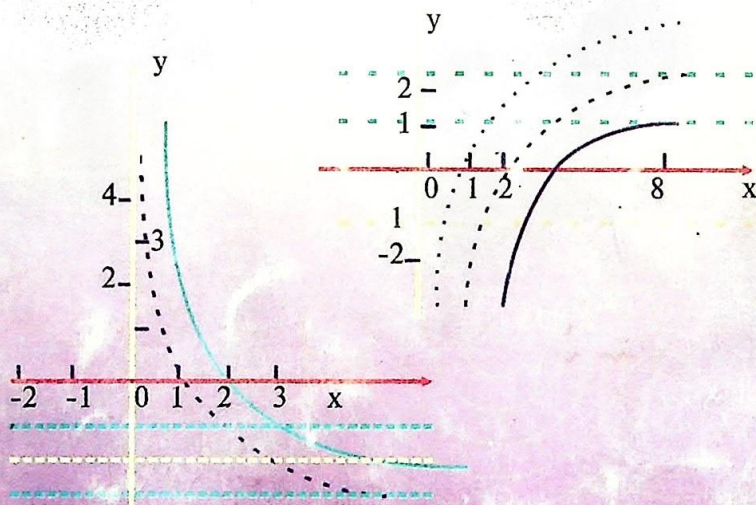


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KO'RSATKICHLI, LOGARIFMIK TENGLAMA VA TENGSIZLIKLAR



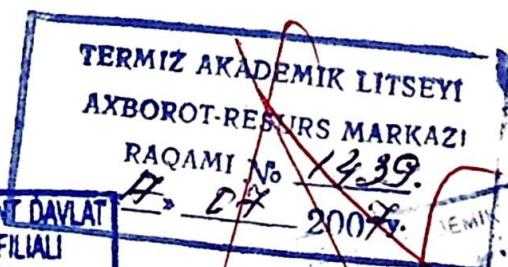
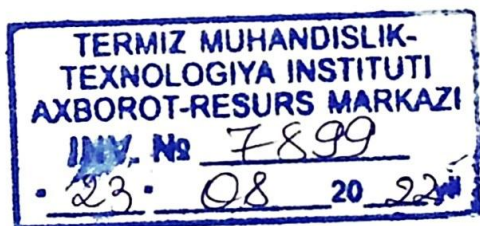
O'ZBEKISTON RESPUBLIKASI OLIY VA O'RTA
MAXSUS TA'LIM VAZIRLIGI

O'RTA MAXSUS, KASB-HUNAR TA'LIMI
MARKAZI

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KO'RSATKICHLI, LOGARIFMIK TENGLAMA VA TENGSIZLIKLAR

*Akademik litsey va kasb-hunar kollejlari
uchun o'quv qo'llanma*



„ILM – ZIYO“ – 2005

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O'quv qo'llanmada logarifmik va ko'rsatkichli tenglama, tengsizliklar va ularning sistemalariga oid ma'lumotlar ko'plab misollarning to'liq yechimlari bilan berilgan. Mustaqil yechish uchun berilgan misollar olingan bilimlarni mustahkamlash uchun yordam beradi.

Ushbu kitob keng o'quvchilar ommasiga mo'ljallangan bo'lib, undan akademik litsey, kasb-hunar kollejlari, oliy o'quv yurtlari talabalari, o'qituvchilar foydalanishlari mumkin.

SO'ZBOSHI

Mazkur o'quv qo'llanma akademik litsey va kasb-hunar kollejlari Davlat Ta'lim Standartida belgilangan hamda matematik ta'lim mazmuniga kiritilgan Ko'rsatkichli va logarifmik funksiyalar. Ko'rsatkichli tenglama va tengsizliklar. Logarifmik tenglama va tengsizliklar. Ko'rsatkichli va logarifmik tenglamalar sistemalari mavzularini o'zlashtirishda o'quvchilarga va o'qituvchilarga, pedagogika oliy o'quv yurtlari o'qituvchilari hamda talabalariga metodik yordam sifatida tayyorlangan.

Ushbu kitobda yuqorida ko'rsatilgan mavzularga oid ko'pgina misollar to'liq yechimlari bilan berilgan bo'lib, o'quv qo'llanma oliy o'quv yurtiga kirish uchun mustaqil tayyorlanayotgan keng yoshlar ommasi uchun ham mo'ljallangan.

O'quv qo'llanmasida tenglamalar, ularning yechimlari, teng kuchli tenglamalar, logarifmik va ko'rsatkichli funksiyalar, ularning grafiqlari, logarifmik va ko'rsatkichli ifodalar qatnashgan tenglama va tengsizliklar, ularning sistemalariga oid tushuncha va ma'lumotlar misollar yordamida yoritilgan.

O'quv qo'llanmaning har bir bobi bir nechta paragraflardan tashkil topgan bo'lib, har bir paragrafda mustaqil ishlash uchun mashqlar keltirilgan.

O'quv qo'llanmani yozishda rus va o'zbek tillarida chop etilgan mavjud adabiyotlardan keng foydalanildi.

Mualliflar — Nizomiy nomidagi Toshkent Davlat pedagogika universiteti dotsentlari A.Yunusov, D.Yunusova, O.Musurmonov hamda 310- iqtisodiyotga yo'naltirilgan litseymaktab matematika o'qituvchisi Yu.Nizomovalar o'quv qo'llanma yuzasidan o'z fikr-mulohazalarini bildirganlarga oldindan minnatdorchiliklarini izhor etadilar.

I BOB. KO'RSATKICHLI VA LOGARIFMIK FUNKSIYALAR

1- §. Ko'rsatkichli funksiyalar

Birdan farqli musbat haqiqiy a son uchun $y = a^x$ ko'rinishdagi funksiya *ko'rsatkichli funksiya* deyiladi.

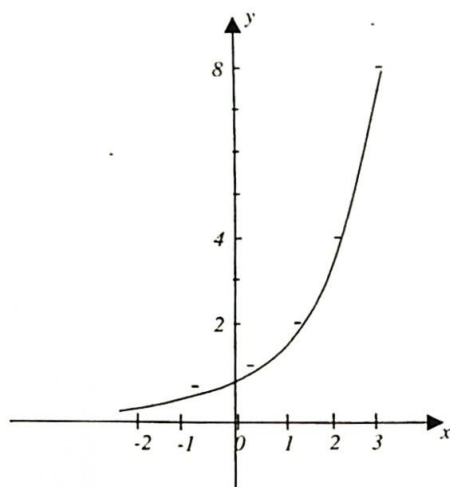
Agar $a > 1$ bo'lsa, u holda $y = a^x$ funksiya $(0, 1)$ nuqtadan o'tuvchi monoton o'suvchi funksiya bo'ladi.

1- m i s o l. $y = 2^x$ funksiyaning taxminiy grafigini chizish uchun x ning

$-2 ; -1,5 ; -1 ; -0,5 ; 0 ; 0,5 ; 1 ; 1,5 ; 2 ; 2,5 ; 3$ qiymatlarida y uchun $0,01$ aniqlikdagi qiymatlarni topib, quyidagi jadval to'ldiriladi:

X	-2	-1,5	-1	-0,5	0	0,5	1	1,5	2	2,5	3
Y	0,25	0,35	0,5	0,71	1	1,41	2	2,83	4	5,76	8

Koordinatalari jadvaldagi qiymatlarga teng bo'lgan nuqtalarni koordinatalar tekisligida tasvirlansa, quyidagi taxminiy grafik hosil bo'ladi (1- rasm):

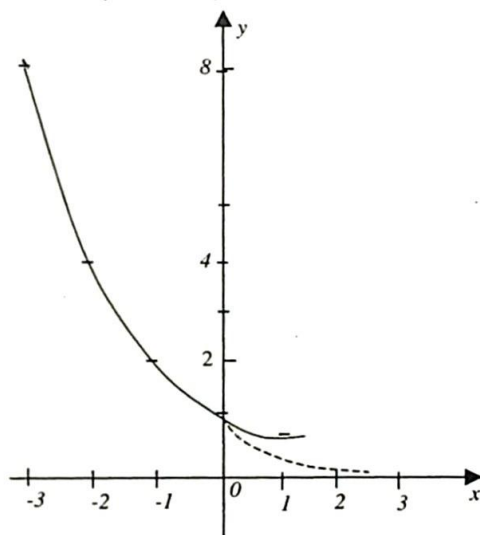


1- rasm.

2- m i s o l. $y = \left(\frac{1}{2}\right)^x$ funksiyaning grafigini quyidagi jadvaldagi kattaliklardan foydalanib chizish mumkin:

X	-2	-1,5	-1	-0,5	0	0,5	1	1,5	2
Y	4	3,83	2	1,41	1	0,71	0,5	0,35	0,25

U holda bu funksiyaning taxminiy grafigi quyidagi ko'rinishda bo'ladi (2- rasm):



2- rasm.

Ko'rsatkichli funksiyaning ba'zi xossalari ko'rib chiqaylik.

1°. $a > 0$ haqiqiy sonning har qanday haqiqiy darajasi, yana haqiqiy son bo'lganligi sabab, $y = a^x$ funksiyaning aniqlanish sohasi barcha haqiqiy sonlar to'plamidan iboratdir.

2°. $a > 0$ bo'lgani sababli har qanday α – haqiqiy son uchun $a^\alpha > 0$. Haqiqatan, $\alpha > 0$ bo'lsa, $a^\alpha > 0$; $\alpha = 0$ bo'lsa, $a^0 = 1 > 0$.

Agar $\alpha < 0$ bo'lsa, $-\alpha > 0$ va, demak, $a^{-\alpha} > 0$. U holda, $a^\alpha = \frac{1}{a^{-\alpha}} > 0$ bo'ladi. Shunday qilib, $y = a^x$ funksiyaning o'zgarish sohasi barcha musbat haqiqiy sonlar to'plamidan iboratdir.

3°. $a > 1$ bo'lsa, $x_1 > x_2$ ekanligidan, $a^{x_1} > a^{x_2}$ kelib chiqadi. Ya'ni, $a > 1$ bo'lsa, $y = a^x$ monoton o'suvchi funksiyadir.

4°. $0 < a < 1$ bo'lsa, $x_1 > x_2$ ekanligidan, $a^{x_1} < a^{x_2}$. Ya'ni, $u = a^x$ monoton kamayuvchi funksiyadir.

Mashqlar

Berilgan funksiyalarning grafiklarini chizing (1–26).

1. $y = 2^x$.

14. $y = 2^{-x^2}$.

2. $y = 2^{-x}$.

15. $y = \frac{a^x + a^{-x}}{2}$.

3. $y = 2^{x+2}$.

16. $y = \frac{a^x - a^{-x}}{2}$.

4. $y = 3 \cdot 2^x$.

17. $y = \frac{a^x - a^{-x}}{a^x + a^{-x}}$.

5. $y = \frac{1}{3} \cdot 2^{x-1}$.

18. $y = \frac{a^x + 1}{a^x - 1}$.

6. $y = \frac{1}{3} \cdot 2^{x+2} + 3$.

19. $y = \frac{a^x - 1}{a^x + 1}$.

7. $y = 2^x + 2^{-x}$.

20. $y = \frac{3^x + 3^{-x}}{2}$.

8. $y = 3 \cdot 2^x - 2^{-x}$.

21. $y = \frac{2^x + 3 \cdot 2^{-x}}{5}$.

9. $y = 3^x + 3 \cdot 2^x$.

22. $y = \frac{a^x}{2} + 4$.

10. $y = 5 \cdot 2^x - 3^{-x}$.

23. $y = \frac{5^x + 2^{-x}}{3} + 2$.

11. $y = 3 \cdot 4^x - 5^{-x}$.

24. $y = \frac{a^{-x}}{2} + 2^x$.

12. $y = \left(\frac{1}{3}\right)^{x+2} + 3$.

25. $y = \frac{3^x + 5^{-x}}{4}$.

13. $y = \left(\frac{1}{2}\right)^{x+2} - 1$.

26. $y = \left(\frac{1}{2}\right)^{x+2} - \left(\frac{1}{3}\right)^{2x}$.

2- §. Logarifm tushunchasi

Ma'lumki, $a \neq 1$, $a > 0$, $b > 0$ haqiqiy sonlar uchun $a^x = b$ tenglama yagona ildizga ega. Bu ildizni $\log_a b$ ko'rinishda belgilaymiz. Masalan, $2^4 = 16$. U holda $4 = \log_2 16$ bo'lishi ravshan.

$\log_a b$ sonni b sonining a asosga ko'ra logarifmi deyiladi. Ya'ni, b sonni hosil qilish uchun a sonini ko'tarish kerak bo'lgan daraja ko'rsatkichi b sonining a asosga ko'ra logarifmi deyiladi.

Demak, $a^x = b$ bo'lsa, $x = \log_a b$. U holda $a^{\log_a b} = b$ ayniyat hosil bo'ladi. Bu ayniyat *asosiy logarifmik ayniyat* deb ataladi.

Endi bir nechta misol ko'rib chiqamiz.

$$2^{-2} = \frac{1}{4}, \text{ demak, } \log_2 \frac{1}{4} = -2. \quad 3^{-2} = \frac{1}{9}, \text{ demak, } \log_3 \frac{1}{9} = -2.$$

$$3^3 = 27, \text{ demak, } \log_3 27 = 3. \quad 3^4 = 81, \text{ demak, } \log_3 81 = 4.$$

Logarifmning asosiy xossalari

Har doim $\log_a b$ ifoda yozilganda $a > 0$, $b > 0$, $a \neq 1$ shartlar yozilmagan bo'lsa ham, bu shartlarni bor deb hisoblanadi.

Logarifmning asosiy xossalari:

$$1^0. b = a^{\log_a b} \text{ (asosiy logarifmik ayniyat).}$$

$$2^0. \log_a a = 1. \quad 7^0. \log_a b = \frac{\log_c b}{\log_c a}.$$

$$3^0. \log_a 1 = 0. \quad 8^0. \log_a b = \frac{1}{\log_b a}.$$

$$4^0. \log_a b \cdot c = \log_a b + \log_a c. \quad 9^0. \frac{\log_c b}{\log_c a} = \frac{\log_d b}{\log_d a}.$$

$$5^0. \log_a \frac{b}{c} = \log_a b - \log_a c. \quad 10^0. \log_{a^\alpha} b^\alpha = \log_a b.$$

$$6^0. \forall \alpha \in R \text{ uchun } \log_a b^\alpha = \alpha \cdot \log_a b.$$

$$11^0. \log_{a^\alpha} b = \frac{1}{\alpha} \cdot \log_a b.$$

1^o ning isboti yuqorida berildi. 2^o, 3^o lar bevosita logarifmning ta'rifidan kelib chiqadi.

4^o ni isbotlash uchun asosiy ayniyatdan foydalaniladi, ya'ni

$b \cdot c = a^{\log_a b \cdot c}$, $b = a^{\log_a b}$, $c = a^{\log_a c}$, u holda $a^{\log_a b \cdot c} = a^{\log_a b} \cdot a^{\log_a c}$ yoki $a^{\log_a b \cdot c} = a^{\log_a b + \log_a c}$.

Demak, $\log_a b \cdot c = \log_a b + \log_a c$.

5^o. Yuqoridagidek, $\frac{b}{c} = a^{\log_a \frac{b}{c}} = \frac{a^{\log_a b}}{a^{\log_a c}}$ yoki $a^{\log_a \frac{b}{c}} = a^{\log_a b - \log_a c}$,

u holda $\log_a \frac{b}{c} = \log_a b - \log_a c$.

6^o. $b^\alpha = a^{\log_a b^\alpha}$, $b = a^{\log_a b}$ ayniyatlarni hisobga olsak, $b^\alpha = (a^{\log_a b})^\alpha = a^{\alpha \cdot \log_a b}$. U holda $\log_a b^\alpha = \alpha \cdot \log_a b$.

7^o. $\log_c b = \log_c a^{\log_a b} = (\log_a b) \cdot (\log_c a)$ tenglikdagi birinchi va oxirgi ifodalarni tenglashtirilsa, $\log_c b = (\log_a b) \cdot (\log_c a)$ tenglik hosil bo'ladi. Bu tenglikning ikkala tomonini $\log_c a$ ga

bo'lib, $\log_a b = \frac{\log_c b}{\log_c a}$ tenglikni hosil qilamiz.

8^o. $\log_a b = \frac{\log_b b}{\log_b a} = \frac{1}{\log_b a}$.

9^o. $\frac{\log_c b}{\log_c a} = \frac{\frac{\log_d b}{\log_d c}}{\frac{\log_d a}{\log_d c}} = \frac{\log_d b}{\log_d c} \cdot \frac{\log_d c}{\log_d a} = \frac{\log_d b}{\log_d a}$.

10^o. $\log_{a^\alpha} b^\alpha = \frac{\log_a b^\alpha}{\log_a a^\alpha} = \frac{\alpha \cdot \log_a b}{\alpha} = \log_a b$.

11^o. $\log_{a^\alpha} b = \frac{\log_a b}{\log_a a^\alpha} = \frac{\log_a b}{\alpha} = \frac{1}{\alpha} \cdot \log_a b$.

Yuqorida keltirilgan xossalarni qo'llanilishiga oid misollar bilan tanishib chiqamiz.

1- m i s o l. $\sqrt{25^{\frac{1}{\log_8 5}} + 81^{\frac{1}{\log_6 9}}}$ ifodaning qiymatini jadvalsiz hisoblang.

Y e c h i s h. 8^0 - xossaga ko'ra $\frac{1}{\log_8 5} = \log_5 8$ va $\frac{1}{\log_6 9} = \log_9 6$. U holda $25^{\frac{1}{\log_8 5}} = 25^{\log_5 8} = (5^2)^{\log_5 8} = 5^{2 \cdot \log_5 8}$. 6^0 - xossaga asosan $2 \cdot \log_5 8 = \log_5 8^2 = \log_5 64$, u holda $5^{2 \cdot \log_5 8} = 5^{\log_5 64}$ hosil bo'lgan ifodaga asosiy logarifmik ayniyatni qo'llasak, $5^{\log_5 64} = 64$ tenglik hosil bo'ladi. Shunday qilib, $25^{\frac{1}{\log_8 5}}$ ifoda 64 ga teng ekan. Shunga o'xshash $81^{\frac{1}{\log_6 9}} = 81^{\log_9 6} = (9^2)^{\log_9 6} = 9^{2 \cdot \log_9 6} = 9^{\log_9 6^2} = 9^{\log_9 36} = 36$. U holda yuqoridagi ifoda qiymati $\sqrt{64+36}$ ga yoki 10 ga teng bo'lishi ravshan.

2- m i s o l. $49^{\frac{1}{\log_2 7}} + 9^{\log_{729} 8}$ ifoda qiymatini toping.

Y e c h i s h. 8^0 - xossaga ko'ra, $\frac{1}{\log_2 7} = \log_7 2$. U holda $49^{\frac{1}{\log_2 7}} = (7^2)^{\log_7 2} = 7^{\log_7 4} = 4$. 10^0 - xossaga ko'ra $\log_{729} 8 = \log_9 2^3 = \log_9 2$. Asosiy logarifmik ayniyatga asosan $9^{\log_{729} 8} = 9^{\log_9 2} = 2$. Demak, berilgan ifodaning qiymati 6 ga teng ekan.

3- m i s o l. $121^{\log_{11} 5} + 9^{1-\log_9 2} + 9^{\log_{81} 36}$ ifodani soddalashtiring.

Y e c h i s h. Birinchi qo'shiluvchini soddalashtirish uchun $121 = 11^2$ tenglikni hisobga olib, asosiy logarifmik ayniyatdan va 6^0 - xossadan foydalanib, $121^{\log_{11} 5} = (11^2)^{\log_{11} 5} = 11^{2 \cdot \log_{11} 5} = 11^{\log_{11} 5^2} = 11^{\log_{11} 25} = 25$ hosil qilinadi. Shunga o'xshash ikkinchi qo'shiluvchini soddalashtiriladi: $9^{1-\log_9 2} = \frac{9}{9^{\log_9 2}} = \frac{9}{2} = 4,5$. Uchinchi qo'shiluvchini 10^0 - xossadan foydalanib soddalashti-

riladi: $9^{\log_{81} 36} = 9^{\log_{9^2} 6^2} = 9^{\log_9 6} = 6$. U holda berilgan ifoda $25 + 4,5 + 6$ yoki $35,5$ ga teng bo'ladi.

4- m i s o l. $\log_3(a^{\frac{1}{\log_3 a}} \cdot a^{\frac{1}{\log_9 a}} \cdot \dots \cdot a^{\frac{1}{\log_{729} a}})$, $a > 0$, $a \neq 1$ ifoda qiymatini toping.

Y e c h i s h. $\frac{1}{\log_b a} = \log_a b$ ayniyatni hisobga olinsa,

logarifmini topish lozim bo'lgan ifoda $a^{\log_a 3} \cdot a^{\log_a 9} \cdot \dots \cdot a^{\log_a 729}$ ko'rinishga keladi. U holda asosiy logarifmik ayniyatga ko'ra oxirgi ifodani $3 \cdot 9 \cdot \dots \cdot 729 = 3 \cdot 3^2 \cdot \dots \cdot 3^6 = 3^{1+2+\dots+6} = 3^{21}$ ko'rinishga keltirish mumkin. U holda berilgan ifoda qiymati 21 ga teng bo'lishi ayon.

Mashqlar

Quyidagi ifodalarning qiymatini aniqlang (1–25):

1. $\log_2 \sqrt[3]{16}$.

11. $\lg \frac{1}{a^2} \sqrt{a^3}$.

2. $\log_3(27\sqrt{3})$.

12. $\lg_{a^4} \sqrt[7]{a^{-9}}$.

3. $\log_5 \sqrt{5\sqrt{5}}$.

13. $\lg_{\sqrt[3]{a}} \left(\frac{1}{\sqrt[5]{a^2}} \right)$.

4. $\log_{\sqrt[3]{173}} 9$.

14. $\lg \frac{1}{9} \sqrt[16]{27}$.

5. $\log_{1/\sqrt{2}} \sqrt[3]{128\sqrt{2}}$.

15. $\lg \frac{1}{8} \lg_{\sqrt[3]{2}} \sqrt[3]{4}$.

6. $\lg_{1/16} \sqrt[3]{4}$.

16. $\lg_{32} 4$.

7. $\lg_{0,32} \left(\frac{2}{5} \sqrt{2} \right)$.

17. $\lg_{25} \sqrt[13]{625}$.

8. $\lg_7 \frac{1}{7}$.

18. $\lg_{\sqrt[3]{a}} \left(\frac{\sqrt[4]{a}}{\sqrt[7]{a}} \right)$.

9. $\lg_5 5$.

19. $12^{\lg_{144} 4 + \lg_{12} 2}$.

10. $\lg_{\sqrt{3}} 9$.

20. $7^{\lg_7 3 + \lg_{49} 4}$.

$$21. 256^{\log_5 4} + 27^{\log_{81} 216} + 3^{\frac{4}{\log_{\sqrt{7}} 3}}.$$

$$22. -\log_2 \log_3 \sqrt[4]{3}.$$

$$23. \left(a^{\frac{1}{\log_7 a}} \cdot a^{\frac{1}{\log_{49} a}} \cdot \dots \cdot a^{\frac{1}{\log_{7^n} a}} \right)^{\frac{4}{n(n+1)}}.$$

$$24. (\log_7 8 + \log_8 7 + 2) \cdot (\log_7 8 - \log_{56} 8) \cdot \log_8 7.$$

$$25. 7^{\frac{2}{\log_5 7} + 1} \cdot 5 - 2 \cdot 7^{\log_7 5 + 1} \cdot 5^{\log_5 7 + 1} + 35^{\frac{2}{\log_7 5} + 1}.$$

3- §. Logarifmik funksiyalar

$a > 0$, $a \neq 1$, $x > 0$ sonlar uchun $y = \log_a x$ funksiya *logarifmik* funksiya deyiladi. Bundagi a soni logarifmning *asosi* deyiladi. Asosi 10 ga teng logarifmik funksiya $y = \lg x$, asosi e ($e = 2,7183\dots$) ga teng logarifmik funksiya $y = \ln x$ ko'rinishda yoziladi.

Logarifmik funksiyaning xossalari

1°. Logarifmik funksiyaning aniqlanish sohasi $(0; +\infty)$ to'plamdan, ya'ni logarifmik funksiyaning aniqlanish sohasi barcha musbat haqiqiy sonlar to'plamidan iborat.

2°. Logarifmik funksiyaning o'zgarish sohasi barcha haqiqiy sonlar to'plamidan iborat.

3°. $a > 1$ bo'lganda, logarifmik funksiya *monoton o'suvchi* funksiya bo'ladi.

4°. $a < 1$ bo'lganda, logarifmik funksiya *monoton kamayuvchi* funksiya bo'ladi.

5°. Logarifmik funksiya grafigi Ox o'qini $x = 1$ nuqtada kesib o'tadi.

6°. $y = \log_a x$ funksiya $y = a^x$ funksiyaga teskari funksiyadir.

7°. Agar $a > 1$ bo'lsa, $x \rightarrow +\infty$ da $y \rightarrow +\infty$; $x \rightarrow -\infty$ da, $y \rightarrow -\infty$ bo'ladi.

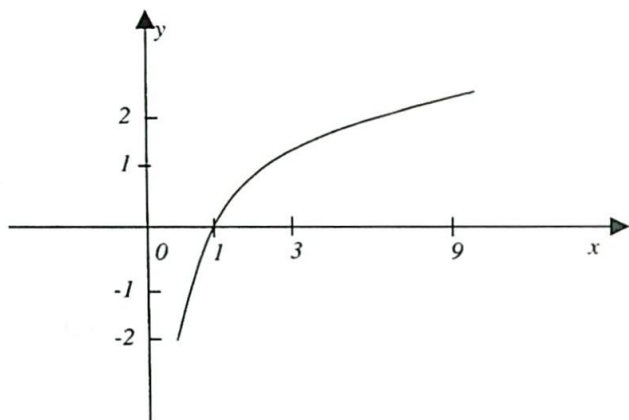
8°. $a < 1$ uchun esa $x \rightarrow +\infty$ da $y \rightarrow -\infty$; $x \rightarrow -\infty$ da $y \rightarrow +\infty$ bo'ladi.

Logarifmik funksiya grafigi

Quyida logarifmik funksiya grafiklarini chizishga doir misollarni ko'rib chiqamiz.

1- m i s o l. $y = \log_3 x$ funksiya grafigini chizish uchun quyidagi jadvaldan foydalanamiz (3- rasm):

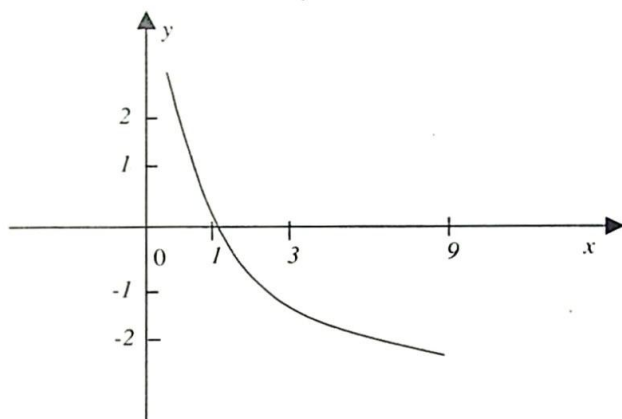
x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
y	-2	-1	0	1	2



3- rasm.

2- m i s o l. $y = \log_{\frac{1}{3}} x$ funksiya grafigini chizamiz. Buning uchun quyidagi jadvaldan foydalanamiz (4- rasm):

x	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
y	2	1	0	-1	-2



4- rasm.

Logarifmik va ko'rsatkichli funksiyalarning grafiklarini chizishda qo'l keladigan funksiyalarning grafigini chizishda ko'p ishlatiladigan sxema bilan tanishib chiqamiz. Ya'ni, $y = f(x)$ funksiya grafigi orqali $y = a \cdot f(bx + c) + d$ ko'rinishdagi funksiya grafigi qanday chizilishini ko'rib chiqamiz.

1. $y = f(x)$ funksiya grafigi chizib olinadi.

2. $y = f(x)$ funksiya grafigini Ox o'qi bo'ylab b marta siqib, $y = f(bx)$ funksiya grafigi hosil qilinadi.

3. $y = f(bx)$ funksiya grafigi c musbat son bo'lsa, Ox o'qi bo'ylab chap tomonga c birlik, agar c manfiy son bo'lsa, u holda c birlik o'ng tomonga parallel ko'chiramiz. Natijada $y = f(bx + c)$ funksiya grafigi hosil bo'ladi.

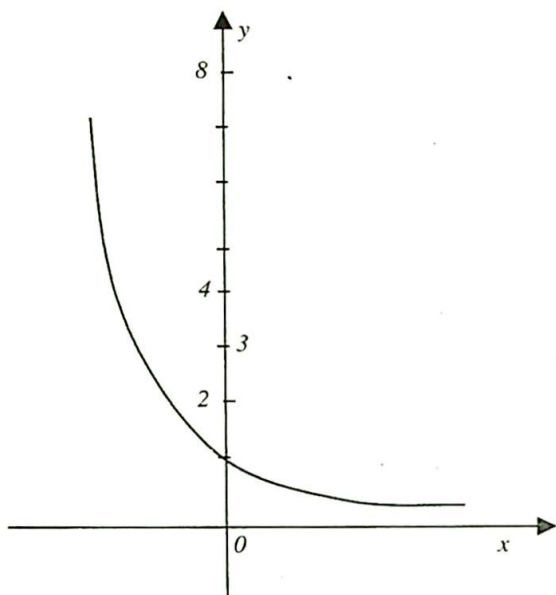
4. $y = f(bx + c)$ funksiya grafigini Oy o'qi bo'ylab a marta cho'zgak, hosil bo'lgan grafik $y = a \cdot f(bx + c)$ — funksiya grafigi bo'ladi.

