M. N. SAMATOVA

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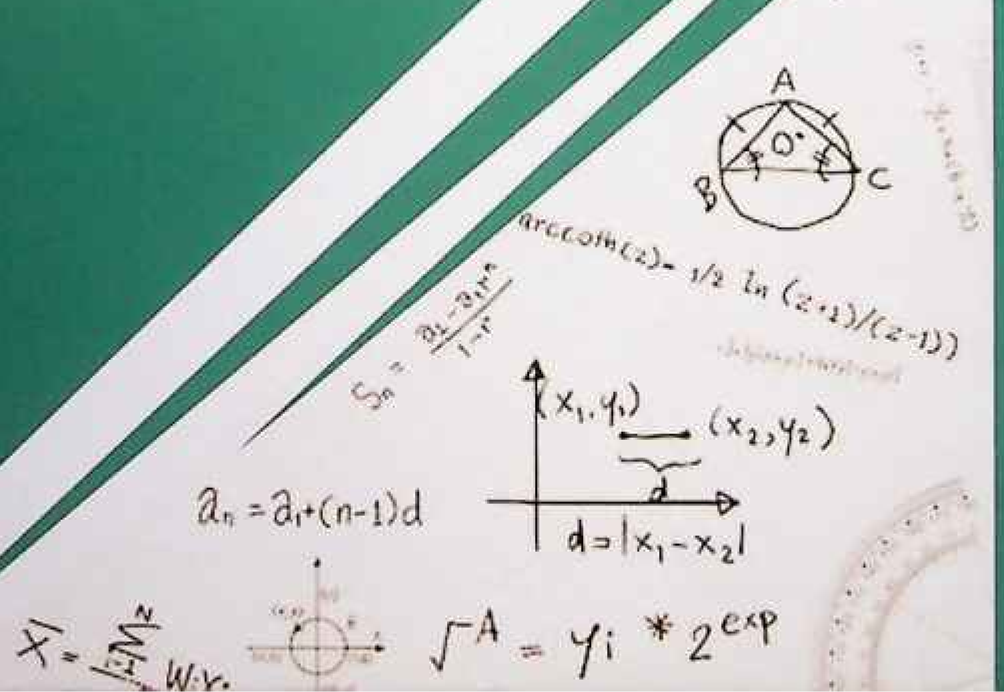

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M. N. Samatova

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O‘quv Qo‘llanma

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Taqrizchilar:
fizika matematika fanlari falsafa doktori
S. Choriyeva,
pedagogika fanlari doktori, professor
O‘. Sultonova

Ushbu o‘quv qo‘llanma oliy ta’lim muassasasining texnika sohasidagi yo’l harakatini tashkil etish va transport vositalari muhandisligi bakalavriat ta’lim yo‘nalishlari uchun mo‘ljallangan. O’quv qo‘llanmada nazariy tushunchalar, ta’riflar, teoremalar, formulalar va ularning mohiyati, misol va masalalarning yechimi tushuntirilgan, mustahkamlash uchun va mustaqil bajarish uchun mashqlar, nazorat testlari va ularning javoblari bilan keltirilgan.

O‘quv qo‘llanmadan oliy ta’lim muassasalarining texnika yo‘nalishlarining kunduzgi, kechki va sirtqi bo‘lim talabalari ham foydalanishlari mumkin.

Mazkur O‘quv qo‘llanma O‘zbekiston respublikasi Oliy ta’lim, fan va innovatsiyalar vazirligining “21” avgust 2025-yildagi 316-son qaroriga asosan nashr etishga tavsiya etilgan. № 900500

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SO‘Z BOSHI

O‘zbekiston Respublikasi Prezidenti Sh.M.Mirziyoyevning “Matematika hamma aniq fanlarga asos. Bu fanni yaxshi bilgan bola aqlli, keng tafakkurli bo‘lib o‘sadi, istalgan sohada muvaffaqiyatli ishlab ketadi” deb takidlagan fikrlarida matematika fanining ahamiyatliligini ko‘rishimiz mumkin.

Texnika oliy ta’lim muassasalarida oliy matematika fani talabalarning mantiqiy fikrlash malakalarini oshirish hamda o‘z fikrlarini tushunarli va aniq bayon etish uchun zamin yaratadi.

O‘zbekiston Respublikasi Prezidenti Sh.M.Mirziyoyevning 2020 yil 24-yanvardagi murojatnomasida mamlakatimizni ijtimoiy-iqtisodiy rivojlantirishda munosib hissa qo‘shuvchi, zamonaviy bilim ko‘nikmaga ega bo‘lgan, yuqori malakali mutaxassislar tayyorlash hamda sohada ilg‘or ta’lim texnologiyalarini joriy qilish orqali ta’lim sohasini yanada takomillashtirish borasida tizimli tadbirlar belgilab berildi.

Ushbu o`quv qo`llanma 6 bobga ajratilgan. Har bir mavzuga asosiy tushunchalar va muhum formulalar keltirilgan, mavzuga oid misollar yechib ko‘rsatilgan. Mustaqil bajarish uchun misol va masalalar berilgan va har bir bob oxirida mavzulashtirilgan nazorat testlari keltirilgan. Yechilgan va berilgan misol va masalalar dasturda ko`zda tutilgan matematikaning fundamental tushunchalari: matritsa, determinant, chiziqli tenglamalar sistemasi, tekislikda va fazoda tekislik va to‘gri chiziq tenglamalari, vektor, funksiya tushunchasi, limit, bir o‘zgaruvchili funksiya hosilasi, tadbiqlari, aniqmas integral, aniq integral va unga keltiriluvchi masalalar yechimini berishga bag‘ishlangan.

O`quv qo`llanmada Oliy o‘quv yurtlarining muhandis-texnika sohasidagi bakalavriat ta’lim yo‘nalishlari talabalari bilim olishlari uchun zarur bo`ladigan matematik bilim va malakalar sistemasini chuqur hamda ongli ravishda o`zlashtirishlarini ta’minlovchi imkoniyatlarni ochib berish ko`zda tutilgan.

Ushbu kitob haqidagi tanqidiy fikr – mulohazalarni muallif chuqur mamnuniyat bilan qabul qiladi va oldindan tashshakkur bildiradi.

I-BOB.CHIZIQLI ALGEBRA.

1-§. Ikkinchi va uchunchi tartibli determinantlar. Asosiy xossalari. Determinantlarni hisoblash. Yuqori tartibli determinantlar.

Quyidagi to'rtta sondan iborat bo'lgan jadvalga $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$ kvadrat matritsa deyiladi. Shu kvadrat matritsaning elementlaridan tuzilgan songa ikkinchi tartibli determinant deyiladi va quyidagicha yoziladi:

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{21}a_{12}.$$

Uchinchi tartibli determinant deb 3x3 o'lchamli kvadrat matritsaning elementlaridan tuzilgan songa aytiladi va quyidagicha hisoblanadi.

Uchburchak qoidasi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}a_{22}a_{33} + a_{13}a_{21}a_{32} + a_{31}a_{12}a_{23}$$
$$-(a_{13}a_{22}a_{31} + a_{11}a_{23}a_{32} + a_{33}a_{12}a_{21})$$

Sariyus usuli:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \begin{matrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{matrix} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} -$$
$$-(a_{13}a_{22}a_{31} + a_{11}a_{23}a_{32} + a_{12}a_{21}a_{33})$$

n-tartibli

$$A=\begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

kvadrat matritsaning elementlaridan tuzilgan songa n-tartibli determinant deyiladi va quyidagi ko'rinishda yoziladi:

$$\Delta=\begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix}$$

Minor va algebraik to'ldiruvchilar:

Determinant a_{ij} elementining M_{ij} minori deb, shu determinantdan bu element turgan qator va ustunni o'chirish natijasida hosil bo'lgan determinantga aytiladi.

Masalan, $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ uchunchi tartibli determinantning minorlari quyidagiga teng:

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}, \quad M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}, \quad M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix},$$

$$M_{21} = \begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix}, \quad M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}, \quad M_{23} = \begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix},$$

$$M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}, \quad M_{32} = \begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix}, \quad M_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}.$$

a_{ij} elenentining algebraik to'ldiruvchisi deb unga mos minorning musbat yoki manfiy ishora bilan olingan kattaligiga aytiladi, bunda $i+j$ juft bo'lsa, musbat ishora bilan, $i+j$ toq bo'lsa, manfiy ishora bilan olinadi.

a_{ij} elementnini algebraik to'ldiruvchisi A_{ij} bilan belgilanadi va

$$A_{ij} = (-1)^{i+j} M_{ij}$$

munosabat bilan aniqlanadi.

Masalan,

$$A_{11} = M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}, \quad A_{12} = -M_{12} = -\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix},$$

$$A_{13} = M_{13} = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}, \quad A_{21} = -M_{21} = -\begin{vmatrix} a_{12} & a_{13} \\ a_{32} & a_{33} \end{vmatrix},$$

$$A_{22} = M_{22} = \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix}, A_{23} = -M_{23} = -\begin{vmatrix} a_{11} & a_{12} \\ a_{31} & a_{32} \end{vmatrix},$$

$$A_{31} = M_{31} = \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix}, \quad A_{32} = -M_{32} = -\begin{vmatrix} a_{11} & a_{13} \\ a_{21} & a_{23} \end{vmatrix},$$

$$A_{33} = M_{33} = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}.$$

Ixtiyoriy tartibli determinantni qator elementlari bo'yicha yoyish usulida hisoblashimiz mumkin.

$$\Delta = \begin{vmatrix} a_{11}\, a_{12} \dots a_{1n} \\ a_{21}\, a_{22} \dots a_{2n} \\ \dots\dots\dots\dots\dots\dots\dots \\ a_{n1}\, a_{n2} \dots a_{nn} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12} + \dots + a_{1n}A_{1n}$$

Bu yerda $A_{11}, A_{12}, \dots A_{1n}$ lar algebraik to'ldiruvchilar.

Determinantlarni asosiy xossalari.

1. Agar determinantning barcha satrlari mos ustunlari bilan almashtirilsa uning qiymati o'zgarmaydi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{vmatrix}$$

2. Agar determinant nollardan iborat qatorga ega bo'lsa, uning qiymati nolga teng bo'ladi:

$$\begin{vmatrix} a_{11} & 0 & a_{13} \\ a_{21} & 0 & a_{23} \\ a_{31} & 0 & a_{33} \end{vmatrix} = 0$$

3. Agar determinant ikkita bir xil parallel qatorga ega bo'lsa, uning qiymati nolga teng bo'ladi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

4. Agar determinant ikkita parallel qatorining o'rinlari almashtirilsa, determinant ishorasi o'zgaradi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = -\begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

5. Agar determinant ikkita parallel qatorining mos elementlari proportsional bo'lsa, uning qiymati nolga teng bo'ladi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ ka_{11} & ka_{12} & ka_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

6.Biror qator elementlarining umumiy ko'paytuvchisini determinant belgisidan tashqariga chiqarish mumkin:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ ka_{21} & ka_{22} & ka_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = k \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

7. Determinantning qiymati biror qator elementlari bilan shu elementlarga tegishli algebraik to'ldiruvchilari ko'paytmalari yig'indisiga teng, bu xossa determinantni qator elementlari bo'yicha yoyish deyiladi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = A_{11}a_{11} + A_{12}a_{12} + A_{13}a_{13}.$$

8. Agar determinantning biror qatori elementlariga parallel qatorning mos elementlarini biror o'zgarmas songa ko'paytirib qo'shilsa, determinantning qiymati o'zgarmaydi:

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} + ka_{11} & a_{22} + ka_{12} & a_{23} + ka_{13} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

9. Biror qator elementlari bilan parallel qator mos elementlari algebraik to'ldiruvchilari ko'paytmalarining yig'indisi nolga teng.

$$A_{11}a_{21} + A_{12}a_{22} + A_{13}a_{23} = 0$$

10. Agar determinantning biror qatorining har bir elementi ikki qo'shiluvchining yig'indisidan iborat bo'lsa, u holda determinant ikki

determinant yig'indisiga teng bo'lib, ularning biri tegishli qator birinchi qo'shiluvchilaridan, ikkinchisi ikkinchi qo'shiluvchilaridan iborat bo'ladi.

$$\begin{vmatrix} a_{11} & a_{12}+b_1 & a_{13} \\ a_{21} & a_{22}+b_2 & a_{23} \\ a_{31} & a_{32}+b_3 & a_{33} \end{vmatrix} = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} + \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}$$

1-misol. Ikkinchi tartibli determinantni hisoblang.

$$\begin{vmatrix} -3 & 1 \\ -2 & 5 \end{vmatrix} = -3\cdot 5 - 1\cdot(-2) = -15 + 2 = -13.$$

2-misol.

Uchinchi tartibli determinantni uchburchak qoidasidan foydalanib hisoblang.

$$\begin{vmatrix} 2 & 1 & -2 \\ -4 & 3 & 1 \\ 1 & 2 & 0 \end{vmatrix} = 2\cdot 3\cdot 0 + (-2)\cdot(-4)\cdot 2 + 1\cdot 1\cdot 1 - (-2\cdot 3\cdot 1 +$$

$+0\cdot 1\cdot(-4) + 2\cdot 1\cdot 2)$=0+16+1-(-6+0+4)=17+2=19.

3-misol. Quyidagi matritsaning minorlarini toping.

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 1 & 0 & -4 \\ -2 & 3 & 5 \end{pmatrix}$$

Yechish.

$M_{11} = \begin{vmatrix} 0 & -4 \\ 3 & 5 \end{vmatrix} = 12$ $\qquad$ $M_{12} = \begin{vmatrix} 1 & -4 \\ -2 & 5 \end{vmatrix} = 5 - 8 = -3$

$M_{13} = \begin{vmatrix} 1 & 0 \\ -2 & 3 \end{vmatrix} = 3$ $\qquad$ $M_{21} = \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -5 - 6 = -11$

$M_{22} = \begin{vmatrix} 3 & 2 \\ -2 & 5 \end{vmatrix}$=15+4=19 $\qquad$ $M_{23} = \begin{vmatrix} 3 & -1 \\ -2 & 3 \end{vmatrix} = 9 - 2 = 7$

$M_{31} = \begin{vmatrix} -1 & 2 \\ 0 & -4 \end{vmatrix} = 4 \qquad M_{32} = \begin{vmatrix} 3 & 2 \\ -2 & 5 \end{vmatrix} = 15 + 4 = 19$

$M_{33} = \begin{vmatrix} 3 & -1 \\ 1 & 0 \end{vmatrix} = 1$

4-misol. Quyidagi determinantni qator elementlari bo'yicha yoyish usuli bilan hisoblang.

$$\begin{vmatrix} 2 & 0 & -2 \\ 1 & -3 & 4 \\ 2 & 1 & 5 \end{vmatrix} = 2 \cdot \begin{vmatrix} -3 & 4 \\ 1 & 5 \end{vmatrix} - 2 \cdot \begin{vmatrix} 1 & -3 \\ 2 & 1 \end{vmatrix} = 2(-15-4) - 2(1+6) =$$
$= -38 - 14 = -52.$

Ixtiyoriy tartibli determinantni hisoblash usullari:

I-usul. Determinant tartibini pasaytirish usuli-biror qator elementlarining bittasidan boshqalarini nolga aylantirib, shu qator bo'yicha yoyish usuli.

5-misol.To'rtinchi tartibli determinantni tartibini pasaytirish usuli bilan yeching.

$$\begin{vmatrix} 2 & 4 & -1 & 2 \\ -1 & 2 & 3 & 1 \\ 2 & 5 & 1 & 4 \\ 1 & 2 & 0 & 3 \end{vmatrix}$$

Yechish: $\begin{vmatrix} 2 & 4 & -1 & 2 \\ -1 & 2 & 3 & 1 \\ 2 & 5 & 1 & 4 \\ 1 & 2 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 4 & -1 & 2 \\ 5 & 14 & 0 & 7 \\ 4 & 9 & 0 & 6 \\ 1 & 2 & 0 & 3 \end{vmatrix} =$

$$= -(-1)^4 \cdot \begin{vmatrix} 5 & 14 & 7 \\ 4 & 9 & 6 \\ 1 & 2 & 3 \end{vmatrix} = -\begin{vmatrix} 5 & -4 & 8 \\ 4 & -1 & 6 \\ 1 & 0 & 0 \end{vmatrix} = -1 \cdot (-1)^{1+3} \begin{vmatrix} -4 & 8 \\ -1 & 6 \end{vmatrix} =$$
$-(-24+8) = 16.$

II-usul. Determinantni uchburchak ko'rinishga keltirish usuli - determinantning bosh diagonalidan bir tomonida yotuvchi hamma elementlari nolga aylantirilib uchburchaksimon shaklga keltiriladi.

$$\Delta=\begin{vmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ 0 & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & a_{nn} \end{vmatrix} \quad \text{yoki} \quad \Delta=\begin{vmatrix} a_{11} & 0 & \dots & 0 \\ a_{21} & a_{22} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{vmatrix}$$

Uchburchak shaklidagi determinantning qiymati diagonallari elementlari ko'paytmasiga teng.

$\Delta= a_{11}\cdot a_{22}\cdot \dots \cdot a_{nn}.$

6-misol. Determinantlarni uchburchak shakliga keltirib hisoblang:

$$\Delta=\begin{vmatrix} 1 & 2 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 7 & 5 & 2 \\ -1 & 4 & -6 & 3 \end{vmatrix}$$

Yechish.

$$\begin{vmatrix} 1 & 2 & 2 & 1 \\ 1 & 2 & 4 & 2 \\ 2 & 7 & 5 & 2 \\ -1 & 4 & -6 & 3 \end{vmatrix}=\begin{vmatrix} 1 & 2 & 2 & 1 \\ 0 & 0 & 2 & 1 \\ 0 & 3 & 1 & 0 \\ 0 & 6 & -4 & 4 \end{vmatrix}=-\begin{vmatrix} 1 & 2 & 2 & 1 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 6 & -4 & 4 \end{vmatrix}=$$

$$=-\begin{vmatrix} 1 & 2 & 2 & 1 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 6 & -4 \end{vmatrix}=-\begin{vmatrix} 1 & 2 & 2 & 1 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 7 \end{vmatrix}=-42$$

Mustaqil bajarish uchun topshiriqlar.

1. Determinantlarni hisoblang:

1) $\begin{vmatrix} \sqrt{2} & -1 \\ 3 & \sqrt{8} \end{vmatrix}$; 2) $\begin{vmatrix} -2 & 4 \\ 3 & 5 \end{vmatrix}$; 3) $\begin{vmatrix} \sqrt{a} & -b \\ b & \sqrt{a} \end{vmatrix}$; 4) $\begin{vmatrix} 1 & 0 \\ 13 & 8 \end{vmatrix}$;

5) $\begin{vmatrix} -1 & 4 \\ -5 & 2 \end{vmatrix}$; 6) $\begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$; 7) $\begin{vmatrix} 5 & 3 \\ 6 & 4 \end{vmatrix}$; 8) $\begin{vmatrix} a+b & a-b \\ a-b & a+b \end{vmatrix}$;

9) $\begin{vmatrix} a & 1 \\ a^2 & a \end{vmatrix}$; 10) $\begin{vmatrix} 3 & -4 \\ 1 & 2 \end{vmatrix}$; 11) $\begin{vmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{vmatrix}$; 12) $\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\beta & \cos\beta \end{vmatrix}$;

13) $\begin{vmatrix} 5 & -3 & 2 \\ 4 & 1 & 0 \\ -2 & 3 & 1 \end{vmatrix}$; 14) $\begin{vmatrix} -1 & -4 & -2 \\ 0 & -3 & 0 \\ -2 & -3 & 2 \end{vmatrix}$; 15) $\begin{vmatrix} 9 & -5 & 6 \\ 2 & -1 & 3 \\ -1 & 2 & 3 \end{vmatrix}$;

16) $\begin{vmatrix} 3 & -1 & -5 \\ 3 & 0 & 4 \\ -2 & -3 & 4 \end{vmatrix}$; 17) $\begin{vmatrix} 3 & -2 & 1 \\ -2 & 1 & 3 \\ 2 & 0 & -2 \end{vmatrix}$; 18) $\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \\ 5 & 0 & 1 \end{vmatrix}$;

19) $\begin{vmatrix} 5 & -1 & 0 & 2 \\ 2 & 3 & 4 & 1 \\ 2 & -2 & -3 & 0 \\ 1 & 3 & 2 & -1 \end{vmatrix}$; 20) $\begin{vmatrix} 3 & 2 & -1 & 6 \\ 4 & -2 & 3 & 1 \\ 7 & 3 & 0 & 1 \\ 0 & 4 & -2 & -1 \end{vmatrix}$; 21) $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & 0 & 1 \\ 1 & 0 & 2 & -3 \\ 3 & 2 & 3 & 1 \end{vmatrix}$;

22) $\begin{vmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{vmatrix}$; 23) $\begin{vmatrix} 3 & -2 & 1 \\ -2 & 1 & 3 \\ 2 & 0 & -2 \end{vmatrix}$; 24) $\begin{vmatrix} 2 & 0 & 5 \\ 1 & 3 & 16 \\ 0 & -1 & 10 \end{vmatrix}$;

25) $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$;

2. Quyidagi matritsaning minorlarini toping.

$$A = \begin{pmatrix} 2 & -2 & 2 \\ 3 & 1 & -1 \\ 2 & -3 & 4 \end{pmatrix}; \quad B = \begin{pmatrix} -2 & 2 & -3 \\ -4 & 2 & 1 \\ -2 & 5 & 3 \end{pmatrix}; \quad C = \begin{pmatrix} 0 & -5 & 1 \\ 5 & -1 & 3 \\ 1 & -2 & -4 \end{pmatrix};$$

3. Quyidagi determinantlarni tartibini pasaytirish usuli bilan yeching.

1) $\begin{vmatrix} -1 & 3 & 2 \\ -2 & 5 & -2 \\ -2 & 4 & -1 \end{vmatrix}$; 2) $\begin{vmatrix} -4 & -5 & -3 \\ 2 & 0 & 1 \\ -3 & -3 & 2 \end{vmatrix}$;

3) $\begin{vmatrix} 7 & -1 & -6 \\ 4 & -2 & 0 \\ -5 & 2 & 1 \end{vmatrix}$; 4) $\begin{vmatrix} 0 & -1 & -5 \\ 3 & 3 & 4 \\ -4 & -3 & 2 \end{vmatrix}$;

5) $\begin{vmatrix} 4 & -3 & 3 \\ -2 & -2 & -2 \\ 1 & 5 & -1 \end{vmatrix}$; 6) $\begin{vmatrix} -1 & -2 & -3 \\ 3 & 2 & 1 \\ -2 & -1 & 0 \end{vmatrix}$;

7) $\begin{vmatrix} 2 & -1 & -6 \\ -3 & -1 & 0 \\ -2 & 0 & 1 \end{vmatrix}$; 8) $\begin{vmatrix} 0 & -2 & -5 \\ 3 & 3 & 3 \\ -3 & 0 & 2 \end{vmatrix}$;

9) $\begin{vmatrix} 1 & -1 & 0 & 2 \\ 2 & 3 & 4 & 2 \\ 2 & -2 & -4 & 0 \\ 3 & 4 & 2 & -3 \end{vmatrix}$; 10) $\begin{vmatrix} 3 & 2 & -1 & 3 \\ 4 & -2 & 3 & -9 \\ 7 & 3 & 0 & 0 \\ 0 & 14 & -2 & 6 \end{vmatrix}$; 11) $\begin{vmatrix} 1 & 2 & 0 & 2 \\ 2 & 3 & 4 & 2 \\ 2 & 2 & 0 & 0 \\ 3 & 4 & 1 & 3 \end{vmatrix}$;

12) $\begin{vmatrix} -3 & -2 & 1 & 2 \\ -2 & 0 & 4 & 3 \\ 1 & -2 & -4 & 0 \\ 6 & 2 & 2 & -1 \end{vmatrix}$; 13) $\begin{vmatrix} 1 & 2 & -2 & 5 \\ 4 & -3 & 3 & -9 \\ 2 & 3 & 0 & 0 \\ -1 & 7 & -3 & 0 \end{vmatrix}$; 14) $\begin{vmatrix} -1 & 2 & 1 & 0 \\ 4 & 3 & 4 & 2 \\ 0 & 2 & 0 & 3 \\ -3 & 1 & 1 & 2 \end{vmatrix}$;

4.Tenglamalarni yeching.

1) $\begin{vmatrix} y & 2 \\ -7 & y \end{vmatrix} = 23$; 2) $\begin{vmatrix} 2t-1 & t+1 \\ t+2 & t-1 \end{vmatrix} = -6$;

3) $\begin{vmatrix} sin5x & -cos3x \\ cos5x & sin3x \end{vmatrix} = 0$; 4) $\begin{vmatrix} t+2 & 6-2t \\ t+2 & t+3 \end{vmatrix} = 0$;

5) $\begin{vmatrix} 1 & -1 & -1 \\ t^2 & -2 & 0 \\ t & 2 & 1 \end{vmatrix} = 0$; 4) $\begin{vmatrix} 6 & 4 & 2t \\ 3 & t+2 & 1 \\ t-1 & 2 & 0 \end{vmatrix} = 0$;

2-§. Matritsalar va ular ustida amallar. Teskari matritsa va uni tuzish.

1.Ta'rif: m ta satr va n ta ustundan iborat bo'lgan to'g'ri to'rtburchak shaklidagi sonli jadvalga $m \times n$ o'lchamli matritsa, uni tashkil etgan sonlar matritsani elementlari deyiladi va quyidagi ko'rinishda yoziladi

$$A=\begin{pmatrix} a_{11}\ a_{12} \dots a_{1n} \\ a_{21}\ a_{22} \dots a_{2n} \\ \dots\dots\dots\dots\dots\dots\dots \\ a_{m1}\ a_{m2} \dots a_{mn} \end{pmatrix}$$

$a_{11}, a_{12}, \dots a_{1n}, a_{21}, a_{22}, \dots a_{2n}, \dots a_{m1}, a_{m2}, \dots a_{mn}$-matritsaning elementlari.

Agar $m = 1$ bo'lsa, satr matritsa hosil bo'ladi

A=$(a_{11} \quad a_{12} \cdots \quad a_{1n})$

Agar $n = 1$ bo'lsa, ustun matritsa hosil bo'ladi

$$A=\begin{pmatrix} a_{11} \\ a_{21} \\ \dots . \\ a_{m1} \end{pmatrix}$$

Agar $n = m$ bo'lsa, kvadrat matritsa hosil bo'ladi

$$A=\begin{pmatrix} a_{11}\ a_{12} \dots a_{1n} \\ a_{21}\ a_{22} \dots a_{2n} \\ \dots\dots\dots\dots\dots\dots\dots \\ a_{n1}\ a_{n2} \dots a_{nn} \end{pmatrix}$$

$a_{11}, a_{22}, \dots a_{nn}$-matritsaning bosh diagonali deyiladi.

Bosh diagonalidan tashqari barcha elementlari 0 lardan iborat bo'lgan kvadrat matritsaga diagonal matritsa deyiladi

$$A=\begin{pmatrix} a_{11}\ 0 \dots 0 \\ 0\ a_{22} \dots 0 \\ \dots \dots \dots \dots \\ 0\ 0 \dots a_{nn} \end{pmatrix}$$

Masalan: $A_{2\times2}=\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$ yoki $A_{3\times3}=\begin{pmatrix} 5 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Bosh diagonali elementlari 1 ga qolgan elementlari 0 ga teng bo'lgan matritsaga birlik matritsa deyiladi va E bilan belgilanadi

$$E=\begin{pmatrix} 1\ 0 \dots 0 \\ 0\ 1 \dots 0 \\ \dots \dots \dots \dots \\ 0\ 0 \dots 1 \end{pmatrix}$$

Birlik matritsaning determinanti 1 ga teng (detE=1).

Barcha elementlari nolga teng bo'lgan matritsaga nol matritsa deyiladi va u 0 bilan belgilanadi.

$0_{2\times2}=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ $\quad 0_{2\times3}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ $\quad 0_{3\times3}=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

Agar A va B matritsalar bir xil tartibli va ularning mos elementlari o'zaro teng bo'lsa , ya'ni $a_{ij}=b_{ij}$ bo'lsa, teng matritsalar deyiladi va A=B ko'rinishda yoziladi.

$A=\begin{pmatrix} a+a & a-a \\ \frac{a}{a} & a\cdot a \end{pmatrix}$ va $B=\begin{pmatrix} 2a & 0 \\ 1 & a^2 \end{pmatrix}$ matritsalar o'zaro teng.

Matritsalar ustida amallar:

➢ $m\times n$ o'lchamli ikkita martitsani qo'shishda mos elementlari bir biriga qo'shiladi

Xususiy holda 3x3 o'lchamli matritsani qo'shamiz,

Masalan,

$$A=\begin{pmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{pmatrix} \text{ va } B=\begin{pmatrix}b_{11} & b_{12} & b_{13}\\ b_{21} & b_{22} & b_{23}\\ b_{31} & b_{32} & b_{33}\end{pmatrix}$$ matritsalar berilgan bo'lsin,

U holda, bu matritsalarning yig'indisi quyidagi ifodaga teng:

$$A+B=\begin{pmatrix}a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13}\\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23}\\ a_{31}+b_{31} & a_{32}+b_{32} & a_{33}+b_{33}\end{pmatrix}$$

1-misol: $A=(3 \quad -2 \quad 0)$ va $B=(-1 \quad 5 \quad 6)$ $A+B$ ni toping.
Yechish:

$$A+B=(2 \quad 3 \quad 6)$$

➢ $m\times n$ o'lchamli matritsani $k\neq 0$ songa ko'paytirish uchun, matritsa elementlarini har birini shu k songa ko'paytiriladi.

$$kA=\begin{pmatrix}ka_{11} & ka_{12} & ka_{13}\\ ka_{21} & ka_{22} & ka_{23}\\ ka_{31} & ka_{32} & ka_{33}\end{pmatrix}.$$

2-misol: $A=\begin{pmatrix}2 & -2 & 2\\ 3 & 1 & -1\\ 2 & -3 & 4\end{pmatrix}$; B=$\begin{pmatrix}-2 & 2 & -3\\ -4 & 2 & 1\\ -2 & 5 & 3\end{pmatrix}$ matritsalar berilgan $2A+3B$ matritsani toping.

Yechish. $2A+3B=2\begin{pmatrix}2 & -2 & 2\\ 3 & 1 & -1\\ 2 & -3 & 4\end{pmatrix}+3\begin{pmatrix}-2 & 2 & -3\\ -4 & 2 & 1\\ -2 & 5 & 3\end{pmatrix}=$

$$=\begin{pmatrix}4 & -4 & 4\\ 6 & 2 & -2\\ 4 & -6 & 8\end{pmatrix}+\begin{pmatrix}-6 & 6 & -9\\ -12 & 6 & 3\\ -6 & 15 & 9\end{pmatrix}=\begin{pmatrix}-2 & 2 & -5\\ -6 & 8 & 1\\ -2 & 9 & 17\end{pmatrix}.$$

➢ $m\times q$ o'lchamli A matritsaning $q\times n$ o'lchamli B matritsaga ko'paytmasi deb, $m\times n$ o'chamli S matritsaga aytiladi, S matritsaning s_{ij}

elementi A matritsaning i- satri elementlarini B matritsaning j- ustunidagi mos elementlariga ko'paytmalari yig'indisiga teng.

Ko'paytma matritsa mavjud bo'lishi uchun A matritsani ustunlari soni B matritsaning satrlari soniga teng bo'lishi kerak.

$A=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}$ $\quad B=\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}$ berilsin , u holda AB matritsa quyidagicha aniqlanadi:

$$AB=\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{pmatrix}\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}=\begin{pmatrix} a_{11}b_{11}+a_{12}b_{21} & a_{11}b_{12}+a_{12}b_{22} \\ a_{21}b_{11}+a_{22}b_{21} & a_{21}b_{12}+a_{22}b_{22} \\ a_{31}b_{11}+a_{32}b_{21} & a_{31}b_{12}+a_{32}b_{22} \end{pmatrix}.$$

3-misol: Ushbu $A=\begin{pmatrix} 2 & -1 \\ 0 & 3 \\ 1 & 4 \end{pmatrix}$ $\quad B=\begin{pmatrix} 2 & 1 \\ -2 & 3 \end{pmatrix}$ matritsalarning AB ko'paytmasini toping.

Yechish. $AB=\begin{pmatrix} 2 & -1 \\ 0 & 3 \\ 1 & 4 \end{pmatrix}\cdot\begin{pmatrix} 2 & 1 \\ -2 & 3 \end{pmatrix}=$

$$=\begin{pmatrix} 2\cdot 2+(-1)\cdot(-2) & 2\cdot 1-1\cdot 3 \\ 0\cdot 2-2\cdot 3 & 0\cdot 1+3\cdot 3 \\ 1\cdot 2-2\cdot 4 & 1\cdot 1+4\cdot 3 \end{pmatrix}=\begin{pmatrix} 4+2 & 2-3 \\ 0-6 & 0+9 \\ 2-8 & 1+12 \end{pmatrix}=$$
$$\begin{pmatrix} 6 & -1 \\ -6 & 9 \\ -6 & 13 \end{pmatrix}.$$

4-misol: $A=\begin{pmatrix} 2 & 4 & 3 \\ 3 & 5 & 2 \\ 2 & 3 & 4 \end{pmatrix}$ va $B=\begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$ matritsalar berilgan AB matritsa topilsin.

Yechish:

$$AB = \begin{pmatrix} 2 & 4 & 3 \\ 3 & 5 & 2 \\ 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2\cdot 3 + 4\cdot 2 + 3\cdot 4 \\ 3\cdot 3 + 5\cdot 2 + 2\cdot 4 \\ 2\cdot 3 + 3\cdot 2 + 4\cdot 4 \end{pmatrix} =$$

$$= \begin{pmatrix} 6+8+12 \\ 9+10+8 \\ 6+6+16 \end{pmatrix} = \begin{pmatrix} 26 \\ 27 \\ 28 \end{pmatrix}$$

A matritsaga qarama qarshi matritsa deb $(-1)\cdot A$ matritsaga aytiladi.

Ya'ni

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \; bo'lsa, \qquad -A = \begin{pmatrix} -a_{11} & -a_{12} & -a_{13} \\ -a_{21} & -a_{22} & -a_{23} \\ -a_{31} & -a_{32} & -a_{33} \end{pmatrix}$$

bo'ladi.

➢ **Teskari matritsa**. Berilgan n-tartibli A kvadrat matritsaga teskari matritsa deb A^{-1} bilan belgilanuvchi va $AA^{-1} = A^{-1}A = E$ shartni qanoatlantiruvchi kvadrat matritsaga aytiladi.

Berilgan A matritsaga teskari matritsa mavjud bo'lishi uchun uning determinanti nolga teng bo'lmasligi kerak.

Berilgan n-tartibli A matritsaga teskari matritsa quyidagicha aniqlanadi:

$$A^{-1} = \frac{1}{|A|} \begin{pmatrix} A_{11}\, A_{21} \dots A_{n1} \\ A_{12}\, A_{22} \dots A_{n2} \\ \dots\dots\dots\dots\dots\dots\dots \\ A_{1n}\, A_{2n} \dots A_{nn} \end{pmatrix}$$

Berilgan 3-tartibli A matritsaga teskari matritsa quyidagicha aniqlanadi:

$$A^{-1} = \frac{1}{|A|} \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}$$

Quyidagi ikkinchi tartibli A matritsa

$$A = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

uchun teskari matritsa quyidagiga teng:

$$A^{-1} = \frac{1}{|A|}\begin{pmatrix} A_{11} & A_{21} \\ A_{12} & A_{22} \end{pmatrix}$$

5-misol: $A = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}$ matritsa berilgan . Bu matritsaga teskari matritsani toping. $AA^{-1} = E$ ekanligini tekshiring.

Yechish: A matritsaning determinantini hisoblaymiz.

$$detA = \begin{vmatrix} 3 & 1 \\ 2 & 0 \end{vmatrix} = -2 \neq 0$$

Demak, $detA = -2 \neq 0$ ekanligidan matritsaga teskari matritsa mavjud.
Algebraik to'ldiruvchilarini topamiz.

$A_{11} = (-1)^{1+1}0=0$ $\qquad$ $A_{12} = (-1)^{1+2}2 = -2$

$A_{21} = (-1)^{2+1}1 = -1$ $\qquad$ $A_{22} = (-1)^{2+2}3 = 3$

$$A^{-1} = \frac{1}{-2}\begin{pmatrix} 0 & -1 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} \\ 1 & -\frac{3}{2} \end{pmatrix}.$$

Endi $AA^{-1} = E$ ekanligini tekshiramiz,

$$AA^{-1} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 & \frac{1}{2} \\ 1 & -\frac{3}{2} \end{pmatrix} = \begin{pmatrix} 3\cdot 0 + 1\cdot 1 & 3\cdot\frac{1}{2} + 1\cdot\left(-\frac{3}{2}\right) \\ 2\cdot 0 + 0\cdot 1 & 2\cdot\frac{1}{2} + 0\cdot(-\frac{3}{2} \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Demak, $A^{-1} = \begin{pmatrix} 0 & \frac{1}{2} \\ 1 & -\frac{3}{2} \end{pmatrix}$ matritsa A matritsaga teskari matritsa ekan.

6-misol: $A = \begin{pmatrix} 3 & -1 & 0 \\ -2 & 1 & 1 \\ 2 & -1 & 4 \end{pmatrix}$ matritsa berilgan A^{-1} matritsa topilsin.

$AA^{-1} = E$ ekanligi tekshirilsin.

Yechish: A matritsaning determinantini hisoblaymiz.

$$detA = \begin{vmatrix} 3 & -1 & 0 \\ -2 & 1 & 1 \\ 2 & -1 & 4 \end{vmatrix}$$

$$= 3 \cdot 1 \cdot 4 + 0 \cdot (-2) \cdot (-1) + 2 \cdot 1 \cdot (-1) - 0 \cdot 1 \cdot 2 -$$

$$-4 \cdot (-1) \cdot (-2) - 3 \cdot 1 \cdot (-1) = 12 - 2 - 8 + 3 = 5 \neq 0$$

Demak, $detA = 5 \neq 0$ ekanligidan matritsaga teskari matritsa mavjud.

Algebraik to'ldiruvchilarini topamiz.

$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 1 \\ -1 & 4 \end{vmatrix}$=4+1=5 ; $A_{12} = (-1)^{1+2} \begin{vmatrix} -2 & 1 \\ 2 & 4 \end{vmatrix} =$

$= -(-8 - 2)$=10;

$A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 0 \\ -1 & 4 \end{vmatrix} = -(-4) = 4;$ $A_{22} = (-1)^{2+2} \begin{vmatrix} 3 & 0 \\ 2 & 4 \end{vmatrix} = 12;$

$A_{13} = (-1)^{1+3} \begin{vmatrix} -2 & 1 \\ 2 & -1 \end{vmatrix} = 0;$ $A_{23} = (-1)^{2+3} \begin{vmatrix} 3 & -1 \\ 2 & -1 \end{vmatrix} =$

$= -(-3 + 2) = 1;$

$A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 0 \\ 1 & 1 \end{vmatrix} = -1;$ $A_{32} = (-1)^{3+2} \begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} = -3;$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 3 & -1 \\ -2 & 1 \end{vmatrix} = 3 - 2 = 1 ;$

$$A^{-1} = \frac{1}{|A|} \begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 5 & 4 & -1 \\ 10 & 12 & -3 \\ 0 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \frac{4}{5} & -\frac{1}{5} \\ 2 & \frac{12}{5} & -\frac{3}{5} \\ 0 & \frac{1}{5} & \frac{1}{5} \end{pmatrix}.$$

Endi $AA^{-1} = E$ ekanligini tekshiramiz,

$$AA^{-1}=\begin{pmatrix}3 & -1 & 0\\ -2 & 1 & 1\\ 2 & -1 & 4\end{pmatrix}\cdot\begin{pmatrix}1 & \frac{4}{5} & -\frac{1}{5}\\ 2 & \frac{12}{5} & -\frac{3}{5}\\ 0 & \frac{1}{5} & \frac{1}{5}\end{pmatrix}=$$

$$=\begin{pmatrix}3\cdot1-1\cdot2+0 & 3\cdot\frac{4}{5}-1\cdot\frac{12}{5} & 3\cdot\left(-\frac{1}{5}\right)+1\cdot\frac{3}{5}\\ -2\cdot1+1\cdot2 & -2\cdot\frac{4}{5}+1\cdot\frac{12}{5}+1\cdot\frac{1}{5} & -2\cdot\left(-\frac{1}{5}\right)+1\cdot\left(-\frac{3}{5}\right)+1\cdot\frac{1}{5}\\ 2\cdot1-1\cdot2 & 2\cdot\frac{4}{5}-1\cdot\frac{12}{5}+4\cdot\frac{1}{5} & 2\cdot\left(-\frac{1}{5}\right)+(-1)\cdot\left(-\frac{3}{5}\right)+4\cdot\frac{1}{5}\end{pmatrix}$$

Demak yuqoridagi hisoblashlarni bajarsak $\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$ ekanligi kelib chiqadi.

$$AA^{-1}=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}=E.$$

Mustaqil bajarish uchun topshiriqlar.

1. $A=\begin{pmatrix}-2 & 4\\ 5 & 6\end{pmatrix}$ va $B=\begin{pmatrix}3 & 0\\ 1 & 5\end{pmatrix}$ $3A+2B$=?

2. $A=\begin{pmatrix}-1 & 2 & 3\\ 3 & -1 & 4\end{pmatrix}$ va $B=\begin{pmatrix}-2 & 3 & 0\\ 4 & 2 & 1\end{pmatrix}$ $4A+2B$=?

3. $A=\begin{pmatrix}2 & -3\\ 0 & 3\\ 1 & 5\end{pmatrix}$ va $B=\begin{pmatrix}-3 & 4\\ -2 & 1\\ 3 & -2\end{pmatrix}$ A-3B=?

4. $A=\begin{pmatrix}-2 & 2 & 5\\ -3 & -1 & 1\\ 0 & -4 & 3\end{pmatrix}$; B=$\begin{pmatrix}-1 & 5 & -5\\ -3 & 4 & 3\\ -1 & 3 & 2\end{pmatrix}$ 2A+3B=?

5. Ushbu $A = \begin{pmatrix} 3 & -2 \\ -1 & 0 \\ 4 & -4 \end{pmatrix}$ $\quad B = \begin{pmatrix} -2 & -1 \\ -3 & 3 \end{pmatrix}$ matritsalarning $A \cdot B$ ko'paytmasini toping.

6. $A = \begin{pmatrix} -3 & -3 & 5 \\ -1 & 5 & -2 \\ 1 & 2 & 0 \end{pmatrix}$ va $B = \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$ matritsalar berilgan $A \cdot B$ matritsa toping.

7. $A = \begin{pmatrix} 1 & -3 & 0 \\ 2 & -1 & -2 \\ 3 & -5 & -4 \end{pmatrix}$; B=$\begin{pmatrix} 2 & 2 & -3 \\ 4 & 3 & 1 \\ 2 & 1 & 5 \end{pmatrix}$ matritsalar berilgan $A \cdot B$ matritsani toping.

8. Ushbu matrisalarga teskari matrisalarni toping.

a) $\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $\quad \begin{pmatrix} 3 & 4 \\ 5 & 7 \end{pmatrix}$ $\begin{pmatrix} 3 & 3 \\ 4 & 4 \end{pmatrix}$ $\begin{pmatrix} -5 & 1 \\ 3 & 2 \end{pmatrix}$

b) $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $A = \begin{pmatrix} -2 & 4 \\ 5 & 6 \end{pmatrix}$ $B = \begin{pmatrix} cos\alpha & -sin\alpha \\ sin\alpha & cos\alpha \end{pmatrix}$;

9. $A = \begin{pmatrix} -2 & -1 & 0 \\ -2 & 1 & 1 \\ 2 & -1 & 4 \end{pmatrix}$ matritsa berilgan A^{-1} matritsa topilsin.

10. Quyidagi matritsaning a_{12} elementining algebraik to'ldiruvchisini toping.

$$A = \begin{pmatrix} -3 & -1 & 3 \\ 1 & -2 & -1 \\ -4 & -3 & 6 \end{pmatrix}$$

11 dan 30 gacha C va D matritsalarni ko'paytiring.

11. $C = \begin{pmatrix} 3 & 5 \\ -2 & -4 \end{pmatrix}$, $D = \begin{pmatrix} -5 & 3 \\ 6 & 9 \end{pmatrix}$;

12. $C = \begin{pmatrix} -4 & 2 \\ -7 & 3 \end{pmatrix}$, $D = \begin{pmatrix} -1 & 3 & 1 \\ -2 & 2 & 0 \end{pmatrix}$;

13. $C = \begin{pmatrix} 9 & 3 \\ 1 & 3 \end{pmatrix}$, $D = \begin{pmatrix} 6 & 8 \\ -3 & -5 \end{pmatrix}$;

14. $C = \begin{pmatrix} 0 & -7 \\ 2 & 5 \end{pmatrix}$, $D = \begin{pmatrix} -4 & 3 & 6 \\ 1 & -1 & 2 \end{pmatrix}$

15. $C = \begin{pmatrix} -2 & -1 \\ -4 & 3 \\ 3 & 0 \end{pmatrix}$, $D = \begin{pmatrix} 7 & 1 \\ 3 & 4 \end{pmatrix}$;

16. $C = (6 \quad -4 \quad 2)$, $D = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$;

17. $C = \begin{pmatrix} -1 & -5 & 2 \\ -4 & 4 & -2 \\ 0 & 3 & 3 \end{pmatrix}$, $D = \begin{pmatrix} -2 \\ 1 \\ 5 \end{pmatrix}$;

18. $C = \begin{pmatrix} -1 & 2 & 3 \\ 3 & -1 & 4 \end{pmatrix}$, $D = \begin{pmatrix} 3 & -2 \\ -1 & 0 \\ 4 & -4 \end{pmatrix}$;

19. $C = \begin{pmatrix} 4 & -5 & 1 \\ 1 & 0 & -3 \\ 2 & -5 & -3 \end{pmatrix}$, $D = \begin{pmatrix} 1 & -1 & -3 \\ 2 & 3 & 2 \\ -3 & 0 & 1 \end{pmatrix}$;

20. $C = \begin{pmatrix} -1 & -2 & 5 \\ -3 & 2 & -2 \\ 2 & -3 & 1 \end{pmatrix}$, $D = \begin{pmatrix} 2 & -4 \\ -2 & 4 \\ 0 & -1 \end{pmatrix}$;

21. $C = \begin{pmatrix} 0 & 6 \\ -3 & 1 \\ 5 & -1 \end{pmatrix}$, $D = \begin{pmatrix} -10 & -3 \\ -7 & 5 \end{pmatrix}$;

22. $C=\begin{pmatrix}-13 & -11 & 5\\ -12 & 5 & -3\\ 11 & 12 & 0\end{pmatrix}$, $D=\begin{pmatrix}-3\\ 4\\ 1\end{pmatrix}$;

23. $C=\begin{pmatrix}-10 & 5 & 3\\ 1 & -1 & 4\end{pmatrix}$, $D=\begin{pmatrix}3 & -8\\ -1 & 2\\ 4 & -4\end{pmatrix}$;

24. $C=\begin{pmatrix}7 & -3 & 3\\ 2 & -1 & -2\\ 6 & 0 & -5\end{pmatrix}$, $D=\begin{pmatrix}0 & -3 & -4\\ 2 & 3 & 1\\ 2 & 1 & 1\end{pmatrix}$;

25. $C=\begin{pmatrix}-9 & -13 & 5\\ -1 & 6 & -7\\ 10 & -3 & 2\end{pmatrix}$, $D=\begin{pmatrix}2 & -2\\ -1 & 2\\ 6 & -6\end{pmatrix}$;

26. $C=\begin{pmatrix}1 & -3 & 7\\ -2 & 3 & -2\\ 0 & -1 & 0\end{pmatrix}$, $D=\begin{pmatrix}12 & 2 & 3\\ 4 & 13 & 1\\ -2 & 1 & 15\end{pmatrix}$;

27. $C=\begin{pmatrix}1 & -5 & 3\\ 5 & -10 & -2\\ 3 & 8 & -4\end{pmatrix}$, $D=\begin{pmatrix}-3\\ 11\\ 1\end{pmatrix}$;

28. $C=(13 \quad -12 \quad 10)$, $D=\begin{pmatrix}-2\\ 5\\ 4\end{pmatrix}$;

29. $C=\begin{pmatrix}4 & 7 & 3\\ -4 & -1 & -2\\ -1 & 0 & -1\end{pmatrix}$, $D=\begin{pmatrix}0 & 2 & -3\\ 4 & 3 & 10\\ 9 & 1 & 6\end{pmatrix}$;

30. $C=\begin{pmatrix}5 & -3 & 2\\ 2 & -4 & -3\\ -1 & -5 & -5\end{pmatrix}$, $D=\begin{pmatrix}2 & 2 & -1\\ 9 & 0 & 1\\ -2 & 1 & 1\end{pmatrix}$;

3-§. Chiziqli tenglamalar sistemasi va ularni yechish usullari.

$$a_1x_1 + a_2x_2 + \ldots + a_nx_n = b,$$

ko'rinishdagi tenglamaga chiziqli tenglama deb aytiladi, bu yerda a_i va b_i -sonlar, x_i- nomalumlar.

Chiziqli tenglamada $b = 0$ bo'lsa, u holda bir jinsli, agar $b \neq 0$ bo'lsa, bir jinsli bo'lmagan tenglama deyiladi.

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m \end{cases}$$

ko'rinishdagi sistemaga chiziqli tenglamalar sistemasi deyiladi, bu yerda

a_{ij}, b_i-sonlar, x_j- noma'lumlar, n – noma'lumlar soni, m – tenglamalar soni $(i = \overline{1,m};\ j = \overline{1,n})$.

Kramer qoidasi.

Ikkita chiziqli tenglamalardan iborat quyidagi sistemaga

$$\begin{cases} a_{11}x + a_{12}y = b_1 \\ a_{21}x + a_{22}y = b_2 \end{cases}$$

ikki noma'lumli chiziqli tenglamalar sistemasi deyiladi, bunda $a_{11}, a_{12}, a_{21}, a_{22}$ sonlar tenglamalar sistemasining koeffitsientlari, b_1 va b_2 sonlar ozod hadlar deyiladi.

Ikki noma'lumli chiziqli tenglamalar sistemaning koeffitsientlaridan ushbu

$$\Delta = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}$$

determinantni hosil qilamiz, so'ng bu determinantning birinchi ustunidagi elementlarni ozod hadlar bilan almashtirib

$$\Delta_x = \begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}$$

determinantni hosil qilamiz,

Ikkinchi ustundagi elementlarni ozod hadlar bilan almashtirib

$$\Delta_y = \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}$$

determinantni hosil qilamiz.

Agar $\Delta \neq 0$ bo'lsa va Δ_x, Δ_y lardan aqalli bittasi noldan farqli bo'lsa, u holda chiziqli tenglamalar sistemasi yagona (x, y) yechimga ega bo'lib,

$x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}$ bo'ladi .

Agar $\Delta = 0$ bo'lib, $\Delta_x \neq 0$ va $\Delta_y \neq 0$ bo'lsa, u holda chiziqli tenglamalar sistemasi yechimga ega bo'lmaydi.

