalarini bilish muhimdir.



8.1-shakl

To'sinning deformatsiyalarini istalgan ko'ndalang kesimlarning vaziyatlarini tavsiflay oladigan quyidagi ikkita ko'chishlardan iborat, deb qarash mumkin (8.1-shakl):

- istalgan nuqtalarning solqiligi – chiziqli ko'chish;
- istalgan ko'ndalang kesimlarning aylanish burchaklari – burchakli ko'chish. To'sinning ko'ndalang kesimi og'irlik markazining to'sin o'qiga tik (perpendikulyar) yo'nalishdagi ko'chishi uning mazkur kesimdagi solqiligi deyiladi va v_z yoki F_{max} bilan belgilanadi.

To'sin uzunligi bo'yicha olingan ixtiyoriy ko'ndalang kesimning dastlabki (deformatsiyalanmagan) vaziyatiga nisbatan tekis holda neytral o'q atrofida ma'lum burchakka og'ishiga mazkur kesimning aylanish burchagi deyiladi va θ bilan belgilanadi.

Deformatsiyalangan to'sinning barcha ko'ndalang kesim og'irlik markazlarining geometrik o'rniga egilgan o'q yoki elastik chiziq deyiladi; elastik chiziq tekis egri chiziq bo'lib, u kuch tekisligida yotadi.

Chiziqli ko'chishning gorizonttal tashkil etuvchisi (U_{max}) nisbatan kichik qiymatga ega bo'lgani uchun hisoblash jarayonlarida uni e'tiborga olmasa ham bo'ladi.

Egilishdagi deformatsiyalarni aniqlashning bir qancha usullari mavjud; biz bu bobda faqat ayrim usullar bilan tanishib chiqamiz:

Egilgan o'qning taqribiy differensial tenglamasini bevosita integrallash usuli:

$$\pm \frac{d^2 v}{dz^2} = -\frac{M(z)}{EI_x}. \quad (8.1)$$

Boshlang'ich parametrlar usuli yoki universal formula;

Mor usuli;

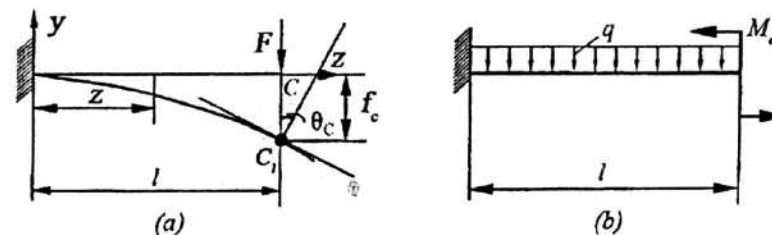
Vereshagin usuli.

8.2-§. Egilgan o'qning taqribiy differensial tenglamasini bevosita integrallash usuli

77-masala

Uzunligi l , bikirligi o'zgarmas ($EI_x = \text{const}$) bo'lgan konsol erkin uchining aylanish burchagi va solqiligi quyidagi uch hol uchun aniqlansin: a) o'ng uchiga F yuk qo'yilgan; b) o'ng uchiga M juft kuch qo'yilgan; d) tekis taralgan q yuk qo'yilgan (8.2-shakl a, b).

Masala egilgan o'qning taqribiy differensial tenglamasini bevosita integrallash usuli yordamida yechilsin.



8.2-shakl

Yechish:

Masalani birinchi hol uchun yechamiz (8.2-shakl, a).

Tayanch kesimni koordinatalar boshi deb hisoblab, Y o'qini yuqoriga va abssissalar o'qini esa o'ng tomonga yo'naltiramiz. Tanlangan koordinatalarga ko'ra, elastik chiziqning taqribiy differensial tenglamasi $EI_x v_z = M_z$ ko'rinishga ega bo'ladi.

Koordinata boshidan ixtiyoriy Z masofadagi kesim uchun eguvchi momentning ifodasini yozamiz: $M_z = -F(l - z)$; u holda yuqoridagi tenglama:

$$EI_x v_z'' = -F(l - z) \text{ ko'rinishida bo'ladi.}$$

Bu tenglamani ketma-ket ikki marta integrallaymiz:

$$EI_x v_z' = EI_x \theta_z = -F \left(lz - \frac{z^2}{2} \right) + C; \quad (1)$$

$$EI_x v_z = -F \left(l \frac{z^2}{2} - \frac{z^3}{6} \right) + Cz + D. \quad (2)$$

Integral o'zgarmlarni aniqlash uchun chegaraviy shartlardan foydalanamiz:

$$Z=0 \text{ bo'lsa, } \theta(l) = 0; \quad v(l) = 0.$$

Bu shartni (1) va (2) tenglamalarga qo'yib, $C = 0$ va $D = 0$ ekanligiga ishonch hosil qilamiz. Shunday qilib, aylanish burchagi va solqilik tenglamasi quyidagicha bo'lar ekan:

$$\theta_z = -\frac{Flz}{2EI_x} \left(2 - \frac{z}{l} \right);$$

$$v_z = -\frac{Flz}{6EI_x} \left(3 - \frac{z}{l} \right).$$

Bu ifodalarning birinchisidagi manfiy ishora C kesimning soat strelkasi yo'nalishida aylanishini, ikkinchisidagi manfiy ishora esa solqilikning pastga qarab yo'nalganligini ko'rsatadi.

Ikkinchi holni qaraymiz (8.2-shakl, b).

Oxirgi tenglamalarga $z=l$ ni qo'yib, so'ralgan kesimning aylanish burchagi va solqiligini topamiz:

$$\theta_c = -\frac{Fl^2}{2EI_x}; \quad v_b = f_b = -\frac{Fl^3}{6EI_x}$$

Yuqoridagi ishlarni yana takrorlab, quyidagi ko'rinishdagi aylanish burchagi va solqilik tenglamalariga ega bo'lamiz:

$$\theta_z = \frac{1}{EI_x} \left(M_e Z - \frac{qZ^3}{6} \right);$$

$$v_z = \frac{1}{EI_x} \left(M_e \frac{z^2}{2} - q \frac{z^4}{24} \right).$$

Bu tenglamalardan konsol erkin uchidagi kesimning aylanish burchagi va solqiligini osongina aniqlash mumkin.

Quyidagi xususiy hollarni qaraymiz:

a) konsolga faqat juft kuch qo'yilgan bo'lsin, ya'ni $q=0$; $M_e \neq 0$ ga teng, u holda:

$$\theta_c = \frac{M_e l}{EI_x}; \quad v_c = f_c = \frac{M_e l^2}{2EI_x};$$

b) konsolga faqat tekis taralgan kuchlar ta'sir etayotgan bo'lsin, ya'ni, $M_e = 0$; $q \neq 0$, u holda:

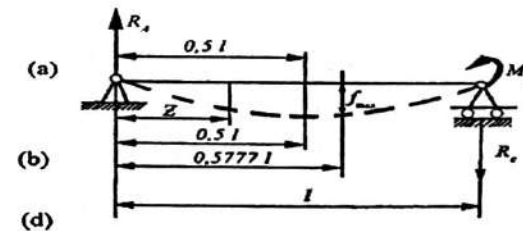
$$\theta_c = -\frac{ql^3}{6EI_x}; \quad v_c = f_c = -\frac{ql^4}{24EI_x}.$$

78-masala

Oddiy to'sinning o'ng tayanchiga juft kuch qo'yilgan (8.3-shakl). To'sin o'zgarma bikirlikka ega bo'lib, uzunligi l ga teng.

To'sinning eng katta solqiligi topilsin va u tayanchlar o'rtasidagi kesimning solqiligi bilan solishtirilsin.

Masala elastik chiziqning taqribiy differensial tenglamasini bevosita integrallash usuli bilan yechilsin.



8.3-shakl

Yechish:

Tayanch reaksiyalarini topamiz: $R_A = R_B = \frac{M_e}{l}$.
O'ng tayanchdan ixtiyoriy Z masofada joylashgan kesimdagi eguvchi moment:

$$M(z) = \frac{M_e}{l} Z \text{ ga teng. U holda,}$$

$$EI_x v_z'' = \frac{M_e}{l} Z. \quad (a)$$

Oxirgi tenglamaning integralini yozamiz:

$$EI_x \theta_z = \frac{M_e}{l} \cdot \frac{z^2}{2} + C. \quad (b)$$

$$EI_x v_z'' = \frac{M_e}{l} \cdot \frac{z^3}{6} + C \cdot Z + D. \quad (d)$$

To'sinning tayanchlariga tiralgan kesimlarida solqilikning 0 ga tengligi quyidagi chegaraviy shartlarni beradi:

$$Z = 0 \text{ da } \theta_{z=0} = 0 \text{ bo'ladi}$$

$$Z = l \text{ da } \theta_{z=l} = 0 \text{ bo'ladi.}$$

Bulardan, $D=0$ va $C=M_e/6$ ekanligi kelib chiqadi.

Nihoyat, aylanish burchagi va solqilik uchun quyidagi tenglamalarga ega bo'lamiz:

$$\theta_z = \frac{M_e Z^2}{2 l \cdot EI_x} - \frac{M_e l}{6 EI_x}; \quad (e)$$

$$v_z = \frac{1}{EI} \cdot \frac{M_e Z^3}{6 l} - \frac{M_e l}{6 EI_x} Z. \quad (f)$$

(d) tenglamaga $Z=0,5l$ ni qo'yib, tayanchlar orasidagi kesimning solqiligini topamiz:

$$v_D = v_{z=0,5l} = -\frac{M_e l}{16 EI_x}$$

(e) tenglamani nolga tenglab, solqiligi eng katta bo'lgan kesimning absissasi Z_0 ni aniqlaymiz:

$$\frac{M_e Z^2}{2 l \cdot EI_x} - \frac{M_e l}{6 EI_x} = 0;$$

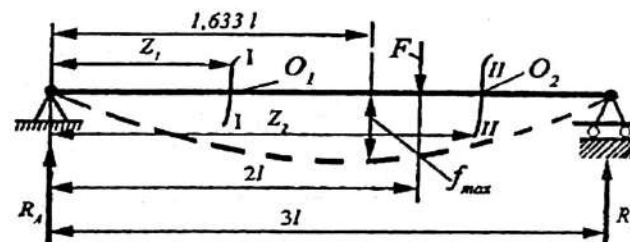
bundan, $Z=0,577l$. Maksimal solqilikni hisoblaymiz:

$$f_{max} = \frac{1}{EI_x} \cdot \frac{M_e (0,577 l)^3}{6 l} - \frac{M_e l}{6 EI_x} \cdot 0,577 l = -\frac{M_e l^2}{15,57 EI_x}$$

Demak, v_D maksimal solqilikdan 2,76 % ga farq qilar ekan.

79-masala

Uzunligi $3l$, bikirligi o'zgarmas bo'lgan oddiy to'singa to'plangan kuch qo'yilgan. Ana shu to'sin uchun aylanish burchagi va solqilik tenglamalarini, maksimal solqilik hamda tayanch kesimlarining aylanish burchaklarini toping (8.4-shakl).



8.4-shakl

Masalani yechish uchun egilgan o'qning taqribiy differensial tenglamasini bevosita integrallash usuli tavsiya etiladi.

Yechish:

Avval to'sinning tayanch reaksiyalarini topamiz:

$$R_A = \frac{F}{3}; \quad R_B = \frac{2F}{3}.$$

F yuk to'sinni ikkita oraliqqa (uchastkaga) ajratganligi sababli, har bir oraliq uchun quyidagi ifodalarni tuzish zarur:

I oraliq ($0 \leq z_1 \leq 2l$):

II oraliq ($2l \leq Z_2 \leq 3l$):

$$M(z_1) = \frac{F}{3} z_1; \quad (a) \quad M(Z_2) = \frac{F}{3} Z_2 - F(Z_2 - 2l); \quad (a')$$

$$EI_x v_1'' = \frac{F}{3} z_1; \quad (b) \quad EI_x v_{z_2}'' = \frac{F}{3} Z_2 - F(Z_2 - 2l); \quad (b')$$

$$EI_x \theta_{z_1} = \frac{F z_1^2}{3 \cdot 2} + C_1; \quad (d) \quad EI_x \theta_{z_2} = \frac{F Z_2^2}{3 \cdot 2} - F \frac{(Z_2 - 2l)^2}{2} + C_2; \quad (d')$$

$$EI_x v_{z_1} = \frac{F z_1^3}{3 \cdot 6} + C_1 z_1 + D_1 \quad (e)$$

$$EI_x v_{z_2} = \frac{F Z_2^3}{3 \cdot 6} - F \frac{(Z_2 - 2l)^3}{6} + C_2 Z_2 + D_2. \quad (e')$$

Yuqoridagi tenglamalar tarkibidagi C_1, D_1, C_2, D_2 ixtiyoriy o'zgarishlarni aniqlash uchun quyidagi to'rtta chegaraviy shartlardan foydalanamiz: $Z_1 = 0$ da $v_{z_1} = 0$, bundan, $D_1 = 0$ kelib chiqadi.

$$Z_2 = 3l \text{ da } v_{z_2} = 0 \text{ yoki } \frac{F (3l)^3}{3 \cdot 6} - F \frac{l^3}{6} + C_2 3l + D_2 = 0 \text{ bo'ladi.}$$

$$Z_1 = Z_2 = 2l \text{ da } \theta_{z_1} = \theta_{z_2} \text{ yoki } \frac{F (2l)^2}{3 \cdot 2} + C_1 = \frac{F (2l)^2}{3 \cdot 2} + C_2, \text{ bundan, } C_1 = C_2 \text{ kelib chiqadi.}$$

$$Z_1 = Z_2 = 2l \text{ da } v_{z_1} = v_{z_2} \text{ yoki } \frac{F (2l)^3}{3 \cdot 6} + C_1 \cdot 2l + D_1 = \frac{F (2l)^3}{3 \cdot 6} + C_2 2l + D_2 = 0 \text{ bo'ladi.}$$

Bundan, $D_1 = D_2$ kelib chiqadi.

$$\text{Shunday qilib, } D_1 = D_2 = 0; \quad C_1 = C_2 = \frac{4Fl^2}{9}$$

I va II uchastkalar uchun aylanish burchagi hamda solqilik qiymatlarini aniqlaymiz.

I oraliq:

II oraliq:

$$\theta_{z_1} = \frac{1}{EI_x} \left(\frac{F z_1^2}{6} - \frac{4Fl^2}{9} \right) = \frac{F}{6EI_x} \left(Z_1^2 - \frac{8l^2}{3} \right); \quad (f)$$

$$\theta_{z_2} = \frac{F}{6EI_x} \left[Z_2^2 - 3(Z_2 - 2l)^2 - \frac{8l^2}{3} \right]; \quad (f')$$

$$v_{z_1} = \frac{F}{18EI_x} (Z_1^3 - 8l^2 Z_1). \quad (g)$$

$$v_{z_2} = \frac{F}{18EI_x} [Z_2^3 - 3(Z_2 - 2l)^3 - 8l^2 Z_2]. \quad (g')$$

Endi solqilikning eng katta qiymati f_{max} ni aniqlashga o'tamiz. Odatda, solqiligi f_{max} bo'lgan kesimda aylanish burchagi nolga teng bo'ladi; tekshirilayotgan to'sinda bunday kesim $0 \leq z_1 \leq 2l$ oraliqda yotishi tabiiy. Shuning uchun:

$$\frac{dv_{z_1}}{dz_1} = \frac{F}{6EI_x} \left(Z_1^2 - \frac{8l^2}{3} \right) = 0.$$

Bundan,

$$Z_1 = 1,633l, \text{ demak, } f_{max} = \frac{F}{18EI_x} [(1,633l)^3 - 8l^2 \cdot 1,633l] = -0,4838 \frac{Fl^3}{EI_x}.$$

(f) tenglamaga $Z = 0$ ni, (f') tenglamaga esa $Z_2 = 3l$ ni qo'yib, tayanch kesimlaridagi aylanish burchagini topamiz:

$$\theta_B = -\frac{4Fl^2}{9EI_x}; \quad \theta_c = \frac{5Fl^2}{9EI_x}.$$

8.3-§. Universal tenglama yordamida deformatsiyalarni aniqlash (boshlang'ich parametrlar usuli)

Egillishdagi deformatsiyalarni aniqlash universal tenglamasi quyidagi ko'rinishga ega bo'ladi:

$$v_z = v_0 + \theta_0 \cdot Z + \frac{1}{EI_x} \left[\sum \frac{M(z-a)^2}{2} + \sum \frac{F(z-b)^3}{6} + \sum \frac{q(z-c)^4}{24} - \sum \frac{q(z-d)^4}{24} \right]. \quad (8.2)$$

Bundan bir marta hosila olinsa, kesimning aylanish burchagi tenglamasi chiqadi:

$$\theta = \theta_0 + \frac{1}{EJ_x} \left[\sum M(z-a) + \sum \frac{F(z-b)^2}{2} + \sum \frac{q(z-c)^3}{6} - \sum \frac{q(z-d)^3}{6} \right], \quad (7.3)$$

bu yerda, v_0, θ_0 – koordinata boshidagi solqilik va aylanish burchaklari;

a, b, c, d – koordinata boshidan mos ravishda juft kuch, to'plangan kuch va yoyilgan kuchlarga bo'lgan masofa.

Bu usul quyidagi asosiy qoidalarga tayanadi:

1) koordinata boshi to'sinning chap uchidan tanlanadi va u hamma oraliqlar uchun umumiy hisoblanadi;

2) qirqimdan chap tomonda joylashgan tashqi kuchlardan eguvchi moment ifodasi tuziladi;

3) $(z-a)^n$ ko'rinishdagi ko'phadlarni integrallashda qavslarni ochmaslik lozim:

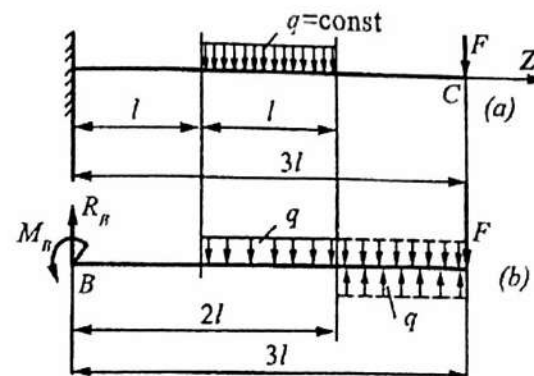
$$\int (z-a)^n dz = \frac{(z-a)^{n+1}}{n+1} + C;$$

4) to'singa qo'yilgan juft kuchlardan eguvchi moment ifodasini tuzayotganda, uni $(z-a_i)^0 = 1$ binomga ko'paytirish tavsiya qilinadi; bunda, a_i koordinata boshidan juft kuchlar qo'yilgan kesimgacha bo'lgan masofa;

5) agar tekis yoyilgan yuk $q=\text{const}$ to'sinning oxirgi uchiga yetmagan bo'lsa, u "sun'iy" ravishda davom ettiriladi.

80- masala

Uzunligi $3l$ va bikirligi o'zgarmas bo'lgan konsol $F=2ql$ to'plangan kuch va tekis taralgan yoyilgan kuchlar bilan yuklangan (8.5-shakl, a). Universal formula yordamida konsol erkin uchining solqiligi va aylanish burchagi aniqlansin.



8.5-shakl

Yechish:

1. Tayanch reaksiyalarini aniqlaymiz:

$$\sum M_{B_i} = 0; -M_B + ql(l + 0,5l) + F \cdot 3l = 0; \text{ bunda, } M_B = 7,5 ql^2;$$

$$\sum Y_i = 0; R_B - ql - F = 0; \text{ bundan, } R_B = 3 ql.$$

2. Konsolni uchta oraliqqa ajratamiz, keyin esa 8.3-§ da aytib o'tilgan I- va 5- qoidalarga tayanib (8.5-shakl, b):

a) koordinatalar boshini chap tayanch ustiga qo'yamiz;

b) III oraliqni sun'iy ravishda tekis taralgan kuchlar bilan yuklaymiz va ularni muvozanatlaydigan kuchlarni ham kiritamiz.

3. Har bir oraliqning boshlanishi uchun quyidagi ifodalarni tuzib chiqamiz:

$$\text{I oraliq: } v_0 = 0; \theta_0 = 0; m_1 = -7,5 ql^2$$

$$F_1 = 3ql; a_1 = 0; b_1 = 0;$$

$$c_1 = 0; q_1 = 0.$$

$$\text{II oraliq: } m_2 = 0; q_2 = -q; F_2 = 0; c_2 = l.$$

$$\text{III oraliq: } m_3 = 0; q_3 = q; F_3 = 0; c_3 = 2l.$$

Bu ifodalarni (8.3), (8.2) universal formulaga qo'yib, tegishli aylanish burchagi va solqilikning tenglamalarini umumiy holda yozib olamiz:

$$\theta_z = \frac{1}{EI_x} \left(-7,5ql^2 \cdot z + 3ql \frac{z^2}{2} \right) \Big|_{z>0}^I + \frac{1}{EI_x} \left[-q \frac{(z-l)^3}{6} \right] \Big|_{z>R}^{II} + \frac{1}{EI_x} \left[q \frac{(z-2l)^3}{6} \right] \Big|_{z=2l}^{III}; \quad (a)$$

$$v_z = \frac{1}{EI_x} \left(-7,5ql^2 \cdot \frac{z^2}{2} + 3ql \frac{z^3}{6} \right) \Big|_{z>0}^I + \frac{1}{EI_x} \left[-q \frac{(z-l)^4}{24} \right] \Big|_{z>l}^{II} + \frac{1}{EI_x} \left[q \frac{(z-2l)^4}{24} \right] \Big|_{z=2l}^{III}. \quad (b)$$

So'ralgan kesimning aylanish burchagi va solqiligini topish uchun oxirgi ifodalarni $z = 3l$ da hisoblaymiz:

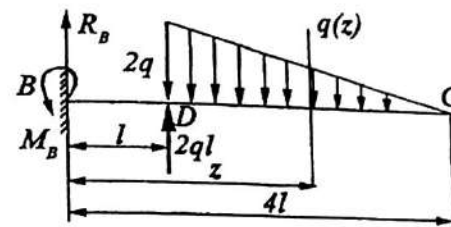
$$\theta_C = \theta_{z=3l} = \frac{1}{EI_x} \left[-7,5ql^2 \cdot 3l + 3ql \frac{(3l^2)}{2} \right] + \frac{1}{EI_x} \left[-q \frac{(2l)^3}{6} \right] + \frac{1}{EI_x} \left(q \frac{l^3}{6} \right) = -10,17 \frac{ql^3}{EI_x};$$

$$v_C = v_{z=3l} = \frac{1}{EI_x} \left[-7,5ql^2 \frac{(3l^2)}{2} + 3ql \frac{(3l)^3}{3} \right] + \frac{1}{EI_x} \left[-q \frac{(2l)^4}{24} \right] + \frac{1}{EI_x} \left(q \frac{l^4}{24} \right) = +20,875 \frac{ql^4}{EI_x};$$

Izoh: bordi-yu, so'ralgan kesim II oraliqda yotsa, (a) va (b) ifodalardagi oxirgi had e'tiborga olinmaydi.

81- masala

Bikirligi o'zgarmas konsol 8.6-shaklda ko'rsatilganidek, to'plangan kuch va yoyilgan kuchlar bilan yuklangan. Konsol erkin uchidagi aylanish burchagi va to'plangan kuch qo'yilgan kesimining solqiligi topilsin.



8.6-shakl

Yechish:

1) Tayanch reaksiyalarini topamiz:

$$R_B = ql; \quad M_B = 2ql^2.$$

2) Konsolni ikkita oraliqqa ajratib, yoyilgan kuchning ta'sir etish qonuniyatini aniqlaymiz:

$$\frac{q(z)}{2q} = \frac{4l - z}{3l} \quad \text{yoki} \quad q(z) = \frac{(4l - z)}{3l} \cdot 2q.$$

$$\text{Bunda,} \quad q'(Z) = -\frac{2q}{3l} = \text{const.}$$

3) Har bir oraliqning boshlanishi uchun quyidagi ifodalarni tuzib olamiz:

$$\text{I oraliq: } v_0 = 0; \quad \theta_0 = 0; \quad m_1 = -2ql^2; \quad F_1 = ql;$$

$$q_1 = 0; \quad a_1 = 0; \quad b_1 = 0; \quad c_1 = 0.$$

$$\text{II oraliq: } m_2 = 0; \quad F_2 = 2ql; \quad q_2 = -2q;$$

$$q'_2 = -(-2q/3l) = 2q/3l; \quad b_2 = l; \quad c_2 = l; \quad d_2 = l.$$

Bundan chiqdi, aylanish burchagi va solqilik tenglamalari quyidagi ko'rinishda:

$$\theta_z = \frac{1}{EI_x} \left(-2ql^2 \cdot z + ql \frac{z^2}{2} \right) \Big|_{z>0}^I + \frac{1}{EI_x} \left[2ql \frac{(z-l)^2}{2} - 2q \frac{(z-l)^3}{6} + \frac{2q}{3l} \cdot \frac{(z-l)^4}{24} \right] \Big|_{z>l}^{II};$$

$$v_z = \frac{1}{EI_x} \left(-2ql^2 \cdot \frac{z^2}{2} + ql \frac{z^3}{6} \right) \Big|_{z>0} + \\ + \frac{1}{EI_x} \left[2ql \frac{(z-l)^3}{6} - 2q \frac{(z-l)^4}{24} + \frac{2q}{3l} \cdot \frac{(z-l)^5}{120} \right] \Big|_{z>l}$$

C kesimning aylanish burchagini topamiz:

$$\theta_c = \theta_{z=4l} = \frac{1}{EI_x} \left(-2ql^2 \cdot 4l + ql \frac{16l^2}{2} \right) + \\ + \frac{1}{EI_x} \left[2ql \cdot \frac{(3l)^2}{2} - 2q \frac{(3l)^3}{6} + \frac{2q}{3l} \cdot \frac{(3l)^4}{24} \right] = -\frac{87}{96} \cdot \frac{ql^3}{EI_x}$$

D kesimning solqiligini hisoblaymiz:

$$v_c = v_{z=l} = \frac{1}{EI_x} \left(-2ql^2 \cdot \frac{l^2}{2} + ql \cdot \frac{l^3}{6} \right) = -\frac{5}{6} \cdot \frac{ql^4}{EI_x}$$

82-masala

Uzunligi $l=3a$ bo'lgan oddiy to'sin tekis taralgan kuchlar bilan yuklangan (8.7-shakl). Mustahkamlik va bikirlik shartlaridan foydalanib, qo'shtavrli kesim tanlansin. Hisoblashlarda

$$\sigma_{adm} = 160 \text{ MPa}, \quad E = 2 \cdot 10^5 \text{ MPa}, \quad f_{adm} = \frac{1}{600}, \quad a = 1,2 \text{ m}, \\ q = 20 \text{ kN/m} \quad \text{deb qabul qilinsin:}$$

Yechish:

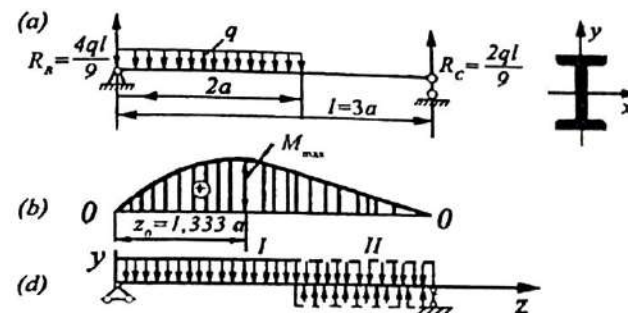
1) Tayanch reaksiyalarini aniqlab, eguvchi moment epyurasini quramiz (8.7-shakl, b).

Epyuradan $M_{max} = 9,87 \cdot 10^{-2} ql^2$ ekanligi ma'lum.

2) 8.7-shakl (b) ni chizib, to'sinning har bir oralig'i uchun tegishli ifodalarni yozib olamiz.

I oralig: $v_0 = 0$; $\theta_0 \neq 0$; $F_1 = 4ql/9$; $q_1 = -q$.

II oralig: $m_2 = 0$; $F_2 = 0$; $q_2 = q$; $c_2 = 2a$.



8.7-shakl

Bu ifodalardan foydalanib, aylanish burchagi va solqilikning tenglamalarini umumiy holda yozamiz:

$$\theta_z = \theta_0 + \frac{1}{EI_x} \left(\frac{4}{9} ql \cdot \frac{Z^2}{2} - q \frac{Z^3}{6} \right) \Big|_I + \frac{1}{EI_x} \left[q \frac{(Z-2a)^3}{6} \right] \Big|_{II};$$

$$v_z = v_0 z + \frac{1}{EI_x} \left(\frac{4}{9} ql \cdot \frac{Z^3}{6} - q \frac{Z^4}{24} \right) \Big|_I + \frac{1}{EI_x} \left[q \frac{(Z-2a)^4}{24} \right] \Big|_{II}.$$

C tayanchda solqilikning nolga tenglik shartidan θ_0 ni aniqlaymiz:

$$\theta_0 l + \frac{1}{EI_x} \left(\frac{4}{9} ql \cdot \frac{l^3}{6} - q \frac{l^4}{24} \right) + \frac{1}{EI_x} q \frac{l^4}{24 \cdot 81} = 0; \quad \text{bundan, } \theta_0 = -\frac{8}{243} \frac{ql^3}{EI_x}$$

Nihoyat quyidagilarga ega bo'lamiz:

$$\theta_z = -\frac{8}{243} \frac{ql^3}{EI_x} + \frac{1}{EI_x} \left(\frac{4}{18} ql \cdot Z^2 - \frac{q}{6} Z^3 \right) \Big|_I + \frac{q(Z-\frac{2l}{3})^3}{6EI_x} \Big|_{II};$$

$$v_z = -\frac{8}{243} \frac{ql^3}{EI_x} \cdot Z + \frac{1}{EI_x} \left(\frac{4}{27} ql \cdot Z^3 - \frac{q}{24} Z^4 \right) \Big|_I + \frac{q(Z-\frac{2l}{3})^4}{24EI_x} \Big|_{II};$$

Eng katta solqilikni aniqlaymiz – tekshirilayotgan to'sinda f_{max} I oraliqda yotadi. Shuning uchun:

$$-\frac{8}{243} \frac{ql^2}{EI_x} + \frac{1}{EI_x} \left(\frac{4}{18} ql \cdot Z^2 - \frac{q}{6} Z^3 \right) = 0.$$

Tenglamani yechib, $Z = 0,482l$ ni hosil qilamiz
Demak,

$$f_{max} = -\frac{8}{243} \frac{ql^3}{EI_x} \cdot 0,482l + \frac{1}{EI_x} \left[\frac{2}{27} ql \cdot (0,482l)^3 - \frac{q}{24} (0,482l)^4 \right] =$$

$$= -9,876 \cdot 10^{-3} \frac{ql^4}{EI_x}.$$

3) To'sin uchun bikirlik shartini yozamiz:

$$9,876 \cdot 10^{-3} \frac{ql^4}{Ef_{adm}} \leq f_{adm};$$

bundan, inersiya momentining qiymati $I_x \geq \frac{9,876 \cdot 10^{-3} ql^4}{Ef_{adm}}$ yoki

$$I_x = \frac{9,876 \cdot 10^{-3} \cdot 20 \cdot 10^3 (3 \cdot 1,2)^4 \cdot 600}{2 \cdot 10^{11} \cdot 3 \cdot 1,2} =$$

$$= 2764,6510^{-8} m^4 = 2764,65 cm^4$$

Inersiya momentining bu qiymatiga qarab, ГОСТ 8239-72 dan profil raqami 22^a bo'lgan qo'shtavrni tanlaymiz. Bu qo'shtavr uchun quyidagilarni jadvaldan olish mumkin: $I_x = 2790 cm^4$; $W_x = 254 cm^3$; endi tanlab olingan qo'shtavrni mustahkamlikka tekshiramiz:

$$\sigma_{adm} \geq \frac{M_{max}}{W_x} = \frac{9,876 \cdot 10^{-2} \cdot 20 \cdot 10^3 (3 \cdot 1,2)^2}{254 \cdot 10^{-6}} = 100,72 \cdot 10^6 \frac{N}{m^2} =$$

$$= 100,72 MPa < \sigma_{adm} = 160 MPa.$$

To'sinning mustahkamligi ta'minlangan.

8.4-§. Egilishda deformatsiyalarni aniqlash Mor usuli

Egilishda deformatsiyalarni aniqlash Mor integrali quyidagi ko'rinishga ega:

$$\Delta = \sum_{i=1}^n \int_0^1 \frac{M_F \cdot \bar{M}_i}{EJ} dz, \quad (8.4)$$

bu yerda, M_F - tashqi kuchlardan olingan eguvchi moment tenglamasi; M_i - birlik kuchdan olingan eguvchi moment tenglamasi.

Mor usulida ko'chishlar quyidagi tartibda aniqlanadi:

1. Elastik deformatsiyalanuvchi brus (to'sin, sterjen, rama, arka) oraliqlarga ajratilib, tayanch reaksiyalari topiladi.

2. Brusning yordamchi holati qurilib, u ko'chishi aniqlanadigan kesimga qarab birlik kuch bilan yuklanadi:

- chiziqli ko'chishni aniqlashda kesimga birlik to'plangan kuch qo'yiladi;
- burchakli ko'chishni aniqlashda kesimga birlik juft kuch (moment) qo'yiladi.

Brusning har bir oraliqi uchun kesish usulini qo'llab, M_F , hamda $\bar{M}_i$, larning analitik ifodasi tuziladi.

83-masala

Bikirligi o'zgarmas bo'lgan oddiy to'sinning o'rtasiga $2F$ kuch qo'yilgan (8.8-shakl, a). Kuch qo'yilgan nuqtaning solqiligi va B kesimining aylanish burchagi Mor va Vereshagin usullarida aniqlansin.

Yechish:

Masalani Mor usuli yordamida yechish uchun to'sinning asosiy (8.8-shakl, a) va yordamchi (8.8-shakl, b, d) holatlari oraliqlarga ajratilib, tayanch reaksiyalari topiladi (tayanch reaksiyalarining qiymati va yo'nalishlari chizmalarda ko'rsatilgan).

Kesish usulini qo'llab, har bir oraliq uchun asosiy va yordamchi holatlar bo'yicha eguvchi momentlarning ifodalarini tuzamiz:

I oraliq: ($0 \leq Z_1 \leq l$): $M_F^I = FZ_1$; $M_1^I = 0,5Z_1$; $M_2^I = 1 - \frac{1}{2l}Z_1$.

II oraliq: ($l \leq Z_2 \leq 2l$): $M_F^{II} = FZ_2 - 2F(Z_2 - l)$;
 $M_1^{II} = 0,5Z_2 - (Z_2 - l)$; $M_2^{II} = 1 - \frac{1}{2l}Z_2$.

K kesimning solqiligini topamiz:

$$v_k = \frac{1}{EI_x} \int_0^l M_F^I \cdot M_1^I dZ_1 + \frac{1}{EI_x} \int_l^{2l} M_F^{II} \cdot M_1^{II} dZ_2 =$$

$$= \frac{1}{EI_x} \int_0^l FZ_1 \cdot \frac{Z_1}{2} dZ_1 + \frac{1}{EI_x} \int_l^{2l} [FZ_2 - 2F(Z_2 - l)] \cdot \left[\frac{Z_2}{2} - 1(Z_2 - l) \right] dZ_2 = \frac{Fl^3}{3EI_x}.$$

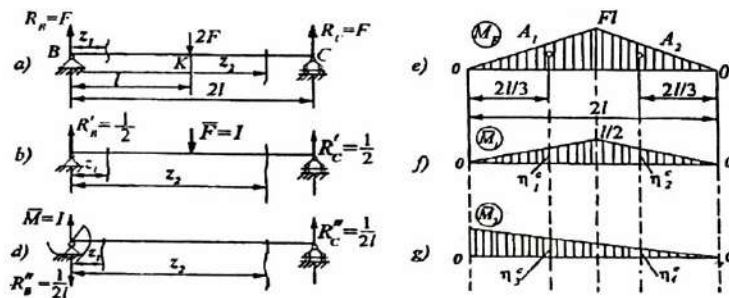
Kuch to'sinning o'rtasiga qo'yilgani uchun eng katta solqilik V_k ga teng bo'ladi

$$f_{max} = \frac{Fl^3}{3EI_x}.$$

Xuddi shu tarzda B kesimning aylanish burchagi topiladi:

$$\theta_k = \frac{1}{EI_x} \int_0^l M_F^I \cdot M_2^I dZ_1 + \frac{1}{EI_x} \int_l^{2l} M_F^{II} \cdot M_2^{II} dZ_2 = \frac{1}{EI_x} \int_0^l FZ_1 \left(1 - \frac{Z_1}{2l} \right) dZ_1 +$$

$$+ \frac{1}{EI_x} \int_l^{2l} [FZ_2 - 2F(Z_2 - l)] \cdot \left(1 - \frac{Z_2}{2l} \right) dZ_2 = \frac{Fl^2}{2EI_x}.$$



8.8-shakl

Vereshagin usulida ko'chishlarni topish uchun tashqi kuch va birlik kuchdan eguvchi moment epyuralarini quramiz (8.8-shakl, e, f, g).

Chiziqli ko'chishni aniqlash uchun 8.8-shakl (e) (f) va (g) larda ko'rsatilgan epyuralar ko'paytiriladi (odatda, Vereshagin usuli «epyuralarni ko'paytirish usuli» deb ham yuritiladi):

$$v_k = v_k(M_F, \bar{M}_I) = \frac{1}{EI_x} (A_1 \cdot \eta_1^c + A_2 \eta_2^c),$$

bu yerda, $A_1 = \frac{1}{2} \cdot Fl \cdot l = \frac{Fl^2}{2}$; $\eta_1^c = \frac{l}{3}$; $A_2 = \frac{1}{2} \cdot Fl \cdot l = \frac{Fl^2}{2}$; $\eta_2^c = \frac{l}{3}$.

Bularni e'tiborga olsak, $v_k = \frac{l}{EI_x} \left(\frac{Fl^2}{2} \cdot \frac{l}{3} + \frac{Fl^2}{2} \cdot \frac{l}{3} \right) = \frac{Fl^3}{3EI_x}$.

B kesimning aylanish burchagi quyidagicha topiladi:

$$\theta_k = \theta_k(M_F, M_2) = \frac{l}{EI_x} (A_1 \eta_3^c + A_2 \eta_4^c),$$

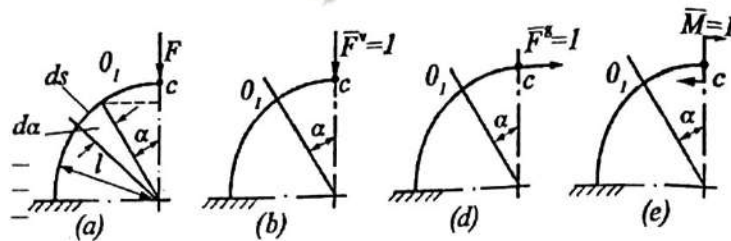
bu yerda, $\eta_3^c = 2/3$, $\eta_4^c = 1/3$.

Demak, $\theta_k = \frac{1}{EI_x} \left(\frac{Fl^2}{2} \cdot \frac{2}{3} + \frac{Fl^2}{2} \cdot \frac{1}{3} \right) = \frac{Fl^2}{2EI_x}$.

Ikkala usulda ham bir xil natijaga erishdik. Ammo yuqorida ko'rdikki, Vereshagin usulida ko'chishlarni aniqlash bir muncha osonroq ekan.

84- masala

Egriligi kichik brus F kuch bilan yuklangan (8.9-shakl, a). Kuch qo'yilgan kesimning vertikal, gorizontal va burchakli ko'chishlari aniqlansin. Brusning bikirligi o'zgarmas deb hisoblansin.



8.9-shakl

Yechish:

Bu masala Mor usuli yordamida yechiladi.

Brusning asosiy va uchala yordamchi holatlari uchun eguvchi moment ifodasini

$$\bar{M}^F = -Fl \sin \alpha \quad (8.9 - \text{shakl, a} - \text{asosiy holat});$$

$$\bar{M}_1^v = -l \sin \alpha \quad (8.9 - \text{shakl, b} - \text{yordamchi holat});$$

$$\bar{M}_1^g = -(l - l \sin \alpha) = -l(1 - \sin \alpha) \quad (8.9 - \text{shakl, d yordamchi});$$

$$M_2 = -1 \quad (8.9 - \text{shakl, e} - \text{yordamchi holat}).$$

Yoy differensialini $l d\alpha$ bilan almashtirib, quyidagilarni aniqlaymiz:

a) vertikal ko'chish:

$$v_c^v = \frac{1}{EI_x} \int_0^{\pi/2} (-Fl \sin \alpha)(-l \sin \alpha) \cdot l d\alpha = \frac{\pi Fl^3}{4 EI_x}$$

b) gorizonttal ko'chish:

$$v_c^g = \frac{1}{EI_x} \int_0^{\pi/2} (-Fl \sin \alpha)[-l(1 - l \cos \alpha)] \cdot l d\alpha = \frac{Fl^3}{2 EI_x};$$

C kesimning to'liq ko'chishi

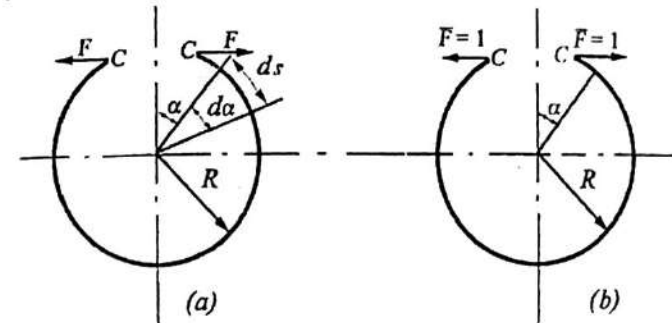
$$v_c = \sqrt{(v_c^v)^2 + (v_c^g)^2} \approx 0,93 \frac{Fl^3}{EI_x};$$

d) C kesimning aylanish burchagi-burchakli ko'chish:

$$\theta_c = \frac{1}{EI_x} \int_0^{\pi/2} (-Fl \sin \alpha)(-1) l d\alpha = \frac{Fl^2}{EI_x}.$$

85-masala

F kuch ta'sirida kesik halqadagi tirqish qanchaga ochiladi (8.10-shakl, a)? Halqaning bikirligi $EI = \text{const}$.



8.10-shakl

Yechish:

Masala Mor integrali yordamida yechiladi.

Asosiy va yordamchi holatlar uchun mos ravishda, tashqi kuch va birlik kuchdan paydo bo'luvchi eguvchi momentlarni aniqlaymiz:

$$0 \leq \alpha \leq 2\pi: M_F = FR(1 - \cos \alpha); M_1 = R(1 - \cos \alpha).$$

Kuch qo'yilgan kesimlarning o'zaro uzoqlashishini hisoblaymiz:

$$\begin{aligned} \delta_C &= \frac{1}{EI} \int_{(1)} M_F \cdot \bar{M}_1 \cdot ds = \frac{1}{EI} \int_0^{2\pi} FR(1 - \cos \alpha) \cdot R(1 - \cos \alpha) R d\alpha = \\ &= \frac{FR^3}{EI} \int_0^{2\pi} (1 - \cos \alpha)^2 d\alpha = 3\pi \frac{FR^3}{EI}. \end{aligned}$$

8.5-§. Egilishda deformatsiyalarni aniqlash Vereshagin usuli

Egilishda deformatsiyalarni aniqlashning Vereshagin formulasi quyidagi ko'rinishga ega:

$$\Delta = \sum_{i=1}^n \frac{A_i \cdot \eta_{1i}^C}{EI_i}, \quad (8.5)$$

bu yerda, A_i – tashqi kuchlardan olingan eguvchi moment epyura yuzasi;

η_i^C – A_i og'irlik markaziga to'g'ri kelgan birlik kuch (yoki birlik moment) dan olingan eguvchi moment epyura ordinatasi.

