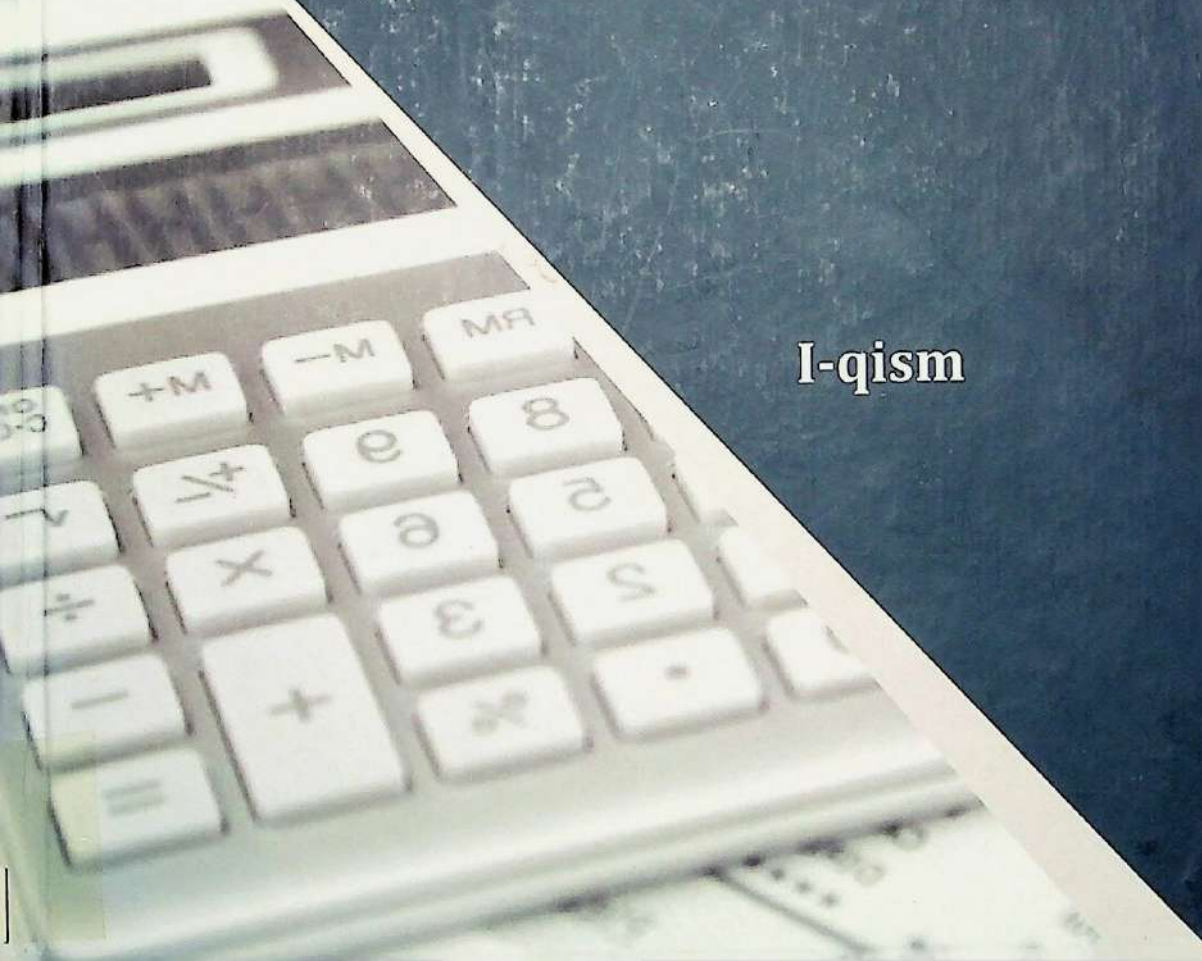


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OLIY MATEMATIKADAN MISOL VA MASALALAR TO'PLAMI

I-qism



UO'K 51.0(093)
KBBK 22.1

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Oliy matematikadan misol va masalalar to'plami. I-qism. O'quv qo'llanma.
S.A.Mingyasharova, J.H.Mamanov. – T.: "Demal" nashriyoti, 2024-y.
– 194 b.

Ushbu o'quv qo'llanma oliy ta'lim muassasasining texnika sohasidagi yengil sanoat, tabiiy tola va texnologik mashinalar va jihozlar bakalavriat ta'lim yo'nalishlari uchun mo'ljallangan. Qo'llanmada nazariy tushunchalar, ta'riflar, teoremlar, formulalar va ularning mohiyati, misol va masalalarning yechimi tushuntirilgan, mustahkamlash uchun mashqlar, nazorat testlari va ularning javoblari bilan keltirilgan.

O'quv qo'llanmadan oliy ta'lim muassasalarining texnika yo'nalishlarining kunduzgi, kechki va sirtqi bo'lim talabalari ham foydalanishlari mumkin.

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Ushbu o'quv qo'llanma Termiz muhandislik-texnologiya instituti o'quv-uslubiy Kengashining 2023-yil 3/4-1-sonli qarori va O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligining 2024-yil 4-martdagi 55-sonli buyrug'iga asosan o'quv qo'llanma sifatida nashrga tavsiya qilingan (№ 55-770).

ISBN 978-9910-7307-9-5

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SO‘Z BOSHI

O‘zbekiston Respublikasi Prezidenti Sh.M.Mirziyoyevning **“O‘zbekiston Respublikasi oliy ta‘lim tizimini 2030-yilgacha rivojlantirish konsepsiyasini tasdiqlash to‘g‘risida”**gi farmoniga ko‘ra O‘zbekiston Respublikasida oliy ta‘limni tizimli isloh qilishning ustuvor yo‘nalishlarini belgilash, zamonaviy bilim va yuksak ma‘naviy-axloqiy fazilatlariga ega, mustaqil fikrlaydigan yuqori malakali kadrlar tayyorlash jarayonini sifat jihatdan yangi bosqichga ko‘tarish eng ustuvor vazifa sifatida belgilangan. Mamlakatimiz mustaqillikka erishgach, ta‘lim tizimini tubdan isloh qilishga katta ahamiyat berildi. Ayniqsa, matematika fanining o‘rni, uning barcha fanlarni o‘zlashtirishga va ularning yaratilishiga asos ekanligini hisobga olib, uning rivojiga alohida e‘tibor qaratilmoqda. Yuqori malakali, raqobatbardosh, zamonaviy talablarga javob bera oladigan kadrlar tayyorlashda ularga chuqur matematik bilimlar berish va bu bilimlarni amaliy masalalarni yechishga tatbiq eta olishga o‘rgatish katta ahamiyatga ega. Shu sababli yengil sanoat va texnologik mashinalar va jihozlar yo‘nalishlari bo‘yicha ta‘lim oluvchi bakalavrlarning o‘quv rejalarida **“Oliy matematika”** fanini o‘qitish ko‘zda tutilgan.

“Oliy matematikadan misol va masalalar to‘plami” (I-qism) o‘quv qo‘llanmasi xorijiy adabiyotlardan foydalanilgan va o‘quv dasturida keltirilgan mavzular bilan to‘ldirilgan holda yozilgan. Oliy o‘quv yurtlarining yengil sanoat va texnologik mashinalar va jihozlar yo‘nalishlari bo‘yicha bakalavrlar tayyorlash uchun bu fanning tasdiqlangan namunaviy o‘quv dasturiga mos keladi.

O‘quv qo‘llanma 9 ta bob, 15 ta mavzudan iborat bo‘lib matematikaga doir 2500 dan ortiq mavzulashtirilgan amaliy mashqlar hamda uslubiy ko‘rsatmalar keltirilgan. Mavzular bo‘yicha 1000 dan ortiq testlar va ularning javoblari bilan berilgan.

Mualliflar ushbu o‘quv adabiyoti kamchiliklardan holi degan fikrdan uzoqda bo‘lganligi tufayli uni takomillashtirish bo‘yicha kitobxonlarning taklif va mulohazalarini minnatdorchilik bilan qabul qiladilar.

Mualliflar

I-BOB. CHIZIQLI ALGEBRA

1-§. MATRITSALAR. MATRITSALAR USTIDA AMALLAR. TESKARI MATRITSA

1.1. Matritsa haqida tushuncha

Matritsa tushunchasi birinchi marta ingliz matematigi U. Gamilton (1805-1865-y.y) va A. Kel (1821-1895-y.y) ishlarida uchraydi. Matritsa tushunchasi oliy matematikaning asosiy tushunchalaridan biri bo'lib, u nafaqat fan nuqtai nazaridan muhim ahamiyatga ega bo'libgina qolmay, balki hayotimizning turli sohalarida uchraydi.

Masalan:

Hafta davomida talabalar uchta fan bo'yicha necha soat davomida shug'ullanishini quyidagi jadval ko'rinishida belgilaymiz:

	Dush.	Sesh.	Chorsh.	Paysh.	Juma	Shanba	Yaksh.
Matem.	4	3	2	2	1	5	4
Fizika	2	1	1	4	6	2	1
Ingliz tili	1	4	3	5	2	1	1

Satrlar va ustunlar nomlanishlarini yo'qotsak 3 ta qator 7 ta ustunli matritsa hosil bo'ladi.

$$\begin{pmatrix} 4 & 3 & 2 & 2 & 1 & 5 & 4 \\ 2 & 1 & 1 & 4 & 6 & 2 & 1 \\ 1 & 4 & 3 & 5 & 2 & 1 & 1 \end{pmatrix}$$

Uumumlashgan quyidagi ta'rifni keltiramiz.

1-ta'rif. m ta satr va n ta ustundan iborat bo'lgan qavsga olingan to'rtburchakli sonli jadvalga *matritsa* deyiladi.

Matritsaning o'lchami undagi qatorlar (gorizontal chiziqlar) va ustunlar (vertikal chiziqlar) soni bilan aniqlanadi. Masalan, $\begin{pmatrix} 3 & 2 \\ -1 & 0 \\ 6 & 5 \end{pmatrix}$ 3 ta qator va 2 ta ustundan iborat. Uning o'lchami 3×2 .

Matritsalar lotin alifbosining bosh harflari bilan belgilanadi. Masalan, $m \times n$ o'lchamli matritsa esa quyidagicha yoziladi:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & & a_{2n} \\ \vdots & & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \quad (1.1)$$

Bu yerda: A -matritsa; a_{ij} -matritsa elementlari; i -berilgan element joylashgan satr raqami; j -unga mos ustun raqami; m - matritsada satrlar soni; n -undagi ustunlar soni. A matritsa i -satri, j -ustuni elementi uchun ham ij belgi ishlatiladi. Umumiy holda u $(A)_{ij}=a_{ij}$ ko'rinishida yoziladi.

Masalan, $\begin{pmatrix} 2 & 4 & 5 \end{pmatrix}$ 1×3 o'lchamli matritsa, $\begin{pmatrix} 1 \\ 0 \\ 5 \end{pmatrix}$ 3×1 o'lchamli matritsa, $\begin{pmatrix} a & b \\ c & d \\ v & g \end{pmatrix}$ ko'rinishidagi matritsa 3×2 o'lchamli va h.k.

2-ta'rif. Satrlar soni ustunlar soniga teng bo'lgan $n \times n$ o'lchamli matritsaga n -tartibli kvadrat matritsa deyiladi.

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{pmatrix} \quad (1.2)$$

Kvadrat matritsaning bosh dioganali deb, indekslari bir xil bo'lgan elementlarni o'z ichiga olgan $a_{11} \ a_{22} \dots a_{nn}$ yozuvga aytiladi.

Masalan, $\begin{pmatrix} 2 & 6 \\ 5 & 4 \end{pmatrix}$ 2×2 o'lchamli kvadrat matritsa, $\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \\ 0 & 1 & 7 \end{pmatrix}$ 3×3 o'lchamli kvadrat matritsa.

Agar matritsaning bosh dioganalida turgan elementlari birga, qolgan barcha elementlari nolga teng bo'lsa, bunday matritsa birlik matritsa deyiladi va E harfi bilan belgilanadi.

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (1.3)$$

O'lchamlari va undagi barcha mos elementlar o'zaro teng bo'lsa, bu matritsalar teng matritsalar deyiladi.

1.2. Matritsalar ustida amallar.

Ikkita matritsani bir-biriga qo'shish (ayirish) uchun ularning o'lchamlari o'zaro teng bo'lishi kerak.

3-ta'rif. Ikkita $m \times n$ o'lchamli matritsalarini bir-biriga qo'shish (ayirish) uchun ularning mos elementlarini bir-biriga qo'shish (ayirish) kifoya.

1-misol. $A = \begin{pmatrix} 3 & 2 \\ 1 & 0 \\ 4 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 0 & -1 \end{pmatrix}$ matritsalar berilgan. $A+B$ va $A-B$ ni toping.

Yechish. $A + B = \begin{pmatrix} 4 & 6 \\ 3 & 5 \\ 4 & 0 \end{pmatrix}$, $A - B = \begin{pmatrix} 2 & -2 \\ -1 & -5 \\ 4 & 2 \end{pmatrix}$

4-ta'rif. $m \times n$ o'lchamli matritsani k songa ko'paytirish uchun, k sonni matritsaning barcha elementlariga ko'paytirish kerak.

$$kA = \begin{pmatrix} ka_{11} & ka_{12} & ka_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ ka_{31} & ka_{23} & ka_{33} \end{pmatrix} \quad (1.4)$$

2-misol. $A = \begin{pmatrix} 2 & 4 & 5 \\ 1 & 3 & 2 \\ 0 & 6 & 5 \end{pmatrix}$ bo'lsa, $3A$ ni toping.

Yechish. $3A = \begin{pmatrix} 6 & 24 & 15 \\ 3 & 9 & 6 \\ 0 & 18 & 15 \end{pmatrix}$.

5-ta'rif. $m \times q$ o'lchamli matritsani $q \times n$ o'lchamli matritsaga ko'paytirganda shunday $m \times n$ o'lchamli c matritsa hosil bo'ladiki, bunda $m \times q$ o'lchamli matritsaning satrida turgan elementlarni $q \times n$ o'lchamli matritsaning ustunidagi mos elementlarga ko'paytirib qo'shish kifoya. Ya'ni

$$C_{ij} = \sum_{k=1}^q a_{ik} b_{kj}, i = 1, \dots, m, j = 1, \dots, n. \quad (1.5)$$

A va B matritsalar uchun komutativlik qonuni o'rinli emas, ya'ni $AB \neq BA$. Ikki xil o'lchamli A va B matritsalar uchun AB mavjud bo'lsa ham, o'lchamlari mos kelmaganligi tufayli BA ko'paytma umuman mavjud bo'lmasligi mumkin. Agar AB va BA mavjud bo'lgan taqdirda ham uning o'lchamlari turlicha bo'lishi mumkin.

A va B matritsalarining o'lchamlari bir xil bo'lib, AB va BA ko'paytmalar mavjud bo'lsa ham, ammo mos elementlari bir-biriga teng bo'lmaydi.

3-misol. $A = \begin{pmatrix} 2 & 3 & 5 \\ 1 & 0 & 3 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 2 \\ 4 & 5 \\ 6 & 2 \end{pmatrix}$ matritsalar berilgan.

AB va BA ni toping.

Yechish. Berilgan A matritsa 2×3 o'lchamli, B matritsa esa 3×2 o'lchamli bo'lganligidan A matritsani B matritsaga ko'paytirsak 2×2 o'lchamli yangi matritsa hosil bo'ladi.

$$\begin{aligned} A \cdot B &= \begin{pmatrix} 2 & 3 & 5 \\ 1 & 0 & 3 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ 4 & 5 \\ 6 & 2 \end{pmatrix} = \\ &= \begin{pmatrix} 2 \cdot 1 + 3 \cdot 4 + 5 \cdot 6 & 2 \cdot 2 + 3 \cdot 5 + 5 \cdot 2 \\ 1 \cdot 1 + 0 \cdot 4 + 3 \cdot 6 & 1 \cdot 2 + 0 \cdot 5 + 3 \cdot 2 \end{pmatrix} = \begin{pmatrix} 44 & 29 \\ 19 & 8 \end{pmatrix} \end{aligned}$$

B matritsa 3×2 o'lchamli, A matritsa esa 2×3 o'lchamli bo'lganligi uchun B matritsani A matritsaga ko'paytirsak 3×3 o'lchamli matritsa hosil bo'ladi.

$$\begin{aligned} B \cdot A &= \begin{pmatrix} 1 & 2 \\ 4 & 5 \\ 6 & 2 \end{pmatrix} \cdot \begin{pmatrix} 2 & 3 & 5 \\ 1 & 0 & 3 \end{pmatrix} = \\ &= \begin{pmatrix} 1 \cdot 2 + 2 \cdot 1 & 1 \cdot 3 + 2 \cdot 0 & 1 \cdot 5 + 2 \cdot 3 \\ 4 \cdot 2 + 5 \cdot 1 & 4 \cdot 3 + 5 \cdot 0 & 4 \cdot 5 + 5 \cdot 3 \\ 6 \cdot 2 + 2 \cdot 1 & 6 \cdot 3 + 2 \cdot 0 & 6 \cdot 5 + 2 \cdot 3 \end{pmatrix} = \\ &= \begin{pmatrix} 4 & 3 & 11 \\ 13 & 12 & 35 \\ 14 & 18 & 36 \end{pmatrix} \end{aligned}$$

A matritsani B matritsaga ko'paytirishda A matritsaning satrlar soni B matritsaning ustunlar soniga teng bo'lishi kerak. Agar bu shart bajarilmasa matritsalarining ko'paytmasi mavjud emas.

1.2. Teskari matritsa

6-ta'rif. Agar $AA^{-1} = A^{-1}A = E$ kabi bo'lsa, A^{-1} kvadrat matritsa, o'shanday tartibli A kvadrat matritsaga teskari matritsa deyiladi va A matritsa xosmas matritsa deyiladi. Berilgan matritsaga teskari matritsa mavjud bo'lmasa, A matritsa xos matritsa deyiladi.

Masalan. $A = \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix}$ matritsalar berilgan.

B matritsaning A ga teskari matritsa ekanini isbotlang.

Isbot. Ta'rifga binoan $AB = E$ bo'lishi kerak.

$$AB = A \cdot B = \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = E$$

$$BA = B \cdot A = \begin{pmatrix} 3 & 5 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = E$$

Kvadrat matritsaning teskarisini topish uchun avvalo berilgan matritsaning determinanti hisoblanadi. Determinanti noldan farqli bo'lsa uning teskarisi mavjud va ular xosmas matritsa deyiladi. Determinanti nolga teng bo'lsa, bu matritsa xos matritsa deyiladi. Determinanti mavjud bo'lgan matritsaning har bir elementi uchun algebraik to'ldiruvchilar topiladi, so'ngra ularni transponerlaymiz. Transponerlangan matritsaning elementlarini uning determinantiga bo'lib A matritsaga teskari matritsa hosil qilinadi.

4-misol. A^T bo'yicha A matritsaning teskari matritsasini toping.

$A \cdot A^{-1} = I$ tenglik orqali tekshiring.

$$A = \begin{pmatrix} 1 & 5 & 4 \\ 3 & 2 & 9 \\ -1 & 1 & 0 \end{pmatrix}$$

Yechish. Berilgan matritsaning determinanti

$$\det A = |A| = -34 \neq 0$$

0 ga teng emas, demak A ning teskari matritsasi mavjud

$$A^{-1} = \frac{A^T}{|A|} = \begin{pmatrix} \frac{9}{34} & -\frac{2}{17} & -\frac{37}{34} \\ \frac{9}{34} & -\frac{2}{17} & -\frac{3}{34} \\ -\frac{5}{32} & \frac{3}{17} & \frac{13}{34} \end{pmatrix}.$$

Mustaqil yechish uchun misollar

1.1-1.5 gacha. A va B matrisalarning yig'indisini va ayirmasini hisoblang.

$$1.1. A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}, B = \begin{pmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{pmatrix}.$$

$$1.2. A = \begin{pmatrix} 1 & 2 \\ 3 & 5 \\ 4 & 1 \end{pmatrix}, B = \begin{pmatrix} 2 & 5 \\ 3 & 4 \\ 1 & 6 \end{pmatrix}.$$

$$1.3. A = \begin{pmatrix} 2 & 1 \\ 4 & -6 \\ 7 & 3 \end{pmatrix}, B = \begin{pmatrix} -4 & 3 \\ 0 & 4 \\ 2 & -8 \end{pmatrix} \text{ bo'lsa, } C = 2A + B \text{ ni toping.}$$

$$1.4. A = \begin{pmatrix} -2 & 3 & 3 \\ 1 & 0 & -4 \\ -3 & 1 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & -3 & 0 \\ -1 & 2 & 2 \\ 6 & -1 & 5 \end{pmatrix}, C = 2A + B.$$

1.5. $A = \begin{pmatrix} 1 & -1 & 2 \\ 4 & 0 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 3 \\ 3 & 2 & 4 \end{pmatrix}$ bo'lsa, $3A - 2B$ ni hisoblang.

1.6-1.14 gacha. AB va BA matritsalarini ko'paytiring (agar ular mavjud bo'lsa).

1.6. $A = \begin{pmatrix} 1 & 2 \\ 2 & 5 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

1.7. $A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 1 & 1 \\ -4 & 2 & 0 \\ 1 & 2 & 1 \end{pmatrix}$.

1.8. $A = (3 \ 2 \ -1)$, $B = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

1.9. $A = \begin{pmatrix} 2 & 0 \\ 1 & -1 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 0 & 4 \\ 4 & 1 & -3 \end{pmatrix}$.

1.10. $A = \begin{pmatrix} 1 & 0 & 2 \\ 3 & -1 & 0 \\ 1 & 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 1 & 0 \\ 3 & 0 & 4 \\ 1 & -1 & 2 \end{pmatrix}$.

1.11. $A = \begin{pmatrix} 2 & 1 & 2 \\ 3 & 0 & 2 \\ 1 & 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 & 2 \\ 4 & 0 & 1 \\ -1 & -2 & 1 \end{pmatrix}$.

1.12. $A = \begin{pmatrix} 3 & 2 \\ -1 & 0 \\ 5 & 8 \\ 6 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 1 & 3 & 7 \\ 2 & 8 & 0 & 1 \end{pmatrix}$.

1.13. $A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 2 \end{pmatrix}$.

1.14. $A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 2 & 1 \\ 5 & 1 & 1 \end{pmatrix}$.

1.15-1.29 gacha. Hisoblang.

1.15. $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 4 \\ -2 & 3 \end{pmatrix}$ va $C = \begin{pmatrix} 2 & 5 \\ 3 & -1 \end{pmatrix}$ bo'lsa, $A^2 + 3B - C = ?$

1.16. $A = \begin{pmatrix} 1 & 2 \\ 5 & 6 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 6 \\ -7 & -1 \end{pmatrix}$ va $C = \begin{pmatrix} 3 & 5 \\ 6 & 1 \end{pmatrix}$ bo'lsa,

$$A^2 + AB - 2C = ?$$

1.17. $A = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $C = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ bo'lsa,

$A^2 + 2B - 5C$ ni hisoblang.

1.18. $A = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ bo'lsa, $A^4 - B^4$ ni hisoblang.

1.19. $A = \begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix}^3$ ni toping.

1.20. b) $A = \begin{pmatrix} -4 & 5 & 1 \\ 2 & -3 & 8 \end{pmatrix}$, $B = \begin{pmatrix} -7 & 4 \\ 10 & -5 \\ 3 & 16 \end{pmatrix}$, $C = B - 2A^T$.

1.21. $A = \begin{pmatrix} 4 & 1 & 3 \\ 0 & -1 & 2 \\ 2 & 1 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 & 3 \\ 2 & 1 & 0 \\ 1 & 1 & -1 \end{pmatrix}$, $C = (3A)^T - B$.

1.22. $A = \begin{pmatrix} 5 & -2 \\ 4 & 3 \\ 0 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -4 & -1 & 5 \\ 2 & 3 & -5 \end{pmatrix}$, $C = A + B^T$.

1.23. $A = \begin{pmatrix} 2 & 3 & 5 \\ -1 & 2 & -4 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -2 & -1 \\ 4 & 3 & 2 \end{pmatrix}$, $C = 2A + 3B$.

1.24. $A = \begin{pmatrix} 3 & 0 \\ -3 & 1 \\ 1 & 3 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 1 \\ -3 & 1 \\ 2 & 1 \end{pmatrix}$, $C = A^T - B^T$.

1.25. $A = \begin{pmatrix} 2 & -4 \\ 3 & 7 \\ 6 & -1 \end{pmatrix}$, $B = \begin{pmatrix} 4 & 0 & 1 \\ 6 & 1 & -3 \end{pmatrix}$, $C = 2A - B^T$.

1.26. $A = \begin{pmatrix} 3 & 1 & 0 \\ -2 & -1 & 4 \\ 7 & 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & -1 & 3 \\ 2 & -1 & 4 \\ 4 & 6 & 2 \end{pmatrix}$, $C = (2B)^T + A$.

1.27. $A = \begin{pmatrix} 3 & -4 & 2 \\ 7 & 5 & 1 \end{pmatrix}$, $B = \begin{pmatrix} -2 & 4 \\ 6 & 6 \\ -3 & 1 \end{pmatrix}$, $C = A - B^T$.

1.28. $A = \begin{pmatrix} 7 & -4 \\ 0 & -3 \\ 1 & 2 \end{pmatrix}$, $B = \begin{pmatrix} -1 & 4 \\ -1 & 2 \\ 6 & 3 \end{pmatrix}$, $C = 3B - 2A$.

1.29. $A = \begin{pmatrix} 6 & 1 & -3 \\ 2 & -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 7 & 3 & 1 \\ -5 & 1 & 4 \end{pmatrix}$, $C = A^T + B^T$.

1.30-1.49 gacha. A va B matritsalarini ko'paytmasini hisoblang.

$$1.30. a) A_{3 \times 2} = \begin{pmatrix} 2 & 0 \\ -1 & -2 \\ 3 & -5 \end{pmatrix}, B_{2 \times 2} = \begin{pmatrix} -2 & 3 \\ 4 & -1 \end{pmatrix}.$$

$$b) A = (1 \ -1 \ 2 \ 3), B = \begin{pmatrix} 1 & 2 \\ 3 & 6 \\ 5 & -1 \\ 1 & -3 \end{pmatrix}$$

$$1.31. A = \begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & 0 \\ 5 & 7 & 1 \\ 1 & 4 & 3 \end{pmatrix}, B = \begin{pmatrix} 5 & -1 & 2 & 1 \\ 2 & 2 & 0 & 1 \\ 1 & -1 & 5 & 4 \end{pmatrix}.$$

$$1.32. A = \begin{pmatrix} -2 & 3 & 0 & 1 \\ 1 & 1 & 2 & -1 \end{pmatrix}, B = \begin{pmatrix} 2 & 0 \\ 1 & -1 \\ -1 & 2 \\ 1 & 3 \end{pmatrix}.$$

$$1.33. A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ 1 & 3 \\ 1 & 2 \end{pmatrix}.$$

$$1.34. A = \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 3 & 2 & 1 \\ 5 & 1 & 1 \end{pmatrix}.$$

$$1.35. A = \begin{pmatrix} 2 & 0 \\ 1 & -1 \\ -1 & 2 \\ 1 & 3 \end{pmatrix}, B = \begin{pmatrix} -2 & 3 & 0 & 1 \\ 1 & 1 & 2 & -1 \end{pmatrix}.$$

$$1.36. A = \begin{pmatrix} 5 & -1 & 3 & 1 \\ 2 & 0 & -1 & 4 \end{pmatrix}, B = \begin{pmatrix} -1 & 3 & 0 \\ -2 & 1 & 1 \\ 3 & 0 & -2 \\ 4 & 1 & 2 \end{pmatrix}.$$

$$1.37. A = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, B = (3 \ 2 \ 1).$$

$$1.38. A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & -2 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & 5 & 3 \\ 6 & 8 & 2 \\ 1 & 2 & -1 \\ 3 & 0 & 1 \end{pmatrix}.$$

$$1.39. A = \begin{pmatrix} 3 & 2 & 1 \\ 5 & 1 & 1 \end{pmatrix}, B = \begin{pmatrix} 1 & 2 \\ 1 & 1 \\ 0 & 1 \end{pmatrix}.$$

$$1.40. A = \begin{pmatrix} 3 & 2 & 1 & 2 \\ 4 & 1 & 1 & 3 \end{pmatrix}, B = \begin{pmatrix} 3 & 4 \\ 2 & 1 \\ 1 & 1 \\ 2 & 3 \end{pmatrix}.$$

$$1.41. A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 & -1 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}.$$

$$1.42. A = (5 \quad 1 \quad 0 \quad -3), B = \begin{pmatrix} 2 & 0 \\ 1 & -4 \\ 3 & 1 \\ 0 & -1 \end{pmatrix}.$$

$$1.43. A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{pmatrix}. \quad 1.44.$$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ -2 & 3 & 1 \\ 4 & 2 & 4 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 2 & 3 & -1 & 3 \\ -2 & -1 & 5 & -2 \end{pmatrix}.$$

$$1.45. A = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & -2 \end{pmatrix}, B = \begin{pmatrix} 1 & 1 \\ -3 & 5 \\ 3 & -5 \\ -1 & 1 \end{pmatrix}. \quad 1.46. A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}, B = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}.$$

$$1.47. A = \begin{pmatrix} 3 & 1 \\ 2 & 1 \\ 1 & 0 \end{pmatrix}, B = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 0 & 1 \end{pmatrix}.$$

$$1.48. A = (1 \quad 2 \quad -1), B = \begin{pmatrix} 2 & 4 & -2 & 1 \\ 1 & 2 & 1 & 2 \\ 3 & 1 & -3 & 1 \end{pmatrix}.$$

$$1.49. A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}, B = \begin{pmatrix} -1 & -2 & -4 \\ -1 & -2 & -4 \\ 1 & 2 & 4 \end{pmatrix}.$$

Mavzu bo'yicha nazorat testlari

1.1. Matritsalar va ular ustida amallar.

- Matritsa mazmuni qayerda to'g'ri ko'rsatilgan?
A) sonlar yig'indisi B) sonlar ko'paytmasi. C) sonlar to'plami. D) sonlar jadvali. E) sonlar birlashmasi.
- $\begin{pmatrix} -1 & 0 & 5 \\ 2 & 7 & 3 \end{pmatrix}$ matritsaning tartibini aniqlang.
A) 2×2 B) 2×3 C) 3×2 D) 3×3 E) $3 \times 2 = 6$

3. $A = \begin{pmatrix} -2 & 5 & 0 \\ 4 & 1 & -3 \\ 6 & -1 & 7 \end{pmatrix}$ matritsa uchun bo'yicha $a_{13} + a_{21}$ yig'indini

toping.

- A) 7 B) 4 C) 6 D) -5 E) 5

4. Elementlari a_{ij} bo'lgan matritsa qachon nol matritsa deyiladi?

- A) Barcha a_{ij} elementlarning yig'indisi nolga teng bo'lsa.
 B) Barcha a_{ij} elementlari nolga teng bo'lsa.
 C) Barcha a_{ij} elementlarning ko'paytmasi nolga teng bo'lsa.
 D) Biror satridagi barcha a_{ij} elementlar nolga teng bo'lsa.
 E) Biror ustundagi barcha a_{ij} elementlar nolga teng bo'lsa.

5. Quyidagi matritsalarining qaysi biri nol matritsa bo'lmaydi?

- A) $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ B) $\begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$ C) $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ D) $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

E) Keltirilgan barcha matritsalar nol matritsa bo'ladi.

6. Elementlari a_{ij} bo'lgan kvadrat matritsa qachon birlik matritsa deyiladi?

- A) Barcha a_{ij} elementlar birga teng bo'lsa.
 B) $a_{ii} = 1$ va $a_{ij} = 0$ ($i \neq j$) bo'lsa.
 C) Barcha a_{ii} diagonal elementlar birga teng bo'lsa.
 D) Biror satridagi barcha a_{ij} elementlar birga teng bo'lsa.
 E) Biror ustundagi barcha a_{ij} elementlar birga teng bo'lsa.

7. Birlik matritsani ko'rsating.

- A) $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ C) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ D) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ E) $\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

8. Birlik matritsani ko'rsating.

- A) $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ C) $\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ D) $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

E) bu yerda birlik matritsa keltirilmagan.

9. Qaysi shartda A va B matritsalar teng deyiladi?

- A) A va B bir xil tartibli matritsalar bo'lsa.
 B) A va B bir xil tartibli kvadrat matritsalar bo'lsa.
 C) A va B bir xil tartibli kvadrat matritsalar va ularning mos elementlari o'zaro teng bo'lsa.

D) A va B matritsalarining diagonal elementlari o'zaro teng bo'lsa.

E) A va B matritsalarining tartiblari bir xil va mos elementlari teng bo'lsa.

10. $A = \begin{pmatrix} 1 & 3 & 7 \\ 5 & 2 & 9 \end{pmatrix}$ matritsa bo'yicha $2A$ matritsani toping.

- A) $\begin{pmatrix} 2 & 6 & 14 \\ 10 & 4 & 18 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 3 & 7 \\ 10 & 4 & 18 \end{pmatrix}$ C) $\begin{pmatrix} 2 & 3 & 7 \\ 10 & 2 & 9 \end{pmatrix}$ D) $\begin{pmatrix} 1 & 6 & 7 \\ 5 & 4 & 9 \end{pmatrix}$ E) $\begin{pmatrix} 2 & 3 & 14 \\ 10 & 2 & 18 \end{pmatrix}$.

11. Qanday matritsalar ni qo'shish va ayirish mumkin?

- A) har qanday matritsalar ni.
 B) satrlar soni bir xil bo'lgan matritsalar ni.
 C) ustunlar soni bir xil bo'lgan matritsalar ni.
 D) tartibi bir xil bo'lgan matritsalar ni.
 E) faqat kvadrat matritsalar ni.

12. $A = \begin{pmatrix} 3 & -2 \\ 5 & 0 \end{pmatrix}$ va $B = \begin{pmatrix} 1 & 4 & 0 \\ 0 & 3 & 2 \end{pmatrix}$ matritsalar ning yig'indisini toping.

- A) $\begin{pmatrix} 4 & 2 & 0 \\ 5 & 3 & 2 \end{pmatrix}$ B) $\begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}$ C) $\begin{pmatrix} 1 & 7 & -2 \\ 0 & 8 & 2 \end{pmatrix}$ D) $\begin{pmatrix} 7 & -2 \\ 8 & 2 \end{pmatrix}$ E) yig'indi aniqlanmagan.

13. Ushbu M va N matritsalar ning $M+N$ yig'indisini toping:

$$M = \begin{pmatrix} 0 & 1 \\ -1 & 2 \end{pmatrix}, N = \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}.$$

- A) $\begin{pmatrix} -1 & 0 \\ -2 & 4 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 0 \\ 2 & -4 \end{pmatrix}$ C) $\begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$ D) $\begin{pmatrix} -1 & 2 \\ 2 & 4 \end{pmatrix}$ E) $\begin{pmatrix} 3 & -2 \\ 0 & 2 \end{pmatrix}$.

14. Berilgan A va B matritsalar uchun $C=2A+3B$ matritsani toping.

$$A = \begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 4 & 2 \\ 2 & 0 & 0 \end{pmatrix}$$

- A) $C = (1.0.12)$ B) $C = (9.4.8)$ C) $C = (6.0.0)$

- D) C matritsa aniqlanmagan E) $C = \begin{pmatrix} 9 & 12 \\ 8 & 0 \end{pmatrix}$.

15. Quyidagi A va B matritsalar ning AB va BA ko'paytmalarini toping:

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0 \\ 2 & 1 \end{pmatrix}$$

- A) $AB = \begin{pmatrix} -1 & 0 \\ 4 & 3 \end{pmatrix}, BA = \begin{pmatrix} 5 & 2 \\ -2 & 0 \end{pmatrix}$ B) $AB = \begin{pmatrix} 4 & 3 \\ -1 & 0 \end{pmatrix}, BA = \begin{pmatrix} -2 & -3 \\ 5 & 6 \end{pmatrix}$

- C) $AB = \begin{pmatrix} -2 & 7 \\ -1 & 2 \end{pmatrix}, BA = \begin{pmatrix} -2 & -1 \\ 7 & 2 \end{pmatrix}$ D) $AB = \begin{pmatrix} 0 & 1 \\ -3 & 0 \end{pmatrix}, BA = \begin{pmatrix} 0 & -3 \\ 1 & 0 \end{pmatrix}$

E) to'g'ri javob keltirilmagan.

16. $A = \begin{pmatrix} -3 & 4 \\ 5 & -2 \end{pmatrix}$ matritsa bo'yicha A^2 matritsani toping.

A) $A^2 = \begin{pmatrix} 9 & 16 \\ 25 & 4 \end{pmatrix}$. B) $A^2 = \begin{pmatrix} -9 & 16 \\ 25 & -4 \end{pmatrix}$. C) $A^2 = \begin{pmatrix} 29 & -20 \\ -25 & 24 \end{pmatrix}$.

D) $A^2 = \begin{pmatrix} 25 & -23 \\ -23 & 29 \end{pmatrix}$. E) $A^2 = \begin{pmatrix} 34 & -22 \\ -22 & 20 \end{pmatrix}$.

1.2. Teskari matritsa.

1. A va unga teskari A^{-1} matritsalar ko'paytmasi uchun qaysi tasdiq o'rinli?

A) AA^{-1} faqat 0 lardan iborat matritsa bo'ladi.

B) AA^{-1} faqat 1 lardan iborat matritsa bo'ladi.

C) AA^{-1} diagonal elementlari 0, qolgan barcha elementlari 1 bo'lgan matritsa bo'ladi.

D) AA^{-1} diagonal elementlari 1, qolgan barcha elementlari 0 bo'lgan matritsa bo'ladi. E) AA^{-1} ixtiyoriy kvadrat matritsa bo'ladi.

2. Qaysi shartda A matritsa maxsus deyiladi?

A) $|A| < 0$ B) $|A| > 0$ C) $|A| \neq 0$ D) $|A| = 0$ E) $|A| \leq 0$.

3. Qaysi shartda A matritsa maxsusmas deyiladi?

A) $|A| < 0$ B) $|A| > 0$ C) $|A| \neq 0$ D) $|A| = 0$ E) $|A| \leq 0$.

4. Quyidagi matritsalaridan qaysi biri maxsusmas?

A) $\begin{pmatrix} 1 & -2 \\ 0 & 0 \end{pmatrix}$ B) $\begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix}$ C) $\begin{pmatrix} 1 & 0 \\ 5 & 0 \end{pmatrix}$ D) $\begin{pmatrix} 0 & 2 \\ 1 & 0 \end{pmatrix}$ E) $\begin{pmatrix} 0 & 7 \\ 0 & 3 \end{pmatrix}$.

5. Quyidagi matritsalaridan qaysi biri maxsus?

A) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ B) $\begin{pmatrix} 3 & 2 \\ 6 & 4 \end{pmatrix}$ C) $\begin{pmatrix} 1 & 2 \\ 5 & 0 \end{pmatrix}$ D) $\begin{pmatrix} 0 & 2 \\ 1 & 3 \end{pmatrix}$ E) $\begin{pmatrix} 0 & 7 \\ 3 & 0 \end{pmatrix}$.

6. A va unga teskari A^{-1} matritsalar uchun quyidagi tengliklardan qaysi biri o'rinli emas (E – birlik matritsa, O – nol matritsa)?

A) $A \cdot A^{-1} = A^{-1} \cdot A$ B) $A \cdot A^{-1} = E$ C) $A^{-1} \cdot A = E$ D) $A A^{-1} - A^{-1} A = O$
E) $A - A^{-1} = O$.

7. Agar A kvadrat matritsaning determinanti Δ bo'lsa, qaysi shartda A^{-1} teskari matritsa mavjud bo'lmaydi?

A) $\Delta = 0$ B) $\Delta < 0$ C) $\Delta > 0$ D) $\Delta \neq 0$ E) $\Delta = 1$.

8. Qanday matritsaga teskari matritsa mavjud?

A) har qanday matritsaga B) har qanday kvadrat matritsaga.

C) determinanti 0 ga teng bo'lgan matritsaga.

D) faqat determinanti musbat bo'lgan matritsaga.

E) determinanti 0 ga teng bo'lgan kvadrat matritsaga.

9. $A = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$ matritsaga teskari A^{-1} matritsani toping.

A) $\begin{pmatrix} 1/2 & 1/3 \\ 1/4 & 1/6 \end{pmatrix}$ B) $\begin{pmatrix} 4 & 6 \\ 2 & 3 \end{pmatrix}$ C) $\begin{pmatrix} 6 & 4 \\ 3 & 2 \end{pmatrix}$ D) $\begin{pmatrix} 6 & -4 \\ -3 & 2 \end{pmatrix}$ E) A^{-1} mavjud emas.

10. $A = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$ matritsaga teskari A^{-1} matritsa topilsin:

A) $\begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$ B) $\begin{pmatrix} 4 & 0.3 \\ 1 & -2 \end{pmatrix}$ C) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ D) $\begin{pmatrix} 2 & 3 \\ 3 & 0 \end{pmatrix}$ E) $\begin{pmatrix} 0 & 1 \\ 1/3 & -2/3 \end{pmatrix}$.

2-§. IKKINCHI VA UCHINCHI TARTIBLI DETERMINANTLAR VA ULARNING XOSSALARI

2.1. Ikkinchi va uchinchi tartibli determinanatlarni hisoblash usullari

Matematikaning qator masalalarini yechishda ma'lum xossalarga ega bo'lgan ifodalardan foydalaniladi. Bunday maxsus ifodalardan biri determinantlardir.

Aytaylik, 4 ta a, b, c, d haqiqiy sonlar berilgan bo'lsin. Ushbu $ad - bc$ ayirma (son)ni berilgan sonlarni yo'l va ustun ko'rinishida joylashtirib, quyidagicha

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

ifodalaymiz. Demak,

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc. \quad (2.1)$$

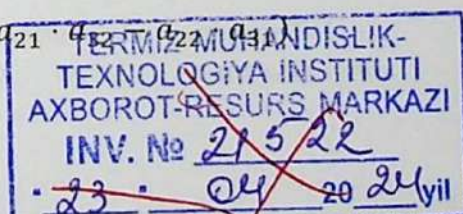
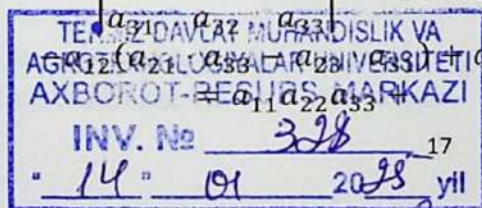
(2.1) ifoda 2-tartibli determinant deyiladi. Bunda a, b, c, d -determinantning elementlari, a, b va c, d sonlar mos ravishda determinantning birinchi va ikkinchi yo'llari, a, c va b, d sonlar determinantning mos ravishda birinchi va ikkinchi ustunlari, a, d sonlar determinantning bosh diagonali, b, c sonlar determinantning yordamchi diagonali deyiladi. Ikkinchi tartibli determinant umumiy holda $\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$ ko'rinishda yoziladi. Determinantni Δ yoki $\det A$ ko'rinishida belgilanadi.

1-misol. $\det A = \begin{vmatrix} 2 & 4 \\ 6 & 8 \end{vmatrix} = 8 \cdot 2 - 4 \cdot 6 = 16 - 24 = -8$

Uchinchi tartibli matritsaning determinanti quyidagi shaklda yoziladi

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad (2.2)$$

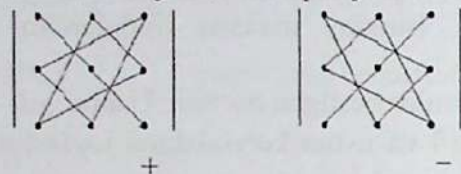
$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}(a_{22} \cdot a_{33} - a_{23} \cdot a_{32}) -$$



$$+a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} - \\ -a_{13}a_{22}a_{31} \quad (2.3)$$

Demak, uchinchi tartibli determinant 6 ta had yig'indisidan iborat bo'lib, ularning uchta musbat ishorali, uchta manfiy ishorali bo'ladi.

Musbat va manfiy ishorali hadlarni yozishda quyidagi tasvirlangan sxemalardan foydalanish qulay bo'ladi



2-misol. Quyidagi determinantni (2.3) formuladan foydalanib hisoblang

$$\begin{vmatrix} 1 & 3 & 4 \\ 2 & 6 & 5 \\ 0 & 1 & 2 \end{vmatrix} = 1 \cdot 6 \cdot 2 + 3 \cdot 5 \cdot 0 + 2 \cdot 1 \cdot 4 -$$

$$-(4 \cdot 6 \cdot 0 + 3 \cdot 2 \cdot 2 + 5 \cdot 1 \cdot 1) = 12 + 8 - 12 - 5 = 3.$$

Determinantning a_{ij} elementining M_{ij} minori deb, shu element turgan satr va ustunni o'chirishdan hosil bo'lgan determinantga aytiladi. Aytaylik a_{23} elementning M_{23} minorini topaylik.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \quad M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix}.$$

Ushbu

$(-1)^{i+k} \cdot M_{ik}$ miqdor (2.2) determinant a_{ik} elementining algebraik to'ldiruvchisi deyiladi. U A_{ik} orqali belgilanadi. Demak,

$$A_{ik} = (-1)^{i+k} \cdot M_{ik}. \quad (2.4)$$

3-misol. Ushbu

$$\begin{vmatrix} 2 & 0 & 3 \\ 1 & 2 & 0 \\ 3 & 0 & 1 \end{vmatrix}$$

determinantning $a_{13} = 3$ elementining algebraik to'ldiruvchisi

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} = 1 \cdot (1 \cdot 0 - 2 \cdot 3) = -6 \text{ bo'ladi.}$$

1-Teorema. Determinantning biror yo'lida joylashgan barcha elementlarning ularga mos algebraik to'ldiruvchilari bilan

ko'paytmasidan tashkil topgan yig'indi shu determinantning qiymatiga teng bo'ladi.

◀ Bu teoremani

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

determinantning birinchi yo'lida joylashgan a_{11}, a_{12}, a_{13} elementlaridan foydalanib isbotlaymiz.

Ravshanki, bu a_{11}, a_{12}, a_{13} elementlarning algebraik to'ldiruvchilari

$$A_{11} = (-1)^{1+1} \cdot M_{11} = 1 \cdot \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix},$$

$$A_{12} = (-1)^{1+2} \cdot M_{12} = - \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix},$$

$$A_{13} = (-1)^{1+3} \cdot M_{13} = 1 \cdot \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix},$$

bo'ladi. Unda

$$a_{11} \cdot A_{11} + a_{12} A_{12} + a_{13} A_{13} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

bo'lib, bu tenglikning o'ng tomonidagi ifoda (2.3) ga ko'ra uchinchi tartibli determinant teng ekanini topamiz. Demak,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11} \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12} \begin{vmatrix} a_{21} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} + a_{13} \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}. \blacktriangleright$$

2.2. Determinantning xossalari

1-xossa. Determinantning biror satri (yoki ustuni) nollardan iborat bo'lsa, bu determinantning qiymati nolga teng.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ 0 & 0 & 0 \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

2-xossa. Determinantning qiymatini biror songa ko'paytirish uning biror satri (ustuni) elementlarini shu songa ko'paytirishga teng kuchli.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = k \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

3-xossa. Matritsani transponirlash natijasida uning determinantning qiymati o'zgarmaydi, ya'ni berilgan A matritsa uchun $|A| = |A^T|$.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

4-xossa. Determinantda ikkita satr yoki ikkita ustunning o'rnini almashtirilsa, determinantning qiymati o'z ishorasini almashtiradi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = - \begin{vmatrix} a_2 & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

5-xossa. Ikkita bir xil satrga (ustunga) ega bo'lgan determinant nolga teng.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

6-xossa. Determinantning ikkita satri (ustuni) o'zaro proporsional bo'lsa, bu determinantning qiymati nolga teng.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ ka_{11} & ka_{12} & ka_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

7-xossa. Agar determinantning biror satri (yoki ustuni) to'laligicha ikki qo'shiluvchining yig'indisidan iborat bo'lsa, uning qiymatini ikkita determinantlar qiymatlari yig'indisi tarzida hisoblash mumkin.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} + b_1 & a_{22} + b_2 & a_{23} + b_3 \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ b_1 & b_2 & b_3 \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

8-xossa. Agar determinantning biror satri elementlarini bir xil songa ko'paytirib boshqa biror satriga qo'shilsa uning qiymati o'zgarmaydi.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ ka_{11} + a_{21} & ka_{12} + a_{22} & ka_{13} + a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

9-xossa. Matritsa biror satri elementlarini boshqa biror satr elementlarining algebraik to'ldiruvchilariga ko'paytmalarining yig'indisi nolga teng.

$$a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33} = 0; \quad a_{21}A_{31} + a_{22}A_{32} + a_{23}A_{33} = 0$$

10-xossa. Kvadrat matritsalar ko'paytmasining determinanti har bir matritsa determinantlari ko'paytmasiga teng, ya'ni $|A \cdot B| = |A| \cdot |B|$

Mustaqil bajarish uchun misollar

2.1-2.16 gacha. Ikkinchi tartibli determinantlarni hisoblang.

$$\begin{aligned}
 &2.1. \begin{vmatrix} 4 & 2 \\ 5 & 8 \end{vmatrix}, \quad 2.2. \begin{vmatrix} 2 & -2 \\ 6 & 1 \end{vmatrix}, \quad 2.3. \begin{vmatrix} 45 & 21 \\ 52 & 18 \end{vmatrix}, \quad 2.4. \begin{vmatrix} 1 & 2 \\ 5 & 8 \end{vmatrix}, \\
 &2.5. \begin{vmatrix} -4 & 2 \\ 5 & -8 \end{vmatrix}, \quad 2.6. \begin{vmatrix} -3 & -5 \\ 4 & 5 \end{vmatrix}, \quad 2.7. \begin{vmatrix} 0, (3) & -0, (42) \\ 99 & 3 \end{vmatrix}, \quad 2.8. \begin{vmatrix} 3 & -6 \\ -5 & 7 \end{vmatrix}, \\
 &2.9. \begin{vmatrix} \sqrt{2} & -3 \\ \frac{1}{3} & \sqrt{2} \end{vmatrix}, \quad 2.10. \begin{vmatrix} \sqrt{a} & -1 \\ a & 1 + \sqrt{2} \end{vmatrix}, \quad 2.11. \begin{vmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{vmatrix}, \quad 2.12. \begin{vmatrix} a^{\frac{3}{4}} & a^{\frac{1}{2}} \\ -a^{\frac{1}{2}} & a^{\frac{1}{4}} \end{vmatrix}, \\
 &2.13. \begin{vmatrix} 2\sin^2 \alpha & \cos^2 \alpha \\ 2\sin^2 \beta & \cos^2 \beta \end{vmatrix}, \quad 2.14. \begin{vmatrix} \sqrt[4]{5} & \sqrt{2} \\ \sqrt{2} & \sqrt[4]{125} \end{vmatrix}, \quad 2.15. \begin{vmatrix} \cos \frac{x}{2} & \sin \frac{x}{2} \\ \sin \frac{x}{2} & \cos \frac{x}{2} \end{vmatrix}, \quad 2.16. \begin{vmatrix} \sqrt{a} & -1 \\ a & \sqrt{a} \end{vmatrix}.
 \end{aligned}$$

2.17-2.46 gacha. Uchinchi tartibli determinantlarni uchburchak usulidan foydalanib hisoblang.

$$\begin{aligned}
 &2.17. a) \begin{vmatrix} 3 & -1 & -1 \\ -3 & 1 & 5 \\ 2 & -2 & 4 \end{vmatrix} \quad b) \begin{vmatrix} 2 & 0 & 4 \\ 5 & -2 & 1 \\ -1 & -2 & 3 \end{vmatrix}, \quad 2.18. \begin{vmatrix} 1 & 2 & -1 \\ 3 & 7 & 2 \\ 2 & 3 & -7 \end{vmatrix}, \quad 2.19. \begin{vmatrix} 3 & 2 & -1 \\ 1 & 2 & 9 \\ 1 & 1 & 2 \end{vmatrix}, \\
 &2.20. \begin{vmatrix} 5 & 3 & 2 \\ -1 & 2 & 4 \\ 7 & 3 & 6 \end{vmatrix}, \quad 2.21. \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 4 & 9 & 16 \end{vmatrix}, \quad 2.22. \begin{vmatrix} 1 & 2 & 3 \\ 3 & -4 & 7 \\ -3 & 12 & -15 \end{vmatrix}, \quad 2.23. \begin{vmatrix} 1 & -3 & -1 \\ 0 & -3 & 2 \\ 0 & 0 & -3 \end{vmatrix}, \\
 &2.24. \begin{vmatrix} 2 & -1 & -3 \\ 1 & 3 & 2 \\ 3 & 2 & 2 \end{vmatrix}, \quad 2.25. \begin{vmatrix} 1 & 2 & 0 \\ 5 & 1 & 2 \\ 0 & 3 & 1 \end{vmatrix}, \quad 2.26. \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & 1 \\ 1 & 1 & 2 \end{vmatrix}, \quad 2.27. \begin{vmatrix} 2 & 1 & 3 \\ 5 & 3 & 2 \\ 1 & 4 & 3 \end{vmatrix}, \\
 &2.28. \begin{vmatrix} 4 & 2 & 1 \\ 5 & 3 & -2 \\ 3 & 2 & -1 \end{vmatrix}, \quad 2.29. \begin{vmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix}, \quad 2.30. \begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}, \quad 2.31. \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 0 \end{vmatrix}, \\
 &2.32. \begin{vmatrix} 4 & -3 & 5 \\ 3 & -2 & 8 \\ 1 & -7 & -5 \end{vmatrix}, \quad 2.33. \begin{vmatrix} 3 & 2 & -4 \\ 4 & 1 & -2 \\ 5 & 2 & -3 \end{vmatrix}, \quad 2.34. \begin{vmatrix} 5 & 6 & 3 \\ 0 & 1 & 0 \\ 7 & 4 & 5 \end{vmatrix}, \quad 2.35. \begin{vmatrix} 3 & 4 & -5 \\ 8 & 7 & -2 \\ 2 & -1 & 8 \end{vmatrix}
 \end{aligned}$$

$$2.36. \begin{vmatrix} 1 & 5 & 25 \\ 1 & 7 & 49 \\ 1 & 8 & 64 \end{vmatrix}, \quad 2.37. \begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix}, \quad 2.38. \begin{vmatrix} 0 & a & 0 \\ b & c & d \\ 0 & e & 0 \end{vmatrix}.$$

$$2.39. \begin{vmatrix} \cos \alpha & \sin \alpha \cos \beta & \sin \alpha \sin \beta \\ -\sin \alpha & \cos \alpha \cos \beta & \cos \alpha \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{vmatrix}, \quad 2.40. \begin{vmatrix} \sin \alpha & \cos \alpha & 1 \\ \sin \beta & \cos \beta & 1 \\ \sin \gamma & \cos \gamma & 1 \end{vmatrix}.$$

$$2.41. \begin{vmatrix} 2 & 0 & 3 \\ 7 & 1 & 6 \\ 6 & 0 & 5 \end{vmatrix}, \quad 2.43. \begin{vmatrix} 4 & 1 & -3 \\ 8 & 3 & -6 \\ 1 & 1 & -1 \end{vmatrix}, \quad 2.44. \begin{vmatrix} 2 & -5 & 0 \\ 3 & 5 & -1 \\ 1 & -4 & 2 \end{vmatrix}.$$

$$2.45. \begin{vmatrix} 1 & 1 & 4 \\ 2 & 1 & 6 \\ 3 & 3 & 13 \end{vmatrix}.$$

2.47-2.61 gacha. Tenglamani yeching.

$$2.47. \begin{vmatrix} 3x+2 & 4 \\ x+4 & 3 \end{vmatrix} = 0, \quad 2.48. \begin{vmatrix} 2x+1 & x-1 \\ 8+2x & x+1 \end{vmatrix} = 0.$$

$$2.49. \begin{vmatrix} x+3 & x+2 \\ -x-2 & x-3 \end{vmatrix} = 0, \quad 2.50. \begin{vmatrix} \sin 3x & \sin 2x \\ \cos 2x & \cos 3x \end{vmatrix} = 0.$$

$$2.51. \begin{vmatrix} 2x+2 & 5 \\ x+5 & 3 \end{vmatrix} = 0, \quad 2.51. \begin{vmatrix} 3x+2 & x-4 \\ x+4 & 3x-2 \end{vmatrix} = 0.$$

$$2.52. \begin{vmatrix} x^2-4 & -1 \\ x-2 & x+2 \end{vmatrix} = 0, \quad 2.53. \begin{vmatrix} 4 \sin x & 1 \\ 1 & \cos x \end{vmatrix} = 0, \quad 2.54. \begin{vmatrix} 2 & -1 & 2 \\ 3 & 5 & 3 \\ 1 & 6 & x+5 \end{vmatrix} = 0$$

$$2.55. \begin{vmatrix} 1 & 2 & 3 \\ 1 & 3-x & 3 \\ 1 & 2 & 5+x \end{vmatrix} = 0$$

$$2.56. \begin{vmatrix} \cos^2 \varphi - \sin^2 \varphi & 2 \cos \varphi \sin \varphi \\ 2 \sin \varphi \cos \varphi & \cos^2 \varphi - \sin^2 \varphi \end{vmatrix} = 0$$

$$2.57. \begin{vmatrix} 3-x & 2 \\ 2 & 4-x \end{vmatrix} = 0$$

$$2.58. \begin{vmatrix} 5-x & 2 \\ 2 & 8-x \end{vmatrix} = 0$$

$$2.59. \begin{vmatrix} x^2 & 4 & 9 \\ x & 2 & 3 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

$$2.60. \begin{vmatrix} x^2 & 3 & 2 \\ x & -1 & 1 \\ 0 & 1 & 4 \end{vmatrix} = 0$$

$$2.61. \begin{vmatrix} 3-x & -1 & 1 \\ -1 & 5-x & -1 \\ 1 & -1 & 3-x \end{vmatrix} = 0.$$

2.62-2.103 gacha. Quyidagi determinantlarning barcha minorlarini yozing.

$$2.62. \begin{vmatrix} -2 & 1 & 0 \\ 4 & 3 & 7 \\ 4 & 2 & 5 \end{vmatrix}$$

$$2.63. \begin{vmatrix} 3 & 6 & 5 \\ -1 & 5 & 4 \\ 0 & 6 & 1 \end{vmatrix}$$

$$2.64. \begin{vmatrix} 2 & -2 & 1 & 0 \\ 0 & 2 & 2 & -1 \\ 3 & -2 & 2 & 3 \\ 3 & 2 & 6 & 1 \end{vmatrix}$$

$$2.65. \begin{vmatrix} 2 & 3 & -3 & -4 \\ 2 & 1 & -1 & 2 \\ 6 & 2 & 1 & 0 \\ 2 & 3 & 0 & -5 \end{vmatrix}$$

$$2.66. \begin{vmatrix} 3 & -1 & 4 & 2 \\ 5 & 2 & 0 & 1 \\ 0 & 2 & 1 & -3 \\ 6 & -2 & 9 & 8 \end{vmatrix}$$

$$2.67. \begin{vmatrix} 2 & -1 & 3 & 0 \\ -5 & 3 & 1 & -2 \\ -3 & -2 & 0 & 4 \\ 1 & 5 & -4 & 2 \end{vmatrix}$$

Determinantni satr bo'yicha yoyish usulidan foydalanib yeching.

Namuna. $\begin{vmatrix} 1 & 2 & 3 \\ 4 & 3 & 1 \\ 6 & 2 & 3 \end{vmatrix} = 1 \cdot (-1)^{1+1} \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} + 2 \cdot (-1)^{1+2} \begin{vmatrix} 4 & 1 \\ 6 & 3 \end{vmatrix} +$
 $+ 5 \cdot (-1)^{1+3} \begin{vmatrix} 4 & 3 \\ 6 & 2 \end{vmatrix} = 7 - 12 + 50 = 45.$

$$2.68. \begin{vmatrix} 3 & 2 & -1 \\ 4 & 1 & 0 \\ 2 & 8 & 4 \end{vmatrix}$$

$$2.69. \begin{vmatrix} 5 & 11 & 9 \\ -2 & 3 & -4 \\ 9 & 12 & 5 \end{vmatrix}$$

$$2.70. \begin{vmatrix} 1 & 3 & 5 \\ 4 & 0 & 5 \\ 9 & 6 & 7 \end{vmatrix}$$

$$2.71. \begin{vmatrix} 5 & -1 & 3 \\ 2 & 7 & 6 \\ 11 & 8 & 5 \end{vmatrix}$$

$$2.72. \begin{vmatrix} 11 & 10 & -2 \\ -5 & 3 & 6 \\ 2 & -1 & 4 \end{vmatrix}$$

$$2.73. \begin{vmatrix} 6 & 5 & 4 \\ 0 & 7 & 9 \\ 3 & 2 & 1 \end{vmatrix}$$

$$2.74. \begin{vmatrix} 2 & -1 & 3 & 0 \\ -5 & 3 & 1 & -2 \\ -3 & -2 & 0 & 4 \\ 1 & 5 & -4 & 2 \end{vmatrix}$$

$$2.75. \begin{vmatrix} 4 & -3 & 2 & -4 \\ 0 & 5 & 3 & 2 \\ -3 & 2 & 6 & 1 \\ 13 & 3 & -2 & 0 \end{vmatrix}$$

$$2.76. \begin{vmatrix} 3 & -1 & 4 & 2 \\ 5 & 2 & 0 & 1 \\ 0 & 2 & 1 & -3 \\ 6 & -2 & 9 & 8 \end{vmatrix}$$

$$2.77. \begin{vmatrix} 3 & -3 & -5 & 8 \\ -3 & 2 & 4 & -6 \\ 2 & -5 & -7 & 5 \\ -4 & 3 & 5 & -6 \end{vmatrix}$$

$$2.78. \begin{vmatrix} 3 & 2 & 2 & 2 \\ 9 & -8 & 5 & 10 \\ 5 & -8 & 5 & 8 \\ 6 & -5 & 4 & 7 \end{vmatrix}$$

$$2.79. \begin{vmatrix} 1 & 2 & -1 & 1 \\ 2 & -1 & 0 & 1 \\ -1 & 1 & -2 & 2 \\ 1 & 4 & 1 & 1 \end{vmatrix}$$

$$2.80. \begin{vmatrix} 1 & -1 & 2 & 1 \\ 3 & 4 & -1 & 0 \\ 2 & 1 & 3 & 1 \\ -5 & 0 & 1 & -2 \end{vmatrix}$$

$$2.81. a) \begin{vmatrix} 1 & 2 & -1 & 2 \\ 2 & -1 & 2 & 3 \\ -1 & -3 & -2 & 4 \\ 3 & -4 & -3 & 6 \end{vmatrix}, b) \begin{vmatrix} -1 & 6 & 5 & 1 \\ -2 & 8 & 6 & 2 \\ 2 & 16 & 7 & 3 \\ -3 & 9 & 3 & 4 \end{vmatrix}$$

$$2.82. a) \begin{vmatrix} 3 & 0 & 2 & 0 \\ 2 & 3 & -1 & 4 \\ 0 & 4 & -2 & 3 \\ 5 & 2 & 0 & 1 \end{vmatrix}, e) \begin{vmatrix} 5 & 62 & -79 & 4 \\ 0 & 2 & 3 & 0 \\ 6 & 183 & 201 & 5 \\ 0 & 3 & 4 & 0 \end{vmatrix}$$

$$2.83. \begin{vmatrix} -5 & 6 & 10 & 6 \\ -9 & 8 & 8 & 5 \\ -8 & 5 & 9 & 5 \\ -11 & 7 & 7 & 4 \end{vmatrix}$$

$$2.84. \begin{vmatrix} 9 & 7 & 9 & 7 \\ 8 & 6 & 8 & 6 \\ -9 & -7 & 9 & 7 \\ -8 & -6 & 8 & 6 \end{vmatrix}, 2.84. \begin{vmatrix} 6 & 8 & -9 & -12 \\ 4 & 6 & -6 & -9 \\ -3 & -4 & 6 & 8 \\ -2 & -3 & 4 & 6 \end{vmatrix}$$

$$2.85. \begin{vmatrix} 6 & -5 & 1 & 2 \\ -4 & 7 & -1 & 4 \\ 5 & -9 & 2 & 7 \\ 4 & -6 & 1 & 2 \end{vmatrix},$$

$$2.86. \begin{vmatrix} -3 & 9 & 3 & 6 \\ -5 & 8 & 2 & 7 \\ 4 & -5 & -3 & -2 \\ 7 & -8 & -4 & -5 \end{vmatrix}, 2.87. \begin{vmatrix} 2 & -5 & 4 & 3 \\ 3 & -4 & 7 & 5 \\ 4 & -9 & 8 & 5 \\ -3 & 2 & -5 & 3 \end{vmatrix}$$

$$2.88. \begin{vmatrix} 3 & -5 & 2 & -4 \\ -3 & 4 & -5 & 3 \\ -5 & 7 & -7 & 5 \\ 8 & -8 & 5 & -6 \end{vmatrix}$$

$$2.89. \begin{vmatrix} 7 & 6 & 3 & 7 \\ 3 & 5 & 7 & 2 \\ 5 & 4 & 3 & 5 \\ 5 & 6 & 5 & 4 \end{vmatrix}, 2.90. \begin{vmatrix} 3 & -3 & -5 & 8 \\ -3 & 2 & 4 & -6 \\ 2 & -5 & -7 & 5 \\ -4 & 3 & 5 & -6 \end{vmatrix}$$

$$2.91. \begin{vmatrix} 3 & 2 & 2 & 2 \\ 9 & -8 & 5 & 10 \\ 5 & -8 & 5 & 8 \\ 6 & -5 & 4 & 7 \end{vmatrix}$$

$$2.92. \begin{vmatrix} 1 & 2 & -1 & 1 \\ 2 & -1 & 0 & 1 \\ -1 & 1 & -2 & 2 \\ 1 & 4 & 1 & 1 \end{vmatrix}, 2.93. \begin{vmatrix} 1 & -1 & 2 & 1 \\ 3 & 4 & -1 & 0 \\ 2 & 1 & 3 & 1 \\ -5 & 0 & 1 & -2 \end{vmatrix}$$

$$2.94. \begin{vmatrix} -1 & 3 & 1 & 2 \\ -5 & 8 & 2 & 7 \\ 3 & -2 & -2 & 0 \\ 1 & 0 & -1 & 1 \end{vmatrix}$$

$$2.95. \begin{vmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 1 & 4 \\ 3 & 1 & 3 & 3 \\ 4 & 4 & 3 & 4 \end{vmatrix}, 2.96. \begin{vmatrix} 4 & 3 & -1 & 2 \\ 2 & 1 & 2 & -1 \\ 3 & 1 & -1 & 1 \\ 1 & 2 & 1 & 2 \end{vmatrix}$$

$$2.97. \begin{vmatrix} 1 & 1 & 3 & 2 \\ 2 & -3 & 1 & 2 \\ 2 & 2 & 4 & 4 \\ -1 & 1 & 5 & 3 \end{vmatrix}$$

$$2.98. \begin{vmatrix} 3 & 1 & -1 & 2 \\ -5 & 1 & 3 & -2 \\ 6 & 0 & 3 & -3 \\ 1 & -5 & 3 & -3 \end{vmatrix}, 2.99. \begin{vmatrix} 2 & 2 & -3 & -3 \\ 4 & 6 & -6 & -9 \\ -3 & -4 & 6 & 8 \\ -5 & -7 & 10 & 14 \end{vmatrix}$$

$$2.100. \begin{vmatrix} -4 & 10 & 0 & -2 & 6 \\ 1 & 0 & 3 & 7 & -2 \\ 3 & -1 & 0 & 5 & -5 \\ 2 & 6 & -4 & 1 & 2 \\ 0 & -3 & -1 & 2 & 3 \end{vmatrix}, 2.101. \begin{vmatrix} 6 & 6 & 10 & -5 \\ 5 & 8 & 8 & -9 \\ 5 & 5 & 9 & -8 \\ 4 & 7 & 7 & -11 \end{vmatrix}$$

2.102.
$$\begin{vmatrix} -1 & 2 & -2 & -1 \\ 3 & -4 & 7 & 5 \\ 4 & -9 & 8 & 5 \\ -3 & 2 & -5 & 3 \end{vmatrix}.$$

Mavzu bo'yicha nazorat testlari

Determinantlar va ularning xossalari.

1. Quyidagi $|A|$ determinantning a_{12} va a_{32} elementlari yig'indisini toping:

$$|A| = \begin{vmatrix} 1 & -4 & 7 \\ 0 & 3 & 5 \\ -2 & 6 & 3 \end{vmatrix}.$$

A) 5. B) 2. C) 7. D) -6. E) 6.

2. Quyidagi $|A|$ determinantning diagonal elementlari yig'indisini toping:

$$|A| = \begin{vmatrix} 1 & -4 & 7 \\ 0 & 9 & 5 \\ -2 & 6 & -10 \end{vmatrix}.$$

A) 14. B) 0. C) 20. D) -6. E) 4.

3. $\begin{vmatrix} 5 & -2 \\ 3 & 4 \end{vmatrix}$ determinantni hisoblang.

A) 14. B) -26. C) 26. D) -14. E) 0.

4. $\begin{vmatrix} x-1 & 3 \\ 2 & 1 \end{vmatrix} = 0$ tenglamani yeching.

A) $x=7$. B) $x=-1$. C) $x=2$. D) $x=4$. E) $x=8$.

5. Ushbu determinantni hisoblang:

$$\begin{vmatrix} 1 & 3 & 2 \\ 2 & 2 & 4 \\ 3 & 1 & 6 \end{vmatrix}$$

A) 1. B) 0. C) -2. D) 4. E) 12.

6. Quyidagi tenglamani yeching:

$$\begin{vmatrix} x & 1 & 2 \\ x & 1 & 1 \\ 1 & 2 & 2 \end{vmatrix} = 0$$

- A) $x=1$. B) $x=2.5$. C) $x=0.5$. D) $x=0$. E) $x=-1$.