Agar $\Delta = \Delta_x = \Delta_y = 0$ bo'lsa, u holda chiziqli tenglamalar sistemasi cheksiz ko'p yechimga ega bo'ladi.

1-misol. $\begin{cases} x - 2y = -5 \\ 3x + 4y = 15 \end{cases}$ tenglamalar sistemasini Kramer usulida yeching.

Yechish:

$\Delta = \begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix} = 4 + 6 = 10$ $\qquad$ $\Delta_x = \begin{vmatrix} -5 & -2 \\ 15 & 4 \end{vmatrix} = -20 + 30 = 10$

$\Delta_y = \begin{vmatrix} 1 & -5 \\ 3 & 15 \end{vmatrix} = 15 + 15 = 30$ $\qquad$ $x = \frac{10}{10} = 1, \quad y = \frac{30}{10} = 3.$

2-misol. $\begin{cases} 3x + 6y = 5 \\ x + 2y = 4 \end{cases}$ Kramer usulida yeching.

Yechish:

$\Delta = \begin{vmatrix} 3 & 6 \\ 1 & 2 \end{vmatrix} = 6 - 6 = 0$ $\qquad$ $\Delta_x = \begin{vmatrix} 5 & 6 \\ 4 & 2 \end{vmatrix} = 10 - 24 = -14$

$\Delta_y = \begin{vmatrix} 3 & 5 \\ 1 & 4 \end{vmatrix} = 12 - 5 = 7$

Demak, $\Delta = 0$ bo'lib, $\Delta_x \neq 0$ va $\Delta_y \neq 0$ bo'lgani uchun, chiziqli tenglamalar sistemasi yechimga ega emas.

3-misol. $\begin{cases} 2x + y = 1 \\ 4x + 2y = 2 \end{cases}$ Kramer usulida yeching.

Yechish:

$\Delta = \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} = 4 - 4 = 0$ $\qquad \Delta_x = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 2 - 2 = 0$

$\Delta_y = \begin{vmatrix} 2 & 1 \\ 4 & 2 \end{vmatrix} = 4 - 4 = 0$

Demak, $\Delta = \Delta_x = \Delta_y = 0$ bo'lgani uchun, chiziqli tenglamalar sistemasi cheksiz ko'p yechimga ega.

$$\begin{cases} a_{11}x + a_{12}y + a_{13}z = b_1 \\ a_{21}x + a_{22}y + a_{23}z = b_2 \\ a_{31}x + a_{32}y + a_{33}z = b_3 \end{cases}$$

ushbu uch noma'lumli tenglamalar sistemasi berilgan bo'lsin. Bu chiziqli tenglamalar sistemasining koeffitsientlaridan ushbu

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

determinantni hosil qilamiz, so'ng bu determinantning birinchi ustunidagi elementlarni ozod hadlar bilan almashtirib

$$\Delta_x = \begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}$$

determinantni hosil qilamiz, ikkinchi ustundagi elementlarni ozod hadlar bilan almashtirib,

$$\Delta_y = \begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}$$

determinantni hosil qilamiz.

Uchunchi ustundagi elementlarni ozod hadlar bilan almashtirib

$$\Delta_z = \begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}$$

determinantni hosil qilamiz.

Agar $\Delta \neq 0$ bo'lsa, u holda chiziqli tenglamalar sistemasi (x, y, z) yechimga ega bo'lib,

$x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}, z = \frac{\Delta_z}{\Delta}$ yechimlarga ega bo'lamiz.

Agar $\Delta = 0$ bo'lib, $\Delta_x \neq 0$ va $\Delta_y \neq 0$, $\Delta_z \neq 0$ bo'lsa, u holda chiziqli tenglamalar sistemasi yechimga ega bo'lmaydi.

Agar $\Delta = \Delta_x = \Delta_y = \Delta_z = 0$ bo'lsa, u holda chiziqli tenglamalar sistemasi cheksiz ko'p yechimga ega bo'ladi.

4-Misol. $\begin{cases} x + 2y + z = 8 \\ 3x + 2y + z = 10 \\ 4x + 3y - 2z = 4 \end{cases}$ tnglamalar sistemasini Kramer usulida yeching.

$$\Delta = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 2 & 1 \\ 4 & 3 & -2 \end{vmatrix} = 1 \cdot 2 \cdot (-2) + 1 \cdot 3 \cdot 3 +$$

$$+4 \cdot 2 \cdot 1 - 1 \cdot 2 \cdot 4 - 1 \cdot 1 \cdot 3 + 2 \cdot 2 \cdot 3 = -4 + 9 + 8 - 8 - 3 + 12 = 14 \neq 0$$

$$\Delta_x = \begin{vmatrix} 8 & 2 & 1 \\ 10 & 2 & 1 \\ 4 & 3 & -2 \end{vmatrix} = -32 + 8 + 30 + 40 - 24 - 4 = 14$$

$$\Delta_y = \begin{vmatrix} 1 & 8 & 1 \\ 3 & 10 & 1 \\ 4 & 4 & -2 \end{vmatrix} = -26 + 12 + 32 - 40 - 4 + 48 = 28$$

$$\Delta_z = \begin{vmatrix} 1 & 2 & 8 \\ 3 & 2 & 10 \\ 4 & 3 & 4 \end{vmatrix} = 8 + 80 + 72 - 64 - 24 - 30 = 42$$

$x = \frac{\Delta_x}{\Delta} = \frac{14}{14} = 1$, $y = \frac{\Delta_y}{\Delta} = \frac{28}{14} = 2$, $z = \frac{\Delta_z}{\Delta} = \frac{42}{14} = 3.$

Gauss usuli: n nomalumli n ta chiziqli tenglamalar sistemasini $n \geq 4$ bo'lganda Kramer qoidasi bo'yicha yechish qiyinchilik tug'diradi, shu sababli bunday holatlarda Gauss usulidan foydalanish qulaylik tug'diradi. Gauss usulida noma'lumlarni ketma-ket yo'qotib uchburchaksimon shaklga keltiriladi va oxirgi tenglamadan boshlab ketma-ket noma'lumlar topiladi.

5-Misol. $\begin{cases} x - 2y + 3z = 6 \\ 2x + 3y - 4z = 20 \\ 3x - 2y - 5z = 6 \end{cases}$ Gauss usulida yeching.

$$\begin{cases} x - 2y + 3z = 6 \\ 7y - 10z = 8 \\ 4y - 14z = -12 \end{cases} \Rightarrow \begin{cases} x - 2y + 3z = 6 \\ 7y - 10z = 8 \\ 2y - 7z = -6 \end{cases} \Rightarrow$$

$$\begin{cases} x - 2y + 3z = 6 \\ 7y - 10z = 8 \\ -\frac{29}{7}z = -\frac{58}{7} \end{cases}$$

$$\begin{cases} x - 2y + 3z = 6 \\ 7y - 10z = 8 \\ z = 2 \end{cases} \Rightarrow \begin{cases} x = 8 \\ y = 4 \\ z = 2 \end{cases}$$ Javob: $x = 8$, $y = 4$, $z = 2$

Matritsa usuli: Quyidagi n ta noma'lumli n ta chiziqli tenglamalar sistemasini qaraylik:

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ a_{n1}x_1 + a_{n2}x_2 + \cdots + a_{nn}x_n = b_n \end{cases}$$

Ushbu belgilarni kiritamiz:

$$A=\begin{pmatrix} a_{11}\ a_{12} \dots a_{1n} \\ a_{21}\ a_{22} \dots a_{2n} \\ \dots\dots\dots\dots \\ a_{n1}\ a_{n2} \dots a_{nn} \end{pmatrix}, B = \begin{pmatrix} b_1 \\ b_2 \\ \dots \\ b_n \end{pmatrix}, X = \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{pmatrix}.$$

$AX = B.$

Agar A matritsa determinant $detA \neq 0$ bo'lsa, u holda A^{-1} teskari matritsa mavjud.

Tenglamaning ikkala qismini A^{-1} ga chapdan ko'paytirib, quyidagi ifodani hosil qilamiz:

$A^{-1}AX = A^{-1}B.$

$$A^{-1}A = E \; va \;\; EX = X$$

ekanligidan, $X = A^{-1}B$ kelib chiqadi.

6-misol: $\begin{cases} 3x + 4y - z = 8 \\ 2x + y + z = 2 \\ 3x - y + 2z = 0 \end{cases}$ chiziqli tenglamalar sistemasini matritsa usulida yeching.

Yechish: $A = \begin{pmatrix} 3 & 4 & -1 \\ 2 & 1 & 1 \\ 3 & -1 & 2 \end{pmatrix}$ $X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ $\quad B = \begin{pmatrix} 8 \\ 2 \\ 0 \end{pmatrix}$

$$AX = B \;\; \Rightarrow X = A^{-1} \cdot B$$

A matritsaning determinantini hisoblaymiz,

$$|A| = (6 + 2 + 12) - (-3 - 3 + 16) = 10 \neq 0$$

A matritsaning algebraik to'ldiruvchilarini topamiz,

$A_{11} = \begin{vmatrix} 1 & 1 \\ -1 & 2 \end{vmatrix} = 3 \qquad A_{12} = -\begin{vmatrix} 2 & 1 \\ 3 & 2 \end{vmatrix} = -1 \qquad A_{13} = \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = -5$

$A_{21} = \begin{vmatrix} 4 & -1 \\ -1 & 2 \end{vmatrix} = -7 \qquad A_{22} = \begin{vmatrix} 3 & -1 \\ 3 & 2 \end{vmatrix} = 9 \qquad A_{23} = -\begin{vmatrix} 3 & 4 \\ 3 & -1 \end{vmatrix} = 15$

$A_{31} = \begin{vmatrix} 4 & -1 \\ 1 & 1 \end{vmatrix} = 5 \qquad A_{32} = -\begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} = -5 \qquad A_{33} = \begin{vmatrix} 3 & 4 \\ 2 & 1 \end{vmatrix} = -5$

$$X = A^{-1} \cdot B = \frac{1}{10}\begin{pmatrix} 3 & -7 & 5 \\ -1 & 9 & -5 \\ -5 & 15 & -5 \end{pmatrix} \cdot \begin{pmatrix} 8 \\ 2 \\ 0 \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 24 - 14 + 0 \\ -8 + 18 - 0 \\ -40 + 30 - 0 \end{pmatrix} =$$

$= \frac{1}{10}\begin{pmatrix} 10 \\ 10 \\ -10 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$. $\qquad$ J: $X = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$

Mustaqil bajarish uchun topshiriqlar.

1. Chiziqli tenglamalar sistemasini Kramer usulida yeching.

a) $\begin{cases} 3x + y = 7 \\ -5x + 2y = 3 \end{cases}$ b) $\begin{cases} 2x - y = 5 \\ 3x + 2y = 4 \end{cases}$ v) $\begin{cases} 6x + 5y = 8 \\ 4x - 3y = 18 \end{cases}$

g) $\begin{cases} 2x + 5y = 3 \\ 4x + 10y = 6 \end{cases}$; d) $\begin{cases} 5x + 3y = 7 \\ 10x + 6y = 2 \end{cases}$ e) $\begin{cases} 2x + 3y = 7 \\ 4x - 5y = 2 \end{cases}$

k) $\begin{cases} 4x_1 - x_2 + x_3 = 3 \\ x_1 - 2x_2 + 2x_3 = 1 \\ 3x_1 + x_2 - x_3 = 4 \end{cases}$ l) $\begin{cases} x_1 - 2x_2 - x_3 = -2 \\ 2x_1 - x_2 = -1 \\ x_2 + x_3 = -2 \end{cases}$ m) $\begin{cases} 2x_1 + x_2 - x_3 = 0 \\ 3x_2 + 4x_3 = -6 \\ x_1 + x_3 = 1 \end{cases}$

2. Chiziqli tenglamalar sistemasini Gauss usulida yeching.

a) $\begin{cases} x_1 + 5x_2 + x_3 = -7 \\ 2x_1 - x_2 - 3x_3 = -1 \\ 3x_1 + 4x_2 + 2x_3 = 8 \end{cases}$ b) $\begin{cases} x_1 - 4x_2 - 2x_3 = -7 \\ 3x_1 + x_2 - x_3 = 5 \\ -3x_1 + 5x_2 + 6x_3 = 7 \end{cases}$

v) $\begin{cases} x_1 + 7x_2 - 10x_3 = -5 \\ 2x_1 + 5x_2 + x_3 = -1 \\ x_1 + 7x_2 - 10x_3 = -5 \end{cases}$ g) $\begin{cases} 3x_1 - x_2 + 5x_3 = 12 \\ 2x_1 - x_2 + 5x_3 = 10 \\ 5x_1 + 2x_2 - 13x_3 = 21 \end{cases}$

d) $\begin{cases} 3x_1 + 5x_2 + 2x_3 = 8 \\ x_1 - 3x_2 + x_3 = 3 \\ 4x_1 + 2x_2 + x_3 = 4 \end{cases}$ e) $\begin{cases} 2x_1 + 3x_2 + 4x_3 + 2x_4 = 3, \\ x_1 + 2x_2 + 2x_3 + x_4 = 2, \\ 2x_1 + 3x_2 + 4x_3 + x_4 = -1, \\ x_1 + 4x_2 + 3x_3 + 2x_4 = 5 \end{cases}$

3. Chiziqli tenglamalar sistemasini matritsa usulida yeching.

a) $\begin{cases} 2x_1 - x_2 = -1 \\ x_1 + 2x_2 - x_3 = -2 \\ x_2 + x_3 = -2 \end{cases}$ b) $\begin{matrix} 4x_1 + 2x_2 - x_3 = 0 \\ x_1 + 2x_2 + x_3 = 1 \\ x_2 - x_3 = -3 \end{matrix}$

v) $\begin{cases} x_1+x_2+x_3=4, \\ x_1+2x_2+4x_3=4, \\ x_1+3x_2+9x_3=2 \end{cases}$ g) $\begin{cases} 3x_1+5x_2+x_3=2 \\ 4x_1+x_2+2x_3=8 \\ 5x_1-3x_2+3x_3=4 \end{cases}$

d) $\begin{cases} x_1+2x_2+x_3=-1 \\ 3x_1-5x_2+3x_3=-14 \\ 2x_1+7x_2-x_3=10 \end{cases}$ e) $\begin{cases} 3x+y-z=2 \\ 2x-3y+z=-1 \\ x-y+2z=5 \end{cases}$

Nazorat testlari.

1. Determinantni hisoblang . $\begin{vmatrix} -4 & 3 \\ -4 & -5 \end{vmatrix}$

A) -32 B)32 C)10 D)8

2.Berilgan determinantlarni taqqoslang: $\Delta_1=\begin{vmatrix} 5 & 1 \\ -6 & 4 \end{vmatrix}$; $\Delta_2=\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}$;

A) $\Delta_1>\Delta_2$ B) $\Delta_1=2\Delta_2$; C) $\Delta_1-\Delta_2<1$; D) $\Delta_2+5=\Delta_2$;

3. Berilgan determinantlarni taqqoslang: $\Delta_1=\begin{vmatrix} -8 & 7 \\ -1 & 3 \end{vmatrix}$; $\Delta_2=\begin{vmatrix} 5 & 3 \\ -2 & 6 \end{vmatrix}$;

A) $\Delta_1<\Delta_2$

B) $\Delta_1=3\Delta_2$; C) $\Delta_1-\Delta_2<1$; D) $\Delta_2+5=\Delta_2$;

4. Agar $\Delta=\begin{vmatrix} a_3 & a_4 \\ a_1 & a_2 \end{vmatrix}$ bo'lsa a_4 elmentning minorini toping.

A) a_3 B) a_1 C) a_2 D) a_4

5. Agar $\Delta=\begin{vmatrix} a_3 & a_4 \\ a_1 & a_2 \end{vmatrix}$ bo'lsa a_3 elmentining minorini toping.

A) a_3 B) a_1 C) a_2 D) a_4

6. Agar $\Delta=\begin{vmatrix} b_1 & b_3 \\ b_2 & b_4 \end{vmatrix}$ bo'lsa b_4 elmentining algebraik to'ldiruvchisi toping.

A) $(-1)^3 b_3$ B) $(-1)^2 b_4$ C) $(-1)^2 b_2$ D) b_1

7. Determinantni hisoblang $\begin{vmatrix} 3 & 2 & 1 \\ -2 & 0 & 3 \\ 0 & 3 & 2 \end{vmatrix}$

A) 4 B) -29 C) 29 D)13

8. Determinantni hisoblang $\begin{vmatrix} 5 & 4 & 3 \\ 0 & -1 & 2 \\ 6 & -1 & 1 \end{vmatrix}$

A) 71 B)-56 C)15 D)12

9. Quyidagi matritsaning M_{12} minorlarini toping.

$$A = \begin{pmatrix} 2 & -2 & 2 \\ 3 & 1 & -1 \\ 2 & -3 & 4 \end{pmatrix}$$

A) -14 B) -10 C) 10 D) 14

10. Quyidagi matritsaning a_{22} elementining algebraik to'ldiruvchisini toping.

$$A = \begin{pmatrix} 2 & -2 & 2 \\ 3 & 1 & -1 \\ 2 & -3 & 4 \end{pmatrix}$$

A)3 B) 3 C) 4 D) 12

11. $B=\begin{pmatrix} 4 & -3 \\ 2 & 1 \end{pmatrix}$ matritsa uchun $detB$ ni hisoblang.

A)22 B) 10 C)12 D)2

12. $A=\begin{pmatrix} 4 & 2 \\ 5 & 3 \end{pmatrix}$ matritsa uchun $detA$ ni toping

A)10 B) -3 C) 22 D) 2

13. $B=\begin{pmatrix} 5 & 3 \\ 3 & 1 \end{pmatrix}$ matritsa uchun $detB$ ni toping.

A)-4 B) 5 C) 22 D) 6

14. $C=\begin{pmatrix} 3 & 4 \\ 3 & 5 \end{pmatrix}$ matritsa uchun $detC$ ni toping.

A)-4 B) 3 C) 4 D) 6

15. A matritsa o'lchovini aniqlang.

$$A=\begin{pmatrix}0 & 6\\ 5 & 1\\ 2 & 3\\ 2 & 0\end{pmatrix}$$

A) 4×3 B) 5×2 C) 4×2 D) 8×4

16. B matritsa o'lchovini aniqlang

$$B=\begin{pmatrix}3 & 2 & 1 & 0 & 1\\ 4 & 2 & 3 & -4 & 1\end{pmatrix}.$$

A) 2×5 B) 5×2 C) 4×2 D) 8×4

17. Quyidagilardan qaysi biri birlik matritsa hisoblanadi.

A) $\begin{pmatrix}1 & 0 & 0\\ 1 & 0 & 0\\ 1 & 1 & 1\end{pmatrix}$ B) $\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$ C) $\begin{pmatrix}0 & 1 & 0\\ 1 & 1 & 1\\ 0 & 1 & 0\end{pmatrix}$ D) $\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$

18. $A=\begin{pmatrix}2 & 1\\ 4 & -6\\ 7 & 3\end{pmatrix}$ $B=\begin{pmatrix}-4 & 3\\ 0 & 4\\ 2 & -8\end{pmatrix}$ bo'lsa, $2A+B$ ni toping.

A) $\begin{pmatrix}0 & 5\\ 8 & -8\\ 16 & -2\end{pmatrix}$ B) $\begin{pmatrix}0 & 5\\ 4 & -2\\ 16 & -2\end{pmatrix}$

C) $\begin{pmatrix}0 & 5\\ 4 & -8\\ 16 & -2\end{pmatrix}$ D) $\begin{pmatrix}0 & 5\\ 4 & -8\\ 16 & -12\end{pmatrix}$

19. Agar $A=\begin{pmatrix}1 & -2\\ 2 & 0\\ 5 & 1\end{pmatrix}$ va $B=\begin{pmatrix}0 & 3\\ 6 & 7\\ -8 & 2\end{pmatrix}$ bulsa A+B yig'indini hisoblang:

A) $\begin{pmatrix} 1 & 1 \\ 9 & 19 \\ 13 & 11 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 1 \\ 8 & 7 \\ -3 & 3 \end{pmatrix}$

C) $\begin{pmatrix} 1 & 1 \\ 9 & 19 \\ -13 & -11 \end{pmatrix}$ D) $\begin{pmatrix} 1 & 1 \\ 19 & 19 \\ 3 & -11 \end{pmatrix}$

20. Ko'paytmani toping. $\begin{pmatrix} 2 & 1 & -3 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix}$

A) (8) B) $\begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ C) $\begin{pmatrix} 4 & 3 & -3 \end{pmatrix}$ D) (10)

21. Agar $A=\begin{pmatrix} 1 & -2 \\ 2 & 0 \\ 5 & 1 \end{pmatrix}$ va $B=\begin{pmatrix} 0 & 3 \\ 6 & 7 \\ -8 & 2 \end{pmatrix}$ bo'lsa A+B yig'indini hisoblang:

A) $\begin{pmatrix} 1 & 3 \\ 8 & 7 \\ -3 & 3 \end{pmatrix}$ B) $\begin{pmatrix} 1 & 1 \\ 9 & 19 \\ 13 & 11 \end{pmatrix}$

C) $\begin{pmatrix} 1 & 1 \\ 9 & 19 \\ -13 & -11 \end{pmatrix}$ D) $\begin{pmatrix} 4 & -6 \\ -2 & 0 \\ 1 & 2 \end{pmatrix}$

22. Agar $M=\begin{pmatrix} 0 & -3 \\ 3 & 7 \\ 5 & 9 \end{pmatrix}$ va $L=\begin{pmatrix} 0 & 3 \\ 6 & 7 \\ -8 & 2 \end{pmatrix}$ bo'lsa $M-L$ ayirmani hisoblang:

A) $\begin{pmatrix} 0 & -6 \\ -3 & 0 \\ 13 & 7 \end{pmatrix}$ B) $\begin{pmatrix} 1 & -5 \\ 3 & 6 \\ 13 & 7 \end{pmatrix}$

C) $\begin{pmatrix} 1 & -5 \\ -3 & -6 \\ -13 & 7 \end{pmatrix}$ D) $\begin{pmatrix} 0 & -6 \\ -3 & 0 \\ 1 & 2 \end{pmatrix}$

23. $A=\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}$ A^2 ni aniqlang.

A) $A=\begin{pmatrix}0 & -2\\ 2 & 0\end{pmatrix}$ B) $\begin{pmatrix}3 & 0\\ 10 & 8\end{pmatrix}$ C) $\begin{pmatrix}0 & 2\\ 16 & 0\end{pmatrix}$

24. $A=\begin{pmatrix}3 & 2 & 1\\ 2 & 1 & 0\\ -1 & -2 & -3\end{pmatrix}$ matritsani birlik matritsaga ko'paytiring.

A) $\begin{pmatrix}5 & 2 & -1\\ 4 & 1 & -2\\ 3 & 0 & -4\end{pmatrix}$; B) $\begin{pmatrix}-5 & -4 & -3\\ -2 & -1 & 1\\ 1 & 2 & -3\end{pmatrix}$;

C) $\begin{pmatrix}3 & 2 & 1\\ 2 & 1 & 0\\ -1 & -2 & -3\end{pmatrix}$; D) $\begin{pmatrix}2 & 2 & 3\\ -2 & -1 & 0\\ -5 & -4 & -3\end{pmatrix}$;

25. Ko'paytmani hisoblang.

$(3 \quad 0 \quad -5)\begin{pmatrix}4\\ 3\\ 1\end{pmatrix}$

A) $\begin{pmatrix}2 & 1 & -3\\ 4 & 3 & 1\end{pmatrix}$; B) 7 ; C) $\begin{pmatrix}8\\ 3\\ -3\end{pmatrix}$; D) $\begin{pmatrix}18\\ 8\\ -24\end{pmatrix}$;

26. $B=\begin{pmatrix}-5 & 4\\ -4 & 3\end{pmatrix}$ matritsa uchun B^{-1} ni aniqlang.

A) $B^{-1}=\begin{pmatrix}-5 & 4\\ -4 & -5\end{pmatrix}$; B) $B^{-1}=\begin{pmatrix}4 & 5\\ -5 & -4\end{pmatrix}$;

C) $B^{-1}=\begin{pmatrix}5 & -5\\ 4 & -3\end{pmatrix}$; D) $B^{-1}=\begin{pmatrix}3 & -4\\ 4 & -5\end{pmatrix}$;

27. $\left.\begin{matrix}5x-3y=2\\ x+2y=3\end{matrix}\right\}$ tenglamalar sistemasi yeching.

A) $x = 1; y = 1$ B) $x = -1; y = 2$ C) $x = 2; y = -1$

28. $\left.\begin{matrix} 4x+3y=-1 \\ 3x-2y=-5 \end{matrix}\right\}$ tenglamalar sistemasi yeching.

A) $x=-1; y=1$ B) $x=-2; y=1$ C) $x=1; y=2$

29. $\begin{cases} 3x - y + z = 12 \\ 5x + y + 2z = 3 \\ x + 2y + 4z = 6 \end{cases}$ tenglamalar sistemasi yeching.

A) $x = 2, y = -2, z = 7$ B) $x = 2, y = -7, z = 3$

C) $x = 0, y = -7, z = 5$ D) $x = 7, y = -1, z = 5$

30. $\begin{cases} x - 3y + z = 5 \\ x + 2z = 5 \\ 4x + 2y - z = -9 \end{cases}$ tenglamalar sistemasi yeching.

A) $x = -2, y = -7, z = 3$ B) $x = -1, y = -1, z = 3$

C) $x = 2, y = 7, z = 3$ D) $x = 2,\ y = 0,\ z = 3$

II-BOB. VEKTORLAR ALGEBRASI.

4-§. Vektor ular ustida amallar. Vektorlarning chiziqli erkliligi. Vektor uzunligi. Yo'naltiruvchi kosinuslari.

1.Vektor haqida umumiy tushuncha.

Matematika, fizika, mexanika kabi fanlarni o'rganishda ikki xil kattalik bilan ishlashga to'g'ri keladi:

1) faqat son qiymati bilan aniqlanadigan kattalik bu-skalyar kattalik;
2) son qiymatidan tashqari yo'nalishga ham ega bo'lgan kattalik -vektor kattalik deyiladi.

1-ta'rif: Vektor kattalik deb, tayin uzunlikka va yo'nalishiga ega bo'lgan kesmaga aytiladi.

$\vec{a}$

Vektorlar ikkita lotin alifbosining bosh harflari bilan yoki bitta kichkina harf bilan belgilanadi.

Vektorning uzunligiga uning moduli deyiladi va $|\overrightarrow{AB}|$ yoki $|\vec{a}|$ ko'rinishida belgilanadi.

Agar vektorning uzunligi bir birlikka teng bo'lsa, u **birlik vektor** yoki **ort** deyiladi.

Uzunligi nolga teng $|\vec{a}| = 0$ bo'lgan vektor **nol vektor** deb yuritiladi.

2-ta'rif: Kollinear vektorlar deb ataladi , agar noldan farqli ikkita vektor bir to'g'ri chiziqda yoki parallel to'g'ri chiziqlarda yotsa, komplanar vektorlar deyiladi, bitta tekislikda yotuvchi yoki shu tekislikka parallel bo'lgan vektorlarga.

Kollinear so'zi lotincha «com» ya'ni birgalikda yoki umumiy ma'nosidagi va «linia» ya'ni chiziq ma'nosidagi so'zlardan tuzilgan bo'lib, «chiziqdosh» degan ma'noni bildiradi.

3-ta'rif: Bir xil yo'nalgan va uzunliklari teng bo'lgan vektorlar **teng vektorlar** deyiladi.

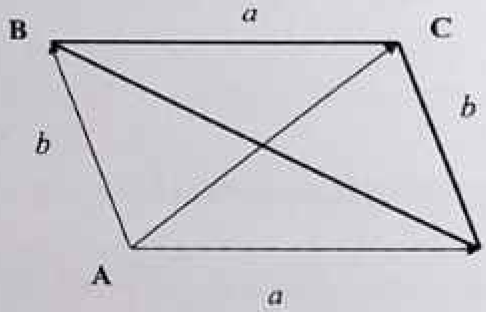
4-ta'rif: Qarama-qarshi yo'nalgan uzunliklari teng bo'lgan vektorlar **qarama-qarshi vektorlar** deb ataladi.

2.Vektorlar ustida amallar.

Vektorlarni qo'shish, ayirish amallari o'rta maktab dasturidan ma'lum bo'lgan uchburchak va parallelogramm qoidalariga asosan amalga oshiriladi.

5-ta'rif: Boshlari biror A nuqtada yotgan ikki $\overrightarrow{AB}$ va $\overrightarrow{AD}$ vektorning yig'indisi deb shu ikki vektordan yasalgan ABCD parallelogramning A uchidan C uchiga yo'nalgan va uzunligi AC dioganalning uzunligiga teng bo'lgan $\overrightarrow{AC}$ vektorga aytiladi, ya'ni

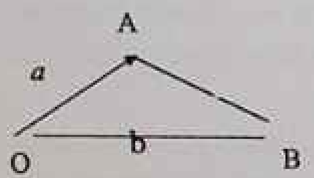
$$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC}$$



1-shakl

Berilgan $\vec{a}$ vektordan $\vec{b}$ vektorni ayirish uchun ixtiyoriy O nuqtadan boshlab $\overrightarrow{OA} = \vec{a}$ va $\overrightarrow{OB} = \vec{b}$ vektorlar yasaladi, so'ngra $\overrightarrow{OB}$ vektorning vektor uchidan $\overrightarrow{OA}$ vektorning A uchiga yo'nalgan $\overrightarrow{BA}$ vektor yasaladi. Keyingi vektor izlangan ayirma vektor bo'ladi.

$$\overrightarrow{OA} - \overrightarrow{OB} = \overrightarrow{AB}$$



2-shakl

6-ta'rif: $\vec{a}$ vektor va skalyar $\lambda \neq 0$ son berilgan bo'lsa, $\vec{a}$ vektorning λ songa ko'paytmasi deb quyidagi ikkita shartlarni qanoatlantiradigan $\vec{b}$ vektorga aytiladi.

1) Agar $\lambda > 0$ bo'lsa, $\vec{a}$ vektor $\vec{b}$ vektor bir xil yo'nalgan vektorlar bo'ladi, ($a \neq 0$) $\lambda < 0$ bo'lsa, $\vec{a}$ va $\vec{b}$ vektorlar qarama-qarshi yo'nalgan vektorlar bo'ladi.

2) $\vec{b}$ vektorning uzunligi $|b|=|\lambda|\ |a|$ formula asosida topiladi.

Vektorlarni qo'shish songa ko'paytirish xossalari.

1. $\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$ (assotsiativlik).
2. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ (kommutativlik).
3. Ixtiyoriy $\vec{c}$ vektorga nol vektor qo'shilsa, $\vec{c}$ vektor hosil bo'ladi, ya'ni

$$\vec{c} + 0 = \vec{c}$$

4. $\forall\vec{a}$ vektor uchun $\vec{a} \cdot 0 = 0$;
5. $\forall\vec{a}$ vektor uchun $\vec{a} \cdot 1 = \vec{a}$; $\vec{a} \cdot (-1) = -\vec{a}$
6. $\forall\vec{a}$ vektor va har qanday $\alpha\ \beta \in$ R sonlar uchun $\alpha\ (\beta\ \vec{a})=(\alpha\ \beta)\ \vec{a}$
7. $\forall\vec{a}$ va $\alpha, \beta \in R$ uchun $(\alpha + \beta)\bar{a} = \alpha\ \bar{a} + \beta\ \bar{a}$.
8. $\forall\vec{a}\ \vec{b}$ lar va $\alpha, \beta \in R$ uchun $(\vec{a} + \vec{b})\alpha = \vec{a}\alpha + \vec{b}\alpha$

3. Vektorlarning chiziqli bog'liqligi.

$\vec{a}_1, \vec{a}_2, \vec{a}_3, \ldots, \vec{a}_n$ vektorlar va $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ haqiqiy sonlar berilgan bo'lsin. $\vec{a}_1, \vec{a}_2, \vec{a}_3, \ldots, \vec{a}_n$ vektorlarning $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ koeffitsientli chiziqli kombinatsiyasi deb $\lambda_1\vec{a}_1, \lambda_2\vec{a}_2, \lambda_3\vec{a}_3, \ldots, \lambda_n\vec{a}_n$ ifodaga aytiladi.

Agar biror $\vec{a}$ vektor $\vec{a}_1, \vec{a}_2, \vec{a}_3, \ldots, \vec{a}_n$ vektorlarning chiziqli kombinatsiyasi shaklida ifodalangan bo'lsa, $\vec{a}$ vektor shu vektorlar bo'yicha yoyilgan deyiladi, ya'ni quyidagi tenglik o'rinli bo'ladi.

$$\vec{a} = \lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 + \lambda_3\vec{a}_3 + \cdots + \lambda_n\vec{a}_n \qquad (1)$$

7-ta'rif: Agar kamida bittasi noldan farqli Agar kamida bittasi noldan farqli $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_n$ sonlar tanlab olinganda

$$\lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 + \lambda_3\vec{a}_3 + \cdots + \lambda_n\vec{a}_n = 0 \qquad (2)$$

tenglik bajarilsa, u holda $\vec{a}_1, \vec{a}_2, \vec{a}_3, \ldots, \vec{a}_n$ vektorlar chiziqli bog'liq deyiladi.

8-ta'rif: Agar (2) munosabat faqat $\lambda_1 = \lambda_2 = \lambda_3 = \cdots = \lambda_n = 0$ bo'lgandagina o'rinli bo'lsa, u holda $\vec{a}_1, \vec{a}_2, \vec{a}_3, \ldots, \vec{a}_n$ vektorlar chiziqli bo'lmagan yoki chiziqli erkli deb ataladi.

Tekislikdagi har qanday ikkita vektorning chiziqli bog'liq bo'lishi uchun ularning kollinear vektorlar bo'lishi zarur va kifoya. Fazodagi har qanday uchta vektorning chiziqli bog'liq bo'lishi uchun, ularning komplanar vektorlar bo'lishi shart. Tekislikdagi har qanday ikkita vektorning va fazodagi har qanday uchta vektorning chiziqli bog'liqsiz vektorlar bo'lishi uchun ularning mos ravishda kollinear va komplanar vektorlar bo'lmasliklari zarur va kifoya.

4.Vektorni bazislar bo'yicha yoyish.

9-ta'rif:. *Tekislikdagi bazis* deb ikkita kollinear bo'lmagan, ya'ni chiziqli bog'liqsiz $\vec{a}_1, \vec{a}_2$ vektorlarga aytiladi.

1-teorema: Tekislikdagi biror $\vec{a}$ vektorning $\vec{a}_1$ va $\vec{a}_2$ bazislar orqali yoyilmasi yagona va u quyidagi ko'rinishda bo'ladi: $\vec{a} = \lambda_1\vec{a}_1 + \lambda_2\vec{a}_2$

10-ta'rif: *Fazodagi bazis* deb, undagi har qanday uchta komplanar bo'lmagan, ya'ni chiziqli bog'liqsiz bo'lgan $(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ vektorlarga aytiladi.

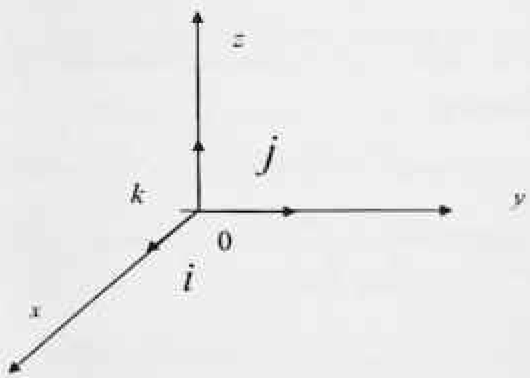
2-teorema: Fazodagi biror $\vec{a}$ vektorning $(\vec{a}_1, \vec{a}_2, \vec{a}_3)$ bazislar orqali yoyilmasi yagona va u quyidagi ko'rinishda bo'ladi:

$$\vec{a} = \lambda_1\vec{a}_1 + \lambda_2\vec{a}_2 + \lambda_3\vec{a}_3 + \cdots + \lambda_n\vec{a}_n \quad (3)$$

Endi dekart koordinata sistemasidagi bazis va ular bo'yicha vektorlarni yoyishni ko'raylik. Dekart koordinata sistemasida Ox, Oy, Oz o'qlar yo'nalishida mos ravishda uzunliklari birga teng bo'lgan $\vec{i}, \vec{j}, \vec{k}$ vektorlarni $|\vec{i}|=|\vec{j}|=|\vec{k}|=1$ olaylik. Uzunliklari birga teng bo'lgan vektorlarga birlik vektor yoki ort deyiladi. Bu vektorlar o'zaro perpendikulyar bo'lib komplanar bo'lmagani uchun, ya'ni chiziqli bog'liqsiz vektorlar bo'lgani uchun bazislarni tashkil qiladi. Shuning uchun ularga dekart ortogonal bazislar deyiladi.

5.Vektorlar koordinatasi. Koordinata shaklida berilgan vektorlar ustida chiziqli amallar .

O XYZ to'g'ri burchakli koordinata sistemasini olaylik, bu sistema Dekart koordinatalar sistemasi deyiladi.



3-shakl

$\vec{a}$ vektor $\overline{OA}$ vektorning o'qlaridagi proeksiyalari mos ravishda a_x, a_y, a_z , $\vec{i}, \vec{j}, \vec{k}$ birlik –bazis vektorlar ortlar deb ataladi

$$\vec{a}(a_x, a_y, a_z) = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

Bu ifodani $\vec{a}$ vektorning $\vec{i}, \vec{j}, \vec{k}$ bazis vektorlar yoki koordinata o'qlari bo'yicha yoyilmasi deyiladi

Bizga quyidagi vektorlar berilgan bo'lsin

$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$; $\vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$

1) $\vec{a} + \vec{b} = (a_x + b_x)\vec{i} + (a_y + b_y)\vec{j} + (a_z + b_z)\vec{k}$

2) $\gamma\vec{a} = \gamma a_x\vec{i} + \gamma a_y\vec{j} + \gamma a_z\vec{k}$

6. Vektorning uzunligi va uning yo'naltiruvchi kosinuslari .

Fazoda Dekart koordinata sistemasidagi Ox Oy Oz o'qlari bo'yicha yo'nalgan $\vec{i}, \vec{j}, \vec{k}$ ortlar ya'ni bazis vektorlar berilgan bo'lsin, $\vec{a}$ vektor o'zining a_x, a_y, a_z

koordinata bilan berilgan bo'lib, u shu bazisda $\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$

ko‘rinishda ifodalanadi. Uning uzunligi

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

ko‘rinishda aniqlanadi.

$\vec{a}$ vektorning koordinata o‘qlari bilan hosil qiladigan burchaklarni mos ravishda α, β, γ burchaklar bilan belgilaymiz.

$\vec{a}$ vektorni yo‘naltiruvchi kosinuslari quyidagi formulalar bilan hisoblanadi

$$cos\alpha = \frac{a_x}{\sqrt{a_x^2 + a_y^2 + a_z^2}}; \quad cos\beta = \frac{a_y}{\sqrt{a_x^2 + a_y^2 + a_z^2}}; \quad cos\gamma = \frac{a_z}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$$

Bu formulalar quyidagi tenglikni qanoatlantiradi.

$$cos^2\alpha + cos^2\beta + cos^2\gamma = 1$$

1-misol: Boshi va oxiri mos ravishda A(4, -5, 3) va B(1, 2, -1) nuqtalarda bo‘lgan vektor koordinatalarini va uzunligini toping.

Yechish: $\overrightarrow{AB} = (1-4; 2+5; -1-3) = (-3; 7; -4);$

$|\overrightarrow{AB}| = \sqrt{9+49+16} = \sqrt{74}.$

2-misol: $\vec{a} = (5; 1; -2) va\, \vec{b} = (-3; 2; -1)$ vektorlarga qurilgan parallelogramm diagonallari uzunliklarini toping.

Yechish: $\vec{a} + \vec{b} = \{2; 3; -3\}; \quad \vec{a} - \vec{b} = \{8; -1; -1\};$

$|\vec{a} + \vec{b}| = \sqrt{4+9+9} = \sqrt{22}; \qquad |\vec{a} - \vec{b}| = \sqrt{64+1+1} = \sqrt{66}.$

3-misol: $\vec{a} = (2; 1; 3) va\, \vec{b} = (-2; 4; -1)$ vektorlar berilgan. Quyidagilarni toping:

a) $\vec{a} - 2\vec{b}$ vektorning koordinata o‘qlaridagi proeksiyalarini;

b) $\vec{a} - 2\vec{b}$ vektorning uzunligini;

c) $\vec{a}-2\vec{b}$ vektorning yo'naltiruvchi kosinuslarini;

Yechish.

a) $\vec{a}-2\vec{b}=\{2-2\cdot(-2);1-2\cdot 4;3-2\cdot(-1)\}=\{6;-7;5\}$.

b) $|\vec{a}-2\vec{b}|=\sqrt{6^2+(-7)^2+5^2}=\sqrt{36+49+25}=\sqrt{110}$.

c) Yonaltiruvchi kosinuslarini topish uchun

$$cos\alpha=\frac{a_x}{\sqrt{a_x^2+a_y^2+a_z^2}};\quad cos\beta=\frac{a_y}{\sqrt{a_x^2+a_y^2+a_z^2}};\ cos\gamma=\frac{a_z}{\sqrt{a_x^2+a_y^2+a_z^2}}$$

dan foydalanib ,

$cos\alpha=\frac{6}{\sqrt{110}}$, $cos\beta=\frac{-7}{\sqrt{110}}$, $cos\gamma=\frac{5}{\sqrt{110}}$ ekanligini topamiz.

Mustaqil bajarish uchun topshiriqlar.

1.$M(4,-5,3)$, $N(1,2,-1)$ va $L(-2,3,1)$ nuqtalar berilgan.

$\overrightarrow{ML},\overrightarrow{NL},\overrightarrow{MN}$ vektorlarni koordinatalarini va uzunligini toping.

2. Ikkita $\vec{a}=(-3;2;4) va\ \vec{b}=(-1;0;3)$ vektorlar berilgan .Quyidagi vektorlarning koordinata o'qlaridagi proeksiyalarini toping:

k) $3\vec{a}-2\vec{b}$; n) $2\vec{a}+4\vec{b}$; m) $\vec{a}+2\vec{b}$; l) $5\vec{a}-3\vec{b}$;

3. Uchlari $A(4,-5,3)$, $B(1,2,-1)$ va $C(-2,3,1)$nuqtalarda bo'lgan ABC uchburchakning A uchidan o'tkazilgan medianasi bilan ustma-ust tushuvchi

$\overrightarrow{AD}$ vektor uzunligini toping.

4. $\vec{a}=(0;4;-5) va\ \vec{b}=(-3;-8;2)$ vektorlarga qurilgan parallelogramm diagonallari uzunliklarini toping.

5. K(-2, 5, -1) va L(6, -3, 4) nuqtalar berilgan. $\overrightarrow{KL}$ vektor uzunligini va yo'nalishini toping.

6. $\vec{a}=(3;4;5)$ vektorning yo'naltiruvchi kosinuslarini toping.

5-§. Ikki vektorning skalyar va vektor ko‘paytmasi va ularning xossalari. Uchta vektorning aralash ko‘paytmasi , xossalari.

1.Ikki vektorning skalyar ko‘paytmasi va uning xossalari.

Tekislikda ikkita $\vec{a}$ va $\vec{b}$ vektorlarni qaraylik. Ularni biror O nuqtaga qo‘yamiz.

Ta’rif: $\vec{a}$, $\vec{b}$ vektorlarning skalyar ko‘paytmasi deb shu vektorlarning uzunliklari bilan ular hosil qilgan burchak kosinusini ko‘paytirishdan hosil qilingan songra aytiladi.

$$\vec{a}\cdot\vec{b}=|\vec{a}|\cdot|\vec{b}|cos\varphi \qquad (1)$$

1-misol: $|\vec{a}|=3$, $|\vec{b}|=2$ va $\varphi=\frac{\pi}{3}$ ga teng bo‘lsa, $\vec{a}$ va $\vec{b}$ vektorlarning skalyar ko‘paytmasini toping.

Yechish: $\vec{a}\cdot\vec{b}=3\cdot2\cdot cos\frac{\pi}{3}=6\cdot\frac{1}{2}=3.$

Natija: Nol vektorni har qanday vektorga skalyar ko‘paytmasi nolga teng.

$\vec{a}$ vektor koordinatalari (a_x, a_y, a_z) va $\vec{b}$ vektor koordinatalari (b_x, b_y, b_z) bo‘lsa,

bu vektorlarning skalyar ko‘paytmasi quyidagicha topiladi:

$$\vec{a}\cdot\vec{b}=a_xb_x+a_yb_y+a_zb_z. \qquad (2)$$

Skalyar ko‘paytmaning xossalari.

1. $\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ o‘rin almashtirish xossalari.

2. $(\vec{a}+\vec{b})=(\vec{b}+\vec{a})\vec{c}=\vec{a}\vec{c}+\vec{b}\vec{c}$ taqsimot xossalari.

3. $\lambda(\vec{a}\cdot\vec{b})=\vec{b}\cdot(\lambda\cdot\vec{a})=\vec{a}\cdot(\lambda\cdot\vec{b})$ guruhlash xossasi.

Agar $\vec{a}$ va $\vec{b}$ vektorlar bir xil yo‘nalishdagi kollinear vektorlar

bo‘lsa, $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|$, chunki cos0=1 .

Agar qarama-qarshi yo‘nalgan bo‘lsa, $\vec{a}\cdot\vec{b}=|\vec{a}||\vec{b}|$, chunki $\cos180^0=-1$.

5. $\vec{a}\cdot\vec{a} = |\vec{a}||\vec{a}|cos0 = |\vec{a}|^2 \Rightarrow \ \vec{a}^2 = |\vec{a}|^2$

6. $\vec{a}\ va\ \vec{b}$ vektorlar perpendikulyar bo'lsa, $\vec{a}\cdot\vec{b} = 0$ bo'ladi.

$\vec{i}, \vec{j}, \vec{k}$ birlik vektorlarning skalyar ko'paytmalari:

$$\vec{i}\cdot\vec{i} = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k} = 1$$

$$\vec{i}\cdot\vec{j} = \vec{j}\cdot\vec{k} = \vec{i}\cdot\vec{k} = 0$$

tengliklarning o'rinli bo'lishi ravshan.(1) formuladan

$cos\varphi = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}$ ekanligi kelib chiqadi,

$$|\vec{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2} \quad \text{va} \qquad \vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z$$

ekanligidan, $\quad cos\varphi = \frac{a_x b_x + a_y b_y + a_z b_z}{\sqrt{a_x^2+a_y^2+a_z^2}\sqrt{b_x^2+b_y^2+b_z^2}}$ (3)

bo'lishi kelib chiqadi.

3) $\vec{a} = (a_x, a_y, a_z)$ va $\vec{b} = (b_x, b_y, b_z)$ vektorlarning perpendikulyarlik sharti quyidagicha aniqlanadi: $\quad a_x b_x + a_y b_y + a_z b_z = 0$

4) $\vec{a} = (a_x, a_y, a_z)$ va $\vec{b} = (b_x, b_y, b_z)$ vektorlarning paralellik sharti:

$\frac{a_x}{b_x} = \frac{a_y}{b_y} = \frac{a_z}{b_z}$.

1-misol: $\vec{a} = (2; -1)$ va $\vec{b} = (4; -2)$ vektorlarning skalyar ko'paytmasini aniqlang.

Yechish: $\vec{a}\cdot\vec{b} = 2\cdot 4 + (-1)\cdot(-2) = 8 + 2 = 10.$

2-misol: $\vec{a} = (-2; 3; -1)$ va $\vec{b} = (-4; 0; 5)$ vektorlarning skalyar ko'paytmasini aniqlang.

Yechish: $\vec{a}\cdot\vec{b} = -2\cdot(-4) + 3\cdot 0 + (-1)\cdot 5 = 8 - 5 = 3.$

3-misol: $\vec{a}(1,2)$, $\vec{b}(1,-\frac{1}{2})$ vektorlar tashkil qilgan burchakni aniqlang.

$$\cos\varphi = \frac{(\bar{a}\cdot\bar{b})}{|\bar{a}|\cdot|\bar{b}|} = \frac{1\cdot 1 + 2\cdot\left(-\frac{1}{2}\right)}{\sqrt{1^2+2^2}\sqrt{1^2+\left(-\frac{1}{2}\right)^2}} = \frac{0}{\sqrt{5}\sqrt{\frac{5}{4}}} = 0 .$$

4-misol: $|\vec{a}| = 4$, $|\vec{b}| = 6 \; va \; \alpha = \frac{\pi}{3}$ (α-ular orasidagi burchak) bo'lsa, $|\vec{a}+\vec{b}|$ ni toping.

Yechish:

$$|\vec{a}+\vec{b}| = \sqrt{(\vec{a}+\vec{b})^2} = \sqrt{\vec{a}^2 + 2\vec{a}\vec{b} + \vec{b}^2} = \sqrt{|\vec{a}|^2 + 2|\vec{a}||\vec{b}|cos\varphi + |\vec{b}|^2}$$
$$=$$

$$= \sqrt{16 + 2\cdot 4\cdot 6\cdot cos\frac{\pi}{3} + 36} = \sqrt{52+24} = \sqrt{76}.$$

2.Ikki vektorning vektor ko'paytmasi va uning xossalari.

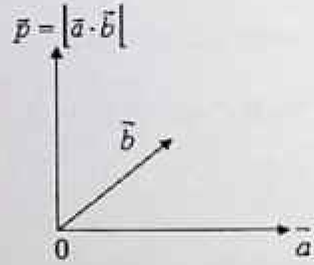
Ta'rif: $\bar{a}$ va $\vec{b}$ vektorlarning vektor ko'paytmasi deb quyidagi uchta shartni qanoatlantiruvchi $\vec{c} = \vec{a}\times\vec{b}$ ga aytiladi:

1) $|\vec{c}| = |\vec{a}||\vec{b}|\sin(\vec{a}\cdot\vec{b})$;

$\vec{c}$ vektorning moduli $\vec{a} \; va \; \vec{b}$ vektorlarga qurilgan parallelogrammning S yuziga teng.

2) $\vec{c}\perp\vec{a}, \vec{c}\perp\vec{b}$;

c vektor a va b vektorlarga perpendikulyar



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3) $\bar{a}, \bar{b}, \vec{c}$ vektorlar umumiy boshga keltirilib, $\vec{p}$ ning uchidan $\bar{a}$ va $\vec{b}$ vektorlar yotgan tekislikka qaraganda $\bar{a}$ vektordan $\vec{b}$ vektor tomonga qarab eng qisqa yo'l bilan burilish soat strelkasi yo'nalishiga teskari bo'lsin, (o'ng sistema). Vektor ko'paytmani $\vec{c} = \vec{a}\times\vec{b}$ bilan belgilaymiz.

2-shartdan $[\bar{a},\vec{b}]$ vektor ko'paytma parallelogramm tekisligiga perpendikulyar vektor ekanligini aniqlaydi.

Vektor ko'paytma quyidagi xossalarga ega.

a) $\vec{a} \times \vec{b} = -\vec{b} \times \overline{a};$

b) $k\vec{a} \times \vec{b} = \vec{a} \times k\vec{b} = k(\vec{a} \times \overline{b})$

c) $\vec{a} \times \overline{(b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$

d) Agar

Birlik vektorning vektor ko'paytmalari quyidagicha bo'ladi:

$\vec{i} \times \vec{i} = 0$, $\vec{i} \times \vec{j} = \vec{k}$, $\vec{i} \times \vec{k} = -\vec{j}$, $\vec{j} \times \vec{i} = -\vec{k}$, $\vec{j} \times \vec{j} = 0$,

$\vec{j} \times \vec{k} = \vec{i}$, $\vec{k} \times \vec{i} = \vec{j}$, $\vec{k} \times \vec{j} = -\vec{i}$, $\vec{k} \times \vec{k} = 0$.