Shuni ta'kidlab o'tish lozimki, o'zgaruvchan kesimli bruslarning ko'chishlarini Vereshagin usulida aniqlab bo'lmaydi, bunday hollarda Mor integralidan foydalanish ma'qul. Bundan tashqari, η_i^C ordinalarni faqat to'g'ri chiziq bilan chegaralangan eguvchi moment epyuralaridan olishni unutmashlik kerak.

Agar tashqi va birlik kuchlardan qurilgan eguvchi moment epyuralari brusning bo'ylama o'qidan bir tomonda yotsa, ko'paytma musbat, aksincha, turli tomonlarda yotsa, manfiy ishorali hisoblanadi.

86-masala

Oddiy to'sinning chap tayanchidan qoq o'rtasigacha intensivligi q bo'lgan tekis taralgan yuk qo'yilgan (8.11-shakl, a). K kesimning solqiligi aniqlansin. To'sin o'zgarmas bikirlikka ega.

Yechish:

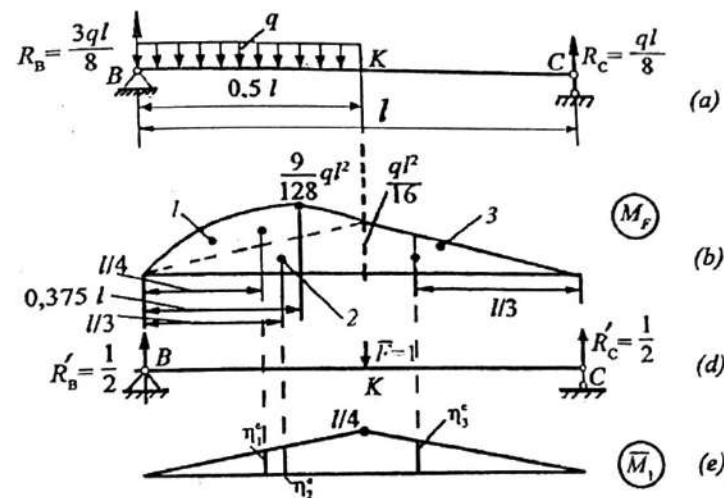
Masalani Vereshagin usulida yechamiz.

Asosiy holat bo'yicha yuklangan to'sinning tayanchlardagi reaksiyalari aniqlangach, tashqi yukdan eguvchi moment epyurasini quramiz (8.11-shakl, b).

K kesimga $F = 1$ birlik kuch qo'yib, to'sinning yordamchi holati quriladi (8.11-shakl, d). keyin esa tayanch reaksiyalarini aniqlab, yordamchi holatga tegishli bo'lgan M_1 , epyurani quramiz (8.11-shakl, e). M_F epyurani uchta bo'lakka ajratamiz va ularning yuzalarini hisoblaymiz:

$$A_1 = \frac{2}{3} \cdot \frac{l}{2} \cdot \frac{q(\frac{l}{2})^2}{8} = \frac{ql^3}{96} - \text{parabolik segmentning yuzasi};$$

$$A_2 = A_3 = \frac{1}{2} \cdot \frac{l}{2} \cdot \frac{ql^2}{16} = \frac{ql^3}{64} - \text{uchburchakning yuzasi}.$$



8.11-shakl

Bu yuzalarning og'irlik markazlariga to'g'ri keluvchi ordinatalar quyidagicha:

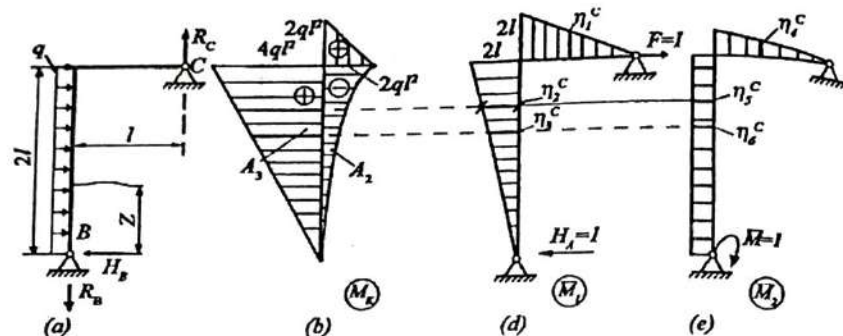
$$\eta_1^C = \frac{l}{2}; \quad \eta_2^C = \eta_3^C = \frac{l}{6}.$$

Vereshagin formulasidan v_K ni aniqlaymiz:

$$v_K = \frac{1}{EI_x} (A_1 \cdot \eta_1^C + A_2 \cdot \eta_2^C + A_3 \cdot \eta_3^C) = \frac{1}{EI_x} \left(\frac{ql^3}{96} \frac{l}{8} + 2 \frac{ql^3}{64} \frac{l}{6} \right) = \frac{5}{768} \cdot \frac{ql^4}{EI_x}.$$

87-masala

8.12-shakl (a) da tasvirlangan tekis rama C kesimining gorizonttal ko'chishi va B tayanch kesimining aylanish burchagini aniqlang. Rama o'zgarmas bikirlikka ega: $EY = \text{const}$



8.12-shakl

Yechish:

Masalani Vereshagin usulida yechamiz.

Berilgan ramaning muvozanatini tekshirib, tayanchlarda paydo bo'luvchi reaksiyalarning o'zaro tengligiga ishonch hosil qilamiz:

$$H_B = R_B = R_C = 2ql.$$

Tashqi kuchlardan hosil bo'lgan eguvchi moment epyurasi (yuk epyurasi)ni «qatlamlarga ajratilgan» ko'rinishda quramiz (8.12-shakl, b).

Izoh: epyuralarni qiymatlarga ajratib chizish Vereshagin usulini ishlatishda ancha qulaylik tug'diradi. Chunki eguvchi moment tenglamasi ko'phadlardan iborat bo'lishiga qaramasdan (masalan, ramaning vertikal qismi uchun yozilgan tenglama ikkita haddan iborat: $M_y = H_B \cdot Z - 0,5 qZ^2$), epyura ko'phad uchun qurilmaydi, balki har bir had (qo'shiluvchi) uchun alohida quriladi.

Yuk epyurasining yuzalarini hisoblaymiz:

$$A_1 = \frac{1}{2} 2ql^2 \cdot l = ql^3; A_2 = \frac{1}{3} 2ql^2 \cdot 2l = \frac{4}{3} ql^3; A_3 = \frac{1}{2} 4ql^2 \cdot 2l = 4ql^3.$$

C tayanchga $F = 1$ birlik gorizontaal kuch qo'yib, tayanchlardagi reaksiyalarni aniqlaymiz hamda M_1 birlik epyurani quramiz (8.12-shakl, d).

B tayanchga M_1 birlik moment qo'yib, ramaning 8.12-shakl (e) da tasvirlangan yordamchi holati quriladi. Bu holat uchun tayanch reaksiyalarini topib, ikkinchi birlik epyura M_2 ni quramiz.

Birlik epyuralardan η_i^C ordinalarni hisoblaymiz:

$$\eta_1^C = \frac{4l}{3}; \eta_2^C = \frac{3l}{2}; \eta_3^C = \frac{4l}{3}; \eta_4^C = \frac{2}{3}; \eta_5^C = \eta_6^C = 1.$$

C kesimning gorizontaal ko'chishi quyidagiga teng:

$$v_C^g = \frac{1}{EI_x} (A_1 \eta_1^C + A_2 \eta_2^C + A_3 \eta_3^C) = \frac{1}{EI_x} \left(ql^3 \frac{4}{3} l - \frac{4}{3} ql^3 \frac{3}{2} l + 4ql^3 \frac{4}{3} l \right) = 4,667 \frac{ql^4}{EI_x}.$$

B kesimning aylanish burchagi quyidagiga teng:

$$\theta_B = \frac{1}{EI_x} (A_1 \eta_4^C + A_2 \eta_5^C + A_3 \eta_6^C) = \frac{1}{EI_x} \left(ql^3 \frac{2}{3} - \frac{4}{3} ql^3 \cdot 1 + 4ql^3 \cdot 1 \right) = 3,333 \frac{ql^3}{EI_x}.$$

8.6-§. Egilishda berilgan balkalar uchun deformatsiyalarni aniqlash bo'yicha topshiriqlar

14- topshiriq

Berilgan: keltirilgan shakllarda tashqi tomondan yuklangan balkalar va siniq sterjenlar berilgan.

Topish kerak:

- 1) 6.19- va 6.20-shakllardagi nuqtaning solqiligi va B kesimning burilish burchagi Vereshagin usulida aniqlansin;
- 2) 6.18-shakldagi C nuqtaning solqiligi va A kesimning burilish burchagi Mor usulida aniqlansin.

Kerakli ma'lumotlar 6.1-jadvalda, hisoblash sxemalari esa 6.18- va 6.19-shakllarda berilgan.

IX BOB MURAKKAB DEFORMATSIYA

9.1-§. Asosiy mulohazalar

Inshoot, mashina va mexanizm qismlarining barcha ko'ndalang kesim yuzalariga ta'sir ko'rsatuvchi ikki yoki undan ortiq ichki zo'riqishlar natijasida vujudga keladigan deformatsiyalar murakkab deformatsiya yoki murakkab qarshilik deyiladi.

Eng ko'p uchraydigan murakkab deformatsiya turlari quyidagilardir:

- qiyshiq egilish;
- markaziy bo'lmagan cho'zilish yoki siqilish;
- egilish va buralishning birgalikdagi ta'siri.

Murakkab deformatsiyaga ishlayotgan bruslar quyidagi tartibda hisoblanadi:

- brus ko'ndalang kesimlaridagi ichki zo'riqish kuchlari aniqlanib, ularning epyuralari quriladi;
- eng katta yuklangan kesim vaziyati aniqlanadi;
- har bir zo'riqish kuchidan hosil bo'lgan kuchlanishni aniqlab, ular algebraik ravishda qo'shiladi va eng katta kuchlanish hosil bo'lgan xavfli nuqta aniqlanadi;
- brus materiali uchun uning mustahkamlik sharti tekshiriladi.

9.2-§. Qiyshiq egilish

Qiyshiq egilishda natijaviy eguvchi moment tekisligi bilan egilish tekisligi ustma-ust tushmaydi.

Qiyshiq egilishda normal kuchlanish quyidagiga teng:

$$\sigma_z = \pm \frac{M_y}{J_y} \cdot x \pm \frac{M_x}{J_x} \cdot y. \quad (9.1)$$

Nuqta cho'zilayotgan yuzachada bo'lsa, M_x va M_y larning ishorasi musbat, siqilayotgan yuzachada bo'lsa, manfiy olinadi.

Neytral o'qning holati quyidagi formula asosida aniqlanadi:

$$\operatorname{tg} \varphi = \frac{J_x}{J_y} \cdot \operatorname{tg} \alpha, \quad (9.2)$$

bu yerda, α – kuch ta'sir chizig'i bilan bosh markaziy o'q o'rtasidagi burchak.

88-masala

Ko'ndalang kesimi burchaklikdan iborat bo'lgan brusning xavfli kesimidagi kuchlanish aniqlansin.

Berilgan: $P=200\text{H}$; $l=1\text{m}$; $E=2 \cdot 10^5 \text{ MPa}$.

Teng tomonli burchaklikni nomeri 5 bo'lib, sortament bo'yicha qiymatlari quyidagicha:

$$J_u = 17,8 \text{ sm}^4, \quad J_v = 4,63 \text{ sm}^4, \quad U_A = 1,52 \text{ sm}, \quad U_B = 2,01 \text{ sm}, \\ V_A = 3,53 \text{ sm}, \quad V_B = 0.$$

Yechish:

Bu yerda kesimning bosh markaziy o'qlari asosga nisbatan o'tgan o'qlarga nisbatan 45° ostida yo'nalgan bo'lib, kuch esa vertikal ta'sir etmoqda. Shuning uchun bu yerda qiyshiq egilish hosil bo'lmoqda.

Bosh yuzalarga nisbatan eguvchi momentlar teng bo'ladi.

$$M_v = P_u \cdot l = P \cdot l \sin 45^\circ = \frac{P \cdot l \sqrt{2}}{2};$$

$$M_u = P_v \cdot l = P \cdot l \cos 45^\circ = \frac{P \cdot l \sqrt{2}}{2};$$

A va B nuqtalardagi kuchlanishlar teng:

$$\sigma_A = \frac{M_v}{J_v} U_A + \frac{M_u}{J_u} V_A = \frac{200 \cdot 1 \cdot \frac{\sqrt{2}}{2}}{4,63 \cdot 10^{-8}} \cdot 1,52 \cdot 10^{-2} + \frac{200 \cdot 1 \cdot \frac{\sqrt{2}}{2}}{17,8 \cdot 10^{-8}} \cdot 3,53 \cdot 10^{-2} = 74,5 \text{ MPa}.$$

$$\sigma_B = \frac{M_v}{J_v} U_B + \frac{M_u}{J_u} V_B = -\frac{200 \cdot 1 \cdot \frac{\sqrt{2}}{2}}{4,63 \cdot 10^{-8}} \cdot 2,01 \cdot 10^{-2} + \frac{200 \cdot 1 \cdot \frac{\sqrt{2}}{2}}{17,8 \cdot 10^{-8}} \cdot 0 = 61,4 \text{ MPa}.$$

Konsol balka uchun bosh yuzalari bo'yicha og'irlik markazi ko'chishi quyidagiga teng:

$$\delta_u = \frac{P_u \cdot l^3}{3E} = \frac{200 \cdot 1 \cdot \frac{\sqrt{2}}{2}}{3 \cdot 2 \cdot 10^{11} \cdot 4,63 \cdot 10^{-8}} = 0,00509 \text{ m};$$

$$\delta_v = \frac{P_v \cdot l^3}{3EI_v} = \frac{200 \cdot 1 \cdot \frac{\sqrt{2}}{2}}{3 \cdot 2 \cdot 10^{11} \cdot 17,8 \cdot 10^{-8}} = 0,00132 \text{ m.}$$

Unda umumiy ko'chish teng bo'ladi:

$$\delta = \sqrt{\delta_u^2 + \delta_v^2} = \sqrt{(0,00509)^2 + (0,00132)^2} = 0,0053 \text{ m.}$$

89- masala

Uzunligi $l = 1$, profil raqami 24 (ГОСТ 8239-72) bo'lgan qo'shtavrlil konsolning erkin uchiga α burchak ostida F kuch qo'yilgan (9.1-shakl, a).

Quyidagilarni aniqlash talab etiladi:

- 1) konsolni mustahkamlikka tekshirish va uning erkin uchi solqiligin hisoblash;
- 2) $\alpha = 0$ hol uchun (to'g'ri ko'ndalang egilish) kesimdagi eng katta normal kuchlanish hamda eng katta solqilikni aniqlash.

Quyidagilar ma'lum: $\alpha = 12^\circ$; $F = 15 \text{ kH}$; $\sigma_{adm} = 160 \text{ MPa}$; $E = 2 \cdot 10^5 \text{ MPa}$; $W_x = 289 \cdot 10^{-6} \text{ m}^3$; $W_y = 34,5 \cdot 10^{-6} \text{ m}^3$; $I_x = 3460 \cdot 10^{-8} \text{ m}^4$; $I_y = 198 \cdot 10^{-8} \text{ m}^4$.

Yechish:

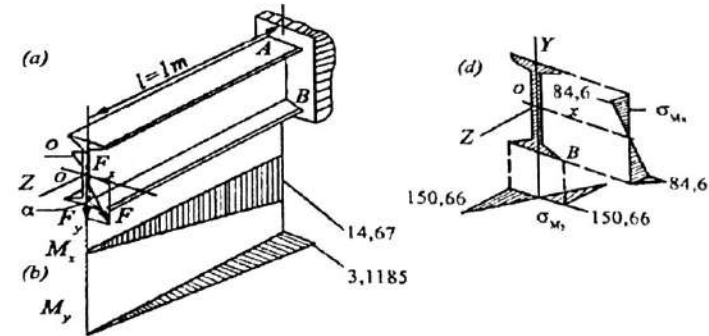
$\alpha \neq 0$ bo'lganligi sababli, brus qiyshiq egilishga qarshilik ko'rsatadi. F kuchni tashkil etuvchilarga ajratib, xavfli kesimdagi eguvchi momentlarni aniqlaymiz (9.1-shakl, b):

$$\begin{aligned} M_{x(\max)} &= F_y \cdot l = \\ &= F \cdot l \cos \alpha = 15 \cdot 10^3 \cdot 1 \cdot 0,9781 = 14,67 \cdot 10^3 \text{ kN} \cdot \text{m}; \\ M_{y(\max)} &= F_x \cdot l = F \cdot l \sin \alpha = 15 \cdot 10^3 \cdot 1 \cdot 0,2079 = 3,1185 \cdot 10^3 \text{ kN} \cdot \text{m}. \end{aligned}$$

U holda, normal kuchlanishlar:

$$\sigma_{M_x}^C = \frac{M_{x(\max)}}{W_x} = \frac{24,45 \cdot 10^3}{289 \cdot 10^{-6}} = 50,77 \cdot 10^6 \frac{\text{N}}{\text{m}^2} = 50,77 \text{ MPa};$$

$$\sigma_{M_y}^B = \frac{M_{y(\max)}}{W_y} = \frac{3,1185 \cdot 10^3}{34,5 \cdot 10^{-6}} = 90,39 \cdot 10^6 \frac{\text{N}}{\text{m}^2} = 90,39 \text{ MPa}.$$



9.1-shakl

Bu qiymatlar yordamida qurilgan kuchlanish epyurasidan (9.1-shakl, b) ko'rinib turibdiki, material cho'zilish va siqilishga bir xil qarshilik ko'rsatar ekan. U holda, C nuqtada natijaviy kuchlanish quyidagiga teng bo'ladi:

$$\sigma_{\max} = \sigma_{M_x}^C + \sigma_{M_y}^C = 50,77 + 90,39 = 141,16 \text{ MPa}.$$

Brusning yuklanganlik darajasini ko'rib o'tamiz:

$$(160 - 141,16) / 160 \cdot 100\% = 11,77\%.$$

Demak, brus 11,77% ga yuklanmagan. Endi konsol erkin uchi solqiligining bosh o'qlardagi proyeksiyalarini aniqlaymiz:

$$f_y = \frac{F_y l^3}{3EI_x} = \frac{F \cdot \cos \alpha \cdot l^3}{3EI_x} = \frac{15 \cdot 10^3 \cdot 0,9781 \cdot 1^3}{3 \cdot 2 \cdot 10^5 \cdot 10^6 \cdot 3460 \cdot 10^{-8}} = 0,7067 \cdot 10^{-3} \text{ m} = 0,7067 \text{ mm};$$

$$f_x = \frac{F_x l^3}{3EI_y} = \frac{F \cdot \sin \alpha \cdot l^3}{3EI_y} = \frac{15 \cdot 10^3 \cdot 0,2079 \cdot 1^3}{3 \cdot 2 \cdot 10^5 \cdot 10^6 \cdot 198 \cdot 10^{-8}} = 2,625 \cdot 10^{-3} \text{ m} = 2,625 \text{ mm}.$$

To'liq solqilik esa quyidagiga teng bo'ladi:

$$f = \sqrt{f_x^2 + f_y^2} = 2,7185 \cdot 10^{-3} \text{ m} = 2,7185 \text{ mm}.$$

$\alpha = 0$ bo'lganda, to'g'ri ko'ndalang egilish sodir bo'ladi. Shuning uchun:

$$\sigma_{\max} = \frac{M_{x(\max)}}{W_x} = \frac{F \cdot l}{W_x} = \frac{15 \cdot 10^3 \cdot 1}{289 \cdot 10^{-6}} = 51,9 \cdot 10^6 \frac{\text{N}}{\text{m}^2} = 51,9 \text{ MPa};$$

$$f = \frac{F \cdot l^3}{3EI_x} = \frac{15 \cdot 10^3 \cdot 1}{3 \cdot 2 \cdot 10^{11} \cdot 3460 \cdot 10^{-8}} = 0,72254 \cdot 10^{-3} \approx 0,723 \text{ mm.}$$

Yuqoridagi hisoblashlardan shunday xulosa chiqadi: kuch tekisligiga nisbatan bikirligi eng katta bo'lgan bosh inersiya tekisligiga atigi 120° ga og'dirilganda normal kuchlanish $(141-51,9)/51,9 \cdot 100\% = 171,98\%$ ga, solqiligi esa $(2,7185-0,723)/0,723 \cdot 100\% = 276,3\%$ (deyarli 3,76 marta) ga oshar ekan.

90- masala

Qiyshiq egilishga qarshilik ko'rsatayotgan yog'och konsol uchun (9.2-shakl):

- ko'ndalang kesim o'lchamlari — b va h lar topilsin;
- kesimning tomonlari bo'yicha normal kuchlanish epyuralari qurilsin;
- neytral o'qning holati aniqlansin;
- konsol erkin uchining to'liq solqiligi aniqlansin.

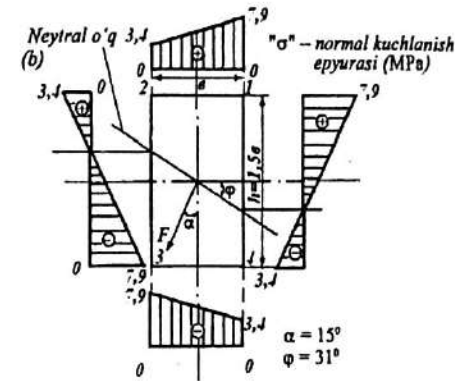
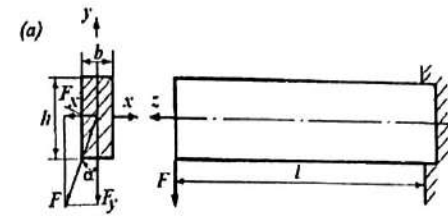
Quyidagilar ma'lum: $F = 12 \text{ kN}$; $\alpha = 15^\circ$; $l = 0,85 \text{ m}$;

$$\frac{h}{b} = 1,5; \quad \sigma_{adm} = 8 \text{ MPa}; \quad E = 1 \cdot 10^4 \text{ MPa.}$$

Yechish:

- F kuchni tashkil etuvchilarga ajratib, xavfli kesimdagi eguvchi momentlarni aniqlaymiz:

$$M_{x(max)} = F \cdot l \cos \alpha = 12 \cdot 0,85 \cdot 0,9659 = 9,8524 \text{ kN} \cdot \text{m};$$



9.2-shakl

$$M_{y(max)} = F \cdot l \sin \alpha = 12 \cdot 0,85 \cdot 0,2588 = 2,6399 \text{ kN} \cdot \text{m};$$

$$K_0 = w_x/w_y = h/b = 1,5$$

ekanligini nazarda tutib, qarshilik momentini hisoblaymiz:

$$W_x = \frac{(9,8524 + 1,5 \cdot 2,6399) \cdot 10^3}{8 \cdot 10^6} = 1726,54 \cdot 10^{-6} \cdot \text{m}^3$$

Bundan tashqari, qarshilik momentini yana quyidagicha yozish ham mumkin:

$$W_x = \frac{bh^2}{6} = \frac{3b^3}{8}.$$

Bulami tenglasak,

$$b = \sqrt[3]{\frac{8 \cdot 1726,54 \cdot 10^{-6}}{3}} = 0,1664 \text{ m}; \quad h = 1,5b = 0,2496 \text{ m}; \quad \text{ekanligi}$$

kelib chiqadi.

b) Xavfli kesimning istalgan nuqtasidagi normal kuchlanishni aniqlaymiz:

$$\sigma_z = \frac{M_{x(max)}}{I_y} \cdot x + \frac{M_{x(max)}}{I_x} \cdot y.$$

Bunda,

$$I_x = \frac{bh^3}{12} = \frac{0,1664 \cdot (0,249)^3}{12} = 21,563 \cdot 10^{-5} m^4;$$

$$I_y = \frac{bh^3}{12} = \frac{0,2496 \cdot (0,1664)^3}{12} = 9,5835 \cdot 10^{-5} m^4;$$

$$\sigma_z = \frac{2,6399 \cdot 10^3}{9,5835 \cdot 10^{-5}} \cdot x + \frac{9,5824 \cdot 10^3}{21,563 \cdot 10^{-5}} \cdot y =$$

$$= 0,27546 \cdot 10^8 x + 0,4569 \cdot 10^8 y \text{ N/m}^2$$

Endi kesim yuzalarida joylashgan xarakterli nuqtalardagi kuchlanishlarni hisoblaymiz (9.2-shakl, b):

$$\begin{aligned} \sigma_{1,3} &= \pm 0,27546 \cdot 10^8 \cdot \frac{0,1664}{2} \pm 0,4596 \cdot \frac{0,2496}{2} 10^8 = \\ &= \pm (2,2918 + 5,7022) \cdot 10^6 \frac{N}{m^2} = \pm 7,994 \text{ MPa}; \end{aligned}$$

$$\begin{aligned} \sigma_{2,4} &= \mp 0,27546 \cdot 10^8 \cdot \frac{0,1664}{2} \pm 0,4595 \cdot \frac{0,2496}{2} 10^8 = (\mp 2,2918 \pm \\ &\pm 5,7022) \cdot 10^6 \frac{N}{m^2} = \pm 3,4104 \text{ MPa}. \end{aligned}$$

d) (9.2) formuladan foydalanib, neytral o'qning holatini aniqlaymiz:

$$\operatorname{tg} \varphi = \frac{21,563 \cdot 10^{-5}}{9,5835 \cdot 10^{-5}} \cdot \operatorname{tg} 15^\circ \approx 0,6029 \approx \operatorname{tg} 31^\circ 08' \text{ yoki}$$

$$\operatorname{tg} \varphi = \left| \frac{y}{x} \right| = \frac{0,2754}{0,4569} = 0,603.$$

e) Kuch qo'yilgan kesimning gorizontal va vertikal solqiliklarini hisoblaymiz:

$$f_x = \frac{F_x \cdot l^3}{3EI_y} = \frac{F \cdot \sin \alpha \cdot l^3}{3EI_y} = \frac{12 \cdot 0,2588 \cdot (0,85)^3 \cdot 10^3}{3 \cdot 10^{10} \cdot 9,5835 \cdot 10^{-5}} = 6,634 \cdot 10^{-4} \text{ m} = 0,0663 \text{ sm};$$

$$f_y = \frac{F_y \cdot l^3}{3EI_x} = \frac{F \cdot \cos \alpha \cdot l^3}{3EI_x} = \frac{12 \cdot 0,9659 \cdot (0,85)^3 \cdot 10^3}{3 \cdot 10^{10} \cdot 21,563 \cdot 10^{-5}} = 11,004 \cdot 10^{-4} \text{ m} = 0,11 \text{ sm}.$$

To'liq solqilik:

$$f = \sqrt{f_x^2 + f_y^2} = 1,2489 \cdot 10^{-3} \text{ m} = 0,1285 \text{ sm}.$$

To'liq solqilikning yo'nalishi neytral o'qqa tik bo'lib, vertikal OY o'qi bilan $\varphi = 31^\circ 08'$ burchak hosil qiladi.

9.3-§. Markaziy bo'lmagan cho'zilish yoki siqilish

Markaziy bo'lmagan cho'zilish yoki siqilishda normal kuchlanish quyidagicha aniqlanadi:

$$\sigma_z = \frac{N}{A} + \frac{M_y}{J_y} \cdot x + \frac{M_x}{J_x} \cdot y. \quad (9.3)$$

Neytral o'qning bosh o'qlaridan ajratgan kesmalari quyidagicha aniqlanadi:

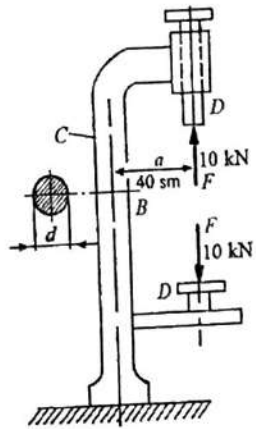
$$a_x = -\frac{i_y^2}{X_F}; \quad (9.4)$$

$$a_y = -\frac{i_x^2}{Y_F}; \quad (9.5)$$

bu yerda, X_F va Y_F lar – kuch qo'yilgan nuqtadan bosh markaziy o'qlargacha olingan masofa.

Markaziy bo'lmagan cho'zilish yoki siqilishda qarshilik markazi atrofida shunday yopiq soha mavjudki, agar kuch shu sohaning ichkarisiga yoki uning konturiga qo'yilsa, kesimda faqat bir xil ishorali kuchlanish paydo bo'ladi. Bu yopiq sohaga kesim yadrosi deyiladi.

91-masala



9.3-shakl

Detallarni teshishda dastgoh (stanok) shpindeli D ga shu shpindel uchi bo'ylab F kuch ta'sir qiladi. Doiraviy kesimli cho'yan ustun C ning diametri aniqlansin (9.3-shakl). Cho'yan uchun cho'zilishda ruxsat etilgan kuchlanish $\sigma'_{adm} = 40 \text{ MPa}$ ga teng. Hisoblashlarda $F = 10 \text{ kN}$ va $a = 0,4 \text{ m}$ deb olinsin.

Yechish:

Ustun markaziy bo'lmagan cho'zilishga qarshilik ko'rsatadi. (9.3) formulaga ko'ra, mustahkamlik sharti:

$$\sigma = \frac{N}{A} + \frac{M_{max}}{W_x} \leq \sigma'_{adm};$$

bu ifodada, $N=F=10 \text{ kN}$; $A=\pi d^2/4$; $W_x=\pi d^3/32$; $M_{max}=Fa=4 \text{ kNm}$. Bularni e'tiborga olib:

$$4 \cdot 10^3 / \pi d^2 + 32 \cdot 10 \cdot 10^3 / \pi d^3 \leq 4 \cdot 10^6 \text{ (N/m}^2\text{)}$$

munosabatni hosil qilamiz. Tanlash yo'li bilan ustunning diametri topiladi: $d = 10,1 \cdot 10^{-2} \text{ m} = 10,1 \text{ sm}$.

92-masala

Kesimi to'g'ri to'rtburchakli kalta (qisqa) yog'och ustunning K (X_F, Y_F) nuqtasiga siquvchi bo'ylama kuch F qo'yilgan (9.4-shakl). Bu masala uchun:

- kesim yadrosi qurilsin;
- xolis o'qning holati aniqlansin;
- kesimning eng chetki nuqtalaridagi kuchlanishlar topilsin;
- kesimning tomonlari bo'yicha normal kuchlanish epyuralari qurilsin.

Hisoblashlar uchun ushbu ma'lumotlar ma'lum: $F=120 \text{ kN}$;

$X_F=0,04 \text{ m}$; $Y_F=0,06 \text{ m}$; $b=0,2 \text{ m}$; $h=0,25 \text{ m}$.

Yechish:

Ustun nomarkaziy siqilishga qarshilik ko'rsatmoqda. Uning ko'ndalang kesimida quyidagi ichki kuchlar paydo bo'ladi:

$N_z = -120 \text{ kN}$ — bo'ylama kuch;

$M_x = -120 \cdot Y_F = -7,2 \text{ kN} \cdot \text{m}$ — eguvchi moment;

$M_y = 120 \cdot X_F = 4,8 \text{ kN} \cdot \text{m}$ — eguvchi moment;

(Eslatma: qisqa ustunlar uchun bu ichki zo'riqishlar uning uzunligi bo'yicha o'zgarmasdan qoladi). Ko'ndalang kesimning geometrik xarakteristikalarini hisoblaymiz:

$$I_x = \frac{bh^3}{12} = 2,6 \cdot 10^{-4} \text{ m}^4; \quad I_y = \frac{bh^3}{12} = 1,667 \cdot 10^{-4} \text{ m}^4.$$

Kesim yuzasi $A=bh=0,05 \text{ m}^2$ bo'lganligi uchun inersiya radiuslarining kvadratlari:

$$i_x^2 = \frac{I_x}{A} = 5,2 \cdot 10^{-3} \text{ m}^2; \quad i_y^2 = \frac{I_y}{A} = 3,334 \cdot 10^{-3} \text{ m}^2.$$

ga teng bo'ladi.

a) (9.4), (9.5) formulalardan foydalanib, neytral o'qning bosh o'qlardan ajratgan kesmalarini aniqlaymiz:

$$a_x = -\frac{i_y^2}{(-X_F)} = \frac{3,334 \cdot 10^{-3}}{0,04} = 8,335 \cdot 10^{-2} \text{ m};$$

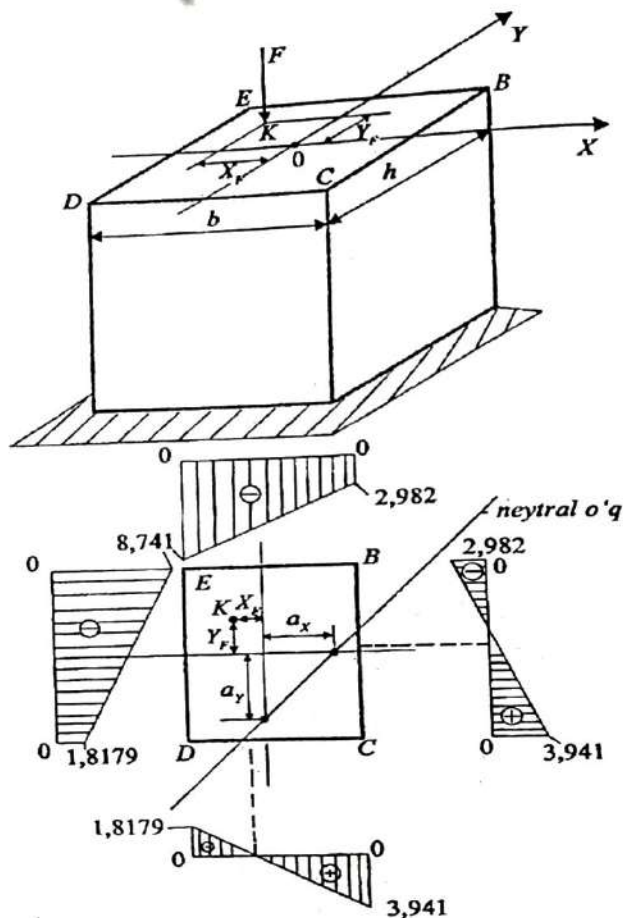
$$a_y = -\frac{i_x^2}{(-Y_F)} = \frac{5,2 \cdot 10^{-3}}{0,06} = 8,667 \cdot 10^{-2} \text{ m}.$$

Bu kesmalarni bosh inersiya o'qlari ustiga masshtab bilan qo'yib, hosil bo'lgan nuqtalar tutashtirilsa, neytral o'qning holati aniqlanadi (9.4-shakl, b):

b) (9.3) formulaga binoan, kesimning istalgan nuqtasidagi kuchlanishni aniqlash mumkin:

$$\sigma_z = -\frac{F}{A} \left(1 - \frac{X_F}{i_y^2} \cdot X + \frac{Y_F}{i_x^2} \cdot Y \right) \text{ yoki}$$

$$\begin{aligned} \sigma_z &= -\frac{120 \cdot 10^3}{0,05} \left(1 - \frac{0,04}{3,334 \cdot 10^{-3}} \cdot X + \frac{0,06}{5,2 \cdot 10^{-3}} \cdot Y \right) = \\ &= -2,4 \cdot 10^6 (1 - 11,9976 \cdot X + 11,53852 \cdot Y) = \\ &= -2,4(1 - 11,9976 \cdot X + 11,5385 \cdot Y) \text{ (MPa)}. \end{aligned}$$



9.4-shakl

So'ralayotgan nuqtalardagi kuchlanishlarni hisoblaymiz:

$$\sigma_B = -2,4 \left(1 - 11,9976 \cdot \frac{0,2}{2} + 11,5385 \cdot \frac{0,25}{2} \right) = -2,982 \text{ MPa};$$

$$\sigma_C = -2,4 \left(1 - 11,9976 \cdot \frac{0,2}{2} - 11,5385 \cdot \frac{0,25}{2} \right) = 3,941 \text{ MPa};$$

$$\sigma_D = -2,4 \left(1 + 11,9976 \cdot \frac{0,2}{2} - 11,5385 \cdot \frac{0,25}{2} \right) = -1,8179 \text{ MPa};$$

$$\sigma_E = -2,4 \left(1 + 11,9976 \cdot \frac{0,2}{2} + 11,5385 \cdot \frac{0,25}{2} \right) = -8,741 \text{ MPa};$$

c) Topilgan qiymatlar yordamida aniq masshtab bilan normal kuchlanishlarning epyuralari quriladi (9.4-shakl, b).

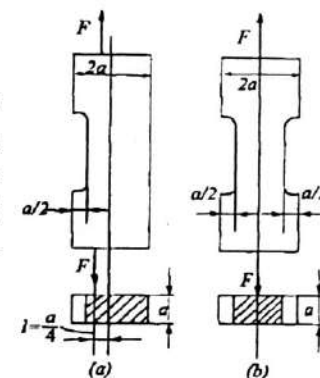
93-masala

9.5-shakl, (a) va (b) da ko'rsatilgan sterjenlarning qaysi biri katta yukka bardosh bera oladi? Hisoblashlarda F va a lar ma'lum deb hisoblansin.

Yechish:

a) bu holatda sterjen nomarkaziy cho'zilishga qarshilik ko'rsatmoqda; kuch qo'yilgan nuqtaning kesim og'irlik markazidan uzoqligi $l = \frac{a}{4}$ (ekssentrisitet) ga teng. Shu sababli eng katta cho'zuvchi kuchlanish quyidagicha bo'ladi:

$$\sigma_{max} = \frac{F}{a \cdot 1,5a} + \frac{F \cdot \frac{a}{4}}{a \cdot \frac{(1,5a)^2}{6}} = \frac{4}{3} \cdot \frac{F}{a^2}.$$



9.5-shakl

b) ikkinchi holatda sterjenga markaziy kuch qo'yilgan; kesimdagi eng katta kuchlanish

$$\sigma_{max} = \frac{F}{a^2},$$

bundan chiqdi, simmetrik sterjen ko'proq yukni ko'tara olar ekan.

9.4-§. Buralish va egilishning birgalikdagi ta'siri

Ko'ndalang kesimi aylanadan iborat bo'lgan sterjen buralish va egilishning birgalikdagi ta'sirida bo'lsa, uning ko'ndalang kesimida (xavfli nuqtasida) tekis kuchlanish holati ro'y beradi.

Bunda valning mustahkamligi mustahkamlik nazariyalari bo'yicha quyidagicha tekshiriladi:

$$\sigma_{ekv} = \frac{M_{ekv}}{W_x} \leq \sigma_{adm}, \quad (9.6)$$

bu yerda, M_{ekv} — hisobiy ekvivalent moment.

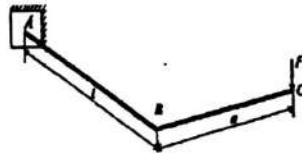
III mustahkamlik nazariyasi bo'yicha

$$M_{ekv}^{III} = \sqrt{M_{um}^2 + M_b^2}; \quad (9.7)$$

bu yerda, M_{um} — umumiy eguvchi moment; M_b — burovchi moment.

Valning xavfli kesimi M_x , M_y va M_b larning qiymatlarini solishtirish yo'li bilan amalga oshiriladi. M_{um} va M_b larning birgalikdagi eng katta qiymatiga ega bo'lgan kesimlar xavfli kesimlar hisoblanadi.

94-masala



9.6-shakl

9.6-shaklda bir uchi bilan devorga mahkamlangan, ikkinchi uchiga esa BC krivoship o'rnatilgan yaxlit val tasvirlangan. Krivoshipning uchiga kuch qo'yilgan. Mustahkamlikning uchinchi nazariyasi bo'yicha valning mustahkamlikdagi ehtiyot koeffitsiyentini aniqlang. Quyidagi ma'lumotlar ma'lum: $F=5\text{kN}$; $l=50 \cdot 10^{-2}\text{m}$; $a=30 \cdot 10^{-2}\text{m}$; $d=6 \cdot 10^{-2}\text{m}$; $\sigma_T=240\text{MPa}$.

Yechish:

Ko'rinib turibdiki, valning tayanch kesimi xavfli kesim hisoblanadi, unga eng katta $M_e=Fl$ eguvchi va $M_b=Fa$ burovchi momentlar ta'sir ko'rsatadi.

Xavfli kesimdagi birorta A nuqtaning normal va urinma kuchlanishlarini aniqlaymiz:

$$\sigma_A = \frac{M_e}{W_x} = \frac{Fl}{\frac{\pi d^3}{32}} = \frac{32 \cdot 5 \cdot 10^3 \cdot 50 \cdot 10^{-2}}{3,14 \cdot (0,06)^3} = 117,95 \cdot 10^6 \frac{\text{N}}{\text{m}^2} = 117,95 \text{MPa};$$

$$\tau_a = \frac{M_b}{W_p} = \frac{Fa}{0,5W_x} = \frac{16 \cdot 5 \cdot 10^3 \cdot 30 \cdot 10^{-2}}{3,14 \cdot (0,06)^3} = 35,386 \cdot 10^6 \frac{\text{N}}{\text{m}^2} = 35,386 \text{MPa}.$$

Uchinchi mustahkamlik nazariyasi bo'yicha ekvivalent kuchlanishini aniqlaymiz:

$$\sigma_{ud} = \sqrt{\sigma_A^2 + 4 \cdot \tau_a^2} = \sqrt{18920,878} = 137,553 \text{MPa}.$$

Demak, mustahkamlikning ehtiyot koeffitsiyenti

$$K = \frac{\sigma_T}{\sigma_{ud}} = \frac{240}{137,553} = 1,745$$

ga teng.

9.5-§. Murakkab deformatsiya bo'yicha topshiriqlar

15- topshiriq

Murakkab kuchlanish. Qiyshiq egilish.

Berilgan: o'zgarmas kesimga ega bo'lgan po'lat to'sin bosh tekisliklar bo'yicha yuklangan. Ruxsat etilgan normal kuchlanish $[\sigma] = 210\text{MPa}$ ga teng.

Topish kerak:

- 1) eguvchi moment epyuralari qurilsin;
- 2) to'sinning mustahkamligi tekshirilsin;
- 3) xavfli kesimdagi nol chiziqning holati aniqlansin va normal kuchlanishlar epyurasi qurilsin.

Kerakli ma'lumotlar 9.1-jadvalda, hisoblash sxemalari esa 9.8-shaklda berilgan.

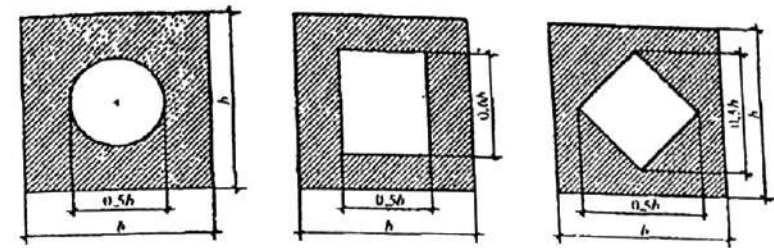
Ko'ndalang kesim o'qituvchi tomonidan beriladi (9.7-shakl).