7. Quyidagi determinantlar uchun qaysi tasdiq o'rinli?

$$\Delta_1 = \begin{vmatrix} 2 & 3 & 1 \\ 0 & 0 & 0 \\ 7 & 4 & -3 \end{vmatrix} \quad \Delta_2 = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 0 \end{vmatrix} \quad \Delta_3 = \begin{vmatrix} 6 & 2 & -7 \\ 1 & 3 & 5 \\ 1 & 3 & 5 \end{vmatrix} \quad \Delta_4 = \begin{vmatrix} 1 & 4 & 5 \\ 1 & 4 & -2 \\ 1 & 4 & 0 \end{vmatrix}$$

- A) $\Delta_1=\Delta_3$, $\Delta_2 \neq \Delta_4$. B) $\Delta_1=\Delta_2$, $\Delta_3 \neq \Delta_4$. C) $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ turli qiymatli. D) $\Delta_1=\Delta_2=\Delta_3=\Delta_4$. E) $\Delta_1=\Delta_2=\Delta_4 \neq \Delta_3$.

8. **Tasdiqni yakunlang:** Determinantda barcha satrlar ustun ko'rinishida yozilsa uning qiymati

- A) o'zgaradi. B) o'zgarmaydi. C) faqat ishorasini o'zgartiradi. D) 0 ga teng bo'ladi. E) 1 ga teng bo'ladi.

9. **Tasdiqni to'ldiring:** Determinantning ikkita satr yoki ustun elementlari proporsional bo'lsa uning qiymati albatta bo'ladi.

- A) musbat. B) manfiy. C) nolga teng. D) noldan farqli. E) birga teng.

10. Ushbu IV tartibli determinant qiymatini toping:

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 7 \\ 0 & 0 & 8 & 9 \\ 0 & 0 & 0 & 10 \end{vmatrix}$$

- A) 0. B) 400. C) 72. D) -100. E) -200.

11. Ushbu determinantning M_{23} minori qayerda to'g'ri ifodalangan:

$$\begin{vmatrix} 3 & -1 & 5 \\ 2 & 0 & 1 \\ -1 & 9 & 4 \end{vmatrix}$$

- A) $\begin{vmatrix} 3 & 5 \\ 2 & 1 \end{vmatrix}$. B) $\begin{vmatrix} -1 & 5 \\ 0 & 1 \end{vmatrix}$. C) $\begin{vmatrix} 3 & -1 \\ -1 & 9 \end{vmatrix}$. D) $\begin{vmatrix} 3 & -1 \\ 2 & 0 \end{vmatrix}$. E) $\begin{vmatrix} 0 & 1 \\ 9 & 4 \end{vmatrix}$.

12. A_{ij} algebraik to'ldiruvchi M_{ij} minor orqali qayerda to'g'ri aniqlangan?

- A) M_{ij} . B) $|M_{ij}|$. C) $-M_{ij}$. D) $(-1)^{i+j}M_{ij}$. E) $(-1)^{i+j}|M_{ij}|$.

13. Ushbu matritsaning A_{21} algebraik to'ldiruvchisi qayerda to'g'ri ko'rsatilgan?

$$\begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 1 \\ 3 & -1 & 2 \end{pmatrix}$$

A) $\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix}$. B) $-\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}$. C) $-\begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix}$. D) $\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix}$. E) $-\begin{vmatrix} 2 & 1 \\ -1 & 2 \end{vmatrix}$.

14. Ushbu determinantning A_{23} algebraik to'ldiruvchisini hisoblang:

$$\begin{vmatrix} 1 & 2 & 7 \\ 5 & -2 & 0 \\ 3 & -1 & 4 \end{vmatrix}$$

A) -35. B) 35. C) -7. D) 7. E) 0.

3-§. CHIZIQLI TENGLAMALAR SISTEMASI VA UNI YECHISH USULLARI. KRAMER, GAUSS VA MATRITSA USULI

3.1. Chiziqli tenglamalar sistemasini yechishning Kramer usuli

Ikkita chiziqli tenglamalardan iborat ushbu

$$\begin{cases} a_{11}x + a_{12}y = b_1, \\ a_{21}x + a_{22}y = b_2 \end{cases} \quad (3.1)$$

sistema ikki x va y noma'lumli chiziqli tenglamalar sistemasi deyiladi, bunda $a_{11}, a_{12}, a_{21}, a_{22}$ sonlar tenglamalar sistemasining noma'lumlar oldidagi koeffitsiyentlari, b_1 va b_2 sonlar ozod sonlar deyiladi.

(3.1) sistemaning koeffitsiyentlaridan ushbu

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

determinantni, so'ng bu determinantning birinchi ustunidagi elementlarni ozod sonlar bilan almashtirib

$$\Delta_x = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}$$

determinantni, ikkinchi ustundagi elementlarni ozod sonlar bilan almashtirib

$$\Delta_y = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}$$

determinantlar hosil qilamiz.

Demak, (3.1) sistema berilgan holda har doim $\Delta, \Delta_x, \Delta_y$ determinantlarga ega bo'lamiz.

1) $\Delta \neq 0$ bo'lsa, u holda (3.1) sistema yagona (x, y) yechimga ega bo'lib,

$$x = \frac{\Delta_x}{\Delta}, \quad y = \frac{\Delta_y}{\Delta}$$

bo'ladi;

2) $\Delta = 0$ bo'lib, $\Delta_x \neq 0, \Delta_y \neq 0$ bo'lsa, u holda (3.1) sistema yechimga ega bo'lmaydi;

3) $\Delta = \Delta_x = \Delta_y = 0$ bo'lsa, u holda (3.1) sistema cheksiz ko'p yechimga ega bo'ladi.

1-misol. Ushbu

$$\begin{cases} 4x - 5y = 2 \\ 6x - 3y = 12 \end{cases}$$

tenglamani Kramer usulidan foydalanib yeching.

Yechish.

$$\Delta = \begin{vmatrix} 4 & -5 \\ 6 & -3 \end{vmatrix} = 4 \cdot (-3) - 6 \cdot (-5) = 18, \quad \Delta = 18 \neq 0$$

$$\Delta_x = \begin{vmatrix} 2 & -5 \\ 12 & -3 \end{vmatrix} = 2 \cdot (-3) - 12 \cdot (-5) = 54$$

$$\Delta_y = \begin{vmatrix} 4 & 2 \\ 6 & 12 \end{vmatrix} = 4 \cdot 12 - 2 \cdot 6 = 36$$

$$x = \frac{\Delta_x}{\Delta} = \frac{54}{18} = 3 \quad y = \frac{\Delta_y}{\Delta} = \frac{36}{18} = 2 \quad J: \{3, 2\}.$$

Uchta chiziqli tenglamalardan iborat ushbu

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1, \\ a_{21}x + a_{22}y + a_{23}z = b_2, \\ a_{31}x + a_{32}y + a_{33}z = b_3, \end{cases} \quad (3.2)$$

sistema uchta x, y va z noma'lumli chiziqli tenglamalar sistemasi deyiladi, bunda $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}, a_{31}, a_{32}, a_{33}$ sonlar tenglamalar sistemasining koeffitsiyentlari, b_1, b_2 va b_3 sonlar ozod sonlar deyiladi.

(3.2) sistemaning koeffitsiyentlaridan quyidagi

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

uchinchi tartibli determinantni hosil qilamiz. So'ng bu determinantning birinchi, ikkinchi va uchinchi ustunlarini mos ravishda ozod sonlar bilan almashtirib quyidagi determinantlarni tuzamiz:

$$\Delta_x = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}, \quad \Delta_y = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}, \quad \Delta_z = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}.$$

Demak, (3.2) sistema berilgan holda har doim $\Delta, \Delta_x, \Delta_y, \Delta_z$ determinantlarga ega bo'lamiz.

1) $\Delta \neq 0$ bo'lsa, u holda (3.2) sistema yagona (x, y, z) yechimga ega bo'lib,

$$x = \frac{\Delta_x}{\Delta}, \quad y = \frac{\Delta_y}{\Delta}, \quad z = \frac{\Delta_z}{\Delta}$$

bo'ladi;

2) $\Delta = 0$ bo'lib, $\Delta_x \neq 0$, $\Delta_y \neq 0$, $\Delta_z \neq 0$ bo'lsa, u holda (3.2) sistema yechimga ega bo'lmaydi;

3) $\Delta = \Delta_x = \Delta_y = \Delta_z = 0$ bo'lsa, u holda (3.2) sistema cheksiz ko'p yechimga ega bo'ladi.

2-misol. Ushbu

$$\begin{cases} 2x + 3y + 2z = 14 \\ -x + 5y - z = 6 \\ 3x - y + z = 4 \end{cases}$$

tenglamani Kramer usulidan foydalanib yeching.

Yechish.

$$\Delta = \begin{vmatrix} 2 & 3 & 2 \\ -1 & 5 & -1 \\ 3 & -1 & 1 \end{vmatrix} = 2 \cdot 5 \cdot 1 + 3 \cdot (-1) \cdot 3 + (-1) \cdot (-1) \cdot 2 - \\ - (2 \cdot 5 \cdot 3 + 3 \cdot (-1) \cdot 1 + (-1) \cdot (-1) \cdot 2) = \\ = 10 - 9 + 2 - 30 + 3 - 2 = -26$$

$$\Delta_x = \begin{vmatrix} 14 & 3 & 2 \\ 6 & 5 & -1 \\ 4 & -1 & 1 \end{vmatrix} = 14 \cdot 5 \cdot 1 + 3 \cdot (-1) \cdot 4 + 6 \cdot (-1) \cdot 2 - \\ - (2 \cdot 5 \cdot 4 + 3 \cdot 6 \cdot 1 + (-1) \cdot (-1) \cdot 14) = \\ = 70 - 12 - 12 - 40 - 18 - 14 = -26$$

$$\Delta_y = \begin{vmatrix} 2 & 14 & 2 \\ -1 & 6 & -1 \\ 3 & 4 & 1 \end{vmatrix} = 2 \cdot 6 \cdot 1 + 14 \cdot (-1) \cdot 3 + (-1) \cdot 4 \cdot 2 - \\ - (2 \cdot 6 \cdot 3 + 14 \cdot (-1) \cdot 1 + (-1) \cdot 4 \cdot 2) = \\ = 12 - 42 - 8 - 36 + 14 + 8 = -52$$

$$\Delta_z = \begin{vmatrix} 2 & 3 & 14 \\ -1 & 5 & 6 \\ 3 & -1 & 4 \end{vmatrix} = 2 \cdot 5 \cdot 4 + 3 \cdot 6 \cdot 3 + (-1) \cdot (-1) \cdot 14 + \\ + 3 \cdot 6 \cdot 3 - (14 \cdot 5 \cdot 3 + 3 \cdot (-1) \cdot 4 + 6 \cdot (-1) \cdot 2) = 40 + \\ + 54 + 14 - 210 + 12 + 12 = -78$$

$$x = \frac{\Delta_x}{\Delta} = \frac{-26}{-26} = 1 \quad y = \frac{\Delta_y}{\Delta} = \frac{-52}{-26} = 2 \quad z = \frac{\Delta_z}{\Delta} = \frac{-78}{-26} = 3.$$

$J: \{1; 2; 3\}.$

3.2. Chiziqli tenglamalar sistemasini yechishning Gauss usuli

Endi chiziqli tenglamalar sistemasini yechishning Gauss usulini ko'rib chiqamiz. Bu usulda tenglamalardan noma'lumlarni ketma-ket yo'qotib, so'ngi tenglamada faqat bitta noma'lum qoldiriladi.

Bizga n o'zgaruvchili quyidagi tenglamalar sistemasi berilgan bo'lsin:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases}$$

Bu sistemaning kengaytirilgan matritsasi deb quyidagi matritsaga aytiladi:

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} & b_1 \\ a_{21} & a_{22} & \dots & a_{2n} & b_2 \\ \vdots & \vdots & \dots & \vdots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} & b_m \end{pmatrix}$$

Biz hozir berilgan sistemaning kengaytirilgan matritsasi qanday qurilishini ko'rsatamiz. Quyidagi sistema berilgan bo'lsin:

$$\begin{cases} x_1 + 3x_2 + 7x_3 = 8 \\ 2x_1 + 4x_2 + 9x_3 = 1 \\ 5x_1 + 2x_2 + 6x_3 = 4 \end{cases}$$

Uning kengaytirilgan matritsasi quyidagi ko'rinishda bo'ladi:

$$\begin{pmatrix} 1 & 3 & 7 & 8 \\ 2 & 4 & 9 & 1 \\ 5 & 2 & 6 & 4 \end{pmatrix}$$

Kengaytirilgan matritsani qurish uchun noma'lumlar koeffitsiyentlaridan tuzilgan matritsaning o'ng tomoniga ozod hadlardan tuzilgan yangi ustun qo'shiladi. Usulning asosiy g'oyasi berilgan sistemani unga teng kuchli bo'lgan, lekin yechish oson bo'lgan sistema bilan almashtirib, keyin hosil bo'lgan sistemani yechishdan iborat. Yangi sistema odatda quyidagi amallarni bajarish natijasida bo'ladigan bir nechta qadamlardan keyin hosil bo'ladi:

1. Tenglamani 0 dan farqli o'zgarmas songa ko'paytirish.
2. Ikkita tenglamaning o'rnini almashtirish.
3. Bir tenglamaga karrali tenglamani ikkinchisiga qo'shish.

Kengaytirilgan matritsaning satrlari sistemadagi tenglamalarga mos kelgani uchun yuqoridagi uchta amal kengaytirilgan matritsa uchun quyidagicha bo'ladi:

1. Satrni 0 dan farqli o'zgarmas songa ko'paytirish.
2. Ikkita satrning o'rnini almashtirish.
3. Bir satrga karrali satrni ikkinchisiga qo'shish.

Bu amallar satrlar ustidagi elementar almashtirishlar deyiladi. Quyidagi misolni yechish orqali bu amallarni qanday qo'llanilishini ko'rsatamiz.

3-misol. Ushbu

$$\begin{cases} x + y + 2z = 5 \\ 2x - 3y - z = 5 \\ 4x - y - 5z = -9 \end{cases}$$

tenglamani Gauss usulida yeching.

Yechish. Tenglamani yechish uchun 1-satrni (-2) ga ko'paytirib,

$$2\text{-satrga hadma-had qo'shsak} \begin{cases} x + y + 2z = 5 \\ -5y - 5z = -5 \\ 4x - y - 5z = -9 \end{cases}$$

hosil bo'ladi.

Yana shu 1-satrni (-4) ga ko'paytirib 3-satrga hadma-had qo'shsak

$$\begin{cases} x + y + 2z = 5 \\ -5y - 5z = -5 \\ -5y - 13z = -29 \end{cases}$$

hosil bo'ladi.

Endi 2- va 3- satrlarni bir-biridan ayirib 3-noma'lumni topib olamiz.

$$\begin{cases} x + y + 2z = 5 \\ -5y - 5z = -5 \\ 8z = 24 \end{cases}$$

Bu yerdan $z = 3$ ekanligi kelib chiqadi. z ni 2-tenglamaga qo'yib, y ni topib olamiz.

$$-5y - 5z = -5 \Rightarrow -5y - 5 \cdot 3 = -5 \Rightarrow y = -2$$

z va y larni 1-tenglamaga qo'yib noma'lum x ni topib olamiz.

$$x + y + 2z = 5 \Rightarrow x - 2 + 2 \cdot 3 = 5 \Rightarrow x = 1$$

$$x = 1, y = -2, z = 3.$$

3.3. Chiziqli tenglamalar sistemasi yechishning matritsa usuli

Chiziqli tenglamalar sistemasini qaraylik

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n = b_2 \\ \dots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n = b_m \end{cases} \quad (3.3)$$

va quyidagicha belgilashlar kiritaylik:

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} - \text{sistemaning matritsasi,}$$

$$X = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix} - \text{noma'lumlar ustuni}, B = \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix} - \text{ozod sonlar ustuni. U holda}$$

(3.3) sistemani matritsaviy tenglama ko'rinishida quyidagicha yozish mumkin:

$$A \cdot X = \begin{pmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_m \end{pmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \\ a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \end{pmatrix} \quad (3.4)$$

$$AX = B.$$

Faraz qilaylik A - xosmas matritsa bo'lsin, u holda unga teskari A^{-1} matritsa mavjud bo'ladi. (3.4) tenglamaning har ikki tomonini A^{-1} ga chapdan ko'paytiraylik.

$$A^{-1}AX = A^{-1}B.$$

Ma'lumki $A^{-1}A = E$, u holda $EX = A^{-1}B$, $EX = X$ ekanligidan $X = A^{-1}B$.

Shunday qilib, (3.4) - matritsaviy tenglamaning yechimi, A matritsaga teskari matritsaning (3.3) sistemaning ozod sonlaridan iborat ustun matritsaga ko'paytmasiga teng ekan.

4-misol. Ushbu

$$\begin{cases} 3x - y + z = 12 \\ x + 2y + 4z = 6 \\ 5x + y + 2z = 3 \end{cases}$$

tenglamani matritsa usulidan foydalanib yeching.

Yechish.

$$A^{-1} \cdot B = X$$

$$A = \begin{pmatrix} 3 & -1 & 1 \\ 1 & 2 & 4 \\ 5 & 1 & 2 \end{pmatrix} \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \quad B = \begin{pmatrix} 12 \\ 6 \\ 3 \end{pmatrix}$$

Bu yerda A matritsaning teskarisini topib olamiz. Buning uchun A matritsaning determinantini hisoblab, so'ng uning algebraik to'ldiruvchilarini topib olamiz.

$$\begin{aligned} \det A &= \begin{vmatrix} 3 & -1 & 1 \\ 1 & 2 & 4 \\ 5 & 1 & 2 \end{vmatrix} = 3 \cdot 2 \cdot 2 + (-1) \cdot 4 \cdot 5 + 1 \cdot 1 \cdot 1 - \\ &- (1 \cdot 2 \cdot 5 + (-1) \cdot 1 \cdot 2 + 4 \cdot 1 \cdot 3) = 12 - 20 + 1 - 10 + 2 - 12 = \\ &= -27. \end{aligned}$$

$$A_{ij} = (-1)^{i+j} M_{ij}$$

formuladan algebraik to'ldiruvchilarni topib olamiz.

$$A_{11} = 0, \quad A_{12} = 18, \quad A_{13} = -9, \quad A_{21} = 3, \quad A_{22} = 1,$$

$$A_{23} = -8,$$

$$A_{31} = -6, \quad A_{32} = -11, \quad A_{33} = 7.$$

$$A = \begin{pmatrix} 0 & 18 & -9 \\ 3 & 1 & -8 \\ -6 & -11 & 7 \end{pmatrix}$$

Algebraik to'ldiruvchilardan iborat bo'lgan matritsani transponerlaymiz ya'ni satrdagi elementlarni ustundagi elementlar bilan almashtirib yozamiz.

$$A^T = \begin{pmatrix} 0 & 3 & -6 \\ 18 & 1 & -11 \\ -9 & -8 & 7 \end{pmatrix}$$

Transponerlangan matritsaning har bir elementini uning determinantiga bo'lib yuborsak A matritsaga teskari matritsa

$$A^{-1} = \frac{1}{-27} \begin{pmatrix} 0 & 3 & -6 \\ 18 & 1 & -11 \\ -9 & -8 & 7 \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{9} & \frac{2}{9} \\ -\frac{2}{3} & -\frac{1}{27} & \frac{11}{27} \\ \frac{1}{3} & \frac{8}{27} & -\frac{7}{27} \end{pmatrix}.$$

Endi topilgan teskari matritsani B matritsaga ko'paytirib noma'lumlarni topib olamiz.

$$A^{-1} \cdot B = \begin{pmatrix} 0 & -\frac{1}{9} & \frac{2}{9} \\ -\frac{2}{3} & -\frac{1}{27} & \frac{11}{27} \\ \frac{1}{3} & \frac{8}{27} & -\frac{7}{27} \end{pmatrix} \cdot \begin{pmatrix} 12 \\ 6 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 - \frac{2}{3} + \frac{2}{3} \\ -8 - \frac{2}{9} + \frac{11}{9} \\ 4 + \frac{16}{9} - \frac{7}{9} \end{pmatrix} = \begin{pmatrix} 0 \\ -7 \\ 5 \end{pmatrix}.$$

Ravshanki, $x = 0$, $y = -7$, $z = 5$ ekanligi kelib chiqadi.

Mustaqil bajarish uchun misollar

Chiziqli tenglamalar sistemasini Kramer usulida yeching.

$$3.1. \begin{cases} 3x - 4y = 0 \\ x - 5y = -4 \end{cases}$$

$$3.2. \begin{cases} 3x - 4y = 11 \\ x + 5y = -9 \end{cases}$$

$$3.3. \begin{cases} 8x + 5y = 13 \\ 5x - y = 4 \end{cases}$$

$$3.4. \begin{cases} 5x + 3y = 12 \\ 2x - y = 7 \end{cases}$$

$$3.5. \begin{cases} 2x + 3y = 7 \\ 4x - 5y = 2 \end{cases}$$

$$3.7. \begin{cases} 3x + 2y + z = 3 \\ 5x - 2y - 2z = 3 \\ x + y - z = -2 \end{cases}$$

$$3.9. \begin{cases} 2x - 3y + z = -7 \\ x + 4y + 2z = -1 \\ x - 4y = -5 \end{cases}$$

$$3.11. \begin{cases} 2x_1 + 3x_2 + 4x_3 = 1 \\ x_1 + 2x_2 + 3x_3 = 0 \\ 3x_1 + 4x_2 + 6x_3 = 1 \end{cases}$$

$$3.13. a) \begin{cases} x_1 + x_2 + 4x_3 = 1 \\ 2x_1 + x_2 + 6x_3 = 2 \\ 3x_1 + 3x_2 + 13x_3 = 2 \end{cases}, b) \begin{cases} x_1 + 2x_2 - x_3 = 7 \\ 2x_1 - x_2 + x_3 = 2 \\ 3x_1 + 5x_2 + 2x_3 = -7 \end{cases}$$

$$3.14. a) \begin{cases} -2x_1 + 3x_2 - 2x_3 = 2 \\ 3x_1 - 2x_2 + x_3 = 2 \\ -5x_1 + 10x_2 - 7x_3 = 10 \end{cases}, b) \begin{cases} x_1 + 2x_2 + 3x_3 - 13 = 0 \\ 3x_1 + 2x_2 + 2x_3 - 16 = 0 \\ 4x_1 - 2x_2 + 5x_3 - 5 = 0 \end{cases}$$

$$3.15. \begin{cases} x_1 - 2x_2 + 4x_3 = 6 \\ 2x_1 - x_2 + 3x_3 = 11 \\ 4x_1 + x_2 - 5x_3 = 9 \end{cases} \quad 3.16. \begin{cases} 2x_1 - 3x_2 + x_3 = 2 \\ 2x_1 + x_2 - 4x_3 = 9 \\ 6x_1 - 5x_2 + 2x_3 = 17 \end{cases} \quad 3.17. \begin{cases} x_1 + 2x_2 - x_3 = 9 \\ 2x_1 - x_2 + 3x_3 = 13 \\ 3x_1 + 2x_2 - 5x_3 = -1 \end{cases}$$

$$3.18. \begin{cases} 2x_1 + x_2 - 3x_3 = -1 \\ x_1 - 3x_2 + 2x_3 = 10 \\ 3x_1 - 4x_2 - x_3 = 5 \end{cases} \quad 3.19. \begin{cases} 2x_1 + x_2 - x_3 = 6 \\ 3x_1 - x_2 + 2x_3 = 5 \\ 4x_1 + 2x_2 - 5x_3 = 9 \end{cases} \quad 3.20. \begin{cases} 6x_1 + 2x_2 - x_3 = 2 \\ 4x_1 - x_2 + 3x_3 = -3 \\ 3x_1 + 2x_2 - 2x_3 = 3 \end{cases}$$

Tenglamalar sistemasini Gauss usulida yeching.

$$3.21. \begin{cases} 3x - 5y + 3z = -3 \\ x + 2y + z = -1 \\ x - 7y - 2z = 5 \end{cases}$$

$$3.23. \begin{cases} x - 2y + 3z = 6 \\ 2x + 3y - 4z = 20 \\ 3x - 2y - 5z = 6 \end{cases}$$

$$3.25. \begin{cases} x + y + 2z = -1 \\ 2x - y + 2z = -4 \\ 4x + y + 4z = -2 \end{cases}$$

$$3.22. \begin{cases} 2x + 3y + 4z = 8 \\ 4y + 11z = 8 \\ 7x - 5y = -3 \end{cases}$$

$$3.24. \begin{cases} x - 9y + 2z = 14 \\ x + 3y - 8z = -18 \\ x - 3y - 2z = 0 \end{cases}$$

$$3.26. \begin{cases} 3x + 2y + z = 5 \\ 2x + 3y + z = 1 \\ 2x + y + 3z = 11 \end{cases}$$

$$3.27. \begin{cases} x + 2y + 3z = 8 \\ 2x + y + 2z = 5 \\ 3x + 2y + z = 4 \end{cases}$$

$$3.29. \begin{cases} 2x_1 + x_2 + 3x_3 = 13 \\ x_1 + x_2 + x_3 = 6 \\ 3x_1 + x_2 + x_3 = 8 \end{cases}$$

$$3.31. \begin{cases} x_1 + 2x_2 + 3x_3 = 3 \\ 3x_1 + x_2 + 2x_3 = 7 \\ 2x_1 + 3x_2 + x_3 = 3 \end{cases}$$

$$3.33. a) \begin{cases} x_1 + 2x_2 - 3x_3 - 2x_4 = 1 \\ -2x_1 - 3x_2 + x_3 + 3x_4 = 3 \\ 5x_1 + 9x_2 - 10x_3 - 9x_4 = 0 \end{cases}$$

$$3.34. \begin{cases} 2x_1 - x_2 - x_3 = 7 \\ -4x_1 + 2x_2 = -2 \\ 6x_1 - 3x_2 + x_3 = -3 \end{cases}$$

$$3.36. \begin{cases} 5x_1 + 3x_2 + 5x_3 + 12x_4 = 10 \\ 2x_1 + 2x_2 + 3x_3 + 5x_4 = 4 \\ x_1 + 7x_2 + 9x_3 + 4x_4 = 2 \end{cases}$$

$$3.28. \begin{cases} x + 2y - z = 4 \\ x + 5y + z = 5 \\ 7x - 2y + z = 4 \end{cases}$$

$$3.30. \begin{cases} 2x_1 + x_2 + x_3 = 7 \\ x_1 + 2x_2 + x_3 = 8 \\ x_1 + x_2 + 2x_3 = 9 \end{cases}$$

$$3.32. \begin{cases} \lambda x_1 + x_2 + x_3 = 1 \\ x_1 + \lambda x_2 + x_3 = 1 \\ x_1 + x_2 + \lambda x_3 = 1 \end{cases}$$

$$b) \begin{cases} 2x_1 + x_2 - x_3 + 3x_4 = 20 \\ 5x_1 - x_2 + 2x_3 - x_4 = 17 \\ -3x_1 + 2x_2 - x_3 + 2x_4 = 1 \\ x_1 - x_2 + 4x_3 - 2x_4 = -4 \end{cases}$$

$$3.35. \begin{cases} x_1 + 2x_2 = 2 \\ 2x_1 + x_2 + 3x_3 = 1 \\ 2x_1 + 7x_2 - 3x_3 = 31 \end{cases}$$

$$3.37. \begin{cases} x_1 + 2x_2 - 2x_3 + 5x_4 = 3 \\ -3x_1 - 2x_2 + 12x_3 - 7x_4 = -5 \\ x_2 + 3x_3 + 4x_4 = 2 \end{cases}$$

Tenglamalar sistemasini matritsa usulida yeching.

$$3.39. \begin{cases} x + y = 4 \\ x - y = 2 \end{cases}$$

$$3.40. \begin{cases} 2x + 5y = 153 \\ x - 2y = 3 \end{cases}$$

$$3.41. \begin{cases} 3x + 5y = 21 \\ 2x - y = 1 \end{cases}$$

$$3.42. \begin{cases} \frac{1}{2}x - \frac{1}{3}y = 1 \\ 3x - 5y = -3 \end{cases}$$

$$3.43. \begin{cases} 2x - 3y = -1 \\ y = 0,75x \end{cases}$$

$$3.44. \begin{cases} \frac{1}{4}x - y = -5 \\ \frac{1}{2}x - \frac{1}{7}y = 3 \end{cases}$$

$$3.44. \begin{cases} x - y - 2z = -3 \\ 2x + y + 2z = 6 \\ 4x - 9y + 4z = 12 \end{cases}$$

$$3.45. \begin{cases} 3x - 2y - z = 0 \\ 2x - 3y - z = -2 \\ 2x + y + 3z = 6 \end{cases}$$

$$3.46. \begin{cases} x - 2y - 3z = -1 \\ 2x - y + 2z = 0 \\ 3x + 2y + z = 7 \end{cases}$$

$$3.47. \begin{cases} x + 2y - z = 2 \\ x - 5y + z = -1 \\ 7x - y + z = 15 \end{cases}$$

$$3.47. \begin{cases} x - 4y - 2z = 7 \\ 3x + y - z = 16 \\ -3x + 5y - 6z = -9 \end{cases}$$

$$3.48. \begin{cases} 3x + y - z = -2 \\ x - 2y + 2z = 9 \\ 4x - y + z = 9 \end{cases}$$

$$3.49. \begin{cases} 3x - 3y - z = -3 \\ 3x + 5y + 4z = -2 \\ 4x - 2y + 3z = -13 \end{cases} \quad 3.50. \begin{cases} 2x + y - 5z = 3 \\ 3x - 5y + 2z = 1 \\ 5x - 6y + 3z = 6 \end{cases}$$

Mavzu bo'yicha nazorat testlari

Chiziqli tenglamalar sistemasi.

1. Quyidagi sistemalardan qaysi biri chiziqli tenglamalar sistemasini ifodalaydi?

$$\begin{aligned} \text{A)} \begin{cases} a_{11}x_1^2 + a_{12}x_2^2 = b_1 \\ a_{21}x_1^2 + a_{22}x_2^2 = b_2 \end{cases} & \quad \text{B)} \begin{cases} \frac{a_{11}}{x_1} + \frac{a_{12}}{x_2} = b_1 \\ \frac{a_{21}}{x_1} + \frac{a_{22}}{x_2} = b_2 \end{cases} & \quad \text{C)} \begin{cases} a_{11}x_1^2 + a_{12}x_2^2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases} \\ \text{D)} \begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1^2 + a_{22}x_2^2 = b_2 \end{cases} & \quad \text{E)} \begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases} \end{aligned}$$

2. Ushbu chiziqli tenglamalar sistemasi koeffitsiyentlarining yig'indisini toping:

$$\begin{cases} 3x_1 - 2x_2 = 5 \\ 2x_1 - 3x_2 = 0 \end{cases}$$

A) 10. B) 0. C) 5. D) 15. E) to'g'ri javob keltirilmagan.

3. Ushbu chiziqli tenglamalar sistemasi ozod hadlarining ko'paytmasini toping:

$$\begin{cases} 4x_1 + 7x_2 = 6 \\ 3x_1 - 5x_2 = 3 \end{cases}$$

A) 28. B) -15. C) 18. D) 12. E) -35.

4. Ushbu ikki noma'lumli chiziqli tenglamalar sistemasining asosiy determinanti qayerda to'g'ri ko'rsatilgan?

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

$$\text{A)} \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}. \quad \text{B)} \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}. \quad \text{C)} \begin{vmatrix} a_{11} & b_1 \\ a_{12} & b_2 \end{vmatrix}. \quad \text{D)} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}. \quad \text{E)} \begin{vmatrix} a_{21} & a_{12} \\ b_1 & b_2 \end{vmatrix}.$$

5. Chiziqli tenglamalar sistemasining asosiy Δ determinantining qiymati qanday bo'lganda uni yechish uchun matritsalar usulini qo'llab bo'lmaydi?

A) $\Delta \neq 0$. B) $\Delta = 0$. C) $\Delta > 0$. D) $\Delta < 0$.
E) ko'rsatilgan barcha hollarda qo'llab bo'ladi.

6. Matritsaviy ko'rinishda yozilgan $AX=B$ chiziqli tenglamalar sistemasi yechimining formulasi qayerda to'g'ri ko'rsatilgan?

A) $X = B \cdot A$. B) $X = A \cdot B^{-1}$. C) $X = B^{-1} \cdot A$. D) $X = A^{-1} \cdot B$. E) $X = B \cdot A^{-1}$.

7. Matritsaviy ko'rinishdagi $AX=B$ tenglamani yeching. Bunda

$$A = \begin{pmatrix} 2 & 3 \\ 5 & 4 \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, B = \begin{pmatrix} 18 \\ 31 \end{pmatrix}.$$

A) $X = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. B) $X = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. C) $X = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$. D) $X = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ E) sistema yechimga

ega emas.

8. Chiziqli tenglamalar sistemasining asosiy Δ determinantining qiymati qanday bo'lganda uni yechish uchun Kramer usulini qo'llab bo'lmaydi?

A) $\Delta \neq 0$. B) $\Delta = 0$. C) $\Delta > 0$. D) $\Delta < 0$. E) ko'rsatilgan barcha hollarda qo'llab bo'ladi.

9. Ushbu ikki noma'lumli ikkita chiziqli tenglamalar sistemasi berilgan:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

Quyidagilardan qaysi biri sistemaning asosiy determinantiga teng emas?

$$A) \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}. B) \begin{vmatrix} a_{11} & a_{22} \\ a_{21} & a_{12} \end{vmatrix}. C) \begin{vmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{vmatrix}. D) - \begin{vmatrix} a_{21} & a_{22} \\ a_{11} & a_{12} \end{vmatrix}.$$

E) Hamma ko'rsatilgan determinantlar asosiy determinantga teng.

10. Ushbu ikki noma'lumli ikkita chiziqli tenglamalar sistemasi berilgan:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

Agar bu sistemaning asosiy determinanti

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} \neq 0$$

bo'lsa, quyidagi tasdiqlardan qaysi biri to'g'ri?

A) sistema yechimga ega emas.

B) sistema yagona yechimga ega.

C) sistema cheksiz ko'p yechimga ega.

D) sistema faqat nol yechimga ega.

E) sistemaning yechimi yo'q yoki cheksiz ko'p.

11. Ushbu ikki noma'lumli ikkita chiziqli tenglamalar sistemasi berilgan:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

Quyidagilardan qaysi biri sistemaning yordamchi determinantiga teng emas?

A) $\begin{vmatrix} b_1 & b_2 \\ a_{12} & a_{22} \end{vmatrix}$. B) $\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}$. C) $\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}$. D) $\begin{vmatrix} a_{11} & a_{21} \\ b_1 & b_2 \end{vmatrix}$.

E) Hamma ko'rsatilgan determinantlar yordamchi determinantdir.

12. Agar

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

ikkita noma'lumli ikkita chiziqli tenglamalar sistemasining asosiy determinanti $\Delta=0$ va yordamchi determinantlari $\Delta_1=\Delta_2=0$ bo'lsa sistemaning yechimi to'grisidagi qaysi tasdiq o'rinli bo'ladi?

A) yechim yagona.

B) yechim cheksiz ko'p.

C) yechim mavjud emas.

D) yechimlar o'zaro teng. E) yechim nolga teng.

13. Agar ikki noma'lumli ikkita chiziqli tenglamalar sistemasida

$$\begin{cases} a_{11}x_1 + a_{12}x_2 = b_1 \\ a_{21}x_1 + a_{22}x_2 = b_2 \end{cases}$$

asosiy determinant $\Delta \neq 0$ bo'lsa, u holda sistemaning yechimi to'grisidagi qaysi tasdiq doimo o'rinli bo'ladi?

A) yechim yagona. B) yechim cheksiz ko'p.

C) yechim mavjud emas. D) yechimlar o'zaro teng.

E) yechimlar proporsional.

14. Tenglamalar sistemasini yeching va $x_1^2 + x_2^2$ ifodaning qiymatini aniqlang:

$$\begin{cases} 2x_1 + x_2 = 3 \\ x_1 - 2x_2 = -1 \end{cases}$$

A) 5. B) 1. C) 4. D) 2. E) 3.

15. Gauss usulining mazmuni qayerda to'g'ri ifodalangan?

A) determinantlar hisoblanadi. B) matritsalar topiladi.

C) teskari matritsa aniqlanadi. D) noma'lumlar birin-ketin yo'qotiladi.

E) to'g'ri javob keltirilmagan.

4-§. TEKISLIKDA BERILGAN TO'G'RI CHIZIQ TENGLAMASI VA UNING TURLARI. IKKI TO'G'RI CHIZIQ ORASIDAGI BURCHAK. NUQTADAN TO'G'RI CHIZIQQACHA BO'LGAN MASOFA

4.1. Tekislikda to'g'ri chiziqli tenglamalari va uning turlari.

Tekislikda to'g'ri chiziqli sodda, ayni paytda muhim geometrik tushunchalaridan biri. Uni tekislikdagi nuqtalar to'plami (nuqtalarning geometrik o'rni) sifatida tushuniladi.

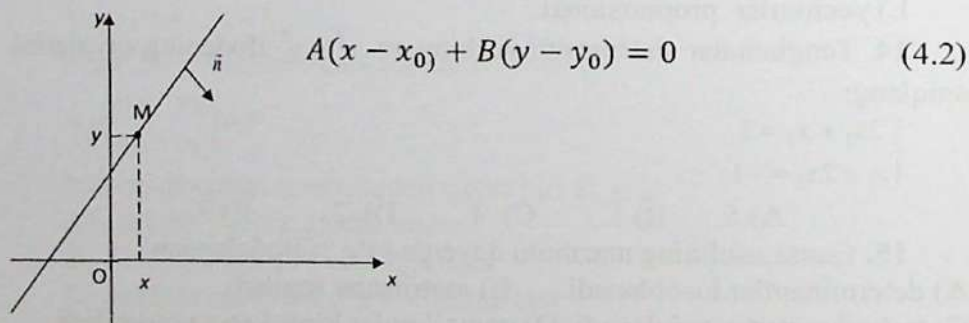
Ma'lumki, tekislikdagi nuqta o'zining x va y koordinatalari bilan to'liq aniqlanadi. Bu x va y sonlar turli qiymatlarni qabul qilganda (x, y) juftliklar turlicha bo'lib, ular turli nuqtalarni tasvirlaydi. Odatda, bunday nuqtalar o'zgaruvchi nuqta deyiladi. Agar o'zgaruvchi nuqtaning koordinatalari x va y lar biror bog'lanishda bo'lsa, umuman aytganda bunday nuqtalar to'plami (geometrik o'rni) biror geometrik shaklni ifodalashi mumkin. Bog'lanish esa geometrik shaklning tenglamasi deyiladi.

Ushbu

$$Ax + By + C = 0 \quad (4.1)$$

tenglama tekislikda to'g'ri chiziqli umumiy tenglamasi deyiladi. Berilgan to'g'ri chiziqli perpendikulyar bo'lgan $\vec{n} = \{A, B\}$ vektor *normal vektor* deyiladi. Tekislikda to'g'ri chiziqli quyidagi tenglamalari mavjud.

Berilgan $P_0(x_0, y_0)$ nuqtadan o'tuvchi va berilgan $\vec{n} = \{A, B\}$ normal vektorga ega to'g'ri chiziqli tenglamasi



4.1-rasm

1-misol. $M(3,4)$ nuqtadan o'tuvchi va berilgan $\vec{a} = \{-2,5\}$ vektorga perpendikulyar bo'lgan to'g'ri chiziqli tenglamasini tuzing.

Yechish. Bu tenglamani hosil qilish uchun A va B nuqtaning o'rniga $\vec{a}$ vektorning koordinatalarini qo'yamiz.

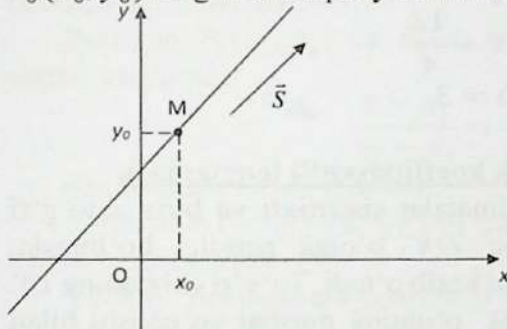
$$\begin{aligned} -2(x-3) + 5(y-4) &= 0 \\ -2x + 6 + 5y - 20 &= 0 \\ -2x + 5y - 14 &= 0 \end{aligned}$$

tenglama hosil bo'ladi ▲

Berilgan to'g'ri chiziqli tenglamasi.

$$\frac{x-x_0}{l} = \frac{y-y_0}{m} \quad (4.3)$$

bunda $\vec{s} = \{l, m\}$ — to'g'ri chiziqli tenglamani yo'naltiruvchi vektor. $P_0(x_0, y_0)$ to'g'ri chiziqli tenglamada yotuvchi berilgan nuqta.



4.2-rasm

2-misol. $M(1, -3)$ nuqtadan o'tuvchi va $\vec{b} = \{4,6\}$ vektorga parallel bo'lgan to'g'ri chiziqli tenglamasini tuzing.

Yechim. To'g'ri chiziqli tenglamaga parallel bo'lgan tenglama kanonik tenglama bo'lib, M nuqtani va $\vec{b}$ vektorini (4.3) tenglamaga qo'yib hisoblaymiz.

$$\frac{x-1}{4} = \frac{y+6}{-3}$$

tenglikni proporsiya qilib, quyidagi natijani olamiz.

$$\begin{aligned} -x + 3 &= 4y + 24 \\ -x - 4y - 21 &= 0 \end{aligned}$$

tenglikni (-1) ga ko'paytirib,

$$x + 4y + 21 = 0$$

hosil bo'ladi. ▲

To'g'ri chiziqli tenglamani burchak koeffitsiyentli tenglamasi.

1-hol. Agar to'g'ri chiziqli tenglamani umumiy tenglamasi

$$Ax + By + C = 0$$

da $B \neq 0$ bo'lganda, uni to'g'ri chiziqning burchak koeffitsiyentli tenglamasiga keltirish uchun (4.1) tenglamani y ga nisbatan yechsak,

$$y = -\frac{A}{B}x - \frac{C}{B}$$

so'ng

$$k = -\frac{A}{B}, \quad b = -\frac{C}{B}$$

deyilsa, unda (4.1) tenglama ushbu

$$y = kx + b \quad (4.4)$$

ko'rinishga keladi. Bu to'g'ri chiziqning burchak koeffitsiyentli tenglamasidir.

3-misol. $8x - 4y + 12 = 0$ to'g'ri chiziqning burchak koeffitsiyentli tenglamasini tuzing.

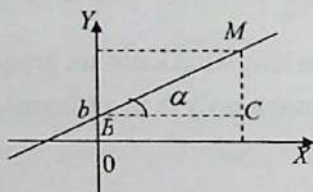
Yechim. Berilgan tenglamani y ga nisbatan yechib olamiz.

$$y = \frac{8}{4}x + \frac{12}{4}$$

$$y = 2x + 3, \quad k = 2, \quad b = 3.$$

To'g'ri chiziqning burchak koeffitsiyentli tenglamasi.

2-hol. Tekislikda Dekart koordinatalar sistemasi va biror L to'g'ri chiziqni olaylik. Bu to'g'ri chiziq OX o'qiga parallel bo'lmasin. Binobarin, L to'g'ri chiziq OX o'qini kesib o'tadi. To'g'ri chiziqning OY o'qi bilan kesishgan nuqtani B , OX o'qining musbat yo'nalishi bilan tashkil etgan burchakni α deylik. (4.3-chizma)



4.3-chizma

Ravshanki, $B = B(0; b)$ bo'lib, b esa OB kesmaning uzunligi.

To'g'ri chiziqda ixtiyoriy $M = M(x, y)$ nuqtani olamiz.

Keltirilgan chizmadan ko'rinadiki, BMC -to'g'ri burchakli uchburchak, $\angle CBM = \alpha$ $BC = x$, $MC = y - b$.

BMC uchburchakdan

$$\operatorname{tg} \alpha = \frac{y - b}{x}$$

bo'lishini topamiz. Bu miqdor to'g'ri chiziqning burchak koeffitsiyenti deyiladi va k bilan belgilanadi:

$$k = \operatorname{tg} \alpha.$$

Natijada

$$k = \frac{y-b}{x} \text{ bo'lib, undan}$$

$$y = kx + b \quad (4.5)$$

bo'lishi kelib chiqadi. Bu tenglama to'g'ri chiziqlarning burchak koeffitsiyentli tenglamasi deyiladi.

4-misol. $M(2,3)$ nuqtadan o'tib, OX o'qi bilan $\alpha = 45^\circ$ burchak hosil qiluvchi to'g'ri chiziq tenglamasini tuzing.

Yechim. $k = \operatorname{tg} \alpha = \operatorname{tg} 45^\circ = 1$ bu yerdan $k = 1$ ekanligi kelib chiqadi.

$y = kx + b$ tenglamaga qo'yib, b ozod sonni topib olamiz.

$$3 = 1 \cdot 2 + b \quad b = 1.$$

Bundan so'ralayotgan burchak koeffitsiyentli tenglama

$$y = x + 1$$

ekanligi kelib chiqadi.

Berilgan $P_1(x_1, y_1)$ va $P_2(x_2, y_2)$ nuqtalar orqali o'tuvchi to'g'ri chiziq tenglamasi

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} \quad (4.6)$$

shu formula orqali topiladi.

5-misol. $P_1(-1,4)$ va $P_2(5,9)$ nuqtalar orqali o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Yechim. Berilgan nuqtalardan o'tuvchi tenglamani tuzish uchun (4.6) formulaga qo'llaymiz.

$$\frac{x - (-1)}{5 - (-1)} = \frac{y - 4}{9 - 4}$$

$$\frac{x + 1}{6} = \frac{y - 4}{5}$$

$$5x + 5 = 6y - 24$$

$$5x - 6y + 29 = 0$$

tenglama hosil bo'ladi.

To'g'ri chiziqlarning kesmalarga nisbatan tenglamasi.

To'g'ri chiziqlarning kesmalarga nisbatan tenglamasini hosil qilish uchun (4.1) tenglamadagi tenglikning ikkala tomonini ham C ozod songa bo'lib yuboramiz.

$$-\frac{Ax}{C} - \frac{By}{C} = 1$$

$$-\frac{x}{\frac{C}{A}} - \frac{y}{\frac{C}{B}} = 1$$

$$a = -\frac{C}{A}, \quad b = -\frac{C}{B}$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

ko'rinishdagi tenglama hosil bo'ladi. Bu tenglama to'g'ri chiziqlarning kesmalarga nisbatan tenglamasi deyiladi.

6-misol. Ushbu $y = 3x + 5$ to'g'ri chiziqlarning kesmalar bo'yicha tenglamasini yozing.

Yechim. Bu tenglamani hosil qilish uchun, berilgan tenglamadagi ozod son 5 ni tenglikning ikkala tomoniga ham bo'lib yuboramiz.

$$\frac{y}{5} - \frac{3x}{5} = \frac{5}{5}$$

$$\frac{x}{-\frac{5}{3}} + \frac{y}{5} = 1$$

$$a = \frac{-5}{3}, \quad b = 5.$$

$P_1(x_1, y_1)$ nuqtadan $P_2(x_2, y_2)$ nuqtagacha bo'lgan masofa

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (4.7)$$

formula orqali topiladi.

7-misol. $P_1(2, 5, 7)$ nuqtadan $P_2(4, 6, 8)$ nuqtagacha bo'lgan masofani toping.

Yechim. Bu yerda masofani topish uchun P_2 nuqtaning koordinatasidan P_1 nuqtaning koordinatalarini ayiramiz va ularni kvadratga ko'tarib, yig'indisini ildizdan chiqaramiz.

$$d = \sqrt{(4 - 2)^2 + (6 - 5)^2 + (8 - 7)^2} = \sqrt{4 + 1 + 1} = \sqrt{6}.$$

Demak, berilgan ikki nuqta orasidagi masofa $\sqrt{6}$ ga teng ekan.

Ikki to'g'ri chiziq orasidagi burchak. Berilgan ikki

$$\begin{cases} A_1x + B_1y + C_1 = 0 \\ A_2x + B_2y + C_2 = 0 \end{cases}$$

to'g'ri chiziqlar orasidagi burchak ularning normal vektorlari orasidagi burchakka teng kuchlidir. Bu yerda birinchi tenglama uchun normal vektor

$\vec{n}_1 = (A_1, B_1)$ ga, ikkinchi tenglama uchun $\vec{n}_2 = (A_2, B_2)$ ga teng.

Ular orasidagi burchakni topish uchun quyidagi formuladan foydalanamiz.

$$\cos\varphi = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| \cdot |\vec{n}_2|} = \frac{A_1 \cdot A_2 + B_1 \cdot B_2}{\sqrt{A_1^2 + B_1^2} \cdot \sqrt{A_2^2 + B_2^2}} \quad (4.8)$$

yuqoridagi formula berilgan ikki to'g'ri chiziqli orasidagi burchakni ifodalaydi.

Agarda to'g'ri chiziqli burchak koeffitsiyentli ko'rinishda berilgan bo'lsa, ya'ni

$y_1 = k_1x + b_1$ va $y_2 = k_2x + b_2$ tenglamalar orasidagi burchak ushbu formula orqali topiladi.

$$\operatorname{tg}\varphi = \frac{k_2 - k_1}{1 + k_1 \cdot k_2} \quad (4.9)$$

8-misol. Berilgan to'g'ri chiziqli orasidagi burchakni toping.

$$\begin{cases} 5x - y + 7 = 0 \\ 2x - 3y + 1 = 0 \end{cases}$$

Yechish. Bu tenglamalar orasidagi burchakni topish uchun, ularning normal vektorlarini aniqlab olish zarur. Birinchi va ikkinchi tenglamalar uchun normal vektorlar $\vec{n}_1 = (5, -1)$ va $\vec{n}_2 = (2, -3)$ ga teng.

$$\cos\varphi = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| \cdot |\vec{n}_2|} = \frac{5 \cdot 2 + (-1) \cdot (-3)}{\sqrt{5^2 + (-1)^2} \cdot \sqrt{2^2 + (-3)^2}} = \frac{13}{\sqrt{26} \cdot \sqrt{13}} = \frac{1}{\sqrt{2}}$$

Demak, berilgan tenglamalar orasidagi burchak $\varphi = 45^\circ$ ga teng.

❖ Parallellik va perpendikulyarlik shartlari.

Agar tenglamalar

$$\begin{cases} A_1x + B_1y + C_1 = 0 \\ A_2x + B_2y + C_2 = 0 \end{cases}$$

umumiy ko'rinishda berilgan tenglamalar uchun:

➤ Parallellik sharti: $\frac{A_1}{A_2} = \frac{B_1}{B_2}$

➤ Perpendikulyarlik sharti: $A_1A_2 + B_1B_2 = 0$

➤ Ustma-ust tushsa: $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$.

✓ Agar tenglamalar burchak koeffitsiyentli ko'rinishda berilgan bo'lsa,

$y_1 = k_1x + b_1$ va $y_2 = k_2x + b_2$

➤ Parallel bo'lsa: $k_1 = k_2$

➤ Perpendikulyar bo'lsa: $k_1 \cdot k_2 = -1$

➤ Ustma-ust tushsa: $k_1 = k_2, \quad b_1 = b_2$.

Mustaqil bajarish uchun misollar

4.1. a) Quyidagi to'g'ri chiziqlar tenglamalari berilgan. Bu to'g'ri chiziqlar koordinata o'qlariga nisbatan joylashganini aytib bering.

1) $2x - y = 0$; 2) $x - 3 = 0$; 3) $x - 5 = 0$; 4) $x = 0$; 5) $y = 0$.

b) $y = 2x - 1$, $y = \frac{x-3}{3}$, $y = \frac{3x+1}{2}$, $y = x + 2$, $y = 4x - 2$ chiziqlarning qaysi birlari Oy o'qi bilan nuqtani kesib o'tadi.

4.2. a) To'g'ri chiziqning umumiy ko'rinishdagi tenglamalari berilgan. Ularni burchak koeffitsiyentli tenglamalar shakliga keltiring:

1) $x - 2y + 1 = 0$, 2) $3x + 5y - 1 = 0$, 3) $8x + 3y = 0$.

4.3. a) Absissalar o'qidan ajratgan kesmaning miqdori 2 birlik, ordinatalar o'qidan ajratgan kesmaning miqdori 3 birlik bo'lgan chiziqning tenglamasini tuzing.

4.4. a) Absissalar o'qidan ajratgan kesmaning miqdori -5 birlik, ordinatalar o'qidan ajratgan kesmasining miqdori 5 birlik bo'lgan to'g'ri chiziqning tenglamasini tuzing.

b) $C(2; -6)$ nuqta orqali $y = 2x + 7$ chizig'iga parallel chiziq o'tkazing.

4.5. a) Ushbu

1) $2x + 3y - 6 = 0$; 2) $3x - 5y - 15 = 0$; 3) $ax + by - ab = 0$; 4) $y = 5x - 2$;
5) $y = x + 1$; 6) $ay = -bx + c$.

to'g'ri chiziqlarning tenglamalarini ularning kesmalarga nisbatan tenglamalari shaklida yozing.

b). Quyidagicha chiziqlarning burchak koeffitsiyentlarini aniqlang.

a) $2x - y + 7 = 0$ b) $x + 3y - 4 = 0$ v) $2x + 5y = 6$ g) $3x - 2y = 1$

d) $5x + 5y - 8 = 0$ e) $\frac{1}{2}x - y + 1 = 0$

4.6. a) $O(0;0)$ va $A(-5,0)$ nuqtalar berilgan bo'lib, OA kesmada diagonallari $B(0,3)$ nuqtada kesishuvchi parallelogramm yasalgan. Bu parallelogrammning tomonlari hamda diagonallarining tenglamalarini tuzing.

b) Uchburchakning tomonlarining tenglamasini $2x + y - 4 = 0$, $3x - y - 1 = 0$, $x + y - 1 = 0$ bo'lsa, uning uchlarining koordinatalarini toping.

4.7. $M(5,2)$ nuqtadan o'tib, koordinata burchagidan yuzi 20 kv. birlikka uchburchak kesuvchi to'g'ri chiziq tenglamasini tuzing.

4.8. BC tomon AE bissektrisani kesish nuqtasi E ning koordinatalari;

c) $A(3;2)$ nuqtadan o'tuvchi va quyidagi keltirilgan shartlarni qanoatlantiruvchi to'g'ri chiziq tenglamasini tuzing:

4.9. 1. Ox o'qi bilan 135° burchak hosil qiluvchi; 2. Oy o'qiga parallel; 3. $B(-2;-1)$ nuqtadan o'tuvchi; burchakni toping.

4.10. $A(1,-2)$, $B(2,3)$, $C(5,6)$, $D(0,-3)$, $E(2,-2)$ nuqtalardan qaysilari $x + y = 1$ tenglamada yotadi.

4.11. Quyidagi tenglamalar orqali qanday nuqtalar to'plami aniqlanadi. Ularning chizmasini tasvirlang.

1) $x - 2y = 0$, 2) $2x + y = 0$, 3) $y - 3 = 0$, 4) $xy = 0$,
5) $x^2 + 7x + 10 = 0$, 6) $y^2 - 8y + 15 = 0$, 7) $x + 4 = 0$.

4.12. Koordinatalar boshidan va $A(3,-2)$ nuqtadan o'tuvchi to'g'ri chiziqli tenglamasini tuzing.

4.13. 1) $2x + 3y + 5 = 0$, 2) $3x - 2y = 4$, 3) $x - y - 1 = 0$,
4) $y = -2$, 5) $x = 3$, 6) $5x + 4y - 3 = 0$, 7) $2x - 4y + 8 = 0$
tenglamalarning k va b parametrlarini toping va burchak koeffitsiyentli tenglama ko'rinishida yozing.

4.14. Ushbu to'g'ri chiziqlar orasidagi burchakni toping.

1) $\begin{cases} y = 3x - 2 \\ y = 2x + 1 \end{cases}$; 2) $\begin{cases} x - 5y + 7 = 0 \\ 3x - 2y + 1 = 0 \end{cases}$; 3) $\begin{cases} y = -2x \\ 3x - y - 4 = 0 \end{cases}$
5) $\begin{cases} 2x + 3y = 0 \\ 6x + 4y - 9 = 0 \end{cases}$; 6) $\begin{cases} 3x - 4y - 6 = 0 \\ 6x + 8y - 11 = 0 \end{cases}$; 7) $\begin{cases} y + \sqrt{3}x - 1 = 0 \\ y - \sqrt{3}x + 5 = 0 \end{cases}$.

4.15. $2x - 3y + 6 = 0$, $4x - 6y - 5 = 0$, $4x + 6y - 3 = 0$,
 $3x + 2y - 6 = 0$

to'g'ri chiziqlardan parallel va perpendikulyar bo'lganlari ko'rsatilsin.

4.16. Koordinatalar boshidan o'tib, $y = -2x + 4$ to'g'ri chiziqli bilan 45° burchak tashkil qiluvchi to'g'ri chiziqli tenglamasini tuzing.

4.17. $P(-1,1)$ nuqtadan o'tib $2x + 3y = 6$ to'g'ri chiziqli bilan 45° burchak tashkil qiluvchi to'g'ri chiziqli tenglamasini tuzing.

4.18. Uchlari $A(2,4)$, $B(5,7)$, $C(1,3)$ nuqtalarda bo'lgan uchburchakning tomonlari tenglamasini tuzing.

4.19. $3x + y - 6 = 0$ va $x + 2y + 1 = 0$ to'g'ri chiziqlarning kesishish nuqtalarini toping.

4.20. $M(-3,-1)$ nuqta orqali o'tuvchi va $2x + y - 3 = 0$ to'g'ri chiziqli perpendikulyar bo'lgan to'g'ri chiziqli tenglamasini tuzing.

4.21. $M(-3,-1)$ nuqta orqali o'tuvchi $2x + y - 3 = 0$ to'g'ri chiziqli parallel bo'lgan to'g'ri chiziqli tenglamasini tuzing.

4.22. ABC uchburchak uchlarining koordinatlari berilgan:

$A(-1,4)$, $B(11,-5)$, $C(15,17)$. AB va BC tomonlarning tenglamasi tuzilsin.

4.23. To'g'ri chiziq $4x-3y-12=0$ tenglama bilan berilgan. Uning koordinata o'qlari bilan kesishgan nuqtalarini toping.

4.24. $M(3,2)$ va $N(4,3)$ nuqtalar orqali o'tuvchi to'g'ri chiziqning Ox o'qi bilan hosil qilgan burchagini toping.

4.25. $3x+y-4y=0$ va $2x-y+1=0$ to'g'ri chiziqlarning kesishish nuqtasi orqali o'tib, $x+y-5=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4.26. Uchlari $A(2,1)$, $B(3,1)$ va $C(1,2)$ nuqtada bo'lgan uchburchak tomonlarining uzunliklari va ichki burchaklarini toping

4.27. Koordinatalar boshidan o'tib: 1) $y=2x+3$ to'g'ri chiziqqa parallel bo'lgan, 2) $x-3y-1=0$ to'g'ri chiziqqa perpendikulyar bo'lgan, 3) $y=3x-5$ to'g'ri chiziq bilan 45° li burchak hosil qiladigan to'g'ri chiziq tenglamasini tuzing.

4.28. Uchburchakning ikkita uchi $A(3,4)$ va $B(5,1)$ nuqtada bo'lib, uning balandliklari $(4,1)$ nuqtada kesishadi. Uchburchakning uchlari C uchini toping.

4.29. $2x+3y-6=0$ to'g'ri chiziq Ox , Oy o'qlarni A, B nuqtalarda kesib o'tadi. C nuqta AB kesmani $AC:CB=1:2$ nisbatda bo'ladi. C nuqtadan AB to'g'ri chiziqqa tushirilgan perpendikulyarning tenglamasini tuzing.

4.30. Uchburchakning $3x+4y-12=0$ va $3x-7y+21=0$ tomonlari berilgan. $M(4;3)$ nuqta uning medianalarining kesishish nuqtasi ekanligi ma'lum. Uchburchakning uchinchi tomoni tenglamasini tuzing.

4.31. Birinchi chorakda joylashgan va tomonining uzunligi 5 uzunlik birligiga teng bo'lgan rombning ikki tomoni absissa o'qi va $4x-3y=0$ to'g'ri chiziq bilan ustma-ust tushadi. Rombning qolgan tomonlarining tenglamalari va dioganallari tenglamalari tuzilsin.

4.32. Koordinatalari mos ravishda $(1;-1)$, $(5;2)$ va $(4;5)$ bo'lgan A, B, C nuqtalar berilgan. AC to'g'ri chiziqqa nisbatan B nuqtaga simmetrik bo'lgan D nuqtaning koordinatalari topilsin va shu nuqtadan A, C nuqtalar orqali o'tkazilgan to'g'ri chiziqlarning tenglamalari tuzilsin.

4.33. Parallelogramm ikki uchining koordinatalari mos ravishda $(1;1)$ va $(2;-2)$ nuqtalarda bo'lib, dioganallari $(-1;0)$ nuqtada kesishadi. Parallelogramm tomonlarining tenglamalari tuzilsin.

4.39. $2x+3y-12=0$ to'g'ri chiziqda $(4;5)$ va $(1;-2)$ nuqtalardan teng uzoqlikda yotgan nuqtaning koordinatalari topilsin.

4.40. $A(-3;5)$ nuqtadan o'tuvchi va ordinata hamda absissa o'qlaridan kesgan kesmalarining uzunliklari nisbati $1:2$ kabi bo'lgan to'g'ri chiziqning tenglamasi tuzilsin.

4.41. $A(-1;3)$ nuqtadan o'tuvchi shunday to'g'ri chiziqlarning tenglamasi tuzilsinki, bu to'g'ri chiziqlarning $x+2y+5=0$ va $x+2y-2=0$ parallel to'g'ri chiziqlar orasidagi kesmaning o'rta nuqtasi $2x-3y-11=0$ to'g'ri chiziqda yotsin.

4.42. Tekislikdagi $x-2y+2=0$ to'g'ri chiziqqa $x+2y-6=0$ to'g'ri chiziqqa nisbatan ikki marta yaqin joylashgan nuqtalar geometrik o'rinlarining tenglamalari tuzilsin.

4.43. Uchlari $A(-4,2)$, $B(0,-1)$ va $C(3,-3)$ nuqtalarda bo'lgan uchburchakning perimetrini toping.

4.44. $A(2,1)$ nuqtadan ham, OY o'qdan ham 5 birlikka uzoqlashgan nuqta topilsin.

4.45. Ox o'qida $A(8,4)$ nuqtadan va koordinatlar boshidan baravar uzoqlikda turgan nuqta topilsin.

4.46. Uchlari $A(4,3)$, $B(-3,2)$ va $C(1,-6)$ nuqtalarda bo'lgan uchburchakka tashqi chizilgan doiraning markazi va radiusi topilsin.

4.47. Ordinatalar o'qida koordinatlar boshidan va $A(-2,5)$ nuqtadan baravar uzoqlikda turgan nuqta topilsin.

4.48. Absissalar o'qida $A(-2,3)$ nuqtadan $3\sqrt{5}$ birlikka uzoqlashgan nuqta topilsin.

4.49. $A(-3,-1)$ va $B(5,3)$ nuqtalar orasidagi masofani toping.

4.50. $A(5,3)$ va $B(6,-4)$ nuqtalar orasidagi masofani toping.

4.51. $A(-2;1)$ va $B(3;6)$ nuqtalar yasalsin. AB kesmani $AN:NB=2:1$ nisbatda bo'luvchi $N(x,y)$ nuqta topilsin.

4.52. $A(-2;3)$ va $B(3;6)$ nuqtalar yasalsin. AB kesmani $AM:MB=2:1$ nisbatda bo'luvchi $M(x,y)$ nuqta topilsin.

4.53. Uchlari $A(2;-1)$, $B(4;3)$ va $C(-2,1)$ nuqtalarda bo'lgan uchburchak tomonlarining o'rtalari aniqlansin.

4.54. Uchlari $O(0;0)$, $A(8,0)$ va $B(0,6)$ nuqtalarda bo'lgan uchburchakda OS medianasi va OD bissektrisa uzunliklari aniqlansin.

4.55. Uchlari $A(2;0)$, $B(5;3)$ va $C(2;6)$ nuqtalarda bo'lgan uchburchakning yuzi hisoblansin.

4.56. Uchlari $A(3;1)$, $B(5;3)$ va $C(2;6)$ nuqtalarda bo'lgan to'rtburchakning yuzi hisoblansin.

4.57. Oy o'qidan $b=3$ kesma ajratib, Ox o'q bilan $1) 45^\circ$, $2) 135^\circ$ burchak tashkil qiluvchi to'g'ri chiziqlar yasalsin. Bu to'g'ri chiziqlarning tenglamalari yozilsin.

4.58. Oy o'qdan $b=-3$ kesma ajratib, Ox o'q bilan 1) 60° , 2) 120° burchak tashkil qiluvchi to'g'ri chiziqlar yasalsin. Bu to'g'ri chiziqlarning tenglamalari yozilsin.

4.59. Koordinatlar boshidan o'tib, Ox o'q bilan: 1) 45° ; 2) 60° ; 3) 90° ; 4) 120° ; 5) 135° burchak tashkil qiluvchi to'g'ri chiziqlarning tenglamalari yozilsin.

4.60. Koordinatlar boshidan va $(-2;3)$ nuqtadan o'tuvchi to'g'ri chiziq yasalsin va uning tenglamasi yozilsin.

4.61. $A(2;-1)$ va $B(3,4)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasi yozilsin.

$$1) 2x-3y=6 \quad 2) 3x+3y=0 \quad 3) y=-3 \quad 4) \frac{x}{4} + \frac{y}{3} = 1$$

to'g'ri chiziqlarning har qaysisi uchun k va b parametrlar aniqlansin.

4.62. 1) $3x+4y=12$ 2) $3x-4y=0$ 3) $2x-5=0$ 4) $2y+5=0$ to'g'ri chiziqlar yasalsin.

4.63. $A(2;3)$ nuqtadan o'tib, Ox o'q bilan 45° burchak tashkil qiluvchi to'g'ri chiziqning k va b parametrlari aniqlansin. Bu to'g'ri chiziqning tenglamasi yozilsin.

4.64. 1) $2x-3y=6$ 2) $3x-2y+4=0$ to'g'ri chiziqning tenglamalari o'qlardan ajratgan kesmalariga nisbatan yozilsin.

4.65. Quyidagi to'g'ri chiziqlar orasidagi burchak aniqlansin:

$$1) \begin{cases} y = 2x - 3 \\ y = \frac{1}{2}x + 1 \end{cases} \quad 2) \begin{cases} 5x - y + 7 = 0 \\ 2x - 3y + 1 = 0 \end{cases} \quad 3) \begin{cases} 2x + y = 0 \\ y = 3x - 4 \end{cases}$$

4.66. $A(-4;-3)$, $B(-5;0)$, $C(5;6)$ va $D(1;0)$ nuqtalar trapetsiyaning uchlari bo'lishi tekshirilsin va uning balandligi topilsin.

4.67. $y = \frac{1}{3}x + 6$ to'g'ri chiziq berilgan. Uning koordinata o'qlari bilan kesishish nuqtalarini toping.

4.68. $3x+y-6=0$ va $x+2y+1=0$ to'g'ri chiziqlarning kesishish nuqtalarini toping.

4.69. $M(-4;-1)$ nuqta orqali o'tuvchi va $4x+y-3=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4.70. $M(-3;-5)$ nuqta orqali o'tuvchi $2x+2y-3=0$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziq tenglamasini tuzing.

4.71. ABC uchburchak uchlarning koordinatlari berilgan:

$A(-1;0)$, $B(15;-5)$, $C(1;16)$. AB va BC tomonlarning tenglamasi tuzilsin.

4.72. To'g'ri chiziq $3x-4y+14=0$ tenglama bilan berilgan. Uning koordinata o'qlari bilan kesishgan nuqtalarini toping.

4.73. $M(4;6)$ va $N(3;-2)$ nuqtalar orqali o'tuvchi to'g'ri chiziqlarning Ox o'qi bilan xosil qilgan burchagini toping.

4.74. $4x-3y+1=0$ va $2y+y+4=0$ to'g'ri chiziqlarning kesishish nuqtasi orqali o'tib, $x-2y+5=0$ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

4.75. $x-4y+3=0$ va $2x-y+5=0$ to'g'ri chiziqlarning tekislikdagi o'zaro qanday joylashishini tekshiring.

4.76. $A(3;3)$ nuqtadan $12x-16y-6=0$ to'g'ri chiziqqacha bo'lgan masofani toping.

4.77. $M(1;4)$ va $N(3;-2)$ nuqtalardan o'tuvchi to'g'ri chiziqlarning burchak koeffitsiyentini toping va tenglamasini tuzing.

4.78. Burchak koeffitsiyenti $k=-3$ bo'lgan va $(-1,4)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

4.79. Koordinata o'qlaridan $a=3$, $b=2$ kesmalar ajratuvchi to'g'ri chiziq tenglamasini toping.

4.80. $M(3;-2)$ nuqtadan $2x-4y+6=0$ to'g'ri chiziqqacha bo'lgan masofani toping.

4.81. $A(-6;5)$ nuqtadan hamda $x+4y-5=0$ va $2x-5y-1=0$ to'g'ri chiziqlarning kesishish nuqtasidan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Mavzu bo'yicha testlar

To'g'ri chiziq tenglamalari va ularga doir masalalar.

1. Analitik geometriyada chiziq nima asosida o'rganiladi?

A) tenglama. B) chizma. C) proektsiya. D) ta'rif. E) nuqtalar.

2. Tekislikdagi to'g'ri chiziqlarning umumiy tenglamasini ko'rsating:

A) $y=kx+b$. B) $\frac{x}{a} + \frac{y}{b} = 1$. C) $Ax+By+C=0$.

D) $x \cos \alpha + y \sin \alpha - p = 0$. E) $\frac{x-x_0}{m} = \frac{y-y_0}{n}$.

3. Tekislikdagi to'g'ri chiziqlarning umumiy $3x+5y+2=0$ tenglamasi bo'yicha uning n normal vektorini toping.

A) $n=(5,2)$. B) $n=(3,5)$. C) $n=(3,2)$. D) $n=(2,5)$. E) $n=(5,3)$.

4. $Ax+By=0$ ($A, B \neq 0$) tenglama qanday to'g'ri chiziqni ifodalaydi?

A) Ox koordinata o'qiga parallel bo'lgan.

B) Ox koordinata o'qiga perpendikulyar bo'lgan.

C) Oy koordinata o'qiga parallel bo'lgan.

D) Oy koordinata o'qiga perpendikulyar bo'lgan.

E) koordinata boshidan o'tuvchi.

5. Tekislikdagi to'g'ri chiziqlarning burchak koeffitsiyentli tenglamasini ko'rsating. A) $y = kx + b$. B) $\frac{x}{a} + \frac{y}{b} = 1$. C) $Ax + By + C = 0$.

D) $x \cos \alpha + y \sin \alpha - p = 0$. E) $\frac{x - x_0}{m} = \frac{y - y_0}{n}$.