Agar dekart koordinatalar sistemasida $a \, va \, b$ vektorlari bilan berilgan, ya'ni

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

$$\vec{b} = b_x\vec{i} + b_y\vec{j} + b_z\vec{k}$$

bo'lsa, u holda

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$

$$= (a_y b_z - a_z b_y)\vec{i} + (a_z b_x - a_x b_z)\vec{j} + (a_x b_y - a_y b_x)\,\vec{k}$$

Vektor ko'paytma yordamida uchburchakning yuzini hisoblash uchun formula hosil qilish mumkin. ABC uchburchak uchlarining koordinatalri bilan berilgan bo'lsin:

$A(x_1, y_1, z_1,)$, $B(x_2, y_2, z_2,)$, $C(x_3, y_3, z_3,)$.

Vektor ko'paytma ta'rifiga ko'ra hosil bo'lgan vektorning moduli parallelogrammning yuziga teng. Uning yarmi esa uchburchakning yuziga teng bo'ladi:

$S_{\Delta ABC} = \frac{1}{2}|\overline{AB} \times \overline{AC}|$.

5-misol. Uchlari A(2; 3; 1), B(4; 0; 2), C(7; 0; 2) nuqtalarda bo'lgan uchburchak yuzini toping.

Yechish:

$\overline{AB}$={2;-3;1}, $\overline{AC}$ ={5;-3;1}

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ 2 & -3 & 1\\ 5 & -3 & 1\end{vmatrix}=-3\vec{i}-6\vec{k}+5\vec{j}\text{-}(-15\,\vec{k}\text{ - }-3\vec{i}+2\vec{j})\text{=}3\vec{j}\text{+}9\vec{k};$$

$S_{\Delta ABC}=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\frac{1}{2}\sqrt{81+9}=\frac{3\sqrt{10}}{2}$ kv. bir.

6-misol: $\vec{a}=\overline{2\imath}+\vec{j}+\overline{3k}\ ;\ \vec{b}=-\vec{\imath}+2\vec{j}-2\vec{k}$

vektorlarga yasalgan parallelogrammning yuzini va uning diagonallari uzunliklarini toping.

Yechish.

$$S=\left|\vec{a}\times\vec{b}\right|$$

$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{\imath} & \vec{j} & \vec{k}\\ 2 & 1 & 3\\ -1 & 2 & -2\end{vmatrix}=(-2\cdot1-2\cdot3)\vec{\imath}+(-1\cdot3-(-2)\cdot(-1)\vec{j}+$
$+(2\cdot2-(-1)\cdot1)\vec{k})=(-2-6)\vec{\imath}+(-3-2)\vec{j}+(4+1)\,\vec{k}=-8\vec{\imath}-$
$5\vec{\imath}+5\vec{k}.$

$$S=\left|\vec{a}\times\vec{b}\right|=\sqrt{(-8)^2+(-5)^2+5^2}=\sqrt{64+25+25}=\sqrt{114}$$

$$d_1=\left|\vec{a}+\vec{b}\right|=\sqrt{(2-1)^2+(1+2)^2+(3-2)^2}=\sqrt{1+9+1}=\sqrt{11}$$

$d_2=\left|\vec{a}-\vec{b}\right|=\sqrt{(2+1)^2+(1-2)^2+(3+2)^2}=$
$\sqrt{9+1+25}=\sqrt{35}.$

7-misol: Uchlari A(3,0,5), B(3,-2,2), S(1,2,4) nuqtalarda bo'lgan ABS uchburchak yuzini toping.

Yechish: $S_{\Delta ABC}=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AS}\right|$

$$\overrightarrow{AB}=\{3-3;-2-0;2-5\}=\{0;-2;-3\};$$

$$\overrightarrow{AS}=\{1-3;2-0;4-5\}=\{-2;2;-1\};$$

$$\overrightarrow{AB}\times\overrightarrow{AS}=\begin{vmatrix}\vec{\imath} & \vec{\jmath} & \vec{k}\\ 0 & -2 & -3\\ -2 & 2 & -1\end{vmatrix}=2\vec{\imath}+6\vec{\jmath}-\left(4\vec{k}-6\vec{\imath}\right)=8\vec{\imath}+6\vec{\jmath}-4\vec{k};$$

$$|\overrightarrow{AB}\times\overrightarrow{AS}|=\sqrt{64+36+16}=\sqrt{116}$$

$S_{\Delta ABC}=\frac{1}{2}|\overrightarrow{AB}\times\overrightarrow{AS}|=\frac{\sqrt{116}}{2}=\sqrt{29}.$

3.Uchta vektorning aralash ko'paytmasi , xossalari.

Bizga $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlar berilgan bo'lsin .

Ta'rif:

$$\vec{a}=a_x\vec{\imath}+a_y\vec{\jmath}+a_z\vec{k}$$

$$\vec{b}=b_x\vec{\imath}+b_y\vec{\jmath}+b_z\vec{k}$$

$$\vec{c}=c_x\vec{\imath}+c_y\vec{\jmath}+c_z\vec{k}$$

bu vektorlarning aralash ko'paytmasi deb , $\vec{a}\times\vec{b}$ vektor ko'paytma bilan $\vec{c}$ vektorning skalyar ko'paytmasiga aytiladi va odatda $(\vec{a}\times\vec{b})\vec{c}$ ko'rinishda yoziladi

$$\vec{a}\times\vec{b}=\begin{vmatrix}\vec{\imath} & \vec{\jmath} & \vec{k}\\ a_x & a_y & a_z\\ b_x & b_y & b_z\end{vmatrix}=\begin{vmatrix}a_y & a_z\\ b_y & b_z\end{vmatrix}\vec{\imath}+\begin{vmatrix}a_z & a_x\\ b_z & b_x\end{vmatrix}\vec{\jmath}+\begin{vmatrix}a_x & a_y\\ b_x & b_y\end{vmatrix}\vec{k},$$

$$(\vec{a}\times\vec{b})\vec{c}=\left(\begin{vmatrix}a_y & a_z\\ b_y & b_z\end{vmatrix}\vec{\imath}+\begin{vmatrix}a_z & a_x\\ b_z & b_x\end{vmatrix}\vec{\jmath}+\begin{vmatrix}a_x & a_y\\ b_x & b_y\end{vmatrix}\vec{k}\right)\left(c_x\vec{\imath}+c_y\vec{\jmath}+c_z\vec{k}\right)=$$

$$=\begin{vmatrix}a_x & a_y & a_z\\ b_x & b_y & b_z\\ c_x & c_y & c_z\end{vmatrix}.$$

Aralash ko'paytmaning geometrik ma'nosi qirralari berilgan $\vec{a}$, $\vec{b}$, $\vec{c}$ vektorlarning modullaridan tashkil topgan parallelopepedning hajmini ifodalaydi.

$$V=(\overrightarrow{OA}\times\overrightarrow{OB})\cdot\overrightarrow{OC}$$

8-Misol. Uchlari $O(0;0;0)$, $A(5;2;0)$, $B(2;5;0)$, $C(1;2;4)$ nuqtalarda bo'lgan parallelopipedning hajmini toping.

$$V=(\vec{OA}\times\vec{OB})\cdot\vec{OC}=\begin{vmatrix}5 & 2 & 0\\ 2 & 5 & 0\\ 1 & 2 & 4\end{vmatrix}=100-16=84 \text{ kub birlik.}$$

Aralash kopaytmada har doim skalyar son chiqadi.

Uchta vektordan qurilgan paralelopipedning hajmi uchta vektorning aralash ko'paytmasiga ishora aniqligida teng.

$$V_{par}=|\vec{a}\vec{b}\vec{c}|$$

Pramida hajmini ham aralash ko'paytma yordamida hisoblash mumkin .

$$V_{pir}=\frac{1}{3}S_{as}\cdot h=\frac{1}{3}\left(\frac{1}{2}S_{as}\right)\cdot h=\frac{1}{6}S_{as}\cdot h$$

$$V_{pir}=\frac{1}{6}S_{as}\cdot h=\frac{1}{6}[a\times b]\cdot c$$

$$V_{pir}=\pm\frac{1}{6}abc.$$

Aralash ko'paytmaning xossalari:

a) $\vec{a}(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b})\vec{c}$

b) Ko'paytiriluvchi vektorlar o'rinlari doiraviy almashtirilsa aralash ko'paytma qiymati o'zgarmaydi, ya'ni

$$\vec{a}\vec{b}\vec{c}=\vec{c}\vec{a}\vec{b}=\vec{b}\vec{c}\vec{a}.$$

c) Qo'shni ikkita vektorlarning o'rinlari almashtirilganda aralash ko'paytma ishorasini o'zgartiradi:

$$\vec{a}\vec{b}\vec{c}=-\vec{a}\vec{c}\vec{b},\ \vec{a}\vec{b}\vec{c}=-\vec{c}\vec{b}\vec{a},\ \vec{a}\vec{b}\vec{c}=-\vec{a}\vec{c}\vec{b}.$$

d) Agar $\vec{a}\vec{b}\vec{c}=0$ bo'lsa, u holda bu uchta vektor komplanar vektor bo'ladi.

Aralash ko'paytmani determinant yordamida hisoblash .

Agar bizga $\begin{cases} \vec{a} = \vec{i}a_x + \vec{j}a_y + \vec{k}a_z \\ \vec{b} = \vec{i}b_x + \vec{j}b_y + \vec{k}b_z \\ \vec{c} = \vec{i}c_x + \vec{j}c_y + \vec{k}c_z \end{cases}$ vektorlar berilgan bo'lsin, u holda

aralash ko'paytma quyidagi detetminantga teng bo'ladi,

$$\vec{a}\vec{b}\vec{c} = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix}.$$

Agar uchta vektor komplanar vektor bo'lsa,

$$\begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = 0\,.$$

9-Misol. $\vec{a}(-3;2;4)\ \vec{b}(0;-2;1)\ \vec{c}(4;-2;5)$ berilgan, aralash ko'paytmasini toping.

Yechish.

$$\vec{a}\vec{b}\vec{c} = \begin{vmatrix} -3 & 2 & 4 \\ 0 & -2 & 1 \\ 4 & -2 & 5 \end{vmatrix} = 30 + 8 + 32 - 6 = 64.$$

10-Misol. $\vec{a}(2;5;8)\ \vec{b}(1;-3;-7)\ \vec{c}(0;5;10)$ berilgan vektorlar komplanar vektorlar ekanligini isbotlang.

Yechish: Vektorlar komplanar vektorlar bo'lishi uchun aralash ko'paytmasi 0 ga teng bo'ladi.

$$\vec{a}\vec{b}\vec{c} = \begin{vmatrix} 2 & 5 & 8 \\ 1 & -3 & -7 \\ 0 & 5 & 10 \end{vmatrix} = -60 + 40 + 0 - 0 - 50 + 70 = 0.$$

Demak, vektorlar komplanar ekan.

Mustaqil bajarish uchun topshiriqlar.

1.KLMN pramidaning uchlari berilgan.

a) Pramidaning berilgan qirralari orasidagi burchak kosinusini toping;

b) Pramidaning berilgan yog'i yuzini toping:

1.1. $K(-9,2,6), L(6,-4,2), M(2,5,7),\ N(-3,-2,3)$;

a) KL va KM; b) LMN.

1.2. $K(-5,1,3), L(2,-3,0), M(1,5,4),\ N(3,4,7)$;

a) LM va LN; b) NLM.

1.3. $K\ (-2,4,1), L(-6,3,-3), M(0,-5,5),\ N(-2,-4,-1)$;

a) KL va KM; b) KLM.

1.4. $K(4,-3,5), L(6,-5,2), M(-1,4,2),\ N(0,-3,-1)$;

a) LM va LN; b) NLM.

1.5. $K(-2,3,1), L(4,-4,-5), M(-2,-5,2),\ N(3,2,5)$;

a) KL va KM; b) LKN.

1.6. $K(-4,-1,6), L(-2,3,5), M(-1,5,-4),\ N(8,1,0)$;

a) LM va KL; b) KLM.

1.7. $K\ (5,2,1), L(0,3,4), M(3,2,5),\ N(-4,-3,-1)$;

a) KL va KM; b) LMK.

1.8. $K(2,-1,-1), L(2,-5,-2), M(1,-2,2),\ N(1,3,-1)$;

a) LM va LN; b) NLM.

1.9. $K(4,3,-6), L(2,3,5), M(1,5,4),\ N(8,1,0)$;

a) LM va KL; b) KLM.

1.10. $K(-3,2,5), L(0,-4,6), M(4,-2,5),\ N(-1,-3,7)$;

a) KL va KM; b) LMN.

1.11. $K(-3,1,5), L(6,-3,3), M(4,5,3),\ N(-3,4,-2)$;

a) LM va LN; b) NLM.

1.12. $K\ (2,4,-1), L(0,3,-3), M(4,-5,5),\ N(2,-4,3)$;

a) KL va KM; b) KLM.

1.13. $K(-4,3,8), L(7,5,-2), M(1,-3,2),\ N(0,3,-5)$;

a) LM va LN; b) NLM.

1.14. $K(-4,3,5), L(4,-7,-5), M(-2,-7,2),\ N(-3,2,-5)$;

a) KL va KM; b) LKN.

1.15. $K(-3,-1,7), L(-1,3,3), M(-4,5,-5),\ N(3,2,0)$;

a) LM va KL; b) KLM.

1.16. $K\ (-5,2,4), L(0,3,1), M(3,3,-5),\ N(0,-2,-4)$;

a) KL va KM; b) LMK.

1.17. $K(-2,1,-1), L(2,5,-2), M(-1,2,2),\ N(-1,3,0)$;

a) LM va LN; b) NLM.

1.18. $K(2,-3,6), L(-2,2,3), M(-1,-5,3),\ N(-8,3,0)$;

a) LM va KL; b) KLM.

1.19 . $K(-3,2,4), L(-2,4,2), M(-2,4,3),\ N(-1,-5,0)$;

a) KL va KM; b) LMN.

1.20. $K(-3,1,4), L(1,-3,0), M(1,5,-4),\ N(3,-4,7)$;

a) LM va LN; b) NLM.

1.21. $K\ (-2,4,1), L(-6,3,-3), M(0,-3,5),\ N(-2,-4,-1)$;

a) KL va KM; b) KLM.

1.22. $K(0,-1,5), L(6,-1,2), M(-1,5,2),\ N(0,-1,-1)$;

a) LM va LN; b) NLM.

1.23. $K(-4,3,0), L(3,-4,-2), M(-1,-5,1),\ N(4,2,-2)$;

a) KL va KM; b) LKN.

1.24. $K(-5,-1,0), L(-2,3,0), M(-1,0,-2),\ N(3,1,9)$;

a) LM va KL; b) KLM.

1.25. $K\ (0,-2,1), L(0,3,4), M(3,2,5),\ N(-4,-3,-1)$;

a) KL va KM; b) LMK.

1.26. $K(2,-6,-5), L(2,-4,-2), M(1,-3,2),\ N(4,3,-1)$;

a) LM va LN; b) NLM.

1.27. $K(4,3,-6), L(0,3,5), M(3,-5,4),\ N(8,1,3)$;

a) LM va KL; b) KLM.

1.28.$K\ (0,-2,1), L(0,3,4), M(-1,2,5),\ N(-6,0,-5)$;

a) KL va KM; b) LMK.

1.29. $K(2,-1,-5), L(-2,-4,-1), M(4,-3,2),\ N(2,3,-1)$;

a) LM va LN; b) NLM.

1.30. $K(-2,3,-6), L(1,3,5), M(5,5,-4),\ N(-3,1,3)$;

a) LM va KL; b) KLM.

2. $\vec{n}$ va $\vec{l}$ vektorlarning skalyar ko'paytmasini aniqlang.

2.1. $\vec{n}(-2;3;-3)$; $\vec{l}(-1;-3;4)$;

2.2. $\vec{n}(5;-5;-1)$; $\vec{l}(1;-2;3)$;

2.3. $\vec{n}(-2-1;3)$; $\vec{l}(1;-1;5)$;

2.4. $\vec{n}(-4;5;-2)$; $\vec{l}(1;4;-2)$;

2.5. $\vec{n}(3;-2;-6)$; $\vec{l}(-1;-2;1)$;

2.6. $\vec{n}(1;-2;3)$; $\vec{l}(-3;1;-2)$;

2.7. $\vec{n}(2;-5;4)$; $\vec{l}(4;-3;5)$;

2.8. $\vec{n}(2;-5;4)$; $\vec{l}(4;-3;5)$;

2.9. $\vec{n}(4;-1;6)$; $\vec{l}(0;-3;-7)$;

2.10. $\vec{n}(2;5;8)$; $\vec{l}(1;-3;-5)$;

2.11. $\vec{n}(-2;5;3)$; $\vec{l}(3;-3;0)$;

2.12. $\vec{n}(4;-5;0)$; $\vec{l}(1;-2;-7)$;

2.16. $\vec{n}(0;5;8)$, $\vec{l}(1;-3;2)$;

2.17. $\vec{n}(2;-5;4)$, $\vec{l}(1;-3;-7)$;

2.18. $\vec{n}(3;5;2)$, $\vec{l}(1;-3;3)$;

2.19. $\vec{n}(2;5;-1)$, $\vec{l}(1;0;-7)$;

2.20. $\vec{n}(2;5;8)$, $\vec{l}(1;-3;5)$;

2.21. $\vec{n}(2;5;3)$; $\vec{l}(1;-3;-1)$;

2.21. $\vec{n}(-1;0;-4)$; $\vec{l}(-4;3;2)$;

2.22. $\vec{n}(-2;-1;-3)$, $\vec{l}(-1;3;7)$;

2.23. $\vec{n}(0;-3;4)$; $\vec{l}(1;3;1)$;

2.24. $\vec{n}(2;5;-5)$; $\vec{l}(2;-3;3)$;

2.25. $\vec{n}(2;-3;-8)$; $\vec{l}(1;0;-2)$;

2.26. $\vec{n}(-1;5;8)$; $\vec{l}(1;-3;5)$;

2.13. $\vec{n}(3;5;-6)$; $\vec{l}(1;-3;-2)$;

2.14. $\vec{n}(-2;0;8)$; $\vec{l}(1;2;-4)$;

2.15. $\vec{n}(2;5;8)$; $\vec{l}(0;-3;2)$;

2.27. $\vec{n}(-4;5;6)$; $\vec{l}(-1;0;3)$;

2.29. $\vec{n}(4;0;-3)$; $\vec{l}(1;-3;-4)$;

2.30. $\vec{n}(2;5;8)$; $\vec{l}(0;-3;-7)$;

3. $\vec{k}$ va $\vec{m}$ vektorlarning vektor ko'paytmasini toping.

3.1. $\vec{k}(-1;0;-3)$ $\vec{m}(0;-3;4)$

3.2. $\vec{k}(-2;3;0)$ $\vec{m}(-1;-3;0)$

3.3. $\vec{k}(-3;5;-1)$ $\vec{m}(0;-3;1)$

3.4. $\vec{k}(-5;0;-3)$ $\vec{m}(-1;-3;4)$

3.5. $\vec{k}(-2;3;-2)$; $\vec{m}(-1;0;2)$

3.6. $\vec{k}(-1;2;-1)$; $\vec{m}(-2;-3;0)$

3.7. $\vec{k}(-1;3;-3)$; $\vec{m}(-1;-3;2)$

3.8. $\vec{k}(-3;3;0)$; $\vec{m}(-1;-3;0)$

3.9. $\vec{k}(-2;3;-1)$; $\vec{m}(-1;-2;4)$

3.10. $\vec{k}(-2;2;-3)$; $\vec{m}(-3;-1;3)$

3.11. $\vec{k}(-4;3;-2)$; $\vec{m}(-4;-3;0)$

3.12. $\vec{k}(-2;0;-3)$; $\vec{m}(2;-1;0)$

3.13. $\vec{k}(-1;3;-4)$; $\vec{m}(0;-2;4)$

3.14. $\vec{k}(-2;3;-5)$; $\vec{m}(-1;0;-4)$

3.15. $\vec{k}(-1;2;-3)$; $\vec{m}(-1;-2;0)$

3.16. $\vec{k}(0;-5;2)$; $\vec{m}(-1;3;-2)$

3.17. $\vec{k}(0;-5;3)$; $\vec{m}(-1;-2;1)$

3.18. $\vec{k}(0;5;-3)$; $\vec{m}(-1;3;3)$

3.19. $\vec{k}(-2;-1;8)$, $\vec{m}(1;-2;2)$

3.20. $\vec{k}(-1;-1;4)$; $\vec{m}(1;-3;1)$

3.21. $\vec{k}(-3;5;-3)$, $\vec{m}(1;-1;0)$

3.22. $\vec{k}(1;-3;8)$; $\vec{m}(1;-3;2)$

3.23. $\vec{k}(0;1;5)$; $\vec{m}(1;-2;0)$

3.24. $\vec{k}(4;5;-2)$; $\vec{m}(1;-3;-1)$

3.25. $\vec{k}(-2;-5;3)$ $\vec{m}(1;3;2)$

3.26. $\vec{k}(0;5;-1)$; $\vec{m}(1;0;-2)$

3.27. $\vec{k}(-1;3;-2)$; $\vec{m}(1;-1;0)$

3.28. $\vec{k}(5;3;8)$; $\vec{m}(4;-3;2)$

3.29. $\vec{k}(2;4;-3)$; $\vec{m}(-1;-5;0)$

3.30. $\vec{k}(0;3;-1)$; $\vec{m}(1;-3;4)$

4. $\vec{l}$, $\vec{m}$ *va* $\vec{n}$ vektorlarning aralash ko'paytmasini toping.

4.1. $\vec{l}(-2;2;1)$; $\vec{m}(2;-2;1)$; $\vec{n}(-2;-2;3)$;

4.2. $\vec{l}(-1;1;4)$; $\vec{m}(3;-3;3)$; $\vec{n}(-1;2;-3)$;

4.2. $\vec{l}(-3;2;2)$; $\vec{m}(0;-1;2)$; $\vec{n}(4;0;-5)$;

4.3. $\vec{l}(-1;2;6)$; $\vec{m}(1;-2;4)$; $\vec{n}(-4;-2;0)$;

4.4. $\vec{l}(0;-3;5)$; $\vec{m}(2;-4;1)$; $\vec{n}(4;-2;3)$;

4.5. $\vec{l}(2;-1;3)$; $\vec{m}(1;-2;-1)$; $\vec{n}(0;-2;-3)$;

4.6. $\vec{l}(-2;2;-3)$; $\vec{m}(4;-4;3)$; $\vec{n}(-1;-3;-2)$;

4.7. $\vec{l}(-2;3;0)$; $\vec{m}(2;-3;1)$; $\vec{n}(1;-2;3)$;

4.8. $\vec{l}(-4;0;4)$; $\vec{m}(2;-1;1)$; $\vec{n}(0;-4;2)$;

4.9. $\vec{l}(3;-1;3)$; $\vec{m}(-2;-5;1)$; $\vec{n}(-3;-2;0)$;

4.10. $\vec{l}(-1;2;-1)$; $\vec{m}(0;3;-1)$; $\vec{n}(-2;-3;5)$;

4.11. $\vec{l}(-5;2;2)$; $\vec{m}(3;-2;1)$; $\vec{n}(-5;-2;0)$;

4.12. $\vec{l}(-3;2;-3)$; $\vec{m}(-4;-2;0)$; $\vec{n}(-4;2;5)$;

4.12. $\vec{l}(-4;-1;0)$; $\vec{m}(5;-1;1)$; $\vec{n}(3;-2;0)$;

4.13. $\vec{l}(-3;2;5)$; $\vec{m}(1;-2;-3)$; $\vec{n}(-2;2;5)$;

4.14. $\vec{l}(-1;3;4)$; $\vec{m}(1;2;-1)$; $\vec{n}(0;-2;-4)$;

4.15. $\vec{l}(0;2;-1)$; $\vec{m}(-4;-2;3)$; $\vec{n}(3;-2;6)$;

4.16. $\vec{l}(-2;2;4)$; $\vec{m}(0;-2;2)$; $\vec{n}(4;-2;-2$;

4.17. $\vec{l}(-3;-2;4)$; $\vec{m}(0;-2;0)$; $\vec{n}(4;-1;5)$;

4.18. $\vec{l}(-1;2;-3)$; $\vec{m}(0;-2;3)$; $\vec{n}(3;-2;4)$;

4.19. $\vec{l}(-3;2;0)$; $\vec{m}(0;-2;4)$; $\vec{n}(2;-2;-5)$;

4.20. $\vec{l}(-3;2;4)$; $\vec{m}(0;-2;1)$; $\vec{n}(-4;-1;3)$

4.21. $\vec{l}(-3;2;4)$; $\vec{m}(0;-6;-1)$; $\vec{n}(-7;-2;7)$

4.22. $\vec{l}(-1;3;2)$ $\vec{m}(0;-3;4)$ $\vec{n}(0;-2;5)$

4.23. $\vec{l}(-5;4;1)$ $\vec{m}(3;-5;5)$ $\vec{n}(4;-3;1)$

4.24. $\vec{l}(3;-2;-2)$ $\vec{m}(2;-4;3)$ $\vec{n}(7;-2;6)$

4.25. $\vec{l}(0;1;-1)$ $\vec{m}(7;-5;4)$ $\vec{n}(3;-1;3)$

4.26. $\vec{l}(-2;-2;0)$ $\vec{m}(5;-3;-1)$ $\vec{n}(4;-1;4)$

4.27. $\vec{l}(-1;-3;2)$ $\vec{m}(4;-1;4)$ $\vec{n}(0;-2;7)$

4.28. $\vec{l}(1;-5;3)$ $\vec{m}(3;0;1)$ $\vec{n}(4;-1;4)$

4.29. $\vec{l}(7;0;-1)$ $\vec{m}(0;-3;1)$ $\vec{n}(3;-2;1)$

4.30. $\vec{l}(6;-1;-2)$ $\vec{m}(0;-2;0)$ $\vec{n}(0;-2;2)$

II-BOB ga oid test.

1. $\vec{a}=-\vec{i}+3\vec{j}-2\vec{k}$ vektorning uzunligini hisoblang.

A)$\sqrt{14}$ B) $\sqrt{15}$ C) 6 D) 2

1. $\vec{a}=(3;1;-2)$ va $\vec{b}=(0;-2;-1)$ vektorlarning skalyar ko'paytmasini toping.

A)1 B)0 C) -1 D) 2

2. $\vec{a}=(-3;-1;0)$ va $\vec{b}=(1;-4;1)$ vektorlarning skalyar ko'paytmasini toping.

A)1 B)0 C) -1 D) 2

3. Qaysi javobda kolleniar vektorlar berilgannini toping.

1) $\vec{a}=(2;-6;8)$, $\vec{b}=(-4;12;-16)$, 2) $\vec{a}=(3;5;8)$, $\vec{b}=(-3;-5;8)$,

3) $\vec{a}=(-3;-6;8)$, $\vec{b}=(3;6;-8)$, 4) $\vec{a}=(-3;6;8)$, $\vec{b}=(3;6;-8)$,

A)1 va 3 B)1 va 4 C) 1 va 2 D) 2 va 4

4. Agar $|\bar{a}|=4\sqrt{3}$, $|\bar{b}|=\frac{7}{\sqrt{6}}$ va $(\bar{a}\hat{\ }\bar{b})=\frac{\pi}{4}$ bo'lsa, $\bar{a}\bar{b}$ ni toping.

A)14 B)15 C) 13 D) 2

5. Agar $|\bar{c}|=100$, $|\bar{b}|=20$ va $\cos(c\hat{\ }b)=\frac{1}{10}$ bo'lsa, $\bar{c}\bar{b}$ ni toping.

A)210 B)150 C) 200 D) 120

6. Agar $|\bar{a}|=2$, $|\bar{b}|=13$ va $(a\hat{\ }b)=120^{\circ}$ bo'lsa, $\bar{a}\bar{b}$ ni toping.

A)15 B)-15 C) 13 D) -13

7. $\bar{a}=j+2k$ $\bar{b}=2i-j$ $\bar{c}=2k$ bo'lsa, $\bar{a}+2\bar{b}-\bar{c}$ ni hisoblang.

A) $4i-j$ B) $4i+2j$ C) $4i+7k$ D) $2i+5j+2k$

8. $\bar{a}=i,\ \ \bar{b}=3j\ \ \bar{c}=5k$, bo'lsa, $2\bar{a}+3\bar{b}-\bar{c}$ ni hisoblang.

A) $4i-j$ B) $2i+9j-5k$ C) $4i+7k$ D) $2i+5j+2k$

9. $\bar{a}=(4;8;x),\ \bar{b}=(1;\ y;2)$ bo'lsa, x va y ning qanday qiymatlarida $\bar{a}$ va $\bar{b}$ vektorlar kollinear hisoblanadi?

A) $x=8\ \ y=2$ B) $x=8\ \ y=8$ C) $x=2\ \ y=9$ D) $x=5\ \ y=-5$

10. $\bar{c}=(x;-12;-6),\ \bar{d}=(-2;\ 4;y)$ bo'lsa, x va y ning qanday qiymatlarida $\bar{c}$ va $\bar{d}$ vektorlar kollinear hisoblanadi?

A) $x=8\ \ y=2$ B) $x=8\ \ y=8$ C) $x=2\ \ y=9$ D) $x=6\ \ y=2$

11. $\bar{a}=(6;x;-4),\ \bar{b}=(3;\ 2;y)$ bo'lsa, x va y ning qanday qiymatlarida $\bar{a}$ va $\bar{b}$ vektorlar kollinear hisoblanadi?

A) $x=8\ \ y=2$ B) $x=8\ \ y=8$ C) $x=2\ \ y=9$ D) $x=6\ \ y=2$

12.Agar $C(1;1;1),\ D(-1;-1;-1)$ bo'lsa, $\overline{DC}$ vektor uzunligini aniqlang.

A) 2 B) 6 C) 7 D) $2\sqrt{3}$

13. Agar $E(1;0;-1),\ \ F(0;2;1)$ bo'lsa, $\overline{EF}$ vektor uzunligini hisoblang.

A) 2 B) 6 C) 7 D) 3

14. Agar $B(2;1;2),\ \ C(-2;4;2)$ bo'lsa, $\overline{BC}$ vektor uzunligini aniqlang.

A) 5 B) 6 C) 7 D) 3

15. Agar $A(2;1;1),\ B(3;1;4)$ bo'lsa, $\overline{AB}$ vektor uzunligini aniqlang.

A) 5 B) $\sqrt{10}$ C) 7 D) 3

6. $A(2;-1;0),\ B(3;1;1)$ bo'lsa, $\overline{BA}$ vektor kordinatalarini aniqlang.

) $\overline{BA}=(-1;-2;-1)$ B) $\overline{BA}=(1;0;4)$

C) $\overline{BA}=(0;-1;4)$ D) $\overline{BA}=(-1;0;-4)$

17. $B(6;\ 2;\ -1),\ D(4;\ -5;\ -1)$ bo'lsa, $\overline{BD}$ vektor koordinatasini aniqlang.

A) $\overline{BD}=(8;4;1)$ B) $\overline{BD}=(8;8;-2)$ C) $\overline{BD}=(0;\ 8;\ 3)$ D) $\overline{BD}=(-2;-7;0)$

18. $M(4;2;1),\ N(3;-1;1)$ bo'lsa, $\overline{NM}$ vektor koordinatasini aniqlang.

A) $\overline{MN}=(5;-3;\ 5)$ B) $\overline{MN}=(0,5;\ -3;\ 6)$

C) $\overline{NM}=(1;\ 3;0)$ D) $\overline{MN}=(6;\ -3;\ 4)$

19. $B(4;-1;2),\ C(5;-3;1)$ bo'lsa, $\overline{BC}$ vektor koordinatasini aniqlang.

A) $\overline{BC}=(1;-2;-1)$ B) $\overline{BC}=(8;-4;3)$ C) $\overline{BC}=(15;0;2)$ D) $\overline{BC}=(-2;-4;3)$

20. Quyidagi $\vec{n}$ va $\vec{l}$ vektorlarning skalyar ko'paytmasini aniqlang.

$\vec{n}(-2;3;-3);\ \vec{l}(-1;-3;4)$

A) 19 B) -19 C) 23 D) -23

III-BOB.TEKISLIKDA VA FAZODA ANALITIK GEOMETRIYA.

6-§. Tekislikda to‘g‘ri chiziq tenglamalari.To‘g‘ri chiziqlarning o‘zaro joylashishi.To‘g‘ri chiziqlar orasidagi burchak.

1.Tekislikda Dekart koordinatalar sistemasi va asosiy masalalar.

To‘g‘ri chiziqdagi nuqtaning vaziyati bitta son bilan aniqlanadi. Tekislikdagi nuqtaning vaziyati ikkita son bilan aniqlanadi.

Tekislikda ikkita o‘zaro perpendikulyar Oxva Oy o‘qlar berilgan bo‘lib, ular umumiy sanoq boshiga va umumiy masshtab birligiga ega bo‘lsin.

Oxy tekislikdan $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalar berilgan bo‘lsa, bu nuqtalar orasidagi masofa deb AB kesma uzunligi tushuniladi va quyidagi formula bilan topiladi:

$$d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \qquad (6.1)$$

Ushbu $$d=\sqrt{x^2+y^2}$$

formula A(x;y) nuqtadan koordinatlar boshigacha bo‘lgan masofani ifodalaydi.

Tekislikda boshi $A(x_1;y_1)$ nuqtada va oxiri $B(x_2;y_2)$ nuqtada bo‘lgan AB kesmani $\lambda=\frac{AC}{AB}$ nisbatda bo‘luvchi C nuqtaning koordinatalari ushbu

$$x=\frac{x_1+\lambda x_2}{1+\lambda},\quad y=\frac{y_1+\lambda y_2}{1+\lambda} \qquad (6.2)$$

formulalar bilan aniqlanadi.

gar $C(x_0;y_0)$ nuqta AB kesmani teng ikkiga bo‘lsa, u holda $\lambda=\frac{AC}{CB}=1$ o‘lib, (6.2) formuladan

$$x_0=\frac{x_1+x_2}{2};\ y_0=\frac{y_1+y_2}{2} \qquad (6.3)$$

kelib chiqadi.

To‘g‘ri burchakli koordinatlar sistemasida uchlari $A(x_1;y_1)$ *va* $B(x_2;y_2)$, $C(x_3;y_3)$ nuqtalarda bo‘lgan uchburchak yuzi quyidagi formula orqali topiladi:

$$S=\frac{1}{2}[(x_1y_2-x_2y_1)+(x_2y_3-x_3y_2)+(x_3y_1-x_1y_3)] \qquad (6.4)$$

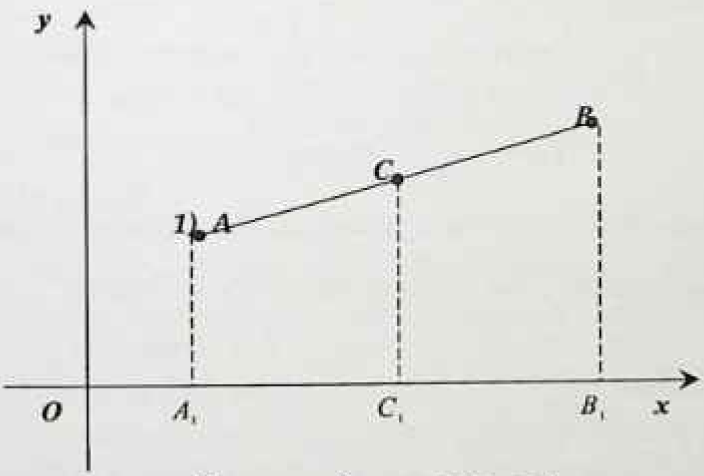
1-misol. $K(8;-3)$ *va* $L(2;5)$ nuqtalar orasidagi masofani toping.

Yechish.

$$KL=\sqrt{(2-8)^2+(5+3)^2}=\sqrt{36+64}=\sqrt{100}=10$$

hosil bo‘ladi.

2-misol. Tekislikda $A(5;-2)$ *va* $B(1;4)$ nuqtalar berilgan. AB kesmani $\lambda=\frac{AC}{CB}=0{,}2$ nisbatda bo‘luvchi C(x;y) nuqtaning koordinatlarini toping.



5-shakl

$$x_1=5, y_1=-2, x_2=1, y_2=4$$

(6.2) formulaga asosan:

$$x=\frac{x_1+\lambda x_2}{1+\lambda}=\frac{5+0{,}2\cdot 1}{1+0{,}2}=\frac{5{,}2}{1{,}2}=\frac{52}{12}=\frac{13}{3}.$$

$$y=\frac{y_1+\lambda y_2}{1+\lambda}=\frac{-2+0{,}2\cdot 4}{1+0{,}2}=\frac{-2+0{,}8}{1{,}2}=\frac{-12}{12}=-1.$$

3-misol. ΔLMN uchburchak uchlarining koordinatlari berilgan:

$$L(2,-1), M(4,2), N(-1,3)$$

Uchburchak tomonlari uzunliklarini toping.

Yechish. Ikki nuqta orasidagi masofani topish formulasidan foydalanib uchburchak tomonlarini topib chiqamiz:

$$LM = \sqrt{(4-2)^2 + (2+1)^2} = \sqrt{4+9} = \sqrt{13}.$$

$$LN = \sqrt{(-1-2)^2 + (3+1)^2} = \sqrt{9+16} = \sqrt{25} = 5.$$

$$MN = \sqrt{(-1-4)^2 + (3+2)^2} = \sqrt{25+25} = \sqrt{50}.$$

4-misol . L(-2;4) va M(4;5) nuqtalarni tutashtiruvchi LM kesma o'rtasining koordinatlarini toping.

Yechish. $x_1 = 5, y_1 = -2, x_2 = 1, y_2 = 4, \lambda = 1$

AB kesmaning o'rtasini $C(x_0; y_0)$ nuqta orqali belgilasak,

$$x_0 = \frac{x_1 + x_2}{2} = \frac{5+1}{2} = 3;$$

$$y_0 = \frac{y_1 + y_2}{2} = \frac{-2+4}{2} = 1;$$

5-misol. ΔLMN uchburchak uchlarining koordinatlari berilgan.

$L(2,3), M(4,2), N(5,3)$, shu uchburchakning yuzini toping.

Yechish. (2.4) formulaga ko'ra

$$x_1 = 2, y_1 = 3, x_2 = 4, y_2 = 2, x_3 = 5, y_3 = 3,$$

$$S = \frac{1}{2}[(x_1y_2 - x_2y_1) + (x_2y_3 - x_3y_2) + (x_3y_1 - x_1y_3)]$$

$$S = \frac{1}{2}[(2\cdot 2 - 4\cdot 3) + (4\cdot 3 - 5\cdot 2) + (5\cdot 3 - 2\cdot 3)] = \frac{1}{2}(-8 + 2 + 9) = \frac{3}{2}.$$

2.Tekislikda to'g'ri chiziqning tenglamalari.

1. Ushbu $y = kx + b$ (6.5) ko'rinishdagi tenglamaga to'g'ri chiziqning burchak koeffitsientli tenglamasi deyiladi. Bunda b-

to'g'ri chiziqning OY o'qdan ajratgan kesmasi, k- to'g'ri chiziqning burchak koeffitsenti: $k = tg\alpha$, bu yerda α-to'g'ri chiziq bilan OX oqning musbat yo'nalishi hosil qilgan burchak.

$b=0$ bo'lsa, to'g'ri chiziq koordinatalar boshidan o'tib, tenglamasi $y=kx$ bo'ladi.

$k=1$ bo'lsa, $y=x$ bo'lib, bu birinchi koordinatalar burchagining bissektrisasi bo'ladi.

2. To'g'ri chiziqning umumiy tenglamasi

$$Ax+By+C=0 \qquad (6.6)$$

ko'rinishda bo'lib, berilgan to'g'ri chiziqqa perpendikulyar bo'lgan $\vec{n} = \{A, B\}$

vector to'g'ri chiziqning normal vektori deyiladi.

3. $\vec{n} = \{A, B\}$ normal vektorga ega va $M_0(x_0, y_0)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasi:

$$A(x - x_0) + B(y - y_0) = 0 \qquad (6.7)$$

4. Kanonik tenglamasi:

$$\frac{x-x_0}{l} = \frac{y-y_0}{m} \qquad (6.8)$$

bo'ladi, bu erda $\vec{s} = (l, m)$ - to'g'ri chiziqning yo'naltiruvchi vektorlari. $M_0(x_0, y_0)$ nuqtadan o'tuvchi to'g'ri chiziq.

5. Kesmalarga nisbatan tenglamasi: $\frac{x}{a} + \frac{y}{b} = 1$ (6.9)

Bu yerda a va b -to'g'ri chiziqning koordinata o'qlarigan ajratgan kesmasi.

6. Ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasi: $M_1(x_1, y_1)$ va $M_2(x_2, y_2)$

nuqtalar berilgan,

$$\frac{x-x_1}{x_2-x_1} = \frac{y-y_1}{y_2-y_1}. \qquad (6.10)$$

7. Ikkita $y = k_1x + b_1$ va $y = k_2x + b_2$ to'g'ri chiziqlar orasidagi burchak quyidagi formula bilan topiladi:

$$tg\varphi = \frac{k_2 - k_1}{1 + k_1k_2}. \qquad (6.11)$$

8. $A_1x + B_1y + C_1 = 0$ va $A_2x + B_2y + C_2 = 0$ to'g'ri chiziq tenglamalari bilan berilgan bo'lsa, ular orasidagi burchak quyidagi formula bilan topiladi:

$$cos\varphi = \frac{A_1A_2 + B_1B_2}{\sqrt{A_1^2 + B_1^2} \cdot \sqrt{A_2^2 + B_2^2}} \qquad (6.12)$$

9. $A_1x + B_1y + C_1 = 0$ va $A_2x + B_2y + C_2 = 0$ to'g'ri chiziqlarning

paralellik va perpendikulyarlik sharti:

Parallellik sharti: $\frac{A_1}{A_2} = \frac{B_1}{B_2}$.

Perpendikulyarlik sharti: $A_1A_2 + B_1B_2 = 0$.

10. Ikkita $y = k_1x + b_1$ va $y = k_2x + b_2$ to'g'ri chiziqlarning paralellik va perpendikulyarlik sharti:

Parallellik sharti: $k_1 = k_2$; Perpendikulyarlik sharti: $k_1k_2 = -1$.

11. $M_0(x_0, y_0)$ nuqtadan berilgan $Ax + By + C = 0$ to'g'ri chiziqqacha bo'lgan masofa quyidagi formula orqali topiladi.

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} \qquad (6.13)$$

12. Parallel bo'lmagan ikki $A_1x + B_1y + C_1 = 0$ va $A_2x + B_2y + C_2 = 0$ to'g'ri chiziqning kesishish nuqtasini topish uchun ularning

$$\begin{cases} A_1x + B_1y + C_1 = 0 \\ A_2x + B_2y + C_2 = 0 \end{cases}$$

tenglamalarini birgalikda yechib, kesishish nuqtasining koordinatlari topiladi.

1-misol. $2x - y + 3 = 0$ to'gri chiziqning burchak koeffitsentini va to'g'ri chiziqning koordinata o'qlarigan ajratgan a va b kesmalarini toping.

Yechish: Tenglamadan y ni topamiz, $y = 2x + 3$ demak $k = 2$.

$2x - y = -3$ har ikkala tomonini -3 da bo'lamiz,

$\frac{2x}{-3} + \frac{y}{3} = 1$, $\frac{x}{\frac{-3}{2}} + \frac{y}{3} = 1$ demak, $a = \frac{-3}{2}, b = 3$.

2-misol. $\vec{n} = \{3,2\}$ normal vektorga ega va $M_0(2, -1)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Yechish: (6.7) formulaga ko'ra $3(x - 2) + 2(y - (-1)) = 0$

$3x - 6 + 2y + 2 = 0$, $3x + 2y - 4 = 0$. Demak, izlanayorgan to'g'ri chiziq tenglamasi , $3x + 2y - 4 = 0$ ko'rinishda bo'ladi.

3-misol. $\vec{s} = (-2,1)$ vektorga parallel va $M_0(3,4)$ nuqtadan o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Yechish: (6.8) formulaga ko'ra $\frac{x-3}{-2} = \frac{y-4}{1}$ bo'ladi, bundan

$x - 3 = -2y + 8$ $x + 2y - 11 = 0$ ko'rinishdagi to'g'ri chiziqning umumiy tenglamasi hosil bo'ladi.

4-misol. Ikki nuqta $M_1(-4,5)$ va $M_2(3, -2)$ o'tuvchi to'g'ri chiziq tenglamasini tuzing.

Yechish: (6.10) tenglamaga $\frac{x+4}{3+4} = \frac{y-5}{-2-5}$, $\frac{x+4}{7} = \frac{y-5}{-7}$ bundan tenglamaning quyidagi umumiy ko'rinishini hosil qilamiz,

$-7x - 28 = 7y - 35$, $7x + 7y - 7 = 0$.

5-misol. ΔABC uchburchak uchlarining koordinatlari berilgan, AB va BC tomonlarning tenglamasini tuzing.

$A(-2;3), B(1;-5), C(5;7)$

Yechish: (6.10) tenglamaga ko'ra AB tomon tenglamasini tuzamiz:

$\frac{x+2}{1+2} = \frac{y-3}{-5-3}$, $\frac{x+2}{3} = \frac{y-3}{-8}$, $-8x - 16 = 3y - 9$, $8x + 3y + 7 = 0$.

BC tomon tenglamasini tuzamiz:

$\frac{x-1}{5-1} = \frac{y+5}{7+5}$, $\frac{x-1}{4} = \frac{y+5}{12}$, $12x - 12 = 4y + 20$, $12x - 4y - 32 = 0$.

6-misol. Ikkita $y = k_1x + b_1$ va $y = k_2x + b_2$ to'g'ri chiziqlar orasidagi burchakni toping.

Yechish. (6.11) formulaga ko'ra $tg\varphi = \frac{3-2}{1+3\cdot 2} = \frac{1}{7}$. $\varphi = arctg\frac{1}{7}$ ga teng.

7-misol. $x + 5y + 9 = 0 \; va \; 2x - 3y + 1 = 0$ to'g'ri chiziqlar orasidagi burchakni toping.

Yechish. (6.12) formulaga ko'ra:

$$\cos\varphi = \frac{1\cdot 2 + 5\cdot(-3)}{\sqrt{1^2+5^2}\cdot\sqrt{2^2+(-3)^2}} = \frac{2-15}{\sqrt{26}\sqrt{13}} = -\frac{\sqrt{2}}{2}$$

bo'ladi. Demak, $\varphi = 135^0$.

8-misol. Quyidagi berilgan to'g'ri chiziqlarning o'zaro parallelligi yoki perpendikulyarligini tekshiring.

a) $y = 3x - 5 \; va \; y = 3x + 6$

b) $y = -4x - 2 \; va \; y = \frac{1}{4}x + 5$

c) $6x - 3y - 8 = 0 \; va \; 2x + 4y + 12 = 0$

d) $6x + 9y + 18 = 0 \; va \; 2x + 3y - 7 = 0$

Echish. Parallellik sharti: $k_1 = k_2$ va $k_1k_2 = -1$ perpendikulyarlik shartiga ko'ra $y = 3x - 5 \; va \; y = 3x + 6$ to'g'ri nchiziqlar parallel ,chunki $k_1 = k_2 = 3$.

$y = -4x - 2 \; va \; y = \frac{1}{4}x + 5$ to'g'ri chiziqlar esa perpendikulyar, chunki

$k_1k_2 = -4\cdot\frac{1}{4} = -1$.

Parallellik sharti: $\frac{A_1}{A_2} = \frac{B_1}{B_2}$ va $A_1A_2 + B_1B_2 = 0$ perpendikulyarlik shartiga ko'ra

$6x - 3y - 8 = 0 \; va \; 2x + 4y + 12 = 0$ to'g'ri chiziqlar perpendikulyar, chunki $A_1A_2 + B_1B_2 = 6\cdot 2 - 3\cdot 4 = 0$.

$6x + 9y + 18 = 0 \; va \; 2x + 3y - 7 = 0$ parallel, chunki $\frac{6}{2} = \frac{9}{3}$ va $3 = 3$.

9-misol. $M_0(4{,}2)$ nuqtadan $3x - 4y + 11 = 0$ to'g'ri chiziqqacha bo'lgan masofani toping.

Yechish. (6.13) formulaga ko'ra topamiz:

$$d = \frac{|3 \cdot 4 - 2 \cdot 4 + 11|}{\sqrt{3^2 + (-4)^2}} = \frac{15}{\sqrt{25}} = \frac{15}{5} = 3.$$

10-misol. $3x + y + 5 = 0 \ va \ x + 4y - 7 = 0$ to'g'ri chiziqlarning tekislikda o'zaro qanday joylashishini tekshiring.

Yechish. Berilgan to'g'ri chiziqlarning tekislikda joylashishini aniqlash uchun ushbu

$$\begin{cases} 3x + y + 5 = 0 \\ x + 4y - 7 = 0 \end{cases}$$

sistemani tekshiramiz. Bu to'g'ri chiziqlarning kesishish nuqtasini topish uchun bu sistemani yechamiz, chunki izlangan nuqta bir vaqtning o'zida berilgan to'g'ri chiziqlarning xar birida yotadi.

Tenglamalar sistemasini yechib $(\frac{181}{11}, -\frac{26}{11})$ nuqtaga kesishishini topamiz.

Mustaqil bajarish uchun mashqlar.