5. Agar d musbat son bo'lsa, $y = a \cdot f(bx + c)$ funksiya grafigini Oy o'qi bo'ylab d birlik yuqoriga, d manfiy son bo'lsa, d birlik pastga parallel ko'chiramiz. Natijada $y = a \cdot f(bx + c) + d$ funksiya grafigi hosil bo'ladi.

3- misol. $y = 2 \cdot \left(\frac{1}{2}\right)^{x+1} - 3$ funksiya grafigini chizing.

1. $y = \left(\frac{1}{2}\right)^x$ funksiya grafigini quyidagi jadval yordamida chizib olamiz (5- rasm):

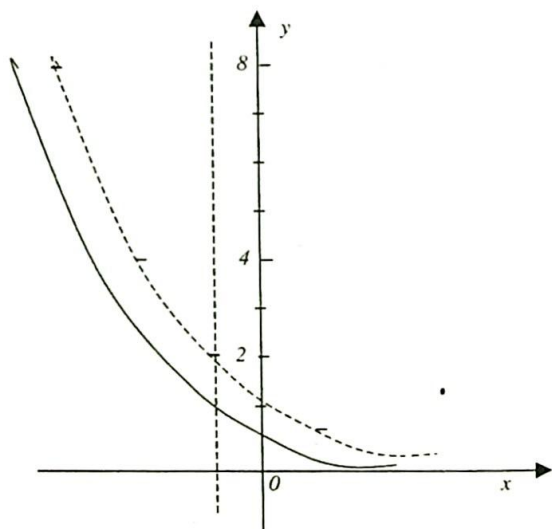
x	-3	-2	-1	0	1	2	3
y	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$



5- rasm.

2. $y = \left(\frac{1}{2}\right)^{x+1}$

funksiya grafigini chizamiz. Buning uchun hosil bo'lgan grafikni Ox o'qi bo'ylab 1 birlik chap tomonga parallel ko'chiramiz (6- rasm):

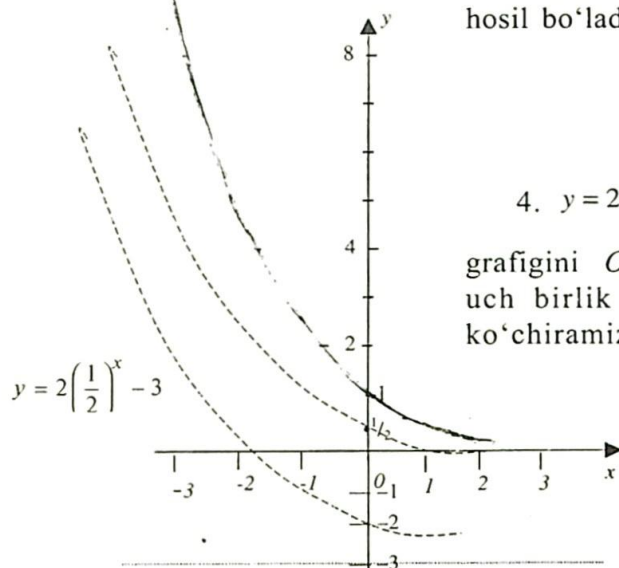


6- rasm.

3. Hosil bo'lgan grafikni Oy o'qi bo'ylab 2 marta uzaytiramiz, natijada

$$y = 2 \cdot \left(\frac{1}{2}\right)^{x+1} \text{ funksiya grafigi}$$

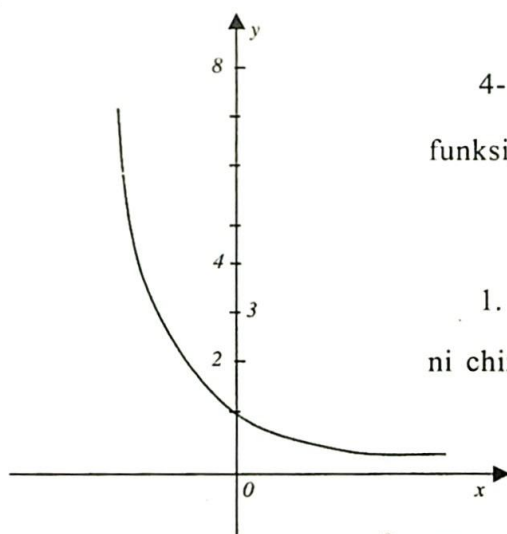
hosil bo'ladi (7- rasm):



7- rasm.

$$4. y = 2 \cdot \left(\frac{1}{2}\right)^{x+1} \text{ funksiya}$$

grafigini Oy o'qi bo'yicha uch birlik pastga parallel ko'chiramiz:



8- rasm.

$$4- m i s o l. y = \left(\frac{1}{3}\right)^{\frac{1}{2}x} + 1$$

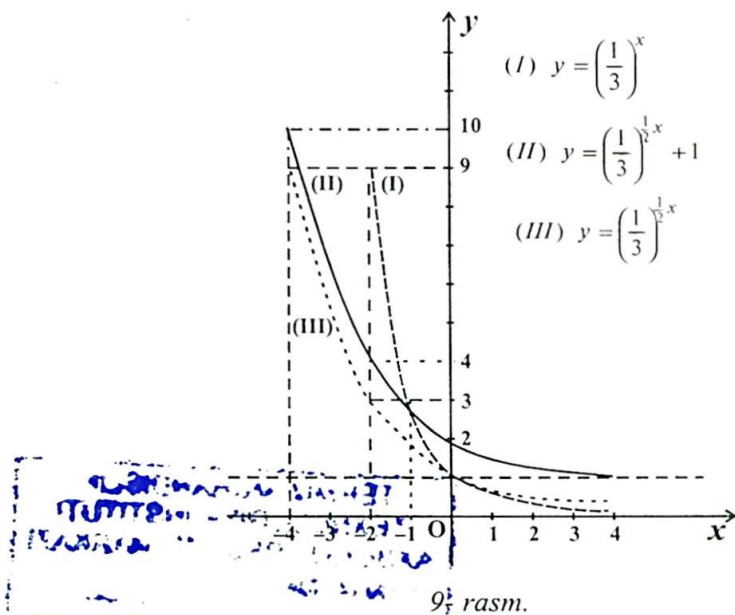
funksiya grafigini chizing.

$$1. y = \left(\frac{1}{3}\right)^x \text{ funksiya grafigi-}$$

ni chizamiz (8- rasm):

2. $y = \left(\frac{1}{3}\right)^x$ funksiya grafigini Ox o'qi bo'yicha $1/2$ marta siqamiz. Boshqacha qilib aytganda, 2 marta cho'zamiz.

3. $y = \left(\frac{1}{3}\right)^{\frac{1}{2}x}$ funksiya grafigini Oy bo'yicha 1 birlik yuqoriga parallel ko'chiramiz (9- rasm).



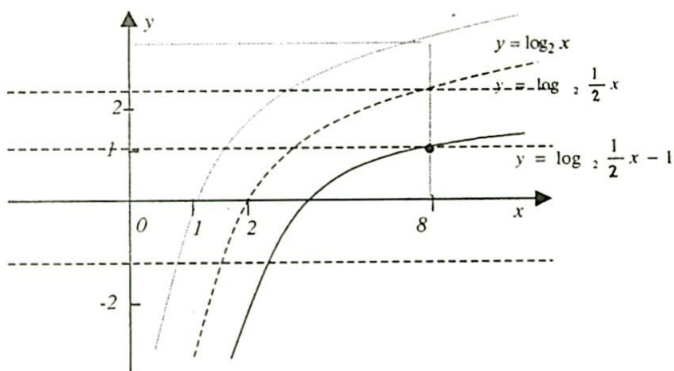
5- m i s o l. $y = \log_2 \frac{1}{2}x - 1$ funksiya grafigini chizing.

1. $y = \log_2 x$ funksiya grafigini chizamiz.

2. Hosil bo'lgan grafikni Ox o'qi bo'yicha $\frac{1}{2}$ marta siqamiz.

3. $y = \log_2 \frac{1}{2}x$ funksiya grafigini Oy o'qi bo'yicha pastga qarab bir birlik parallel ko'chiramiz (10- rasm):

x	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8
y	-3	-2	-1	0	1	2	3



10- rasm.

6- m i s o l. $y = 2\log_{\frac{1}{2}}(x-1)$ funksiya grafigini chizing.

1. $y = \log_{\frac{1}{2}} x$ funksiya grafigini chizamiz.

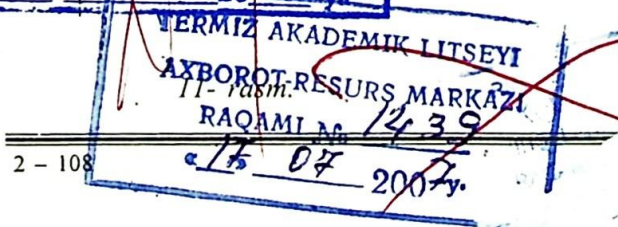
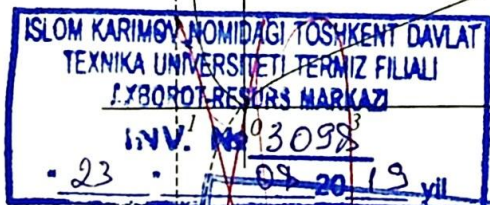
2. Hosil bo'lgan grafikni Ox o'qi bo'ylab o'ng tomonga 1 birlik parallel ko'chiramiz.

3. $y = \log_{\frac{1}{2}}(x-1)$ funksiya grafigini Oy o'qi bo'ylab 2 marta cho'zamiz:



7- m i s o l. $y = |\log_2(x+1)|$ funksiya grafigini chizing (11- rasm).

Bu funksiya grafigini chizish uchun $y = \log_2(x+1)$ funksiya grafigini chizib olib, so'ngra grafikning Ox o'qdan pastda joylashgan qismini Ox o'qqa nisbatan simmetrik akslantiramiz.



Mashqlar

Quyidagi funksiyalar grafiklarini chizing (1–20):

1. $y = |\log_3 x|$.

11. $y = \log_3 x$.

2. $y = \left| \log_{\frac{1}{3}} x \right|$.

12. $y = \log_2 x + \log_{\frac{1}{3}} x$.

3. $y = \log_{\frac{1}{2}} x$.

13. $y = \log_5 \sqrt{x}$.

4. $y = \log_2 x$.

14. $y = \log_7 x^5$.

5. $y = \log_2 (x+1)$.

15. $y = x + \ln x$.

6. $y = (\log_2 x)^2 + 3$.

16. $y = x \ln x$.

7. $y = \log_{\frac{1}{2}} (x-3) + 1$.

17. $y = \sqrt{\log_2 x}$.

8. $y = |\log_2 x| + 3$.

18. $y = \frac{1 - \ln x}{1 + \ln x}$.

9. $y = \log_{\frac{1}{3}} (x+2) + 1$.

19. $y = \log_2 (x + \sqrt{x^2 + 1})$.

10. $y = \left| \log_{\frac{1}{2}} (x+1) \right| - 2$.

20. $y = \ln \frac{1+x}{1-x}$.

4- §. Logarifmik va ko'rsatkichli tenglamalarni yechishda ishlatiladigan asosiy ayniyatlar

a, b, c, α, β haqiqiy sonlar berilgan bo'lsin.

1. $a^0 = 1$.

2. $a > 0, b > 0$ bo'lsa, u holda $(a \cdot b)^\alpha = a^\alpha \cdot b^\alpha$.

3. $a > 0, b > 0, b \neq 0$ bo'lsa, u holda $\left(\frac{a}{b}\right)^\alpha = \frac{a^\alpha}{b^\alpha}$.

4. $a > 0$ bo'lsa, u holda $\frac{a^\alpha}{a^\beta} = a^{\alpha-\beta}$.

5. $a \geq 0$ bo'lsa, u holda $(a^\alpha)^\beta = a^{\alpha \cdot \beta}$.

6. $a > 0$ bo'lsa, u holda $a^{-\alpha} = \frac{1}{a^{\alpha}}$.
7. $a \neq 0, a^{\frac{\alpha}{\beta}} = \sqrt[\beta]{a^{\alpha}}$.
8. $a > 0, a \neq 1$ bo'lsa, u holda $b > 0, a^{\log_a b} = b$.
9. $a > 0, a \neq 1$ bo'lsa, u holda $\log_a a = 1$.
10. $a > 0, a \neq 1$ bo'lsa, u holda $\log_a 1 = 0$.
11. $a > 0, a \neq 1, b \cdot c > 0$ bo'lsa, u holda $\log_a b \cdot c = \log_a |b| + \log_a |c|$.
12. $a > 0, a \neq 1, b > 0, c > 0$ bo'lsa, u holda $\log_a b \cdot c = \log_a b + \log_a c$.
13. $a > 0, a \neq 1, \frac{b}{c} > 0$ bo'lsa, u holda $\log_a \frac{b}{c} = \log_a |b| - \log_a |c|$.
14. $a > 0, a \neq 1, b > 0, c > 0$ bo'lsa, u holda $\log_a \frac{b}{c} = \log_a b - \log_a c$.
15. $a > 0, a \neq 1$ bo'lsa, u holda $\log_a b^{2m} = 2m \cdot \log_a |b|$.
16. $a > 0, a \neq 1$ bo'lsa, u holda $(a^{\alpha} = a^{\beta}) \Leftrightarrow (\alpha = \beta)$.
17. $a > 0, a \neq 1, b > 0, c > 0$ bo'lsa, u holda $\log_a b = \log_a c \Leftrightarrow b = c$.
18. $a > 0, a \neq 1, b > 0$ bo'lsa, u holda $a^{\alpha} = b \Leftrightarrow \alpha = \log_a b$.
19. $a > 0, a \neq 1, c > 0$ bo'lsa, u holda $\log_a c = b \Leftrightarrow c = a^b$.
20. $a > 1$ bo'lsa, u holda $a^{\alpha} > a^{\beta} \Leftrightarrow \alpha > \beta$.
21. $a > 1$ bo'lsa, u holda $a^{\alpha} = a^{\beta} \Leftrightarrow \alpha = \beta$.
22. $a > 1$ bo'lsa, u holda $a^{\alpha} < a^{\beta} \Leftrightarrow \alpha < \beta$.
23. $a > 1$ bo'lsa, u holda $a^{\alpha} \leq a^{\beta} \Leftrightarrow \alpha \leq \beta$.
- 24^o. $0 < a < 1$ bo'lsa, u holda $a^{\alpha} < a^{\beta} \Leftrightarrow \alpha > \beta$.
- 25^o. $0 < a < 1$ bo'lsa, u holda $a^{\alpha} \leq a^{\beta} \Leftrightarrow \alpha = \beta$.
- 26^o. $0 < a < 1$ bo'lsa, u holda $a^{\alpha} > a^{\beta} \Leftrightarrow \alpha < \beta$.
- 27^o. $0 < a < 1$ bo'lsa, u holda $a^{\alpha} = a^{\beta} \Leftrightarrow \alpha \leq \beta$.
28. $a > 1, b > 0, c > 0$ bo'lsa, u holda $\log_a b > \log_a c \Leftrightarrow b > c$.
29. $a > 1, b > 0, c > 0$ bo'lsa, u holda $\log_a b \geq \log_a c \Leftrightarrow b \geq c$.
30. $a > 1, b > 0, c > 0$ bo'lsa, u holda $\log_a b < \log_a c \Leftrightarrow b < c$.

$$31. a > 1, b > 0, c > 0 \text{ bo'lsa, u holda } \log_a b \leq \log_a c \Leftrightarrow b \leq c.$$

$$32. 0 < a < 1, b > 0, c > 0 \text{ bo'lsa, u holda } \log_a b > \log_a c \Leftrightarrow b < c.$$

$$33. 0 < a < 1, b > 0, c > 0 \text{ bo'lsa, u holda } \log_a b \geq \log_a c \Leftrightarrow b \leq c.$$

$$34. 0 < a < 1, b > 0, c > 0 \text{ bo'lsa, u holda } \log_a b < \log_a c \Leftrightarrow b > c.$$

$$35. 0 < a < 1, b > 0, c > 0 \text{ bo'lsa, u holda } \log_a b \leq \log_a c \Leftrightarrow b = c.$$

Mashqlar

1. Agar $\lg_k x, \lg_m x, \lg_n x$ arifmetik progressiyanini tashkil etsa, u holda $n^2 = (nk)^{\lg_k m}$ ekanligini isbotlang.

2. Tenglikni isbotlang: $\frac{\lg_n m}{\lg_{nk} m} = 1 + \lg_n k.$

3. Agar $a = \lg 64$ bo'lsa, u holda $\lg \sqrt[3]{25}$ ni toping.

4. Agar $a = \log_6 2$ bo'lsa, u holda $\log_6 9$ ni toping.

5. Agar $a = \log_{36} 8$ bo'lsa, u holda $\log_{36} 9$ ni toping.

6. Agar $a = \lg 122,5$ va $b = \lg 7$ bo'lsa, u holda $\lg 5$ ni toping.

7. Agar $a = \lg 3$ va $b = \lg 2$ bo'lsa, u holda $\log_5 6$ ni toping.

8. Agar $a = \log_5 4$ va $b = \log_5 3$ bo'lsa, u holda $\log_{25} 12$ ni toping.

9. Isbotlang : $\lg_{a^k} a^n = \frac{n}{k}.$

10. Isbotlang: $\lg_{b a^n} a^{n+1} = \frac{(n+1) \lg_b a}{1 + n \lg_b a}.$

5- §. Algebraik ifodalarni logarifmlash

Birhad ko'rinishidagi algebraik ifoda yoki birhadlar nisbati ko'rinishidagi ratsional ifodaning logarifmini hisoblash amali *logarifmlash* amali deyiladi.

1- misol. $A = 5x^2y^2\sqrt{z}$ ifodaning a ($a > 0, a \neq 1$) asosga ko'ra logarifmini toping.

Yechish.

$$\begin{aligned}\log_a A &= \log_a 5x^2y^2\sqrt{z} = \log_a 5 + \log_a x^2 + \log_a y^2 + \log_a \sqrt{z} = \\ &= \log_a 5 + 2\log_a |x| + 2\log_a |y| + \frac{1}{2}\log_a z.\end{aligned}$$

2- misol. $B = \frac{x(a+b)^2}{y(c-d)^3}$ ifodani a ($a > 0, a \neq 1$) asosga ko'ra logarifmini toping.

Yechish.

$$\begin{aligned}\log_a B &= \log_a \frac{x(a+b)^2}{y(c-d)^3} = \log_a x(a+b)^2 - \log_a y(c-d)^3 = \\ &= \log_a x + \log_a (a+b)^2 - \log_a |y| - \log_a |(c-d)^3| = \log_a x + \\ &+ 2\log_a |(a+b)| - \log_a |y| - 3\log_a |c-d|.\end{aligned}$$

6- §. Potensirlash

Logarifmlashga teskari amal *potensirlash*dir. Agar birorta sonning logarifmi berilgan bo'lib, shu sonning o'zini topish talab qilingan bo'lsa, masalan, $\log_a x = 3\log_a b - \frac{1}{2}\log_a a + 2\log_a y - 5$, u holda logarifmning xossalaridan foydalanib, quyidagini hosil qilamiz:

$$\begin{array}{ccccccc}\log_a x & = & \log_a b^3 & - & \log_a \sqrt{a} & + & \log_a y^2 - \log_a a^5. \\ \text{Hosil} & & \text{bo'lgan} & & \text{tenglikni} & & \text{potensirlaymiz:}\end{array}$$

$$\log_a x = \log_a \frac{b^3}{\sqrt{a}} \cdot \frac{y^2}{a^5}. \text{ Natijada } x = \frac{b^3 \cdot y^2}{a^5 \sqrt{a}} \text{ hosil bo'ladi.}$$

Amaliyotda asosi 10 ga teng bo'lgan logarifmlar juda ko'p ishlatiladi. Bunday logarifmlar *o'nli logarifmlar* deb ataladi va $\log_{10} x$ o'rniga $\lg x$ deb yoziladi.

O'nli logarifmlar quyidagi ajoyib xossaga ega: $-\lg 10^\alpha = \alpha$.

$$\text{Masalan, } \lg 10000 = 4, \lg \frac{1}{100000} = -5 \text{ va hokazo.}$$

Bundan tashqari, asosi $e = 2,71\dots$ ga teng bo'lgan logarifmlar ham ko'p qo'llaniladi. $\log_e a$ o'rniga $\ln a$ deb yozish qabul qilingan. Bunday logarifm *natural logarifm* deyiladi.

1- m i s o l. $\log_c x = \log_c a + 2\log_c b$, $a > 0$, $b > 0$, $c > 0$, $c \neq 1$ bo'lsa, x ni toping.

Ye c h i s h. Logarifmning asosiy xossalaridan foydalanib, $2\log_c b = \log_c b^2$, $\log_c a + \log_c b^2 = \log_c ab^2$. U holda $\log_c x = \log_c ab^2$ tenglikdan $x = ab^2$ hosil bo'ladi.

2- m i s o l. $\log_{\frac{1}{2}} x = \log_{\frac{1}{2}} 2,4 + \log_{\frac{1}{2}} 15$ bo'lsa, x ni toping.

Ye c h i s h. Logarifm xossalaridan foydalanib, $\log_{\frac{1}{2}} x = \log_{\frac{1}{2}} 2,4 \cdot 15$, ya'ni, $\log_{\frac{1}{2}} x = \log_{\frac{1}{2}} 36$ ni hosil qilamiz.

Bundan: $x = 36$.

3- m i s o l. $\log_3 2 + \log_3 4 + \log_3 0,016$ ifoda musbat yoki manfiy son bo'lishini aniqlang.

Ye c h i s h. Logarifmlar yig'indisini ko'paytmaning logarifmiga keltirib $\log_3 2 \cdot 4 \cdot 0,016 = \log_3 0,064$ dan $0,064 < 1$ bo'lgani uchun $\log_3 0,064 < 0$ kelib chiqadi.

4- m i s o l. Qaysi ifodaning qiymati katta: $\log_{\frac{1}{2}} 6 + \log_{\frac{1}{2}} 12$ va $\log_{\frac{1}{2}} 7$?

Ye c h i s h. Logarifmlar yig'indisini ko'paytmaning logarifmiga keltirib, $\log_{\frac{1}{2}} 6 + \log_{\frac{1}{2}} 12 = \log_{\frac{1}{2}} 72$, logarifmning asosi birdan kichik bo'lganligi e'tiborga olinsa, $\log_{\frac{1}{2}} 72 < \log_{\frac{1}{2}} 7$ kelib chiqadi.

5- m i s o l. $\log_{\frac{1}{2}} (x^2 - x + 6)$ ifoda -3 ga teng bo'ladigan x ning qiymatini toping.

Ye c h i s h. Logarifm ta'rifiga ko'ra, $x^2 - x + 6 = (\frac{1}{2})^{-3}$
 yoki $x^2 - x - 2 = 0$. Viyet formulalariga asosan
 $x_1 = -1$ $x_2 = 2$. x ning topilgan qiymatlarida $x^2 - x + 6$ ifoda
 musbat bo'lganligi uchun $x_1 = -1$ $x_2 = 2$ x ning izlangan
 qiymatlari bo'ladi.

Mashqlar

Tenglamani yeching (1-4).

1. $\lg x = \lg 1,4 + \lg 15$.
2. $\log_{\frac{1}{2}} x = \log_{\frac{1}{2}} 9,6 - \log_{\frac{1}{2}} 2,4$.
3. $\log_2 x = 2\log_2 12 - \log_2 18$.
4. $\log_2 x = \log_2 50 + 3\log_2 0,5$.

Quyidagi ifodalarning ishorasini aniqlang (1-5):

1. $\lg \frac{1}{2} + \lg \frac{3}{2} + \lg 0,25$.
2. $\frac{1}{2} \lg 5 + \frac{1}{2} \lg 4 - \log_{\frac{1}{2}} \frac{1}{4}$.
3. $\log_2 6 + \log_2 0,7 - \frac{1}{2} \log_2 16$.
4. $\lg 7 - \frac{1}{2} \lg 3 - 3 \lg 2$.
5. $\log_{\frac{1}{2}} (x^2 - x + 7)$ ifoda -2 ga teng bo'ladigan x ning
 qiymati mavjudmi?

1- §. Tenglama haqida umumiy tushunchalar

Ma'lumki, tarkibida noma'lum o'zgaruvchi qatnashgan tenglik *tenglama* deyiladi. Noma'lum o'zgaruvchining tenglikni ayniyatga aylantiradigan qiymatlari tenglamaning ildizlari deyiladi.

Tenglamaning barcha ildizlari to'plami tenglamaning *yechimi* deyiladi. Bu to'plam bo'sh to'plam bo'lishi ham mumkin. Bunday holatda tenglama yechimga ega emas deyiladi. Tenglamani shartli ravishda

$$h(x) = g(x) \quad (1)$$

tenglik ko'rinishida belgilab olaylik va

$$h_1(x) = g_1(x) \quad (2)$$

undan farqli tenglama bo'lsin. Agar (1) tenglamaning yechimi (2) tenglama yechimining to'plamostisi bo'lsa, (2) tenglama (1) tenglamaning *natijasi* deyiladi. Agar ikkala tenglama birbirining natijasi bo'lsa, bu tenglamalarni *teng kuchli* tenglamalar deymiz. Demak, teng kuchli tenglamalarning yechimlari bir xil, ya'ni teng bo'lar ekan.

(2) tenglama (1) tenglamaning natijasi bo'lsa, u holda (1) tenglamaning ildizlari bo'lmagan (2) tenglamaning ildizlari (1) tenglama uchun *chet ildizlar* deb ataladi.

$h(x) = g(x)$ va $h(x) - g(x) = 0$ tenglamalar teng kuchli tenglamalar bo'lishi ravshan, shuning uchun $h(x) - g(x)$ ifodani $f(x)$ orqali belgilab, har qanday bir noma'lumli tenglamani $f(x) = 0$ ko'rinishda yozish mumkin.

Agar $f(x) = 0$ ko'rinishdagi tenglama ba'zi almashtirishlar yordamida

$f(x) = 0$ tenglamaning natijasi bo'lgan $f_1(x) = 0$ ko'rinishdagi soddaroq tenglamaga keltirib olingan bo'lsa, u holda $f_1(x) = 0$ tenglamaning barcha yechimlari to'plamidan $y = f(x)$ funksiyaning aniqlanish sohasiga kirmagan barcha yechimlarini $f(x) = 0$ tenglamaga qo'yib tekshirib chiqamiz

va $f(x) = 0$ tenglamani qanoatlantiradigan ildizlar to'plamini tenglamaning yechimi sifatida olamiz.

Agar $f(x)$ ifoda tarkibida transsendent (trigonometrik, logarifmik va boshqa) funksiyalar qatnashgan bo'lsa, u holda $f(x) = 0$ tenglama *transsendent tenglama* bo'ladi.

$3^x + 3^{-x} = 2$; $2^x = 8x$; $\lg x = \frac{1}{3}x$; $\sin x = \frac{1}{2}$ ko'rinishdagi tenglamalar transsendent tenglamalarga misol bo'la oladi. Umumiy holda transsendent tenglamalarni yechish usulini oldindan ko'rsatish mumkin emas. Lekin, ba'zi xususiy hollarda transsendent tenglamalarni yechish mumkin.

Ma'lumki, tenglama tarkibida noma'lum o'zgaruvchi faqat daraja ko'rsatkichidagina qatnashsa, bunday tenglama *ko'rsatkichli tenglama* deyiladi.

Bir hil noma'lum qatnashgan bir nechta tenglamalar berilgan bo'lib, noma'lumning berilgan tenglamalardan hech bo'lmaganda bittasini qanoatlantiradigan barcha qiymatlari to'plamini topish talab qilinsin. Hosil bo'lgan to'plamni *berilgan tenglamalar birlashmasining yechimi* deyiladi.

Berilgan tenglamalarning birlashmasini kvadrat qavs orqali birlashtirib yozamiz.

Masalan,
$$\begin{cases} 2x+1=3 \\ x^2-4=0 \end{cases}$$

Ba'zi hollarda tenglamalarning birlashmasini $2x+1=3$, $x^2-4=0$ yoki $(2x+1=3) \vee (x^2-4=0)$ ko'rinishda ham yozish mumkin.

Birlashmaning yechimi birlashmani tashkil etgan barcha tenglamalar yechimlarining birlashmasidan iboratdir.

Yuqorida keltirilgan birlashma misolida birinchi tenglama yechimi $\{1\}$ va ikkinchi tenglama yechimi $\{-2; 2\}$ to'plamlardan iborat. U holda birlashmaning yechimi $\{1\} \cup \{-2; 2\}$ yoki $\{-2; 1; 2\}$ to'plamdan iborat bo'ladi.

Tenglamalarni yechish jarayonida $f_1(x)=0$ tenglamani ketma-ket almashtirishlar yordamida berilgan tenglama natijasi bo'lgan $f_2(x)=0$ tenglama hosil bo'lsin. U holda $f_2(x)=0$ tenglamaning $f_1(x)=0$ tenglamani qanoatlantirmaydigan ildizlari $f_1(x)=0$ tenglamaning *chet ildizlari* deyiladi.