6. Tekislikdagi to'g'ri chiziq OX o'qi bilan 45° burchak tashkil etadi. Quyidagi tenglamalardan qaysi biri uning burchak koeffitsiyentli tenglamasi bo'la oladi?

A) $y = 3x + 2$. B) $y = 2x + 3$. C) $y = x + 2$. D) $y = -x + 7$. E) $y = -3x + 1$.

7. Umumiy tenglamasi $3x - 2y + 4 = 0$ bo'lgan to'g'ri chiziqlarning burchak koeffitsiyentli tenglamasini yozing.

A) $y = 0,75x + 1$. B) $y = 0,5x + 2$. C) $y = 1,75x + 1$. D) $y = 1,5x + 2$.
E) $y = 3x + 4$.

8. Tekislikdagi to'g'ri chiziqlarning kesmalardagi tenglamasini ko'rsating. A) $y = kx + b$. B) $\frac{x}{a} + \frac{y}{b} = 1$. C) $Ax + By + C = 0$.

D) $x \cos \alpha + y \sin \alpha - p = 0$. E) $\frac{x - x_0}{m} = \frac{y - y_0}{n}$.

9. Umumiy tenglamasi $5x + 2y - 20 = 0$ bo'lgan to'g'ri chiziqlarning kesmalardagi tenglamasini yozing.

A) $\frac{x}{5} + \frac{y}{2} = 1$. B) $\frac{x}{2} + \frac{y}{5} = 1$. C) $\frac{x}{4} + \frac{y}{10} = 1$. D) $\frac{x}{10} + \frac{y}{4} = 1$. E) $\frac{x}{5} - \frac{y}{2} = 1$.

10. Tekislikdagi to'g'ri chiziqlarning normal tenglamasini ko'rsating.

A) $y = kx + b$. B) $\frac{x}{a} + \frac{y}{b} = 1$. C) $Ax + By + C = 0$.

D) $x \cos \alpha + y \sin \alpha - p = 0$. E) $\frac{x - x_0}{m} = \frac{y - y_0}{n}$.

11. Qaysi tenglama $M(x_0, y_0)$ nuqtadan o'tuvchi to'g'ri chiziqlar dastasini ifodalaydi?

A) $y - y_0 = k(x - x_0)$. B) $\frac{y - y_0}{x - x_0} = k$. C) $\frac{x - x_0}{y - y_0} = \frac{1}{k}$.

D) $A(x - x_0) + B(y - y_0) = 0$. E) $y - y_0 = k(x + x_0)$.

12. $M(-1, 4)$ nuqtadan o'tuvchi barcha to'g'ri chiziqlar tenglamasini ko'rsating.

A) $(x + 1)^2 + (y + 4)^2 = 0$. B) $y = k(x - 1) + 4$. C) $y = k(x - 1) - 4$.
D) $y = k(x + 1) + 4$. E) $y = k(x + 1) - 4$.

13. Qaysi tenglama $M(-3,1)$ va $N(0,7)$ nuqtalardan o'tuvchi to'g'ri chiziqda yotuvchi $P(1,y)$ nuqtaning ordinatasini toping.

- A) $y=0$. B) $y=-4$. C) $y=9$. D) $y=-7$. E) $y=\pm 5$.

14. Ikki $y=k_1x+b_1$ va $y=k_2x+b_2$ to'g'ri chiziqlar orasidagi α burchak tangensi formulasini toping.

A) $\operatorname{tg} \alpha = \frac{k_2 - k_1}{1 + k_1 k_2}$. B) $\operatorname{tg} \alpha = \frac{k_1 + k_2}{1 - k_1 k_2}$.

C) $\operatorname{tg} \alpha = \frac{k_1 \cdot k_2}{1 + k_1 k_2}$. D) $\operatorname{tg} \alpha = \frac{k_1 - k_2}{1 - k_1 k_2}$. E) $\operatorname{tg} \alpha = \frac{1 + k_1 k_2}{k_2 - k_1}$.

15. $y=2x-3$ va $y=-3x+5$ to'g'ri chiziqlar orasidagi burchakni toping.

- A) 90° . B) 60° . C) 45° . D) 30° . E) 120° .

16. $y=k_1x+b_1$ va $y=k_2x+b_2$ to'g'ri chiziqlarning parallellik shartini ko'rsating.

- A) $k_1 \cdot k_2 = -1$. B) $k_1 + k_2 = 0$. C) $k_1 \cdot k_2 = 1$. D) $k_1 - k_2 = 0$. E) $k_1 \cdot k_2 = 0$.

17. $A_1x+B_1y+C_1=0$ va $A_2x+B_2y+C_2=0$ to'g'ri chiziqlarning perpendikulyarlik shartini ko'rsating.

- A) $A_1B_1+A_2B_2=0$. B) $A_1A_2+B_1B_2=0$. C) $A_1B_1-A_2B_2=0$.

- D) $A_1A_2-B_1B_2=0$. E) $A_1A_2+B_1B_2+C_1C_2=0$.

18. $M(-1,3)$ va $N(4,-2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamasini ko'rsating.

- A) $y=x-2$. B) $y=-x+2$. C) $y=3-x$. D) $y=4-x$. E) $y=1-x$.

19. Umumiy tenglamalari $2x-3y-37=0$ va $3x-4y-51=0$ bo'lgan to'g'ri chiziqlarning kesishish nuqtasi koordinatalarining yig'indisini toping.

- A) 3. B) -6. C) 5. D) -4. E) 0.

20. $M(1,1)$ nuqtadan $12x+5y+22=0$ to'g'ri chiziqgacha bo'lgan masofani toping.

- A) 3. B) 2. C) 13. D) $\sqrt{13}$. E) 4.

5-§. IKKINCHI TARTIBLI EGRI CHIZIQNING KANONIK TENGLAMASI. TEKISLIKDA BERILGAN CHIZIQ TENGLAMASI.

5.1. Aylana

Bizga ma'lumki, tekislikda berilgan (tayin) nuqtadan baravar uzoqlikda joylashgan nuqtalar (tekislik nuqtalari) to'plami aylana, berilgan nuqta esa aylana markazi deyiladi.

Endi aylananing tenglamasini keltirib chiqarish maqsadida tekislikda Dekart koordinatalar sistemasini va $M_0 = M_0(x_0, y_0)$ nuqtani olamiz. Ravshanki, bu nuqtadan r masofada ($r > 0$) joylashgan nuqtalar (bunday nuqtalar to'plami aylana bo'ladi) o'zgaruvchi nuqtalar bo'ladi. Bunday nuqtalardan birini $M = M(x, y)$ deylik. $M_0 = M_0(x_0, y_0)$ va $M = M(x, y)$ nuqtalar orasidagi masofa

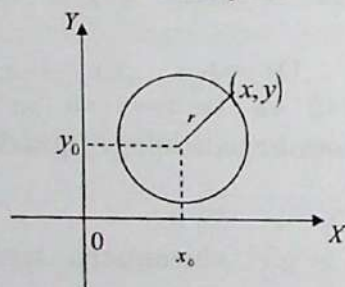
$$M_0M = \sqrt{(x-x_0)^2 + (y-y_0)^2}$$

bo'ladi.

Keyingi tenglikdan

$$(x-x_0)^2 + (y-y_0)^2 = r^2 \quad (5.1)$$

bo'lishi kelib chiqadi.



5.1-chizma

Shunday qilib, aylanada joylashgan o'zgaruvchi $M = M(x, y)$ nuqtaning koordinatalari x va y larni bog'lovchi tenglamaga keldik. Bu (5.1) tenglama aylananing sodda tenglamasi deyiladi, r esa aylana radiusi deyiladi.

Demak, aylananing tenglamasi markaz deb atalgan $M_0 = M_0(x_0, y_0)$ nuqtaga hamda r radiusga bog'liq bo'lib, ular yordamida

aylananing tekislikdagi holati to'liq aniqlanadi. Xususiyl holda markazi koordinatalar boshida bo'lgan aylana tenglamasi quyidagicha

$$x^2 + y^2 = r^2 \quad (5.2)$$

bo'ladi.

Masalan, markazi $(-1, 2)$ radiusi 5 ga teng bo'lgan aylananing tenglamasi

$$(x+1)^2 + (y-2)^2 = 25$$

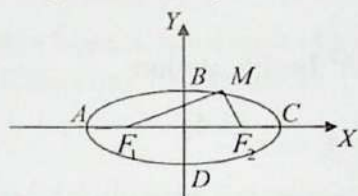
bo'ladi.

5.2. Ellips

Tekislikda ikkita tayin nuqtalarni olaylik. Tekislikning bu nuqtalargacha bo'lgan masofalari yig'indisi o'zgarmas songa teng bo'ladigan nuqtalari to'plami (nuqtalarning geometrik o'rni) ellips deyiladi.

Endi ellipsning tenglamasini keltirib chiqaramiz. Ta'rifda keltirilgan tayin nuqtalardan birini F_1 , ikkinchisini F_2 orqali belgilaymiz. Tekislikda Dekart koordinatalar sistemasini quyidagicha quramiz:

F_1 va F_2 nuqtalardan o'tuvchi to'g'ri chiziqni absissa o'qi (OX o'qi), $F_1 F_2$ kesmaning o'rtasidan o'tuvchi hamda absissa o'qiga perpendikulyar bo'lgan to'g'ri chiziqni ordinata o'qi (OY o'qi) deb olamiz. (5.2-chizma)



5.2-chizma

Aytaylik, F_1 va F_2 nuqtalar orasidagi masofa $2c$ ga ($c > 0$) teng bo'lsin. U holda bu nuqtalarning koordinatalari mos ravishda $(-c, 0)$ va $(c, 0)$ bo'ladi:

$$F_1(-c, 0), \quad F_2(c, 0).$$

Odatda, F_1 va F_2 nuqtalar ellipsning fokuslari deyiladi.

Ellipsda ixtiyoriy $M = M(x, y)$ nuqtani olaylik. Unda ellips ta'rifiga binoan $F_1 M$ va $F_2 M$ masofalar yig'indisi o'zgarmas songa teng bo'ladi. Bu o'zgarmas sonni $2a$ deylik ($a > 0$).

Demak,

$$F_1 M + F_2 M = 2a. \quad (5.3)$$

Ikki nuqta orasidagi masofa formulasidan foydalanib topamiz:

$$F_1 M = \sqrt{(x - (-c))^2 + (y - 0)^2} = \sqrt{(x + c)^2 + y^2},$$

$$F_2 M = \sqrt{(x - c)^2 + (y - 0)^2} = \sqrt{(x - c)^2 + y^2}.$$

Unda (5.3) ga ko'ra $\sqrt{(x + c)^2 + y^2} + \sqrt{(x - c)^2 + y^2} = 2a$ bo'ladi.

Bu tenglikni quyidagicha

$$\sqrt{(x + c)^2 + y^2} = 2a - \sqrt{(x - c)^2 + y^2}$$

yo'zib, uning ikki tomonini kvadratga ko'tarsak, unda

$$(x + c)^2 + y^2 = 4a^2 - 4a\sqrt{(x - c)^2 + y^2} + (x - c)^2 + y^2$$

bo'ladi. Bunda esa

$$x^2 + 2cx + c^2 = 4a^2 + 4a\sqrt{(x-c)^2 + y^2} + x^2 - 2cx + c^2$$

ya'ni

$$4cx - 4a^2 = -4a\sqrt{(x-c)^2 + y^2},$$

$$a^2 - cx = a\sqrt{(x-c)^2 + y^2}$$

bo'lishi kelib chiqadi. Keyingi tenglikning ikki tomonini kvadratga ko'tarish natijasida

$$(a^2 - cx)^2 = a^2((x-c)^2 + y^2),$$

ya'ni $x^2(a^2 - c^2) + a^2y^2 = a^2(a^2 - c^2)$ hosil bo'ladi.

Ravshanki, $2a > 2c$ ya'ni $a > c$ bo'lganligi uchun $a^2 - c^2 > 0$ bo'ladi.

Uni b^2 bilan belgilaymiz:

$b^2 = a^2 - c^2$. Natijada $x^2 \cdot b^2 + y^2 a^2 = a^2 b^2$ bo'lib, undan

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad (5.4)$$

bo'lishi kelib chiqadi.

Shunday qilib ellipsdagi o'zgaruvchi $M = M(x, y)$ nuqtaning koordinatalari x va y larni bog'lovchi tenglama hosil bo'ldi. Bu (5.4) tenglama ellipsning sodda tenglamasi deyiladi.

Ellips tenglamasi (5.4) da x ni $-x$ ga, y ni $-y$ ga almashtirilganda (5.4) tenglama o'zgarmaydi. Demak, ellips (yopiq egri chiziq) koordinata o'qlariga nisbat simmetrik joylashgan. Agar (5.4) tenglamada $y = 0$ deyilsa, unda $x^2 = a^2$, $x = \pm a$ bo'ladi. Demak, ellips OX o'qini ikki $A(-a, 0)$, $C(a, 0)$ nuqtalarda kesadi. Agar (5.4) tenglamada $x = 0$ deyilsa, unda $y^2 = b^2$, $y = \pm b$ bo'ladi.

Demak, ellips OY o'qini ikki $B(0, b)$, $D(0, -b)$ nuqtalarda kesadi.

Odatda, $A(-a, 0)$, $B(0, b)$, $C(a, 0)$, $D(0, -b)$ nuqtalar ellipsning uchlari deyiladi. AC kesma ellipsning katta o'qi, BD kesma ellipsning kichik o'qi deyiladi. Ravshanki, AC kesmaning uzunligi $2a$, BD kesmaning uzunligi esa $2b$ ga teng. Demak, (5.4) tenglamada a ellips katta yarim o'qi, b esa kichik yarim o'qi bo'ladi.

$$\text{Ushbu } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

tenglama bilan berilgan ellipsni qaraylik. Bu ellipsning fokuslari orasidagi masofa $2c$ ga teng.

$$\text{Ushbu } \varepsilon = \frac{2c}{2a} = \frac{c}{a} \quad (5.5)$$

miqdor ellipsning *ekstsentrisseti* deyiladi. Ma'lumki, $a > c$. Demak, ellipsning ekstsentrisseti uchun $0 < \varepsilon < 1$ bo'ladi. (agar $\varepsilon = 0$ bo'lsa, $c = 0$ bo'lib, ellips aylana bo'lib qoladi).

Ellipsning ekstsentrisseti ellipsning siqilish darajasini bildiradi. Haqiqatdan ham, (5.4) munosabatdan, $b^2 = a^2 - c^2$ bo'lishini e'tiborga olib topamiz:

$$\varepsilon^2 = \frac{c^2}{a^2} = \frac{a^2 - b^2}{a^2} = 1 - \left(\frac{b}{a}\right)^2, \quad \frac{b}{a} = \sqrt{1 - \varepsilon^2}$$

Bu tenglikdan ko'rinadiki, ε ning ortib borishi bilan $\frac{b}{a}$ nisbat kamaya boradi, binobarin ellips tortila boradi.

Ellipsning direktrisalari

$$x = \pm \frac{a}{\varepsilon} = \pm \frac{a^2}{c}, \quad a > b,$$

to'g'ri chiziqli tenglamalari orqali topiladi va ushbu xossalarga ega:

$$\frac{r_1}{d_1} = \frac{r_2}{d_2} = \varepsilon$$

1-misol. Katta o'qi 10 ga, ekstsentrisseti 0,8 ga teng bo'lgan ellipsning tenglamasi topilsin.

Yechish. Shartga ko'ra $2a = 10$. Demak, $a = 5$. Ma'lumki ekstsentrisset

$$\varepsilon = \frac{c}{a}, \quad c = a \cdot \varepsilon.$$

Unda $c = 5 \cdot 0,8 = 4$ bo'ladi. $b^2 = a^2 - c^2$ bo'lishidan $b^2 = 25 - 16 = 9$, $b = 3$

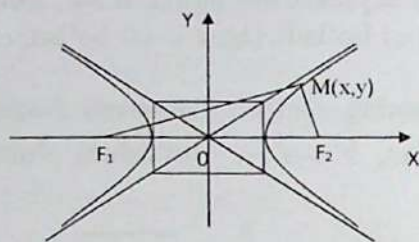
ekanligi kelib chiqadi. Izlanayotgan ellipsning tenglamasi

$$\frac{x^2}{25} + \frac{y^2}{9} = 1 \text{ bo'ladi. } \blacktriangleright$$

5.3. Giperbola

Tekislikda ikkita tayin nuqtalarni olaylik. Tekislikning bu nuqtalargacha bo'lgan masofalari ayirmasi o'zgarmas songa teng bo'ladigan nuqtalar to'plami (nuqtalarning geometrik o'rni) giperbola deyiladi. Endi giperbolaning tenglamasini keltirib chiqaramiz. Ta'rifda

keltirilgan nuqtalarni F_1 va F_2 orqali belgilaymiz. F_1 va F_2 nuqtalardan o'tuvchi to'g'ri chiziqni absissa o'qi, F_1F_2 kesmaning o'rtasidan o'tuvchi hamda absissa o'qiga perpendikulyar bo'lgan to'g'ri chiziqni ordinata o'qi deb koordinatalar sistemasini quramiz. (5.3-chizma)



5.3-chizma

Agar F_1 va F_2 nuqtalar orasidagi masofani $2c$ ($c > 0$) deyilsa, unda bu nuqtalarning koordinatalari mos ravishda $(-c, 0)$ va $(c, 0)$ bo'ladi:

$$F_1(-c, 0) \quad F_2(c, 0).$$

Bu F_1 va F_2 nuqtalar giperbolaning fokuslari deyiladi.

Giperbolada ixtiyoriy $M = M(x, y)$ nuqtani olaylik. Unda giperbola ta'rifiga binoan F_1M va F_2M masofalar ayirmasi o'zgarmas songa (uni $2a$ deyilsa) teng bo'lib, $F_1M - F_2M = 2a$, $F_2M - F_1M = 2a$, umuman

$F_1M - F_2M = \pm 2a$ bo'ladi. Ravshanki,

$$F_1M = \sqrt{(x+c)^2 + y^2}, \quad F_2M = \sqrt{(x-c)^2 + y^2}.$$

Demak,

$$\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = \pm 2a.$$

Endi $\sqrt{(x+c)^2 + y^2} - \sqrt{(x-c)^2 + y^2} = \pm 2a$ tenglikni (xuddi ellipsning tenglamasini keltirib chiqarishdagi qilingan ishlar kabi) ikki tomonini kvadratga ko'tarib, so'ng lozim bo'lgan soddalashtirishlarni bajarib, hosil bo'lgan tenglikni ya'na bir bor kvadratga ko'tarib, natijada

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad (5.6)$$

tenglamaga kelimiz, bunda $b^2 = c^2 - a^2$, ($a < c$).

Shunday qilib, giperboladagi o'zgaruvchi $M = M(x, y)$ nuqtaning koordinatalari x va y larni bog'lovchi tenglama hosil bo'ldi. Bu (5.6) tenglama giperbolaning sodda tenglamasi deyiladi.

Giperbola ham koordinata o'qlariga nisbatan simmetrik joylashgan, u 5.3-chizmada tasvirlangan. Giperbola ikki qismdan iborat bo'lib, bu qismlar uning shoxchalari deyiladi. Agar (5.6) tenglamada $y = 0$ deyilsa, unda

$x^2 = a^2$, $x = \pm a$ bo'ladi. Demak, giperbola OX o'qini $A(-a, 0)$ va $B(a, 0)$ nuqtalarda kesadi. Bu nuqtalar giperbolaning uchlari deyiladi. Giperbola OY o'qi bilan kesishmaydi.

Ushbu $\varepsilon = \frac{c}{a}$ miqdor giperbolaning *ekstsentrisseti* deyiladi.

Giperbolaning ekstsentrisseti ham uning shaklini xarakterlaydigan miqdordir. Giperbola tenglamasi

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ ni } y \text{ ga nisbatan yechib}$$

$$y = \pm \frac{b}{a} \sqrt{x^2 - a^2},$$

uni quyidagicha yozamiz:

$$y = \pm \frac{b}{a} x \sqrt{1 - \frac{a^2}{x^2}}.$$

Bu tenglikdan ko'rinadiki, x yetarlicha katta bo'lganda, $\frac{a^2}{x^2}$ nisbat 0 ga yaqin bo'lib,

$$\sqrt{1 - \frac{a^2}{x^2}} \text{ miqdor } 1 \text{ ga yaqin bo'ladi.}$$

$$\text{Natijada ushbu } y = \pm \frac{b}{a} x \sqrt{1 - \frac{a^2}{x^2}} \approx \pm \frac{b}{a} x \text{ munosabat hosil bo'ladi.}$$

Demak, x yetarlicha katta bo'lganda giperbola nuqtalarining ordinatalari ushbu

$$y = \pm \frac{b}{a} x$$

to'g'ri chiziqlar nuqtalarining ordinatalariga yetarlicha yaqin bo'ladi. Bu

$$y = \pm \frac{b}{a} x \quad (5.7)$$

to'g'ri chiziqlar giperbolaning *asimptotalari* deyiladi (5.3-chizma)

Giperbolaning *direktrisalari*

$$x = \pm \frac{a}{\varepsilon} = \pm \frac{a^2}{c}, \quad (5.8)$$

to'g'ri chiziq tenglamalari orqali topiladi va ushbu xossalarga ega:

$$\frac{r_1}{d_1} = \frac{r_2}{d_2} = \varepsilon. \quad (5.9)$$

2-misol. Ushbu $16x^2 - 25y^2 = 400$ giperbolaning fokuslari, ekstsentrisseti va asimptotalari topilsin.

Yechish. Agar tenglamaning ikki tomonini 400 ga bo'lsak, unda giperbolaning tenglamasi quyidagi

$$\frac{x^2}{25} - \frac{y^2}{16} = 1 \text{ ko'rinishga keladi.}$$

Demak,

$$a^2 = 25, \quad b^2 = 16, \quad c = \sqrt{a^2 + b^2} = \sqrt{25 + 16} = \sqrt{41},$$

$$F_1 = F_1(-\sqrt{41}, 0), \quad F_2 = F_2(\sqrt{41}, 0),$$

$$e = \frac{c}{a} = \frac{\sqrt{41}}{5}$$

asimptotalari esa

$$y = \frac{4}{5}x, \quad y = -\frac{4}{5}x$$

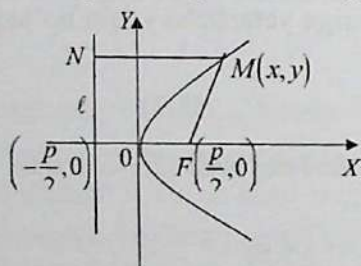
bo'ladi. ►

5.4. Parabola

Tekislikda tayin ℓ to'g'ri chiziq va bu to'g'ri chiziqda yotmagan tayin F nuqtani olaylik. Tekislikning ℓ to'g'ri chiziq hamda F nuqtadan baravar uzoqlikda bo'lgan nuqtalari to'plami (nuqtalarning geometrik o'rni) parabola deyiladi.

Endi parabolaning tenglamasini keltirib chiqaramiz.

F nuqtadan o'tuvchi va ℓ to'g'ri chiziqqa perpendikulyar bo'lgan to'g'ri chiziqni absissa o'qi (OX o'qi), F nuqta va ℓ to'g'ri chiziq orasidagi kesmaning o'rtasidan o'tuvchi va absissa o'qiga perpendikulyar bo'lgan to'g'ri chiziqni ordinata o'qi (OY o'qi) deb koordinatalar sistemasini quramiz (5.4-chizma).



5.4-chizma

F nuqta bilan ℓ to'g'ri chiziq orasidagi masofani p deylik. Unda

F nuqtaning koordinatasi $\left(\frac{p}{2}, 0\right)$

$$F = F\left(\frac{p}{2}, 0\right)$$

bo'lib, ℓ to'g'ri chiziqning tenglamasi

$$x = -\frac{p}{2} \text{ bo'ladi.}$$

Bu $F\left(\frac{p}{2}, 0\right)$ nuqta parabolaning fokusi, ℓ to'g'ri chiziq esa parabolaning direktrissi deyiladi. Parabola ixtiyoriy $M(x, y)$ nuqtani olaylik. Unda parabola ta'rifiga binoan

$NM = FM$ bo'ladi. Ravshanki,

$$NM = x + \frac{p}{2}, \quad FM = \sqrt{\left(x - \frac{p}{2}\right)^2 + y^2}.$$

demak,

$$x + \frac{p}{2} = \sqrt{\left(x - \frac{p}{2}\right)^2 + y^2}.$$

Bu tenglikning ikki tomonini kvadratga ko'tarib, so'ng lozim bo'lgan soddalashtirishlarni bajarib

$$y^2 = 2px \quad (5.10)$$

bo'lishini topamiz.

Shunday qilib, paraboladagi o'zgaruvchi $M(x, y)$ nuqtaning koordinatalari x va y larni bog'lovchi tenglama hosil bo'ladi. Bu (5.10) tenglama parabolaning sodd tenglamasi deyiladi.

Ravshanki, $x = 0$ da $y = 0$ bo'ladi. Demak, parabola koordinata boshidan o'tadi. Ayni paytda, uning tenglamasida y kvadratda qatnashgani uchun parabola OX o'qiga nisbatan simmetrik, x esa har doim musbat bo'lgani uchun parabola OY o'qining o'ng tomonida joylashgan bo'ladi (5.4-chizma).

3-misol. Ushbu $A(1, 2)$ nuqtadan o'tuvchi parabola tenglamasi topilsin.

Yechish. Modomiki izlanayotgan parabola $y^2 = 2px$ $A(1, 2)$ nuqtadan o'tishi lozim ekan, unda bu nuqtaning koordinatalari parabola tenglamasini qanoatlantiradi:

$$2^2 = 2p \cdot 1$$

Bu tenglamadan $p = 2$ ekani kelib chiqadi. Demak,

$$y^2 = 4x. \blacktriangleright$$

Mustaqil bajarish uchun misollar

Aylana.

5.1. Markazi $A(a,b)$ nuqtada va radiusi R ga teng bo'lgan aylananing umumiy tenglamasini tuzing.

1) $A(2,-3)$ va $R=6$, 2) $A(-4,-3)$ va $R=5$, 3) $A(5,0)$ va $R=2$, 4) $A(-3,-2)$ va $R=\sqrt{5}$, 5) $A(0,-3)$ va $R=11$, 6) $A(3,5)$ va $R=7$.

5.2. Radiusi $\sqrt{15}$ ga teng hamda $(2; 0)$ va $(0; -2)$ nuqtalardan o'tuvchi aylana tenglamasini tuzing.

5.3. $4x^2 + 4y^2 - 12x + 16y = 0$ tenglama bilan berilgan aylana radiusini va markazini aniqlang.

5.4. $x^2 + y^2 - 8x + 12y - 10 = 0$ tenglama bilan berilgan aylana radiusini va markazini aniqlang.

5.7. $x^2 + y^2 - 6x + 8y - 24 = 0$ tenglama bilan berilgan aylana radiusini va markazini aniqlang.

5.8. a) Markazi $C(-4,3)$, radiusi $R=5$ bo'lgan aylana tenglamasi yozilsin va u yasalsin.

5.9. $A(-1,-1)$, $B(3,2)$, $O(0,0)$ nuqtalar bu aylanada yotadimi?

5.10. a) Aylananing ushbu $2x^2 + 2y^2 - 3x + 4y + 2 = 0$ tenglamasiga ko'ra, uning markazi va radiusini aniqlang. b). Aylana diametrlaridan biri uchlarining koordinatalari berilgan: $A(1;4)$ va $B(-3;2)$. Shu aylananing tenglamasini tuzing.

5.11. $A(-4,6)$ nuqta berilgan. Diametri OA kesmadan iborat aylana tenglamasi yozilsin.

5.12. 1) $x^2 + y^2 - 4x + 6y - 3 = 0$; 2) $x^2 + y^2 - 8x = 0$; 3) $x^2 + y^2 + 4y = 0$ aylanalar yasalsin.

5.13. $x^2 + y^2 + 5x = 0$ aylana $x+y=0$ to'g'ri chiziq yasalgan va ularning kesishgan nuqtalari topilsin.

5.14. $A(1,2)$ nuqtadan o'tuvchi va koordinata o'qlariga urinuvchi aylana tenglamasi yozilsin.

5.15. $x^2 + y^2 + 4x - 6y = 0$ aylananing Oy o'q bilan kesishgan nuqtalariga o'tkazilgan radiuslari orasidagi burchak topilsin.

5.12. $A(-1,3)$, $B(0,2)$ va $C(1,-1)$ nuqtalardan o'tuvchi aylana tenglamasi yozilsin.

5.13. $A(4,4)$ nuqtadan va $x^2 + y^2 + 4x - 4y = 0$ aylana bilan $y=-x$ to'g'ri chiziqning kesishgan nuqtalaridan o'tuvchi aylana tenglamasi yozilsin.

5.14. $y = -\sqrt{-x^2 - 4x}$ egri chiziqning joylashish sohasi aniqlab, shakli chizilsin.

5.15. $x^2 + y^2 - 8x - 4y + 16 = 0$ aylanaga koordinatalar boshidan o'tkazilgan urinmalarning tenglamalari yozilsin.

5.16. $A(a, 0)$ nuqta berilgan. M nuqta shunday harakat qiladiki, $\triangle OMA$ da OMA burchak doimo to'g'ri burchak bo'lib qoladi. M nuqta trayektoriyasining tenglamasi yozilsin.

5.17. $A(-6, 0)$ va $B(2, 0)$ nuqtalar berilgan. Shunday nuqtalarning geometrik o'rni topilsinki, ulardan OA va OB kesmalar teng burchaklar ostida ko'rinsin.

5.18. $M(x, y)$ nuqta shunday harakatlanadiki, undan $A(-a, 0)$, $B(0, a)$ va $C(a, 0)$ nuqtalargacha bo'lgan masofalar kvadratlarining yig'indisi $3a^2$ ga teng bo'lib qolaveradi. Nuqta trayektoriyasining tenglamasi yozilsin.

5.19. $M(x, y)$ nuqta shunday harakatlanadiki, undan koordinata burchaklarining bissektrisalarigacha bo'lgan masofalar kvadratlarining yig'indisi a^2 ga teng bo'lib qolaveradi. Nuqta trayektoriyasining tenglamasi yozilsin.

5.20. $x^2 + y^2 = a^2$ aylana berilgan. Uning $A(a, 0)$ nuqtasidan mumkin bo'lgan barcha vatarlar o'tkazilgan. Bu vatarlar o'rtalarining geometrik o'rni aniqlansin.

5.21. $A(-3, 0)$ va $B(3, 6)$ nuqtalar berilgan. Diametrik AB kesmadan iborat aylana tenglamasi yozilsin.

Ellips.

5.22. a) $x^2 + 4y^2 = 16$ ellips yasalsin, uning fokuslari va ekstsentrisiteti topilsin. **b)** Uchlari $(\pm 5; 0)$ va $(0; \pm 3)$ bo'lgan ellipsning kanonik tenglamasi va fokus nuqtalarining koordinatlarini toping.

5.23. a) Agar ellipsning: 1) fokuslari orasidagi masofa 8 ga teng bo'lib, kichik yarim o'qi $b = 3$; 2) katta yarim o'qi $a = 6$, ekstrensiteti $\varepsilon = 0,5$ bo'lsa, uning kanonik tenglamasi yozilsin. **b)** Ellipsning fokuslari absissa o'qida hamda o'qlari 16 va 10 bo'lsa kanonik tenglamani tuzing. **c)** Ellipsning fokus nuqtalarining koordinatalari $(\pm 3; 5)$ va katta o'qi 10 ga teng kanonik tenglamasini yozing.

5.24. a) Ellipsning katta yarim o'qi $a = 5$ va c parametri: 1) 4,8; 2) 4; 3) 3; 4) 1,4; 5) 0 ga teng bo'lsa, uning kichik yarim o'qi b va ekstrensiteti ε topilsin. **b)** Ellipsning $4x^2 + 9y^2 = 36$ tenglamasini kanonik ko'rinishiga olib keling va o'qlar uzunliklarini hisoblang. **c)** Ellips $a) x^2 + 4y^2 - 4x + 8y - 8 = 0$, $b) 9x^2 + 4y^2 + 54x - 40y + 145 = 0$ larning markazi fokus nuqtalarining koordinatlari hamda o'qlarining uzunliklarini toping.

5.25. a) Yer fokuslaridan birida quyosh joylashgan ellips bo'yicha harakat qiladi. quyoshdan yergacha bo'lgan eng kichik masofa taxminan

147,5 million kilometrğa, eng katta masofa 152,5 million kilometrğa teng bo'lsa, yer orbitasining katta yarim o'qi va ekstsentrisiteti topilsin.

b) Ellipsning $64x^2 + 9y^2 - 576 = 0$ tenglamasini kanonik ko'rinishga olib keling va fokus nuqtasini koordinatalarini toping.

5.26. a) Koordinata o'qlariga nisbatan simmetrik bo'lgan ellips

$M(2, \sqrt{3})$ va $B(0, 2)$ nuqtalardan o'tadi. Uning tenglamasi yozilsin va M nuqtadan fokuslargacha bo'lgan masofa topilsin. **b)** Ellipsning $16x^2 + 25y^2 - 400 = 0$ tenglamasini kanonik ko'rinishga olib keling va o'q uchlarini koordinatalarini toping.

5.27. a) Fokuslari Ox o'qda yotuvchi ellips koordinata o'qlariga nisbatan simmetrik bo'lib, $M(-4, \sqrt{21})$ nuqtadan o'tadi va $e = \frac{3}{4}$ ekstsentrisitetiga ega. Ellips tenglamasi yozilsin va M nuqtaning fokal radiuslari topilsin. **b)** Ellipsning fokus nuqtalari koordinatalari $(\pm 8, 0)$ va ekstsentrisiteti $e = 0,8$. Kanonik tenglamasini tuzing.

5.28. a) $x^2 + 2y^2 = 18$ ellipsning o'qlari orasidagi burchagini teng ikkiga bo'luvchi vatar uzunligi topilsin. **b)** Quyidagi egri chiziq tenglamasi qanday chiziqqa tegishli ekanligini aniqlang va grafigini chizing. $2x^2 + 2xy + 2y^2 + \sqrt{2}x - \sqrt{2}y - 5 = 0$

5.29. a) $9x^2 + 25y^2 = 225$ ellipsda shunday $M(x, y)$ nuqta topilsinki, undan o'ng fokusgacha bo'lgan masofa chap fokusgacha bo'lgan masofadan 4 marta katta bo'lsin. **b)** Ellips o'qlarining qanday nisbatida ekstsentrisitet

a) $\frac{1}{2}$, b) $\frac{1}{4}$, v) $\frac{1}{10}$, g) $\frac{1}{12}$ teng bo'ladi.

Giperbola.

5.30. a) $x^2 - 4y^2 = 16$ giperbola va uning asimptotalari yasalsin. Giperbolaning fokuslari, ekstsentrisiteti va asimptotalari orasidagi burchak topilsin. **b)** Quyidagi tenglamalarni kanonik ko'rinishga olib keltirib.

a) $9x^2 - 16y^2 - 144 = 0$, **b)** $2x^2 - y^2 = 6$, **c)** Giperbolaning fokusi $(0, \pm 3)$ va asimptotasi $y = \pm 2x$ berilgan. Giperbolaning tenglamasi va ekstsentrisitetini toping

5.31. a) $x^2 - 4y^2 = 16$ giperbolada ordinatasi 1 ga teng M nuqta olingan.

Undan fokuslargacha bo'lgan masofalar topilsin. **b)** Giperbola $\frac{x^2}{144} + \frac{y^2}{25} = 1$ ning fokus nuqtaning koordinatalari va o'q uzunligini toping?

c) Quyidagi tenglamalardan qaysilari giperbolani tenglamasini ifoda qiladi:

a) $3x^2 - 5y^2 + 10x + 12y - 78 = 0$, v) $3x^2 + 7y^2 - 140x + 11x - 500 = 0$,

b) $13x^2 - 17x + 10y - 24 = 0$, g) $4x^2 - 4y^2 - 4x + 7y - 120 = 0$.

5.32. a) Giperbola koordinata o'qlariga nisbatan simmetrik bo'lib, $M(6; 2\sqrt{2})$ nuqtadan o'tadi va $b=2$ mavhum yarim o'qqa ega. Uning tenglamasi yozilsin hamda M nuqtadan fokuslargacha bo'lgan masofalar topilsin. **b)** Teng tomonli giperbolaning ekstsentrissetini toping?

5.33. a) Uchlari $\frac{x^2}{25} + \frac{y^2}{9} = 1$ ellipsning fokuslari esa uning uchlarida bo'lgan giperbolaning tenglamasi yozilsin. **b)** $\frac{y^2}{64} - \frac{x^2}{17} = 1$ giperbolaning ekstsentrisseti fokus koordinatasini toping.

5.34. a) $y^2 = a^2 + x^2$ giperbola yasalsin, uning fokuslarining koordinatalari va asimptotalari orasidagi burchak topilsin.

b) $4x^2 - 25y^2 - 100 = 0$ giperbolaning asimptotalarini toping. **c)** Giperbola $\frac{(y+7)^2}{25} - \frac{(x-3)^2}{16} = 1$ ni 3 birlikga absissa o'qini manfiy yo'nalishida va 4 birlik ordinata o'qini musbat yo'nalishi bo'yicha parallel ko'chiring va tenglamasini tuzing.

5.35. a) $x^2 - 4y^2 = 16$ giperbolaga $A(0, -2)$ nuqtadan o'tkazilgan urinmalarning tenglamalari yozilsin. **b)** Giperbola $3x^2 - 5y^2 = 28$ va to'g'ri chiziq $2x - y = 8$ ning kesim nuqtasining koordinatalarini toping. **c)** Giperbola $\frac{(x+1)^2}{4} - \frac{(y-2)^2}{3} = 1$ ni ordinata o'qi musbat yo'nalishi parallel 2 birlikka siljiting.

5.36. a) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ giperbolaning fokusidan asimptotalarigacha bo'lgan masofalar va asimptotalari orasidagi burchak topilsin. **b)** $\frac{x^2}{25} - \frac{y^2}{4} = 1$ giperbola $A(5, 0)$, $B(\frac{5\sqrt{5}}{2}, -1)$, $C(5\sqrt{2}, -2)$ va $D(2, -3)$ nuqtalar orqali o'tadimi?

5.37. a) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ giperbolaga ichki chizilgan kvadratning tomoni topilsin va qanday giperbolalarga kvadartni ichki chizish mumkinligi tekshirilsin. **b)** Giperbolaning fokus nuqtasini koordinatalari $(\pm 2\sqrt{2}, 0)$ va asimptotasi $y = \pm x$ berilgan. Giperbolaning tenglamasini toping.

Parabola.

5.38. a) $F(0,2)$ nuqtadan va $y = 4$ chiziqdan bir xil uzoqlashgan nuqtalar geometrik o'rnining tenglamasi tuzilsin. Bu egri chiziqning koordinata o'qlari bilan kesishgan nuqtalari topilsin va u yasalsin. **b)** $y^2 = 12x$ parabola grafigi $A(3; -6)$, $B(0.5, \sqrt{6})$ va $C(1;3)$ nuqtalar orqali o'tadimi?

5.39. a) Koordinatalar boshidan va $x = -4$ to'g'ri chiziqdan bir xil uzoqlikda bo'lgan nuqtalar geometrik o'rnining tenglamasi tuzilsin. Bu egri chiziqning koordinata o'qlari bilan kesishgan nuqtalari topilsin va u yasalsin. **b)** Har bir parabolaning fokus va direktrissasini toping:

c) $y^2 = -12x$, b) $y^2 + x = 0$, d) $3x - y^2 = 0$.

5.40. a) $(0,0)$ va $(1,-3)$ nuqtalardan o'tuvchi va Ox o'qqa nisbatan simmetrik; **b)** $(0,0)$ va $(2,-4)$ nuqtalardan o'tuvchi Oy o'qqa simmetrik bo'lgan parabola tenglamasi yozilsin. **c)** Uchi koordinata boshida va fokusi a) $(3,0)$, b) $(-2,0)$, c) $(0,4)$, d) $(0; -\frac{1}{4})$ bo'lgan parabolalarning tenglamasini toping.

5.41. a) Markazi $y^2 = 2px$ parabolaning fokusida bo'lib, parabola direktrissasiga urinuvchi aylana tenglamasi yozilsin parabola va aylananing kesishgan nuqtalari topilsin. **b)** Uchi koordinata boshida va direktrissasi

a) $x = 2$, b) $x = -5$, v) $y = 3$, g) $y = -2$ bo'lgan parabola tenglamasini toping.

5.42. a) $x^2 + y^2 + 4y = 0$ aylana $x + y = 0$ to'g'ri chiziqning kesishgan nuqtalaridan o'tib, Oy o'qqa nisbatan simmetrik bo'lgan parabolaning va uning direktrissasining tenglamalari yozilsin. Aylana, to'g'ri chiziq va parabola yasalsin. **b)** Radius vektori 10 bo'lgan $y^2 = 8x$ parabolaning fokus nuqtasini koordinatasi topilsin.

5.43. a) $y^2 = 6x$ parabolada fokal radius vektori 4,5 ga teng bo'lgan nuqta topilsin. **b)** $y^2 = 36x$ parabolaning $(4; y)$ nuqtasini fokal radius vektorini toping.

5.44. a) Projektorning oynali sirti parabolaning o'z simmetrik o'qi atrofida aylanishidan hosil bo'lgan oynaning diametri 80 sm, chuqurligi 10 sm. Nurlarning parallel dasta sharidan qaytishi uchun yorug'lik manbai parabolaning fokusida o'rnatilishi kerak bo'lsa, yorug'lik manbai parabola uchidan qanday masofada o'rnatilishi kerak?

b) Parabolani uchi va o'qini toping. a) $y = x^2 + 5$, b) $y = 12 - x^2$,

v) $y = x^2 + 2x - 3$, g) $x = y^2 + 8y$, d) $y = x^2 - x + 4$.

5.45. $y = -\sqrt{x}$ egri chiziqning joylashish sohasi aniqlansin. Bu egri chiziq yasalsin.

5.46. $y^2 = 6x$ parabola uchidan o'tishi mumkin bo'lgan barcha vatarlar o'tkazilgan, bu vatarlar o'rtalari geometrik o'rning tenglamasi yozilsin.

Mavzu bo'yicha testlar

1. II tartibli tenglama va chiziqlar. Aylana va ellips.

1. Markazi $M(a,b)$ nuqtada va radiusi R bo'lgan aylana tenglamasini ko'rsating.

A) $(x+a)^2 + (y+b)^2 = R^2$. B) $(x+b)^2 + (y+a)^2 = R^2$.

C) $(x-a)^2 + (y-b)^2 = R^2$. D) $(x-b)^2 + (y-a)^2 = R^2$.

E) $(x-a) + (y-b) = R$.

2. Markazi $M_0(x_0, y_0)$ nuqtada joylashgan R radiusli aylana tenglamasini ko'rsating.

A) $x_0 + y_0 = R$.

B) $|x - x_0| + |y - y_0| = R$.

C) $(x - x_0)^2 + (y - y_0)^2 = R^2$.

D) $(x + x_0)^2 + (y + y_0)^2 = R^2$.

E) $xx_0 + yy_0 = R^2$.

3. Umumiy tenglamasi $x^2 + y^2 - 4x + 2y + 1 = 0$ bo'lgan aylananing R radiusini toping.

A) $R=2$.

B) $R=3$.

C) $R=4$.

D) $R=5$.

E) $R=6$.

4. Umumiy tenglamasi $x^2 + y^2 - 6x - 4y - 3 = 0$ bo'lgan aylananing $M_0(x_0, y_0)$ markazini toping.

A) $M_0(-2, -3)$. B) $M_0(2, 3)$. C) $M_0(-3, -2)$. D) $M_0(3, 2)$. E) $M_0(-3, 2)$.

5. Ta'rifni to'ldiring: Berilgan ikkita nuqtalargacha masofalar ... o'zgarish son bo'lgan tekislikdagi nuqtalarning geometrik o'rni ellips deyiladi.

A) ayirmasi.

B) ko'paytmasi.

C) yig'indisi.

D) bo'linmasi.

E) kvadratlarining yig'indisi.

6. Yarim o'qlari $a=5$ va $b=6$ bo'lgan ellipsning kanonik tenglamasi qayerda to'g'ri ko'rsatilgan?

A) $\frac{x^2}{5} + \frac{y^2}{6} = 1$.

B) $\frac{x^2}{5} - \frac{y^2}{6} = 1$.

C) $\frac{x^2}{25} + \frac{y^2}{36} = 1$.

D) $\frac{x^2}{25} - \frac{y^2}{36} = 1$.

E) $\frac{x}{25} + \frac{y}{36} = 1$.

7. Kanonik tenglamasi $(x/a)^2 + (y/b)^2 = 1$ ($a > b > 0$) bo'lgan ellipsning e eksentrisiteti formulasini ko'rsating:

$$A) \varepsilon = \frac{a^2 - b^2}{a}$$

$$B) \varepsilon = \frac{a^2 - b^2}{b}$$

$$C) \varepsilon = \frac{\sqrt{a^2 - b^2}}{a}$$

$$D) \varepsilon = \frac{\sqrt{a^2 - b^2}}{b}$$

$$E) \varepsilon = \frac{\sqrt{a^2 - b^2}}{ab}$$

8. Kanonik tenglamasi $(x/a)^2 + (y/b)^2 = 1$ ($a > b > 0$) bo'lgan ellipsning fokuslari $F(\pm c, 0)$ bo'lsa, c qanday formula bilan topiladi?

$$A) c = a^2 - b^2$$

$$B) c = b^2 - a^2$$

$$C) c = \sqrt{a^2 - b^2}$$

$$D) c = \sqrt{a^2 + b^2}$$

$$E) c = \sqrt{a - b}$$

9. Agar ellips $2x^2 + 5y^2 = 10$ tenglama bilan berilgan bo'lsa, uning o'ng fokusini toping.

$$A) F(\sqrt{5}, 0) \quad B) F(\sqrt{2}, 0) \quad C) F(2, 0) \quad D) F(1, 0) \quad E) F(\sqrt{3}, 0)$$

10. Kichik yarim o'qi 3 va fokuslari orasidagi masofa 8 bo'lgan ellipsning kanonik tenglamasini yozing.

$$A) \frac{x^2}{9} + \frac{y^2}{7} = 1$$

$$B) \frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$C) \frac{x^2}{9} + \frac{y^2}{25} = 1$$

$$D) \frac{x^2}{5} + \frac{y^2}{3} = 1$$

$$E) \frac{x^2}{16} + \frac{y^2}{9} = 1$$

11. Ellipsning eksentrisiteti ε qanday shartni qanoatlantiradi?

$$A) \varepsilon > 0$$

$$B) \varepsilon < 0$$

$$C) \varepsilon \neq 0$$

$$D) \varepsilon > 1$$

$$E) 0 < \varepsilon < 1$$

12. Kanonik tenglamasi $(x/10)^2 + (y/6)^2 = 1$ bo'lgan ellips direktrisasining tenglamasini toping.

$$A) x = \pm 12,5 \quad B) x = \pm 14,5 \quad C) x = \pm 16,5 \quad D) x = \pm 18,5 \quad E) x = \pm 20,5$$

2. Giperbola va parabola.

1. **Ta'rifni to'ldiring:** Berilgan ikkita F_1 va F_2 nuqtalargacha masofalari ... o'zgarmas son bo'lgan tekislikdagi nuqtalarning geometrik o'rni giperbola deyiladi.

A) ayirmasining moduli.

B) ko'paytmasi.

C) yig'indisi.

D) bo'linmasi.

E) kvadratlarining yig'indisi.

2. Yarim o'qlari a va b ($a, b > 0$) bo'lgan giperbolaning kanonik tenglamasi qayerda to'g'ri ko'rsatilgan?

$$A) \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$B) \frac{x^2}{a} - \frac{y^2}{b} = 1$$

$$C) \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$D) a^2x^2 + b^2y^2 = 1$$

$$E) ax^2 - by^2 = 1$$

3. $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ giperbolaning asimptotalari tenglamasini ko'rsating.

A) $y = \pm \frac{ab}{a+b}x$. B) $y = \pm \frac{a}{b}x$. C) $y = \pm \frac{b}{a}x$. D) $y = (a \pm b)x$. E) $y = \pm abx$.

4. Agar $x^2 - 4y^2 = 4$ giperbolaga tegishli nuqtaning ordinatasi 0 ga teng bo'lsa, uning absissasini toping.

A) $x=1$. B) $x=1$. C) $x=2$. D) $x=\pm 2$. E) $x=\pm 1$.

5. $9x^2 - 4y^2 = 36$ giperbola yarim o'qlarining yig'indisini toping.

A) 5. B) 13. C) 9. D) 45. E) 32.

6. Agar giperbolaning tenglamasi $x^2 - 4y^2 = 16$ ko'rinishda bo'lsa, uning asimptotalari tenglamalarini toping.

A) $y = \pm \frac{1}{2}x$. B) $y = \pm \frac{1}{4}x$. C) $y = \pm 4x$. D) $y = \pm 2x$. E) $y = \pm x$.

7. Kanonik tenglamasi $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ bo'lgan giperbolaning ekstsentrismati ε qaysi formula bilan hisoblanadi?

A) $\varepsilon = \sqrt{1 + \frac{b^2}{a^2}}$. B) $\varepsilon = \sqrt{1 - \frac{b^2}{a^2}}$. C) $\varepsilon = \sqrt{1 + \frac{b^2}{a^2}}$.

D) $\varepsilon = \sqrt{1 - \frac{b^2}{a^2}}$. E) $\varepsilon = \sqrt{\frac{b^2}{a^2} - 1}$.

8. Giperbolaning ekstsentrismati ε qanday shartni qanoatlantiradi?

A) $\varepsilon > 1$. B) $\varepsilon < 1$. C) $\varepsilon \neq 1$. D) $\varepsilon = 1$. E) $0 < \varepsilon < 1$.

9. $9x^2 - 16y^2 = 144$ giperbolaning ε ekstsentrismatini aniqlang.

A) $\varepsilon = 1,1$. B) $\varepsilon = 1,15$. C) $\varepsilon = 1,25$. D) $\varepsilon = 1,5$. E) $\varepsilon = 1,75$.

10. Parametri $p > 0$ bo'lgan parabolaning kanonik tenglamasini ko'rsating:

A) $y^2 = x + 2p$. B) $y^2 = x/2p$. C) $y^2 = x - 2p$.

D) $y^2 = 2p/x$. E) $y^2 = 2px$.

11. Quyidagi tenglamalardan qaysi biri parabola ni ifodalamaydi ($p > 0$)?

A) $y^2 = x + p$. B) $y^2 = x/p$. C) $y^2 = x - p$. D) $y^2 = p/x$. E) $y^2 = px$.

12. Kanonik tenglamasi $y^2 = 32x$ bo'lgan parabola fokusining absissasini toping.

A) 2. B) 4. C) 6. D) 8. E) 10.

13. Agar parabola $y^2 = 8x$ tenglama bilan berilgan bo'lsa, uning direktrisi tenglamasini toping.

A) $x = -8$. B) $x = 8$. C) $x = 4$. D) $x = 2$. E) $x = -2$.

14. Parabolaning ekstsentrismati ε qanday shartni qanoatlantiradi?

A) $\varepsilon > 1$. B) $\varepsilon < 1$. C) $\varepsilon \neq 1$. D) $\varepsilon = 1$. E) $0 < \varepsilon < 1$.

6-§. VEKTORLAR USTIDA CHIZIQLI AMALLAR. VEKTORLARNING SKALYAR, VEKTOR VA ARALASH KO'PAYTMALARI.

6.1 Vektorlar haqida tushuncha va ular ustida amallar.

Tayin uzunlikka va yo'nalishga ega bo'lgan kesma *vektor* deb ataladi va $\overline{AB}$ yoki $\vec{a}$ kabi belgilanadi.

Bunda A nuqtaga vektorning boshlang'ich nuqtasi, B nuqtaga uning oxirgi nuqtasi deyiladi.

$\overline{BA}$ vektor $\overline{AB}$ vektorga qarama-qarshi vektor hisoblanadi. $\vec{a}$ vektorga qarama-qarshi vektor ($-\vec{a}$) bilan belgilanadi.

AB kesmaning uzunligiga $\overline{AB}$ vektorning *uzunligi* yoki *moduli* deyiladi va $|\overline{AB}|$ ko'rinishda belgilanadi.

Boshlang'ich va oxirgi nuqtalari ustma-ust tushadigan vektor *nol* vektor deb ataladi va $\vec{0}$ bilan belgilanadi.

Uzunligi birga teng vektorga birlik vektor deyiladi va $\vec{e}$ orqali belgilanadi. $\vec{a}$ vektor bilan bir xil yo'nalgan birlik vektorga $\vec{a}$ vektorning *orti* deyiladi va $\vec{a}^\circ$ bilan belgilanadi.

Bir to'g'ri chiziqli yoki parallel to'g'ri chiziqlarda yotuvchi vektorlar *kollinear* vektorlar deb ataladi.

$\vec{a}$ va $\vec{b}$ vektorlar kollinear, bir xil yo'nalgan va uzunliklari teng bo'lsa, ularga teng vektorlar deyiladi va $\vec{a}=\vec{b}$ kabi yoziladi. Teng vektorlar erkin vektorlar deb yuritiladi. Vektorni fazoning ixtiyoriy nuqtasiga o'z-o'ziga parallel ko'chirish mumkin.

Bir tekislikda yoki parallel tekisliklarda yotuvchi vektorlar *komplanar* vektorlar deb ataladi.

$\vec{a}$ va $\vec{b}$ vektorlar *yig'indisi* deb $\vec{a}$ va $\vec{b}$ vektorlar bilan komplanar bo'lgan $\vec{a}+\vec{b}$ vektorga aytiladi. Ikki vektorning yig'indisi uchburchak yoki parallelogramm qoidalari bilan topiladi. $\vec{a}$ va $\vec{b}$ vektorlarning ayirmasi deb, $\vec{b}$ vektor bilan yig'indisi $\vec{a}$ vektorni beradigan $\vec{a}-\vec{b}$ vektor tushuniladi (6.1-rasm).

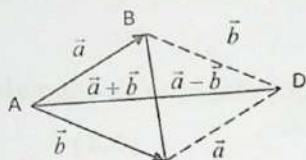
$$\vec{a}+\vec{b}=\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}+\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}=\begin{pmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{pmatrix}; \quad \vec{a}-\vec{b}=\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}-\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}=\begin{pmatrix} x_1-x_2 \\ y_1-y_2 \\ z_1-z_2 \end{pmatrix}$$

Bir nechta vektorni uchburchak usuli bilan ketma-ket qo'shib borish mumkin. Bir nechta vektorni bunday qo'shish usuliga ko'pburchak qoidasi deyiladi.

$\vec{a}$ vektorning $k \neq 0$ songa ko'paytmasi deb, $\vec{a}$ vektorga kollinear, uzunligi $|k| \cdot |\vec{a}|$ ga teng bo'lgan, $k > 0$ bo'lsa $\vec{a}$ vektor bilan bir xil yo'nalgan, $k < 0$ bo'lganda $\vec{a}$ vektorga qarama-qarshi yo'nalgan $k\vec{a}$ vektorga aytiladi.

Agar $\vec{b} = k\vec{a}$ bo'lsa, u holda $\vec{a}$ va $\vec{b}$ vektorlar kollinear bo'ladi

$\vec{a} = |\vec{a}| \cdot \vec{a}^0$, ya'ni har bir vektor uzunligi bilan ortining ko'paytmasiga teng bo'ladi.



6.1- rasm

Fazodagi vektor tasviri. Umumiy O nuqtaga ega va o'zaro perpendikulyar bo'lgan uchta koordinata o'qi va M nuqta berilgan bo'lsin. Bu nuqtaning radius vektori $\overrightarrow{OM} = \vec{r}$ ning koordinata o'qlaridagi proyeksiyalari $OM_1 = x$, $OM_2 = y$, $OM_3 = z$ $\vec{r} = \overrightarrow{OM}$ vektorning to'g'ri burchakli koordinatalari deyiladi.

Fazodagi nuqtaning radius vektori $\overrightarrow{OM} = \vec{r}$, radius vektorning moduli yoki uzunligi ushbu:

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} \quad (6.1)$$

formula bilan aniqlanadi. Radius vektor $\vec{i}, \vec{j}, \vec{k}$ ortlar orqali quyidagicha ifodalanadi.

$$\vec{r} = x\vec{i} + y\vec{j} + z\vec{k} \quad (6.2)$$

Agar $\vec{u} = \overrightarrow{AB}$ vektor koordinata o'qlari bilan α, β va γ burchaklar tashkil etsa, u holda $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$, munosabat o'rinli.

6.2. Vektorlar ustida skalyar va vektor ko'paytmalar

Ikki vektorning skalyar ko'paytmasi deb shu vektorlar modullarining ular orasidagi burchak kosinusi bilan ko'paytmasiga aytiladi. Quyidagi $\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko'paytmasi $\vec{a} \cdot \vec{b}$ ko'rinishida belgilanadi. Demak,

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cos \varphi \quad (6.3)$$

- 1) $(\alpha \vec{a}, \vec{b}) = (\vec{a}, \alpha \vec{b}) = \alpha (\vec{a}, \vec{b}), \alpha = \text{const.}$
- 2) $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$ o'rin almashtirish qonuni.
- 3) $\vec{a}(\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$ tarqatish qonuni.
- 4) Agar $\vec{a} \parallel \vec{b}$ bo'lsa $\vec{a} \cdot \vec{b} = \pm a \cdot b$ xususiyl holda $|\vec{a}|^2 = \vec{a} \cdot \vec{a} = |\vec{a}| \cdot |\vec{a}| \cos 0 = a^2$,
 $|\vec{a}| = \sqrt{a^2}$.
- 5) agar $a \perp b$ bo'lsa $a \cdot b = a \cdot b \cdot \cos 90^\circ = 0$.
- 6) ortlari skalyar ko'paytmasi $\vec{i} \cdot \vec{j} = 0, \vec{j} \cdot \vec{k} = 0, \vec{i} \cdot \vec{k} = 0, \vec{i} \cdot \vec{i} = 1, \vec{j} \cdot \vec{j} = 1, \vec{k} \cdot \vec{k} = 1$
- 7) agar vektorlar $\vec{a} \{a_x, a_y, a_z\}$ va $\vec{b} \{b_x, b_y, b_z\}$ koordinatalar orqali berilgan bo'lsa, $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$.

Ikki vektor orasidagi burchak kosinusini quyidagicha aniqlanadi:

$$\cos \varphi = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2 + a_y^2 + a_z^2} \cdot \sqrt{b_x^2 + b_y^2 + b_z^2}}. \quad (6.4)$$

Vektorlarning o'zaro parallel sharti: $\vec{b} = m\vec{a}$ yoki
 $\frac{b_x}{a_x} = \frac{b_y}{a_y} = \frac{b_z}{a_z} = m$;

Vektorlarning o'zaro perpendikulyar sharti $\vec{a} \cdot \vec{b} = 0$ yoki
 $a_x b_x + a_y b_y + a_z b_z = 0$

Vektorni boshqa vektor yo'nalishidagi proyeksiyasi. Berilgan $\vec{a}$ vektorni $\vec{b}$ vektor yo'nalishidagi proyeksiyasi quyidagi formula orqali topiladi.

$$\text{Pr}_{\vec{b}} \vec{a} = |\vec{a}| \cos \alpha = \frac{(\vec{a}, \vec{b})}{|\vec{b}|}$$

Vektorlarni vektor ko'paytmasi. Ikkita $\vec{a}$ va $\vec{b}$ vektorlarning **vektor ko'paytmasi** deb shunday uchinchi $\vec{c}$ vektorga aytiladiki

$$(6.2 - \text{rasm}), \vec{c} = [\vec{a}; \vec{b}] \text{ yoki } \vec{c} = \vec{a} \times \vec{b}.$$

1) $\vec{c}$ vektor son qiymati bo'yicha berilgan $\vec{a}$ va $\vec{b}$ vektorlardan yasalgan parallelogram yuziga teng moduliga ega;

2) $\vec{c}$ vektor parallelogram tekisligiga perpendikulyar;

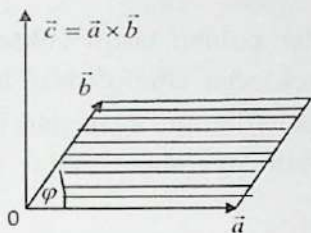
Ikkita vektordan qo'shilgan parallelogramning yuzi quyidagicha topiladi.

$$|\vec{c}| = |\vec{a}| |\vec{b}| \sin \alpha = |[\vec{a}, \vec{b}]| \quad (6.5)$$

Bu $\vec{a}$ va $\vec{b}$ vektorlardan yasalgan uchburchakning yuzi

$$S_{nap} = \frac{1}{2} |\vec{a} \times \vec{b}| \quad (6.6)$$

3) $\vec{c}$ vektorning yo'nalishi uch o'lchovli fazoda (R^3) $\vec{a} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ va $\vec{b} = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$ vektorlarni vektor ko'paytmasi.



6.2-rasm. Vektorlarni vektor ko'paytirishga doir.

$[\vec{a}, \vec{b}] = (y_1 z_2 - y_2 z_1) \vec{i} - (x_1 z_2 - x_2 z_1) \vec{j} + (x_1 y_2 - x_2 y_1) \vec{k}$
formula orqali yoki

$$[\vec{a}, \vec{b}] = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix} \quad (6.7)$$

topiladi.

Vektor ko'paytma quyidagi xossalarga ega.

- 1) $[\vec{a}, \vec{b}] = -[\vec{b}, \vec{a}]$
- 2) $[\vec{a} + \vec{b}, \vec{c}] = [\vec{a}, \vec{c}] + [\vec{b}, \vec{c}]$
- 3) $[\alpha \vec{a}, \vec{b}] = [\vec{a}, \alpha \vec{b}] = \alpha [\vec{a}, \vec{b}]$
- 4) Agar $\vec{a}, \vec{b}$ nol bo'lingan vektorlar bo'lsa, $[\vec{a}, \vec{b}] = 0$, ekanligidan, bu vektorlarni parallel ($\vec{a} \parallel \vec{b}$) ekanligi kelib chiqadi.

6.3. Aralash ko'paytma va uning xossalari

$\vec{a}, \vec{b}$ va $\vec{c}$ vektorlarni R^3 fazoda aralash ko'paytmasi sonidan iborat bo'lib, $\vec{a} \cdot \vec{b} \cdot \vec{c}$ ko'rinishda belgilanadi.

$$\vec{a} \cdot \vec{b} \cdot \vec{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} \quad (6.8)$$

bu yerda $\vec{a} = \begin{pmatrix} a_x \\ a_y \\ a_z \end{pmatrix}$; $\vec{b} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix}$; $\vec{c} = \begin{pmatrix} c_x \\ c_y \\ c_z \end{pmatrix}$

vektorlar $\vec{a}, \vec{b}$ va $\vec{c}$ vektorni aralash ko'paytmasi $\vec{a}$ vektorni $[\vec{b}, \vec{c}]$ vektorga yoki $[\vec{a}, \vec{b}]$ vektorni $\vec{c}$ vektorga skalyar ko'paytmasidan iborat bo'ladi $\vec{a} \cdot \vec{b} \cdot \vec{c} = (\vec{a}, [\vec{b}, \vec{c}]) = ([\vec{a}, \vec{b}], \vec{c})$

Aralash ko'paytma quyidagi xossalarga ega:

1. Aralash ko'paytmaning ikkita ko'paytmasini o'zni o'zaro almashtirilsa, ko'paytmaning ishorasi o'zgaradi $\vec{a} \cdot \vec{b} \cdot \vec{c} = -\vec{a} \cdot \vec{c} \cdot \vec{b} = -\vec{b} \cdot \vec{a} \cdot \vec{c} = -\vec{c} \cdot \vec{b} \cdot \vec{a}$.

2. Aralash ko'paytma hadlarini aylanma usulda almashtirishda o'z ishorasini o'zgartirmaydi $\vec{a} \cdot \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} \cdot \vec{a}$.

3. Agar $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlar noldan farqli vektorlar bo'lsa, u holda $\vec{a} \cdot \vec{b} \cdot \vec{c} = 0$ bo'lishi uchun bu vektorlar chiziqli bog'liq (yoki komplanar) bo'lishi kerak. $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlardan qurilgan parallelopipedning hajmi $V_{\text{nap}} = |\vec{a} \cdot \vec{b} \cdot \vec{c}|$ teng. Xuddi shunday shu uchta vektordan qurilgan tetrayedrning hajmi $V_t = \frac{1}{6} |\vec{a} \cdot \vec{b} \cdot \vec{c}|$.

Mustaqil bajarish uchun misollar

6.1-6.20 gacha. $\vec{AB}$ vektorning koordinatasi va uzunligini toping.

6.1. $A(1; 1; 3), B(2; 2; 3)$.

6.2. $A(0; 1; 3), B(1; 2; 3)$.

6.3. $A(0; 1; -1), B(1; 2; 0)$.

6.4. $A(2; 2; 3), B(3; 2; 4)$.

6.5. $A(2; 1; 2), B(3; 2; 2)$.

6.6. $A(0; 1; 1), B(1; 2; 2)$.

6.7. $A(0; 1; 4), B(1; 2; 4)$.

6.8. $A(1; 1; 1), B(1; 2; 2)$.

6.9. $A(0; -4; 3), B(1; -3; 4)$.

6.10. $A(1; 2; 1), B(0; 1; 2)$.

6.11. $A(2; 1; 3), B(3; 2; 4)$.

6.12. $A(0; 1; 1), B(1; 2; 2)$.

6.13. $A(2; 1; 3), B(3; 2; 4)$.

6.14. $A(2; 0; 7), B(0; 2; 4)$.

6.15. $A(8; 2; -5), B(7; 1; 4)$.

6.16. $A(-2; 1; 3), B(5; 1; 2)$.

6.17. $A(2; -1; 4), B(5; 2; 3)$.

6.18. $A(3; 1; 3), B(2; 2; -1)$.

6.19. $A(2; 2; 3), B(3; 1; 3)$.

6.20. $A(0; 7; 3), B(4; 7; -5)$.

6.21. Koordinatalar boshidan $M(12; -3; 4)$ nuqttagacha bo'lgan masofani aniqlang.

6.22. $\vec{r}(0; 2; -3)$ radius vektorning ortlar bo'yicha yoyilmasini yozing va modulini hisoblang.

6.23. $A(-2; +1; +3)$ va $B(0; -1; +2)$ nuqtalar orasidagi masofani toping.

6.24. $\vec{a}\{3; 2; 7\}$ va $\vec{b}\{4; 1; -5\}$ vektorlar yig'indisini va ayirmasini toping.

6.25. Uchlari $A(5; 2; 6), B(6; 4; 4), C(4; 3; 2)$ va $D(3; 1; 4)$ nuqtalarda bo'lgan to'rtburchakning kvadrat ekanligini tekshiring.

6.26. α va β larning qanday qiymatlarida $\vec{a} = 2\vec{i} + \alpha\vec{j} + \vec{k}$, $\vec{b} = 3\vec{i} - 6\vec{j} + \beta\vec{k}$ vektorlar kollinear bo'ladi?

6.27. Uchlari $A(2;1;-4)$, $B(1;3;5)$, $C(7;2;3)$ va $D(8;0;-6)$ nuqtalarda bo'lgan to'rtburchakning parallelogramm ekanligini isbotlang va parallelogramm tomonlarining uzunliklarini toping.

6.28. $A(-1;2;3)$, $B(2;-1;1)$, $C(1;-3;-1)$ va $D(-5;3;3)$ nuqtalar trapetsiyaning uchlari bo'lishini tekshiring.

6.29. AB kesmaning boshlang'ich nuqtasi $A(-1;2;4)$ va uni $1/2$ nisbada bo'luvchi $C(2;0;2)$ nuqta berilgan. B uchining koordinatalarini toping.

6.30. Uchlari $A(1;2;3)$ va $B(4;2;-1)$ bo'lgan AB kesmani teng ikkiga bo'luvchi M nuqtaning koordinatalarini toping.

6.31. Uchlari $A(5;3;-10)$, $B(0;1;4)$ va $C(-1;3;2)$ nuqtalarda bo'lgan uchburchak berilgan. B ichki burchak bissektrissasining yo'naltiruvchi kosinuslarini toping.

6.32. a va b vektorlar orasidagi burchak $\varphi = \frac{\pi}{4}$ ga teng va $|\vec{a}| = \sqrt{2}$; $|\vec{b}| = 3$ ekanligi ma'lum. $\vec{c} = 2\vec{a} + 3\vec{b}$ vektorning uzunligini hisoblang.

6.33. $\vec{a}$ va $\vec{b}$ vektorlar orasidagi burchak $\varphi = \frac{\pi}{3}$ ga teng. $|\vec{a}| = 3$; $|\vec{b}| = 4$. $\vec{c} = 3\vec{a} + 2\vec{b}$ vektorning uzunligini hisoblang.

6.34. Agar $|\vec{a}| = 7\sqrt{2}$, $|\vec{b}| = 4$ va $\left(\vec{a}, \vec{b}\right) = \frac{\pi}{4}$ bo'lsa? $3\vec{a} + \alpha\vec{b}$ va $\vec{a} - 2\vec{b}$ vektorlar α ning qanday qiymatlarida o'zaro perpendikulyar bo'ladi?

6.35. $\vec{a}$ va $\vec{b}$ vektorlarning koordinatalari berilgan:
 $\vec{a} = 7\vec{i} + 2\vec{j} + 3\vec{k}$, $\vec{b} = 2\vec{i} - 2\vec{j} + 4\vec{k}$. Bu vektorlarning skalyar ko'paytmasini toping.

6.36. Uchlari $A(-1;5;1)$, $B(1;1;-2)$ va $C(-3;3;2)$ nuqtalarda bo'lgan uchburchak berilgan. AC tomonni davom ettirishdan hosil bo'lgan tashqi burchakni aniqlang.

6.37. Uchlari $A(-2;3;1)$, $B(-2;-1;4)$ va $C(-2;-4;0)$ nuqtalarda bo'lgan uchburchak berilgan. Bu uchburchakning C ichki burchagini hisoblang.

6.38. $\vec{a}, \vec{b}$ va $\vec{c}$ vektorlarning koordinatalari berilgan:
 $\vec{a}\{1;-4;8\} = \vec{i} - 4\vec{j} + 8\vec{k}$, $\vec{b}\{4;4;-2\} = 4\vec{i} + 4\vec{j} - 2\vec{k}$, $\vec{c}\{2;3;6\} = 2\vec{i} + 3\vec{j} + 6\vec{k}$.
 $(\vec{b} + \vec{c})$ vektor $\vec{a}$ vektordagi proyeksiyasini toping.