1. Berilgan to'g'ri chiziqlarni chizing, burchak koeffitsentini va to'g'ri chiziqning koordinata o'qlarigan ajratgan a va b kesmalarini toping.

a) $-4x + y - 3 = 0$; b) $x - 4y + 7 = 0$; v) $5x + 3y + 2 = 0$;

g) $2x - 3y + 6 = 0$; d) $2x - 3y + 12 = 0$; e) $8x - y = 0$;

j) $-x + 5y + 15 = 0$; i) $4x - 5y + 20 = 0$; k) $6x - 4y - 12 = 0$;

l) $x - y = 0$; m) $-5x + 7y - 35 = 0$; n) $2x - 10y + 20 = 0$;

2. $M_0(x_0, y_0)$ nuqtadan o'tuvchi va $\overrightarrow{n} = \{A, B\}$ normal vektorga ega to'g'ri chiziq tenglamasini tuzing.

a) $M_0(3, -1)$ va $\overrightarrow{n} = \{2{,}4\}$; b) $M_0(-3{,}2)$ va $\overrightarrow{n} = \{-2{,}4\}$;

v) $M_0(5{,}2)$ va $\overrightarrow{n} = \{-3{,}1\}$; g) $M_0(-4{,}3)$ va $\overrightarrow{n} = \{-3{,}1\}$;

d) $M_0(-3{,}4)$ va $\overrightarrow{n} = \{-5{,}3\}$; e) $M_0(-6{,}7)$ va $\overrightarrow{n} = \{-6{,}5\}$;

j) $M_0(6,-4)$ va $\vec{n}=\{-2,6\}$; i) $M_0(-3,5)$ va $\vec{n}=\{-4,4\}$;

k) $M_0(-1,5)$ va $\vec{n}=\{-1,2\}$; l) $M_0(6,3)$ va $\vec{n}=\{-2,2\}$;

m) $M_0(-6,-3)$ va $\vec{n}=\{-2,7\}$; n) $M_0(-5,5)$ va $\vec{n}=\{-1,4\}$;

3. $M_0(x_0, y_0)$ nuqtadan o'tuvchi va $\vec{s}=(l,m)$ vektorga parallel bo'lgan to'g'ri chiziq tenglamasini tuzing.

a) $M_0(3,-1)$ va $\vec{s}=\{2,4\}$; b) $M_0(-8,-3)$ va $\vec{s}=\{-4,2\}$;

v) $M_0(5,-3)$ va $\vec{s}=\{-3,2\}$; g) $M_0(3,4)$ va $\vec{s}=\{-3,3\}$;

d) $M_0(-3,-2)$ va $\vec{s}=\{5,3\}$; e) $M_0(-6,-4)$ va $\vec{s}=\{-7,1\}$;

j) $M_0(7,-6)$ va $\vec{s}=\{-2,1\}$; i) $M_0(-1,2)$ va $\vec{s}=\{5,-4\}$;

k) $M_0(-1,-4)$ va $\vec{s}=\{-5,2\}$; l) $M_0(-2,-3)$ va $\vec{s}=\{-6,-2\}$;

m) $M_0(-7,-3)$ va $\vec{s}=\{-1,1\}$; n) $M_0(-2,3)$ va $\vec{s}=\{-5,-3\}$;

4. $y=5x+3$ va $y=-4x+12$ to'g'ri chiziqlar orasidagi burchakni toping.

5. $2x-3y+19=0\ va\ 6x+4y+13=0$ to'g'ri chiziqlar orasidagi burchakni toping.

6. ΔABC uchburchak uchlarining koordinatlari berilgan, tomonlarning tenglamasini tuzing.

a) $A(-1;-3), B(2;\ -3), C(-5;4)$; b) $A(4;-5), B(-1;\ 6), C(-3;4)$;

v) $A(2;-4), B(6;\ -5), C(-4;3)$; g) $A(5;-3), B(4;\ -3), C(7;6)$;

d) $A(-1;-6), B(5;\ -7), C(-2;4)$; e) $A(-3;-3), B(2;\ -3), C(-5;4)$;

j) $A(-1;-4), B(4;\ -2), C(-5;3)$; i) $A(-5;-3), B(6;\ -1), C(-3;2)$;

) $A(-1;-6), B(5;\ -7), C(-2;4)$; l) $A(-3;-3), B(2;\ -3), C(-5;4)$;

) $A(-1;-4), B(4;\ -2), C(-5;3)$; n) $A(-5;-3), B(6;\ -1), C(-3;2)$;

$M_0(-3,2)$ nuqtadan $6x+8y+7=0$ to'g'ri chiziqqacha bo'lgan masofani toping.

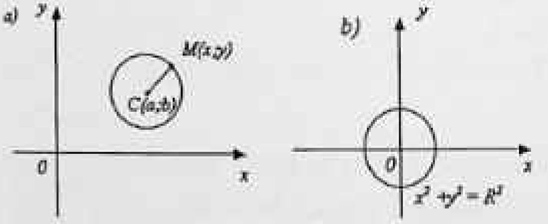
7-§. Ikkinchi tartibli egri chiziqlar. Aylana, ellips, giperbola va parabola.

7.1. Aylana deb berilgan markaz deb ataluvchi nuqtadan bir xil uzoqlikdagi nuqtalarni tutashtirishdan hosil bo'lgan geometrik shaklga aytiladi.
Markazi (a,b) nuqtada va radiusi r ga teng bo'lgan aylana tenglamasi: (1-a-rasm)

$$(x-a)^2+(y-b)^2=r^2 \qquad (7.1)$$

Markazi koordinatalar boshida va radiusi r ga teng bo'lgan aylana tenglamasi:
(1-b rasm)

$$x^2+y^2=r^2 \qquad (7.2)$$



6-shakl

1-misol. $x^2+y^2+2x-4y-4=0$ tenglama aylananing tenglamasi ekanligi ko'rsatilsin va aylananing markazi hamda radiusi topilsin.

Yechish. $A=C=1$, $B=0$, $\left(\frac{D}{2}\right)^2+\left(\frac{E}{2}\right)^2-F=1^2+2^2-(-4)=9>0$,

demak berilgan tenglama aylanani umumiy tenglamasi ekan. Tenglamani

$$(x^2+2x+1)+(y^2-4y+4)-1-4-4=0$$

ko'rinishda yozib undan

$$(x+1)^2+(y-2)^2=3^2$$

aylananing kanonik tenglamasiga ega bo'lamiz.

Shunday qilib aylananing markazi $0_1(-1;2)$ nuqta va radiusi R=3 ekan.

2-misol. $x^2-2x+y^2+2y+4=0$ tenglama hech qanday egri chiziqni aniqlamasligi ko'rsatilsin.

Yechish. Tenglamani $(x^2-2x+1)+(y^2+2y+1)-1-1+4=0$

ko'rinishda yozsak undan $(x-1)^2+(y+1)^2=-2$

tenglikka ega bo'lamiz. Koordinatalari bu tenglamani qanoatlantiruvchi nuqta mavjud emas. Demak berilgan tenglama hech qanday egri chiziqni tenglamasi emas.

7.2. Ellips deb tekislikdagi shunday nuqtalar to'plamiga aytiladiki, bu nuqtalarning har biridan shu tekislikning fokuslar deb ataluvchi ikkita nuqtasigacha bo'lgan masofalar yigindisi o'zgarmas miqdordir.
Kanonik tenglamasi quyidagi ko'rinishga ega:

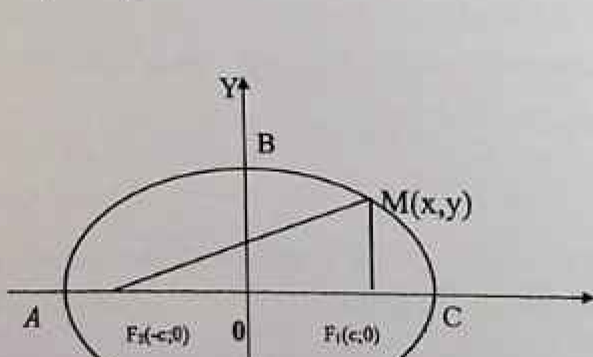
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a>b.$$

Bu yerda a va b ellipsning katta va kichik yarim o'qlari uzunliklari. 2c fokuslar orasidagi masofa, $c^2=a^2-b^2$ tenglik o'rinli bo'ladi.

Ellipsning ekstrentrisiteti: $\varepsilon=\frac{c}{a}=\frac{a^2-b^2}{a}<1.$

Ellipsning M(x,y) nuqtasidan fokuslarigacha bo'lgan masofalar uning fokal radiuslari deyiladi.

$x=\pm\frac{a}{\varepsilon}=\pm\frac{a^2}{c}, a>b,$



7-shakl

3-Misol. Kichik yarim o'qi $b=4$ va ekssentrisiteti $\varepsilon=0,6$ bo'lgan ellipsning kanonik tenglamasi yezilsin.

Yechish. Shartga ko'ra $\varepsilon=\frac{c}{a}=0,6;\ c=0,6a,\ a^2-c^2=b^2$

tenglikka c va b ning qiymatlarini qo'yib a ni aniqlaymiz.

$$a^2-(0,6a)^2=4^2; a^2(1-0,36)=16;\ 0,64a^2=16; a^2=\frac{16}{0,64}=25.$$

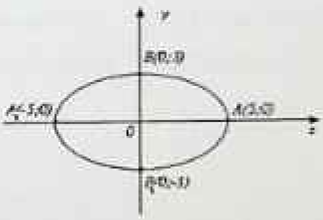
Shunday qilib ellipsning kanonik tenglamasi $\frac{x^2}{25}+\frac{y^2}{16}=1$ ko'rinishda bo'lar ekan.

4-misol. $9x^2+25y^2-225=0$ tenglamaga ko'ra ikkinchi tartibli egri chiziqning turi aniqlansin va egri chiziq chizilsin.

Yechish. Berilgan tenglamani $9x^2+25y^2=225$ ko'rinishda yozib buni 225 ga bo'lsak

$$\frac{9x^2}{225}+\frac{25y^2}{225}=1 \quad \text{yoki} \quad \frac{x^2}{5^2}+\frac{y^2}{3^2}=1$$

kelib chiqadi. Demak berilgan tenglama yarim o'qlari $a=5$, $b=3$ bo'lgan ellipsni tenglamasi ekan
(8-shakl)



8-shakl

5-misol. $4x^2-16x+9y^2-54y+61=0$ egri chiziq chizilsin.

Yechish. Tenglamani $4(x^2-4x)+9(y^2-6y)+61=0;$

$4(x^2-4x+4)+9(y^2-6y+9)-16-81+61=0;\ 4(x-2)^2+9(y-3)^2=36$

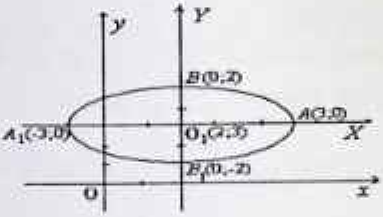
ko'rinishda yozib buni 36 ga bo'lsak $\frac{(x-2)^2}{9}+\frac{(y-3)^2}{4}=1$ yoki

$\frac{(x-2)^2}{3^2}+\frac{(y-3)^2}{2^2}=1$ tenglama hosil bo'ladi. x-2=X; y-3=Y almashtirish olsak

$\frac{X^2}{3^2}+\frac{Y^2}{2^2}=1$ kelib chiqadi.

Bu ellipsning 0_1XY sistemaga nisbatan kanonik tenglamasi.

Shunday qilib berilgan tenglama ellipsning umumiy tenglamasi ekan. Agar $0xy$ "eski" sistemani $0_1(2,3)$ nuqtaga parallel kuchirilsa ya'ni 0_1XY sistemaga nisbatan ellipsning tenglamasi kanonik ko'rinishga ega bo'lar ekan (9-shakl).



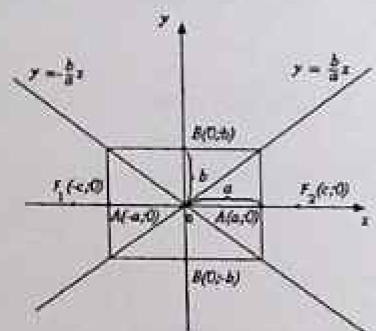
9-shakl

7.3. Giperbola deb, tekislikning har bir nuqtasidan tayin ikkita F_1 va F_2 fokuslargacha bo'lgan masofalar ayirmasi o'zgarmas songa teng.

Kanonik tenglamasi:

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1\,,$$

Bu yerda a-haqiqiy yarim oqi uzunligi, b- mavhum yarim oqi uzunligi. Fokuslar orasidagi masofani 2c desak $b^2=c^2-a^2$ bo'ladi.



10-shakl

Giperbola $0x$ o'qini ikkita $A_1(-a;0)$ va $A_2(a;0)$ nuqtalarda kesib 'tadi. Ular giprbolaning uchlari deyiladi. Ular orasidagi $|A_1\,A_2|=2a$ asofa giperbolaning haqiqiy o'qi deyiladi. $B_1(0;-b)$ va $B_2(0;b)$ nuqtalar perbolaning mavhum uchlari, ular orasidagi $|B_1B_2|=2b$ masofa esa perbolaning mavhum o'qi deb ataladi. a va b sonlariga giperbolaning haqiqiy va mavhum yarim o'qlari deb ataladi. Giperbolaning o'qlari kesishadigan nuqta uning markazi deyiladi.

Giperbolaning ekssentrisiteti: $\varepsilon = \frac{c}{a} = \frac{a^2+b^2}{a} > 1$.

$M(x,y)$ nuqtasidan fokuslarigacha bo'lgan masofalariga giperbolaning fokal radiuslari deyiladi va r_1 va r_2 bilan belgilanadi.

$x = \pm\frac{a}{\varepsilon} = \pm\frac{a^2}{c}$ tenglamagadan iborat giperbolaning direktrisasi quyidagi xossalarga ega: $\frac{r_1}{d_1} = \frac{r_2}{d_2} = \varepsilon$.

Giperbola tenglamalari $y = \pm\frac{b}{a}x$dan iborat ikkita asimptotaga ega.

Giperbolaning $M(x;y)$ nuqtasidan uning F_1 va F_2 fokuslarigacha bo'lgan masofalar shu nuqtaning fokal radiuslari deyiladi. Ular uchun

$$r_1 = \pm(a + \varepsilon x) \text{ va } r_2 = \pm(a - \varepsilon x)$$

formulalar o'rinlidir.

Agar giperbolaning kanonik tenglamasida $a = b$ bo'lsa, u holda tenglama

$x^2 - y^2 = a^2$ ko'rinishga keladi va u teng tomonli giperbola deyiladi.

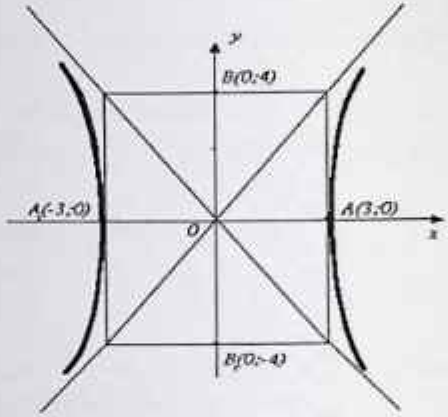
6-misol. $16x^2-9y^2=144$ egri chiziq chizilsin.

Yechish. Uni har ikkala tomonini 144 ga bo'lsak

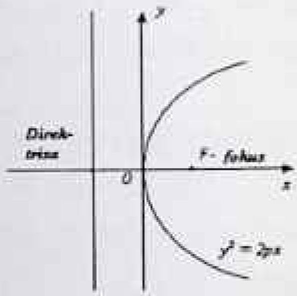
$\frac{16x^2}{144} - \frac{9y^2}{144} = 1$ yoki $\frac{x^2}{9} - \frac{y^2}{16} = 1$; $\frac{x^2}{3^2} - \frac{y^2}{4^2} = 1$

kelib chiqadi. Demak qaralayotgan egri chiziq yarim o'qlari a=3 va b=4 bo'lgan giperbola ekan. Markazi koordinatalar boshida bo'lib tomonlari koordinata o'qlariga parallel hamda asosi 6 balandligi 8 bo'lgan to'g'ri to'rtburchak yasaymiz.

Uning diagonallarini cheksiz davom ettirib giperbolaning asimptotalarini hosil qilamiz. Giperbolaning uchlari A_1(-3;0) va A(3;0) nuqtalar orqali asimptotalarga nihoyatda yaqinlashib boruvchi silliq chiziqni o'tkazamiz. Hosil bo'lgan egri chiziq giperbolaning grafigi bo'ladi (11-shakl).



11-shakl



12-shakl

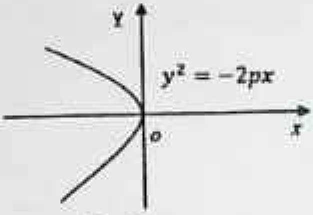
7.4. Parabola deb fokus nuqtadan va direktrisa deb ataluvchi l to'g'ri chiziqdan bir xil uzoqlikda yotuvchi tekislikdagi nuqtalar to'plamiga aytiladi.

Bunda F nuqta fokus, l to'g'ri chiziq esa direktrisa deyiladi.

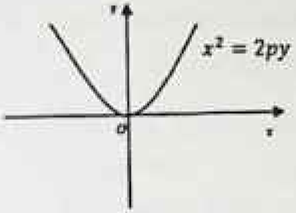
$y^2 = 2px$ tenglamaga parabolaning kanonik tenglamasi, $p(p > 0)$ uning parametri deyiladi.

$r = d = x + \frac{p}{2}$ parabolaning fokal radiusi deb ataladi. Parabola uchun $\varepsilon = \frac{r}{d} = 1$ bo'ladi .

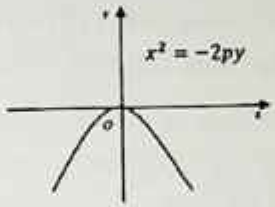
$y^2 = -2px, x^2 = 2py$ va $x^2 = -2py$ lar ham parabolaning tenglamalari bo'lib, ular mos ravishda Ox o'qining chap qismida, Oy o'qining yuqori qismida va Oy o'qining quyi qismida joylashgan bo'ladi (2,3,4-chizmalar).



2-chizma



3-chizma



4-chizma

7-misol. Markazi S(-1;1) nuqtada, radiusi 3 birlik bo'lgan aylana tenglamasini tuzing.

Yechish. Shartga ko'ra aylana markazining koordinatlari $a = -1; b = 1$ va R=3. Berilganlarni (7.1) formulaga qo'yib, aylana tenglamasini tuzamiz.

$$(x+1)^2 + (y-1)^2 = 3^2$$
$$(x+1)^2 + (y-1)^2 = 9 \ .$$

8-misol. $x^2 + y^2 - 6x + 4y - 23 = 0$ aylananing markazi va radiusi topilsin.

Yechish. $x^2 + y^2 - 6x + 4y - 23 = 0$ berilgan tenglamaning chap tomonini to'la kvadratdan iborat ifodalarga ajratamiz.

$$(x^2 - 6x + 9) + (y^2 + 4x + 4) - 13 - 23 = 0$$
$$(x-3)^2 + (y+2)^2 - 36 = 0$$
$$(x-3)^2 + (y+2)^2 = 6^2$$

bu tenglamani (7.1) tenglama bilan solishtirsak , (3; -2) nuqta aylana markazi, radiusi R = 6 bo'ladi.

9- misol. Katta yarim o'qi $a = 5$ va ekssentrisiteti $\varepsilon = 0,6$ bo'lgan holda ellipsning kanonik tenglamasini toping.

Yechish. Shartga ko'ra $\varepsilon = \frac{c}{a} = 0,6$ $\quad c^2 = a^2 - b^2$ dan $b^2 = a^2 - c^2$

$c = 0,6 \cdot 5 = 3$. Demak, fokuslar orasidagi masofaning yarmi 3 bo'ladi. Ellips kichik yarim o'qining kvadrati $b^2 = a^2 - c^2 = 25 - 9 = 16$ bo'lib, ellipsning izlanayotgan kanonik tenglamasi quyidagicha bo'ladi:

$$\frac{x^2}{25} + \frac{y^2}{16} = 1 \ .$$

10- misol. $M(2;3)$ nuqtadan o'tuvchi, katta yarim o'qi $a = 4$ bo'lgan ellipsning kanonik tenglamasini tuzing.

Yechish. $a = 4$ bo'lganda ellipsning kanonik tenglamasi

$\frac{x^2}{16}+\frac{y^2}{b^2}=1$ bo'ladi. $M(2;3)$nuqtaning koordinatlari bu tenglamani qanoatlantirishi kerak. Demak, $\frac{2^2}{16}+\frac{(-3)^2}{b^2}=1$. Bundan $b^2 = 12$ ni bo'lib va uni yuqoridagi tenglamaga qo'ysak, ellipsning izlangan kanonik tenglamasini hosil qilamiz

$$\frac{x^2}{16}+\frac{y^2}{12}=1.$$

11–misol. Fokuslari orasidagi masofa 26 ga, ekssentrisiteti esa $\frac{13}{12}$ ga tengligini bilgan holda giperbolaning kanonik tenglamasini tuzing.

Yechish. Shartga ko'ra $2c=26$ va $\varepsilon=\frac{c}{a}=\frac{13}{12}$. Demak, giperbolaning katta yarim o'qi $a=12$, $c=\frac{26}{2}=13$; $c^2=a^2+b^2$ formulaga ko'ra giperbolaning kichik yarim o'qi $b=\sqrt{c^2-a^2}=\sqrt{13^2-12^2}=5$ bo'ladi. Giperbola tenglamasining ko'rinishi quyidagicha bo'ladi:

$$\frac{x^2}{144}-\frac{y^2}{25}=1.$$

12-misol. O'qlari koordinat o'qlari bilan ustma–ust tushadigan giperbola $M_1(-3;\frac{\sqrt{2}}{2})$ va $M_1(4;-2)$ nuqtalar orqali o'tadi. Uning kanonik tenglamasini toping.

Yechish. Giperbolaning kanonik tenglamasini yozamiz: $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Bu tenglamani $M_1(-3;\frac{\sqrt{2}}{2})$ va $M_1(4;-2)$ nuqtalarning koordinatalari qanoatlantiradi. Demak,

$$\frac{(-3)^2}{a^2}-\frac{(\frac{\sqrt{2}}{2})^2}{b^2}=1 \quad ; \quad \frac{4^2}{a^2}-\frac{(-2)^2}{b^2}=1 \quad , \quad \frac{9}{a^2}-\frac{\frac{1}{2}}{b^2}=1 \quad ; \quad \frac{16}{a^2}-\frac{4}{b^2}=1$$

Bundan $a^2=8$ va $b^2=4$ ni topamiz va uni giperbolaning kanonik tenglamasiga qo'yamiz hamda quyidagini hosil qilamiz:

$$\frac{x^2}{8}-\frac{y^2}{4}=1$$

13-misol. $y^2=6x$ parabola berilgan. Uning direktrisasi tenglamasini tuzing va fokusini toping.

Yechish. Berilgan tenglamani parabolaning kanonik tenglamasi $y^2=2px$ bilan taqqoslab ko'ramizki, $2p=6\,, p=3$. Parabola direktrisasining tenglamasi $x=-\frac{p}{2}$ va fokusi $\frac{p}{2}$ va 0 koordinatlarga ega bo'lganidan, ko'rilayotgan xol uchun direktrisa tenglamasi $x=-\frac{3}{2}$ va fokus $F(\frac{3}{2};0)$ bo'ladi.

Mustaqil bajarish uchun mashqlar.

1. Quyidagi tenglamalar aylananing tenglamasi ekanligi ko'rsatilsin va aylananing markazi hamda radiusi topilsin.

a) $x^2+y^2-6x+2y+1=0$; b) $x^2+y^2+10x-6y+30=0$

2. Quyidagi tenglamalar hech qanday egri chiziqni aniqlamasligi ko'rsatilsin.

a) $x^2+y^2-4x+6y+17=0;$ b) $x^2+y^2-8x+2y+26=0$

3. Katta yarim o'qi a=10 va ekssentrisiteti ε=0,8 bo'lgan ellipsning kanonik tenglamasi topilsin.

4. $y^2=8x$ parabola berilgan. Uning direktrisasi tenglamasini tuzing va fokusini toping.

5. Fokuslari orasidagi masofa 10 ga, ekssentrisiteti esa $\frac{5}{3}$ ga tengligini bilgan holda giperbolaning kanonik tenglamasini tuzing.

8-§. Fazoda tekislikning tenglamalari.

8.1. Fazoda Dekart koordinatalar sistemasi.

Tekislikdagi Dekart koordinatalariga o'xshash fazodagi koordinatalar ham aniqlanadi, o'zaro perpendikulyar OX, OY, OZ son o'qlari, umumiy 0 nuqtadan o'tsin. Fazoda A nuqtaga uchta haqiqiy son (x, y, z) va aksincha uchta haqiqiy songa bitta nuqta mos keladi. Bu moslik ham bir qiymatlidir. Bu sonlarga nuqtaning fazodagi koordinatalari deyiladi. x abtsissasi, y ordinatasi, z *aplikatasi* deb ataladi. Koordinat o'qlaridan o'tuvchi tekisliklarga *koordinat tekisliklari* deyiladi va ular fazoni 8 ta bo'laklarga - oktantlarga ajratadi. $A\ (x, y, z)$ nuqtaning koordinatalari OA radius vektorning ham koordinatalari bo'ladi.

Fazoda ikki nuqta orasidagi masofa, kesmani berilgan nisbatda bo'lish.

Fazoda ikki A va B nuqta berilgan bo'lib, ularning koordinatlari mos ravishda $(x_1; y_1; z_1)$ va $(x_2; y_2; z_2)$ bo'lsin. Bu $A(x_1, y_1, z_1)$ va $B(x_2, y_2, z_2)$ nuqtalar orasidagi masofani topish formulasi:

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2} \quad (8.1)$$

bo'ladi.

Fazoda $A(x_1, y_1, z_1), B(x_2, y_2, z_2)$ nuqtalar berilgan bo'lib, ularni tutashtirish natijasida AB kesmani berilgan $\frac{AC}{CB} = \lambda$ nisbatda bo'luvchi nuqtaning koordinatalari quyidagi

$$x = \frac{x_1 + \lambda x_2}{1+\lambda}, y = \frac{y_1 + \lambda y_2}{1+\lambda}, z = \frac{z_1 + \lambda z_2}{1+\lambda} \qquad (8.2)$$

formulalar bilan topiladi. Xususan, C nuqta AB kesmani teng ikkiga bo'lsa $AC = CB$, u holda $\frac{AC}{CB} = \lambda = 1$ bo'lib, izlanayotgan C nuqtaning (x, y, z) koordinatlari

$$x = \frac{x_1 + x_2}{2}, \quad y = \frac{y_1 + y_2}{2}, \quad z = \frac{z_1 + z_2}{2} \qquad (8.3)$$

bo'ladi.

Fazoda tekislik tenglamalari.

8.2. $M(x_1, y_1, z_1)$ nuqtadan o'tuvchi va $\vec{n}(A, B, C)$ vektorga perpendikulyar tekislik tenglamasi.

$M(x, y, z)$ tekislikning ixtiyoriy nuqtasi bo'lsin. U xolda $M_1M \perp \vec{n}$ va ikki vektorning perpendikulyarlik shartiga ko'ra

$$A(x - x_1) + B(y - y_1) + C(z - z_1) = 0 \qquad (8.4)$$

8.3. Tekislikning umumiy tenglamasi quyidagicha yoziladi:

$$Ax + By + Cz + D = 0 \qquad (8.5)$$

$\vec{n}(A, B, C)$ vektor (4) yoki (5) tekislikka normal vektor deyiladi.

$Ax + By + Cz + D = 0$ tenglamaning xususiy xollari:

1. $D=0$ bo'lganda, $Ax + By + Cz = 0$ - tekislik koordinatlar boshidan o'tadi.

2. $C=0$ bo'lganda, $Ax + By + D = 0$ - tekislik Oz o'qqa parallel bo'ladi.

3. $B = 0$ bo'lganda, $Ax + Cz + D = 0$ - tekislik Oy o'qqa parallel bo'ladi.

4. $A = 0$ bo'lganda, $By + Cz + D = 0$ - tekislik Ox o'qqa parallel bo'ladi.

5. $C=D=0$ bo'lganda, $Ax + By = 0$ - tekislik Oz o'qdan o'tadi.

6. $B = C = 0$ bo'lganda, $Ax + D = 0$ - tekislik yOz tekislikka parallel.

7. $A = 0, B = 0$ bo'lganda, $Cz + D = 0$-tekislik Oxy tekislikka parallel.

8. $A = C = 0$ bo'lganda, $By + D = 0$ -Oxz tekisligiga parallel bo'ladi.

9. $A = B = D = 0$ bo'lganda, $Cz = 0$, $z = 0$ bo'lib, u Oxy tekisligini ifodalaydi.

10. $A = C = D = 0$ bo'lganda, $By = 0$, $y = 0$ bo'lib, u Oxz tekisligini ifodalaydi.

11. $B = C = D = 0$ bo'lganda, $Ax = 0$, $x = 0$ bo'lib, u Oyz tekisligini ifodalaydi.

8.4. Agar $A \neq 0, B \neq 0, C \neq 0, D \neq 0$ bo'lsa, u hiolda

$Ax + By + Cz + D = 0$ tekislik tenglamasini $Ax + By + Cz = -D$ ko'rinishga keltirib -D ga bo'lib, quyidagi tenglamani hosil qilamiz:

$$\frac{x}{-\frac{D}{A}}x + \frac{y}{-\frac{D}{B}} + \frac{z}{-\frac{D}{C}} = 1$$

$a = -\frac{D}{A}, b = -\frac{D}{B}, c = -\frac{D}{C}.$

Tekislikning koordinata o'qlaridan ajratgan kesmalar bo'yicha tenglamasini hosil qilamiz:

$$\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1 \qquad (8.6)$$

8.5. Fazoda bir to'g'ri chiziqda yotmagan uchta $M_1(x_1; y_1; z_1)$, $M_2(x_2; y_2; z_2)$ va $M_3(x_3; y_3; z_3)$ nuqtalardan o'tuvchi tekislik tenglamasi

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \quad (8.7)$$

ko'rinishda bo'ladi.

8.6. $M(x_1; y_1; z_1)$ nuqtadan o'tuvchi va $\frac{x-a}{m} = \frac{y-b}{n} = \frac{z-c}{p}$ to'g'ri chiziqqa perpendikulyar bo'lgan tekislik tenglamasi

$m(x - x_1) + n(y - y_1) + p(z - z_1) = 0$ ko'rinishda bo'ladi. (8.8)

8.7. Fazoda $A_1x + B_1y + C_1z + D_1 = 0$ va $A_2x + B_2y + C_2z + D_2 = 0$ tekisliklarlar berilgan:

Tekisliklar parallelik sharti: $\frac{A_1}{A_2} = \frac{B_1}{B_2} = \frac{C_1}{C_2}$.

Tekisliklar perpendikulyarlik sharti: $A_1A_2 + B_1B_2 + C_1C_2 = 0$

Fazodagi ikkita tekislik orasidagi φ burchak

$$cos\varphi = \frac{A_1A_2 + B_1B_2 + C_1C_2}{\sqrt{A_1^2 + B_1^2 + C_1^2} \cdot \sqrt{A_2^2 + B_2^2 + C_2^2}} \qquad (8.9)$$

formula yordamida topiladi.

8.8. $M_0(x_0;y_0;z_0)$ nuqtadan $Ax+By+Cz+D=0$ tekislikkacha bo'lgan masofa

$$d=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}} \text{ formuladan topiladi.} \qquad (8.10)$$

1-misol. $A(2;5;0), B(5;1;12)$ nuqtalar orasidagi masofa topilsin.

Yechish. Bu nuqtalar orasidagi masofani (8.1) formuladan foydalanib topamiz:

$$AB=\sqrt{(5-2)^2+(1-5)^2+(12-0)^2}=\sqrt{3^2+(-4)^2+12^2}=\sqrt{169}=13.$$

2-misol. M(2;-3;4) nukta orkali o'tuvchi va $\vec{n}(1;-1;4)$ vektorga perpendikulyar tekislik tenglamasi tuzing.

Yechish. Ma'lumki, berilgan $M(x_1,y_1,z_1)$ nuqtadan o'tib, berilgan $\vec{n}(A,B,C)$ vektorga perpendikulyar bo'lgan tekislikning tenglamasi

$$A(x-x_1)+B(y-y_1)+C(z-z_1)=0$$

ko'rinishga ega. Masala shartidan

$$x_1=2;\ y_1=-3;\ z_1=4;$$
$$A=1;\ B=-1;\ C=4$$

bularni yuqoridagi tenglamaga qo'yib, izlangan tekislik tenglamasini hosil qilamiz:

$$1(x-2)-1\cdot(y+3)+4(z-4)=0 \Rightarrow x-2-y3+4z-16=0 \Rightarrow$$
$$\Rightarrow x-y+4z-21=0$$

3-misol. $x+3y-5z-30=0$ tekislik berilgan. Bu tekislikning koordinat o'qlari bilan kesishish nuqtalarining koordinatlarini toping.

Yechish. Tekislikning berilgan tenglamasini uning koordinata o'qlaridan kesgan kesmalari bo'yicha tenglamasi ko'rinishiga keltiramiz:

$$\frac{2x}{30}+\frac{3y}{30}-\frac{5z}{30}=1 \quad \text{yoki} \quad \frac{x}{15}+\frac{y}{10}+\frac{z}{-6}=1.$$

Demak, tekislik Ox o'qini (15;0;0) , Oy o'qini (0;10;0) , Oz o'qini (0;0;-6) nuqtalarda kesadi.

9-§. Fazoda to'g'ri chiziq tenglamalari. To'g'ri chiziq bilan tekislikning o'zaro joylashishi.

9.1. $A(a;b;c)$ nuqtadan o'tuvchi va $l(m;n;l)$ vektorga parallel bo'lgan to'g'ri chiziq tenglamalari. $N(x;y;z)$ - to'g'ri chiziqning ixtiyoriy nuqtasi bo'lsin. U holda ikki vektorning parallellik shartiga ko'ra:

$$\frac{x-a}{m}=\frac{y-b}{n}=\frac{z-c}{p} \qquad (9.1)$$

(9.1) tenglamalar to'g'ri chiziqning kanonik tenglamalari deyiladi. $\vec{l}(m,n,p)$ vektor to'g'ri chiziqning yo'naltiruvchi vektori deyiladi.

9.2. (9.1) tenglamadagi har bir nisbatni t parametrga tenglab, to'g'ri chiziqning

$$\begin{cases} x = mt + a, \\ y = nt + b, \\ z = pt + c \end{cases} \qquad (9.2)$$

ko'rinishdagi parametrik tenglamalariga ega bo'lamiz.

9.3. Ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamalari:

$A(x_1;y_1;z_1)$ va $B(x_2;y_2;z_2)$

$$\frac{x-x_1}{x_2-x_1}=\frac{y-y_1}{y_2-y_1}=\frac{z-z_1}{z_2-z_1} \qquad (9.3)$$

9.4. To'g'ri chiziqning umumiy tenglamasi:

$$\begin{cases} A_1x + B_1y + C_1z + D_1 = 0, \\ A_2x + B_2y + C_2z + D_2 = 0. \end{cases} \qquad (9.4)$$

9.5. (9.4) tenglamalardan bir marta y ni, ikkinchi marta x ni yo'qotib, to'g'ri chiziqning proekuiyalari bo'yicha yozilgan tenglamalariga ega bo'lamiz:

$$\begin{cases} x = mz + a, \\ y = nz + b \end{cases} \qquad (9.5)$$

(9.5) tenglamalarni ushbu

$$\frac{x-a}{m}=\frac{y-b}{n}=\frac{z-0}{t} \qquad (9.6)$$

ınonik ko'rinishida yozish mumkin.

9.6. Fazodagi ikkita $\frac{x-x_1}{m_1}=\frac{y-y_1}{n_1}=\frac{z-z_1}{p_1}$ va $\frac{x-x_2}{m_2}=\frac{y-y_2}{n_2}=\frac{z-z_2}{p_2}$ to'g'ri chiziq berilgan bo'lsin.

Ushbu to'g'ri chiziqlar orasidagi burchak

$$cos\varphi=\frac{m_1m_2+n_1n_2+p_1p_2}{\sqrt{{m_1}^2+{n_1}^2+{p_1}^2}\cdot\sqrt{{m_2}^2+{n_2}^2+{p_2}^2}}$$

formuladan topiladi.

Ushbu to'g'ri chiziqlarning parallellik sharti:

$$\frac{m_1}{m_2}=\frac{n_1}{n_2}=\frac{p_1}{p_2}$$

Perpendikulyarlik sharti:

$$m_1m_2+n_1n_2+p_1p_2=0$$

1-misol. $(1;4;3)$ nuqtadan o'tgan va yo'naltiruvchi vektori $\vec{l}(2,3,1)$ bo'lgan to'g'ri chiziq tenglamasini tuzing.

Yechish. (9.1) tenglamadan foydalanamiz. Masala shartiga ko'ra:

$a=1;\ b=4;\ c=3;\ \ m=2;\ n=3;\ \ p=1.$

U holda izlanayotgan to'g'ri chiziq tenglamasi quyidagicha bo'ladi:

$\frac{x-1}{2}=\frac{y-4}{3}=\frac{z-3}{1}.$

2-misol. $A(-3;1;2)$ va $B(8;-2;5)$

nuqtalardan o'tuvchi to'g'ri chiziq tenglamalarini tuzing.

Yechish. Berilgan ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasi (3) ko'rinishda bo'lib, unga A,B nuqtalarning koordinatlarini qo'ysak,

$\frac{x+3}{8+3}=\frac{y-1}{-2-1}=\frac{z-2}{5-2}$ yoki $\frac{x+3}{11}=\frac{y-1}{-3}=\frac{z-2}{3}$

to'g'ri chiziq tenglamalariga ega bo'lamiz.

2. Tekislik bilan to'g'ri chiziqning kesishgan nuqtasi. To'g'ri chiziq tenglamalarini $x = mt + a, y = nt + b, z = pt + c$ parametrik ko'rinishda yozib, tekislikning $Ax + By + Cz + D = 0$ tenglamasidagi $x; y; z$ larning o'rniga ularning t ga nisbatan yozilgan qiymatlarini qo'yamiz. Xosil bo'lgan tenglamadan t_0 ni, so'ngra kesishgan nuqta koordinatlari x_0, y_0, z_0 ni topamiz.

3. Ikki to'g'ri chiziqning bir tekislikda yotish sharti:

$$\begin{vmatrix} a-a_1 & b-b_1 & c-c_1 \\ m & n & p \\ m_1 & n_1 & p_1 \end{vmatrix} = 0. \qquad (9)$$

3-misol. Berilgan $\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{-2}$ to'g'ri chiziq va $2x + y - 2z - 6 = 0$ tekislik orasidagi burchakni va ularning kesishish nuqtasini toping.

Echish. To'g'ri chiziq va tekislik orasidagi burchak (6) formula yordamida aniqlanadi. Shuning uchun berilgan $A = 2; B = 1; C = -2;\ m = 1; n = 2;\ p = -2$ larni bu formulaga qo'yib topamiz:

$$\sin\theta = \frac{2\cdot 1 + 1\cdot 2 + (-2)\cdot(-2)}{\sqrt{2^2 + 1^2 + (-2^2)}\cdot\sqrt{1^2 + 2^2 + (-2)^2}} = \frac{8}{3\cdot 3} = \frac{8}{9}.$$

Demak, $\theta = \arcsin\frac{8}{9}.$

Endi ularning kesishish nuqtasini topamiz, uning uchun to'g'ri chiziq tenglamasini parametrik ko'rinishga keltiramiz:

$$\frac{x-1}{1} = \frac{y-1}{2} = \frac{z-1}{-2} = t, \quad \frac{x-1}{1} = t;\ \frac{y-1}{2} = t;\ \frac{z-1}{-2} = t,$$

$$\left.\begin{aligned} x &= t + 1, \\ y &= 2t + 1, \\ z &= -2t + 1 \end{aligned}\right\} \ (*)$$

ni tekislik tenglamasiga qo'yamiz:

$$2(t+1)+2(t+1)-2(-2t+1)-6=0;$$
$$2t+2+2t+1+4t-2-6=0;$$
$$8t+3-8=0;$$
$$8t=5; \qquad t=\frac{5}{8}.$$

t ning bu qiymatini $(*)$ ga qo'ymiz:

$$x=\frac{5}{8}+1=\frac{13}{8}; \quad y=\frac{10}{8}+1=\frac{9}{4}; \qquad z=-\frac{10}{8}+1=-\frac{2}{8}=-\frac{1}{4}.$$

Demak, berilgan to'g'ri chiziq va tekislikning kesishish nuqtasi $(\frac{13}{8};\frac{9}{4};\frac{1}{4})$ dan iborat.

4-misol. $M(-1;3;0)$ nuqtadan o'tib, $2x-y-2z-4=0$ tekislikka perpendikulyar bo'lgan to'g'ri chiziq tenglamasini tuzing.

Yechish. $M_1(x_1;y_1;z_1)$ nuqtadan o'tib, $Ax+By+Cz+D=0$ tekislikka perpendikulyar to'g'ri chizik tenglamasi

$$\frac{x-x_1}{A}=\frac{y-y_1}{B}=\frac{z-z_1}{C}$$

formula yordamida aniqlanadi.

Demak,

$$\frac{x+1}{2}=\frac{y-3}{-1}=\frac{z}{-2}.$$

8-9-§ larga oid mustaqil bajarish uchun mashqlar.

1.A nuqtadan o'tib $\vec{n}$ vektorga perpendikulyar bo'lgan tekislik tenglamasini yozing.

a) A(2,-2,3), $\vec{n}=3\vec{i}+2\vec{j}-4\vec{k}$

b) A(-1,3,4), $\vec{n}=-2\vec{i}+2\vec{j}-3\vec{k}$

c) A(-3,4,2), $\vec{n}=\vec{i}-3\vec{j}$

d) A(5,3,-1), $\vec{n}=4\vec{i}+2\vec{j}-2\vec{k}$

e) A(-2,1,-4), $\vec{n} = 3\vec{\imath} - \vec{k}$

2. Tekislikning umumiy tenglamasi berilgan, bu tekislikni yasang.

a) $2x + 3y + 4z - 12 = 0$

b) $x + 3y + 6z - 6 = 0$

c) $3x - 6y + 9z - 36 = 0$

d) $4x - 2y + 8z - 16 = 0$

3. Tekisliklar orasidagi burchakni toping.

a) $x + 5y + 4z - 7 = 0$ va $2x + y - 5z - 1 = 0$

b) $4x + 3y + 4z - 3 = 0$ va $x + 2y - 4z - 3 = 0$

c) $2x + y + 4z - 7 = 0$ va $2x + y - 5z - 1 = 0$

d) $2x - y + 3z - 1 = 0$ va $7x - y - 5z = 0$

5. To'g'ri chiziqning kanonik tenglamalarini yozing.

a) $\begin{cases} 3x - y + 2z - 4 = 0 \\ 2x + 3y - 2z + 6 = 0 \end{cases}$

b) $\begin{cases} 3 + 4y - 2z + 1 = 0 \\ x - 4y - 2z + 3 = 0 \end{cases}$

:) $\begin{cases} x - y - z - 2 = 0 \\ x + 3y + 2z - 6 = 0 \end{cases}$

l) $\begin{cases} 2x - y + z + 6 = 0 \\ 3x + y + 2z - 4 = 0 \end{cases}$

) $\begin{cases} x - y + z + 2 = 0 \\ 7x + y + z - 5 = 0 \end{cases}$

$x + 5y + 2z - 5 = 0$
$x - 5y + z + 6 = 0$

$- y + 2z - 1 = 0$
$x + y + z + 11 = 0$

Uchburchakning uchlari $A(2,-3,2), B(3,2,-5)$ va $C(4,5,2)$ berilgan. BD mediananing kanonik tenglamasini yozing.

7. To‘g‘ri chiziqlar orasidagi burchakni toping.

a) $\frac{x+2}{3}=\frac{y-1}{-2}=\frac{z}{1}$ va $\frac{x-3}{-1}=\frac{y+2}{-2}=\frac{z-3}{3}$

b) $\frac{x-1}{3}=\frac{y-4}{4}=\frac{z+2}{5}$ va $\frac{x+3}{6}=\frac{y}{-2}=\frac{z+2}{-2}$

c) $\frac{x}{-3}=\frac{y+3}{6}=\frac{z-4}{-9}$ va $\frac{x+2}{6}=\frac{y-5}{-2}=\frac{z-8}{-2}$

d) $\frac{x-2}{-3}=\frac{y+1}{6}=\frac{z-4}{2}$ va $\frac{x+3}{-4}=\frac{y-2}{-1}=\frac{z+3}{5}$

e) $\begin{cases}2x+y-4z+2=0\\4x-y-5z+4=0\end{cases}$ va $\begin{cases}x=2t+1,\\y=3t-2,\\z=-6t+1\end{cases}$

l) $\begin{cases}2x-3y+2z+2=0\\x+y+z-3=0\end{cases}$ va $\begin{cases}x-y+z+2=0\\3x+4y+2z-1=0\end{cases}$

k) $\begin{cases}x-3y+4z+7=0\\2x+3y-z-3=0\end{cases}$ va $\begin{cases}2x-2y+3z+12=0\\-x+3y-3z-11=0\end{cases}$

m) $\begin{cases}2x-y-7=0\\2x-z+5=0\end{cases}$ va $\begin{cases}3x-2y+8=0\\z=3x\end{cases}$

8. $M_0(-2,\ 3,-3)$ nuqtadan o‘tib, $\frac{x+3}{6}=\frac{y}{-2}=\frac{z+2}{-2}$ to‘g‘ri chiziqqa parallel to‘g‘ri chiziqning kanonik tenglamasini yozing.

9. A(-3,1,3) va B(-1,2,5) nuqtalardan o‘tuvchi to‘g‘ri chiziq bilan $2x+y+2z-6=0$ tekislik orasidagi burchakni toping.

10. Berilgan to‘g‘ri chiziq bilan tekislikning kesishish nuqtasini toping:

a) $\frac{x-3}{6}=\frac{y+2}{-2}=\frac{z-5}{-2}$ va $2x+3y-z-3=0$

b) $\frac{x}{-3}=\frac{y+3}{6}=\frac{z-4}{-9}$ va $x+2y+3z-3=0$

c) $\frac{x-3}{-1}=\frac{y+2}{-2}=\frac{z-3}{-3}$ va $3x-2y-2z+1=0$

d) $\frac{x+3}{-4}=\frac{y-2}{-4}=\frac{z+3}{5}$ va $x+y+z-2=0$

e) $\frac{x-1}{5}=\frac{y-5}{-1}=\frac{z}{3}$ va $x-2y+2z+4=0$

f) $\frac{x-3}{3}=\frac{y-5}{-3}=\frac{z-5}{-4}$ va $2x-4y-z+5=0$

k) $\frac{x}{3}=\frac{y+5}{-1}=\frac{z-1}{-2}$ va $-x-2y+z+3=0$

11. M nuqtadan A,B,C nuqtalar orqali o'tuvchi tekislikkacha bo'lgan masofani toping:

a) M(-2,1,3), A(-3,2,5), B(3,1,4) C(2,2,1)

b) M(1,3,-3), A(-2,2,4), B(1,1,3) C(-1,2,-1)

c) M(-1,-1,2), A(-2,1,3), B(-2,1,2) C(1,-1,-2)

d) M(-2,-1,2), A(-1,2,-2), B(-1,1,2) C(-2,-2,1)

e) M(-2,1,-1), A(-3,2,-4), B(2,1,-4) C(-2,2,1)

f) M(-2,1,3), A(-3,1,0), B(2,1,-2) C(-3,2,-1)

k) M(1,1,3), A(-3,1,4), B(2,1,3) C(1,2,-2)

12.ABCD pramidaning uchlari berilgan.Quyidagilarni toping:

a) ABC, ABD tekislik tenglamalarini;

b) AB, AC, AD qirra tenglamasini;

c) ABC va ABD yoqlar orasidagi burchak kosinusini;

d) AD qirra va ABC yoq orasidagi burchak sinusini;

e) C uchdan ABD yoqqacha bo'lgan masofani.

12.1. A(-7,3,5), B(3,2,5), C(10,2,4) D(7,-2,1)

12.2. A(8,3,2), B(4,-2,2), C(3,1,-4) D(3,-1,3)

12.3. A(-4,2,-5), B(-3,4,-5), C(-2,3,-4) D(6,-1,1)

12.4. A(-1,1,0), B(1,-2,-3), C(1,2,4) D(1,-2,3)

12.5. A(2,-3,3), B(0,2,-2), C(-1,4,-2) D(2,1,4)

12.6. A(-4,2,-2), B(-3,2,4), C(-4,3,4) D(3,-1,5)

12.7. A(3,-2,4), B(4,0,-3), C(-3,2,-1) D(-1,-4,2)

12.8. A(-5,0,1), B(-3,-2,1), C(2,1,1) D(-2,-3,0)

12.9. A(6,-2,3), B(-1,0,4), C(4,-2,3) D(5,-1,2)

12.10. A(1,-3,-3), B(8,-3,-4), C(5,2,3) D(4,5,5)

12.11. A(4,-1,2), B(3,-2,-3), C(9,2,-4) D(3,3,7)

12.12. A(-2,5,4), B(-3,1,-4), C(-7,-2,-1) D(2,2,-8)

13. Quyidagi tekisliklar orasidagi burchak topilsin.

1) $x-2y+2z-8=0;$ va $x+z-6=0$

2) $x+2z-6=0;$ va $x+2y-4=0$.

14. $(2;2;-2)$ nuqtadan o'tuvchi va $x-2y-3z=0$ tekislikka parallel tekislik tenglamasi topilsin.

15. $(-1;-1;\ 2)$ nuqtadan o'tuvchi va $x-2y-3z=0$ hamda $x+2y-2z+4=0$ tekisliklarga perpendikulyar tekislikning tenglamasi yozilsin.

16. $K_1(-1;-2;0)$ va $K_2(1;1;2)$ nuqtalardan o'tuvchi hamda $x+2y+2z-4=0$ tekislikka pependikulyar tekislikning tenglamasi yozilsin.

17. A(4;3;0) nuqtadan $K_1(1;3;0)$, $K_2(4;-1;2)$ va $K_1(3;0;1)$ nuqtalardan o'tuvchi tekislikkacha bo'lgan masofa topilsin.

18. $4x+3y-5z-8=0$ va $4x+3y-5z+12=0$ parallel teksiliklar orasidagi masofa topilsin.

19 $3x-1=y,\ -3x+2=2z,$

to'g'ri chiziq bilan $2x+y+z-4=0$ tekislik orasidagi burchak topilsin.

20. $\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z-1}{3}$ to'g'ri chiziq $2x+y-z=0$ tekislikka paralel ekanligi,

$\frac{x+1}{2}=\frac{y+1}{-1}=\frac{z+3}{3}$ to'g'ri chiziq esa shu tekislik ustida yotishi ko'rsatilsin.

21. $(1;2;-3)^{(-1;2;-3)}$ nuqtadan o'tuvchi va x=2, y-z=1 to'g'ri chiziqqa perpendikulyar tekislikning tenglamasi yozilsin.

22. $\frac{x-2}{1}=\frac{y-3}{2}=\frac{z+1}{3}$ to'g'ri chiziqdan va $(3;4;0)$ nuqtadan o'tuvchi tekislikning tenglamasi yozilsin.

23. $\frac{x-1}{1}=\frac{y+1}{2}=\frac{z+2}{2}$ to'g'ri chiziqdan o'tuvchi va $2x+3y-z=4$ tekislikka perpendikulyar tekislikning tenglamasi yozilsin.

24. $\frac{x}{2}=\frac{y-1}{1}=\frac{z+1}{2}$ to'g'ri chiziqdan o'tuvchi va $x+2y+3z-29=0$ tekislik bilan kesishgan nuqtasi topilsin.

25. M(2;-3;4) nukta orqali o'tuvchi va $\vec{n}=\{1;-1;4\}$ vektorga perpendikulyar tekislik tenglamasi tuzing.

26. Ushbu tekisliklar yasalsin:

1) $5x-2y+3z-10=0$; 2) $3x+2y-z=0$;

3) $3x+2z=6$; 4) $2z-7=0$

27. $K_1(0;-1;3)$ va $K_2(1;3;5)$ nuqtalar berilgan. K_1nuqtadan o'tuvchi va $P=\overrightarrow{K_1K_2}$ vektorga perpendikulyar tekislik tenglamasi yozilsin.

28. Berilgan $2x+3y-z+2=0$ va $x+y+5z-1=0$ tekisliklar orasidagi burchakni toping.

29. M(3;-2;1) nuqtadan $3x+6y-5z+2=0$ tekislikkacha bo'lgan masofani toping.

30. A(3;-1;4) va B(1;1;2) nuqtalardan o'tuvchi to'g'ri chiziqning nglamalari yozilsin.

IV-BOB. MATEMATIK ANALIZGA KIRISH.