9.1-jadval

№	a, m	c, m	d, m	F, kN	q, kN/m	M, kN·m	h, sm	b, sm	Profillar nomeri	
									qo'shtavr	shveller
1	1	3	2	30	20	10	20	15	18	14
2	2	2	2	10	20J	30	24	12	16	16
3	1	3	2	20	20	20	30	15	36	14 a
4	2	2	2	20	20	30	24	18	18	1614a
5	1	3	2	10	20	10	28	12	18a	18
6	2	2	2	10	10	30	18	16	16	20
7	1	3	2	20	30	10	24	12	20	1814a

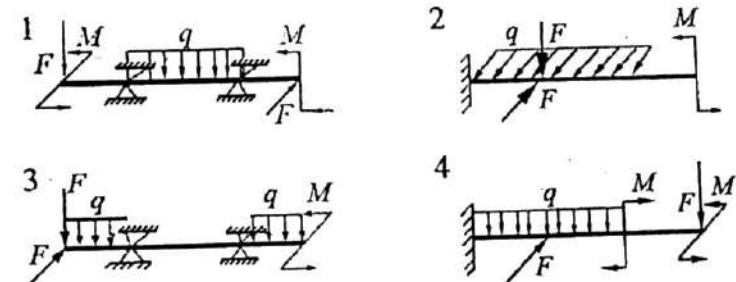
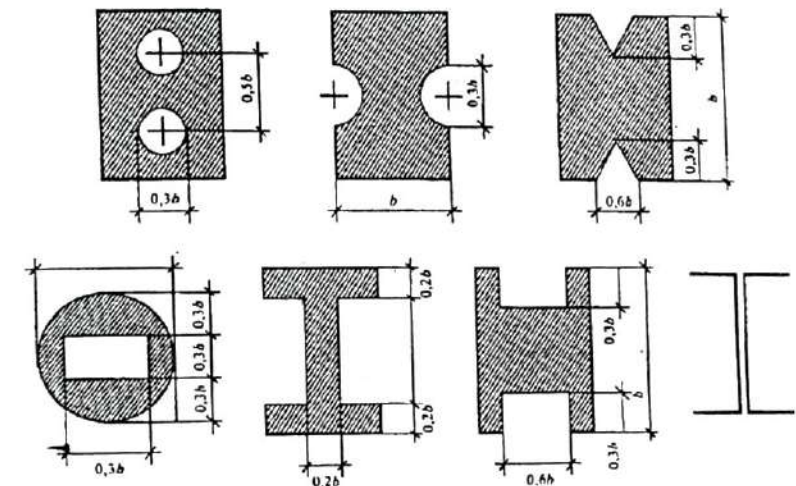
9.1-jadval (davomi)

№	a, m	c, m	d, m	F, kN	q, kN/m	M, kN·m	h, sm	b, sm	Profillar nomeri	
									qo'shtavr	shveller
8	2	2	2	20	10	20	20	12	30	2014 a
9	1	3	2	20	10	20	27	16	27	22
10	2	2	2	10	30	10	21	12	27 a	2214 a
11	1	3	2	10	30	20	20	12	24	24
12	2	2	2	10	30	20	18	12	24 a	2414 a
13	1	3	2	20	10	10	24	16	27	14
14	2	2	2	20	10	30	24	16	24	16
15	1	3	2	10	20	30	30	18	24 a	1614a
16	2	2	2	20	20	10	30	18	30	27
17	1	3	2	10	30	10	30	18	30 a	18
18	2	2	2	20	20	20	28	12	20	16
19	1	3	2	20	20	20	28	15	20 a	20
20	2	2	2	10	30	10	24	18	18	22

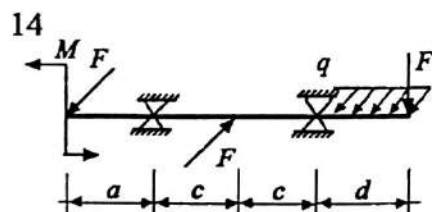
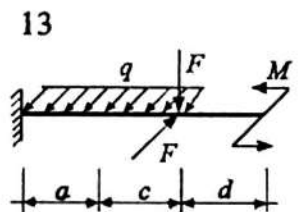
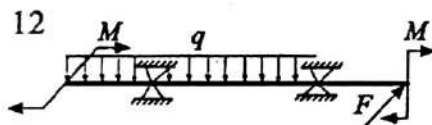
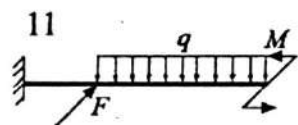
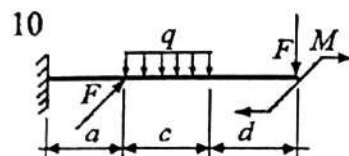
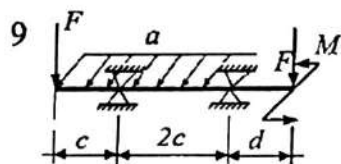
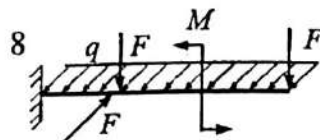
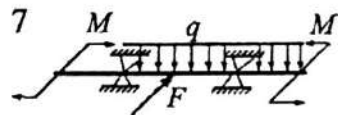
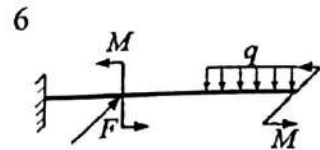
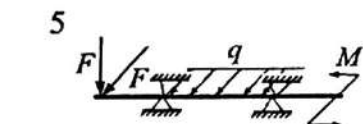


9.7-shakl

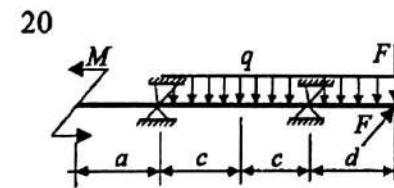
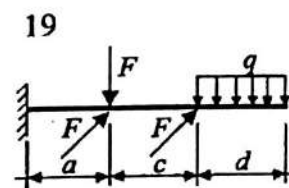
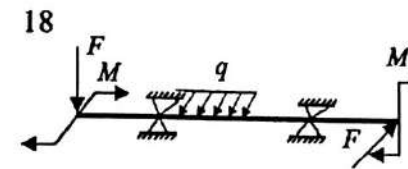
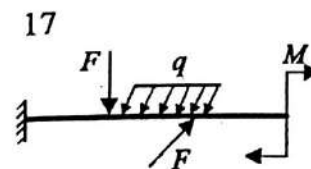
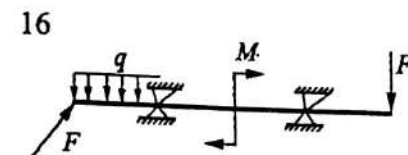
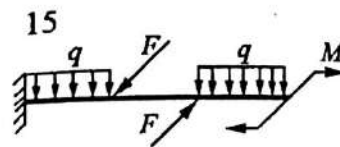
9.7-shakl (davomi)



9.8-shakl



9.8-shakl (davomi)



9.8-shakl (davomi)

16-topshiriq

Markaziy bo'lmagan siqilish.

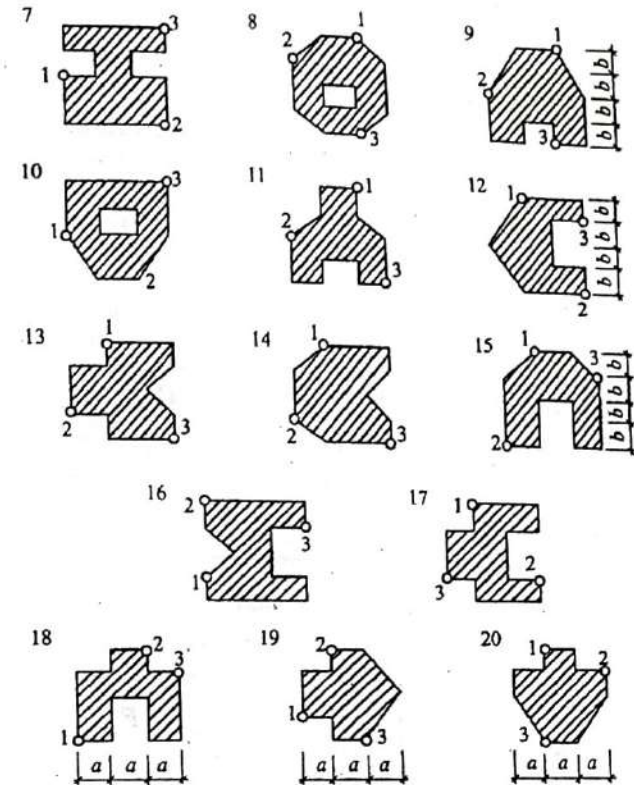
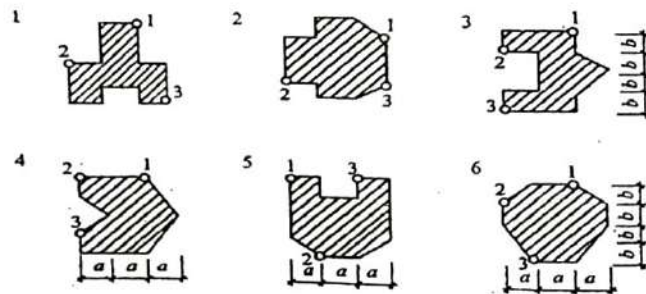
Berilgan: Bikir kolonna o'qiga parallel yo'nalgan bo'lib, kesimning sxemada (9.8-shakl) ko'rsatilgan nuqtasiga qo'yilgan. Material uchun ruxsat etilgan normal kuchlanish cho'zilishda $[\sigma] = 3 \text{ MPa}$; siqilishda esa 30 MPa ga teng.

Topish kerak:

- 1) nol chiziqning vaziyati aniqlansin;
- 2) eng katta siquvchi va cho'zuvchi kuchlanishlarning epyurasi qurilsin; kolonnaning mustahkamligi haqida xulosa berilsin;
- 3) berilgan kesim uchun ruxsat etilgan F_{adm} kuch aniqlansin;
- 4) kesim yadrosi aniqlansin.

Kerakli ma'lumotlar 9.2-jadvalda, hisoblash sxemalari esa 9.9-shaklda berilgan.

№	F, kN	Kesim o'lchamlari		Kuchning qo'yilish nuqtasi
		a	b	
1	12	30	12	1
2	100	24	10	2
3	160	30	12	3
4	100	24	14	1
5	180	36	15	2
6	200	40	18	3
7	320	50	20	1
8	220	28	16	2
9	240	36	18	3
10	260	40	16	1



17-topshiriq

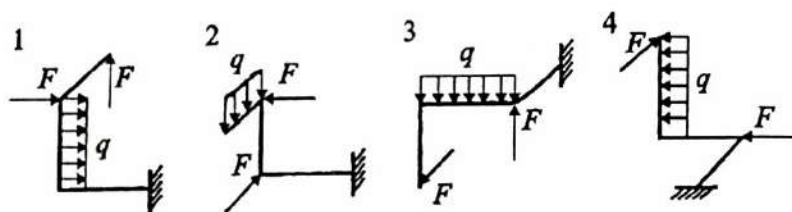
Fazoviy sterjenlarni hisoblash.

Berilgan: Bir xil uzunlikdagi o'zaro perpendikulyar ko'ndalang kesimi aylanadan iborat uchta sterjen o'zaro bikir qilib mahkamlangan. Sterjenlarga gorizontal va vertikal tekisliklar bo'yicha kuchlar ta'sir etmoqda. Sterjenli po'latdan yasalgan bo'lib, ruxsat etilgan normal kuchlanishlar cho'zilishga $[\sigma_2] = 210 \text{ MPa}$; siqilishga $[\sigma] = 130 \text{ MPa}$.

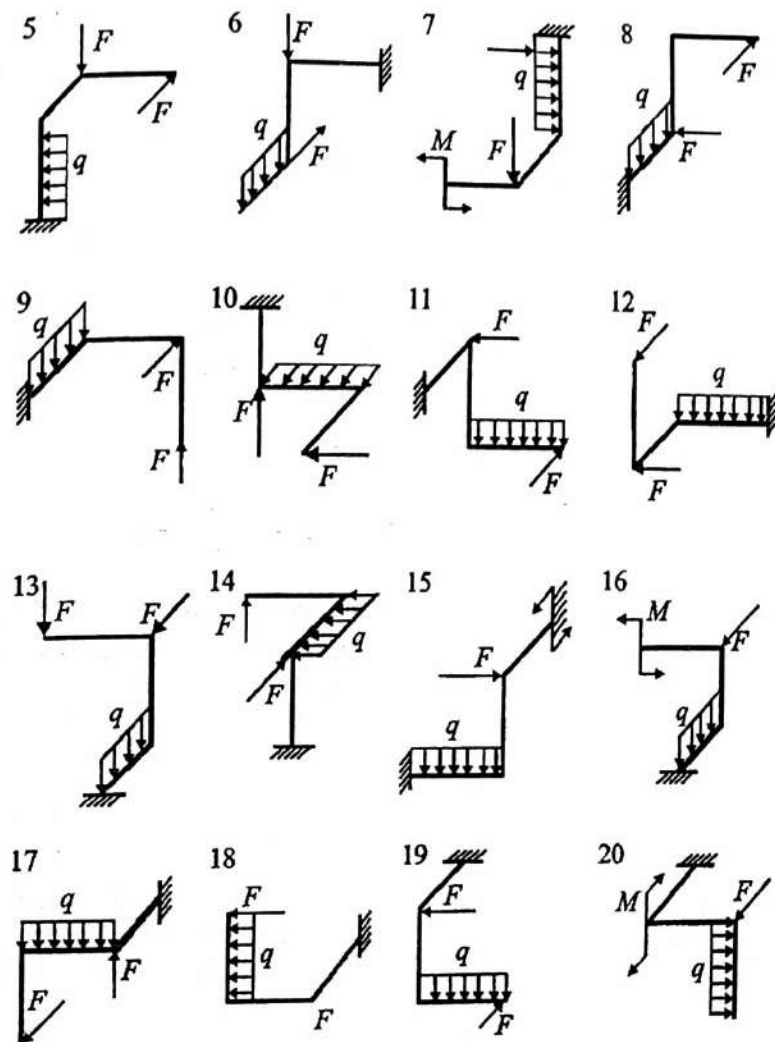
Topish kerak:

- 1) aksonometriyada bo'ylama kuch, kesuvchi kuch va eguvchi moment epyuralari qurilsin;
- 2) xavfli kesim aniqlansin va uning mustahkamligi haqida xulosa berilsin. Kerakli ma'lumotlar 9.3-jadvalda, hisoblash sxemalari 9.10-shaklda berilgan.

№	F , kN	q , kN/m	M , kN/m	l , m	d , sm
1	8	6	4	0,6	5
2	10	4	1	0,4	6
3	6	10	2	0,5	8
4	4	10	6	0,6	6
5	4	6	8	0,4	8
6	8	10	8	0,6	5
7	4	4	4	0,5	8
8	6	4	6	0,4	7
9	6	8	4	0,6	8
10	5	6	4	0,4	10



9.10-shakl



9.10-shakl (davomi)

X BOB

STATIK NOANIQ RAMALARNI KUCH USULIDA HISOBLASH

10.1-§. Asosiy mulohazalar

Statik noaniq ramalar deb, tayanchlarda paydo bo'luvchi reaksiya kuchlari hamda barcha kesimlardagi ichki zo'riqish kuchlarini statikaning muvozanat tenglamalari yordamida aniqlash mumkin bo'lmagan ramalarga aytiladi.

Statik noaniq ramalar «ortiqcha» bog'lanishlar soniga teng bo'lgan statik noaniqlik darajasi bilan xarakterlanadi. Odatda, tekis ramalarning statik noaniqlik darajasi quyidagi formuladan topiladi:

$$S = n - 3, \quad (10.1)$$

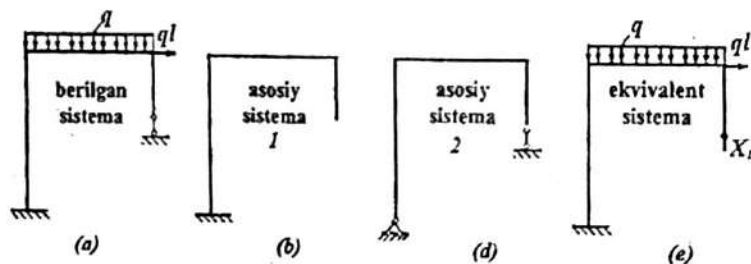
bu yerda, n - ramaning tayanchlardagi noma'lum reaksiya kuchlari soni.

Masalan, 10.1-shakl (a)da tasvirlangan rama bir marta, 10.2-shakl (a)dagi rama esa uch marta statik noaniqdir.

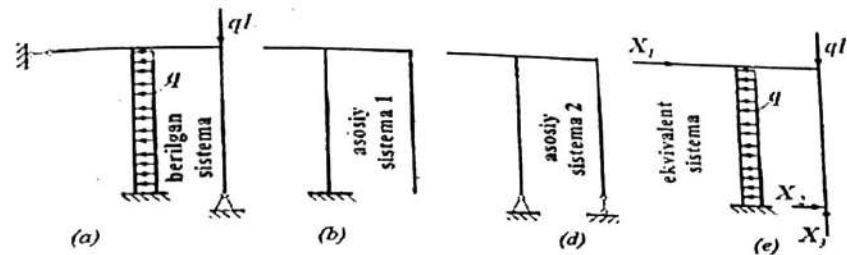
Ramadagi ortiqcha bog'lanishlarni olib tashlash yo'li bilan hosil qilingan statik aniq va geometrik o'zgarmas sistemaga asosiy sistema deyiladi.

10.1-shakl (b), (d) va 10.2-shakl (b), (d)larda berilgan statik noaniq rama uchun turli xil variantlarda asosiy sistemalar tanlanishi tasvirlangan.

Ramaga qo'yilgan tashqi kuchlar hamda noma'lum reaksiya kuchlari bilan yuklatilgan asosiy sistema teng kuchli (ekvivalent) sistema deb ataladi (10.1-shakl, e, 10.2-shakl, e).



10.1-shakl



10.2-shakl

n marta statik noaniq rama uchun kuch usulining kanonik tenglamalar sistemasi quyidagi ko'rinishga ega:

$$\left. \begin{aligned} \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \dots + \delta_{1n} \cdot X_n + \Delta_{1p} &= 0 \\ \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 + \dots + \delta_{2n} \cdot X_n + \Delta_{2p} &= 0 \\ &\dots \\ \delta_{n1} \cdot X_1 + \delta_{n2} \cdot X_2 + \dots + \delta_{nn} \cdot X_n + \Delta_{np} &= 0 \end{aligned} \right\} \quad (10.2)$$

bu yerda, δ_{11} – birlik kuchlardan hosil bo'lgan bosh ko'chishlar;

$\delta_{ik} = \delta_{ki}$ – birlik kuchlardan hosil bo'lgan yordamchi ko'chishlar;

Δ_{ip} – tashqi kuchlardan hosil bo'lgan ko'chishlar.

(10.2) kanonik tenglamalar sistemasi tarkibidagi ko'chishlar Mor integrali va Vereshagin formulasi yordamida aniqlanadi.

Statik noaniq ramalarni kuch usulida hisoblash tartibi quyidagicha:

- ramalarning statik noaniqlik darajasini topish;
- asosiy va teng kuchli (ekvivalent) sistemalarni tanlash;
- kanonik tenglama yoki tenglamalar sistemasini tuzish;
- birlik va tashqi kuchlardan hosil bo'lgan ko'chishlarni aniqlash;
- kanonik sistema yoki tenglamalar sistemasini yechish;
- eguvchi moment, kesuvchi kuch va bo'ylama kuchlarning tugallangan epyuralarini qurish;
- tugallangan epyuralarning qanchalik to'g'ri chizilganligini deformatsion tekshirish. Agar eguvchi momentning tugallangan M epyurasi to'g'ri chizilgan bo'lsa, u holda har bir $X_1, X_2, \dots, X_n$ noma'lum kuchlarning yo'nalishi bo'yicha umumiy ko'chish nolga teng bo'lishi mumkin.

Bu ko'chishlarni Mor integrali va Vereshagin usulida eguvchi momentning tugallangan M epyurasini M_1 va M_2 birlik kuch epyuralariga ko'paytirish orqali topish mumkin:

$$\delta_{um} = \sum \int_e \frac{M \cdot \bar{M}_i}{E_i I_i} dz = 0 \quad \text{yoki} \quad \delta_{um} = \sum \frac{\omega_i \cdot \eta_i}{E_i I_i} = 0.$$

10.2-§. Statik noaniq ramalarga doir masalalar

95-masala

10.3-shaklda tasvirlangan rama uchun tugallangan eguvchi moment, kesuvchi kuch va bo'ylama kuch epyuralari qurilsin. Berilgan: $a = 2l$, $b = 3$, $EI = \text{const}$.

Yechish:

Masalani kuch usulida yechamiz.

1. Berilgan ramaning statik noaniqlik darajasini topamiz:

$$S = n - 3 = 5 - 3 = 2.$$

Demak, berilgan rama ikki marta statik noaniq bo'lib, ikkita «ortiqcha» bog'lanishga ega ekan.

2. Ramadagi ortiqcha bog'lanishlarni yo'qotib, asosiy sistemani tanlaymiz (10.4-shakl, a), keyin uni tashqi kuchlar hamda noma'lum reaksiya kuchlari X_1 va X_2 lar bilan yuklab, ekvivalent (teng kuchli) sistemani tuzamiz (10.4-shakl, b).

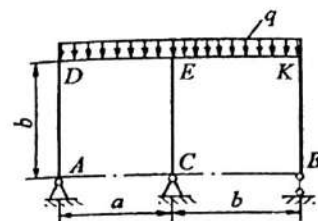
3. Ekvivalent sistema uchun kanonik tenglamalar sistemasi quyidagicha yoziladi:

$$\left. \begin{aligned} \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \Delta_{1p} &= 0 \\ \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 + \Delta_{2p} &= 0 \end{aligned} \right\}$$

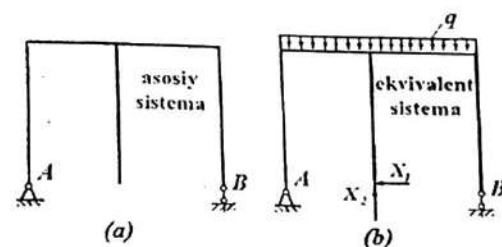
Asosiy sistemani alohida-alohida tashqi kuch (taqsimlangan kuch q , 10.5-shakl, a) hamda noma'lum X_1 va X_2 reaksiya kuchlari o'rniga qo'yilgan $\bar{X}_1 = 1$ va $\bar{X}_2 = 1$ birlik kuchlar bilan yuklab (10.5-shakl, b va d), tayanchlarda hosil bo'luvchi reaksiya kuchlarini topamiz.

10.5-shakl (a) uchun:

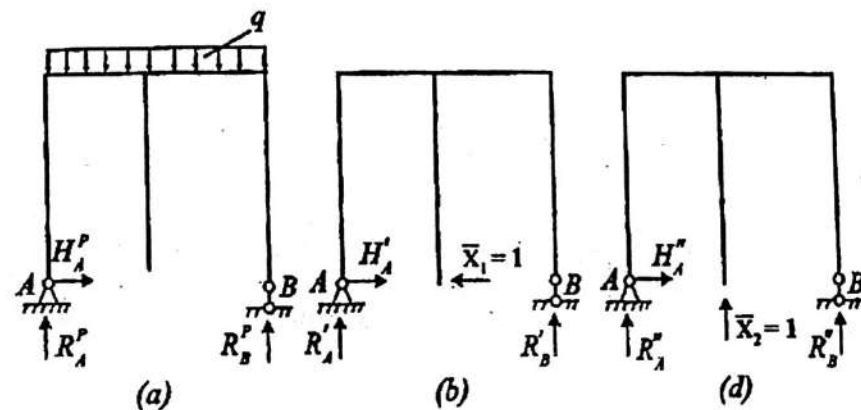
$$\sum M_{Ai} = 0; \quad q \frac{(a+b)^2}{2} - R_B^p(a+b) = 0; \quad R_B^p = \frac{q(a+b)^2}{2(a+b)} = \frac{5}{2}ql;$$



10.3-shakl



10.4-shakl



10.5-shakl

$$\sum M_{Bi} = 0; \quad R_A^p(a+b) - q \frac{(a+b)^2}{2} = 0; \quad R_A^p = \frac{q(a+b)^2}{2(a+b)} = \frac{5}{2}ql;$$

$$\sum X_i = 0; \quad H_A^p = 0;$$

10.5-shakl (b) uchun:

$$\sum M_{Ai} = 0; \quad R_B' = (a+b) = 0; \quad R_B'' = 0;$$

$$\sum M_{B_i} = 0; \quad R'_A = (a+b) = 0; \quad R'_A = 0;$$

$$\sum X_i = 0; \quad H'_A - \bar{X}_1 = 0; \quad H'_A = 1.$$

10.5-shakl (d) uchun:

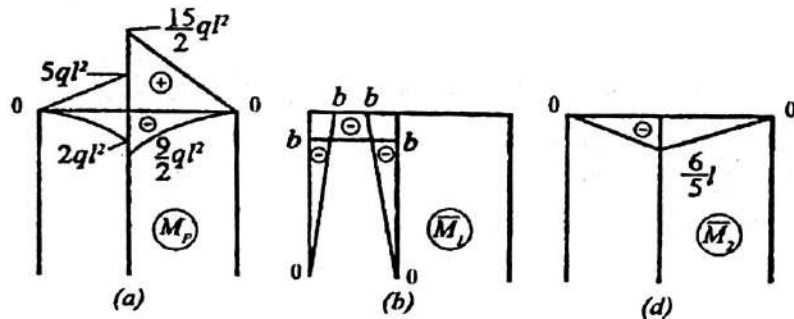
$$\sum M_{A_i} = 0; \quad \bar{X}_2 \cdot a = R''_B(a+b) = 0; \quad R''_B = \frac{\bar{X}_2 \cdot a}{a+b} = -\frac{2}{5};$$

$$\sum M_{B_i} = 0; \quad -R''_A(a+b) - \bar{X}_2 \cdot b = 0; \quad R''_A = \frac{\bar{X}_2 \cdot b}{a+b} = -\frac{3}{5};$$

$$\sum X_i = 0; \quad H''_A = 0;$$

Kesish usulidan foydalanib, tashqi kuchdan M_p , birlik $\bar{X}_1$ va $\bar{X}_2$ kuchlardan esa $\bar{M}_1$ va $\bar{M}_2$ eguvchi moment epyuralarini quramiz (10.6-shakl). Bu epyuralardan foydalanib, Vereshagin usulida kanonik tenglamalar sistemasi tarkibidagi δ_{ii} , δ_{ij} va Δ_{ip} ko'chishlarni aniqlaymiz:

$$\delta_{11} = \delta(\bar{M}_1 \cdot \bar{M}_1) = \frac{1}{EI} \left(2 \cdot \frac{b \cdot b}{2} \cdot \frac{2}{3} b + b \cdot a \cdot b \right) = \frac{36l^3}{EI};$$



10.6-shakl

$$\delta_{22} = \delta(\bar{M}_2 \cdot \bar{M}_2) = \frac{1}{EI} \left(\frac{1}{2} \cdot \frac{6l}{5} \cdot a \cdot \frac{2}{3} \cdot \frac{6l}{5} + \frac{1}{2} \cdot \frac{6l}{5} \cdot b \cdot \frac{2}{3} \cdot \frac{6l}{5} \right) = \frac{12l^3}{5EI};$$

$$\delta_{12} = \delta_{21} = \delta(\bar{M}_1 \cdot \bar{M}_2) = \frac{1}{EI} \left[\frac{1}{2} \left(-\frac{6l}{5} \right) \cdot a \cdot (-b) \right] = \frac{18l^3}{5EI};$$

$$\Delta_{1p} = \Delta(\bar{M}_1 \cdot \bar{M}_p) = \frac{1}{EI} \left[\frac{1}{2} \cdot 5ql^2 \cdot a \cdot (-b) + \frac{1}{3} (-2ql^2) \cdot a \cdot (-b) \right] = -\frac{11ql^4}{EI};$$

$$\Delta_{2p} = \Delta(\bar{M}_2 \cdot \bar{M}_p) = \frac{1}{EI} \left[\frac{1}{2} \cdot 5ql^2 \cdot a \cdot \frac{2}{3} \cdot \left(-\frac{6l}{5} \right) + \frac{1}{3} \cdot (-2ql^2) \cdot a \cdot \frac{3}{4} \cdot \left(-\frac{6l}{5} \right) + \frac{1}{2} \cdot \frac{15}{2} ql^2 \cdot b \cdot \frac{2}{3} \cdot \left(-\frac{6l}{5} \right) + \frac{1}{3} \cdot \left(-\frac{9}{5} ql^2 \right) \cdot b \cdot \frac{3}{4} \cdot \left(-\frac{6l}{5} \right) \right] = -\frac{31ql^4}{4EI};$$

$$\begin{cases} \frac{36l^3}{EI} \cdot X_1 + \frac{18l^3}{5EI} \cdot X_2 - \frac{11ql^4}{EI} = 0 \\ \frac{18l^3}{5EI} \cdot X_1 + \frac{12l^3}{5EI} \cdot X_2 - \frac{31ql^4}{4EI} = 0 \end{cases} \begin{cases} \times \frac{5EI}{l^3} \\ \times \frac{20EI}{l^3} \end{cases}$$

$$\begin{cases} 180X_1 + 18X_2 = 55ql \\ 72X_1 + 48X_2 = 155ql \end{cases} \text{ bundan,}$$

$$X_1 = -\frac{150}{7344} ql = -0,02ql; \quad X_2 = \frac{23940}{7344} ql = 3,25ql.$$

1. M , Q va N larning tugallangan epyuralarini chizish. Tashqi yuk hamda topilgan X_1 va X_2 kuchlarni ramaga qo'yib, ularda hosil bo'lgan tayanch reaksiyalarini topamiz (10.7-shakl):

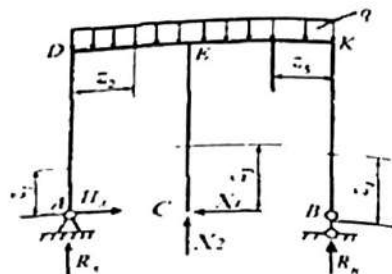
$$\sum M_{A_i} = 0; \quad -q \frac{(a+b)^2}{2} - R_B(a+b) + X_2 \cdot a = 0;$$

$$R_B = \frac{\frac{q(a+b)^2}{2} - X_2 \cdot a}{(a+b)} = 1,196ql;$$

$$\sum M_{B_l} = 0; \quad q \frac{(a+b)^2}{2} - R_A(a+b) - X_2 \cdot b = 0;$$

$$R_A = \frac{q(a+b)^2 - X_2 \cdot b}{(a+b)} = 0,544ql;$$

$$\sum X_l = 0; \quad H_A - X_1 = 0; \\ H_A = X_1 = -0,02ql.$$



10.7-shakl

Endi ramani uchastkalarga ajratib, har bir oraliq uchun N , Q va M larning analitik ifodasini tuzamiz:

AD oraliq: $0 \leq z_1 \leq b$

$$N_{z_1} = -R_A = -0,544ql; \quad Q_{z_1} = -H_A = 0,02ql;$$

$$M_{z_1} = -H_A \cdot z_1; \quad M_{z_1=0} = 0; \quad M_{z_1=b} = -H_A \cdot b = 0,06ql^2.$$

DE oraliq: $0 \leq z_2 \leq a$

$$N_{z_2} = -H_A = 0,02ql; \quad Q_{z_2} = R_A - qz_2;$$

$$Q_{z_2=0} = R_A = 0,544ql; \quad Q_{z_2=a} = R_A - q \cdot 2l = -1,456ql;$$

(agar $Q_{z_2} = 0$ bo'lsa u holda, $z_2 = R_A/q = 0,544l$)

$$M_{z_2} = R_A \cdot z_2 - q \frac{z_2^2}{2} - H_A \cdot b; \quad M_{z_2=0} = -H_A \cdot b = 0,06ql^2;$$

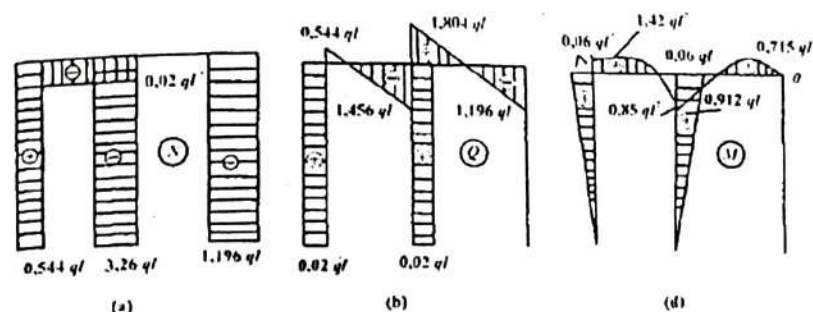
$$M_{z_2=0,544l} = R_A \cdot 0,544l - q \frac{(0,544l)^2}{2} - H_A \cdot b = 1,42ql^2;$$

$$M_{z_2=a} = -0,852ql^2$$

CE oraliq: $0 \leq z_3 \leq b$

$$N_{z_3} = -X_2 = -3,26ql; \quad Q_{z_3} = X_1 = -0,02ql;$$

$$M_{z_3} = X_1 \cdot z_3; \quad M_{z_3=0} = 0; \quad M_{z_3=b} = X_1 \cdot b = 0,06ql^2.$$



10.8-shakl

BK oraliq: $0 \leq z_2 \leq b$

$$N_{z_4} = -R_B = -1,196ql; \quad Q_{z_4} = 0; \quad M_{z_4} = 0;$$

KE oraliq: $0 \leq z_5 \leq b$

$$N_{z_5} = 0; \quad Q_{z_5} = -R_B + qz_5;$$

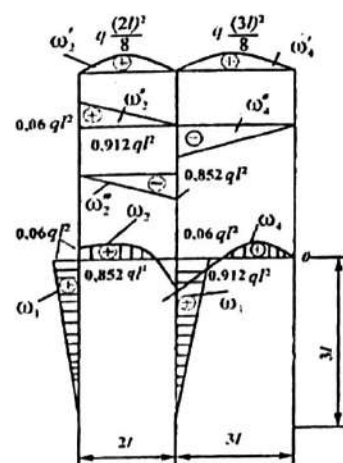
$$Q_{z_5=0} = -R_B;$$

$$Q_{z_5=b} = -R_B + qb = 1,804ql;$$

(agar $Q_{z_5} = 0$ bo'lsa, $z_5 = R_B/q = 1,196l$)

$$M_{z_5} = R_B \cdot z_5 - q \frac{z_5^2}{2};$$

$$M_{z_5=0} = 0; \quad M_{z_5=b} = -0,912ql^2.$$



10.9-shakl

$$M_{z_5=1,196l} = R_B \cdot 1,196l - q \frac{(1,196l)^2}{2} = 0,715ql^2.$$

Hosil qilingan qiymatlar asosida M , Q va N larning tugallangan epyuralarini chizamiz (10.8-shakl).

5. Epyuralarni deformatsion tekshirish.

Buning uchun tugallangan M epyurasini (10.8-shakl, d) $\overline{M}_1$ va $\overline{M}_2$ epyuralarga (10.6-shakl) ko'paytiramiz. Bunda tugallangan M epyurasini oddiy shakllarga ajratish ko'chishlarni aniqlashda qulaylik tug'diradi:

$$\sum \int_e \frac{M_l \overline{M}_l}{EI} = 0 \text{ yoki}$$

$$\begin{aligned}\delta(M \cdot \bar{M}_1) &= \frac{1}{EI} (\omega_1 \eta'_1 + \omega'_2 \eta'_2 + \omega''_2 \eta'_2 + \omega'''_2 \eta'_2 + \omega_3 \eta'_3) = \\ &= \frac{1}{EI} \left[\frac{0,06ql^2 \cdot 3l}{2} \cdot \frac{2}{3} \cdot (-3l) + \frac{\frac{1}{2}ql^2 \cdot 2l}{3} \cdot (-3l) + \frac{0,06ql^2 \cdot 2l}{2} \cdot (-3l) - \right. \\ &\quad \left. - \frac{0,852ql^2 \cdot 2l}{2} \cdot (-3l) + \frac{0,06ql^2 \cdot 3l}{2} \cdot \frac{2}{3} \cdot (-3l) \right] = \\ &= \frac{ql^4}{EI} (-0,18 - 2 - 0,18 + 2,556 - 0,18) = \frac{0,016ql^4}{EI} \approx 0.\end{aligned}$$

Bu qiymat $\Delta_{1p} = \frac{11ql^4}{EI}$ ga nisbatan 3,7% ni tashkil etganligi uchun uni nolga teng deb qabul qilish mumkin:

$$\begin{aligned}\delta(M \cdot \bar{M}_2) &= \frac{1}{EI} (\omega'_2 \eta'_2 + \omega''_2 \eta'_2 + \omega'''_2 \eta'_2 + \omega'_4 \eta'_4 + \omega''_4 \eta'_4) = \\ &= \frac{1}{EI} \left[\frac{2}{3} \cdot \frac{ql^2}{2} \cdot 2l \cdot \frac{1}{2} \cdot \left(-\frac{6}{5}l\right) + \frac{0,06ql^2 \cdot 2l}{2} \cdot \frac{1}{3} \cdot \left(-\frac{6}{5}l\right) - \frac{0,852ql^2 \cdot 2l}{2} \cdot \right. \\ &\quad \left. \cdot \frac{2}{3} \cdot \left(-\frac{6}{5}l\right) + \frac{2}{3} \cdot \frac{9ql^2 \cdot 3l}{8} \cdot \frac{1}{2} \cdot \left(-\frac{6}{5}l\right) - \frac{0,912ql^2 \cdot 3l}{2} \cdot \frac{2}{3} \cdot \left(-\frac{6}{5}l\right) \right] = \\ &= \frac{0,089ql^4}{EI} \approx 0.\end{aligned}$$

96-masala

10.10-shaklda tasvirlangan rama uchun tugallangan eguvchi moment, kesuvchi va bo'ylama kuch epyuralari qurilsin.

Berilgan: $a = l$; $b = 2l$; $EI = \text{const}$; $P = ql$.

Yechish:

Masalani kuch usulida yechamiz.

1. Berilgan ramaning statik noaniqlik darajasini aniqlaymiz:

$$S = n - 3 = 5 - 3 = 2.$$

Demak, ramani hisoblash uchun statikaning muvozanat tenglamalaridan tashqari, yana ikkita qo'shimcha deformatsiya tenglamasini tuzish zarur ekan.

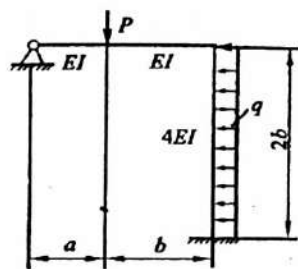
2. Ramani qo'zg'almas sharnirli tayanchdan ozod qilib, asosiy sistemani tanlaymiz (10.11-shakl, a), keyin uni tashqi kuchlar hamda

noma'lum reaksiya kuchlari X_1 va X_2 lar bilan yuklab, ekvivalent sistemani tuzamiz (10.11-shakl, b).

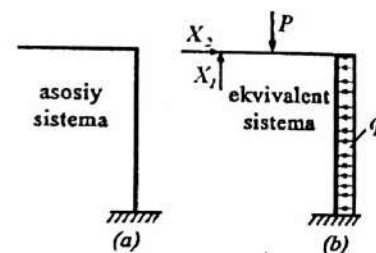
3. Ekvivalent sistema uchun kanonik tenglamalar sistemasi quyidagicha yoziladi:

$$\begin{cases} \delta_{11} \cdot X_1 + \delta_{12} \cdot X_2 + \Delta_{1p} = 0 \\ \delta_{21} \cdot X_1 + \delta_{22} \cdot X_2 + \Delta_{2p} = 0 \end{cases}$$

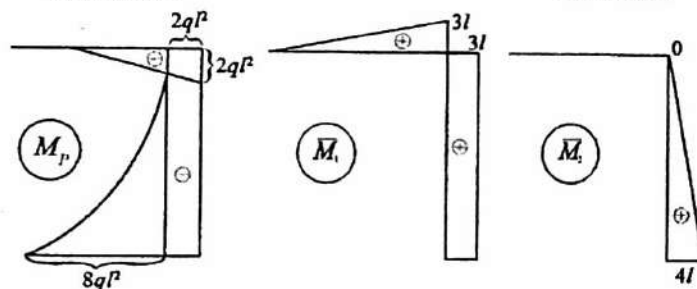
Asosiy sistemani alohida-alohida tashqi kuch hamda noma'lum X_1 va X_2 reaksiya kuchlari o'rniga qo'yilgan $\bar{X}_1 = 1$ va $\bar{X}_2 = 1$ birlik kuchlar bilan yuklaymiz va ular uchun M_p , $\bar{M}_1$ va $\bar{M}_2$ eguvchi moment epyuralarini quramiz (10.12-shakl). Bunda asosiy sistema bitta tayanchga ega



10.10-shakl



10.11-shakl



10.12-shakl

bo'lgani uchun uning reaksiya kuchlari aniqlanmaydi.

$$\delta_{11} = (\bar{M}_1, \bar{M}_1) = \frac{1}{EI} \cdot \frac{3l \cdot 3l}{2} \cdot \frac{2}{3} \cdot 3l + \frac{1}{4EI} \cdot 3l \cdot 2b \cdot 3l = \frac{18l^3}{EI};$$

$$\delta_{22} = (\overline{M}_2, \overline{M}_2) = \frac{1}{4EI} \cdot \frac{1}{2} \cdot 4l \cdot 2b \cdot \frac{2}{3} \cdot 4l = \frac{16l^3}{3EI};$$

$$\delta_{12} = \delta_{21} = (\overline{M}_1, \overline{M}_2) = \frac{1}{EI} \cdot \frac{1}{2} \cdot 4l \cdot 2b \cdot 3l = \frac{6l^3}{EI};$$

$$\Delta_{1P}(\overline{M}_1, M_P) =$$

$$= \frac{1}{EI} \cdot \frac{1}{2} \cdot (-2ql^2) \cdot \frac{7l}{3} + \frac{1}{4EI} \left[(-2ql^2) \cdot 2b \cdot 3l + \frac{1}{3} (-8ql^2) \cdot 2b \cdot 3l \right]$$

$$= -\frac{56}{3} \cdot \frac{ql^4}{EI}$$

$$\Delta_{2P}(\overline{M}_2, M_P) = \frac{1}{4EI} \left[(-2ql^2) \cdot 2b \cdot \frac{1}{2} \cdot 4l + (-8ql^2) \cdot 2b \cdot \frac{1}{3} \cdot \frac{3}{4} \cdot 4l \right] =$$

$$= -\frac{12ql^4}{EI};$$

$$\begin{cases} \frac{18l^3}{EI} \cdot X_1 + \frac{6l^3}{EI} \cdot X_2 - \frac{56}{3} \cdot \frac{ql^4}{EI} = 0 \\ \frac{6l^3}{EI} \cdot X_1 + \frac{16l^3}{3EI} \cdot X_2 - \frac{12ql^4}{EI} = 0 \end{cases} \begin{matrix} \times \frac{3EI}{l^3} \\ \times \frac{3EI}{l^3} \end{matrix}$$

$$\begin{cases} 54X_1 + 18X_2 = 56ql \\ 18X_1 + 16X_2 = 36ql \end{cases}, \text{ bunda, } X_1 = 0,46 ql; X_2 = 1,73 ql.$$

4. Eguvchi moment, kesuvchi kuch va bo'ylama kuchlarning tugallangan epyuralarini chizish. Kesish usulidan foydalanib (10.13-shakl), quyidagi ifodalarni yozamiz:

1-uchastka: $0 \leq z_1 \leq a$

$$N_{z_1} = -X_2 = -1,73 ql; Q_{z_1} = -X_1 = 0,46 ql;$$

$$M_{z_1} = X_1 \cdot z_1; M_{z_1=0} = 0; M_{z_1=a} = X_1 a = 0,46 ql^2$$

2-uchastka: $0 \leq z_2 \leq b$

$$N_{z_2} = -X_2 = -1,73 ql;$$

$$Q_{z_2} = X_1 - P = -0,54 ql.$$

$$M_{z_2} = X_1(a + z_2) - Pz_2;$$

$$M_{z_2=0} = 0,46 ql^2; M_{z_2=b} = X_1 a = 0,62 ql^2.$$

3-uchastka: $0 \leq z_3 \leq b$

$$N_{z_3} = X_1 - P = -0,54 ql; Q_{z_3} = X_2 - qz_3;$$

$$Q_{z_3=0} = 1,73 ql; Q_{z_3=2b} = -2,27 ql;$$

$$M_{z_3} = X_1(a + b) - Pb + X_2 z_3 - \frac{qz_3^2}{2};$$

$$M_{z_3=0} = -0,62 ql^2; M_{z_3=1,73a} = 0,88 ql^2;$$

$$Q_{z_3=2b} = -1,69 ql^2$$

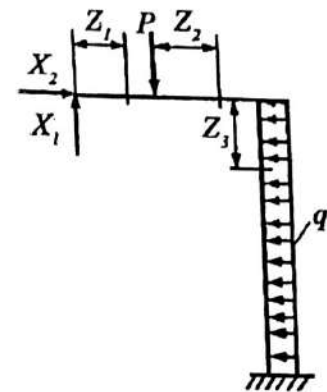
(agar $Q_{z_3} = 0$ bo'lsa, u holda, $z_3 = X_2/q = 1,73a$).