6.39. $|\vec{a}| = 8$, $|\vec{b}| = 15$, $\vec{a} \cdot \vec{b} = 96$ berilgan. $\vec{a}$ vektorning $\vec{b}$ vektor bilan vektor ko'paytmasining uzunligini toping.

6.40. Uchlari $A(1;2;0)$, $B(3;0;-3)$ va $C(5;2;6)$ nuqtalarda bo'lgan uchburchak yuzini hisoblang.

6.41. $\overrightarrow{AB} = -3\vec{i} - 2\vec{j} + 6\vec{k}$ va $\overrightarrow{BC} = -2\vec{i} + 4\vec{j} + 4\vec{k}$ vektorlar $\triangle ABC$ tomonlari. $\overrightarrow{AD}$ balandlikning uzunligini hisoblang.

6.42. $\vec{a}\{6;0;2\}$, $\vec{b}\{1,5;2;1\}$ vektorlardan tuzilgan parallelogramning yuzini va diagonalarining uzunliklarini toping.

6.43. Vektorning koordinatalarini ayting: 1) $3\vec{i} + 2\vec{j} - 5\vec{k}$; 2) $2\vec{i} - \vec{k}$; 3) $0,5\vec{i} + \sqrt{2}\vec{j}$; 4) $3\vec{k}$; 5) $-4\vec{i}$; 6) $\vec{0}$.

6.44. Quyidagi vektorlar berilgan: 1) $\vec{a} = 2\vec{i} + 3\vec{j} - 5\vec{k}$; 2) $\vec{b} = -\vec{i} - 2\vec{j} + 3\vec{k}$. Ularning koordinatalarini toping.

6.45. $A(4;-3;2)$ va $B(-2;4;-3)$, $M(0;5;1)$ va $N(-4;0;-3)$ nuqtalarning koordinatalarini bilgan holda $\overrightarrow{AB}$ va $\overrightarrow{MN}$ vektorlarning koordinatalarini toping.

6.46. $\vec{a} = (2;3;-4)$, $\vec{b} = (-1;2;1)$ va $\vec{c} = (3;0;2)$ vektorlarning koordinatalarini bilgan holda quyidagi vektorlarning koordinatalarini toping: 1) $\vec{a} + \vec{b}$;

2) $\vec{a} + \vec{c}$; 3) $\vec{a} + \vec{b} - \vec{c}$; 4) $3\vec{a}$; 5) $-\vec{a} + 2\vec{c}$; 6) $2\vec{a} + 3\vec{b} - 2\vec{c}$.

6.47. Ikki vektorning kollinearlik shartidan foydalanib, quyidagi vektorlarning kollinear emasligini tekshiring: 1) $\vec{a} = (2/5; -1/3; 4/5)$ va $\vec{b} = (3/5; -1/2; 6/5)$; 2) $\vec{c} = (-6; 1/3; 3)$ va $\vec{d} = (-2; 1/9; -1/3)$.

6.48. n va p ning qanday qiymatlarida $\vec{a} = (-3; n; 4)$ va $\vec{b} = (-2; 4; p)$ vektorlar kollinear bo'ladi?

6.49. Vektorning uzunligini hisoblang:

1) $\vec{a} = -\vec{i} - 2\vec{j} + 2\vec{k}$;

2) $\vec{b} = \vec{i} + 2\vec{j} - 3\vec{k}$;

3) $\vec{c} = \vec{i} - \vec{k}$; 4) $\vec{d} = -3\vec{k}$.

6.50-6.63 gacha. Quyidagi $\vec{a}$ va $\vec{b}$ vektorlarni skalyar ko'paytmasini toping.

6.50. $\vec{a}\{-2;1;1\}$, $\vec{b}\{3;-2;4\}$.

6.51. $\vec{a}\{0;1;1\}$, $\vec{b}\{-1;-3;0\}$.

6.52. $\vec{a}\{-2;1;1\}$, $\vec{b}\{0;-2;-5\}$.

6.53. $\vec{a}\{0;1;1\}$, $\vec{b}\{3;-1;0\}$.

6.54. $\vec{a}\{0;-1;-1\}$, $\vec{b}\{1;-3;8\}$.

6.55. $\vec{a}\{0;-1;-1\}$, $\vec{b}\{2;0;2\}$.

6.56. $\vec{a}\{0;-1;-1\}$, $\vec{b}\{1;2;-1\}$.

6.57. $c_1 \cdot c_2 = -40$, $\vec{c} \times \vec{c} = -5\vec{k}$.

6.58. $\vec{a}\{-2;1;2\}$, $\vec{b}\{1;0;-1\}$.

6.59. $\vec{a}\{0;1;1\}$, $\vec{b}\{-3;-1;1\}$.

6.60. $\vec{a}\{-2;1;-2\}$, $\vec{b}\{-1;0;3\}$.

6.61. $\vec{a}\{1;-1;-1\}$, $\vec{b}\{-2;3;-1\}$.

$$6.62. \vec{a}\{2; -1; 3\}, \vec{b}\{0; 1; 1\}.$$

$$6.63. \vec{a}\{2; 1; -2\}, \vec{b}\{-1; 0; -2\}.$$

Quyidagi $\vec{a}$ va $\vec{b}$ vektorlarni vektor ko'paytmasini toping.

$$6.64. \vec{a}\{-2; 1; 1\}, \vec{b}\{3; -2; 4\}.$$

$$6.65. \vec{a}\{0; 1; 1\}, \vec{b}\{-1; -3; 0\}.$$

$$6.66. \vec{a}\{-2; 1; 1\}, \vec{b}\{0; -2; -5\}.$$

$$6.67. \vec{a}\{0; 1; 1\}, \vec{b}\{3; -1; 0\}.$$

$$6.68. \vec{a}\{0; -1; -1\}, \vec{b}\{1; -3; 8\}.$$

$$6.69. \vec{a}\{0; -1; -1\}, \vec{b}\{2; 0; 2\}.$$

$$6.70. \vec{a}\{0; -1; -1\}, \vec{b}\{1; 2; -1\}.$$

$$6.71. c_1 \cdot c_2 = -40, \vec{c} \times \vec{c} = -5\vec{k}.$$

$$6.72. \vec{a}\{-2; 1; 2\}, \vec{b}\{1; 0; -1\}.$$

$$6.73. \vec{a}\{0; 1; 1\}, \vec{b}\{-3; -1; 1\}.$$

$$6.74. \vec{a}\{-2; 1; -2\}, \vec{b}\{-1; 0; 3\}.$$

$$6.75. \vec{a}\{1; -1; -1\}, \vec{b}\{-2; 3; -1\}.$$

$$6.76. \vec{a}\{2; -1; 3\}, \vec{b}\{0; 1; 1\}.$$

$$6.77. \vec{a}\{2; 1; -2\}, \vec{b}\{-1; 0; -2\}.$$

$$6.78. \vec{a}\{2; 0; 0\}, \vec{b}\{-3; 1; 1\}.$$

$$6.79. \vec{a}\{2; 1; 0\}, \vec{b}\{1; 1; 3\}.$$

Quyidagi $\vec{a}, \vec{b}, \vec{c}$ vektorlarni komplanarligini tekshiring.

$$6.80. \vec{a}\{-2; 1; 1\}, \vec{b}\{0; -2; -5\}, \vec{c}\{2; -1; -1\}.$$

$$6.81. \vec{a}\{0; 1; 1\}, \vec{b}\{0; 4; -2\}, \vec{c}\{2; 1; 0\}.$$

$$6.82. \vec{a}\{2; 0; 1\}, \vec{b}\{2; 0; -1\}, \vec{c}\{-2; -1; 4\}.$$

$$6.83. \vec{a}\{1; -1; -1\}, \vec{b}\{-2; 3; -1\}, \vec{c}\{0; 1; 0\}$$

$$6.84. \vec{a}\{1; 1; 1\}, \vec{b}\{2; 3; 0\}, \vec{c}\{3; -1; -1\}.$$

$$6.85. \vec{a}\{-1; 0; -2\}, \vec{b}\{-3; 2; -1\}, \vec{c}\{2; 0; -2\}$$

$$6.86. \vec{a}\{1; 0; 3\}, \vec{b}\{0; 1; 1\}, \vec{c}\{2; -1; 3\}.$$

$$6.87. \vec{a}\{-3; 1; 4\}, \vec{b}\{2; 0; 0\}, \vec{c}\{-3; 1; 1\}.$$

$$6.88. \vec{a}\{1; 0; -1\}, \vec{b}\{0; -1; -1\}, \vec{c}\{0; 0; -2\}.$$

$$6.89. \vec{a}\{-1; 0; -2\}, \vec{b}\{0; 0; -1\}, \vec{c}\{-1; 0; 3\}$$

$$6.90. \vec{a}\{-1; 0; -2\}, \vec{b}\{1; 0; -4\}, \vec{c}\{2; 0; -2\}$$

$$6.91. \vec{a}\{1; 0; -2\}, \vec{b}\{-3; 2; -1\}, \vec{c}\{4; 2; -3\}$$

$$6.92. \vec{a}\{1; 2; 4\}, \vec{b}\{-3; 6; 4\}, \vec{c}\{3; -6; 4\}$$

$$6.93. \vec{a}\{1; -1; 1\}, \vec{b}\{1; 1; 1\}, \vec{c}\{2; 3; 4\}$$

$$6.94. \vec{a}\{5; 3; -1\}, \vec{b}\{1; -2; 3\}, \vec{c}\{2; 0; -4\}$$

$$6.95. \vec{a}\{-3; 3; 3\}, \vec{b}\{2; 1; 1\}, \vec{c}\{19; 11; 17\}$$

$$6.96. \vec{a}\{1; 6; 5\}, \vec{b}\{3; -2; 4\}, \vec{c}\{7; -18; 2\}.$$

$$6.97. \vec{a}\{7; -3; 2\}, \vec{b}\{3; -7; 8\}, \vec{c}\{1; -1; 1\}$$

$$6.98. \vec{a}\{2; 1; -1\}, \vec{b}\{1; -4; 1\}, \vec{c}\{3; -2; 2\}.$$

$$6.99. a) \vec{a}\{3; 1; -1\}, \vec{b}\{-2; -1; 0\}, \vec{c}\{5; 2; -1\}.$$

6.100. Uchlari A(2;2;2), B(4;3;3), C(4;5;4) va D(5;5;6) bo'lgan uch burchakli piramidaning hajmini toping.

Mavzu bo'yicha nazorat testlari

1. Vektorlar va ular ustida amallar.

1. Skalyar deb nimaga aytiladi?

A) Faqat yo'nalishi bilan aniqlanadigan kattalikka skalyar deb aytiladi.

B) Faqat son qiymati bilan aniqlanadigan kattalikka skalyar deb aytiladi.

C) Ham son qiymati, ham yo'nalishi bilan aniqlanadigan kattalikka skalyar deb aytiladi.

D) Yo'nalgan kesmaga skalyar deb aytiladi.

- E) Har qanday kattalik skalyar deyiladi.
2. Quyidagi kattaliklardan qaysi biri skalyar emas?
- A) Sirt yuzasi. B) Kesma uzunligi. C) Jism hajmi.
E) kuch momenti. D) Burchak kattaligi.
3. Vektor deb nimaga aytiladi?
- A) Faqat yo'nalishi bilan aniqlanadigan kattalikka vektor deb aytiladi.
B) Faqat son qiymati bilan aniqlanadigan kattalikka vektor deb aytiladi.
C) Ham son qiymati, ham yo'nalishi bilan aniqlanadigan kattalikka vektor deb aytiladi.
D) Har qanday kesmaga vektor deb aytiladi.
E) Har qanday kattalik vektor deyiladi.
4. Quyidagilardan qaysi biri vektor bo'ladi?
- A) Sirt yuzasi. B) Jism hajmi. C) Kesma uzunligi. D) Moddiy nuqta harakati. E) Modda massasi.
5. Kollinear vektorlar ta'rifi qayerda to'g'ri ifodalangan?
- A) Bir xil yo'nalgan vektorlar kollinear deb aytiladi.
B) Har qanday a va b vektorlar kollinear vektorlar deb aytiladi.
C) Bir xil yo'nalgan va uzunliklari teng bo'lgan vektorlar kollinear deb aytiladi.
D) Bitta to'g'ri chiziqli yoki parallel to'g'ri chiziqlarda yotuvchi a va b vektorlarga kollinear vektor deb aytiladi.
E) Bitta tekislikda yoki parallel tekisliklarda yotuvchi a va b vektorlarga kollinear vektor deb aytiladi.
6. Vektorlar tengligini ifodalovchi ta'rifni ko'rsating.
- A) Bir xil yo'nalgan a va b vektorlar teng deb aytiladi.
B) Bir xil uzunlikli a va b vektorlar teng deb aytiladi.
C) Turli yo'nalgan, ammo uzunliklari teng bo'lgan a va b vektorlar teng deb aytiladi.
D) a va b vektorlar kollinear, bir xil yo'nalgan va uzunliklari teng bo'lsa, ular teng deb aytiladi.
E) Kollinear va bir xil yo'nalgan a va b vektorlar teng deb aytiladi.
7. Fazodagi i, j va k ort vektorlar uchun quyidagi tasdiqlardan qaysi biri o'rinli emas?
- A) Bu vektorlarning modullari birga teng.
B) Bu vektorlar o'zaro kollinear.
C) Bu vektorlar o'zaro perpendikulyar.
D) Bu vektorlar koordinata o'qlarida joylashgan.

E) Bu vektorlar koordinata o'qlari bo'yicha musbat yo'nalishga ega.

8. $a=(2, -5)$ vektorning $\bar{i}$ va $\bar{j}$ ortlar bo'yicha yoyilmasi qayerda to'g'ri ko'rsatilgan ?

A) $a=2\bar{i}-5\bar{j}$. B) $a=-5\bar{i}+2\bar{j}$. C) $a=-2\bar{i}+5\bar{j}$. D) $a=5\bar{i}-2\bar{j}$. E) $a=2\bar{i}+5\bar{j}$.

9. $a=(2, -5, 1)$ vektorning $\bar{i}$, $\bar{j}$, $\bar{k}$ ortlar bo'yicha yoyilmasi qayerda to'g'ri ko'rsatilgan ?

A) $a=\bar{i}-5\bar{j}+2\bar{k}$. B) $a=-5\bar{i}+\bar{j}+2\bar{k}$. C) $a=2\bar{i}-5\bar{j}+\bar{k}$. D) $a=\bar{i}+2\bar{j}-5\bar{k}$.

E) $a=2\bar{i}+\bar{j}-5\bar{k}$.

10. $a=(0, 0, z)$ vektor qanday xususiyatga ega?

A) Bu vektor OZ koordinata o'qiga parallel joylashgan.

B) Bu vektor OZ koordinata o'qida yotadi.

C) Bu vektor OZ koordinata o'qiga perpendikulyar joylashgan.

D) Bu vektor koordinata boshidan o'tadi.

E) To'g'ri tasdiq keltirilmagan.

11. $a=(3, 5)$ vektorning $\lambda=-4$ songa ko'paytmasini toping.

A) $-4a=(-12, 5)$. B) $-4a=(3, -20)$. C) $-4a=(-12, -20)$.

D) $-4a=-4\cdot 3+(-4)\cdot 5=-32$. E) To'g'ri javob keltirilmagan.

12. Noldan farqli $a=(x_1, y_1, z_1)$ va $b=(x_2, y_2, z_2)$ vektorlar kollinear bo'lishining zaruriy va yetarli sharti qayerda to'g'ri ko'rsatilgan?

A) $x_1=x_2, y_1=y_2, z_1=z_2$.

B) $x_1=\lambda x_2, y_1=\lambda y_2, z_1=\lambda z_2$.

C) $x_1x_2=y_1y_2=z_1z_2$.

D) $x_1+x_2=0, y_1+y_2=0, z_1+z_2=0$.

E) To'g'ri javob keltirilmagan.

13. Quyidagi vektorlar juftliklarining qaysi biri kollinear bo'ladi?

A) $a=(2, 5, 1)$, $b=(3, 6, 2)$. B) $a=(4, 8, 6)$, $b=(2, 4, 3)$.

C) $a=(3, 4, 5)$, $b=(6, 8, 15)$. D) $a=(5, 0, 2)$, $b=(10, 0, 6)$.

E) $a=(1, 1, 0)$, $b=(1, 0, 1)$.

14. Uchlari $A(-3, 1, 0)$ va $B(1, 7, 4)$ nuqtalarda joylashgan kesmaning o'rta nuqtasi koordinatalarining yig'indisi nimaga teng ?

A) 0. B) 1. C) -4. D) 5. E) -2.

2. Vektorlarning skalyar ko'paytmasi, uning xossalari va tatbiqlari.

1. a va b vektorlarning skalyar ko'paytmasi qayerda to'g'ri ifodalangan ?

A) $a \cdot b = |a| \cdot |b|$. B) $a \cdot b = |a| \cdot |b| \cos \varphi$. C) $a \cdot b = |a| \cdot |b| \sin \varphi$.

D) $a \cdot b = |a| \cdot |b| \operatorname{tg} \varphi$. E) $a \cdot b = |a| \cdot |b| \operatorname{ctg} \varphi$.

2. Qaysi holda a va b vektorlarning skalyar ko'paytmasi $|a \cdot b| = \pm |a| \cdot |b|$ shartni qanoatlantiradi?

- A) a va b bir xil uzunlikka ega bo'lsa. B) a va b ort vektorlar bo'lsa.
C) a va b ortogonal bo'lsa. D) a va b kollinear bo'lsa.
E) hech qaysi a va b vektorlar uchun bu shart bajarilmaydi.

3. a va b vektorlarning skalyar ko'paytmasining xossasi qayerda noto'g'ri ifodalangan?

- A) $a \cdot b = b \cdot a$. B) $a \cdot a = |a|^2$. C) $a \cdot (b + c) = a \cdot b + a \cdot c$.
D) $(\lambda a, b) = (a, \lambda b) = \lambda(a, b)$. E) Barcha xossalar to'g'ri.

4. i, j, k ort vektorlarning skalyar ko'paytmalari bo'yicha quyidagi tasdiqlardan qaysi biri o'rinli emas?

- A) $i \cdot i = 1, i \cdot j = 0, i \cdot k = 0$. B) $j \cdot j = 1, j \cdot i = 0, j \cdot k = 0$.
C) $k \cdot k = 1, k \cdot i = 0, k \cdot j = 0$. D) $j \cdot (i + k) = 0, i \cdot (k + j) = 0, k \cdot (i + j) = 0$.
E) $j \cdot (i + j) = 0, i \cdot (k + i) = 0, k \cdot (j + k) = 0$.

5. Tekislikda koordinatalari bilan berilgan $a = (x_1, y_1)$ va $b = (x_2, y_2)$ vektorlarning $a \cdot b$ skalyar ko'paytmasini hisoblash formulasini ko'rsating.

- A) $a \cdot b = (x_1 + y_1) \cdot (x_2 + y_2)$. B) $a \cdot b = x_1 y_1 + x_2 y_2$.
C) $a \cdot b = x_1 x_2 + y_1 y_2$. D) $a \cdot b = x_1 y_2 + x_2 y_1$.
E) $a \cdot b = (x_1 + y_1) + (x_2 + y_2)$.

6. Tekislikda koordinatalari bilan berilgan $a = (3, 4)$ va $b = (-5, 2)$ vektorlarning $a \cdot b$ skalyar ko'paytmasini hisoblang.

- A) 2. B) 23. C) -14. D) -7. E) 0.

7. $a = (-8, 6)$ vektorning $|a|$ modulini hisoblang.

- A) $|a| = 2$. B) $|a| = 14$. C) $|a| = 48$. D) $|a| = 10$. E) $|a| = \sqrt{14}$

8. $a = (-2, 6, 3)$ vektorning $|a|$ modulini hisoblang.

- A) $|a| = 11$. B) $|a| = 121$. C) $|a| = 7$. D) $|a| = 49$. E) $|a| = \sqrt{7}$

9. $a = (x_1, y_1, z_1)$ va $b = (x_2, y_2, z_2)$ vektorlar orasidagi φ burchakning kosinusini topish formulasini ko'rsating.

- A) $\frac{\sqrt{x_1^2 + y_1^2 + z_1^2}}{\sqrt{x_2^2 + y_2^2 + z_2^2}}$. B) $\frac{x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2} \cdot \sqrt{x_2^2 + y_2^2 + z_2^2}}$.
C) $\frac{(x_1 + y_1 + z_1)(x_2 + y_2 + z_2)}{\sqrt{x_1^2 + y_1^2 + z_1^2} \cdot \sqrt{x_2^2 + y_2^2 + z_2^2}}$. D) $\frac{x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2} + \sqrt{x_2^2 + y_2^2 + z_2^2}}$.
E) $\frac{x_1 \cdot x_2 + y_1 \cdot y_2 + z_1 \cdot z_2}{\sqrt{x_1^2 + y_1^2 + z_1^2} - \sqrt{x_2^2 + y_2^2 + z_2^2}}$.

10. $a = (\lambda, 0, -1)$ va $b = (1, -1, 0)$ vektorlar orasidagi burchak $\varphi = 60^\circ$ bo'ladigan λ parametrning qiymatlar to'plamini toping.

- A) $\{0\}$. B) $\{\pm 1\}$. C) $\{-1\}$. D) $\{1\}$. E) $\emptyset$.

3. Vektor ko'paytma, uning xossalari va tatbiqlari.

1. Agar $c=a \times b$ bo'lsa, quyidagi tasdiqlardan qaysi biri o'rinli emas?

- A) $|c|=|a||b|\sin\varphi$ (φ — a va b vektorlar orasidagi burchak).
B) $c \perp a$. C) $c \perp b$. D) c , a va b vektorlar bir tekislikda yotadi.
E) Keltirilgan barcha tasdiqlar o'rinli.

2. Qanday a va b vektorlarning skalyar va vektorial ko'paytmalari o'zaro teng bo'ladi?

- A) a va b vektorlar teng bo'lsa.
B) a va b vektorlar kollinear bo'lsa.
C) a va b vektorlar ortogonal bo'lsa.
D) a va b vektorlar qarama-qarshi bo'lsa.
E) bunday vektorlar mavjud emas.

3. Agar $|a|=4$, $|b|=5$ va $\varphi=30^\circ$ bo'lsa, $|a \times b|=?$

- A) 20. B) 10. C) $10\sqrt{3}$. D) 41. E) $5\sqrt{2}$.

4. Agar $|a|=4$, $|b|=5$ va $a \cdot b = 10$ bo'lsa, $|a \times b|=?$

- A) 12. B) 6. C) $6\sqrt{3}$. D) $10\sqrt{3}$. E) $6\sqrt{2}$.

5. Vektorial ko'paytmaning xossasi qayerda xato yozilgan?

- A) $a \times b = b \times a$. B) $a \times \lambda b = \lambda a \times b$ (λ — o'zgarmas son).
C) $a \times (b+c) = a \times b + a \times c$. D) $a \times a = 0$. E) $(a+b) \times c = a \times c + b \times c$.

6. $(2a+b) \times (2a-b)$ ko'paytmani hisoblang.

- A) $4a \times a - b \times b$. B) 0. C) $4a \times b$. D) $4|a|^2 - |b|^2$. E) $4b \times a$.

7. Agar i, j va k ortlar bo'lsa, ularning vektorial ko'paytmasi qayerda to'g'ri ko'rsatilgan?

- A) $i \times i = 1$. B) $i \times k = j$. C) $i \times j = k$. D) $k \times j = i$. E) $k \times k = k$.

8. $a=(0, -2, 3)$ va $b=(1, 4, 0)$ bo'lsa, $a \times b = (x, y, z)$ vektorial ko'paytmaning koordinatalarini toping.

- A) $(-12, 3, 2)$. B) $(2, -5, 0)$. C) $(0, -8, 0)$. D) $(1, 2, 3)$. E) $(0, 8, 0)$.

9. $a=(0, -2, 3)$ va $b=(1, 4, 0)$ vektorlardan yasalgan parallelogramm yuzasi hisoblansin.

- A) $\sqrt{13}$. B) $\sqrt{17}$. C) $\sqrt{157}$. D) $\sqrt{129}$. E) 1.

10. $a=(0, -2, 4)$ va $b=(2, 4, 0)$ vektorlardan yasalgan ucburchak yuzasi hisoblansin.

- A) $5\sqrt{21}$. B) $4\sqrt{21}$. C) $3\sqrt{21}$. D) $2\sqrt{21}$. E) $\sqrt{21}$.

4. Vektorlarning aralash ko'paytmasi doir testlar.

1. Ta'rif bo'yicha uchta vektorning aralash ko'paytmasi abc qanday hisoblanadi?

A) Dastlab $a \cdot b$ skalyar ko'paytma, so'ngra $(a \cdot b) \cdot c$ ko'paytma hisoblanadi.

B) Dastlab $a \times b$ vektorial ko'paytma, so'ngra $(a \times b) \cdot c$ skalyar ko'paytma hisoblanadi.

C) Dastlab $a \times b$ vektorial ko'paytma, so'ngra $(a \times b) \times c$ vektorial ko'paytma hisoblanadi.

D) Dastlab $|a||b|$ ko'paytma, so'ngra $(|a||b|)|c|$ ko'paytma hisoblanadi.

E) To'g'ri javob keltirilmagan.

2. Uchta vektorning aralash ko'paytmasi abc qanday geometrik ma'noga ega?

A) a , b va c vektorlardan hosil qilingan piramida hajmi.

B) a , b va c vektorlarga yasalgan parallelepiped hajmi.

C) a va b vektorlardan hosil qilingan parallelogramm yuzasini $|c|$ modulga ko'paytmasi.

D) a va c vektorlardan hosil qilingan parallelogramm yuzasini $|b|$ modulga ko'paytmasi.

E) c va b vektorlardan hosil qilingan parallelogramm yuzasini $|a|$ modulga ko'paytmasi.

3. Ta'rifni yakunlang: Fazodagi uchta vektor komplanar deyiladi, agar ...

A) ular o'zaro kollinear bo'lsa.

B) ular o'zaro perpendikulyar bo'lsa.

C) ular koordinata o'qlariga parallel bo'lsa.

D) ular koordinata tekisliklarida joylashgan bo'lsa.

E) ular bitta yoki parallel tekisliklarda joylashgan bo'lsa.

4. Fazodagi uchta a , b va c vektorlar komplanar bo'lishining zaruriy va yetarli sharti nimadan iborat?

A) $abc > 0$. B) $abc < 0$. C) $abc \neq 0$. D) $abc = 0$. E) $abc = \pm 1$.

5. Aralash ko'paytmaning xossasi qayerda xato ko'rsatilgan?

A) $abc = cab$. B) $abc = -bac$. C) $(a \times b) \cdot c = a \cdot (b \times c)$.

D) $a \parallel b \Rightarrow abc = 0$. E) $|abc| = |a| \cdot |b| \cdot |c|$.

6. a , b va c vektorlar o'zaro perpendikulyar va $|a|=6$, $|b|=3$ va $|c|=4$ bo'lsa, abc aralash ko'paytma qiymatini hisoblang.

A) 18. B) 24. C) 12. D) 72. E) 1.

7. Koordinatalari bilan berilgan $a=(2,-3,1)$, $b=(1,0,4)$ va $c=(5,-2,0)$ vektorlarning aralash ko'paytmasi abc hisoblansin.

A) 0. B) 23. C) -46. D) -23. E) 1.

8. m parametrning qanday qiymatlarida $a=(-1, 0, 1)$, $b=(1, -1, m)$ va $c=(-1, 3m, 1)$ vektorlar komplanar bo'ladi?

A) 1 va -2. B) -1 va 2. C) 0 va 1. D) -1 va 0. E) ± 1 .

9. $a=(1,1,0)$, $b=(1,0,1)$ va $c=(0,1,1)$ vektorlarga yasalgan parallelopiped hajmini hisoblang.

A) 10. B) 1. C) 2. D) 3. E) 4.

10. $a=(3,2,1)$, $b=(1,-3,2)$ va $c=(-1,1,0)$ vektorlarga yasalgan piramida hajmini hisoblang.

A) 1. B) 2. C) 3. D) 4. E) 5.

7-§. FAZODA TO'G'RI CHIZIQ TENGLAMALARI. IKKI TO'G'RI CHIZIQ ORASIDAGI BURCHAK. PARALLELLIK VA PERPENDIKULARLIK SHARTLARI.

1. Tekislik tenglamalari:

-Umumiy tenglamasi:

$$Ax + By + Cz + D = 0, \quad (7.1)$$

bu yerda A,B,C,D lar haqiqiy sonlar;

- $M_0(x_0, y_0, z_0)$ nuqtadan o'tuvchi va $\vec{n} = (A, B, C)$ normal vektorlarga perpendikulyar bo'lgan tekislik tenglamasi

$$A(x - x_0) + B(y - y_0) + C(z - z_0) = 0; \quad (7.2)$$

- koordinata o'qlarini mos ravishda a,b,c, nuqtalardan kesib o'tgan tekislik tenglamasi

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1; \quad (7.3)$$

-berilgan uchta $M_1(x_1, y_1, z_1), M_2(x_2, y_2, z_2), M_3(x_3, y_3, z_3)$, nuqtadan o'tuvchi tekislik tenglamasi:

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \quad (7.4)$$

2. $A(x_0, y_0, z_0)$ nuqtadan $Ax + By + Cz + D = 0$ tekislikgacha bo'lgan d masofa

$$d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}} \quad (7.5)$$

3. Berilgan ikkita $A_1x + B_1y + C_1z + D_1 = 0$ va $A_2x + B_2y + C_2z + D_2 = 0$ tekisliklar orasidagi burchak (φ) quyidagicha topiladi

$$\cos \varphi = \pm \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \sqrt{A_2^2 + B_2^2 + C_2^2}}. \quad (7.6)$$

4. Ikki tekislikka parallellik

$$\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}. \quad (7.7)$$

va perpendikulyarlik shartlari:

$$A_1A_2 + B_1B_2 + C_1C_2 = 0. \quad (7.8)$$

5. Tekislikdagi to'g'ri chiziq ikki tekislikni kesishidan hosil bo'ladi

$$\begin{cases} A_1x + B_1y + C_1z + D_1 = 0 \\ A_2x + B_2y + C_2z + D_2 = 0 \end{cases} \quad (7.9)$$

Bu to'g'ri chiziqning yo'naltiruvchi vektori $\vec{S}$ ushbu

$$\vec{S} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix} \quad (7.10)$$

formula bo'yicha aniqlanadi.

6. $M(x_1; y_1; z_1)$ nuqtadan o'tuvchi va $\vec{S} = (m, n, p)$ yo'naltiruvchi vektorlarga parallel bo'lgan to'g'ri chiziq tenglamasi

$$\frac{x - x_1}{m} = \frac{y - y_1}{n} = \frac{z - z_1}{p} \quad (7.11)$$

to'g'ri chiziqning kanonik tenglama yoki

$$x = x_1 + mt; \quad y = y_1 + nt; \quad z = z_1 + pt \quad (7.12)$$

to'g'ri chiziqning parametrik tenglamasi deyiladi.

7. $M_1(x_1; y_1; z_1)$ va $M_2(x_2; y_2; z_2)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1} \quad (7.13)$$

8. Agar ikki to'g'ri chiziqning yo'naltiruvchi vektori berilgan bo'lsa, $\vec{S}_1 = (m_1, n_1, p_1)$ va $\vec{S}_2 = (m_2, n_2, p_2)$, u holda

$$\cos \varphi = \pm \frac{m_1 m_2 + n_1 n_2 + p_1 p_2}{\sqrt{m_1^2 + n_1^2 + p_1^2} \sqrt{m_2^2 + n_2^2 + p_2^2}} \quad (7.14)$$

9. Ikki to'g'ri chiziqning parallelligi

$$\frac{m_1}{m_2} = \frac{n_1}{n_2} = \frac{p_1}{p_2} \quad (7.15)$$

bo'ladi.

10. Ikki to'g'ri chiziqning perpendikulyarlik sharti

$$m_1 m_2 + n_1 n_2 + p_1 p_2 = 0 \quad (7.16)$$

11. Berilgan $\frac{x - x_1}{m} = \frac{y - y_1}{n} = \frac{z - z_1}{p}$ to'g'ri chiziq va $Ax + By + Cz + D = 0$ tekisliklar orasidagi burchak

$$\sin \varphi = \frac{|Am + Bn + Cp|}{\sqrt{A^2 + B^2 + C^2} \sqrt{m^2 + n^2 + p^2}} \quad (7.17)$$

12. To'g'ri chiziq va tekislikning ikkinchi parallellik sharti

$$Am + Bn + Cp = 0 \quad (7.18)$$

13. To'g'ri chiziq va tekislikning parallellik sharti

$$\frac{A}{m} = \frac{B}{n} = \frac{C}{p} \quad (7.19)$$

bo'ladi.

Mustaqil bajarish uchun misollar

7.1. $A\left(a; -\frac{a}{2}; a\right)$ va $B\left(0; \frac{a}{2}; 0\right)$ nuqtalardan teng uzoqlikda bo'lgan nuqtalar geometrik o'rning tenglamasi yozilsin.

7.2. a) $M_1(-4; 0; 4)$ nuqtadan o'tuvchi Ox va Oy o'qlardan $a=4$ va $b=3$ kesmalar ajratuvchi tekislikning tenglamasi yozilsin.

b) Oz o'q hamda $M(4; -2; 7)$ nuqta orqali o'tuvchi tekislik tenglamasini tuzing.

c) $N(4; -5; 7)$ nuqta orqali o'tib, Oyz tekislikka parallel bo'lgan tekislik tenglamasini tuzing.

7.3. a) 1) $x-2y+2z-8=0$ va $x+z-6=0$; 2) $x+2z-6=0$ va $x+2y-4=0$ tekisliklar orasidagi burchak topilsin.

b) $M_1(2; 3; -1)$ va $M_2(1; 5; 3)$ nuqtalardan o'tib $3x-y+3z+15=0$ tekislikka perpendikulyar bo'lgan tekislik tenglamasini tuzing.

c) $2x+3y-5z+30=0$ tekislikning koordinata o'qlari bilan kesishgan nuqtalarining koordinatalarini toping.

7.4. $(-1; -1; 2)$ nuqtadan o'tuvchi va $x-2y+z-4=0$ hamda $x+2y-2z+4=0$ tekisliklarga perpendikulyar tekislikning tenglamasi yozilsin.

7.5. $4x+3y-5z-8=0$ va $4x+3y-5z+12=0$ parallel tekisliklar orasidagi masofa topilsin.

7.6. $2x-y+3z-9=0$; $x+2y+2z-3=0$; $3x+y-4z+6=0$ tekislikning kesishgan nuqtasi topilsin.

7.7. 1) $\begin{cases} x=z+5 \\ y=4-2z \end{cases}$ va 2) $\begin{cases} \frac{x-3}{1} = \frac{y-2}{2} = \frac{z-3}{1} \end{cases}$ to'g'ri chiziqlarning xOy va xOz tekisliklardagi izlari topilsin va to'g'ri chiziq yasalsin.

7.8. $\begin{cases} x+2y+3z-13=0 \\ 3x+y+4z-14=0 \end{cases}$ to'g'ri chiziq tenglamalarini: 1) proyeksiyalari bo'yicha; 2) kanonik ko'rinishda yozilsin. To'g'ri chiziqning koordinata tekisliklaridagi izlari topilsin hamda to'g'ri chiziq va uning proyeksiyalari yasalsin.

7.9. 1) $\begin{cases} y=3 \\ z=2 \end{cases}$ 2) $\begin{cases} y=2 \\ z=x+1 \end{cases}$ 3) $\begin{cases} x=4 \\ z=y \end{cases}$ to'g'ri chiziq yasalsin va ularning vektorlari aniqlansin.

7.10. $A(-1; 2; 3)$ va $B(2; 6; -2)$ nuqtalardan o'tuvchi to'g'ri chiziq tenglamalari yozilsin va uning yo'naltiruvchi kosinuslari topilsin.

7.11. $A(2; -1; 3)$ va $B(2; 3; 3)$ nuqtalardan o'tuvchi to'g'ri chiziq yasalsin va uning tenglamalari yozilsin.

$$7.12. \begin{cases} x-y+z-4=0 \\ 2x+y-2z+5=0 \end{cases} \text{ va } \begin{cases} x+y+z-4=0 \\ 2x+3y-z-6=0 \end{cases} \text{ to'g'ri chiziqlar orasidagi}$$

burchak topilsin.

7.13. $\frac{x}{2} = \frac{y}{3} = \frac{z}{1}$ to'g'ri chiziqning $x=z+1$, $y=1-z$ to'g'ri chiziqqa perpendikulyar ekanini ko'rsating.

7.14. $(-4; 3; 0)$ nuqtadan o'tuvchi va $\begin{cases} x-2y+z=4 \\ 2x+y-z=0 \end{cases}$ to'g'ri chiziqqa parallel bo'lgan to'g'ri chiziq tenglamalari yozilsin.

7.15. $(2; -3; 4)$ nuqtadan Oz o'qqa tushilgan perpendikulyarning tenglamalari yozilsin.

7.16. $N(2; -1; 3)$ nuqtadan $\frac{x+1}{3} = \frac{y+2}{4} = \frac{z-1}{5}$ to'g'ri chiziqqacha bo'lgan masofa topilsin.

7.17. $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z+3}{2}$ va $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z+1}{2}$ parallel to'g'ri chiziqlar orasidagi masofa topilsin.

7.18. $y=3x-1$, $2z=-3x+2$ to'g'ri chiziq bilan $2x+y+z-4=0$ tekislik orasidagi burchak topilsin.

7.19. $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z-1}{3}$ to'g'ri chiziq $2x+y-z=0$ tekislikka parallel ekanligi, $\frac{x+1}{2} = \frac{y+1}{-1} = \frac{z+3}{3}$ to'g'ri chiziq esa shu tekislik ustida yotishi ko'rsatilgan.

7.20. $(-1; 2; -3)$ nuqtadan o'tuvchi va $x=2$, $y-z=1$ to'g'ri chiziqqa perpendikulyar tekislikning tenglamasi yozilsin.

7.21. $\frac{x-2}{1} = \frac{y-3}{2} = \frac{z+1}{3}$ to'g'ri chiziqdan va $(3; 4; 0)$ nuqtadan o'tuvchi tekislikning tenglamasi yozilsin.

7.22. $\frac{x-1}{1} = \frac{y+1}{2} = \frac{z+2}{2}$ to'g'ri chiziqdan o'tuvchi va $2x+3y-z=4$ tekislikka perpendikulyar tekislikning tenglamasi yozilsin.

7.23. $\frac{x-3}{2} = \frac{y}{1} = \frac{z-1}{2}$ va $\frac{x+1}{1} = \frac{y-1}{1} = \frac{z}{2}$ parallel to'g'ri chiziqlardan o'tuvchi tekislikning tenglamasi yozilsin.

7.24. Koordinatalar boshidan o'tuvchi va $4y=3x$, $y=0$ va $z=0$ tekisliklar bilan teng burchaklar tashkil etuvchi to'g'ri chiziq tenglamalari yozilsin va o'sha burchaklar topilsin.

7.25. $x=2t-1$, $y=t+2$, $z=1-t$ to'g'ri chiziqning $3x-2y+z=3$ tekislik bilan kesishgan nuqtasi topilsin.

7.26. (2; 3; 4) nuqtaning $x=y=z$ to'g'ri chiziqdagi proyeksiyasi topilsin.

7.27. Quyidagi tekislik tenglamasini tuzing va tegishli shaklni chizing.

a) $M_0(4, -5, 7)$ nuqtadan o'tuvchi va Oxz koordinatalar tekisligiga parallel tekislik;

b) Ox o'qqa parallel hamda ikki $M_1(4, 3, -6)$ va $M_2(5, 7, 1)$ nuqtadan o'tuvchi tekislik;

v) Oz o'q va $M_0(-3, 2, -1)$ nuqta orqali o'tuvchi tekislik;

g) $M_0(2, -1, 1)$ nuqtadan o'tuvchi va normal vektori $\vec{n} = \{1, -2, 3\}$ bo'lgan tekislik;

7.28. $M_1(-1, 3, 4)$, $M_2(2, 3, -3)$ va $M_3(3, 4, 2)$ nuqtalardan o'tuvchi tekislik tenglamasini toping.

7.29-7.40 gacha. Fazoda umumiy tenglamalari berilgan to'g'ri chiziqlarning kanonik tenglamasini tuzing.

$$7.29. \begin{cases} x - y + 2z + 4 = 0 \\ 3x + y - 5z - 8 = 0 \end{cases}$$

$$7.31. \begin{cases} 2x + 3y - 2z + 6 = 0 \\ 3x + 3y + z - 1 = 0 \end{cases}$$

$$7.33. \begin{cases} 3x + 4y + 3z + 1 = 0 \\ 6x - 5y + 3z + 8 = 0 \end{cases}$$

$$7.35. \begin{cases} x - 3y + 3z + 1 = 0 \\ 5x + 3y + 2z - 4 = 0 \end{cases}$$

$$7.37. \begin{cases} x + 3y + 2z + 14 = 0 \\ 5x + 3y + 2z - 4 = 0 \end{cases}$$

$$7.39. \begin{cases} x + 5y + 2z - 5 = 0 \\ 2x - 5y + z + 6 = 0 \end{cases}$$

$$7.30. \begin{cases} x - 2y + 3z - 4 = 0 \\ 3x + 2y - 5z - 4 = 0 \end{cases}$$

$$7.32. \begin{cases} x - 3y + z + 3 = 0 \\ 2x - 3y - 2z + 6 = 0 \end{cases}$$

$$7.34. \begin{cases} 2x - 4y - 2z + 4 = 0 \\ 6x + 5y - 4z + 4 = 0 \end{cases}$$

$$7.36. \begin{cases} x + 5y - z + 11 = 0 \\ 8x - 5y - 3z - 1 = 0 \end{cases}$$

$$7.38. \begin{cases} x - y + 2z - 1 = 0 \\ x + y + z + 11 = 0 \end{cases}$$

$$7.40. \begin{cases} x + y - 2z - 2 = 0 \\ 6x - y - 4z - 3 = 0 \end{cases}$$

7.41. Ikkita $M_1(3, -1, 2)$ va $M_2(4, -2, -1)$ nuqta berilgan. M_1 nuqtadan o'tuvchi va M_1M_2 vektorga perpendikulyar tekislik tenglamasini tuzing.

7.42. Uchburchakning uchlari berilgan: $M(3, 5, -2)$, $N(-5, 2, 3)$ va $P(-3, 4, 2)$ P uchidan o'tkazilgan mediananing parametrik tenglamasini tuzing.

7.43. Koordinatalar boshidan o'tuvchi $2x - y + 3z - 1 = 0$ va $x + 2y + z = 0$ tekisliklarga perpendikulyar tekislik tenglamasini tuzing.

7.44. $P(2, -1, 4)$ nuqtadan $M_1(1, -2, 1)$, $M_2(2, 3, -4)$ va $M_3(3, 4, 1)$ nuqtalar orqali o'tuvchi tekislikkacha bo'lgan d masofani toping.

7.45. $M_1(4,5,6)$ va $M_2(-3,-1,2)$ nuqtalardan o'tuvchi to'g'ri chiziqqa nisbatan $P(2,-5,7)$ nuqtaga simmetrik Q nuqtani toping.

7.46. Uchburchakning $A(2,-2,4)$ va $B(-2,-3,4)$ uchlari berilgan. Uning C uchidan AB tomonga o'tkazilgan medianasiga B uchidan tushirilgan perpendikulyar uzunligini hisoblang.

7.47. $M_1(1,-1,2)$ va $M_2(-3,-2,1)$ nuqtalardan o'tuvchi $x - 2y + 3z + 5 = 0$ tekislikka perpendikulyar tekislik tenglamasini tuzing.

7.48-7.57 gacha. Berilgan to'g'ri chiziq bilan tekislikning kesishish nuqtasini toping.

$$7.48. \frac{x-3}{0} = \frac{y+2}{3} = \frac{z-5}{10}, \quad x + 2y - 2z + 25 = 0.$$

$$7.49. \frac{x+1}{1} = \frac{y+3}{0} = \frac{z-2}{-2}, \quad 2x - 7y - 3z + 21 = 0.$$

$$7.50. \frac{x-1}{2} = \frac{y-2}{-3} = \frac{z-3}{1}, \quad 5x - 2y - z - 13 = 0.$$

$$7.51. \frac{x+5}{3} = \frac{y-3}{1} = \frac{z-1}{6}, \quad 5x - 2y + 3z - 3 = 0.$$

$$7.52. \frac{x+1}{3} = \frac{y}{0} = \frac{z+1}{-2}, \quad 5x - 2y + 3z - 3 = 0.$$

$$7.53. \frac{x-8}{0} = \frac{y+2}{-1} = \frac{z-3}{1}, \quad 4x + 9y + 5z = 0.$$

$$7.54. \frac{x+1}{3} = \frac{y}{0} = \frac{z+1}{-2}, \quad 5x - 2y + 3z - 3 = 0.$$

$$7.55. \frac{x+8}{7} = \frac{y-2}{1} = \frac{z-1}{-1}, \quad 6x - y - 4z - 3 = 0.$$

$$7.56. \frac{x-1}{2} = \frac{y+3}{5} = \frac{z-5}{-1}, \quad 5x - 7y - 3z + 11 = 0.$$

$$7.57. \frac{x+5}{3} = \frac{y-1}{0} = \frac{z-1}{-2}, \quad 3x + 7y - z - 6 = 0.$$

7.58-7.77 gacha. $ABCD$ piramidaning uchlari berilgan. Quyidagilarni toping.

a) ABC tekislik tenglamasini;

b) AB qirra tenglamasini;

c) D uchidan o'tuvchi ABC o'qqa perpendikulyar to'g'ri chiziq tenglamasini;

d) C uchidan o'tuvchi AB qirrara parallel to'g'ri chiziq tenglamasini;

e) D uchidan o'tuvchi AB qirrara perpendikulyar tekislik tenglamasini

f) ABC va ABD yoqlar orasidagi burchak kosinusini;

g) D uchidan ABC yoqqacha bo'lgan masofani.

7.58. $A(7,3,5)$, $B(5,3,2)$, $C(10,2,4)$, $D(7,-2,1)$.

7.59. $A(-8,-6,-3)$, $B(4,2,1)$, $C(0,5,2)$, $D(0,2,5)$.

7.60. $A(7,-3,14)$, $B(-6,0,5)$, $C(1,2,1)$, $D(-2,-1,2)$.

- 7.61. $A(5,5,-6)$, $B(-4,-8,4)$, $C(1,7,-1)$, $D(-4,0,-2)$
 7.62. $A(-5,4,6)$, $B(-3,7,4)$, $C(1,5,-2)$, $D(-4,4,-3)$.
 7.63. $A(16,-8,-13)$, $B(6,2,5)$, $C(-3,0,3)$, $D(0,2,1)$.
 7.64. $A(6,4,-6)$, $B(-7,-8,4)$, $C(10,14,-2)$, $D(-13,0,-1)$.
 7.65. $A(15,3,-2)$, $B(-5,-7,4)$, $C(1,6,-2)$, $D(-4,9,-3)$.
 7.66. $A(3,6,-7)$, $B(-8,-4,3)$, $C(2,8,-3)$, $D(-9,1,-5)$.
 7.67. $A(5,4,-12)$, $B(-4,16,4)$, $C(11,7,1)$, $D(-4,2,-12)$.
 7.68. $A(15,5,6)$, $B(11,-8,4)$, $C(10,7,21)$, $D(4,1,-0)$.
 7.69. $A(1,4,16)$, $B(7,-8,-4)$, $C(0,7,-1)$, $D(-5,0,2)$.
 7.70. $A(11,5,6)$, $B(-5,9,2)$, $C(10,6,-1)$, $D(-4,0,-5)$.
 7.71. $A(13,5,5)$, $B(-4,4,3)$, $C(1,5,-1)$, $D(-8,1,-2)$.
 7.72. $A(1,6,-2)$, $B(-3,-6,4)$, $C(1,4,-2)$, $D(-4,2,-3)$.
 7.73. $A(5,5,-6)$, $B(-4,-8,4)$, $C(1,7,-1)$, $D(-3,0,-2)$.
 7.74. $A(0,8,-4)$, $B(-14,-2,-3)$, $C(2,5,-6)$, $D(-5,0,0)$.
 7.75. $A(5,5,-6)$, $B(-4,-8,4)$, $C(1,7,-1)$, $D(-4,0,-2)$.
 7.76. $A(4,3,7)$, $B(-3,-4,5)$, $C(12,7,4)$, $D(22,0,12)$.
 7.77. $A(3,2,1)$, $B(-5,4,3)$, $C(2,9,-1)$, $D(0,0,-3)$.

Mavzu bo'yicha nazorat testlari

Tekislik va uning tenglamalari.

1. Tekislikning umumiy tenglamasi qayerda to'liq va to'g'ri ifodalangan?

- A) $Ax+By+Cz+D=0$. B) $Ax+By+CDz=0$. C) $Ax+By+(C+D)z=0$.
 D) $Ax^{-1}+By^{-1}+Cz^{-1}+D=0$. E) $Ax^2+By^2+Cz^2+D=0$.

2. Umumiy tenglamasi $2x-5y+4z+9=0$ bo'lgan tekislikka tegishli va koordinatalarining yig'indisi 15 bo'lgan $M(x,y,5)$ nuqtaning abssissasini toping.

- A) 4. B) -7. C) 3. D) 0. E) -1.

3. Umumiy tenglamasi $2x-5y+4z-9=0$ bo'lgan tekislikka tegishli, OZ o'qda yotuvchi va koordinatalarining yig'indisi birga teng bo'lgan nuqtaning ordinatasini toping.

- A) 4. B) -7. C) 3. D) 0. E) -1.

4. Quyidagilardan qaysi biri $Ax+By+Cz+D=0$ tenglamali tekislikning n normal vektori bo'ladi?

- A) $n=(B,C,D)$. B) $n=(A,C,D)$. C) $n=(A,B,C)$. D) $n=(A,B,D)$. E) $n=(C,A,B)$.

5. **Tasdiqni yakunlang:** Umumiy tenglamasi $Ax+By+Cz+D=0$ bo'lgan tekislikning $n=(A,B,C)$ normal vektori shu ...

- A) tekislikda yotadi. B) tekislikka parallel bo'ladi.
 C) tekislikka perpendikulyar bo'ladi. D) tekislikka og'ma bo'ladi.
 E) to'g'ri javob keltirilmagan.

6. $3x+4y+7z-81=0$ tenglama bilan berilgan tekislik normalini aniqlang.

- A) $n=(3,4,-81)$. B) $n=(3,-4,-81)$. C) $n=(4,7,-81)$.
 D) $n=(-3,-4,81)$. E) $n=(3,4,7)$.

7. Quyidagi tenglamalardan qaysi biri koordinatalar boshidan o'tuvchi tekislikni ifodalaydi?

- A) $Ax+By+D=0$. B) $Ax+By+Cz=0$. C) $By+Cz+D=0$.
 D) $Ax+By+Cz+D=0$. E) $Ax+Cz+D=0$.

8. $x+y-z=0$ tenglamali P tekislik to'g'risidagi quyidagi tasdiqlardan qaysi biri o'rinli?

- A) P koordinatalar boshidan o'tadi. B) P OXY tekisligiga parallel.
 C) P OYZ tekisligiga parallel. D) P OXY tekisligiga parallel.
 E) P tekislik OZ koordinata o'qiga perpendikulyar.

9. $Ax+By+Cz+D=0$ tenglama $A=D=0$ holda qanday P tekislikni ifodalaydi?

- A) P OX o'qiga parallel. B) P OX o'qiga perpendikulyar.
 C) P OX o'qi orqali o'tadi. D) P OY o'qiga perpendikulyar.
 E) P OY o'qiga parallel.

10. Quyidagi tenglamalardan qaysi biri OZ koordinata o'qidan o'tuvchi tekislikni ifodalaydi?

- A) $Ax+By+Cz=0$. B) $Ax+Cz+D=0$. C) $Ax+By=0$.
 D) $By+Cz+D=0$. E) $By+D=0$.

11. Quyidagi tenglamalardan qaysi biri OY koordinata o'qiga parallel tekislikni ifodalaydi?

- A) $Ax+By+Cz=0$. B) $Ax+Cz+D=0$. C) $Ax+By+D=0$.
 D) $By+Cz+D=0$. E) $Ax+D=0$.

12. Quyidagi tenglamalardan qaysi biri XOZ koordinata tekisligiga parallel tekislikni ifodalaydi?

- A) $Ax+By+Cz=0$. B) $Ax+Cz+D=0$. C) $Ax+By=0$.
 D) $By+Cz+D=0$. E) $By+D=0$.

13. Quyidagi tenglamalardan qaysi biri YOZ koordinata tekisligini ifodalaydi?

- A) $By=0$. B) $Cz=0$. C) $Ax=0$. D) $By+Cz=0$. E) $Ax+D=0$.

14. Tekislikning kesmalardagi tenglamasi qayerda to'g'ri yozilgan?

- A) $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 1$. B) $ax + by + cz = 1$. C) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$.

$$D) \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1. \quad E) \frac{x}{a} \cdot \frac{y}{b} \cdot \frac{z}{c} = 1.$$

15. **Tasdiqni yakunlang:** Koordinata boshidan o'tmaydigan, koordinata o'qlariga parallel bo'lmagan va $Ax+By+Cz+D=0$ umumiy tenglama bilan berilgani tekislikning kesmalardagi tenglamasiga o'tish uchun umumiy tenglama ... soniga bo'linadi.

$$A) -C. \quad B) -A. \quad C) -B. \quad D) -D. \quad E) -(A^2+B^2+C^2).$$

16. $3x-4y+12z-24=0$ tekislikning kesmalardagi tenglamasini toping.

$$A) \frac{x}{8} + \frac{y}{6} + \frac{z}{12} = 1. \quad B) \frac{x}{4} + \frac{y}{-6} + \frac{z}{1} = 1. \quad C) \frac{x}{3} - \frac{y}{2} - \frac{z}{4} = 1.$$

$$D) \frac{x}{8} + \frac{y}{3} - \frac{z}{4} = 1. \quad E) \frac{x}{8} + \frac{y}{-6} + \frac{z}{2} = 1.$$

17. Tekislikning normal tenglamasi qayerda to'g'ri yozilgan?

$$A) x \cos^{-1} \alpha + y \cos^{-1} \beta + z \cos^{-1} \gamma - p = 0.$$

$$B) x^{-1} \cos \alpha + y^{-1} \cos \beta + z^{-1} \cos \gamma - p = 0. \quad E) x \cos \alpha - y \cos \beta - z \cos \gamma + p = 0.$$

$$C) x \cos \alpha + y \cos \beta + z \cos \gamma - p = 0. \quad D) x^2 \cos \alpha + y^2 \cos \beta + z^2 \cos \gamma - p = 0.$$

18. **Tasdiqni to'ldiring:** Tekislikning umumiy $Ax+By+Cz+D=0$ tenglamasidan uning normal tenglamasiga o'tish uchun bu tenglama ... ifodaga bo'linadi.

$$A) \pm \sqrt{A^2 + B^2 + C^2}. \quad B) \pm \sqrt{A^2 + B^2 + D^2}. \quad C) \pm \sqrt{B^2 + C^2 + D^2}.$$

$$D) \pm \sqrt{A^2 + B^2 + C^2 + D^2}. \quad E) \pm \sqrt{|A| + |B| + |C| + |D|}.$$

19. Umumiy tenglamasi $2x+2y+z-18=0$ bo'lgan tekislikning normal tenglamasini toping.

$$A) \frac{2}{\sqrt{5}}x + \frac{2}{\sqrt{5}}y + \frac{1}{\sqrt{5}}z - \frac{18}{\sqrt{5}} = 0. \quad B) \frac{2}{\sqrt{3}}x + \frac{2}{\sqrt{3}}y + \frac{1}{\sqrt{3}}z - \frac{18}{\sqrt{3}} = 0.$$

$$C) \frac{2}{3}x + \frac{2}{3}y + \frac{1}{3}z - 6 = 0. \quad D) \frac{2}{5}x + \frac{2}{5}y + \frac{1}{5}z - \frac{18}{5} = 0.$$

$$E) x \cdot \cos 60^\circ + y \cdot \cos 30^\circ + z \cdot \cos 45^\circ - 9 = 0.$$

20. Quyidagilardan qaysi biri tekislikning normal tenglamasi bo'ladi?

$$A) \frac{2}{13}x + \frac{5}{13}y + \frac{7}{13}z - 5 = 0. \quad B) \frac{4}{13}x - \frac{3}{13}y - \frac{6}{13}z - 13 = 0.$$

$$C) -\frac{5}{13}x + \frac{1}{13}y + \frac{8}{13}z - 9 = 0. \quad D) \frac{3}{13}x - \frac{4}{13}y + \frac{12}{13}z - 2 = 0.$$

$$E) x \cdot \cos 60^\circ + y \cdot \cos 30^\circ + z \cdot \cos 45^\circ - 9 = 0.$$

Tekislikka doir asosiy masalalar

1. Berilgan $M_0(x_0, y_0, z_0)$ nuqtadan o'tuvchi tekisliklar dastasining tenglamasi qayerda to'g'ri ifodalangan?

$$A) A(x+x_0)+B(y+y_0)+C(z+z_0)=0.$$

$$B) Axx_0+Byy_0+Czz_0=0.$$

$$C) \frac{x-x_0}{A} + \frac{y-y_0}{B} + \frac{z-z_0}{C} = 0.$$

$$D) \frac{x+x_0}{A} + \frac{y+y_0}{B} + \frac{z+z_0}{C} = 0.$$

$$E) A(x-x_0)+B(y-y_0)+C(z-z_0)=0.$$

2. Fazoning $M(1,2,-3)$ nuqtasidan o'tuvchi tekisliklar dastasi tenglamasini ko'rsating.

$$A) Ax+2By-3Cz=0.$$

$$B) A(x-1)+B(y-2)+C(z+3)=0.$$

$$C) A(x+1)+B(y+2)+C(z-3)=0.$$

$$D) Ax+2By-3Cz+D=0.$$

$$E) A(x-1)+B(y-2)+C(z-3)+D=0.$$

3. $M_1(3,2,-1)$, $M_2(0,3,1)$ va $M_3(4,5,0)$ nuqtalardan o'tuvchi tekislik tenglamasini ko'rsating.

$$A) 2x-y+z-3=0.$$

$$B) x-2y+z+2=0.$$

$$C) x-y+2z+1=0.$$

$$D) x-y+z=0.$$

$$E) x-2y+2z+3=0.$$

4. $M_1(3,2,-1)$, $M_2(0,3,1)$ va $M_3(4,5,0)$ nuqtalardan o'tuvchi tekislikda yotuvchi $M_0(x,-4,7)$ nuqtaning absissasini toping.

$$A) x_0=-3.$$

$$B) x_0=5.$$

$$C) x_0=-1,5.$$

$$D) x_0=9.$$

$$E) x_0=0.$$

5. Berilgan $M_0(x_0,y_0,z_0)$ nuqtadan o'tuvchi va $n=(A,B,C)$ vektorga perpendikulyar tekislik tenglamasini ko'rsating.

$$A) A(x+x_0)+B(y+y_0)+C(z+z_0)=0.$$

$$B) Axx_0+Byy_0+Czz_0=0.$$

$$C) \frac{x-x_0}{A} + \frac{y-y_0}{B} + \frac{z-z_0}{C} = 0.$$

$$D) \frac{x+x_0}{A} + \frac{y+y_0}{B} + \frac{z+z_0}{C} = 0.$$

$$E) A(x-x_0)+B(y-y_0)+C(z-z_0)=0.$$

6. $M(1,2,3)$ nuqtadan o'tuvchi va $n=(3,2,1)$ normal vektorga ega tekislik tenglamasini yozing.

$$A) 3x+2y+z-10=0.$$

$$B) x+2y+3z-14=0.$$

$$C) 2x+3y+z-11=0.$$

$$D) x+3y+2z-13=0.$$

$$E) 3x+y+2z-11=0.$$

7. Berilgan $M_0(3,-4,0)$ nuqtadan o'tuvchi va $n=(1,2,-3)$ normal vektorga ega bo'lgan tekislikda yotuvchi $N(-3,8,z)$ nuqtaning aplikatasini toping.

$$A) z=0.$$

$$B) z=5.$$

$$C) z=-1.$$

$$D) z=6.$$

$$E) z=-3,5.$$

8. $M_0(x_0,y_0,z_0)$ nuqtadan $Ax+By+Cz+D=0$ tekislikkacha bo'lgan masofani topish formulasini ko'rsating.

$$A) d = |Ax_0 + By_0 + Cz_0 + D|.$$

$$B) d = \sqrt{|Ax_0 + By_0 + Cz_0 + D|}.$$

$$C) d = \frac{|Ax_0 + By_0 + Cz_0 + D|}{\sqrt{A^2 + B^2 + C^2}}.$$

$$D) d = \frac{|A + x_0 + B + y_0 + C + z_0 + D|}{\sqrt{A^2 + B^2 + C^2}}.$$

$$E) d = \sqrt{\frac{Ax_0 + By_0 + Cz_0 + D}{A^2 + B^2 + C^2}}.$$

9. $3x+4y-2\sqrt{6}z+14=0$ tekislikdan koordinata boshigacha bo'lgan masofani toping.

- A) 14. B) 7. C) 2. D) 1. E) $2\sqrt{6}$.

10. Ushbu $4x+3y-5z-8=0$ va $4x+3y-5z-12=0$ parallel tekisliklar orasidagi masofani toping.

- A) 4. B) 20. C) $\sqrt{2}$. D) $\frac{2\sqrt{2}}{5}$. E) $2\sqrt{2}$.

11. $x+y-z-1=0$ va $2x-2y-2z+1=0$ tekisliklar orasidagi burchak kosinusini toping.

- A) 0. B) 1. C) $\sqrt{3}/4$. D) $1/3$. E) $3/4$.

12. $x+y-18=0$ va $y+z-72=0$ tenglamalar bilan berilgan tekisliklar orasidagi burchak topilsin.

- A) 30° . B) $\arccos \frac{\sqrt{3}}{4}$. C) 45° . D) $\arccos \frac{\sqrt{3}}{5}$. E) 60° .

13. Normal vektorlari $n_1=(-1, -1, 0)$ va $n_2=(0, -1, -1)$ bo'lgan tekisliklar orasidagi burchak topilsin.

- A) 45° . B) 30° . C) $\arccos \frac{\sqrt{2}}{3}$. D) 60° . E) 90° .

14. $A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2=0$ tekisliklarning parallellik sharti qayerda to'g'ri ko'rsatilgan?

- A) $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{D_1}{D_2}$. B) $\frac{A_1}{A_2} = \frac{C_1}{C_2} = \frac{D_1}{D_2}$. C) $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$.
D) $\frac{B_1}{B_2} = \frac{C_1}{C_2} = \frac{D_1}{D_2}$. E) $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = \frac{D_1}{D_2}$.

15. $kx-2y+5z+10=0$ va $6x-(1+k)y+10z-2=0$ tekisliklar k parametrning qanday qiymatida parallel bo'ladi?

- A) $k=3$. B) $k=-4$. C) $k=2$. D) $k=-5$. E) $k=\pm 1$.

16. Umumiy tenglamalari $A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2=0$ bilan berilgan tekisliklarning perpendikulyarlik shartini ko'rsating.

- A) $A_1A_2+B_1B_2+C_1C_2+D_1D_2=0$. B) $A_1A_2+B_1B_2+D_1D_2=0$.
C) $A_1A_2+B_1B_2+C_1C_2=0$. D) $B_1B_2+C_1C_2+D_1D_2=0$. E) $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$.

17. $kx-2y-5z+10=0$ va $6x-(1+k)y+10z-2=0$ tekisliklar k parametrning qanday qiymatida parallel bo'ladi?

- A) $k=3$. B) $k=-4$. C) $k=6$. D) $k=-5$. E) $k=\pm 1$.

18. $x-y-1=0$, $y+z=0$ va $x-z-1=0$ tekisliklarning kesishish nuqtasi koordinatalarining yig'indisini toping.

- A) 0. B) 1. C) -1. D) 4. E) -4.

19. $2x-3y+4z-12=0$ tenglama bilan berilgan tekislikning koordinata o'qlari bilan kesishgan nuqtalarining koordinatalari topilsin.

- A) $(-1,0,0)$, $(0,-5,0)$, $(0,0,4)$. B) $(-6,0,0)$, $(0,3,0)$, $(0,0,4)$.
C) $(5,0,0)$, $(0,-4,0)$, $(0,0,-3)$. D) $(6,0,0)$, $(0,-4,0)$, $(0,0,3)$.
E) $(1,0,0)$, $(0,3,0)$, $(0,0,5)$.

20. $M(2, -1, 1)$ nuqtadan o'tib, $3x+2y-z+4=0$ tekislikka parallel bo'lgan tekislikning koordinata o'qlari bilan kesishish nuqtalarining koordinatalarini toping.

- A) $(-1,0,0)$, $(0,-5,0)$, $(0,0,4)$. B) $(-6,0,0)$, $(0,3,0)$, $(0,0,4)$.
C) $(5,0,0)$, $(0,-4,0)$, $(0,0,-3)$. D) $(6,0,0)$, $(0,-4,0)$, $(0,0,3)$.
E) $(1,0,0)$, $(0,3/2,0)$, $(0,0,-3)$.

21. Kesmalardagi tenglamasi $\frac{x}{5} + \frac{y}{-2} + \frac{z}{9} = 1$ bo'lgan P tekislik va koordinata tekisliklari bilan chegaralangan piramida hajmini toping.

- A) 90 B) 60. C) 45. D) 30. E) 15.

22. Kesmalardagi tenglamasi $\frac{x}{a} + \frac{y}{4} + \frac{z}{-9} = 1$ bo'lgan P tekislik va koordinata tekisliklari bilan chegaralangan piramida hajmi 72 kub birlikka teng. Noma'lum a parametr qiymatini toping.

- A) 2. B) -2. C) ± 2 . D) ± 12 . E) 6.

23. $M(3, -2, 1)$ nuqtadan o'tib, $a_1=(0,1,-2)$ va $a_2=(5,0,2)$ vektorlarga parallel bo'lgan tekislik tenglamasini toping.

- A) $3x+2y-z-4=0$. B) $3x-5y+2z-21=0$. C) $2x-10y-5z-21=0$.
D) $2x+5y-z+5=0$. E) $5x-12y-3z-36=0$.

Fazodagi to'g'ri chiziq tenglamalari.

1. $M_0(x_0, y_0, z_0)$ nuqtadan o'tuvchi va $a=(m, n, p)$ vektorga parallel to'g'ri chiziqning kanonik tenglamasini ko'rsating.

- A) $\frac{x-m}{x_0} = \frac{y-n}{y_0} = \frac{z-p}{z_0}$. B) $\frac{x+m}{x_0} = \frac{y+n}{y_0} = \frac{z+p}{z_0}$.
C) $\frac{x-x_0}{m} = \frac{y-y_0}{n} = \frac{z-z_0}{p}$. D) $\frac{x+x_0}{m} = \frac{y+y_0}{n} = \frac{z+z_0}{p}$.

E) To'g'ri javob keltirilmagan.

2. Kanonik tenglamasi $\frac{x}{m} = \frac{y-y_0}{n} = \frac{z-z_0}{p}$ ko'rinishda bo'lgan L to'g'ri chiziq qanday xususiyatga ega?

9. $3x+4y-2\sqrt{6}z+14=0$ tekislikdan koordinata boshigacha bo'lgan masofani toping.

- A) 14. B) 7. C) 2. D) 1. E) $2\sqrt{6}$.

10. Ushbu $4x+3y-5z-8=0$ va $4x+3y-5z-12=0$ parallel tekisliklar orasidagi masofani toping.

- A) 4. B) 20. C) $\sqrt{2}$. D) $\frac{2\sqrt{2}}{5}$. E) $2\sqrt{2}$.

11. $x+y-z-1=0$ va $2x-2y-2z+1=0$ tekisliklar orasidagi burchak kosinusini toping.

- A) 0. B) 1. C) $\sqrt{3}/4$. D) $1/3$. E) $3/4$.

12. $x+y-18=0$ va $y+z-72=0$ tenglamalar bilan berilgan tekisliklar orasidagi burchak topilsin.

- A) 30° . B) $\arccos \frac{\sqrt{3}}{4}$. C) 45° . D) $\arccos \frac{\sqrt{3}}{5}$. E) 60° .

13. Normal vektorlari $n_1=(-1, -1, 0)$ va $n_2=(0, -1, -1)$ bo'lgan tekisliklar orasidagi burchak topilsin.

- A) 45° . B) 30° . C) $\arccos \frac{\sqrt{2}}{3}$. D) 60° . E) 90° .

14. $A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2=0$ tekisliklarning parallellik sharti qayerda to'g'ri ko'rsatilgan?

- A) $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{D_1}{D_2}$. B) $\frac{A_1}{A_2} = \frac{C_1}{C_2} = \frac{D_1}{D_2}$. C) $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$.
D) $\frac{B_1}{B_2} = \frac{C_1}{C_2} = \frac{D_1}{D_2}$. E) $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2} = \frac{D_1}{D_2}$.

15. $kx-2y+5z+10=0$ va $6x-(1+k)y+10z-2=0$ tekisliklar k parametrning qanday qiymatida parallel bo'ladi?

- A) $k=3$. B) $k=-4$. C) $k=2$. D) $k=-5$. E) $k=\pm 1$.

16. Umumiy tenglamalari $A_1x+B_1y+C_1z+D_1=0$ va $A_2x+B_2y+C_2z+D_2=0$ bilan berilgan tekisliklarning perpendikulyarlik shartini ko'rsating.

- A) $A_1A_2+B_1B_2+C_1C_2+D_1D_2=0$. B) $A_1A_2+B_1B_2+D_1D_2=0$.
C) $A_1A_2+B_1B_2+C_1C_2=0$. D) $B_1B_2+C_1C_2+D_1D_2=0$. E) $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$.

17. $kx-2y-5z+10=0$ va $6x-(1+k)y+10z-2=0$ tekisliklar k parametrning qanday qiymatida parallel bo'ladi?

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- A) 0. B) 1. C) -1. D) 4. E) -4.

19. $2x-3y+4z-12=0$ tenglama bilan berilgan tekislikning koordinata o'qlari bilan kesishgan nuqtalarining koordinatalari topilsin.