1- m i s o l. $\sqrt{x-1}=1-x$ tenglamani kvadratga oshirilsa $x-1=(1-x)^2$ yoki $(x-1)(1-x+1)=0$ hosil bo'ladi. Bundan $x_1=1$, $x_2=2$ ildizlar kelib chiqadi. Lekin $x_2=2$ tenglamaning aniqlanish sohasiga tegishli emas. Shuning uchun u chet ildiz.

Ba'zi hollarda $f_1(x)=0$ tenglamani $f_2(x)=0$ ko'rinishdagi boshqa tenglama bilan almashtirish natijasida ba'zi ildizlar yo'qolib qolishi ham mumkin.

2- m i s o l. $x^2=x$ tenglamadan $x=1$ tenglamaga o'tish natijasida $x=0$ ildiz yo'qolib qoladi. Bunday holat $x^2=x$ tenglamaning ikkala tomonini $x \neq 0$ shart bilan, x ga bo'lish natijasida yuzaga keladi. Shuning uchun $x=0$ qiymat tenglamaning ildizi bo'lish yoki bo'lmasligi alohida tekshiriladi. Berilgan tenglama uchun $x=0$ ildiz bo'lishi ayon, ya'ni

$$x^2 = x \Leftrightarrow \begin{cases} x = 0, \\ x = 1. \end{cases}$$

Tenglama yechimini topishda tenglama ustida turli hil almashtirishlar bajarishga to'g'ri keladi. Shuning uchun, tenglamani qanday almashtirish natijasida o'ziga teng kuchli bo'lgan tenglama hosil bo'lishini yoki berilgan tenglamaning natijasi bo'lgan tenglama hosil bo'lishini yoki bo'lmas qanday almashtirish natijasida ba'zi ildizlar yo'qolib qolishini bilish muhim ahamiyatga ega.

Tenglamalarni yechishda ko'p ishlatiladigan teng kuchli almashtirishlar quyidagilar:

1^o. $f(x)=g(x)$ ko'rinishdagi tenglamaning ikkala tarafiga tenglamaning aniqlanish sohasiga tegishli x ning barcha qiymatlarida aniqlangan $\varphi(x)$ ifoda qo'shilsa, hosil bo'lgan $f(x)+\varphi(x)=g(x)+\varphi(x)$ tenglama berilgan tenglamaga teng kuchli tenglama bo'ladi.

3- m i s o l. $x^2+3x=-2$ tenglama $x^2+3x+2=-2+2$ tenglama yoki $x^2+3x+2=0$ tenglamaga teng kuchli.

4- m i s o l. $x^4-4=0$ tenglama $x^4-4+\sqrt[4]{x}=\sqrt[4]{x}$ tenglamaga teng kuchli bo'la olmaydi. Chunki $\sqrt[4]{x}$ ifoda x ning tenglama aniqlanish sohasidagi manfiy qiymatlari uchun ma'noga ega emas.

2°. $\varphi(x)$ ifoda berilgan $f(x) = g(x)$ tenglamaning aniqlanish sohasidagi barcha x lar uchun aniqlanib, bu qiymatlarning hech birida nolga teng bo'lmasin. U holda

$f(x) \cdot \varphi(x) = g(x) \cdot \varphi(x)$ va $\frac{f(x)}{\varphi(x)} = \frac{g(x)}{\varphi(x)}$ tenglamalar berilgan tenglamaga teng kuchli tenglamalardir.

5- m i s o l. $x-9 = \sqrt{x}+3$ tenglamaning ikkala tomonini $\sqrt{x}+3$ ifodaga bo'lish natijasida berilgan tenglamaga teng kuchli bo'lgan tenglama hosil bo'ladi, chunki $\sqrt{x}+3$ ifoda x ning tenglamaning aniqlanish sohasiga kirgan barcha qiymatlarida aniqlangan bo'lib, bu qiymatlar uchun musbat.

6- m i s o l. $(x-1)(x+1) = x+1$ tenglamaga $x+1 \neq 0$ shart qo'yib uning ikkala tomonini $x+1$ ifodaga bo'linsa, hosil bo'lgan $x-1=1$ tenglama berilgan tenglamaga teng kuchli bo'la olmaydi, chunki $x+1$ ifoda tenglama aniqlanish sohasiga kirgan x ning -1 ga teng bo'lgan qiymatida nolga teng. Lekin

berilgan tenglama $\begin{cases} x-1=1 \\ x+1=0 \end{cases}$ birlashmaga teng kuchli bo'ladi.

3°. Berilgan $f(x) = g(x)$ tenglamaning ikkala tomonini toq $(2m+1)$ darajaga ko'tarsak, hosil bo'lgan $[f(x)]^{2m+1} = [g(x)]^{2m+1}$ tenglama berilgan tenglamaga teng kuchli bo'ladi.

7- m i s o l. $\sqrt[3]{x-2} = 1$ tenglama $x-2=1$ tenglamaga teng kuchli.

4°. Ikkala tarafi musbat bo'lgan yoki ikkala tarafi ham manfiy bo'lgan yoki bitta tarafi nolga teng bo'lgan $f(x) = g(x)$ tenglamalarning ikkala tarafini bir hil natural darajaga ko'tarish natijasida hosil bo'lgan $[f(x)]^n = [g(x)]^n$ tenglama berilgan tenglamaga teng kuchli bo'ladi.

8- m i s o l. $2x-1 = \sqrt{x-3}$ tenglamaning ikkala tarafini kvadratga ko'tarilsa, hosil bo'lgan $(2x-1)^2 = x-3$ tenglama berilgan tenglamaga teng kuchli, chunki berilgan tenglama-

ning aniqlanish sohasi $x > 3$ da tenglamaning ikkala tarafi ham musbat.

9- m i s o l. $\sqrt{x-1} = x-3$ tenglamaning ikkala tarafini kvadratga oshirishdan hosil bo'lgan $x-1 = (x-3)^2$ tenglamani berilgan tenglamaga teng kuchli deb ayta olmaymiz, chunki masalan $x=2$ qiymatda $\sqrt{x-1} > 0$, $x-3 < 0$ bo'lib, tenglamaning ikkala tarafi ikki hil ishoraga ega. Hosil bo'lgan $x-1 = (x-3)^2$ tenglamani soddalashtirib $x^2 - 7x + 10 = 0$ Viyet formulalari qo'llansa, $x_1 = 2$, $x_2 = 5$ ildizlar hosil bo'ladi. Ulardan $x_2 = 5$ berilgan tenglamaning ildizi, $x_1 = 2$ esa tenglamaning chet ildizi bo'ladi.

Shunday qilib, ba'zan tenglamaning ikkala tarafidagi ifodalarni bir hil darajaga ko'tarish, berilgan tenglamaga teng kuchli tenglamaga olib kelmasa ham, tenglamalarni yechishda muhim ahamiyatga ega ekan.

Umuman tenglamalar ustida ba'zi almashtirishlarni bajarganimizda tenglamaning aniqlanish sohasining kengayib ketishi chet ildizlarning kelib chiqishiga sabab bo'lishi mumkin. Aksincha, tenglama aniqlanish sohasining torayishi esa, ba'zi ildizlarning yo'qolib qolishiga olib keladi. Shuning uchun berilgan tenglamani boshqa tenglama bilan almashtirganda, tenglamaning aniqlanish sohasini o'zgarishi albatta e'tiborga olinishi lozim.

Agar almashtirish natijasida tenglama aniqlanish sohasi kengayib, chet ildizlar hosil bo'lsa, ular tenglama yechimiga kiritilmaydi. Buning uchun hosil bo'lgan barcha ildizlarni tenglamaga qo'yib ko'rib, tenglamani qanoatlantirganlari qoldiriladi.

Agar tenglamani boshqa tenglama bilan almashtirganda tenglamaning aniqlanish sohasi torayib qolsa, o'zgaruvchining berilgan tenglama aniqlanish sohasiga kirib, almashtirish natijasida hosil bo'lgan tenglamaning aniqlanish sohasiga kirmagan barcha qiymatlari berilgan tenglamaning ildizi bo'lish bo'lmasligi tekshiriladi.

10- m i s o l. $\sqrt{4x+5} = 8 - \sqrt{2x-1}$ tenglamani yeching.

Ye ch i sh. Berilgan tenglamada o'zgaruvchining yo'l

$$\text{qo'yishi mumkin bo'lgan qiymatlari sohasi } \begin{cases} 4x + 5 \geq 0, \\ 2x - 1 \geq 0, \\ 8 - \sqrt{2x-1} \geq 0 \end{cases}$$

tengsizliklar sistemasining yechimidan iborat. Bu sistema qu-

$$\text{yidagi } \begin{cases} x \geq \frac{1}{2} \\ \sqrt{2x-1} \leq 8 \end{cases} \text{ sistemaga teng kuchli. Bu sistemani}$$

qanoatlantiruvchi qiymatlarda berilgan tenglamaning ikkala tarafi musbat. Shuning uchun tenglamani kvadratga ko'tarib, soddalashtirilsa $8\sqrt{2x-1} = 29 - x$ tenglama kelib chiqadi. Bunda $29 - x \geq 0$ deb faraz qilib, tenglamani kvadratga ko'tarilsa, $x^2 - 186x + 905 = 0$ tenglama, undan $x_1 = 5$, $x_2 = 181$ ildizlar kelib chiqadi.

Hosil bo'lgan ildizlardan $x_1 = 5$ tenglamaning ildizi, $x_2 = 181$ tenglamaning chet ildizi bo'ladi.

11- m i s o l. $\sqrt{3x+2} = 3\sqrt{2} + \sqrt{x-2}$ tenglamani eching.

$$\text{Ye ch i sh. Tenglamaning aniqlanish sohasi } \begin{cases} 3x + 2 \geq 0 \\ x - 2 \geq 0 \end{cases}$$

sistema yechimidan iborat.

$$\text{Demak, } \begin{cases} x \geq -\frac{2}{3} \\ x \geq 2 \end{cases} \text{ bundan } x \geq 2 \text{ berilgan tenglamaning}$$

aniqlanish sohasi.

Tenglama unga teng kuchli tenglamalar sistemasi bilan almashtirish yordamida yechiladi:

$$\begin{aligned} \sqrt{3x+2} &= 3\sqrt{2} + \sqrt{x-2} \Leftrightarrow \begin{cases} x \geq 2 \\ 3x+2 = 18 + 6\sqrt{2(x-2)} + x-2 \end{cases} \Leftrightarrow \\ \Leftrightarrow \begin{cases} x \geq 2 \\ 2x-14 = 6\sqrt{2(x-2)} \end{cases} \Leftrightarrow \begin{cases} x \geq 2 \\ x \geq 7 \\ x^2 - 14x + 49 = 9(2x-4) \end{cases} \Leftrightarrow \end{aligned}$$

$$\Leftrightarrow \begin{cases} x \geq 7 \\ x^2 - 32x + 85 = 0 \end{cases} \Leftrightarrow \begin{cases} x \geq 7 \\ x_{1,2} = 16 \pm 3\sqrt{19} \end{cases} \Leftrightarrow \begin{cases} x \geq 7 \\ \begin{cases} x_1 = 16 + 3\sqrt{19} \\ x_2 = 16 - 3\sqrt{19} \end{cases} \end{cases}$$

Hosil bo'lgan $x_2 = 16 - 3\sqrt{19}$ ildiz $x \geq 4\frac{2}{3}$ shartni qanoatlantirmaganligi uchun tenglamaning ildizi $x = 16 + 3\sqrt{19}$.

12- m i s o l. $\sqrt{3-2x} = x$ tenglamani yeching.

Y e c h i s h. Misolda qatnashayotgan $u = \sqrt{3-2x}$ funksiyaning aniqlanish sohasi $2x - 3 \leq 0$ bundan $x \leq 1,5$; $u = x$ funksiyaning aniqlanish sohasi esa $(-\infty, +\infty)$ dan iborat. U holda tenglamaning aniqlanish sohasi $x \leq 1,5$ bo'ladi.

Endi tenglamaning ikkala tomoni kvadratga ko'tarilsa, $x^2 + 2x - 3 = 0$ kvadrat tenglama hosil bo'ladi. Viyet formulalariga asosan, bu tenglama ildizlari $x_1 = -3$ va $x_2 = 1$. Ikkala ildiz ham tenglamaning aniqlanish sohasida yotadi. Lekin $x_1 = -3$ chet ildiz, chunki $\sqrt{3-2x} \geq 0$ bundan $x \geq 0$. Agar $x \geq 0$ bo'lishini oldindan hisobga olinganda, x qabul qilishi mumkin bo'lgan qiymatlari to'plamini $[0; 1,5]$ segment sifatida olish mumkinligi kelib chiqar edi va bu holda faqat $x = 1$ ildiznigina tenglamaga qo'yib tekshirish yetarli bo'ladi. Berilgan tenglama uchun $[0; 1,5]$ to'plamni x ning *yo'l qo'yishi mumkin bo'lgan qiymatlari sohasi* deb ataladi. Qisqacha (qiymatlar sohasi) Q.S. deb belgilab olamiz.

13- m i s o l. $\sqrt{x+\sqrt{x+2}} + \sqrt{x-\sqrt{x+2}} = 2$ tenglamani yeching.

Y e c h i s h. Bu ko'rinishdagi tenglamalar uchun x ning Q.S. ni topish ko'p vaqtni talab etadi. Shuning uchun oldin tenglamani almashtirishlar yordamida berilgan tenglamaning natijasi bo'lgan soddaroq tenglama ko'rinishiga keltirib olamiz. Hosil bo'lgan tenglama ildizlarini topib, berilgan tenglamaga qo'yib tekshirib chiqiladi.

Berilgan tenglamaning ikkala tomonini kvadratga ko'tarib, soddalashtiriladi. Hosil bo'lgan $\sqrt{x^2 - x - 2} = 2 - x$ tenglamani yana kvadratga ko'tarib, soddalashtirilsa $x = 2$ ildizi hosil

bo'ladi. x ning bu qiymatini berilgan tenglamaga qo'yib, $x = 2$ tenglamaning ildizi bo'lishiga ishonch hosil qilamiz.

14- m i s o l. $\sqrt{x-7} + \sqrt{7-x} = 5$ tenglamani yeching.

Y e c h i s h. Bunday ko'rinishdagi tenglamalarda x ning Q.S. ni aniqlash tenglamaning yechimlarini topish imkoniyati

tini beradi. Haqiqatan ham, $\begin{cases} 7-x \geq 0 \\ x-7 \geq 0 \end{cases}$ dan $x = 7$ kelib

chiqadi. Demak, x ning yo'l qo'yishi mumkin bo'lgan qiymati faqat 7 ekan. Bu qiymat berilgan tenglamani qanoatlantirmaganligi sababli tenglama yechimi bo'sh to'plam.

15- m i s o l. $x^2 + 4x \cos x + 4 = 0$ tenglamani yeching.

Y e c h i s h. Tenglamadagi $4\cos x$ ifodani x ning koef-fitsiyenti sifatida qarab, kvadrat tenglama yechimlari topiladi:

$$x_{1,2} = \frac{-4\cos x \pm \sqrt{16\cos^2 x - 16}}{2}. \text{ Bunda } 16\cos^2 x - 16 \geq 0$$

shartdan $\cos^2 x = 1$ yoki $\sin^2 x = 0$ ni hosil qilinadi. U holda

$x = k\pi$, $k \in \mathbb{Z}$ va $x = \frac{-4(\pm 1)}{2} = \pm 2$. Demak, berilgan tenglama yechimi bo'sh to'plam.

Mashqlar

Tenglamalarni yeching (1-20):

- $\sqrt{x+3} = \sqrt{a-x}$.
- $x + 12\sqrt{x} - 64 = 0$.
- $\sqrt{4+2x-x^2} = x-2$.
- $\frac{x-4}{\sqrt{x}+2} = x-8$.
- $\sqrt{x+10} + \sqrt{x-2} = 6$.
- $(x-3)^2 + 3x - 22 = \sqrt{x^2 - 3x + 7}$.
- $\sqrt{x} - \sqrt{x+3} = 1$.
- $2x^2 + 3x - 5\sqrt{2x^2 + 3x + 9} + 3 = 0$.
- $\sqrt{15-x} + \sqrt{3-x} = 6$.
- $x\sqrt{x^2+15} - 2 = \sqrt{x}\sqrt{x^2+15}$.

$$11. \sqrt{\frac{20+x}{x}} + \sqrt{\frac{20-x}{x}} = \sqrt{6}.$$

$$12. \sqrt{\frac{x+1}{x-1}} - \sqrt{\frac{x-1}{x+1}} = \frac{3}{2}.$$

$$13. \sqrt{x^2 + 5x + 3} - \sqrt{x^2 + 5x - 2} = 1.$$

$$14. \sqrt{\frac{x+a}{x-a}} + 2\sqrt{\frac{x-a}{x+a}} = 3.$$

$$15. \sqrt{2x+3} + \sqrt{x+1} = 5.$$

$$16. \sqrt{2-x} + \frac{4}{\sqrt{2-x}+3} = 2.$$

$$17. \sqrt{2x-3} + \sqrt{4x+1} = 4.$$

$$18. 4x^2 + 12x\sqrt{1+x} = 27(1+x).$$

$$19. \sqrt{x+4} + \sqrt{2x+6} = 7.$$

$$20. \sqrt{5x+7} - \sqrt{2x+3} = \sqrt{3x+4}.$$

Quyidagi tenglamalarning ildizlari mavjud emasligini isbotlang (1-4):

$$1. \sqrt{x-1} + \sqrt{2-x} = \sqrt{x-5}.$$

$$2. \sqrt{x^2+1} + \sqrt{x^4+1} = 1.$$

$$3. \sqrt[3]{x^2-144} = \sqrt{x-6} + \sqrt{6-x}.$$

$$4. \sqrt[4]{x^2-144} = \sqrt{x-8} + \sqrt{8-x}.$$

Berilgan tenglamalar teng kuchli yoki yo'qligini aniqlang (1-4):

$$1. x^2 + 1 = \sqrt{x}; \quad x^2 + 1 + \sqrt{1+x} = \sqrt{x} + \sqrt{1+x}.$$

$$2. x^3 + x = 0; \quad \frac{x^3 + x}{x} = 0.$$

$$3. \frac{2\sqrt{x} - 7x^2}{2} = x + \sqrt{x}; \quad 2\sqrt{x} - 7x^2 = 2x + 2\sqrt{x}.$$

$$4. \sqrt{x} - 2 = \sqrt{2x+1}; \quad (\sqrt{x} - 2)^2 = (\sqrt{2x+1})^2.$$

2- §. $a^{f(x)} = a^{g(x)}$ ko'rinishdagi tenglamalar

$a^{f(x)} = a^{g(x)}$, $a > 0$, $a \neq 1$ ko'rinishdagi tenglamalarni yechishda ularning $f(x) = g(x)$ ko'rinishdagi tenglamaga teng kuchli ekanligidan foydalaniladi.

1- m i s o l. $2^{x^2-4} = 1$ tenglamani yeching.

Y e c h i s h. $1 = 2^0$ bo'lganligi uchun berilgan tenglamani $2^{x^2-4} = 2^0$ ko'rinishda yozib olinadi. U holda $x^2 - 4 = 0$, bundan $x_{1,2} = \pm 2$.

2- m i s o l. $4^{\cos^2 x - 1} = 1$ tenglamani yeching.

Y e c h i s h. Bu tenglama $\cos^2 x - 1 = 0$ tenglamaga teng kuchli. U holda $\cos^2 x = 1$ yoki $\cos x = \pm 1$, bundan $x = k\pi$, $k \in \mathbb{Z}$.

3- m i s o l. $3^{x^2 - \frac{1}{2}x} = \sqrt[3]{9}$ tenglamani yeching.

Y e c h i s h. Berilgan tenglamani $3^{x^2 - \frac{1}{2}x} = 3^{\frac{2}{3}}$ ko'rinishida yozib olinadi. U holda $x^2 - \frac{1}{2}x = \frac{2}{3}$ yoki $6x^2 - 3x - 4 = 0$ tenglama hosil bo'ladi. Kvadrat tenglamani yechib, $x_{1,2} = \frac{3 \pm \sqrt{105}}{12}$ topiladi.

4- m i s o l. $\left(\frac{1}{3}\right)^6 \cdot 3^{x^2} - \frac{1}{\sqrt[3]{243}} = 0$ tenglamani yeching.

Y e c h i s h. Agar $\left(\frac{1}{3}\right)^6 \cdot 3^{x^2} = 3^{-6} \cdot 3^{x^2} = 3^{x^2-6}$ va

$\frac{1}{\sqrt[3]{243}} = \frac{1}{\sqrt[3]{3^5}} = \frac{1}{3^{\frac{5}{3}}} = 3^{-\frac{5}{3}}$ tengliklarni hisobga olinsa, tenglama

$3^{x^2-6} = 3^{-\frac{5}{3}}$ ko'rinishga keladi. U holda $x^2 - 6 = -\frac{5}{3}$ yoki $x^3 - 6x + 5 = 0$. Agar $x = 1$ ildiz tenglamani qanoatlantirishini hisobga olib $x^3 - 6x + 5$ ni $x - 1$ ga bo'lsak, $x^2 + x - 5$ ga ega bo'lamiz. Bundan $(x - 1)(x^2 + x - 5) = 0$ u holda $x_1 = 1$,

$x_{2,3} = \frac{-1 \pm \sqrt{21}}{2}$ lar keltirib chiqariladi.

5- m i s o l. $x^{2\lg^2 x} = x^{3\lg x + 2}$ tenglamani yeching.

Y e c h i s h. Berilgan tenglamaning aniqlanish sohasi $(0, +\infty)$ to'plamdan iborat.

Tenglamani shakl almashtirib, $2\lg^2 x = 3\lg x + 2$ tenglamaga ega bo'lamiz, ya'ni $2\lg^2 x - 3\lg x - 2 = 0$. Hosil bo'lgan tenglamaga $\lg x = t$ belgilashni qo'llasak, $2t^2 - 3t - 2 = 0$ tenglama hosil bo'ladi. Bu tenglamaning

$t_1 = 2$, $t_2 = -\frac{1}{2}$ ildizlaridagi t ning o'rniga $\lg x$ ni qo'ysak,

$$\lg x = 2, x_1 = 100; \lg x = -\frac{1}{2}, x = 10^{-\frac{1}{2}}, x_2 = \frac{\sqrt{10}}{10}.$$

Bu ildizlardan tashqari $x = 1$ ham tenglama ildizi bo'lishini ko'rish qiyin emas. Shunday qilib, tenglama uchta

ildizga ega $x_1 = 100; x_2 = \frac{\sqrt{10}}{10}, x_3 = 1$.

6- m i s o l. $2^{\frac{x}{2}} \cdot 4^{\frac{x}{6}} \cdot 2^{-\frac{1}{2x}} = 2^{\frac{1}{3}}$ tenglamani yeching.

Y e c h i s h. Bu tenglamani $2^{\frac{x}{2}} \cdot 2^{\frac{2x}{6}} \cdot 2^{-\frac{1}{2x}} = 2^{\frac{1}{3}}$ yoki $2^{\frac{x}{2} + \frac{x}{3} - \frac{1}{2x}} = 2^{\frac{1}{3}}$ ko'rinishga keltirib, daraja ko'rsatkichlarini o'zaro

tenglashtirib olamiz. U holda $\frac{x}{2} + \frac{x}{3} - \frac{1}{2x} = \frac{1}{3}$ tenglamada

$x \neq 0$ bo'lishini hisobga olgan holda umumiy mahrajga keltiraylik. Natijada $3x^2 + 2x^2 - 3 = 14x$ yoki $5x^2 - 14x - 3 = 0$ kvadrat tenglamani hosil qilamiz. Bu kvadrat tenglamadan $x_1 = 3, x_2 = -0,2$ ildizlarni topamiz.

Mashqlar

Tenglamalarni yeching (1-24):

$$1. \sqrt[5]{5^{2x-8}} - 25 = 0. \quad 2. \sqrt[5]{5^{x^2-20x+55}} = 1.$$

$$3. \sqrt{5^{x(13-x)}} \cdot 2^{11} = 10^{11}. \quad 4. 81 \cdot \sqrt[3]{3^{x^2-8x}} = 1.$$

$$5. 2^2 \cdot 2^5 \cdot 2^8 \cdot \dots \cdot 2^{3n-1} = 8^5. \quad 6. 3^x - 8 = 3^{2-x}.$$

$$7. 3^{\frac{1}{2}} \cdot 3^{\frac{x}{1+x^2}} \cdot 3^{\frac{2+x+x^2}{2(1+x^2)}} = 9^2. \quad 8. 2^x \cdot \sqrt[3]{4^x \cdot 8^{-\frac{1}{x}}} = 16\sqrt[3]{4}.$$

9. $\sqrt{2} \cdot 2^{\frac{5}{4\sqrt{x+10}}} = 4^{\frac{1}{\sqrt{x+1}}}$. 10. $2^{\frac{3x-9}{3x-7}} \cdot (\sqrt[3]{4^{\frac{1-3x}{x-1}}})^{\frac{1}{3}} = 1$.
11. $10^{x^2-3} = 0,01 \cdot ((0,1)^{1-x})^3$. 12. $\left(\frac{3}{5}\right)^x \left(2\frac{7}{9}\right)^{x^2-13} = \left(\frac{3}{5}\right)^9$.
13. $\left(\frac{1}{5}\right)^{\frac{1}{\sqrt{x-x}}} \cdot 0,2^{(\sqrt{x})^{-1}} = 5^{\frac{2}{3}}$. 14. $\left(\frac{1}{2}\right)^{\frac{1}{1-\sqrt{x}}} \cdot 2^{\frac{1}{-1-\sqrt{x}}} = 4^{\frac{\sqrt{x}}{x+\sqrt{x}}}$.
15. $\frac{7^x}{4^{\frac{x^2}{2}-\frac{3}{2}}} - \left(\frac{4}{7}\right)^{-x} = 0$. 16. $(4 - \sqrt{15})^{\frac{x}{2}} + (4 + \sqrt{15})^{\frac{x}{2}} = 8$.
17. $2^{\sqrt{x}-4} = 4^{\frac{1}{8}(x-21)}$. 18. $(0,75)^{x-2} \cdot \left(1\frac{1}{3}\right)^{\frac{1}{2}} = \frac{1}{2} \cdot 3^{\frac{3x-7}{4}}$.
19. $\left(\frac{9}{8}\right)^x \cdot \left(\frac{3}{2}\right)^{-x} = \left(\sqrt{\frac{4}{3}}\right)^{-6}$. 20. $10^{x^2} \cdot 1000 = (0,1)^{2x-6}$.
21. $\sqrt{2^x} \cdot 4^{\frac{x}{3}} \cdot \sqrt[3]{2^{\frac{3}{x}}} = \sqrt[3]{128}$. 22. $\left(\frac{3}{5}\right)^x \cdot \left(\frac{9}{25}\right)^{12-x^2} = \left(\frac{125}{27}\right)^{-3}$.
23. $\left(\frac{5}{2}\right)^{\frac{4+\sqrt{9-x}}{\sqrt{9-x}}} \cdot \left(\frac{2}{5}\right)^{1-\sqrt{9-x}} = 10^{-5} \cdot 5^{10}$.
24. $2^{2\sqrt{x+1}} - 4^{\frac{1}{2}\sqrt{x+1}+1} = 0$.

3- §. $a^{f(x)} = b^{g(x)}$ ko'rinishdagi tenglamalar

$a^{f(x)} = b^{g(x)}$ (1), $a > 0$, $b > 0$, $a \neq 1$, $b \neq 1$ ko'rinishdagi tenglamalarni yechish uchun tenglamaning ikkala tomonini c ($c > 0$, $c \neq 1$) asosga ko'ra logarifmlab, berilgan tenglamaga teng kuchli bo'lgan $f(x) \cdot \log_c a = g(x) \cdot \log_c b$ (2) tenglama hosil bo'ladi. Bu tenglamaning ildizlari (1) tenglamaning ildizlaridan iborat bo'ladi.

(1) ko'rinishdagi tenglama namunalari bilan tanishib chiqaylik.

1- m i s o l. $5^{x-3} = 3^{1-2x}$ tenglamani yeching.

Y e c h i s h. Tenglamaning ikkala tomonini 10 asosga

ko'ra logarifmlab $(x-3) \cdot \lg 5 = (1-2x) \cdot \lg 3$ tenglamani hosil qilamiz. U holda $x \cdot \lg 5 - 3 \cdot \lg 5 = \lg 3 - 2x \cdot \lg 3$ yoki

$$x(\lg 5 + 2 \cdot \lg 3) = \lg 3 + 3 \cdot \lg 5, \text{ bundan esa } x = \frac{\lg 3 + 3 \lg 5}{2 \lg 3 + \lg 5}$$

yechim hosil bo'ladi.

2- m i s o l. $\sqrt[12]{\left(\frac{17}{25}\right)^{9(x+2)}} \cdot 36^{\frac{2x-1}{2}} = 216 \cdot \sqrt{\left(\frac{17}{25}\right)^{3x}}$ tenglamani

yeching.