10-§. To`plamlar va ular ustida amallar.

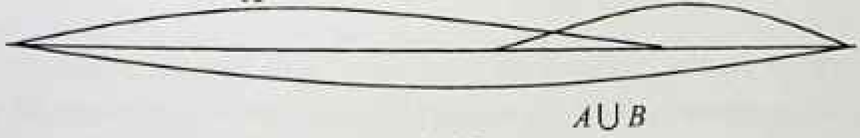
To`plamlar lotin alifbosining $A,B,C,...,X,Y,Z$ kabi bosh harflari bilan, to`plamning elementlari esa $a,b,c,...,x,y,z$ kabi kichik harflari bilan belgilanadi. Berilgan to`plamni tashkil qiluvchi ob'ektlar (narsalar) uning elementlari deyiladi.

$a \in M$ yozuv a element M to`plamga tegishli ekanligini, $a \overline{\in} M$ (yoki $a \notin M$) yozuv esa $a$ element $M$ to`plamga tegishli emasligini bildiradi.

Agar A to`plamning har bir elementi $B$ to`plamning ham elementi bo`lsa, $A$ to`plam B to`plamga qism to'plam deyiladi va $A \subset B$ ko'rinishda yoziladi.

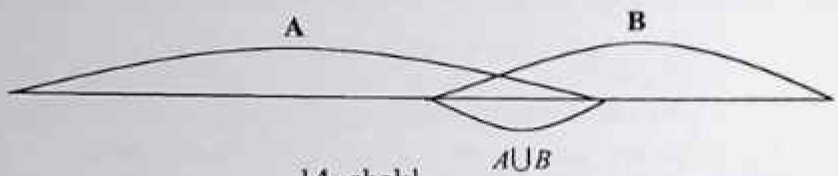
Agar A to`plam $B$ to`plamning qismi va B to`plam $A$ to`plamning qismi bo`lsa, bu to'plamlar **teng to'plamlar** deyiladi va bu munosabat $A=B$ shaklda yoziladi; Ya'ni $A \subset B$ va $B \subset A$ munosabatlarning birgalikda bajarilganidan $A=B$ ekanligi kelib chiqadi.

A va B to'plamlarning birlashmasi deb, A va B to`plamlarning elementlaridan tashkil topgan C to'plamga aytiladi va $A \cup B=C$ ko`rinishda yoziladi (13-shakl). Agar biror element A to`plamga ham $B$ to`plamga ham tegishli bo`lsa, bu element $C$ to`plamda bir marta ishtirok etadi.



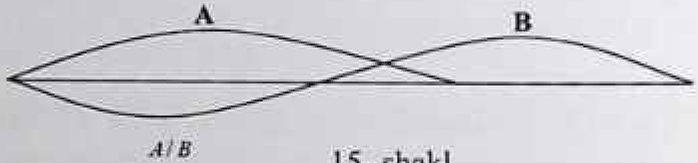
13-shakl

Ikkita A va B to`plamlarning umumiy elementlaridan tuzilgan $C$ to`plamga A va B to`plamlarning kesishmasi (umumiy qismi) deyiladi va $C=A \cap B$ ko`rinishda yoziladi (14-shakl). Bu yerda $\cap$ to`plamlar kesishmasining belgisi.



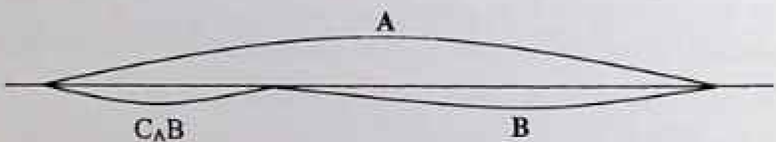
14- shakl

A to`plamning $B$ to`plamga kirmagan barcha elementlaridan tuzilgan C to`plamga $A$ va $B$ to`plamlarning ayirmasi deyiladi va u $C = A/B$ ko`rinishda yoziladi (15-shakl).



15- shakl

Agar $B \subset A$ bo`lsa, u holda $A/B$ ayirma to`plam B ni A gacha to`ldiradi deyiladi va $C_A B$ ko`rinishda yoziladi (16-shakl).



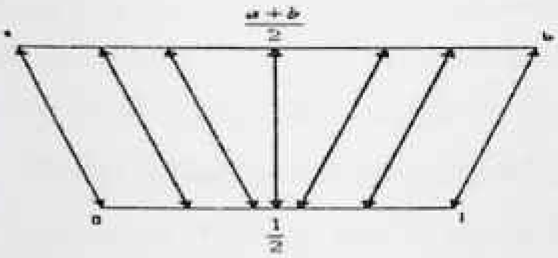
16- shakl

A va B to`plamlar berilgan bo`lsin. Agar A to`plamning har bir elementiga $B$ to`plamning bitta va faqat bitta elementi va aksincha, B to`plamning har bir elementiga $A$ to`plamning bitta va faqat bitta elementi mos qo`yilgan bo`lsa, u holda A va B to`plamlar orasida o`zaro bir qiymatli moslik o`rnatilgan deyiladi.

1-misol. Tomosha zalida stullar to`plami bilan tomoshabinlar to`plamini qaraylik. Agar bitta tomoshabinga bitta stul va aksincha bitta stulga bitta tomoshabin mos kelsa, ya'ni ortiqcha tomoshabinlar (bo`sh joylar) bo`lmasa, u holda stullar to`plami bilan tomoshabinlar to`plami orasida o`zaro bir qiymatli moslik o`rnatilgan bo`ladi.

2-misol. $A=[a;b]$ va $B=[0;1]$ to`plamlar orasidagi o`zaro bir qiymatli moslikni o`rnatuvchi formulani toping.

Yechish: $y\in[a;b]$ va $x\in[0;1]$ bo`lsin. Bu to`plamlar orasidagi o`zaro bir qiymatli moslikni o`rnatuvchi formula $y=a+(b-a)x$ ko`rinishga ega. Haqiqatan bu formuladan $x=0$ bo`lsa, $y=a$; $x=1$ bo`lsa, $y=b$ bo`lishligi kelib chiqadi. Agar $x=\frac{1}{2}$ bo`lsa, $y=\frac{a+b}{2}$ bo`lib, $[0;1]$ kesmaning o`rtasini $[a;b]$ kesmaning o`rtasiga mos qo`yadi. Bu bir qiymatli moslikni geometrik ham tasvirlash mumkin (17-shakl).



17-shakl

3-misol. $A=\{x|0\le x\le 2\}$ va $B=\{y|0\le y\le 3\}$ to`plamlar berilgan. Yangi $A\cup B$, $A\cap B$, $A|B$ va $C_AB$ to`plamlar topilsin.

Yechish. $A\cup B=\{z|0\le z\le 3\}$; $A\cap B=\{u|0\le u\le 2\}$;
$A|B=\{\upsilon|2<\upsilon\le 3\}$; $C_AB=\{k|2<k\le 3\}$.

4-misol. $A=\{a,b,c\}$ to`plamning barcha qism to`plamlaridan tashkil topgan to`plamda nechta element bo`ladi?

Yechish. Bir elementli to`plamlar 3 ta: $\{a\},\{b\},\{c\}$ ; ikki elementli to`plamlar ham 3 ta: $\{a;b\},\{a;c\},\{c;b\}$; uch elementli to`plam bitta: $\{a;b;c\}$; bo`sh to`plam ham bitta element hisoblanadi. U $\theta$($\theta$–bo`sh to`plam belgisi) orqali belgilanadi.

Shunday qilib, uch elementli to`plamning barcha qism to`plamlaridan tashkil topgan to`plamda $2^3=8$ ta element bo`ladi.

Bu to`plam $B=\{\emptyset,\{a\},\{b\},\{c\},\{a;b\},\{a;c\},\{c;b\},\{a;b;c\}\}$ ko`rinishga ega.

Mustaqil bajaish uchun misollar va masalalar.

1. Quyidagi keltirilgan to`plamlardan qaysilari chekli, qaysilari cheksiz:

1) kollejdagi talabalar to`plami; 2) manfiy butun sonlar to`plami;

3) 3 ga karrali natural sonlar to`plami; 4) yetishtirilgan g'alladagi donlar soni; 5) ko`phadning ildizlari to`plami;

6) berilgan nuqtadan o`tuvchi to`g'ri chiziqlar to`plami.

Javob: 1), 4), 5)-chekli to`plamlar; 2), 3), 6) cheksiz to`plamlar.

2. Quyidagi to`plamlarni ularning xarakteristik xossasidan foydalanib, to`plam ko`rinishida ifodalab yozing:

1) barcha natural sonlar to`plami; 2) juft natural sonlar to`plami;

3) toq natural sonlar to`plami; 4) natural sonlarning kvadratlaridan tuzilgan to`plam;

5) natural sonlarning kublaridan tuzilgan to`plam; 6) barcha butun sonlar o`plami;

) $(x^4-13x^2+36)(x^2-12x+35)=0$ tenglamaning ildizlari to`plami;

$x^3-6x^2+11x-6=0$ tenglamaning ildizlari to`plami;

$^2+y^2=100$ tenglamaning nomanfiy butun yechimlar to`plami.

b: 6) $z=\{\dots,-2,-1,0,1,2,\dots\}$; 8) $\{1;2;3\}$; 9) $\{(0;10),(10;0),(6,8),(8;6)\}$.

uyidagi to`plamlarning qaysi biri bo`sh to`plam?

) balandliklari teng bo`lmagan uchburchaklar to`plami;

2) $x^2-2\pi x+3=0$ tenglamaning butun yechimlari to`plami;

3) $\sin 2x = \frac{1}{2}$ tenglamaning musbat yechimlari to`plami;

4) $x^2 - 5x + 6 = 0$ tenglamaning manfiy butun yechimlari to`plami;

5) $\{x | 1 \le x \le 100,\ x \in N \text{ ва } x - \text{туб сон}\}$

Javob: 2) $\pi \pm \sqrt{\pi - 3} \in Z$; 3) bo`sh to`plam emas; 4) bo`sh to`plam;

5) $1 \le x \le 100$ tengsizlikni qanoatlantiruvchi tub natural sonlar 25 ta.

4. $\sin x = a$ va $\cos x = a$ tenglamalarning yechimlari qanday to`plamlar bo`ladi?

Javob: 1) $a = 0$ va $0 < |a| \le 1$ bo`lganda berilgan tenglamalar cheksiz ko`p yechimlarga ega; 2) $|a| > 1$ bo`lganda berilgan tenglamalar yechimga ega emas, ya'ni tenglamalar yechimlari bo`sh to`plam tashkil qiladi.

5. $A = \{x | 0 \le x \le 3,5\}$ va $B = \{y | 0,5 \le y \le 4,5\}$ to`plamlar berilgan.

$A \cup B,\ A \cap B,\ A / B$ va $C_A B$ to`plamlarni toping va ularni son to`g'ri chizig'ida tasvirlang.

6. Quyidagi to`plamlar bo`sh to`plam ekanligini isbotlang.

$A = \{x | x \in N, x < 0\}$; $B = \{x | x \in N,\ 10, x < 11\}$, $C = \{x | x^2 = -9\}$.

7. Quyidagi to`plamlar berilgan:

$A = \{0,3,6,9,12,15,18\}$, $B = \{1,4,7,10,13,16,19\}$,

$C = \{2,5,8,11,14,17,20\}$, $D = \{x | x \in N, x \le 20, x$ ni 3 ga bo`lganda 1 qoldiq qoladi $\}$

a) A, B, C to`plamlardan qaysi biri $D$ to`plamga teng?

b) Shunday E va K to`plamlarni tuzingki, ular uchta $A, B, C$ to`plamlardan ikkitasiga teng bo`lsin.

Javob: $D = B$.

8. Quyidagi to`plamlardan qaysilari o`zaro teng bo`ladi:

A-kvadratlar to`plami; $B$-to`g'ri to`rtburchaklar to`plami;

C-to`g'ri burchakli to`rtburchaklar to`plami;

D-tomonlari teng bo`lgan to`g'ri to`rtburchaklar to`plami;

E-to`g'ri burchakli romblar to`plami.

Javob: $A=D=E;\quad B=C.$

9. $A=\{20,5,60\}$, $B=\{20,60,8,7\}$ va $C=\{8,20,60,7,5\}$ to`plamlar berilgan.

quyidagi munosabatlar o`rinlimi?

a) $A\subset B$ b) $B\subset C$ v) $A\subset C$ g) $C\subset A$.

10. $A=\{26,39,5,58,17\}$, $B=\{14,5,17,37,26,80\}$, $C=\{80,14,37\}$ to`plamlar berilgan. Bu to`plamlar har bir juftining umumiy qismini (kesishmasini) toping.

11. $P=\{x|x\in N,\ x<20$ va x-toq son $\}$

$Q=\{x|x\in N,\ x<20$ va x-juft son $\}$,

$S=\{x|x\in N,\ x<20$ va x son 3 ga bo`linadi $\}$,

$T=\{x|x\in N,\ x<20$ va x son 4 ga bo`linadi $\}$ to`plamlar berilgan. Berilgan to`plamlarning va $Q\cap T$, $Q\cap S$, $P\cap S$, $P\cap Q$, $Q\cap S\cap T$ to`plamlarning elementlarini sanang.

12.Butun sonlarning to`plamlari berilgan:

$=\{-8,-7,-6,-5,-4\}$, $B=\{-6,-5,-4,-3,-2,-10\}$, $C=\{-4,-3,-2,-1,0,1,2\}$,

$7,-6,-5,-4\}$.

agi to`plamlar topilsin:

$\cup B\cup C\cup D$ 2) $A\cap B\cap C\cap D$ 3) $(A\cup B)\cap C$ 4) $(A\cap C)\cup(B\cap C)$

$(A\cap B)\cup(C\cap D)$ 6) $(A\cup B)\cap(C\cup D)$.

11-§. Funksiyalar. Elementar funksiyalar, monoton funksiyalar va teskari funksiyalar.

1.Funksiya haqida tushuncha va uning ta'rifi.

1-*Ta'rif* : Agar x miqdorning X sohadagi har bir qiymatiga biror f qonuniyatga ko'ra y miqdorning Y-sohadan aniq bir qiymati mos keltirilsa, y miqdor x miqdorning X-sohadagi funksiyasi deyiladi va $y=f(x)$ kabi yoziladi. Bu holda x - argument yoki erkli o'zgaruvchi, y - esa funksiya yoki erksiz o'zgaruvchi deyiladi. $y = f(x)$ funksiyada erkli o'zgaruvchi x ni qabul qiladigan qiymatlar sohasi aniqlanish sohasi y ni qabul qiladigan qiymatlar to'plami o'zgarish(qiymatlar) sohasi deyiladi, ya'ni X soha aniqlanish soha, Y soha esa o'zgarish(qiymatlar) sohasi.

Masalan, doiraning yuzi radiusning funksiyasi, kvadratning yuzi tomonining funksiyasi bo'ladi.

1-misol. Funksiyalarni aniqlanish sohasini toping.

1) $y=\frac{3}{x}$; 2) $y=\frac{1}{2}(x-1)^{-1}$; 3) $y=\sqrt{3x+2}$;

4) $y=\frac{1}{\sqrt{4x-5}}$; 5) $y=\lg(2x-1)$; 6) $y=\frac{1}{\lg(2x-1)}$;

Yechish:

1. $y=\frac{3}{x}$ funksiyalarni aniqlanish sohasi, kasr ma'noga ega bo'lishi uchun uning maxraji noldan farqli bo'lishi kerak. Demak, $x\neq 0$ yoki $x\in(-\infty;0)\cup(0;+\infty)$.

2. $y=\frac{1}{2}(x-1)^{-1}$ yuqoridagidek muhokama yuritsak, $2x-1\neq 0$ yoki $2x\neq 1$, $x\neq\frac{1}{2}$. Demak, aniqlanish sohasi $(-\infty;\frac{1}{2})\cup(\frac{1}{2};+\infty)$ dan iborat.

3. $y=\sqrt{3x+2}$ kvadrat ildiz ma'noga ega bo'lishi uchun ildiz ostidagi ifoda manfiy bo'lmasligi kerak, ya'ni $3x+2\geq 0$, bunda $x\geq-\frac{2}{3}$. Demak, aniqlanish sohasi $[-\frac{2}{3},+\infty)$ dan iborat.

4. $y=\frac{1}{\sqrt{4x-5}}$ Agar yuqoridagidek muhokama yuritsak, u holda $4x-5>0$ bo'ladi. Bundan $x>\frac{5}{4}$. Demak, aniqlanish sohasi $(\frac{5}{4}, +\infty)$ dan iborat.

5. $y=\lg(2x-1)$ Logarifmik funksiya faqat musbat sonlar uchun aniqlangan. Demak, $(2x-1)>0$ bo'lishi kerak. Bundan $x>\frac{1}{2}$. Demak, aniqlanish sohasi $(\frac{1}{2}, +\infty)$ dan iborat.

6. $y=\frac{1}{\lg(2x-1)}$ yuqoridagidek muhokama yuritsak, $2x-1>0$, $2x-1\neq 1$ bo'ladi. Bundan $x>\frac{1}{2}$, $x\neq 1$ kelib chiqadi. Demak, aniqlanish sohasi $(\frac{1}{2}; +1)\cup(1; +\infty)$ dan iborat.

3. Chegaralangan va chegaralanmagan funksiyalar.

2- **Ta'rif.** $y=f(x)$ funksiyaning o'zgarish sohasidagi har qanday qiymati uchun shunday o'zgarmas chekli B sonni ko'rsatish mumkin bo'lib, $f(x)\leq B$ bo'lsa, $f(x)$ *yuqoridan chegaralangan* funksiya deyiladi.

3- **Ta'rif.** $y=f(x)$ funksiyaning o'zgarish sohasidagi har qanday qiymati uchun shunday o'zgarmas chekli A sonni ko'rsatish mumkin bo'lib, $f(x)\geq A$ bo'lsa, $f(x)$ *quyidan chegaralangan* deyiladi.

2-Misol. 1. $y=x^2-4x+6$ funksiya $-\infty<x<+\infty$ oraliqda aniqlangan bo'lib, u quyidan chegaralangan. Haqiqatdan ham, $y=(x-2)^2+2$. Demak, $y\geq 2$ ya'ni funksiyaning eng katta qiymati yo'q. Eng kichik qiymati 2.

2. $y=-3x^2+4x+1$ funksiya yuqoridan chegaralangan. Haqiqatdan ham,

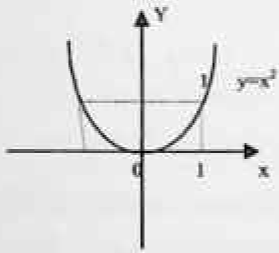
$$-3x^2+4x+1=-3(x^2-\frac{4}{3}x-\frac{1}{3})=-3(x-\frac{2}{3})^2-\frac{7}{3},$$

ya'ni funksiyaning eng katta qiymati bor. Eng kichik qiymati yo'q. Demak, $y\leq\frac{7}{3}$.

4-Ta'rif. Agar $y=f(x)$ funksiya yuqoridan ham, quyidan chegaralangan bo'lsa, ya'ni $A \le f(x) \le B$ bo'lsa, bunday funksiyaga **chegaralangan** funksiya deyiladi.

Masalan, $y=sinx$, $y=cosx$ funksiyalar chegaralangandir, chunki $-1 \le sinx \le 1$ va $-1 \le cos \le 1$ shartlari bajariladi.

5-Ta'rif. $y=f(x)$ funksiyaning aniqlanish sohasiga tegishli x o'zgaruvchining har bir qiymati bilan $-x$ qiymat ham shu funksiyaning aniqlanish sohasiga tegishli bo'lsa va bunda $f(-x)=f(x)$ tenglik bajarilsa, $y=f(x)$ funksiya **juft funksiya** deyiladi. Bundan tashqari, $f(-x)=(-x)^2=x^2=f(x)$ tenglik bajariladi. Juft funksiya grafigi ordinata o'qiga nisbatan simmetrik bo'ladi (1-chizma).



18-shakl

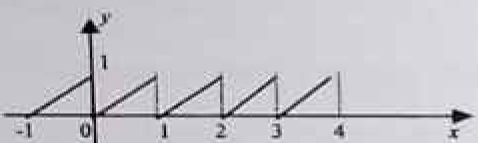
$y=cos\alpha$ juft funksiyadir, $cos\alpha=cos(-\alpha)$.

6-Ta'rif. $y=f(x)$ funksiyaning aniqlanish sohasiga tegishli x ning har bir qiymati bilan $-x$ qiymat ham shu funksiyaning aniqlanish sohasiga tegishli bo'lsa va bunda $f(-x)=-f(x)$ tenglik bajarilsa, $y=f(x)$ funksiya **toq funksiya** deyiladi.

Toq funksiyaning grafigi koordinata boshiga nisbatan simmetrik joylashadi. Masalan, $f(x)=x^3$ funksiya toq funksiyadir. $sin(-\alpha)=-sin\alpha$.

$y=2x+5, y=x^2+x^3$, $y=sinx+cosx$ juft ham, toq ham emas. Demak funksiyalar har doim juft yoki toq bo'lishi shart emas ekan.

7-Ta'rif. Agar $f(x)$ funksiya uchun shunday $T>0$ son mavjud va funksiyaning aniqlanish sohasidan olingan har bir x uchun $x+T$ va $x-T$ lar aniqlanish sohasiga joylashgan bo'lib, $f(x+T)=f(x)$ tenglik o'rinli bo'lsa, u holda $f(x)$ **davriy funksiya** deb ataladi. T sonlarni eng kichigi funksiyaning *davri* deyiladi.



19-shakl

Masalan, y=sinx , y=cosx, y=tgx, y=x-[x] davriy funksiyalardir.

Davriy funksiyaning grafigini hosil qilish uchun uning bir davr ichidagi grafigini chizib, so'ngra uni chapga va o'ngga cheksiz ko'p marta ko'chirish kerak.

Masalan, $f(x)=x-[x]=x-E(x)$ funksiya berilgan. Bunda $E(x)=[x]$ ifoda x ning butun qismini bildiradi. (E – fransuzcha Entier -ante- butun so'zining birinchi harfi). Masalan, $[x]=m$ $(m\le x<m+1)$ m butun son.

$f(x)=x-E(x)=\{x\}$. Bu funksiya x ning kasr qismini bildiradi, ya'ni $f(1)=0;\ f(1{,}05)=0{,}05;\ \ldots$. $f(x)$ funksiya davriydir va uning davri $t=1$ dir. Haqiqatdan,

$$f(x+1)=x+1-E(x+1)=x+1-E(x)-1=x-E(x)=f(x).$$

Demak, har qanday butun son ham davr bo'ladi. Funksiyaning grafigi 19-shaklda ko'rsatilgan.

Monoton funksiyalar

8-Ta'rif: $y=f(x)$ funksiyaning X sohadagi ixtiyoriy ikkita (x_1,x_2) qiymatlari uchun $x_1\le x_2$ bo'lganda $f(x_1)\le f(x_2)$ tengsizlik o'rinli bo'lsa, u holda $y=f(x)$ funksiyasi (x_1,x_2) oralig'ida **o'suvchi** funksiya deyiladi.

Yuqoridagi ta'rifdan ko'rinadiki, funksiya biror oraliqda o'suvchi bo'lishi uchun shu oraliqdagi argumentning kichik qiymatiga funksiyaning kichik qiymati, argumentning katta qiymatiga funksiyaning katta qiymati mos kelar ekan.

1) $y=2^x$ funksiyasi butun son o'qida o'suvchi.
2) $y=tgx$ funksiya ham o'suvchi funksiyadir.

9-Ta'rif: $y=f(x)$ funksiyaning X sohadagi ixtiyoriy ikkita (x_1,x_2) qiymatlari uchun $x_1 \le x_2$ bo'lganda $f(x_1) \ge f(x_2)$ tengsizlik o'rinli bo'lsa, u holda $y=f(x)$ funksiyasi (x_1,x_2) oralig'ida **kamayuvchi** funksiya deyiladi.

Ta'rif: Biror (x_1, x_2) oralig'ida o'suvchi va kamayuvchi funksiyalar monoton funksiyalar deyiladi.

2. Elementar funksiyalar.

Bu yerda elementar funksiyalar deb atalgan funksiyalarning ba'zi bir sinflarini ko'rsatib o'taylik.

1. **Darajali funksiya**. Quyidagi $y=x^{\mu}$ ko'rinishdagi funksiyani darajali funksiya deyiladi, bu yerda μ ixtiyoriy o'zgarmas haqiqiy son. Agar μ kasr bo'lsa, biz ildizga ega bo'lamiz. Masalan, m natural son bo'lsin va:

$$y=x^{\frac{1}{m}}=\sqrt[m]{x}.$$

Bu funksiya m toq bo'lganda, x ning hamma qiymatlari uchun va m juft bo'lganda, x ning faqat musbat qiymatlari uchun aniqlanadi. Bu holda biz ildizning faqat arifmetik qiymatini hisobga olamiz. Nihoyat, μ irratsional son bo'lsa, $x>0$ deb faraz etamiz ($x=0$ qiymat $\mu>0$ bo'lgandagina olinadi).

3.**Ko'rsatkichli funksiya**, $y=a^x$ ko'rinishdagi funksiyaga ko'rsatkichli funksiya deyiladi, bu yerda $a>0, a\neq 1$. x istalgan haqiqiy qiymat qabul qila oladi. $a>1$ bo'lganda o'suvchi, $0<a<1$ bo'lganda kamayuvchi.

4.**Logarifmik funksiya**, ya'ni $y=\log_a x$ ko'rinishdagi funksiya, bu yerda $a>0, a\neq 1$; x faqat musbat qiymatlar qabul qiladi.$(0,\infty)$ da aniqlangan.

$a>1$ bo'lganda o'suvchi, $0<a<1$ bo'lganda kamayuvchi.

5.**Trigonametrik funksiyalar.** Ushbu

$y=sinx, y=cosx, y=tgx, y=ctgx$ funksiyalar trigonametrik funksiyalar deyiladi.

Ushbu $y = arcsinx$, $y = arccosx, y = arctgx, y = arcctgx$ funksiyalarga teskari trigonametrik funksiyalar deyiladi.

Teskari funksiyalar.

Faraz qilaylik , biror Y to'plamdan olingan har bir y ga X to'plamdan yagona x biror qoidaga ko'ra mos qo'yilgan bo'lsa, funksiya hosil bo'ladi. Bu funksiya $y = f(x)$ funksiyaga teskari funksiya deyiladi va quyidagicha yoziladi: $x = f^{-1}(y)$ kabi belgilanadi. Masalan,$y = \frac{1}{5}x - 2$ funksiyaga teskari funksiya $x = 5(y + 2)$ bo'ladi.

Mustaqil bajarish uchun misol va masalalar.

1. Quyidagi funksiyalarni aniqlanish sohasini toping.

1) $y = \sqrt{x^2 - 4}$; 4) $y = \frac{x}{x+3}$;

2) $y = \frac{1}{x^2-6x+8} + \sqrt[3]{2x + 1}$; 5) $y = \sqrt{x^2 - 8x + 15}$

3) $y = \ln(x^2 - 1)$; 6) $y = \lg(x^2 - 7x + 12)$

2. Quyidagi funksiyalarni juft toqligini ko'rsating.

1) $f(x) = x^2 - 7$; 2) $f(x) = x^3 - 7x$;

3) $f(x) = 2x^2 - 5x^4$; 4) $f(x) = cosx - sinx$;

5) $f(x) = 22^x - 5x^4$; 6) $f(x) = 6cosx - x^2$

3. Berilgan funksiyalar uchun quyidagi murakkab funksiyalarni toping:

1)$f(x) = x^2,\ g(x) = x + 3,$ $f(f(x))$, $f(g(x))$, $g(f(x))$, $g(g(x))$.

2) $u(x) = x^3,\ v(x) = sinx,$ $v(u(x))$, $u(v(x))$, $u(u(x))$.

12-§. Limitlar. Limitlar haqidagi teoremalar: Ajoyib limitlar.

Sonli ketma-ketlik va uning limiti

1-Ta'rif. Agar har bir n natural son uchun biror x_n son mos qo'yilgan bo'lsa, u holda

$$x_1, x_2, x_3, \dots, x_n, \dots$$

ketma-ketlik berilgan deyiladi, u $\{x_n\}$ orqali belgilanadi. x_n-ketma-ketlikning umumiy hadi deb ataladi.

Ketma-ketliklarga misollar. $\left\{\frac{1}{n}\right\} = 1, \frac{1}{2}, \frac{1}{3}, \dots, \frac{1}{n}, \dots.$

$$\{2^n\} = 2,4,8, \dots\dots, 2^n, \dots.$$

$$\{1\} = 1,1,1, \dots\dots,1, \dots..$$

$$\{\sin n\} = sin1, sin2, sin3, \dots\dots, \sin n, \dots..$$

Ketma-ketlikning limiti. $\{x_n\}$ ketma-ketlik va a soni berilgan bo'lsin.

2-Ta'rif. Agar ixtiyoriy yetarlicha kichik $\varepsilon > 0$ son uchun shunday musbat butun $N = N(\varepsilon)$ son topilib, $n \geq N$ tengsizlikni qanoatlantiruvchi barcha n natural sonlarda $|x_n - a| < \varepsilon$ tengsizlik bajarilsa, a soni $\{x_n\}$ ketma-ketlikning limiti deb ataladi va $a = \lim\limits_{n\to\infty} x_n$ ko'rinishda belgilanadi.

3-Ta'rif. Ixtiyoriy $M > 0$ son uchun shunday N natural son topilib, barcha $n > N$ uchun $|x_n| > M$ tengsizlik o'rinli bo'lsa, $\{x_n\}$ ketma-ketlik cheksiz katta deb ataladi va u $\lim\limits_{n\to\infty} x_n = \infty$ ko'rinishda yoziladi.

Agar ixtiyoriy $M > 0$ son uchun shunday N natural son topilib, barcha $n > N$ uchun $x_n > M$ (mos ravishda $x_n < -M$) tengsizlik o'rinli bo'lsa, bu holda ham ketma-ketlik cheksiz katta deb ataladi va u $\lim\limits_{n\to\infty} x_n = +\infty$ (mos ravishda $\lim\limits_{n\to\infty} x_n = -\infty$) ko'rinishda yoziladi.

Agar ketma-ketlik (chekli) limitga ega bo'lsa, uni biz *yaqinlashuvchi*, limitga ega bo'lmasa, *uzoqlashuvchi* ketma-ketlik deb ataymiz.

Yaqinlashuvchi ketma-ketliklarning xossalari.

4-Ta'rif. Agar ketma-ketlik qiymatlari to'plami yuqoridan (quyidan) chegaralangan bo'lsa, u holda bu ketma-ketlik yuqoridan (quyidan) chegaralangan deyiladi.

5-Ta'rif. Agar ketma-ketlik yuqoridan ham, quyidan ham chegaralangan bo'lsa, u chegaralangan ketma-ketlik deb ataladi.

1-Teorema. Agar $\{x_n\}$ va $\{y_n\}$ ketma – ketliklar yaqinlashuvchi va $\lim\limits_{n\to\infty} x_n = a, \lim\limits_{n\to\infty} y_n = b$ bo'lsa, $\{x_n \pm y_n\}$ ketma – ketlik ham yaqinlashuvchi bo'ladi va

$$\lim_{n\to\infty}(x_n \pm y_n) = \lim_{n\to\infty} x_n \pm \lim_{n\to\infty} y_n = a \pm b$$

tenglik o'rinli bo'ladi.

2-Teorema. Agar $\{x_n\}$ ketma – ketlik yaqinlashuvchi bo'lsa, $\{c \cdot x_n\}$ ketma – ketlik ham yaqinlashuvchi hamda $\lim\limits_{n\to\infty} c \cdot x_n = c \cdot \lim\limits_{n\to\infty} x_n$ tenglik o'rinli bo'ladi.

3-Teorema. Agar $\{x_n\}$ va $\{y_n\}$ ketma – ketliklar yaqinlashuvchi bo'lsa, $(x_n \cdot y_n)$ ketma – ketlik ham yaqinlashuvchiva $\lim\limits_{n\to\infty}(x_n \cdot y_n) = \lim\limits_{n\to\infty} x_n \cdot \lim\limits_{n\to\infty} y_n$ tenglik o'rinli bo'ladi.

4-Teorema. Agar $\{x_n\}$ va $\{y_n\}$ ketma – ketliklar yaqinlashuvchi, hamda ixtiyoriy n uchun $y_n \neq 0$ va $\lim\limits_{n\to\infty} y_n \neq 0$ bo'lsa, u holda $\left\{\frac{x_n}{y_n}\right\}$ ketma – ketlik ham yaqinlashuvchi va $\lim\limits_{n\to\infty}\left(\frac{x_n}{y_n}\right) = \frac{\lim\limits_{n\to\infty} x_n}{\lim\limits_{n\to\infty} y_n}$ tenglik o'rinli bo'ladi.

?.Monoton ketma – ketliklar. $\{x_n\}$ ketma – ketlik berilgan bo'lsin.

a'if. Agar $x_1 \leq x_2 \leq x_3 \ldots . \leq x_n \leq x_{n+1} \leq \cdots$ o'rinli bo'lsa. $\{x_n\}$ ıymaydigan ketma-ketlik deb ataladi. Agar $x_1 < x_2 < x_3 \ldots . < x_n <$ $< \cdots$ o'rinli bo'lsa, $\{x_n\}$ qat'iy o'suvchi ketma-ketlik

, ataladi.

-Ta'if. Agar $x_1 \geq x_2 \geq x_3 \ldots . \geq x_n \geq x_{n+1} \geq \cdots$ o'rinli bo'lsa. $\{x_n\}$ o'smaydigan ketma-ketlik deb ataladi. Agar $x_1 > x_2 > x_3 \ldots . > x_n >$

$x_{n+1} > \cdots$ o'rinli bo'lsa, $\{x_n\}$ qat'iy kamayuvchi ketma-ketlik deb ataladi.

O'smaydigan va kamaymaydigan ketma – ketliklar monoton ketma-ketliklar deb ataladi.

Monoton o'suvchi ketma- ketliklar o'zining birinchi hadi bilan quyidan chegaralangan, monoton kamayuvchi ketma ketliklar esa o'zining birinchi hadi bilan yuqoridan chegaralangan bo'ladi.

Agar kamayadigan ketma –ketlik yuqoridan chegaralangan bo'lsa, ya'ni shunday M soni topilib, bunda ixtiyoriy n sonda $x_n < M$ bo'lsa, u holda $\{x_n\}$ chegaralangan bo'ladi.

5-Teorema (Veyershtrass alomati). Har qanday monoton chegaralangan ketma-ketlik limitga ega.

***e* soni.** $x_n = (1 + 1/n)^n$ ketma-ketlikning limiti mavjud va bu limitni biz e orqali belgilaymiz va uning qiymati

$$e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n = 2{,}7182818459 \ldots.$$

ko'rinishida bo'ladi.

1-Misol. Ketma-ketlikning limiti ta'rifidan foydalanib $\lim\limits_{n\to\infty} 1/n = 0$ ekanligini isbotlang.

Yechish: 1) Ta'rifga ko'ra agar ixtiyoriy $\varepsilon > 0$ soni uchun shunday $N(\varepsilon)$ soni topilib, $n > N(\varepsilon)$ tengsizlikni qanoatlantiruvchi barcha n natural sonlarda

$$\left|\frac{1}{n} - 0\right| = \frac{1}{n} < \varepsilon$$

tengsizlik bajarilsa 0 soni $1/n$ ketma-ketlikning limiti deyiladi.

2) Endi qanday n lar uchun $1/n < \varepsilon$ tengsizlik bajarilishini tekshiramiz, ya'ni n ga nisbatan bu tengsizlikni yechamiz

$1/n < \varepsilon \leftrightarrow n > N(\varepsilon) = 1/\varepsilon$ tengsizlikni hosil qilamiz.

2-Misol. $\lim\limits_{n\to\infty}\frac{n^2-2n+5}{2n^2-3n+2}$ Limitni hisoblang:

Yechish: Limit ostidagi kasrning surat va maxrajini n^2 ga bo'lamiz:

$$\lim_{n\to\infty}\frac{n^2-2n+5}{2n^2-3n+2}=\lim_{n\to\infty}\frac{1-\frac{2}{n}+\frac{5}{n^2}}{2-\frac{3}{n}+\frac{2}{n^2}}=\frac{\lim\limits_{n\to\infty}\left(1-\frac{2}{n}+\frac{5}{n^2}\right)}{\lim\limits_{n\to\infty}\left(2-\frac{3}{n}+\frac{2}{n^2}\right)}=$$

$$=\frac{\lim\limits_{n\to\infty}1-\lim\limits_{n\to\infty}\frac{2}{n}+\lim\limits_{n\to\infty}\frac{5}{n^2}}{\lim\limits_{n\to\infty}2-\lim\limits_{n\to\infty}\frac{3}{n}+\lim\limits_{n\to\infty}\frac{2}{n^2}}=\frac{1-2\lim\limits_{n\to\infty}\frac{1}{n}+5\lim\limits_{n\to\infty}\frac{1}{n^2}}{2-3\lim\limits_{n\to\infty}\frac{1}{n}+2\lim\limits_{n\to\infty}\frac{1}{n^2}}=\frac{1-2\cdot 0+5\cdot 0}{2-3\cdot 0+2\cdot 0}$$

$$=\frac{1}{2}.$$

3-Misol. $\lim\limits_{n\to\infty}\frac{n^4-5n+1}{n^3-n^2+7}$ limitni hisoblang:

Yechish: Limit ostidagi kasrning surat va maxrajini n^3 ga bo'lamiz:

$$\lim_{n\to\infty}\frac{n^4-5n+1}{n^3-n^2+7}=\lim_{n\to\infty}\frac{n-\frac{5}{n^2}+\frac{1}{n^3}}{1-\frac{1}{n}+\frac{7}{n^3}}=\frac{\lim\limits_{n\to\infty}\left(n-\frac{5}{n^2}+\frac{1}{n^3}\right)}{\lim\limits_{n\to\infty}\left(1-\frac{1}{n}+\frac{7}{n^3}\right)}=$$

$$=\frac{\lim\limits_{n\to\infty}n-\lim\limits_{n\to\infty}\frac{5}{n^2}+\lim\limits_{n\to\infty}\frac{1}{n^3}}{\lim\limits_{n\to\infty}1-\lim\limits_{n\to\infty}\frac{1}{n}+\lim\limits_{n\to\infty}\frac{7}{n^3}}=\frac{\infty-0+0}{1-0+0}=\infty.$$

13-§. Funksiyaning limiti. Cheksiz katta va cheksiz kichchik funksiyalar.

1. $y = f(x)$ funksiya X to'plamda aniqlangan bo'lsin.

8.Ta'rif. Agar har bir $\varepsilon > 0$ *uchun shunday* $\delta > 0$ $(\delta = \delta(\varepsilon))$ topilib ,to'plamning $0 < |x - a| < \delta$ tengsizlikni qanoatlantiruvchi barcha x nuqtalarida $|f(x) - A| < \varepsilon$ tengsizlik o'rinli bo'lsa, u holda A f(x) funksiyaning x=a nuqtadagi limiti deyiladi va $\lim_{x \to n} f(x) = A$ ko'rinishda yoziladi.

9.Ta'rif. Agar har bir $\varepsilon > 0$ *uchun shunday* $\delta > 0$ *topilib*, $|x| > \delta$ tengsizlikni qanoatlantiruvchi x larda $|f(x) - A| < \varepsilon$ tengsizlik o'rinli bo'lsa, A f(x) funksiyaning $x \to \infty$ dagi limiti deyiladi va $\lim_{x \to \infty} f(x) = A$ ko'rinishda yoziladi.

10.Ta'rif. Agar $y = f(x)$ funksiyaning x=a nuqtadagi yoki $x \to a$ dagi limiti ta'rifida x o'zgaruvchi a dan kichik bo'lganicha qolsa, u holda funksiyaning A_1 limiti funksiyaning x=a nuqtadagi (yoki $x \to a - 0$ *dagi*) chap tomonlama limiti deb ataladi.

$A_1 = \lim_{x \to a-0} f(x)$ agar a=0 bo'lsa , u holda $A_1 = \lim_{x \to -0} f(x)$ ko'rinishda yoziladi.

11.Ta'rif. Agar $y = f(x)$ funksiyaning x=a nuqtadagi yoki $x \to a$ dagi limiti ta'rifida x o'zgaruvchi a dan katta(x>a) bo'lganicha qolsa, u holda funksiyaning A_2 limiti funksiyaning x=a nuqtadagi (yoki $x \to a + 0$ *dagi*) o'ng tomonlama limiti deb ataladi.

$A_1 = \lim_{x \to a+0} f(x)$ agar a=0 bo'lsa , u holda $A_1 = \lim_{x \to +0} f(x)$ ko'rinishda yoziladi.

12.Ta'rif. Agar $y = f(x)$ funksiyaning a nuqtaning biror atrofida aniqlangan va istalgan M>0 son uchun shunday yoki $\delta > 0$ son mavjud bo'lsaki $|x - a| < \delta$

tengsizliknin qanoatlantiradigan barcha $x \neq a$ lar uchun $|f(x)| > M$ tengsizlik bajarilsa , $x \to a$ da f(x) funksiya cheksizlikka intiladi deb aytiladi va bu quyidagicha yoziladi: $\lim_{x \to a} f(x) = \infty$.

13.Ta'rif. Agar

$\lim_{x \to a} f(x) = \infty$ yoki $\lim_{x \to \infty} f(x) = \infty$ bo'lsa, u holda f(x) funksiya $x \to a$ da cheksiz katta funksiya deb ataladi.

14.Ta'rif. Agar $\lim_{x \to a} f(x) = 0$ yoki $\lim_{x \to \infty} f(x) = 0$ bo'lsa, u holda f(x) funksiya $x \to a$ da cheksiz kichik funksiya deb ataladi.

6.Teorema. 1. Agar f(x) funksiya $x \to a$ da ($x \to \infty$ da) cheksiz kichik funksiya bo'lsa, u holda $\frac{1}{f(x)}$ funksiya $x \to a$ da ($x \to \infty$ da) cheksiz katta funksiyadir.

2. Agar f(x) funksiya $x \to a$ da ($x \to \infty$ da) cheksiz katta funksiya bo'lsa, u holda $\frac{1}{f(x)}$ funksiya $x \to a$ da ($x \to \infty$ da) cheksiz kichik funksiyadir.

7.Teorema. Cheksiz kichik funksiyaning algebraik yig'indisi, ko'paytmasi, chegaralangan funksiyaga ko'paytmasi, noldan farqli limitga ega bo'lgan funksiyaga bo'linmasi cheksiz kichik funksiyadir.

4-misol: Limitni hisoblang: $\lim_{x \to 2}(3x - 5)$.

Yechish. $\lim_{x \to 2}(3x - 5) = 3 \cdot 2 - 5 = 1$.

misol: Limitni hisoblang: $\lim_{x \to 0}\left(\frac{3x-1}{x^3-5}\right)$.

h. $\lim_{x \to 0}\left(\frac{3x-1}{x^3-5}\right) = \frac{-1}{-5} = 0{,}2$

: Limitni hisoblang: $\lim_{x \to 2} \frac{x^2-6x+8}{x^2-8x+12}$.

h.

$$\lim_{x \to 2} \frac{x^2-6x+8}{x^2-8x+12} = \lim_{x \to 2} \frac{(x-2)(x-4)}{(x-2)(x-6)} = \lim_{x \to 2} \frac{(x-4)}{(x-6)} = \frac{2-4}{2-6} = \frac{-2}{-4} = 0{,}5$$

7-misol: Limitni hisoblang: $\lim_{x \to 2} \frac{\sqrt{6-x}-2}{\sqrt{8x-7}-3}$

Yechish. $\lim\limits_{x\to2}\frac{\sqrt{6-x}-2}{\sqrt{8x-7}-3}=\lim\limits_{x\to2}\frac{(\sqrt{6-x}-2)(\sqrt{8x-7}+3)(\sqrt{6-x}-2)}{(\sqrt{8x-7}-3)(\sqrt{8x-7}+3)(\sqrt{6-x}-2)}=$

$=\lim\limits_{x\to2}\frac{(6-x-4)(\sqrt{8x-7}+3)}{(8x-7-9)(\sqrt{6-x}+2)}=\lim\limits_{x\to2}\frac{(2-x)(\sqrt{8x-7}+3)}{(8x-16)(\sqrt{6-x}+2)}=$

$\lim\limits_{x\to2}\frac{(2-x)(\sqrt{8x-7}+3)}{-8(2-x)(\sqrt{6-x}+2)}=\frac{\sqrt{8\cdot2-7}+3}{-8(\sqrt{6-2}+2)}=\frac{6}{-32}=-\frac{3}{16}.$

Birinchi va ikkinchi ajoyib limitlar.

Birinchi ajoyib limit .

8.Teorema. $\frac{\sin x}{x}$ funktsiya $x\to0$ da 1 ga teng limitga ega.

8-misol.a) $\lim\limits_{x\to0}\frac{\sin 3x}{x}=\lim\limits_{x\to0}\frac{3\sin 3x}{3x}=3\lim\limits_{x\to0}\frac{\sin 3x}{3x}=3.$

b) $\lim\limits_{x\to0}\frac{\sin 2x}{\sin 5x}=\lim\limits_{x\to0}\frac{2\sin 2x}{2x}\cdot\frac{1}{\frac{5\sin 5x}{5x}}=\frac{2}{5}.$

Ikkinchi ajoyib limit.

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n=e$$

9.Teorema.$(1+\frac{1}{x})^x$ $funksiya$ $x\to\infty$ da e songa teng limitga ega:

10-misol: Quyidagi limitlarni hisoblang:

$\lim\limits_{x\to\infty}(\frac{5x-1}{5x+2})^{2x-1}.$

Yechish. Kasrning suratini maxrajiga bo'lib, butun qismini ajratib olamiz:

$$\frac{5x-1}{5x+2}=\frac{(5x+2)-3}{5x+2}=1+\frac{-3}{5x+2}.$$

Funksiyani ikkinchi ajoyib limitdan foydalanish mumkin bo'ladigan qilib o'zgartiramiz:

$$(\frac{5x-1}{5x+2})^{2x-1}=(1+\frac{-3}{5x+2})^{2x-1}=\left[(1+\frac{-3}{5x+2})^{\frac{5x+2}{-3}}\right]^{\frac{-3(2x-1)}{5x+2}}=$$

$$=\left[\left(1+\frac{-3}{5x+2}\right)^{\frac{5x+2}{-3}}\right]^{\frac{-3\left(2-\frac{1}{x}\right)}{5+\frac{2}{x}}}.$$

$x \to \infty$ da $\frac{-3}{5x+2} \to 0$ bo'lgani sababli ikkinchi ajoyib limitga ko'ra:

$\lim\limits_{x\to\infty}\left(1+\frac{-3}{5x+2}\right)^{\frac{5x+2}{-3}} = e$ *va* $\lim\limits_{x\to\infty}\frac{-3\left(2-\frac{1}{x}\right)}{5+\frac{2}{x}} = \frac{-6}{5}$ ekanligidan,

$\lim\limits_{x\to\infty}\left(\frac{5x-1}{5x+2}\right)^{2x-1} = e^{\frac{-6}{5}}$ kelib chiqadi.

Mustaqil bajarish uchun misol va masalalar.

1.Ketma-ketlik limitlarini hisoblang:

1). $\lim\limits_{n\to\infty}\frac{5n^3-3+2n^2}{3-4n^2-10n^3}$. 2). $\lim\limits_{n\to\infty}\frac{5n^3-2n+1}{n^3-7n^2-4}$

3). $\lim\limits_{n\to\infty}\frac{4n^2-3n-1}{3n^2-2n-2}$. 4). $\lim\limits_{n\to\infty}\frac{n^2-2n+7}{-2n^3+2n^2+5}$

5). $\lim\limits_{n\to\infty}\frac{3n^2-7n+5}{3n^3-3n^2-4}$ 6). $\lim\limits_{n\to\infty}\frac{n^4-2n^3+5n^2-2n+8}{2n^3+7n^2+5n-6}$

7). $\lim\limits_{n\to\infty}\frac{5n^2-8+5n^4}{3n^3-2n^2-4n}$. 8). $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{2n}$

9). $\lim\limits_{n\to\infty}\frac{n^3-6n+5n^2}{6n^3-2n^2-n^4}$ 10). $\lim\limits_{n\to\infty}\frac{5n^3-2n+5n^4}{9n^3-2n^2-4n^4}$

2.Funksiya limitlarini toping:

1. $\lim\limits_{x\to\infty}\frac{x^4-6x+2}{3x^2+2x^4+x}$. 2. $\lim\limits_{x\to\infty}\frac{5x^3-3x+1}{7x^3-9x^2-1}$.

$\lim\limits_{x\to\infty}\frac{3x^2-4x-1}{3x^3-3x-2}$. 4. $\lim\limits_{x\to\infty}\frac{x^2-2x+7x^3}{-2x^3+2x+5}$.

$\lim\limits_{x\to\infty}\frac{8x^2-7x+5}{3x-3x^2-2}$. 6. $\lim\limits_{x\to\infty}\frac{2x-x^3+5x^2-2x}{7x^3+7x^2+5x}$.

7. $\lim\limits_{x\to\infty}\frac{6x^2-8+x^4}{2x-2x^2-5x^4}$. 8. $\lim\limits_{x\to\infty}\frac{4x^3-8+3x^4}{x^3-2x^2-6x^4}$.

9. $\lim\limits_{x\to\infty}\frac{5x^2-8+5x^4}{3x^3-2x^2-4x^4}$. 10. $\lim\limits_{x\to\infty}\frac{3x^2-6+8x^4}{9x^3-3x^2-4x^4}$.

3.Ko'rsatilgan limitlarni toping.

1. $\lim\limits_{x\to 1}\frac{x^2-6x+5}{x^2+2x-3}$. 4. $\lim\limits_{x\to 2}\frac{20+x^2-12x}{x^2+x-6}$.

2. $\lim\limits_{x\to 3}\frac{x^2-5x+6}{x^2-10x+21}$. 5. $\lim\limits_{x\to -2}\frac{4x^2-7x-2}{5x^2+3x-14}$.

3. $\lim\limits_{x\to -1}\frac{-x^2+x+2}{-x^3-3x^2+2}$. 6. $\lim\limits_{x\to -3}\frac{x^2-6x-27}{3x^2+10x+3}$.

4.Ko'rsatilgan limitlarni toping.

1. $\lim\limits_{x\to 2}\frac{\sqrt{2x+1}-\sqrt{9-2x}}{3x^2-2x-8}$. 5. $\lim\limits_{x\to -4}\frac{\sqrt{x+5}-\sqrt{9+2x}}{x^3+64}$.

2. $\lim\limits_{x\to -5}\frac{\sqrt{3x+17}-\sqrt{7+x}}{x^2+4x-5}$. 6. $\lim\limits_{x\to 3}\frac{\sqrt{1+x}-\sqrt{-5+3x}}{x^2-9}$.

3. $\lim\limits_{x\to 1}\frac{3x^2+4x-7}{\sqrt{x+8}-\sqrt{5+4x}}$. 7. $\lim\limits_{x\to -2}\frac{x^2-x-6}{\sqrt{2-x}-\sqrt{x+6}}$.