- A) $(-1,0,0)$, $(0,-5,0)$, $(0,0,4)$. B) $(-6,0,0)$, $(0,3,0)$, $(0,0,4)$.
C) $(5,0,0)$, $(0,-4,0)$, $(0,0,-3)$. D) $(6,0,0)$, $(0,-4,0)$, $(0,0,3)$.
E) $(1,0,0)$, $(0,3,0)$, $(0,0,5)$.

20. $M(2, -1, 1)$ nuqtadan o'tib, $3x+2y-z+4=0$ tekislikka parallel bo'lgan tekislikning koordinata o'qlari bilan kesishish nuqtalarining koordinatalarini toping.

- A) $(-1,0,0)$, $(0,-5,0)$, $(0,0,4)$. B) $(-6,0,0)$, $(0,3,0)$, $(0,0,4)$.
C) $(5,0,0)$, $(0,-4,0)$, $(0,0,-3)$. D) $(6,0,0)$, $(0,-4,0)$, $(0,0,3)$.
E) $(1,0,0)$, $(0,3/2,0)$, $(0,0,-3)$.

21. Kesmalardagi tenglamasi $\frac{x}{5} + \frac{y}{-2} + \frac{z}{9} = 1$ bo'lgan P tekislik va koordinata tekisliklari bilan chegaralangan piramida hajmini toping.

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- A) 2. B) -2. C) ± 2 . D) ± 12 . E) 6.

23. $M(3, -2, 1)$ nuqtadan o'tib, $a_1=(0,1,-2)$ va $a_2=(5,0,2)$ vektorlarga parallel bo'lgan tekislik tenglamasini toping.

- A) $3x+2y-z-4=0$. B) $3x-5y+2z-21=0$. C) $2x-10y-5z-21=0$.
D) $2x+5y-z+5=0$. E) $5x-12y-3z-36=0$.

Fazodagi to'g'ri chiziqli tenglamalari.

1. $M_0(x_0, y_0, z_0)$ nuqtadan o'tuvchi va $a=(m,n,p)$ vektorga parallel to'g'ri chiziqli tenglamasini ko'rsating.

- A) $\frac{x-m}{x_0} = \frac{y-n}{y_0} = \frac{z-p}{z_0}$. B) $\frac{x+m}{x_0} = \frac{y+n}{y_0} = \frac{z+p}{z_0}$.
C) $\frac{x-x_0}{m} = \frac{y-y_0}{n} = \frac{z-z_0}{p}$. D) $\frac{x+x_0}{m} = \frac{y+y_0}{n} = \frac{z+z_0}{p}$.

E) To'g'ri javob keltirilmagan.

2. Kanonik tenglamasi $\frac{x}{m} = \frac{y-y_0}{n} = \frac{z-z_0}{p}$ ko'rinishda bo'lgan L to'g'ri chiziqli qanday xususiyatga ega?

- A) L to'g'ri chiziqli YOZ koordinata tekisligiga parallel joylashgan.
 B) L to'g'ri chiziqli YOZ koordinata tekisligiga perpendikulyar joylashgan.
 C) L to'g'ri chiziqli YOZ koordinata tekisligini kesib o'tadi.
 D) L to'g'ri chiziqli YOZ koordinata tekisligini kesib o'tmaydi.
 E) To'g'ri javob keltirilmagan.

3. Fazodagi L to'g'ri chiziqli tenglamasi $\frac{x-x_0}{m} = \frac{y-y_0}{n} = \frac{z-z_0}{p}$ kanonik

tenglamasida $m=0$ bo'lsa, L qanday xususiyatga ega bo'ladi?

- A) L to'g'ri chiziqli OX koordinata o'qiga parallel joylashgan.
 B) L to'g'ri chiziqli OX koordinata o'qiga perpendikulyar joylashgan.
 C) L to'g'ri chiziqli OX koordinata o'qiga parallel emas.
 D) L to'g'ri chiziqli OX koordinata o'qiga perpendikulyar emas.
 E) To'g'ri javob keltirilmagan.

4. Kanonik tenglamasi $\frac{x-3}{2} = \frac{y+1}{5} = \frac{z}{-3}$ bo'lgan to'g'ri chiziqli

yo'naltiruvchi vektorining koordinatalarini toping.

- A) $(-3, 1, 0)$. B) $(3, -1, 0)$. C) $(2, 5, -3)$. D) $(-2, -5, 3)$. E) $(3, 1, 0)$.

5. Kanonik tenglamasi $\frac{x-3}{12} = \frac{y}{-4} = \frac{z+4}{3}$ bo'lgan to'g'ri chiziqli

yo'naltiruvchi vektorining modulini toping.

- A) 1 B) $\sqrt{7}$. C) 5. D) 13. E) $\sqrt{11}$.

6. Kanonik tenglamasi $\frac{x-3}{2} = \frac{y+1}{5} = \frac{z}{-3}$ bo'lgan to'g'ri chiziqli

boshlang'ich nuqtasining koordinatalarini toping.

- A) $(-3, 1, 0)$. B) $(3, -1, 0)$. C) $(2, 5, -3)$. D) $(-2, -5, 3)$. E) $(3, 5, 0)$.

7. Quyidagilardan qaysi biri fazodagi $M_0(x_0, y_0, z_0)$ boshlang'ich nuqta va $a=(m, n, p)$ yo'naltiruvchi vektorga ega to'g'ri chiziqli parametrik tenglamasiga teng kuchli emas?

- A) $xx_0=mt, yy_0=nt, zz_0=pt$. B) $x-mt=x_0, y-nt=y_0, z-pt=z_0$.
 C) $x-x_0=mt, y-y_0=nt, z-z_0=pt$. D) $x-x_0-mt=0, y-y_0-nt=0, z-z_0-pt=0$.
 E) $\frac{x-x_0}{m} = \frac{y-y_0}{n} = \frac{z-z_0}{p} = t$.

8. Kanonik tenglamasi $\frac{x-3}{2} = \frac{y+1}{5} = \frac{z}{-3}$ bo'lgan to'g'ri chiziqli

parametrik tenglamasini toping.

- A) $x=2-3t, y=5+t, z=-3$. B) $x=2+3t, y=5-t, z=-3$.
 C) $x=3+2t, y=-1+5t, z=-3t$. D) $x=3-2t, y=-1-5t, z=3t$.

E) $x=3+2t, y=5+t, z=-3t$.

9. To'g'ri chiziqning $x=3+2t, y=2+5t, z=4-3t$ parametrik tenglamasi bo'yicha uning kanonik tenglamasini toping.

A) $\frac{x-3}{2} = \frac{y+1}{5} = \frac{z}{-3}$.

B) $\frac{x-3}{2} = \frac{y-5}{3} = \frac{z+1}{-3}$.

C) $\frac{x-3}{2} = \frac{y-2}{5} = \frac{z-4}{-3}$. D) $\frac{x-3}{2} = \frac{y-2}{5} = \frac{z-4}{-3}$. E) $\frac{x-2}{3} = \frac{y+1}{5} = \frac{z+4}{3}$.

10. Fazodagi to'g'ri chiziqning umumiy tenglamasini ko'rsating.

A) $Ax+By+C=0$. B) $Ax+By+Cz+D=0$.

C) $A_1x+B_1y+C_1z+D_1=A_2x+B_2y+C_2z+D_2$.

D) $\begin{cases} A_1x+B_1y+C_1z+D_1=0 \\ A_2x+B_2y+C_2z+D_2=0 \end{cases}$.

E) $\begin{cases} A_1x+B_1y+C_1z=0 \\ A_2x+B_2y+C_2z=0 \end{cases}$.

11. Umumiy tenglamasi $\begin{cases} 2x+3y+5z-7=0 \\ x-y+2z+5=0 \end{cases}$ bo'lgan to'g'ri

chiziqning parametrik tenglamasini toping.

A) $x=\frac{5}{3}+\frac{2}{3}t, y=\frac{4}{5}-\frac{7}{5}t, z=t$. B) $x=-\frac{8}{5}-\frac{11}{5}t, y=\frac{17}{5}-\frac{1}{5}t, z=t$.

C) $x=\frac{17}{3}+\frac{2}{3}t, y=-\frac{8}{3}+\frac{7}{3}t, z=t$. D) $x=\frac{6}{5}-\frac{4}{5}t, y=\frac{7}{3}-\frac{41}{3}t, z=t$.

E) $x=5+2t, y=4-7t, z=t$.

12. Umumiy tenglamasi $\begin{cases} A_1x+B_1y+C_1z+D_1=0 \\ A_2x+B_2y+C_2z+D_2=0 \end{cases}$ bilan berilgan

to'g'ri chiziqning a yo'naltiruvchi vektori qaysi formula orqali aniqlanadi?

A) $a=\begin{vmatrix} i & j & k \\ A_1 & C_1 & D_1 \\ A_2 & C_2 & D_2 \end{vmatrix}$. B) $a=\begin{vmatrix} i & j & k \\ A_1 & B_1 & D_1 \\ A_2 & B_2 & D_2 \end{vmatrix}$. C) $a=\begin{vmatrix} i & j & k \\ A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \end{vmatrix}$.

D) $a=\begin{vmatrix} i & j & k \\ B_1 & C_1 & D_1 \\ B_2 & C_2 & D_2 \end{vmatrix}$. E) $a=\begin{vmatrix} i & j & k \\ A_1 & D_1 & C_1 \\ A_2 & D_2 & C_2 \end{vmatrix}$.

13. Umumiy tenglamasi $\begin{cases} 2x+3y+5z-7=0 \\ x-y+2z+5=0 \end{cases}$ bo'lgan to'g'ri chiziqning

a yo'naltiruvchi vektorini toping.

A) $a=(2,-7,1)$. B) $a=(-5,13,4)$. C) $a=(0,-4,-14)$.

D) $a=(11,1,-5)$. E) $a=(-12,-6,-1)$.

14. Fazodagi to'g'ri chiziqning qanday tenglamasi bo'lmaydi?

A) parametrik. B) normal. C) umumiy. D) kanonik.

E) keltirilgan barcha tenglamalarga ega bo'ladi.

15. Quyidagi sistemalardan qaysi biri ushbu kanonik tenglama bilan berilgan fazodagi L to'g'ri chiziqning umumiy tenglamasini ifodalamaydi:

$$\frac{x-5}{2} = \frac{y-4}{-7} = \frac{z-1}{-3}.$$

A) $\begin{cases} 7x + 2y - 43 = 0, \\ 3x - 2z - 17 = 0 \end{cases}$ B) $\begin{cases} 7x + 2y - 43 = 0, \\ 3y - 7z - 5 = 0 \end{cases}$ C) $\begin{cases} 3x - 2z - 17 = 0, \\ 3y - 7z - 5 = 0 \end{cases}$

D) barcha sistemalar L to'g'ri chiziqning umumiy tenglamasini ifodalaydi.

E) barcha sistemalar L to'g'ri chiziqning umumiy tenglamasini ifodalamaydi.

8-§. KETMA-KETLIK VA FUNKSIYANING LIMITI. BIRINCHI VA IKKINCHI MUHIM LIMITLAR. FUNKSIYANING NUQTADAGI UZLUKSIZLIGI.

8.1. Sonlar ketma-ketligi tushunchasi

Aytaylik, biror qoidaga ko'ra har bir natural n songa ($n \in N$) bitta x_n haqiqiy son mos qo'yilgan bo'lsin ($f: n \rightarrow x_n$). Ravshanki, bu holda argumenti n bo'lgan funksiyaga ega bo'lamiz. Bunday funksiya natural argumentli funksiya deyiladi:

$$x_n = f(n).$$

Bu funksiya qiymatlaridan iborat ushbu

$$x_1, x_2, x_3, \dots, x_n, \dots \quad (8.1)$$

to'plam sonlar ketma-ketligi deyiladi va $\{x_n\}$ kabi belgilanadi. $x_1, x_2, \dots$ sonlar (8.1) ketma-ketlikning hadlari, x_n esa (8.1) ketma-ketlikning umumiy yoki n -hadi deyiladi.

Masalan, har bir natural n songa $\frac{1}{n}$ sonni mos qo'yish bilan ($n \rightarrow \frac{1}{n}$) ushbu

$$1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}, \dots$$

ketma-ketlik hosil bo'ladi. Bu ketma-ketlikning umumiy hadi $x_n = \frac{1}{n}$ bo'ladi.

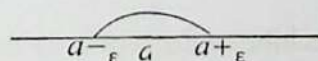
8.2. Sonlar ketma-ketligining limiti

Biror $\{x_n\}$: $x_1, x_2, x_3, \dots, x_n, \dots$

ketma-ketlik va a soni berilgan bo'lsin. Ushbu

$$(a - \varepsilon, a + \varepsilon) = U_\varepsilon(a)$$

interval (sonlar to'plami) a nuqtaning atrofi (ε -atrofi) deyiladi, bunda ε -ixtiyoriy musbat son (8.1-chizma)



8.1-chizma

1-Ta'rif. Agar a nuqtaning ixtiyoriy $U_\varepsilon(a)$ atrofi olinganda ham (8.1) ketma-ketlikning biror hadidan boshlab keyingi barcha hadlari shu atrofga tegishli bo'lsa, a son x_n ketma-ketlikning limiti deyiladi va

$$\lim_{n \rightarrow \infty} x_n = a \quad \text{yoki} \quad n \rightarrow \infty \quad \text{äà} \quad x_n \rightarrow a$$

kabi yoziladi.

Ta'rifdagi "biror hadidan boshlab, keyingi barcha hadidan" iborasi shunday natural n_0 topilib, $\forall n > n_0$ uchun deb aytilishini bildiradi.

Demak, $\forall n > n_0$ uchun $x_n \in U_\varepsilon(a) = (a - \varepsilon, a + \varepsilon)$ bo'lishi bunday hadlarning ushbu

$$a - \varepsilon < x_n < a + \varepsilon, \quad -\varepsilon < x_n - a < \varepsilon$$

ya'ni

$$|x_n - a| < \varepsilon$$

tengsizlikning bajarilishini keltirib chiqaradi.

Unda yuqorida keltirilgan ta'rifni quyidagicha ham aytsa bo'ladi: agar ixtiyoriy $\varepsilon > 0$ son olinganda ham shunday natural n_0 son topilib, barcha $n > n_0$ uchun $|x_n - a| < \varepsilon$

tengsizlik bajarilsa, a son x_n ketma-ketlikning limiti deyiladi.

1-misol. Ushbu

$$x_n = \frac{1}{n}:$$

$$1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}, \dots$$

ketma-ketlikning limiti 0 bo'lishi isbotlansin.

Yechish. Ixtiyoriy $\varepsilon > 0$ sonni olamiz. Ravshanki berilgan ketma-ketlikning limiti 0 bo'lishi uchun

$$|x_n - 0| = \left| \frac{1}{n} - 0 \right| < \varepsilon \quad (8.2)$$

tengsizlikning n ning biror qiymatidan boshlab o'rinli bo'lishini ko'rsatish yetarli. Keyingi tengsizlik

$$\frac{1}{n} < \varepsilon \quad (8.3)$$

ni yechib,

$$n > \frac{1}{\varepsilon}$$

bo'lishini topamiz. Agar n_0 sifatida $\left[\frac{1}{\varepsilon} \right]$ ($[a] - a$ soninig a dan katta bo'lmagan butun qismi) olinsa, $n_0 = \left[\frac{1}{\varepsilon} \right]$ unda barcha $n > n_0$ da (8.3) demak, (8.2) tengsizlik bajariladi. Ta'rifga binoan

$$\lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

bo'ladi. ►

Endi ketma-ketliklarning limitini hisoblashga misollar keltiramiz.
2-misol.

$$\lim_{n \rightarrow \infty} \frac{n+2}{n-3} = \lim_{n \rightarrow \infty} \frac{1 + \frac{2}{n}}{1 - \frac{3}{n}} = \frac{\lim_{n \rightarrow \infty} \left(1 + \frac{2}{n} \right)}{\lim_{n \rightarrow \infty} \left(1 - \frac{3}{n} \right)} = 1.$$

3-misol.

$$\lim_{n \rightarrow \infty} \frac{n^2 - n + 2}{n^3 + 1} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n} - \frac{1}{n^2} + \frac{2}{n^3}}{1 + \frac{1}{n^3}} = \frac{\lim_{n \rightarrow \infty} \left(\frac{1}{n} - \frac{1}{n^2} + \frac{2}{n^3} \right)}{\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n^3} \right)} = \frac{0}{1} = 0.$$

4-misol.

$$\begin{aligned} \lim_{n \rightarrow \infty} \left(\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \dots + \frac{n-1}{n^2} \right) &= \lim_{n \rightarrow \infty} \frac{1}{n^2} (1 + 2 + 3 + \dots + (n-1)) = \\ &= \lim_{n \rightarrow \infty} \frac{1}{n^2} \cdot \frac{(n-1) \cdot n}{2} = \lim_{n \rightarrow \infty} \frac{1}{2} \cdot \left(1 - \frac{1}{n} \right) = \frac{1}{2}. \end{aligned}$$

5-misol.

$$\begin{aligned} \lim_{n \rightarrow \infty} (\sqrt{n+1} - \sqrt{n}) &= \lim_{n \rightarrow \infty} \frac{(\sqrt{n+1} - \sqrt{n})(\sqrt{n+1} + \sqrt{n})}{\sqrt{n+1} + \sqrt{n}} = \\ &= \lim_{n \rightarrow \infty} \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n+1} + \sqrt{n}} = 0 \end{aligned}$$

8.3. Funksiyaning aniqlanish va o'zgarish sohasi

Agar X to'plamning har bir x elementiga Y to'plamning ma'lum bir y elementi biror f qonun-qoida asosida mos qo'yilgan bo'lsa, u holda X to'plamda $y = f(x)$ **funksiya** berilgan deyiladi. Bunda x -**erкли o'zgaruvchi yoki argument**, y -**erksiz o'zgaruvchi yoki funksiya** deyiladi.

1. $y = f(x)$ funksiyada x argument qabul qila oladigan barcha qiymatlar to'plami funksiyaning **aniqlanish sohasi** deyiladi va $D\{f\}$ kabi belgilanadi. $x \in D\{f\}$ bo'lganda $y = f(x)$ funksiya qabul qiladigan qiymatlar to'plami funksiyaning **o'zgarish sohasi** deb ataladi va $E\{f\}$ kabi belgilanadi.

2. Agar $y = f(x)$ funksiya uchun $f(-x) = f(x)$ yoki $f(-x) = -f(x)$ shart bajarilsa u mos ravishda **juft yoki toq funksiya** deyiladi.

3. Agar $y = f(x)$ funksiyada argumentning har qanday $x_1 < x_2$ qiymatlari uchun $f(x_1) < f(x_2)$ yoki $f(x_1) > f(x_2)$ shart bajarilsa, u mos ravishda **o'suvchi yoki kamayuvchi funksiya** deyiladi. O'suvchi va kamayuvchi funksiyalar birgalikda **monoton funksiyalar** deyiladi.

4. Agarda ixtiyoriy $x \in D\{f\}$ va biror chekli M soni uchun $|f(x)| < M$ tengsizlik bajarilsa, unda $y = f(x)$ **chegaralangan funksiya** deyiladi. Aks holda $y = f(x)$ chegaralanmagan funksiyani ifodalaydi.

Namunaviy masalalarning yechilishiga e'tobor bering.

1. Quyidagi funksiyaning $D\{f\}$ aniqlanish sohasini toping:

$$y = \frac{\sqrt{x^2 - 4}}{2^x(x-6)} + \ln(x+10)$$

Yechish:

$$\begin{cases} x^2 - 4 \geq 0 \\ 2^x(x-6) \neq 0 \\ x+10 > 0 \end{cases} \Rightarrow \begin{cases} (x-2)(x+2) \geq 0 \\ x \neq 6 \\ x > -10 \end{cases} \Rightarrow \begin{cases} x \in (-\infty; -2] \cup [2; +\infty) \\ x \neq 6 \\ x \in (-10; +\infty) \end{cases}$$

Bu yerdan $D\{f\} = x \in (-10; -2] \cup [2; 6) \cup (6; +\infty)$ ekanligini topamiz.

2. $y = 10^{-2x^2}$ funksiyaning $E\{f\}$ o'zgarish sohasini toping.

Yechish: Berilgan $y = 10^{-2x^2} = f(x)$ funksiyaga teskari bo'lgan $f^{-1}(x)$ funksiyani va uning aniqlanish sohasini topamiz:

$$\begin{aligned} y = 10^{-2x^2} &\Rightarrow \ln y = -2x^2 \Rightarrow x^2 = -\frac{1}{2} \ln y \geq 0 \Rightarrow x = \sqrt{-\frac{1}{2} \ln y} \Rightarrow \\ &\Rightarrow f^{-1}(x) = \sqrt{-\frac{1}{2} \ln x} \Rightarrow \ln x \leq 0 \Rightarrow D\{f^{-1}\} = (0, 1] \end{aligned}$$

Bu yerdan, teskari funksiyalarning xossasiga asosan, $E\{f\} = D\{f^{-1}\} = (0, 1]$ ekanligini topamiz.

8.4. Funksiyaning limitini aniqlash

1. Agar ixtiyoriy kichik $\varepsilon > 0$ soni uchun shunday $\delta = \delta(\varepsilon) > 0$ son topilsaki, $|x - a| < \delta$ shartni qanoatlantiruvchi barcha x va biror chekli A soni uchun $|f(x) - A| < \varepsilon$ tengsizlik bajarilsa, u holda A soni $y = f(x)$ **funksiyaning $x \rightarrow a$ holdagi limiti** deyiladi va $\lim_{x \rightarrow a} f(x) = A$ kabi ifodalanadi.

2. Agar ixtiyoriy kichik $\varepsilon > 0$ soni uchun shunday $N = N(\varepsilon) > 0$ son topilsaki, $|x| > N$ shartni qanoatlantiruvchi barcha x va biror chekli A soni uchun $|f(x) - A| < \varepsilon$ tengsizlik bajarilsa, u holda A soni $y = f(x)$ **funksiyaning $x \rightarrow \pm\infty$ holdagi limiti** deyiladi va $\lim_{x \rightarrow \pm\infty} f(x) = A$ kabi ifodalanadi.

3. Agar $\lim_{x \rightarrow a} \alpha(x) = 0$ yoki $\lim_{x \rightarrow \pm\infty} \alpha(x) = 0$ bo'lsa, unda $\alpha(x)$ funksiya mos ravishda $x \rightarrow a$ yoki $x \rightarrow \pm\infty$ holda **cheksiz kichik miqdor** deyiladi.

4. Cheksiz kichik miqdorlar quyidagi xossalarga ega:

a) Agar $\alpha(x)$ cheksiz kichik miqdor bo'lsa, unda ixtiyoriy C o'zgarmas son uchun $C\alpha(x)$ ham cheksiz kichik miqdor bo'ladi.

b) Agar $\alpha(x)$ va $\beta(x)$ cheksiz kichik miqdorlar bo'lsa, unda $\alpha(x) \pm \beta(x)$, $\alpha(x) \cdot \beta(x)$ ham va $\beta(x)$ cheksiz kichik miqdorlar bo'ladi.

c) $\alpha(x)$ va $\beta(x)$ cheksiz kichik miqdorlarning nisbati $\alpha(x)/\beta(x)$ cheksiz kichik miqdor bo'lishi shart emas.

Agar $\lim_{x \rightarrow a} \frac{\alpha(x)}{\beta(x)} = 1$ (a — chekli son yoki cheksiz) bo'lsa, $\alpha(x)$ va $\beta(x)$ cheksiz kichik miqdorlar **ekvivalent** deb ataladi va $\alpha(x) \sim \beta(x)$ kabi belgianadi. Masalan, agar $\alpha(x)$ cheksiz kichik miqdor bo'lsa

$$\sin \alpha(x) \sim \alpha(x); \quad \arcsin \alpha(x) \sim \alpha(x); \quad \operatorname{tg} \alpha(x) \sim \alpha(x); \quad \operatorname{arctg} \alpha(x) \sim \alpha(x);$$

$$1 - \cos \alpha(x) \sim \frac{\alpha^2(x)}{2}; \quad \log_a(1 + \alpha(x)) \sim \frac{\alpha(x)}{\ln a}; \quad \ln(1 + \alpha(x)) \sim \alpha(x);$$

$$a^{\alpha(x)} - 1 \sim \alpha(x) \ln a; \quad e^{\alpha(x)} - 1 \sim \alpha(x), \quad (1 + \alpha(x))^n \sim 1 + n\alpha(x).$$

d) Agar $\alpha(x)$ cheksiz kichik miqdor bo'lsa, unda $\lim_{x \rightarrow a} \frac{1}{\alpha(x)} = \infty$, ya'ni

$\frac{1}{\alpha(x)}$ funksiya cheksiz katta bo'ladi.

8.5. Funksiya limitining xossalari

6. Turli funksiyalarning limitlarini hisoblashda

$$\lim_{x \rightarrow a} Cf(x) = C \lim_{x \rightarrow a} f(x) = C \cdot A \quad (C - \text{const}), \quad \lim_{x \rightarrow a} C = C$$

formulalardan va

$$\lim_{x \rightarrow 0} x^\alpha = \begin{cases} 0, & \text{agar } \alpha > 0 \text{ bo'lsa;} \\ 1, & \text{agar } \alpha = 0 \text{ bo'lsa;} \\ \infty, & \text{agar } \alpha < 0 \text{ bo'lsa.} \end{cases}, \quad \lim_{x \rightarrow \infty} x^\alpha = \begin{cases} \infty, & \text{agar } \alpha > 0 \text{ bo'lsa;} \\ 1, & \text{agar } \alpha = 0 \text{ bo'lsa;} \\ 0, & \text{agar } \alpha < 0 \text{ bo'lsa.} \end{cases}$$

ekanligidan foydalanish mumkin.

7. $\lim_{x \rightarrow a} f(x) = A, \lim_{x \rightarrow a} g(x) = B$ bo'lganda $f(x)$ va $g(x)$ funksiyalar nisbatlarining limiti A va B chekli sonlar ($B \neq 0$) holda

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)} = \frac{A}{B}$$

formula bilan aniqlanadi.

8. $\lim_{x \rightarrow a} f(x) = A, \lim_{x \rightarrow a} g(x) = B$ bo'lganda $f(x)$ va $g(x)$ funksiyalar

ko'paytmasining limiti A va B chekli sonlar bo'lgan holda

$$\lim_{x \rightarrow a} [f(x) \cdot g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x) = A \cdot B$$

formula bilan aniqlanadi.

9. $\lim_{x \rightarrow a} f(x) = A, \lim_{x \rightarrow a} g(x) = B$ bo'lganda $f(x)$ va $g(x)$ funksiyalar

algebraik yig'indisining limiti A va B chekli sonlar bo'lgan holda

$$\lim_{x \rightarrow a} [f(x) \pm g(x)] = \lim_{x \rightarrow a} f(x) \pm \lim_{x \rightarrow a} g(x) = A \pm B$$

formula bilan aniqlanadi.

8.6. Ajoyib limitlar.

Limitlarni hisoblashda quyidagi tengliklardan foydalanish mumkin:

1) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ - *birinchi ajoyib limit*;

2) $\lim_{x \rightarrow \pm\infty} \left(1 + \frac{1}{x}\right)^x = \lim_{x \rightarrow 0} (1+x)^{1/x} = e = 2.71828\dots$ - *ikkinchi ajoyib limit*;

3) $\lim_{x \rightarrow 0} \frac{\log_a(1+x)}{x} = \log_a e$;

4) $\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$;

6) $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$;

6) $\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1$;

7) $\lim_{x \rightarrow 0} \frac{(1+x)^\alpha - 1}{x} = \alpha$;

8) $\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - 1}{x} = \frac{1}{2}$.

Namunaviy masalalarning yechilishiga e'tobor bering.

1. $\lim_{x \rightarrow 0} \frac{\sqrt{4-x} - \sqrt{4+x}}{2x} = ?$

Yechish: Bu limit qiymati $\frac{0}{0}$ ko'rinishdagi aniqmaslikni ochish orqali

quyidagicha aniqlanadi:

$$\lim_{x \rightarrow 0} \frac{\sqrt{4-x} - \sqrt{4+x}}{2x} = \lim_{x \rightarrow 0} \frac{(\sqrt{4-x} - \sqrt{4+x})(\sqrt{4-x} + \sqrt{4+x})}{2x(\sqrt{4-x} + \sqrt{4+x})} =$$

$$= \lim_{x \rightarrow 0} \frac{(\sqrt{4-x})^2 - (\sqrt{4+x})^2}{2x(\sqrt{4-x} + \sqrt{4+x})} = \lim_{x \rightarrow 0} \frac{-2x}{2x(\sqrt{4-x} + \sqrt{4+x})} = -\lim_{x \rightarrow 0} \frac{1}{\sqrt{4-x} + \sqrt{4+x}} = -\frac{1}{4}$$

2. $\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+2x} - 1}{x} = ?$

Yechish: Bu limit qiymati $\frac{0}{0}$ ko'rinishdagi aniqmaslikni ochish orqali

quyidagicha aniqlanadi:

$$\lim_{x \rightarrow 0} \frac{\sqrt[3]{1+2x} - 1}{x} = \left[\frac{0}{0} \right] = \left| \frac{1+2x = t^3, x = (t^3 - 1)/2}{x \rightarrow 0 \Rightarrow t \rightarrow 1} \right| = \lim_{t \rightarrow 1} \frac{t - 1}{(t^3 - 1)/2} =$$

$$= \lim_{t \rightarrow 1} \frac{2(t - 1)}{(t - 1)(t^2 + t + 1)} = \lim_{t \rightarrow 1} \frac{2}{t^2 + t + 1} = \frac{2}{3}.$$

3. $\lim_{x \rightarrow \infty} \frac{4x^3 - 3x^2 + 2x - 1}{x^3 + 3x^2 - 5x + 4} = ?$

Yechish: Bu limit qiymati $\frac{\infty}{\infty}$ ko'rinishdagi aniqmaslikni ochish orqali

quyidagicha aniqlanadi:

$$\lim_{x \rightarrow \infty} \frac{4x^3 - 3x^2 + 2x - 1}{x^3 + 3x^2 - 5x + 4} = \left[\frac{\infty}{\infty} \right] = \lim_{x \rightarrow \infty} \frac{x^3[4 - (3/x) + (2/x^2) - (1/x^3)]}{x^3[1 + (3/x) - (5/x^2) + (4/x^3)]} =$$

$$= \lim_{x \rightarrow \infty} \frac{4 - (3/x) + (2/x^2) - (1/x^3)}{1 + (3/x) - (5/x^2) + (4/x^3)} = \frac{4 - 0 + 0 - 0}{1 + 0 - 0 + 0} = 4.$$

4. $\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^3 + 4x^2 + 3x} = ?$

Yechish: Bu limit qiymati $\frac{0}{0}$ ko'rinishdagi aniqmaslikni ochish orqali

quyidagicha aniqlanadi:

$$\lim_{x \rightarrow 1} \frac{x^2 - 3x + 2}{x^3 + 4x^2 + 3x} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 1} \frac{(x-1)(x-2)}{x(x-1)(x-3)} = \lim_{x \rightarrow 1} \frac{x-2}{x(x-3)} = \frac{1-2}{1 \cdot (1-3)} = \frac{-1}{-2} = \frac{1}{2}.$$

5. $\lim_{x \rightarrow 1} \left[\frac{1}{x-1} - \frac{2x}{x^2-1} \right] = ?$

Yechish: Bu limit qiymati $\infty - \infty$ ko'rinishdagi aniqmaslikni ochish orqali quyidagicha aniqlanadi:

$$\lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{2x}{x^2-1} \right) = [\infty - \infty] = \lim_{x \rightarrow 1} \frac{x+1-2x}{x^2-1} = \lim_{x \rightarrow 1} \frac{1-x}{x^2-1} = \left[\frac{0}{0} \right] =$$

$$= -\lim_{x \rightarrow 1} \frac{x-1}{(x-1)(x+1)} = -\lim_{x \rightarrow 1} \frac{1}{x+1} = -\frac{1}{1+1} = -\frac{1}{2}.$$

$$6. \lim_{x \rightarrow +\infty} (\sqrt{x^2+x} - \sqrt{x^2-x}) = ?$$

Yechish: Bu limit qiymati $\infty - \infty$ ko'rinishdagi aniqmaslikni ochish orqali quyidagicha aniqlanadi:

$$\begin{aligned} \lim_{x \rightarrow +\infty} (\sqrt{x^2+x} - \sqrt{x^2-x}) &= [\infty - \infty] = \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2+x} - \sqrt{x^2-x})(\sqrt{x^2+x} + \sqrt{x^2-x})}{\sqrt{x^2+x} + \sqrt{x^2-x}} = \\ &= \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2+x})^2 - (\sqrt{x^2-x})^2}{\sqrt{x^2+x} + \sqrt{x^2-x}} = \lim_{x \rightarrow +\infty} \frac{(x^2+x) - (x^2-x)}{\sqrt{x^2(1+1/x)} + \sqrt{x^2(1-1/x)}} = \\ &= \lim_{x \rightarrow +\infty} \frac{2x}{x(\sqrt{1+1/x} + \sqrt{1-1/x})} = \lim_{x \rightarrow +\infty} \frac{2}{\sqrt{1+1/x} + \sqrt{1-1/x}} = \frac{2}{\sqrt{1+0} + \sqrt{1-0}} = 1. \end{aligned}$$

$$7. \lim_{x \rightarrow 0} (x \operatorname{ctg} x) = ?$$

Yechish: Bu limit qiymati $0 \cdot \infty$ ko'rinishdagi aniqmaslikni ochish va I ajoyib limitdan foydalanish orqali quyidagicha aniqlanadi:

$$\lim_{x \rightarrow 0} (x \operatorname{ctg} x) = [0 \cdot \infty] = \lim_{x \rightarrow 0} \left(\frac{x}{\sin x} \cos x \right) = \lim_{x \rightarrow 0} \frac{x}{\sin x} \cdot \lim_{x \rightarrow 0} \cos x = 1^{-1} \cdot 1 = 1$$

$$8. \lim_{x \rightarrow \pi/2} (1 + \operatorname{ctg} x)^{\operatorname{ctg} x} = ?$$

Yechish: Bu limit qiymati 1^∞ ko'rinishdagi aniqmaslikni ochish va II ajoyib limitdan foydalanish orqali quyidagicha aniqlanadi:

$$\begin{aligned} \lim_{x \rightarrow \pi/2} (1 + \operatorname{ctg} x)^{\operatorname{ctg} x} &= [1^\infty] = \lim_{x \rightarrow \pi/2} (1 + \operatorname{ctg} x)^{1/\operatorname{ctg} x} = \\ &= [\operatorname{ctg} x = t, x \rightarrow \pi/2 \Rightarrow t \rightarrow 0] = \lim_{t \rightarrow 0} (1+t)^{1/t} = e. \end{aligned}$$

$$9. \lim_{x \rightarrow \infty} \left(\frac{x-1}{x+1} \right)^{2x} = ?$$

Yechish: Dastlab bu limit qaysi ko'rinishdagi aniqmaslikka kelishini aniqlaymiz:

$$\lim_{x \rightarrow \infty} \left(\frac{x-1}{x+1} \right)^{2x} = \left[\frac{\infty}{\infty} \right]^\infty = \lim_{x \rightarrow \infty} \left(\frac{x(1-1/x)}{x(1+1/x)} \right)^{2x} = \lim_{x \rightarrow \infty} \left(\frac{1-1/x}{1+1/x} \right)^{2x} = \left(\frac{1-0}{1+0} \right)^\infty = 1^\infty.$$

Demak, bu limit qiymati 1^∞ ko'rinishdagi aniqmaslikni ochish va II ajoyib limitdan foydalanish orqali quyidagicha aniqlanadi:

$$\lim_{x \rightarrow \infty} \left(\frac{x-1}{x+1} \right)^{2x} = [1^\infty] = \lim_{x \rightarrow \infty} \left(1 + \frac{x-1}{x+1} - 1 \right)^{2x} = \lim_{x \rightarrow \infty} \left(1 + \frac{-2}{x+1} \right)^{2x \cdot \frac{-2}{x+1} \cdot \frac{x+1}{-2}} =$$

$$= \lim_{x \rightarrow \infty} \left(\left(1 + \frac{-2}{x+1} \right)^{\frac{x+1}{-2}} \right)^{\frac{-4x}{x+1}} = \lim_{x \rightarrow \infty} \left(\left(1 + \frac{-2}{x+1} \right)^{\frac{x+1}{-2}} \right)^{\lim_{x \rightarrow \infty} \frac{-4x}{x+1}} = e^{\lim_{x \rightarrow \infty} \frac{-4x}{x+1}} = e^{-4}.$$

10. $\lim_{x \rightarrow 0} \frac{e^{x^2} - e^{2x}}{\cos 4x - \cos 2x} = ?$

Yechish:

$$\lim_{x \rightarrow 0} \frac{e^{x^2} - e^{2x}}{\cos 4x - \cos 2x} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow 0} \frac{e^{2x}(e^{x^2-2x} - 1)}{-2 \sin \frac{4x+2x}{2} \sin \frac{4x-2x}{2}} = \lim_{x \rightarrow 0} \frac{e^{2x}(e^{x^2-2x} - 1)}{-2 \sin 3x \sin x}$$

Cheksiz kichik miqdorlarning ekvivalentlik shartidan $x \rightarrow 0$ bo'lganda

$$e^{x^2-2x} - 1 \sim x^2 - 2x \sim -2x, \quad \sin 3x \sim 3x, \quad \sin x \sim x$$

bo'ladi. Demak,

$$\lim_{x \rightarrow 0} \frac{e^{2x}(e^{x^2-2x} - 1)}{-2 \sin 3x \sin x} = \lim_{x \rightarrow 0} \frac{e^{2x}(-2x)}{-2 \cdot 3x \cdot x} = \lim_{x \rightarrow 0} \frac{e^{2x}}{3x} = \left[\frac{1}{0} \right] = \infty.$$

11. $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 3x}{\cos x} = ?$

Yechish: Keltirish formulalari va $\sin \alpha x \sim \alpha x$ ekvivalentlikdan foydalanib quyidagi javobni olamiz:

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos 3x}{\cos x} &= \left[\frac{0}{0} \right] = \left[\frac{t = \frac{\pi}{2} - x, x = \frac{\pi}{2} - t}{x \rightarrow \frac{\pi}{2} \Rightarrow t \rightarrow 0} \right] = \lim_{t \rightarrow 0} \frac{\cos 3\left(\frac{\pi}{2} - t\right)}{\cos\left(\frac{\pi}{2} - t\right)} = \\ &= \lim_{t \rightarrow 0} \frac{\cos\left(\frac{3\pi}{2} - 3t\right)}{\cos\left(\frac{\pi}{2} - t\right)} = \lim_{t \rightarrow 0} \frac{-\sin 3t}{\sin t} = \lim_{t \rightarrow 0} \frac{-3t}{t} = -3. \end{aligned}$$

12. $\lim_{x \rightarrow e} \frac{\ln x - 1}{x - e} = ?$

Yechish: Bu limit qiymati $\frac{0}{0}$ ko'rinishdagi aniqlanmaslikni ochish va II ajoyib limitdan foydalanish orqali quyidagicha aniqlanadi:

$$\lim_{x \rightarrow e} \frac{\ln x - 1}{x - e} = \left[\frac{0}{0} \right] = \lim_{x \rightarrow e} \frac{\ln x - \ln e}{x - e} = \lim_{x \rightarrow e} \frac{\ln \frac{x}{e}}{\frac{x}{e} - 1} = \left[\frac{t = x - e}{x = e + t} \right] = \lim_{x \rightarrow e} \frac{\ln \frac{e+t}{e}}{\frac{e+t}{e} - 1} = \lim_{x \rightarrow e} \frac{\ln \frac{e+t}{e}}{t} =$$

$$= \lim_{t \rightarrow 0} \frac{\ln\left(1 + \frac{t}{e}\right)}{t} = \lim_{t \rightarrow 0} \ln\left(1 + \frac{t}{e}\right)^{\frac{1}{t}} = \left| \frac{t}{e} = z, t \rightarrow 0 \Rightarrow z \rightarrow 0 \right| =$$

$$= \lim_{z \rightarrow 0} \ln\left[\left(1 + z\right)^{\frac{1}{z}}\right] = \frac{1}{e} \ln \lim_{z \rightarrow 0} \left[\left(1 + z\right)^{\frac{1}{z}}\right] = \frac{1}{e} \cdot \ln e = \frac{1}{e}.$$

13. $\lim_{x \rightarrow \infty} x^2 \left(\cos \frac{1}{x} - 1 \right) = ?$

Yechish: Bu limit qiymati $0 \cdot \infty$ ko'rinishdagi aniqmaslikni ochish va $x \rightarrow 0 \Rightarrow 1 - \cos x \sim x^2/2$ ekvivalentlikdan foydalanish orqali quyidagicha aniqlanadi:

$$\lim_{x \rightarrow \infty} x^2 \left(\cos \frac{1}{x} - 1 \right) = [\infty \cdot 0] = - \lim_{x \rightarrow \infty} x^2 \left(1 - \cos \frac{1}{x} \right) = \left| \frac{1}{x} = t, x \rightarrow \infty \Rightarrow t \rightarrow 0 \right| =$$

$$= - \lim_{t \rightarrow 0} (1 - \cos t) / t^2 = - \lim_{t \rightarrow 0} (t^2 / 2) / t^2 = - \lim_{t \rightarrow 0} 1/2 = -1/2.$$

Mustaqil bajarish uchun misollar

8.1. Ushbu $-\frac{1}{2} + \frac{3}{4} - \frac{5}{8} + \frac{7}{16} - \frac{9}{32} + \dots$ sonli ketma-ketlikning umumiy a_n hadini toping.

8.2. Quyidagi ketma-ketliklardan qaysi birlari yaqinlashuvchi?

$$x_n = \frac{1}{n^2 + 1}, \quad y_n = \frac{1}{2n^2 - 1}, \quad z_n = (-1)^n.$$

8.3. Quyidagi ketma-ketliklardan qaysi birlari yaqinlashuvchi?

$$x_n = \frac{n^2 + 1}{2n}, \quad y_n = (-1)^{n^2}, \quad z_n = \frac{1}{n+1}$$

8.4. Umumiy hadi $a_n = \frac{3n}{1-2n}$ bo'lgan sonli ketma-ketlikning limitini hisoblang.

8.5. $\left\{ \frac{2009n}{n-2008} \right\}$ sonli ketma-ketlik limitini hisoblang.

Agar $a_n = 1/n$, $b_n = 1/(1-2n)$ bo'lsa $\{a_n/b_n\}$ sonli ketma-ketlik limitini hisoblang.

8.6. Sonli ketma-ketlik limitini hisoblang:

$$\lim_{n \rightarrow \infty} \frac{n^2 + 3n + 12}{3n^2 - 2n + 12}$$

8.7. Sonli ketma-ketlik limitini hisoblang:

$$\lim_{n \rightarrow \infty} (1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots + \frac{1}{3^n})$$

8.8. Sonli ketma-ketlik limitini hisoblang:

$$\lim_{n \rightarrow \infty} \frac{n^2 + 2n + 1}{n + 12}$$

8.9. Sonli ketma-ketlik limitini hisoblang:

$$\lim_{n \rightarrow \infty} \frac{n + 1}{3n^2 - 1}$$

8.10. Sonli ketma-ketlik limitini hisoblang:

$$\lim_{n \rightarrow \infty} \frac{n}{\sin n}$$

8.11. Sonli ketma-ketlik limitini hisoblang:

$$\lim_{n \rightarrow \infty} \frac{\sin n}{n}$$

8.12. Sonli ketma-ketlik limitini hisoblang:

$$\lim_{n \rightarrow \infty} (\sqrt{n + 2009} - \sqrt{n + 2008}).$$

8.13. $e = 2.71\dots$ sonni ifodalovchi II ajoyib limit qayerda to'g'ri ifodalangan?

A) $\lim_{n \rightarrow \infty} n^n = e$, B) $\lim_{n \rightarrow \infty} (\frac{1}{1-n})^n = e$, C) $\lim_{n \rightarrow \infty} (\frac{1}{1+n})^n = e$,
 D) $\lim_{n \rightarrow \infty} (1 + \frac{1}{n})^n = e$, E) $\lim_{n \rightarrow \infty} (1 - \frac{1}{n})^n = e$.

8.14. Ushbu limitni hisoblang:

$$\lim_{n \rightarrow \infty} (1 + \frac{2}{n})^{2n}$$

8.15. Ushbu limitni hisoblang (C – noldan farqli ixtiyoriy o'zgarmas son):

$$\lim_{n \rightarrow \infty} (1 + \frac{C}{n})^n$$

8.16-8.45 gacha. Ko'phadlar nisbatidan iborat funksiyaning limitini hisoblang.

$$8.16) \lim_{x \rightarrow -2} \frac{x^3 + 2x^2 - 4x - 8}{x^3 + 8};$$

$$8.17) \lim_{x \rightarrow 6} \frac{x^2 - 8x + 12}{x^3 - 7x^2 + 6x};$$

$$8.18) \lim_{x \rightarrow -2} \frac{2x^2 + x - 1}{x^3 - 3x - 2};$$

$$8.19) \lim_{x \rightarrow 2} \frac{x^3 - 3x - 2}{x^2 - 5x + 6};$$

$$8.20) \lim_{x \rightarrow 0} \frac{(1+x)^3 - (1+3x)}{x + x^5};$$

$$8.21) \lim_{x \rightarrow -1} \frac{x^3 + 2x^2 - x - 2}{x^2 + x};$$

$$8.22) \lim_{x \rightarrow \infty} \frac{(x^2 + 2x - 3)^2}{x^3 + 4x^2 + 3x};$$

$$8.24) \lim_{x \rightarrow 1} \frac{x^4 - x^3 + x - 1}{2x^2 - x - 1};$$

$$8.26) \lim_{x \rightarrow -2} \frac{x^2 + 2x - 3}{x^3 + 4x^2 + 3x};$$

$$8.28) \lim_{x \rightarrow 1} \frac{x^4 - 1}{2x^4 - x^2 - 1};$$

$$8.30) \lim_{x \rightarrow -2} \frac{x^3 + 4x^2 + 3x}{x^2 + x - 6};$$

$$8.32) \lim_{x \rightarrow 2} \frac{x^3 - 8}{x^3 + 2x^2 - 4x - 8};$$

$$8.34) \lim_{x \rightarrow -1} \frac{x^4 + 2x + 1}{x^2 - x - 2};$$

$$8.36) \lim_{x \rightarrow -2} \frac{3x^3 - 4x^2 - 2}{5x^3 + 8x^2 + 1};$$

$$8.38) \lim_{x \rightarrow 2} \frac{4x^2 - 5x - 6}{3x^3 - 7x^2 + 2x};$$

$$8.40) \lim_{x \rightarrow 2} \frac{x^3 - 8}{x^3 - 3x - 2};$$

$$8.42) \lim_{x \rightarrow 2} \frac{3x^3 - 7x^2 + 2x}{4x^2 - 5x - 6};$$

$$8.44) \lim_{x \rightarrow -\infty} \frac{x^3 - 2x - 1}{(x^2 - x - 2)^2};$$

$$8.23) \lim_{x \rightarrow 1} \frac{(2x^2 - x - 1)^2}{x^3 + 2x^2 - x - 2};$$

$$8.25) \lim_{x \rightarrow -2} \frac{x^3 - 3x + 2}{x^3 - x^2 - x + 1};$$

$$8.27) \lim_{x \rightarrow -3} \frac{(x^2 + 2x - 3)^2}{x^3 + 4x^2 + 3x};$$

$$8.29) \lim_{x \rightarrow \infty} \frac{x^3 - 3x - 2}{x^3 + 2x^2 - x - 2};$$

$$8.31) \lim_{x \rightarrow 1} \frac{x^4 - 1}{x^3 - x^2 - x + 1};$$

$$8.33) \lim_{x \rightarrow -3} \frac{x^3 - 2x - 1}{x^4 + 2x + 1};$$

$$8.35) \lim_{x \rightarrow 2} \frac{x^3 - 3x^2 + 4}{x^4 - 4x^2};$$

$$8.37) \lim_{x \rightarrow 2} \frac{x^3 - 3x - 2}{x^3 - 2x^2 - x + 2};$$

$$8.39) \lim_{x \rightarrow -1} \frac{x^4 + 2x^3 + x^2}{x^4 + 2x + 1};$$

$$8.41) \lim_{x \rightarrow \infty} \frac{(1+x)^3 - (1+3x)}{x^2 + x^5};$$

$$8.43) \lim_{x \rightarrow -1} \frac{x^3 - 3x - 2}{x + x^2};$$

$$8.45) \lim_{x \rightarrow -1} \frac{x^3 - 3x - 2}{x^2 - x - 2};$$

8.46-8.75 gacha. Irratsional funksiyalar qatnashgan limitlarni hisoblang.

$$8.46) \lim_{x \rightarrow \pm\infty} (\sqrt{x^2 + 2x - 1} - \sqrt{x^2 - 5x + 3});$$

$$8.47) \lim_{x \rightarrow 5} \frac{x^2 - 25}{2 - \sqrt{x - 1}};$$

$$8.48) \lim_{x \rightarrow 16} \frac{\sqrt[4]{x} - 2}{\sqrt{x} - 4};$$

$$8.49) \lim_{x \rightarrow \infty} (x + \sqrt[3]{1 - x^3});$$

$$8.50) \lim_{x \rightarrow 3} \frac{\sqrt{x+13} - 2\sqrt{x+1}}{x^2 - 9};$$

$$8.51) \lim_{x \rightarrow \infty} (\sqrt{x^2 + 4} - \sqrt{x+3});$$

$$8.52) \lim_{x \rightarrow 0} \frac{\sqrt[3]{8+3x-x^2}}{\sqrt[3]{x^2+x^3}};$$

$$8.54) \lim_{x \rightarrow 0} \frac{\sqrt[3]{x+1}-\sqrt{1-x}}{\sqrt{x}};$$

$$8.56) \lim_{x \rightarrow -2} \frac{\sqrt[3]{x-6}+2}{x+2};$$

$$8.58) \lim_{x \rightarrow 0} \frac{\sqrt{25-2x}-3}{\sqrt[3]{x}-2};$$

$$8.60) \lim_{x \rightarrow -8} \frac{\sqrt{1-x}-3}{2+\sqrt[3]{x}};$$

$$8.62) \lim_{x \rightarrow 0} \frac{\sqrt{1+3x^2}-(1+x)}{\sqrt[3]{x}};$$

$$8.64) \lim_{x \rightarrow 8} \frac{\sqrt{9+2x}-5}{\sqrt[3]{x}-2};$$

$$8.66) \lim_{x \rightarrow \infty} (x + \sqrt[3]{1-x^3});$$

$$8.68) \lim_{x \rightarrow +\infty} x^2 (\sqrt{x^3+2} - \sqrt{x^3-2});$$

$$8.70) \lim_{x \rightarrow 1} \left(\frac{3}{1-\sqrt{x}} - \frac{2}{1-\sqrt[3]{x}} \right);$$

$$8.72) \lim_{x \rightarrow 0} \frac{\sqrt[3]{27+x} - \sqrt[3]{27-x}}{\sqrt[3]{x^2} - \sqrt[5]{x}};$$

$$8.74) \lim_{x \rightarrow 8} \frac{\sqrt{9+2x}-5}{\sqrt[3]{x^2}-4};$$

$$8.53) \lim_{x \rightarrow 0} \frac{\sqrt{1-2x+x^2}-(1+x)}{x};$$

$$8.55) \lim_{x \rightarrow 1} \frac{\sqrt[3]{x}-1}{\sqrt{1+x}-\sqrt{2x}};$$

$$8.57) \lim_{x \rightarrow \infty} \left(\sqrt[3]{(x+1)^2} - \sqrt[3]{(x-1)^2} \right);$$

$$8.59) \lim_{x \rightarrow -\infty} \left(\sqrt[3]{(x+2)^2} - \sqrt[3]{(x-2)^2} \right);$$

$$8.61) \lim_{x \rightarrow \infty} (\sqrt{(x+3)(x-2)} - x);$$

$$8.63) \lim_{x \rightarrow -2} \frac{\sqrt[3]{x-6}+2}{x^3+8};$$

$$8.65) \lim_{x \rightarrow +\infty} (\sqrt{x^2+5x+4} - \sqrt{x^2+x});$$

$$8.67) \lim_{x \rightarrow 0} \frac{\sqrt[3]{1+x}-1}{\sqrt{1+x}-1};$$

$$8.69) \lim_{x \rightarrow +\infty} (\sqrt{x^2-5x+6} - x);$$

$$8.71) \lim_{x \rightarrow 1} \frac{\sqrt[4]{x}-1}{\sqrt[3]{x}-1};$$

$$8.73) \lim_{x \rightarrow +\infty} x(\sqrt{x^2+2} - x);$$

$$8.75) \lim_{x \rightarrow 0} \frac{\sqrt[3]{8+3x+x^2}-2}{x+x^2}.$$

8.76-8.103 gacha. Trigonometrik ifodali funksiya limitini hisoblang.

$$8.76) \lim_{x \rightarrow \frac{\pi}{6}} \frac{2 \sin x - 1}{\sin 6x}; \quad 8.77) \lim_{x \rightarrow \pi} \frac{1 - \sin \frac{x}{2}}{\pi - x}; \quad 8.78) \lim_{x \rightarrow 0} \frac{\ln(1-7x)}{\sin(\pi(x+7))};$$

$$8.79) \lim_{x \rightarrow \frac{\pi}{2}} \frac{2^{\cos^2 x} - 1}{\ln \sin x}; \quad 8.80) \lim_{x \rightarrow 0} \frac{\sin(\alpha+x) - \sin(\alpha-x)}{x};$$

$$\begin{aligned}
 8.81) \lim_{x \rightarrow 0} \frac{\sin 3x - \sin 2x}{\ln(1 + 2\lg 3x)}; & \quad 8.82) \lim_{x \rightarrow 0} \frac{\cos 2x - \cos x}{1 - \cos x}; \\
 8.83) \lim_{x \rightarrow 0} \frac{e^{\sin 2x} - e^{\sin x}}{\lg 3x}; & \quad 8.84) \lim_{x \rightarrow 0} \frac{\cos\left(x + \frac{5\pi}{2}\right) \cdot \lg x}{\arcsin 2x^2}; \\
 8.85) \lim_{x \rightarrow 0} \frac{1 - \cos 2x}{\cos 7x - \cos 3x}; & \quad 8.86) \lim_{x \rightarrow 0} \frac{x^2 - 2x}{\lg[2\pi(x + 0.5)]}; \quad 8.87) \lim_{x \rightarrow 0} \frac{\ln \cos 2x}{\sin^2 3x}; \\
 8.88) \lim_{x \rightarrow 2} \frac{\sin 7\pi x}{\lg 8\pi x}; & \quad 8.89) \lim_{x \rightarrow \frac{\pi}{3}} \frac{2\sin x - \sqrt{3}}{\cos \frac{3x}{2}}; \quad 8.90) \lim_{x \rightarrow 0} \frac{1 + x \sin x - \cos 2x}{\sin^2 x}; \\
 8.91) \lim_{x \rightarrow 1} \frac{1 + \cos \pi x}{\lg^2 \pi x}; & \quad 8.92) \lim_{x \rightarrow 0} \frac{1 - \sqrt{\cos x}}{x \sin^2 \sqrt{x}}; \quad 8.93) \lim_{x \rightarrow 1} \frac{1 - \sqrt{\cos x}}{1 - \cos \sqrt{x}}; \\
 8.94) \lim_{x \rightarrow 0} \frac{1 - \cos 2x + \lg^2 x}{x \sin^2 3x}; & \quad 8.95) \lim_{x \rightarrow \frac{\pi}{3}} \frac{1 - 2\cos x}{\pi - 3x}; \quad 8.96) \lim_{x \rightarrow \frac{\pi}{3}} \frac{1 - 2\cos x}{\sin(\pi - 3x)}; \\
 8.97) \lim_{x \rightarrow 0} \frac{\sqrt{\cos 4x} - 1}{\sin^2 8x}; & \quad 8.98) \lim_{x \rightarrow \frac{1}{2}} \frac{(2x - 1)^2}{e^{\sin \pi x} - e^{-\sin 3\pi x}}; \quad 8.99) \lim_{x \rightarrow \frac{\pi}{2}} \frac{2^{\cos^2 x} - 1}{\ln \sin x}; \\
 8.100) \lim_{x \rightarrow 0} \frac{\sin 4x - 2\sin 2x}{x \ln \cos 6x}; & \quad 8.101) \lim_{x \rightarrow \pi} \frac{\lg(3^x - 3)}{\frac{\cos \frac{3x}{2} - 1}{3}}; \\
 8.102) \lim_{x \rightarrow 0} \frac{\sin 2x - \sin x}{x \ln(1 - x \sin x)}; & \quad 8.103) \lim_{x \rightarrow \pi} \frac{\cos 5x - \cos 3x}{\sin^2 x}.
 \end{aligned}$$

Mavzu bo'yicha nazorat testlari

1. Sonli to'plamlar. Sonning absolut qiymati va uning xossalari. Sonli ketma-ketlik va uning limiti.

1. Umumiy holda sonli to'plam elementlari ... sonlardan iborat bo'ladi. A) butun. B) natural. C) haqiqiy. D) ratsional. E) irratsional.

2. N (natural sonlar), Z (butun sonlar), Q (ratsional sonlar), R (haqiqiy sonlar) sonli to'plamlar orasidagi munosabat qayerda to'g'ri ko'rsatilgan?

- A) $N \subset Z \subset Q \subset R$. B) $Z \subset N \subset Q \subset R$. C) $R \subset Q \subset Z \subset N$.
D) $Q \subset Z \subset N \subset R$. E) $n \subset Z \subset R \subset Q$.

3. Asosiy sonli to'plamlar uchun quyidagi munosabatlardan qaysi biri to'g'ri? A) $N \cap Z = N$. B) $Q \cap Z = Q$. C) $Z \cap R = R$.

D) barcha munosabatlar to'g'ri. E) barcha munosabatlar noto'g'ri.

4. Quyidagi sonli to'plamlardan qaysi biri oraliq deb ataladi?

- A) (a, b) . B) $[a, b)$. C) $(a, b]$. D) $[a, b]$. E) $\{a, b\}$.

5. Quyidagi sonli to'plamlardan qaysi biri kesma deb ataladi?

- A) (a,b) . B) $[a,b)$. C) $(a,b]$. D) $[a,b]$. E) $\{a,b\}$.

6. Quyidagi sonli to'plamlardan qaysi birlari yarim oraliq deb ataladi?

- 1) (a,b) 2) $[a,b)$ 3) $(a,b]$ 4) $[a,b]$ 5) $\{a,b\}$

- A) 1) va 2). B) 2) va 3). C) 3) va 4). D) 4) va 5). E) 1) va 4).

7. (a,b) , $[a,b)$, $(a,b]$ va $[a,b]$ sonli to'plamlarning d uzunligi qayerda to'g'ri ko'rsatilgan? A) $a-b$. B) $a+b$. C) $b-a$. D) $a \cdot b$. E) $\sqrt{b^2 - a^2}$.

8. $[-3,5)$ yarim oraliq uzunligini toping.

- A) 2. B) 5. C) 3. D) 4. E) 8.

9. Quyidagilardan qaysi biri sonli ketma-ketlikni tashkil etmaydi?

- A) Barcha juft sonlar. B) Barcha toq sonlar. C) Barcha tub sonlar. D) Barcha natural sonlar. E) Barcha musbat sonlar.

10. Ushbu $-\frac{1}{2} + \frac{3}{4} - \frac{5}{8} + \frac{7}{16} - \frac{9}{32} + \dots$ sonli ketma-ketlikning umumiy a_n

hadi qayerda to'g'ri ko'rsatilgan?

- A) $a_n = \pm \frac{2n+1}{2^n}$. B) $a_n = \pm \frac{2n-1}{2^n}$. C) $a_n = (-1)^n \frac{2n+1}{2^n}$.

- D) $a_n = (-1)^n \frac{2n-1}{2^n}$. E) $a_n = (-1)^n \frac{2n \pm 1}{2^n}$.

11. Quyidagi ketma-ketliklardan qaysi birlari yaqinlashuvchi?

$$x_n = \frac{1}{n^2 + 1}, \quad y_n = \frac{1}{2n^2 - 1}, \quad z_n = (-1)^n.$$

- A) faqat x_n . B) faqat x_n va y_n . C) faqat z_n .

- D) faqat y_n va z_n E) uchala ketma-ketlik ham yaqinlashuvchi.

12. Limitlar haqidagi quyidagi tasdiqlardan qaysi biri to'g'ri?

A) Har qanday yuqoridan chegaralangan ketma-ketlik limitga ega.

B) Har qanday quyidan chegaralangan ketma-ketlik limitga ega.

C) Har qanday chegaralangan ketma-ketlik limitga ega.

D) Keltirilgan barcha tasdiqlar to'g'ri.

E) Keltirilgan barcha tasdiqlar noto'g'ri.

13. Umumiy hadi $a_n = \frac{3n}{1-2n}$ bo'lgan sonli ketma-ketlikning limitini

hisoblang.

- A) $\frac{3}{2}$. B) $-\frac{3}{2}$. C) 3. D) $-\frac{1}{2}$. E) 0.

14. $\left\{ \frac{2009n}{n-2008} \right\}$ sonli ketma-ketlik limitini hisoblang.

- A) $-\frac{1}{2008}$. B) 2009. C) $-\frac{2009}{2008}$. D) -1. E) 0.

15. Agar $a_n = 1/n$, $b_n = 1/(1-2n)$ bo'lsa $\{a_n/b_n\}$ sonli ketma-ketlik limitini hisoblang.

- A) -2. B) -0.5. C) 0. D) 1. E) -3.

16. Sonli ketma-ketlik limitini hisoblang:

$$\lim_{n \rightarrow \infty} \frac{n^2 + 3n + 12}{3n^2 - 2n + 12} = ? \quad \text{A) 1. B) 3. C) 1/2. D) 1/3. E) 3/2.}$$

17. Sonli ketma-ketlik limitini hisoblang:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots + \frac{1}{3^n}\right) \quad \text{A) 3/2. B) } \infty. \text{ C) 3. D) 2. E) 1.}$$

18. Sonli ketma-ketlik limitini hisoblang:

$$\lim_{n \rightarrow \infty} (\sqrt{n+2009} - \sqrt{n+2008}).$$

- A) 1. B) $+\infty$. C) $-\infty$. D) 0. E) mavjud emas.

19. $e = 2.71$ sonni ifodalovchi II ajoyib limit qayerda to'g'ri ifodalangan?

- A) $\lim_{n \rightarrow \infty} n^n = e$. B) $\lim_{n \rightarrow \infty} \left(\frac{1}{1-n}\right)^n = e$. C) $\lim_{n \rightarrow \infty} \left(\frac{1}{1+n}\right)^n = e$.
D) $\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$. E) $\lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right)^n = e$.

20. Ushbu limitni hisoblang:

$$\lim_{n \rightarrow \infty} \left(1 + \frac{2}{n}\right)^{2n} = ? \quad \text{A) } e^2. \text{ B) } e^4. \text{ C) } e. \text{ D) } e^2/2. \text{ E) } e/4.$$

2. Funksiya va u bilan bog'liq tushunchalar.

1. **Ta'rifni to'ldiring:** $y=f(x)$ funksiya deb x o'zgaruvchining har bir $x \in D$ qiymatiga y o'zgaruvchining $y \in E$ qiymatini mos qo'yilishiga aytiladi.

- A) bir nechta. B) kamida bitta. C) faqat bitta.
D) ikkita. E) kamida ikkita.

2. **Ta'rifni to'ldiring:** $y=f(x)$ funksiyaning aniqlanish sohasi deb x argumentning $y=f(x)$ funksiya bo'ladigan qiymatlar to'plamiga aytiladi.

- A) Musbat. B) manfiy. C) nol. D) ma'noga ega. E) cheksiz.

3. $f(x) = \sqrt{2x+1} - \lg x$ funksiyaning aniqlanish sohasini toping.

- A) $(-1, +\infty)$. B) $(0, +\infty)$. C) $(2, 11)$. D) $(-\infty, +\infty)$. E) $(1, +\infty)$.

4. $f(x) = \ln \sqrt{x-1}$ funksiyaning aniqlanish sohasini toping.

- A) $[1, +\infty)$. B) $[0, +\infty)$. C) $(-\infty, +\infty)$. D) $(-\infty, 1)$. E) $(1, +\infty)$.

5. $y = \sqrt{4 - 5 \sin x}$ funksiyaning qiymatlar sohasini toping.

A) $[0, 4]$. B) $[0, 2]$. C) $[0, 5]$. D) $[-1, 3]$. E) $[0, 3]$.

6. Qaysi shartda $y=f(x)$ funksiya albatta juft bo'ladi?

A) $f(x)/f(-x) > 1$. B) $f(x) - f(-x) = 0$. C) $f(x) + f(-x) = 0$.

D) $f(x)/f(-x) \leq 0$. E) $|f(x)| = |f(-x)|$.

7. Quyidagi funksiyalardan qaysi biri juft emas?

A) $y = \cos^3 x$. B) $y = -x^4$. C) $y = |x|$. D) $y = \sin^3 x$. E) $y = \sin x^2$.

8. Ushbu funksiya orasidan juft funksiyalarni ko'rsating.

$$f(x) = \frac{\sin x}{x}, \quad g(x) = |x + 6|, \quad y(x) = \sqrt{x}$$

A) $f(x)$. B) $g(x)$. C) $y(x)$. D) $f(x), g(x)$. E) $g(x), y(x)$.

9. Quyidagi funksiyalardan qaysi biri toq?

A) $y = \sin^3 x$. B) $y = \cos^3 x$. C) $y = |x|^3$. D) $y = (x - 1)^3$. E) $y = (x + 1)^3$.

10. $f(x) = x^2 - 18x$ funksiya haqidagi to'g'ri tasdiqni aniqlang.

A) $f(x)$ toq funksiya. B) $f(x)$ juft funksiya. C) $f(x)$ na juft, na toq.

D) $x/f(x)$ toq funksiya. E) $x/f(x)$ juft funksiya.

3. Funksiya limiti va uning xossalari.

1. **Ta'rifini to'ldiring:** $y=f(x)$ funksiya $x \rightarrow a$ bo'lganda A soniga teng limitga ega deyiladi, agarda ixtiyoriy kichik $\varepsilon > 0$ soni uchun shunday $\delta > 0$ son topilsaki, $|x - a| < \delta$ shartni qanoatlantiruvchi barcha x uchun ... bo'lsa.

A) $|f(x) + A| < \varepsilon$. B) $|f(x) - A| > \varepsilon$. C) $|f(x) + A| > \varepsilon$.

D) $|f(x) - A| < \varepsilon$. E) $|f(x) - A| = \varepsilon$.

2. **Teoremani yakunlang:** Agarda $y=f(x)$ funksiya $x \rightarrow a$ bo'lganda limitga ega bo'lsa, bu limit.....

A) kamida bitta qiymatga ega bo'ladi. B) cheksiz ko'p qiymatga ega bo'ladi. C) faqat bitta qiymatga ega bo'ladi. D) kamida ikkita qiymatga ega bo'ladi. E) bittadan ortiq qiymatga ega bo'ladi.

3. $y = 2x^2 + 5x - 1$ funksiyaning $x \rightarrow 2$ bo'lgandagi limiti toping.

A) 10 B) 12. C) 17. D) 21. E) -1.

4. $\lim_{x \rightarrow 1} \frac{x-1}{1-\sqrt{2-x}}$ limitni hisoblang.

A) 0. B) ∞ . C) $-\infty$. D) 2. E) mavjud emas.

5. $\lim_{x \rightarrow \infty} (\sqrt{x+1} - \sqrt{x})$ limitni hisoblang.

A) 0. B) ∞ . C) -2. D) 2. E) mavjud emas.

6. Agar $\lim_{x \rightarrow a} f_1(x) = A_1$ va $\lim_{x \rightarrow a} f_2(x) = A_2$ (A_1, A_2 —chekli sonlar) bo'lsa, quyidagi tengliklardan qaysi biri har doim ham o'rinli bo'lmaydi?

A) $\lim_{x \rightarrow a} [C_1 f_1(x) + C_2 f_2(x)] = C_1 A_1 + C_2 A_2$, C_1, C_2 — const.

B) $\lim_{x \rightarrow a} [f_1(x) \pm f_2(x)] = A_1 \pm A_2$.

C) $\lim_{x \rightarrow a} [f_1(x) \cdot f_2(x)] = A_1 \cdot A_2$.

D) $\lim_{x \rightarrow a} \frac{f_1(x)}{f_2(x)} = \frac{A_1}{A_2}$.

E) Barcha tengliklar har doim o'rinli bo'ladi.

7. I ajoyib limitni ko'rsating.

A) $\lim_{x \rightarrow \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$.

B) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.

C) $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$.

D) $\lim_{x \rightarrow 0} \frac{\ln(1 + ax)}{x} = a$.

E) $\lim_{x \rightarrow 0} \frac{(1 + x)^a - 1}{x} = a$.

8. II ajoyib limitni ko'rsating.

A) $\lim_{x \rightarrow \pm\infty} \left(1 + \frac{1}{x}\right)^x = e$.

B) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.

C) $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \ln a$.

D) $\lim_{x \rightarrow 0} \frac{\ln(1 + ax)}{x} = a$.

E) $\lim_{x \rightarrow 0} \frac{(1 + x)^a - 1}{x} = a$.

9. Quyidagi limitlardan qaysi biri notog'ri hisoblangan?

A) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.

B) $\lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$.

C) $\lim_{x \rightarrow 0} \frac{\sin^2 x}{x^2} = 1$.

D) $\lim_{x \rightarrow 0} \frac{\sin x^2}{x^2} = 1$.

E) Barcha limitlar to'g'ri hisoblangan.

10. $\lim_{x \rightarrow 0} \frac{\sin 5x}{x}$ limitni hisoblang. A) 1. B) 0. C) ∞ . D) 5. E) 1/5.

11. $\lim_{x \rightarrow 0} \frac{2x^2 + 5x + 4}{5x^2 + 4x + 7}$ limitni hisoblang.

A) 4/7.

B) 2/5.

C) ∞ .

D) 5/4.

E) 0.

12. $\lim_{x \rightarrow 1} \frac{x^2 + 4x - 5}{x^2 + x - 2}$ limitni hisoblang.

A) 1.

B) 4.

C) 5/2.

D) 2.

E) mavjud emas.

13. $\lim_{x \rightarrow \infty} \left(\frac{3x+2}{3x+1} \right)^{3x+1}$ limitni hisoblang.

A) 1.

B) ∞

C) e .

D) mavjud emas.

E) 0.