Y e c h i s h. Berilgan tenglamani teng kuchli almashtirishlar orqali

$$\left(\frac{17}{25}\right)^{\frac{3(x+2)}{4}} \cdot 6^{2x-1} = 6^3 \cdot \left(\frac{17}{25}\right)^{\frac{3x}{2}} \text{ ko'rinishga keltirib olamiz.}$$

Tenglamaning ikkala tomonini $\left(\frac{17}{25}\right)^{\frac{3x}{2}} \cdot 6^{2x-1}$ ifodaga bo'lib

yuborsak, $\left(\frac{17}{25}\right)^{\frac{3(x+2)}{4} - \frac{3x}{2}} = 6^{3-(2x-1)}$ tenglamani hosil qilamiz.

Oxirgi tenglamaning ikkala tomonini logarifmlaymiz:

$$\left(\frac{6-3x}{4}\right) \cdot \lg \frac{17}{25} = (4-2x) \cdot \lg 6 \text{ va tenglamani soddalashtiramiz:}$$

$$\left(8 = \lg 6 - 3 \lg \frac{17}{25}\right) \cdot x = 2 \left(8 \lg 6 - 3 \lg \frac{17}{25}\right). \text{ Bundan esa } x = 2$$

yechim kelib chiqadi.

3- m i s o l. $40 \cdot 3^{x^2} - 40 \cdot 7^{2x} - 3^{x^2} + 7^{2x} = 0$ tenglamani yeching.

Y e c h i s h. O'xshash hadlarni guruhlasak, $(40 \cdot 3^{x^2} - 3^{x^2}) + (-40 \cdot 7^{2x} + 7^{2x}) = 0$ hosil bo'ladi. U holda $3^{x^2} \cdot 39 - 39 \cdot 7^{2x} = 0$ yoki $3^{x^2} = 7^{2x}$. Ikkala tenglamani o'n asosga ko'ra logarifmlasak, $x^2 \lg 3 - 2x \lg 7 = 0$ tenglamaga ega bo'lamiz. Bu tenglamadagi x ni qavs tashqarisiga chiqarib, $x(x \lg 3 - 2 \lg 7) = 0$ tenglamani hosil qilamiz. Bundan $x_1 = 0$, $x_2 = \log_3 7$ ildizlarni topamiz.

4- m i s o l. $|x-3|^{x^2-3x} = |x-3|^{4x-10}$ tenglamani yeching.

Y e c h i s h. Berilgan tenglamaning qiymatlar sohasi $|x-3| \neq 0$ yoki $x \neq 3$. Agar $|x-3|=1$ bo'lsa, berilgan tenglik o'rinli. U holda $x-3 = \pm 1$ yoki $x_1 = 4, x_2 = 2$. Agar $|x-3| \neq 1$ shart o'rinli bo'lsa, $x^2 - 3x = 4x - 10$ yoki $x^2 - 7x + 10 = 0$ tenglamalar hosil bo'ladi. Viyet formulalariga asosan 2 va 5 sonlar tenglama ildizlaridir. Shunday qilib, $x_1 = 2, x_2 = 4, x_3 = 5$.

Mashqlar

Tenglamalarni yeching (1-25):

1. $8^{x-7} = 11^{7-x}$.
2. $16^{x-3} = 81^{x-3}$.
3. $a^{x+1} = b^{2-x}$.
4. $3^x - 2^{4-x} = 0$.
5. $5^{\frac{1}{2x}} - 0,04^{1-x} = 0$.
6. $2^{x^2-1} - 3^{x^2} = \left(\frac{1}{3}\right)^{1-x^2} - 2^{x^2+2}$.
7. $\sqrt[3]{100} + \sqrt[3]{25} = \frac{5}{4} \cdot \sqrt[3]{50}$.
8. $3^{2(2x+5)} \cdot 5^{2(3x+1)} = 15^{5x+6}$.
9. $2^{3x} \cdot 7^{x-2} = 4^{x+1}$.
10. $7^y - 5^{y+1} = 3 \cdot 5^{y-1} - 13 \cdot 7^{y-1}$.
11. $6^{2x+4} = 3^{3x} \cdot 2^{x+8}$.
12. $\sqrt{2^x \sqrt[3]{4^x \cdot 0,125^{1/x}}} = 4\sqrt[3]{2}$.
13. $8^{\frac{x-3}{3x-7}} \sqrt[3]{0,25^{\frac{3x-1}{x-1}}} = 1$.
14. $\frac{1}{2^{\sqrt{x-1}}} \cdot 0,5^{\frac{1}{\sqrt{x+1}}} = 4^{\frac{\sqrt{x}}{x+\sqrt{x}}}$.
15. $|x-2|^{x^2-5x} = |x-2|^{-6}$.
16. $2^{x^2-1} - 3^{x^2} = 3^{x^2-1} - 2^{x^2+2}$.
17. $\frac{1}{5 \cdot 0,2^{\frac{1}{x}}} = 0,0016^{x-1}$.
18. $|x-3|^{3x^2-10x+3} = 1$.
19. $35 \cdot 3^{x^2} - 35 \cdot 5^{2x} - 3^{x^2} + 5^{2x} = 0$.
20. $3 \cdot 4^x + 3^{2x+3} = 6 \cdot 4^{x+1} - \frac{9}{2} \cdot 3^{2x+1}$.
21. $\sqrt{2} \cdot 0,5^{\frac{5}{4\sqrt{x+10}}} - 16^{\frac{1}{2(\sqrt{x+1})}} = 0$.
22. $2^{x^2-3} \cdot 5^{x^2-3} = 0,01 \cdot (10^{x-1})^3$.
23. $2,5^{\frac{4+\sqrt{9-x}}{\sqrt{9-x}}} \cdot 0,4^{1-\sqrt{9-x}} = 5^{10} \cdot 0,1^5$.

$$24. |x-2|^{x^2-2x} = |x-2|^{5x-10}.$$

$$25. 53 \cdot 3^{x^2} - 53 \cdot 5^{2x} - 3^{x^2} + 5^{2x} = 0.$$

$$4- \S. A_0 a^{f(x)+k_0} + A_1 a^{f(x)+k_1} + \dots + A_n a^{f(x)+k_n} = M$$

ko‘rinishdagi tenglamalar

$A_0 a^{f(x)+k_0} + A_1 a^{f(x)+k_1} + \dots + A_n a^{f(x)+k_n} = M$ (1) bunda $A_1, A_2, \dots, A_n, a, M$ — haqiqiy sonlar bo‘lib, albatta $a > 0$ va $a \neq 1$ bo‘lsin.

(1) ko‘rinishdagi tenglamalarni yechish uchun qavsdan tashqariga $a^{f(x)}$ ni chiqarib olib, tenglamani (1) tenglamaga teng kuchli bo‘lgan

$a^{f(x)}(A_0 a^{k_0} + A_1 a^{k_1} + \dots + A_n a^{k_n}) = M$ tenglama ko‘rinishiga keltirib olib, hosil bo‘lgan tenglama ildizlarini topamiz.

1- m i s o l. $2^{3y} + 3 \cdot 2^{3y-1} = 20$ tenglamani yeching.

Y e c h i s h. $2^{3y}(1 + 3 \cdot 2^{-1}) = 20$. U holda $2^{3y} = 8$. Bundan esa $y = 1$ yechimni topamiz.

2- m i s o l. $5^{2x} - 7^x - 5^{2x} \cdot 35 + 7^x \cdot 35 = 0$ tenglamani yeching.

Y e c h i s h. Yig‘indi hadlarining o‘rnini almashtirib

$$5^{2x} - 5^{2x} \cdot 35 + 7^x \cdot 35 - 7^x = 0 \text{ tenglamani, undan esa}$$

$$5^{2x}(1 - 35) + 7^x(35 - 1) = 0 \text{ tenglamaga o‘tamiz. U holda}$$

$5^{2x} = 7^x$ yoki $\left(\frac{25}{7}\right)^x = 1$ tenglamani hosil qilib, $x = 0$ yechimni topamiz.

3- m i s o l. $(3^{x^2-54})^{\frac{1}{2}} - 7(3^{x^2-58})^{\frac{1}{2}} = 162$ tenglamani yeching.

$$Y e c h i s h. \text{ Berilgan tenglamani avval } 3^{\frac{x^2}{2}-27} - 7 \cdot 3^{\frac{x^2}{2}-29} = 162$$

ko‘rinishga, keyin $3^{\frac{x^2}{2}-27}(1 - 7 \cdot 3^{-2}) = 162$ ko‘rinishga keltirib

olamiz. Hosil bo‘lgan tenglamani soddalashtirib $3^{\frac{x^2}{2}-27} = 3^6$

tenglamani, undan $\frac{x^2}{2} - 27 = 6$ va nihoyat $x^2 = 66$ tenglamani olamiz. Bundan $x = \pm 8$ yechimlarni hosil qilamiz.

4- misol. $3^{x-1} + 3^{x-4} + 3^{x-2} = 18,5 + 9,25 + 4,625 + \dots$ tenglamani yeching.

Tenglamaning chap tarafi $3^x \left(\frac{1}{3} + \frac{1}{81} + \frac{1}{9} \right) = \frac{3^x \cdot 37}{81}$ ko'rinishga keltiriladi. Tenglamaning o'ng tarafidagi ifoda esa birinchi hadi 18,5 ga, mahraji $1/2$ teng cheksiz kamayuvchi geometrik progressiya barcha hadlarining yig'indisidan iboratdir. U holda tenglamaning o'ng tarafini $S = \frac{b_1}{1-q}$ formula orqali hisoblab,

$S = \frac{18,5}{1 - \frac{1}{2}} = 37$ ni hosil qilamiz. Shunday qilib,

berilgan tenglama $\frac{3^x \cdot 37}{81} = 37$ ko'rinishga keladi. Bu tenglamadan $x = 4$ yechim hosil bo'ladi.

Mashqlar

Tenglamalarni yeching (1-25):

1. $3^{x^2+2} - 3^{x^2} = 144$.
2. $2^{x+3} - 2^x = 56$.
3. $2^x - 2^{x+1} = -4$.
4. $3^{2x+2} + 3^{2x} = 90$.
5. $4^x - 3^{x+0,5} = 3^{x+0,5} - 2^{2x-1}$.
6. $5^{\frac{(2x-8)x}{5}} - 25 = 0$.
7. $\frac{2^{x+2} + 2^x}{2^x - 2} = 5$.
8. $3^{4x+1} + 3^{4x+2} + 3^{4x+3} = 9620$.
9. $\frac{3^{1+x}}{3^x - 13} = 4$.
10. $3^{2\sqrt{x}} + 3^{2\sqrt{x}-1} - 3^{2\sqrt{x}-2} = 11$.
11. $9^x - 2^{x+\frac{1}{2}} = 2^{x+\frac{7}{2}} - 3^{2x-1}$.
12. $2^{3x} \cdot 3^x - 2^{3(x-1)} \cdot 3^{x-1} = 552$.
13. $4^x - 7^{x+2} = 7^{x+1} - 2 \cdot 4^{x+1}$.
14. $2 \cdot 3^{x+2} - 5 \cdot 3^{x-3} = 1443$.
15. $4^x - 3^{x-\frac{1}{2}} = 3^{x+\frac{1}{3}} - 2^{2x-1}$.
16. $\frac{7^{x^2+1}}{7 \cdot 7^{x^2+1} - 6,75} = 4$.
17. $2^{x-1} + 2^{x-4} + 2^{1^{x-2}} = 13$.
18. $5^{x-1} : (5 \cdot 0,2)^{x-1} + 1 = 0$.

$$19. 3 \cdot 4^x + \frac{1}{3} \cdot 9^{x+2} = 6 \cdot 4^{x+1} - \frac{1}{2} \cdot 9^{x+1}.$$

$$20. 2^x + 2^{x+1} + 2^{x+2} + 2^{x+3} = 3^x + 3^{x+1} + 3^{x+2} + 3^{x+3}.$$

$$21. 3^{7x} - 4 \cdot 3^{7x-1} + 5 \cdot 3^{7x-2} + 3^{7x-4} = 57.$$

$$22. 2^{x+4} + 2^{x+1} + 3 \cdot 2^{x+2} = 120.$$

$$23. 9 \cdot 16^x + 64 \cdot 16^{x-1} - 256 \cdot 16^{x-2} = 384.$$

$$24. \sqrt{7^{2x+6}} - \sqrt{49^{x+2}} - 2^{x+5} + 2 \cdot 0,25^{-(1+0,5x)} = 0.$$

$$25. 121^{x-1} - 169^{x-1} + 11^{2x-3} + 13^{2x-3} = 0.$$

5- §. Kvadrat tenglamaga keltiriladigan ko'rsatkichli tenglamalar

$A \cdot a^{2 \cdot f(x)} + B \cdot a^{f(x)} + C = 0$ tenglamada A, B, C, a lar haqiqiy sonlar bo'lib, $a > 0$, $a \neq c$ bo'lsin. Bunday tenglamalar-ni $a^{f(x)} = t$ belgilash kiritib, $A \cdot t^2 + B \cdot t + C = 0$ kvadrat tenglama ko'rinishiga keltirib olib yechiladi.

1- m i s o l. $4^x - 5 \cdot 2^x + 4 = 0$ tenglamani yeching.

Y e c h i s h. 2^x ni t orqali belgilab olsak, tenglama $t^2 - 5t + 4 = 0$ ko'rinishga keladi. Bu tenglamani yechib, $t_1 = 1$,

$t_2 = 4$ ga ega bo'lamiz, u holda $\begin{cases} 2^x = 1, \\ 2^x = 4 \end{cases}$ sistema hosil bo'ladi.

Demak, $x_1 = 0$, $x_2 = 2$

2- m i s o l. $\frac{1}{2^x} + 2^x = 2$ tenglamani yeching.

Y e c h i s h. $2^x = t$ belgilash kiritaylik, u holda $\frac{1}{t} + t = 2$

tenglamaga ega bo'lamiz. Bu tenglama $\begin{cases} t^2 - 2t + 1 = 0 \\ t \neq 0 \end{cases}$ sistemaga

teng kuchli. Bu sistemani yechib, $t = 1$ ni hosil qilamiz, u holda $2^x - 1 = 0$ tenglamadan $x = 0$ yechim hosil bo'ladi.

3- m i s o l. $10^{1+2x^2} - 10 = 99 \cdot 10^{x^2}$ tenglamani yeching.

Y e c h i s h. $10^{1+2x^2} = 10 \cdot 10^{2x^2} = 10 \cdot (10^{x^2})^2$ tenglikni hisobga

olsak, $10^{x^2} = t$ belgilash kiritib, $10t^2 - 10 = 99t$ kvadrat tenglamani hosil qilamiz. Bundan $t^2 - 9,9t - 1 = 0$ tenglama va Viyet teoremasiga asosan $t_1 = 10, t_2 = -0,1$ ildizlarga ega bo'lamiz. U holda $10^{x^2} > 0$ bo'lganligi uchun $10^{x^2} = 10$, $x^2 = 1, x_{1,2} = \pm 1$ yechimlar hosil bo'ladi.

4- m i s o l. $\frac{3 \cdot 5^{2x}}{5} - 2 \cdot 5^{x-1} = 0,2$ tenglamani yeching.

Y e c h i s h. Tenglamada umumiy mahraj berib, ixchamlashtirsak $3 \cdot 5^{2x} - 2 \cdot 5^x - 1 = 0$ tenglama hosil bo'ladi. Bu tenglamada 5^x ni t orqali belgilab, $3t^2 - 2t - 1 = 0$ kvadrat tenglamaga keltiramiz. Uning ildizlari $t_{1,2} = \frac{2 \pm \sqrt{16}}{6}$,

$t_1 = 1, t_2 = -\frac{1}{3}$ dan iborat bo'ladi. $5^x > 0$ bo'lgani uchun $x = 0$ yechim hosil bo'ladi.

5- m i s o l. $\sqrt[6]{3^x} + (\sqrt[12]{3})^{x-12} = 84$ tenglamani yeching.

Y e c h i s h. Bu tenglamani $\sqrt[6]{3^x} + \frac{(\sqrt[12]{3})^x}{3} = 84$ ko'rinishga keltirib olamiz. $\sqrt[12]{3^x} = z$ belgilash kiritsak, $z^2 + \frac{z}{3} = 84$ yoki $3z^2 + z - 252 = 0$ kvadrat tenglama hosil bo'ladi. Bu kvadrat tenglamani yechib, $z_1 = 9, z_2 = -\frac{28}{3}$ ildizlarni hosil qilamiz. $\sqrt[12]{3^x} > 0$ bo'lgani uchun $\sqrt[12]{3^x} = 9$ yoki $x = 24$ yechim hosil bo'ladi.

Mashqlar

Quyidagi tenglamalarning yechimlarini toping (1-25):

1. $4^x - \frac{5}{4} \cdot 2^{x+2} + 4 = 0.$

2. $\frac{1}{2^x - 1} + 2^{1-x} = 2.$

3. $\frac{5^x}{5} + 5 \cdot 5^{2-x} = 26.$

4. $(16^{\sin x})^{\cos x} + \frac{6}{4^{\sin^2(x - \frac{\pi}{4})}} - 4 = 0.$

5. $5^{1+x^2} - 5^{1-x^2} = 24.$

6. $10^x + 5^{x-1} \cdot 2^{x-2} = 950.$

$$7. 3 \cdot 5^{2x-1} - 2 \cdot 5^{x-1} = \frac{1}{5}.$$

$$8. \frac{2^x + 10}{4} = \frac{9}{2^{x-2}}.$$

$$9. 2 \cdot 7^{3x} - 5 \cdot 49^{3x} + 3 = 0.$$

$$10. 4^{(x+1)^{\frac{1}{2}}} - 2^{(x+1)^{\frac{1}{2}}} \cdot 4 = 0.$$

$$11. 2^x - \frac{2}{4^x} - (0,5)^x + 1 = 0.$$

$$12. (2 - \sqrt{3})^x + (2 + \sqrt{3})^x = 4.$$

$$13. 3 \cdot 5^{4x-1} - 2 \cdot 5^{2x-1} = 0,2.$$

$$14. \sqrt[5]{3^{1,5x}} + \sqrt[10]{3^{1,5x-10}} = 84.$$

$$15. 4^{\frac{3x}{2}} - 10 \cdot 2^{\frac{3x}{2}-1} - 24 = 0.$$

$$16. 9^{\sqrt{2x-5}} - 27 = 6 \cdot \sqrt{3^{2x-5}}.$$

$$17. 4^{x-1} - \frac{17}{8} \cdot 2^x = -1.$$

$$18. (\sqrt{4 + \sqrt{15}})^x + (\sqrt{4 - \sqrt{15}})^x = 8.$$

$$19. 2^{x+\sqrt{x^2-4}} - 5 \cdot 2^{\frac{1}{2}x-1+\sqrt{\frac{x^2}{4}-1}} - 6 = 0.$$

$$20. (\sqrt{7} - 4\sqrt{3})^x + (\sqrt{7 + \sqrt{48}})^x = 14.$$

$$21. 10 \cdot 4^x - 9 \cdot 2^x(4^x + 1) + 2(16^x + 2 \cdot 4^x + 1) = 0$$

$$22. 3^{3x+1} - 4 \cdot 27^{x-3} + \frac{1}{3^{3x-1}} - 80 = 0.$$

$$23. (2 + \sqrt{3})^{x^2-2x+1} + (2 - \sqrt{3})^{x^2-2x-1} = 8 + 4\sqrt{3}.$$

$$24. 2^{2x} \cdot 9^x - 2 \cdot 6^{3x-1} + 4^{2x-1} \cdot 3^{4x-2} = 0.$$

$$25. 17 \cdot 2^{\sqrt{9x^2-24x}} - 8 = 2 \cdot 4^{\sqrt{9x^2-24x}}.$$

6- §. Bir jinsli ko'rsatkichli tenglamalar

Ozod hadi nolga teng bo'lgan ba'zi ko'rsatkichli tenglamalarning yechilish usullarini ko'rib chiqaylik.

1. $A \cdot a^{2x} + B \cdot a^x \cdot b^x + C \cdot b^{2x} = 0$ (1) tenglamadagi A , B , C lar haqiqiy sonlar; a va b sonlar esa musbat haqiqiy sonlar bo'lsin. (1) tenglamani yechish uchun, tenglikning ikkala tomo-

nini b^{2x} ga bo'lib, $A \cdot \left(\frac{a}{b}\right)^{2x} + B \cdot \left(\frac{a}{b}\right)^x + C = 0$ (2) ko'rinishdagi

tenglamaga ega bo'lamiz. (2) tenglamada $\left(\frac{a}{b}\right)^x$ ni t orqali belgilab olsak, 5- § dagi $A \cdot t^2 + B \cdot t + C = 0$ tenglamani hosil qilamiz.

1- m i s o l. $4 \cdot 3^{2x} - 9 \cdot 2^{2x} = 5 \cdot 6^x$ tenglamani yeching.

Y e c h i s h. Berilgan tenglamaning ikkala tomonini ham

2^{2x} ga bo'lib yuborsak, $4 \cdot \left(\frac{3}{2}\right)^{2x} - 5 \cdot \left(\frac{3}{2}\right)^x - 9 = 0$ tenglamani

hosil qilamiz. $\left(\frac{3}{2}\right)^x = t$ belgilashni kiritsak, u holda quyidagi

$4t^2 - 5t - 9 = 0$ kvadrat tenglamani hosil qilamiz. Bu

tenglamaning ildizlari $t_1 = \frac{9}{4}$, $t_2 = -1$ lardan iborat. Lekin

$\left(\frac{3}{2}\right)^x > 0$ bo'lganligi uchun faqat $\left(\frac{3}{2}\right)^x = \frac{9}{4}$ tenglamanigina qaraymiz. Natijada $x = 2$ yechim hosil bo'ladi.

2- m i s o l. $A \cdot a^{f(x)} + B \cdot a^{g(x)} = 0$ yoki $A \cdot a^{f(x)} + B \cdot b^{f(x)} = 0$ ko'rinishdagi tenglamalar ham tenglamaning ikkala tomonini $B \cdot a^{f(x)}$ ga bo'lish yordamida ishlanadi.

3- m i s o l. $5^{2x-1} + 2^{2x} - 5^{2x} + 2^{2x+2} = 0$ tenglamani yeching.

Y e c h i s h. Berilgan tenglamani $\frac{5^{2x}}{5} - 5^{2x} + 2^{2x} + 4 \cdot 2^{2x} = 0$

yoki $-4 \cdot 5^{2x} + 5^2 \cdot 2^{2x} = 0$ ko'rinishda yozib olib, ikkala tarafini

$5 \cdot 5^{2x}$ ga bo'lsak, $\left(\frac{2}{5}\right)^{2x} = \left(\frac{2}{5}\right)^2$ tenglamaga ega bo'lamiz.

Undan esa $x = 1$ javobni topamiz.

4- m i s o l. $4 \cdot 5^x - 5 \cdot 4^x = 0$ tenglamani yeching.

Y e c h i s h. Berilgan tenglamaning ikkala tomonini ham

$4 \cdot 4^x$ ga bo'lib, $\left(\frac{5}{4}\right)^x = \frac{5}{4}$ yoki $x = 1$ ga ega bo'lamiz.

5- m i s o l. $2 \cdot 4^x + 25^{x+1} = 15 \cdot 10^x$ tenglamani yeching.

Y e c h i s h. Berilgan tenglamani $2 \cdot 2^{2x} + 25 \cdot 5^{2x} - 15 \cdot 2^x \cdot 5^x = 0$ ko'rinishda yozib olamiz va uning ikkala tomo-

nini 5^{2x} ga bo'lamiz. Hosil bo'lgan $2 \cdot \left(\frac{2}{5}\right)^{2x} - 15 \cdot \left(\frac{2}{5}\right)^x + 25 = 0$

tenglamaga $\left(\frac{2}{5}\right)^x = t$ belgilashni kiritsak, $2t^2 - 15t + 25 = 0$

kvadrat tenglama hosil bo'ladi. Bu tenglamadan $t_{1,2} = \frac{15 \pm 5}{4}$

yoki $\begin{cases} t_1 = \frac{5}{2} \\ t_2 = 5 \end{cases}$ larni hosil qilamiz. U holda $\begin{cases} \left(\frac{2}{5}\right)^x = \frac{5}{2} \\ \left(\frac{2}{5}\right)^x = 5 \end{cases}$ dan

$\begin{cases} x_1 = -1 \\ x_2 = \frac{\lg 5}{1 - 2 \lg 5} \end{cases}$ larni topamiz.

5- m i s o l. $5^{x+6} - 3^{x+7} = 43 \cdot 5^{x+4} - 19 \cdot 3^{x+5}$ tenglamani yeching.

Y e c h i s h. Bir hil asosli darajalar qatnashgan hadlarni guruhlab olamiz va hosil bo'lgan $(5^{x+6} - 43 \cdot 5^{x+4}) + (19 \cdot 3^{x+5} - 3^{x+7}) = 0$ tenglamada umumiy ko'paytuvchilarni qavslardan tashqariga chiqarib $5^{x+4}(25 - 43) = 3^{x+5}(3^2 - 19)$

yoki $\frac{5^{x+4}}{3^{x+4}} = \frac{-3 \cdot 10}{-18}$ bundan $\left(\frac{5}{3}\right)^{x+4} = \frac{5}{3}$ tenglama hosil bo'ladi.

U holda $x = -3$ kelib chiqadi.

6- m i s o l. $|x-4|^{x^2-x} = (x-4)^2$ tenglamani yeching.

Y e c h i s h. Berilgan tenglamani $|x-4|^{x^2-x} = |x-4|^2$ ko'rinishda yozib olib, modulning va darajaning xossalariidan foydalanish natijasida quyidagi holatlar kelib chiqadi:

1. $|x-4| = 0$. U holda $x_1 = 4$.

2. $|x-4| = 1$. U holda $x_2 = 5, x_3 = 3$.

3. $x-4 \neq 0$ va $x-4 \neq 1$. Bu holatda $x^2 - x - 2 = 0$ kvadrat tenglama berilgan tenglamaga teng kuchli bo'lib, $x_4 = -1, x_5 = 2$ ildizlar kelib chiqadi. Shunday qilib, tenglama yechimi $\{-1, 2, 3, 4, 5\}$ to'plamdan iborat.

Mashqlar

Quyidagi tenglamalarning yechimlarini toping (1-84):

1. $2^{x^2} \cdot 5^{x^2} = 10^{-3} \cdot (10^{3-x})^2$. 2. $4^x + 6^x = 2 \cdot 9^x$.

3. $4^x - \frac{3^x}{\sqrt{3}} = \sqrt{3} \cdot 3^x - \frac{2^{2x}}{2}$. 4. $2^{4x} + 6^{2x} = 2 \cdot 81^x$.

5. $4^x - 2 \cdot 6^x = 3 \cdot 9^x$.

6. $3 \cdot 4^x + 2 \cdot 25^x = 5 \cdot 10^x$.

7. $2^{\frac{4}{x}} = 12^{\frac{1}{x}} + 3 \cdot 2^{\frac{2}{x}}$.

8. $\frac{2^{2x}}{4} - 3^{x-1,5} = 3^{0,5x} - \frac{4^x}{8}$.

9. $5^{3x-1} = 1$.

10. $4^{2\sqrt{2}} = 2^{2x-6}$.

11. $(0,25)^{x-14} = \frac{128}{2^{x-16}}$.

12. $3^{x-1} = 5$.

13. $\left(\frac{2}{3}\right)^{x-2} = \left(\frac{3}{2}\right)^{2x-1}$.

14. $\left(\frac{1}{8} \cdot 4^x\right)^x = 2^{3x-4}$.

15. $\left[\frac{1}{3} \left(3^{\sqrt{x+1}}\right)^{\frac{2}{\sqrt{x}}}\right]^{\frac{2}{\sqrt{x+2}}} = 9$.

16. $\sqrt{(0,25)^{x^2}} \cdot 2^{2x+2} = \frac{1}{64}$.

17. $2^{3x+1} \cdot 5^x = 3200$.

18. $3^{5x-1} \cdot 7^{1-5x} = 1$.

19. $3^{x-2} + 2 \cdot 3^{x+1} = 55$.

20. $3^{x+1} - 2 \cdot 3^{x-1} - 4 \cdot 3^{x-2} = 17$.

21. $2 \cdot 4^{2x} - 17 \cdot 4^x + 8 = 0$.

22. $4^{x-2} - 17 \cdot 2^{x-4} + 1 = 0$.

23. $3 \cdot \sqrt[5]{81} - 10 \cdot \sqrt[5]{9} + 3 = 0$.

24. $64^{1/x} - 2^{3+3/x} + 12 = 0$.

25. $9^{x^2-1} - 36 \cdot 3^{x^2-3} + 3 = 0$.

26. $(\sqrt[5]{3})^x + (\sqrt[10]{3})^{x-10} = 84$.

27. $12^2\sqrt[3]{3} - \sqrt[3]{3} = 27$.

28. $10^{1+x^2} - 10^{1-x^2} = 99$.

29. $5^{x-1} + 5 \cdot 0,2^{x-2} = 26$.

30. $49^{1+\sqrt{x-2}} - 344 \cdot 7^{\sqrt{x-2}} = -7$.