Hosil qilingan qiymatlar asosida M , Q va N larning tugallangan epyuralarini chizamiz (10.14-shakl).

2. Epyuralarni deformatsion tekshirish. Buning uchun tugallangan M epyurasini (10.14-shakl, a). $\overline{M}_1$ va $\overline{M}_2$ epyuralarga (10.12-shakl) ko'paytiramiz. Tugallangan M epyurasini oddiy shakllarga ajratish 10.15-shaklda ko'rsatilgan.

$$\sum_e \int \frac{M_i \overline{M}_i}{EI} dz = 0 \text{ yoki}$$

$$\delta(M, \overline{M}_1) = \frac{1}{EI} (\omega_1 \eta'_1 + \omega'_2 \eta'_2 + \omega''_3 \eta''_3) + \frac{1}{4EI} (\omega'_3 \eta'_3 + \omega''_3 \eta''_3 + \omega'''_3 \eta'''_3) =$$



10.13-shakl

10.3-§. Statik noaniq ramalarni kuch usulida hisoblash bo'yicha topshiriqlar

18-topshiriq

Statik noaniq ramalar uchun ichki kuch faktorlar epyuralarini qurish.

Berilgan: keltirilgan shakllarda (10.16-shakl) tashqi tomondan yuklangan statik noaniq rama berilgan.

Topish kerak:

- 1) ramaning statik noaniqlik darajasi aniqlansin;
- 2) asosiy va ekvivalent sistemalar tuzilsin;
- 3) noaniq tenglamalar tuzilib ortiqcha reaksiya kuchlari topilsin;
- 4) rama uchun kesuvchi kuch, bo'ylama kuch va eguvchi moment epyuralari qurilsin;

5) topilgan reaksiya kuchlari to'g'riligi tekshirilsin;

6) mustahkamlik shartiga binoan, ramaning ko'ndalang kesimi tanlansin (kesim ko'rinishi - o'qituvchi ko'rsatmasi bo'yicha).

Kerakli ma'lumotlar 6.1-jadvalda, hisoblash sxemalari esa 10.16-shaklda berilgan.

$$= \frac{1}{EI} \left[\frac{0,46ql^2}{2} \cdot \frac{2}{3}l - \frac{0,62ql^2 \cdot 2l}{2} \cdot \left(1 + \frac{2}{3}\right)l + \frac{0,46ql^2 \cdot 2l}{2} \cdot \left(1 + \frac{1}{3}\right)l \right] +$$

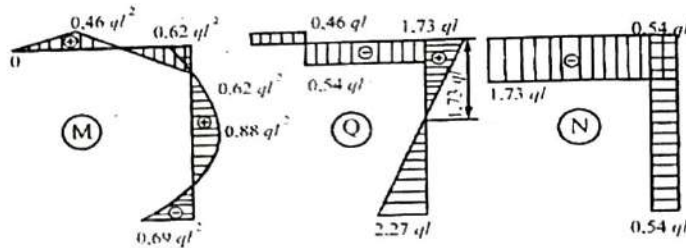
$$+ \frac{1}{4EI} \left[-\frac{1,69ql^2 \cdot 4l}{2} \cdot 3l - \frac{0,62ql^2 \cdot 4l}{2} \cdot 3l + \frac{2}{3} \cdot 2ql^2 \cdot 4l \cdot 3l \right] =$$

$$= \frac{0,26ql^4}{EI}.$$

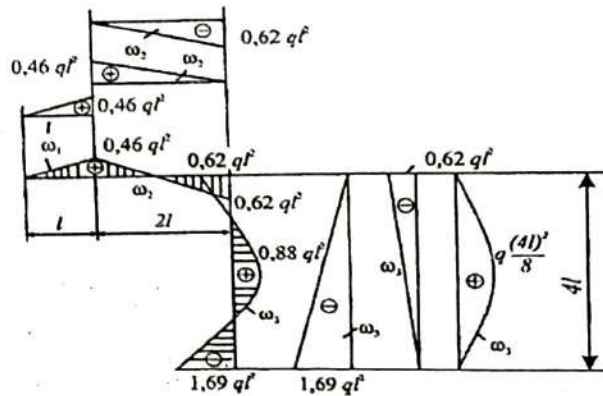
$$\delta(M \cdot \bar{M}_2) = \frac{1}{4EI} (\omega_3 \eta_3^2 + \omega_3'' \eta_3^2 + \omega_3''' \eta_3^2)$$

$$= \frac{1}{4EI} \left[-\frac{1,69ql^2 \cdot 4l}{2} \cdot \frac{2}{3} \cdot 4l - \frac{0,62ql^2 \cdot 4l}{2} \cdot 4l + \frac{2}{3} \cdot 2ql^2 \cdot 4l \cdot \frac{1}{2} \cdot 4l \right] = 0.$$

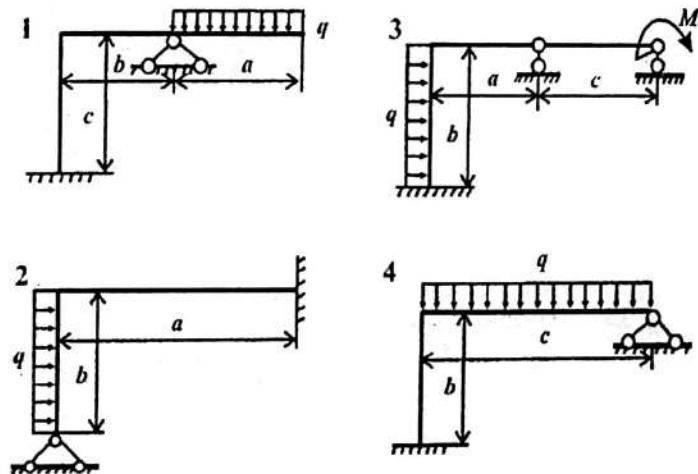
Bu qiymat, $\Delta_{1P} = -\frac{56}{3} \cdot \frac{ql^4}{EI}$ ga nisbatan 1,4% ni tashkil etganligi uchun uni nolga teng deb qabul qilish mumkin.



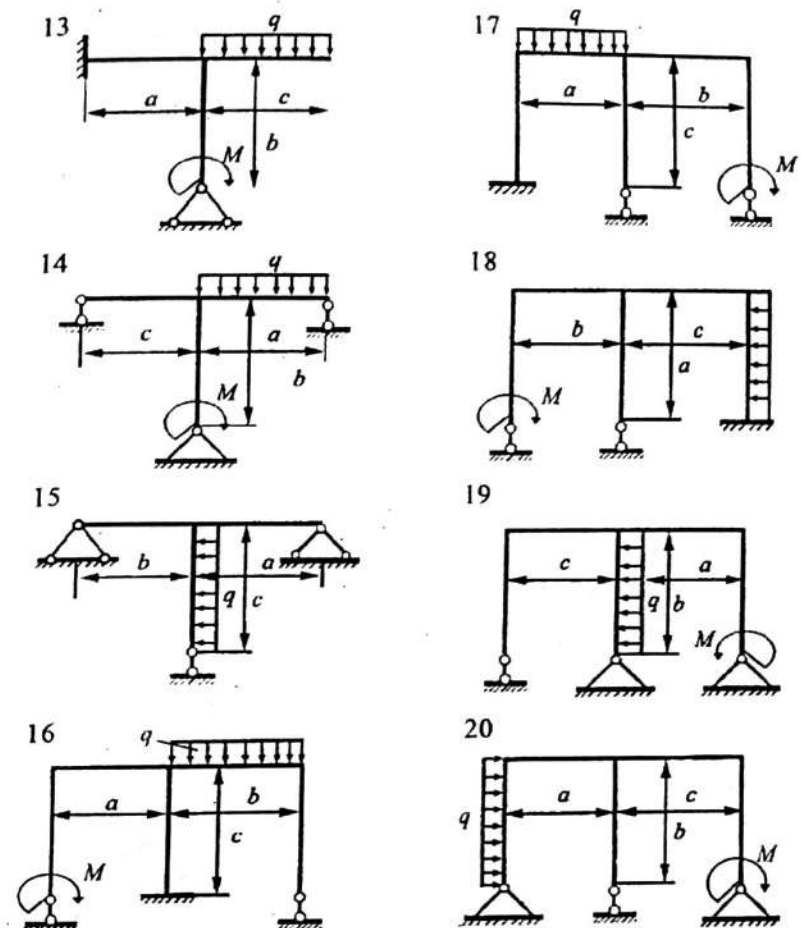
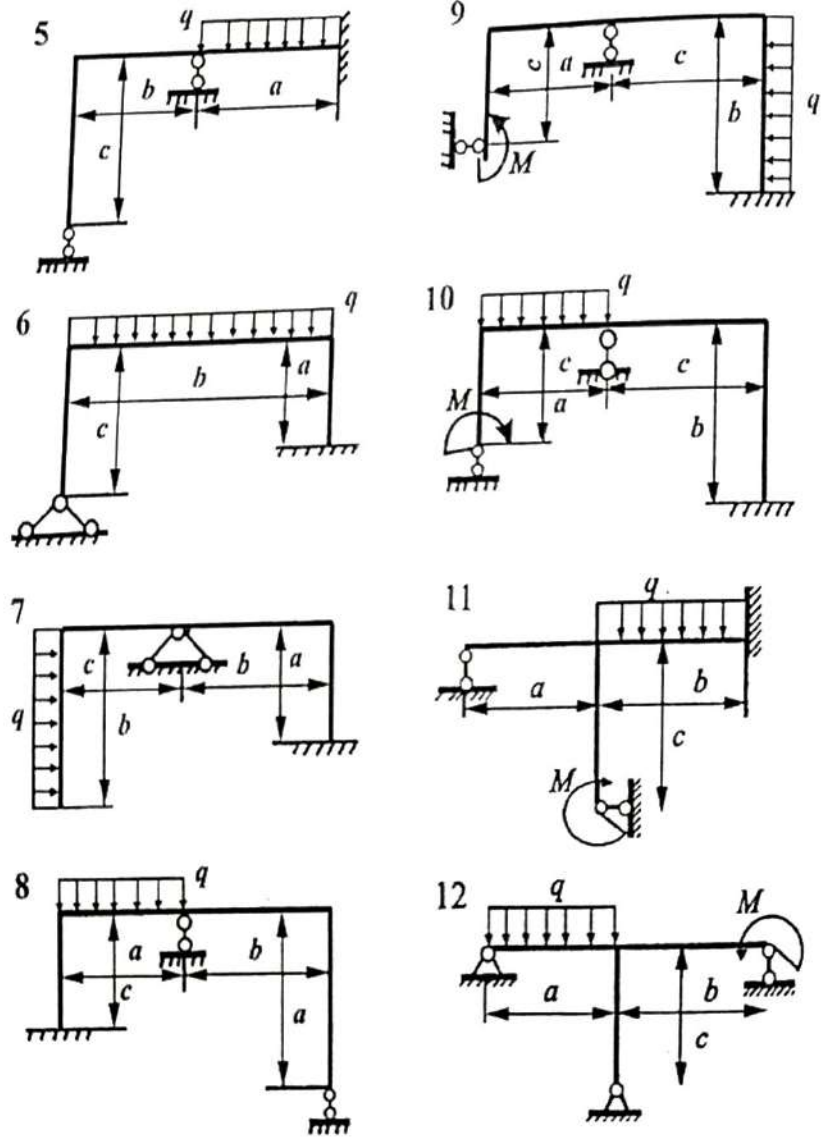
10.14-shakl



10.15-shakl



10.16-shakl



XI BOB

STATIK NOANIQ TO'SINLARNI HISOBLASH

11.1-§. Asosiy mulohazalar

Statik noaniq to'sinlarni kuch usulida hisoblash tartibi statik noaniq ramalarni hisoblash kabi amalga oshiriladi:

- masalaning statik noaniqlik darajasi aniqlanadi;
- masalaning statik noaniqlik darajasiga teng deformatsiya tenglamalari tuziladi;
- deformatsiya tenglamalaridagi birlik kuch, hamda tashqi kuchlardan hosil bo'lgan ko'chishlar Vereshagin usulida topiladi;
- deformatsiya tenglamalarini birgalikda yechib, undagi ortiqcha reaksiya kuchlari aniqlanadi va masala statik aniq masalaga aylantiriladi.

11.2-§. Statik noaniq to'sinlarni kuch usulida hisoblash

97-masala

Berilgan bikirligi ozgarmas bo'lgan statik noaniq balkaning ko'ndalang kesim yuzalari to'g'ri to'rtburchak shaklida aniqlansin. K nuqtaning ko'chishi topilsin: $[\sigma] = 10 \text{ MPa}$.

Yechish:

Berilgan balka bir marta statik noaniqdir, chunki B nuqtada uchta va A nuqtada esa bitta reaksiya kuchlari mavjud. Bunda, $S=1$.

A nuqtadagi bog'lanishni tashlab yuborib, uni X_1 , noma'lum reaksiya kuchi bilan almashtiramiz (11.1-shakl). A nuqtaning ko'chishi nolga teng bo'lgani uchun quyidagi deformatsiya tenglamasini yozamiz:

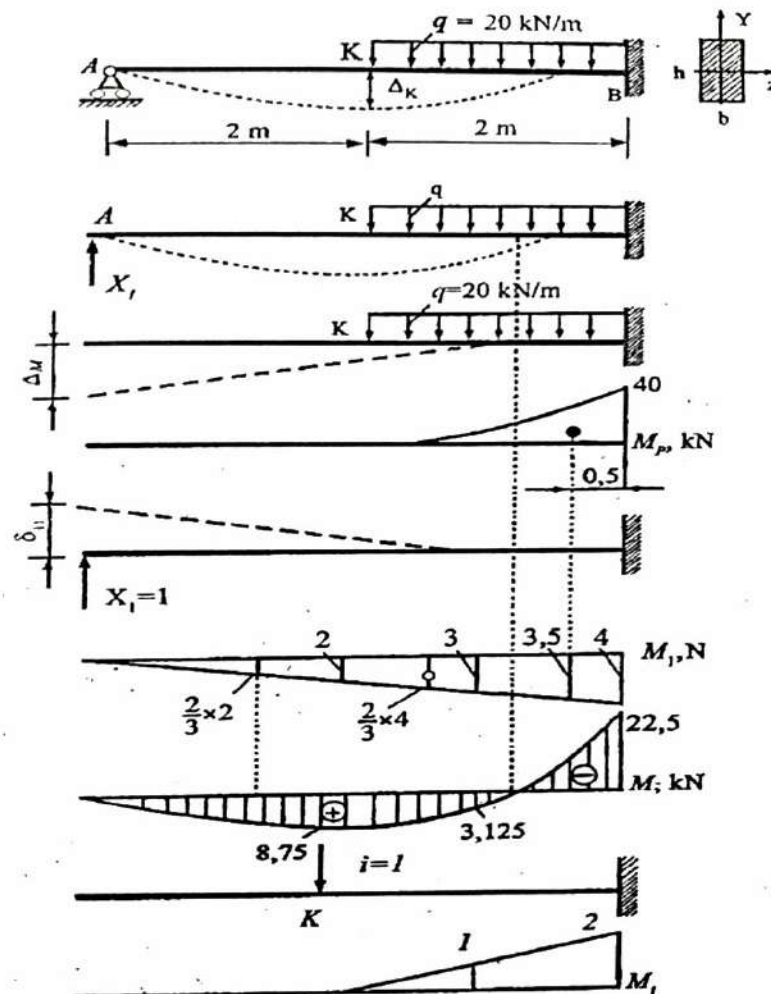
$$\Delta_A = \Delta_{AX_1} + \Delta_{AP} = \delta_{11} \cdot X_1 + \Delta_{1P} = 0 \quad (a)$$

bu yerda, δ_{11} – A nuqtaning birlik $X_1=1$ ga teng bo'lgan kuchdan hosil bo'lgan ko'chishi;

Δ_{1P} – A nuqtaning tashqi kuchlardan hosil bo'lgan ko'chishidir.

δ_{11} , Δ_{1P} larni Vereshagin usulidan foydalangan holda aniqlaymiz:

$$\delta_{11} = \frac{\omega_1^0 \eta_1^0}{EI} = \frac{1}{EI} \cdot \frac{4 \cdot 4}{2} \cdot \frac{2}{3} \cdot 4 = \frac{64}{3EI};$$



11.1-shakl

$$\Delta_{1P} = \frac{\omega_1 \eta_1}{EI} = -\frac{1}{EI} \cdot \frac{1}{3} \cdot 40 \cdot 2 \cdot 3.5 = -\frac{280}{3EI}.$$

Topilgan qiymatlarni deformatsiya tenglamasiga (a) qo'yib, X_1 reaksiya kuchini aniqlaymiz:

$$\frac{64}{3EI} \cdot X_1 - \frac{280}{3EI} = 0,$$

bu yerdan, $X_1 = 4,375 \text{ kN}$.

Masala statik aniq masalaga aylangandan so'ng, kesuvchi kuch va eguvchi moment epyuralari oddiy balkalarning yechimi singari amalga oshiriladi.

Topilgan X_1 ning qiymati to'g'riligini tekshirish uchun $\Delta_A = 0$ dan foydalanamiz (M epyura yuzasini M_1 epyura ordinatalariga ko'paytiramiz):

$$\Delta_A = \sum \frac{\omega \cdot \eta_1}{EI} = \frac{1}{EI} \cdot \left[\frac{1}{2} \cdot 8,75 \cdot 2 \cdot \frac{2}{3} \cdot 2 + \frac{2}{6} (8,75 \cdot 2 + 4 \cdot 3,125 \cdot 3 - 22,5 \cdot 4) \right] = 0$$

A nuqta ko'chishining nolga tengligi masalani to'g'ri yechilganligini ko'rsatadi. Egilishdagi mustahkamlik sharti bo'yicha balka kesim yuzasi o'lchamlarini aniqlaymiz:

$$\sigma_{\max} = \frac{M_{\max}}{W_x} \leq [\sigma],$$

bu yerda, $W_x = \frac{bh^2}{6}$; $M_{\max} = 22,5 \text{ kN} \cdot \text{m}$. Unda $h = 3b$ ekanligini e'tiborga olib, quyidagini hosil qilamiz:

$$\frac{M_{\max}}{\frac{3}{2}b^3} \leq [\sigma]. \text{ U holda, } b \geq \sqrt[3]{\frac{2M_{\max}}{3R}} = \sqrt[3]{\frac{2 \cdot 22500}{3 \cdot 10 \cdot 10^6}} = 11,5 \cdot 10^{-2} \text{ m} = 11,5 \text{ sm},$$

$$\text{bunda, } h = 3b = 34,5 \text{ sm}; I_z = \frac{bh^3}{12} = \frac{11,5 \cdot 34,5^3}{12} = 39350 \text{ sm}^3$$

K nuqta ko'chishini aniqlash uchun bu nuqtaga $i=1$ ga teng kuch qo'yamiz va undan birlik eguvchi moment epyurasi quramiz (M_1). M va (M_1) epyuralarini o'zaro ko'paytirib (Vereshagin usuli), K nuqta ko'chishini aniqlaymiz:

$$V_k = \sum \frac{\omega \eta_1}{EI} = \frac{2}{6EI} (0 - 4 \cdot 3,125 \cdot 1 + 22,5 \cdot 2) = \frac{65}{6EI} = \frac{65000}{6 \cdot 1 \cdot 10^{10} \cdot 39350 \cdot 10^{-8}} = 2,75 \cdot 10^{-3} \text{ m} = 2,75 \text{ mm}.$$

11.3-§. Statik noaniq to'sinlarni uch moment tenglamasi yordamida yechish

➤ Uch moment tenglamasi quyidagi ko'rinishga ega:

$$M_{n-1} \cdot l_n + 2M_n(l_n + l_{n+1}) + M_{n+1} \cdot l_{n+1} = -6 \cdot \left(\frac{\omega_n \cdot a_n}{l_n} + \frac{\omega_{n+1} \cdot b_{n+1}}{l_{n+1}} \right), \quad (11.1)$$

bu yerda, $M_n, M_{n+1}, \dots$ – tayanchlardagi eguvchi momentlar; $l_n, l_{n+1}, \dots$ – tayanchlar orasidagi masofalar; a_n, b_{n+1} – tashqi kuchlardan hosil bo'lgan eguvchi moment epyuralari (ω_n, ω_{n+1}) yuzalarining og'irlik markazidan chap va o'ng tayanchlarga bo'lgan masofa; ω_n, ω_{n+1} – berilgan tashqi kuchlardan n va $n+1$ prolyotlarda hosil bo'lgan eguvchi moment epyurasining yuzasi.

98-masala

Bir nechta tayanchlardan iborat bo'lgan statik noaniq balka uchun (11.2-shakl) eguvchi moment va kesuvchi kuch epyuralari qurilsin ($q = 2P/e$).

Vechish:

Balkaning qistirib mahkamlangan chap qismini fikran davom ettirib, qo'shimcha tayanch (0) hosil qilamiz (11.2-shakl, b). Bu shakldan $S = m - 2 = 5 - 2 = 3$, demak, masala uch marta statik noaniqdir. 0,1; 1,2; 2,3 va 3,4 tayanchlarda yotgan balkalarni alohida olib, tashqi kuchlardan hosil bo'lgan eguvchi moment epyuralarini quramiz (11.2-shakl, d). Ulardan hosil bo'lgan epyura yuzalari va masofalar teng bo'ladi.

$$\omega_1 = 0; \omega_2 = \frac{Pl^2}{8}; a_2 = b_2 = \frac{l}{2}; \omega_3 = 0; \omega_4 = \frac{ql^2}{12} = \frac{Pl^2}{6}; a_4 = b_4 = \frac{l}{2}.$$

Balkaning uch oraliq tayanchi uchun uch moment tenglamasini tuzamiz ($n = 1, 2, 3$):

$$\left. \begin{aligned} 2M_1 + M_2 &= -\frac{3}{8}Pl & (n=1) \\ M_1 + 4M_2 + M_3 &= -\frac{3}{8}Pl & (n=2) \\ M_2 + 4M_3 + M_4 &= -\frac{3}{8}Pl & (n=3) \end{aligned} \right\} \quad (a)$$

Shakldan ko'rinib turibdiki, oxirgi tayanchdan M_4 moment quyidagiga teng bo'ladi:

$$M_4 = -\frac{Pl}{4} \quad (b)$$

(a) tenglamalarni (b) ni e'tiborga olgan holda yechib, quyidagilarni olamiz:

$$M_1 = -0,168Pl; \quad M_2 = -0,038Pl; \quad M_3 = -0,053Pl.$$

Tayanchlarda hosil bo'lgan eguvchi momentlar qiymati grafik ravishda 11.2-shakl (b) da ko'rsatilgan. Bu momentlarning manfiy ishoralari ularning haqiqiy yo'nalishi shakldan teskari tomonda ekanligini ko'rsatadi.

Balkaning tayanch reaksiya kuchlarini aniqlash uchun har bir oraliq masofani alohida olib (oddiy balka ko'rinishida), unga tashqi hamda topilgan moment qiymatlari qo'yiladi. Shundan so'ng statika tenglamalari yordamida oddiy balka reaksiya kuchlari topiladi:

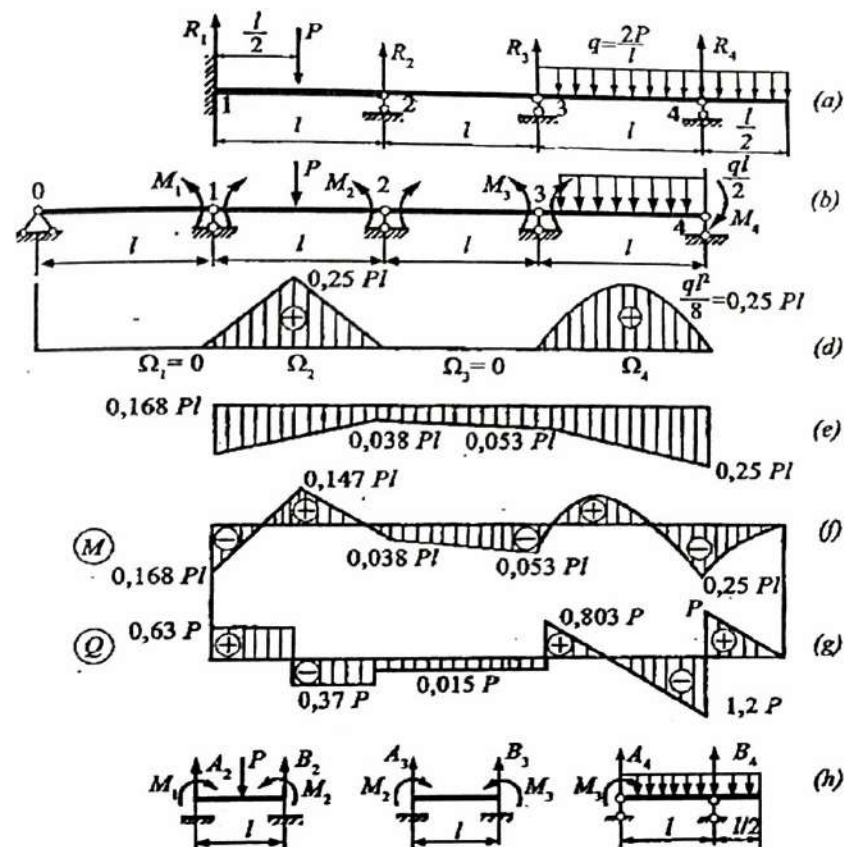
$$A_2 = 0,63P; \quad B_2 = 0,37P; \quad A_3 = \frac{M_3 - M_2}{l} = 0,015P; \quad B_3 = -\frac{M_3 - M_2}{l} = 0,015P$$

$$A_4 = \frac{3}{8}ql - \frac{M_3}{l} = 0,803P; \quad B_4 = \frac{9ql}{8} + \frac{M_3}{l} = 2,20P.$$

Unda tayanchlardagi haqiqiy reaksiya kuchlari quyidagilarga teng bo'ladi:

$$R_1 = A_2 = 0,63P; \quad R_2 = B_2 + A_3 = 0,36P; \quad R_3 = B_3 + A_4 = 0,82P; \\ R_4 = B_4 = 2,20P.$$

Reaksiya kuchlari topilgandan so'ng, har bir oddiy balka uchun Q va M epyuralari quriladi va ular umumlashtiriladi (11.2-shakl, f, g)



11.2-shakl

11.4-§. Statik noaniq to'sinlarni hisoblash bo'yicha topshiriqlar

19-topshiriq

Statik noaniq to'sinlarni hisoblash.

Berilgan: bir necha tayanchlardan iborat bo'lgan uzluksiz to'singa tashqi kuchlar ta'sir etyapti. To'sin po'latdan yasalgan bo'lib, ruxsat etilgan normal kuchlanish $[\sigma]=160 \text{ MPa}$ ga teng.

Topish kerak:

1) noma'lum tayanch reaksiya kuchlari uch moment tenglamasi yordamida topilsin;

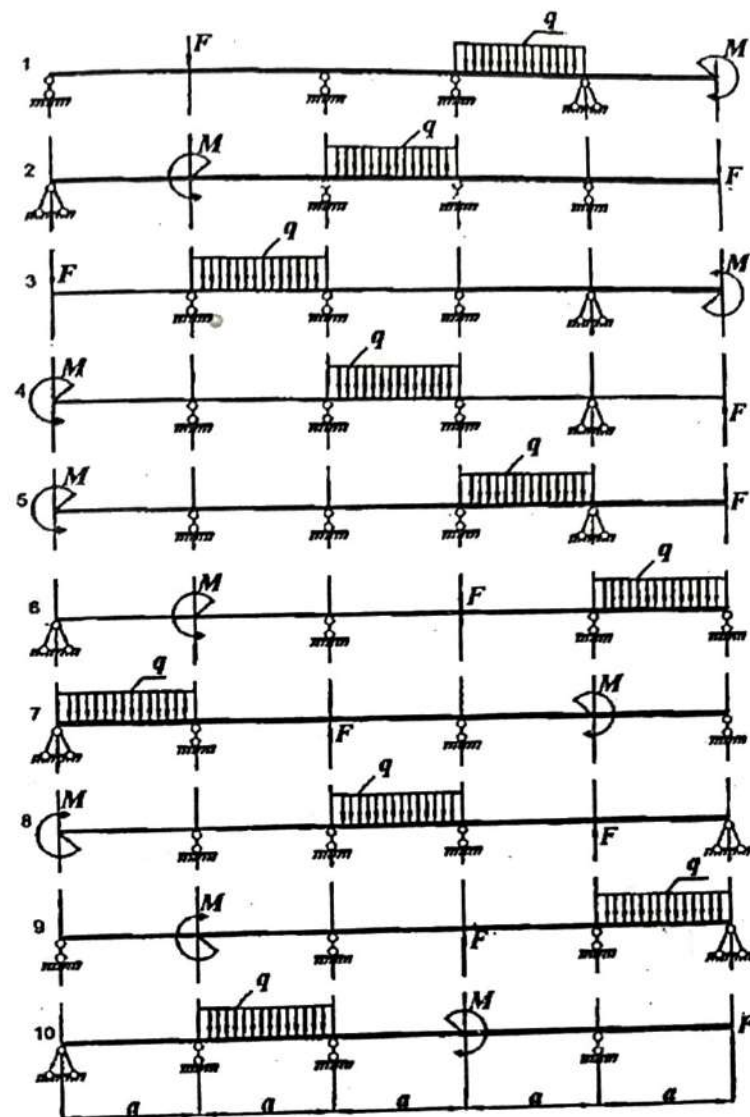
2) eguvchi moment va kesuvchi kuch epyuralari qurilsin;

3) mustahkamlik shartiga asosan, to'sinning qo'shtavr ko'rinishdagi ko'ndalang kesim yuzasi tanlansin.

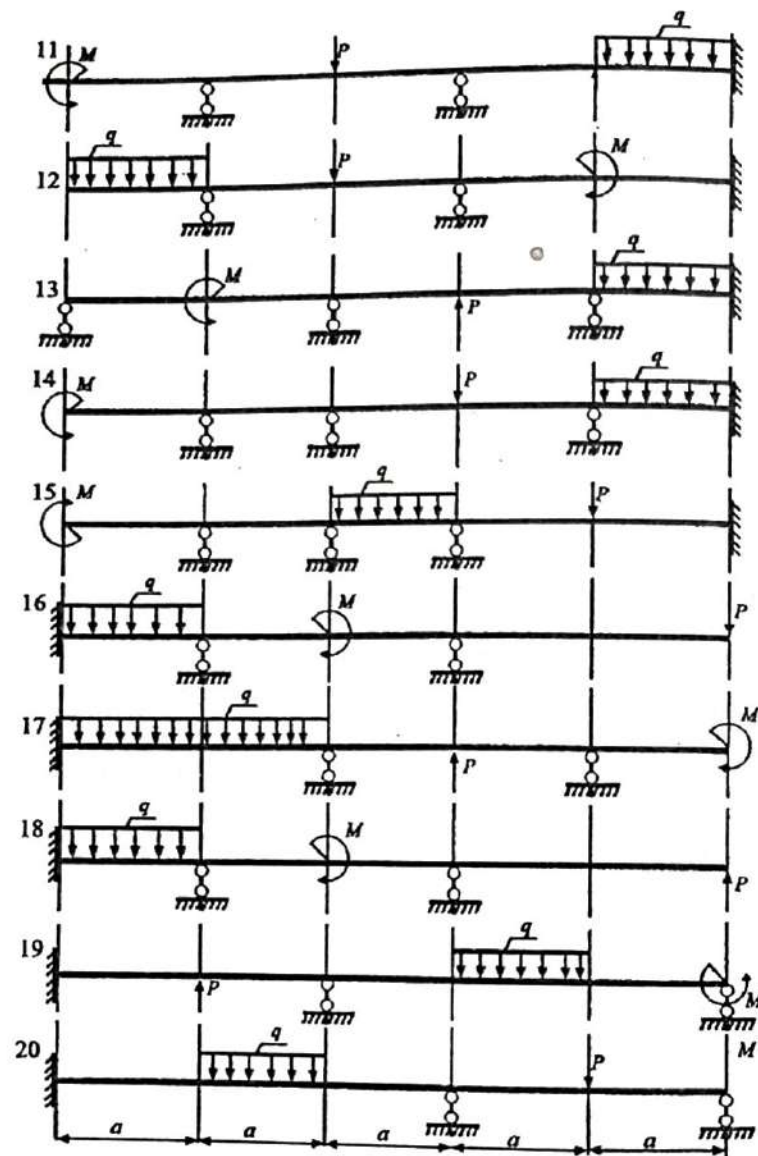
Kerakli ma'lumotlar 11.1-jadvalda, hisoblash sxemalari esa 11.3-shaklda berilgan.

11.1-jadval

№	O'chamlar	Yuklanish		
	a, m	$q, \text{kN/m}$	F, kN	$M, \text{kN} \cdot \text{m}$
1	2	20	30	18
2	1,2	28	28	20
3	1,4	26	32	24
4	1,6	24	40	30
5	1,8	30	42	32
6	1	32	36	24
7	12	34	48	20
8	1,4	28	42	26
9	1,6	20	36	32
10	1,8	30	28	34



11.3-shakl



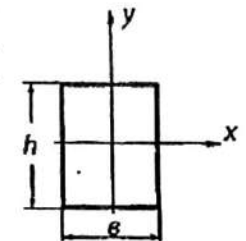
XII BOB

BO'YLAMA EGILISH
(SIQILGAN STERJENLARNING USTUVORLIGI)

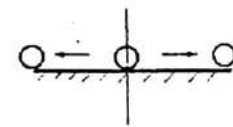
12.1-§. Asosiy mulohazalar

- Uzun ingichka sterjen deb, ko'ndalang o'lchamlari uzunligidan bir necha (8–10) marta kichik bo'lgan sterjenga aytiladi. Bunday sterjenni markaziy kuch bilan siqqanimizda u ham siqilishga, ham egilishga ishlaydi.
- Uzun ingichka sterjen siqilganida, ko'ndalang kesim yuzasining inersiya momenti qaysi o'qqa nisbatan kichik bo'lsa, o'sha o'qqa nisbatan egiladi. Shuning uchun hisob formulalariga ko'ndalang kesim yuzasining minimal inersiya moment $J_{\min}$ kiritiladi:

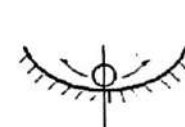
$$J_x > J_y; J_x = \frac{bh^3}{12}; J_y = \frac{hb^3}{12}; h > l.$$



- Ustuvor muvozanat haqida tushuncha:



befarq holat



ustuvor holat



noustuvor holat

- Kritik kuch deb, sterjenning ham to'g'ri chiziqli, ham egri chiziqli muvozanat holatlari ustuvor bo'lgan vaqtga to'g'ri kelgan siquvchi kuchga aytiladi. Bunda sterjenning egri chiziqli holati ustuvor bo'ladi.
- Siquvchi kuch kritik qiymatga yetib bormaganda sterjen faqat sof siqilishga ishlaydi. Kritik qiymatdan ortganidan keyin esa ham siqilish, ham egilishga ishlaydi.
- Kritik kuchni topish Eyler formulasi quyidagiga teng:

$$F_{kr} = \frac{\pi^2 EJ_{\min}}{(\mu l)^2}, \quad (12.1)$$

bu yerda, E – elastiklik moduli;
 $J_{\min}$ – kesimning minimal inersiya momenti;
 l – sterjen uzunligi;
 μ – sterjenning mahkamlanish turini hisobga olish koeffitsiyenti.

➤ Kritik kuchlanish quyidagi qiymatga teng;

$$\sigma_{kr} = \frac{\pi^2 E}{\lambda^2}, \quad (12.2)$$

bu yerda, λ – sterjen egiluvchanligi.

➤ Euler formulasidan hamma vaqt ham foydalanib bo'lmaydi. Undan foydalanish uchun quyidagi shart bajarilishi zarur:

$$\sigma_{kr} = \frac{\pi^2 E}{\lambda^2} \leq \sigma_n \quad (12.3)$$

yoki

$$\lambda \geq \sqrt{\frac{E}{\sigma_n}}, \quad (12.4)$$

bu yerda, σ_n – sterjen materialining proporsionallik chegarasi.

Shunga ko'ra, Euler formulasini St.3 navli po'latdan yasalgan sterjenlar uchun egiluvchanlik 100 dan katta bo'lgandagina tatbiq etish mumkin.

$\lambda \leq 100$ bo'lganda, kritik kuch Yasinskiy formulasi yordamida aniqlanadi:

po'lat uchun, $\sigma_{kr} = a - b \cdot \lambda$,

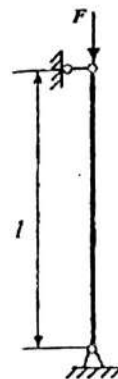
cho'yan uchun, $\sigma_{kr} = a - b \cdot \lambda + c\lambda^2$, bu yerda, a, b, c – materiallarning xossasiga bog'liq bo'lgan o'zgarmaslar bo'lib, ular tajribalardan aniqlanadi.

12.2-§. Bo'ylama egilishga doir masalalar

99-masala

Uzunligi $l=1,85$ m bo'lgan sterjenga ta'sir etuvchi siquvchi kuchning ruxsat etilgan qiymati aniqlansin (12.1-shakl). Sterjen St.3 po'latdan yasalgan bo'lib, ko'ndalang kesim profili 12-raqamli qo'shtavrdan iborat.

Hisoblashda $E = 2,1 \cdot 10^5$ MPa; $n_{adm}^s = 3$ deb olinsin.



Yechish:

Dastlab, Euler formulasini qo'llash haqidagi masalani hal qilish zarur. Buning uchun sterjenning egiluvchanligini hisoblaymiz:

$$\lambda = \frac{\mu l}{i_{\min}} = \frac{1 \cdot 1,85}{1,38 \cdot 10^{-2}} = 134,06,$$

bu yerda, inersiya radiusi $i_{\min}$ ning qiymati GOST 8239-72 bo'yicha sortament jadvalidan olindi.

12.1-shakl Demak, $\lambda > \lambda_{cheg} = 100$ bo'lgani uchun kritik kuch

Euler formulasi bo'yicha aniqlanadi:

$$F_{kr} = \frac{\pi^2 E I_{\min}}{(\mu l)^2} = \frac{(3,14)^2 2,1 \cdot 10^5 \cdot 10^6 \cdot 27,9 \cdot 10^{-8}}{(1 \cdot 1,85)^2} = 168,958 \text{ kN},$$

bu yerda, $I_{\min} = 27,9 \text{ cm}^4$ bo'lib, sortament jadvalidan olindi.

U holda siquvchi kuchning ruxsat etilgan qiymati quyidagicha hisoblanadi:

$$F_{adm}^s = \frac{F_{kr}}{n_{adm}^s} = \frac{168,958}{3} = 56,319 \text{ kN}.$$

100-masala

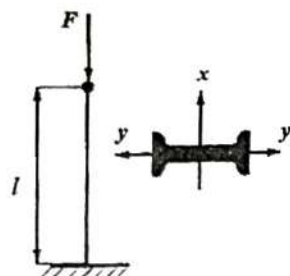
Uzunligi $l = 2,5$ m bo'lgan, St.3 navli po'latdan yasalgan qo'shtavr kesimli ustun $F=110$ kN siquvchi kuch bilan yuklangan (12.2-shakl). Ustunning talab etilgan profil nomeri aniqlansin. Hisoblashlarda $E = 2 \cdot 10^5$ MPa; $n_{adm}^s = 2,8$ deb olinsin.

Yechish:

Hisoblashni Euler formulasi bo'yicha olib boramiz. Ustunning ustuvorlik shartini

$$F \leq \frac{F_{sch}}{n_{adm}^s} \quad (a)$$

ko'rinishda yozib olamiz.



Formulani nazarda tutib, yuqoridagi shartdan ko'ndalang kesimning talab etilgan minimal inersiya momentini aniqlaymiz:

$$I_{\min} \geq \frac{F \cdot n_{adm}^2 (\mu l)^2}{\pi^2 E} = \frac{110 \cdot 10^3 \cdot 2,8 \cdot (2 \cdot 2,5)^2}{3,14^2 \cdot 2 \cdot 10^{11}} = 390,48 \cdot 10^{-8} \text{ m}^4 = 390,48 \text{ sm}^4.$$

Bu qiymatga qarab, sortament jadvalidan 30^a nomerli qo'shtavr tanlanadi, bu qo'shtavr uchun

12.2-shakl
 $I_y = I_{\min} = 436 \text{ sm}^4, i_{\min} = 2,95 \text{ sm}.$

Endi ustunning qabul qilingan profili uchun Eyer formulasini tatbiq qilish mumkin yoki mumkin emasligini tekshirib ko'ramiz. Buning uchun ustunning egiluvchanligini aniqlab, uni $\lambda_{cheg} = 100$ bilan solishtirish kifoya:

$$\lambda = \frac{\mu l}{i_{\min}} = \frac{2 \cdot 2,5}{2,95 \cdot 10^{-2}} = 169,49 > 100.$$

Demak, Eyer formulasini qo'llash mumkin ekan.

101-masala

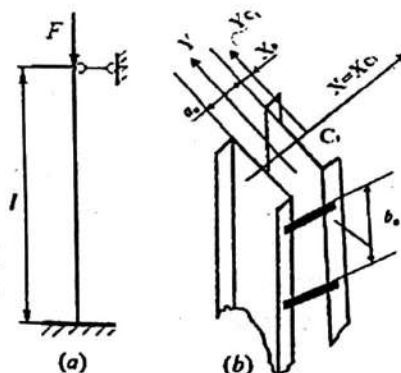
Pastki uchi qistirib tiralgan, yuqori uchi esa qo'zg'aluvchan sharnirli tayanchga mahkamlangan ustunga $F=120 \text{ kN}$ yuk qo'yilgan (12.3- shakl, a). Ustunning uzunligi $l = 5,5 \text{ m}$ bo'lib, ko'ndalang kesimi yonma-yon o'rnatilgan ikkita shvellardan iborat (12.3-shakl, b).

Quyidagilarni aniqlash talab etiladi:

a) ustunning zaruriy profil nomeri tanlansin;

b) shvellarlardan ratsional foydalanish uchun ularning orasidagi a_0 masofa qanchaga teng bo'lishi kerak?

d) belbog' polosalarining oraliqlari aniqlansin. Hisoblashda ustuvorlikning zaruriy ehtiyot koeffitsiyenti $n_{adm}^s = 4$ qilib olinsin.