14. $\lim_{x \rightarrow 0} (1 + 2x)^{3/x}$ limitni hisoblang. A) 1. B) e^2 . C) e . D) e^6 . E) 0.

**9-§. FUNKSIYANING HOSILASI. MURAKKAB
FUNKSIYANING HOSILASI. PARAMETRIK VA OSHKORMAS
FUNKSIYALARDAN OLINGAN HOSILA. ANIQMASLIKLARNI
OCHISHDA LOPITAL QOIDALARI.**

9.1. Funksiyaning hosilasi

Aytaylik, $y = f(x)$ funksiya (a, b) intervalda berilgan bo'lib, $x_0 \in (a, b)$ bo'lsin. x_0 nuqta bilan birga shu (a, b) ga tegishli bo'lgan $x_0 + \Delta x$ ni ($\Delta x \neq 0$ qaraymiz. Natijada funksiya ushbu

$$\Delta y = \Delta f(x_0) = f(x_0 + \Delta x) - f(x_0) \quad (9.1)$$

orttirmaga ega bo'ladi. Ravshanki,

$$\frac{\Delta y}{\Delta x} = \frac{\Delta f(x_0)}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \quad (9.2)$$

nisbat muayyan $f(x)$ va tayin x_0 da Δx ning funksiyasiga aylanadi. $\Delta x \rightarrow 0$ da bu nisbat limiti funksiya hosilasi tushunchasiga olib keladi.

1-ta'rif. Agar

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \rightarrow 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \quad (9.3)$$

limit mavjud bo'lsa, bu limit $y = f(x)$ funksiyaning x_0 nuqtadagi hosilasi deyiladi va $f'(x_0)$ yoki $\frac{df(x_0)}{dx}$ yoki $y'_{x=x_0}$ kabi belgilanadi.

Agar (9.3) limit chekli bo'lsa, hosila chekli deyiladi, (9.3) limit cheksiz bo'lsa, hosila cheksiz deyiladi.

1-eslatma. Funksiyaning tayin nuqtadagi chekli hosilasi sonni ifodalaydi.

Agar (a, b) oraliqning har bir x nuqtasida funksiyaning chekli hosilasi mavjud bo'lsa, unda hosila x ning funksiyasiga aylanadi.

Funksiyaning o'ng va chap limitlari singari funksiyaning o'ng va chap hosilalari ta'riflanadi. Ushbu

$$\lim_{\Delta x \rightarrow +0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}, \quad \lim_{\Delta x \rightarrow -0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x} \quad (9.4)$$

limitlar mavjud bo'lsa, ular mos ravishda funksiyaning x_0 nuqtadagi o'ng va chap hosilalari deyiladi va $f'(x_0+0)$, $f'(x_0-0)$ kabi belgilanadi:

$$\begin{aligned} f'(x_0+0) &= \lim_{\Delta x \rightarrow +0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}, \\ f'(x_0-0) &= \lim_{\Delta x \rightarrow -0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}. \end{aligned} \quad (9.5)$$

Xususan, $[a, b]$ segmentda berilgan $f(x)$ funksiyaning a nuqtadagi hosilasi deganda uning shu nuqtadan o'ng hosilasi, b nuqtadagi hosilasi deganda uning shu nuqtadagi chap hosilasi tushiniladi.

1-misol. Ushbu

$$f(x) = x^2$$

funksiyaning $x_0 = 2$ nuqtadagi hosilasi topilsin.

Yechish. Ta'rifdan foydalanib topamiz. Ravshanki, berilgan funksiyaning $x_0 = 2$ nuqtadagi orttirmasi

$$\Delta f(2) = f(2 + \Delta x) - f(2) = (2 + \Delta x)^2 - 2^2 = 4 + 4\Delta x + \Delta x^2 - 4 = 4\Delta x + \Delta x^2$$

bo'ladi. Unda

$$\frac{\Delta f(2)}{\Delta x} = \frac{4\Delta x + \Delta x^2}{\Delta x} = 4 + \Delta x$$

bo'lib,

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta f(2)}{\Delta x} = \lim_{\Delta x \rightarrow 0} (4 + \Delta x) = 4$$

bo'ladi. Demak,

$$f'(2) = 4 \blacktriangleright$$

9.2. Hosila hisoblash qoidalari

Aytaylik, $f(x)$ va $g(x)$ funksiyalar (a, b) da berilgan bo'lib,

$x \in (a, b)$ nuqtada $f'(x)$ va $g'(x)$ hosilalarga ega bo'lsin. U holda:

1) ixtiyoriy o'zgarmas c da $y = c \cdot f(x)$ funksiya hosilaga ega bo'lib,

$$y' = (c \cdot f(x))' = c \cdot f'(x)$$

bo'ladi;

2) funksiyalar yig'indisi $y = f(x) + g(x)$ funksiya hosilaga ega bo'lib,

$$y' = (f(x) + g(x))' = f'(x) + g'(x)$$

bo'ladi;

3) funksiyalar ko'paytmasi $y = f(x) \cdot g(x)$ funksiya hosilaga ega bo'lib,

$$y' = (f(x) \cdot g(x))' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

bo'ladi;

4) funksiyalar nisbati $y = \frac{f(x)}{g(x)}$ funksiya ($g(x) \neq 0$) hosilaga ega bo'lib,

$$y' = \left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{g^2(x)}$$

bo'ladi;

Bu tasdiqlarning birini, masalan 2-ning isbotini keltiramiz.

◀Shartga ko'ra $f(x)$ va $g(x)$ funksiyalar $f'(x)$ va $g'(x)$ hosilalarga ega. Unda ta'rifga binoan

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}, \quad g'(x) = \lim_{\Delta x \rightarrow 0} \frac{g(x + \Delta x) - g(x)}{\Delta x}$$

bo'ladi. Ravshanki, $y = f(x) + g(x)$ funksiyaning orttirmasi

$$\begin{aligned} \Delta y &= [f(x + \Delta x) + g(x + \Delta x)] - [f(x) + g(x)] = \\ &= [f(x + \Delta x) - f(x)] + [g(x + \Delta x) - g(x)] \end{aligned}$$

bo'lib,

$$\frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} + \frac{g(x + \Delta x) - g(x)}{\Delta x}$$

bo'ladi. Keyingi tenglikdan topamiz:

$$\begin{aligned} \lim_{\Delta x \rightarrow 0} \frac{\Delta y}{\Delta x} &= \lim_{\Delta x \rightarrow 0} \left[\frac{f(x + \Delta x) - f(x)}{\Delta x} + \frac{g(x + \Delta x) - g(x)}{\Delta x} \right] = \\ &= \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} + \lim_{\Delta x \rightarrow 0} \frac{g(x + \Delta x) - g(x)}{\Delta x} = f'(x) + g'(x). \end{aligned}$$

Demak,

$$y' = (f(x) + g(x))' = f'(x) + g'(x). \blacktriangleright$$

Misollar:

$$1. y = \frac{3}{2}x \text{ bo'lsa, } y' = \left(\frac{3}{2}x\right)' = \frac{3}{2}(x)' = \frac{3}{2} \cdot 1 = \frac{3}{2} \text{ bo'ladi}$$

$$2. y = x + \frac{1}{x} \text{ bo'lsa, } y' = \left(x + \frac{1}{x}\right)' = (x)' + \left(\frac{1}{x}\right)' = 1 - \frac{1}{x^2} \text{ bo'ladi.}$$

$$3. y = \frac{2x+1}{3x+1} \text{ bo'lsa,}$$

$$y' = \left(\frac{2x+1}{3x+1}\right)' = \frac{(2x+1)' \cdot (3x+1) - (2x+1) \cdot (3x+1)'}{(3x+1)^2} =$$

$$= \frac{2 \cdot (3x+1) - 3(2x+1)}{(3x+1)^2} = -\frac{1}{(3x+1)^2}$$

bo'ladi.

9.3. Hosilalar jadvali

Yuqorida funksiya hosilalari uchun topilgan formulalarni jamlab, ularni jadval ko'rinishida yozamiz:

$$1) (x^a)' = a \cdot x^{a-1}, \quad (u^a)' = a u^{a-1} \cdot u'$$

$$2) (\alpha^x)' = \alpha^x \cdot \ln \alpha, \quad (\alpha^u)' = \alpha^u \cdot \ln \alpha \cdot u'$$

$$3) (e^x)' = e^x, \quad (e^u)' = e^u \cdot u'$$

$$4) (\log_a x)' = \frac{1}{x} \log_a e, \quad (\log_a u)' = \frac{1}{u} \log_a e \cdot u'$$

$$5) (\ln x)' = \frac{1}{x}, \quad (\ln u)' = \frac{1}{u} \cdot u'$$

$$6) (\sin x)' = \cos x, \quad (\sin u)' = \cos u \cdot u'$$

$$7) (\cos x)' = -\sin x, \quad (\cos u)' = -\sin u \cdot u'$$

$$8) (\operatorname{tg} x)' = \frac{1}{\cos^2 x}, \quad (\operatorname{tg} u)' = \frac{1}{\cos^2 u} \cdot u'$$

$$9) (\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}, \quad (\operatorname{ctg} u)' = -\frac{1}{\sin^2 u} \cdot u'$$

$$10) (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, \quad (\arcsin u)' = \frac{1}{\sqrt{1-u^2}} \cdot u'$$

$$11) (\arccos x)' = -\frac{1}{\sqrt{1-x^2}}, \quad (\arccos u)' = -\frac{1}{\sqrt{1-u^2}} \cdot u'$$

$$12) (\operatorname{arctg} x)' = \frac{1}{1+x^2}, \quad (\operatorname{arctg} u)' = \frac{1}{1+u^2} \cdot u',$$

$$13) (\operatorname{arcctg} x)' = -\frac{1}{1+x^2}, \quad (\operatorname{arcctg} u)' = -\frac{1}{1+u^2} \cdot u'$$

9.4. Murakkab va oshkormas funksiyalarning hosilalari

Murakkab funksiyaning hosilasi: Agar $y = f(u)$, $u = u(x)$, ya'ni $y = f[u(x)]$ bo'lsa, u holda

$$y' = f'(u)u' \quad (9.6)$$

bu yerda $f(u)$ va $u(x)$ hosilaga ega bo'lgan funksiyalar

Logarifmik funksiyaning hosilasi: $y = \ln y$ funksiyani hosilasi

$$(\ln y)' = \frac{y'}{y} = \frac{f'(x)}{f(x)} \quad (9.7)$$

ko'rinishida bo'ladi.

Parametrik ko'rinishida berilgan funksiyaning hosilasi. Agar $y = f(x)$ funksiya parametrik ko'rinishda $x = \varphi(t)$ va $y = \psi(t)$ berilgan bo'lsa, u holda uning hosilasi

$$y'_x = \frac{y'_t}{x'_t}; \quad (9.8)$$

ko'rinishida bo'ladi.

Agar y ga nisbatan yechilmagan $F(x; y) = 0$ tenglama y ni x ning bir qiymatli funksiyasi sifatida aniqlasa, u holda y o'zgaruvchi x ning oshkormas funksiyasi deyiladi. Bu oshkormas funksiyadan y' hosilani topish uchun y ni x ning funksiyasi deb, $F(x; y) = 0$ tenglamaning ikki tomonini x bo'yicha differensiallash kerak. Hosil bo'lgan tenglamadan izlangan y' ni topamiz. y'' ni topish uchun $F(x; y) = 0$ tenglamani x bo'yicha ikki marta differensiallash kerak va hokazo.

9.5. Aniqmasliklarni ochishning Lopital qoidalarini

Tegishli funksiyalarning hosilalari mavjud bo'lganda $\frac{0}{0}, \frac{\infty}{\infty}, 0 \cdot \infty,$

$\infty - \infty, 1^\infty, 0^0, \infty^0$ ko'rinishdagi aniqmasliklarni ochish masalasi yengillashadi. Odatda hosilalardan foydalanib, aniqmasliklarni ochish *Lopital qoidalarini* deb ataladi. Biz quyida *Lopital qoidalarining* bayoni bilan shug'ullanamiz.

$f(x)$ va $\varphi(x)$ funksiyalar x_0 nuqtaning biror atrofida (x_0 nuqtaning o'zidan tashqari) differensiallanuvchi va $\varphi'(x) \neq 0$ bo'lsin.

Agar $\lim_{x \rightarrow x_0} f(x) = \lim_{x \rightarrow x_0} \varphi(x) = 0$ yoki $\lim_{x \rightarrow x_0} f(x) = \lim_{x \rightarrow x_0} \varphi(x) = \infty$

bo'lib, $\lim_{x \rightarrow x_0} \frac{f'(x)}{\varphi'(x)}$ mavjud bo'lsa, u holda $\lim_{x \rightarrow x_0} \frac{f(x)}{\varphi(x)} = \lim_{x \rightarrow x_0} \frac{f'(x)}{\varphi'(x)}$ bo'ladi.

$x \rightarrow \infty$ da ham Lopital qoidasi o'rinli.

$0 \cdot \infty$ va $\infty - \infty$ shaklidagi aniqmasliklar algebraik almashtirishlar orqali $\frac{0}{0}$ yoki $\frac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarga keltirilib, so'ngra Lopital qoidasidan foydalaniladi.

1^∞ , 0^0 , ∞^0 ko'rinishdagi aniqmasliklar logarifmlash orqali $\frac{0}{0}$ yoki $\frac{\infty}{\infty}$ ko'rinishdagi aniqmasliklarga keltirilib, so'ngra Lopital qoidasidan foydalaniladi.

2-misol. Ushbu $\lim_{x \rightarrow 2} \frac{\ln(x^2-3)}{x^2+3-10}$ limitni hisoblang.

Yechish. Bu holda $f(x) = \ln(x^2-3)$, $g(x) = x^2+3-10$ bo'lib, ular uchun teoremaning barcha shartlari bajariladi.

Haqiqatan ham, 1) $\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \ln(x^2-3) = \ln 1 = 0$;

$\lim_{x \rightarrow 2} g(x) = \lim_{x \rightarrow 2} (x^2+3-10) = 0$;

2) $f'(x) = \frac{2x}{x^2-3}$, $g'(x) = 2x+3$, $x \neq \pm\sqrt{3}$;

3) $\lim_{x \rightarrow 2} \frac{f'(x)}{g'(x)} = \lim_{x \rightarrow 2} \frac{2x}{(x^2-3)(2x+3)}$ bo'ladi.

Demak, 1-teoremaga binoan $\lim_{x \rightarrow 2} \frac{\ln(x^2-3)}{x^2+3-10} = \frac{4}{7}$.

3-misol. Ushbu $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$ limitni hisoblang.

Yechish. $f(x) = \ln x$, $g(x) = x$ funksiyalar uchun Lopital qoidasi shartlarini tekshiramiz: 1) bu funksiyalar $(0, +\infty)$ da differensiallanuvchi;

2) $f'(x) = \frac{1}{x}$, $g'(x) = 1$; 3) $\lim_{x \rightarrow +\infty} \frac{f'(x)}{g'(x)} = \lim_{x \rightarrow +\infty} \frac{1/x}{1} = 0$, ya'ni mavjud.

Demak, izlanayotgan limit ham mavjud va $\lim_{x \rightarrow \infty} \frac{\ln x}{x} = 0$ tenglik o'rinli.

4-misol. Ushbu $\lim_{x \rightarrow 0} \left(\frac{\lg x}{x}\right)^{\frac{1}{x^2}}$ limitni hisoblang.

Yechish. Ravshanki, $x \rightarrow 0$ da $\left(\frac{\lg x}{x}\right)^{\frac{1}{x^2}}$ ifoda 1^∞ ko'rinishdagi aniqmaslik bo'ladi. Uni logarifmlab, $\frac{0}{0}$ aniqmaslikni ochishga keltiramiz:

$$\begin{aligned}
 \lim_{x \rightarrow 0} \ln y &= \lim_{x \rightarrow 0} \frac{\ln \frac{tgx}{x}}{x^2} = \lim_{x \rightarrow 0} \frac{\left(\ln \frac{tgx}{x}\right)'}{(x^2)'} = \lim_{x \rightarrow 0} \frac{\frac{x}{tgx} \cdot \frac{\frac{x}{\cos^2 x} - tgx}{x^2}}{2x} = \\
 &= \frac{1}{2} \lim_{x \rightarrow 0} \frac{x - \sin x \cos x}{x^3} = \frac{1}{2} \lim_{x \rightarrow 0} \frac{(x - \sin x \cos x)'}{(x^3)'} = \\
 &= \frac{1}{2} \lim_{x \rightarrow 0} \frac{1 - \cos^2 x + \sin^2 x}{3x^2} = \frac{1}{6} \lim_{x \rightarrow 0} \frac{2\sin^2 x}{x^2} = \frac{1}{6} \cdot 2 = \frac{1}{3}
 \end{aligned}$$

Demak, $\lim_{x \rightarrow 0} \left(\frac{tgx}{x}\right)^{\frac{1}{x^2}} = e^{\frac{1}{3}} = \sqrt[3]{e}$.

Mustaqil bajarish uchun misollar

Differensiallashning qoidalaridan foydalanib quyidagi funksiyalarning hosilalari topilsin.

9.1. $y = 3x^3 - 4x^2 + 9x - 9$; 9.2. $y = \frac{2}{x} + \ln x$; 9.3. $y = 3\sin x + 5^x$;

9.4. $y = -2\sin x + tgx$; 9.5. $y = \frac{1}{x} - 1000$; 9.6. $y = 25t^{10} + 2\sqrt{t}$; 9.7. $y = \sin x + \cos x$;

9.8. $y = 6x^{15} - 9x^{20}$;

9.9. $y = 100x^2 - 100$; 9.10. $y = e - 3^x + 4$; 9.11. $y = 3x^7 - 6x^6 + 5x^2 - 7$;

9.12. $y = (x^3 - 2x^2 - 1)(x^5 + x^2)$; 9.13. $y = \sqrt[3]{3x} + \sqrt{2x} + 10$; 9.14. $y = \frac{1}{x\sqrt{2x}}$;

9.15. $y = 10^{x+3} - 7$; 9.16. $y = 5\sqrt{x^2} + 7$; 9.17. $y = \frac{11 - 5x + x^6}{\sqrt{x}}$;

9.18. $y = e^{2x^2 + 3x - 5}$; 9.19. $y = \frac{1}{2x} - \ln x + e^{x+2}$; 9.20. $y = e^{2x^3 - \frac{1}{\sqrt{x}} + 1}$.

9.21. $y = (ax^2 + bx + c)^3$; 9.22. $y = \left(e^{2x} - \frac{2}{x^3} + \ln \sqrt{x}\right)^2$; 9.23. $y = \sin 3(ax + b)$;

9.24. $y = \cos(e^x - e^{-x})$; 9.25. $y = tg^2(\ln 2x)$; 9.26. $y = ctg^3(tg 2x)$;

9.27. $y = \ln^2(\ln 4x)$; 9.28. $y = (e^{\sin x} + 3^{4x})^5$; 9.29. $y = 7^{\cos^3 4x}$;

9.30. $y = \sin^m x \cdot \sin mx$; 9.31. 1) $y = \frac{x^3}{3} - 2x^2 + 4x - 5$; 2) $y = \frac{bx + c}{a}$.

9.32. 1) $y = \frac{x^5}{5} - \frac{2x^3}{3} + x$; 2) $y = \left(1 - \frac{x^2}{2}\right)^2$

9.33. 1) $y = x + 2\sqrt{x}$; 2) $y = (\sqrt{a} - \sqrt{x})^2$; 9.34. 1) $y = \frac{10}{x^3}$; 2) $y = \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$.

9.35. 1) $y = x + \frac{1}{x^2} - \frac{1}{5x^5}$; 2) $y = 3x - 6\sqrt{x}$

$$9.36. 1) y = 6\sqrt[3]{x} - 4\sqrt[4]{x} \quad 2) y = \left(1 - \frac{1}{\sqrt[3]{x}}\right)^2$$

$$9.37. 1) y = \frac{1}{2x^2} - \frac{1}{3x^3} \quad 2) y = \frac{8}{\sqrt[4]{x}} - \frac{6}{\sqrt[3]{x}}$$

$$9.38. 1) y = x - \sin x \quad 2) y = x - \operatorname{tg} x$$

$$9.39. 1) y = x^2 \cos x \quad 2) y = y^2 \operatorname{ctg} x$$

$$9.40. 1) y' = \frac{\cos x}{x^2} \quad 2) y' = \frac{x^2}{x^2 + 1}$$

Ko'rsatkichli va logarifmik funksiyalarning hosilalari topilsin.

$$9.41. 1) y = x \ln x \quad 2) y' = \frac{1 + \ln x}{x};$$

$$9.42. 1) y = \ln x - \frac{2}{x} - \frac{1}{2x^2} \quad 2) y = \ln(x^2 + 2x)$$

$$9.43. 1) y = \ln(1 + \cos x). \quad 2) y = \ln \sin x - \frac{1}{2} \sin^2 x. \quad 9.44.$$

$$y = \ln(\sqrt{x} + \sqrt{x+1}) \text{ Quyidagi funksiyalarning hosilalari topilsin.}$$

$$9.45.1) y = x^2 + 3^x. \quad 2) y = x^2 2^x. \quad 3) y = x^2 e^x. \quad 9.46.1) y = a^{\sin x}. \quad 2) y = e^{-x^2}.$$

$$3) y = x^2 e^{-2x}$$

$$9.47. y = 2 \left(e^{\frac{x}{2}} - e^{-\frac{x}{2}} \right). \quad 9.48. y = \sqrt{x} e^{\sqrt{x}} \quad 9.49. y = \frac{1+e^x}{1-e^x}. \quad 9.50. y = e^{\frac{x}{a}} \cos \frac{x}{a}.$$

Murakkab, oshkormas va parametrik ko'rinishdagi funksiyalarni integrallang.

$$9.51. 1) y = \sin 6x \quad 2) y = \cos(a - bx)$$

$$3). a) y = 2x^5 - 5 \cdot 2^x + 4x - 7 \log_2 x - \ln 2; \quad b) y = (1+x^2) \cdot \operatorname{arctg} x;$$

$$v) y = \frac{\sin x + \cos x}{\sin x - \cos x}; \quad g) y = \log_3(2x^3 + 1).$$

$$9.52. 1) y = \sin \frac{x}{2} + \cos \frac{x}{2} \quad 2) y = 6 \cos \frac{x}{3}$$

$$9.53. 1) y = (1-5x)^4; \quad 2) y = \sqrt[3]{(4+3x)^2}.$$

$$9.54. 1) y = \frac{1}{(1-x^2)^5}; \quad 2) y = \sqrt{1-x^2}; \quad 3) y = \sqrt{\cos 4x}.$$

$$9.55. y = \sqrt{2x - \sin 2x}. \quad 9.56. y = \sin^4 x = (\sin x)^4. \quad 9.57. y = \sqrt{4x + \sin 4x}.$$

$$9.58. y = x^2 \sqrt{1-x^2}. \quad 9.59. y = \sin^4 x + \cos^4 x. \quad 9.60. y = \sqrt[3]{1 + \cos 6x}.$$

$$9.61. 1) y = \operatorname{tg} x + \frac{2}{3} \operatorname{tg}^3 x + \frac{1}{5} \operatorname{tg}^5 x; \quad 2) y = \sin^2 x^3.$$

3) $y = x^2 + e^x \ln x$ funksiyaga teskari bo'lgan funksiya uchun x'_y ni toping.

4) Hosilani tekshiring. a) $y = x^{x^3}$; b) $y = \frac{(3x+2)^4 \sqrt[3]{5x-1}}{(1-2x)^3 \sqrt{1-x^2}}$.

Quyidagi tenglamalardan y' topilsin.

9.62. 1) $x^2 + y^2 = a^2$; 2) $y^2 = 2px$; 3) $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$;

4) $x^2 + 9y^2 = 16$ oshkormas funksiya uchun $y'_x = \frac{dy}{dx}$ ni toping.

9.63. 1) $x^2 + xy + y^2 = 6$; 2) $x^2 + y^2 - xy = 0$.

Quyidagi funksiyalarning differensiallari topilsin:

9.64 1) $y = x^n$; 2) $y = x^3 - 3x^2 + 3x$;

3) $x = a^2 \cos^2 t$, $y = a \sin^2 t$ berilgan bo'lsa, $y'_x = \frac{dy}{dx}$ ni toping.

9.65 1) $y = \sqrt{1+x^2}$; 2) $S = \frac{gt^2}{2}$.

9.66 1) $r = 2\varphi - \sin 2\varphi$; 2) $x = \frac{1}{t^2}$.

9.67 $y = |x|$ funksiyani $x_0 = 0$ nuqtadagi hosilasini toping.

9.68 $y = \cos^2 x \cdot \sqrt{1-x^2}$ funksiyaning ixtiyoriy x nuqtasi uchun differensialini hisoblang.

9.69. $y = \ln \cos \frac{1}{x}$. 9.70. $y = \frac{x^2 - 1}{2(1 + 2x)}$. 9.71. $y = \cos 2x + 4 \cos x$.

9.72. $f(x) = e^{x^2} + e^{-x^2}$. 9.72. $f(x) = \ln(x \sin x)$.

9.73. $f(x) = \ln(x + \sqrt{x^2 + 1})$ 9.74. $y = \operatorname{arctg} \frac{1}{x}$. 9.75. $y = \operatorname{arctg} \frac{2}{x}$.

9.76. $y = \arccos \sqrt{1-x^2}$. 9.77. $y = x^2 \operatorname{ctg} 2x$; $y = \frac{3 \operatorname{ctg} 2x}{\sin 4x}$.

9.78. $y = \ln \ln x$. 9.79. $y = \frac{x^3 - x^2}{\sqrt{1-x}}$.

9.80. $y = \arcsin x + \sqrt{1-x^2}$. 9.81 $y = \ln \sqrt{4x^2 + 1} + \frac{1}{2} \operatorname{arctg} 2x$.

9.82. $y = \arcsin\left(\frac{x}{2} - 1\right) - \sqrt{x(4-x)}$. 9.83. $y = \frac{\cos x}{\sin x + \cos x}$

9.84. $y = \frac{\ln \cos x}{\cos x}$ 9.85. $y = e^x \ln x$ berilgan $y'(1)$ ni toping.

9.86. $y = 2^x \cos 2^x$ berilgan $y'(0)$ ni toping.

9.87. $y = \ln^3 \sin^2 \left(\frac{\pi}{4} - 2x \right)$ berilgan $y' \left(\frac{\pi}{4} \right)$ ni toping.

9.88. $y = 3^{\ln x} + 3 \ln x + \frac{3}{\ln x}$ berilgan $y'(e)$ ni toping.

Lopital qoidasidan foydalanib limitlarni toping.

9.89. $\lim_{x \rightarrow 1} \frac{\sin \pi x}{\ln x}.$

9.90. $\lim_{x \rightarrow 0} \frac{\operatorname{tg} x - \sin x}{x - \sin x}.$

9.91. $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right).$

9.92. $\lim_{x \rightarrow 0} \operatorname{tg} x \cdot \ln x.$

9.93. $\lim_{x \rightarrow \infty} \left(1 + \frac{3}{x} \right)^x.$

9.94. $\lim_{x \rightarrow 0} x^2 \ln x$

9.95. $\lim_{x \rightarrow 0} \frac{x - \arctg x}{x^3}.$

9.96. $\lim_{x \rightarrow 0} \frac{\ln x}{\operatorname{ctg} x}.$

Mavzu bo'yicha nazorat testlari

Funksiya hosilasi va uning talqinlari.

1. $y=f(x)$ funksiyaning Δx argument orttirmasiga mos keladigan Δf funksiya orttirmasi qayerda to'g'ri ifodalangan?

- A) $\Delta f=f(x)-f(\Delta x).$ B) $\Delta f=f(x+\Delta x)-f(\Delta x).$ C) $\Delta f=f(x+\Delta x)-f(x).$
D) $\Delta f=f(x+\Delta x)-f(x-\Delta x).$ E) $\Delta f=f(x)\Delta x.$

2. $y=1$ o'zgarish funksiyani Δy orttirmasi qayerda to'g'ri ko'rsatilgan?

- A) $\Delta x.$ B) $-\Delta x.$ C) 1. D) -1. E) 0.

3. $y=x^3$ funksiyaning Δy orttirmasi qayerda to'g'ri ko'rsatilgan?

- A) $3x^2\Delta x + (\Delta x)^3.$ B) $3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3.$

4. $y=f(x)$ funksiya hosilasini ta'rif bo'yicha hisoblashda quyidagi amallardan qaysi biri bajarilmaydi?

- A) x argumentga Δx orttirma beriladi.
B) funksiya orttirmasi Δf hisoblanadi.
C) orttirmalar nisbati $\Delta f/\Delta x$ hisoblanadi.
D) $\Delta f/\Delta x$ nisbatning $\Delta x \rightarrow 0$ bo'lgandagi limiti hisoblanadi.
E) ko'rsatilgan amallarning barchasi bajariladi.

5. $y=f(x)$ funksiya hosilasining ta'rif qayerda to'g'ri ko'rsatilgan?

- A) $f'(x) = \lim_{\Delta x \rightarrow x} \frac{\Delta f}{\Delta x}.$ B) $f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta x}{\Delta f}.$

- C) $f'(x) = \lim_{\Delta x \rightarrow 0} \frac{\Delta f}{\Delta x}.$ D) $f'(x) = \lim_{\Delta x \rightarrow x} \frac{\Delta x}{\Delta f}.$ E) $f'(x) = \lim_{\Delta f \rightarrow 0} \frac{\Delta x}{\Delta f}.$

6. Hosilaning mexanik ma'nosi qayerda to'g'ri ko'rsatilgan?

- A) harakatda bosib o'tilgan masofa. B) harakatda sarflangan vaqt.
C) harakatda oniy tezlik. D) harakatda to'xtash holati.
E) harakatni boshlash holati.

7. Hosilaning geometrik ma'nosi qayerda to'g'ri ko'rsatilgan?

A) chiziqqa o'tkazilgan kesuvchining burchak koeffitsienti.

B) chiziqqa o'tkazilgan urinmaning burchak koeffitsienti.

C) chiziqqa o'tkazilgan normalning burchak koeffitsienti.

D) chiziqqa o'tkazilgan urinma va OX o'q orasidagi burchak tangensi.

E) chiziqqa o'tkazilgan urinma va OY o'q orasidagi burchak tangensi.

8. Quyidagilardan qaysi biri $y=f(x)$ funksiya grafigiga (x_0, y_0) nuqtada o'tkazilgan urinma tenglamasini ifodalaydi?

A) $y - y_0 = f'(x_0)(x - x_0)$. B) $y - y_0 = f'(x_0)x$. C) $y - y_0 = f'(x_0)x_0$.

D) $x - x_0 = f'(x_0)(y - y_0)$. E) $y - y_0 = \frac{1}{f'(x_0)}(x - x_0)$.

9. $y=x^2$ parabolaning $x_0=2$ abssissali nuqtasiga o'tkazilgan urinma tenglamasini aniqlang.

A) $y=x+2$. B) $y=2x+2$. C) $y=3x-2$. D) $y=4x-4$. E) $y=2x$.

10. **Ta'rifni to'ldiring:** Berilgan $y=f(x)$ funksiya berilgan x_0 nuqtada differensiallanuvchi deyiladi, agar u bu nuqtada qiymatli hosilaga ega bo'lsa?

A) musbat. B) manfiy. C) noldan farqli. D) chekli. E) nolga teng.

Funksiyaning differentsiallashtirish qoidalarini.

1. Differentsiallashtirish qoidasi qayerda xato ko'rsatilgan?

A) $(Cu)' = Cu'$ ($C = \text{const.}$). B) $(u \pm v)' = u' \pm v'$. C) $(u \cdot v)' = u'v + uv'$.

D) $\left(\frac{u}{v}\right)' = \frac{u'v + uv'}{v^2}$. E) $(f(u))' = f'(u)u'$.

2. Differentsiallashtiruvchi u va v funksiyalar u/v nisbatining hosilasini hisoblash formulasi to'g'ri yozilgan javobni ko'rsating.

A) $\frac{u'v + uv'}{v}$. B) $\frac{u'v + uv'}{v^2}$. C) $\frac{u'v - uv'}{v^2}$.

D) $\frac{u'v - uv'}{v}$. E) $\frac{u'v' - uv'}{v^2}$.

3. $y=x^2/\sin x$ funksiyaning y' hosilasini hisoblang.

A) $y' = x^2/\cos x$. B) $y' = 2x/\sin x$. C) $y' = 2x/\cos x$.

D) $y' = x(2\sin x + x\cos x)/\sin^2 x$. E) $y' = x(2\sin x - x\cos x)/\sin^2 x$.

4. Differentsiallashtiruvchi u va v funksiyalar $u \cdot v$ ko'paytmasining hosilasini hisoblash formulasi qayerda to'g'ri yozilgan?

- A) $u'v'$. B) $u'v'+uv$. C) $u'v+uv'$. D) $u'v-uv'$. E) $u'v'-uv$.

5. $y=x^2\sin x$ funksiyani y' hosilasini hisoblang.

- A) $y'=x^2\cos x$. B) $y'=x(x\sin x-2\cos x)$. C) $y'=2x\sin x$.
D) $y'=x(x\cos x+2\sin x)$. E) $y'=x(x\cos x-2\sin x)$.

6. $f(x)=\sin x$, $u(x)=\ln x$ funksiyalar bo'yicha tuzilgan $y=f(u)=\sin \ln x$ murakkab funksiya hosilasini hisoblang.

- A) $y'=\cos \ln x$. B) $y'=\sin(1/x)$. C) $y'=(\sin \ln x)/x$.
D) $y'=(\cos \ln x)/\ln x$. E) $y'=(\cos \ln x)/x$.

7. $y=\cos(x^2+1)$ funksiyani y' hosilasini hisoblang.

- A) $y'=\sin(x^2+1)$. B) $y'=-\sin(x^2+1)$. C) $y'=\sin 2x$.
D) $y'=2x\sin(x^2+1)$. E) $y'=-2x\sin(x^2+1)$.

8. $y=e^{x^4}$ funksiyani hosilasi to'g'ri yozilgan javobni toping.

- A) $x^4 \cdot e^{x^4-1}$. B) $4x^3 \cdot e^{x^4}$. C) $4x^3 \cdot e^{x^4} \cdot \lg e$. D) $4x^3 \cdot e^{x^4-1} \cdot \lg e$. E) e^{x^4}

9. $y=a^x$ ko'rsatkichli funksiya hosilasini toping.

- A) $y'=a^x$. B) $y'=xa^{x-1}$. C) $y'=a^x \ln a$. D) $y'=a^x \log_a e$.
E) to'g'ri javob ko'rsatilmagan.

10. $y=\log_a x$ funksiya hosilasini toping.

- A) $y'=\frac{1}{x}$. B) $y'=\frac{1}{x} \ln a$. C) $y'=\frac{1}{x} a$. D) $y'=\frac{1}{x \ln a}$.

E) to'g'ri javob ko'rsatilmagan.

11. Qaysi trigonometrik funksiya hosilasi xato ko'rsatilgan?

- A) $(\sin x)'=\cos x$. B) $(\cos x)'=\sin x$. C) $(\operatorname{tg} x)'=\frac{1}{\cos^2 x}$.

- D) $(\operatorname{ctg} x)'=-\frac{1}{\sin^2 x}$. E) barcha hosilalar to'g'ri ko'rsatilgan.

12. Qaysi teskari trigonometrik funksiyani hosilasi to'g'ri ko'rsatilgan?

- A) $(\arcsin x)'=1/(1-x^2)$. B) $(\arccos x)'=1/\sqrt{x^2-1}$.
C) $(\operatorname{arctg} x)'=1/\sqrt{x^2+1}$. D) $(\operatorname{arcctg} x)'=-1/\sqrt{x^2+1}$.

E) barcha hosilalar xato ko'rsatilgan.

13. Qaysi funksiyani hosilasi noto'g'ri ko'rsatilgan?

- A) $(x^\alpha)'=\alpha x^{\alpha-1}$. B) $(a^x)'=a^x$. C) $(\sin x)'=\cos x$. D) $(\ln x)'=1/x$.
E) Barcha hosilalar to'g'ri ko'rsatilgan.

14. Agar $f(x)=x^4-7x^3+2x^2+\pi$ bo'lsa, $f'(1)$ nimaga teng?

- A) 0. B) -13. C) -7. D) $\pi-4$. E) 3.

15. Quyidagi funksiyalardan qaysi biri murakkab

darajali-ko'rsatkichli funksiya bo'ladi?

- A) $y = \cos^2 x$. B) $y = 2^{\cos x}$. C) $y = \cos x^2$. D) $y = x^{2 \cos x}$.

E) keltirilgan funksiyalar orasida darajali-ko'rsatkichli funksiya yo'q.

16. $y = x^{\ln x}$ funksiya hosilasini toping.

- A) $y = x^{\ln x} \cdot \ln x$. B) $y = \ln x \cdot x^{\ln x - 1}$. C) $y = 2 \ln x \cdot x^{\ln x - 1}$.

- D) $y = \ln x \cdot x^{\ln x} (1 + x^{-1})$. E) $y = \ln x \cdot x^{\ln x} (1 - x^{-1})$.

VI-BOB. KOMPLEKS SONLAR VA ULAR USTIDA AMALLAR. KOMPLEKS O'ZGARUVCHILI FUNKSIYALAR

10-§. KOMPLEKS SONLARNI TASVIRLASH. KOMPLEKS SONNING MODULI VA ARGUMENTI. KOMPLEKS SONLAR USTIDA AMALLAR. KOMPELKS SONNING SHAKLLARI. KOMPLEKS SONNI DARAJAGA KO'TARISH VA ILDIZ CHIQARISH.

10.1. Kompleks sonlar ustida amallar

Ta'rif: $z=a+bi$ ko'rinishidagi son kompleks son deyiladi. Bu yerda $a=\operatorname{Re}z$; $b=\operatorname{Im}z$ $i^2=-1$ yoki $\sqrt{-1}=i$. $\bar{z}=a-ib$ -berilgan kompleks sonning qo'shmasi deyiladi. E- kompleks sonlar to'plami bo'lsin. $z=x+iy$ kompleks son shu to'plam ixtiyoriy elementi bo'lishi mumkin. $z_1=a_1+ib_1$, $z_2=a_2+ib_2$ kompleks sonlar berilgan bo'lsa, ular ustida quyidagi amallarni bajarish mumkin:

1) Qo'shish: $z_1+z_2=(a_1+b_1)+i(b_1+b_2)$

2) Ayirish: $z_1-z_2=(a_1-b_1)+i(b_1-b_2)$

3) Ko'paytirish: $z_1z_2=(a_1b_2-b_1b_2)+i(a_1b_2+a_2b_1)$

4) Bo'lish: $\frac{z_1}{z_2}=\frac{a_1+ib_1}{a_2+ib_2}=\frac{a_1+ib_1}{a_2+ib_2} \cdot \frac{a_2-ib_2}{a_2-ib_2}=\frac{(a_1a_2+b_1b_2)+i(a_2b_1-a_1b_2)}{a_2^2+b_2^2}$

10.2. Kompleks sonning trigonometrik va ko'rsatkichli shakli

$z=a+bi$ kompleks son $\overline{OA}$ vektor bilan tasvirlangan bo'lsin. $\overline{OA}$ vektorning OA uzunligini r deb, bu vektor bilan OX o'qning musbat yo'nalishi orasidagi burchakni φ deb belgilasak $a=r\cos\varphi$; $b=r\sin\varphi$.

Bu qiymatlarni $z=a+bi$ ga qo'yib, r ni qavsdan tashqariga chiqarsak: $z=r(\cos\varphi+i\sin\varphi)$ (10.1) ni $z=a+bi$ kompleks sonning trigonometrik shakli deyiladi, bu yerda $r=|z|$,
 $r=\sqrt{a^2+b^2}$, $\arg z=\varphi=\arctg\frac{b}{a}$.

Agar Eyler formulasini $e^{i\varphi}=\cos\varphi+i\sin\varphi$ e'tiborga olsak, unda
$$z=re^{i\varphi} \quad (10.2)$$

(10.2) kompleks sonning ko'rsatkichli formasi deyiladi.

10.3. Trigonometrik shakldagi kompleks sonlarni ko'paytirish va bo'lish

Ushbu: $z_1=r_1(\cos\varphi_1+i\sin\varphi_1)$, $z_2=r_2(\cos\varphi_2+i\sin\varphi_2)$ kompleks sonlarni ko'paytirib, quyidagini hosil qilamiz:

$z_1 z_2 = r_1 r_2 [\cos \varphi_1 \cos \varphi_2 - \sin \varphi_1 \sin \varphi_2 + i(\sin \varphi_1 \cos \varphi_2 + \cos \varphi_1 \sin \varphi_2)]$
 yoki $z_1 z_2 = r_1 r_2 [\cos(\varphi_1 + \varphi_2) + i \sin(\varphi_1 + \varphi_2)]$. Umuman, matematik
 induksiya metodi bilan, n ta
 $z_1 = r_1 (\cos \varphi_1 + i \sin \varphi_1), z_2 = r_2 (\cos \varphi_2 + i \sin \varphi_2), \dots, z_n = r_n (\cos \varphi_n + i \sin \varphi_n)$

kompleks sonlar uchun:

$$z_1 z_2 \dots z_n = r_1 r_2 \dots r_n [\cos(\varphi_1 + \varphi_2 + \dots + \varphi_n) + i \sin(\varphi_1 + \varphi_2 + \dots + \varphi_n)]$$

ekanligini ko'rsatish mumkin. Shunday qilib, bu hol uchun

$$z_1 z_2 \dots z_n = r_1 r_2 \dots r_n [\cos(\varphi_1 + \varphi_2 + \dots + \varphi_n) + i \sin(\varphi_1 + \varphi_2 + \dots + \varphi_n)] \text{ tenglikdan}$$

$$z^n = r^n (\cos n\varphi + i \sin n\varphi) \text{ yoki}$$

$$[r(\cos n\varphi + i \sin n\varphi)]^n = r^n (\cos n\varphi + i \sin n\varphi) \quad (10.3)$$

kelib chiqadi. Bu formula Muavr formulasi deyiladi.

10.4. Kompleks sondan ildiz chiqarish

$a+bi$ kompleks sonning kvadrat ildizi deb, kvadrati $a+bi$ songa teng bo'lgan $x+yi$ kompleks songa aytiladi. Shunday qilib, $a+bi = (x+yi)^2$.

$a+bi$ kompleks sonning kvadrat ildizi $\sqrt{a+bi}$ ko'rinishida belgilanadi.

Demak, x va y ning $x = \pm \sqrt{\frac{a + \sqrt{a^2 + b^2}}{2}}$ va $y = \pm \sqrt{\frac{-a + \sqrt{a^2 + b^2}}{2}}$ qiymatlarini $a+bi = x+yi$ ga qo'yib, $b>0$ va $b<0$ ga mos quyidagi ikki formulani hosil qilamiz:

$$\sqrt{a+bi} = \pm \left(\sqrt{\frac{a + \sqrt{a^2 + b^2}}{2}} + i \sqrt{\frac{-a + \sqrt{a^2 + b^2}}{2}} \right) \quad b>0 \text{ uchun,}$$

$$\sqrt{a+bi} = \pm \left(\sqrt{\frac{a + \sqrt{a^2 + b^2}}{2}} - i \sqrt{\frac{-a + \sqrt{a^2 + b^2}}{2}} \right) \quad b<0 \text{ uchun.}$$

Ushbu $z = r(\cos \varphi + i \sin \varphi)$ kompleks sonning n - darajali ildizini

$$\sqrt[n]{r(\cos \varphi + i \sin \varphi)} = \sqrt[n]{r} \left(\cos \frac{\varphi + 2k\pi}{n} + i \sin \frac{\varphi + 2k\pi}{n} \right) \quad (10.4)$$

ko'rinishida yozish mumkin. (10.4) formulada k istalgan butun sonni ifodalaydi. Lekin k ga $0, 1, 2, 3, \dots, n-1$ qiymatlarni berish kifoya.

Mustaqil bajarish uchun misollar

10.1. Quyidagilarni toping va chizmada tasvirlang.

1) $z_1 + z_2$; 2) $z_1 - z_2$; 3) $\overline{z_1}$; 4) $z_1 \cdot z_2$; 5) $z_1 : z_2$; 6) z_2^3 ; 7) $z_1 + z_2$;

8) $z_1 - z_2$ va chizmada tasvirlang. $z_1 + z_2$, $z_1 - z_2$, $z_1 \cdot z_2$, $\frac{z_1}{z_2}$ larni

hisoblang.

a) $z_1 = -2$, $z_2 = 2i$. b) $z_1 = 2i$, $z_2 = -2i$. c) $z_1 = -2i$, $z_2 = 1+i$.

d) $z_1 = 2i$, $z_2 = -2i$. e) $z_1 = \sqrt{15} - i$, $z_2 = 1 - i\sqrt{15}$.

f) $z_1 = -3 + 4i$, $z_2 = 3 - 4i$

g) $z_1 = 6 + 11i$, $z_2 = 7 + 3i$

h) $z_1 = \frac{75}{58} + \frac{59}{58}i$; $z_2 = 154 + 414i$. j) $z_1 = 3 - i$, $z_2 = 4 + 5i$.

k) $z_1 = 4 - 5i$, $z_2 = 2 + 3i$

m) $z_1 = 23 + 11i$, $z_2 = 4 - 3i$

n) $z_1 = 2 + 10i$, $z_2 = 14 - 3i$.

10.2. Quyidagi kompleks sonlarni tasvirllovchi vektorlar yasalsin:

a) $3 + i$; b) $-4 + 5i$; v) $7 - 4i$; g) $-2 - 6i$; d) 1 ; e) -1 ;

j) $i\sqrt{2}$; z) $1 + i\sqrt{3}$; i) $2 - \sqrt{3} + 4i$; k) 8 ; l) $1 - 2i\sqrt{2}$; m) -7 ; n) $-5i$.

10.3. Quyidagi ildizlarni hisoblang:

a) $\sqrt{2i}$; b) $\sqrt{-8i}$; v) $\sqrt{3 - 4i}$; g) $\sqrt{-15 + 8i}$;

d) $\sqrt{-11 + 60i}$; e) $\sqrt{2 - 3i}$; j) $\sqrt{1 - i\sqrt{3}}$; z) $\sqrt{4 + i} + \sqrt{4 - i}$;

i) $\sqrt[4]{-7 + 24i}$; k) $\sqrt[4]{-4}$; l) $\sqrt[4]{-1}$; m) $\sqrt[4]{2 + i\sqrt{12}}$. n) $\sqrt{15 + 8i}$ ni

hisoblang.

o) $x^2 - (5 - i)x + (8 - i) = 0$ kvadrat tenglamani yeching.

10.4. $\sqrt{7 + 3i}$ ni hisoblang.

10.5. Ushbu tengliklardan x va y lar topilsin.

1) $3y + 5xi = 15 - 7i$; 2) $(2x + 3y) + (x - y)i = 7 + 6i$;

3) $(2x + y) - i = 5 + (y - x)i$; 4) $(3i - 1)x + (2 - 3i)y = 2 - 3i$.

10.6. Ushbu kvadrat tenglamalarni yeching.

a) $x^2 - (3 + 2i)x + (1 + 3i) = 0$; b) $(7 + i)x^2 - (7 + i)x + 2 = 0$;

v) $(1 + i)x^2 - (7 + 2i)x + (8 - i) = 0$.

10.7. Ushbu kompleks sonlarni trigonometrik shaklga keltiring:

a) 3 ; b) -2 ; v) $\frac{1}{2} - i\frac{\sqrt{3}}{2}$; g) $2i$; d) $\frac{\sqrt{2}}{2}(1 - i)$; e) $-1 + i\sqrt{3}$; j) $3 + i$;

z) $4 - i$; i) $-2 + i$; k) $-1 - 2i$; l) $1 + i\sqrt{3}$; m) i ; n) $-i$.

Quyidagi ildizlarning barcha qiymatlarini toping. Ildiz ostida turgan sonlarni ko'rsatkichli ko'rinishda tasvirlang.

10.8. $\sqrt{1 + i}$. 10.9. $\sqrt{3 + 4i}$. 10.10. $\sqrt{-1 + i}$. 10.11. $\sqrt[3]{\sqrt{15} - i}$.

10.12. $\sqrt[3]{-3 + 4i}$. 10.13. $\sqrt[3]{3 - 4i}$. 10.14. $\sqrt[4]{-1}$. 10.15. $\sqrt[3]{1 - i}$.

10.16. $\sqrt[3]{-1 - i}$. 10.17. $\sqrt{-2 + 2i}$

10.18. Quyidagi ildizlarni hisoblang.

a) $\sqrt{-1 + i}$; b) $\sqrt[4]{-1}$; v) $\sqrt[4]{32}$; g) $\sqrt{i}$; d) $\sqrt{-i}$; e) $\sqrt[4]{2}$.

10.19. Quyidagi ifodalarni sinus va kosinusning darajalari orqali ifodalang.

1) $\sin 3x$ 2) $\cos 3x$ 3) $\sin 4x$ 4) $\cos 4x$ 5) $\sin 5x$

10.20. Quyidagi ifodalarni sinus va kosinusning karrali yoylari orqali ifodalang.

1) $\sin^2 x$ 2) $\sin^3 x$ 3) $\sin^4 x$ 4) $\cos^4 x$ 5) $\cos^5 x$ 6) $\cos^6 x$

10.21. Chiziq tenglamasini kompleks ko'rinishda yozing.

a) $3x + y = 1$ b) $(x-1)y = 2$ v) $2(x-1)^2 + 3(y+2)^2 = 6$

10.22. $z = 2 + 2i$ sonni trigonometrik shaklda ifodalang va z^6 darajaga ko'taring

10.23. $z = 3\left(\cos \frac{2\pi}{3} - i \sin \frac{2\pi}{3}\right)$ sonning moduli va argumentini toping.

10.24. Ushbu $z_1 = r_1(\cos \varphi_1 + i \sin \varphi_1)$ kompleks sonni $z_2 = r_2(\cos \varphi_2 + i \sin \varphi_2)$ kompleks soniga bo'ling.

10.25. $z_1 = 3(\cos 30^\circ + i \sin 30^\circ)$, $z_2 = 2(\cos 60^\circ + i \sin 60^\circ)$ berilgan.

a) $z_1 + z_2$, b) $z_1 - z_2$, c) $z_1 \cdot z_2$, g) $\frac{z_1}{z_2}$ larni hisoblang.

10.26. $\sin 2\varphi$ va $\cos 2\varphi$ ni $\sin \varphi$ va $\cos \varphi$ orqali ifodalang.

10.27. $\left[2\left(\cos \frac{\pi}{6} + i \sin \frac{\pi}{6}\right)\right]^6$ ni hisoblang.

10.28. $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{20}$ ni hisoblang.

10.29. Hisoblang.

- 1) i^{65} ; i^{134} ; i^{105} ; i^{45} ; i^{206} ; i^{304} ; 2) $i^{43} + i^{48} - i^{64} + i^{45} - i^{101}$;
- 3) $(i^{36} + i^{25} + i^{17} + i^{14})(i^{23} + i^{11})$;
- 4) $(i^{64} + i^{17} + i^{26} + i^{82})(i^{72} - i^{34})$;
- 5) $(i^{12} - i^{45} + i^{67} - i^{17})(i^{36} - i^{98})$.

10.30. Quyidagi kompleks sonlarni trigonometrik shaklga keltirib, so'ngra ko'rsatilgan amallarni bajaring:

a) $(-5 + 5i\sqrt{3})(1-i)$; b) $\frac{-5 + 5i\sqrt{3}}{(1-i)^2}$; v) $(1+i)^{25}$;

g) $\left(\frac{1+i\sqrt{3}}{1-i}\right)^{20}$; d) $\frac{(-1+i\sqrt{3})^{15}}{(1-i)^{20}} + \frac{(-1-i\sqrt{3})^{15}}{(1+i)^{20}}$

e) $(1+i)(-2+2\sqrt{3}i)$; l) $-3i(-3+3i)$ m) $(2\sqrt{2}-2i\sqrt{6}) \cdot 6i$.

10.31. Ushbu $(\cos 15^\circ + i \sin 15^\circ)(\cos 30^\circ + i \sin 30^\circ) = \cos 45^\circ + i \sin 45^\circ$ ayniyatdan foydalanib, $\sin 15^\circ$ va $\cos 15^\circ$ ning qiymatlarini toping.

10.32. $\frac{(1-i\sqrt{3})(\cos \alpha + i \sin \alpha)}{2(1-i)(\cos \varphi - i \sin \varphi)}$ ni hisoblang.

10.33. Quyidagi ayniyatlarni isbotlang (n - butun son):

a) $(1+i)^n = 2^{\frac{n}{2}}\left(\cos \frac{n\pi}{4} + i \sin \frac{n\pi}{4}\right)$, b) $(\sqrt{3}-i)^n = 2^{\frac{n}{2}}\left(\cos \frac{n\pi}{6} - i \sin \frac{n\pi}{6}\right)$.

10.34. $(1 + \cos \varphi + i \sin \varphi)^n$ ni bajaring.

10.35. $(\cos \varphi - i \sin \varphi)^n = \cos n\varphi - i \sin n\varphi$ ayniyatni isbotlang.

10.36. $\left(\frac{1+ig\varphi}{1-ig\varphi}\right)^n = \frac{1+ig\varphi}{1-ig\varphi}$ ayniyatni isbotlang.

10.37. $|\alpha^{-n}| = |\alpha|^{-n}$ tenglikni isbotlang.

10.38. Quyidagi modullarni hisoblang:

a) $\left|\left(\frac{1}{2}-\frac{1}{3}i\right)(3+2i)\right|$; b) $\left|\frac{\sqrt{2}-i}{\sqrt{3}+i}\right|$; v) $\left|8+\frac{1}{\sqrt{5}}i\right|^5$;

g) $\left|\frac{\left(2+\frac{1}{2}i\right)^4 \cdot (\sqrt{2}-2i) \cdot (1-\sqrt{3}i)}{(2+i\sqrt{2})^2 \cdot (\sqrt{3}+i) \cdot \left(\frac{1}{2}-2i\right)^6}\right|$; d) $\left|\left(\sqrt{5}+\frac{1}{\sqrt{3}}i\right)^{-2}\right|$;

e) $\left|\sqrt{(2+7i)^2 + (1-i)^4} \cdot (5+3i)\right|$; j) $\left|\frac{\sqrt[3]{(3-4i)^3} \cdot \sqrt{(1+2i)(3-i)}}{\sqrt{(4+5i)^3} (1+i)\sqrt{2+i}}\right|^2$.

Mavzu bo'yicha nazorat testlari

Kompleks sonlar va ular ustida.

1. $2-i$ sonini $3+4i$ soniga qo'shgandagi natijani toping.
A) $5+3i$; B) $4+i$; C) $2+7i$; D) $6-4i$.
2. $11+0,7i$ sonidan $0,3-2i$ soni ayrilgandagi natijani toping.
A) $10,7+2,7i$; B) $3,3-1,4i$; C) $1,7+9,2i$; D) $11-2i$.
3. $5,7-3i$ sonini $6+2,1i$ soniga qo'shgandagi natijani toping.
A) $12,7+3i$; B) $4,6+2i$; C) $11,7-0,9i$; D) $7,5-2,5i$.
4. $4-2i$ sonini $1,5+2i$ soniga ko'paytirgandagi natijani toping.
A) $5+10i$; B) $6-2i$; C) $10+5i$; D) $4-1,5i$.
5. $3+0,1i$ sonini $1+i$ soniga ko'paytirgandagi natijani toping.
A) $3+i$. B) $2,9+3,1i$. C) $2,7+0,4i$. D) $5-1,7$.
6. 16 sonini $4+4i$ soniga bo'lgandagi natijani toping.
A) $2+8i$. B) $4-4i$. C) $2-2i$. D) $16-4i$.
7. $1-i$ sonini $1+i$ soniga bo'lgandagi natijani toping.
A) i . B) $1-i$. C) $-i$. D) -1 .
8. $3+i$ sonini kubga oshirgandagi natijani toping.
A) 27 . B) $8i$. C) 9 . D) $7,5i$.
9. $2-2i$ sonini 8 -darajaga oshirgandagi natijani toping.
A) 1024 . B) 2048 . C) 4096 . D) $8192i$.
10. $z^8+16=0$ tenglama nechta kompleks yechimga ega?
A) Yechimi yo'q. B) 16 . C) 8 . D) 4 .

11-§. KO'P O'ZGARUVCHILI FUNKSIYANING XUSUSIY HOSILALARI. YUQORI TARTIBLI XUSUSIY HOSILALAR

11.1. Ko'p o'zgaruvchili funktsiyaning aniqlanish sohasi

Agar biror A to'plamning har bir (x, y) haqiqiy sonlar juftligi biror qoida bilan B to'plamdagi yagona z haqiqiy songa mos qo'yilgan bo'lsa, u holda A to'plamda ikki o'zgaruvchining funktsiyasi z aniqlangan deyiladi va quyidagi ko'rinishlarda belgilanadi: $z = f(x, y)$, $z = z(x, y)$ va h.k.

A to'plam funktsiyaning aniqlanish sohasi deyiladi.

1-misol. $z = \sqrt{1 - x^2 - y^2}$ funktsiyaning aniqlanish sohasini toping.

Yechish. Berilgan funktsiya $1 - x^2 - y^2 \geq 0$, ya'ni $x^2 + y^2 \leq 1$ shartda haqiqiy qiymatlar qabul qiladi. Demak, funktsiyaning aniqlanish sohasi markazi koordinatalar boshida bo'lgan, radiusi 1 ga teng doiradan iborat.

11.2. Birinchi tartibli xususiy hosilalar

$z = f(x, y)$ funktsiyada y ni o'zgarimas deb qarab, undan x bo'yicha olingan hosila z ning x bo'yicha xususiy hosilasi deyiladi va u $\frac{\partial z}{\partial x}$ yoki $f'_x(x, y)$ ko'rinishida belgilanadi $f'_x(x, y) = \frac{\partial f}{\partial x} = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$ z ning y bo'yicha xususiy hosilasi ham shunga o'xshash ta'riflanadi va quyidagicha belgilanadi

$$\frac{\partial z}{\partial y} = f'_y(x, y) \text{ ya'ni } f'_y(x, y) = \lim_{\Delta y \rightarrow 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}.$$

Agar $z = f(x, y)$ funktsiya (x, y) nuqtada uzluksiz xususiy hosilalarga ega bo'lsa, uning to'liq orttirmasi $\Delta z = \frac{\partial z}{\partial x} \Delta x + \frac{\partial z}{\partial y} \Delta y + \varepsilon \cdot \rho$ ko'rinishda yoziladi, bunda $\rho = \sqrt{|\Delta x|^2 + |\Delta y|^2}$ nolga intilganda $(\rho \rightarrow 0) \varepsilon \rightarrow 0$. U holda $\frac{\partial z}{\partial x} \Delta x + \frac{\partial z}{\partial y} \Delta y$ ifoda to'liq orttirma Δz ning bosh qismi bo'ladi.

To'liq differensial. $z = f(x, y)$ funktsiyaning to'liq differensiali quyidagi formula orqali belgilanadi: $dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$

11.3. Yuqori tartibli xususiy hosilalar

$z = f(x, y)$ funksiyaning birinchi tartibli xususiy hosiladan olingan xususiy hosila ikkinchi tartibli xususiy hosila deyiladi va uni quyidagicha yoziladi:

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial z}{\partial x} \right) = f''_{xx}(x, y)$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial y} \right) = f''_{yy}(x, y)$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x} \right) = f''_{xy}(x, y)$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial z}{\partial y} \right) = f''_{yx}(x, y)$$

Xuddi shuningdek, uch va undan yuqori tartibli xususiy hosilalar ham yuqoridagi kabi aniqlanadi. $f''_{xy}(x, y)$ va $f''_{yx}(x, y)$ xususiy hosilalar $z = f(x, y)$ funksiyaning aralash hosilalari deyiladi.

Mustaqil bajarish uchun misollar

11.1-11.30 gacha. Quyidagi birinchi tartibli xususiy hosilalarni hisoblang.

11.1. $a) z = x^2 e^{y^2};$

$b) z = x^3 + 3x^2 y - y^3.$

11.2. $z = \ln(x^2 + y^2).$

11.3. $z = \frac{y}{x}.$

11.4. $z = \arctg \frac{y}{x}.$

11.5. $z = \frac{xy}{x-y}.$

11.6. $z = x e^{-yx}.$

11.7. $z = \sqrt{x^2 - y^2}.$

11.8. $z = \frac{x}{\sqrt{x^2 + y^2}}.$

11.9. $z = \arctg \frac{y}{x}.$

11.10. $z = x^y.$

11.11. $z = e^{\frac{\sin y}{x}}.$

11.12. $z = \ln \sin \frac{x+a}{\sqrt{y}}.$

11.13. $u = (xy)^z.$

11.14. $z = \frac{3u}{g} + \frac{2g}{u}.$

11.15. $z = \frac{x^3 + y^3}{x^2 + y^2}.$

11.16. $z = (5x^2 y - y^3 + 7)^3.$

11.17. $z = x\sqrt{y} + \frac{y}{\sqrt{x}}.$

11.18. $z = \frac{1}{\arctg \frac{x}{y}}.$

11.19. $z = \arcsin \frac{\sqrt{x^2 - y^2}}{\sqrt{x^2 + y^2}}.$

11.20. $z = \ln \frac{\sqrt{x^2 + y^2} - x}{\sqrt{x^2 + y^2} + x}.$

11.21. $z = \ln \lg \frac{x}{y}.$

11.22. $z = (1 + \log_x x)^3.$

11.23. $z = xye^{\sin x y}.$

11.24. $z = (x^2 + y^2) \frac{1 - \sqrt{x^2 + y^2}}{1 + \sqrt{x^2 + y^2}}.$

11.25. $z = \arctg \sqrt{x y}.$

11.26. $z = 2 \sqrt{\frac{1 - \sqrt{x y}}{1 + \sqrt{x y}}}.$

11.27. $u = (\sin x)^{y z}.$

$$11.28. u = \arctg(x - y)^z.$$

11.29. $u = \sqrt{az^3 - bt^3}$. $\frac{\partial u}{\partial z}$ va $\frac{\partial u}{\partial t}$ ni $z = b$, $t = a$ dagi qiymatini toping.

11.30. $z = \frac{x \cos y - y \cos x}{1 + \sin x + \sin y}$. $\frac{\partial z}{\partial x}$ va $\frac{\partial z}{\partial y}$ ni $x = y = 0$ dagi qiymatini hisoblang.

11.31-11.55 gacha. Quyidagi funksiyani to'liq hosilasini toping.

$$11.31. z = e^{xy}(x + y). \quad 11.32. z = \ln(1 + e^x + y^2).$$

$$11.33. z = \frac{\sqrt{x} - \sqrt{y}}{x + y}. \quad 11.34. z = \sin\left(\frac{x}{y}\right). \quad 11.35. z = \frac{x \arcsin y}{y}.$$

$$11.36. z = x^y + y^x. \quad 11.37. z = \ln(x^2 + y^2). \quad 11.38. z = \ln \lg(y/x).$$

$$11.39. z = \sin(x^2 + y^2). \quad 11.40. z = x^y. \quad 11.41. u = \ln(x + \sqrt{x^2 + y^2}).$$

$$11.42. u = e^x(\cos y + x \sin y). \quad 11.43. u = e^{xy}(x \cos y + y \sin x).$$

$$11.44. z = \arctg \frac{2(x + \sin y)}{4 - x \sin y}. \quad 11.45. u = e^{xz}.$$

$$11.46. z = 5x^3 y^2. \quad 11.47. u = (xy)^z. \quad 11.48. z = (\sin x)^{\cos y}$$

$$11.49. z = \frac{y}{x} - \frac{x}{y}. \quad 11.50. z = e^{x^2 + y^2}. \quad 11.51. z = \sin^2 x + \cos^2 y$$

$$11.52. z(x, y) = \frac{y}{x^2}; \quad df(1;1) = ? \quad 11.53. z(x, y, z) = e^{x^2 + y^2 + z^2}; \quad dz(0;1;2) = ?$$

$$11.54. z = \ln(x + \sqrt{x^2 + y^2}). \quad 11.55. u = x^{y^2 z}.$$

11.57-11.66 gacha. Quyidagi funksiyalardan ikkinchi tartibli xususiy hosilalarini oling.

$$11.57. a) z = y \ln x; \quad b) u = 4x^3 + 3x^2 y + 3xy^2 - y^3. \quad \frac{\partial^2 u}{\partial x \partial y} \text{ -ni toping.}$$

$$11.58. a) z = \ln(1 + x + 2y); \quad b) u = xy + \sin(x + y) \quad \frac{\partial^2 u}{\partial x^2} \text{ - ni toping.}$$

$$11.59. u = \ln \lg(x + y). \quad \frac{\partial^2 u}{\partial x \partial y} \text{ -ni toping.} \quad 11.60. z = \arctg \frac{x + y}{1 - xy}. \quad \frac{\partial^2 z}{\partial x \partial y} \text{ -ni}$$

toping.

$$11.61. z = x^2 \ln(x + y) \quad \frac{\partial^2 z}{\partial x \partial y} \text{ -ni toping.}$$

$$11.62. u = x \sin xy + y \cos xy. \quad \frac{\partial^2 u}{\partial x^2} \text{ - ni toping.}$$

$$11.63. u = \sin x(x + \cos y) \quad \frac{\partial^3 z}{\partial x^2 \partial y} \text{ -ni toping.}$$

$$11.64. z = 0,5 \ln(x^2 + y^2) \quad d^2 z \text{ -ni toping.}$$

$$11.65. z = \cos(x + y) \quad d^2 z \text{ -ni toping.}$$

11.3. Yuqori tartibli xususiy hosilalar

$z = f(x, y)$ funksiyaning birinchi tartibli xususiy hosiladan olingan xususiy hosila ikkinchi tartibli xususiy hosila deyiladi va uni quyidagicha yoziladi:

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} \left(\frac{\partial z}{\partial x} \right) = f''_{xx}(x, y)$$

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$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y} \left(\frac{\partial z}{\partial x} \right) = f''_{xy}(x, y)$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x} \left(\frac{\partial z}{\partial y} \right) = f''_{yx}(x, y)$$

Xuddi shuningdek, uch va undan yuqori tartibli xususiy hosilalar ham yuqoridagi kabi aniqlanadi. $f''_{xy}(x, y)$ va $f''_{yx}(x, y)$ xususiy hosilalar $z = f(x, y)$ funksiyaning aralash hosilalari deyiladi.

Mustaqil bajarish uchun misollar

11.1-11.30 gacha. Quyidagi birinchi tartibli xususiy hosilalarni hisoblang.

11.1 $a) z = x^2 e^{y^2}; \quad \text{b) } z = x^3 + 3x^2 y - y^3. \quad 11.2. z = \ln(x^2 + y^2).$

11.3. $z = \frac{y}{x}. \quad 11.4. z = \arctg \frac{y}{x}. \quad 11.5. z = \frac{xy}{x-y}.$

11.6. $z = x e^{-yx}. \quad 11.7. z = \sqrt{x^2 - y^2}. \quad 11.8. z = \frac{x}{\sqrt{x^2 + y^2}}.$

11.9. $z = \arctg \frac{y}{x}. \quad 11.10. z = x^y. \quad 11.11. z = e^{\frac{\sin^2 y}{x}}.$

11.12. $z = \ln \sin \frac{x+a}{\sqrt{y}}. \quad 11.13. u = (xy)^z. \quad 11.14. z = \frac{3u}{g} + \frac{2g}{u}.$

11.15. $z = \frac{x^3 + y^3}{x^2 + y^2}. \quad 11.16. z = (5x^2 y - y^3 + 7)^3. \quad 11.17. z = x\sqrt{y} + \frac{y}{\sqrt{x}}.$

11.18. $z = \frac{1}{\arctg \frac{x}{y}}. \quad 11.19. z = \arcsin \frac{\sqrt{x^2 - y^2}}{\sqrt{x^2 + y^2}}. \quad 11.20. z = \ln \frac{\sqrt{x^2 + y^2} - x}{\sqrt{x^2 + y^2} + x}.$

11.21. $z = \ln \lg \frac{x}{y}. \quad 11.22. z = (1 + \log_x x)^3. \quad 11.23. z = xye^{\sin \pi xy}.$

11.24. $z = (x^2 + y^2) \frac{1 - \sqrt{x^2 + y^2}}{1 + \sqrt{x^2 + y^2}}. \quad 11.25. z = \arctg \sqrt{x^y}.$

11.26. $z = 2 \sqrt{\frac{1 - \sqrt{xy}}{1 + \sqrt{xy}}}. \quad 11.27. u = (\sin x)^{yz}.$

$$11.28. u = \arctg(x - y)^z.$$

11.29. $u = \sqrt{az^3 - bt^3}$. $\frac{\partial u}{\partial z}$ va $\frac{\partial u}{\partial t}$ ni $z = b$, $t = a$ dagi qiymatini toping.

11.30. $z = \frac{x \cos y - y \cos x}{1 + \sin x + \sin y}$. $\frac{\partial z}{\partial x}$ va $\frac{\partial z}{\partial y}$ ni $x = y = 0$ dagi qiymatini hisoblang.