4. $\lim\limits_{x\to 5}\frac{\sqrt{4x+5}-\sqrt{-5+6x}}{\sqrt{x+4}-\sqrt{-1+2x}}$. 8. $\lim\limits_{x\to 4}\frac{\sqrt{5-x}-\sqrt{x-3}}{\sqrt{x-2}-\sqrt{-10+3x}}$.

5.Ko'rsatilgan limitlarni toping.

1. $\lim\limits_{x\to 0}\frac{\sin 7x}{9x}$. 2. $\lim\limits_{x\to 0}\frac{\sin 12x}{\cos 3x}$ 3. $\lim\limits_{x\to 0}\frac{\cos 5x}{\sin 10x}$ 4. $\lim\limits_{x\to 0}\frac{\sin 6x}{tg 3x}$

5. $\lim\limits_{x\to\infty}\left(\frac{3x-1}{3x+4}\right)^{5x-1}$. 6. $\lim\limits_{x\to\infty}\left(\frac{2x+5}{2x+4}\right)^{5x-2}$. 7. $\lim\limits_{x\to\infty}\left(\frac{-1+4x}{3+4x}\right)^{x-1}$

8. $\lim\limits_{x\to\infty}\left(\frac{5x-1}{5x+2}\right)^{3x-2}$. 9. $\lim\limits_{x\to\infty}\left(\frac{3-2x}{5-2x}\right)^{2x-3}$ 10. $\lim\limits_{x\to\infty}\left(\frac{3-5x}{2-5x}\right)^{x-5}$.

Nazorat testlari.

1. $\lim\limits_{n\to\infty}\frac{n^3-5n^2-7n+9}{n^4-2n+13}$ ni toping. A) $\frac{9}{13}$ B)0 C)∝

2. $\lim\limits_{x\to 2}\frac{x^2-4}{x^2-5x+6}$ ni toping. A) -4 B) 1 C) 2

3. $\lim\limits_{x\to 0}\frac{\sin 7x}{9x}=?$ A) $\frac{7}{9}$ B) 0 C) $\frac{1}{9}$

4. $\lim\limits_{x\to -3}\frac{x^2+x-6}{x^2-9}$ ni toping. A) $\frac{5}{6}$ B) 1 C) -6

5. $\lim\limits_{x\to 0}\frac{\sin 12x}{\cos 3x}=?$ A) 0 B) 4 C) 9

6. $\lim\limits_{n\to\infty}\frac{4n^{12}+5n^{10}+17}{18n^{10}-7n^{12}}=?$ A) $-\frac{4}{7}$ B) $\frac{5}{18}$ C) 0

7. Limitni toping. $\lim\limits_{x\to 8}\dfrac{x^2-4x-32}{x^2-9x+8}$ A) $\frac{12}{7}$ B) $\frac{5}{9}$ C) 1

8. Birinchi ajoyib limitni ko‘rsating.

1) $\lim\limits_{x\to x_0}\dfrac{\sin x}{x}=1$ 2) $\lim\limits_{x\to x_0}\dfrac{\sin x}{x}=0$ 3) $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=0$ 4) $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$

A) 4 B)2 C) 3 D)1

9. Limitni hisoblang $\lim\limits_{x\to 0}\dfrac{\sin 3x}{2x}$ A) $\frac{3}{2}$ B) $\frac{1}{7}$ C) $\frac{1}{8}$ D) $\frac{-2}{7}$

10. Limitni hisoblang $\lim\limits_{n\to\infty}\dfrac{n+2}{2n-7}$ A) $\frac{1}{2}$ B $\frac{1}{7}$ C) $\frac{1}{8}$ D) $\frac{-2}{7}$

11. Limitni hisoblang $\lim\limits_{x\to 6}\dfrac{x^2-36}{x^2-6x}$ A)2 B)0 C)1 D)3

12.Limitni hisoblang $\lim\limits_{x\to\infty}\dfrac{3x^2-1}{x^2+1}$ A)1 B) 0 C) 2 D)3

13. Limitni hisoblang $\lim\limits_{x\to 0}\dfrac{e^{x^2}-1}{x^2}$ A)1 B) 0 C)2 D)3

14. Limitni hisoblang $\lim\limits_{n\to\infty}\dfrac{n^2+2n-1}{2n^3+3n-3}$. A) 0 B) $\frac{1}{2}$ C) $\frac{1}{3}$ D) $\frac{2}{3}$

15. $\lim\limits_{x\to\infty}\dfrac{10x^2+2x+1}{1+3x-5x^2}$ ni hisoblang. A) -2 B) -4 C) 4

16. $\lim\limits_{x\to 3}\dfrac{x^2-7x+12}{3-4x+x^2}$ ni hisoblang. A) $-\frac{1}{2}$ B) $-\frac{1}{3}$ C) -5

7. $\lim\limits_{x\to 0}\dfrac{\sin\frac{x}{3}}{x}$ ni hisoblang. A) $\frac{1}{3}$ B) $\frac{3}{2}$ C) -5

$\lim\limits_{x\to -4}\dfrac{x^2+3x-4}{x+4}$ ni hisoblang. A) -5 B) -7 C) 6

· $\lim\limits_{x\to 0}\dfrac{\sin\frac{2x}{5}}{x}$ ni hisoblang. A) $\frac{2}{5}$ B) $\frac{4}{9}$ C) $-\frac{4}{9}$

20. $\lim\limits_{x\to 0}\dfrac{\sin\frac{x}{3}}{2x}$ ni hisoblang. A) $\frac{1}{6}$ B) $\frac{1}{16}$ C) $-\frac{3}{5}$

V-BOB. BIR O'ZGARUVCHILI FUNKSIYANING DIFFERENSIAL HISOBI

14-§. Hosilaning ta'rifi va uni hisoblash qoidalari.

y=f(x) funktsiya (a,b) oraliqda aniqlangan bo'lib, x va $x+\Delta x$ shu oraliqdagi nuqtalar bo'sin. Bu holda argumentning Δx orttirmasiga funktsiyaning Δf=f($x+\Delta x$)-f(x) orttirmasi mos keladi.

TA'RIF: y=f(x) funktsiya Δf orttirmasining Δx argument orttirmasiga nisbati $\Delta x \to 0$ bo'lganda chekli limitga ega bo'lsa, bu limit qiymati funktsiyaning x nuqtadagi hosilasi deb ataladi va quyidagicha

y' *yoki* $f'(x)$ *yoki* $\frac{dy}{dx}$ *yoki* $\frac{df}{dx}$. belgilanadi.

Ta'rifga asosan $f'(x)=\lim\limits_{\Delta x\to 0}\frac{\Delta f}{\Delta x}=\lim\limits_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$

1-misol: f(x)=x^2 funktsiya hosilasini ta'rifga asosan topamiz:

Funksiya ortirmasini topamiz,

Δf=f($x+\Delta x$)-f(x) = $(x+\Delta x)^2 - x^2 = 2x\Delta x + (\Delta x)^2$,

Funksiya ortirmasining argument ortirmasiga nisbatining limitini topamiz

$f'(x)=\lim\limits_{\Delta x\to 0}\frac{\Delta f}{\Delta x}=\lim\limits_{\Delta x\to 0}(2x+\Delta x)=2x$ Demak, $(x^2)'=2x$ bo'lar ekan.

2-misol: $f(x) = x^3$ funksiya hosilasini ta'rifiga asosan topamiz:

$$\Delta f = f(x+\Delta x) - f(x) = (x+\Delta x)^3 - x^3$$

$$= x^3 + 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3 - x^3 = 3x^2\Delta x + 3x(\Delta x)^2 + (\Delta x)^3,$$

$$f'(x) = \lim_{\Delta x\to 0}\frac{\Delta f}{\Delta x} = \lim_{\Delta x\to 0}(3x^2 + 3x\Delta x + (\Delta x)^2) = 3x^2.$$

Demak hosilani hisoblash qoidasi uning ta'rifidan kelib chiqadi. Biror $y=f(x)$ funktsiyaning $x_0 \in [a,b]$ nuqtadagi hosilasini quyidagi tartibda topish mumkin:

1) funktsiyaning x_0 va $x_0+\Delta x$ nuqtalardagi qiymatlari topiladi:

$y=f(x_0),\ y+\Delta y=f(x_0+\Delta x)$;

2) funktsiyaning orttirmasi hisoblanadi.

$$\Delta y=(y+\Delta y)-y=f(x_0+\Delta x)-f(x_0)$$

3) funktsiya orttirmasining argument orttirmasiga bo'lgan nisbati tuziladi.

$$\frac{\Delta y}{\Delta x}=\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x};$$

4) $\Delta x\to 0$ da nisbatning limiti topiladi

$$y'=f'(x)=\lim_{\Delta x\to 0}\frac{\Delta y}{\Delta x}=\lim_{\Delta x\to 0}\frac{f(x_0+\Delta x)-f(x_0)}{\Delta x};$$

3-misol. Hosilani hisoblash qoidasidan foydalanib $y=3x^2+5x+6$ funktsiyaning $x=x_0=2$ nuqtadagi hosilasini toping.

Yechish. 1) berilgan funktsiyaning x_0 va $x_0+\Delta x$ nuqtalardagi qiymatlarini topamiz:

$$y=3x_0^2+5x_0+6;\ y+\Delta y=3(x_0+\Delta x)^2+5(x_0+\Delta x)+6=$$
$$3x_0+6x_0\Delta x+3\Delta x^2+5x_0+5\Delta x+6=3x_0^2+5x_0+6+(5+6x_0)\ \Delta x+3\Delta x;$$

2) Δy funktsiyaning orttirmasini topamiz.

$$\Delta y=(y+\Delta y)-y=3x_0^2+5x_0+6+(6x_0+5)\Delta x+3\Delta x_2-\left(3x_0^2+5x_0+6\right)=$$
$$=(6x_0+5)\ \Delta x+3\Delta x^2$$

$$\frac{\Delta x}{\Delta y}=\frac{(6x_0+5)\ \Delta x+3\Delta x^2}{\Delta x}=6x_0+5+3\Delta x;$$

4) $\Delta x\to 0$ da $\frac{\Delta y}{\Delta x}$ nisbatning limitini hisoblaymiz

$$\left(3x^2+5x+6\right)'_{x=x_0}=\lim_{\Delta x\to 0}(6x_0+5+3\Delta x)=\lim_{\Delta x\to 0}(6x_0+5)+\lim_{\Delta x\to 0}3\Delta x=6x_0+5$$
$$\left(3x^2+5x+6\right)'\Big|_{x_0=2}=(6x_0+5)\Big|_{x_0=2}=6\cdot 2+5=17.$$

3. Ba'zi bir funktsiyalarning hosilalari.

№	Funktsiya	Funktsiyaning hosilasi
1.	u^p	$(u^p)' = pu^{p-1} \cdot u'$
2.	$\sqrt{u}$	$(\sqrt{u})' = \frac{1}{2\sqrt{u}} \cdot u'$
3.	$\frac{1}{u}$	$(\frac{1}{u})' = -\frac{1}{u^2} \cdot u'$
4.	a^u	$(a^u)' = a^u \cdot lna \cdot u'$
5.	e^u	$(e^u)' = e^u \cdot u'$
6.	$log_a u$	$(log_a u)' = \frac{1}{ulna} \cdot u'$
7.	lnu	$(lnu)' = \frac{1}{u} \cdot u'$
8.	$\sin u$	$(\sin u)' = cosu \cdot u'$
9.	$cosu$	$(cosu)' = -\sin u \cdot u'$
10.	tgu	$(tgu)' = \frac{1}{cos^2 u} \cdot u'$
11.	$ctgu$	$(ctgu)' = -\frac{1}{sin^2 u} \cdot u'$
12.	$arcsinu$	$(arcsinu)' = \frac{1}{\sqrt{1-u^2}} \cdot u'$
13.	$arccosu$	$(arccosu)' = -\frac{1}{\sqrt{1-u^2}} \cdot u'$
14.	$arctgu$	$(arctgu)' = \frac{1}{1+u^2} \cdot u'$
15.	$arcctgu$	$(arcctgu)' = -\frac{1}{1+u^2} \cdot u'$

3. Differentsiallash qoidalari. c–o`zgarmas son, $f(x)$ va $g(x)$ differentsiallanuvchi funktsiyalar bo`lsin. Bunday holda:

1. $(c)' = 0$ (c-o'zgarmas son).

2. $(cf(x))' = c \cdot f'(x)$ (c-o'zgarmas son).

3. $(f(x) \pm g(x))' = f'(x) \pm g'(x)$,

4. $(f(x) \cdot g(x))' = f'(x) \cdot g(x) + g'(x) \cdot f(x)$.

5. $\left(\frac{f(x)}{g(x)}\right)' = \frac{f'(x) \cdot g(x) - g'(x) \cdot f(x)}{g^2(x)}$ $(g(x) \neq 0)$formulalar o`rinli.

4. **Murakkab funktsiyaning hosilasi**. Faraz qilaylik $y = f(x)$ funktsiya $x = x_0$ nuqtada hosilaga ega bo`lib, $g(y)$ funktsiya $y_0 = f(x_0)$ nuqtada hosilaga ega bo`lsin; bunday holda $F(x) = g[f(x)]$ murakkab funktsiya $x = x_0$ nuqtada $F'(x_0) = g'(y_0) \cdot f'(x_0)$ (5) hosilaga ega bo`ladi.

4-misol. $F(x) = e^{3x}$ funktsiyaning hosilasini hisoblang.

Yechish. $y = f(x) = e^x$ desak, $g(y) = y^3$. $g'(y) = 3y^2, f'(x) = e^x$.

(5) formulaga ko`ra berilgan murakkab funktsiyaning hosilasini topamiz:

$F'(x) = e^x \cdot 3e^{2x} = 3e^{3x}$.

5.Yuqori tartibli hosila:

$y = f(x)$ funksiyaning ikkinchi tartibli hosila deyiladi, agar $y = f(x)$ funksiyaning hosilasi $f'(x)$ o'z navbatida yana hosilaga ega bo'lsa, va $f''(x)$ ko'rinishda belgilanadi.

$y = f(x)$ funksiyaning uchunchi tartibli hosila deyiladi, agar $y = f(x)$ funksiyaning ikkinchi tartibli hosilasi $f''(x)$ o'z navbatida yana hosilaga ega bo'lsa, va $f'''(x)$ ko'rinishda belgilanadi.

uddi shunday to'rtinchi, beshinchi va xakazo n –tartibli hosilalarga ta'rif rish mumkin.

5-misol. $y = \frac{1}{3}x^6 + 2x^5 - 4x + 10$ funksiyaning beshinchi tartibli hosilasi topilsin.

Yechish: $y' = \left(\frac{1}{3}x^6 + 2x^5 - 4x + 10\right)' = 2x^5 + 10x^4 - 4$;

$y^{II} = (2x^5 + 10x^4 - 4)' = 10x^4 + 40x^3$; $\quad y^{III} = (10x^4 + 40x^3)' = 40x^3 + 120x^2$; $\quad y^{IV} = (40x^3 + 120x^2)' = 120x^2 + 240x$;

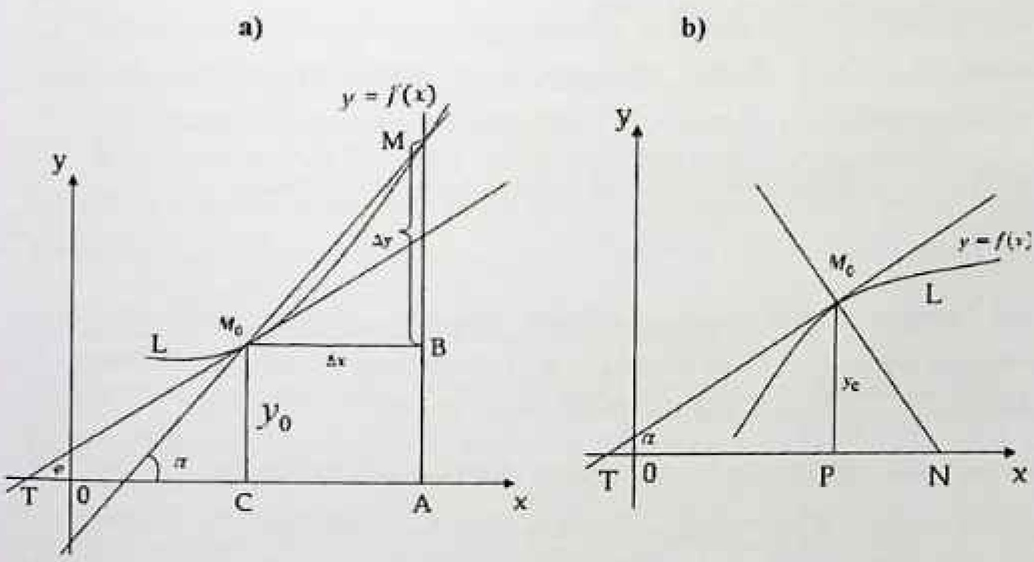
$$y^{V} = (120x^2 + 240x)' = 240x + 240 = 240(x+1).$$

6-misol. $y = cos4x$ funksiyaning to‘rtinchi tartibli hosilasi topilsin.

Yechish: $y' = (cos4x)' = -4sin4x; y'' = (-4sin4x)' = -16cos4x;$

$$y''' = (-16cos4x)' = 64sin4x; y^{IV} = (64sin4x)' = 256cos4x.$$

Hosilaning geometrik ma'nosi. Egri chiziqqa o`tkazilgan urinma va normalning tenglamalari. $y = f(x)$ tenglama bilan aniqlanadigan egri chiziqda tayin $M_0(x_0;y_0)$ va boshqa ixtiyoriy $M(x;y)$ nuqtani qaraymiz.



20-shakl

$y = f(x)$ funktsiya hosilasining geometrik ma'nosini quyidagicha ta'riflash mumkin: $y' = f'(x)$ hosilaning $x = x_0$ nuqtadagi qiymati funktsiya grafigiga $x = x_0$ abstsissali nuqtada o`tkazilgan urinmaning burchak koeffitsientiga teng

$$k = tg\ \varphi = \lim_{\Delta x \to 0} tg\ \alpha = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = f'(x_0).$$

$y = f(x)$ funktsiya grafigining $M_0(x_0, f(x_0))$ nuqtasi orqali unga o`tkazilgan urinma $k = f'(x_0)$ burchak koeffitsientiga ega bo`lib, bu urinmaning tenglamasi (20-shakl, a))

$$y - f(x_0) = f'(x_0)(x - x_0) \qquad (6)$$

ko`rinishga ega.

$f'(x_0) \neq 0$ bo`lganda, $k_n = -\frac{1}{k} = -\frac{1}{f'(x_0)}$ bo`lishini e'tiborga olsak, normal to`g`ri chiziq tenglamasi $y - f(x_0) = -\frac{1}{f'(x_0)}(x - x_0)$ (7)

ko`rinishga ega bo`ladi (1-shakl, b)).

7-misol. $y = x^2 - 6x + 5$ egri chiziqqa $x_0 = 2$ abstsissali nuqtasi orqali o`tkazilgan urinma va normal to`g`ri chiziqlar tenglamalarini toping.

Yechish. $x_0 = 2$ abstsissali nuqtaning ordinatasini hisoblaymiz.

$y_0 = x_0^2 - 6x_0 + 5 = 4 - 6 \cdot 2 + 5 = -3$

Endi berilgan funktsiyaning hosilasini topamiz: $y' = 2x - 6$. Hosilaning geometrik ma'nosiga ko`ra urinma to`g`ri chiziqning burchak koeffitsienti hosilasining $x_0 = 2$ nuqtadagi qiymatiga teng: $k = y' = 2 \cdot 2 - 6 = -2$.

) tenglamadan foydalanib, urinmaning tenglamasini tuzamiz:

$$= -2(x-2) \Rightarrow y + 3 = -2x + 4 \Rightarrow 2x + y - 1 = 0$$

to`g`ri chiziqning burchak koeffitsientini topamiz:

$$\frac{1}{k} = -\frac{1}{-2} = \frac{1}{2}$$

tenglamadan foydalanib, normal to`g`ri chiziq tenglamasini topamiz:

$$y + 3 = \frac{1}{2}(x-2) \Rightarrow y + 3 = \frac{1}{2}x - 1 \Rightarrow 2y + 6 = x - 2 \Rightarrow x - 2y - 8 = 0.$$

Hosilaning mexanik ma'nosi. M moddiy nuqta $S=f(t)$ qonun bo'yicha to'g'ri chiziqli harakat qilayotgan bo'lsin. Nuqtaning $[t_0; t_0+\Delta t]$ oraliqdagi o'rtacha tezligi $\vartheta_{\text{уртача}}=\frac{\Delta f(t_0)}{\Delta t}=\frac{f(t_0+\Delta t)-f(t_0)}{\Delta t}$

formula bo'yicha hisoblanadi. Bu nuqtaning $t=t_0$ paytdagi $\vartheta(t_0)$ oniy tezligi esa $S(t)$ yo'ldan vaqti bo'yicha olingan hosilaning $t=t_0$ nuqtadagi qiymatiga teng:

$$\vartheta(t_0)=\lim_{\Delta t\to 0}\vartheta_{\text{уртача}}=\lim_{\Delta t\to 0}\frac{\Delta s}{\Delta t}=S'(t_0)=\frac{ds(t_0)}{dt} \qquad (8)$$

Ikkinchi tartibli hosilaning mexanik ma'nosi.

To'g'ri chiziq bo'ylab harakat qiluvchi jismning o'tgan s yo'li bilan t vaqt orasidagi bog'lanish $y=f(t)$ formula bilan ifodalansin.

Hosilaning mexanik ma'nosiga binoan jismning oniy tezligi yo'ldan vaqt bo'yicha olingan hosilaga teng, ya'ni $v(t)=s'(t)=\frac{ds}{dt}$.

Biror t momentda jismning tezligi v ga teng bo'lsin. Agar harakat tekis bo'lmasa, u holda t paytdan keyin o'tgan Δt vaqt oralig'ida tezlik Δv ga o'zgaradi. $a_{o'rt}=\frac{\Delta v}{\Delta t}$

nisbat Δt vaqtdagi **o'rtacha tezlanish** deyiladi.

O'rtacha tezlanishning vaqt orttirmasi Δt nolga intilgandagi limiti berilgan momentdagi yoki oniy tezlanish deb ataladi: $a=\lim_{\Delta t\to 0}\frac{\Delta v}{\Delta t}=\frac{dv}{dt}$

Demak, oniy tezlanish tezlikdan vaqt bo'yicha olingan hosilaga teng. Ammo $v=\frac{ds}{dt}$ bo'lgani uchun $a=\frac{d}{dt}\left(\frac{ds}{dt}\right)=\frac{d^2s}{dt^2}$, ya'ni **to'g'ri chiziqli harakat tezlanishi yo'ldan vaqt bo'yicha olingan ikkinchi hosilaga teng.** $s=f(t)$ ga asosan: $a=f''(t)$.

Bu ikkinchi tartibli hosilaning mexanik ma'nosidir.

8-misol. Jism $S=\left(\frac{1}{3}t^3+\frac{1}{2}t^2-8\right)\mu$ qonun bo`yicha harakat qiladi. Jismning $t=10$ sek paytdagi tezligini toping.

Yechish. Hosilaning mexanik ma'nosiga ko`ra jismning oniy tezligi $S(t)$ yo`ldan t vaqt bo`yicha olingan birinchi tartibli hosilasining qiymatiga teng:

$$\vartheta(t)=\frac{ds(t)}{dt}=\frac{d}{dt}\left(\frac{1}{3}t^3+\frac{1}{2}t^2-8\right)=t^2+t$$

$$\vartheta(10)=10^2+10=100+10=110\, m/cek.$$

9-misol. Quyidagi funktsiyalarning hosilalarini toping:

1) $y=3x^4-5x^3+4x^2-7x+4;$ 2) $y=x\sqrt{x}(3\ln x-2);$ 3) $y=\left(3x^3+5\right)^{16};$

4) $y=x^x;$ 5) $y=\ln\left(x+\sqrt{x^2+1}\right);$

6) $y=\frac{x}{2}\sqrt{x^2+a}+\frac{a}{2}\cdot\ln\left(x+\sqrt{x^2+a}\right);$ 7) $y=\frac{2x^2+3}{7x^2+2}.$

Yechish. 1) $y'=\left(3x^4\right)'-\left(5x^3\right)'+\left(4x^2\right)-(7x)'+(4)'=3\left(x^4\right)'-5\left(x^3\right)'4\left(x^2\right)'-$
$-7x'+4'=3\cdot4x^3-5\cdot3x^2+4\cdot2x-7=12x^3-15x^2+8x-7.$

2) Berilgan funktsiyaning biroz ko`rinishini o`zgartirib yozamiz:

$$y=x^{3/2}(3\ln x-2);\quad y'=\left(x^{3/2}\right)'(3\ln x-2)+(3\ln x-2)'\ x^{3/2}=\frac{3}{2}x^{1/2}(3\ln x-2)+$$
$$+x^{3/2}\cdot\frac{3}{x}=\frac{9}{2}x^{1/2}\ln x-3x^{1/2}+3x^{1/2}=\frac{9}{2}\sqrt{x}\ln x.$$

3) Agar $3x^3+5=u$ desak, $y=u^{16}$ bo`ladi. Murakkab funktsiyani differentsiallash qoidasiga ko`ra:

$$\left(u^{16}\right)'_u\cdot\left(3x^3+5\right)'_x=16u^{15}\left(9x^2\right)=144x^2\left(3x^2+5\right)^{15}$$

erilgan funktsiyaning ko`rinishini asosiy logarfmik ayniyatdan lanib, o`zgartiramiz: $y=x^x=e^{\ln x^x}=e^{x\ln x}$.

. yerda $x\ln x=u$ desak, $y=e^u$ bo`ladi. Murakkab funktsiyani differentsiallash qoidasidan foydalanamiz:

$$y'=\left(e^u\right)'_u\left(x\ln x\right)'_x=e^u\left[x'\ln x+x\left(\ln x\right)'\right]=e^{x\ln x}\left(\ln x+x\cdot\frac{1}{x}\right)=e^{\ln x^x}\left(\ln x+1\right)=x^x\left(\ln x+1\right).$$

$$5.\ y'=\frac{1}{x+\sqrt{x^2+1}}\cdot\left(x+\sqrt{x^2+1}\right)'=\frac{1}{x+\sqrt{x^2+1}}\left(1+\frac{2x}{2\sqrt{x^2+1}}\right)=$$

$$=\frac{1}{x+\sqrt{x^2+1}}\cdot\frac{x+\sqrt{x^2+1}}{\sqrt{x^2+1}}=\frac{1}{\sqrt{x^2+1}}$$

$$6.\ y'=\frac{x}{2}\cdot\left(\sqrt{x^2+a}\right)'+\left(\frac{x}{2}\right)'\cdot\sqrt{x^2+a}+\frac{a}{2}\left[\ln\left(x+\sqrt{x^2+a}\right)\right]'=\frac{x}{2}\cdot\frac{2x}{2\sqrt{x^2+a}}+$$

$$+\frac{1}{2}\sqrt{x^2+a}+\frac{a}{2}\cdot\frac{1}{x+\sqrt{x^2+a}}\left(1+\frac{2x}{2\sqrt{x^2+a}}\right)=\frac{x^2}{2\sqrt{x^2+a}}+\frac{\sqrt{x^2+a}}{2}+\frac{a}{2}\cdot\frac{1}{x+\sqrt{x^2+a}}\cdot$$

$$\cdot\frac{\sqrt{x^2+a}+x}{\sqrt{x^2+a}}=\frac{x^2}{2\sqrt{x^2+a}}+\frac{\sqrt{x^2+a}}{2}+\frac{a}{2\sqrt{x^2+a}}=\frac{x^2+a}{2\sqrt{x^2+a}}+\frac{\sqrt{x^2+a}}{2}=\frac{x^2+a+x^2+a}{2\sqrt{x+a}}=$$

$$=\frac{2\left(x^2+a\right)}{2\sqrt{x^2+a}}=\sqrt{x^2+a}.$$

7) Berilgan funktsiyaning hosilasini topish uchun (4) formuladan foydalanamiz:

$$y'=\left(\frac{2x^2+3}{7x^2+2}\right)'=\frac{\left(2x^2+3\right)'\left(7x^2+2\right)-\left(7x^2+2\right)'\left(2x^2+3\right)}{\left(7x^2+2\right)^2}=\frac{4x\left(7x^2+2\right)-14x\left(2x^2+3\right)}{\left(7x^2+2\right)^2}=$$

$$=\frac{28x^3+8x-28x^3-42x}{\left(7x^2+2\right)^2}=-\frac{34x}{\left(7x^2+2\right)^2}.$$

$y=\left[u(x)\right]^{\vartheta(x)}$ ko`rinishdagi funktsiyaning hosilasini topish uchun, avvalo berilgan funktsiyada $y(x)>0$ va $u(x)>0$ deb, u logarifmlanadi, keyin esa murakkab funktsiyaning hosilasi topiladi.

$$\ln y=\vartheta\ln u\Rightarrow\frac{1}{y}\cdot y'=\vartheta\ln u+\vartheta\left(\ln u\right)'\Rightarrow y'=y\left(\vartheta\ln u+\vartheta\left(\ln u\right)'\right)$$

4) banddagi $y=x^x$ funktsiyaning hosilasini yuqoridagi formula bo`yicha topamiz. Bizning holda $u(x)=x,\vartheta(x)=x$

$$y'=x^x\left(x'\ln x+x\left(\ln x\right)'\right)=x^x\left(1\ln x+x\cdot\frac{1}{x}\right)=x^x\left(\ln x+1\right).$$

Chiqarilgan formula bo`yicha $y=(\sin x)^{\cos}\ (\sin x>0)$ funktsiyaning hosilasini topsak, $y'=(\sin x)^{\cos x-1}\left(\cos^2 x-\sin^2 x\ln\sin x\right)$ bo`ladi.

Oshkormas va parametrik holda berilgan funksiyalarning hosilasi:

1.$F(x,y)=0$ (9) tenglama bilan berilgan funksiyaga oshkormas koʻrinishda berilgan funksiya deyiladi. Oshkormas funksiyaning hosilasini quyidagicha topish mumkin:

1) y ni x ning funksiyasi sifatida qarab hosila olamiz va uni 0 ga tenglaymiz.
2) Hosil boʻlgan tenglamadan y' ni $y'=f(x,y)$ koʻrinishda topamiz.

10- Misol: $x^3y^2+5xy+4=0$ berilgan, y' ni toping.

Berilgan munosabatni y ni x ning funksiyasi deb qarab hosila olamiz:

$$3x^2y^2+2x^3y\cdot y'+5y+5xy'=0.$$

Hosil boʻlgan tenglamadan y' ni topamiz:

$$y'=-\frac{3x^2y^2+5y}{2x^3y+5x}.$$

2. Agar $y=f(x)$ funksiya parametrik koʻrinishda berilgan boʻlsa, ya'ni

$\begin{cases} x=\varphi(t), \\ y=f(t), \end{cases}$ $(\alpha\le t\le\beta)$, boʻlsa, u holda bu funksiyaning hosilasi

quyidagicha topiladi: $f'(x)=\frac{y'_t}{x'_t}=\frac{u'(t)}{\varphi'(t)}$,

$$f''(x)=\frac{u''(t)\varphi'(t)-\varphi''(t)u'(t)}{\varphi'(t)}.\quad (10)$$

isol: $\begin{cases} x=a\cos^3 t \\ y=a\sin^3 t \end{cases}$ berilgan, $f'(x)$ *va* $f''(x)$ ni toping.

ish: $f'(x)=\frac{u'(t)}{\varphi'(t)}$ formuladan, $f'(x)=-\frac{3a\sin^2 t\cdot\cos t}{3a\cos^2 t\cdot\sin t}=-tgt;$

$$'(x)=\frac{u''(t)\varphi'(t)-\varphi''(t)u'(t)}{\varphi'(t)}=\frac{3a\cdot\sin t(2\cos^2 t-\sin^2 t)(-3a\cos^2 t\cdot\sin t)}{(-3a\cos^2 t\cdot\sin t)^3}-$$

$$-\frac{3a\cdot\cos t(2\sin^2 t-\cos^2 t)(3a\sin^2 t\cdot\cos t)}{(-3a\cos^2 t\cdot\sin t)^3}=\frac{1}{3a\sin t\cos^4 t}.$$

Mustaqil bajarish uchun misol va masalalar

1. Hosilaning ta'rifidan foydalanib, y' ni toping.

1) $y=3x+4$; 2) $y=\frac{3}{4}x+\frac{1}{2}$; 3) $y=3x^2-5x+6$; 4) $y=-5x^2+3$;

5) $y=x^2-3x$; 6) $y=1-8x$; 7) $y=(x-2)^2$; 8) $y=3x^3$.

2. $(x^3)'=3x^2$ tenglikni hosila ta'rifidan foydalanib isbotlang.

3. Berilgan harakat qonuniga ko'ra harakatning oniy tezligini toping:

1) $S(t)=2t+1$; 2) $S(t)=3-5t$; 3) $S(t)=0,25t^4+2$; 4) $S(t)=\frac{3}{2}t^2+5t+3$.

4. $f(x)=3x^2+x+1$ $f'(1)$, $f'(-2)$ ni toping.

5. $f(x)=\frac{2+3x}{2-3x}$ *bo'lsa,* $f'(0), f'(7), f'(3)$ ni toping.

6. $\vartheta(t)=\frac{t^2}{t+5}\cdot\vartheta(0)$, $\vartheta(1)$ ni toping.

7. $z(x)=\frac{7(x-2)}{(x+1)(x-1)}\cdot z'(1)$ ni toping.

8. $g(x)=\frac{ax+b}{cx-d}\cdot g'(x), g'(m), g'(0), g'(2)$ ni toping.

9. $z(x)=\frac{8(x+5)}{(x-1)\ (x+3)}\cdot z'(0)$, $z'(1)$ ni toping.

Funktsiyalarning hosilalarini toping:

10. $y=\frac{1}{5}x^5+\frac{1}{4}x^4-\frac{1}{3}x^3-\frac{1}{2}x^2+1$. Javob: $y'=x(x+1)(x^2-1)$.

11. $y=\sqrt{x}+\sqrt[3]{x^2}-\sqrt[4]{x^3}+5$ Javob: $y'=\frac{1}{2\sqrt{x}}+\frac{2}{3\sqrt[3]{x}}-\frac{3}{4\sqrt[4]{x}}$.

12. $y=x^3(x-8)$. 13. $y=2x^2\sqrt{x}$. 14. $y=5x^3\sqrt[3]{x}$

Javob: 12. $y'=4x^2(x-6)$; 13. $y'=5\sqrt{x^3}$; 14. $y'=\frac{50}{3}\sqrt[3]{x^7}$

$15.\ y=\dfrac{1}{\sqrt{x}}.$ $\quad 16.\ y=\dfrac{\sqrt[3]{x^4}}{2}.$ $\quad 17.\ y=\dfrac{3}{\sqrt{x^3}}.$ $\quad 18.\ y=12x\left(x^2-8\right).$

Javob: $15.\ y'=-\frac{1}{2\sqrt{x^3}}$; $16.\ y'=\frac{2}{3}\sqrt[3]{x}$; $17.\ y'=-\frac{9}{2\sqrt{x^5}}$; $18.\ y'=36x^2-96$

$19.\ y=\sqrt{x}\left(2x+5\right).$ $\quad 20.\ y=\dfrac{\sqrt{x}}{3\sqrt[5]{x^2}}$

Javob: $19.\ y'=3\sqrt{x}+\frac{5}{2\sqrt{x}}$; $20.\ y'=\frac{1}{30\sqrt[10]{x^9}}$;

$21.\ y=\left(3x^3+8x^2-5\right)^2$ $\quad$ Javob: 21. $2\left(3x^3+8x^2-5\right)\left(9x^2+16x\right).$

$22.\ y=\left(\dfrac{1}{5}x^5+7\right)^2\cdot 4\sqrt[3]{x^2}.$ $\quad 23.\ y=\sqrt{ax+b}.$

Javob: $22.\ y'=\frac{128}{125}\sqrt[3]{x^{29}}+\frac{952}{15}\sqrt[3]{x^{14}}+\frac{392}{3\sqrt[3]{x}}$; $23.\ y'=\frac{a}{2\sqrt{ax+b}}$;

$24.\ y=\sqrt{\left(3x^2+5x^3\right)^4}.$ $\quad$ Javob: $\dfrac{12\left(2x+5x^2\right)}{7\sqrt[7]{\left(3x^2+5x^3\right)^3}}.$

$25.\ y=e^{-x}.$ $\quad 26.\ y=e^{3-4x}.$ $\quad$ Javob: $25.\ y'=-e^{-x}$; $\quad 26.\ -4e^{3-4x}.$

$27.\ y=\left(x^2+2x+2\right)e^{-x}$ $\quad$ Javob: $-x^2e^x.$

$28.\ y=3x^3\ln x-x^3$ $\quad$ Javob: $9x^2\ln x.$

$29.\ y=x^2e^{x^2}\ln x.$ $\quad$ Javob: $xe^{x^2}\left(2x^2\ln x-2\ln x+1\right).$

$y=\dfrac{2^{3x}}{3^{2x}}$ $\quad$ Javob: $\left(\dfrac{8}{9}\right)^x\ln\dfrac{8}{9}.$

$y=\dfrac{\ln x}{x}+\dfrac{1}{5x^5}$ $\quad$ Javob: $-\dfrac{5\ln x}{x^6}$

.. $y=\log_7 x$ $\quad$ Javob: $\dfrac{1}{x\ln 7}.$

33. $y=\log_{0,25} x$ Javob: $\frac{1}{x\ln 0\cdot 25}$.

34. $y=\left(x^3+5\right)\log_7(2x+1)$ Javob: $\frac{2\left(x^3+5\right)}{(2x+1)\ln 7}+3x^2\log_7(2x+1)$.

35. $y=\lg(2x+3)$. Javob: $\frac{2}{(2x+3)\ \ln 10}$.

36. $y=\frac{1}{3}x^3-\frac{1}{4}x^4+x^2$ 37. $y=\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}$ 38. $y=(1-x)^2$

Javob: 36. $y'=x^2-x^3+2x$;

39) $y=\left(x+\sqrt{x}\right)^3$ 40) $y=2^x+3^x+4^{-x}+2^x3^x$ 41) $y=\log_2 x+2\log_4 x-\ln x$

42. $y=\frac{(1+2x)^2}{\sqrt{x}}$ 43. $y=\left(x+\frac{1}{x}+2\sqrt{x}\right)^3$ 44. $y=\left(\ln\sqrt{1-x^2}\right)\cdot\sqrt{1+x^2}$

45. $y=\log_x 2\log_x 3\log_x 4$ 46. $y=2^{\log_2 2^x}+x^2+x$ 47. $y=\frac{e^x-e^{-x}}{e^x+e^{-x}}$

48) $y=\ln\left(e^x+\sqrt{1+e^{2x}}\right)$ 49) $y=e^x+e^{e^x}+e^{e^{e^x}}$

50. $y=\frac{x-e^{2x}}{x+e^{2x}}$ Javob: $y'=\frac{2e^{2x}(1-2x)}{\left(x+e^{2x}\right)}$.

51. $y=\frac{x\ln x-1}{x\ln x+1}$ Javob: $y'=\frac{2(\ln x+1)}{x^2\ln^2 x-1}$

52. $y=\ln\frac{\sqrt{x^2+1}-x^2}{\sqrt{x^2+1}+x^2}$ Javob: $y'=-\frac{4x}{\sqrt{x^4+1}}$

53. $y=\log_{x^2} 2$ Javob: $y'=-\frac{\ln 2}{2x\ln^2 x}$.

54. $y=\left(\frac{x-1}{x+1}\right)^4$ Javob: $y'=\frac{8(x-1)^3}{(x+1)^3}$.

55. $y=x^{\ln x}$ Javob: $y'=2x^{\ln x-1}\ln x$

56. $y=e^x-x$ 57. $y=x\ln 2-2^x$ 58. $y=e^x x^2$.

59. $y=e^x\sqrt{x}$ 60. $y=\sqrt{\dfrac{2x-1}{3}}+\ln\dfrac{2x+3}{5}$ 61. $y=\dfrac{\sqrt{3x}}{3^x+1}$.

62. $y=\sqrt{\dfrac{1-x}{6}}-2\ln\dfrac{2-5x}{3}$ Javob: $y'=\dfrac{1}{2\sqrt{6-6x}}+\dfrac{10}{2-5x}$.

63. $y=\dfrac{e^x-e^{-x}}{x}$ 64. $y=e^{2x}\ln(2x-1)$

65. $y=\dfrac{2^x-\log_2 x}{\ln 2\cdot x}$ Javob: $\dfrac{1}{x^2\ln 2}\left(x2^x\ln 2-\dfrac{1}{\ln 2}-2^x+\log_2 x\right)$

$y=f(x)$ funktsiya grafigining $x=x_0$ abstsissali nuqtasiga o`tkazilgan urinma tenglamasini tuzing:

66. $f(x)=x^2-2x,\ x_0=0{,}5$ 67. $f(x)=\dfrac{1}{x^4}+2,\ x_0=1$ 68. $f(x)=\sqrt{x^2+1},\ x_0=2$

69. $f(x)=x\ln,\ x_0=e$ 70. $f(x)=x^2+x+1,\ x_0=1$ 71. $f(x)=\dfrac{1}{x}\ x_0=3$

72. $f(x)=x-3x^2,\ x_0=2$ Javob: $y=-11x+12$.

73. $f(x)=e^x,\ x_0=0$ Javob: $y=x+1$

74. $f(x)=\dfrac{1}{x^2},\ \tilde{o}_0=-2$ Javob: $y=\dfrac{1}{4}x+\dfrac{3}{4}$.

75. $f(x)=\ln x,\ x_0=1$

5. $f(x)=\sqrt{x},\ x_0=1$ Javob: $y=\dfrac{1}{2}x+\dfrac{1}{2}$.

$f(x)$ funktsiya grafigiga berilgan M nuqta orqali o`tuvchi urinma lamasini tuzing:

$f(x)=-x^2+1,\ M(1;1)$ 78. $f(x)=x^3, M(2;4)$ 79. $f(x)=x^2-x,\ M(-1;-1)$

80. $f(x)=\sqrt{x},\ M\left(\dfrac{1}{4};\dfrac{1}{2}\right)$ 81. $f(x)=\dfrac{1}{x},\ M(-1;1)$

$y=f(x)$ funktsiya grafigida shunday nuqtalarni topingki, bu nuqtalarda funktsiya grafigiga o`tkazilgan urinmalar berilgan to`g`ri chiziqqa parallel bo`lsin:

82. $f(x)=x^3-3x^2+2,\ y=3x$ 83. $f(x)=x^2-7x-3,\ 5x+y-3=0$

84. $f(x)=\ln(4x-1), y=x$ 85. $y=x^3,\ y=4x$

$y=f(x)$ funktsiya grafigida shunday nuqtalarni topingki, bu nuqtalarda funktsiya grafigiga o`tkazilgan urinmalar berilgan to`g`ri chiziqlarga perpendikulyar bo`lsin:

86. $f(x)=x^3+2x-1,\ x+y=0$ 87. $f(x)=x^2+x+1, y+5=0$

88. $f(x)=\ln x,\ 2y+x+1=0$ 89. $y=x^2, 2x-y+1$ Javob: $\left(-\frac{1}{4};\frac{1}{16}\right)$

90. Moddiy nuqta $x(t)=pt^2+qt+r\ (p\neq 0)$ qonun bo`yicha harakatlanadi. Uning tezligi va tezlanishini toping: Javob: $a(t)=v'(t)=2p$.

91. Jism o`q atrofida $\varphi(t)=3t^2-4t+2$ (rad) qonunga ko`ra aylanadi. Uning $t=4$ *сек.* paytdagi $\omega(t)$ burchak tezligini toping. Javob: 20 *рад/сек.*

92. Moddiy nuqta to`g`ri chiziq bo`ylab $S(t)=2t^3+t-1$ (sm) qonunga ko`ra harakatlanadi. Vaqtning qaysi paytida uning tezlanishi 2sm/sek bo`ladi? Javob: $\frac{1}{6}$ *сек.*

93. Moddiy nuqta $S(t)=-\frac{1}{6}t^3+3t^2-5(m)$ qonunga ko`ra to`g`ri chiziqli harakat qilmoqda. $t$ vaqtning qanday momentida uning tezlanishi 0 ga teng bo`ladi va bu paytda nuqta qanday tezlik bilan harakat qiladi. Javob: 6 sek., 18 m/sek.

94. $y=x$ to`g`ri chiziqqa (1; 1) nuqtada urinuvchi $y=x^2+bx+c$ parabola tenglamasini toping. Javob: $y=x^2-x+1$.

95. Agar $y=2x+2b$ to`g`ri chiziq $y=x^2+bx+c$ parabolaga (2; 0) nuqtada urinsa, b va c parametrlarning barcha qiymatlarini toping. Javob: $b=-2, c=0$.

96. $y=x$ to`g`ri chiziq $y=a^x\,(a>0, a>1)$ funktsiya grafigiga urinsa, a parametrning barcha qiymatlarini toping. Javob: $a=e^{1/e}$.

97. $y=e^x+e^{-x}$ funktsiya grafigining gorizontal urinmasi tenglamasini toping. Javob: $y=2$.

98. $\left(\frac{1}{2};2\right)$ nuqtadan o`tib, $y=-\frac{x^2}{2}+2$ parabola grafigiga urinuvchi va $(x-2)^2+y^2=1$ aylanani turli nuqtalarda kesuvchi to`g`ri chiziq tenglamasini toping. Javob: $y=-x+\frac{5}{2}$.

99. $f(x)=\frac{1}{3}x^3-x^2-\frac{18}{5}x+\frac{14}{5}$ funktsiya grafigida shunday nuqtalarni topingki, y nuqtalarda grafikka o`tkazilgan urinmalar $5x-3y+2=0$ to`g`ri chiziqqa perpendikulyar bo`lsin. Javob: $\left(-1;\frac{76}{16}\right)$ va $(3;-8)$.

100. $y=3x^2-4x-17$ parabolaga $x_0=\frac{5}{2}$ abstsissali nuqtada o`tkazilgan urinma abstsissalar o`q bilan qanday burchak tashkil qiladi? Javob: $\frac{\pi}{4}$.

101. $y=3x^2-4x+7$ funktsiya grafigida shunday nuqtalarni topingki, u nuqtalardan egri chiziqqa o`tkazilgan urinma $ox$ o`qi bilan $\frac{\pi}{4}$ burchak tashkil qilsin. Javob: (1; 6) va $\left(-\frac{1}{9};\ 6\frac{230}{243}\right)$.

102. $y=-\sqrt{2x^3}$ funktsiya grafigining qanday nuqtasiga o`tkazilgan urinma $-3y+2=0$ to`g`ri chiziqqa perpendikulyar bo`ladi? Javob: $\left(\frac{1}{8};-\frac{1}{16}\right)$.

$y=\sqrt{2x}$ àà $y=\frac{x^2}{2}$ funktsiyalar grafiklarining kesishish nuqtalari orqali ri chiziqlarga o`tkazilgan urinmalarning tenglamalarini toping. Javob: $\frac{1}{2}x+1$, $y=2x-2$.

15-§. Lopital qoidasi. Teylor formulasi va asosiy Makloren kengaytmalari.

Agar $x \to a$ da $f(x)$ va $g(x)$ funksiyalar cheksiz kichik miqdorlar ya'ni, $\lim_{x \to a} f(x) = 0$ $\lim_{x \to a} g(x) = 0$ bo'lsa, u holda ularning $\frac{f(x)}{g(x)}$ nisbati $x \to a$ da $\frac{0}{0}$ ko'rinishdagi aniqmaslik deb ataladi.

Agar $x \to a$ da $f(x)$ va $g(x)$ funksiyalar cheksiz katta miqdorlar bo'lsa, ya'ni $\lim_{x \to a} f(x) = \pm\infty$ $\lim_{x \to a} g(x) = \pm\infty$ bo'lsa, u holda $\frac{f(x)}{g(x)}$ nisbat $x \to a$ da $\frac{\infty}{\infty}$ ko'rinishidagi aniqmaslik deyiladi.

Berilgan $\frac{0}{0}$ yoki $\frac{\infty}{\infty}$ ko'rinishdagi $\frac{f(x)}{g(x)}$ aniqmaslikning $x \to a$ dagi limitini topish shu aniqmaslikni ochish deyiladi.

<u>**Lopitalning birinchi qoidasi**</u>. $f(x)$ va $g(x)$ funksiyalar $x = a$ nuqta atrofida aniqlangan, diferensiyallanuvchi va $g'(x) \neq 0$ bo'lsin. Bundan tashqari $f(x)$ va $g(x)$ funksiyalar $x \to a$ shartda cheksiz kichik miqdorlar bo'lsin, ya'ni

$$\lim_{x \to a} f(x) = 0 \quad \lim_{x \to a} g(x) = 0 \quad , \quad \lim_{x \to a} f(x) = 0 \quad \lim_{x \to a} g(x) = 0$$

tengliklar bajarilsin. Bu holda, agar $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ mavjud bo'lsa, u holda $\lim_{x \to a} \frac{f(x)}{g(x)}$ ham mavjud bo'lib,

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

tenglik o'rinli bo'ladi.

Lopitalning bu qoidalaridan foydalanib, muhim limitlar deb ataluvchi ushbu limitlarni isbotlash qiyin emas.

1. $\lim_{x \to 0} \frac{sinx}{x} = 1$. 2. $\lim_{x \to 0} \frac{\ln(1+x)}{x} = \alpha$. 3. $\lim_{x \to 0} \frac{a^x - 1}{x} = lna$

4. $\lim_{x \to 0} \frac{(1+x)^\alpha - 1}{x} = \alpha$.

Lopitalning ikkinchi qoidasi. $f(x)$ va $g(x)$ funksiyalar $x = a$ nuqta atrofida aniqlangan, differensiyallanuvchi va $g'(x) \neq 0$ bo'lsin. Bundan tashqari, $f(x)$ va $g(x)$ funksyalar $x \to a$ shartda cheksiz katta miqdorlar bo'lsin, ya'ni

$$\lim_{x \to a} f(x) = \infty, \ \lim_{x \to a} g(x) = \infty,$$

bo'lsin. Bu holda, agar $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ mavjud bo'lsa, u holda $\lim_{x \to a} \frac{f(x)}{g(x)}$ ham mavjud bo'ladi va quyidagi tenglik o'rinli bo'ladi:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}.$$

Lopitalning birinchi yoki ikkinchi qoidasidagi $\frac{f'(x)}{g'(x)}$ nisbat $x \to a$ da yana $\frac{0}{0}$ yoki $\frac{\infty}{\infty}$ ko'rinishdagi aniqmasliklardan iborat bo'lib, $f'(x)$ va $g'(x)$ hosilalar yana Lopitalning birinchi va ikkinchi qoidalari shartlarini qanoatlantirsa, hamda

$$\lim_{x \to a} \frac{f''(x)}{g''(x)}$$

mavjud bo'lsa, u holda, quyidagi tenglik o'rinli bo'ladi:

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} = \lim_{x \to a} \frac{f''(x)}{g''(x)}$$

Shunday qilib, aniqmaslikni ochish uchun Lopital qoidasini bir necha marta ketma-ket qo'llash mumkin ekan. Buning uchun har gal Lopital qoidasi shartlarining bajarilishini tekshirish kerak bo'ladi.

Lopital qoidasiga teskari tasdiq ham doimo o'rinli bo'lmasligi umkin. Ya'ni

$\lim_{x \to a} \frac{f(x)}{g(x)}$ mavjud, ammo $\lim_{x \to a} \frac{f'(x)}{g'(x)}$ mavjud bo'lmasligi mumkin.