12.3-shakl

Yechish:

Butun ko'ndalang kesimning zaruriy minimal inersiya momentini quyidagi shartdan aniqlaymiz (oldingi masalaga qarang):

$$I_{\min} \geq \frac{F n_{adm}^2 (\mu l)^2}{\pi^2 E} = \frac{120 \cdot 10^3 \cdot 4 \cdot (0,7 \cdot 5,5)^2}{\pi^2 \cdot 2 \cdot 10^{11}} = 360,81 \cdot 10^{-8} \text{ m}^4 = 360,81 \text{ sm}^4.$$

Bitta shveller uchun esa

$$I_{\min}^C = \frac{I_{\min}}{2} = 180,405 \text{ sm}^4 \quad \text{bo'lgani uchun } \text{ГОСТ } 8239-72 \text{ bo'yicha profili } 22^a \text{ raqamli shvellar tanlanadi. Bu shvellar uchun:}$$

$$I_{\min}^C = 187 \text{ sm}^4; A^C = 28,8 \text{ sm}^2$$

Ikkita shvellar uchun esa: $I_{\min} = 374 \text{ sm}^4; A = 57,6 \text{ sm}^2.$

Tanlab olingan profil uchun Eyer formulasini qo'llash mumkinligini tekshiramiz:

$$\lambda = \frac{\mu l}{\sqrt{\frac{I_{\min}}{A}}} = \frac{0,7 \cdot 5,5 \cdot 100}{\sqrt{\frac{374}{57,6}}} = 151,1 > \lambda_{cheg} = 100.$$

Bundan chiqdi, Eyer formulasini ishlatish mumkin ekan.

Endi shvellarlar oralig'ini aniqlaymiz.

Agar $I_y \geq I_x$ shart bajarilsa, shvellarlardan ratsional foydalanish mumkin. Bu shartdan quyidagi tenglama kelib chiqadi:

$$I_y = 2 \left[I_y^0 + A^0 \left(\frac{a_0}{2} + X_0 \right)^2 \right] \geq 2 I_x^0.$$

Bunda, $I_x^0 = 2330 \text{ sm}^4; I_y^0 = 187 \text{ sm}^4; A^C = 28,8 \text{ sm}^2;$

$X_0 = 2,46 \text{ sm}$ bo'lib, sortament jadvalidan olinadi. Bu qiymatlarni e'tiborga olib, tegishli soddalashtirishlardan keyin a_0 ga nisbatan kvadrat tenglamaga ega bo'lamiz:

$a_0^2 + 9,84 a_0 - 2 \cdot 73,4 = 0;$ bundan $a_0 = 12,33$ ekanligi kelib chiqadi.

Ustun balandligi bo'yicha oraliqni ta'minlash uchun shvellarlar albatta 12.3-shakl (b) da tasvirlangandek, polosalar bilan bog'lab qo'yilishi lozim.

1-usul. Quyidagi mulohazaga tayanib, belbog' polosalar orasidagi b_0 masofani aniqlaymiz. Har bir shveller mustaqil holda $0,5 \cdot F$ kuchni qabul qilib, ustuvorlik shartini bajarishi kerak:

$$\frac{F}{2} = \frac{\pi^2 E I_{min}^c}{n_{adm}^2 b_0^2}, \text{ bunda, } b_0 = \pi \sqrt{\frac{2 E I_{min}^c}{n_{adm}^2 F}} = 3,14 \sqrt{\frac{2 \cdot 2 \cdot 10^{11} \cdot 187 \cdot 10^{-8}}{4 \cdot 120 \cdot 10^3}} = 3,92 \text{ m.}$$

2-usul. Quyidagi mulohazalarga tayanib, belbog' polosalar orasidagi b_0 masofani aniqlaymiz. Har bir shvellerning b_0 uzunlikdagi egiluvchanligi butun ustunning egiluvchanligidan kichik bo'lish shart:

$$\lambda_c = \frac{b_0}{i_{min}^c} \leq \lambda_{ustun}; \text{ bundan,}$$

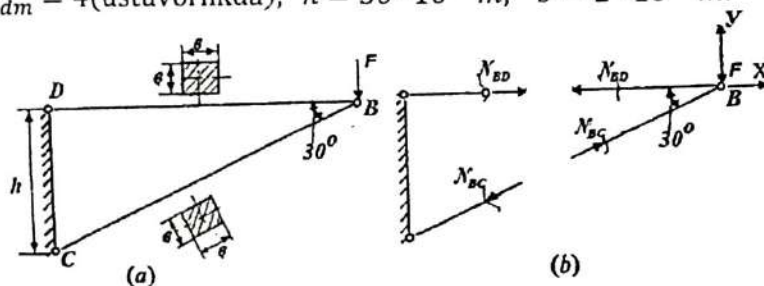
$$b_0 = \lambda_{ustun} \cdot i_{min}^c = \frac{\mu l}{\sqrt{\frac{2 I_{min}^c}{2 A^c}}} \cdot \sqrt{\frac{I_{min}^c}{A^c}} = 0,7 \cdot 5,5 = 3,85 \text{ m}$$

102-masala

Xrom-molibden po'latidan yasalgan kronshteyn B tuguniga qo'yilgan kuchning ruxsat etilgan qiymati aniqlansin (12.4-shakl, a). Hisoblashlar uchun quyidagilar ma'lum:

$$\sigma_{pr} = 540 \text{ MPa}; \sigma_{0,r} = 600 \text{ MPa}; E = 2 \cdot 10^5 \text{ MPa} \quad n_{adm} = 2,5 \text{ (mustahkamlikda);}$$

$$n_{adm}^s = 4 \text{ (ustuvorlikda); } h = 30 \cdot 10^{-2} \text{ m}; b = 2 \cdot 10^{-2} \text{ m.}$$



12.4-shakl

Yechish:

B tugunni qirqib, uning muvozanatini tekshiramiz (12.4-shakl, b):

$$\sum X_i = N_{BC} \cos 30^\circ - N_{BD} = 0; \quad \sum Y_i = N_{BC} \cos 60^\circ - F = 0.$$

Bulardan sterjendagi zo'riqlishlarni topamiz:

$$N_{BC} = 2F \quad (BC \text{ sterjen siqiladi});$$

$$N_{BD} = 1,73 \quad (BD \text{ sterjen cho'ziladi}).$$

Oddiy cho'zilishdagi mustahkamlik shartini yozamiz:

$$\sigma_{BD} = \frac{1,73F}{b^2} \leq \sigma_{adm}.$$

Bunda, ruxsat etilgan kuchlanish: $\sigma_{adm} = \frac{\sigma_{0,r}}{n_{adm}} = 240 \text{ MPa}$ ga teng.

Demak, cho'zuvchi kuchning ruxsat etilgan qiymati:

$$F_{adm} = \frac{240 \cdot (2 \cdot 10^{-2})^2 \cdot 10^3}{1,73} = 55,49 \text{ kN.}$$

BC sterjenni ustuvorlikka tekshiramiz. Eyler formulasini qo'llash mumkinligi haqidagi masalani hal qilamiz. 12.4-shakldan egiluvchanlikning chegaraviy qiymatini aniqlaymiz:

$$\lambda_{cheg} = \pi \sqrt{\frac{E}{\sigma_{pr}}} = 3,14 \cdot \sqrt{\frac{2 \cdot 10^5}{540}} = 62.$$

Sterjenning egiluvchanligi:

$$\lambda = \frac{\mu l_{BC}}{i_{min}} = \frac{1 \cdot 2 \cdot h}{\sqrt{\frac{b^4}{12 \cdot b^2}}} = 103,92 > \lambda_{cheg}. \text{ Shunday qilib, Eyler formulasi bo'yicha:}$$

$$(N_{BC})_{cr} = \frac{\pi^2 E I_{min}}{(\mu l_{BC})^2} = \frac{(3,14)^2 \cdot 2 \cdot 10^5 \cdot 10^6 \cdot (2 \cdot 10^{-2})^4}{(1 \cdot 2 \cdot 30 \cdot 10^{-2})^2 \cdot 12} = 76,76 \cdot 10^3 \text{ N} = 76,76 \text{ kN.}$$

Ustuvorlik shartidan sterjen uchun ruxsat etilgan kuchni aniqlaymiz:

$$(N_{BC})_{adm} = \frac{(N_{BC})_{cr}}{n_{adm}^s} = 19,19; \quad F_{adm} = 0,5(N_{BC})_{adm} = 9,595 \text{ kN.}$$

Demak, F kuchning ruxsat etilgan qiymati 9,595 kN ga teng ekan.

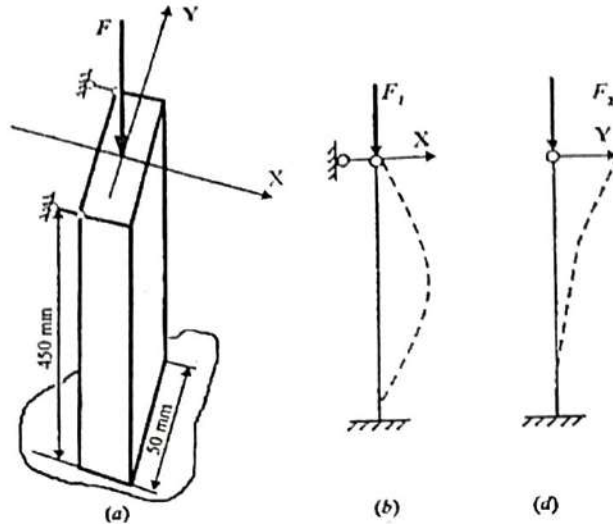
103-masala

12.5-shakl (a) da tasvirlangan sterjen uchun ustuvorlikning haqiqiy ehtiyoj koeffitsiyenti aniqlansin.

Sterjenning o'lchamlari mm larda berilgan bo'lib, u St.3 navli po'latdan yasalgan. $l = 450 \text{ mm}; F = 44 \cdot 10^3 \text{ N.}$

Yechish:

Sterjenning ustuvorligi yo'qoladigan tekislikni aniqlash uchun egiluvchanliklarni o'zaro taqqoslash zarur.



12.5-shakl

a) XOZ tekisligida (12.5-shakl, b) $\lambda_x = \frac{\mu_1 l}{i_y} = \frac{0,7 \cdot 450}{\sqrt{\frac{I_y}{A}}} = \frac{0,7 \cdot 450}{\sqrt{\frac{50 \cdot 15^3}{12 \cdot 15 \cdot 50}}} = 72,75$.

b) YOZ tekisligida (12.5-shakl, d) $\lambda_y = \frac{\mu_2 l}{i_x} = \frac{2 \cdot 450}{\sqrt{\frac{I_x}{A}}} = \frac{2 \cdot 450}{\sqrt{\frac{15 \cdot 50^3}{12 \cdot 15 \cdot 50}}} = 62,35$.

Bundan chiqdi, ustuvorlikning yo'qolishi XOZ tekisligida sodir bo'lar ekan.

Sterjenning egiluvchanligi $40 < \lambda_x < 100$ oraliqda bo'lganligi sababli, kritik kuchlanish F. S. Yasinskiy formulasidan topiladi:

$$\sigma_{cr} = 310 - 1,14 \cdot 72,75 = 227,065 \text{ MPa}$$

Siquvchi kuchlanishning haqiqiy qiymati:

$$\sigma = \frac{F}{A} = \frac{44 \cdot 10^3}{15 \cdot 50 \cdot 10^{-6} \text{ m}^2} = 58,67 \cdot \frac{10^6 \text{ N}}{\text{m}^2} = 58,67 \text{ MPa} \text{ ni tashkil etadi.}$$

Bunga ruxsat beriladi.

Ustuvorlikning haqiqiy (hisoblab topiladigan) ehtiyot koeffitsiyenti quyidagiga teng:

$$n^s = \frac{\sigma_{cr}}{\sigma} = \frac{227,065}{58,67} = 3,87.$$

104-masala

12.6-shaklda tasvirlangan sterjen ustuvorlikka tekshirilsin. Sterjen cho'yan materialdan yasalgan bo'lib, uning barcha o'lchamlari mm larda berilgan.

Hisoblashlar uchun quyidagi ma'lumotlar berilgan: $F = 80 \text{ kN}$; $\sigma_{adm}^- = 135 \text{ MPa}$.

Yechish:

Quyidagi miqdorlarni aniqlaymiz:

Ko'ndalang kesim yuzasi:

$$A = \frac{\pi D^2}{4} \left(1 - \frac{d^2}{D^2}\right) = \frac{3,14 \cdot 80^2}{4} \left(1 - \frac{50^2}{80^2}\right) = 3061,5 \text{ mm}^2 = 3061,5 \cdot 10^{-6} \text{ m}^2.$$

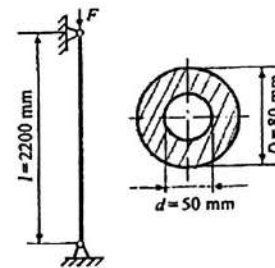
Inersiya momenti:

$$I_x = I_y = \frac{\pi D^4}{64} \left(1 - \frac{d^4}{D^4}\right) = \frac{3,14 \cdot 80^4}{64} \left(1 - \frac{50^4}{80^4}\right) = 1702959,3 \text{ mm}^4 = 170,296 \cdot 10^{-8} \text{ m}^4$$

Minimal inersiya radiusi: $i_{min} = \sqrt{\frac{I_x}{A}} = 23,585 \text{ mm} = 23,585 \cdot 10^{-3} \text{ m}.$

Sterjenning egiluvchanligi:

$$\lambda = \frac{\mu l}{i_{min}} = \frac{1 \cdot 2,2}{23,585 \cdot 10^{-3}} = 93,28.$$



12.6-shakl

Bo'ylama egilish koeffitsiyenti φ ni jadvaldan topish uchun $\lambda = 90$ va $\lambda = 100$ orasidagi qiymatlarni interpolatsiya qilamiz:

$$\varphi_{\lambda=93,28} = \varphi_{\lambda=90} - \frac{\varphi_{\lambda=90} - \varphi_{\lambda=100}}{10} \cdot (93,28 - 90) =$$

$$= 0,2 - \frac{0,2 - 0,16}{10} \cdot 3,28 = 0,1869.$$

Ruxsat etilgan kritik kuchlanish:

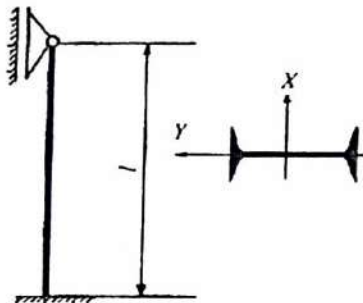
$$\sigma_{adm}^s = \varphi \cdot \sigma_{adm}^- = 0,1869 \cdot 135 \text{ MPa} = 25,23 \text{ MPa}.$$

Siquvchi kuchlanishning haqiqiy qiymati esa:

$$\sigma = \frac{F}{A} = \frac{80 \cdot 10^3}{3061,5 \cdot 10^{-6}} = 26,13 \cdot 10^6 \text{ N/m}^2 = 26,13 \text{ MPa}$$

Shunday qilib, $\sigma < \sigma_{adm}^s$ sharti saqlanmagan, ya'ni sterjen noustuvor muvozanat holatida. Ammo sterjenning o'ta yuklanganligi:

$$\frac{26,13 - 25,23}{25,23} \cdot 100\% = 3,57\% \text{ ni tashkil etadi. Bunga ruxsat etiladi.}$$



12.7-shakl

105-masala

12.7-shaklda ko'rsatilgan ustun uchun sortament jadvalidan qo'shtavrli kesim nomeri tanlansin.

Ustunning uzunligi $l=2,6$ m; unga qo'yilgan siquvchi kuch $F=210$ kN; materiali St.4 navli po'lat; siqilishdagi ruxsat etilgan kuchlanish $\sigma_{adm}^- = 160 \text{ MPa}$.

Yechish:

Ustuvorlikni ta'minlay oladigan kesim yuza tanlash uchun ketma-ket yaqinlashish usulidan foydalanamiz.

Birinchi galda $\varphi_i = 0,5$ deb qabul qilamiz va hisob tenglamasidan zaruriy ko'ndalang kesim yuzani aniqlaymiz:

$$A_1 = \frac{F}{\varphi_1 \cdot \sigma_{adm}^-} = \frac{210 \cdot 10^3}{0,5 \cdot 160 \cdot 10^6} = 26,25 \cdot 10^{-4} \text{ m}^2$$

Sortament jadvali (ГОСТ 8239-72) dan 20-raqamli qo'shtavrni tanlaymiz; bu qo'shtavrmning yuzasi $A_1 = 26,8 \cdot 10^{-4} \text{ m}^2$, minimal inersiya radiusi $i_{min} = 2,07 \cdot 10^{-2} \text{ m}$ ga teng.

Ustunning egiluvchanligini hisoblaymiz:

$$\lambda_1 = \frac{\mu l}{i_{min}} = \frac{0,7 \cdot 2,6}{2,07 \cdot 10^{-2}} = 87,92.$$

Egiluvchanlikning bu qiymatiga mos keluvchi φ ni aniqlash uchun jadvalga murojaat qilamiz. $\lambda=80$ va $\lambda=90$ orasidagi qiymatlarni chiziqli interpolatsiyalaymiz:

$$\begin{aligned} \varphi_1^1 &= \varphi_{\lambda=87,92} = \varphi_{\lambda=80} - \frac{\varphi_{\lambda=80} - \varphi_{\lambda=90}}{10} (87,92 - 80) = \\ &= 0,75 - \frac{(0,75 - 0,69)}{10} \cdot 7,92 = 0,7025. \end{aligned}$$

φ_1 va φ_1^1 lar orasidagi farq ancha katta, shu sababli hisoblashni yana takrorlaymiz:

$$\varphi_2 = \frac{\varphi_1 + \varphi_1^1}{2} = 0,6013.$$

Yangi kesim yuzasini topamiz:

$$A_2 = \frac{F}{\varphi_2 \sigma_{adm}^-} = \frac{210 \cdot 10^3}{0,6013 \cdot 160 \cdot 10^6} = 21,83 \cdot 10^{-4} \text{ m}^2,$$

Bu kesim yuzasiga profil nomeri 18 bo'lgan qo'shtavr to'g'ri keladi. Ustunning egiluvchanligini aniqlaymiz:

$$\lambda_2 = \frac{\mu l}{i_{min}} = \frac{0,7 \cdot 2,6}{1,88 \cdot 10^{-2}} = 96,81.$$

Chiziqli interpolatsiyalab, $\varphi_2^1 = 0,6287$ ni hosil qilamiz. Shu koeffitsiyent bilan kesimda paydo bo'ladigan haqiqiy kuchlanishni aniqlaymiz va uni σ_{adm}^- bilan taqqoslaymiz:

$$\begin{aligned} \sigma_x &= \frac{F}{\varphi_2^1 A_{21}} = \frac{210 \cdot 10^3}{0,6287 \cdot 32,4 \cdot 10^{-4}} = 142,74 \cdot \frac{10^6 \text{ N}}{\text{m}^2} = \\ &= 142,74 \text{ MPa} < 160 \text{ MPa}. \end{aligned}$$

Ustun $\delta_\sigma = \frac{|142,74 - 160|}{160} \cdot 100\% = -10,78\%$ ga yuklanmagan.

Bu farq 5% dan sezilarli darajada katta bo'lganligi uchun hisoblashni yana davom ettiramiz:

$$\varphi_3 = \frac{0,6013 + 0,6287}{2} = 0,615.$$

Bu koeffitsiyent bo'yicha ustunning kesim yuzasini hisoblaymiz:

$$A_3 = \frac{F}{\varphi_3 \sigma_{adm}} = \frac{210 \cdot 10^3}{0,615 \cdot 160 \cdot 10^6} = 21,34 \cdot 10^{-4} m^2.$$

Sortament jadvalidan yana 18-raqamli qo'shtavrni olishga to'g'ri keladi. Boshqacha aytganda, ustun 10,78 % ga yuklanmagan bo'lishiga qaramasdan, hisoblashni to'xtatishga majburmiz.

Nihoyat, ustunning kesimi uchun profil nomeri 18 bo'lgan qo'shtavrni tanlaymiz.

12.3-§. Bo'ylama egilish (siqilgan sterjenlarning ustuvorligi) bo'yicha topshiriq

20-topshiriq

Siqilgan sterjenlarni ustuvorlikka hisoblash.

Berilgan:

l uzunligiga teng bo'lgan po'lat sterjen F kuch ta'sirida simmetrik o'qi bo'ylab siqilyapti. Ruxsat etilgan normal kuchlanish $\sigma_{adm} = 210$ MPa ga teng.

Topish kerak:

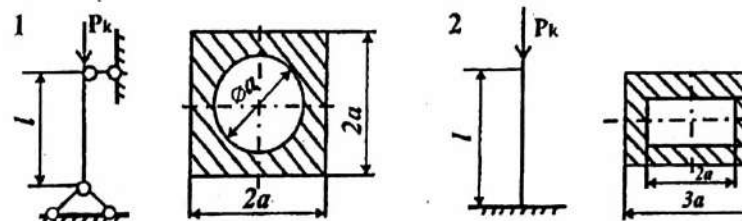
1) ustuvorlik shartiga binoan, sterjenning ko'ndalang kesim yuzasi tanlansin;

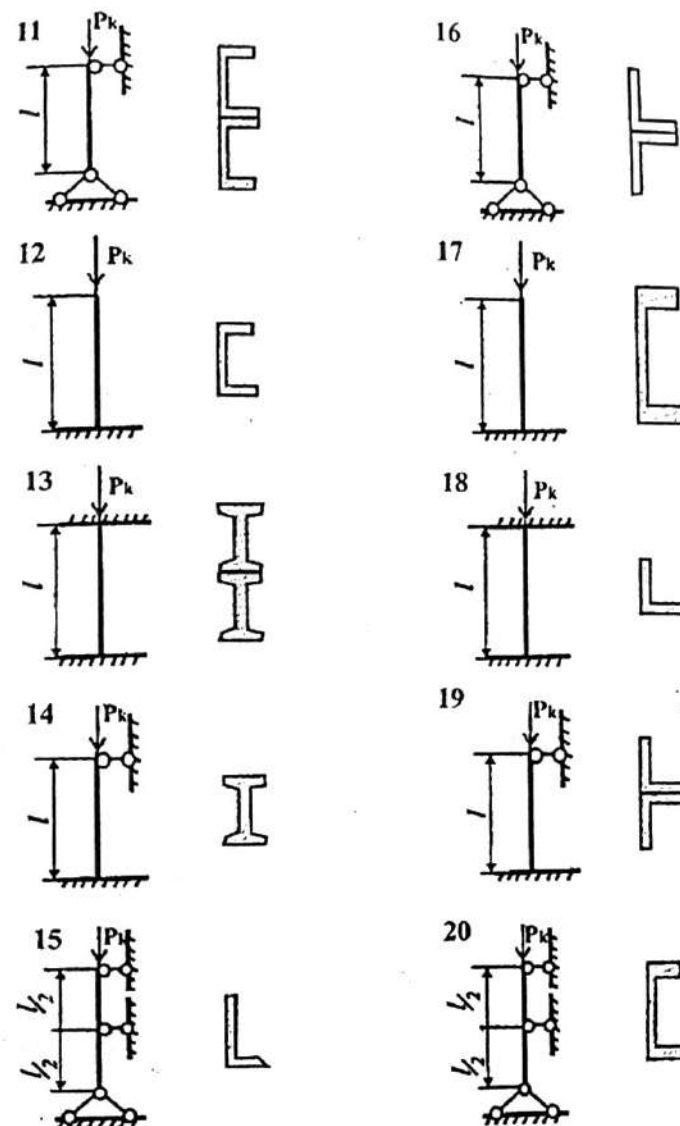
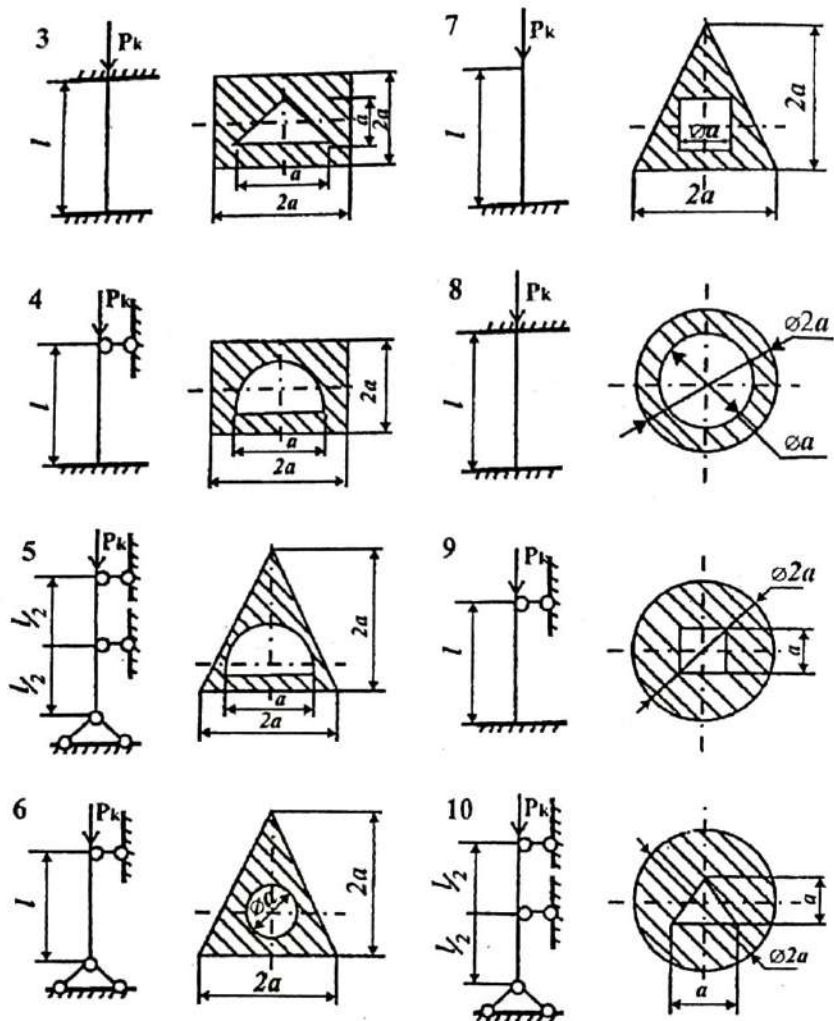
2) berilgan sterjen uchun kritik kuch miqdori aniqlansin.

Kerakli ma'lumotlar 12.1-jadvalda, hisoblash sxemalari esa 12.8-shaklda keltirilgan.

12.1-jadval

№	P, kN	l, m
1	160	3,5
2	210	3,0
3	176	3,4
4	124	3,6
5	110	4,0
6	210	3,2
7	172	2,8
8	140	3,6
9	100	4,0
10	200	3,5





XIII BOB

ZARBA

13.1-§. Asosiy mulohazalar

Ma'lumki statik yuklar inshoot va mashina qismlariga sekin-asta qo'yiladi, natijada ularda hosil bo'ladigan harakat tezlanishi juda kichik bo'lib, loyihalash paytida ular e'tiborga olinmaydi. Agarda kuchlar ta'sirida konstruktsiya elementlari katta tezlanishga ega bo'lsa, unda inersiya kuchlari hosil bo'ladi va bunday yuklanishga zarbiy (dinamik) yuklanish deyiladi.

➤ Juda ham qisqa vaqt ichida qo'yilgan va tezligi bir lahzada nolga tenglashuvchi yuk zarbali yuk deyiladi.

Odatda, zarbali yuklarning ta'siri amalda dinamik koeffitsiyent yordamida baholanadi:

$$k_d = \frac{\lambda_d}{\lambda_{st}} \quad (13.1)$$

bu yerda, λ_{st} – yuk statik ravishda qo'yilganda hosil bo'ladigan ko'chish;

λ_d – dinamik (zarbali) yuk ta'siridan paydo bo'lgan ko'chish.

Xuddi shu tarzda, zarbali kuchlardan hosil bo'ladigan deformatsiya (kuchlanish)larni tegishli statik deformatsiya (kuchlanish)lar orqali ifodalash mumkin:

$$\Delta l_d = k_d \Delta l_{st}; \quad (13.2)$$

$$\sigma_d = k_d \sigma_{st}. \quad (13.3)$$

Yuqoridagi formulalardan ko'rinib turibdiki, zarba hodisasini o'rganishda k_d ni aniqlash muhim ahamiyatga ega ekan.

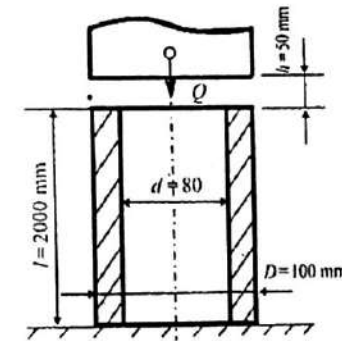
13.2-§. Zarbiy yuklanishga doir masalalar

106-masala

Pastki uchi bilan yerga bikir qilib mahkamlangan $l=2,0$ m uzunlikdagi halqasimon kesmali po'lat sterjindagi $h=5$ sm balandlikdagi og'irligi $Q=1000$ kg bo'lgan yuk zarb bilan tushmoqda (13.1-shakl).

Quyidagi hollar uchun sterjen ko'ndalang kesimida hosil bo'luvchi eng katta siquvchi kuch aniqlansin:

- sterjenning xususiy hollari hisobga olinmaganda;
- sterjenning xususiy og'irligi ($\gamma=7,85 \cdot 10^4$ N/m³) hisobga olinganda.



13.1-shakl

Yechish:

Statik ko'chishni aniqlaymiz:

$$\Delta l_{st} = \frac{Ql}{EA} = \frac{1000 \cdot 200}{2 \cdot 10^6 \cdot \frac{3,14}{4} (10^2 - 8^2)} = 3,54 \cdot 10^{-3} \text{ sm.}$$

Bu yerda, $h > \Delta l_{st}$ bo'lgani uchun:

$$k_d = \sqrt{\frac{2h}{\Delta l_{st}}} = \sqrt{\frac{2 \cdot 5}{3,54 \cdot 10^{-3}}} = 53.$$

U holda, $\sigma_d = k_d \cdot \sigma_{st} = k_d \cdot \frac{Q}{A} = 53 \cdot \frac{1000}{28,2} = 1880 \text{ kg/sm}^2$.

Sterjenning xususiy og'irligini hisobga olgandagi dinamik koeffitsiyentini hisoblaymiz:

$$k_d = \sqrt{\frac{2h}{\Delta l_{st} (1 + \alpha \cdot \frac{G}{Q})}} = \sqrt{\frac{2 \cdot 5}{3,54 \cdot 10^{-3} (1 + \frac{1}{3} \cdot \frac{44}{1000})}} = 52,6.$$

bu yerda, $G = \gamma \cdot A \cdot l = 7,85 \cdot 10^{-3} \cdot 28,2 \cdot 200 = 44 \text{ kg}$.

U holda, $\sigma_d = 52,6 \cdot \frac{1000}{28,2} \approx 1865 \text{ kg/sm}^2$

Hisoblash natijalarini taqqoslab, zarblanuvchi jism xususiy og'irligining dinamik koeffitsiyentga ta'siri sezilarsiz ekanligini ko'rsatish mumkin.

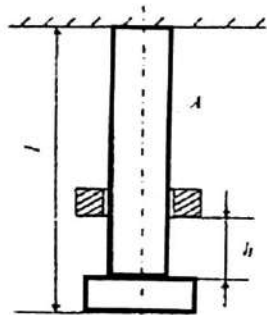
107-masala

Uzunli $l = 2\text{m}$ bo'lgan po'lat sterjenning pastki uchiga «deformatsiyalanmaydigan» disk o'rnatilgan (13.2-shakl).

$Q = 8\text{kg}$ yuk qancha balandlikdan unga zarb bilan tushganda, sterjenning ko'ndalang kesimida qiymati $\sigma = 1000 \text{ kg/sm}^2$ dinamik kuchlanish paydo bo'ladi?

Ishqalanish kuchi, sterjen va disklarning massalari e'tiborga olinmasin. Masala sterjen massasi e'tiborga olingan va olinmagan hollar uchun yechilsin.

Berilgan: $\gamma = 8\text{g/sm}^3$; $A = 4\text{sm}^2$; $E = 2 \cdot 10^6 \text{ kg/sm}^2$.



13.2-shakl

Yechish:

Q kuchning statik ta'siridan hosil bo'ladigan kuchlanishni aniqlaymiz:

$$\sigma_{st} = \frac{Q}{A} = \frac{8}{4} = 2 \text{ kg/sm}^2.$$

Unda dinamik koeffitsiyent quyidagiga teng bo'ladi:

$$k_d = \frac{\sigma_d}{\sigma_{st}} = \frac{1000}{2} = 500.$$

Agarda Q yukning boshlang'ich tezligini $v_0 = \sqrt{2gh}$, sterjenning cho'zilishdagi bikirligini $c = \frac{EA}{l}$, sterjen og'irligini $Q_0 = \gamma \cdot A \cdot l$ va $k_m = \frac{1}{3}$ deb olsak, unda dinamik koeffitsiyent quyidagiga teng bo'ladi:

$$k_d = 1 + \sqrt{1 + \frac{2h \cdot EA}{Q \cdot l} \cdot \frac{1}{1 + \frac{1}{3} \cdot \frac{Q_0}{Q}}} = 1 + \sqrt{1 + \frac{2h \cdot 2 \cdot 10^6 \cdot 4}{8 \cdot 2 \cdot 10^2} \cdot \frac{1}{1 + \frac{1}{3} \cdot \frac{64}{8}}} = 1 + \sqrt{1 + \frac{3 \cdot 10^4}{3,8} \cdot h}.$$

Unda, $500 = 1 + \sqrt{1 + \frac{3 \cdot 10^4}{3,8} h}$; bu erdan, $h = \frac{(499^2 - 1) \cdot 3,8}{3 \cdot 10^4} \approx 31,5 \text{ sm}$.

Sterjen massasini e'tiborga olmaganda:

$$k_d = 500 = \sqrt{\frac{v_0^2 \cdot c}{Q \cdot g}} = \sqrt{\frac{2h \cdot EF}{Q \cdot l}} \sqrt{\frac{2h \cdot 2 \cdot 10^6 \cdot 4}{8 \cdot 2 \cdot 10^2}} = 10^2 \sqrt{h};$$

bu yerdan, $h = 25 \text{ sm}$.

Demak, sterjen xususiy og'irligi e'tiborga olinmaganda; yukning xavfsiz tushish balandligi kamaytirilgan holda bo'lar ekan, ya'ni

$$\frac{31,5 - 25}{31,5} \cdot 100 = 21\%.$$

108- masala

$Q = 500\text{kg} = 5\text{kN}$ og'irlikdagi yuk $v = 1,5 \text{ m/sek}$ gorizonta tezlik bilan harakatlanib, o'z yo'lida uzunligi $l = 3\text{m}$, ko'ndalang kesimining yuzasi $d = 15 \text{ sm}$ bo'lgan va pastki uchi bilan yerda bikir mahkamlab qo'yilgan po'lat ustunning yuqori uchida to'qnashadi (13.3-shakl). Ustunning xususiy og'irligi e'tiborga olinmagan holda unda hosil bo'luvchi eng katta kuchlanish aniqlansin.

Yechish:

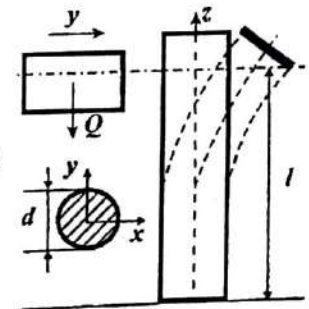
Zarba ta'sirida ustun egiladi. Boshqacha aytganda, yuk sterjenga urilishi bilanoq, uning to'la energiyasiga aylanadi:

$$T = U_g \text{ yoki } \frac{Qv^2}{2q} = \int_0^l \frac{M_g^2 dz}{2El_x};$$

Bu yerda, $T = \frac{Qv^2}{2q}$ — harakatdagi jismning energiyasi.

Kesish usuliga asosan: $M_g = P_g z$.

$$\text{U holda, } U_g = \int_0^l \frac{P_g^2 z^2 dz}{2El_x} = \frac{P_g^2 l^3}{6El_x}.$$



13.3-shakl

Energiyalarni solishtirib, sterjenga ta'sir etuvchi dinamik kuchni keyin esa dinamik kuchlanishni hisoblaymiz:

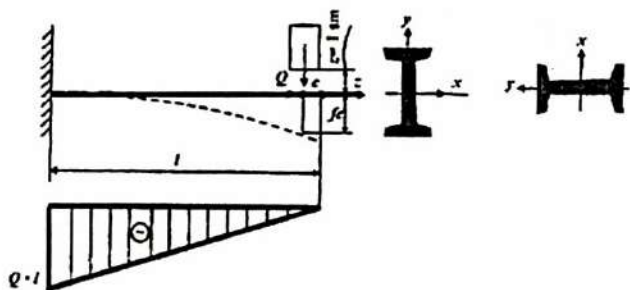
$$P_k = \frac{v}{l} \sqrt{\frac{3QE I_x}{ql}} = \frac{vd^2}{8l} \sqrt{\frac{3\pi QE}{ql}} = 78925 \text{ N};$$

$$\sigma_k = \frac{P_k l}{W_x} = \frac{32 P_k l}{\pi d^3} = \frac{32 v d^2 l}{\pi d^3 8l} \sqrt{\frac{3\pi QE}{ql}} = \frac{4v}{\pi d} \sqrt{\frac{3\pi QE}{ql}} = 714 \cdot 10^5 \frac{\text{kg}}{\text{m}^2};$$

bu yerda, $I_x = \frac{\pi d^4}{64}$; $W_x = \frac{\pi d^3}{32}$.

109-masala

Qo'shtavr kesimli (№ 20, ГОСТ 8239-72), $l = 1,0 \text{ m}$ uzunlikdagi konsol $Q = 75 \text{ kg}$ og'irlikdagi yukka ikki xil ko'rinishda, ya'ni bikirligi eng katta va eng kichik tekisliklar bo'yicha qarshilik ko'rsata oladigan qilib mahkamlangan (13.4-shakl). Ikkita holat uchun ham eng katta statik va dinamik kuchlanish (solqilik)larni topib, natijalarni taqqoslang. Konsolning o'z og'irligi hisobga olinmasin.



13.4-shakl

Yechish:

1- hol. Dastlab, konsolning eng katta tekislikda egilishini ko'rib chiqamiz. Sortament jadvalidan quyidagi ma'lumotlarni olamiz:

$$I_x = 1810 \text{ sm}^4; I_y = 112 \text{ sm}^4; W_x = 181 \text{ sm}^3; W_y = 22,4 \text{ sm}^3$$

Kuch statik ravishda qo'yilganda, kesimdagi eng katta normal kuchlanish quyidagicha topiladi:

$$\sigma_{st} = \frac{M_{max}}{W_x} = \frac{75 \cdot 100}{181} = 41,5 \text{ kg/sm}^2.$$

Dinamik koeffitsiyent:

$$k_d = 1 + \sqrt{1 + \frac{2h}{\Delta l_{st}}}; \text{ bu yerda, } \Delta l_{st} = \frac{Ql^3}{3EI_x} = f_{st} \text{ ga teng bo'ladi.}$$

Balka Q yuk ta'sirida statik yuklanganda:

$$\Delta l_{st} = \frac{75 \cdot 100^3}{3 \cdot 2 \cdot 10^5 \cdot 1810} = 6,91 \cdot 10^{-3} \text{ sm};$$

$$k_d = 1 + \sqrt{1 + \frac{2 \cdot 1,0}{6,91 \cdot 10^{-3}}} = 18,1.$$

Dinamik solqilik va eng katta dinamik kuchlanish quyidagiga teng bo'ladi:

$$\Delta l_d = f_d = k_d \cdot f_{st} = 18,1 \cdot 6,91 \cdot 10^{-3} = 0,125 \text{ sm};$$

$$\sigma_d = k_d \cdot \sigma_{st} = 18,1 \cdot 41,5 = 750 \text{ kg/sm}^2.$$

2- hol. Xuddi shu tartibda hisoblashni davom ettirib, konsolning bikirligi eng kichik tekislikda egilishini tekshiramiz:

$$\sigma_{st} = \frac{Ql}{W_y} = \frac{75 \cdot 100}{22,4} = 335 \text{ kg/sm}^2;$$

$$\Delta l_{st} = f_{st} = \frac{Ql^3}{3EI_y} = \frac{75 \cdot 100^3}{3 \cdot 2 \cdot 10^6 \cdot 112} = 112 \cdot 10^{-3} \text{ sm};$$

$$k_d = 1 + \sqrt{1 + \frac{2 \cdot 1,0}{112 \cdot 10^{-3}}} = 5,35;$$

$$\Delta l_d = f_d = k_d \cdot f_{st} = 5,35 \cdot 112 \cdot 10^{-3} = 0,6;$$

$$\sigma_d = k_d \cdot \sigma_{st} = 5,35 \cdot 335 = 1790 \text{ kg/sm}^2.$$

Demak, ikkinchi holatdagi kuchlanish yukning statik ta'sirida birinchi holatidan 8,08 marta, zarbiy ta'sirida esa 2,39 marta ko'p. Buning sababi – ikkinchi holatda balka bikirligi birinchi holatga nisbatan ancha kam. Bu dinamik koeffitsiyentning kamayishiga olib keladi.

110- masala

Qo'shtavr kesimli (№ 20) ikki tayanchdagi konsol balkaning (13.5-shakl, a) C kesimiga $H=6$ sm balandlikdan $F=1,2$ kN og'irlikdagi yuk ta'sir etadi.

Quyidagilar berilgan: $l=2$ m; $a=1$ m; $\sigma_{adm}=210$ MPa; $E=2 \cdot 10^5$ MPa.

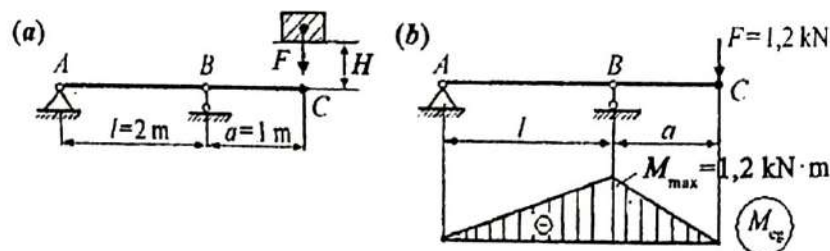
Quyidagilar aniqlansin:

1. Balkaning mustahkamligi;
2. Dinamik kuch ta'siridagi C kesimning solqiligi.

Yechish:

Sortament jadvalidan 20-raqamli qo'shtavr uchun quyidagilarni olamiz: $J_x=1840$ sm⁴; $W_x=184$ sm³. Konsol balkaning C kesimiga ta'sir qiladigan F kuchning statik holda ta'sir etishidagi solqilikni Mor yoki Vereshagin usulida aniqlaymiz. Uning qiymati quyidagiga teng bo'ladi (bu qiymatni topish talabga havola):

$$\Delta_n = \frac{F \cdot a^2}{3EJ_x} (l+a) = \frac{1,2 \cdot 10^3 \cdot 1}{3 \cdot 2 \cdot 10^{11} \cdot 1840 \cdot 10^{-8}} (2+1) = 3,26 \cdot 10^{-4} \text{ m} = 0,0326 \text{ sm}.$$



13.5-shakl

Dinamik koeffitsiyentni aniqlaymiz:

$$k_D = 1 + \sqrt{1 + \frac{2H}{\Delta_{st}}} = 1 + \sqrt{1 + \frac{2 \cdot 6 \cdot 10^{-2}}{0,0326 \cdot 10^{-2}}} = 20,21.$$

Statik kuch ta'sirida konsol balkaning B tayanchida hosil bo'ladigan kuchlanishni aniqlaymiz (13.5-shakl, b):

$$\sigma_{st} = \frac{M_{max}}{W_x} = \frac{1,2 \cdot 10^{-3}}{184 \cdot 10^{-6}} = 6,52 \cdot 10^6 \text{ Pa} = 6,52 \text{ MPa}.$$

Konsol balkaning B tayanchida hosil bo'ladigan dinamik kuchlanishni aniqlaymiz:

$$\sigma_D = \sigma_{st} \cdot k_D = 6,52 \cdot 20,21 = 131,8 \text{ MPa} < [\sigma] = 210 \text{ MPa}.$$

Demak, konsol balkaning mustahkamligi ta'minlangan.