11.31-11.55 gacha. Quyidagi funksiyani to'liq hosilasini toping.

$$11.31. z = e^{xy}(x + y). \quad 11.32. z = \ln(1 + e^x + y^2).$$

$$11.33. z = \frac{\sqrt{x} - \sqrt{y}}{x + y}. \quad 11.34. z = \sin\left(\frac{x}{y}\right). \quad 11.35. z = \frac{x \arcsin y}{y}.$$

$$11.36. z = x^y + y^x. \quad 11.37. z = \ln(x^2 + y^2). \quad 11.38. z = \ln \lg(y/x).$$

$$11.39. z = \sin(x^2 + y^2). \quad 11.40. z = x^y. \quad 11.41. u = \ln(x + \sqrt{x^2 + y^2}).$$

$$11.42. u = e^x(\cos y + x \sin y). \quad 11.43. u = e^{xy}(x \cos y + y \sin x).$$

$$11.44. z = \arctg \frac{2(x + \sin y)}{4 - x \sin y}. \quad 11.45. u = e^{xyz}.$$

$$11.46. z = 5x^3 y^2. \quad 11.47. u = (xy)^z. \quad 11.48. z = (\sin x)^{\cos y}$$

$$11.49. z = \frac{y}{x} - \frac{x}{y}. \quad 11.50. z = e^{x^2 + y^2}. \quad 11.51. z = \sin^2 x + \cos^2 y$$

$$11.52. z(x, y) = \frac{y}{x^2}; \quad df(1;1) = ? \quad 11.53. z(x, y, z) = e^{x^2 + y^2 + z^2}; \quad dz(0;1;2) = ?$$

$$11.54. z = \ln(x + \sqrt{x^2 + y^2}). \quad 11.55. u = x^{y^2 z}.$$

11.57-11.66 gacha. Quyidagi funksiyalardan ikkinchi tartibli xususiy hosilalarini oling.

$$11.57. a) z = y \ln x; \quad b) u = 4x^3 + 3x^2 y + 3xy^2 - y^3. \quad \frac{\partial^2 u}{\partial x \partial y} \text{-ni toping.}$$

$$11.58. a) z = \ln(1 + x + 2y); \quad b) u = xy + \sin(x + y). \quad \frac{\partial^2 u}{\partial x^2} \text{-ni toping.}$$

$$11.59. u = \ln \lg(x + y). \quad \frac{\partial^2 u}{\partial x \partial y} \text{-ni toping.} \quad 11.60. z = \arctg \frac{x + y}{1 - xy}. \quad \frac{\partial^2 z}{\partial x \partial y} \text{-ni}$$

toping.

$$11.61. z = x^2 \ln(x + y). \quad \frac{\partial^2 z}{\partial x \partial y} \text{-ni toping.}$$

$$11.62. u = x \sin xy + y \cos xy. \quad \frac{\partial^2 u}{\partial x^2} \text{-ni toping.}$$

$$11.63. u = \sin x(x + \cos y). \quad \frac{\partial^3 z}{\partial x^2 \partial y} \text{-ni toping.}$$

$$11.64. z = 0.5 \ln(x^2 + y^2). \quad d^2 z \text{-ni toping.}$$

$$11.65. z = \cos(x + y). \quad d^2 z \text{-ni toping.}$$

11.66. $z = \cos(ax + e^y) \cdot \frac{\partial^3 z}{\partial x \partial y^2}$ -ni toping.

Mavzu bo'yicha nazorat testlari

1. Ko'p o'zgaruvchili funksiyalar.

1. Tomonlari x va y bo'lgan to'g'ri to'rtburchakka doir qaysi masalaning javobi ikki o'zgaruvchili funksiya bilan ifodalanmaydi?

- A) To'g'ri to'rtburchak yuzasini topish;
- B) To'g'ri to'rtburchak perimetrini topish;
- C) To'g'ri to'rtburchak diagonalini topish;
- D) To'g'ri to'rtburchakning ikkita qarama-qarshi tomonining yig'indisini topish;

E) Barcha masalalarning javoblari ikki o'zgaruvchili funksiya bilan ifodalanadi.

2. Ikki o'zgaruvchili funksiyaning ko'rsating.

A) $f = ax^2 + bx + c$. B) $f = x + \frac{1}{y}$. C) $f = y + \frac{1}{y}$.

D) $f = t^2 + t + 1$. E) $f = \sin(ax^2 + b)$.

3. Ikki o'zgaruvchili $z = f(x, y)$ funksiya aniqlanish sohasidagi har bir $M(x, y)$ nuqtaga nimani mos qo'yadi?

- A) to'g'ri chiziqdagi biror nuqtani. B) tekislikdagi biror nuqtani.
- C) fazodagi biror nuqtani. D) biror to'plamni. E) biror chiziqni.

4. Ushbu ikki o'zgaruvchili funksiyaning $D\{f\}$ aniqlanish sohasini toping:

$$f(x, y) = \sqrt{9 - x^2 - y^2}.$$

- A) Tomoni $a=3$ va markazi $O(0,0)$ nuqtada joylashgan kvadrat.
- B) Tomoni $a=3$ va markazi $O(0,0)$ nuqtada joylashgan muntazam uchburchak.

- C) Radiusi $R=3$ va markazi $O(0,0)$ nuqtada joylashgan doira.
- D) Radiusi $R=3$ va markazi $O(0,0)$ nuqtada joylashgan aylana.
- E) $[-3,3]$ kesmadan iborat soha.

5. Ushbu ikki o'zgaruvchili funksiyaning qiymatlar sohasini toping:

$$f(x, y) = \sqrt{9 - x^2 - y^2}.$$

- A) $[-3,3]$. B) $(-3,3)$. C) $[0,3]$. D) $(0,3)$. E) $[0,9]$.

6. Ushbu ikki o'zgaruvchili funksiyaning $D\{f\}$ aniqlanish sohasini toping:

$$f(x, y) = \sqrt{16 + x^2 + y^2}.$$

A) Tomoni $a=4$ va markazi $O(0,0)$ nuqtada joylashgan kvadrat;
 B) Tomoni $a=4$ va markazi $O(0,0)$ nuqtada joylashgan muntazam uchburchak;

C) Radiusi $R=4$ va markazi $O(0,0)$ nuqtada joylashgan doira;
 D) Radiusi $R=4$ va markazi $O(0,0)$ nuqtada joylashgan aylana;
 E) R^2 tekislik.

7. Ushbu ikki o'zgaruvchili funksiyaning qiymatlar sohasini toping:

$$f(x, y) = \sqrt{16 + x^2 + y^2}.$$

A) $[0, 4]$. B) $(0, 4)$. C) $[4, +\infty)$. D) $(-\infty, 4]$. E) $[0, 16]$.

8. Ikki o'zgaruvchili $z = f(x, y)$ funksiya grafigi qanday geometrik ob'ektni ifodalaydi?

A) tekislikdagi to'g'ri chiziqni. B) tekislikdagi egri chiziqni.
 C) fazodagi tekislikni. D) fazodagi sirtini. E) fazodagi jismni.

9. **Ta'rifni to'ldiring:** Ikki o'zgaruvchili $z = f(x, y)$ funksiyaning sath chizig'i deb ... tenglama bilan aniqlanadigan chiziqqa aytiladi.

A) $f(C, y) = 0$. B) $f(x, C) = 0$. C) $f(0, y) = C$.

D) $f(x, 0) = C$. E) $f(x, y) = C$.

10. Ikki o'zgaruvchili $z = 9x^2 + 9y^2$ funksiyaning sath chiziqlari nimadan iborat?

A) parabola. B) giperbola. C) ellips. D) aylana.
 E) kesishuvchi ikkita to'g'ri chiziq.

2. Ko'p o'zgaruvchili funksiyaning hosila va differensiallari.

1. $z = f(x, y)$ funksiyaning x argumenti bo'yicha f'_x xususiy hosilasini ta'rif bo'yicha aniqlashda quyidagi amallardan qaysi biri bajarilmaydi?

A) x argumentga Δx orttirma beriladi.
 B) funksiyaning $\Delta_y f = f(x + \Delta x, y) - f(x, y)$ xususiy orttirmasi hisoblanadi.
 C) orttirmalar nisbati $\Delta_y f / \Delta x$ aniqlanadi.
 D) $\Delta x \rightarrow 0$ bo'lganda $\Delta_y f / \Delta x$ nisbat limiti topiladi.
 E) Keltirilgan barcha amallar bajariladi.

2. $z = f(x, y)$ funksiyaning x argumenti bo'yicha f'_x xususiy hosilasi qanday aniqlanadi?

A) $f'_x = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x, y + \Delta y) - f(x, y)}{\Delta x}$.

B) $f'_x = \lim_{\Delta x \rightarrow 0} \frac{f(x, y + \Delta x) - f(x, y)}{\Delta x}$.

C) $f'_x = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x, y + \Delta y) - f(x + \Delta x, y)}{\Delta x}$.

$$D) f'_x = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}.$$

$$E) f'_x = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x, y) + f(x, y)}{\Delta x}.$$

3. $z=f(x,y)$ funksiyaning y argumenti bo'yicha f'_y xususiy hosilasini ta'rif bo'yicha aniqlashda quyidagi amallardan qaysi biri bajarilmaydi?

- A) y argumentga Δy orttirma beriladi.
- B) funksiyaning $\Delta_y f = f(x, y + \Delta y) - f(x, y)$ xususiy orttirmasi hisoblanadi.
- C) orttirmalar nisbati $\Delta_y f / \Delta y$ aniqlanadi.
- D) $\Delta y \rightarrow 0$ bo'lganda $\Delta_y f / \Delta y$ nisbat limiti topiladi.
- E) Keltirilgan barcha amallar bajariladi.

4. $z=f(x,y)$ funksiyaning y argumenti bo'yicha f'_y xususiy hosilasi qanday aniqlanadi?

$$A) f'_y = \lim_{\Delta y \rightarrow 0} \frac{f(x + \Delta x, y + \Delta y) - f(x, y)}{\Delta y}.$$

$$B) f'_y = \lim_{\Delta y \rightarrow 0} \frac{f(x, y + \Delta y) - f(x, y)}{\Delta y}.$$

$$C) f'_y = \lim_{\Delta y \rightarrow 0} \frac{f(x + \Delta x, y + \Delta y) - f(x, y + \Delta y)}{\Delta y}.$$

$$D) f'_y = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x, y) - f(x, y + \Delta y)}{\Delta x}.$$

$$E) f'_y = \lim_{\Delta y \rightarrow 0} \frac{f(x, y + \Delta y) + f(x, y)}{\Delta y}.$$

5. $z=x^2+y^3+xy$ funksiyaning x bo'yicha f'_x xususiy hosilasini toping.

- A) $f'_x = 2x + 3y^2 + y$. B) $f'_x = 2x + y^3 + x$. C) $f'_x = 2x + y$.
- D) $f'_x = 2x + 3y^2 + x$. E) $f'_x = 2x + xy$.

6. $z=x^2+y^3+xy$ funksiyaning y bo'yicha f'_y xususiy hosilasini toping.

- A) $f'_y = 2x + 3y^2 + y$. B) $f'_y = x^2 + 3y^2 + y$. C) $f'_y = 2x + 3y^2$.
- D) $f'_y = 2x + 3y^2 + x$. E) $f'_y = 3y^2 + x$.

7. $z=f(x,y)$ funksiya $M_0(x_0, y_0)$ nuqtada differentsiallanuvchi bo'lishi uchun qaysi shart talab etilmaydi?

A) $z=f(x,y)$ funksiya $M_0(x_0, y_0)$ nuqta va uning biror atrofida aniqlangan.

B) $M_0(x_0, y_0)$ nuqta va uning biror atrofida f'_x, f'_y xususiy hosilalar mavjud.

C) $M_0(x_0, y_0)$ nuqta va uning biror atrofida f'_x, f'_y xususiy hosilalar uzluksiz.

D) $M_0(x_0, y_0)$ nuqta va uning biror atrofida xususiy hosilalar $f'_x \neq 0, f'_y \neq 0$.

E) Keltirilgan barcha shartlar talab etiladi.

8. Ikki o'zgaruvchili $z=f(x, y)$ funksiyaning df to'liq differensial formulasi ko'rsating.

A) $df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$. B) $df = \frac{\partial f}{\partial x} dy + \frac{\partial f}{\partial y} dx$. C) $df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$.

D) $df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial y} dy$. E) $df = \frac{\partial f}{\partial x} dx - \frac{\partial f}{\partial y} dy$.

9. $z=x^2+y^3+xy$ funksiyaning df to'liq differensialini toping.

A) $df = (x^2 + x)dx + (y^3 + y)dy$.

B) $df = (2x + 3y^2 + x)dx + (2x + 3y^2 + y)dy$.

C) $df = (2x + y)dx + (3y^2 + x)dy$. D) $df = 2xdx + 3y^2dy$.

E) $df = (3y^2 + x)dx + (2x + y)dy$.

10. $z=y^x$ funksiyaning df to'liq differensialini toping.

A) $df = y^x (\ln y dx - \frac{x}{y} dy)$. B) $df = y^x (\ln y dx - \frac{y}{x} dy)$.

C) $df = y^x (\ln y dx + \frac{y}{x} dy)$. D) $df = y^x (\ln y dx + \frac{x}{y} dy)$.

E) $df = y^x (\ln y dy - \frac{x}{y} dx)$.

12-§. BOSHLANG'ICH FUNKSIYA VA ANIQMAS INTEGRALNING TA'RIFI, XOSSALARI. INTEGRALLASH USULLARI

12.1. Boshlang'ich funksiya. Aniqmas integral

1. Agar $[a, b]$ kesmaning istalgan ichki nuqtasida $F(x)$ funksiyaning hosilasi $f(x)$ ga teng bo'lsa, bu $F(x)$ funksiya $f(x)$ uchun boshlang'ich funksiya deyiladi.

$$F'(x) = f(x) \Rightarrow dF(x) = f(x)dx, \quad x \in [a, b].$$

Boshlang'ich funksiyaning uning hosilasi $f(x)$ yoki differensial $f(x)dx$ bo'yicha izlash differensiallashga teskari amaldir, bu amal integrallash deyiladi. $f(x)$ funksiya yoki $f(x)dx$ differensialning boshlang'ich funksiyalari to'plami aniqmas integral deyiladi va $\int f(x)dx$ simvol bilan belgilanadi. Shunday qilib, agar $d[F(x) + C] = f(x)dx$ bo'lsa,

$$\int f(x)dx = F(x) + C \quad (12.1)$$

bo'ladi.

Bu yerda $f(x)$ integral ostidagi funksiya; $f(x)dx$ - integral ostidagi ifoda; C - integrallash o'zgarmasi, x -integrallash o'zgaruvchisi, $\int$ - integral belgisi.

12.2. Aniqmas integralning asosiy xossalari:

a) funksiya differensialining aniqmas integrali shu funksiya bilan ixtiyoriy o'zgarmaning yig'indisiga teng:

$$\int dF(x) = F(x) + C$$

b) Aniqmas integralning differensial $f(x)dx$ integral ostidagi ifodaga, aniqmas integralning hosilasi esa integral ostidagi funksiya ga teng:

$$d\left(\int f(x)dx\right) = f(x)dx, \quad \left(\int f(x)dx\right)' = f(x) \quad (12.2)$$

c) Funksiyalar algebraik yig'indisining (ayirmasining) aniqmas integrali bu funksiyalar aniqmas integrallarining yig'indisiga (ayirmasiga) teng:

$$\int (f(x) \pm g(x))dx = \int f(x)dx \pm \int g(x)dx \quad (12.3)$$

d) o'zgarma ko'paytuvchi aniqmas integral belgisidan tashqariga chiqarish mumkin ($a = const$):

$$\int af(x)dx = a \int f(x)dx, \quad (a = const) \quad (12.4)$$

e) agar $\int f(x)dx = F(x) + C$ bo'lib, $u = \varphi(x)$ uzluksiz hosilaga ega bo'lgan istalgan ma'lum funksiya bo'lsa,

$$\int f(u)du = F(u) + C \quad (12.5)$$

bo'ladi.

3. Integrallashning asosiy formulalari.

$$\int 0 dx = C \quad (12.6)$$

$$\int dx = x + C \quad (12.7)$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C; \quad (n \neq -1) \quad (12.8)$$

$$\int \frac{dx}{x} = \ln|x| + C \quad (12.9)$$

$$\int a^x dx = \frac{a^x}{\ln a} + C; \quad a > 0; \quad a \neq 1 \quad (12.10)$$

$$\int e^x dx = e^x + C \quad (12.11)$$

$$\int \sin x dx = -\cos x + C \quad (12.12)$$

$$\int \cos x dx = \sin x + C \quad (12.13)$$

$$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + C \quad (12.14)$$

$$\int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + C \quad (12.15)$$

$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C; \quad -a < x < a; \quad a > 0 \quad (12.16)$$

$$\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C \quad (12.17)$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C \quad (12.18)$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + C \quad (12.19)$$

$$\int \frac{dx}{\sin x} = \ln \left| \operatorname{tg} \frac{x}{2} \right| + C \quad (12.20)$$

$$\int \frac{dx}{\cos x} = \ln \left| \operatorname{tg} \left(\frac{x}{2} + \frac{\pi}{4} \right) \right| + C \quad (12.21)$$

$$\int \operatorname{sh} x dx = \operatorname{ch} x + C \quad (12.22)$$

$$\int \frac{dx}{\operatorname{ch}^2 x} = \operatorname{th} x + C \quad (12.23)$$

$$\int \frac{dx}{\operatorname{sh}^2 x} = -\operatorname{cth} x + C \quad (12.24)$$

Yoyish yo'li bilan integrallash berilgan integralni sodda integrallarning yig'indisiga keltirishdan iboratdir.

12.3. Integrallash usullari

Bevosita integrallash. Bevosita integrallash jadval integrallaridan bevosita foydalanishga asoslangandir. Bu yerda quyidagi hollar ro'y berishi mumkin:

- a) berilgan integral tegishli jadval integrali yordamida topiladi;
- b) berilgan integralga (12.3) va (12.4) xossalarni qo'llanilgandan so'ng bir yoki bir necha jadval integraliga keltiriladi;
- v) berilgan integral integral osti funksiya ustida elementar shakl almashtirishdan so'ng (12.3) va (12.4) xossalar qo'llanilgandan bir yoki bir nechta jadval integraliga keltiriladi.

1-Misol. Integralni toping:

$$\int \frac{7x - 5x^2 + 1}{x^2} dx$$

Yechimi: Suratni maxrajga bo'lib, keyin (12.3) va (12.4) xossalarni qo'llab, integral ostidagi funksiyani almashtiramiz va integrallar jadvalidan foydalanamiz:

$$\int \frac{7x - 5x^2 + 1}{x^2} dx = \int \left(\frac{7}{x} - 5 + \frac{1}{x^2} \right) dx = 7 \int \frac{dx}{x} - 5 \int dx + \int \frac{dx}{x^2} = 7 \ln|x| - 5x - \frac{1}{x} + c.$$

Differensial belgisi ostiga kiritish usuli. Differensial belgisi ostiga kiritish usuli integral ostidagi ifodani almashtirishdan iborat.

$$2\text{-Misol } 1) \int (x+4)^5 dx = \int (x+4)^5 d(x+4) = \frac{(x+4)^6}{6} + c.$$

Bu yerda $dx = d(x+4)$ ligidan foydalandik.

$$2) \int \frac{x dx}{1+x^2} = \frac{1}{2} \int \frac{2x dx}{1+x^2} = \frac{1}{2} \int \frac{d(1+x^2)}{1+x^2} = \frac{1}{2} \ln(1+x^2) + c.$$

Bu yerda oldin integralni 2 songa ko'paytirdik va ayni shu paytda uni 2 songa bo'ldik. Undan keyin $2x dx = d(1+x^2)$ ekanligidan foydalandi.

$$3) \int \frac{dx}{4x-11} = \frac{1}{4} \int \frac{4 dx}{4x-11} = \frac{1}{4} \int \frac{d(4x-11)}{4x-11} = \ln|4x-11| + c.$$

O'zgaruvchini almashtirish (o'rniga qo'yish) usuli bilan integrallashning mohiyati shundan iboratki, $\int f(x) dx$ integralni asosiy integrallash formulalarining birortasi oson integrallanadigan $\int F(u) du$ integralga keltirishdan iboratdir. Faraz qilamiz, $x = \varphi(u)$ o'rganiladigan oraliqda uzluksiz, differensiullanuvchi funksiya bo'lsin, u holda

$$\int f(x) dx = \int f(\varphi(u)) \varphi'(u) du = \int F(u) du. \quad (12.25)$$

Yangi o'zgaruvchi u ga nisbatan integral topilgandan so'ng $u = \varphi(x)$ o'rniga qo'yish yordamida uni x o'zgaruvchiga keltiriladi.

Bo'laklab integrallash. $d(u\vartheta) = u d\vartheta + \vartheta du$ tenglikning ikkala tomonini integrallab, quyidagini hosil qilamiz:

$$\int d(u\vartheta) = \int u d\vartheta + \int \vartheta du, \quad u\vartheta = \int u d\vartheta + \int \vartheta du$$

bu yerdan

$$\int u d\vartheta = u\vartheta - \int \vartheta du \quad (12.26)$$

Mustaqil bajarish uchun misollar

12.1-12.10 gacha. Ushbu aniqmas integrallarni topib, to'g'riligini differensiallash yordamida tekshiring.

$$12.1. \int \left(3x^4 + \frac{2}{x^3} - \sqrt[4]{x^3} \right) dx.$$

$$12.2. \int \left(5x^5 + 2\sqrt[3]{x^2} - \frac{8}{x^2} \right) dx.$$

$$12.3. \int \left(4x^2 + 3\sqrt{x} - \frac{3}{\sqrt{x}} \right) dx.$$

$$12.4. \int \left(3x^2 + 4\sqrt{x} - \frac{4}{\sqrt[3]{x}} \right) dx.$$

$$12.5. \int \left(7x^6 - 3\sqrt[3]{x} + \frac{5}{\sqrt[3]{x^2}} \right) dx.$$

$$12.6. \int \left(1 - 5\sqrt[4]{x^3} + \frac{3}{\sqrt[6]{x}} \right) dx.$$

$$12.7. \int \left(\sqrt[3]{x^2} + \frac{2x^3}{3} + \frac{1}{x^3} \right) dx.$$

$$12.8. \int \left(1 + 2\sqrt{x} - \frac{3x^4}{5} \right) dx.$$

$$12.9. \int \left(\sqrt{x} - \frac{2}{7}x^6 + \frac{8}{x^4} \right) dx.$$

$$12.10. \int \left(2\sqrt[6]{x} - \frac{5x^6}{7} + \frac{3}{\sqrt[6]{x}} \right) dx.$$

12.11-12.30 gacha. Aniqmas integrallarni toping.

$$12.11 \int \sqrt{\cos x} \sin x dx.$$

$$12.12 \int \frac{\sin x}{\cos^2 x} dx.$$

$$12.13 \int (\ln x)^3 \frac{dx}{x}.$$

$$12.14 \int \frac{x^2}{2x^3 + 3} dx.$$

$$12.15 \int \frac{\arctg x}{1+x^2} dx.$$

$$12.16 \int \sqrt{5x^4 + 3} x^3 dx.$$

$$12.17 \int \frac{\cos x}{\sqrt[3]{\sin x}} dx.$$

$$12.18 \int x^2 e^{x^3+1} dx.$$

$$12.19 \int e^{-x^2} x dx.$$

$$12.20 \int \frac{x^3}{\sqrt{8x^4 - 1}} dx.$$

$$12.21 \int \frac{x}{2+x^4} dx.$$

$$12.22 \int \frac{x}{2x^2 + 3} dx.$$

$$12.23 \int \sqrt{\ln x} \cdot \frac{dx}{x}.$$

$$12.24 \int \arcsin^2 x \frac{dx}{\sqrt{1-x^2}}.$$

$$12.25 \int \frac{x}{\sqrt{1-2x^2}} dx.$$

$$12.26 \int \frac{\sqrt{\arctg x}}{1+x^2} dx.$$

$$12.27 \int \frac{x}{2x^4 + 5} dx. \quad 12.28 \int \frac{\ln x + 3}{x} dx.$$

$$12.29 \int \frac{dx}{x \ln x}.$$

$$12.30 \int \sqrt{1+2x^2} x dx.$$

12.31-12.60 gacha. Integrallarni toping.

12.31 a) $\int \frac{dx}{\sqrt{25-9x^2}}$; b) $\int \frac{dx}{\sqrt{x+4}+\sqrt{x-1}}$; c) $\int \frac{(2x^2-3)dx}{x^2(x^2-3)}$;

d) $\int \frac{dx}{\sqrt[3]{x^2} \cos^2 \sqrt[3]{x}}$; e) $\int \frac{x^4 dx}{x^2+2}$.

12.32 a) $\int \frac{x^3-5x^2+1}{\sqrt{x}} dx$;

b) $\int \left(\sqrt{t^3} + \frac{1}{\sqrt[3]{t^2}} \right) dt$.

12.33 $\int \sqrt{x^3} (5\sqrt[3]{x^2}-1) dx$.

12.34. $\int \frac{d\varphi}{\sqrt{16-\varphi^2}}$.

12.35 $\int \left(y\sqrt{y} - \frac{1}{\sqrt{y}} + \frac{1}{(y+2)^2} \right) dy$.

12.36. $\int \frac{x-16}{\sqrt{x}+4} dx$.

12.37. $\int \frac{dx}{\sqrt{9-4x^2}}$.

12.38. $\int \frac{dx}{\sqrt{x^2+5}}$.

12.39. $\int \frac{dx}{16x^2-9}$

12.40. $\int \frac{2^{x-1}+3^{x+1}}{6^x} dx$

12.41. $\int 5^{x-1} e^x dx$

12.42. $\int (3^x-2)(3^{-x}+2) dx$

12.43. $\int \frac{dx}{\sqrt{x+2}-\sqrt{x-1}}$

12.44. $\int \frac{\sqrt{2-x^2}+\sqrt{2+x^2}}{\sqrt{4-x^4}} dx$

12.45. $\int \frac{\sqrt{x^2+4}-6}{x^2+4} dx$

12.46. $\int \frac{x^4 dx}{x^2+1}$

12.47. $\int \frac{x^3-1}{x+1} dx$

12.48. $\int \sin^2 \frac{x}{2} dx$

12.49. $\int \operatorname{ctg}^2 x dx$

12.50. $\int \operatorname{cth}^2 x dx$

12.51. $\int \frac{5-\cos x}{\cos x} dx$

12.52. $\int \frac{e^{2x}-4}{e^x+2} dx$

12.53. $\int \frac{dx}{\sin^2 x + \cos 2x}$

12.54. $\int \frac{1-5x^2}{x^2(1-x^2)} dx$

12.55. $\int \frac{8-\sqrt{x}}{\sqrt[3]{x}+2\sqrt[6]{x}+4} dx$

12.56. $\int e^{-x}(e^x+5) dx$

12.57. $\int (\sqrt{x}-1)(x+\sqrt{x}+1) dx$

12.58. $\int (3\operatorname{ctg} x-1)(3\operatorname{tg} x+1) dx$

12.59. $\int \frac{1+2\cos^2 x}{1+\cos 2x} dx$

12.60. $\int \sqrt{x}\sqrt{x}\sqrt{x} dx$

12.61-12.111 gacha. O'zgaruvchilarni almashtirish usuli yordamida integrallarni toping.

12.61 a) $\int \frac{5x^4-6}{x^5-6x} dx$; b) $\int \frac{\operatorname{arctg}^3 x}{1+x^2} dx$; c) $\int \frac{x^2 dx}{\sqrt{1+x^3}}$;

d) $\int \frac{\operatorname{arctg}^3 x}{1+x^2} dx$; e) $\int \frac{\ln(1+\sqrt{x})}{x+\sqrt{x}} dx$.

- 12.62. a) $\int \left(\sin \frac{\pi}{20} x - \cos \frac{x}{20} \right) dx$; б) $\int (11x-7)^7 dx$.
- 12.63. $\int \frac{d(\arctg x)}{\arctg^2 x}$. 12.64. $\int \frac{dx}{\sin^2 x (1 + \operatorname{ctg} x)^{3/2}}$. 12.65. $\int \sqrt{3x+5} dx$.
- 12.66. $\int \frac{dx}{(2x-3)^2}$. 12.67. $\int 4^{3x+7} dx$. 12.68. $\int \pi^{2x-1} dx$.
- 12.69. $\int \frac{3 \operatorname{ctg}^2 x - 1}{\cos^2 x} dx$. 12.70. $\int \frac{dx}{x(9 + \ln^2 x)}$. 12.71. $\int \frac{x dx}{\sqrt{x^2 - 5}}$.
- 12.72. $\int 2^{x^3+1} x^4 dx$. 12.73. $\int \frac{dx}{\sqrt{1-x^2} \arcsin x}$. 12.74. $\int e^x \cos e^x dx$.
- 12.75. $\int \sec^2(1 + \ln^2 x) \ln(1 + \ln^2 x)$. 12.76. $\int \frac{\pi^{1/x^2} dx}{x^3}$. 12.77. $\int \frac{2x^3 + x}{x^4 + x^2} dx$.
- 12.78. $\int \frac{\sqrt{\ln z}}{z} dz$. 12.79. $\int \frac{e^{3x} dx}{\sqrt{1 + e^{6x}}}$. 12.80. $\int \frac{5 \cos x - \operatorname{ctg} x}{\sin^2 x} dx$.
- 12.81. $\int x^{-2} e^{-1/x} dx$. 12.82. $\int \frac{u du}{\cos^2 u^2}$. 12.83. $\int \frac{5^{\sqrt{x}}}{\sqrt{x}} dx$.
- 12.84. $\int \frac{dx}{25 \cos^2 x + 9 \sin^2 x}$. 12.85. $\int 5^{\sin^2 x} \sin 2x dx$.
- 12.86. $\int \frac{dx}{\sqrt{1-x^2} \sqrt{\arccos^2 x - 4}}$.
- 12.87. $\int \frac{\sin x dx}{\cos^2 x \sqrt{\cos x}}$. 12.88. $\int \frac{dx}{\sqrt[3]{\operatorname{tg}^5 x \sin^2 x}}$. 12.89. $\int \frac{dx}{(x^2+1) \sqrt{\arctg x + 1}}$.
- 12.90. $\int e^{5 \sin x - 2} \cos x dx$. 12.91. $\int \frac{3^x dx}{7 + 3^{2x}}$. 12.92. $\int \frac{dx}{e^{2x} + 5}$.
- 12.93. $\int \frac{(x-1) dx}{\sqrt{2x-x^2}}$. 12.94. $\int \sqrt{2-3 \sin 2x} \cos 2x dx$. 12.95. $\int \frac{e^{5x} dx}{(e^{5x} + 11)^2}$.
- 12.96. $\int \frac{x^2 dx}{\sin^2 x^3}$. 12.97. $\int \frac{dx}{x \sqrt{9 - \ln^2 2x}}$. 12.98. $\int \frac{t^2 dt}{t^6 - 1}$.
- 12.99. $\int \frac{\sin 2x dx}{1 + \cos^2 2x}$. 12.100. $\int \frac{x dx}{16 + x^4}$. 12.101. $\int \frac{x dx}{1 - x^4}$.
- 12.102. $\int \frac{dx}{\sqrt{x}(1+x)}$. 12.103. $\int \cos^2 \left(\sin \frac{x}{2} \right) \cos \frac{x}{2} dx$. 12.104. $\int \ln 2x dx$.
- 12.105. $\int x \operatorname{tg}(x^2 - 5) dx$. 12.106. $\int \frac{x^2 e^{\sqrt{x^3+1}}}{\sqrt{x^3+1}} dx$. 12.107. $\int \left[\cos \left(3x - \frac{\pi}{6} \right) \right]^{-2} dx$.
- 12.108. $\int \frac{x dx}{(x^2+2) \sqrt{x^2+2}}$. 12.109. $\int \frac{\sin 2x dx}{\sin^2 x + 5}$.
- 12.110. $\int (1-x)^{99} x dx$. 12.111. $\int x^2 (1+x^3)^4 dx$.

12.112-12.138 gacha. Bo'laklab integrallash usuli yordamida toping.

12.112. a) $\int (x+3)\sin x dx$; b) $\int x^2 \arctg x dx$; c) $\int \arcsin x dx$;

d) $\int (x^2 + 3x)e^x dx$; e) $\int e^{2x} \sin 3x dx$.

12.113. a) $\int x \cos 2x dx$; b) $\int \arcsin 2x dx$.

12.114. $\int \ln x dx$.

12.115. $\int x \arctg x^2 dx$.

12.116. $\int \sqrt{x} \ln \sqrt{x} dx$.

12.117. $\int x \ln(x^2 + 4) dx$.

12.118. $\int x^3 \arctg x dx$.

12.119. $\int \frac{\ln x}{x^2} dx$.

12.120. $\int \frac{x \arcsin x}{\sqrt{1-x^2}} dx$.

12.121. $\int \frac{x \cos x}{\sin^3 x} dx$.

12.122. $\ln(x^2 + 1) dx$.

12.123. $\int x^2 5^x dx$.

12.124. $\int x^{99} (x \cos x + 100 \sin x) dx$.

12.125. $\int x^{3/2} \cos \sqrt{x} \frac{dx}{\sqrt{x}}$.

12.126. $\int \frac{\ln^3 x}{\sqrt{x}} dx$.

12.127. $\int x^{1/3} \ln^2 x dx$.

12.128. $\int \sec^2 x \ln(\sec x) dx$.

12.129. $\int x \operatorname{ch} x dx$.

12.130. $\int (3x^2 - 8x)e^{2x} dx$.

12.131. $\int (x^2 + 3x + 5)e^{3x+1} dx$.

12.132. $\int (x+2)\cos(3x+7) dx$.

12.133. $\int (1+x)^2 \sin x dx$.

12.134. $\int (3x^2 + 6x + 5) \arctg x dx$.

12.135. $\int (x-2)\arcsin x dx$.

12.136. $\int e^{3x} \cos 2x dx$.

12.137. $\int (1-3x-x^2)\ln x dx$.

12.138. $\int (x^3 - x - 1)\ln^2 x dx$.

Mavzu bo'yicha nazorat testlari

1. Boshlang'ich funksiya va aniqlamas integral.

1. Quyidagi shartlarning qaysi birida $F(x)$ berilgan $f(x)$ funksiyaning boshlang'ich funksiyasi deyiladi?

A) $F(x) = f(x) + C$ ($C = \text{const}$).

B) $\lim_{t \rightarrow x} F(t) = f(x)$.

C) $F'(x) = f(x)$.

D) $F''(x) = f(x)$.

E) $F(x) = f'(x)$.

2. Quyidagilardan qaysi biri $f(x) = \ln x$ uchun boshlang'ich funksiya bo'ladi?

A) $\frac{1}{x}$.

B) $x \ln x$.

C) $x \ln x + x$.

D) $x \ln x - x$.

E) $\frac{1}{x} \ln x - x$.

3. Quyidagilardan qaysi birining boshlang'ich funksiyasi $F(x) = x \cos x$ bo'ladi?

A) $x^2 \sin x$. B) $-\sin x$. C) $\cos x - x \sin x$.

D) $\sin x + x \cos x$. E) $x(\sin x + \cos x)$.

4. Teoremani to'ldiring: Agar $F(x)$ biror $f(x)$ funksiya uchun boshlang'ich funksiya bo'lsa, unda ixtiyoriy C o'zgarmas soni uchun ... funksiya ham $f(x)$ uchun boshlang'ich funksiya bo'ladi.

A) $C \cdot F(x)$. B) $C - F(x)$. C) $C + F(x)$.

D) $C/F(x)$. E) $F(x+C)$.

5. Agar $F(x)$ biror $f(x)$ funksiya uchun boshlang'ich funksiya bo'lsa, unda ta'rif bo'yicha $\int f(x)dx$ aniqmas integral qanday aniqlanadi?

A) $C \cdot F(x)$. B) $C - F(x)$. C) $C + F(x)$.

D) $C/F(x)$. E) $F(x+C)$.

6. Aniqmas integralning geometrik ma'nosi qayerda to'g'ri va to'liq ko'rsatilgan?

A) Qandaydir to'g'ri chiziq.

B) Qandaydir egri chiziq.

C) Qandaydir chiziqlar sinfi.

D) OX o'qi bo'yicha o'zaro parallel chiziqlar sinfi.

E) OY o'qi bo'yicha o'zaro parallel chiziqlar sinfi.

7. Qayerda aniqmas integralning xossasi xato ko'rsatilgan?

A) $\left(\int f(x)dx \right)' = f(x)$. B) $d\left(\int f(x)dx \right) = f(x)dx$. C) $\int dF(x) = F(x) + C$.

D) $\int F'(x)dx = F(x) + C$. E) Barcha xossalar to'g'ri ko'rsatilgan.

8. Aniqmas integral uchun qaysi tenglik bajarilmaydi?

A) $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$.

B) $\int f(x)g(x)dx = \int f(x)dx \cdot \int g(x)dx$.

C) $\int [f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$.

D) $\int kf(x)dx = k \int f(x)dx$ ($k - \text{const}$).

E) keltirilgan barcha tengliklar bajariladi.

9. Quyidagi tengliklardan qaysi biri integralning chiziqlilik xossasini ifodalamaydi?

A) $\int [f(x) + g(x)]dx = \int f(x)dx + \int g(x)dx$.

B) $\int kf(x)dx = k \int f(x)dx$ ($k - \text{const}$).

C) $\int [f(x) - g(x)]dx = \int f(x)dx - \int g(x)dx$.

D) $\int [A \cdot f(x) + B \cdot g(x)]dx = A \int f(x)dx + B \int g(x)dx$.

E) Barcha tengliklar integralni chiziqlilik xossasini ifodalaydi.

10. Qaysi darajali funksiya uchun aniqmas integrali noto'g'ri yozilgan?

$$\begin{aligned} \text{A)} \int \sqrt{x} dx &= \frac{2x\sqrt{x}}{3} + C. & \text{B)} \int \frac{1}{x^2} dx &= -\frac{1}{x} + C. & \text{C)} \int \frac{1}{\sqrt{x}} dx &= 2\sqrt{x} + C. \\ \text{D)} \int \sqrt[3]{x} dx &= \frac{3x\sqrt[3]{x}}{4} + C. & \text{E)} \int \frac{1}{x} dx &= -\frac{1}{x^2} + C. \end{aligned}$$

11. Integrallar jadvalidan keltirilgan quyidagi tengliklardan qaysi biri to'g'ri yozilgan?

$$\begin{aligned} \text{A)} \int \frac{dx}{\cos^2 x} &= \operatorname{ctg} x + C. & \text{B)} \int \frac{dx}{\sin^2 x} &= -\operatorname{tg} x + C. & \text{C)} \int e^x dx &= \frac{e^x}{\lg e} + C. \\ \text{D)} \int \frac{dx}{1+x^2} &= \arcsin x + C. & \text{E)} \int \frac{dx}{x^2-a^2} &= \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + C. \end{aligned}$$

12. $\int \cos(10x+7) dx$ aniqmas integral javobi qayerda to'g'ri ifodalangan?

$$\begin{aligned} \text{A)} \sin(10x+7)+C. & \quad \text{B)} 10\sin(10x+7)+C. & \text{C)} 7\sin(10x+7)+C. \\ \text{D)} 10^{-1}\sin(10x+7)+C. & \quad \text{E)} 7^{-1}\sin(10x+7)+C. \end{aligned}$$

13. $\int (3x+5)^4 dx$ integralni hisoblang.

$$\begin{aligned} \text{A)} C+(3x+5)^5/5. & \quad \text{B)} C+(3x+5)^5/15. & \text{C)} C+(3x+5)^5/25. \\ \text{D)} C+3(3x+5)^5/5. & \quad \text{E)} C+3(3x+5)^5/25. \end{aligned}$$

14. $\int 4\cos^2 x dx$ integralni hisoblang.

$$\begin{aligned} \text{A)} 2\sin^2 x + C. & \quad \text{B)} -2\sin 2x + C. & \text{C)} 2\cos 2x + C. \\ \text{D)} 2x - \cos 2x + C. & \quad \text{E)} 2x + \sin 2x + C. \end{aligned}$$

2. Aniqmas integralni hisoblash usullari.

1. Aniqmas integralni hisoblashning qaysi usuli mavjud emas?

- A) ko'paytirish usuli. B) o'zgaruvchini almashtirish usuli.
C) differensial ostiga kiritish usuli. D) yoyish usuli.
E) bo'laklab integrallash usuli.

2. Qaysi tenglik aniqmas integralni yoyish usulida hisoblashni ifodalaydi?

$$\begin{aligned} \text{A)} \int f(x) dx &= \int u dv = uv - \int v du. & \text{B)} \int f(x) dx &= \int f(\varphi(t)) \varphi'(t) dt. \\ \text{C)} \int f(x) dx &= \int \sum_{k=1}^n a_k f_k(x) dx = \sum_{k=1}^n a_k \int f_k(x) dx. \\ \text{D)} \int f(ax+b) dx &= \frac{1}{a} F(ax+b) + C. & \text{E)} \int f(x) dx &= \int f[\varphi(t)] \varphi'(t) dt. \end{aligned}$$

3. $\int \frac{2x^2-3x+5}{x^2} dx$ integralni yoyish usulida hisoblang.

$$\begin{aligned} \text{A)} 2 - \frac{3}{x} + \frac{5}{x^2} + C. & \quad \text{B)} 2x - 3\ln|x| + \frac{5}{x^2} + C. & \text{C)} 2x - \frac{3}{x} - \frac{5}{3x^3} + C. \end{aligned}$$

$$D) 2x - 3\ln|x| - \frac{5}{x} + C. \quad E) 2x + \frac{3}{x^2} - \frac{5}{x} + C.$$

4. Quyidagi integrallardan qaysi biriga differensial ostiga kiritish usulini qo'llab bo'lmaydi?

$$A) \int f(x)f'(x)dx. \quad B) \int \frac{f'(x)dx}{f(x)}.$$

$$C) \int [f(x) \pm f'(x)]dx. \quad D) \int \sqrt{f(x)}f'(x)dx.$$

E) barcha integrallarga differensial ostiga kiritish usulini qo'llab bo'ladi.

5. Quyidagi integrallardan qaysi biri differensial ostiga kiritish usulida xato hisoblangan?

$$A) \int f(x)f'(x)dx = \frac{1}{2}f^2(x) + C. \quad B) \int \cos[f(x)]f'(x)dx = \sin[f(x)] + C.$$

$$C) \int \sin[f(x)]f'(x)dx = \cos[f(x)] + C. \quad D) \int \frac{f'(x)dx}{f(x)} = \ln|f(x)| + C.$$

E) keltirilgan barcha integrallar to'g'ri hisoblangan.

6. Qaysi tenglik aniqmas integralni o'zgaruvchilarni almashtirish usulida hisoblashni ifodalaydi?

$$A) \int f(x)dx = \int u dv = uv - \int v du. \quad B) \int f(x)dx = \int f(\varphi(t))\varphi'(t)dt.$$

$$C) \int f(x)dx = \int \sum_{k=1}^n a_k f_k(x)dx = \sum_{k=1}^n a_k \int f_k(x)dx.$$

$$D) \int f(ax+b)dx = \frac{1}{a}F(ax+b) + C. \quad E) \int f(x)dx = \int f[\varphi(t)]\varphi'(t)dt.$$

7. $\int f(x)dx$ aniqmas integralda $x=\varphi(t)$ almashtirma bajarilganda u qanday ko'rinishga keladi?

$$A) \int f[\varphi(t)]\varphi(t)dt. \quad B) \int f[\varphi'(t)]\varphi'(t)dt. \quad C) \int f[\varphi(t)]dt.$$

$$D) \int f[\varphi(t)]\varphi'(t)dt. \quad E) \int f[\varphi'(t)]\varphi(t)dt.$$

8. $\int \frac{x^4 dx}{\sqrt{x^{10}-1}}$ integral qaysi almashtirma orqali jadval integraliga keltiriladi? A) $t=x^2$. B) $t=x^3$. C) $t=x^4$. D) $t=x^5$. E) $t=x^6$.

9. $\int \frac{\sin x dx}{\sqrt{\cos x}}$ integral qaysi almashtirma orqali jadval integraliga keltiriladi?

$$A) t=\sin x. \quad B) t=\cos x. \quad C) t=\sqrt{\cos x}. \quad D) t=\operatorname{tg} x. \quad E) t=\operatorname{ctg} x.$$

10. Qaysi tenglik bo'laklab integrallash usulini ifodalaydi?

$$A) \int f(x)dx = \int u dv = uv - \int v du. \quad B) \int f(x)dx = \int f(\varphi(t))\varphi'(t)dt.$$

$$C) \int f(x)dx = \int \sum_{k=1}^n a_k f_k(x)dx = \sum_{k=1}^n a_k \int f_k(x)dx.$$

$$D) \int f(ax+b)dx = \frac{1}{a} F(ax+b) + C. \quad E) \int f(x)dx = \int f[\varphi(t)]\varphi'(t)dt.$$

11. Aniqmas integralni bo'laklab integrallash formulasini ko'rsating.

$$A) \int u dv = \int v du. \quad B) \int u dv = u - \int v du. \quad C) \int u dv = uv - \int v du.$$

$$D) \int u dv = uv + \int v du. \quad E) \int u dv = u + \int v du.$$

12. $\int x^2 \ln x dx$ integralni hisoblash uchun integral ostidagi ifodani qanday bo'laklash maqsadga muvofiq bo'ladi?

$$A) u=x, dv=x \ln x dx. \quad B) u=x^2, dv=\ln x dx. \quad C) u=\ln x, dv=x^2 dx.$$

$$D) u=x \ln x, dv=x dx. \quad E) u=x^2 \ln x, dv=dx.$$

13. Ushbu integrallardan qaysi birini bo'laklab integrallash usulida hisoblab bo'lmaydi?

$$A) \int x^n a^x dx. \quad B) \int x^n \ln x dx. \quad C) \int x^n \sin x dx. \quad D) \int x^n \cos x dx.$$

E) keltirilgan barcha integrallarni bo'laklab integrallash mumkin.

14. $\int \frac{dx}{x^2 + 4x - 5}$ integralni hisoblang.

$$A) \frac{1}{6} \ln \left| \frac{x-1}{x+5} \right| + C. \quad B) \frac{1}{6} \ln \left| \frac{x+5}{x-1} \right| + C. \quad C) \frac{1}{6} \ln \left| \frac{x+5}{x+1} \right| + C.$$

$$D) \frac{1}{6} \ln \left| \frac{x-5}{x-1} \right| + C. \quad E) \ln |x^2 + 4x - 5| + C.$$

13-§. KVADRAT UCHHAD QATNASHGAN BA'ZI FUNKSIYALARNI INTEGRALLASH. RATSIONAL FUNKSIYALARNI INTEGRALLASH.

13.1. Eng sodda ratsional kasrlarni integrallash

Quyidagi ratsional kasrlarni eng sodda ratsional kasrlar deb aytamiz:

$$\text{I. } \frac{A}{x-a}, \quad \text{II. } \frac{A}{(x-a)^k}, \quad \text{III. } \frac{Ax+B}{x^2+px+q}, \quad \text{IV. } \frac{Ax+B}{(x^2+px+q)^n}$$

I va II turdagi oddiy kasrlarni integrallash jadval integrallariga oson keltiriladi:

$$\text{I. } \int \frac{A dx}{x-a} = A \int \frac{d(x-a)}{x-a} = A \ln|x-a| + C$$

$$\begin{aligned} \text{II. } \int \frac{A dx}{(x-a)^k} &= A \int (x-a)^{-k} d(x-a) = \\ &= A \frac{(x-a)^{-k+1}}{-k+1} + C = \frac{A}{(1-k)(x-a)^{k-1}} + C \end{aligned}$$

III. turdagi oddiy kasrning integralini qaraymiz: $\frac{p^2}{4} - q < 0$ bo'lsin,

unda

$$\begin{aligned} \int \frac{(Ax+B)dx}{x^2+px+q} &= \int \frac{\frac{A}{2}(2x+p) - \frac{Ap}{2} + B}{x^2+px+q} dx = \\ &= \frac{A}{2} \int \frac{(2x+p)dx}{x^2+px+q} + (B - \frac{Ap}{2}) \int \frac{dx}{x^2+px+q} = \\ &= \left[\frac{x^2+px+q}{(2x+p)dx} = dt \right] = \frac{A}{2} \int \frac{dt}{t} + (B - \frac{Ap}{2}) \int \frac{dx}{x^2+px+q} = \\ &= \frac{A}{2} \ln|x^2+px+q| + (B - \frac{Ap}{2}) \int \frac{d(x+\frac{p}{2})}{(x+\frac{p}{2})^2 + q - \frac{p^2}{4}} = \\ &= \frac{A}{2} \ln|x^2+px+q| - (B - \frac{Ap}{2}) \cdot \frac{1}{\sqrt{q - \frac{p^2}{4}}} \arctg \frac{x + \frac{p}{2}}{\sqrt{q - \frac{p^2}{4}}} + C \end{aligned}$$

Endi IV turdagi oddiy kasrning integralini hisoblaymiz.

$$\text{IV. } \int \frac{(Ax+B)dx}{(x^2+px+q)^n} = \int \frac{\frac{A}{2}(2x+p) + B - \frac{Ap}{2}}{(x^2+px+q)^n} dx = \frac{A}{2} \int \frac{(2x+p)dx}{(x^2+px+q)^n} +$$

$$+ (B - \frac{Ap}{2}) \int \frac{d(x + \frac{p}{2})}{((x + \frac{p}{2})^2 + q - \frac{p^2}{4})^n} = \frac{A}{2} I + (B - \frac{Ap}{2}) \cdot \bar{I}$$

Bu yerda

$$I = \int \frac{(2x + p)dx}{(x^2 + px + q)^n}, \quad \bar{I} = \int \frac{d(x + \frac{p}{2})}{((x + \frac{p}{2})^2 + q - \frac{p^2}{4})^n}$$

bo'ladi.

$$1) \quad I = \int \frac{(2x + p)dx}{(x^2 + px + q)^n} = \left[\frac{x^2 + px + q = t}{(2x + p)dx = dt} \right] = \int \frac{dt}{t^n} = \frac{1}{(1-n)(x^2 + px + q)^{n-1}} + C$$

$$2) \quad \bar{I} = \int \frac{d(x + \frac{p}{2})}{((x + \frac{p}{2})^2 + q - \frac{p^2}{4})^n} = \left[\begin{array}{l} x + \frac{p}{2} = t \\ dx = dt \\ q - \frac{p^2}{4} = a^2 \end{array} \right] =$$

$$= \int \frac{dt}{(t^2 + a^2)^n} = \frac{1}{a^2} \int \frac{t^2 + a^2 - t^2}{(t^2 + a^2)^n} dt =$$

$$= \frac{1}{a^2} \int \frac{dt}{(t^2 + a^2)^{n-1}} - \frac{1}{a^2} \int \frac{t^2 dt}{(t^2 + a^2)^n} \quad (*)$$

Oxirgi integralga bo'laklab integrallash formulasini qo'llaymiz:

$$\int \frac{t^2 dt}{(t^2 + a^2)^n} = \left[\begin{array}{l} u = t, dv = \frac{t dt}{(t^2 + a^2)^n} \\ du = dt, v = \frac{1}{2(1-n)(t^2 + a^2)^{n-1}} \end{array} \right] =$$

$$= \frac{t}{2(1-n)(t^2 + a^2)^{n-1}} + \frac{1}{2(n-1)} \int \frac{dt}{(t^2 + a^2)^{n-1}}$$

Agar

$$I_n = \int \frac{dt}{(t^2 + a^2)^n}$$

belgi kiritsak, u holda (*)dagi formula quyidagi ko'rinishni oladi:

$$I_n = \frac{t}{2(n-1)a^2(t^2 + a^2)^{n-1}} + \frac{2n-3}{2(n-1)a^2} I_{n-1}$$

yoki

$$I_n = \frac{1}{2(n-1)a^2} \cdot \left(\frac{t}{(t^2 + a^2)^{n-1}} + (2n-3)I_{n-1} \right) \quad (**)$$

Bu formula bo'yicha I_{n-1}, I_{n-2} orqali ifodalaymiz, so'ngra I_{n-2} ni I_{n-3} orqali ifodalaymiz va hokazo. Bu jarayon quyidagi integralni xosil qilgunchimizcha davom etadi:

$$I_1 = \int \frac{dt}{t^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{t}{a} + C$$

(**) formula keltirish yoki rekurent formula deyiladi.

misol $\int \frac{(x-1)dx}{(x^2 + 2x + 3)^2}$ integralni hisoblang.

Yechimi:

$$\int \frac{(x-1)dx}{(x^2 + 2x + 3)^2} =$$

$$\begin{aligned} & \int \frac{\frac{1}{2}(2x+2) - 2}{(x^2 + 2x + 3)^2} dx = \frac{1}{2} \int \frac{(2x+2)dx}{(x^2 + 2x + 3)^2} - 2 \int \frac{dx}{(x^2 + 2x + 3)^2} = \\ & = -\frac{1}{2} \frac{1}{(x^2 + 2x + 3)} - 2 \int \frac{dx}{(x^2 + 2x + 3)^2} = -\frac{1}{2} \frac{1}{(x^2 + 2x + 3)} - 2I_2. \quad (***) \end{aligned}$$

I_2 integralni hisoblaymiz:

$$\begin{aligned} I_2 &= \int \frac{dx}{(x^2 + 2x + 3)^2} = \int \frac{dx}{((x+1)^2 + 2)^2} = \left[\begin{matrix} x+1=t \\ dx=dt \end{matrix} \right] = \\ &= \int \frac{dt}{(t^2 + 2)^2} = \frac{1}{2} \int \frac{t^2 + 2 - t^2}{(t^2 + 2)^2} dt = \frac{1}{2} \int \frac{dt}{t^2 + 2} - \\ &- \frac{1}{2} \int \frac{t^2 dt}{(t^2 + 2)^2} = \frac{1}{2\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} - \frac{1}{2} \int \frac{t^2 dt}{(t^2 + 2)^2} \end{aligned}$$

Oxirgi integralni qaraymiz:

$$\begin{aligned} \int \frac{t^2 dt}{(t^2 + 2)^2} &= \frac{1}{2} \int \frac{td(t^2 + 2)}{(t^2 + 2)^2} = -\frac{1}{2} \int td\left(\frac{1}{t^2 + 2}\right) = \\ &= \left[\begin{matrix} u=t, dv=d\left(\frac{1}{t^2 + 2}\right) \\ du=dt, v=\frac{1}{t^2 + 2} \end{matrix} \right] = -\frac{1}{2} \frac{t}{t^2 + 2} + \frac{1}{2} \int \frac{dt}{t^2 + 2} = \\ &= -\frac{1}{2} \cdot \frac{t}{t^2 + 2} + \frac{1}{2 \cdot \sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + C = \frac{x+1}{2(x^2 + 2x + 3)} + \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{x+1}{\sqrt{2}} + C \end{aligned}$$

Demak,

$$I_2 = \frac{1}{2\sqrt{2}} \operatorname{arctg} \frac{x+1}{\sqrt{2}} - \frac{1}{2} \left[-\frac{x+1}{2(x^2+2x+3)} + \frac{1}{2\sqrt{2}} \operatorname{arctg} \frac{x+1}{\sqrt{2}} \right] + C$$

bo'ldi.

I_2 integral qiymatini (***) tenglikka qo'ysak, berilgan integralning qiymati kelib chiqadi:

$$\int \frac{(x-1)dx}{(x^2+2x+3)^2} = -\frac{x+2}{2(x^2+2x+3)} - \frac{\sqrt{2}}{4} \operatorname{arctg} \frac{x+1}{\sqrt{2}} + C$$

13.2. Ratsional kasrlarni eng sodda ratsional kasrlarga ajratish

Ma'lumki, $P_n(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots + a_{n-1}x + a_0x^n + a_n$

Funksiya darajali ko'phad deyiladi, bunda $a_0, a_1, a_2, \dots$, ko'phadning koeffitsiyentlari, n esa daraja kursatgichi.

1-ta'rif: Ikki ko'phad nisbati ratsional kasr deyiladi.

$$R(x) = \frac{Q_m(x)}{P_n(x)} = \frac{e_0x^m + e_1x^{m-1} + \dots + e_{m-1}x + e_m}{a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n} \quad (13.1)$$

Agar $m < n$ bo'lsa, u holda ratsional kasr to'g'ri, agar $m \geq n$ bo'lsa, u holda ratsional kasr noto'g'ri kasr deb aytiladi. $R(x)$ ratsional kasr noto'g'ri bo'lgan hollarda kasrning $Q(x)$ suratini $R_n(x)$ maxrajiga bo'lish yo'li bilan uning butun qismini ajratish kerak:

$$\frac{Q_m(x)}{P_n(x)} = q(x) + \frac{r(x)}{P_n(x)}$$

Bu yerda $\frac{r(x)}{P_n(x)}$ to'g'ri kasr, chunki $r(x)$ qoldiqning darajasi $R_n(x)$

ning darajasidan kichik.

1-misol Ushbu

$$R(x) = \frac{2x^4 - 3x^3 + 1}{x^2 + x - 2}$$

noto'g'ri ratsional kasrning butun qismini ajratish.

Yechimi:

$$2x^4 - 3x^3 + 1$$

$$\underline{x^2 + x - 2}$$

$$2x^4 + 2x^3 - 4x^2$$

$$-5x^3 + 4x^2 + 1$$

$$\underline{2x^2 - 5x + 9}$$

$$-5x^3 - 5x^2 + 10x$$

$$9x^2 - 10x + 1$$

$$\underline{9x^2 + 9x - 18}$$

$$-19x + 19$$

Demak,

$$\frac{2x^4 - 3x^3 + 1}{x^2 + x - 2} = 2x^2 - 5x + 9 + \frac{-19x + 19}{x^2 + x - 2}.$$

Ushbu $R(x) = \frac{Q_m(x)}{P_n(x)}$

to'g'ri ratsional kasrni qarab chiqamiz, bu kasrning $R_n(x)$ maxraji $(x - \alpha)^k, (x^2 + px + q)^s$

ko'rinishdagi chiziqli va kvadratik ko'paytuvchilarga yoyiladi, bunda $(x - \alpha)^k$ ko'rinishdagi ko'paytuvchi q karrali haqiqiy ildizga mos keladi, $(x^2 + px + q)^s$ ko'rinishdagi ko'paytuvchi s karralidagi kompleks ildizlarga mos keladi:

$$P_n(x) = a_0(x - \alpha_1)^{k_1} \cdot (x - \alpha_2)^{k_2} \cdot \dots \cdot (x - \alpha_r)^{k_r} \cdot (x^2 + p_1x + q_1)^{s_1} \cdot (x^2 + p_2x + q_2)^{s_2} \cdot \dots \cdot (x^2 + p_lx + q_l)^{s_l}. \quad (13.2)$$

1-teorema: Har qanday $R(x) = \frac{Q_m(x)}{P_n(x)}$ ratsional kasrni, bunda $P_n(x)$

(13.2) formula bo'yicha ko'paytuvchilarga ajratilgan, $\frac{A}{x - \alpha}, \frac{A}{(x - \alpha)^k},$

$(k \geq 2$ va butun), $\frac{Ax + B}{x^2 + px + q}, \frac{Ax + B}{(x^2 + px + q)^s}, (s \geq 2$ va butun) turdagi oddiy

kasrlarning yig'indisi ko'rinishida ifodalash mumkin. Bunda:

1) (13.2) yoyilmaning $(x - \alpha)$ ko'rinishdagi ko'paytuvchisiga bitta $\frac{A}{x - \alpha}$ kasr mos keladi;

2) (13.2) yoyilmaning $(x - \alpha)^k$ ko'rinishdagi ko'paytuvchisiga

$$\frac{A_1}{(x - \alpha)^k} + \frac{A_2}{(x - \alpha)^{k-1}} + \dots + \frac{A_k}{(x - \alpha)},$$

kasrlar yig'indisi mos keladi;

3) (13.2) yoyilmaning $x^2 + px + q$ ko'rinishdagi ko'paytuvchisiga bitta $\frac{Ax + B}{x^2 + px + q}$ kasr mos keladi;

4) (13.2) yoyilmaning $(x^2 + px + q)^s$ ko'rinishdagi ko'paytuvchisiga

$$\frac{A_1x + B_1}{(x^2 + px + q)^s} + \frac{A_2x + B_2}{(x^2 + px + q)^{s-1}} + \dots + \frac{A_sx + B_s}{(x^2 + px + q)}$$

kasrlar yig'indisi mos keladi;

Bu teoremani isbotsiz qabul qilamiz.

Yuqorida ko'rsatilgan ratsional kasrning oddiy kasrlar yig'indisi yoyilmasidagi $A, B, A_1, A_2, \dots, B_1, B_2, \dots$ koeffitsiyentlarni aniqlash uchun turli usullar mavjud. Ulardan bittasi noma'lum koeffitsiyentlar usulidir. Uni misolda ko'ramiz.

2-misol: Ushbu

$$R(x) = \frac{x+2}{x^3-3} \quad \text{ratsional kasrni oddiy kasrlar yigindisiga ajrating.}$$

Yechimi: Maxrajni ko'paytuvchilarga ajratamiz:

$$x^3 - x = x(x^2 - 1) = x(x-1)(x+1)$$

keltirilgan teorema asosan berilgan kasrni oddiy kasrlarga ajratish bunday ko'rinishda bo'lishi mumkin:

$$\frac{x+2}{x^3-x} = \frac{A}{x} + \frac{B}{x-1} + \frac{D}{x+1}$$

A, V, D noma'lum koeffitsiyentlarni topishga kirishamiz. Buning uchun oxirgi tenglikning o'ng qismini umumiy maxrajga keltiramiz va hosil qilingan tenglikning ikkala qismida maxrajni tashlab yuboramiz. Bu amallar natijasida quyidagi ayniyat kelib chiqadi:

$$x+2 \equiv A(x-1)(x+1) + Bx(x+1) + Dx(x-1),$$

$$x+2 \equiv (A+B+D)x^2 + (B-D)x - A$$

ko'phadlarning tengligidan quyidagi A, V, D larga nisbatan tenglamalar sistemasi hosil qilamiz va ildizlarini topamiz:

$$\begin{cases} A+B+D=0 \\ B-D=1 \\ -A=2 \end{cases} \Rightarrow \begin{cases} A=-2 \\ B=\frac{3}{2} \\ D=\frac{1}{2} \end{cases}$$

$$\text{Demak, } \frac{x+2}{x^3-x} = -\frac{2}{x} + \frac{3}{2(x-1)} + \frac{1}{2(x+1)}$$

3-misol Ushbu $\frac{x+1}{x^3+x^2-x+2}$ ratsional kasrni oddiy kasrlar yig'indisiga ajrating.

Yechimi: Berilgan ratsional kasrning maxrajini ko'paytuvchilarga ajratamiz:

$$\frac{x+1}{x^3+x^2-x+2} = \frac{x+1}{(x+2)(x^2-x+1)}$$

Yuqorida keltirilgan teorema asosan

$$\frac{x+1}{(x+2)(x^2-x+1)} = \frac{A}{x+2} + \frac{Bx+C}{x^2-x+1}$$

yuqoridagi misoldagiday, bu tenglikni ham o'ng qismini umumiy maxrajga keltiramiz va hosil qilingan tenglikning ikkala qismidagi maxrajlarni tashlab yuboramiz. Natijada quyidagi ayniyat hosil bo'ladi:

$$x+1 \equiv (A+B)x^2 + (-A+2B+C)x + A+2C$$

ko'phadlarning tengligidan foydalanib A, V, C larga nisbatan tenglamalar sistemasini tuzib uning ildizlarini topamiz:

$$\begin{cases} A+B=0 \\ -A+2B+C=1 \\ A+2C=1 \end{cases} \Rightarrow \begin{cases} A=-\frac{1}{7} \\ B=\frac{1}{7} \\ C=\frac{4}{7} \end{cases}$$

Shunday qilib, ratsional kasr quyidagi sodda ratsional kasrlarga yig'indisiga ajraldi:

$$\frac{x+1}{x^3+x^2-x+2} = -\frac{1}{7(x+2)} + \frac{x+4}{7(x^2-x+1)}.$$

13.3. Ratsional kasrlarni integrallash

Ratsional kasrni integrallash uchun quyidagilarni bajarish kerak:

1) uning to'g'ri yoki noto'g'ri kasr ekanligini tekshirib va aks holda ya'ni, noto'g'ri kasr bo'lganda, oldin uning butun qismini ajratib, shundan keyin ko'phad butun qismi va ratsional kasr qismlari yig'indisi ko'rinishida yoziladi;

2) To'g'ri ratsional kasr oddiy ratsional kasrlar yig'indisiga ajratiladi;

3) Yoyilmaning koeffitsiyentlari topiladi;

4) Ratsional kasrning integrali hisoblanadi.

4-misol Integralni hisoblang:

$$\int \frac{(x^2+3)dx}{x(x-1)(x+2)}$$

Yechimi: Yuqoridagi ko'rsatmalarni birin-ketin bajarib, quyidagilarni hosil qilamiz:

$$\frac{x^2+3}{x(x-1)(x+2)} = \frac{A}{x} + \frac{B}{x-1} + \frac{D}{x+2},$$

$$x^2+3 \equiv (A+B+D)x^2 + (A+2B-D)x - 2A,$$

$$\begin{cases} A+B+D=1 \\ A+2B-D=0 \\ -2A=3 \end{cases} \Rightarrow \begin{cases} A=-\frac{3}{2} \\ B=\frac{4}{3} \\ D=\frac{7}{6} \end{cases}$$

$$\begin{aligned} \int \frac{(x^2+3)dx}{x(x-1)(x+2)} &= -\frac{3}{2} \int \frac{dx}{x} + \frac{4}{3} \int \frac{dx}{x-1} + \frac{7}{6} \int \frac{dx}{x+2} = \\ &= -\frac{3}{2} \ln|x| + \frac{4}{3} \ln|x-1| + \frac{7}{6} \ln|x+2| + c. \end{aligned}$$

Mustaqil bajarish uchun misollar

Ratsional kasrni integrallang.

13.1. a) $\int \frac{x^2 dx}{(x-1)^2(x+1)},$

b) $\int \frac{dx}{(x-1)(x^2-x+1)}.$

13.2. a) $\int \frac{dx}{(x-1)(x-3)},$

b) $\int \frac{x dx}{(2+x)(x-3)}.$

13.3. $\int \frac{2x^2+3x+6}{(1+x)^2(4-x)} dx.$

13.4 $\int \frac{x^2 dx}{(x+5)^2(x+4)}.$

13.5. $\int \frac{(x^2-5x)dx}{x^2-5x+6}.$

13.6. $\int \frac{dx}{x^3+2x^2+x}.$

13.7. $\int \frac{dx}{x(x^2+1)}.$

13.8. $\int \frac{dx}{(x+1)(x^2+4)}.$

13.9. $\int \frac{dx}{(x+1)(x^2+x+1)^2}.$

13.10. $\int \frac{(x^3+1)dx}{(x^2-4x+5)^2}.$

13.11. $\int \frac{1-2x+5x^2-2x^3}{(1-x)^4(x^2+1)} dx.$

13.12. $\int \frac{x^3-10x+25}{x^4-10x^3+25x^2} dx.$

13.13. $\int \frac{x^5 dx}{8-x^3}.$

13.14. $\int \frac{x dx}{(1-x^3)^2}.$

13.15. $\int \frac{x dx}{(x^2+2x+2)^2}.$

13.16. $\int \frac{dx}{x^7+x^5}.$

13.17. $\int \frac{x^2 dx}{(2-3x)^7}.$

13.18. $\int \frac{dx}{x(1+x)^5}.$

13.15. $\int \frac{dx}{(1+x^2)^5}.$

13.16. $\int \frac{9x^2-14x+1}{x^3-2x^2-x+2} dx.$

13.17. $\int \frac{6dx}{x(x-1)(x^2-5x+6)}.$

13.18. $\int \frac{(x^4+1)dx}{x^3-x^2+x-1}.$

13.19. $\int \frac{dx}{(x^2+4)^4}.$

13.20. $\int \frac{x^3+4x}{(x+1)^2(x^2+x+1)} dx.$

$$13.21. \int \frac{3x^3 + x + 4}{(x^2 + 9)(x^2 - 1)} dx.$$

$$13.23. \int \frac{2x^3 + x^2 + 3x}{(x+2)^2(x^2+2x+2)} dx.$$

$$13.25. \int \frac{x^3 + 6x^2 + 8x}{(x+2)^2(x^2+4)} dx.$$

$$13.27. \int \frac{2x^3 - x^2 - 4x}{(x-1)^2(x^2+4x+5)} dx.$$

$$13.29. \int \frac{x^3 + 4x^2 + 3x + 2}{(x-1)^2(x^2+1)} dx$$

Quyidagi ratsional kasrlarni integrallang:

$$13.31. \int \frac{2x^3 + x^2 + 3x}{(x+2)^2(x^2+2x+2)} dx$$

$$13.33. \int \frac{x^3 + 6x^2 + 9x + 2}{(x+1)^2(x^2+1x+1)} dx$$

$$13.35. \int \frac{x^3 + 6x^2 + 8x}{(x+2)^2(x^2+4)} dx$$

$$13.37. \int \frac{x^3 + 6x^2 + 8x + 4}{(x-2)(x+1)^3} dx$$

$$13.22. \int \frac{x^3 + 4x^2 + 5x}{x(x+1)^3} dx.$$

$$13.24. \int \frac{x^3 + x + 1}{(x^2 - x + 1)(x^2 + 1)} dx.$$

$$13.26. \int \frac{x^2 + 2x + 3}{(x+1)(x^2+1)} dx.$$

$$13.28. \int \frac{x^3 - 4x^2 - 6x}{x^3(x+1)} dx.$$

$$13.30. \int \frac{x^3 + 6x^2 + 8x + 4}{(x-2)(x+1)^3} dx$$

$$13.32. \int \frac{x^3 + x + 1}{(x^2 - x + 1)(x^2 + 1)} dx$$

$$13.34. \int \frac{x^3 + 4x^2 + 5x}{x(x+1)^3} dx$$

$$13.36. \int \frac{x^3 + 4x^2 + 3x + 2}{(x-1)^2(x^2+1)} dx$$

$$13.38. \int \frac{x^2 + 2x + 3}{(x+1)(x^2+1)} dx$$

Maxrajida kvadrat uchhad qatnashgan integrallarni toping.

$$13.39. \int \frac{dx}{15 - 9x^2 - 6x}$$

$$13.41. \int \frac{dx}{\sqrt{x^2 - 8x + 25}}$$

$$13.43. \int \frac{dx}{x^2 + \sqrt{3}x + 1}$$

$$13.45. \int \frac{(4x-5)dx}{x^2 - 2x + 5}$$

$$13.47. \int \frac{e^x dx}{e^{2x} - 2e^x + 6}$$

$$13.49. \int \frac{(2x-1)dx}{\sqrt{5+4x-x^2}}$$

$$13.51. \int \frac{dx}{x\sqrt{x^2 - 5x + 4}}$$

$$13.53. \int \frac{dx}{(x^2 - 3x + 3)^2}$$

$$13.40. \int \frac{dx}{x^2 + 2x + 17}$$

$$13.42. \int \frac{dx}{\sqrt{3+2x-x^2}}$$

$$13.44. \int \frac{(x-2)dx}{x^2 - 4x + 13}$$

$$13.46. \int \frac{(3x-5)dx}{\sqrt{2x^2 - 12x + 15}}$$

$$13.48. \int \frac{xdx}{\sqrt{x^2 - 6x + 1}}$$

$$13.50. \int \frac{6x+1}{\sqrt{x-x^2}} dx$$

$$13.52. \int \frac{dx}{x^2\sqrt{1+x^2}}$$

$$13.54. \int \frac{dx}{(x^2 + 2x)^2}$$

Mavzu bo'yicha nazorat testlari

Ratsional funksiyalar va ularni integrallash.

1. Quyidagi yig'indilardan qaysi biri ko'phadni ifodalaydi?

A) $\sum_{k=0}^n a_k \sin kx$. B) $\sum_{k=0}^n a_k \sin^k x$. C) $\sum_{k=0}^n a_k x^k$. D) $\sum_{k=0}^n a_k x^{-k}$. E) $\sum_{k=0}^n a_k x_k$.