Agar $\lim\limits_{x\to a} f(x) = 0$, $\lim\limits_{x\to a} g(x) = \infty$ bo'lsa, $f(x) \cdot g(x)$ ko'paytma $x \to a$ da $0 \cdot \infty$ ko'rinishdagi aniqmaslik deyiladi. Bunday aniqmasliklarni ochish uchun ularni

$$f(x) \cdot g(x) = \frac{f(x)}{\frac{1}{g(x)}} \text{ yoki } f(x) \cdot g(x) = \frac{g(x)}{\frac{1}{f(x)}}$$

ko'rinishda yozib, $\frac{0}{0}$ yoki $\frac{\infty}{\infty}$ ko'rinishidagi aniqmaslikka keltiriladi va Lopital qoidasi qo'llaniladi.

Agar $\lim\limits_{x\to a} f(x) = 1$, $\lim\limits_{x\to a} g(x) = \infty$ bo'lsa, $f(x)^{g(x)} (f(x) > 0)$ ifoda $x \to a$ da 1^∞ ko'rinishdagi aniqmaslik deyiladi. Bunday ko'rinishdagi aniqmasliklarni ochish uchun $u = f(x)^{g(x)}$ deb belgilaymiz va uning har ikkala tomonini e asosga ko'ra logarifim- laymiz. Natijada

$\ln u = \ln f(x)^{g(x)}$, $\ln u = g(x) \cdot \ln f(x)$ ni hosil qilamiz. Bu $0 \cdot \infty$ ko'rinishdagi aniqmaslik bo'lib, uni ochishni yuqorida ko'rib o'tdik.

Agar berilgan $f(x)$ va $g(x)$ funksiyalar uchun $\lim\limits_{x\to a} f(x) = 0$, $\lim\limits_{x\to a} g(x) = 0$ yoki $\lim\limits_{x\to a} f(x) = \infty$, $\lim\limits_{x\to a} g(x) = 0$ bo'lsa, u holda $f(x)^{g(x)}$ $(f(x) > 0)$ ifoda $x \to a$ da 0^0 yoki ∞^0 ko'rinishdagi aniqmaslik deyiladi. Bunday ko'rinishdagi aniqmasliklar yuqorida 1^∞ ko'rinishdagi aniqmasliklar uchun ko'rib o'tilgan usulda ochiladi.

Umuman olganda $\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, 1^∞, 0^0, ∞^0, $\infty - \infty$

kabi yoziladigan aniqmasliklar mavjud bo'lib, ular Lopital qoidasidan va ba'zi bir qo'shimcha formulalardan foydalanib ochiladi.

1-misol: Lopitalning birinchi qoidasidan foydalanib,

$$\lim_{x\to a} \frac{\sin x}{x} = 1$$

ekanligi isbotlansin.

Isbot. Bu yerda $f(x) = \sin x$ *va* $g(x) = x$ bo'lib, ular Lopital qoidasi shartlarini qanoatlantiradi. Demak,

$$\lim_{x\to 0}\frac{\sin x}{x}\left(\frac{0}{0}\right)=\lim_{x\to 0}\frac{(\sin x)'}{(x)'}=\lim_{x\to 0}\frac{\cos x}{1}=\lim_{x\to 0}\cos x=1.$$

2-misol: $\lim\limits_{x\to -1}\frac{x^3-5x^2+2x+8}{x^4-2x^3-16x^2+2x+15}$ limit hisoblansin.

Yechish: Agar berilgan kasrda x o'rniga -1 ni qo'ysak, u holda $\frac{0}{0}$ ko'rinishdagi aniqmaslik paydo bo'ladi. Berilgan kasrning surat va mahrajlari Lopital qoidasi shartlarini qanoatlantiradi. Demak,

$$\lim_{x\to -1}\frac{x^3-5x^2+2x+8}{x^4-2x^3-16x^2+2x+15}=\lim_{x\to -1}\frac{3x^2-10x+2}{4x^3-6x^2-32x+2}=\frac{15}{24}=\frac{5}{8}.$$

3-misol. $\lim\limits_{x\to 1}\frac{\ln(1-x^2)}{\ln(1-x)}$ hisoblansin.

Yechish: Bu $\frac{\infty}{\infty}$ ko'rinishdagi aniqmaslikdir.

Bu yerda $f(x)=\ln(1-x^2)$ va $g(x)=\ln(1-x)$ bo'lib ular lopitalning ikkinchi qoidasi shartlarini qanoatlantiradi. Demak,

$$\lim_{x\to 1}\frac{\ln(1-x^2)}{\ln(1-x)}=\lim_{x\to 1}\frac{\frac{-2x}{1-x^2}}{\frac{-1}{1-x}}=\lim_{x\to 1}\frac{2x(1-x)}{1-x^2}=2\lim_{x\to 1}\frac{x}{1+x}=2\cdot\frac{1}{2}=1.$$

4-misol. $\lim\limits_{x\to\infty}\frac{x^\alpha}{\ln x}(\alpha>0)$ hisoblansin.

Yechish: Bu ham $\frac{\infty}{\infty}$ ko'rinishdagi aniqmaslikdir. Unga Lopital qoidasini qo'llaymiz:

$$\lim_{x\to\infty}\frac{x^\alpha}{\ln x}=\lim_{x\to\infty}\frac{(x^\alpha)'}{(\ln x)'}=\lim_{x\to\infty}\frac{\alpha x^{\alpha-1}}{\frac{1}{x}}=\alpha\lim_{x\to\infty}x^\alpha=\infty.$$

5-misol. $\lim\limits_{x\to\frac{\pi}{2}}\frac{\mathrm{tg}\,3x}{\mathrm{tg}\,5x}$ ni hisoblang.

Yechish: $x\to\frac{\pi}{2}$ da $tg3x$ va $tg5x\ lar$ cheksiz katta miqdorlar bo'lgani uchun bu holda ham $\frac{\infty}{\infty}$ ko'rinishdagi aniqmaslikka kelamiz. Uni ochish uchun Lopital qoidasini qo'llaymiz:

$$\lim_{x\to\frac{\pi}{2}}\frac{\operatorname{tg}3x}{\operatorname{tg}5x}=\lim_{x\to\frac{\pi}{2}}\frac{\frac{3}{\cos^2 3x}}{\frac{5}{\cos^2 5x}}=\lim_{x\to\frac{\pi}{2}}\frac{3\cos^2 5x}{5\cos^2 3x}=\frac{3}{5}\lim_{x\to\frac{\pi}{2}}\left(\frac{\cos 5x}{\cos 3x}\right)^2=$$

$$=\frac{3}{5}\left(\lim_{x\to\frac{\pi}{2}}\frac{-5\sin 5x}{-3\sin 3x}\right)^2=\frac{3}{5}\cdot\frac{25}{9}\cdot\left(\lim_{x\to\frac{\pi}{2}}\frac{\sin 5x}{\sin 3x}\right)^2=\frac{5}{3}\left(\frac{1}{-1}\right)^2=\frac{5}{3}.$$

6-misol. $\lim\limits_{x\to 0} x\cdot\ln|x|$ ni hisoblang.

Yechish: $f(x)\cdot g(x)=x\cdot\ln|x|$ bo'lib, bu $0\cdot\infty$ ko'rinishdagi aniqmaslikdir. Uni ochish uchun $f(x)\cdot g(x)$ ning ko'rinishini o'zgartiramiz:

$$\lim_{x\to 0} x\cdot\ln|x|(0\cdot\infty)=\lim_{x\to 0}\frac{\ln|x|}{\frac{1}{x}}\left(\frac{\infty}{\infty}\right)=\lim_{x\to 0}\frac{(\ln|x|)'}{\left(\frac{1}{x}\right)'}=\lim_{x\to 0}\frac{\frac{1}{|x|}}{-\frac{1}{x^2}}=$$

$$=-\lim_{x\to 0}|x|=0.$$

7-misol. $\lim\limits_{x\to\infty} xe^{-x}$ limitni hisoblang.

Yechish: $\lim\limits_{x\to\infty} e^{-x}=0$ bo'lgani uchun $x\to\infty$ da xe^{-x} ifoda $0\cdot\infty$ ko'rinishdagi aniqmaslikdan iborat. $e^{-x}=\frac{1}{e^x}$ bo'lgani uchun

$$\lim_{x\to\infty} xe^{-x}(0\cdot\infty)=\lim_{x\to\infty}\frac{x}{e^x}\left(\frac{\infty}{\infty}\right)=\lim_{x\to\infty}\frac{(x)'}{(e^x)'}=\lim_{x\to\infty}\frac{1}{e^x}=0.$$

8-misol. $\lim\limits_{x\to 0+0} x^x$ hisoblansin.

Yechish: x^x ifoda 0^0 ko'rinishidagi ko'rinishidagi aniqmaslikdir. Uni ochish uchun $x^x= e^{x\ln x}$ dan foydalanamiz. Demak,

$$\lim_{x\to 0+0} x^x(0^0)=\lim_{x\to 0+0} e^{x\ln x}=e^{\lim\limits_{x\to 0+0} x\ln x}.$$

Buni hisoblash uchun dastlab $\lim\limits_{x\to 0+0} x\ln x$ ni topamiz:

$$\lim_{x\to 0+0} x\ln x(0\cdot\infty)=\lim_{x\to 0+0}\frac{\ln x}{\frac{1}{x}}\left(\frac{\infty}{\infty}\right)=\lim_{x\to 0+0}\frac{\frac{1}{x}}{-\frac{1}{x^2}}=-\lim_{x\to 0+0} x=0.$$

Demak, $\lim\limits_{x\to 0+0} x^x = e^{\lim\limits_{x\to 0+0} x\ln x} = e^0 = 1.$

9-misol. $\lim\limits_{x\to 0}(1 + m\,x)^{\frac{1}{x}}$ hisoblansin.

Yechish: $(1 + m\,x)^{\frac{1}{x}}$ ifoda 1^{∞} ko'rinishdagi aniqmaslikdir. Uni ochish uchun $(1 + m\,x)^{\frac{1}{x}} = e^{\frac{1}{x}\ln(1+m\,x)}$ tenglikdan foydalanamiz.

$$\lim_{x\to 0}(1 + m\,x)^{\frac{1}{x}}(1^{\infty}) = \lim_{x\to 0} e^{\frac{1}{x}\ln(1+m\,x)} = e^{\lim\limits_{x\to 0}\frac{1}{x}\ln(1+m\,x)}.$$

$$\lim_{x\to 0}\frac{1}{x}\ln(1 + m\,x)(0\cdot\infty) = \lim_{x\to 0}\frac{\ln(1 + m\,x)}{x}\left(\frac{0}{0}\right) = \lim_{x\to 0}\frac{\frac{m}{1+mx}}{1} = m.$$

Bu qiymatni o'z o'rniga qo'yamiz va $\lim\limits_{x\to 0}(1 + m\,x)^{\frac{1}{x}}(0\cdot\infty) = e^m$ ni hosil qilamiz.

10-misol. $\lim\limits_{x\to\frac{\pi}{2}}\left(\frac{1}{\cos x} - tgx\right)$ hisoblansin.

Yechish: $x\to\frac{\pi}{2}$ da $\frac{1}{\cos x} - tgx$ ifoda $\infty - \infty$ ko'rinishdagi aniqmaslikdir. Aniqmaslikni ochish uchun $\frac{1}{\cos x} - tgx$ ifodani shakl almashtirish yordamida o'zgartiramiz.

$$\lim_{x\to\frac{\pi}{2}}\left(\frac{1}{\cos x} - tgx\right)(\infty - \infty) = \lim_{x\to\frac{\pi}{2}}\frac{1}{\cos x}(1 - \cos x\cdot \mathrm{tg}x)(0\cdot\infty) = \quad =$$

$$\lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\cos x}\left(\frac{0}{0}\right) = \lim_{x\to\frac{\pi}{2}}\frac{(1-\sin x)'}{(\cos x)'} = \lim_{x\to\frac{\pi}{2}}\frac{-\cos x}{-\sin x} = \lim_{x\to\frac{\pi}{2}}\mathrm{ctg}\,x = 0.$$

Teylor va Makloren formulasi:

funksiya $x_0 \epsilon R$ nuqtaning biror atrofi $(x_0 - \delta, x_0 + \delta)$ da aniqlangan bu atrofda $f'(x), f''(x), \dots, f^{(n+1)}(x)$ hosilalarga ega va $f^{(n+1)}(x)$ $_0$ nuqtada uzluksiz bo'lsa, u holda quyidagi $f(x) = f(x_0) +$ $- x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \dots + \frac{f^{(n)}(x_0)}{n!}\cdot$ $\cdot(x - x_0)^n +$

$\frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1}$ formula o'rinli bo'ladi. Bunda $\xi = x_0 + \theta(x - x_0), 0 < \theta < 1$.

Bu formula Teylor formulasi deyiladi. $R_{n+1}(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1}$ ni Lagranj ko'rinishidagi qoldiq had deyiladi.

Teylor formulasida $x_0 = 0$ bo'lgan hol alohida ahamiyatga ega:

$$f(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(x_0)}{2!}x^2 + \cdots + \frac{f^{(n)}(0)}{n!}x^n + R_{n+1}(x)$$

Odatda, bu formula Makloren formulasi deyiladi. Bu formuladan funksiya limitini topishda, taqribiy hisoblash masalalarida foydalaniladi.

Quyida ba'zi bir elementar funksiyalar uchun Makloren formulasini keltiramiz:

a) $f(x) = e^x$ bo'lsin. Bu funksiya uchun
$f'(x) = e^x, f''(x) = e^x, f'''(x) = e^x, \ldots f^{(n)}(x) = e^x$ ekanligidan

$f(0) = 1,\ f'(0) = e^0 = 1, f''(x) = 1, f'''(x) = 1, \ldots, f^{(n)}(0) = 1,$ $(n = 1,2..)$. U holda $f(x) = e^x$ uchun quyidagi Makloren formulani yozish mumkin:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + R_{n+1}(x), R_{n+1}(x) = \frac{x^{n+1}}{(n+1)!}e^{\theta x}\ (0 < \theta < 1)$$

Agar $n \to \infty$ da $R_{n+1} \to 0(x) \to 0$ bo'lishini etiborga olsak, $f(x) = e^x$ uchun $e^x \approx 1 + \frac{x}{1!} + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}$ taqribiy formulaga ega bo'lamiz.

b) $f(x) = sinx$ bo'lsin. Bu funksiyaning n-tartibli hosilasi uchun $f^{(n)}(x) = (sinx)^{(n)} = \sin\left(x + n \cdot \frac{\pi}{2}\right)$ formula o'rinli ekanini topish qiyin emas.

$f(0) = sin0 = 0$ ekanligi ravshan. $f^{(n)}(0)$ ni aniqlaymiz:

$$f^{(n)}(0) = sin\frac{n\pi}{2} = \begin{cases} 0, agar\ n - juft\ bo'lsa, \\ (-1)^{\frac{n-1}{2}}, agar\ n - toq\ bo'lsa. \end{cases}$$

Demak $f(x) = sinx$ funksiya uchun Makloren formulasi

$$sinx = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^n \cdot \frac{x^{2n}}{(2n)!} + 0(x^{n+1}),$$

ko'rinishda bo'ladi.

c) $f(x) = cosx$ bolsin. Funksiyaning $n-$ tartibli hosilasi uchun

$$f^{(n)}(x) = (cosx)^{(n)} = \cos\left(x + n \cdot \frac{\pi}{2}\right)$$

formula o'rinlidir. $f(0) = 1$ bo'lishi ravshan.

$$f^{(n)}(0) = cos\frac{n\pi}{2} = \begin{cases} 0, agar\ n - toq\ son\ bo'lsa, \\ (-1)^{\frac{n}{2}}, agar\ n - juft\ bolsa. \end{cases}$$

Demak, $f(x) = cosx$ funksiya uchun Makloren formulasi

$$cosx = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots + (-1)^n \cdot \frac{x^{2n}}{(2n)!} + 0(x^{n+1}),$$

ko'rinishda bo'ladi. Bu yerda $\theta(x^{n+1})$ qoldiq had.

d) $f(x) = \ln(1+x)$ bo'lsin. Bu funksiyaning $n-$ tartibli hosilasini topamiz:

$f^{I}(x) = \frac{1}{1+x} = (1+x)^{-1}$, $\qquad f^{II}(x) = (-1)(1+x)^{-2}$,

$f^{III}(x) = (-1)(-2)(1+x)^{-3}$, $\qquad f^{IV}(x) = (-1)(-2)(-3)(1+x)^{-4}$

bulardan foydalanib, $n-$tartibli hosilani topish uchun quyidagi formulani hosil qilamiz:

$$f^{(n)}(x) = (-1)^{n-1}(n-1)!\,(1+x)^{-n} = (-1)^{n-1}\frac{(n-1)!}{(1+x)^n}.$$

$f(0) = \ln(1+0) = ln1 = 0$ va $f^{(n)}(x) = (-1)^{n-1}(n-1)!$ ekanligini e'tiborga olsak, $f(x) = \ln(1+x)$ funksiya uchun Makloren formulasi:

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots + (-1)^{n-1}\frac{x^n}{n} + 0(x^{n+1}),$$

ko'rinishda bo'ladi.

11-misol. $G(x) = x^5 - 2x^4 + x^3 - x^2 + 2x - 1$ ko'phadni Teylor formulasi yordamida $(x-1)$ ning darajalari bo'yicha yoying.

Yechish: Ko'phadni $x-1$ darajalari bo'yicha yoyish uchun ko'phadni va uning hosilalarini $x=1$ nuqtadagi qiymatlarini topish kerak.

$G(x)=1-2+1-1+2-1=0.$

$G^{I}(x)=5x^4-8x^3+3x^2-2x+2;\ G^{II}(x)=20x^3-24x^2+6x-2;$

$G^{III}(x)=60x^2-48x+6;\quad G^{IV}(x)=120x-48;\quad G^{V}(x)=120;$

$G^{VI}(x)=0.$

$G^{I}(1)=5-8+3-2+2=0;\quad G^{II}(1)=20-24+6-2=0;$

$G^{III}(1)=60-48+6=18;\ G^{IV}(1)=120-48=72;\ G^{V}(1)=120;$

$G^{VI}(1)=0;\ \ldots,\ G^{(n)}(x)=0\ (n\geq 6).$

Topilganlarni Teylor formulasiga qo'yamiz:

$G(x)=\frac{18}{3!}(x-1)^3+\frac{72}{4!}(x-1)^4+\frac{120}{5!}(x-1)^5$ yoki

$G(x)=x^5-2x^4+x^3-x^2+2x-1=3(x-1)^3+3(x-1)^4+$
$+(x-1)^5.$

Mustaqil yechish uchun topshiriqlar.

1. Lopital qoidasidan foydalanib limitlarni hisoblang.

1) $\lim\limits_{x\to 3}\frac{x^3-27}{x^3-6x-9}$; 2) $\lim\limits_{t\to a}\frac{t^n-a^n}{t^m-a^m}$; 3) $\lim\limits_{x\to 0}\frac{-cosnx+1}{-cosmx+1}$;

4) $\lim\limits_{x\to 0}\frac{-cosnx+1}{-cosmx+1}$; 5) $\lim\limits_{x\to\frac{\pi}{2}}\frac{1-sinx}{1-sinx}$; 6) $\lim\limits_{x\to 0}\frac{\ln sinx}{\ln sin2x}$; 7) $\lim\limits_{x\to 0}\frac{\ln sin\,x}{\ln sin2x}$;

8 $\lim\limits_{x\to 0}\frac{\cos x-1}{e^x-1}$; 9) $\lim\limits_{x\to 0}x^{\sin x}$;

2. $f(x)=3x^4-4x^3-x^2+5$ ko'phadni $(x-2)$ ikkihadning darajalari bo'yicha yoying.

3. $f(x)=\frac{1}{\sqrt{x}}$ funksiya uchun $x_0=2$ nuqtada to'rtinchi tartibli Teylor formulasini yozing.

16-§. Differensiallanuvchi funksiyalar haqida ba'zi bir teoremalar.

Roll teoremasi. Agar $f(x)$ funksiya: 1) $[a;b]$ kesmada uzluksiz; 2) shu kesmaning ichki nuqtalarida hosilaga ega; 3) $f(a) = f(b)$ bo'lsa, u holda a bilan b orasida shunday $x = c$ nuqta mavjud bo'ladiki, unda

$f'(c) = 0$ bo'ladi.

Lagranj teoremasi. Agar $f(x)$ funksiya: 1) $[a;b]$ kesmada uzluksiz; 2) shu kesmaning ichki nuqtalarida hosilaga ega bo'lsa, u holda a va b orasida shunday $x = c$ nuqta topiladiki, unda

$f(b) - f(a) = f'(c)(b - a)$ bo'ladi.

Koshi teoremasi. Agar $f(x)$ va $\varphi(x)$ funksiya: 1) $[a;b]$ kesmada uzluksiz; 2) shu kesmaning ichki nuqtalarida har ikkala funksiya ham hosilaga ega, shuning bilan birga, $\varphi'(x) \neq 0$ bo'lsa, u holda a bilan b orasida shunday $x = c$ nuqta mavjud bo'ladiki, unda

$\frac{f(b)-f(a)}{\varphi(b)-\varphi(a)} = \frac{f'(c)}{\varphi'(c)}$ bo'ladi.

Bu teoremalarda a bilan b orasida qandaydir o'rta $x = c$ qiymat haqida so'z borganligi uchun, ular o'rta qiymat haqidagi teoremalar deb ataladi.

Ferma teoremasi. $f(x)$ funksiya $(a;b)$ oraliqda berilgan bo'lib, u shu oraliqning biror c nuqtasida o'zining eng katta (kichik) qiymatiga erishsin. Agar funksiya c nuqtada chekli hosilaga ega bo'lsa, u holda bu nuqtada

$f'(c) = 0$ bo'ladi.

1-misol: $f(x) = 3x^2 - 1$ funksiya $[1;2]$ kesmada Ferma asining shartlarini qanoatlantiradimi?

hish: Berilgan funksiya [1;2] kesmada monoton o'suvchi bo'lib, u eng katta yoki eng kichik qiymatiga kesmaning chetlarida, ya'ni uqtada o'zining eng kichik qiymatiga va $x = 2$ nuqtada o'zining eng iymatiga erishadi. Bu esa berilgan funksiya $[1;2]$ kesmada Ferma masining shartlarini qanoatlantirmasligini bildiradi.

2-misol: $f(x) = 1 - \sqrt[3]{x^2}$ funksiya $[-1;1]$ kesmada Roll teoremasi shartlarini qanoatlantiradimi?

Yechish: Funksiya $[-1;1]$ kesmada uzluksiz va $f(-1) = f(1) = 0$. Demak, berilgan funksiya Roll teoremasining ikkita shartini qanoatlantiryapti. Funsiya'ning hosilasi

$$f'(x) = -\frac{2}{3\sqrt[3]{x^2}}$$

bo'lib, u $x = 0$ dan farqli nuqtalarda mavjud. Bu nuqta ichki nuqta bo'lib, u nuqtada Roll teoremasining uchinchi sharti bajarilmayapti. Bu esa berilgan funksiyaga Roll teoremasini qo'llab bo'lmasligini bildiradi.

3-misol: $f(x) = 3x^2 - 5$ funksiya $[-2;0]$ kesmada Lagranj teoremasining shartlarini qanoatlantiradimi? Agar qanoatlantirsa, $f(b) - f(a) = =f'(c)(b-a)$ formulada muhim o'rin tutuvchi c nuqtani toping.

Yechish: Berilgan funksiya $[-2;0]$ kesmada uzluksiz va uning barcha ichki nuqtalarida chekli hosilaga ega bo'lgani uchun Lagranj teoremasini qanoatlantiradi. c nuqtani aniqlash uchun

$f(b) - f(a) = f'(c)(b-a)$ ni $f'(c) = \frac{f(b)-f(a)}{b-a}$ ko'rinishda yozamiz.

$f(b) = f(0) = -5,\ f(a) = f(-2) = 7,\ f'(c) = 6c$ bo'lgani uchun

$6c = \frac{-5-7}{0-(-2)},\ \ 6c = \frac{-12}{2},\ \ c = -1.$

Demak, $c = -1$ bo'lar ekan.

4-misol. $f(x) = x^2 - 2x + 3$ va $\varphi(x) = x^3 - 7x^2 + 20x - 5$ funksiyalar [1;4] kesmada Koshi teoremasi shartlarini qanoatlantirishini tekshiring va unga mos c ning qiymatini toping.

Yechish: $f(x)$ va $\varphi(x)$ funksiyalar [1;4] kesmada uzluksiz hamda ular $f'(x) = 2x - 2$ va $\varphi'(x) = 3x^2 - 14x + 20$ chekli hosilalarga ega. Bundan tashqari, $\varphi'(x)$ x ning har qanday qiymatlarida noldan farqli. Demak, berilgan funksiyalar uchun [1;4] kesmada Koshi teoremasini qo'llash mumkin. Ya'ni,

$$\frac{f(4)-f(1)}{\varphi(4)-f(1)} = \frac{f'(c)}{\varphi'(c)},\quad \frac{11-2}{27-9} = \frac{2c-2}{3c^2-14c+20},\ (1<c<4).$$

Oxirgi tenglamani yechib, $c_1 = 2$ va $c_2 = 4$ larni topamiz. Bulardan $c_1 = 2$ uchki nuqta hisoblanadi. Demak, bu nuqta izlanayotgan nuqta ekan.

Mustaqil yechish uchun topshiriqlar:

1. $[\frac{\pi}{6};\frac{5\pi}{6}]$ kesmada $f(x) = lnsinx$ funksiya Roll teoremasi shartlarini qanoatlantiradimi? **Javob**: Qanoatlantiradi.

2. $f(x) = x^2 - 4x + 3$ funksiya ildizlari orasida uning hosilasini ham ildizi bor ekani tekshirilsin. Bu grafik usulda tushuntirilsin.

Javob: Funksiyaning ildizlari 1; 3. $f'(x) = 2x - 4$ hosilanig ildizi 2 ga teng; 1<2<3.

3. $y = |sinx|$ egri chiziqning $[-\frac{\pi}{2};\frac{\pi}{2}]$ segmentdagi $\overline{AB}$ yoyi yasalsin. Nima uchun bu yoyda AB vatarga parallel urunma yo'q? Roll teoremasining qaysi sharti bu yerda bajarilmaydi?**Javob**: Chunki $x = 0$ sinish nuqtasi (urinma ikkita).

4. $[1; e]$ kesmada $f(x) = lnx$ funksiya uchun Lagranj formulasi yozilsin va formuladagi c ning qiymati topilsin. **Javob**: $c = e - 1$.

5. Quyidagi funksiyalar uchun Lagranj formulasi yozilsin va c topilsin.

1) [0,1] kesmada $f(x) = arctgx$; 2) [0,1] kesmada $f(x) = arcsinx$;

Javob: 1) $\sqrt{\frac{4}{\pi} - 1}$; 2) $\sqrt{1 - \frac{4}{\pi^2}}$

7. $f(x) = x^3$ va $\varphi(x) = x^2$ funsiyalar uchun Koshining

$\frac{f(b)-f(a)}{\varphi(b)-f(a)} = \frac{f'(c)}{\varphi'(c)}$ formulasi yozilsin hamda c topilsin.

Javob: $\frac{b^3-a^3}{b^2-a^2} = \frac{3c^2}{2c}$; $c = \frac{2(a^2+ab+b^2)}{3(a+b)}$.

ıyidagi funksiyalar uchun Koshi formulasi yozilsin va c topilsin:

$0;\frac{\pi}{2}]$ kesmada $sinx$ va $cosx$; 2) [1;4] kesmada x^2 va $\sqrt{x}$;

Javob: 1) $\frac{\pi}{4}$; 2) $\sqrt[3]{(\frac{15}{4})^2}$.

17-§. Funksiyaning monotonligi, kritik va ekstremum nuqtalari. Funksiya grafigining botiqligi va qavariqligi, burilish nuqtalari, asimtotalari. Funksiyani to'la tekshirish.

Funksiyaning o'suvchi hamda kamayuvchiligi:

$y=f(x)$ funksiya (a,b) da berilgan bo'lsin. Ma'lumki, ixtiyoriy $x_1 \in (a,b)$ ixtiyoriy $x_2 \in (a,b)$ lar uchun

Agar $x_1 < x_2$ bo'lganda $f(x_1) \le f(x_2)$ bo'lsa, $f(x)$ funksiya (a,b) da **o'suvchi,**

Agar $x_1 < x_2$ bo'lganda $f(x_1) \ge f(x_2)$ bo'lsa, $f(x)$ funksiya (a,b) da **kamayuvchi** deyiladi.

Oraliqda o'suvchi va kamayuvchi funksiyalar **monoton funksiyalar** deyiladi.

Funksiyaning hosilalari yordamida uning o'suvchiligini hamda kamayuvchiligini aniqlash mumkin.

Monotonlikning zaruriy sharti:

Agar $f(x)$ funksiya (a,b) da $f'(x)$ hosilaga ega bo'lib, funksiya (a,b) da o'suvchi bo'lsa, u holda $f'(x) \ge 0$ $\quad (x \in (a,b))$ bo'ladi.

Agar $f(x)$ funksiya (a,b) da $f'(x)$ hosilaga ega bo'lib, funksiya (a,b) da kamayuvchi bo'lsa, u holda $f'(x) \le 0$ $\quad (x \in (a,b))$ bo'ladi.

Monotonlikning yetarlilik sharti:

Agar $f(x)$ funksiya (a,b) da $f'(x)$ hosilaga ega bo'lib,

$$f'(x) \ge 0 \qquad (x \in (a,b))$$

bo'lsa, u holda funksiya (a,b) da o'suvchi bo'ladi.

Agar $f(x)$ funksiya (a,b) da $f'(x)$ hosilaga ega bo'lib,

$$f'(x) \le 0 \qquad \left(x \in (a,b)\right)$$

bo'lsa, u holda funksiya (a,b) kamayuvchi bo'ladi.

Funksiyaning birinchi tartibli hosilasi nolga teng yoki uzilishga ega bo'ladigan nuqtalari **kritik nuqtalari** deyiladi.

1-misol. Ushbu $y = 2 - 3x + x^3$

funksiyaning monotonlik oraliqlari topilsin.

Yechish: Berilgan funksiyaning aniqlanish sohasi, $(-\infty; \infty)$

bo'ladi. Endi funksiyaning hosilasini topamiz:

$y' = -3 + 3x^2$, hosilani 0 ga tenglashtirib kritik nuqtasini topamiz,

$-3 + 3x^2 = 0,\ 3x^2 = 3,\ x^2 = 1, x = \pm 1.$

$-3 + 3x^2 \ge 0$ da o'suvchi, $\begin{cases} x \le -1 \\ x \ge 1 \end{cases}$ da o'sadi,

$-3 + 3x^2 \le 0$ da kamayuvchi, $-1 \le x \le 1$ da kamayadi.

Funksiya ekstremumi. Funksiya ekstremumga erishishining zaruriy va etarli shartlari.

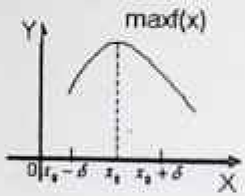
$f(x)$ funksiya (a,b) da berilgan bo'lib, x_0 nuqta o'zining atrofi $U_\delta(x_0) = (x_0 - \delta, x_0 + \delta)$ bilan $(\delta > 0)$ (a,b) intervalga tegishli bo'lsin.

1-ta'rif. Agar ixtiyoriy $x \in (x_0 - \delta, x_0 + \delta)$ uchun

$$f(x) \le f(x_0)$$

bo'lsa, $f(x)$ funksiya x_0 nuqtada maksimumga erishadi deyiladi. x_0 nksiyaning maksimum nuqtasi, $f(x_0)$ ga funksiyaning maksimum mati deyiladi va $\max f(x)$ kabi belgilanadi: (21-shakl)

$$f(x_0) = \max f(x)$$

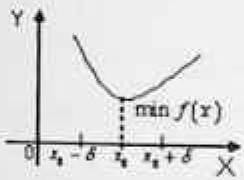


21-shakl

2-ta'rif. Agar ixtiyoriy $x \in (x_0 - \delta, x_0 + \delta)$ uchun

$$f(x) \geq f(x_0)$$

bo'lsa, $f(x)$ funksiya x_0 nuqtada minimumga erishadi deyiladi. x_0 funksiyaning minimum nuqtasi, $f(x_0)$ ga funksiyaning minimum qiymati deyiladi va $\min f(x)$ kabi belgilanadi: (22-shakl) $f(x_0) = \min f(x)$



22-shakl

Funksiya maksimum va minimumlari uning ekstremumlari deyiladi.

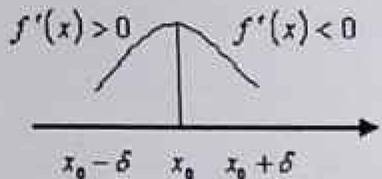
Ekstremumning zaruriy sharti:

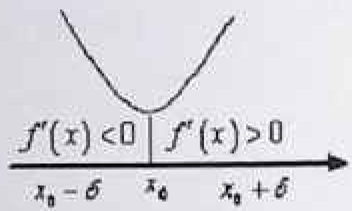
Agar $f(x)$ funksiya $x_0 \in (a,b)$ nuqtada ekstremumga erishsa va bu nuqtada funksiyaning hosilasi mavjud bo'lsa, u holda $f'(x_0) = 0$ bo'ladi yoki mavjud bo'lmaydi.

Endi funksiya ekstremumga erishishining yetarli shartlarini keltiramiz:

Agar $f(x)$funksiya (a,b) da hosilaga ega bo'lib, $x_0 \in (a,b)$ da $f'(x_0) = 0$bo'lsin , $f(x)$funksiyaning hosilasi x_0 nuqtadan o'tishda ishorasini o'zgartirsa, u holda x_0 bu funksiyaning ekstremum nuqtasi bo'ladi:

a) Agar $f(x)$ funksiya hosilasining ishorasi x_0 nuqtadan chapdan o'ngga o'tishda musbatdan manfiyga ("+" dan "-") o'zgartirsa, u holda x_0 nuqtada funksiya maksimumga erishadi.
b) Agar $f(x)$ funksiya hosilasining ishorasi x_0 nuqtadan chapdan o'ngga o'tishda manfiydan musbatga ("-" dan "+") o'zgartirsa, u holda x_0 nuqtada funksiya minimumga erishadi.





23-shakl

Natijada $f(x)$ funksiya ekstremumini topishning quyidagi qoidasiga kelamiz:

1) Funksiya hosilasi topiladi;

2) $f'(x)=0$ tenglama yechiladi. Masalan x_0 tenglama yechimlaridan biri bo'lsin: $f'(x_0)=0$;

3) x_0 nuqtaning chap atrofi $(x_0-\delta,x_0)$ va o'ng atrofi $(x_0,x+\delta)$ da $f'(x)$ hosilaning ishorasi aniqlanadi va yuqorida keltirilgan a), b) qoidalar tatbiq etilib ekstremum qiymati topiladi.

2-misol. Quyidagi funksiyani ekstremumga tekshiring.

$$y=\frac{x^2}{x-2}$$

Yechish: Aniqlanish sohasi $x-2\neq 0, x\neq 2$ bo'ladi.

Berilgan funksiyaning hosilasini topamiz:

$$y'=\frac{2x(x-2)-x^2}{(x-2)^2}=\frac{x^2-4x}{(x-2)^2}=\frac{x(x-4)}{(x-2)^2},$$

hosilani 0 ga tenglashtirib kritik nuqtalarini topamiz,

$x\neq 2, x=0, x=4$ kritik nuqtalarni hosil qilamiz.

x	$(-\infty;0)$	0	$(0;2)$	2	$(2;4)$	4	$(4;\infty)$
y'	+	*0*	-	*uzilish nuqta*	-	*0*	+

y	*o'sadi*	$y_{max} = 0$	*kamayadi*		*kamayadi*	$y_{min} = 8$	*o'sadi*

Funksiyaning $[a,b]$ segmentdagi eng katta va eng kichik qiymatlari.

Ma'lumki, $f(x)$ funksiya $[a,b]$ segmentda uzluksiz bo'lsa, unda uzluksiz funksiyalarning xossasiga ko'ra bu funksiya $[a,b]$ da eng katta va eng kichik qiymatlarga erishadi. Bu qiymatlar quyidagicha topiladi:

1) $f(x)$ funksiyaning hosilasi $f'(x)$ topilib, u nolga tenglanadi: $f'(x)=0$.

2) $f'(x)=0$ tenglama yechiladi. Aytaylik, bu tenglamaning yechimlari x_1, x_2, x_3 bo'lsin.

3) $f(x)$ funksiyaning x_1, x_2, x_3 nuqtalardagi qiymatlari topiladi.

Aytaylik, ular $f(x_1), f(x_2), f(x_3)$ bo'lsin.

4) $f(x)$ funksiyaning $[a,b]$ segmentning chekkalari a va b nuqtalardagi qiymatlari topiladi: $f(a), f(b)$.

Natijada, $f(x_1), f(x_2), f(x_3),\ f(a), f(b)$

qiymatlar hosil bo'ladi. Bu qiymatlar orasidagi kattasi $f(x)$ funksiyaning $[a,b]$ dagi eng katta qiymati, kichigi esa $f(x)$ funksiyaning $[a,b]$ dagi eng kichik qiymati bo'ladi.

3-misol. Ushbu $f(x) = x^2e^{-x}$ funksiyaning $[-1,3]$ segmentdagi eng katta va eng kichik qiymatlari topilsin.

Yechish: Bu funksiyaning hosilasini topamiz,

$$f'(x) = 2xe^{-x} - x^2e^{-x} = (2x - x^2)e^{-x}$$

bo'lib, hosilani 0 ga tenglashtirib kritik nuqtalarini topamiz.

$f'(x)=0$ ya'ni $\quad (2x-x^2)e^{-x}=0$

tenglamaning yechimlari $x_1=0, \quad x_2=2$ bo'ladi, chunki

$$(2x-x^2)e^{-x}=0 \quad x_1=0, \quad x_2=2.$$

Endi berilgan funksiyaning bu $x_1=0$, $x_2=2$ nuqtalardagi hamda

$[-1,3]$ segmentning chetki nuqtalari $x_3=-1$, $x_4=3$ dagi qiymatlarini topamiz:

$$f(x_1)=f(0)=0, \qquad f(x_3)=f(-1)=e$$
$$f(x_2)=f(2)=4e^{-2} \qquad f(x_4)=f(x_3)=e^{-3}$$

Demak, berilgan funksiyaning $[-1,3]$ segmentdagi katta qiymati e , eng kichik qiymati 0 bo'ladi.

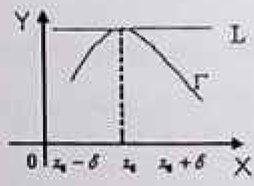
Funksiya grafigining qavariqligi, botiqligi, egilish nuqtasi va asimptotasi

Funksiya grafigining qavariligi va botiqligi.

Faraz qilaylik, $f(x)$ funksiya (a,b) da berilgan, $x_0 \in (a,b)$ va bu nuqtaning $(x_0-\delta, x_0+\delta)$ atrofi $(\delta>0)$ shu (a,b) intervalga tegishli bo'lsin.

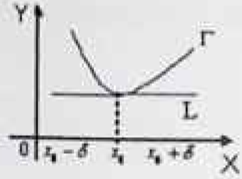
Berilgan $f(x)$ funksiya grafigi – egri chiziqni Γ, unga $x \in (x_0-\delta, x_0+\delta)$ nuqtasida o'tkazilgan urinmani L deylik.

Agar $(x_0-\delta, x_0+\delta)$ da Γ egri chiziq L urinmadan pastda joylashgan bo'lsa, $f(x)$ funksiya grafigi $(x_0-\delta, x_0+\delta)$ da **qavariq** deyiladi. (24-shakl)



24-shakl

Agar $(x_0-\delta, x_0+\delta)$ da Γ egri chiziq L urinmadan yuqorida joylashgan bo'lsa, $f(x)$ funksiya grafigi $(x_0-\delta, x_0+\delta)$ da **botiq** deyiladi.



25-shakl

Funksiya hosilalari yordamida uning grafigini qavariqligini, botiqligini aniqlash mumkin.

Aytaylik, $f(x)$ funksiya $(x_0-\delta, x_0+\delta)$ da ikkinchi tartibli uzluksiz $f''(x)$ hosilaga ega bo'lsin.

6-teorema. Agar $\tilde{o}\in(x_0-\delta, x_0+\delta)$ da $f''(x)<0$ bo'lsa, u holda $f(x)$ funksiya grafigi $(x_0-\delta, x_0+\delta)$ da qavariq bo'ladi, agar $f''(x)>0$ bo'lsa, u holda $f(x)$ funksiya grafigi $(x_0-\delta, x_0+\delta)$ da botiq bo'ladi.

Funksiya grafigining egilish nuqtasi

Agar ixtiyoriy $\tilde{o}\in(x_0-\delta, x_0)$ da funksiya grafigi Γ urinma L dan yuqorida (pastda) joylashgan bo'lib, ixtiyoriy $\tilde{o}\in(x_0, x_0+\delta)$ da funksiya grafigi Γ urinma L dan pastda (yuqorida) joylashgan bo'lsa, x_0 nuqta $f(x)$ funksiya grafigining **egilish nuqtasi** deyiladi.

Boshqacha qilib aytganda $f(x)$ funksiya grafigi $(x_0-\delta, x_0)$ da botiq (qavariq) bo'lib, $(x_0, x_0+\delta)$ da qavariq (botiq) bo'lsa, x_0 nuqta $f(x)$ funksiya grafigining **egilish nuqtasi** deyiladi.

Funksiya hosilalari yordamida uning grafigining egilish nuqtasini topish mumkin.

Natija. $f(x)$ funksiya grafigining egilish nuqtalarini ikkinchi tartibli $f''(x)$ ning nolga aylantiradigan nuqtalar orasida ($f''(x)=0$ tenglamaning yechimlari orasidan) qidirish kerak.

Teorema. Aytaylik, $f(x)$ funksiya $(x_0-\delta, x_0+\delta)$ da ikkinchi tartibli $f''(x)$hosilaga ega bo'lsin.

Agar $f''(x)$ hosila x_0 nuqtani o'tishda ishorasini o'zgartirsa, u holda x_0 nuqta $f(x)$ funksiya grafigining egilish nuqtasi bo'ladi.

4-misol. Ushbu funksiya qavariq va botiqlikka tekshirilsin, $f(x)=x^3-3x^2+1$ egilish nuqtasi topilsin.

Yechish: Berilgan funksiya uchun

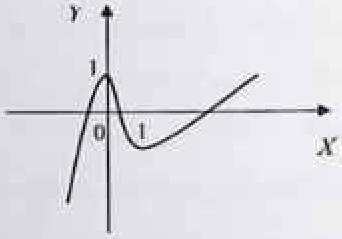
$f'(x)=3x^2-6x,\qquad f''(x)=6x-6$ bo'ladi.

Ravshanki, $f''(x)=6x-6<0,\ x-1<0,\ x<1$. Demak, berilgan funksiya grafigi $(-\infty,1)$ da qavariq bo'ladi.

Shuningdek,

$f''(x)=6x-6>0,\ \ x-1>0,\ \ x>1.$

Demak, berilgan funksiya grafigi $(1,+\infty)$ da botiq bo'ladi. $x=1$ nuqta $f(x)$ funksiya grafigining egilish nuqtasi bo'ladi. (26-shakl).



26-shakl

Funksiya grafigining asimptotalari

Aytaylik, $f(x)$ funksiya $a\in R$ nuqtaning biror $(a-\delta,\ a+\delta)$ atrofida $(\delta>0)$ berilgan bo'lsin.

Aytaylik, $f(x)$ funksiya $a\in R$ nuqtaning biror $(a-\delta,\ a+\delta)$ atrofida $(\delta>0)$ berilgan bo'lsin.

Agar ushbu

$$\lim_{x\to a+0} f(x)=\infty,\ \lim_{x\to a-0} f(x)=\infty,$$

limitlardan biri yoki ikkalasi cheksiz bo'lsa,

$x=a$

to'g'ri chiziq $f(x)$ funksiya **vertikal asimptotikasi** deyiladi.

Masalan, $x=0$ to'g'ri chiziq (ordinatalar o'qi)

$$y = \frac{1}{x}$$

funksiyaning vertikal asimptotikasi bo'ladi, chunki

$$\lim_{x \to +0} \frac{1}{x} = +\infty \qquad \lim_{x \to -0} \frac{1}{x} = -\infty.$$

Agar $k = \lim_{x \to \infty} \frac{f(x)}{x}$ va $b = \lim_{x \to \infty} (f(x) - kx)$ limitlar mavjud bo'lsa, u holda

$$y = kx + b$$

to'g'ri chiziq $y = f(x)$ funksiyaning og'ma asimtotasi deyiladi.

Xususan, $k = 0$ da gorizontal asimtotaga ega bo'lamiz.

Masalan,

$y = x - 4$

to'g'ri chiziq

$f(x) = \frac{x^2 - 3x - 2}{x + 1}$

funksiyaning og'ma asimptotasi bo'ladi, chunki

$$k = \lim_{x \to \infty} \frac{f(x)}{x} = \lim_{x \to \infty} \frac{x^2 - 3x - 2}{x(x+1)} = 1$$

$$b = \lim_{x \to \infty} \frac{x^2 - 3x - 2}{(x+1)} - x = \lim_{x \to \infty} \frac{-4x - 2}{x + 1} = -4$$

bo'ladi.

5-misol. Ushbu

$f(x) = \frac{x^2 - 2x}{x - 1}$

funksiya grafigining asimptotikalari topilsin.

Yechish: Bu funksiya $x = 1$ nuqtadan boshqa barcha nuqtalarda aniqlangan va uzluksiz.

Ravshanki,

$$\lim_{x\to 1-0}\frac{x^2-2x}{x-1}=+\infty,\ \lim_{x\to 1+0}\frac{x^2-2x}{x-1}=-\infty$$

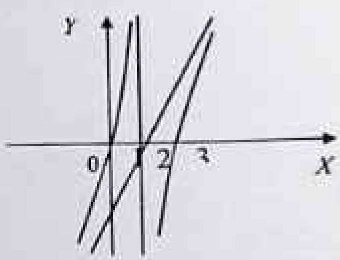
bo'ladi. Demak, $x=1$ to'g'ri chiziq berilgan funksiya grafigining vertikal asimptotasi bo'ladi.

Berilgan funksiyani quyidagicha yozib olamiz:

$$k=\lim_{x\to\infty}\frac{f(x)}{x}=\lim_{x\to\infty}\frac{x^2-2x}{x(x-1)}=1$$

$$b=\lim_{x\to\infty}\frac{x^2-2x}{(x-1)}-x=\lim_{x\to\infty}\frac{-x}{x-1}=-1$$

bo'lib, $y=x-1$ to'g'ri chiziq $f(x)$ funksiyaning og'ma asimptotasi ekanini topamiz. (27-shakl).



27-shakl

Shuni aytish kerakki $y=kx+\hat{a}$ to'g'ri chiziq $f(x)$ funksiyaning og'ma asimptotasi bo'lishi uchun

$$k=\lim_{x\to+\infty}\frac{f(x)}{x},\ \hat{a}=\lim_{x\to+\infty}\left[f(x)-kx\right] \qquad (5)$$

tengliklarning o'rinli bo'lishi zarur va etarli.

Bundan $f(x)$ funksiya grafigining og'ma asimptotasini topish uchun (5) limitlarni hisoblash yetarli bo'ladi.

Funksiya grafigini yasash

Endi funksiya grafigini yasashga o'tish mumkin.U quyidagi sxema asosida bajariladi:

1) Funksiyaning aniqlanish sohasini topish.

2) Funksiyani juft-toqlikka tekshirish.

3) Funksiyani davriylikka tekshirish.

4) Funksiyani uzluksizlikka tekshirish va uzilish nuqtalarini topish.

5) Funksiya grafigining koordinata o'qlari bilan kesishish nuqtalarini topish.

6) Monotonlik oraliqlarini aniqlash.

7) Ekstremumga tekshirish.

8) Botiq va qavariqlikka tekshirish.

9) Funksiyaning asimptotalarini topish.

10) Funksiya grafigini chizish.

6-misol. $y=\frac{2x-1}{(x-1)^2}$ funksiyani to'liq tekshiring va grafigini chizing.

1. Funksiya $x=1$ nuqtadan tashqari sonlar o'qining barcha nuqtalarida aniqlangan.

2. $f(-x)=\frac{-2x-1}{(-x-1)^2}\neq f(x)$ va $f(-x)\neq -f(x)$, demak, funksiya toq ham emas, juft ham emas.

3. Funksiya davriy emas.

4. $x=1$ nuqtada II-tur uzilishga ega.

$$\lim_{x\to 1\pm 0} f(x)=\lim_{x\to 1\pm 0}\frac{2x-1}{(x-1)^2}=+\infty$$

qolgan nuqtalarda funksiya uzluksiz.

5. Agar $x=0$ bo'lsa, u holda $y=-1$ va $y=0$ da $x=\frac{1}{2}$. Bundan kelib chiqadiki, $(0;-1)$ va $\left(\frac{1}{2};0\right)$ nuqtalar funksiya grafigining koordinata o'qlari bilan kesishish nuqtalari.

6. $y' = -\frac{2x}{(x-1)^2}$ funksiya aniqlanish sohasini quyidagi oraliqlarga bo'lamiz: $(-\infty; 0)$, $(0, 1)$, $(1; +\infty)$

$(-\infty; 0)$ oraliqlarda funksiya kamayadi. $(0, 1)$ oraliqda esa funksiya o'sadi. $(1; +\infty)$ oraliqda funksiya kamayadi.

7. $y'(x)$ hosila ishorasini $x = 0$ nuqtani o'tishda manfiydan musbatga o'zgartiradi. Demak, berilgan funksiya $x = 0$ nuqtada minimumga erishadi va $y_{\min} = y(0) = -1$ bo'ladi.

8. Funksiya botiq va qavariqligini tekshirish uchun ikkinchi tartibli hosilani olamiz. $y'' = 2 \cdot \frac{2x+1}{(x-1)^4}$ funksiyaning aniqlanish sohasini quyidagi oraliqlarga ajratamiz.

$\left(-\infty; -\frac{1}{2}\right), \left(-\frac{1}{2}; 1\right)$ $(1; +\infty)$.

$\left(-\infty; \frac{1}{2}\right)$ da $f''(-1) = -\frac{1}{8} < 0$ funksiya qavariq,

$\left(-\frac{1}{2}; 1\right)$ da $f''(0) = 2 > 0$ funksiya botiq, $(1; +\infty)$ oraliqda $f''(0) = 10 > 0$ funksiya botiq. Funksiyaning ikkinchi tartibli hosilasi $x = -\frac{1}{2}$ dan $x = 1$ nuqtaga o'tishda o'z ishorasini o'zgartiradi, bundan kelib chiqadiki, $f\left(-\frac{1}{2}\right) = -\frac{8}{9}$ nuqta egilish nuqtasi bo'ladi.