31. $2 \cdot 16^x + 2 \cdot 81^x = 5 \cdot 36^x$. 32. $10^{2/x} + 25^{1/x} = 4,25 \cdot 50^{1/x}$.
33. $4^{-1/x} + 6^{-1/x} = 9^{-1/x}$. 34. $4^{x+1,5} + 9^x = 6^{x+1}$.
35. $16^x - 5 \cdot 8^x + 6 \cdot 4^x = 0$. 36. $6 \cdot \sqrt[3]{9} - 13 \cdot \sqrt[3]{6} + 6\sqrt[3]{4} = 0$.
37. $5^x + 3 \cdot 4^x = 17^x$. 38. $3^{x-1} + 5^{x-1} = 34$.
39. $7^{6-x} = x + 2$. 40. $2^{x+5} = 32 - x$.
41. $4^x + 5^x = 9^x$. 42. $(x-3)^{x^2-x} = (x-3)^2$.
43. $(x^2 - 5x + 7)^{x-2} = 1$. 44. $(x+2)^{(x+3)^2} = 1$.
45. $|x-3|^{x^2-x} = (x-3)^2$. 46. $|x|^{x^2-2x} = 1$.
47. $14 \cdot 4^x - 53 \cdot 14^x + 2 \cdot 49^{x+\frac{1}{2}} = 0$.
48. $2^{2x} \cdot 3^{2x} - \frac{6^{3x}}{3} + 4^{2x-1} \cdot 3^{4x-2} = 0$.
49. $\frac{3}{4^x} + \frac{1}{3 \cdot 9^{x-2}} = 6 \cdot 4^{1-x} - \frac{1}{2} \cdot 9^{1-x}$.
50. $5^{\frac{x}{\sqrt{x+2}}} \cdot 5^{-\frac{4}{\sqrt{x+2}}} = 125^{x-4} \cdot (0,04)^{x-2}$.
51. $3^{2x+4} + 45 \cdot 2^x \cdot 3^x - 9 \cdot 2^{2x+2} = 0$.
52. $3 \cdot 16^x + 2 \cdot 81^x - 5 \cdot 4^x \cdot 9^x = 0$.
53. $5^{x^2} - 3 \cdot 5^{x^2+1} + 2 \cdot 5^{x^2-1} = -68$.
54. $7^{x+2} - \frac{1}{7} \cdot 7^{x+1} - 14 \cdot 7^{x-1} + 2 \cdot 7^x = 48$.
55. $4 \cdot 3^{x+2} + 5 \cdot 3^x - 7 \cdot 3^{x+1} = 40$.
56. $2^x + 2^{x+1} + 2^{x-1} = 7$.
57. $6^{x^2+2x} - 3 \cdot 6^{x^2+2x+1} + 6 \cdot 6^{x^2+2x-1} = -16$.
58. $(\sqrt{3})^{2x+2} - 2 \cdot 3^{x-1} + 5 \cdot 3^{x+1} = 52$.
59. $4^{\sqrt{x+1}} - 3 \cdot 4^{\sqrt{x+2}} + 2 \cdot 4^{\sqrt{x+3}} = 21$.
60. $3 \cdot 5^{\sqrt[3]{x+1}} - 2 \cdot 5^{\sqrt[3]{x+2}} = -7$.
61. $2^x + 2^{x-1} + 2^{x-2} = 3^x - 3^{x-1} + 3^{x-2}$.

62. $4^x - 3^{x-0.5} = 5^{x+0.5} - 2^{2x-1}$.
63. $2 \cdot 3^{x+1} - 5^{x+2} = 3^{x+4} - 5^{x+3}$.
64. $9^x - 2^{x+0.5} = 2^{x+1.5} - 3^{2x-1}$.
65. $7^{3x} + 9 \cdot 5^{2x} = 5^{2x} + 9 \cdot 7^{3x}$.
66. $3 \cdot 4^x + \frac{1}{3} \cdot 9^{x+2} = 6 \cdot 4^{x+1} - 0.5 \cdot 9^{x+1}$.
67. $2^{x^2-1} - 3^{x^2} = 3^{x^2-1} - 2^{x^2+2}$.
68. $5^{2x-1} + 2^{2x} - 5^{2x} + 2^{2x+2} = 0$.
69. $6^{x+1} + 3^{x+1} = 3^{x+2} - 6^x + 3^x$.
70. $7^x + 6^{x+1} + 3 \cdot 6^{x-1} = 7^{x+1} - 3 \cdot 7^{x-1}$.
71. $4^{\sqrt{3x^2-2x+1}} + 2 = 9 \cdot 2^{\sqrt{3x^2-2x}}$.
72. $3^{2x+4} + 45 \cdot 6^x - 9 \cdot 2^{2x+2} = 0$.
73. $6 \cdot 4^x - 13 \cdot 6^x + 6 \cdot 9^x = 0$.
74. $4^{\lg x+1} - 6^{\lg x} - 2 \cdot 3^{\lg x^2+2} = 0$.
75. $(\sqrt{7+\sqrt{48}})^x + (\sqrt{7-\sqrt{48}})^x = 14$.
76. $(2+\sqrt{3})^{x^2-2x+1} + (2-\sqrt{3})^{x^2-2x-1} = \frac{4}{2-\sqrt{3}}$.
77. $(4+\sqrt{15})^x + (4-\sqrt{15})^x = 62$.
78. $(\sqrt{5+2\sqrt{6}})^x + (\sqrt{5-2\sqrt{6}})^x = 10$.
79. $(\sqrt{5\sqrt{2}-7})^x + 6(\sqrt{5\sqrt{2}+7})^x = 7$.
80. $(\sqrt{2-\sqrt{3}})^{x^2} + (\sqrt{2+\sqrt{3}})^{x^2} = 4$.
81. $(9+\sqrt{80})^{2x-1} + (9-\sqrt{80})^{2x-1} = 18$.
82. $(3+\sqrt{8})^{x^2+2x} + (3-\sqrt{8})^{x^2+2x} = 6$.
83. $(\sqrt{\sqrt{5}+\sqrt{9}})^x + (\sqrt{\sqrt{9}+\sqrt{5}})^x = 2\sqrt{5}$.
84. $(3\sqrt{2}-17)^{x/2} + (3\sqrt{2}+17)^{x/2} = 6\sqrt{2}$.

Noma'lum o'zgaruvchi faqat logarifm belgisi ostida qatnashgan tenglamalarni *logarifmik tenglamalar* deb tushunamiz. Masalan, $\log_a x = b$ yoki $\log_x c = d$ ko'rinishdagi tenglamalar logarifmik tenglamalardir. $\log_c x = x^2$ kabi tenglamalar esa logarifmik tenglamalarga misol bo'la olmaydi.

Quyida logarifmik tenglamalarning ba'zi bir turlari bilan tanishib chiqamiz.

1- § . Logarifm ta'rifidan foydalanib yechiladigan logarifmik tenglamalar

Ma'lumki, $a > 0, a \neq 1, b > 0, b \neq 0$ sonlar uchun $\log_a b$ ifoda b sonini hosil qilish uchun a sonni ko'tarish kerak bo'lgan daraja ko'rsatkichidir. Ya'ni, agar $a^c = b$ bo'lganda $c = \log_a b$ bo'ladi. U holda $a^{\log_a b} = b$. Bu tenglikni *asosiy logarifmik ayniyat* deb atilishi ma'lum.

Ta'rifdan foydalanib yechiladigan misollar namunalari bilan tanishib chiqamiz.

1- m i s o l. $\log_2(-x) = 3$ tenglamani yeching.

Y e c h i s h. Logarifm ta'rifiga ko'ra $2^3 = -x$, bundan $x = -8$ kelib chiqadi.

2- m i s o l. $\log_{\frac{1}{2}} x^2 = 4$ tenglamani yeching.

Y e c h i s h. Ta'rifga ko'ra $2^{-4} = x^2$, bundan $x = \pm \frac{1}{4}$ yechimlarni hosil qilamiz.

3- m i s o l. $\log_{x^3} \sqrt[3]{16} = 2$ tenglamani yeching.

Y e c h i s h. Ta'rifdan foydalanib $x^6 = 2^{\frac{4}{3}}$ ifodani hosil qilamiz. Undan $x = 2^{\frac{2}{9}}$ yoki $x = \sqrt[9]{4}$ yechimni topamiz.

4- m i s o l. $\log_2 x + \log_2 (x-1) = 1$ tenglamani yeching.

Y e c h i s h. Logarifmlar yig'indisi xossasi va logarifm ta'rifidan foydalanib $x^2 - x - 2 = 0$ tenglamani hosil qilamiz. Uning $x_1 = 2$ va $x_2 = -1$ yechimlaridan $x = 2$ javob kelib chiqadi.

5- m i s o l. $\frac{\log_3(9-3^x)}{3+x} = 1$ tenglamani yeching.

Bu tenglama $\begin{cases} 3+x \neq 0, \\ 9-3^x > 0, \\ \log_3(9-3^x) = 3+x \end{cases}$ sistemaga teng kuchli.

$\log_3(9-3^x) = 3+x$ tenglamani logarifm ta'rifidan foydalanib $9-3^x = 3^{3+x}$ yoki $28 \cdot 3^x = 9$ ko'rsatkichli tenglamaga keltirib olamiz. Natijada, $3^x = \frac{9}{28}$ tenglama hosil bo'ladi. Bundan logarifmning ta'rifiga ko'ra $x = \log_3 \frac{9}{28}$ yoki $x = 2 - \log_3 28$ javobni hosil qilamiz.

6- m i s o l. $\log_4 9 + 2\log_4 x = x^{\frac{\log_{27} 64}{\log_9 x^2}}$ tenglamani yeching.

Berilgan tenglamaning qiymatlar sohasi $\begin{cases} x > 0, \\ \log_9 x^2 \neq 0 \end{cases}$

sistema orqali topiladi. Uning $(0,1) \cup (1, +\infty)$ dan iborat ekanligini aniqlash qiyin emas. U holda, logarifm xossalaridan foydalanib, berilgan tenglamaning o'ng tomoni $x^{\log_x 4}$ ko'rinishga keltiriladi. Unga asosiy logarifmik ayniyatni qo'llasak, tenglamaning o'ng tomoni 4 ga tengligi kelib chiqadi. $\log_4 9 = \log_2 3$ va $2\log_4 x = \log_2 x$ ayniyatlarni hisobga olsak, tenglama $\log_2 3x = 4$ ko'rinishga keladi. U holda $x = 5\frac{1}{3}$ ildizga ega bo'lamiz.

7- misol. $\log_2 x + \log_{\sqrt{2}} x + \log_{\sqrt[3]{2}} x + \dots + \log_{\sqrt[30]{2}} x = 930$ tenglamani yeching.

Yechish. $\log_{a^n} b = \frac{1}{n} \log_a b$ ayniyatdan foydalanib, $\log_2 x + 2\log_2 x + \dots + 30\log_2 x = 930$ tenglama hosil qilinadi. Umumiy ko'paytuvchini qavsdan tashqariga chiqarilsa $(1+2+\dots+30)\log_2 x = 930$ hosil bo'ladi. Bundan $465 \cdot \log_2 x = 930$ yoki $\log_2 x = 2$ tenglamaga va undan $x = 4$ javob hosil bo'ladi.

Mashqlar

Quyidagi tenglamalarni yeching (1–44):

1. $\log_4 (1-2x) = 1$.
2. $\log_{\left(-\frac{1}{x}\right)} \frac{1}{2} = 4$.
3. $\log_3 (x-10) = 2$.
4. $\log_{\sqrt[3]{4}} x = 1,5$.
5. $\log_x \left(\frac{1}{5\sqrt{5}} \right) = -1,5$.
6. $\log_x \sqrt[5]{36} = -0,4$.
7. $3^{\log_3 (x-3)} = \log_3 81$.
8. $3^{\log_{1,5} x} = \log_5 125$.
9. $\log_x \sqrt[3]{0, (3)} = \frac{1}{3}$.
10. $2^{\log_8 (-4x)} = \log_3 81$.
11. $3^{\log_2 5x} = \log_5 125$.
12. $10^{\lg(34-4x)} = \log_2 1024$.
15. $x \log_2 x = 0$.
16. $\lg(x^2 - 1) = 0$.
17. $\log_{\sqrt[3]{5}} \sqrt[3]{512} = -\frac{3}{4}$.
18. $2^{\log_2 (x-8,5)} = \log_4 256$.
19. $\log_7 \log_2 \log_3 \log_2 x = 0$.
20. $\log_{100} \log_2 \log_3 \frac{1}{x-3} = 0$.
21. $\log_5 \log_7 \log_2 x = 0$.
22. $\log_2 3 + 2 \log_4 5x = 4$.
23. $\log_x (5x^2) \cdot \log_5^2 x = 1$.
24. $\log_x 2 - \log_4 x + 7/6 = 0$.
25. $\log_2 (x^2 - 5x + 14) = 3$.
26. $\log_{x-2} (2x^2 - 16x + 39) = 2$.

27. $\log_{3-x}(2x^2 + 4x - 2) = 2$. 28. $(1 + \lg x)(\lg x - 1) = \lg x^3 - 3$.
29. $\frac{1}{5 - \lg x} + \frac{2}{1 + \lg x} = 1$. 30. $\frac{1 - \log_3 x}{1 + \log_3 x} + \log_3 x = 1$.
31. $\log_x 2 = \frac{4}{7}(2 - \sqrt{2} + \sqrt{2} - 1 + 1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} - \frac{1}{2} + \dots)$.
32. $\log_s x = \left(\sqrt{\frac{4}{3}} + \sqrt{\frac{3}{4}} + \sqrt{\frac{3}{4}} \cdot \sqrt{\frac{3}{4}} + \dots \right) \cdot \sqrt{\frac{3}{4}}$.
33. $\log_2(x + 14) + \log_2(x + 2) = 6$.
34. $\log_{16} x + \log_4 x + \log_2 x = 7$.
35. $\lg \sqrt{5x - 4} + \lg \sqrt{x + 1} = 2 + \lg 0,18$.
36. $\lg 5 + \lg(x + 10) = 1 - \lg(2x - 1) + \lg(21x - 20)$.
37. $\log_5(x - 2) + \log_{\sqrt{5}}(x^3 - 2) + \log_{0,2}(x - 2) = 4$.
38. $\log_5(3x - 11) + \log_5(x - 27) = 3 + \log_5 8$.
39. $\log_{\sqrt{5}}(4^x - 6) - \log_{\sqrt{5}}(2^x - 2) = 2$.
40. $\log_2 x + \log_4 x + \log_8 x = 11$.
41. $\log_{\sqrt{3}} x + \log_{\sqrt[4]{3}} x + \log_{\sqrt[6]{3}} x + \dots + \log_{\sqrt[16]{3}} x = 36$.
42. $2 \log_x 3 + \log_{3x} 3 + 3 \log_{9x} 3 = 0$.
43. $\frac{1}{\log_2 x} + \log_2 x = 2,5 \cdot \log_2 2$.
44. $\frac{9}{4}x^2 \cdot \log_{1,5x} 27 \cdot \log_9 1,5x = \frac{3x + 8}{2}$.

2- §. Algebraik tenglamalarga keltiriladigan logarifmik tenglamalar

Algebraik tenglamalarga keltiriladigan logarifmik tenglamalarni yechishda logarifmning asosiy xossalaridan foydalanishga to'g'ri keladi. Bu xossalar 4- § da keltirilgan.

Algebraik tenglamalarga keltiriladigan logarifmik tenglamalarning ko'p uchraydigan turlari bilan tanishib chiqamiz.

$$\log_a f(x) = \log_a g(x) \quad (1)$$

ko'rinishdagi tenglamalar berilgan bo'lsin. Bu tenglamani yechish uchun x ning yo'l qo'yiladigan qiymatlari sohasini topib olishimiz lozim. Buning uchun

$$\begin{cases} f(x) > 0, \\ g(x) > 0 \end{cases} \quad (2)$$

tengsizliklar sistemasining yechimlarini topish kerak bo'ladi. Lekin (1) tenglamadagi logarifmlarning asoslari bir xil bo'lganligi sababli, $f(x) = g(x)$ tenglamani hosil qilamiz. Shuning uchun, (2) tengsizliklar sistemasidagi bitta tengsizlikni yechimlarini topib, so'ngra $f(x) = g(x)$ tenglamani yechish yetarli.

2. Agar $\log_a f(x) + \log_a g(x) = \log_a \varphi(x)$ (2) ko'rinishdagi tenglama berilgan bo'lsa, bu tenglamani $(f(x) > 0) \wedge (g(x) > 0) \wedge (\varphi(x) > 0)$ shartni hisobga olgan holda, logarifmning xossalaridan foydalanib $\log_a f(x) \cdot g(x) = \log_a \varphi(x)$ ko'rinishga keltirib olamiz va (1) tenglama kabi yechimlari topiladi.

3. $\log_a f(x) - \log_a g(x) = \log_a \varphi(x)$ (3) ko'rinishdagi tenglama berilgan bo'lsa, $(f(x) > 0) \wedge (g(x) > 0) \wedge (\varphi(x) > 0)$ shartni hamda logarifmning xossalaridan foydalanib

$\log_a \frac{f(x)}{g(x)} = \log_a \varphi(x)$ ko'rinishga keltirib, (1) tenglama kabi yechamiz.

4. $p \cdot \log_a f(x) = \log_a g(x)$ (4) ko'rinishdagi tenglama ham $f(x) > 0$ va $g(x) > 0$ shartlarni hisobga olgan holda $\log_a (f(x))^p = \log_a g(x)$ ko'rinishga keltirib, (1) tenglama kabi yechiladi.

Algebraik tenglamalarga keltiriladigan logarifmik tenglamalar namunalari bilan tanishib chiqamiz.

1- m i s o l. $\frac{\log_2(\sqrt{x+1}+1)}{\log_2 \sqrt{x-1}} = 2$ tenglamani yeching.

Y e c h i s h. Avval berilgan tenglamaning qiymatlar sohasini topamiz:

$$\begin{cases} x+1 \geq 0, \\ x-1 > 0, \\ x-1 \neq 1 \end{cases} \text{ sistemadan } \begin{cases} x \geq -1, \\ x > 0, \\ x \neq 2 \end{cases} \text{ ni hosil qilamiz.}$$

Ya'ni, $x \in (1, 2) \cup (2, +\infty)$.

Endi $x \in (1, 2) \cup (2, +\infty)$ tasdiq o'rinli bo'ladi deb hisoblab, teng kuchli almashtirishlar orqali $\log_2(\sqrt{x+1}+1) = 2 \cdot \log_2 \sqrt{x-1}$ tenglamani hosil qilamiz. Logarifm xossalariiga ko'ra $\log_2(\sqrt{x+1}+1) = \log_2(\sqrt{x-1})^2$ tenglamani, undan esa $\sqrt{x+1}+1 = (\sqrt{x-1})^2$ tenglamani hosil qilamiz. Bu tenglamaning $x_{1,2} = \frac{5 \pm \sqrt{13}}{2}$ yechimlaridan $x_1 = \frac{5 + \sqrt{13}}{2}$ gina x ning yo'l qo'yiladigan qiymatlari sohasiga kiradi:

$$\sqrt{\frac{5+\sqrt{13}}{2}+1} = \sqrt{\frac{7+\sqrt{13}}{2}}$$

$$x_1 = \frac{5+\sqrt{13}}{2} \text{ ildizni berilgan tenglamani qanoatlantirishi-}$$

ni tekshiraylik: $\log_2(\sqrt{\frac{5+\sqrt{13}}{2}+1}+1) = 2 \cdot \log_2 \sqrt{\frac{5+\sqrt{13}}{2}-1}$ dan

logarifm xossalariiga ko'ra $\sqrt{\frac{5+\sqrt{13}}{2}+1}+1 = \frac{3+\sqrt{13}}{2}$ ni va

undan $\sqrt{\frac{7+\sqrt{13}}{2}} = \frac{1+\sqrt{13}}{2}$, $\frac{7+\sqrt{13}}{2} = \frac{(1+\sqrt{13})^2}{4}$ larni va ni-

hoyat $7+\sqrt{13} = \frac{1+2\sqrt{13}+13}{2}$ tenglikdan $7+\sqrt{13} = 7+\sqrt{13}$

ayniyatni hosil qilamiz. Demak, $x = \frac{5+\sqrt{13}}{2}$ tenglamaning ildizi ekan.

2- m i s o l. $\lg(5-x) - \frac{1}{3}\lg(35-x^3) = 0$ tenglamani yeching.

Y e c h i s h. x ning Q.S. $\begin{cases} 5-x > 0 \\ 35-x^3 > 0 \end{cases}$ tengsizliklar sistemasi

orqali topiladi. Demak, $u \in (-\infty, \sqrt[3]{35})$ oraliqdan iborat.

Logarifmning yuqorida keltirilgan xossalariidan foydalanib, berilgan tenglamani yechamiz:

$$5-x = \sqrt[3]{35-x^3} \Leftrightarrow (5-x)^3 = 35-x^3 \Leftrightarrow \\ \Leftrightarrow 125 - 3 \cdot 5^2 \cdot x + 3 \cdot 5 \cdot x^2 - x^3 = 35 - x^3 \Leftrightarrow 15x^2 - 75x + 90 = 0 \text{ tenglamadan } x^2 - 5x + 6 = 0 \text{ tenglamani hosil qilib,}$$

uning $\begin{cases} x_1 = 2, \\ x_2 = 3 \end{cases}$ ildizlarini topamiz. Ikkala ildiz ham x ning

yo'l qo'yiladigan qiymatlari sohasiga kiradi. Bu ildizlarni berilgan tenglamaga qo'yib ko'rib, tenglama yechimi $\{2, 3\}$ to'plamdan iborat bo'lishiga ishonch hosil qilish mumkin.

3- m i s o l. $2\log_3(x-2) + \log_3(x-4)^2 = 0$ tenglamani yeching.

Y e c h i s h. x ning Q.S.ni topamiz: $\begin{cases} x-2 > 0, \\ x-4 \neq 0 \end{cases}$ dan

$x \in (2, 4) \cup (4, +\infty)$ kelib chiqadi.

Logarifmning xossalaridan foydalanib, berilgan tenglamadan $\log_3(x-2)^2(x-4)^2 = 0$ tenglamani hosil qilamiz. Bu tenglamaga logarifmning ta'rifini qo'llasak, $((x-2)(x-4))^2 = 1$

hosil bo'ladi. Tenglamani $\begin{cases} x^2 - 6x + 8 = 1, \\ x^2 - 6x + 8 = -1 \end{cases}$ ko'rinishga kelti-

rib, undan $\begin{cases} x_1 = 3 + \sqrt{2}, \\ x_2 = 3 - \sqrt{2}, \\ x_3 = 3 \end{cases}$ ildizlarni topamiz. Bu ildizlardan

faqat $x_1 = 3 + \sqrt{2}$ va $x_3 = 3$ lar tenglamani qanoatlantiradi.

Demak, $\{3; 3 + \sqrt{2}\}$ to'plam tenglamaning yechimi ekan.

4- m i s o l. $\log_{2x+1}(8x^3 - 18x + 8) \cdot \log_{2x-1}(2x+1) = 3$ tenglamani yeching.

Y e c h i s h. Logarifm ta'rifiga asosan x ning yo'l qo'yiladigan qiymatlari sohasi quyidagi sistemaning yechimidan iborat:

$$\begin{cases} 8x^3 - 18x + 8 > 0, \\ 2x + 1 > 0, \\ 2x + 1 \neq 0, \\ 2x - 1 > 0, \\ 2x - 1 \neq 1. \end{cases} \quad \text{Bu sistema} \quad \begin{cases} 4x^3 - 9x + 4 > 0, \\ x > -\frac{1}{2}, \\ x > \frac{1}{2}, \\ x \neq 1, \\ x \neq 0 \end{cases} \quad \text{yoki}$$

$$\begin{cases} 4x^3 - 9x + 4 > 0, \\ x > \frac{1}{2}, \\ x \neq 1 \end{cases} \quad (*) \text{ sistemaga teng kuchli. Hosil bo'lgan}$$

sistemadagi birinchi tengsizlik ancha murakkab bo'lganligi uchun uning yechimini topmasdan berilgan tenglama yechimlaridan sistema shartlarini qanoatlantiradiganlarini ajratib olish yetarli.

Tenglamadagi ikkinchi ko'paytuvchini $2x+1$ asosga o'tkazib olaylik. Hosil bo'lgan $\log_{2x+1}(8x^3 - 18x + 8) \times$

$\times \frac{1}{\log_{2x+1}(2x-1)} = 3$ tenglamaga umumiy mahraj bersak,

$\log_{2x+1}(8x^3 - 18x + 8) = 3 \cdot \log_{2x+1}(2x-1)$ tenglama kelib chiqadi. Bu tenglama potensirlansa $8x^3 - 18x + 8 = (2x-1)^3$ algebralik tenglama hosil bo'ladi. Uning yechimlari $x_1 = 0,5$; $x_2 = 1,5$ lardan iborat.

Yuqorida keltirilgan (*) sistemani faqat $x = 1,5$ qanoatlantiradi. Demak, javob $x = 1,5$.

$$5- \text{ m i s o l. } \frac{2 - 4 \cdot \log_{12} 2}{\log_{12}(3x+2)} - 1 = \frac{\log_6(8-3x)}{\log_6(3x+2)} \text{ tenglamani}$$

yeching.

Y e c h i s h. Noma'lumning yo'l qo'yiladigan qiymatlari sohasi

$$\begin{cases} \log_{12}(3x+2) \neq 0, \\ \log_6(3x+2) \neq 0, \\ 3x+2 > 0, \\ 8-3x > 0 \end{cases} \quad \text{sistema yechimidan iborat. Bu}$$

$$\text{sistemadan } \begin{cases} x \neq -\frac{1}{3}, \\ x > -\frac{2}{3}, \\ x < \frac{8}{3} \end{cases} \quad \text{tengsizliklar sistemasiga ega bo'lamiz.}$$

U holda x ning yo'l qo'yiladigan qiymatlari to'plami $(-\frac{2}{3}, -\frac{1}{3}) \cup (-\frac{1}{3}, \frac{8}{3})$ to'plamdan iborat.

$$\begin{aligned} &\text{Tenglamani yechish uchun uni avval } \frac{\log_{12} 9}{\log_{12}(3x+2)} - 1 = \\ &= \frac{\log_6(8-3x)}{\log_6(3x+2)} \text{ ko'rinishga keltirib, } \frac{\log_c b}{\log_c a} = \log_a b \text{ ayniyatdan} \end{aligned}$$

foydalanilsa $\log_{3x+2} 9 - 1 = \log_{3x+2} (8-3x)$ yoki $\frac{9}{3x+2} = 8-3x$ algebraik tenglama hosil bo'ladi. Umumiy mahraj bersak, $9x^2 - 18x - 7 = 0$ kvadrat tenglama kelib chiqadi. Uningildizlari $\frac{7}{3}, -\frac{1}{3}$ sonlardan iborat. Ulardan faqat $x = \frac{7}{3}$ noma'lumning yo'l qo'yiladigan qiymatlari sohasiga tegishli. Demak, tenglamaning javobi $x = \frac{7}{3}$.

7- m i s o l. $|\log_2(6x-1) - \log_2 3| = |\log_2(5-4x) - 1|$ tenglamani yeching.

Y e c h i s h. Avval noma'lumning yo'l qo'yiladigan qiymatlar to'plami aniqlanadi, ya'ni $\begin{cases} 6x-1 > 0 \\ 5-4x > 0 \end{cases}$ undan $\begin{cases} x > \frac{1}{6} \\ x < \frac{5}{4} \end{cases}$

kelib chiqadi.

Demak, x ning yo'l qo'yiladigan qiymatlar to'plami $\left(\frac{1}{6}, \frac{5}{4}\right)$ dan iborat.

Endi tenglamaning yechimini topamiz. Buning uchun absolut qiymatning ta'rifidan foydalanilsa

$$\begin{cases} \log_2(6x-1) - \log_2 3 = \log_2(5-4x) - 1 \\ \log_2(6x-1) - \log_2 3 = -(\log_2(5-4x) - 1) \end{cases} \quad \text{birlashma hosil bo'}$$

$$\text{ladi. U holda } \begin{cases} \log_2 \frac{6x-1}{3} = \log_2 \frac{5-4x}{2} \\ \log_2 \frac{6x-1}{3} = \log_2 \frac{2}{5-4x} \end{cases} \Rightarrow \begin{cases} 12x-2=15-12x \\ (6x-1)(5-4x)=6 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} 24x=17 \\ 24x^2-34x+1 \end{cases} \Rightarrow \begin{cases} x_1 = \frac{17}{24}, \\ x_2 = \frac{1}{2}, \\ x_3 = \frac{11}{12}. \end{cases}$$

Topilgan ildizlarning har biri o'zgaruvchining yo'l qo'yiladigan qiymatlari to'plamiga tegishli. Demak, berilgan tenglamaning yechimi $\left\{\frac{1}{2}, \frac{11}{12}, \frac{17}{24}\right\}$ to'plamdan iborat.

8- m i s o l. $\log_2(4x^2) \cdot \log_4(32x) = (\log_4 8x^3)^2$ tenglamani yeching.