2. $P(x)=(x^2+2x-10)^2(3x+5)$ ko'phadning darajasini aniqlang:

A) 1. B) 2. C) 3. D) 4. E) 5.

3. Agar $P_n(x)$ va $Q_m(x)$ ko'phadlar bo'lsa, unda quyidagi funksiyalardan qaysi biri ratsional funksiya deyiladi?

A) $P_n(x)+Q_m(x)$. B) $P_n(x)-Q_m(x)$. C) $P_n(x)/Q_m(x)$.
D) $P_n(x) \cdot Q_m(x)$. E) $P_n[Q_m(x)]$.

4. Qaysi shartda $R(x)=P_n(x)/Q_m(x)$ to'g'ri ratsional funksiya deyiladi?

A) $m \geq n$. B) $m \leq n$. C) $m > n$; D) $m < n$. E) $m \neq n$.

5. Qaysi holda $R(x)=P_n(x)/Q_m(x)$ noto'g'ri ratsional funksiya bo'lmaydi?

A) $m \geq n$. B) $m \leq n$. C) $m = n$. D) $m > n$. E) $m = n - 1$.

6. Quyidagilardan qaysi biri to'g'ri ratsional funksiya bo'ladi?

A) $\frac{x^2+x+1}{x+1}$. B) $\frac{(x+1)^3}{x^2+x+1}$. C) $\frac{x^2+x+1}{x^3+1}$.

D) $\frac{x^2+x+1}{(x+1)^2}$. E) $\frac{x^2+x+1}{x^2+1}$.

7. I tur eng sodda ratsional funksiyaning ko'rsatish.

A) $\frac{Ax+B}{x-a}$. B) $\frac{Ax+B}{(x-a)^k}$. C) $\frac{A}{(x-a)^k}, k \geq 2$. D) $\frac{A}{x-a}$. E) $\frac{x+b}{x-a}$.

8. II tur eng sodda ratsional funksiyaning ko'rsatish.

A) $\frac{Ax+B}{x^2+px+q}$. B) $\frac{Ax+B}{(x-a)^k}$. C) $\frac{A}{x-a}$.

D) $\frac{A}{x^2+px+q}$. E) $\frac{A}{(x-a)^k}, k \geq 2$.

9. III tur eng sodda ratsional funksiyaning ko'rsatish.

A) $\frac{Ax+B}{x^2+px+q}$. B) $\frac{Ax+B}{(x^2+px+q)^k}, k \geq 2$. C) $\frac{Ax+B}{(x-a)^k}, k \geq 3$.

D) $\frac{Ax^2+Bx+C}{x^2+px+q}$. E) $\frac{Ax^2+Bx+C}{(x^2+px+q)^k}$.

10. IV tur eng sodda ratsional funksiyaning ko'rsatish.

A) $\frac{Ax+B}{x^2+px+q}$. B) $\frac{Ax+B}{(x^2+px+q)^k}, k \geq 2$. C) $\frac{Ax+B}{(x-a)^k}, k \geq 3$.

D) $\frac{Ax^2+Bx+C}{x^2+px+q}$. E) $\frac{Ax^2+Bx+C}{(x^2+px+q)^k}$.

11. I tur eng sodda ratsional funksiya integrali qaysi elementar funksiyalar orqali ifodalanadi?

- A) faqat logarifmik funksiya. B) logarifmik va arktangens funksiyalar.
C) faqat ratsional funksiya. D) ratsional va arktangens funksiyalar.
E) faqat arktangens funksiyalar.

12. II tur eng sodda ratsional funksiya integrali qaysi elementar funksiyalar orqali ifodalanadi?

- A) faqat logarifmik funksiya. B) logarifmik va arktangens funksiyalar.
C) faqat ratsional funksiya. D) ratsional va arktangens funksiyalar.
E) logarifmik va ratsional funksiyalar.

13. III tur eng sodda ratsional funksiya integrali qaysi elementar funksiyalar orqali ifodalanadi?

- A) faqat logarifmik funksiya. B) logarifmik va arktangens funksiyalar.
C) faqat ratsional funksiya. D) ratsional va arktangens funksiyalar.
E) logarifmik va ratsional funksiyalar.

14. IV tur eng sodda ratsional funksiya integrali qaysi elementar funksiyalar orqali ifodalanadi?

- A) faqat logarifmik funksiya.
B) logarifmik va arktangens funksiyalar.
C) faqat ratsional funksiya.
D) ratsional va arktangens funksiyalar.
E) logarifmik va ratsional funksiyalar.

15. $R(x) = \frac{3x^2 + 5x - 1}{(x-5)(x-3)(x+1)}$ ratsional funksiyaning yoyilmasi qanday

ko'rinishda bo'ladi?

A) $\frac{A_1x + B_1}{x-5} + \frac{A_2x + B_2}{x-3} + \frac{A_3x + B_3}{x+1}$. B) $\frac{A_1}{x-5} + \frac{A_2}{x-3} + \frac{A_3}{x+1}$.

C) $\frac{A_1}{(x-5)(x-3)} + \frac{A_2}{(x-5)(x+1)} + \frac{A_3}{(x-3)(x+1)}$.

D) $\frac{A_1x + B_1}{(x-5)(x-3)} + \frac{A_2x + B_2}{(x-5)(x+1)} + \frac{A_3x + B_3}{(x-3)(x+1)}$.

E) to'g'ri javob keltirilmagan.

16. $\int \frac{3x+5}{x^2+2x-3} dx$ integralni hisoblang.

A) $\frac{2}{x-1} + \frac{1}{x+3} + C$. B) $\frac{1}{x-1} - \frac{2}{x+3} + C$. C) $2 \ln \left| \frac{x+3}{x-1} \right| + C$.

D) $2 \ln|x-1| + \ln|x+3| + C$. E) $\ln|x-1| - 2 \ln|x+3| + C$.

14-§. TRIGONOMETRIK FUNKSIYALARNI INTEGRALLASH. IRRATSIONAL FUNKSIYALARNI INTEGRALLASH

14.1. Trigonometrik funksiyalarni integrallash usullari

Ushbu $\int R(\sin x, \cos x) dx$ integral berilgan bo'lsin ($R = \sin x$ va $\cos x$ ga nisbatan ratsional finksiya) Bu integral $\operatorname{tg} \frac{x}{2} = t$ almashtirish yordami bilan hammavaqt ratsional funksiyaning integraliga keltirilishi mumkinligini ko'rsatamiz. $\sin x$ va $\cos x$ funksiyalarni $\operatorname{tg} \frac{x}{2}$ bilan, ya'ni t bilan ifodalaymiz:

$$\sin x = \frac{2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}}{1} = \frac{2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}}{\sin^2 \frac{x}{2} + \cos^2 \frac{x}{2}} = \frac{2 \operatorname{tg}^2 \frac{x}{2}}{1 + \operatorname{tg}^2 \frac{x}{2}} = \frac{2t}{1+t^2}$$

$$\cos x = \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{1} = \frac{\cos^2 \frac{x}{2} - \sin^2 \frac{x}{2}}{\cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}} = \frac{1-t^2}{1+t^2}$$

$$\text{shuning uchun, } x = 2 \arctg t, \quad dx = \frac{2dt}{1+t^2}$$

$$\text{demak, } \int R(\sin, \cos x) dx = \int R\left(\frac{2t}{1+t^2}, \frac{1-t^2}{1+t^2}\right) \frac{2dt}{1+t^2}$$

1-misol. Integralni hisoblang: $\int \frac{dx}{\sin x}$

Yechimi: Yuqorida yozilgan formulalarga asosan:

$$\int \frac{dx}{\sin x} = \int \frac{\frac{2dt}{1+t^2}}{\frac{2t}{1+t^2}} = \int \frac{dt}{t} = \ln|t| + C = \ln\left|\operatorname{tg} \frac{x}{2}\right| + C$$

b) Aniqmas integral $\int R(\sin x) \cos x dx$ ko'rinishda, ya'ni $\cos x$ ga nisbatan toq funksiya bo'lsin. U holda $\sin x = t$, $\cos x dx = dt$ bajarib quyidagini hosil qilamiz. $\int R(\sin x) \cos x dx = \int R(t) dt$

2-misol. Integralni hisoblang:

$$\int \sin^5 x \cos x dx$$

$$\text{Yechimi: } \int \sin^5 x \cos x dx = \left[\begin{matrix} \sin x = t \\ \cos x dx = dt \end{matrix} \right] = \int t^5 dt = \frac{t^6}{6} + C = \frac{\sin^6 x}{6} + C$$

v) Agar integral

$$\int R(\cos x) \sin x dx$$

ko'rinishda berilgan bo'lsa, u holda $\cos x = t$, $\sin x dx = -dt$ almashtirish maqsadga muvofiqdir:

$$\int R(\cos x) \cdot \sin x dx = -\int R(t) dt$$

3-misol. Integralni hisoblang: $\int \frac{\sin x}{\cos^4 x} dx$

Yechimi:

$$\int \frac{\sin x dx}{\cos^4 x} = \left[\begin{array}{l} \cos x = t \\ \sin x dx = -dt \end{array} \right] = -\int \frac{dt}{t^4} = -\int t^{-4} dt = \frac{1}{3t^3} + C = \frac{1}{3\cos^3 x} + C$$

g) Agar integral ostidagi funksiya faqat $\operatorname{tg} x$ ga bog'lik bo'lsa, u holda $\operatorname{tg} x = t$, $x = \operatorname{arctg} x$, $dx = \frac{dt}{1+t^2}$ almashtirish yordamida bu integral ratsional funksiyaning integraliga keltiriladi:

$$\int R(\operatorname{tg} x) dx = \int R(t) \cdot \frac{dt}{1+t^2}$$

4-misol. Integralni hisoblang: $\int \operatorname{tg} x dx$

$$\begin{aligned} \text{Yechimi: } \int \operatorname{tg} x dx &= \left[\begin{array}{l} \operatorname{tg} x = t \\ dx = \frac{dt}{1+t^2} \end{array} \right] = \int \frac{t dt}{1+t^2} = \frac{1}{2} \int \frac{d(1+t^2)}{1+t^2} = \frac{1}{2} \ln|1+t^2| + C = \\ &= \frac{1}{2} \ln|1+\operatorname{tg}^2 x| + C = \frac{1}{2} \ln\left(\frac{1}{\cos^2 x}\right) + C. \end{aligned}$$

d) Agar integral ostidagi funksiya $R(\sin x, \cos x)$ ko'rinishda bo'lsa, ammo $\sin x, \cos x$ funksiyalar faqat juft darajalarda kirs, u holda $\operatorname{tg} x = t$ almashtirish tatbiq etiladi, chunki

$$\cos^2 x = \frac{1}{1+\operatorname{tg}^2 x} = \frac{1}{1+t^2}$$

$$\sin^2 x = \frac{\operatorname{tg}^2 x}{1+\operatorname{tg}^2 x} = \frac{t^2}{1+t^2}, \quad dx = \frac{dt}{1+t^2}$$

5-misol. Integralni hisoblang: $\int \frac{dx}{2-\sin^2 x}$

$$\text{Yechimi: } \int \frac{dx}{2-\sin^2 x} = \left[\begin{array}{l} \operatorname{tg} x = t, \quad dx = \frac{dt}{1+t^2} \\ \sin^2 x = \frac{t^2}{1+t^2} \end{array} \right] =$$

$$= \int \frac{dt}{\left(2 - \frac{t^2}{1+t^2}\right)(1+t^2)} = \int \frac{dt}{2+t^2} = \frac{1}{\sqrt{2}} \operatorname{arctg} \frac{t}{\sqrt{2}} + C = \frac{1}{\sqrt{2}} \operatorname{arctg} \left(\frac{\operatorname{tg} x}{\sqrt{2}} \right) + C$$

e) Endi $\int \sin^m x \cos^n x dx$ ko'rinishdagi integral berilgan bo'lsin.

Bunda ikki holni qaraymiz:

1) m va n dan kamida bittasi toq son. Anqlik uchun n toq bo'lsin.
 $n=2p+1$ deb olib integralni almashtiramiz:

$$\begin{aligned} \int \sin^m x \cos^{2p+1} x dx &= \int \sin^m x \cos^{2p} x \cdot \cos x dx = \\ &= \int \sin^m x (1 - \sin^2 x)^p \cos x dx = \left[\begin{matrix} \sin x = t \\ \cos x dx = dt \end{matrix} \right] = \int t^m (1 - t^2)^p dt. \quad (14.1) \end{aligned}$$

Bu esa ratsional funksiyaning integrali.

2) m va n manfiy bo'lmagan juft son. $m=2p$, $n=2q$ deb faraz qilamiz:

$$\sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x, \cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x \quad (14.2)$$

Bularni integralga qo'yamiz:

$$\int \sin^{2p} x \cos^{2q} x dx = \int \left(\frac{1}{2} - \frac{1}{2} \cos 2x \right)^p \left(\frac{1}{2} + \frac{1}{2} \cos 2x \right)^q dx. \quad (14.3)$$

Darajalarni ko'tarib, qavslarni ochib, $\cos 2x$ ning juft va toq darajalarini o'z ichiga olgan hadlarni hosil qilamiz, birinchi punktda ko'rsatilgan usuldan yoki (14.2) formuladan foydalanamiz.

6-misol. Integralni hisoblang: $\int \sin^4 x dx$

Yechimi:

$$\begin{aligned} \int \sin^4 x dx &= \frac{1}{4} \int (1 - \cos 2x)^2 dx = \frac{1}{4} \int (1 - 2 \cos 2x + \cos^2 2x) dx = \\ &= \frac{1}{4} \left[1 - \sin 2x + \frac{1}{2} \int (1 + \cos 4x) dx \right] = \frac{1}{4} \left[\frac{3}{2} x - \sin 2x + \frac{\sin 4x}{8} \right] + c \end{aligned}$$

14.2. Irratsional funksiylarning integrali

Har qanday irratsional funksiylardan olingan integral elementar funksiylar orqali ifodalanavermaydi. O'zgaruvchilarni almashtirish yordamida ratsional funksiyaning integrallariga keltiriladigan irratsional funksiylarning ayrimlarini qaraymiz.

1) $\int R(x, x^{\frac{m}{n}}, \dots, x^{\frac{r}{s}}) dx$ ko'rinishdagi integralni qaraymiz. Bunda R o'zgaruvchilarga nisbatan faqat ratsional amallar bajarilishini ko'rsatadi, $m, n, r, s, \dots$ - natural sonlardir.

Integral ostidagi funksiyaning ratsional funksiyaga keltirish uchun $\frac{m}{n}, \dots, \frac{r}{s}$ kasrlarning umumiy maxraji k ni topish va $x = t^k, dx = kt^{k-1} dt$ almashtirish bajarish maqsadga muvofiqdir.

7-misol. Ushbu integral hisoblansin: $\int \frac{\sqrt{x} dx}{\sqrt[4]{x^3+1}}$

Yechimi: $\int \frac{\sqrt{x} dx}{\sqrt[4]{x^3+1}} = \int \frac{x^{\frac{1}{2}} dx}{x^{\frac{3}{4}} + 1}$

$\frac{1}{2}, \frac{3}{4}$ kasrlarning umumiy maxraji 4 bo'ladi, shuning uchun $x = t^4$, $dx = 4t^3 dt$ almashtirishni bajaramiz:

$$\begin{aligned} \int \frac{x^{\frac{1}{2}} dx}{x^{\frac{3}{4}} + 1} &= 4 \int \frac{t^2}{t^3 + 1} t^3 dt = 4 \int \left(t^2 - \frac{t^2}{t^3 + 1} \right) dt = \\ &= 4 \int t^2 dt - 4 \int \frac{t^2}{t^3 + 1} dt = 4 \cdot \frac{t^3}{3} - \frac{4}{3} \ln|t^3 + 1| + C = \\ &= \frac{4}{3} \left[x^{\frac{3}{4}} - \ln|x^{\frac{3}{4}} + 1| \right] + C \end{aligned}$$

2) $\int R\left[x, \left(\frac{ax+b}{cx+d}\right)^{\frac{m}{n}}, \dots, \left(\frac{ax+b}{cx+d}\right)^{\frac{r}{s}}\right] dx$ ko'rinishdagi integralni qaraymiz

$R, m, n, s, r, \dots$ larga yuqorida qo'yilgan shartlar shakllanadi. Agar k son $\frac{m}{n}, \dots, \frac{r}{s}$ kasrlarning umumiy maxraji bo'lsa, bu integral

$$\frac{ax+b}{cx+d} = t^k$$

almashtirish yordamida ratsional funksiyaning integraliga olib keladi.

(a, v, s, d bir vaqtda nolga aylanmaydigan butun sonlar).

8-misol. Integralni hisoblang:

$$\int \frac{\sqrt{2x-3} dx}{\sqrt[3]{2x-3} + 1}$$

Yechimi: Bu erda $a=2, v=-3, s=0, d=1$ ekanligini va $\frac{1}{2}, \frac{1}{3}$

kasrlarning umumiy maxraji 6 ekanligini nazarga olib

$$2x-3 = t^6, t = \sqrt[6]{2x-3}, dx = 3t^5 dt$$

almashtirish bajaramiz. Natijada:

$$\begin{aligned}\int \frac{\sqrt{2x-3}dx}{\sqrt[3]{2x-3}+1} &= 3 \int \frac{t^3 \cdot t^5 dt}{t^2+1} = 3 \int \frac{t^8 dt}{t^2+1} = 3 \int (t^6 - t^4 + t^2 - 1 + \frac{1}{t^2+1}) dt = \\ &= 3(\frac{t^7}{7} - \frac{t^5}{5} + \frac{t^3}{3} - t + \arctgt) + C = \frac{3}{7} \sqrt[6]{(2x-3)^7} - \frac{3}{5} \sqrt[6]{(2x-3)^5} + \\ &+ \sqrt{2x-3} - 3\sqrt[6]{2x-3} + 3\arctg\sqrt[6]{2x-3} + C.\end{aligned}$$

Mustaqil bajarish uchun misollar

Trigonometrik funksiyalar qatnashgan ba'zi ifodalarni integrallarini toping.

14.1. a) $\int \frac{dx}{3+5\cos x}$;

14.2. a) $\int \frac{\sin x dx}{\sin x - \cos x}$;

14.3. $\int \frac{dx}{1-\operatorname{ctgx}}$

14.5. $\int \frac{dx}{(\sin x - \cos x)^2}$

14.7. $\int \frac{dx}{\sin^2 x - 5 \sin x \cos x}$

14.9. $\int \frac{\sin x dx}{6 \cos^2 x + \sin^2 x}$

14.11. $\int \frac{dx}{\sin^2 x - 2 \operatorname{tg} x + 3 \cos^2 x}$

14.13. $\int \frac{\sin^3 x + 1}{\cos^2 x} dx$

14.15. $\int \frac{dx}{8 - 4 \sin x + 7 \cos x}$

14.17. $\int \frac{\cos x dx}{\sin^3 x - \cos^3 x}$

b) $\int \frac{dx}{\cos^3 x}$.

b) $\int \frac{\cos x dx}{1 + \cos x}$.

14.4. $\int \frac{\sin x + \cos x}{\sin x - \cos x} dx$

14.6. $\int \frac{dx}{1 + \cos x + \sin x}$

14.8. $\int \frac{dx}{\cos x(1 + \sin x)}$

14.10. $\int \frac{(\sin x + \sin^3 x) dx}{\cos 2x}$

14.12. $\int \frac{1 + \operatorname{tg} x}{1 - \operatorname{tg} x} dx$

14.14. $\int \frac{dx}{3 + 5 \cos x}$

14.16. $\int \frac{dx}{4 + 3 \operatorname{tg} x}$

14.18. $\int \frac{1 + \operatorname{tg} x}{\sin x} dx$

Sinus va kosinuslarni ko'paytmasi va darajasi qatnashgan ba'zi ifodalarni integrallarini toping.

14.19. $\int \cos^3 2x dx$ 14.20. $\int \cos^2 5x dx$ 14.21. $\int \sin^2 x \cos^2 x dx$

14.22. $\int \sin^2 2x \cos^3 x dx$ 14.23. $\int \cos^7 x dx$ 14.24. $\int \sin^6 x dx$

14.25. $\int \frac{\sin^3 x}{\cos^4 x} dx$ 14.26. $\int \frac{dx}{\sin^2 x \cos^2 x}$ 14.27. $\int \frac{\sin^2 2x}{\cos^4 2x} dx$

14.28. $\int \sin^2 x \cos^3 x dx$ 14.29. $\int \cos 3x \sin 5x dx$ 14.29. $\int \sin \frac{x}{2} \sin \frac{x}{4} dx$

14.30. $\int \sin\left(5x - \frac{\pi}{2}\right) \cos\left(x + \frac{\pi}{4}\right) dx$ 14.31. $\int \operatorname{tg}^2 x dx$ 14.32. $\int \sin x \cdot \sin 2x \cdot \sin 3x dx$

$$14.33. \int \frac{\cos^2 x}{\sin^3 x} dx \quad 14.34. \int \operatorname{ctg}^3 x dx \quad 14.35. \int \operatorname{tg}^6 3x dx$$

$$14.36. \int \operatorname{tg}^4 \varphi \cdot \sec^4 \varphi d\varphi \quad 14.37. \int \frac{\operatorname{ctg}^{3/2} x dx}{\sin^4 x}$$

Ba'zi bir irratsional ifodalar qatnashgan integrallarni toping.

$$14.38. a) \int \frac{\sqrt{x^2-1}}{x} dx \quad b) \int x(2x-1)^{1/3} dx$$

$$14.39. \int \left(\frac{x}{x+1} \right)^{3/2} dx \quad 14.40. \int \frac{x+1+\sqrt{x+2}}{x+3} dx \quad 14.41. \int \sqrt{\frac{x-3}{x}} \frac{dx}{x}$$

$$14.42. \int \sqrt{\frac{1-x}{1+x}} \frac{dx}{(1+x)^2} \quad 14.43. \int \frac{\sqrt{x+25}}{x} dx \quad 14.44. \int \frac{\sqrt{\left(\frac{x}{1-x}\right)^3}}{x + \sqrt{\frac{x}{1-x}}} dx$$

$$14.45. \int \frac{\sqrt{x} + \sqrt[3]{x}}{\sqrt[4]{x^5} - \sqrt[6]{x^7}} dx \quad 14.46. \int \sqrt{\frac{x+1}{x-1}} dx \quad 14.47. \int \frac{\sqrt[6]{x} dx}{1 + \sqrt[3]{x}}$$

$$14.48. \int \frac{x + \sqrt[3]{x^2} + \sqrt[6]{x}}{x(1 + \sqrt[3]{x})} dx \quad 14.49. \int \frac{dx}{(\sqrt{x+2} - \sqrt[3]{x+2})\sqrt{x-2}} \quad 14.50. \int \frac{\sqrt[3]{1 + \sqrt[4]{x}}}{\sqrt{x}} dx$$

$$14.51. \int \frac{\sqrt[3]{x} dx}{\sqrt{1 + \sqrt[3]{x^2}}} \quad 14.52. \int \sqrt{x}(1 + 2\sqrt[6]{x})^3 dx \quad 14.53. \int x^5 \sqrt{4+x^2} dx$$

$$14.54. \int \frac{dx}{\sqrt[3]{1+x^3}} \quad 14.55. \int \frac{\sqrt[3]{1+x^2}}{x^2} dx \quad 14.56. \int \frac{x^7 dx}{\sqrt{1+x^2}}$$

$$14.57. \int \frac{dx}{x^4 \sqrt{x^2-1}} \quad 14.58. \int \frac{dx}{x(1+x^3)^{1/4}}$$

Eyler almashtirish yordamida integrallarni toping.

$$14.59. \int \frac{dx}{x\sqrt{x^2+4x+1}} \quad 14.60. \int \frac{dx}{x + \sqrt{x^2-x+4}}$$

$$14.61. \int \frac{x dx}{\sqrt{(6x-8-x^2)^2}} \quad 14.62. \int \frac{x^2 dx}{\sqrt{x^2+2x+5}}$$

$$14.63. \int \frac{dx}{x\sqrt{x^2-x-5}} \quad 14.64. \int \frac{dx}{x - \sqrt{x^2-1}} \quad 14.65. \int \frac{dx}{(x-2)\sqrt{(7x-x^2-10)^3}}.$$

Mavzu bo'yicha nazorat testlari

1. Ayrim trigonometrik ifodali integrallarni topish.

1. Trigonometrik funksiyali ifodalarni ratsional funksiyaga ketiruvchi universal almashtirmani ko'rsating.

A) $\sin x = t$. B) $\cos x = t$. C) $\operatorname{tg} x = t$. D) $\operatorname{ctg} x = t$. E) $\operatorname{tg}(x/2) = t$.

2. $t = \operatorname{tg} \frac{x}{2}$ universal almashtirma qatnashgan quyidagi tengliklardan qaysi biri noto'g'ri?

A) $\sin x = \frac{2t}{1+t^2}$. B) $\cos x = \frac{1-t^2}{1+t^2}$. C) $dx = \frac{2dt}{1+t^2}$. D) $x = 2\operatorname{arctg} t$.

E) keltirilgan barcha tengliklar to'g'ri.

3. $\int R(\cos x) \sin x dx$ ko'rinishdagi integrallarni hisoblash uchun qaysi almashtirmadan foydalaniladi?

A) $\cos x = t$. B) $\sin x = t$. C) $\operatorname{tg} x = t$. D) $\operatorname{ctg} x = t$. E) $\operatorname{tg} 2x = t$.

4. Trigonometrik ifodali $\int (1 - \cos^4 x) \sin x dx$ integralni hisoblang.

A) $\cos x - \sin^4 x + C$. B) $\sin x - \frac{\cos^5 x}{5} + C$. C) $-\cos x + \frac{\cos^5 x}{5} + C$.

D) $\sin x - \frac{\sin^5 x}{5} + C$. E) $-\cos x + \frac{\sin^5 x}{5} + C$.

5. $\int R(\sin x) \cos x dx$ ko'rinishdagi integral qanday almashtirma yordamida hisoblanishi mumkin?

A) $\cos x = t$. B) $\sin x = t$. C) $\operatorname{tg} x = t$. D) $\operatorname{ctg} x = t$. E) $\operatorname{tg} 2x = t$.

6. $\int \frac{\cos x}{1 - \sin x} dx$ integral javobi qayerda to'g'ri ko'rsatilgan?

A) $\frac{\sin x}{1 - \cos x} + C$. B) $\frac{\sin x}{1 - \sin x} + C$. C) $\frac{1 + \sin x}{1 - \sin x} + C$.

D) $-\ln|1 - \sin x| + C$. E) $\ln|1 - \cos x| + C$.

7. Quyidagi almashtirmalarning qaysi biridan $\int R(\operatorname{tg} x) dx$ ko'rinishdagi integralni ratsional funksiyali integralga keltirishda foydalanib bo'lmaydi?

A) $t = \operatorname{tg} x$. B) $t = \sin x$. C) $t = \operatorname{ctg} x$. D) $t = \operatorname{tg} \frac{x}{2}$.

E) ko'rsatilgan barcha almashtirmalardan foydalanib bo'ladi.

8. $\int \sin^{2m+1} x \cos^n x dx$ integralni hisoblash uchun qaysi almashtirmadan foydalanish qulay?

A) $\sin x = t$. B) $\cos x = t$. C) $\operatorname{tg} x = t$. D) $\operatorname{ctg} x = t$. E) $\sin^2 x = t$.

9. $\int \sin^3 x \cos^5 x dx$ integralni hisoblang.

A) $\frac{\cos^8 x}{8} - \frac{\cos^6 x}{6} + C$. B) $\frac{\sin^8 x}{8} - \frac{\cos^6 x}{6} + C$. C) $\frac{\cos^8 x}{8} - \frac{\sin^6 x}{6} + C$.

D) $\frac{\sin^8 x}{8} - \frac{\sin^6 x}{6} + C$. E) $\frac{\sin^8 x}{8} + \frac{\sin^6 x}{6} + C$.

10. $\int \sin^m x \cos^{2n+1} x dx$ integralni hisoblash uchun qaysi almashtirmadan foydalanish qulay?

- A) $\sin x = t$. B) $\cos x = t$. C) $\tan x = t$. D) $\cot x = t$. E) $\cos^2 x = t$.

11. $\int \sin^5 x \cos^3 x dx$ integralni hisoblang.

- A) $\frac{\cos^6 x}{6} - \frac{\cos^8 x}{8} + C$. B) $\frac{\sin^6 x}{6} - \frac{\cos^8 x}{8} + C$. C) $\frac{\cos^6 x}{6} - \frac{\sin^8 x}{8} + C$.
D) $\frac{\sin^6 x}{6} - \frac{\sin^8 x}{8} + C$. E) $\frac{\cos^6 x}{6} + \frac{\cos^8 x}{8} + C$.

12. $-10 \int \sin 3x \sin 2x dx$ integralni hisoblang.

- A) $\cos 5x - 5 \cos x + C$. B) $\sin 5x - 5 \cos x + C$. C) $\cos 5x - 5 \sin x + C$.
D) $\sin 5x - 5 \sin x + C$. E) $\cos 3x \cdot \cos 2x + C$.

2. Ayrim irratsional ifodali integrallarni hisoblash.

1. Binomial integralning umumiy ko'rinishi qayerda to'g'ri ifodalangan?

- A) $\int x(a+bx)^p dx$. B) $\int x^r(a+bx)^p dx$. C) $\int x^r(a+bx^s)^p dx$.
D) $\int x^r(a+bx^s) dx$. E) $\int x^r(ax^t+bx^s)^p dx$.

2. Qaysi shartda $\int x^r(a+bx^s)^p dx$ binomial integral albatta elementar funksiyalar orqali ifodalanadi?

- A) p —butun son. B) r —butun son. C) s —butun son.
D) $r+s$ —butun son. E) keltirilgan barcha hollarda.

3. Qaysi shartda $\int x^r(a+bx^s)^p dx$ binomial integral elementar funksiyalarda integrallanuvchi bo'lasligi mumkin?

- A) $\frac{r+1}{s} + p$ —butun son. B) $\frac{r+1}{s}$ —butun son. C) s —butun son.

D) p —butun son. E) keltirilgan barcha hollarda integrallanuvchi bo'ladi.

4. $\int x^r(a+bx^s)^p dx$ binomial integralda $(r+1)/s$ —butun son, $p=k/m$ bo'lsa, qaysi almashtirma orqali undan ratsional funksiyali integralga o'tiladi?

- A) $a+bx^s = t$. B) $a+bx^s = t^k$. C) $a+bx^s = t^m$.
D) $a+bx = t^m$. E) $ax^{-s}+b = t^m$.

5. $\int R \left[x, \left(\frac{ax+b}{cx+d} \right)^m, \dots, \left(\frac{ax+b}{cx+d} \right)^r \right] dx$ irratsional funksiyali integral qanday

almashtirma orqali ratsional funksiyali integralga keltiriladi?

- A) $\frac{ax+b}{cx+d} = t^n$. B) $\frac{ax+b}{cx+d} = t^s$. C) $\frac{ax+b}{cx+d} = t^{ns}$.
 D) $\frac{ax+b}{cx+d} = t^k$, $k = EKUK(n, \dots, s)$. E) $\frac{ax+b}{cx+d} = t^k$, $k = EKUB(n, \dots, s)$

6. $\int R\left(x, \sqrt{\frac{ax+b}{cx+d}}, \sqrt[3]{\frac{ax+b}{cx+d}}\right) dx$ irratsional funksiyali integral qanday almashtirma orqali ratsional funksiyali integralga keltiriladi?

- A) $\frac{ax+b}{cx+d} = t^2$. B) $\frac{ax+b}{cx+d} = t^3$. C) $\frac{ax+b}{cx+d} = t^6$.
 D) $\frac{ax+b}{cx+d} = t^5$. E) $x = t^6$.

7. $\int \frac{1+\sqrt{6x+1}}{1+\sqrt[3]{6x+1}} dx$ integralni hisoblash qaysi ratsional funksiyali integralga keltiriladi?

- A) $\int \frac{1+t^2}{1+t^3} t^4 dt$. B) $\int \frac{1+t^3}{1+t^2} t^5 dt$. C) $\int \frac{1+t^2}{1+t^3} dt$. D) $\int \frac{1+t^3}{1+t^2} dt$.

E) to'g'ri javob keltirilmagan.

8. $\int R(x, \sqrt{ax^2+bx+c}) dx$, $a > 0$, irratsional funksiyali integralni ratsional funksiyali integralga keltiruvchi Eylerning I almashtirmasini ko'rsating.

- A) $\sqrt{ax^2+bx+c} = t$. B) $\sqrt{ax^2+bx+c} = x\sqrt{a} - t$.
 C) $\sqrt{ax^2+bx+c} = xt + a$. D) $\sqrt{ax^2+bx+c} = (x-a)t$.

E) to'g'ri javob keltirilmagan.

9. $\int R(x, \sqrt{ax^2+bx+c}) dx$, $c > 0$, irratsional funksiyali integralni ratsional funksiyali integralga keltiruvchi Eylerning II almashtirmasini ko'rsating.

- A) $\sqrt{ax^2+bx+c} = t+c$. B) $\sqrt{ax^2+bx+c} = \sqrt{c}x+t$.
 C) $\sqrt{ax^2+bx+c} = xt + \sqrt{c}$. D) $\sqrt{ax^2+bx+c} = (x-c)t$.
 E) $\sqrt{ax^2+bx+c} = \sqrt{ax^2+bc} + ct$.

10. Qaysi almashtirma $\int R(x, \sqrt{x^2-10x+9}) dx$ irratsional funksiyali integralni ratsional funksiyali integralga keltirmaydi?

- A) $\sqrt{x^2-10x+9} = x-t$. B) $\sqrt{x^2-10x+9} = (x-1)t$.
 C) $\sqrt{x^2-10x+9} = xt+3$. D) $\sqrt{x^2-10x+9} = (x-9)t$.
 E) barcha almashtirmalar ratsional funksiyali integralga keltiradi.

15-§. ANIQ INTEGRALNI HISOBLASH.

15.1. Aniq integralni hisoblash.

1. $[a, b]$ kesmada $f(x)$ funksiya aniqlangan bo'lsin. $[a, b]$ oralig'ini $a = x_0 < x_1 < \dots < x_n = b$ nuqtalar bilan n ta bo'laklarga ajrataylik. Har bir $[x_{i-1}, x_i]$ kesmadan bittadan $\xi_i (\xi_i \in [x_{i-1}, x_i])$ nuqta olib, $\Delta J = \sum_{i=1}^n f(\xi_i) \Delta x_i$ yig'indi tuzamiz, bunda $\Delta x_i = x_i - x_{i-1}$ ΔJ -ko'rinishdagi yig'indi, integral yig'indi deyiladi. Uning $\max \Delta x_i \rightarrow 0$ dagi limiti, (u mavjud va chekli bo'lsa) $f(x)$ funksiyaning a dan b gacha aniq integrali deyiladi hamda

$$\int_a^b f(x) dx = \lim_{\max \Delta x_i \rightarrow 0} \sum_{i=1}^n f(\xi_i) \Delta x_i \quad (15.1)$$

ko'rinishida yoziladi.

2. Aniq integralning xossalari.

$$1) \int_a^b \alpha f(x) dx = \alpha \int_a^b f(x) dx; \quad (\alpha = \text{const}); \quad (15.2)$$

$$2) \int_a^b [f(x) \pm g(x)] dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx; \quad (15.3)$$

$$3) \int_a^b f(x) dx = - \int_b^a f(x) dx; \quad (15.4)$$

$$4) \int_a^a f(x) dx = 0; \quad (15.5)$$

$$5) \int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx; \quad (15.6)$$

6) Agar $y = f(x)$ funksiya $[a, b]$ kesmada uzluksiz bo'lsa, u holda $\xi \in (a, b)$ topiladiki,

$$\int_a^b f(x) dx = f(\xi)(b - a); \quad (15.8)$$

bo'ladi.

8) Agar $y = f(x)$ juft funksiya bo'lsa, u holda

$$\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx; \quad (15.9)$$

9) Agar $y = f(x)$ toq funksiya bo'lsa, u holda

$$\int_{-a}^a f(x) dx = 0 \quad (15.10)$$

3. Aniq integral Nyuton-Leybnits

$$\int_a^b f(x)dx = F(x)\Big|_a^b = F(b) - F(a) \quad (15.11)$$

formulasi orqali hisoblanadi.

4. $\int_a^b f(x)dx$ integralni hisoblash uchun $x = \varphi(t)$, $\alpha \leq t \leq \beta$ almashtirishni qo'llaymiz. Agar $[\alpha; \beta]$ kesmada $x = \varphi(t)$, $\varphi'(t)$, $f(\varphi(t))$ funksiyalar uzluksiz va $\varphi(\alpha) = a$, $\varphi(\beta) = b$ bo'lsa, quyidagi

$$\int_a^b f(x)dx = \int_{\alpha}^{\beta} f(\varphi(t))\varphi'(t)dt \quad (15.12)$$

tenglik o'rinli.

5. $[a, b]$ kesmada $u = u(x)$, $\vartheta = \vartheta(x)$ funksiyalar uzluksiz hosilalarga ega bo'lsa, quyidagi bo'laklab integrallash formulasi o'rinli bo'ladi:

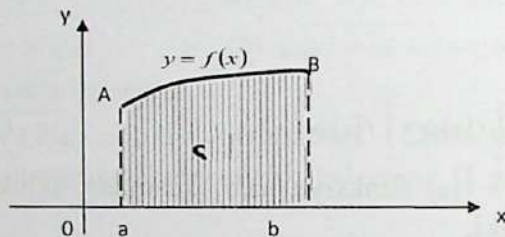
$$\int_a^b u d\vartheta = u\vartheta\Big|_a^b - \int_a^b \vartheta du \quad (15.13)$$

15.2. Yassi figuralar yuzlarini hisoblash

1) Uzluksiz $y = f(x)$ ($f(x) \geq 0$) egri chiziq $x = a$ va $x = b$ to'g'ri chiziqlar hamda Ox o'qining $[a, b]$ kesmasi bilan chegaralangan egri chiziqli trapetsiyaning yuzi

$$S = \int_a^b f(x)dx \quad (15.14)$$

formula bilan hisoblanadi (15.1- rasm).



15.1- rasm.

Agar $[a, b]$ kesmada $f(x) \leq 0$ va uzluksiz bo'lsa, u holda $aABb$ yuzasi

$$S = \left| \int_a^b f(x)dx \right| \quad (15.15)$$

formula bilan topiladi.

2) Uzluksiz $x = \varphi(y)$ ($\varphi(y) \geq 0$) egri chiziq, $y = c_1$ va $y = d$ to'g'ri chiziqlar hamda Oy o'qining $[c, d]$ kesmasi bilan chegaralangan egri chiziqli trapetsiyaning yuzi

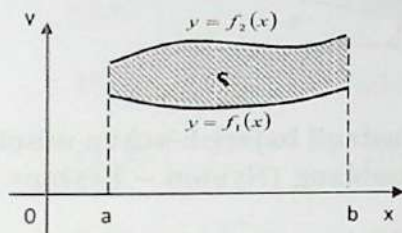
$$S = \int_c^d \varphi(y)dy \quad (15.16)$$

formula bilan hisoblanadi.

3) Uzluksiz $y = f_1(x)$ va $y = f_2(x)$ egri chiziqlar hamda $x = a$, $x = b$ ($a < b$) to'g'ri chiziqlar bilan chegaralangan sohaning yuzi

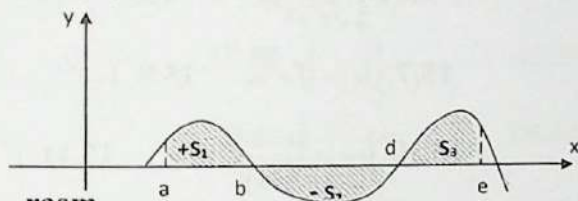
$$S = \int_a^b (f_2(x) - f_1(x)) dx, \quad (15.17)$$

formula bilan hisoblanadi (15.2 - rasm).



15.2 - rasm.

4) Agar $[d, e]$ kesmada $y = f(x)$ funksiya uzluksiz va chekli sonda o'z ishorasini almashtirsin (15.3 - rasm). Masalan, $[a, b]$, $[b, d]$ musbat va $[d, e]$ kesmada manfiy qiymatlarni qabul qilsin.



15.3 - rasm.

U holda

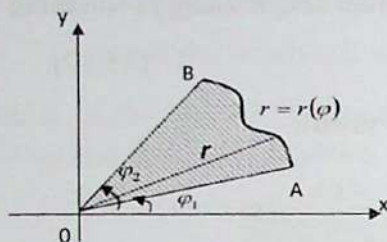
$$S = S_1 + S_2 + S_3 = \int_a^b f(x) dx + \left| \int_b^d f(x) dx \right| + \int_d^e f(x) dx.$$

5) Yuqori chegarasi parametrik ko'rinishda berilgan egri chiziq ($x = \varphi(t)$, $y = \psi(t)$ ($\alpha \leq t \leq \beta$)) va yon tomonlari $a = \varphi(\alpha)$ va $b = \varphi(\beta)$ chiziqlar bilan chegaralangan egri chiziqli trapetsiya yuzasini hisoblash uchun (15.13) formuladan foydalanamiz.

$$S = \int_a^b f(x) dx = \int_a^b y dx = \int_a^b \begin{cases} y = \psi(t) \\ x = \varphi(t) \\ dx = \varphi'(t) dt \\ x = a \Rightarrow t = \alpha \\ x = b \Rightarrow t = \beta \end{cases} = \int_{\alpha}^{\beta} \psi(t) \varphi'(t) dt$$

6) Qutb koordinatalar sistemasida berilgan egri chiziqli OAB sektorning yuzini (15.4- rasm) quyidagi formula yordamida topamiz:

$$S = \lim_{\Delta\varphi \rightarrow 0} \sum \frac{1}{2} r^2 \Delta\varphi = \frac{1}{2} \int_{\varphi_1}^{\varphi_2} r^2 d\varphi \quad (15.18)$$



15.4 – rasm.

Mustaqil bajarish uchun misollar

Integrallarni hisoblang (Nyuton – Leybnis formulasini qo'llash yo'li bilan).

15.1 a) $\int_0^1 \frac{x dx}{\sqrt{1+x^2}}$;

b) $\int_0^1 \sqrt{1+x} dx$.

15.2 $\int_{-2}^1 \frac{dx}{(1+5x)^2}$

15.3 $\int_2^{-13} \frac{dx}{\sqrt[3]{(3-x)^4}}$;

15.4 $\int_0^3 \frac{y-1}{\sqrt{y+1}} dy$

15.5 $\int_0^{T/2} \sin\left(\frac{2\pi}{T} - \varphi_0\right) dt$

15.6 $\int_0^{16} \frac{dx}{\sqrt{x+9} - \sqrt{x}}$

15.7 $\int_0^e (e^x - 1)^4 e^x dx$

15.8 $\int_0^{2\pi} \frac{3dx}{26-x} \quad (b > a > 0)$

15.9 $\int_0^1 \frac{x dx}{(x^2+1)^2}$

15.10 $\int_1^e \frac{dx}{x\sqrt{1-(\ln x)^2}}$

15.11 $\int_1^e \frac{1+\lg x}{x} dx$

15.12 $\int_1^2 \frac{e^{1/x} dx}{x^2}$

15.13 $\int_0^{\sqrt{a/2}} \frac{x^{n-1} dx}{\sqrt{a^2 - x^{2n}}}$

15.14 $\int_1^e \frac{dx}{x\sqrt{1+\ln x}}$

15.15 $\int_{1/2}^{\sqrt{3}/2} \frac{x^3 dx}{\left(\frac{5}{8} - x^4\right)\sqrt{\frac{5}{8} - x^4}}$

15.16 $\int_0^{a/2} \frac{adx}{(x-a)(x-2a)}$

15.18 $\int_2^3 \frac{dx}{2x^2+3x-2}$

15.18 $\int_0^1 \frac{dx}{x^2+4x+5}$

15.19 $\int_1^2 \frac{dx}{x+x^3}$

15.20 $\int_{-0.5}^1 \frac{dx}{\sqrt{8+2x-x^2}}$

15.21 $\int_{-\pi/2}^{\pi/2} \frac{dx}{1+\cos x}$

15.22 $\int_0^{\pi/2} \cos^3 x \sin 2x dx$

15.23 $\int_{-\pi/2}^{\pi/2} \sqrt{\cos x - \cos^3 x} dx$

15.24 $\int_0^{\pi/\omega} \sin^2(\omega t + \varphi_0) dx$

15.25 $\int_{-\pi/2}^{-\pi/4} \frac{\cos^3 x dx}{\sqrt[3]{\sin x}}$

15.26 $\int_0^{\pi/4} \operatorname{ctg}^4 \varphi d\varphi$

15.28 $\int_{1/\pi}^{2/\pi} \frac{\sin \frac{1}{x} dx}{x^2}$

15.28 $\int_{-\pi/2}^{\pi/2} \cos t \sin\left(2t - \frac{\pi}{4}\right) dt$

15.29 $\int_{\pi/12}^{\pi/8} \frac{dx}{\cos^2 2x}$

15.30 $\int_{-1}^0 \sqrt{2-3x} dx$

15.31 $\int_1^3 \frac{dx}{4x-3}$

15.32 $\int_0^1 \left(5 + \frac{1}{1+x^2}\right) dx$

$$15.33 \int_0^{\sqrt{3}/2} \left(\frac{3}{\sqrt{1-x^2}} + \frac{4}{\sqrt{3-x^2}} \right) dx \quad 15.34 \int_{-2}^2 e^{3x+4} dx \quad 15.35 \int_{\pi/2}^{\pi/6} \left(\sin 3x - \frac{1}{2} \cos 4x \right) dx$$

$$15.36 \int_0^2 \frac{2dx}{x^2+4} \quad 15.37 \int_{-3}^{-2} \frac{dx}{9+(x-2)^2} \quad 15.38 \int_5^6 \frac{3dx}{\sqrt{-4x+x^2}}$$

$$15.39 \int_{-\pi/2}^{\pi/2} \frac{\sin x dx}{1+\cos x} \quad 15.40 \int_0^1 \frac{x dx}{\sqrt{3x^2+8}} \quad 15.41 \int_0^4 \left(1+e^{\frac{x}{4}} \right) dx$$

$$15.42 \int_1^4 \frac{x+1}{\sqrt{x}} dx \quad 15.43 \int_{-1}^7 \frac{dx}{\sqrt{3x+4}} \quad 15.44 \int_1^1 (1-\sqrt{x}) dx$$

$$15.45 \int_1^5 \frac{dx}{3x-2} \quad 15.46 \int_0^1 \frac{dz}{(2z+1)^3} \quad 15.47 \int_0^{\pi} \cos \frac{x}{2} \cos \frac{3x}{2} dx$$

$$15.48 \int_0^5 \frac{x dx}{\sqrt{1+3x}} \quad 15.49 \int_{\ln 2}^{\ln 3} \frac{dx}{e^x + e^{-x}} \quad 15.50 \int_0^{\pi/2} \frac{d\phi}{2+\cos \phi}$$

$$15.51 \int_0^1 \frac{x^2 dx}{\sqrt{y-x^2}} \quad 15.52 \int_0^{\sqrt{a}} x^2 \sqrt{a^2-x^2} dx \quad 15.53 \int_0^1 \ln(x+1) dx$$

$$15.54 \int_a^{a\sqrt{3}} \frac{dx}{a^2+x^2} \quad 15.55 \int_0^1 \frac{dx}{\sqrt{x^2+1}} \quad 15.56 \int_1^{\sqrt{e}} \frac{dx}{x\sqrt{1-(\ln x)^2}}$$

$$15.57 \int_{-\pi/2}^{-\pi/4} \frac{\cos^3 x dx}{\sqrt[3]{\sin x}} \quad 15.58 \int_0^{\sqrt{2}/2} \frac{dx}{\sqrt[3]{1-x^2}} \quad 15.59 \int_1^e \frac{\sin(\ln x)}{x} dx$$

$$15.60 \int_{\ln 2}^{\ln 3} \frac{dx}{ch^2 x} \quad 15.61 \int_0^1 \frac{z^3}{z^8+1} dz \quad 15.62 \int_{\pi/6}^{\pi/3} \frac{dx}{\cos^2 x \sin^4 x}$$

$$15.63 \int_2^3 \frac{dx}{\sqrt{5+4x-x^2}} \quad 15.64 \int_1^2 \frac{e^x dx}{e^x-1} \quad 15.65 \int_{\pi/6}^{\pi/3} \operatorname{tg}^4 x dx$$

$$15.66 \int_0^2 \frac{2x-1}{2x+1} dx \quad 15.67 \int_{e^2}^{e^3} \frac{dx}{x \ln x} \quad 15.68 \int_{\pi/4}^{\pi/3} \frac{1+\operatorname{tg}^2 x}{(1+\operatorname{tg} x)^2} dx$$

Integrallarni hisoblang.

$$15.69. \int_4^5 x\sqrt{x^3-16} dx \quad 15.80. \int_1^5 \frac{x dx}{\sqrt{1+3x}} \quad 15.81. \int_1^2 \frac{4x+2}{2x-1} dx$$

$$15.82. \int_4^9 \frac{\sqrt{x} dx}{\sqrt{x}-1} \quad 15.83. \int_e^{e^2} \frac{2 \ln x + 1}{x} dx \quad 15.84. \int_0^4 \frac{dx}{\sqrt{2x+1}}$$

$$15.85. \int_{-2}^1 x^2 \sqrt{1-x^2} dx \quad 15.86. \int_{\ln 3}^{\ln 8} \frac{e^x dx}{\sqrt{e^x+1}}$$

$$15.87. a) \int_0^{2\pi} x \cos x dx; \quad a) \int_0^{\ln 2} x e^x dx. \quad 15.88. \int_1^e x \ln x dx \quad 15.89. \int_1^e \ln^2 x dx$$

$$15.90. \int_0^{\sqrt{3}} \arctg x dx \quad 15.91. \int_{-1}^1 x^2 e^{-x} dx \quad 15.92. \int_0^{\pi} e^x \sin x dx$$

$$15.93. \int_0^{\pi/2} (x+3) \sin x dx \quad 15.94. \int_{-\pi}^{\pi} x \sin x \cos x dx \quad 15.95. \int_0^{\pi/2} \cos^3 x \sin x dx$$

$$\begin{array}{lll}
15.96. \int_{-7}^7 \frac{x^4 \sin x}{x^6 + 2} dx & 15.97. \int_0^{\pi/2} \sin^3 x dx & 15.98. \int_0^{\ln 2} \sqrt{e^x - 1} dx \\
15.99. \int_0^1 \frac{dx}{x^2 + 4x + 5} & 15.100. \int_0^{\pi} \frac{dx}{3 + 2 \cos x} & 15.101. \int_1^3 \frac{dx}{x \sqrt{x^2 + 5x + 1}} \\
15.102. \int_0^{1/2} \sqrt{\frac{1+x}{1-x}} dx & 15.103. \int_1^2 \frac{dx}{x^2 + 2x} & 15.104. \int_{-1}^0 \frac{dx}{1 + \sqrt[3]{x+1}} \\
15.105. \int_0^{\pi/4} t g^3 x dx & 15.106. \int_{-3}^3 x^2 \sqrt{9 - x^2} dx & 15.107. \int_4^9 \frac{dx}{\sqrt{x-1}} \quad (x=t^2) \\
15.108. \int_3^8 \frac{x dx}{\sqrt{1+x}} & 15.109. \int_1^2 \frac{dx}{x \sqrt{1+x^2}} & 15.110. \int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x^2} dx \\
15.111. \int_0^1 \frac{\sqrt{e^x} dx}{\sqrt{e^x + e^{-x}}} & 15.112. \int_3^{26} \frac{\sqrt[3]{(x-2)^2} dx}{3 + \sqrt[3]{(x-2)^2}} & 15.113. \int_3^6 \frac{\sqrt{x^2 - 9}}{x^4} dx \\
15.114. \int_{\sqrt{2}/2}^1 \frac{\sqrt{4-x^2}}{x^2} dx & 15.115. \int_0^a \frac{dx}{x + \sqrt{a^2 - x^2}} & 15.116. \int_0^{\pi/2} \frac{dx}{2 + \cos x}
\end{array}$$

Quyidagi chiziqlar bilan chegaralangan figuralarning yuzini toping.

$$\begin{array}{lll}
15.117. y = 4 - x^2, \quad y = 0 & 15.118. \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 & 15.119. y^2 = 2px, \quad x = h \\
15.120. y = 3 - 2x - x^2, \quad y = 0 & 15.121. xy = 4, \quad x = 1, \quad x = 4, \quad y = 0 & \\
15.122. y = \ln x, \quad x = e, \quad y = 0 & 15.123. y^2 = 2x + 4, \quad x = 0 & \\
15.124. y^2 = x^3, \quad y = 8, \quad x = 0 & 15.125. y^2 = (4-x)^3, \quad x = 0 & \\
15.126. 4(y^2 - x^2) + x^3 = 0 \text{ egri chiziq ilmog'i.} & 15.127. y = x^2, \quad y = 2 - x^2 & \\
15.128. y = x^2 + 4x, \quad y = x + 4 & 15.129. a^2 y^2 = x^3 (2a - x) & \\
15.130. (y-x)^2 = x^3, \quad x = 1 & 15.131. y^2 (2a-x) = (x-a)^2 \text{ atrofidagi ilmog'i} & \\
15.132. y = \frac{a}{2} \left(e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right) \text{ zanjir chiziq } x = \pm a \text{ va } y = 0 & & \\
15.133. y = \arcsin 2x; \quad x = 0; \quad y = -\pi/2 & & \\
15.134. y = x^2 + 1; \quad x = y^2; \quad 3x + 2y - 16 = 0; \quad x = 0 & & \\
15.135. y = (x+1)^2; \quad y^2 = x+1 & & \\
15.136. y = 4 - x^2; \quad y = x^2 - 2x; \quad y = 0 \text{ (2-chorak).} & & \\
15.137. y = -1/2x^2 + 2; \quad x + 2y - 4 = 0; \quad y = 0 \text{ (1-chorak).} & & \\
15.138. y = e^x; \quad y = e^{x/2}; \quad y = e^2 & 15.139. y = x^4 - 2x^2; \quad y = 0 & \\
15.140. y = 3 + 2x - x^2; \quad y = x + 1 & 15.141. y = x^2 + 3; \quad xy = 4; \quad y = 2; \quad x = 0 & \\
15.142. y = x^3; \quad y = -2x^2 + 3x & 15.143. y = \sqrt{1-x}; \quad y = x+1; \quad y = 0 & \\
15.144. y = \cos 2x; \quad y = 0; \quad x = 0; \quad x = \pi/4 & 15.145. y = 2 - x^4; \quad y = x^2 &
\end{array}$$

$$15.146. xy = 1; y = x^2; x = 3; y = 0$$

$$15.147. x = 0; x = 2; y = 2^x; y = 2x - x^2$$

Mavzu bo'yicha nazorat testlari

1. Aniq integral va uning xossalari

1. $[a, b]$ kesma bo'yicha $y=f(x)$ funksiya uchun $S_n(f)$ integral yig'indi tuzishda quyidagi amallardan qaysi biri bajarilmaydi?

A) $[a, b]$ kesma x_i ($i=1, 2, \dots, n-1$) va $x_0=a, x_n=b$ nuqtalar bilan n bo'lakka ajratiladi.

B) $[x_{i-1}, x_i]$ ($i=1, 2, \dots, n$) kesmalardan ixtiyoriy ξ_i nuqtalar tanlanadi.

C) tanlangan ξ_i nuqtalarda $f(x)$ funksiya qiymatlari $f(\xi_i)$ hisoblanadi.

D) $[x_{i-1}, x_i]$ ($i=1, 2, \dots, n$) kesmalarning uzunliklari $x_i - x_{i-1} = \Delta x_i$ topiladi.

E) ko'rsatilgan barcha amallar bajariladi.

2. $[a, b]$ kesmada aniqlangan $y=f(x)$ funksiya uchun tuzilgan

$$S_n(f) = \sum_{k=1}^n f(\xi_k) \Delta x_k$$

integral yig'indi orqali aniq integral qanday aniqlanadi?

$$A) \int_a^b f(x) dx = S_n(f).$$

$$B) \int_a^b f(x) dx = \max S_n(f).$$

$$C) \int_a^b f(x) dx = \min S_n(f).$$

$$D) \int_a^b f(x) dx = \lim_{\substack{n \rightarrow \infty \\ \max \Delta x \rightarrow 0}} S_n(f).$$

E) to'g'ri javob keltirilmagan.

3. $\int_a^b f(x) dx$ aniq integral uchun quyidagi tushunchalardan qaysi biri aniqlanmagan?

A) a - integralning quyi chegarasi.

B) b - integralning yuqori chegarasi.

C) $f(x)$ - integral osti funksiyasi.

D) $f(x) dx$ - integral osti ifodasi.

E) keltirilgan tushunchalarning hammasi aniqlangan.

4. Aniq integralning geometrik ma'nosini ko'rsating.

A) egri chiziqli trapetsiyaga urinma chiziqning burchak koeffitsiyenti.

B) egri chiziqli trapetsiyaning perimetri.

C) egri chiziqli trapetsiyaning o'rta chizigi uzunligi.

D) egri chiziqli trapetsiyaning yuzi. E) to'g'ri javob keltirilmagan.

5. Aniq integralning mexanik ma'nosini ko'rsating.

- A) o'zgaruvchi kuchning eng katta qiymati.
- B) o'zgaruvchi kuchning eng kichik qiymati.
- C) o'zgaruvchi kuchning momenti.
- D) o'zgaruvchi kuchning bajargan ish.
- E) o'zgaruvchi kuchning o'rta qiymati.

6. $[a, b]$ kesmada aniqlangan $y=f(x)$ funksiya qanday shartni

qanoatlantirganda $\int_a^b f(x)dx$ aniq integral doimo mavjud bo'ladi?

- A) yuqoridan chegaralangan. B) quyidan chegaralangan.
- C) o'suvchi. D) kamayuvchi. E) uzluksiz.

7. Aniq integralning xossasi qayerda xato ko'rsatilgan?

A) $\int_a^b [f(x) + g(x)]dx = \int_a^b f(x)dx + \int_a^b g(x)dx.$

B) $\int_a^b [f(x) - g(x)]dx = \int_a^b f(x)dx - \int_a^b g(x)dx.$

C) $\int_a^b f(x)g(x)dx = \int_a^b f(x)dx \cdot \int_a^b g(x)dx.$

D) $\int_a^b kf(x)dx = k \int_a^b f(x)dx$ ($k = \text{const.}$).

E) barcha xossalar to'g'ri ko'rsatilgan.

8. Aniq integral xossasini ifodalovchi

$$\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

tenglik bajarilishi uchun c nuqta qanday shartni qanoatlantirishi kerak?

- A) $c < a$. B) $c > b$, C) $c = a$ yoki $c = b$. D) $a < c < b$.

E) ko'rsatilgan barcha shartlarda tenglik bajariladi.

9. Aniq integral uchun quyidagi tengliklardan qaysi biri o'rinli emas?

A) $\int_a^a f(x)dx = 0$. B) $\int_a^b f(x)dx = \int_b^a f(x)dx$. C) $\int_a^b kf(x)dx = k \int_a^b f(x)dx$.

D) $\int_a^b f(x)dx = \int_a^b f(t)dt$. E) keltirilgan barcha tengliklar o'rinli.

10. $\int_{-1}^2 |x-1|dx$ integralning qiymati qaysi oraliqqa tegishli bo'ladi?

- A) $(-1, 2)$. B) $(0, 6)$. C) $(1, 3)$. D) $(-1, 3)$. E) $(0, 1)$.

11. $\int_a^b dx$ integral qiymati nimaga teng?

- A) $b-a$. B) $a-b$. C) $b+a$. D) b^2-a^2 . E) $(a+b)/2$.

12. $\int_{-1}^4 (-4)dx$ integral qiymati nimaga teng?

- A) 5. B) -5. C) -20. D) 20. E) 12.

2. Aniq integralni hisoblash usullari.

1. Agar $y=f(x)$ funksiya $[a,b]$ kesmada uzluksiz bo'lsa, unda yuqori chegarasi o'zgaruvchi bo'lgan aniq integral orqali aniqlanadigan

$$\Phi(x) = \int_a^x f(t)dt, x \in [a,b]$$

funksiya xossasi qayerda to'g'ri ko'rsatilgan?

- A) $\Phi'(x) = f(a)$. B) $\Phi'(x) = f(x)$. C) $\Phi'(x) = 0$.
D) $\Phi'(x) = xf(x)$. E) $\Phi'(x) = f(x)/x$.

2. $\int_a^b f(x)dx$ aniq integralni Nyuton-Leybnits formulasi bilan

hisoblashda quyidagi amallardan qaysi biri bajarilmaydi?

- A) $f(x)$ funksiyaning biror $F(x)$ boshlang'ich funksiyasi topiladi.
B) boshlang'ich funksiyaning $F(a)$ va $F(b)$ qiymatlari hisoblanadi.
C) $F(b)+F(a)$ yig'indi hisoblanadi. D) $F(b)-F(a)$ ayirma hisoblanadi.
E) ko'rsatilgan barcha amallar bajariladi.

3. Agar $y=F(x)$ berilgan $[a,b]$ kesmada $y=f(x)$ funksiyaning boshlang'ich funksiyasi bo'lsa, unda aniq integral uchun Nyuton-Leybnits formulasi qayerda to'g'ri ifodalangan?

- A) $\int_a^b f(x)dx = F(x)$. B) $\int_a^b f(x)dx = F(a) \cdot F(b)$. C) $\int_a^b f(x)dx = F(b)/F(a)$.
D) $\int_a^b f(x)dx = F(b) - F(a)$. E) $\int_a^b f(x)dx = F(b) + F(a)$.

4. $\int_0^1 xdx$ aniq integralning qiymatini Nyuton-Leybnits formulasi

yordamida toping.

- A) 1. B) 0. C) 1/2. D) -1. E) 2/3.

5. $\int_0^{\pi/3} \sin 3x dx$ aniq integralning qiymatini Nyuton-Leybnits

formulasidan foydalanib aniqlang.

- A) 0. B) 1. C) 2/3. D) 0.75. E) -0.3.

6. $\int_0^{\pi} \cos^2 x dx$ aniq integral qiymatini Nyuton–Leybnits formulasi yordamida toping.

- A) π . B) $\pi/2$. C) $\pi/3$. D) $\pi/4$. E) $\pi/6$.

7. $\int_a^b u dv$ aniq integralni bo'laklab integrallash formulasi yordamida hisoblash jarayonida quyidagi amallardan qaysi biri bajarilmaydi?

- A) $u=u(x)$ funksiyaning du differensial hisoblanadi.
 B) $dv= dv(x)$ differensial bo'yicha $v=v(x)$ funksiya topiladi.
 C) $u(b)v(b) + u(a)v(a)$ yig'indi hisoblanadi.

D) $\int_a^b v du$ integral hisoblanadi. E) ko'rsatilgan barcha amallar

bajariladi.

8. $\int_0^1 x e^x dx$ aniq integralni bo'laklab integrallash formulasi yordamida hisoblang.

- A) e . B) $e/2$. C) 1. D) 0.5. E) 2.

9. $\int_a^b f(x) dx$ aniq integralni o'zgaruvchilarni almashtirish formulasi

orqali hisoblashda quyidagi amallardan qaysi biri bajarilmaydi?

- A) $x=\varphi(t)$ almashtirma tanlanadi.
 B) $\varphi(t)=a$ va $\varphi(t)=b$ tenglamalarning yechimlari α va β topiladi.
 C) $f[\varphi(t)]$ murakkab funksiya tuziladi.
 D) almashtirmaning hosilasi $\varphi'(t)$ hisoblanadi.
 E) ko'rsatilgan barcha amallar bajariladi.

10. $\int_a^b f(x) dx$ aniq integralni $x=\varphi(t)$ almashtirma orqali hisoblash formulasini ko'rsating.

- A) $\int_a^b f(x) dx = \int_a^b \varphi(t) dt$. B) $\int_a^b f(x) dx = \int_a^b f[\varphi(t)] dt$.
 C) $\int_a^b f(x) dx = \int_a^\beta f(t) \varphi'(t) dt$. D) $\int_a^b f(x) dx = \int_a^b f[\varphi(t)] \varphi(t) dt$.
 E) $\int_a^b f(x) dx = \int_a^\beta f[\varphi(t)] \varphi'(t) dt$.

11. $\int_1^5 \frac{dx}{x\sqrt{3x+1}}$ aniq integral qiymatini hisoblash uchun qaysi

almashtirmadan foydalanish mumkin?

A) $t=1/x$. B) $t=\sqrt{3x+1}$. C) $t=1/\sqrt{3x+1}$. D) $t=\sqrt{x}$. E) $t=x\sqrt{3x+1}$.

12. $\int_1^5 \frac{dx}{x\sqrt{3x+1}}$ aniq integral qiymatini o'zgaruvchilarni almashtirish

usulida hisoblang.

A) $2/3$. B) $\ln(2/3)$. C) $9/5$. D) $\ln(9/5)$. E) $\pi/2$.

Mustaqil bajarish uchun testlarning javoblari

1.1. Matritsalar va ular ustida amallar

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
D	B	B	B	E	B	C	E	C	A	D	E	C	D	E	B

1.2. Teskari matritsa

1	2	3	4	5	6	7	8	9	10						
D	D	C	D	B	E	A	E	E	E						

2.1. Determinantlar va ularning xossalari

1	2	3	4	5	6	7	8	9	10	11	12	13	14
B	B	C	A	B	C	D	B	C	B	C	D	C	D

3.1. Chiziqli tenglamalar sistemasi

1	2	3	4	5	6	7	8	9	10	11	12	13	14		
E	B	C	D	B	D	B	B	B	B	E	B	A	D		

4.1. To'g'ri chiziqli tenglamalari va ularga doir masalalar

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	C	B	E	A	C	C	B	C	D	E	D	C	A	C	D
17	18	19	20												
B	B	D	A												

5.1. II tartibli tenglama va chiziqlar. Aylana va ellips

1	2	3	4	5	6	7	8	9	10	11	12				
C	C	A	D	C	C	C	C	E	B	E	A				

5.2. Giperbola va parabola

1	2	3	4	5	6	7	8	9	10	11	12	13	14		
A	A	C	D	A	A	C	A	C	E	D	D	E	D		

6.1. Vektorlar va ular ustida amallar

1	2	3	4	5	6	7	8	9	10	11	12	13	14		
B	E	C	D	D	D	B	A	C	A	C	B	B	D		

6.2. Vektorlarning skalyar ko'paytmasi, uning xossalari va tatbiqlari

1	2	3	4	5	6	7	8	9	10						
B	D	E	E	C	D	D	C	B	D						

6.3. Vektor ko'paytma, uning xossalari va tatbiqlari

1	2	3	4	5	6	7	8	9	10						
D	E	B	D	A	E	C	A	C	B						

6.4. Vektorlarning aralash ko'paytmasi, uning xossalari va tatbiqlari

1	2	3	4	5	6	7	8	9	10						
B	B	E	D	E	D	C	D	C	B						

7.1. Tekislik va uning tenglamalari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	C	E	C	C	E	B	A	C	C	B	E	C	D	D	E
17	18	19	20												
C	A	A	D												

7.2. Tekislikka doir asosiy masalalar

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
E	B	C	D	E	A	D	C	C	D	D	E	D	C	A	C
17	18	19	20	21	22	23									
C	B	D	E	E	D	C									

7.3. Fazodagi to'g'ri chiziqli tenglamalari

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
C	C	B	C	D	B	A	C	C	D	B	C	D	B	D	

8.1. Sonli to'plamlar. Sonning absolut qiymati va uning xossalari.**Sonli ketma-ketlik va uning limiti**

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	A	A	A	D	B	C	A	E	D	B	E	B	B	A	D
17	18	19	20												
A	D	D	B												

8.2. Funktsiya va u bilan bog'liq tushunchalar

1	2	3	4	5	6	7	8	9	10						
C	D	B	E	E	B	D	A	A	C						

8.3. Funktsiya limiti va uning xossalari

1	2	3	4	5	6	7	8	9	10	11	12	13	14		
D	C	C	D	A	D	B	A	E	D	A	D	C	D		

9.1. Funktsiya hosilasi ta'rifi, uning mexanik va geometrik ma'nosi

1	2	3	4	5	6	7	8	9	10						
C	E	B	E	C	C	B	A	D	D						

9.2. Funktsiyani differentsiallash qoidalari. Hosilalar jadvali															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
D	C	E	C	D	E	E	B	C	D	B	E	B	B	D	E

10.1. Kompleks sonlar va ular ustida.															
1	2	3	4	5	6	7	8	9	10						
A	A	C	A	B	C	C	B	C	B						

11.1. Ko'p o'zgaruvchili funktsiyalar															
1	2	3	4	5	6	7	8	9	10						
D	B	C	D	C	E	C	D	E	D						

11.2. Ko'p o'zgaruvchili funktsiyaning hosila va differentsiallari															
1	2	3	4	5	6	7	8	9	10						
E	D	E	B	C	E	D	D	C	D						

12.1. Boshlang'ich funksiya va aniqmas integral. Integrallar jadvali															
1	2	3	4	5	6	7	8	9	10	11	12	13	14		

C	D	C	C	C	E	E	B	E	E	E	D	B	E		
---	---	---	---	---	---	---	---	---	---	---	---	---	---	--	--

12.2. Aniqmas integralni hisoblash usullari															
1	2	3	4	5	6	7	8	9	10	11	12	13	14		
A	C	D	C	C	E	D	D	B	A	C	C	C	A		

13.1. Ratsional funktsiyalar va ularni integrallash															
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
C	E	C	C	D	C	D	E	A	B	A	C	B	D	B	D

14.1. Ayrim trigonometrik ifodali integrallarni hisoblash															
1	2	3	4	5	6	7	8	9	10	11	12				
E	E	A	C	B	D	B	B	A	A	D	D				

14.2. Ayrim irratsional ifodali integrallarni hisoblash															
1	2	3	4	5	6	7	8	9	10						
C	A	C	C	D	C	B	B	C	E						

15.1. Aniq integral va uning xossalari

1	2	3	4	5	6	7	8	9	10	11	12				
E	D	E	D	D	E	C	E	B	B	A	C				

15.2. Aniq integralni hisoblash usullari

1	2	3	4	5	6	7	8	9	10	11	12				
B	C	D	C	C	B	C	C	E	E	B	D				

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ISBN 978-9910-7307-9-5



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