Dinamik kuch ta'sirida konsol balkaning C kesimidagi solqilikni aniqlaymiz:

$$\Delta_D = \Delta_{st} \cdot k_D = 0,0326 \cdot 20,21 = 0,659 \text{ sm}.$$

Demak, dinamik kuch ta'sirida konsol balkaning C kesimidagi solqiligi 20,21 marta katta bo'lar ekan.

13.3-§. Zarbiy yuklanish bo'yicha topshiriqlar

21-topshiriq

Zarba nazariyasi.

Berilgan: elastik sistemaga F yuk H balandlikdan tushib uriladi. Sterjen po'latdan yasalgan. Statik yuklanish uchun ruxsat etilgan normal kuchlanish $[\sigma]=210$ MPa

Topish kerak:

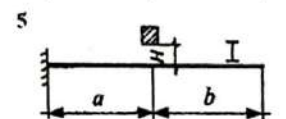
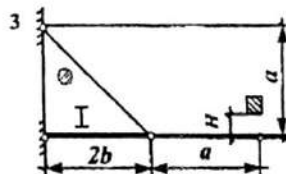
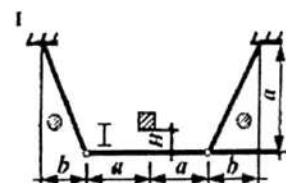
1) sistemaning ko'ndalang kesimlaridagi eng katta normal kuchlanishlar aniqlansin;

2) yuk ta'sir etayotgan nuqtaning dinamik ko'chishi topilsin.

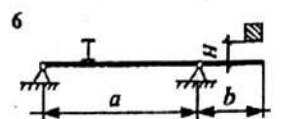
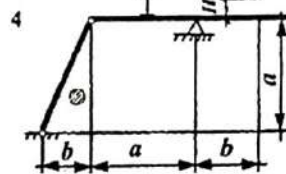
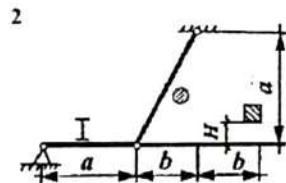
Kerakli ma'lumotlar 13.1-jadvalda, hisoblash sxemalari esa 13.6-shaklda keltirilgan.

13.1-jadval

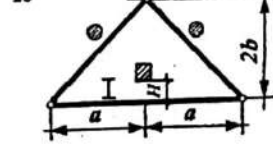
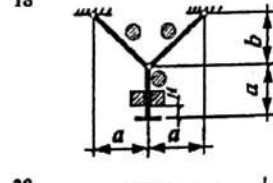
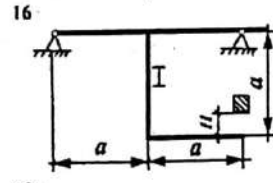
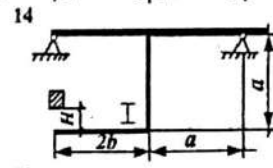
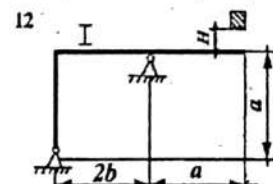
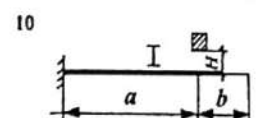
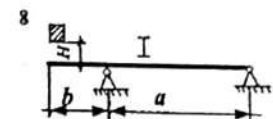
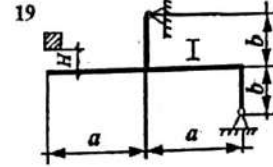
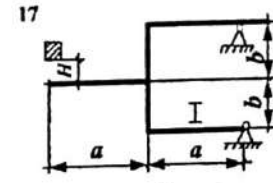
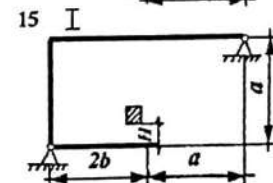
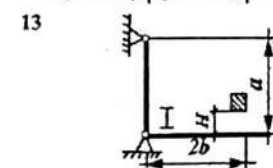
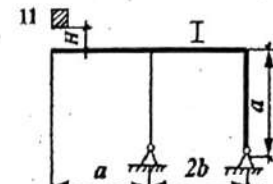
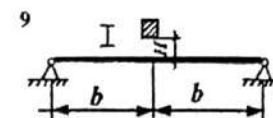
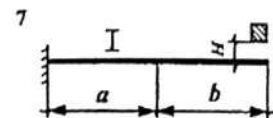
№	F, N	H, sm	a, m	F, m	Qo'shtavr nomeri	d, sm
1	500	6	4,0	1,2	27	1,4
2	450	4	3,8	1,0	24a	1,6
3	600	7	3,4	1,4	24	1,8
4	650	6,2	3,2	1,6	22a	2,0
5	700	5	3,4	1,8	22	2,2
6	740	6	2,6	1,2	20a	2,4
7	600	4	3,4	3,0	20	2,6
8	400	4,6	3,2	2,6	18a	2,8
9	460	4,2	3,0	2,2	18	3,0
10	700	5,2	2,6	2,4	16	3,2



13.6-shakl



13.6-shakl (davomi)



XIV BOB

OLIMPIADA MASALALARI

14.1-§. Cho'zilish va siqilishga doir masalalar

111-masala

Mutlaq bikir AB balka orqali to'rtta ustunga P kuch uzatiladi (14.1-shakl). Ustunlarning loyihaviy uzunligi l ga teng, ko'ndalang kesimlari, materiali bir xil. Ikkinchi ustunning uzunligi loyihadagidan Δ_2 ga, to'rtinchi ustunning uzunligi Δ_4 ga kalta, uchinchi ustunning uzunligi Δ_3 ga uzun. P kuch ta'sir etganda, AB balka gorizontol holatda qolishi uchun Δ_3 nimaga teng bo'lishi kerak?

Berilgan: $\Delta_2 = 0,2 \text{ mm}$; $\Delta_4 = 0,1 \text{ mm}$; $l = 50 \text{ sm}$;

$E = 0,1 \cdot 10^6 \text{ kg/sm}^2$; $F = 12 \times 12 \text{ sm}^2$; $a = 0,5 \text{ m}$; $P = 40 \text{ t}$.

Yechish:

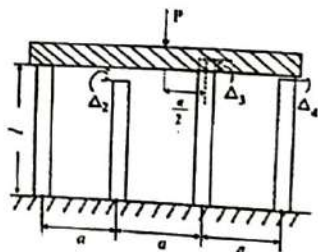
Statika tenglamalarini tuzamiz (14.2-shakl):

$$\sum Y = 0; \quad N_1 + N_2 + N_3 + N_4 - P = 0; \quad (1)$$

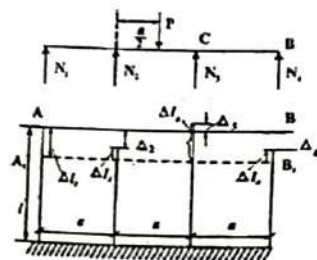
$$\sum mom_c = 0; \quad 2N_1 + N_2 - N_4 = \frac{P}{2}. \quad (2)$$

Deformatsiya tenglamalarini tuzamiz:

$$\Delta l_2 = \Delta l_1 - \Delta_2 \Rightarrow N_2 = N_1 - \Delta_2 \frac{EF}{l}; \quad (3)$$



14.1-shakl



14.2-shakl

$$\Delta l_4 = \Delta l_1 - \Delta_4 \Rightarrow N_4 = N_1 - \Delta_4 \frac{EF}{l}. \quad (4)$$

$\frac{EF}{l} = B$ belgilash kiritamiz va (3), (4) ni (2) ga qo'yamiz:

$$N_1 = (\Delta_2 - \Delta_4) \frac{B}{2} + \frac{P}{4}. \quad (5)$$

(5) ni (3), (4) ga qo'yamiz:

$$\begin{cases} N_2 = -(\Delta_2 + \Delta_4) \frac{B}{2} + \frac{P}{4} \\ N_4 = (\Delta_2 - 3\Delta_4) \frac{B}{2} + \frac{P}{4} \end{cases} \quad (6)$$

$$N_3 = (5\Delta_4 - \Delta_2) \frac{B}{2} + \frac{P}{4}$$

(5), (6) ni (1) ga qo'yib, quyidagilarni aniqlaymiz:

$$\Delta_3 = \Delta l_3 - \Delta l_1.$$

Shunga ko'ra:

$$B = \frac{EF}{l} = \frac{0,1 \cdot 10^6 \cdot 144}{50} = 28,8 \cdot 10^4 \text{ kg/sm}, \quad P = 4 \cdot 10^4 \text{ kg};$$

$$N_1 = 0,01 \frac{28,8 \cdot 10^4}{2} + 10^4 = 11440 \text{ kg};$$

$$N_2 = -0,03 \frac{28,8 \cdot 10^4}{2} + 10^4 = 5680 \text{ kg};$$

$$N_3 = 0,03 \frac{28,8 \cdot 10^4}{2} + 10^4 = 14320 \text{ kg};$$

$$N_4 = -0,01 \frac{28,8 \cdot 10^4}{2} + 10^4 = 8560 \text{ kg};$$

$$\Delta l_1 = \frac{N_1 l}{EF} = \frac{11440 \cdot 50}{0,1 \cdot 10^6 \cdot 144} = 0,039;$$

$$\Delta l_3 = \frac{N_3 l}{EF} = \frac{14320 \cdot 50}{0,1 \cdot 10^6 \cdot 144} = 0,049;$$

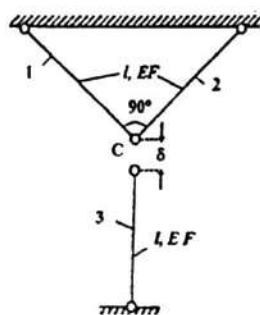
$$\Delta l_3 = 0,049 - 0,039 = 0,01 \text{ sm}; \quad \Delta = 0,1 \text{ mm}.$$

112-masala

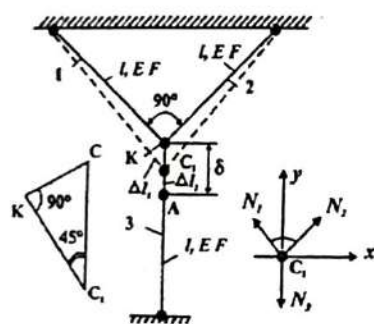
1-, 2-, 3-sterjenlardan uchinchi δ miqdor (δ – kichik miqdor)ga qisqa tayyorlangan (14.3-shakl). Ularning uchlarini C sharnirda tutashtirish uchun 3-sterjen Δt^0 ga qizdirildi.

Topish kerak:

- 1) 3-sterjen qanchaga (Δt^0 –?) qizdirilgan?
- 2) Bu sterjenning harorati avvalgi holiga qaytganda ($\Delta t^0=0$ bo'lganda), barcha sterjenlardagi zo'riqish kuchlari nimaga teng bo'ladi?



14.3-shakl



14.4-shakl

Yechish:

3-sterjen qizdirilganda,

$$\Delta l_t^{(3)} = \alpha \cdot l \cdot \Delta t_c^0 = \delta$$

$$\text{Bundan, } \Delta t_c^0 = \frac{\delta}{\alpha l}.$$

3-sterjen harorati avvalgi holiga qaytganda ($\Delta t^0=0$), C tugun C_1 ga ko'chadi (14.4- shakl).

Unda, C_1 , tugunning muvozanat sharti quyidagiga teng:

$$\sum X = 0; \quad N_1 = N_2;$$

$$\sum Y = 0, \quad N_3 = \sqrt{2} \cdot N_1. \quad (1)$$

Demak, masala statik noaniq. U holda deformatsiya tenglamasi quyidagiga teng bo'ladi:

$$CC_1 = AC - C_1A \text{ yoki } \sqrt{2} \cdot \Delta l_1 = \delta - \Delta l_3$$

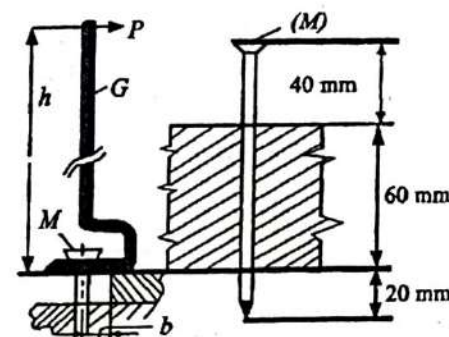
$$\text{bu yerda, } CC_1 = \frac{\Delta l_1}{\cos 45^\circ} = \sqrt{2} \Delta l_1.$$

$$\sqrt{2} \cdot \frac{N_1 l}{EF} = \delta - \frac{N_3 l}{EF}, \text{ bu tenglamaga (1) ni olib kelib qo'yamiz:}$$

$$\frac{\sqrt{2} N_1 l}{EF} = \delta - \frac{\sqrt{2} N_1 l}{EF}; \text{ bu yerdan, } N_1 = \frac{EF \delta}{2\sqrt{2} l}; \quad N_3 = \frac{EF \delta}{2 l}.$$

113-masala

Qalinligi 60 mm bo'lgan yog'ochdan (G) mix sug'urgich yordamida (M) mix sug'uriladi (14.5-shakl). Mixda hosil bo'lgan N bo'ylama kuchning ruxsat etilgan qiymati, mix sug'urgich dastasiga qo'yiladigan P kuch aniqlansin. Mix uchun: $[\sigma] = 1800 \text{ kg/sm}^2$; $l = 120 \text{ mm}$; $d = 5 \text{ mm}$; $h = 35 \text{ sm}$; $b = 2,8 \text{ sm}$.



14.5-shakl

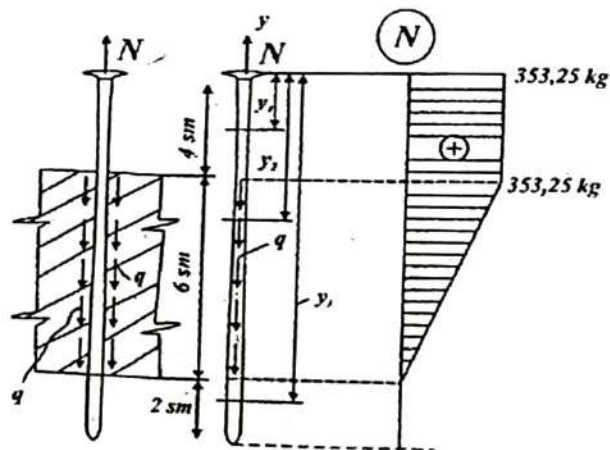
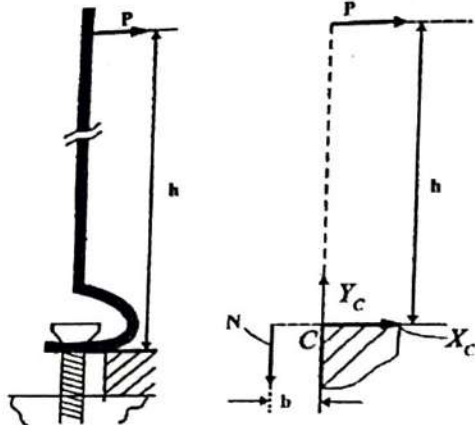
Yechish:

Mixning o'qi bo'ylab qo'yiladigan kuch quyidagiga teng (14.6-shakl):

$$N \leq [\sigma] \cdot F = 1800 \text{ kg/sm}^2 \cdot \frac{3,14 \cdot (0,5)^2}{4} \text{ sm}^2 = 353,25 \text{ kg}.$$

Mix sug'uradigan moslamaning dastasiga qo'yiladigan kuchni aniqlaymiz:

$$\sum \text{mom}_c = P \cdot h - N \cdot b = 0; \quad P = \frac{N \cdot b}{h} = \frac{353,25 \cdot 2,5}{35} = 25,2.$$



14.6-shakl

Mixni sug'urish vaqtida mixning yog'och ichidagi qismida urinma kuchlar hosil bo'ladi. Bu kuchlar mixni yog'och bilan ishqalanishidan hosil bo'ladi, ularni yog'och qalinligi bo'ylab tekis taqsimlangan deb qabul qilamiz ($q = \text{const}$).

$$\text{U holda, } \sum Y = N - q \cdot 6 = 0; \quad q = \frac{N}{6} = 58,87 \frac{\text{kg}}{\text{sm}}.$$

Bo'ylama kuch epyurasini quramiz:

$$\text{I. } 0 \leq Y_1 \leq 4; \quad \sum Y = N - N_1 = 0; \quad N_1 = N = 353,25 \text{ kg}.$$

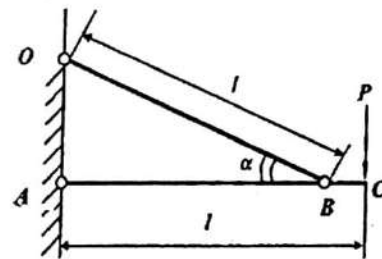
$$\text{II. } 0 \leq Y_2 \leq 10 \text{ da, } N_2 = N - q(Y_2 - 4); \quad Y_2 = 4 \text{ da, } N_2 = 353,25; \quad Y_2 = 10 \text{ da, } N_2 = 0.$$

$$\text{III. } 0 \leq Y_3 < 12; \quad N_3 = 0.$$

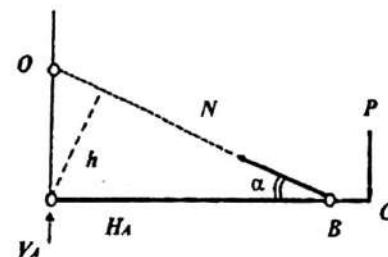
114-masala

OB tortqichning uzunligini o'zgartirmagan holda uning shunday holatini ($\alpha = ?$) aniqlash kerakki, natijada tortqichning massasi minimum qiymatga ega bo'lsin (14.7-shakl).

l, P, ρ – tortqich materialining solishtirma og'irligi va $[\sigma]$ – ruxsat etilgan kuchlanish ma'lum deb qabul qilinsin.



14.7-shakl



14.8-shakl

Yechish:

$$14.8\text{-shakldan: } OB = l; \quad AC = l; \quad h = \frac{1}{2} \sin 2\alpha.$$

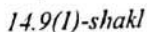
$$\text{U holda, } \sum m_A = 0; \quad p \cdot l - N \cdot h = 0; \quad \text{bu yerdan, } N = \frac{2P}{\sin 2\alpha}.$$

$$\text{Bizga ma'lumki, } \sigma = \frac{N}{F} \leq [\sigma]; \quad \text{u holda, } F \geq \frac{N}{[\sigma]} = \frac{2P}{\sin 2\alpha \cdot [\sigma]}.$$

OB sterjen massasi $m = \frac{F \cdot l \cdot \rho}{g}$ ga teng; bu yerda, m F ga proporsional-dir.

Demak, OB sterjen massasi $\sin 2\alpha = 1$ yoki $\alpha = 45^\circ$ da minimum qiymatga ega bo'ladi.

115-masala



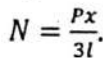
Mutlaq qattiq AB brus C, D roliklar hamda tros vositasida og'ma holda o'rnatilgan (14.9 (1)-shakl). Brusning og'maligi kichik son Δ bilan berilgan. P kuchning qanday holatida ($x=?$) brus gorizontal holatni qabul qiladi? P, l, E, F, Δ - berilgan deb hisoblansin

$$\left(P = 5 \text{ t}; \quad l = 2 \text{ m}; \quad \Delta = 1 \text{ sm}; \quad F = 0,5 \text{ sm}^2; \quad E = 2 \cdot 10^6 \frac{\text{kg}}{\text{sm}^2} \right).$$

Yechish:

Trosda hosil bo'ladigan taranglik kuchini N bilan belgilaymiz (14.9(2)-shakl).

U holda, $\sum m_A = 0$; $3N \cdot l = P \cdot x$;



14.9(2)-shakl

AB brus gorizontol holatga (AB_I) kelishi uchun tros $\Delta + \frac{\Delta}{2} = 1,5\Delta$ ga uzaytirish kerak.

Unda, $\Delta l = \frac{Nl}{EF}$ formuladan: $l = 3l$; $\Delta l = 1,5\Delta$ ekanligi aniqlanadi.
Shunga ko'ra:

$$N = \frac{1,5\Delta EF}{3l}. \quad (2)$$

264

yoki

$$(1) \text{ va } (2) \text{ dan } x = \frac{1,5\Delta EF}{P}$$

$$x = \frac{1,5 \cdot 1 \cdot 2 \cdot 10^6 \cdot 0,5}{0,5 \cdot 10^4} = 300 \text{ sm} = 3\text{m}.$$

116-masala

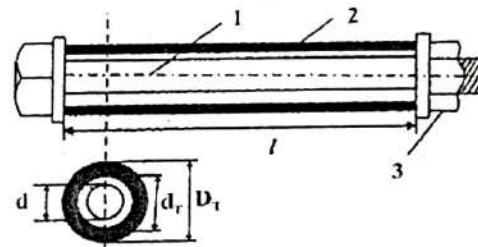
Duraluminiy trubka (2) po'lat bolt (1) ga kiygizilgan va gayka (3) 60° ga aylantirilib, bolt tortib qo'yilgan (14.10 (1)-shakl). Boltning vint qadami $\Delta = 2,4$ mm.

Trubka va bolt kesimlarida hosil bo'ladigan zo'riqishlar aniqlansin. Trubka harorati 30° ga kamaytirilganda zo'riqishlar qanchaga o'zgaradi?

Berilgan: $D_T = 32 \text{ mm}$; $d_T = 26 \text{ mm}$; $d_b = 21 \text{ mm}$;

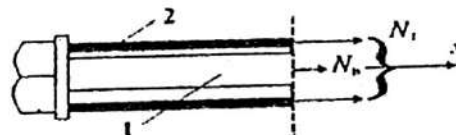
$$l = 400 \text{ mm}; \quad E_1 = 2 \cdot 10^6 \text{ kg/sm}^2;$$

$$E_2 = 0,7 \cdot 10^6 \frac{kg}{sm^2}; \quad \alpha = 12,5 \cdot 10^{-6} \left(\frac{1}{grad} \right); \quad \alpha_2 = 25 \cdot 10^{-6} \left(\frac{1}{grad} \right)$$



14.10(1)-shakl

Yechish:



14.10(2)-shakl

Quyidagi belgilashlarni kiritamiz (14.10(2)-shakl): $N_b = N_1$; $N_T = N_2$.

Muvozanat tenglamasini tuzamiz:

$$\sum X = 0; \quad N_1 + N_2 = 0 \quad (1); \quad N_2 = -N_1.$$

Deformatsiya tenglamasini tuzamiz:

$$\Delta l_1 + |\Delta l_2| = \lambda_0, \quad (2)$$

bu yerda, $\lambda_0 = \frac{\Delta}{6} = 4 \text{ mm}.$

Unda, $\Delta l_1 = \frac{N_1 l}{E_1 F_1}; \quad \Delta l_2 = \frac{N_2 l}{E_2 F_2} - \alpha_2 l \Delta t.$

(1) va (2) dan

$$\frac{N_1 l}{E_1 F_1} + \frac{N_1 l}{E_2 F_2} - \alpha_2 l \Delta t = \lambda_0. \quad (3)$$

U holda,
$$N_1 = \frac{\frac{\lambda_0}{l} - \alpha_2 \Delta t}{\frac{1}{E_1 F_1} + \frac{1}{E_2 F_2}}. \quad (4)$$

Bolt va trubkaning ko'ndalang kesim yuzalarini aniqlaymiz:

$$F_1 = 3,46 \text{ sm}^2; \quad F_2 = 2,73 \text{ sm}^2.$$

Shunga ko'ra,

$$E_1 F_1 = 2 \cdot 10^6 \cdot 3,46 = 6,42 \cdot 10^6;$$

$$E_2 F_2 = 0,7 \cdot 10^6 \cdot 2,79 = 1,91 \cdot 10^6.$$

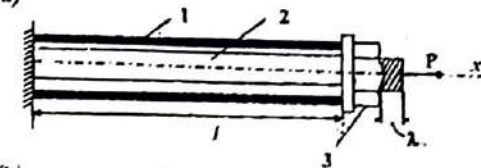
$\Delta t = 30^\circ$ deb topilgan qiymatlarni (4) ga qo'ysak, $N_1 = 374 \text{ kg}$ bo'ladi. $\Delta t = 0$ da esa $N_1 = 1496 \text{ kg}.$

117-masala

Trubka (1) ni gayka bilan qisilganda boltning (2) uchi gaykadan λ_0 ga chiqib qolgan (14.11-shakl). Shundan keyin bolt uchiga P kuch ta'sir etdi.

Trubka va bolt kesimlarida hosil bo'ladigan zo'riqishlar aniqlansin. P ning qanday qiymatida trubkadagi zo'riqish nolga teng bo'ladi? (a)

$P, l, \lambda_0, E_1, F_1, E_2, F_2,$ lar berilgan deb hisoblansin.



Yechish:

14.11-shakl (b)da N_1 - bolt, N - trubkadagi zo'riqish.

X o'qiga nisbatan kuchlardan proyeksiya olamiz:

14.11-shakl



$$\sum X = 0; \quad P + N_2 - N_1 = 0. \quad (1)$$

Bu tenglamadan:

$$N_1 + N_2 = P. \quad (2)$$

Trubka va boltlarning deformatsiyasi esa quyidagiga teng bo'ladi:

$$\Delta l_1 = \frac{N_1 l}{E_1 F_1}; \quad \Delta l_2 = \frac{N_2 l}{E_2 F_2}.$$

Endi quyidagi belgilashlarni kiritamiz:

$$E_1 F_1 = B_1;$$

$$E_2 F_2 = B_2.$$

Unda,

$$\Delta l_1 - \Delta l_2 = \lambda_0 \quad (3)$$

yoki

$$\frac{N_1}{B_1} - \frac{N_2}{B_2} = \frac{\lambda_0}{l}. \quad (4)$$

(2) dan,
$$N_1 = P - N_2 \quad (5)$$

(5) ni (4) ga olib borib qo'yamiz:

$$N_2 = \frac{\frac{P}{B_1} - \frac{\lambda_0}{l}}{\frac{1}{B_1} + \frac{1}{B_2}}; \quad N_1 = \frac{\frac{P}{B_1} + \frac{\lambda_0}{l}}{\frac{1}{B_1} + \frac{1}{B_2}}$$

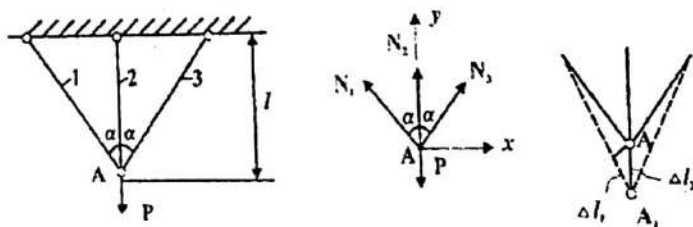
$$P = \frac{\lambda_0 B_1}{l} = \frac{\lambda_0 E_1 F_1}{l} \text{ da } N_2 = 0 \text{ bo'ladi.}$$

118-masala

A tugunda tutashtirilgan sterjenli sistemaga P kuch ta'sir etadi (14.12-shakl). 1- va 3-sterjenlarning kesim yuzalari $F_1 = F_3 = 4\text{sm}^2$, ikkinchisidagi $F_2 = 3\text{sm}^2$. Sterjenlarning materiali po'lat-3.

$$E = 2 \cdot 10^6 \text{ kg/sm}^2 (2 \cdot 10^5 \text{ MPa}); \quad l = 2\text{m}; \quad P = 40\text{t}.$$

Burchak α ning qanday qiymatida barcha sterjenlardagi zo'riqishlar bir xil qiymatga ega bo'ladi?



14.9(1)-shakl

Yechish:

Muvozanat tenglamalarini tuzamiz (14.12-shakl):

$$\sum x = 0; \quad -N_1 \cdot \sin \alpha + N_3 \cdot \sin \alpha = 0; \quad \text{unda, } N_1 = N_3;$$

$$\sum y = 0; \quad 2N_1 \cdot \cos \alpha + N_2 - P = 0. \quad (1)$$

Deformatsiya tenglamasini tuzamiz:

$$\Delta l_1 = \Delta l_2 \cdot \cos \alpha. \quad (2)$$

$$\text{Unda,} \quad l_2 = l; \quad l_1 = \frac{l}{\cos \alpha};$$

$$N_1 = \frac{F_1}{F_2} \cos^2 \alpha N_2 \quad (3)$$

Agarda (1) da $\alpha = 30^\circ$ bo'lsa, $N_1 = N_2$ bo'ladi.

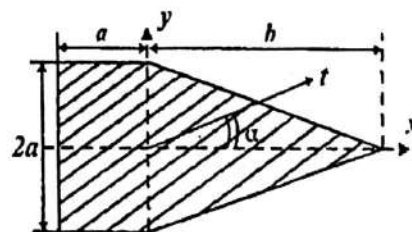
$$\text{Unda, } \frac{F_1}{F_2} \cos^2 \alpha = 1; \quad \text{bu yerda, } \cos \alpha = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}. \quad \text{Demak,}$$

$$\alpha = 30^\circ \text{ da } N_1 = N_2 = N_3 = 14,65\text{t}.$$

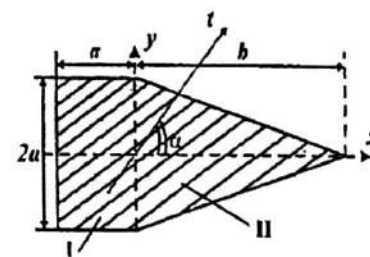
14.2-§. Tekis kesimlarning geometrik xarakteristiklari

119-masala

$\frac{b}{a}$ nisbatning qanday qiymatida 0 nuqtadan o'tuvchi istalgan t o'qqa nisbatan kesimning inersiya moment α burchakka bog'liq bo'lmaydi? α berilgan deb hisoblansin (14.13-shakl).



14.13-shakl



14.14-shakl

Yechish:

Quyidagi belgilashni kiritamiz (14.14-shakl): $\frac{b}{a} = k$, bu yerda, $b = ak$.

$$J_x = J_x^I + J_x^{II} = \frac{a(2a)^3}{12} + 2 \frac{ba^3}{12};$$

$$\text{bu demak,} \quad J_x = \frac{2}{3} a^4 + \frac{k^3 a^4}{6}.$$

$$J_y = J_y^I + J_y^{II} = \frac{(2a)a^3}{3} + \frac{2a(ka)^3}{12};$$

bu demak,

$$J_y = \frac{2}{3}a^4 + \frac{k^3a^4}{6}.$$

U holda,

$$J_x = J_y \Rightarrow \frac{2}{3}a^4 + \frac{k^3a^4}{6} = \frac{2}{3}a^4 + \frac{k^3a^4}{6}. \quad (1)$$

(1) dan $k(k^2 - 1) = 0$; $k_1 = 0$; $k_{2,3} = \pm 1$; $k_1 = 0$; $k_3 = -1$ masala ma'nosiga zid. $k_2 = 1$ yechimni olamiz. Demak, $b=a$ ekan.

U holda:

$$J_x = J_y = \frac{5}{6}a^4; \quad J_{xy} = 0 \quad (X - \text{simmetriya o'qi}).$$

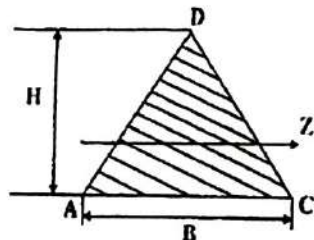
Unda,

$$J_t = J_x \cos^2 \alpha + J_y \sin^2 \alpha = \frac{5a^4}{6} (\cos^2 \alpha + \sin^2 \alpha) = \frac{5}{6}a^4.$$

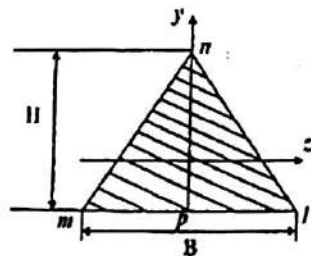
Ushbu qiymat α ga bog'liq emas.

120-masala

$AD = DC = a$, $k = \frac{B}{H}$ nisbatning qanday qiymatida kesimning markaziy z o'qiga nisbatan inersiya momenti ($k = ?$) eng katta qiymatga ega bo'ladi (14.15-shakl)?



14.15-shakl



14.16-shakl

Yechish:

Quyidagicha belgilashlar kiritamiz (14.16-shakl):

$$mn = nl = a; \quad k = \frac{B}{H}; \quad B = kH.$$

Unda,

$$J_z = \frac{BH^3}{36} = \frac{kH^4}{36};$$

Uchburchak pnl dan (14.16-shakl):

$$a^2 = H^2 + \left(\frac{B}{2}\right)^2 = H^2 + \frac{k^2H^2}{4} = H^2 \left(1 + \frac{k^2}{4}\right);$$

$$H = \frac{a}{\sqrt{1 + \frac{k^2}{4}}}; \quad J_z = \frac{a^4}{36} \cdot \frac{k}{\left(1 + \frac{k^2}{4}\right)^2};$$

bu yerda, $J_z = J_z(k)$.

$$\text{U holda, } (J_z)' k = \frac{a^4}{36} \cdot \frac{1 \cdot \left(1 + \frac{k^2}{4}\right)^2 - 2k \left(1 + \frac{k^2}{4}\right) \frac{2k}{4}}{\left(1 + \frac{k^2}{4}\right)^4} = 0.$$

$$\text{Bu yerda, } \frac{a^4}{36} \neq 0; \text{ u holda, } \left(1 + \frac{k^2}{4}\right) \cdot \left(1 + \frac{k^2}{4} - k^2\right) = 0.$$

$$\text{Shunga ko'ra, } 1 + \frac{k^2}{4} - k^2 = 0 \Rightarrow k = \frac{2\sqrt{3}}{3} \approx 1,15.$$

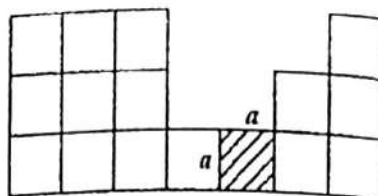
121-masala

Bizning ixtiyorimizda kartondan tayyorlangan, tomonlari a ga teng bo'lgan 16 ta kvadratchalar bor (14.17-shakl). Balka eng katta mustahkamlikka ega bo'lishi uchun mazkur kvadratchalardan terib hosil qilingan kesimning eng maqbul shakli qanday bo'lishi kerak?

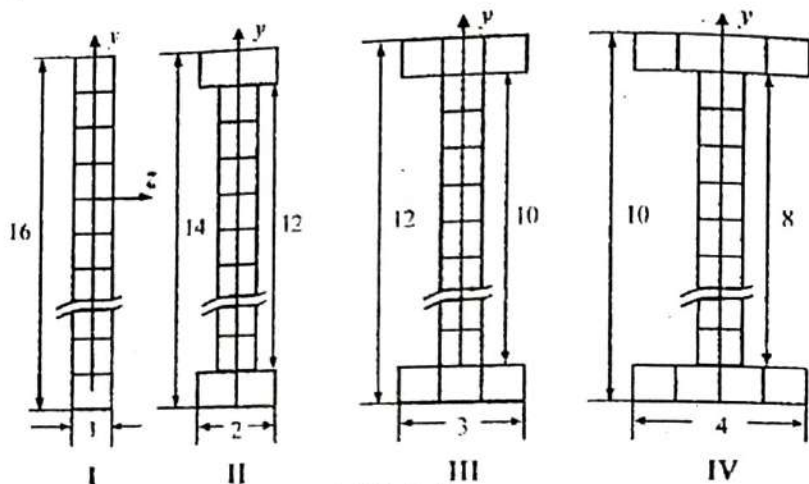
Yechish:

Masalani yechish uchun 2 ta tushuncha asos qilib olinadi:

1. Kesim vertikal o'qqa (y) simmetrik bo'lishi kerak;
2. Balkaning mustahkamligi kesimning qarshilik moment W_z ga to'g'ri proporsional.



14.17-shakl



14.18-shakl

Yuqorida 4 xil kesim ($a = 1$) hosil qilindi (14.18-shakl). z o'qi bo'yicha ularning inersiya momentlarini aniqlaymiz:

$$\text{I. } J_z = \frac{bh^3}{12} = \frac{1 \cdot 16^3}{12}; \quad Y_{\max} = 8; \quad W_z = \frac{J_z}{Y_{\max}} = \frac{16^3}{12 \cdot 8} = 42,6.$$

$$\text{II. } J_z = \frac{2 \cdot 14^3}{12} - \frac{1 \cdot 12^3}{12} = 313; \quad Y_{\max} = 7; \quad W_z = \frac{313}{7} = 44,76.$$

$$\text{III. } J_z = \frac{3 \cdot 12^3}{12} - \frac{2 \cdot 10^3}{12} = 265; \quad Y_{\max} = 6; \quad W_z = 44,22.$$

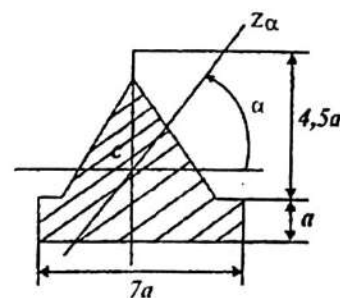
$$\text{IV. } J_z = \frac{4 \cdot 10^3}{12} - \frac{3 \cdot 8^3}{12} = 205,33; \quad Y_{\max} = 5; \quad W_z = 41,06.$$

$$\text{V. Kvadrat (4x4) uchun } W_z = 10,6.$$

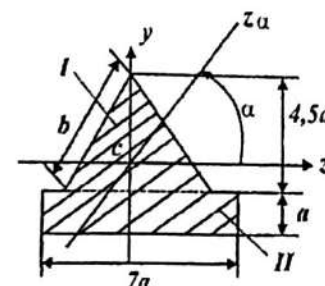
II kesim eng maqbul ekan, chunki $W_z = W_z^{\max} = 44,76$.

122-masala

Kesim teng tomonli uchburchak va to'g'ri to'rtburchakdan tashkil topgan (14.19-shakl). Uchburchakning og'irlik markazidan o'tgan istalgan o'qqa nisbatan kesimning inersiya momenti o'zgarmas qiymatga ega ekanligi isbot etilsin.



14.19-shakl



14.20-shakl

Yechish:

Bizga ma'lumki,

$$b^2 - \left(\frac{b}{2}\right)^2 = \left(\frac{9}{2}a\right)^2, \text{ bu yerdan, } b = 3a\sqrt{3}; \quad h = 4,5a.$$

Kesimlarning z va y o'qlariga nisbatan inersiya momentlarini aniqlaymiz (14.20-shakl).

Teng tomonli uchburchak uchun:

$$J_z^I = \frac{bh^3}{36} = 7,59\sqrt{3}a^4;$$

$$J_y^I = \frac{bh^3}{36} = 7,59\sqrt{3}a^4;$$

To'g'ri burchakli to'rtburchak uchun:

$$J_z^{II} = \frac{7aa^3}{12} + 7a^2 \cdot 4a^2 = \frac{343a^4}{12} = 28,58a^4;$$

$$J_y'' = \frac{a(7a)^3}{12} = \frac{343}{12}a^4 = 28,58a^4;$$

Unda,

$$J_z = J_z' + J_z'' = 7,59\sqrt{3}a^4 + 28,58a^4 = 41,71a^4;$$

$$J_y = J_y' + J_y'' = 7,59\sqrt{3}a^4 + 28,58a^4 = 41,71a^4; \quad J_{xy} = 0.$$

Demak, shakl simmetrikdir.

$$\text{Ya'ni, } \operatorname{tg} 2\alpha_0 = -\frac{J_{zy}}{J_z - J_y} = \frac{0}{0}.$$

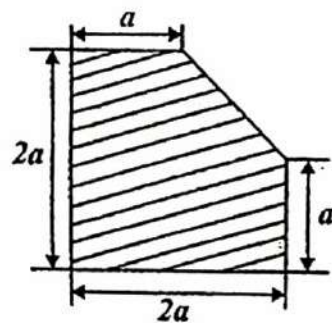
Demak, O nuqtadan o'tuvchi har qanday o'q bosh o'qdan iborat. Shuning uchun,

$$J_{za} = 41,71a^4; \quad J_{za} = J_z \cos^2 \alpha + J_y \sin^2 \alpha = J_z.$$

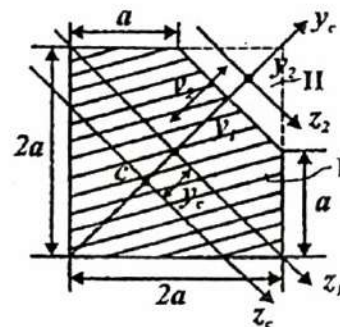
123-masala

Berilgan kesimning markaziy bosh o'qlariga nisbatan inersiya momentlari aniqlansin (14.21-shakl).

a – berilgan deb hisoblansin.



14.21-shakl



14.22-shakl

Yechish:

Kesim simmetriya o'qiga ega. Uning og'irlik markazi C nuqta simmetriya o'qida yotadi. Boshlang'ich o'qlarni kesimning diagonalari bo'yicha yo'naltiramiz (14.22-shakl).

U holda,

$$Z_c = 0; \quad Y_c = \frac{F_1 Y_1 - F_2 \cdot Y_2}{F_1 - F_2},$$

bu yerda,

$$F_1 = 4a^2; \quad F_2 = \frac{a^2}{2}; \quad Y_1 = 0; \quad Y_2 = \frac{2a\sqrt{2}}{3}; \\ Y_c = -0,134a.$$

Bosh o'qlarning bittasi (Y_c) simmetriya o'qi bo'ylab yo'naladi.

Z_c va Y_c o'qlari bo'yicha kesimning inersiya momentlarini aniqlaymiz:

$$J_{Zc}' = \frac{(2a)^4}{12} + (0,134a)^2 \cdot 4a^2 = 1,404a^4;$$

$$J_{Zc}'' = \frac{a\sqrt{2} \left(\frac{a\sqrt{2}}{2}\right)^3}{36} + \frac{a^2}{2} \left(a\sqrt{2} + 0,134 - \frac{a\sqrt{2}}{3}\right)^2 = 0,69a^4;$$

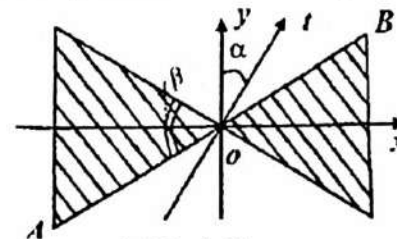
$$J_{Zc} = J_{Zc}' - J_{Zc}'' = 1,404a^4 - 0,69a^4 = 0,714a^4.$$

Unda,

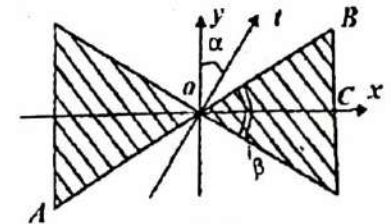
$$J_{Yc} = J_{Yc}' - J_{Yc}'' = \frac{(2a)^4}{14} - \frac{a^4}{12} \cdot \frac{1}{2} = \left(\frac{16}{12} - \frac{1}{24}\right)a^4 = \frac{31}{24}a^4 = 1,291a^4.$$

124-masala

Burchak β ning qanday qiymatida kesimning og'irlik markazi O dan o'tuvchi istalgan t o'qqa nisbatan inersiya momenti α ga bog'liq bo'lmaydi? $AO=OB=l$ berilgan deb hisoblansin (14.23-shakl).