Y e c h i s h. x ning yo'l qo'yiladigan qiymatlari to'plami

$$\begin{cases} \log_2 4x^2 \geq 0 \\ \log_4 32x \leq 0; \\ x > 0, \end{cases} \begin{cases} \log_2 4x^2 \geq 0 \\ \log_4 32x \geq 0; \\ x > 0 \end{cases} \quad \text{sistemalarning yechimlari yig'in}$$

disidan iborat. Demak, avval sistemalar yechiladi. Birinchi sistema yechimi $\left(0, \frac{1}{32}\right]$. Ikkinchi sistemaning yechimini keltiraylik.

$$\begin{cases} \log_2 4x^2 \geq 0 \\ \log_4 32x \geq 0 \\ x > 0 \end{cases} \Leftrightarrow \begin{cases} 4x^2 \geq 1 \\ 32x \geq 1 \\ x > 0 \end{cases} \Leftrightarrow \begin{cases} (x-\frac{1}{2})(x+\frac{1}{2}) \geq 0 \\ x \geq \frac{1}{32} \end{cases} \Leftrightarrow x \in \left[\frac{1}{2}, +\infty\right)$$

Bu shartlarni hisobga olsak, berilgan tenglama

$$(\log_2 4 + \log_2 x^2) \cdot (\log_4 32 + \log_4 x) = (\log_4 8 + \log_4 x^3)^2$$

$$\text{yoki } (2 + 2 \log_2 x) \cdot \left(\frac{5}{2} + \frac{1}{2} \log_2 x \right) = \left(\frac{3}{2} + \frac{3}{2} \log_2 x \right)^2$$

ko'rinishga keladi. $\log_2 x = t$ belgilash kiritilsa, tenglama

$$2(1+t) \cdot \frac{1}{2}(5+t) = \frac{9}{4}(1+t)^2 \text{ ko'rinishga keladi. Shakl almashti-}$$

rish va soddalashtirishlar yordamida $(1+t)(5+t - \frac{9}{4}(1+t)) = 0$

yoki $(1+t)(\frac{11}{4} - \frac{5}{4}t) = 0$. U holda $t_1 = -1, t_2 = \frac{11}{5}$ yechimlar kelib chiqadi.

Kiritilgan belgilashdan $\log_2 x = -1$ va $\log_2 x = \frac{11}{5}$ bundan

$x_1 = \frac{1}{2}, x_2 = 4\sqrt[5]{2}$ hosil bo'ladi. Topilgan ildizlar noma'lumning yo'l qo'yiladigan qiymatlari sohasiga tegishli bo'lganligi uchun berilgan tenglamaning ildizlari $x_1 = \frac{1}{2}, x_2 = 4\sqrt[5]{2}$ lardan iborat bo'ladi.

Mashqlar

Quyidagi tenglamalarni yeching (1-88):

1. $\lg^2 x - 5 \lg x + 6 = 0$. 2. $\lg x^2 = \lg(5x - 4)$.

3. $\lg x = 4(\lg 4 - 1)$. 4. $\frac{\ln x}{1 - \ln 2} = 2$.

5. $\log_{x-3}(x^2 - 5x + 10) = 2$. 6. $\log_{3-x}(x^2 - 5x + 10) = 2$.

7. $\log_2 x^2 = \log_2 \frac{2}{-x^2 + 3}$. 8. $\log_3(6 - x^2) = \log_{0,3} x^2$.

9. $1000^{\lg(2x-1)} = 10^8$. 10. $\lg \sqrt{2x-1} = 1 + \lg \frac{1}{\sqrt{x-9}}$.

11. $\frac{1}{3} \log_2 3^{5x-7} = \frac{2}{x} \log_2 3$. 12. $\log_{\sqrt{10}}(3^{\sqrt{\frac{x^2-4x}{x-3}}} + 1) = 2$.

13. $\lg_{\sqrt{x}} 2 - \lg_{\sqrt{x}} 3 = -0,5$. 14. $\ln 64x^3 = 2,5 \ln 2x$.
15. $\frac{\log_3 2x - 1}{\log_3 2x - \frac{1}{\log_2 3}} = \frac{1}{2}$. 16. $4^{\lg x} - 6^{\lg x} - 2 \cdot 9^{\lg x + 1} = 0$.
17. $\lg(\frac{1}{2} + x) = \lg \frac{1}{2} - \lg x$. 18. $\lg(x-1) + \frac{1}{2} \lg(x+7)^2 = \lg 1$.
19. $\log_2 x + \log_2 2x + \log_2 8x = 10$.
20. $\log_3 x^2 + 2 \log_9 x^2 + \log_{81} x^{16} = 8$.
21. $\lg(x+6) - 0,5 \lg(2x-3) = \lg 4$.
22. $\log_{x-2}(x^2 - 5x + 10) = 2$.
23. $\log_7(\frac{1}{2} + x) + \log_7 x = -\log_7 2$.
24. $2 \lg(\sqrt{2x + \frac{x}{12}} + \sqrt{\frac{x}{12}}) - 1 = \lg 1,5$.
25. $\log_2(\frac{x^3}{8} + 27) - \log_2(\frac{x^2}{4} - 3x - 9) = \log_2 7$.
26. $\frac{\lg(\sqrt{30x+1} + 4) - (\lg x + 1)}{2 - 2 \lg 2 + \lg 15 - \lg 1000} = 1$.
27. $\frac{1}{2} \log_2(5x+3) - \log_4(5x-1) = 2 - \log_2 \sqrt{8}$.
28. $\ln(15x-1) + \ln(15x+1) = 3 \ln 2 + \ln(15x-2)$.
29. $\log_2 x - 1 + \log_2(x+2) = \log_2(5-3x)$.
30. $\log_3(6x-11) + \log_3(2x-27) = \frac{1,5}{\lg 3}$.
31. $\log_5 2(x-3) - \log_5 \sqrt{2} = \frac{1}{\log_3 5} + \log_5 \sqrt{2(x-5)}$.
32. $\log_2 21x - \frac{1}{\log_5 2} = 2(\log_2 21 + \log_2 2) - \log_2 7$.
33. $\log_5 3 + \frac{1}{2} \log_5 2(x-11) = \log_5 \sqrt{2(x+5)}$.

$$34. 2\log_2 5 + \log_2 (x+3) - \log_2 (5x-4) = \frac{2}{\lg 2}.$$

$$35. \log_7 (7x-2) - \frac{1}{2} \log_7 (21x-6) = \frac{1}{\log_2 7}.$$

$$36. \log_3 (21x-5) - \frac{1}{\log_2 3} = 0,5 \log_3 (63x-20).$$

$$37. \lg(0,5x-5) - \lg 2 = 0,5 \lg(1,5x-20).$$

$$38. \lg\left(\frac{x-8}{2}\right) + \lg\left(\frac{x-38}{2}\right) = 3 - \frac{1}{\lg_4 10}.$$

$$39. \lg(x+50) - 1 - \lg(93x+25) = \lg\left(\frac{x-150}{10}\right) - \lg 17.$$

$$40. \ln 2 + \ln(x+1) + \ln 0,2 = \ln \sqrt{2(x-2)}.$$

$$41. \log_2 (x+14) + \log_2 (0,5x+12) = \frac{3}{\lg 2}.$$

$$42. \log_3 \frac{1}{\sqrt{22x-117}} - \frac{1}{\lg 3} = \log_3 5 - \log_3 2x.$$

$$43. \log_4 0,3x - 0,5 \log_4 (0,8x - \frac{63}{4}) = \log_4 3 - \frac{1}{\lg 4}.$$

$$44. \ln 0,5x - \frac{1}{2} \ln(1,2x - 15,75) = \ln 5.$$

$$45. \log_4 2(x-15) - \log_4 \sqrt{3} = \frac{1}{2} (\log_4 (x-24) + \log_4 2) + \frac{1}{\log_5 4}.$$

$$46. \lg \frac{5}{3} + \lg(250 - x^3) = 4 \lg 5.$$

$$47. \lg \sqrt{x} + 0,1505 = 0,7782 - \lg(\sqrt{2x} + 1).$$

$$48. \ln 5 = -\ln \sqrt{3} + \ln \sqrt{4x} + \frac{1}{4} \ln 3.$$

$$49. \log_{\frac{1}{2}} 1,5x - \log_{\frac{1}{2}} (1,5x + 2) = \log_{\frac{1}{2}} 3 - \log_{\frac{1}{2}} (2\frac{1}{4}x^2 - 4).$$

$$50. \log_7 (\sqrt{17}x - 3x - 1) + 0,5 \log_7 (\sqrt{17}x - 3x + 7)^2 = \lg 1.$$

51. $\log_2 |x-5| + \log_2 (x^2 - 6x + 3) = 2\log_2 (x-5) + \log_2 5$.
52. $\ln(x + \sqrt{2}) = -\ln(x - \sqrt{7})$.
53. $\log_2 (128 - x^3) = 3\log_2 x$.
54. $\log_7 (729x^3 + 8) - \frac{1}{2}\log_7 (81x^2 + 36x + 4) = \log_7 (58 + 9x)$.
55. $\log_{\frac{1}{2}} 10 + \log_{\frac{1}{2}} (\frac{1}{2}x + 1) - \log_{\frac{1}{2}} 2\sqrt{2} = 0,5(\log_{\frac{1}{2}} (0,5x + 1) + 2\log_{\frac{1}{2}} 4 + 2\log_{\frac{1}{2}} 5 - \log_{\frac{1}{2}} (0,5x + 5))$.
56. $\ln(2x-1)^3 - 3\ln(2x-3) = 3\ln 2$.
57. $\frac{1}{\lg 2} + 2 = \frac{1}{2}\log_2 (10x + 30) + \log_2 \sqrt{10(x-3)}$.
58. $\log_3 10 - \log_3 5 = \frac{1}{3}(\log_3 0,5 + \log_3 3,2x + \frac{1}{3}\log_3 5)$.
59. $\log_y 10 + \log_y (y-15) = 3\log_y 10 - \frac{2}{\lg y} - \log_y (y-15)^2$.
60. $4(1 + \log_{\frac{1}{3}} (30 + 2x))^{\frac{1}{2}} - 0,51\log_{\frac{1}{3}} (22 - 2x) + \log_{\frac{1}{3}} 5 = \lg \lg^4 (10000)$.
61. $\log_7 (70x-9)^2 + \log_7 (30x-4)^2 = \frac{2}{\lg 7}$.
62. $\ln(75x^2 - 85x + 2) - \ln(25x^2 - 30x + 1) = \ln 2$.
63. $\frac{1}{2}\lg(4x^2 - 20x + 25) + \lg(2x+5)^2 - \lg(4x^2 - 25) = \lg 2 - \lg(x-27) + 2\lg 2 + \lg 5 - 1$.
64. $5\ln 2x - 2\ln 12 - \ln 2 = 3\ln x$.
65. $0,5(1 + \log_5 \frac{x}{2} + \log_5 \sqrt{0,1}) = \log_5 \sqrt{3} + \frac{1}{2}$.
66. $\lg(2x-3)^{\frac{1}{2}} - 0,5(\lg(2x-1)^2 - \lg 2 - \lg(x+1)) = 0$.

$$67. \log_2 11 + \log_2 (x+2) = 2\log_2 10 + \log_2 9.$$

$$68. \log_3 x = 2\log_3 (2x-13).$$

$$69. \frac{\ln(4x^2 + 4x + 1)}{\ln(16x^4 + 32x^3 + 32x^2 - 14x + 13)} = \ln \sqrt{e}.$$

$$70. \log_2 (35 - 8x^3) = 3\log_2 (5 - 2x).$$

$$71. \log_4 (2\log_3 (1 + \log_2 (1 + 3\log_2 \frac{x}{4}))) = \frac{1}{2}.$$

$$72. \lg(3x+14) + \lg(3x+2) = \lg 64.$$

$$73. \log_2 3x + \log_2 (3x+5) - \log_2 50 = 0.$$

$$74. \lg x - \lg 11 = \lg 19 - \lg(30-x).$$

$$75. \lg(x+2) - \lg 5 = \lg(x-6).$$

$$76. \lg(x+6) - \frac{1}{2}\lg(2x-3) = 2 - \lg 25.$$

$$77. 2\lg \sqrt{x + \frac{x}{24}} + \sqrt{\frac{x}{24}} - 1 = \lg 3 - \lg 2.$$

$$78. \lg(x^3 + 27) - 0,5\lg(x^2 + 6x + 9) = 3\lg \sqrt[3]{7}.$$

$$79. 3\lg x = \frac{5}{2}\lg \frac{x}{2}.$$

$$80. \lg(x-1) + \lg(x+1) = 3\lg 2 + \lg(x-2).$$

$$81. \lg x + \lg(x+1) = \lg(5-6x) - \lg 2.$$

$$82. \lg(x-6) - \frac{1}{2}\lg 2 = \lg 3 + \frac{1}{2}\lg(x-10).$$

$$83. \lg 3 + \frac{1}{2}\lg(y-22) = \lg \sqrt{y+10}.$$

$$84. \lg(x-5) - \lg 2 = \frac{1}{2}\lg(3x-20).$$

$$85. \lg(x+5) - \lg(3x+25) = \lg(x-15) - \lg 17.$$

$$86. \lg(x-30) - \frac{1}{2}\lg 3 = \frac{1}{2}\lg(x-48) + \lg 5.$$

$$87. \lg(1250 - 5x^3) = \lg 3 + \lg 208, (3).$$

$$88. \lg x - \lg(x+2) = \lg 3 - \lg(x^2 - 4).$$

IV BOB. KO'RSATKICHLI, LOGARIFMIK TENGLAMALAR VA TENGSIZLIKLAR SISTEMALARI

1- §. Ko'rsatkichli va logarifmik tenglamalarning sistemalari

Logarifmik va ko'rsatkichli tenglamalar sistemasini yechish uchun, odatda, berilgan sistema algebraik tenglamalar sistemasiga keltiriladi.

Bunday sistemalarning ba'zi turlarini ko'rib chiqamiz.

I. Bitta tenglamasi algebraik, ikkinchisi ko'rsatkichli yoki logarifmik tenglama bo'lgan sistemalar.

$$1- \text{ m i s o l. } \begin{cases} \log_y x + \log_x y = 2, & \text{tenglamalar siste-} \\ x^2 - y = 20 & \text{masini yeching.} \end{cases}$$

Y e c h i s h. Berilgan sistemada $x > 0, y > 0, x \neq 1, y \neq 1$ deb faraz qilamiz. $\log_y x = \frac{1}{\log_x y}$ bo'lishini hisobga olib,

$\log_x y$ ni t orqali belgilab olamiz. U holda $\log_y x = \frac{1}{t}$ bo'ladi.

Belgilashlar natijasida $t + \frac{1}{t} = 2$ tenglama hosil bo'ladi. Undan $t^2 - 2t + 1 = 0$ va $t_{1,2} = 1$ hosil bo'ladi.

Demak, $\log_x y = 1$. U holda $x = y$ ekanligini hisobga olsak, sistemadagi ikkinchi tenglama $x^2 - x = 20$ yoki $x^2 - x - 20 = 0$ ko'rinishga kelib, $x_1 = -4, x_2 = 5$ yechimlarni hosil qilamiz.

Natijada $x > 0, y > 0$ ni e'tiborga olib, sistemaning $\begin{cases} x = 5, \\ y = 5 \end{cases}$ yechimi hosil bo'ladi.

$$2- \text{ m i s o l. } \begin{cases} 2^{\frac{x+y}{3}} + 2^{\frac{x+y}{6}} = 6, & \text{tenglamalar sistemasini} \\ x^2 + 5y^2 = 6xy & \text{yeching.} \end{cases}$$

Y e c h i s h. $2^{\frac{x+y}{3}} = (2^{\frac{x+y}{6}})^2, 2^{\frac{x+y}{6}} = z$ belgilashni kiritsak, birinchi tenglama $z^2 + z - 6 = 0$ ko'rinishga keladi va undan $z_1 = -3, z_2 = 2$ yechimlarni hosil qilamiz. U holda $2^{\frac{x+y}{6}} > 0$

bo'lishini e'tiborga olsak, $2^{\frac{x+y}{6}} = 2$ yoki $x + y = 6$ tenglamani hosil qilamiz. Natijada, berilgan sistemaga teng kuchli

bo'lgan $\begin{cases} x + y = 6, \\ x^2 + 5y^2 = 6xy \end{cases}$ algebraik tenglamalar sistemasini hosil qilamiz. Bu sistemaning yechilishi quyidagicha:

$$\begin{aligned} \begin{cases} x + y = 6, \\ x^2 + 5y^2 = 6xy \end{cases} &\Leftrightarrow \begin{cases} x = 6 - y \\ (6 - y)^2 + 5y^2 = (6 - y)y \end{cases} \Leftrightarrow \begin{cases} x = 6 - y, \\ y^2 - 4y + 3 = 0 \end{cases} \Leftrightarrow \\ &\Leftrightarrow \begin{cases} \begin{cases} x = 6 - y, \\ y = 3; \end{cases} \\ \begin{cases} x = 6 - y, \\ y = 1 \end{cases} \end{cases} \Leftrightarrow \begin{cases} \begin{cases} x = 3, \\ y = 3; \end{cases} \\ \begin{cases} x = 5, \\ y = 1. \end{cases} \end{cases} \end{aligned}$$

3- misol. $\begin{cases} \frac{y^2}{x} + \frac{x^2}{y} = 12, \\ 10^{\lg \frac{1}{x}} + 3^{-\log_3 y} = \frac{1}{3}. \end{cases}$ tenglamalar sistemasini yeching.

Y e c h i s h. Logarifm xossalariga asosan quyidagi sistemani hosil qilamiz:

$$\begin{aligned} \begin{cases} \frac{y^2}{x} + \frac{x^2}{y} = 12, \\ \frac{1}{x} + \frac{1}{y} = \frac{1}{3}, \\ x > 0, \\ y > 0 \end{cases} &\Leftrightarrow \begin{cases} x^3 + y^3 = 12xy, \\ x + y = \frac{1}{3}xy, \\ x > 0, \\ y > 0 \end{cases} \Leftrightarrow \begin{cases} (x + y)((x + y)^2 - 3xy) = 12xy, \\ x + y = \frac{1}{3}xy, \\ x > 0, \\ y > 0. \end{cases} \end{aligned}$$

Agar $x + y$ ni t , $x \cdot y$ ni z orqali belgilasak, u holda

$$\begin{cases} t(t^2 - 3z) = 12z, \\ t = \frac{1}{3}z, \\ t > 0, \\ z > 0 \end{cases} \quad \text{sistema hosil bo'ladi. Bu sistema yechimlarini}$$

topaylik:

$$\begin{cases} t^3 - 3tz - 12z = 0, \\ z = 3t, \\ t > 0, \\ z > 0 \end{cases} \Leftrightarrow \begin{cases} t^3 - 9t^2 - 36t = 0, \\ z = 3t, \\ t > 0, \\ z > 0 \end{cases} \Leftrightarrow \begin{cases} t(t^2 - 9t - 36) = 0, \\ z = 3t, \\ t > 0, \\ z > 0. \end{cases}$$

Sistemadagi birinchi tenglamaning yechimlari $t_1 = -3$, $t_2 = 0$, $t_3 = 12$. Lekin, $t > 0$ shartni $t = 12$ qanoatlantiradi. Bu qiymatni ikkinchi tenglamaga qo'ysak, $z = 36$ hosil bo'ladi.

Shunday qilib, oxirgi sistemaning yechimi $\begin{cases} t = 12, \\ z = 36. \end{cases}$ Kiritilgan

$x + y = t$, $x \cdot y = z$ belgilashlarni hisobga olsak, $\begin{cases} x + y = 12, \\ x \cdot y = 36 \end{cases}$

sistemaga ega bo'lamiz. Bu sistemaning yechimlari $\begin{cases} x = 6, \\ y = 6 \end{cases}$

bo'lishi ravshan.

4- m i s o l. $\begin{cases} x - y = 2, \\ x^{x^2 - y^2} = x^{16}. \end{cases}$ tenglamalar sistemasini yeching.

Y e c h i s h. Sistemadagi ikkinchi tenglama $x = 1$ bo'lganda ham o'rinli ekanligini inobatga olsak, berilgan sistema quyidagi birlashmaga teng kuchli ekanligi kelib chiqadi:

$$\begin{cases} \begin{cases} x - y = 2, \\ x = 1; \end{cases} \\ \begin{cases} x - y = 2, \\ x^2 - y^2 = 16 \end{cases} \end{cases} \Leftrightarrow \begin{cases} \begin{cases} x = 1, \\ y = -1; \end{cases} \\ \begin{cases} y = x - 2, \\ x^2 - (x - 2)^2 = 16 \end{cases} \end{cases} \Leftrightarrow \begin{cases} \begin{cases} x = 1, \\ y = -1; \end{cases} \\ \begin{cases} y = x - 2, \\ x^2 - x^2 + 4x - 4 = 16 \end{cases} \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \begin{cases} x = 1, \\ y = -1; \end{cases} \\ \begin{cases} y = x - 2, \\ 4x = 20 \end{cases} \end{cases} \Leftrightarrow \begin{cases} \begin{cases} x = 1, \\ y = -1; \end{cases} \\ \begin{cases} y = 3, \\ x = 5. \end{cases} \end{cases}$$

Shunday qilib, sistema quyidagi ikkita yechimga ega ekan:

$$1) \begin{cases} x = 1, \\ y = -1; \end{cases} \quad 2) \begin{cases} x = 5, \\ y = 3. \end{cases}$$

$$5\text{- m i s o l. } \begin{cases} \log_3(x-y) - \log_2(x+y) + 1 = 0 \\ (x-y)(x+y) = 2 \end{cases} \text{ tenglamalar}$$

sistemasini yeching.

Y e c h i s h. Berilgan sistemada $x-y > 0$ va $x+y > 0$ deb faraz qilamiz. Ya'ni $x > |y|$ shart bajarilsin. U holda

$$\begin{cases} \log_3(x-y) = \log_2(x-y) - 1 \\ \log_2(x+y) + \log_2(x-y) = 1 \end{cases} \Leftrightarrow \begin{cases} \log_3(x-y) = -\log_2(x-y) \\ \log_2(x+y) + \log_2(x-y) = 1. \end{cases}$$

Birinchi tenglamada $x-y=1$ bo'lsa u holda $x=y+1$. Bu qiymatni ikkinchi tenglamaga qo'yilsa $\log_2(2y+1)=1$, ya'ni $y=\frac{1}{2}$ kelib chiqadi.

Demak, $y=\frac{1}{2}$, $x=\frac{3}{2}$ x, y larning topilgan qiymatlari $x > |y|$ shartni va sistemani qanoatlantiradi.

Agar $x-y \neq 1$ bo'lsa, birinchi tenglamadagi logarifmlarni $x-y$ asosga o'tkaziladi. U holda $\frac{1}{\log_{x-y} 3} = \frac{1}{\log_{x-y} 2}$ yoki $\log_{x-y} 3 = \log_{x-y} 2$ ziddiyat kelib chiqadi.

Demak, sistemaning yechimi $\left\{\left(\frac{3}{2}, \frac{1}{2}\right)\right\}$ to'plamdan iborat.

$$6\text{- m i s o l. } \begin{cases} x^4 = y^5 \\ \log_5 x - \log_5 y = \frac{\log_5 x}{\log_5 y} \end{cases} \text{ tenglamalar sistemasi-}$$

ni yeching.

Y e c h i s h. Berilgan sistemada $x > 0, y > 0, y \neq 5$ deb faraz qilamiz. Birinchi tenglamani 5 asosga ko'ra logarifmlasak, u holda

$$4\log_5 x = 5\log_5 y \Rightarrow \log_5 x = \frac{5}{4}\log_5 y. \text{ Hosil qilingan}$$

tenglikni tenglamalar sistemasidagi ikkinchi tenglamaga qo'yilsa $\frac{1}{4} \log_5 y = \frac{5}{4}$, ya'ni $y = 5^5$ kelib chiqadi. U holda

$\log_5 x = \frac{5}{4} \cdot 5$, ya'ni $x = 5^{\frac{25}{4}}$. Shunday qilib, berilgan sistema-

ning yechimi $\begin{cases} x = 5^{\frac{25}{4}}, \\ y = 5^5 \end{cases}$, dan iborat.

7- m i s o l. $\begin{cases} xy = 4, \\ \frac{1}{2} (\lg^2 x + \lg^2 y) = 5 \cdot (\lg 4)^2 \end{cases}$ tenglamalar sistemasini yeching.

Y e c h i s h. Logarifm ta'rifiga ko'ra tenglamalar sistemasida $x > 0, y > 0$ bo'lsin deb faraz qilamiz. $\lg^2 x + \lg^2 y = (\lg x + \lg y)^2 - 2 \lg x \cdot \lg y$ ayniyatdan foydalanib, tenglamalar sistemasidagi ikkinchi tenglamani $\lg^2 xy - 2 \lg x \cdot \lg y = 10(\lg 2)^2$ ko'rinishga keltirib olamiz. U holda birinchi tenglamadan $y = \frac{4}{x}$ kelib chiqishini hisobga olib, ikkinchi tenglamani

$-2 \lg x \cdot (\lg 4 - \lg x) = 6(\lg 2)^2$ ko'rinishga keltiramiz. Yoki $2(\lg x)^2 - 4 \lg 2 \cdot \lg x - 6(\lg 2)^2 = 0$ hosil bo'ladi. $\lg x = t$ belgilash yordamida tenglama $t^2 - 2 \lg 2 \cdot t - 3(\lg 2)^2 = 0$ ko'rinishga keladi. Viyet formulalari yordamida $t_1 = 3 \lg 2$, $t_2 = -\lg 2$, ya'ni $t_1 = \lg 8, t_2 = \lg 0,5$ lar hosil bo'ladi. Bundan

$\begin{cases} x = 8, \\ y = 0,5 \end{cases} \wedge \begin{cases} x = 0,5, \\ y = 8 \end{cases}$ yechimlar kelib chiqadi.

8- m i s o l. $\begin{cases} x^{\log_3 y} + 2y^{\log_3 x} = 27, \\ \frac{y}{x} = 3 \end{cases}$ sistemada $x > 0, y > 0$

deb faraz qilib, $x^{\log_3 y} = y^{\log_3 x}$ ayniyatdan foydalanamiz. U

holda $\begin{cases} x^{\log_3 y} = 9, \\ y = 3x \end{cases}$ sistema hosil bo'ladi. $y = 3x$ ni birinchi tenglamaga qo'yib, shu tenglamani ikkala tomonini 3 asosga ko'ra logarifmlaymiz. Natijada $\log_3 x \cdot \log_3 3x = 2$ yoki $\log_3 x (\log_3 x + 1) = 2$ tenglama hosil bo'ladi.

$\log_3 x$ ni t orqali belgilab, $t^2 + t - 2 = 0$ tenglamani hosil

qilamiz. U holda $t_1 = -2, t_2 = 1$ yoki sistemaning $\begin{cases} x = \frac{1}{9} \\ y = \frac{1}{3} \end{cases}$ va

$\begin{cases} x = 3, \\ y = 9 \end{cases}$ yechimlari kelib chiqadi.

II. Bitta tenglamasi logarifmik, ikkinchi tenglamasi ko'rsatkichli bo'lgan tenglamalar sistemasi

1- misol. $\begin{cases} 3^y \cdot 9^x = 81, \\ \lg(x+y)^2 - \lg x = \lg 9 \end{cases}$ tenglamalar sistemasini yeching.

$$\text{Yechish. } \begin{cases} 3^y \cdot 3^{2x} = 81, \\ \lg \frac{(x+y)^2}{x} = \lg 9 \end{cases} \Leftrightarrow \begin{cases} 3^{y+2x} = 3^4, \\ \frac{(x+y)^2}{x} = 9, \\ x > 0, \\ x+y \neq 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} y+2x=4, \\ (x+y)^2=9x, \\ x>0, \\ x+y \neq 0 \end{cases} \Leftrightarrow \begin{cases} y=4-2x, \\ (x+4-2x)^2=9x, \\ x>0, \\ x+y \neq 0 \end{cases} \Leftrightarrow \begin{cases} y=4-2x, \\ 16-8x+x^2=9x, \\ x>0, \\ x+y \neq 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} y = 4 - 2x, \\ x^2 - 17x + 16 = 0, \\ x > 0, \\ x + y \neq 0 \end{cases} \Leftrightarrow \begin{cases} y = 4 - 2x, \\ x = 1, \\ x > 0, \\ x + y \neq 0; \\ y = 4 - 2x, \\ x = 16, \\ x > 0, \\ x + y \neq 0 \end{cases} \Leftrightarrow \begin{cases} y = 2, \\ x = 1; \\ x = 16, \\ y = -28. \end{cases}$$

2- misol. $\begin{cases} 3^x \cdot 2^y = 972, \\ \log_3(x-y)^2 = 2. \end{cases}$ tenglamalar sistemasini yeching.

Yechish. $\begin{cases} 3^x \cdot 2^y = 972, \\ \log_3(x-y)^2 = 2 \end{cases} \Leftrightarrow \begin{cases} 3^x \cdot 2^y = 972, \\ (x-y)^2 = 9 \end{cases} \Leftrightarrow$

$$\Leftrightarrow \begin{cases} 3^x \cdot 2^y = 972, \\ x - y = 3; \\ 3^x \cdot 2^y = 972, \\ x - y = -3 \end{cases} \Leftrightarrow \begin{cases} 3^x \cdot 2^y = 972, \\ y = x - 3; \\ 3^x \cdot 2^y = 972, \\ y = x + 3 \end{cases} \Leftrightarrow \begin{cases} 3^x \cdot 2^{x-3} = 972, \\ y = x - 3; \\ 3^x \cdot 2^{x+3} = 972, \\ y = x + 3 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} 3^x \cdot 2^x = 8 \cdot 972, \\ y = x - 3; \\ 3^x \cdot 2^x = \frac{1}{8} \cdot 972, \\ y = x + 3 \end{cases} \Leftrightarrow \begin{cases} 6^x = 8 \cdot 972, \\ y = x - 3; \\ 6^x = \frac{1}{8} \cdot 4 \cdot 243, \\ y = x + 3 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x = \log_6 2^5 \cdot 3^5, \\ y = x - 3; \\ x = \log_6 2^{-1} \cdot 3^5, \\ y = x + 3 \end{cases} \Leftrightarrow \begin{cases} x = 5, \\ y = 2; \\ x = 5 \log_6 3 - \log_6 2, \\ y = 5 \log_6 3 - \log_6 2 + 3. \end{cases}$$

Hosil bo'lgan yechimlarning ikkalasi ham $x - y \neq 0$ shartni qanoatlantiradi. Shuning uchun ham ular berilgan sistemaning yechimlari bo'ladi.

$$3\text{- m i s o l. } \begin{cases} 2^{2x-y} - 2^{\frac{2x-y}{2}} - 2 = 0, \\ \lg(3x-y) + \lg(x+y) - \frac{4}{\log_2 10} = 0. \end{cases} \quad \text{tenglamalar}$$

sistemasini yeching.