14.23-shakl



14.24-shakl

Yechish:

OBC uchburchakdan (14.24-shakl):

$$OC = OB \cos \frac{\beta}{2} = l \cos \frac{\beta}{2}; \quad BC = OB \sin \frac{\beta}{2} = l \sin \frac{\beta}{2}.$$

Unda,

$$J_x = \frac{4OC(BC)^3}{12} = \frac{1}{3} l^4 \cos \frac{\beta}{2} \sin^3 \frac{\beta}{2};$$

$$J_y = \frac{4BC(OC)^3}{4} = l^4 \sin \frac{\beta}{2} \cos^3 \frac{\beta}{2};$$

$$J_x = J_y \Rightarrow 4 \cos^2 \frac{\beta}{2} = 1; \quad \cos \frac{\beta}{2} = \pm \frac{1}{2}.$$

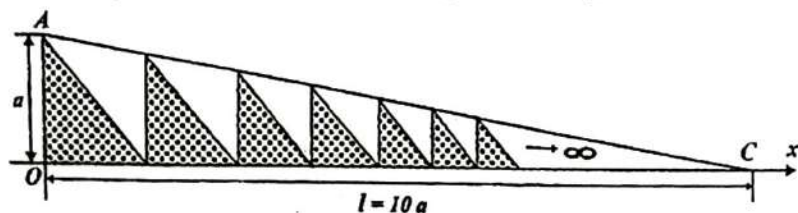
Bu yerda, $\frac{\beta}{2} = 60^\circ$; $\beta = 120^\circ$, unda,

$$J_x = J_y = \frac{l^4 \sqrt{3}}{16}; \quad J_{xy} = 0;$$

$$J_t = J_x \cos^2 \alpha + J_y \sin^2 \alpha - J_{xy} \sin 2\alpha = J_x = \frac{l^4 \sqrt{3}}{16}.$$

125-masala

Rasmdagi bo'yalgan yuzaning *OX* o'qqa nisbatan inersiya momenti *AOC* uchburchak to'liq yuzining mazkur o'qqa nisbatan inersiya momentining necha foizini tashkil etadi (14.25-shakl)?



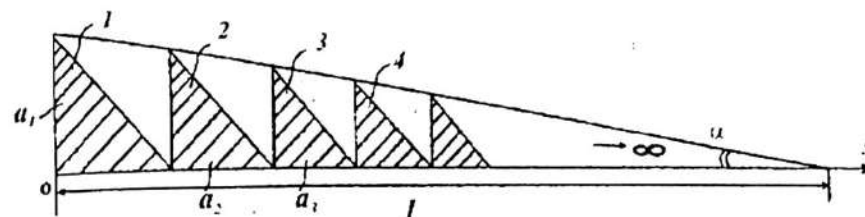
14.25-shakl

Yechish:

Quyidagi belgilashlarni kiritamiz:

$$a_1 = l \operatorname{tg} \alpha = l \cdot k; \quad k = \operatorname{tg} \alpha; \quad a_2 = (l - l \cdot k)k = l \cdot k(k - 1);$$

$$a_3 = a_2 - a_2 k = l \cdot k(1 - k)^2; \quad a_4 = a_3 - a_3 k = l \cdot k(1 - k)^3.$$



14.26-shakl

14.26-shaklda shtrixlangan uchburchaklarning *x* o'qiga nisbatan inersiya momentlari yig'indisi quyidagiga teng bo'ladi:

$$J_x = J_x^{(1)} + J_x^{(2)} + J_x^{(3)} + J_x^{(4)} \dots + J_x^{(n)}. \quad (1)$$

Bu yerda,

$$J_x^{(1)} = \frac{(l \cdot k)^4}{12}; \quad J_x^{(2)} = \frac{(l \cdot k)^4 (1 - k)}{12};$$

$$J_x^{(3)} = \frac{(l \cdot k)^4 (1 - k)^2}{12}; \quad J_x^{(4)} = \frac{(l \cdot k)^4 (1 - k)^3}{12};$$

unda,

$$\frac{J_x^{(2)}}{J_x^{(1)}} = (1 - k); \quad \frac{J_x^{(3)}}{J_x^{(2)}} = (1 - k); \quad \frac{J_x^{(4)}}{J_x^{(3)}} = (1 - k), \dots$$

(1) tenglama cheksiz kamayuvchi geometrik progressiya bo'lib, uning maxraji $q = (1 - k)$ bo'lsa, yig'indi $S = \frac{a_1}{1 - q}$ ga tengdir. Shunga ko'ra,

$$J_x = \frac{J_x^{(1)}}{1 - q} = \frac{(lk)^4}{12(1 - q)} = \frac{(lk)^4}{12[1 - (1 - k)^4]}.$$

U holda to'liq uchburchak yuzining inersiya momenti quyidagiga teng bo'ladi:

$$J_x^T = \frac{l \cdot (lk)^3}{12} = \frac{l^4 k^3}{12}; \quad k = \frac{1}{10}.$$

Unda, $\rho = \frac{J_x}{J_x^T} 100\% = \frac{k}{1-(1-k)} \cdot 100\% = \frac{10^5}{3439} = 29,07\%.$

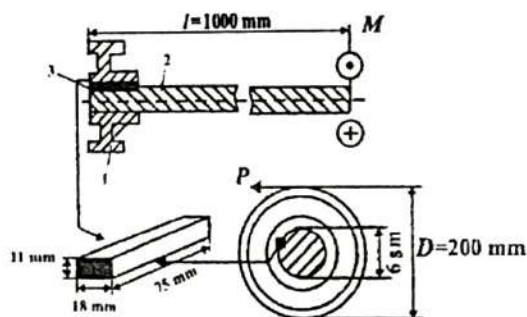
Demak, rasmda bo'yalgan yuzaning OX o'qqa nisbatan inersiya momenti AOC uchburchak to'liq yuzining mazkur o'qqa nisbatan inersiya momentining 29,07 % ini tashkil qilar ekan.

14.3-§. Buralishga doir masalalar

126-masala

Shkiv (1) val (2) bilan shponka (pona) (3) yordamida birlashgan. Shponkaning yarmi val sirtidagi o'ymaga (kanalga), yarmi shkivdagi o'ymaga kirib turadi (14.27-shakl). Val qarshilik ko'rsata oladigan burovchi momentning ruxsat etilgan qiymati aniqlansin. Shponka materiali uchun $[\tau]_c = 600 \text{ kg/sm}^2$; $[\sigma]_{ez} = 2100 \text{ kg/sm}^2$; $[\tau] = 800 \text{ kg/sm}^2$.

$[\tau]$ – val materiali uchun ruxsat etilgan kuchlanish.



14.27-shakl

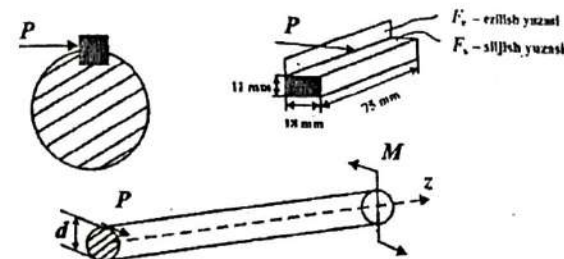
Yechish:

14.28-shakldan foydalanib, siljishdagi urinma kuchlanishni aniqlaymiz:

$$\tau_c = \frac{P}{F_c} = \frac{P}{1,8 \cdot 7,5} \leq [\tau]_c = 600 \text{ kg/sm}^2.$$

U holda,

$$[P] \leq 1,8 \cdot 7,5 \cdot 600 = 8100 \text{ kg};$$



14.28-shakl

$$\sigma_{ez} = \frac{P}{F_{ez}} = \frac{P}{0,55 \cdot 7,5} \leq [\sigma]_c = 2100 \text{ kg/sm}^2,$$

bu yerdan,

$$[P] \leq 8662 \text{ kg}$$

$P = [P] \leq 1,8 \cdot 7,5 \cdot 600 = 8100 \text{ kg}$ ni qabul qilamiz.

Z o'qiga nisbatan valga ta'sir etayotgan kuchlardan moment olamiz:

$$\sum \text{mom}_z = 0; \quad M - P \cdot \frac{d}{2} = 0;$$

$$[M] = [P] \cdot \frac{d}{2} = 8100 \text{ kg} \cdot 3 \text{ sm} = 24300 \text{ kg} \cdot \text{sm}.$$

Valning mustahkamlik shartidan: $\tau_{max} = \frac{M^b}{W_p} \leq [\tau];$

$$[M] = \frac{\pi d^3}{16} \cdot 800 = \frac{3,14 \cdot 6^3 \cdot 800}{16};$$

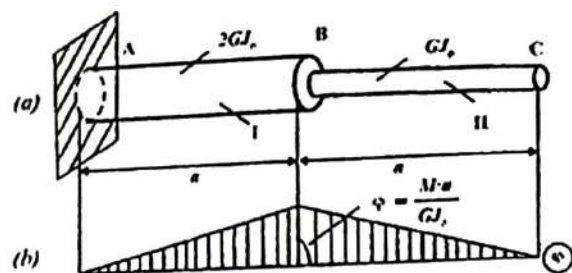
$$[M] = 33912 \text{ kg} \cdot \text{sm}.$$

$[M]$ ning ikkita qiymatidan kichigi olinadi, ya'ni: $[M] = 24,3 \text{ kg} \cdot \text{m}.$

127-masala

Berilgan valning 1-qismi bikirligi 2-qismi bikirligidan 2 marta katta. 14.29-shakl (b) da valning buralish burchaklar φ epyurasi keltirilgan.

Valga qo'yilgan tashqi kuchlar aniqlansin. Burovchi moment M_b epyurasi qurilsin. Valning bikirligi $G \cdot J_p$ va a aniq deb hisoblansin.



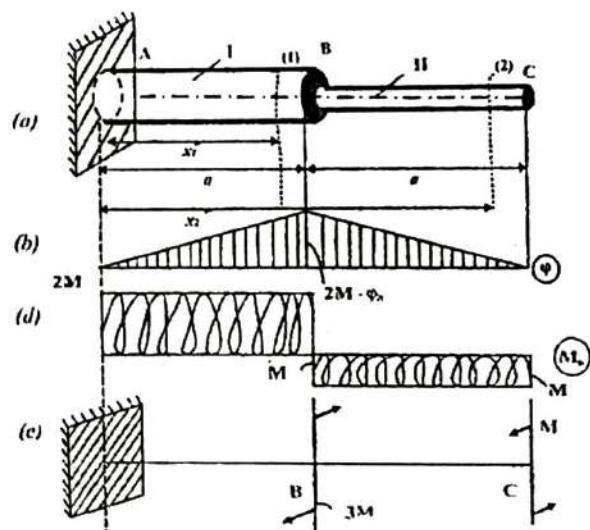
14.29-shakl

Yechish:

Valning II qismidagi bikirlik GJ_p bo'lsa, I qismining bikirligi $2GJ_p$ bo'ladi.

Valning buralish burchagini aniqlash formulasi quyidagiga teng:

$$\varphi = \frac{M \cdot l}{GJ_p}.$$



14.30-shakl

Valning birinchi uchastkasida: $0 \leq x_1 \leq a$ (14.30-shakl).
Unda,

$$\varphi_1 = \frac{M_b^{(1)} \cdot x_1}{2GJ_p},$$

bu yerda, $x_1 = 0$; $\varphi_1 = \varphi_A = 0$.

$$x_1 = a \text{ da } \varphi_1 = \varphi_B = \frac{M_b^{(1)} \cdot a}{2GJ_p} = \frac{M \cdot a}{GJ_p},$$

bu yerda, $M_b^{(1)} = 2M$.

Valning ikkinchi uchastkasida: $a \leq x_2 \leq 2a$;

$$\varphi_2 = \varphi_B + \frac{M_b^{(2)} \cdot (x_2 - a)}{GJ_p};$$

$$x_2 = a \text{ da } \varphi_2 = \varphi_B = \frac{M \cdot a}{GJ_p};$$

$$x_2 = a \text{ da } \varphi_2 = 0; \text{ unda, } \frac{M \cdot a}{GJ_p} + \frac{M_b^{(2)} \cdot a}{GJ_p} = 0.$$

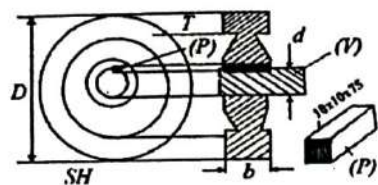
Bu yerda, $M_b^{(2)} = -M$.

Demak (1) uchastkada burovchi moment $2M$ ga, (2) uchastkada esa M ga teng ekan, topilgan qiymatlar bo'yicha M_b epyurasini quramiz.

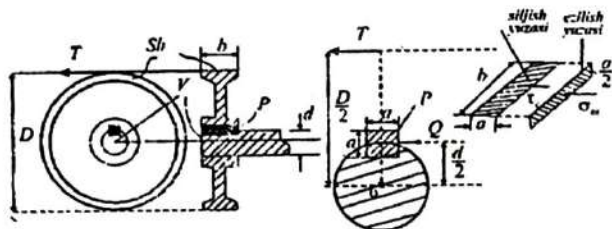
Epyuradan ko'rinib turibdiki, valdagi C nuqtaga M va B nuqtaga esa $3M$ ga teng bo'lgan burovchi momentlar ta'sir etayotgan ekan (14.30-shakl, e).

128-masala

(V) Val (Sh) shkiv bilan (P) pona vositasida birlashtirilgan (14.31-shakl). Ponaning yarmi valga, yarmi shkivga kirib turadi. Ponaning siljish va ezilish bo'yicha mustahkamligi tekshirilsin. $[\tau] = 900 \text{ kg/sm}^2$; $[\sigma]_{ez} = 2200 \text{ kg/sm}^2$; $T = 400 \text{ kg}$ ($D = 900 \text{ mm}$; $d = 60 \text{ mm}$; $b = 75 \text{ mm}$; $a = 10 \text{ mm}$).



14.31-shakl



14.32-shakl

Yechish:

14.32-shakldan val (V) va shkiv (Sh) ni birlashtiruvchi (P) ponaga tushadigan kuchni aniqlaymiz:

$$\sum m_0 = 0; T \cdot \frac{D}{2} - Q \cdot \frac{d}{2} = 0; Q = T \cdot \frac{D}{d} = 400 \cdot \frac{90}{6} = 6000 \text{ kg.}$$

Ponada hosil bo'ladigan kesuvchi urinma kuchlanish quyidagiga teng bo'ladi:

$$\tau_c = \frac{Q}{a \cdot b} = \frac{6000 \text{ kg}}{1 \cdot 7,5 \text{ sm}^2} = 800 \text{ kg/sm}^2 < 900 \text{ kg/sm}^2$$

Eguvchi normal kuchlanish:

$$\sigma_{ez} = \frac{Q}{\frac{a}{2} \cdot b} = \frac{6000 \text{ kg}}{0,5 \cdot 7,5 \text{ sm}^2} = 1600 \text{ kg/sm}^2 < 2200 \text{ kg/sm}^2$$

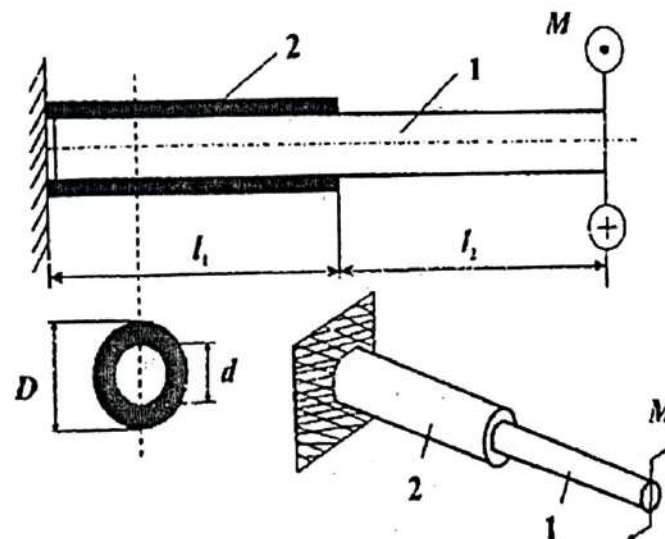
Demak, birikma mustahkam.

129-masala

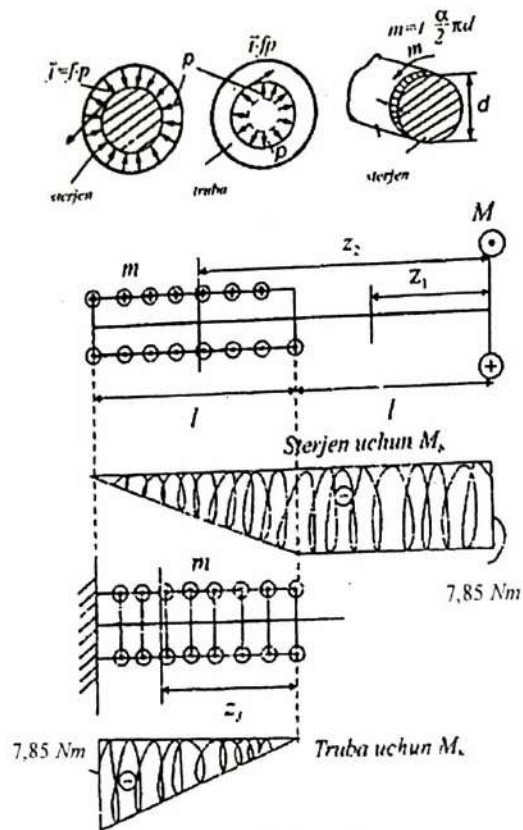
Uzunligi 2 l bo'lgan (1) val I qismining chap kesimi bikir mahkamlangan (2) trubka ichida qisilgan holda joylashgan (14.33-shakl). Trubkaning val sirtiga ko'rsatadigan bosimi p val sirti bo'yicha tekis tarqalgan. Trubka va val orasidagi ishqalanish koeffitsiyenti f ga teng. Valning uchiga ta'sir etuvchi moment M qanday qiymatga erishganda val (1) trubka (2) ga nisbatan aylana boshlaydi?

Mazkur holatda (1) val va (2) trubka uchun burovchi moment epyuralari qurilsin.

Berilgan: D = 12,5 mm; d = 10 mm; p = 2,5 MPa; l = 100 mm; f = 0,2.



14.33-shakl



14.34-shakl

Yechish:

Kesish usulidan foydalanib, 2-uchastka uchun burovchi moment tenglamasini tuzamiz (14.34-shakl):

$$\sum \text{mom}_z = 0; \quad M - ml = 0; \quad \text{unda, } M = \frac{1}{2} p f d^2 \pi l = -7,85 \text{ Nm}.$$

$M \geq 7,85 \text{ Nm}$ da sterjen trubaga nisbatan aylana boshlaydi.

Burovchi moment epyurasini quramiz:

1. Val uchun

I uchastkada: $0 \leq z_1 \leq l$; $M_1 = -M = -7,85 \text{ Nm}$

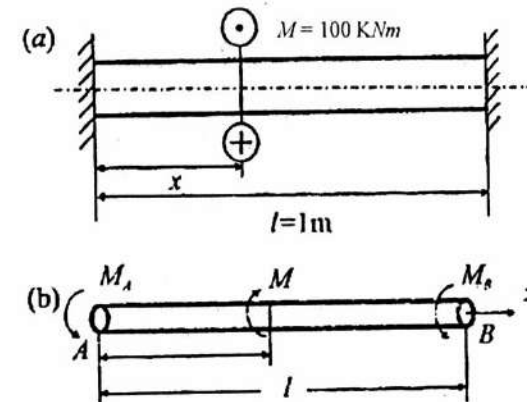
II uchastkada: $l \leq z_2 \leq 2l$; $M_2 = -M + m(z_2 - l)$.

2. Truba uchun $M_3 = -mz_3$; $0 < z_3 < l$.

130-masala

Valni (14.35-shakl) mustahkamlik va bikirlik shartlaridan aniqlangan diametri bir xil qiymatga ega bo'lishi uchun valga ta'sir etuvchi juft kuch M val tayanchidan ($x = ?$) qanday masofaga qo'yilishi kerak?

Berilgan: $[\tau] = 100 \text{ MPa}$; $[\theta] = 0,5^\circ (\text{grad/m})$; $G = 8 \cdot 10^4 \text{ MPa}$



14.35-shakl

Yechish:

Statika tenglamasini tuzamiz:

$$\sum m_z = 0; \quad M_A + M_B = M. \quad (1)$$

$\varphi_B = 0$ shartidan deformatsiya tenglamalarini tuzamiz:

$$\left. \begin{aligned} M_B &= \frac{M \cdot x}{l} \\ M_A &= M \left(1 - \frac{x}{l} \right) \end{aligned} \right\} \quad (2)$$

Mustahkamlik sharti bo'yicha:

$$\tau_{max} = \frac{M}{W_p} \leq [\tau]; \quad D_1 = \sqrt[3]{\frac{10^5 \cdot 16 \left(1 - \frac{x}{l}\right)}{\pi \cdot 10^8}} = 0,72 \sqrt[3]{\left(1 - \frac{x}{l}\right)}.$$

Bikirlilik sharti bo'yicha:

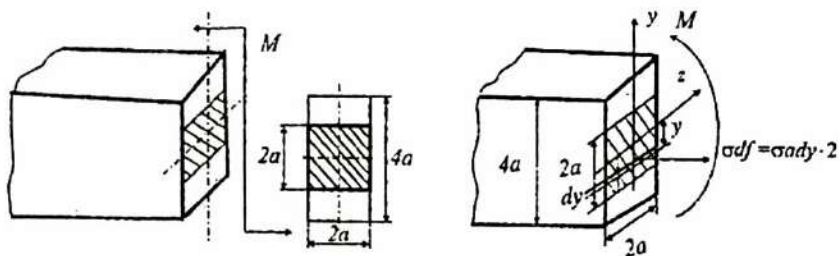
$$\varphi = \frac{M \cdot (l-x)x \cdot 32}{\pi D^4 G l^2} \leq \frac{0,5}{57,3}; \quad D_2 = 0,195 \sqrt[4]{\frac{(l-x)x}{l}}.$$

$D_1 = D_2$ shartiga ko'ra, $x = 0,482l = 0,482$ m.

14.4-§. Egilishga doir masalalar

131-masala

Balka kesimidagi eguvchi moment M ga teng. Mazkur momentning necha foizini kesimning shtrixlangan qismi va necha foizini shtrixlanmagan qismi qabul qiladi? M, a lar berilgan deb hisoblansin (14.36-shakl).



14.36-shakl

Yechish:

Balka kesimidagi normal kuchlanish quyidagi formula yordamida aniqlanadi:

$$\sigma = \frac{M \cdot y}{J_z} \quad (1)$$

Shtrixlangan yuza uchun esa eguvchi moment quyidagiga teng:

$$\bar{M} = \iint_{(\bar{F})} \sigma \cdot y dF = \frac{M \cdot 2a}{J_z} \int_{-a}^a y^2 dy = \frac{M \cdot 2a}{J_z} \cdot \frac{y^3}{3} \Big|_{-a}^a = \frac{M}{J_z} \cdot \frac{4a^3}{3} = \frac{M}{J_z} \cdot \bar{J}_z,$$

bu yerda, $\bar{J}_z$ – kesim shtrixlangan qismining inersiya momenti.

$$\bar{M} = \frac{M}{J_z} \bar{J}_z \text{ tenglikdan:}$$

$$\frac{\bar{M}}{M} = \frac{\bar{J}_z}{J_z}; \quad J_z = \frac{32a^4}{3}; \quad \bar{J}_z = \frac{4a^4}{3}.$$

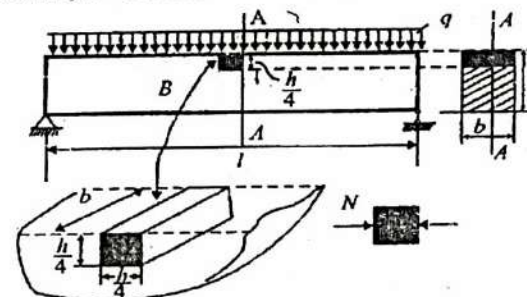
Unda,

$$\frac{\bar{M}}{M} = \frac{1}{8}; \quad \bar{M} = \frac{1}{8} M.$$

Demak, kesimning shtrixlangan qismi M ning $\frac{1}{8}$ yoki 12,5 %, shtrixlanmagan qismi esa $\frac{7}{8}$ yoki 87,3% ni qabul qiladi.

132-masala

Yog'och balka o'rta kesimining yuqorisida kvadrat kesimli $\frac{h}{4} \times \frac{h}{4}$ o'yma hosil qilinib, bu o'ymaga shu o'lchamga ega bo'lgan yog'och brus zich qilib joylashtirilgan (14.37-shakl). Mazkur (B) brusga ta'sir etuvchi (N) kuchning qiymati aniqlansin (brus siqiladi). $q = 0,4$ t/m; $l = 4$ m; $b = 15$ sm; $h = 30$ sm.



14.37-shakl

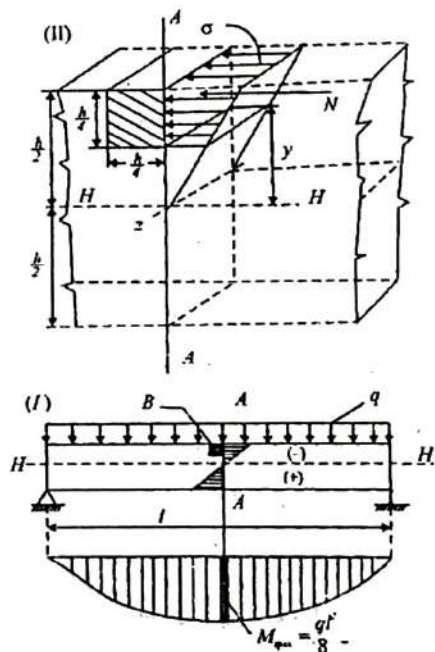
Yechish:

Balka $\frac{h}{4} \times \frac{h}{4}$ oralig'ining o'rtasida eguvchi moment M eng katta qiymatga erishadi:

$M_{max} = \frac{ql^2}{8}$ (14.38-shakl, b). Balkaning neytral o'qidan (H-H) yuqori qismi siqiladi. Demak, (B) brus tomonlariga siquvchi kuch ta'sir etadi.

A-A kesimdagi kuchlanlik holati 14.38-shakl (a) da keltirilgan. Brus (B) tomonlariga ta'sir etuvchi siquvchi kuch N shu tomonga qo'yilgan kuchlanishlarning yig'indisiga tengdir:

$$N = \int_{h/4}^h \sigma dF = \int_{h/4}^h \frac{M_{max} \cdot y}{J_z} dF.$$



14.38-shakl

Bu yerda,

$$dF = b \cdot dy; \quad J_z = \frac{bh^3}{12}.$$

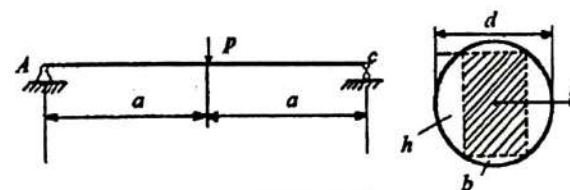
Unda,

$$N = \frac{\frac{ql^2}{8} \cdot b}{\frac{bh^3}{12}} \int_{h/4}^{h/2} y dy = \frac{3ql^2}{2h^3} \int_{h/4}^{h/2} y dy = B \frac{y^2}{2} \Big|_{h/4}^{h/2}$$

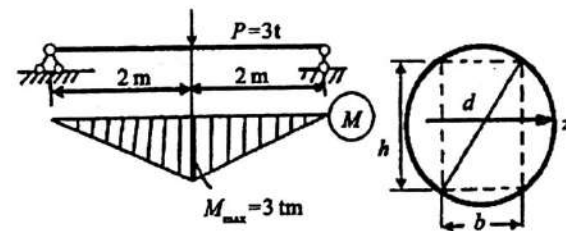
$$N = \frac{B}{2} \left(\frac{h^2}{4} - \frac{h^2}{16} \right) = \frac{B}{2} \cdot \frac{3h^2}{16} = \frac{9ql^2}{64h} = \frac{9 \cdot 0,5 \cdot 4^2}{64 \cdot 0,3} = 2,94t.$$

133-masala

Doira kesimli yog'och balkadan to'g'ri to'rtburchak kesimli balka tayyorlangan (14.39-shakl). Bu balkaning mustahkamligi eng katta bo'lishi uchun to'sin diametri (d) va to'rtburchak tomonlari nimaga teng bo'ladi? Yog'och uchun $[\sigma] = 100 \text{ kg/sm}^2$; $P = 3t$; $a = 2 \text{ m}$



14.39-shakl.



14.40-shakl.

Yechish:

Ma'lumki, $W_2 = W_{max}$ bo'lsa, balkaning mustahkamligi eng katta bo'ladi. Unda,

$$W_z = \frac{bh^2}{6} = \frac{b(d^2 - b^2)}{6}; \quad \frac{dW_z}{db} = d^2 - 3b^2 = 0.$$

$$\text{Bu yerdan, } b = \frac{d}{\sqrt{3}}; \quad h^2 = d^2 - \frac{d^2}{3} = \frac{2}{3}d^2; \quad h = \frac{\sqrt{2}}{\sqrt{3}}d.$$

Unda,

$$\frac{b}{h} = \frac{\sqrt{2}}{2}; \quad W_z = \frac{d^3}{9\sqrt{3}}; \quad (1); \quad W_z = \frac{W_{\max}}{[\sigma]} = 3000 \text{ sm}^3.$$

Bu tenglamadagi W_z ning o'rniga (1) ni qo'yib, quyidagini hosil qilamiz:

$$\frac{d^3}{9\sqrt{3}} = 3000 \text{ sm}^3.$$

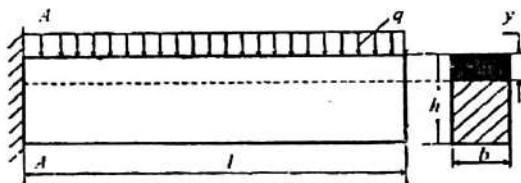
Bu yerdan, $d \approx 30 \text{ sm}$. Unda,

$$b \approx 0,58d = 17,4 \text{ sm};$$

$$h \approx 0,82d = 24,49 \text{ sm}.$$

134-masala

Yog'och balka tayanch kesimi (A-A) ning yuqori qismidan arralangan (14.41-shakl). Qancha arralanganda ($y=?$) balkaning mustahkamligi 50% ga kamayadi. q , l , $h = 2b$ berilgan deb qabul qilinsin.



14.41-shakl

Yechish:

Balka xavfli kesimidan arralangan.

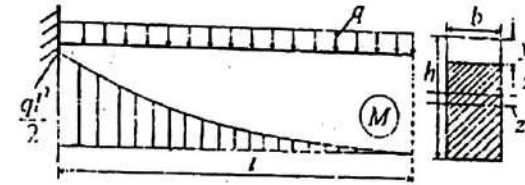
Balkaning mustahkamlik shartidan: $M_{\max} \leq [\sigma] \cdot W_z$. Qarshilik momenti W_z qanchaga kamaysa, mustahkamlik ham shunchaga kamayadi. Demak, arralangan balkaning mustahkamligi 0,5 marta kamayishi uchun:

$$\bar{W}_z = 0,5W_z; \quad \frac{b(h-y)^2}{b} = 0,5 \frac{bh^2}{2}.$$

Bu tenglamadan,

$$y^2 - 2yh + 0,5h^2 = 0.$$

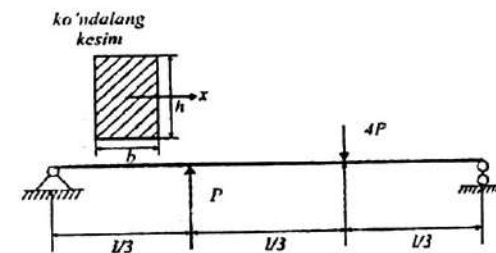
U holda, $y_1 = 1,7h$; $y_2 = 0,3h$. Bu yerda, $y_1 = 1,7h$ ma'noga zid yechim bo'lganligi sababli, $y = 0,3h$ qabul qilinadi.



14.42-shakl

135-masala

$\frac{l}{h}$ nisbatning qanday qiymatlarida balka uchun normal va urinma kuchlanishlar bo'yicha mustahkamlik shartlari bajariladi? Balka materiali uchun ruxsat etilgan normal kuchlanish $[\sigma]$ urinma kuchlanish $[\tau] = 0,1[\sigma]$; P , l , b lar berilgan deb hisoblansin (14.43-shakl).



14.43-shakl

Yechish:

1. Balkaning A va B nuqtalaridagi reaksiya kuchlari quyidagilarga teng:

$$V_A = \frac{2}{3}P; \quad V_B = \frac{7}{3}P.$$

2. Topilgan qiymatlar bo'yicha Q va M epyuralarini quramiz. Bu epyuralardan (14.44-shakl): $M_{max} = \frac{7}{9}Pl$; $Q_{max} = \left| -\frac{7P}{3} \right|$.

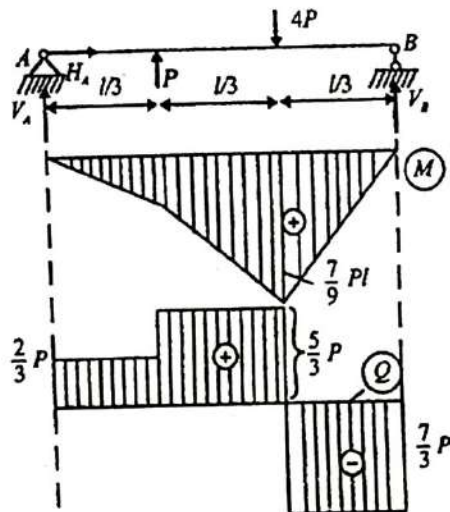
3. Balka uchun mustahkamlik shartlari quyidagiga teng:

$$\sigma_{max} = \frac{M_{max}}{W_z} \leq [\sigma], \quad (1)$$

bu yerda, $W_z = \frac{bh^2}{6} = \frac{h^3}{9}$.

$$\tau_{max} = \frac{Q_{max} \cdot S_z}{J_z \cdot b} \leq [\tau]; \quad (2)$$

$$F = \frac{h^2}{1,5}; \quad \frac{h}{b} = 1,5.$$



14.44-shakl

(1) dan:

$$\frac{7Pl}{h^3} \leq [\sigma]; \quad (3)$$

$$[\tau] = 0,1[\sigma]. \quad (4)$$

U holda,

$$\tau_{max} = \frac{3Q}{2F} = \frac{21P}{4h^2}. \quad (5)$$

(2), (3), (5) dan:

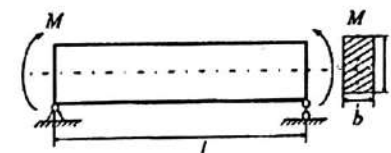
$$\frac{21P}{4h^2} \leq 0,1 \frac{7Pl}{h^3}.$$

Bu yerdan, $\frac{l}{h} \geq \frac{30}{4} = 7,5$.

136-masala

14.45-shaklda berilgan yog'och balkaning tag tomoniga mustahkamligi yuqori bo'lgan yupqa metall tasma yopishtirib qo'yilgan. Yog'och balka uchun $[\sigma] = 100 \text{ kg/sm}^2$; $l = 5 \text{ m}$; $b = 10 \text{ sm}$; $h = 20 \text{ sm}$. Balkaga ta'sir etuvchi M ning ruxsat etilgan qiymati aniqlansin.

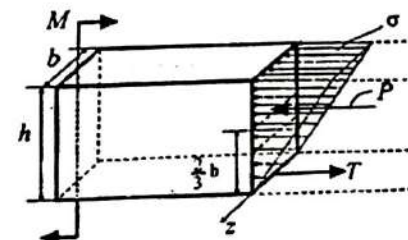
Izoh: metall tasma uzilmaydi, yog'ochdan ko'chib ketmaydi hamda uzunligi o'zgarmaydi.



14.45-shakl

Yechish:

Balka kesimining neytral o'qi Z balka tagidagi tasmadan o'tadi. T – tasmadagi zo'riqish. P – balka kesimidagi normal kuchlanishning teng ta'sir etuvchisi (14.46-shakl).



14.46-shakl

Statika tenglamasini tuzamiz:

$$\sum \text{mom}_z = 0; -M + \frac{\sigma \cdot h}{2} \cdot b \cdot \frac{2}{3} h = 0;$$

bu yerdan,

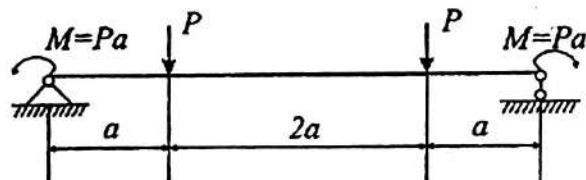
$$M = \frac{\sigma h^2 b}{3}.$$

$$\sigma = [\sigma] \text{ da } [M] = [\sigma] \frac{h^2 b}{3} = \frac{4}{3} \cdot 10^5 \text{ kg.sm} = \frac{4}{3} 10^3 \text{ kg.m.}$$

14.5-§. Egilishda deformatsiyalarni aniqlash

137-masala

Balka egilgan o'qining shakli qurilsin. P , a berilgan deb hisob-lansin (14.47-shakl).



14.47-shakl

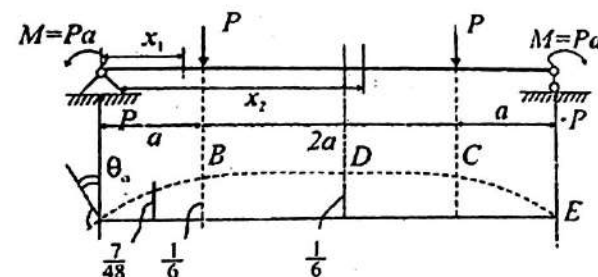
Yechish:

Balka uchastkalari bo'yicha universal tenglamalar tuzamiz (14.48-shakl):

$$Y_1 = Y_0 + \theta_0 x_1 + \frac{1}{EJ} \left(\frac{-Pax_1^2}{2} + \frac{Px_1^3}{6} \right); \quad 0 \leq x_1 \leq a;$$

$$Y_2 = Y_0 + \theta_0 x_2 + \frac{1}{EJ} \left(\frac{-Pax_2^2}{2} + \frac{Px_2^3}{6} - \frac{P(x_2 - a)^3}{6} \right); \quad a \leq x_2 \leq 2a;$$

$$\theta_2 = \frac{dY_2}{dx_2} = \theta_0 + \frac{1}{EJ} \left(-Pax_2 + \frac{Px_2^2}{2} - \frac{P(x_2 - a)^2}{2} \right).$$



14.48-shakl

Bu tenglamalarda:

$$Y_0 = 0. \quad \text{Unda,} \quad \theta_2/x_2=2a = 0 \Rightarrow \theta_0 = \frac{Pa^2}{2EJ};$$

$$Y_{\frac{x_1}{x_1}=a} = \frac{Pa^2}{2EJ} a + \frac{1}{EJ} \left(\frac{-Pa^3}{2} + \frac{Pa^3}{6} \right) = \frac{Pa^3}{6EJ}$$

$$Y_{\frac{x_2}{x_2}=2a} = \frac{Pa^3}{6EJ};$$

$$\theta_1 = \frac{dY_1}{dx_1}; \quad \theta_{\frac{x_1}{x_1}=a} = \frac{Pa^2}{2EJ} + \frac{1}{EJ} \left(-Pa^2 + \frac{Pa^2}{2} \right) = 0;$$

$$x_1 = \frac{a}{2} \text{ da } Y_{1(\frac{a}{2})} = \frac{Pa^2}{2EJ} \cdot \frac{a}{2} + \frac{1}{EJ} \left(\frac{-Pa^3}{8} + \frac{Pa^3}{48} \right) = \frac{7Pa^3}{48EJ}.$$

Demak,

$$\theta_B = \theta_D = \theta_C = 0;$$

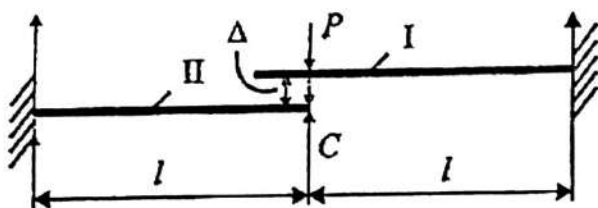
$$Y_B = Y_C = Y_D = \frac{Pa^3}{6EJ}.$$

Egilgan o'qning BC qismi to'g'ri chiziqdan iborat.

138-masala

I balka Δ ga egilganda C nuqtada II balkaga urinadi, so'ng ikkala balka birgalikda egila boshlaydi (14.49-shakl). Balkalar uchun EJ , l , Δ berilgan deb qabul qilinsin.

1) P ning qanday qiymatida balkalar urinadi? 2) Balkalar birgalikda egilganda balkalar (I, II) ning kesimlaridagi ichki kuchlar aniqlansin. Eguvchi moment M epyurasi qurilsin.



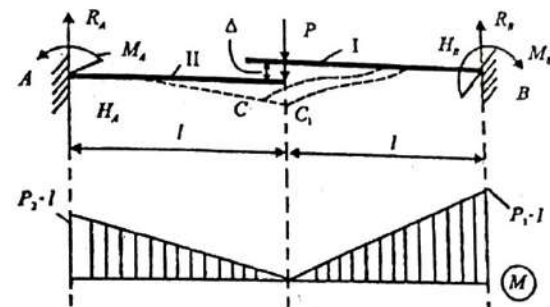
14.49-shakl

Yechish:

Konsol balkaning uchidagi solqiligi $Y = \frac{Pl^3}{3EJ}$ formula bilan aniqlanadi.

Agarda $\frac{Pl^3}{3EJ} = \Delta$ bo'lsa, I balka II balkaning uchiga C nuqtada urinadi (14.50-shakl). Bunda I balka II balkaga uringan vaqtdagi P kuch quyidagiga teng bo'ladi:

$$P = \frac{3EJ\Delta}{l^3}.$$



14.50-shakl

Uringandan so'ng, $P > P$ da ikkala balka birgalikda egila boshlaydi.

Masala statik noaniq bo'lib qoladi, chunki statika tenglamalari soni beshta, noma'lumlar soni esa 6 ta.

$$\left. \begin{aligned} \sum m_A = 0; \quad \sum x = 0 \\ \sum m_B = 0; \quad \sum y = 0 \\ \sum m_C = 0 \end{aligned} \right\} \quad (1)$$

Qo'shimcha tenglamani sistemaning deformatsiyalanish shaklidan hosil qilamiz.

$$Y_C^I = Y_C^{II} + \Delta \quad (2)$$

yoki

$$\frac{P_1 l^3}{3EJ} = \frac{P_2 l^3}{3EJ} + \Delta. \quad (3)$$

C nuqtaga ta'sir etayotgan kuchlardan Y o'qiga proyeksiya olsak:

$$\begin{aligned} \sum Y = 0; \quad P_1 + P_2 = P; \\ P_1 = P - P_2; \end{aligned} \quad (4)$$

(4) ni (3) ga olib borib qo'yamiz:

$$\frac{(P - P_2) l^3}{3EJ} = \frac{P_2 l^3}{3EJ} + \Delta \Rightarrow P_2 = \frac{1}{2} \left(P - \frac{EJ\Delta}{l^3} \right);$$

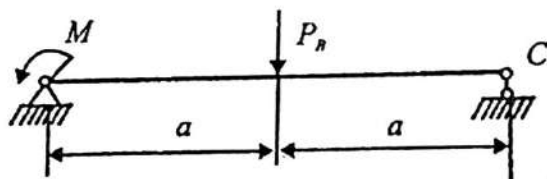
yoki

$$P_1 = P - P_2 = \frac{1}{2} \left(P + \frac{3EJ\Delta}{l^3} \right).$$

P_1 va P_2 ma'lum bo'lgandan so'ng, I va II balkalar uchun M epyurasi quriladi.