Y e c h i s h. Birinchi tenglamadagi $2^{\frac{2x-y}{2}}$ ni t orqali belgilasak, tenglama $t^2 - t - 2 = 0$ ko'rinishga keladi. Bu tenglamaning ildizlari $t_1 = -1$, $t_2 = 2$.

$2^{\frac{2x-y}{2}} > 0$ bo'lgani uchun $2^{\frac{2x-y}{2}} = 2$ bo'lishi ayon. U holda $2x - y = 2$. Natijada quyidagi sistemaga ega bo'lamiz:

$$\begin{cases} y = 2x - 2, \\ \lg(3x - y) + \lg(x + y) - \frac{4}{\log_2 10} = 0. \end{cases}$$

Hosil bo'lgan tenglamalar sistemasini yechamiz:

$$\begin{cases} y = 2x - 2, \\ \lg(x + y) \cdot (3x - y) - \frac{\log_2 16}{\log_2 10} = 0 \end{cases} \Leftrightarrow \begin{cases} y = 2x - 2, \\ \lg(x + y)(3x - y) = \lg 16, \\ x + y > 0, \\ 3x - y > 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} y = 2x - 2, \\ (x + y)(3x - y) = 16, \\ x + y > 0, \\ 3x - y > 0 \end{cases} \Leftrightarrow \begin{cases} y = 2x - 2, \\ (3x - 2)(2 - x) = 16, \\ x + y > 0, \\ 3x - y > 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} x = -3\frac{1}{3}, \\ y = -8\frac{2}{3}, \\ x + y > 0, \\ 3x - y > 0; \end{cases} \Leftrightarrow \begin{cases} x = 2, \\ y = 2. \end{cases}$$

$$\Leftrightarrow \begin{cases} x = 2, \\ y = 2, \\ x + y > 0, \\ 3x - y > 0 \end{cases}$$

III. Aralash sistemalar

1- m i s o l. $\begin{cases} (x+y) \cdot 3^{y-x} = \frac{5}{27}, \\ 3\log_5(x+y) = x-y. \end{cases}$ tenglamalar sistemasini yeching.

Y e c h i s h. $\begin{cases} (x+y) \cdot 3^{y-x} = \frac{5}{27}, \\ 3\log_5(x+y) = x-y \end{cases} \Leftrightarrow \begin{cases} (x+y) = 5 \cdot 3^{x-y-3}, \\ (x+y) = 5^{\frac{x-y}{3}}, \\ x+y > 0 \end{cases} \Leftrightarrow$

$$\Leftrightarrow \begin{cases} (x+y) = 5 \cdot 3^{x-y-3}, \\ 3^{x-y-3} = 5^{\frac{x-y}{3}-1}, \\ x+y > 0 \end{cases} \Leftrightarrow \begin{cases} (x+y) = 5 \cdot 3^{x-y-3}, \\ (x-y-3)\lg 3 = \frac{x-y-3}{3}\lg 5, \\ x+y > 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} (x+y) = 5 \cdot 3^{x-y-3}, \\ (x-y-3)(\lg 3 - \lg \sqrt[3]{5}) = 0, \\ x+y > 0 \end{cases} \Leftrightarrow \begin{cases} (x+y) = 5 \cdot 3^{x-y-3}, \\ (x-y-3) = 0, \\ x+y > 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} (x+y) = 5 \cdot 3^{x-y-3}, \\ x = y+3, \\ x+y > 0 \end{cases} \Leftrightarrow \begin{cases} 2y+3 = 5 \cdot 3^0, \\ x = y+3, \\ x+y > 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} y=1, \\ x=4, \\ x+y>0 \end{cases} \Leftrightarrow \begin{cases} x=4, \\ y=1. \end{cases}$$

2- misol. $\begin{cases} x + \lg y = 1, \\ y^x = 10^{-2} \end{cases}$ tenglamalar sistemasini yeching.

Yechish. $\begin{cases} x + \lg y = 1, \\ y^x = 10^{-2}. \end{cases} \Leftrightarrow \begin{cases} x = 1 - \lg y, \\ x = \frac{-2}{\lg y} \end{cases} \Leftrightarrow$

$$\Leftrightarrow \begin{cases} 1 - \lg y = \frac{-2}{\lg y}, \\ x = 1 - \lg y \end{cases} \Leftrightarrow \begin{cases} \lg y - (\lg y)^2 = -2, \\ x = 1 - \lg y \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} (\lg y)^2 - \lg y - 2 = 0, \\ x = 1 - \lg y \end{cases} \Leftrightarrow \begin{cases} \lg y = 2, \\ x = 1 - \lg y; \\ \lg y = -1, \\ x = 1 - \lg y \end{cases} \Leftrightarrow \begin{cases} x = -1, \\ y = 100; \\ x = 2, \\ y = 0,1. \end{cases}$$

Mashqlar

Tenglamalar sistemasini yeching (1–25):

1. $\begin{cases} (x+y)2^{y-2x} = \frac{25}{4}, \\ (x+y)^{1/(2x-y)} = 5. \end{cases}$ 2. $\begin{cases} x^{x-2y} = 36, \\ 4(x-2y) + \log_6 x = 9. \end{cases}$

3. $\begin{cases} y = 1 + \log_4 x, \\ x^y = 4^6. \end{cases}$ 4. $\begin{cases} y - \log_3 x = 1, \\ x^y = 3^{12}. \end{cases}$

5. $\begin{cases} 4^{-y} \log_2 x = 4, \\ \log_2 x + 2^{-2y} = 4. \end{cases}$ 6. $\begin{cases} 3^{-x} 2^y = 1152, \\ \log_5 (x+y) = 1. \end{cases}$

$$7. \begin{cases} 5^{\lg(x-2y)} = 1, \\ 3^{x-y} - 26 \cdot 3^{0,5(x-y)} = 27. \end{cases} \quad 8. \begin{cases} 2^x \cdot 8^{-y} = 2\sqrt{2}, \\ \log_9 1/x + 0,5 = 0,5 \cdot \log_3 9y. \end{cases}$$

$$9. \begin{cases} 4^{x+y} = 120, \\ 5^{3x-2y-3} = 1. \end{cases} \quad 10. \begin{cases} 4^{x+y} = 2^{y-x}, \\ 4^{\log_{\sqrt{2}} x} = y^4 - 5. \end{cases}$$

$$11. \begin{cases} 5^{\sqrt[3]{x}} \cdot 2^{\sqrt{y}} = 200, \\ 5^{2\sqrt[3]{x}} + 2^{2\sqrt{y}} = 689. \end{cases} \quad 12. \begin{cases} 4^x - 7 \cdot 2^{x-0,5y} = 2^{3-y}, \\ 5^{y-x} = 125. \end{cases}$$

$$13. \begin{cases} \log_2 x + \log_4 y = 4, \\ \log_4 x + \log_2 y = 5. \end{cases} \quad 14. \begin{cases} \log_{xy} (x-y) = 1, \\ \log_{xy} (x+y) = 0. \end{cases}$$

$$15. \begin{cases} \log_3 x + \log_9 y = 1,5, \\ \log_x 3 + \log_y 9 = 3. \end{cases} \quad 16. \begin{cases} \log_x y + \log_y x = 2,5, \\ \log_6 (x^2 + y^2) = 1. \end{cases}$$

$$17. \begin{cases} \log_x y + \log_y x = 2,5, \\ x \cdot y = 8. \end{cases} \quad 18. \begin{cases} \lg x - \lg y = 2, \\ x - 10y = 900. \end{cases}$$

$$19. \begin{cases} 4^x - 7 \cdot 2^{x-0,5y} = 2^{3-y}, \\ y - x = 3. \end{cases} \quad 20. \begin{cases} x + y = 5, \\ 2^x / 3^y = 1152. \end{cases}$$

$$21. \begin{cases} 4^{\frac{x+y}{y} \cdot x} = 32, \\ \log_3 (x-y) = 1 - \log_3 (x+y). \end{cases}$$

$$22. \begin{cases} 8(\sqrt{2})^{x-y} = 0,5^{y-3}, \\ \log_3 (x-2y) + \log_3 (3x+2y) = 3. \end{cases}$$

$$23. \begin{cases} 3^{1+2\log_3(y-x)} = 48, \\ 2\log_5 (2y-x-12) - \log_5 (y-x) = \log_5 (y+x). \end{cases}$$

$$24. \begin{cases} \lg(x^2 + y^y) = 2 - \lg 5, \\ \lg(x+y) + \lg(x-y) = \lg 1,2 + 1. \end{cases}$$

$$25. \begin{cases} \log_2 (x+y) - \log_2 (x-y) = 1, \\ x^2 - y^2 = 2. \end{cases}$$

2- §. Ko'rsatkichli va logarifmik tengsizliklar va tengsizliklar sistemasi

Sodda tengsizliklar 4- § dagi 20–35 tengsizliklar yordamida yechiladi. Murakkabroq tengsizliklar logarifmning xossalari, logarifmik va ko'rsatkichli tenglamalarni yechishda ishlatilgan usullar orqali sodda tengsizliklarga keltiriladi.

Ba'zi bir misollarni ko'rib chiqamiz.

1- m i s o l . $3^{\frac{1}{x}} + 3^{\frac{3x+1}{x}} \leq 252$. tengsizlikni yeching.

. Y e c h i s h . Berilgan tengsizlikni $3^{\frac{1}{x}} + 3^{3+\frac{1}{x}} \leq 252$ ko'rinishda yozib olib, $3^{\frac{1}{x}}$ ni qavsdan tashqariga chiqaramiz. U holda $28 \cdot 3^{\frac{1}{x}} \leq 252$ yoki $3^{\frac{1}{x}} \leq 9$ tengsizlikka ega bo'lamiz. Bu tengsizlik $\frac{1}{x} \leq 2$ yoki $\frac{2x-1}{x} \geq 0$ tengsizlikka teng kuchli. $x \neq 0$ ekanligini hisobga olib, intervallar metodi orqali bu tengsizlikning yechimi $(-\infty, 0) \cup [\frac{1}{2}, +\infty)$ to'plamdan iborat bo'lishini ko'rish mumkin.

2- m i s o l . $(3^{x-1} - 2^{x-1}) + (3^{x-2} - 2^{x-2}) + (3^{x-3} - 2^{x-3}) \geq 0$. tengsizlikni yeching

Y e c h i s h . Berilgan tengsizlikning ikkala tomonini 2^{x-1} ga bo'lamiz va quyidagi tengsizlikni hosil qilamiz:

$$\left(\frac{3^{x-1}}{2^{x-1}} - 1\right) + \left(\frac{1}{3} \cdot \frac{3^{x-1}}{2^{x-1}} - \frac{1}{2}\right) + \left(\frac{1}{9} \cdot \frac{3^{x-1}}{2^{x-1}} - \frac{1}{4}\right) \geq 0. \text{ Hosil bo'lgan}$$

tengsizlikdagi $\left(\frac{3}{2}\right)^{x-1}$ ni t orqali belgilasak, u holda

$t - 1 + \frac{t}{3} - \frac{1}{2} + \frac{t}{9} - \frac{1}{4} \geq 0$ yoki $36t - 36 + 12t - 18 + 4t - 9 = 0$. Bun-

dan $52t \geq 63$ yoki $t = \frac{63}{52}$ hosil qilamiz. Ya'ni, $\left(\frac{3}{2}\right)^{x-1} \geq \frac{63}{52}$.

Tengsizlikning ikkala tarafini $\frac{3}{2}$ asosga ko'ra logarifmlaylik.

$\frac{3}{2} > 1$ bo'lgani sababli, tengsizlik ishorasi o'zgarmasligini

hisobga olsak, $x \geq \log_{\frac{3}{2}} \frac{63}{52} + 1$ ga ega bo'lamiz.

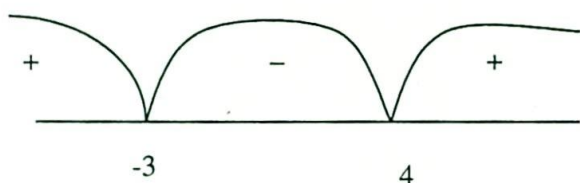
3- m i s o l. $4^{x^{-1}-1} - 2^{x^{-1}-2} - 3 \geq 0$ tengsizlikni yeching.

Y e c h i s h. $4^{x^{-1}-1} - 2^{x^{-1}-2} - 3 \geq 0 \Leftrightarrow \frac{(2^2)^{x^{-1}}}{4} - \frac{2^{x^{-1}}}{2^2} - 3 \geq 0 \Leftrightarrow$

$\Leftrightarrow \frac{(2^{x^{-1}})^2}{4} - \frac{2^{x^{-1}}}{4} - 3 \geq 0$. Agar $2^{x^{-1}} = t$ belgilashdan foydalan-

sak, $\frac{t^2}{4} - \frac{t}{4} - 3 \geq 0$ tengsizlik hosil bo'ladi. U holda

$t^2 - t - 12 \geq 0$ yoki $(t+3)(t-4) \geq 0$ ga ega bo'lamiz. Tengsizlikni intervallar usuli bilan yechamiz (12- rasm):



12- rasm.

Demak, $t \in (-\infty, -3] \cup [4, +\infty)$. $2^{x^{-1}} > 0$ bo'lishini e'tiborga olsak, $2^{x^{-1}} \geq 4$ tengsizlikka ega bo'lamiz. Bu tengsizlikdan $x^{-1} \geq 2$ ni hosil qilib, intervallar usuli yordmida $0 < x \leq \frac{1}{2}$ yechimni hosil qilamiz.

4- m i s o l. $10^{\frac{2}{x}} + 25^{\frac{1}{x}} \geq 4,25 \cdot 50^{\frac{1}{x}}$ tengsizlikni yeching.

Y e c h i s h. Berilgan tengsizlikni $10^{\frac{2}{x}} + 5^{\frac{2}{x}} \geq \frac{17}{4} \cdot 5^{\frac{1}{x}} \cdot 10^{\frac{1}{x}}$

ko'rinishga keltirib, uning ikkala tarafini $5^{\frac{1}{x}} \cdot 10^{\frac{1}{x}} > 0$ ifodaga

bo'lamiz. Natijada $\frac{10^{\frac{1}{x}}}{5^{\frac{1}{x}}} + \frac{5^{\frac{1}{x}}}{10^{\frac{1}{x}}} \geq \frac{17}{4}$ yoki $2^{\frac{1}{x}} + \frac{1}{2^{\frac{1}{x}}} \geq \frac{17}{4}$ hosil

bo'ladi. $2^{\frac{1}{x}} = t$ belgilashni kiritib, $t + \frac{1}{t} \geq \frac{17}{4}$ tengsizlikka ega

bo'lamiz. $2^{\frac{1}{x}} = t > 0$ ekanligini e'tiborga olsak, $4t^2 - 17t + 4 \geq 0$ yoki $4(t-4)(t-\frac{1}{4}) \geq 0$ tengsizlikni hosil qilamiz.

Intervallar usuli orqali tengsizlikni yechib, $\begin{cases} 2^{\frac{1}{x}} \leq 2^{-2}, \\ 2^{\frac{1}{x}} \geq 2^2 \end{cases}$

birlashmaga ega bo'lamiz. U holda $\begin{cases} \frac{1}{x} \leq -2, \\ 2^{\frac{1}{x}} \geq 2 \end{cases}$ yoki $\begin{cases} x \in [-0,5;0), \\ x \in (0;0,5]. \end{cases}$

Demak, $x \in [-0,5;0) \cup (0;0,5]$.

5- m i s o l. $(4 + \sqrt{15})^{\frac{1}{2^x}} + (4 - \sqrt{15})^{\frac{1}{2^x}} > 2$. tengsizlikni yeching.

Y e c h i s h. Berilgan tengsizlikni yechish uchun

$(4 + \sqrt{15})(4 - \sqrt{15}) = 1$ bo'lishini e'tiborga olsak, $(4 + \sqrt{15})^{\frac{1}{2^x}} = t$

va $(4 - \sqrt{15})^{\frac{1}{2^x}} = \frac{1}{t}$ belgilashlarni kiritish mumkin. Natijada

$t + \frac{1}{t} > 2$ yoki $\frac{t^2 - 2t + 1}{t} > 0$ tengsizlikka ega bo'lamiz. $t > 0$

bo'lishini hisobga olsak, $t \neq 1$ yechimni hosil qilamiz. U holda

$(4 + \sqrt{15})^{\frac{1}{2^x}} \neq 1$ yoki $x \neq 0$ yechim hosil bo'ladi.

6- m i s o l. $2^x + 5^x \geq 7^x$ tengsizlikni yeching.

Y e c h i s h. Tengsizlikning ikkala tarafini 7^x ga bo'lib,

$\left(\frac{2}{7}\right)^x + \left(\frac{5}{7}\right)^x \geq 1$ tengsizlikka ega bo'lamiz. $\frac{2}{7} + \frac{5}{7} = 1$ ni hisobga

olib, 3 holni ko'rib chiqamiz.

1- hol. $x = 1$, u holda $\frac{2}{7} + \frac{5}{7} = 1$. Ya'ni $x = 1$ tengsizlikni qanoatlantiradi.

2- hol. $x > 1$, u holda $\left(\frac{2}{7}\right)^x < \frac{2}{7}$ va $\left(\frac{5}{7}\right)^x < \frac{5}{7}$. Demak, $\left(\frac{2}{7}\right)^x + \left(\frac{5}{7}\right)^x < \frac{2}{7} + \frac{5}{7} = 1$, ya'ni $x > 1$ tengsizlikning yechimi bo'la olmaydi.

3- hol. $x < 1$. U holda $\left(\frac{2}{7}\right)^x > \frac{2}{7}$ va $\left(\frac{5}{7}\right)^x > \frac{5}{7}$. Demak, $\left(\frac{2}{7}\right)^x + \left(\frac{5}{7}\right)^x > \frac{2}{7} + \frac{5}{7} = 1$. Shunday qilib, $x \leq 1$ tengsizlikning yechimi bo'lar ekan.

7- m i s o l. $(x-2)^{2(x-2)} > 1$ tengsizlikni yeching.

Y e c h i s h. Bu tengsizlikni $(x-2)^2 \geq 0$ bo'lishini hisobga olib, ikki holni ko'rib chiqamiz.

1- hol. $(x-2)^2 < 1$ va $(x-2)^{2(x-2)} > 1$ bo'lsin. U holda
$$\begin{cases} (x-2)^2 < 1, \\ x-2 < 0 \end{cases} \Leftrightarrow \begin{cases} -1 < x-2 < 1, \\ x-2 < 0 \end{cases} \Leftrightarrow \begin{cases} 1 < x < 3, \\ x < 2 \end{cases} \Leftrightarrow 1 < x < 2$$
 tengsizlikning yechimi bo'ladi.

2- hol. $(x-2)^2 > 1$ va $(x-2)^{2(x-2)} > 1 \Leftrightarrow x-2 > 0$ va $x-2 > 1 \Leftrightarrow x > 3$.

Shunday qilib, tengsizlikning yechimi $(1; 2) \cup (3; +\infty)$ to'plamdan iborat bo'lar ekan.

8- m i s o l. $\log_2(x+27) + \log_{\frac{1}{2}}(16-2x) < \log_2 x$ tengsizlikni yeching.

Y e c h i s h. Avval tengsizlikdagi x ning yo'l qo'yilishi mumkin bo'lgan qiymatlari sohasini topib olaylik,

$$\begin{cases} x+27 > 0, \\ 16-2x > 0, \\ x > 0 \end{cases} \Leftrightarrow \begin{cases} x > -27, \\ -2x > -16, \\ x > 0 \end{cases} \Leftrightarrow \begin{cases} x > -27, \\ x < 8, \\ x > 0 \end{cases} \Leftrightarrow \begin{cases} x < 8, \\ x > 0. \end{cases}$$

Ya'ni, $x \in (0; 8)$.

Berilgan tengsizlikni $\log_2(x+27) - \log_2(16-2x) < \log_2 x$
yoki $\log_2 \frac{x+27}{16-2x} < \log_2 x$ ko'rinishga keltirib olamiz.

Bu tengsizlikdan $\frac{x+27}{16-2x} < x$ ni hosil qilamiz. Agar $16-2x > 0$ bo'lishini hisobga olsak, $2x^2 - 15x + 27 < 0$ tengsizlik hosil bo'ladi. x ning yo'l qo'yilishi mumkin bo'lgan qiymatlari sohasini hisobga olib, berilgan tengsizlik

$$\begin{cases} 2x^2 - 15x + 27 < 0, \\ 0 < x < 8 \end{cases} \text{ sistemaga teng kuchli bo'lishini ko'ramiz.}$$

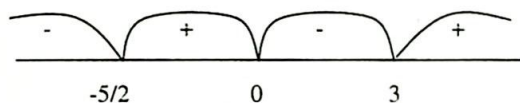
Bu sistemani yechib, $x \in (3; 4,5)$ yechimni hosil qilamiz.

9- m i s o l. $2 \lg x - \frac{15}{\lg x} \geq 1$ tengsizlikni yeching.

Y e c h i s h. x ning yo'l qo'yiladigan qiymatlari sohasi $x > 0$ va $x \neq 1$, ya'ni $(0; 1) \cup (1; +\infty)$ to'plamdan iborat. $\lg x = t$

belgilashni kiritib, $\frac{2t^2 - t - 15}{t} \geq 0$ yoki $\begin{cases} t \cdot (t-3)(2t+5) \geq 0, \\ t \neq 0 \end{cases}$

sistemani hosil qilamiz. Sistemadagi birinchi tengsizlikni intervallar usuli bilan yechamiz (13- rasm):



13- rasm.

Demak, sistema yechimi $t \in [-\frac{5}{2}; 0) \cup [3; +\infty)$.

$x > 0$ ni hisobga olib, t ni $\lg x$ bilan almashtirsak, berilgan

tengsizlik $\begin{cases} x > 0, \\ -2,5 \leq \lg x < 0, \\ \lg x > 3 \end{cases}$ sistemaga teng kuchli bo'lar ekan.

Bu sistemaning yechimini keltiramiz: $\begin{cases} x > 0, \\ -2,5 \leq \lg x < 0, \\ \lg x > 3 \end{cases} \Leftrightarrow$

$$\Leftrightarrow \begin{cases} x > 0, \\ \left[\frac{1}{\sqrt{10^5}} \leq x < 1, \right. \\ \left. x > 1000 \right] \end{cases} \Leftrightarrow \begin{cases} x > 0, \\ \left[\frac{1}{100\sqrt{10}} \leq x < 1, \right. \\ \left. x > 1000 \right] \end{cases} \Leftrightarrow$$

$$\Leftrightarrow x \in \left[\frac{1}{100\sqrt{10}}; 1 \right) \cup (1000; +\infty).$$

10- m i s o l. $x^{\log_3^2 x - 3\log_3 x + 1} > 27$ tengsizlikni yeching.

Y e c h i s h. Tengsizlikning aniqlanish sohasi $x > 0$

bo'lishini e'tiborga olsak, u holda $\begin{cases} x^{\log_3^2 x - 3\log_3 x + 1} > 3^3, \\ x > 0 \end{cases} \Leftrightarrow$

$$\begin{cases} \log_3 x \cdot (\log_3^2 x - 3\log_3 x + 1) > 3, \\ x > 0 \end{cases} \text{ hosil bo'ladi. Bu yerda}$$

$\log_3 x = t$ belgilashni qo'llab, $t^3 - 3t^2 + t - 3 > 0 \Leftrightarrow (t-3)(t^2+1) > 0 \Leftrightarrow t-3 > 0 \Leftrightarrow t > 3$ ni hosil qilamiz.

$\log_3 x = t$ belgilashni e'tiborga olsak, berilgan tengsizlik

$\begin{cases} x > 0, \\ \log_3 x > 3 \end{cases}$ sistemaga teng kuchli ekanligini ko'ramiz. Bu sistemadan $x \in (27; +\infty)$ yechim hosil bo'ladi.

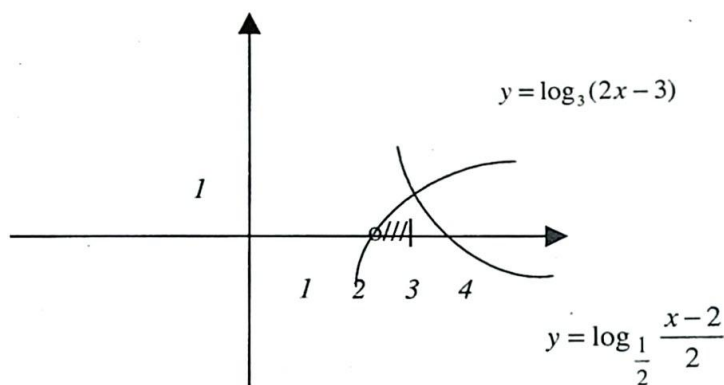
11- m i s o l. $\log_3(2x-3) \leq \log_{\frac{1}{2}} \frac{x-2}{2}$ tengsizlikni yeching.

Y e c h i s h. Avval x ning yo'l qo'yilishi mumkin bo'lgan

$$\text{qiymatlari sohasini topaylik: } \begin{cases} 2x-3 > 0, \\ \frac{x-2}{2} > 0 \end{cases} \Leftrightarrow \begin{cases} x > \frac{3}{2}, \\ x > 2 \end{cases} \Leftrightarrow x > 2$$

$$\Leftrightarrow \Leftrightarrow x \in (2; +\infty).$$

Endi $y = \log_3(2x-3)$ va $y = \log_{\frac{1}{2}} \frac{x-2}{2}$ funksiyalarning grafiklarini chizib, grafiklar kesishish nuqtasining absissasini topamiz (14- rasm).



14- rasm.

chizmaga asosan — $x \in (2, 3]$.

12- m i s o l. $\log_{0.5} \log_5 \frac{x+1}{x-1} \geq 0$ tengsizlikni yeching.

Y e c h i s h . Berilgan tengsizlik quyidagi sistemaga teng

$$\text{kuchli: } \begin{cases} \log_{0.5} \log_5 \frac{x+1}{x-1} \geq 0, \\ \log_5 \frac{x+1}{x-1} > 0, \\ \frac{x+1}{x-1} > 0 \end{cases} . \text{ Bu sistemaning yechimini}$$

topamiz.

$$\begin{aligned} & \begin{cases} \log_{0.5} \log_5 \frac{x+1}{x-1} \geq 0, \\ \log_5 \frac{x+1}{x-1} > 0, \\ \frac{x+1}{x-1} > 0 \end{cases} \Leftrightarrow \begin{cases} \log_5 \frac{x+1}{x-1} \leq 1, \\ \frac{x+1}{x-1} > 1, \\ \frac{x+1}{x-1} > 0 \end{cases} \Leftrightarrow \begin{cases} \frac{x+1}{x-1} \leq 5, \\ \frac{x+1}{x-1} > 1 \end{cases} \Leftrightarrow \\ & \Leftrightarrow \begin{cases} \frac{x+1-5x+5}{x-1} \leq 0, \\ \frac{x+1-x+1}{x-1} > 0 \end{cases} \Leftrightarrow \begin{cases} \frac{x-1.5}{x-1} \geq 0, \\ x > 1 \end{cases} \Leftrightarrow x \geq 1.5. \end{aligned}$$

Mashqlar

Tengsizliklarni yeching (1–24):

$$1. 2^{2x-1} + 2^{2x-3} - 2^{2x-5} > 2^{7-x} + 2^{5-x} - 2^{3-x}.$$

$$2. 5^{x^2-7x+12} > 1.$$

$$3. 3^{x+0,5} + 3^{x-0,5} > 4^{x+0,5} - 2^{2x-1}.$$

$$4. 3^{4-3x} - 35 \cdot 3^{3x-2} + 6 \geq 0.$$

$$5. 4^{1/x-1} - 2^{1/x-2} - 3 \leq 0.$$

$$6. 3^{2x+4} + 45 \cdot 6^x \geq 9 \cdot 2^{2x+2}.$$

$$7. 25^{2\sqrt{x}} + 5 < 25^{\sqrt{x}+0,5} + 25^{\sqrt{x}}.$$

$$8. 7 \cdot 4^{x^2} - 9 \cdot 14^{x^2} + 2 \cdot 49^{x^2} \leq 0.$$

$$9. (3 + 2\sqrt{2})^{2x} + (3 - 2\sqrt{2})^{2x} \leq 6.$$

$$10. (2 + \sqrt{3})^x + (2 - \sqrt{3})^x \geq 4.$$

$$11. 3^x + 4^x < 7^x.$$

$$12. (0,5)^{2x} + (0,3)^{2x} > (0,8)^{2x}.$$

$$13. (x^2 + x + 1)^{\frac{x+5}{x+2}} \geq (x^2 + x + 1)^8.$$

$$14. (4x^2 + 2x + 1)^{x^2-x} > 1.$$

$$15. \log_{0,3}(5x - 2) < \log_{0,3}(3 - 2x).$$

$$16. 0,5 + \log_9 x - \log_3 5x > \log_{\frac{1}{3}}(x + 3).$$

$$17. \log_2(1 + \log_{\frac{1}{9}} x - \log_9 x) < 1.$$

$$18. \log_{\frac{1}{3}}(x - 1) + \log_{\frac{1}{3}}(x + 1) + \log_{\sqrt{3}}(5 - x) < 1.$$

$$19. \frac{1 - \log_4 x}{1 + \log_2 x} \leq 0,5.$$

$$20. \log_2^2(x - 1)^2 - \log_{0,5}(x - 1) > 5.$$

$$21. \log_3(\log_2(2 - \log_4 x) - 1) < 1.$$

$$22. 2,25^{\log_2(x^2-3x-10)} > \left(\frac{2}{3}\right)^{\log_{\frac{1}{2}}(x^2+4x+4)}.$$

$$23. 0,4^{\log_3 \frac{3}{x} \log_3 3x} > 6,25^{\log_3 x^2 + 2}.$$

$$24. \log_2(4x - 2) \leq \log_{0,3}(x - 0,7).$$

3- §. Funksiyaning aniqlanish sohasini topish

$y=f(x)$ funksiyaning aniqlanish sohasi $D(f)$ orqali belgilanadi.