139-masala

Balkaga ta'sir etayotgan kuch – P , masofa – a , balkaning bikirligi – EJ berilgan (14.51-shakl). M ning qanday qiymatida balka egilgan o'qining BC oralig'i to'g'ri chiziqdan iborat bo'ladi?



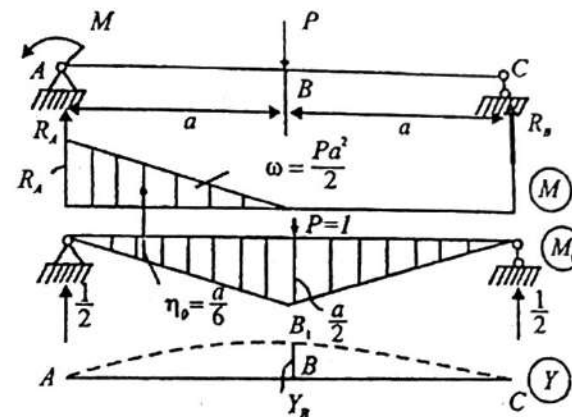
14.51-shakl

Yechish:

Balka egilgan o'qining BC oralig'i ushbu oraliqda $M(x)$ bo'lsa, to'g'ri chiziq bo'ladi. Haqiqatan ham, $EJy'' = 0$ dan $EJy = Cx + D$ – to'g'ri chiziq. $M(x) = 0$ bo'lganligidan $R_B = 0$ ekanligi kelib chiqadi.

Statika tenglamalarini tuzib, M va R_A larni aniqlaymiz:

$$\begin{aligned} \sum \text{mom}_A &= 0; & M - Pa &= 0; & M &= Pa; \\ \sum Y &= 0; & R_A - P &= 0; & R_A &= P. \\ Y_B &= \frac{\omega \cdot \eta_0}{EJ} = \frac{\frac{Pa^2}{2} \cdot \left(-\frac{a}{6}\right)}{EJ} = -\frac{Pa^3}{12EJ}. \end{aligned}$$



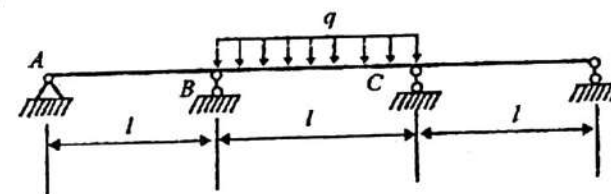
14.52-shakl

Ishora «-» bo'lganligi uchun B kesimdagi solqilik $P = 1$ kuch yo'nalishiga teskari.

140-masala

Balka B va C tayanchlarining vertikal ko'chishlari qanday qiymatga teng bo'lganda, mazkur kesimlarda eguvchi momentlar nolga teng bo'ladi? q, l, EJ lar berilgan deb qabul qilinsin (14.53-shakl).

Izoh: B va C tayanchlar faqat vertikal yo'nalishda ko'chish imkoniyatiga ega.



14.41-shakl

Yechish:

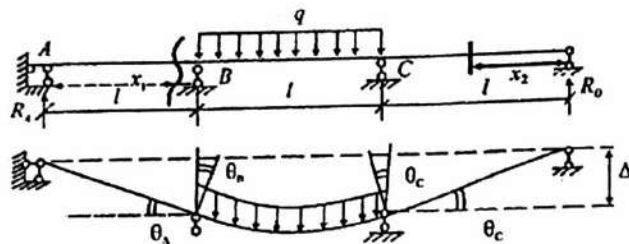
Masala sharti bo'yicha: $M_B = R_A \cdot l = 0$; $M_C = R_D \cdot l = 0$.

Bundan, $R_A = R_D = 0$; $M_1 = R_A \cdot x_1 = 0$; $M_2 = R_D \cdot x_2 = 0$.

Ya'ni, AB, CD uchastkalarda eguvchi moment nolga teng.

Shu uchastkalar uchun $EJy'' = 0$; $EJy = C_1 \cdot x + C_2$. Bu esa to'g'ri chiziqli tenglamasi.

Demak, deformatsiya vaqtida AB va CD uchastkalar to'g'ri chiziqli holida qoladi.



14.54-shakl

Agar A va D tayanchlar bo'lmaganda, balkaning A va D kesimlari Δ ga yuqoriga ko'tarilgan bo'lar edi. Lekin A va D kesimlarda qo'zg'almas tayanch bor. Shuning uchun C va B kesimlarni Δ ga pastga tushiramiz (14.54-shakl).

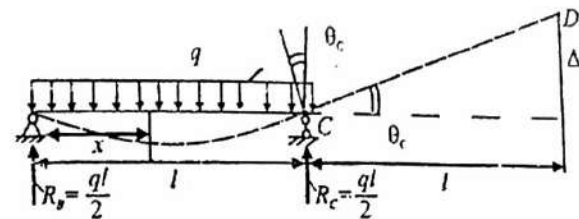
$$y = y_0 + \theta_B x + \frac{1}{EJ} \left(\frac{ql}{2} \cdot \frac{x^3}{6} - \frac{qx^4}{24} \right).$$

Bu yerda, $x = l$ da $y(l) = 0$; u holda,

$$\theta_B l + \frac{1}{EJ} \left(\frac{ql^4}{12} - \frac{ql^4}{24} \right) = 0 \Rightarrow \theta_B = -\frac{ql^3}{24EJ}; \theta_C = |\theta_B| = \frac{ql^3}{24EJ};$$

$$\Delta = l \tan \theta_C \approx l \theta_C = \frac{ql^4}{24EJ},$$

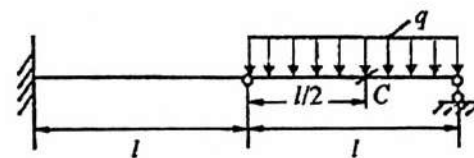
Demak, B va C tayanchlarni vertikal holda $\Delta = \frac{ql^4}{24EJ}$ masofaga pastga ko'chirish kerak.



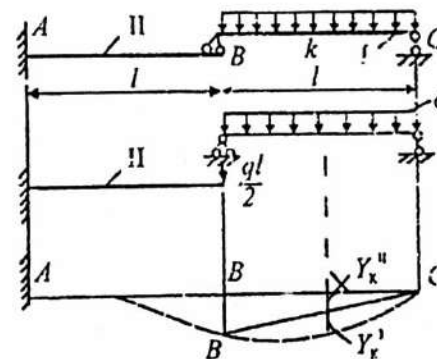
14.55-shakl

141-masala

Balkaning C kesimidagi solqiligi, nazariy usullardan foydalanilmasdan, oldin ma'lum bo'lgan yechimlardan aniqlansin. q , l , EJ lar ma'lum deb qabul qilinsin (14.56-shakl).



14.56-shakl



14.57-shakl

Yechish:

II konsol balkaning egilishi hisobiga I balka absolyut qattiq jism kabi ko'chadi (14.57-shakl), ya'ni:

$$y_k'' = -\frac{5ql^3}{384 EJ}; \quad y_k'' = \frac{BB_1}{2} = -\frac{ql^4}{12 EJ},$$

bu yerda,

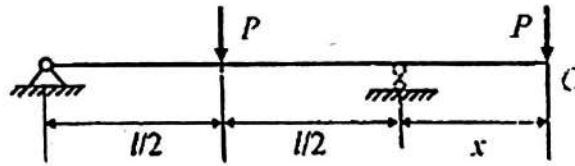
$$BB_1 = y_B'' = -\frac{\frac{ql}{2} \cdot l^3}{3 EJ} = -\frac{ql^4}{6 EJ},$$

unda,

$$y_k = y_k' + y_k'' = -\frac{5ql^4}{384 EJ} - \frac{ql^4}{12 EJ} = -\frac{37ql^4}{384 EJ}.$$

142-masala

Balka konsolining uzunligi x nimaga teng bo'lganda, C kesimdagi solqilik nolga teng bo'ladi? P , l , EJ berilgan deb hisoblansin (14.58-shakl).



14.58-shakl

Yechish:

Balkani ikkita (I va II) balkalar yig'indisi deb qaraymiz (14.59-shakl). U holda,

$$Y_c = Y_c' + Y_c'' \quad (1)$$

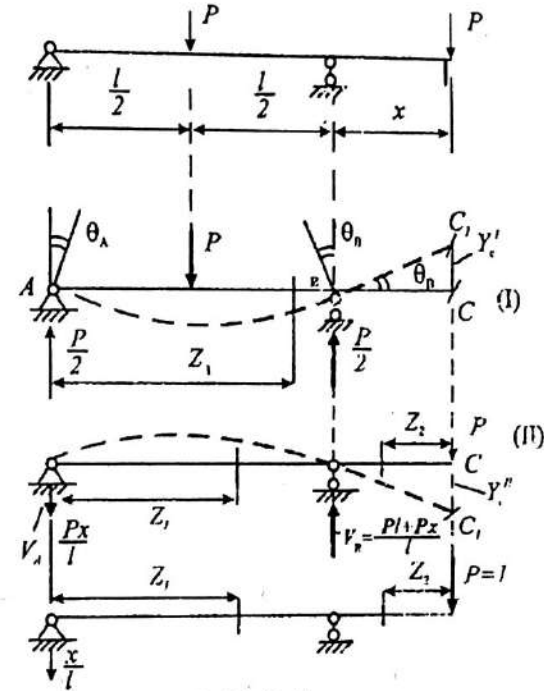
I balka uchun:

$$Y = Y_0 + \theta_0 z + \frac{1}{EJ} \cdot \frac{Px^3}{12},$$

$$Y_0 = 0; \quad \theta = \frac{dy}{dx} = \theta_0 + \frac{Px^2}{4 EJ},$$

bu yerda, $x = \frac{l}{2}$ da $\theta = 0$; $\theta_0 = -\frac{Pl^2}{16 EJ}$; $\theta_A = -\theta_B$;

$$\theta_B = \frac{Pl^2}{16 EJ}; \quad Y_c' = -\frac{Pl^2}{16 EJ} \cdot x. \quad (2)$$



14.59-shakl

II balka uchun Mor integralidan foydalanamiz.

$$Y_c = \int_0^l \frac{M_1 \bar{M}_1}{EJ} \cdot dz_1 + \int_0^x \frac{M_2 \bar{M}_2}{EJ} \cdot dz_2 = \frac{1}{EJ} \left(\frac{Px^2 l}{3} + \frac{Px^3}{3} \right), \quad (3)$$

bu yerda, $M_1 = -\frac{Px}{l} \cdot z_1$; $M_2 = -Pz_2$,

$$\bar{M}_1 = -\frac{x}{l} \cdot z_1; \quad \bar{M}_2 = -z_2.$$

(2) va (3) ni (1) ga qo'yamiz:

$$\frac{Pl^3}{16 \cdot EJ} - \frac{1}{EJ} \left(\frac{Plx^2}{3} + \frac{Px^3}{3} \right) = 0.$$

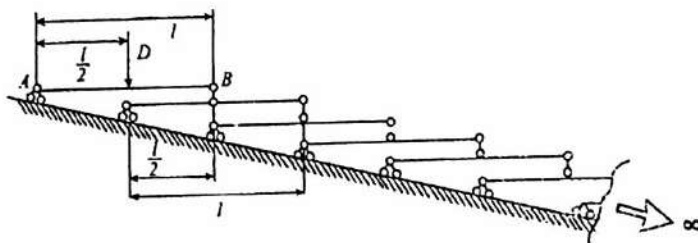
Bu yerdan,

$$16x^2 + 16xl - 3l^2 = 0.$$

$$x_{1,2} = \frac{-16 \pm 21,2}{32} \cdot l; \quad x_1 \approx 0,16l.$$

143-masala

AB balkaning o'rtasidagi solqiligi aniqlansin. P, l , barcha balkalarning egilishidagi bikirligi EJ doimiy va berilgan deb qabul qilinsin (14.60-shakl).



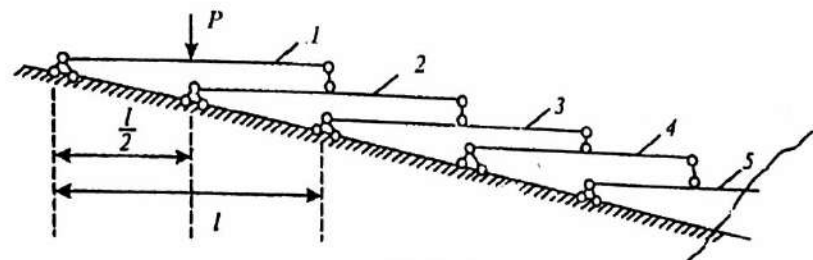
14.60-shakl

Yechish:

Balkalarning maksimal solqiliklari quyidagilarga teng (14.61-shakl):

$$\left. \begin{aligned} \text{1-balkaning maksimal solqiligi:} \quad Y_1 &= \frac{Pl^3}{48EJ}; \\ \text{2-balkaning solqiligi:} \quad Y_2 &= \frac{Pl^3}{2 \cdot 48EJ}; \\ \text{3-balkaga } \frac{P}{4} \text{ kuch uzatiladi, shuning uchun:} \quad Y_3 &= \frac{Pl^3}{4 \cdot 48EJ}; \\ \text{va h.k.} \quad Y_4 &= \frac{Pl^3}{8 \cdot 48EJ}; \end{aligned} \right\} (1)$$

1-balkada 2-balkaning solqiligi hisobiga $Y_{1,2} = \frac{Y_2}{2}$ (2) solqilik hosil bo'ladi.



14.61-shakl

3-balkaning solqiligi hisobiga esa

$$Y_{1,3} = \frac{Y_3}{4}. \quad (3)$$

U holda,

$$Y_{1,4} = \frac{Y_4}{8}; \quad Y_{1,5} = \frac{Y_5}{16}.$$

Demak, 1-balkaning C kesimidagi solqiligi:

$$Y_c = Y_{11} + Y_{12} + Y_{13} + Y_{14} + Y_{15} + \dots \quad (4)$$

(4) formulaga (1), (2) va (3) larni olib kelib qo'yamiz:

$$\begin{aligned} Y_c &= \frac{Pl^3}{48EJ} + \frac{Pl^3}{4 \cdot 48EJ} + \frac{Pl^3}{16 \cdot 48EJ} + \frac{Pl^3}{64 \cdot 48EJ} + \dots \\ &= \sigma_1 + \sigma_1 q + \sigma_1 q^2 + \sigma_1 q^3 + \dots \end{aligned}$$

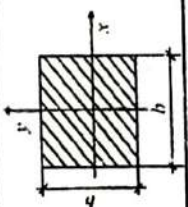
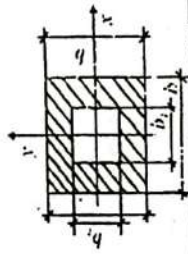
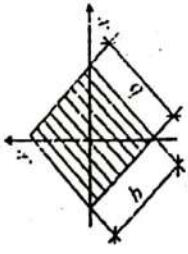
bu yerda, $\sigma_1 = \frac{Pl^3}{48EJ}.$

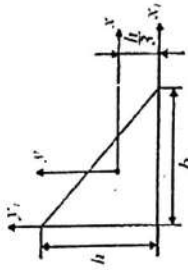
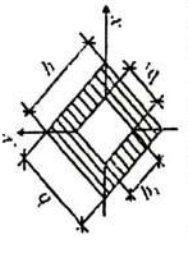
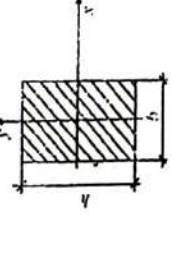
Shunga ko'ra, $q = \frac{1}{4}$ cheksiz kamayuvchi geometrik progressiya maxrajidir.

Yig'indi formulasi, $S = \frac{\sigma_1}{1-q}$

Unda, $Y_c = \frac{\sigma_1}{1-q} = \frac{Pl^3}{48EJ(1-\frac{1}{4})} = \frac{Pl^3}{36EJ}.$

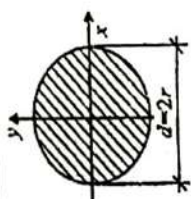
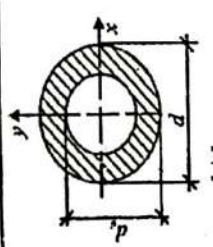
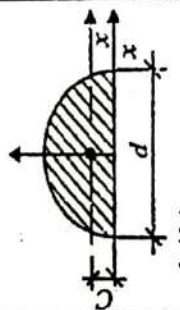
Ba'zi tekis kesimlarning yuzalari, og'irlik markazi va geometrik tavsifnomalari

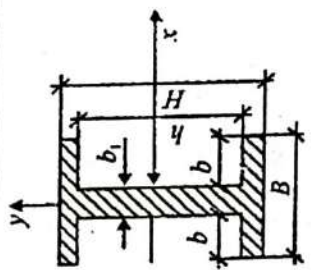
Kesim	Kesim yuzasi	O'q'larga nisbatan inersiya momenti	Qarshilik momenti	Inersiya radiusi
1	2	3	4	5
	b^2	$I_x = I_y = \frac{b^4}{12}$	$W_x = W_y = \frac{b^3}{6}$	$i_x = i_y = \frac{b}{\sqrt{12}}$
	$b^2 - b_1^2$	$I_x = I_y = \frac{b^4 - b_1^4}{12}$	$W_x = W_y = \frac{b^4 - b_1^4}{6b}$	$i_x = i_y = 0,289 \sqrt{b^2 + b_1^2}$
	b^2	$I_x = I_y = \frac{b^4}{12}$	$W_x = W_y = 0,118b^3$	$i_x = i_y = 0,289b$

1	2	3	4	5
	$\frac{hb}{2}$	$J_x = \frac{bh^3}{36}$ $J_{x_1} = \frac{bh^3}{12}$ $J_y = \frac{bh^3}{36}$ $J_{y_1} = \frac{hb^3}{12}$	Yuqoridagi tolalar uchun: $W_x = \frac{bh^2}{24}$ Pastki tolalar uchun: $W_x = \frac{bh^2}{12}$	Markazdan qochirma inersiya momentlari: $J_{xy} = \frac{h^2 b^2}{72}$ $J_{x_1 y_1} = \frac{b^2 h^2}{24}$
	$b^2 - b_1^2$	$I_x = I_y = \frac{b^4 - b_1^4}{12}$	$W_x = W_y = \frac{b^4 - b_1^4}{6\sqrt{2} \cdot b}$	$i_x = i_y = 0,289 \sqrt{b^2 + b_1^2}$
	bh	$I_x = \frac{bh^3}{12}$ $I_y = \frac{b^3 h}{12}$	$W_x = \frac{bh^2}{6}$ $W_y = \frac{b^2 h}{6}$	$i_x = \frac{h}{\sqrt{12}}$ $i_y = \frac{b}{\sqrt{12}}$

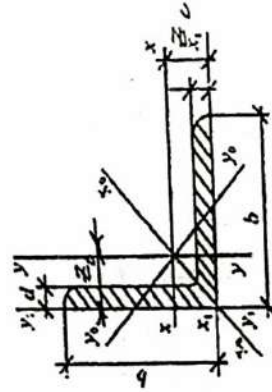
1	2	3	4	5
	$bh - b_1h_1$	$I_x = \frac{bh^3 - b_1h_1^3}{12}$ $I_y = \frac{b^3h - b_1^3h_1}{12}$	$W_x = \frac{bh^3 - b_1h_1^3}{6h}$ $W_y = \frac{b^3h - b_1^3h_1}{6b}$	$i_x = \sqrt{\frac{bh^3 - b_1h_1^3}{12(bh - b_1h_1)}}$ $i_y = \sqrt{\frac{b^3h - b_1^3h_1}{12(bh - b_1h_1)}}$
	$\frac{1}{2}bh$	$I_x = \frac{bh^3}{36}, I_{x1} = \frac{bh^3}{12}$ $I_y = \frac{b^3h}{48}$	Uchburchak uchidagi kuchlanishni aniqlashda: $W_x = \frac{bh^2}{24}$ Uchburchak asosidagi nuqtalarning kuchlanishini aniqlashda: $W_x = \frac{bh^2}{12}$	$i_x = 0,236 h$
	$\frac{1}{2}bh$	$I_x = \frac{bh^3}{48}$	$W_x = \frac{bh^2}{24}$	$i_x = 0,204 b$

1	2	3	4	5
	$\frac{b+b_1}{2}h$	$I_x = \frac{h^3(b^2 + 4bb_1 + b_1^2)}{36(b+b_1)}$	Yuqori asosdagi nuqtalarning kuchlanishini aniqlashda: $W_x = \frac{h^2(b^2 + 4bb_1 + b_1^2)}{12(2b+b_1)}$ Pastki asosdagi nuqtalarning kuchlanishini aniqlashda: $W_y = \frac{h^2(b^2 + 4bb_1 + b_1^2)}{12(b+2b_1)}$	$i_x = \frac{h}{6(b+b_1)} \times \sqrt{2(b^2 + 4bb_1 + b_1^2)}$
	$\frac{b+b_1}{2}h$	$I_x = \frac{h}{48} \cdot \frac{b^4 - b_1^4}{b - b_1}$	$W_x = \frac{h}{24} \cdot \frac{b^4 - b_1^4}{b^2 - bb_1}$	$i_x = \sqrt{\frac{b^2 + b_1^2}{24}}$
	$2,598 a^2$	$I_x = I_y = 0,541 a^4$	$W_x = 0,625 a^3$ $W_y = 0,541 a^3$	$i_x = i_y = 0,4565 a$

1	2	3	4	5
	$\frac{\pi d^2}{4}$	$I_x = I_y = \frac{\pi d^4}{64} = \frac{\pi r^4}{4}$	$W_x = W_y = \frac{\pi d^3}{32} = \frac{\pi r^3}{4}$	$i_x = i_y = \frac{d}{4} = \frac{r}{2}$
	$\frac{\pi(d-d_0^2)}{4}$	$I_x = I_y = \frac{\pi d^4}{64} (1-c^4)$ $C = \frac{d_0}{d}$	$W_x = W_y = \frac{\pi d^3}{32} (1-c^4)$	$i_x = i_y = \frac{1}{4} \sqrt{d^2 + d_0^2}$
	$\frac{\pi d^2}{8}$	$I_x = 0,00686 d^4$ $I_y = \frac{\pi d^4}{128}$	Pastiki tolalar uchun: $W_x = 0,0324 d^3$ Yuqoridagi tolalar uchun: $W_x = 0,0239 d^3$ $W_y = \frac{\pi d^3}{64}$	$i_x = 0,13 d$ $i_y = \frac{d}{4}$

1	2	3	4	5
	$2Bh_1 + b_1h$	$I_x = \frac{Bh^3 - 2bh^3}{12}$ $I_y = \frac{hb_1^3 - 2h_1b^3}{12}$	$W_x = \frac{BH^3 - 2bh^3}{GH}$ $W_y = \frac{hb_1^3 + 2h_1b^3}{6H}$	$i_x = \sqrt{\frac{I_x}{A}}$ $i_y = \sqrt{\frac{I_y}{A}}$

ГОСТ 8509-72 ga muvofiq prokat po'latning sortamenti



Yonlari teng burchakliklar

1-Jadval

Profilar nomet'i	O'lchamlari, mm		Kesim yuzasi A , sm ²	I_x , sm ⁴	I_{x_1} , sm ⁴	I_{x_0} max, sm ⁴	i_{x_0} max, sm ⁴	I_{y_0} min, sm ⁴	I_{y_0} min, sm ⁴	I_{y_0} min, sm ⁴	i_{y_0} min, sm ⁴	I_{x_1} , sm ⁴	i_{x_1} , sm ⁴	Massa 1 m, kg
	b	d												
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
5	50	3	2,96	7,11	1,55	11,3	1,95	2,95	1	12,4	1,33	2,38	2,38	2,38
		4	3,89	9,21	1,54	14,6	1,94	3,8	0,99	16,6	1,38	3,05	3,05	3,05
		5	4,8	11,2	1,53	17,8	1,92	4,63	0,98	20,9	1,42	3,77	3,77	3,77
		6	5,8	13,1	1,73	20,8	2,18	5,41	1,11	23,3	1,52	3,44	3,44	3,44
5,6	56	5	5,41	16	1,72	25,4	2,16	6,54	1,1	29,2	1,57	4,25	4,25	4,25

1-Jadval(davomi)

1	2	3	4	5	6	7	8	9	10	11	12	13
6,3	63	4	4,96	18,9	1,95	29,9	2,45	7,81	1,25	33,1	1,69	3,9
		5	6,13	23,1	1,94	36,6	2,44	9,52	1,25	41,5	1,74	4,81
		6	7,28	27,1	1,93	42,9	2,43	11,2	1,24	50	1,78	5,72
		7	8,42	31,9	2,16	46	2,72	12	1,39	51	1,88	6,87
7	70	8	10,7	48,2	2,13	76,4	2,68	20	1,37	91,9	2,02	8,37
		9	12,8	56,1	2,27	105	2,86	27,5	1,46	127	2,18	10,1
		10	15,1	66,1	2,47	138	3,11	36,6	1,59	166	2,37	12,1
		11	17,8	77,4	2,7	178	3,41	48,1	1,74	219	2,61	14,6
7,5	75	12	20,8	89,6	2,9	219	3,66	60,8	1,88	277	2,89	17,1
		13	24,1	104	3,1	277	4,01	81,2	2,06	349	3,19	20,6
		14	27,8	121	3,3	349	4,41	105	2,29	437	3,54	24,6
		15	31,9	141	3,6	437	4,86	138	2,54	549	3,94	29,6
8	80	16	36,6	166	3,9	549	5,36	178	2,84	687	4,39	35,6
		17	41,5	194	4,2	687	5,96	229	3,18	861	4,94	42,6
		18	46,6	229	4,5	861	6,66	296	3,54	1081	5,54	51,1
		19	52,7	277	4,9	1081	7,46	381	3,94	1381	6,19	61,1

1- Jadvai (davomi)

1	2	3	4	5	6	7	8	9	10	11	12	13
9	90	6	10,6	82,1	2,78	130	3,5	34	1,79	145	2,43	8,33
		7	12,3	94,3	2,77	150	3,49	38,9	1,78	169	2,47	9,65
		8	13,9	106	2,76	168	3,48	43,8	1,77	194	2,51	10,9
		9	15,6	118	2,75	186	3,46	48,6	- 1,77	129	2,55	12,2
10	100	6,5	12,8	122	3,09	193	3,88	50,7	1,99	214	2,68	10,1
		7	13,8	131	3,09	207	3,88	54,2	1,98	231	2,71	10,8
		8	15,6	147	3,07	233	3,87	60,9	1,98	265	2,75	12,2
		10	19,2	179	3,05	284	3,84	74,1	10,96	333	2,83	15,1
		12	22,8	209	3,03	331	3,81	86,9	1,95	402	2,91	17,9
		14	26,3	237	3	371	3,78	99,3	1,94	472	2,99	20,6
11	110	16	29,7	264	2,98	416	3,74	112	1,94	542	3,06	23,3
		7	15,2	176	3,4	279	4,29	72,7	2,19	308	2,96	11,9
12,5	125	8	17,2	198	3,39	315	4,28	81,8	2,18	353	3	13,5
		8	19,7	294	3,87	467	4,87	122	2,49	516	3,36	15,5
		9	22	327	3,86	520	4,86	135	2,48	582	3,4	17,3
		10	24,3	360	3,85	571	4,84	149	2,47	649	3,45	19,1
		12	28,9	422	3,82	570	4,82	174	2,46	782	3,53	22,7
		14	33,4	482	3,8	764	4,78	200	2,45	916	3,61	26,2
		16	37,8	539	3,78	853	4,75	224	2,44	1051	3,68	29,6

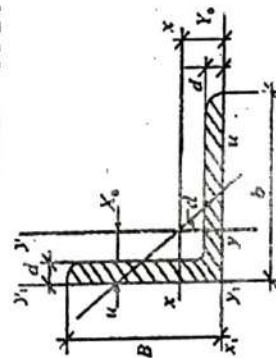
1- Jadvai (davomi)

1	2	3	4	5	6	7	8	9	10	11	12	13
14	140	9	24,7	466	4,34	739	5,47	192	2,79	888	3,78	19,4
		10	27,3	512	4,33	814	5,46	211	2,78	911	3,82	21,5
		12	32,5	602	4,31	957	5,43	248	2,76	1097	3,9	25,5
16	160	10	31,4	774	4,96	1229	6,25	319	3,19	1356	4,3	24,7
		11	34,4	844	4,95	1341	6,24	348	3,18	1494	4,39	27
		12	37,4	913	4,94	1450	6,23	376	3,17	1633	4,47	29,4
		14	43,3	1046	4,92	1662	6,2	431	3,16	1911	4,55	34
		16	49,1	1175	4,89	1866	6,17	485	4,14	2191	4,63	38,5
		18	54,8	1299	4,87	2061	6,13	537	3,13	2472	4,7	43
18	180	20	60,4	1419	4,85	2248	6,10	589	3,12	2756	4,85	47,7
		11	38,8	1216	5,6	1933	7,06	500	3,59	2128	4,89	30,5
		12	42,2	1317	5,59	2093	7,04	540	3,58	2324	5,37	33,1
20	200	12	47,1	1823	6,22	2896	7,84	749	3,99	3182	5,42	37
		13	5,9	1961	6,21	3116	7,83	805	3,98	3452	5,46	39,9
		14	54,6	2094	6,2	3333	7,81	861	3,97	3722	5,54	42,8
		16	62	2363	6,17	3755	7,78	970	3,96	4265	5,7	48,7
		20	76,5	2871	6,12	4560	7,72	1182	3,93	5355	5,89	60,1
		25	94,3	3466	6,06	5494	7,63	1438	3,91	6733	6,07	74
		30	111,5	4020	6	6351	7,55	1688	3,89	8130	6,07	87,6

1-jadval (davomi)

1	2	3	4	5	6	7	8	9	10	11	12	13
22	220	14	60,4	2814	6,83	4470	8,6	1159	4,38	4941	5,93	47,4
		16	68,6	3175	6,81	5045	8,58	1306	4,36	5661	6,02	53,8
25	250	16	78,4	4717	7,76	7492	9,78	1942	4,98	8286	6,75	61,5
		18	87,7	5247	7,73	8337	9,75	2158	4,96	9342	6,83	68,9
		20	97	5765	7,71	9160	9,72	2370	4,94	10401	6,91	76,1
		22	106,1	5270	7,69	9961	9,69	2579	4,93	11464	7	83,3
		25	119,7	7005	7,65	11125	9,64	2887	4,91	13064	7,11	94
		28	133,1	7717	7,61	12244	9,59	3190	4,89	14674	7,23	104,5
		30	142	8177	7,59	12965	9,56	3389	4,89	15753	7,31	111,4

ГОСТ 8510-72 ga muvofiq prokat po'latning sortamenti



Yonlari teng bo'lmagan burchakliklar

2-jadval

Pro- fillar nomeri	O'lchamlari, mm				Kesim yuzasi A, sm ²	I _x , sm ⁴	i _x , sm	I _y , sm	I _{xy} , sm ⁴	tg α	I _{x1} , sm ⁴	I _{y1} , sm ⁴	x ₀ , sm	y ₀ , sm	Mas- sa 1 m, kg		
	B	b	d														
1	2	3	4	5		6	7	8	9	10	11	12	13	14	15	16	17
5,6/3,6	56	36	4	3,58	11,4	1,78	3,7	1,02	2,19	0,78	0,406	23,2	6,25	0,84	1,82	2,81	
			5	4,41	13,8	1,77	4,48	1,01	2,66	0,78	0,404	29,2	7,91	0,88	1,86	3,46	
6,3/4	63	40	4	4,04	16,3	2,01	5,16	1,13	3,07	0,87	0,397	33	8,51	0,91	2,03	3,17	
			5	4,98	19,9	2	6,26	1,12	3,72	0,86	0,396	41,4	10,8	0,95	2,08	3,91	
			6	5,9	23,3	1,99	7,28	1,11	4,36	0,86	0,393	49,9	13,1	0,99	2,12	4,91	
			8	7,68	29,6	2,23	9,15	1,09	5,58	0,85	0,386	66,9	17,9	1,07	2,20	4,63	

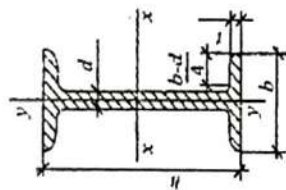
2- ilova (davomi)

2- Jadval (davomi)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
7/4.5	70	45	5	5.95	27.8	2.39	9.05	1.27	5.34	0.98	0.406	56.7	15.2	1.05	2.28	6.03
			5	6.11	34.8	2.38	12.5	1.43	7.24	1.09	0.436	69.7	20.8	1.17	2.39	4.39
7.5/5	75	50	6	7.25	40.9	2.35	14.6	1.42	8.48	1.08	0.435	83.9	25.2	1.21	2.44	4.79
			8	9.47	52.4	2.56	18.5	1.4	10.9	1.07	0.43	112	34.2	1.29	2.52	5.69
8/5	80	50	5	6.36	41.6	2.55	12.7	1.41	7.58	1.09	0.387	84.6	20.8	1.13	2.6	7.43
			6	7.55	49	2.88	14.8	1.4	8.88	1.08	0.386	102	25.2	1.17	2.65	4.99
			5.5	7.86	65.3	2.88	19.7	1.58	11.8	1.22	0.384	132	32.2	1.26	2.92	5.92
9/5.6	90	56	6	8.54	70.6	2.858	21.2	1.58	12.7	1.22	0.384	145	35.2	1.28	2.95	6.17
			8	11.18	90.9	3.2	27.1	1.56	16.3	1.21	0.38	194	47.8	1.36	3.04	6.7
			6	9.59	98.3	3.19	30.6	1.79	18.2	1.38	0.393	198	49.9	1.42	3.23	7.53
10/6.3	100	63	7	11.1	113	3.15	35	1.78	20.8	1.37	0.392	232	58.7	1.46	3.28	8.7
			8	12.6	127	3.53	39.2	1.77	23.4	1.36	0.391	266	67.6	1.5	3.32	9.87
			10	15.5	154	3.50	47.1	1.75	28.3	1.35	0.387	333	85.8	1.58	3.4	12.1
11/7	110	70	6.5	11.4	142	4.01	45.6	2	26.9	1.53	0.402	286	74.3	1.58	3.55	8.98
			8	13.9	172	4	54.6	1.98	32.3	1.52	0.4	353	92.3	1.64	3.61	10.9
			7	14.1	227	3.98	73.7	2.29	43.4	1.76	0.407	452	119	1.8	4.01	11
12.5/8	125	80	8	16	256	3.95	83	2.28	48.8	1.75	0.406	518	137	1.84	4.05	12.5
			10	19.7	312	4.49	100	2.26	59.3	1.74	0.404	649	173	1.92	4.14	15.5
			12	23.4	365	4.47	117	2.24	69.5	1.72	0.4	781	210	2	5.22	18.3

2- Jadval (davomi)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
14/9	140	90	8	18	364	5.15	120	2.58	70.3	1.98	0.411	727	194	2.03	4.49	14.1
			10	22.2	444	5.13	146	2.56	85.5	1.96	0.409	911	245	2.12	4.58	17.5
16/10	160	100	9	22.9	606	5.11	186	2.85	110	2.2	0.391	1212	300	2.23	5.19	18
			10	25.3	667	5.08	204	2.84	121	2.19	0.39	1359	335	2.28	5.23	19.8
			12	30	784	5.8	239	2.82	142	2.18	0.388	1634	405	2.36	5.32	23.6
			14	34.7	897	8.17	272	2.8	162	2.16	0.385	1910	477	2.43	5.4	27.3
18/11	180	110	10	28.3	952	5.7	276	3.12	165	2.42	0.375	1933	444	2.44	5.88	22.2
			12	33.7	1123	5.77	324	3.1	194	2.4	0.374	2324	537	2.52	5.97	26.4
			11	34.9	1449	6.45	446	3.58	264	2.75	0.392	2920	718	2.79	6.5	27.4
20/12.5	200	125	12	37.9	1568	6.43	482	3.57	285	2.74	0.392	3189	786	2.83	6.54	29.7
			14	43.9	1801	6.41	551	3.54	327	2.73	0.39	3726	922	2.91	6.62	34.4
			16	49.8	2026	6.38	617	3.52	367	2.72	0.388	4264	1061	2.99	6.71	39.1
			12	48.3	3147	8.07	1032	4.62	604	3.54	0.41	6212	1634	3.53	7.97	37.9
25/16	250	160	16	62.6	4091	8.02	1333	4.58	781	3.5	0.408	8308	2200	3.69	8.14	49.9
			18	71.7	4545	7.99	1475	4.56	866	3.49	0.407	9358	2487	3.77	8.23	55.8
			20	78.5	4987	7.97	1613	4.53	949	3.48	0.405	10410	2776	3.85	8.31	61.7



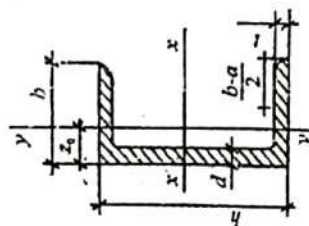
3- Jadval

Profilar raqami	O'lchamlari, mm				Kesim yuzasi A , sm^2	I_x , sm^4	W_x , sm^3	i_x , sm	S_x , sm^3	I_y , sm^4	W_y , sm^3	i_y , sm	massa 1 m, kg
	h	b	d	t									
1	2	3	4	5	6	7	8	9	10	11	12	13	14
10	100	55	4,5	7,2	12	198	39,7	4,06	23	17,9	6,49	1,22	9,46
12	120	64	4,8	7,3	14,7	350	58,4	4,88	33,7	27,9	8,72	1,38	11,5
14	140	73	4,9	7,5	17,4	352	81,7	5,73	46,8	41,9	11,5	1,55	13,7
16	160	81	5	7,8	20,2	873	109	6,57	62,3	58,6	14,5	1,70	15,9
18	180	90	5,1	8,1	23,4	1290	143	7,42	81,4	82,6	18,4	1,88	18,4
18a	180	100	5,1	8,3	25,4	1430	159	7,51	89,8	114	22,8	2,12	19,9
20	200	100	5,2	8,4	26,8	1840	184	8,28	104	115	23,1	2,07	21
20a	200	110	5,2	8,6	28,9	2030	203	8,37	114	155	28,2	2,32	22,7
22	220	110	5,4	8,7	30,6	2550	232	9,13	131	157	28,6	2,27	24
22a	220	120	5,4	8,9	32,8	2790	254	9,22	143	206	34,3	2,5	25,8

3- Jadval (davomi)

1	2	3	4	5	6	7	8	9	10	11	12	13	14
24	240	115	5,6	9,5	34,8	3460	289	9,97	163	198	34,5	2,37	27,3
24a	240	125	5,6	9,8	37,5	3800	317	10,1	178	260	41,6	2,63	29,4
27	270	125	6	9,8	40,2	5010	371	11,2	210	260	41,5	2,54	31,5
27a	270	135	6	10,2	43,2	5500	407	11,3	229	337	50	2,8	33,9
30	300	135	6,5	10,2	46,5	7080	472	12,3	268	337	49,9	2,69	36,5
30a	300	145	6,5	10,7	49,9	7780	518	12,5	292	436	60,1	2,95	39,2
33	330	140	7	11,2	53,8	9840	597	13,5	339	419	59,9	2,79	42,2
36	360	145	7,5	12,3	61,9	13380	743	14,7	423	516	71,1	2,89	48,6
40	400	155	8,3	13	71,6	19062	953	16,2	545	667	86,1	3,03	57
45	450	160	9	14,2	84,7	27696	1231	18,1	708	808	101	3,09	66,5
50	500	170	10	15,2	100	39727	1589	19,9	919	1043	123	3,23	78,5
55	550	180	11	16,5	118	55962	2035	21,8	1181	1356	151	3,39	92,6
60	600	190	12	17,8	138	76806	2560	23,6	1491	1725	182	3,54	108

Shvellerlar (ГОСТ 8240-72)



4- jadval

Profillar nomeri	O'lchamlari, mm				Kesim yuzasi A , sm^2	I_x , sm^4	W_x , sm^3	i_x , sm	S_x , sm^2	I_y , sm^4	W_y , sm^3	i_y , sm	z_0 , sm	massa 1 m, kg
	h	b	d	t										
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
5	50	32	4.4	7	6.16	22.8	9.1	1.92	5.59	5.61	2.75	0.954	1.16	4.84
6,5	65	36	4.4	7.2	7.51	48.6	15	2.54	9	8.7	3.68	1.08	1.24	5.9
8	80	40	4.5	7.4	8.98	89.4	22.4	3.16	13.3	12.8	4.75	1.19	1.31	7.05
10	100	46	4.5	7.6	10.9	174	34.8	3.99	20.4	20.4	6.46	1.37	1.44	8.59
12	120	52	4.8	7.8	13.3	304	50.6	4.78	29.6	31.2	8.52	1.53	1.54	10.4
14	140	58	4.9	8.1	15.6	491	70.2	5.6	40.8	45.4	11	1.7	1.67	12.3
14a	140	62	4.9	8.7	17	545	77.8	5.66	45.1	57.5	13.3	1.84	1.87	13.3

4- jadval (davomi)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
16	160	64	5	8.4	18.1	747	93.4	6.42	54.1	63.6	13.8	1.87	1.8	14.2
16a	160	68	5	9	19.5	823	103	6.49	59.4	78.8	16.4	2.01	2	15.3
18	180	70	5.1	8.7	20.7	1090	121	7.24	69.8	86	17	2.04	1.94	16.3
18a	180	74	5.1	9.3	22.2	1190	132	7.32	76.1	105	20	2.18	2.13	17.4
20	200	78	5.2	9	23.4	1520	152	8.07	87.8	113	20.5	2.2	2.07	18.4
20a	200	80	5.2	9.7	25.2	1670	167	8.1	95.9	139	24.2	2.35	2.28	19.8
22	220	82	5.4	9.5	26.7	2110	192	8.89	110	151	25.1	2.37	2.21	21
22a	220	87	5.4	10.2	28.8	2330	212	8.99	121	187	30	2.55	2.46	22.6
24	240	90	5.6	10	30.6	2900	242	9.73	139	208	31.6	2.6	2.42	24
24a	240	95	5.6	10.7	32.9	3180	265	9.84	151	254	37.2	2.78	2.67	25.8
27	270	95	6	10.5	35.2	4160	308	10.9	178	262	37.3	2.73	2.47	27.7
30	300	100	6.5	11	40.5	5810	387	12	224	327	43.6	2.84	2.52	31.8
33	330	105	7	11.7	46.5	7980	484	13.1	281	410	51.8	2.97	2.59	36.5
36	360	110	7.6	12.6	53.4	10820	601	14.2	350	513	61.7	3.1	2.68	41.9
40	400	115	8	13.5	61.5	15220	761	15.7	444	642	73.4	3.23	2.75	48.3

Bo'ylama egilish koeffitsiyentining qiymatlari.

Elementning egiluvchanligi λ	Po'latdan yasalgan elementlar uchun ϕ qiymati	
	St.3 va St. 4	St.5
0	1,00	1,00
10	0,99	0,98
20	0,97	0,96
30	0,95	0,93
40	0,92	0,89
50	0,89	0,85
60	0,86	0,80
70	0,81	0,74
80	0,75	0,67
90	0,69	0,59
100	0,60	0,50
110	0,52	0,43
120	0,45	0,37
130	0,40	0,32
140	0,36	0,28
150	0,32	0,25
160	0,29	0,23
170	0,26	0,21
180	0,23	0,19
190	0,21	0,17
200	0,19	0,15
210	0,17	0,14
220	0,16	0,13

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