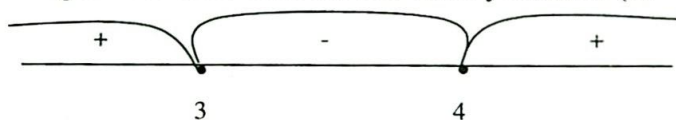
1- m i s o l. $f(x)=\sqrt{\log_2(x^2-7x+13)}$ funksiyaning aniqlanish sohasini toping.

Ye ch i sh. Kvadrat ildiz ta'rifiga ko'ra ildiz ostidagi ifoda manfiy bo'la olmaydi, ya'ni $\log_2(x^2-7x+13) \geq 0$. Logarifm ta'rifidan $x^2-7x+13 > 0$ bo'lishi kelib chiqadi. U holda

$$\log_2(x^2-7x+13) \geq 0 \Leftrightarrow \begin{cases} x^2-7x+13 \geq 1 \\ x^2-7x+13 > 0 \end{cases} \Leftrightarrow x^2-7x+13 \geq 1 \Leftrightarrow$$

$$\Leftrightarrow x^2-7x+12 \geq 0.$$

Viyet formulalariga asosan $x^2-7x+12$ ko'phadning ildizlari 4 va 3 bo'lishiga ishonch hosil qilish qiyin emas. U holda hosil bo'lgan oxirgi tengsizlik $(x-3)(x-4) \geq 0$ ko'rinishga keladi. Bu tengsizlikni intervallar metodi bilan yechamiz (15- rasm):



15- rasm.

Javob: $(-\infty, 3] \cup [4, +\infty)$.

2- m i s o l. $y = \sqrt{\log_{\frac{1}{2}}(x^2-5x+6)}$ funksiyaning aniqlanish sohasini toping.

Ye ch i sh. 1- misolni yechishda keltirilgan mulohazalarni takrorlab, bu funksiyaning aniqlanish sohasi

$$\begin{cases} x^2-5x+6 > 0 \\ \log_{\frac{1}{2}}(x^2-5x+6) \geq 0 \end{cases} \text{ sistema yechimidan iborat bo'lishini}$$

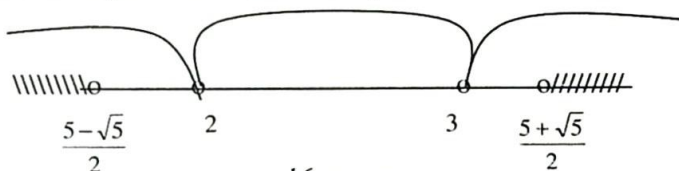
ko'rish qiyin emas.

Logarifm asosi birdan kichik bo'lganligi uchun sistemadagi ikkinchi tengsizlikni $x^2-5x+6 \leq 1$ tengsizlik bilan almashtirilsa

$$\begin{cases} x^2 - 5x + 6 > 0, \\ x^2 - 5x + 6 \leq 0 \end{cases} \text{ sistema kelib chiqadi. U holda}$$

$$\begin{cases} (x-2)(x-3) > 0, \\ x^2 - 5x + 5 \leq 0 \end{cases} \Leftrightarrow \begin{cases} (x-2)(x-3) > 0, \\ (x - \frac{5+\sqrt{5}}{2})(x - \frac{5-\sqrt{5}}{2}) \leq 0. \end{cases}$$

Hosil bo'lgan sistemadagi har bir tengsizlik intervallar metodi bilan yechiladi (16- rasm).



16- rasm.

Javob: $\left[\frac{5-\sqrt{5}}{2}, 2\right) \cup \left[3, \frac{5+\sqrt{5}}{2}\right]$.

3- m i s o l. $f(x) = \lg \frac{x^3 - x^2 - 4x + 4}{x^2 - x + 3} + 3\sqrt{x-3}$ funksiya-ning aniqlanish sohasini toping.

Y e c h i s h. Logarifmning ta'rifiga ko'ra $\frac{x^3 - x^2 - 4x + 4}{x^2 - x + 3} > 0$.

Kvadrat ildiz ostidagi ifoda manfiy bo'la olmasligidan $x-3 \geq 0$. U holda berilgan funksiyaning aniqlanish sohasi

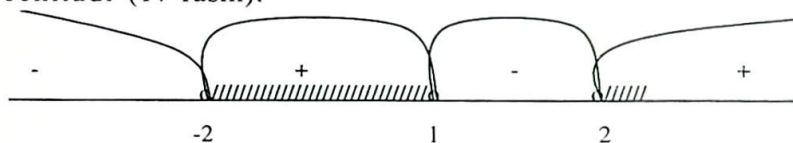
$$\begin{cases} \frac{x^3 - x^2 - 4x + 4}{x^2 - x + 3} > 0 \\ x - 3 \geq 0 \end{cases} \text{ sistema yechimidan iborat.}$$

Sistemadagi birinchi tengsizlikning mahraji x ning har qanday haqiqiy sondan iborat qiymatida musbat son bo'ladi.

Haqiqatan ham $x^2 - x + 3 = (x - \frac{1}{2})^2 + 2\frac{3}{4} > 0$. Shuning uchun

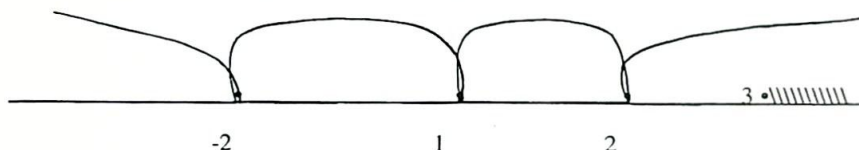
ham sistemaning birinchi tengsizligi $x^3 - x^2 - 4x + 4 > 0$ tengsizlikka teng kuchli. Tengsizlikning chap tomonini ko'paytuvchilarga ajratib $(x-1)(x-2)(x+2) > 0$ tengsiz-

lik hosil qilinadi. Bu tengsizlik intervallar metodi bilan yechiladi (17-rasm):



17- rasm.

$x - 3 \geq 0$ tengsizlik e'tiborga 'olinsa (18- rasm).



18- rasm.

Demak, berilgan funksiyaning aniqlanish sohasi $(3, +\infty)$ dan iborat.

4- m i s o l. $f(x) = \left(\log_{0,2} \frac{x^2 - 6x + 8}{x^2 - 3x + 2} \right)^{\frac{1}{2}}$ funksiyaning

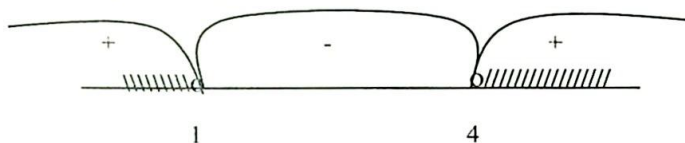
aniqlanish sohasini toping.

Ye ch i sh. Logarifmning ta'rifi va kvadrat ildiz ostidagi ifodaning manfiy bo'la olmasligidan

$$\log_{0,2} \frac{x^2 - 6x + 8}{x^2 - 3x + 2} \geq 0 \Leftrightarrow \begin{cases} \frac{x^2 - 6x + 8}{x^2 - 3x + 2} \leq 1 \\ \frac{x^2 - 6x + 8}{x^2 - 3x + 2} > 0 \end{cases} \Leftrightarrow \begin{cases} \frac{-3x + 6}{x^2 - 3x + 2} \leq 0 \\ \frac{x^2 - 6x + 8}{x^2 - 3x + 2} > 0 \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} \frac{x-2}{(x-1)(x-2)} \geq 0 \\ \frac{(x-2)(x-4)}{(x-1)(x-2)} > 0 \end{cases} \Leftrightarrow \begin{cases} \frac{1}{x-1} \geq 0 \\ \frac{x-4}{x-1} > 0 \\ x \neq 2 \end{cases} \text{ kelib chiqadi.}$$

Hosil bo'lgan sistemadagi ikkinchi tengsizlikni intervallar metodi bilan yechib (19- rasm)



19- rasm.

sistemadagi birinchi tengsizlik yechimi $x > 1$ bo'lishini e'tiborga olinsa, berilgan funktsiyaning aniqlanish sohasi $(4, +\infty)$ ekanligi kelib chiqadi.

5- m i s o l. $f(x) = \sqrt{5^{2x-2} + 25^x - 10}$ funktsiyaning aniqlanish sohasini toping.

Y e c h i s h. Kvadrat ildiz ostidagi ifoda manfiy bo'lolmasligidan $5^{2x-2} + 25^x - 10 \geq 0 \Leftrightarrow \frac{5^{2x}}{25} + 5^{2x} - 10 \geq 0$ kelib chiqadi. Bundan

$$5^{2x} + 25 \cdot 5^{2x} - 250 \geq 0 \Leftrightarrow 26 \cdot 5^{2x} \geq 250 \Leftrightarrow 5^{2x} \geq \frac{125}{13} \Leftrightarrow$$

$$\Leftrightarrow 2x \geq \log_5 125 - \log_5 13 \text{ yoki } x \geq \frac{3}{2} - \log_5 \sqrt{13}, \text{ u holda}$$

$$D(f) = \left[\frac{3}{2} - \log_5 \sqrt{13}; +\infty \right).$$

6- m i s o l. $y = \sqrt[4]{\log_{\frac{2}{3}} \frac{2x}{4x^2 - 1}}$ funktsiyaning aniqlanish sohasini toping.

Y e c h i s h. Juft darajali ildizostidagi ifoda manfiy bo'lolmagani uchun $\log_{\frac{2}{3}} \frac{2x}{4x^2 - 1} \geq 0$. Bu tengsizlik quyidagi

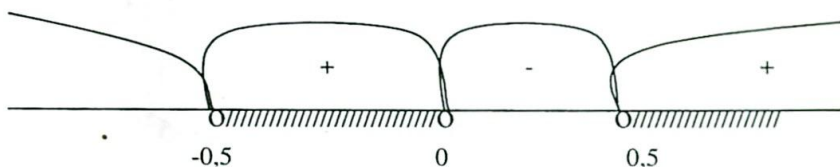
$$\text{sistemaga teng kuchli: } \begin{cases} \frac{2x}{4x^2 - 1} > 0 \\ \frac{2x}{4x^2 - 1} \leq 1 \end{cases}.$$

Hosil bo'lgan sistemani soddalashtiramiz:

$$\begin{cases} \frac{x}{(x-\frac{1}{2}) \cdot (x+\frac{1}{2})} > 0 \\ \frac{x^2 - \frac{1}{2}x - \frac{1}{4}}{(x-\frac{1}{2}) \cdot (x+\frac{1}{2})} \geq 0 \end{cases} \Leftrightarrow \begin{cases} \frac{x}{(x-0,5) \cdot (x+0,5)} > 0 \\ \frac{(x-\frac{\sqrt{5}+1}{4}) \cdot (x+\frac{\sqrt{5}-1}{4})}{(x-0,5) \cdot (x+0,5)} \geq 0 \end{cases}$$

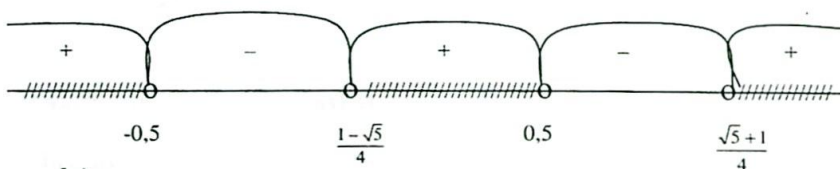
Hosil bo'lgan sistemadagi tengsizliklarning har biriga alohida intervallar metodini qo'llaymiz (20- rasm).

a) $\frac{x}{(x-0,5)(x+0,5)} > 0;$



a)

b) $\frac{(x-\frac{\sqrt{5}+1}{4}) \cdot (x-\frac{\sqrt{5}-1}{4})}{(x-0,5) \cdot (x+0,5)} \geq 0;$



b)

20- rasm.

Tengsizliklarning hosil bo'lgan yechimlari kesishmasini

olsak, $D(f) = \left[\frac{-1-\sqrt{5}}{4}, 0 \right) \cup \left[\frac{\sqrt{5}+1}{4}, +\infty \right).$

Mashqlar

Quyidagi funksiyalarning aniqlanish sohasini toping (1–6):

$$1. y = \log_5(0,64^{2-\log_{\sqrt{2}} x} - 1,25^{8-(\log_2 x)^2}).$$

$$2. y = \sqrt{\log_{\frac{1}{2}}(x-3) - 1}.$$

$$3. y = \sqrt[6]{4^{\frac{x+1}{x}} - 17 \cdot 2^{\frac{1}{x}} + 4}.$$

$$4. y = \log_3(1 - \log_{0,5}(x^2 - 2x - 2,5)).$$

$$5. y = \sqrt{\frac{-\log_{0,5}(x-1)}{\sqrt{-x^2 + 2x + 8}}}.$$

$$6. y = \sqrt{\log_{0,5}(x^2 - 8)}.$$

JAVOBLAR

I BOB

- 2- §. 1. $4/3$. 2. $7/2$. 3. $3/4$. 4. -6 . 5. -5 . 6. $-1/6$. 7. $1/2$. 8. -1 . 9. 1.
 10. 4. 11. $-3/4$. 12. $-9/28$. 13. $-1,2$. 14. $-3/32$. 15. $-2/3$. 16. $0,4$.
 17. $2/13$. 18. $9/28$. 19. 4. 20. 6. 21. 890. 22. 3. 23. 49. 24.
 $\log_7 8 + 1$. 25. 140.
- 4- §. 3. $\frac{2}{3} - \frac{a}{9}$. 4. $2 - 2a$. 5. $1 - \frac{2}{3}a$. 6. $\frac{a - 2b + 1}{2}$. 7. $\frac{a + b}{1 - b}$.
8. $\frac{1}{2}(a + b)$.

II BOB

- 1- §. 1. $\left\{ \frac{a-3}{2} \mid a \in [-3; \infty) \right\}$, $\emptyset$ agar $a \in (-\infty; -3)$. 2. $\{16\}$.
 3. $\{3\}$. 4. 9. 5. $\{6\}$. 6. $\{6; -3\}$. 7. $\emptyset$. 8. $\{-4, 5; 3\}$. 9. $\{-1\}$.
 10. $\{1\}$. 11. $\{12\}$. 12. $\{\frac{5}{3}\}$. 13. $\{-6\}$. 14. $a \neq 0, \{5a/3\}$;
 $a = 0, (-\infty; 0) \cup (0; \infty)$. 15. $\{3\}$. 16. $\{1\}$. 17. $\{2\}$. 18. $\{3\}$. 19. $\{5\}$.
 20. $\{-4/3\}$.
- 2- §. 1. -1 ; 5. 2. 5; 15. 3. 2; 1. 4. 2; 6. 5. 3. 6. 2.
 7. 81. 8. $-0,2$; 3. 9. 25. 10. $5/3$. 11. 1; 2. 12. $-2,5$; 3.
 13. $9/4$. 14. 4. 15. -3 ; 1. 16. $\{-2; 2\}$. 17. $\{25\}$. 18. $\{3\}$. 19. 3.
 20. -3 ; 1. 21. 3; $-1/5$. 22. 3; $-5/2$. 23. -7 ; 8. 24. 3.
- 3- §. 1. 7. 2. 3. 3. $\frac{2 \lg b - \lg a}{\lg a + \lg b} a \neq 0 \wedge b \neq 0; a > 0 \wedge b > 0, a \neq \frac{1}{b}$.
4. $4 \log_6 2$. 5. 1; $1/4$. 6. $-\sqrt{3}, \sqrt{3}$. 7. $-1/2$; $1/2$. 8. 4. 9. 2.
 10. 2. 11. 4. 12. $-0,2$; 3. 13. $5/3$. 14. 4. 15. 1, 3, 5.
 16. $-\sqrt{3}; \sqrt{3}$. 17. $x_1 = 1, x_2 = \frac{1}{4}$. 18. $x_1 = \frac{1}{3}, x_2 = 2, x_3 = 4$. 19. 0;
 $2 \log_3 5$. 20. $-1/2$. 21. 25. 22. 1, 3, 5. 23. -7 ; 8. 24. 1, 3, 5.
 25. $x_1 = 0, x_2 = 2 \log_3 5$.

4- §. 1. ± 2 . 2. 3. 3. 2. 4. 1. 5. 1,5. 6 -1; 5. 7. 1;
 $\log_2(3+\sqrt{29}) - 1$. 8. $\frac{1+\lg 74 - \lg 3}{4 \lg 3}$. 9. $4 \log_3 2$. 10. 1. 11. 1,5.

12. 2. 13. $\log_4 \frac{56}{9}$. 14. 4. 15. 1,5. 16. $\emptyset$. 17. 4. 18. $\emptyset$.

19. -0,5. 20. $\frac{2 \lg 2 - \lg 3}{\lg 2 - \lg 3}$. 21. 5/7. 22. 2. 23. 1,25. 24. -2.

25. 1,5.

5- §. 1. $\{0; 2\}$. 2. $\{-1; 1\}$. 3. 1; 3. 4. $\pi n/2, n \in \mathbb{Z}$. 5. 1.
 6. 3. 7. 0. 8. 3. 9. 0. 10. 3. 11. 0. 12. -1; 1. 13. 0. 14. 3.
 15. 2. 16. 13, (3). 17. $\{-1; 3\}$. 18. $\{-2; 2\}$. 19. 2, 5. 20. -2;
 2. 21. $\{0; 1; -1\}$. 22. 1. 23. 1; $1+\sqrt{2}$; $1-\sqrt{2}$. 24. 1.

25. $x_1 = -\frac{1}{3}, x_2 = 3$.

6- §. 1. -3; 1. 2. 0. 3. -1; 1; $\sqrt{2}$; $-\sqrt{2}$. 4. 0. 5. $\frac{\lg 3}{\lg 2 - \lg 3}$.

6. $\{0, \frac{\lg 1,5}{\lg 2,5}\}$. 7. $\frac{\lg \frac{4}{3}}{\frac{\sqrt{5}-1}{2}}$. 8. 5/2. 9. $x = 1/3$. 10. $x = 3 + 2\sqrt{2}$.

11. $x = 5$. 12. $x = \log_3 15$. 13. $x = 1$. 14. $x = 1$; 2. 15. $x = 1$.
 16. $x = 2$; 4. 17. $x = 2$. 18. $x = 1$. 19. $x = 2$. 20. $x = 2$.
 21. $x = -0,5$; 1,5. 22. $x = 0,4$. 23. $x = -2$; 2. 24. $x = 3$;
 $\log_6 8$. 25. $x = \pm 1$; $\pm \sqrt{2}$. 26. $x = 20$. 27. $x = 0,25$; 0,5.
 28. $x = -1$; 1. 29. $x = 1$; 3. 30. $x = 3$. 31. $x = 0$; 0,5.

32. $x = -0,5$; 0,5. 33. $x = \frac{\lg 1,5}{\lg 2 - \lg(1+\sqrt{5})}$. 34. $x = \frac{\lg 4}{\lg 15}; \frac{\lg 2}{\lg 1,5}$.

35. $x = 1$; $\lg \frac{3}{4}$. 36. $x = -1$; 1. 37. $x = 1$. 38. $x = 3$. 39. $x = 5$.

40. $x = 0$. 41. $x = 1$. 42. $x = 2$; 3; 4. 43. $x = 2$; 3. 44.
 $x = -1$; -3. 45. $x = -1$; 2; 3; 4. 46. $x = -1$; 1; 2. 47. -1; 1.
 48. 1. 49. 1/2. 50. 9. 51. -2. 52. 0; 1/2. 53. $x = -1$; 1.
 54. $x = 0$. 55. $x = \log_3 2$. 56. $x = 1$. 57. $x = -2$; 0. 58. $x = 1$.
 59. $x \in \emptyset$. 60. $x = -1$. 61. $x = 2$. 62. $x = 1,5$. 63. $x = -1$. 64.

$x = \log_{4,5} \frac{9}{2\sqrt{2}}$. 65. $x=0$. 66. $x=-0,5$. 67. $-\sqrt{3}; \sqrt{3}$. 68. $x = 1$.
 69. $x = 0$. 70. $x = 1$. 71. $x = -1/3$; 1. 72. $x = -2$. 73. $x = -1$; 1.
 74. $x = 10^{-2}$. 75. $x = \pm 2$. 76. $x = 1$; $17/12$; $11/6$. 77. $x = \pm 2$.
 78. $x = \pm 2$. 79. $x = 0$; $\log_{\sqrt{5\sqrt{2}-7}} 6$. 80. $x = \pm 2$. 81. $x = 0$; 1.
 82. $x = 1$. 83. $x = 1$. 84. $x = \pm 2$.

III BOB

1- §. 1. 1,5. 2. $-\sqrt{2}$. 3. 19. 4. 2. 5. 5. 6. 6. 7. 7. 8. 2.
 9. 0,3. 10. -16. 11. $2/5$. 12. 6. 13. 1. 14. $\pm \sqrt{2}$. 15. 8.
 16. 12,5. 17. 512. 18. $28/9$. 19. $x = 128$. 20. 2. 21. $x = 0,2$;
 $\sqrt{5}$. 22. $x = 0,5\sqrt{2}$; 8. 23. $x = 2$; 3. 24. $x = 5$; 7. 25. $x = -11$; 1.
 26. $x = 10^{-4}$; 10. 27. $x = 100$; 1000. 28. $x = 1$; 3. 29. 2.
 30. 625. 31. $x = 2$. 32. $x = 16$. 33. $x = 8$. 34. $x = 1,5$.
 35. $x = 3$. 36. $x = 37$. 37. $x = 2$. 38. $x = 64$. 39. $x = 36$;

$\sqrt{3}$. 40. $x = \frac{\sqrt[3]{9}}{9}; \frac{\sqrt{3}}{3}$. 41. 2; 4. 42. $\frac{4}{3}$.

2- §. 1. 100; 1000. 2. 4; 1. 3. $16/625$. 4. $e^2/4$. 5. $\emptyset$.
 6. -1. 7. 1; -1; $-\sqrt{2}; \sqrt{2}$. 8. $\sqrt{2} \pm 1$. 9. 1,5. 10. 10,5. 11. 2;
 $-3/5$. 12. 2; 6; 13. $81/16$. 14. $1/128$. 15. 2,25. 16. 0,01.
 17. 0,5. 18. $\sqrt{17} - 3$. 19. 4. 20. ± 3 . 21. 6; 14. 22. 6. 23. $1/2$.
 24. 5. 25. 2; 4. 26. 4. 27. 1. 28. $1/5$; $1/3$. 29. $\sqrt{26} - 4$.
 30. 18,5. 31. 6; 9. 32. 20. 33. 13. 34. 1; 19. 35. 2. 36. $1/3$;
 $5/7$. 37. 14; 30. 38. 2,5. 39. 1. 40. 4; 6,5. 41. 26. 42. 9;
 13. 43. 35,45. 44. 55; 65. 45. 30; $37\frac{1}{2}$. 46. 5. 47. 2. 48. $25\frac{\sqrt{3}}{4}$.
 49. 2. 50. 1. 51. 7. 52. 8. 53. 4. 54. 1. 55. 6. 56. 2,5.
 57. 5. 58. $\sqrt[3]{25}$. 59. 16. 60. 3. 61. 92. 62. 0; 1. 63. 56,5.
 64. 3. 65. 18. 66. 3,5. 67. $79\frac{10}{11}$. 68. 34. 69. 0,5; 2,5; 2.
 70. 1; 1,5. 71. 8. 72. $2/3$. 73. $5/3$. 74. 11,19. 75. 8. 76. 6,14.
 77. 10. 78. 1; 2. 79. $1/32$. 80. 3,5. 81. $\frac{1}{2}(\sqrt{26} - 4)$. 82. 12,18.
 83. 26. 84. 7,15. 85. 20. 86. 60,75. 87. 5. 88. 3.

IV BOB

- 1- §. 1. (9; 16). 2. (6; 2). 3. $\{(16; 3); (1/64; -2)\}$.
 4. $\{(27; 4); (1/81; -3)\}$. 5. (4; -0,5). 6. (-2; 7). 7. (11; 5).
 8. (2; 1/6). 9. (2; 1,5). 10. (0,5; -0,5). 11. $\{(8; 9);$
 $(27 \log_3^3 2; 4 \log_2^2 5)\}$. 12. (1; 4). 13. [4; 16). 14. $\left(\frac{\sqrt{5}-1}{2}; \frac{3-\sqrt{5}}{2}\right)$.
 15. (3; 3). 16. $\{(2; \sqrt{2}); (\sqrt{2}; 2)\}$. 17. $\{(2; 4); (4; 2)\}$. 18. (1000;
 10). 19. (1; 4). 20. (7; -2). 21. (2; 1). 22. (3; -3). 23. (16;
 20). 24. $\{(4; 2); (4; -2)\}$. 25. (1,5; 0,5).

- 2- §. 1. $x \in (8/3; +\infty)$. 2. $x \in (-\infty; 3) \cup (4; +\infty)$. 3. $x \in (-\infty;$
 1,5). 4. $x \in (-\infty; 1 - \log_3 \sqrt[3]{5}]$. 5. $x \in (-\infty; 0) \cup [0,5; +\infty)$.
 6. $x \in [-2; +\infty)$. 7. $x \in (0; 0,5)$. 8. $x \in [1; -1]$. 9. $x \in [-0,5;$
 0,5]. 10. $x \in [1; +\infty)$. 11. $x \in (-\infty; 1)$. 12. $x \in (-\infty; 0,5)$.
 13. $x \in (-2; -1] \cup [0,5; 0]$. 14. $x \in (-\infty; -0,5) \cup (1; +\infty)$.
 15. $x \in (5/7; 1,5)$. 16. $x \in (0; +\infty)$. 17. $x \in (1/3; 3)$. 18. $x \in (2; 5)$.
 19. $x \in (0; 0,5) \cup (\sqrt{2}; +\infty)$. 20. $x \in (1; 1/2\sqrt{2}) \cup (3; +\infty)$.
 21. $x \in (2^{-28}; 1)$. 22. $x \in (-\infty; -2) \cup [6; +\infty)$. 23. $x \in (0; 1/3) \cup$
 $\cup (243; +\infty)$. 24. $x \in (0,7; 1]$. 25. $x \in (0; 1/64) \cup (4; +\infty)$.
 26. $x \in (-3; 3,5) \cup [5; +\infty)$. 27. $x \in [-0,5; 0) \cup [0; 0,5]$.
 28. $x \in (-\infty; -1) \cup (3; +\infty)$. 29. $x \in [2; 4)$. 30. $x \in [-3;$
 $-2\sqrt{2}) \cup (2\sqrt{2}; 3]$.

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***Ko'rsatkichli, logarifmik tenglama
va tengsizliklar***

*Akademik litsey va kasb-hunar kollejlari
uchun o'quv qo'llanma*

- K98 **Ko'rsatkichli, logarifmik tenglama va teng-
sizliklar: Akademik litsey va kasb-hunar
kollejlari uchun o'quv qo'llanma / Mualliflar:
A.Yunusov D.Yunusova, O.Musurmonova va
boshqalar. — O'zbekiston Respublikasi Oliy
va o'rta maxsus ta'lim vazirligi; O'rta ta'lim,
kasb-hunar ta'limi markazi. —T.: C.,Ilm Ziya",
Toshkent, 2005. —96 b.**

Toshkent — „ILM ZIYO“ — 2005

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2005- yil 30- martda chop etishga ruxsat etildi. Bichimi $84 \times 108 \frac{1}{12}$.

Kegli 10,5 shponli. „Tayms“ harfida terilib, ofset bosma usulida chop etildi. Shartli b.t. 6,0. Nashr. b.t. 5,8. Adadi 2000 nusxa.

Bahosi shartnoma asosida. Buyurtma № 108

„ILM ZIYO“ MChJ. Toshkent — 700129. Navoiy ko'chasi, 30- uy.

Shartnoma № 33-05

O'zbekiston Respublikasi matbuot va axborot agentligining
G'.G'ulom nomidagi nashriyot-matbaa ijodiy uyi bosmaxonasida
chop etildi. Toshkent, 700128. Toshkent, U.Yusupov ko'chasi, 86.