9. $x = 1$ funksiyaning vertikal asimptotasi, $y = 0$ gorizontal asimptotasi, ya'ni

$$\lim_{x\to\pm\infty} f(x) = \lim_{x\to\pm\infty} \frac{2x-1}{(x-1)^2} = \lim_{x\to\pm\infty} \frac{\frac{2}{x} - \frac{1}{x_2}}{\left(1 - \frac{1}{x}\right)^2} = 0$$

Og'ma asimptotasini topamiz:

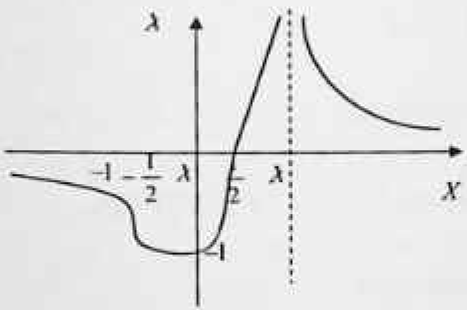
$$k_{1,2}=\lim_{x\to\pm\infty}\frac{f(x)}{x}=\lim_{x\to\pm\infty}\frac{2x-1}{x(x-1)^2}=\lim_{x\to\pm\infty}\frac{2-\frac{1}{x}}{x^2\left(1-\frac{1}{x}\right)^2}=0,$$

u holda $k=0$

$$b_{1,2}=\lim_{x\to\pm\infty}(f(x)-kx)=\lim_{x\to\pm\infty}\frac{2x-1}{(x-1)^2}=0,$$

u holda $b=0$. Bundan kelib chiqadiki $y=kx+b$ og'ma asimptota yo'q.

10. Funksiya grafigi:



28-shakl

Mustaqil yechish uchun topshiriqlar:

1. Quyidagi funktsiyalarning o`sish va kamayish oraliqlarini toping:

1. $f(x)=3x+1$; 2. $f(x)=-4x+2$; 3. $f(x)=\frac{2}{x}$; 4. $y(x)=\frac{3}{2-x}$;

5. $v(x)=x^2$; 6. $f(x)=(x-1)^2$; 7) $y(x)=5x-3x+1$; 8. $f(x)=x^2-2x+5$;

9. $h(x)=x^3-27x$; 10. $u(x)=x^2(x-3)$; 11. $f(x)=x^2-7x+6$; 12. $u(x)=-3x^2+8x+3$;

13. $v(x)=-\frac{1}{3}x^2+2x-3$; 14. $f(x)=x^2-3x+10$; 15. $f(x)=2x^2-8x+11$.

16. $f(x)=x^3-2x^2$; 17. $f(x)=\frac{x}{3}-\sqrt[3]{x}$; 18. $f(x)=x\ln x$;

Javob:18) $\left[0;\frac{1}{e}\right]$ da kamayadi; $\left[\frac{1}{e};+\infty\right]$ da o`sadi.

19. $f(x)=\frac{1}{5}x^5+\frac{1}{3}x^3$ monotonlikka tekshiring.

Javob:19). $[-\infty;-1]$ va; $[1;\infty]$ da o`suvchi, $[-1;1]$ da kamayuvchi.

20. $f(x)=\dfrac{x^2}{x^2-1}$ monotonlikka tekshiring.

Javob: $[-\infty;-1]$ va; $[-1;1]$ va $[1;\infty]$ da kamayuvchi.

21. $f(x)=x^5-5x^4+5x^3+1$ monotonlikka tekshiring.

Javob: $[-\infty;1]$ va$[3;\infty]$da o`sadi, $[1;3]$ kamayadi.

22. $f(x)=\dfrac{(x-2)(1-x)}{x^2}$ monotonlikka tekshiring.

Javob: $[-\infty;0]$ va$\left[\frac{3}{4};\infty\right]$ da kamayadi, $\left[0;\dfrac{3}{4}\right]$ da o`sadi.

23. $f(x)=\dfrac{(1-x)^3}{1-2x}$ monotonlikka tekshiring.

Javob: $\left[0;\frac{1}{2}\right]$ va$\left[\frac{1}{2};1\right]$ da o`sadi, $[-\infty;0]$ va$[1;\infty]$ da kamayadi.

24. $f(x)=(x+1)^{\frac{2}{3}}(x-5)^2$ monotonlikka tekshiring.

Javob: $\left[-1;\frac{1}{2}\right]$ va$[5;\infty]$ da o`sadi, $[-\infty;-1]$ va$\left[\frac{1}{2};5\right]$ da kamayadi.

25. $f(x)=\dfrac{x}{\ln x}$

26. $f(x)=x^2+\dfrac{1}{x}+1$

27. $f(x)=x^2e^{-x^2}$

28. $f(x)=\sqrt{x^2}(1+x)$

29. $f(x)=3x^4-8x^3+6x+1$

Javob: $[0;\infty]$ da o`sadi, $[-\infty;0]$ da kamayadi.

30. $f(x)=-x^3-x^2-7x-\pi\sqrt{3}$

Javob: $[-\infty;0]$ va $[1;\infty]$ o`sadi, $[0;1]$da kamayadi.

2. Funksiyaning ekstremumlarini toping.

a) $f(x) = x^2 - 2x + 3$

b) $y = x + \frac{1}{x-1}$

c) $y = -\frac{1}{3}x^3 - x^2 + 3x - 5$

d) $y = \frac{1}{3}x^3 + \frac{7}{2}x^2 + 12x + 1$

e) $y = x^2 e^{-2x}$

g) $y = \frac{1}{4}x^4 + \frac{5}{3}x^3 + 3x^2 + 10$

h) $y = \frac{3}{4-x}$

k) $y = 2x^3 + 3x^2 - 12x + 7$

f) $y = \frac{1}{5}x^5 - 4x^2$

3. Funksiyani to'la tekshiring va grafigini chizing.

a) $y = \frac{x^3}{3} + \frac{x^2}{2} - 6x$

b) $y = \frac{(x-1)^2+1}{x-1}$

c) $y = \frac{x^2}{x-2}$

d) $y = x + \frac{1}{x}$

e) $y = -4x^3 + 12x$

f) $y = 3x - x^3$

g) $y = 12x - x^3$

VI-BOB. BIR O'ZGARUVCHILI FUNKSIYANING INTEGRAL HISOBI.

18-§. Boshlang'ich funksiya va aniqmas integralning ta'rifi, xossalari. Aniqmas integral jadvali.

Ta'rif: Biror oraliqda aniqlangan $f(x)$ funksiya uchun bu oraliqning hamma qiymatlarida

$$F'(x)=f(x)$$

tenglik o'rinli bo'lsa, u holda $F(x)$ funksiya $f(x)$ funksiyaning **boshlangich funksiyasi** deyiladi.

1-Misol :1) $F(x)=\frac{a^x}{\ln a}$, $(a\neq 1, a>0)$ butun sonlar chizigida $f(x)=a^x$ funksiyaning boshlangich funksiyasi bo'ladi, chunki x ning istalgan qiymatida

$$F'(x)=(\frac{a^x}{\ln a})'=a^x=f(x)$$

tenglik to'g'ri bo'ladi.

2) $F(x)=\frac{x^5}{5}$ funksiya sonlar o'qining hamma nuqtalarida $f(x)=x^4$ funksiyaning boshlang'ich funksiyasi bo'ladi, chunki x ning istalgan qiymatida uning hosilasiga nisbatan

$$F'(x)=(\frac{x^5}{5})'=x^4=f(x)$$

tenglik o'rinli bo'ladi.

Berilgan funksiyaning boshlangich funksiyasini topish masalasi bir qiymatli xal qilinmaydi. Xaqiqatan ham, agar $F(x)$ funksiya $f(x)$ ning boshlangich funksiyasi bo'lsa u holda $F(x)+C$ funksiya ham (bunda C ixtiyoriy o'zgarmas son) $f(x)$ ning boshlangich funksiyasi bo'ladi, chunki C ning istalgan qiymati uchun $(F(x)+C)'=f(x)$ bo'ladi.

2-Misol: $F(x)=\frac{a^x}{\ln a}$ funksiya $f(x)=a^x$ funksiyaning boshlang'ich funksiyasi

$$(F(x)+C)'=\left(\frac{a^x}{\ln a}+C\right)'=a^x=f(x)$$

Tenglikdan esa $\frac{a^x}{\ln a}+C$ funksiya ham a^x funksiyaning boshlangich funksiyasi ekanligi kelib chiqadi.

Aniqmas integral va uning xossalari.

Ta'rif: Agar F(x) funksiya biror oraliqda $f(x)$ funksiyaning boshlangich funksiyasi bo'lsa, u holda F(x)+C (bunda C ixtiyoriy doimiy) funksiyalar to'plami shu kesmada $f(x)$ funksiyaning aniqmas integrali deyiladi va

$$\int f(x)dx=F(x)+C$$

kabi belgilanadi. Bu yerda $f(x)$ integral ostidagi funksiya, $f(x)dx$ integral ostidagi ifoda, x integrallash o'zgaruvchisi deyiladi. Aniqmas integralni topish jarayoni integrallash deyiladi.

3-Misol : 1) $\int a^x dx=\frac{a^x}{\ln a}+C$

2) $\int x^4 dx=\frac{x^5}{5}+C$

3) $\int \frac{dx}{x}=\ln x+C$

Aniqmas integral quyidagi xossalarga ega :

I. Aniqmas integralning hosilasi integral ostidagi funksiyaga teng, ya'ni

$$(\int f(x)dx)'=f(x)$$

Isbot: $(\int f(x)dx)'=(F(x)+C)'=F'(x)=f(x)$

II. Aniqmas integralning differensiali integral ostidagi ifodaga teng, ya'ni

$$d(\int f(x)dx) = f(x)dx$$

III. Biror funksiyaning xosilasidan olingan aniqmas integral shu funksiya bilan ixtiyoriy o'zgarmasning yigindisiga teng, ya'ni

$$\int F'(x)dx = F(x) + C$$

IV. Biror funksiyaning differensialidan olingan aniqmas integral shu funksiya bilan o'zgarmas yigindisiga teng, ya'ni

$$\int dF(x) = F(x) + C$$

V. O'zgarmas ko'paytuvchini integral belgisi tashqarisiga chiqarish mumkin ya'ni

$$\int kf(x)dx = k\int f(x)dx$$

VI. Chekli sondagi funksiyalarning algebraik yigindisidan olingan aniqmas integral shu funksiyalarning har biridan olingan aniqmas integrallarning algebraik yigindisiga teng, ya'ni

$$\int (f_1(x) + f_2(x) - f_3(x))dx = \int f_1(x)dx + \int f_2(x)dx - \int f_3(x)dx$$

Asosiy integrallar jadvali.

1. $\int dx = x + c$
2. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$
3. $\int \frac{dx}{x^2} = -\frac{1}{x} + c$
4. $\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + c$
5. $\int \frac{dx}{x} = \ln|x| + c$
6. $\int a^x dx = \frac{a^x}{\ln a} + c$
7. $\int e^x dx = e^x + c$
8. $\int sin\, xdx = -cos\, x + c$
9. $\int cos\, xdx = sin\, x + c$
10. $\int \frac{dx}{cos^2 x} = tgx + c$
11. $\int \frac{dx}{sin^2 x} = -ctgx + c$
12. $\int \frac{dx}{\sin x} = \ln\left|tg\frac{x}{2}\right| + c$

13. $\int \frac{dx}{\cos x} = \ln\left|tg\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + c$

14. $\int tgx dx = -\ln|\cos x| + c$

15. $\int ctgx dx = \ln|\sin x| + c$

16. $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} arctg \frac{x}{a} + c$

17. $\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln\left|\frac{x-a}{x+a}\right| + c$

18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = arcsin \frac{x}{a} + c$

19. $\int \frac{dx}{\sqrt{x^2 \pm a}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + c$

Bevosita integrallash usuli deb V va VI xossalarni qo'llanilishi, shuningdek, integrallashning asosiy formulalar jadvalidan foydalanib integrallashga aytiladi.

4-M i s o l : Integralni toping: $\int \frac{3x^2-2-x}{x^2-1} dx$

Yechimi: $\int \frac{3x^2-2-x}{x^2-1} dx = \int \frac{3x^2-3+1-x}{x^2-1} dx = \int \left(3 - \frac{x-1}{x^2-1}\right) dx =$

$= \int \left(3 - \frac{1}{x+1}\right) dx = \int 3dx - \int \frac{dx}{x+1} = 3x - ln|x+1| + C.$

Differensial belgisi ostiga kiritish usuli integral ostidagi ifodani almashtirishdan iborat.

Masalan: $dx = \frac{1}{k} d(kx + b)$, agar a,k- o'zgarmas bo'lsa;

$xdx = \frac{1}{2} d(x^2)$, $cosxdx = d(sinx)$, $sinxdx = -d(cosx)$,

$\frac{dx}{x} = d(lnx)$, $\frac{dx}{cos^2 x} = d(tgx)$, $\frac{dx}{1+x^2} = d(arctgx)$, va h.k

5-misol: $\int (x-7)^3 dx = \int (x-7)^3 d(x-7) = \frac{(x-7)^4}{4} + C.$

6-misol: $\int \frac{arctgx\, dx}{1+x^2} = \int arctgx\, d(arctgx) = \frac{arctg^2 x}{2} + C.$

7-misol: $\int \frac{1}{(2x-1)^2} dx = \frac{1}{2} \int \frac{1}{(2x-1)^2} d(2x-1) = -\frac{1}{2(2x-1)} + C.$

19-§. Integrallashning asosiy usullari: o'zgaruvchini almashtirish va bo'laklab integrallash.

Aniqmas integralda o'zgaruvchilarni almashtirish.

Integrallar jadvaliga kirmagan integralni hisoblash kerak bo'lsin. x ni t erkli o'zgaruvchining biror differensiallanuvchi funksiyasi orkali ifodalab, integrallashning yangi t o'zgaruvchisini kiritamiz: $x=\varphi(t)$,bunga teskari $t=\psi(x)$ funksiya mavjud bo'lsin, u holda

$$dx=\varphi'(t)dt$$

bo'lib

$$\int f(x)dx=\int f(\varphi(t))\cdot\varphi'(t)dt$$ bo'ladi.

1-Misol : $\int\frac{x^2dx}{5-x^6}$ integralni toping.

Yechish. $\int\frac{x^2dx}{5-x^6}=\left[t=x^3,\ dt=3x^2dx\right]=\int\frac{dt}{3(5-t^2)}=$
$-\frac{1}{3}\int\frac{dt}{t^2-\sqrt{5}^2}=$

$-\frac{1}{3}ln\frac{t-\sqrt{5}}{t+\sqrt{5}}+C.$

2-Misol : $$\int\frac{dx}{x\sqrt{x+4}}=\begin{bmatrix}\sqrt{x+4}=t,x+4=t^2\\ x=t^2-4\\ dx=2tdt\end{bmatrix}=$$

$$=\int\frac{2tdt}{(t^2-4)\cdot t}=2\int\frac{dt}{t^2-2^2}=\frac{1}{2}\ln\left|\frac{t-2}{t+2}\right|+C=\frac{1}{2}\ln\left|\frac{\sqrt{x+4}-2}{\sqrt{x+4}+2}\right|+C$$

Bo'laklab integrallash usuli.

Faraz qilaylik, $u(x)$ va $v(x)$ funksiyalar x ning differensiallanuvchi funksiyalari bo'lsin. Bu funksiyalar kupaytmasining differensialini topamiz:

$$d(u \cdot v) = vdu + udv ,$$

bunda

$$udv = d(uv) - vdu$$

Oxirgi tenglikning ikkala qismini integrallab, quyidagini xosil qilamiz:

$$\int udv = \int d(uv) - \int vdu$$

yoki

$$\int udv = uv - \int vdu$$

Bu formula bo'laklab integrallash formulasi deyiladi.

Odatda,

$$\int P_n(x)e^{ax}dx, \int P_n(x)cosaxdx, \int P_n(x)sinaxdx, \int P_n(x)arctgxdx,$$

$$\int P_n(x)arcsinxdx, \int P_n(x)lnxdx, \int P_n(x)arccosxdx$$

va shularga o'xshash integrallar bo'laklab integrallash formulasi orqali hisoblanadi.

$\int P_n(x)arctgxdx, \int P_n(x)arcsinxdx, \int P_n(x)lnxdx, \int P_n(x)arccosxdx$

integrallarda u uchun $arctgx$, $arcsinx$, lnx, $arccosx$ lar qabul qilinadi, qolganlarida u uchun $P_n(x)$ ko'phad qabul qilinadi.

3-Misol. $\int xe^{3x}dx$ integralni toping.

$$\int xe^{3x}dx = \left\| \begin{matrix} u = x, & dv = e^{3x}dx \\ du = dx & v = \dfrac{e^{3x}}{3} \end{matrix} \right\| = \frac{x}{3}e^{3x} - \frac{1}{3}\int e^{3x}dx = \frac{x}{3}e^{3x} -$$

$$-\frac{1}{9}\int e^{3x}d(3x) = \frac{x}{3}e^{3x} - \frac{1}{9}e^{3x} + C$$

Ba'zi hollarda bo'laklab integrallash formulasini bir necha marta qo'llash zarur bo'ladi.

4-Misol: $\int e^x cosxdx$ ni toping.

Bu integralni ikki marta bo'laklab integrallaymiz.

$\int e^x cosxdx = \left\| \begin{matrix} u = e^x, & dv = cosxdx \\ du = e^x dx & v = sinx \end{matrix} \right\| = e^x sinx - \int e^x sinxdx =$

$$= \left\| \begin{matrix} u = e^x, & dv = sinxdx \\ du = e^x dx, & v = -cosx \end{matrix} \right\|$$

$$= e^x sinx - (e^x(-cosx) - \int e^x (-cosx)dx =$$

$= e^x sinx + e^x cosx - \int e^x (cosx)dx$

$$2\int e^x cosxdx = e^x sinx + e^x cosx$$

$$\int e^x cosxdx = \frac{e^x}{2}(sinx + cosx).$$

5-Misol: $\int arctgt\, dt$ ni toping.

Yechish:

$$\int arctgt\, dt = \left\| \begin{matrix} u = arctgt, & dv = dt \\ du = \frac{1}{1+t^2}dt; & v = t \end{matrix} \right\| = arctgt \cdot t - \int \frac{t}{1+t^2}dx =$$

$$= arctgt \cdot t - \frac{1}{2}\int \frac{d(t^2+1)}{1+t^2} = arctgt \cdot t - \frac{1}{2}ln|1+t^2| + C.$$

Mustaqil bajarish uchun misollar.

1. $f(x)$ funktsiya uchun boshlang`ich funktsiyalarning umumiy ko`rinishini toping va tekshring.

1. $f(x)=2-x^3+\dfrac{1}{x^3}$

2. $f(x)=x-\dfrac{2}{x^5}+\cos x$

3. $f(x)=\dfrac{1}{x^2}-\sin x$

4. $f(x)=5x^2-1$

5. $f(x)=(2x-3)^5$

6. $f(x)=3\sin 2x$

7. $f(x)=(4-5x)^7$

10. $f(x)=\dfrac{2}{\cos^2\left(\dfrac{\pi}{3}-x\right)}$

11. $f(x)=\dfrac{4}{(3x-1)^2}$

12. $f(x)=-\dfrac{2}{x^5}+\dfrac{1}{\cos^2(3x-1)}$

13. $f(x)=\dfrac{1}{mx}$

14. $f(x)=\dfrac{x}{x^2+1}$

15. $f(x)=\dfrac{1}{\sqrt{3-x^2}}$

16. $f(x)=\left(x^2-3x+1\right)^{10}(2x-3)$

17. $f(x)=\dfrac{(\ln x)^3}{x}$

18. $f(x)=x\sqrt{x}$

19. $f(x)=\dfrac{1}{\sqrt[3]{x^3}}$

20. $f(x)=\dfrac{1}{x\ln x}$

21. $f(x)\sin(ax+b)$

22. $f(x)=\sqrt{\sin x}\ \cos x$

23. $f(x)=(2x+1)^{20}$

24. $f(x)=\cos(bx+a)$

25. $f(x)=f(ax+b)$

26. $f(x)=x\cos\left(x^2\right)$

2. $f(x)$ funktsiya uchun shunday boshlang`ich funktsiya topingki, uning grafigi $M$ nuqtadan o`tsin.

27. $f(x)=4x+\dfrac{1}{x_2},\quad M(1;4)$

28. $f(x)=1-2x,\ M(3;2)$

29. $f(x)=x^3+2,\ M(2;15)$

30. $f(x)=\dfrac{1}{x^3}-10x^4+3,\ M(1;5)$

Javob: $-\dfrac{1}{2x^2}-2x^5+3x-4,5$

31. $f(x)=-x^2+3x,\ M(2;-1)$

Javob: $-\frac{x^3}{3}+\frac{3}{2}x^2-4\frac{1}{3}$

32. $f(x)=\sqrt{x}\,(x\geq 0),\ M(9;7)$

Javob: $\frac{2}{3}x\sqrt{x}-11$

33. $f(x)=\sin x$ funktsiya uchun shunday $F(x)$ boshlang`ich funktsiya topingki, $F(1)=3$ bo`lsin. Javob: $f(x)=3+\cos 1-\cos x$.

34. Agar $F'(x)=6x+3$ va $F(-2)=4$ bo`lsa, $F(x)$ ni toping.

Javob: $F(x)=3x^2+3x-2$.

35. Agar $F'(x)=9x^2+4x-10$ va $F(-1)=10$ bo`lsa, $F(x)$ ni toping.

Javob: $F(x)=3x^3+2x^2-10x+1$

Aniqmas integrallarni toping va tekshiring:

36. $\int\left(x^7+4x\right)dx$

37. $\int\frac{2+x^2}{x}dx$

38. $\int(4x+3)^2dx$

$'2x^2-\cos x)dx$

$+\frac{3}{1+x^2}\Big)dx$

$\frac{lx}{+x^2}$

2. $\int\frac{dx}{\sqrt{b^2-x^2}}$

43. $\int\sqrt[3]{1+3\sin x}\,\cos x dx$

44. $\int(\sin 5x-\cos 5x)\,dx$

46. $\int\frac{\sin x}{\sqrt{\cos x}}dx$

47. $\int\frac{\sin 2x}{3\sin x}dx$

48. $\int\left(x^{k-2}-1\right)dx$

49. $\int(3x-8)\,dx$

50. $\int(2+3\sin x)\,dx$

51. $\int\left(\frac{3}{\sin^2 x}+\frac{1}{x}\right)dx$

52. $\int\left(\frac{4}{\sqrt{1-x^2}}-8x-\frac{4}{x}\right)dx$

52. $\int\frac{dx}{x\ln x}$

53. $\int\frac{dx}{1+e^x}$

45. $\int \frac{xdx}{\sqrt{3-x^4}}$

54. $\int \frac{\sin 2x}{1-\sin^2 x}dx$

55. $\int \frac{3x^2dx}{x^6+4}$; $(t = x^3)$

56. $\int \frac{x^2dx}{5-x^6}$; $(t = x^3$ belgilash bilan)

57. a) $\int \frac{2xdx}{x^4+3}$; $(t = x^2)$

58. $\int \frac{e^x dx}{4e^x+3}$ $(t = 4e^x + 3)$

59. $\int \frac{dx}{1+\sqrt[3]{x+1}}$; $(t = \sqrt[3]{x+1}$ belgilash bilan)

60. $\int x\sqrt{x-1}\,dx$; $(t = \sqrt{x-1})$

61. $\int \frac{cosx}{sin^2x}dx$; 62. $\int \frac{arctgxdx}{1+x^2}$; 63. $\int \frac{lnxdx}{x}$; 64. $\int \frac{xdx}{3-x^2}$; 65. $\int \frac{3xdx}{x^2-5}$;

66. $\int \frac{tgx}{cos^2x}dx$; 67. $\int \frac{arctg^2xdx}{1+x^2}$; 68. $\int \frac{2xdx}{x^2-9}$; 69. $\int \frac{x}{3x^2-7}dx$;

70. a) $\int xcosx\,dx$; b) $\int xsinx\,dx$;

71. a) $\int xlnx\,dx$; b) $\int xe^x\,dx$;

72. a) $\int x^2cosx\,dx$; b) $\int x^2sinx\,dx$;

73. a) $\int \frac{lnx}{x^3}dx$; b) $\int arc\,sinx\,dx$;

74 a) $\int x\,arctgx\,dx$; 76. $\int x^2e^{3x}dx$;

75 a) $\int e^{-x}sinx\,dx$; 77. $\int e^{3x}sin2x\,dx$;

78. $\int\left(5\sin 3x+2\cos\frac{x}{2}\right)dx$

79. $\int\left(\frac{5}{\cos^2 2x}+\frac{3}{\sin^2 3x}-\frac{2}{l^{3x}}\right)dx$

80. $\int\left(\frac{4}{\cos^2(x+4)}+\frac{5}{\sin^2(2x-1)}\right)dx$

81. $\int\left(5^{3-4x}+e^{x+2}\right)dx$

82. $\int\left(3e^{3x}+7\cdot 3^{7x}\right)dx$

83. $\int \frac{dx}{\sqrt{9-x^2}}$

84. $\int \frac{dx}{4+(3x+1)^2}$

85. $\int \frac{dx}{5-x^2}$

86. $\int \frac{dx}{\sqrt{x^2-5}}$

87. $\int tg\,5x\,dx$

88. $\int ctg\,(x+5)\,dx$

89. $\int \frac{dx}{3-2x+x^2}$

90. $\int \frac{dx}{3-2x-x^2}$

91. $\int \frac{dx}{\sqrt{2-6x-9x^2}}$

92. $\int \frac{dx}{\sqrt{4x^2-6x+11}}$

93. $\int \sqrt{2x-3}\,dx$ Javob: $\frac{1}{3}\sqrt{(2x-3)^3}+c$

94. $\int \frac{dx}{\sqrt[3]{2-3x}}$ Javob: $-\frac{1}{2}\sqrt[3]{(2-3x)^2}+c$

95. $\int \frac{3dx}{\sqrt[4]{3x+5}}$ Javob: $\frac{4}{3}\sqrt[4]{(3x+5)^3}+c$

96. $\int \left(x^4+3\right)^5 x^2dx$ Javob: $\frac{1}{24}\left(x^4+3\right)^6+c$

97. $\int \frac{3x^2dx}{\left(2-3^3\right)^4}$ Javob: $\frac{1}{3\left(2-x^3\right)^3}+c$

98. $\int \frac{x^2dx}{\sqrt{2x^3-5}}$ Javob: $\frac{1}{3}\sqrt{3x^3-5}+c$

99. $\int \frac{3xdx}{5+x^2}$ Javob: $\frac{3}{2}\ln\left(5+x^2\right)+c$

1. $\int \frac{x^3dx}{4x^4-2}$ Javob: $\frac{1}{12}\ln\left|3x^4-2\right|+c$

1. $\int \frac{2\cos xdx}{3\sin x+5}$ Javob: $\frac{2}{3}\ln\left|3\sin x+5\right|+c$

102. $\int \frac{3\cos xdx}{\sqrt[3]{1+2\sin x}}$ Javob: $\sqrt[3]{(1+2\sin x)^2}+c$

103. $\int 5^{3x} xdx$ Javob: $\frac{5^{3x^2}}{6\ln 5}+c$

104. $\int e^{-x^3+2}x^2dx$ Javob: $-\frac{1}{2}e^{-x^3+2}+c$

105. $\int x\cos(x^2+3)dx$ Javob: $\frac{1}{2}\sin(x^2+3)+c$

106. $\int \frac{3x^2dx}{\sin^2(x^3-2)}$ Javob: $-ctg(x^3-2)+c$

107. $\int \frac{2e^x dx}{(5+e^x)^2}$ Javob: $-\frac{2}{5+e^x}+c$

108. $\int xe^x dx$ Javob: $e^x(x-1)+c$

109. $\int \ln|x|dx$ Javob: $x(\ln|x|-1)+c$

110. $\int x\ln|x|dx$ Javob: $\frac{1}{2}x^2\ln|x|-\frac{1}{4}x^2+c$

111. $\int arctgx\,dx$ Javob: $x\,arctgx-\frac{1}{2}\ln(1+x^2)+c$

112. $\int x\,arctgx dx$ Javob: $\frac{1}{2}(x^2arctgx+arctgx-x)+c$

113. $\int (x-1)e^{2x}dx$ Javob: $\frac{1}{2}(x-1)\,e^{2x}-\frac{1}{4}e^{2x}+c$

114. $\int x\cos xdx$ Javob: $x\sin x+\cos x+c$

115. $\int x^2e^xdx$ Javob: $e^x(x^2-2x+2)+c$

116. $\int e^x\sin xdx$ Javob: $\frac{1}{2}e^x(\sin x-\cos x)+c$

117. $\int e^{2x}\cos xdx$ Javob: $\frac{1}{5}e^{2x}(\sin x+2\cos x)+c$

118. $\int \cos 3x\cdot\cos xdx$ Javob: $\frac{1}{4}\sin 2x+\frac{1}{8}\sin 4x+c$

119. $\int\left(\sqrt{x}-\frac{1}{\sqrt{x}}\right)^2dx$ Javob: $\frac{x^2}{2}-2x+\ln|x|+c$

120. $\int (2tgx+3ctgx)^2dx$ Javob: $4tgx-9ctgx-x+c$

20-§. Eng sodda ratsional kasrlarni integrallash. Ratsional kasrlarni sodda ratsional kasrlarga ajratish. Ratsional funksiyalarni integrallash algoritmi.

Eng sodda kasrlarni integrallash.

Quyidagi kasrlar eng sodda kasrlar deyiladi:

$$\frac{A}{x-a}, \frac{A}{(x-a)^m} \ (m>1), \frac{Bx+C}{x^2+px+q}, \frac{Bx+C}{(x^2+px+q)^m}$$

bunda A,B,C,a,p,q lar o'zgarmas sonlar, x^2+px+q kvadrat uchhad haqiqiy ildizga ega emas.

Bu sodda kasrlarni integrali quyidagicha hisoblanadi:

1. $\int \frac{A}{x-a}dx = A\int \frac{d(x-a)}{x-a} = A\ln|x-a| + C.$

2. $\int \frac{A}{(x-a)^m}dx = A\int \frac{d(x-a)}{(x-a)^m} = A\int (x-a)^{-m}d(x-a) =$
$A\frac{(x-a)^{-m+1}}{1-m} + C = \frac{A}{1-m}\cdot\frac{1}{(x-a)^{m-1}} + C.$

3. $\int \frac{Bx+C}{x^2+px+q}dx$ integralni hisoblaymiz.

$x^2+px+q = (x+\frac{p}{2})^2 + q - \frac{p^2}{4}$ ko'rinishda yozib olamiz.

Bundan,

$$\int \frac{Bx+C}{x^2+px+q}dx = \int \frac{Bx+C}{(x+\frac{p}{2})^2+q-\frac{p^2}{4}}dx = \int \frac{Bx+C}{(x+\frac{p}{2})^2+a^2}dx$$

bo'ladi, bunda $a = q - \frac{p^2}{4}$ ga teng. $x + \frac{p}{2} = t$ almashtirish bajaramiz:

$$\int \frac{Bx+C}{x^2+px+q}dx$$
$$= B\int \frac{tdt}{t^2+a^2} + \left(C - Bp\frac{1}{2}\right)\int \frac{dt}{t^2+a^2} = \frac{B}{2}\ln(t^2+a^2) +$$

$$+\left(C - Bp\frac{1}{2}\right)\frac{1}{a}arctg\frac{t}{a} + C_1 =$$

$$=\frac{B}{2}\ln\left((x+\frac{p}{2})^2+a^2\right)+\frac{2C-Bp}{2\sqrt{q-\frac{p^2}{4}}}arctg\frac{x+\frac{p}{2}}{\sqrt{q-\frac{p^2}{4}}}+C_1.$$

Demak,

$$\int\frac{Bx+C}{x^2+px+q}dx=$$

$$=\frac{B}{2}\ln(x^2+px+q)+\frac{2C-Bp}{2\sqrt{q-\frac{p^2}{4}}}arctg\frac{x+\frac{p}{2}}{\sqrt{q-\frac{p^2}{4}}}+C_1.$$

4. $\int\frac{Bx+C}{(x^2+px+q)^m}dx$ integralni hisoblaymiz.

Buning uchun ham uchunchi holdagidek almashtirish bajaramiz:

$x+\frac{p}{2}=t.$

$$\int\frac{Bx+C}{(x^2+px+q)^m}dx=\int\frac{Bx+C}{\left(\left(x+\frac{p}{2}\right)^2+q-\frac{p^2}{4}\right)^m}dx$$

$$=\int\frac{Bt+\left(C-\frac{Bp}{2}\right)}{(t^2+a^2)^m}dt=$$

$$=\frac{B}{2}\cdot\frac{1}{1-m}\cdot\frac{1}{(t^2+a^2)^{m-1}}+\left(C-Bp\frac{1}{2}\right)\int\frac{dt}{(t^2+a^2)^m}$$

$J_m=\int\frac{dt}{(t^2+a^2)^m}$ integral quyidagi rekurent formula yordamida topiladi:

$$J_m=\frac{t}{2(m-1)a^2(t^2+a^2)^{m-1}}+\frac{2m-3}{2(m-1)a^2}J_{m-1}.$$

1-Misol. Quyidagi integrallarni hisoblang.

$\int\frac{3}{x-7}dx;\ \int\frac{1}{x+2}dx;\ \int\frac{-10}{x-5}dx;\ \int\frac{2}{x+8}dx;$

Yechish:

$$\int\frac{3}{x-7}dx=3\int\frac{d(x-7)}{x-7}=3ln|x-7|+C.$$

$$\int \frac{1}{x+1}dx = \int \frac{d(x+1)}{x+1} = ln|x+1| + C.$$
$$\int \frac{-10}{x-5}dx = -10\int \frac{d(x-5)}{x-5} = -10ln|x-5| + C.$$
$$\int \frac{2}{x+8}dx = 2\int \frac{d(x+8)}{x+8} = 2ln|x+8| + C.$$

2-misol. Quyidagi integralni hisoblang: $\int \frac{6}{(x-2)^3}dx$

Yechish:

$$\int \frac{6}{(x-2)^3}dx = 6\int \frac{d(x-2)}{(x-2)^3} =$$
$$= 6\int (x-2)^{-3}d(x-2) = 6\frac{(x-2)^{-3+1}}{1-3} + C =$$
$$= \frac{-3}{(x-2)^2} + C.$$

3-misol: $\int \frac{2x+2}{x^2+2x+7}dx$ integralni hisoblang.

Yechish:

$$\int \frac{2x+2}{x^2+2x+7}dx = \int \frac{d(x^2+2x+7)}{x^2+2x+7} = \ln|x^2+2x+7| + C.$$

4-misol:

$\int \frac{2x+3}{x^2+3x-10}dx$ ratsional funksiyani integrallang.

Yechish:

1-usul: $\int \frac{2x+3}{x^2+3x-10}dx = \int \frac{d(x^2+3x-10)}{x^2+3x-10} = \ln|x^2+3x-10| + C.$

2-usul: Yuqoridagi ko'rsatilgan 3-formulaga asosan integrallaymiz:

$x^2+3x-10 = \left(x+\frac{3}{2}\right)^2 - \frac{49}{4}$ deb olib, $x+\frac{3}{2} = t$ belgilab olamiz, natijada

$dx = dt$ bo'ladi va bundan,

$$\int \frac{2x+3}{x^2+3x-10}dx = \int \frac{2x+3}{\left(x+\frac{3}{2}\right)^2 - \frac{49}{4}}dx = \int \frac{2\left(t-\frac{3}{2}\right)+3}{t^2-\frac{49}{4}}dt$$
$$= \int \frac{2tdt}{t^2-\frac{49}{4}} == \ln\left(t^2-\frac{49}{4}\right) + C = ln|x^2+3x-10| + C.$$

Ratsional kasrlarni sodda kasrlarga yoyish.

$P(x)=\frac{Q_k(x)}{R_l(x)}$ ko'rinishdagi ko'phadlar nisbatiga kasr ratsional funksiya deyiladi.

Agar $k\geq l$ bo'lsa kasr noto'g'ri kasr, agar $k\leq l$ bo'lsa kasr to'g'ri kasr bo'ladi. Agar noto'g'ri kasr bo'lsa suratdagi ko'phadni maxrajdagi ko'phadga bo'lib, butun qismi ajratib olinadi va kasr qismini sodda kasrlarga yoyiladi.

I. $\frac{Q(x)}{(x-x_1)(x-x_2)\dots(x-x_n)}=\frac{A}{x-x_1}+\frac{B}{x-x_2}+\cdots\frac{C}{x-x_n}$

5-Misol: $R(x)=\frac{3x+2}{(x^2-1)(x+2)}$ ratsional kasrni sodda kasrlarning yig'indisiga yoying.

Yechish. $\frac{3x+2}{(x^2-1)(x+2)}=\frac{3x+2}{(x-1)(x+1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+1}+\frac{C}{x+2}$

$3x+2=A(x+1)(x+2)+B(x-1)(x+2)+C(x^2-1)$

$$x=1\quad 5=6A,\quad A=\frac{5}{6}$$

$$x=-1\quad -1=-2B,\quad B=\frac{1}{2}$$

$$x=-2\quad -4=3C,\quad C=-\frac{4}{3}$$

$$\frac{3x+2}{(x^2-1)(x+2)}=\frac{\frac{5}{6}}{x-1}+\frac{\frac{1}{2}}{x+1}+\frac{-\frac{4}{3}}{x+2}=\frac{5}{6}\frac{1}{x-1}+\frac{1}{2}\frac{1}{x+1}-\frac{4}{3}\frac{1}{x+2}.$$

II. $\frac{Q(x)}{(x-a)^k}=\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{(x-a)^2}+\cdots+\frac{D}{(x-a)^k}$

6-Misol: $R(x)=\frac{x^3+4x^2-5x-1}{(x-1)^3(x+2)}$ ratsional kasrni sodda kasrlarning yig'indisiga yoying.

Yechish. $\frac{x^3+4x^2-5x-1}{(x-1)^3(x+2)}=\frac{A}{x-1}+\frac{B}{(x-1)^2}+\frac{C}{(x-1)^3}+\frac{D}{x+2}$

$$x^3+4x^2-5x-1=$$
$$=A(x-1)^2(x+2)+B\,(x+2)(x-1)+C(x+2)$$
$$+D(x-1)^3)$$

$x=1,\quad -1=3C,\ C=-\frac{1}{3}$

$x=-2,\quad 9=-27C,\ D=-\frac{1}{3}$

x^3 oldidagi koeffitsentlarni tenglashtiramiz

$$1 = A + D, A = 1 - D = \frac{4}{3}$$

Ozod hadlarni tenglashtiramiz: $-1 = 2A - B + 2C - D$,

$$B = 2A + 2C - D + 1 = \frac{10}{3}$$

$$\frac{x^3 + 4x^2 - 5x - 1}{(x-1)^3(x+2)} = \frac{\frac{4}{3}}{x-1} + \frac{\frac{10}{3}}{(x-1)^2} + \frac{-\frac{1}{3}}{(x-1)^3} + \frac{-\frac{1}{3}}{x+2} =$$

$$= \frac{1}{3}\left(\frac{4}{x-1} + \frac{10}{(x-1)^2} - \frac{1}{(x-1)^3} - \frac{1}{x+2}\right)$$

III. $\frac{Q(x)}{(x^2+px+q)^k} = \frac{Ax+B}{x^2+px+q} + \frac{Cx+D}{(x^2+px+q)^2} + \frac{Lx+K}{(x^2+px+q)^3} + \cdots + \frac{Mx+N}{(x^2+px+q)^k}$

7-misol: $R(x) = \frac{(x+1)}{(x^2+x+2)(x^2+4x+5)}$ ratsional kasrni sodda kasrlarning yig'indisiga yoying.

Yechish:

$$\frac{(x+1)}{(x^2+x+2)(x^2+4x+5)} = \frac{Ax+B}{x^2+x+2} + \frac{Cx+D}{x^2+4x+5}$$

$$(x+1) = (Ax+B)(x^2+4x+5) + (Cx+D)(x^2+x+2)$$

$$x + 1 = Ax^3 + 4Ax^2 + 5Ax + Bx^2 + 4Bx + 5B +$$
$$+Cx^3 + Cx^2 + 2Cx + Dx^2 + Dx + 2D$$

$$\begin{cases} A + C = 0 \\ 4A + B + C + D = 0 \\ 5A + 4B + 2C + D = 1 \\ 5B + 2D = 1 \end{cases} \Rightarrow \begin{cases} A = -\frac{1}{6} \\ B = 0 \\ C = \frac{1}{6} \\ D = \frac{1}{2} \end{cases}$$

$$\frac{(x+1)}{(x^2+x+2)(x^2+4x+5)} = \frac{-\frac{1}{6}x}{x^2+x+2} + \frac{\frac{1}{6}x + \frac{1}{2}}{x^2+4x+5} =$$

$$= -\frac{1}{6}\cdot\frac{x}{x^2+x+2}+\frac{1}{6}\cdot\frac{x+3}{x^2+4x+5}.$$

Ratsional funksiyalarni integrallash.

Ratsional funksiyalarni integrallashdan oldin quyidagi hisoblashlar bajariladi:

1) Berilgan ratsional kasr to‘g‘ri yoki noto‘g‘ri kasr ekanligini tekshirish. Agar noto‘g‘ri kasr bo‘lsa suratdagi ko‘phadni maxrajdagi ko‘phadga bo‘lib,butun qismini ajratib olish.
2) Kasr maxrajini ko‘paytuvchilarga ajratish.
3) To‘g‘ri ratsional kasrni eng sodda kasrlar yig‘indisiga yoyish.
4) Eng sodda ratsional kasrlarni integrallash

8-misol. Ushbu $\int\frac{15x^2-4x-81}{(x-3)(x+4)(x-1)}dx$ integralni hisoblang.

Yechish: Ushbu integralni hisoblash uchun sodda kasrlay yig‘indisiga yoyamiz,

$$\frac{15x^2-4x-81}{(x-3)(x+4)(x-1)}=\frac{A}{x-3}+\frac{B}{x+4}+\frac{C}{x-1}$$

Umumiy maxraj berib, ikkala tomonidan ham maxrajlarini tashlab yuboramiz:

$$15x^2-4x-81=A(x+4)(x-1)+B(x-3)(x-1)+$$

$$+C(x-3)(x+4)$$

Xususiy qiymatlar usulidan foydalanib $x=3$ qiymat bersak,

$$15\cdot 9-12-81=A(3+4)(3-1)$$

$135-93=14A,\ 14A=42,\quad A=3.$

$x=-4$ qiymat bersak, $15\cdot 16+16-81=\ B(-4-3)(-4-1)$

$$35B=175,\ \ B=5.$$

$x=1$ qiymat bersak, $15-4-81=\ C(1-3)(1+4)$

$$-10C = -70, \ \ C = 7.$$

Demak, A, B, C larni qiymatlarini o'rinlariga qo'yib quyidagi sodda kasrlarga yozamiz:

$$\frac{15x^2 - 4x - 81}{(x-3)(x+4)(x-1)} = \frac{3}{x-3} + \frac{5}{x+4} + \frac{7}{x-1}$$

Integralni topamiz: $\int \frac{15x^2-4x-81}{(x-3)(x+4)(x-1)} dx = \int \frac{3}{x-3} dx + \int \frac{5}{x+4} dx + \int \frac{7}{x-1} dx == 3ln|x-3| + 5ln|x+4| + 7ln|x-1| = \ln (x-3)^3(x+4)^5(x-1)^7.$

Mustaqil bajarish uchun misollar.

Quyidagi aniqmas integrallarni toping:

1. $\int \frac{dx}{x-8}$; 2. $\int \frac{5dx}{3x-2}$; 3. $\int \frac{dx}{x^2+4}$;

4. $\int \frac{xdx}{x^2+9}$; 5. $\int \frac{xdx}{x^2-1}$; 6. $\int \frac{dx}{x^2-9}$;

7. $\int \frac{dx}{5x^2+13}$; 8. $\int \frac{3dx}{(2x-3)^5}$; 9. $\frac{xdx}{(x^2-16)^3}$;

10. $\frac{(2x-2)dx}{(x^2-2x-16)^3}$; 11. $\frac{x^2dx}{(x^3-8)^3}$; 12. $\frac{(3x^2-2x)dx}{(x^3-x^2+2)^4}$;

13. $\int \frac{dx}{x^2+5x}$; 14. $\int \frac{dx}{x^2-5x+6}$; 15. $\int \frac{(1+x)dx}{(x^2+1)(x^2+x+1)}$;

16. $\int \frac{dx}{-x^2+x^3}$; 17. $\int \frac{dx}{x(x^2+1)}$; 18. $\int \frac{xdx}{-1+x^3}$; 19. $\int \frac{xdx}{1+x^3}$;

20. $\int \frac{x^2dx}{-x^4+1}$; 21. $\int \frac{x^4dx}{1-2x^2+x^4}$; 22. $\int \frac{7x-15}{5x-2x^2+x^3} dx$; 23. $\int \frac{dx}{(x^2-4x+5)(x^2-4x+4)}$

24. $\int \frac{dx}{(1+x^3)(1+x)(1+x^2)}$; 25. $\int \frac{11x-9}{(x-3)^2(x+1)} dx$; 26. $\int \frac{x^3+3x^2-3x+1}{(1+x^2)(x-1)^2}$;

27. $\int \frac{3x^2+8}{x(x+2)^2}$; 28. $\int \frac{x^6+2x^4+2x^2-1}{x^5+2x^3+x}$;

29. $\int \frac{(5x^3+9x^2-22x-8)dx}{(x^2-4)x}$; 30. $\int \frac{x^4-3x^2-3x-2}{x(x^2-x-2)}$;

21-§. Trigonometrik funksiyalar qatnashgan ba'zi integrallarni integrallash. Ba'zi bir irratsional ifodalarni integrallash.

Trigonometrik funksiyalar qatnashgan integrallar quyidagi ko'rinishda bo'lishi mumkin:

I. $\int sinmx \cdot cosnx\, dx$; $\int cosmx \cdot cosnx\, dx$; $\int sinmx \cdot sinnx\, dx$.

II. $\int sin^m x \cdot cos^n x\, dx$; Bu yerda m va n lar haqiqiy sonlar.

III. $\int sin^{2n} x\, dx$; $\int cos^{2n} x\, dx$; $n > 0, n \in Z$

IV. $\int sin^{2m} x \cdot cos^{2n} x\, dx; n > 0, m > 0, n \in Z, m \in Z$

V. $\int R(sinx)cosxdx$; $\int R(cosx)sinxdx$;

VI. $\int R(sinx, cosx)dx$;

Bu integrallarning har biriga to'xtalamiz.

I. $\boldsymbol{\int sinmx \cdot cosnx\, dx}$; $\boldsymbol{\int cosmx \cdot cosnx\, dx}$; $\boldsymbol{\int sinmx \cdot sin\, nx\, dx}$

Ko'rinishidagi integrallar funksiyalar ko'paytmasini yig'indiga almashtirish formulalari yordamida integrallanadi.

Ular quyidagilardir:

1) $\sin mx \cdot \cos nx = \frac{1}{2}[\sin(m+n)x + \sin(m-n)x]$;

2) $\cos mx \cdot \cos nx = \frac{1}{2}[\cos(m+n)x + \cos(m-n)x]$;

3) $\sin mx \cdot \sin nx = \frac{1}{2}[\cos(m-n)x - \cos(m+n)x]$;

1-misol. $\int sin6x \cdot cos7x\, dx$ integral hisoblansin.

Yechish: Berilgan integralni hisoblash uchun $sin6x \cdot cos7x$ ko'paytmani yig'indi bilan almashtiramiz.

$$\int sin6x \cdot cos7x\, dx = \frac{1}{2}\int [\sin(6x+7x) + \sin(6x-7x)] =$$
$$= \frac{1}{2}\int (\sin 13x - sinx)dx = \frac{1}{2}\int \sin 13x\, dx - \frac{1}{2}\int \sin x\, dx =$$
$$= \frac{1}{2}(-\frac{\cos 13x}{13} + \cos x) + C = -\frac{\cos 13x}{26} + \frac{\cos x}{2} + C.$$

2-misol. $\int \cos 3x \cdot cos9x\, dx$ integral hisoblansin.

Yechish: Berilgan integralni hisoblash uchun $\cos 3x \cdot cos9x$ ko'paytmani yig'indi bilan almashtiramiz.

$$\cos 3x \cdot \cos 9x = \frac{1}{2}[\cos(3+9)\,x + \cos(3-9)\,x] =$$

$$\frac{1}{2}\int (\cos 12x + cos6x)dx = \frac{1}{2}\int \cos 12x\,dx + \frac{1}{2}\int \cos 6x\,dx =$$

$$= \frac{1}{2}(\frac{\sin 12x}{12} + \frac{\sin 6x}{6}) + C = \frac{\sin 12x}{24} + \frac{\sin 6x}{12} + C.$$

3-misol. $\int sin2x \cdot sin5x\,dx$ integral hisoblansin.

Yechish: Berilgan integralni hisoblash uchun $sin2x \cdot sin5x$ ko'paytmani yig'indi bilan almashtiramiz.

$$\sin 2x \cdot \sin 5x = \frac{1}{2}[\cos(2-5)\,x - \cos(2+5)\,x] =$$

$$= \frac{1}{2}\int (\cos 3x - cos7x)dx = \frac{1}{2}\int \cos 3x\,dx - \frac{1}{2}\int \cos 7x\,dx =$$

$$= \frac{1}{2}(\frac{\sin 3x}{3} + \frac{\sin 7x}{7}) + C = \frac{\sin 3x}{6} - \frac{\sin 7x}{14} + C$$

II. $\int \boldsymbol{sin^m x \cdot cos^n x\,dx}$ ko'rinishidagi integrallarda quyidagicha hollar bo'lishi mumkin:

1-hol. Sinusning daraja ko'rsatkichi m toq musbat son, ya'ni $m = 2k+1$.

Bu holda integral ostidagi ifoda quyidagicha o'zgartiriladi:

$$sin^m x = sin^{2k+1}x = sin^{2k}x \cdot sinx = (sin^2 x)^k \cdot sinx$$
$$= (1 - cos^2 x)^k \cdot sinx$$

bo'lganligi uchun integral ostidagi ifoda

$sin^m x \cdot cos^n x\,dx = (1 - cos^2 x)^k \cdot cos^n x \cdot sinx\,dx$ ko'rinishga keladi.

Agar bu yerda $\cos x = t$ deb olsak, u holda $sinx\,dx = -dt$ bo'lib, integral ostidagi ifoda

$$(1 - cos^2 x)^k \cdot cos^n x \cdot sinx\,dx = -(1 - t^2)^k t^n\,dt$$

ko'rinishga keladi va integral darajali funksiyalar yig'indisini integrallashdan iborat bo'ladi.

4-misol. $\int sin^7 x\,dx$ integral hisoblansin.

Yechish. $sin^7x = sin^6x \cdot sinx = (sin^2x)^3 \cdot sinx = (1 - cos^2x)^3 \cdot sinx$ bo‘lganligi uchun

$\int sin^7x\, dx = \int (1 - cos^2x)^3 \cdot sinx\, dx = \{\cos x = t, sinx\, dx = -dt\}$=

$$\int (1-t^2)^3\,(-dt) = -\int (1-t^2)^3\, dt = -\int (1 - 3t^2 + 3t^4 - t^6)dt =$$

$$= -t + t^3 - \frac{3t^5}{5} + \frac{t^7}{7} + C = -\cos x + cos^3x - \frac{3cos^5x}{5} + \frac{cos^7x}{7} + C.$$

2-hol. Kosinusning daraja ko‘rsatkichi n toq manfiy son, ya’ni $n = 2k + 1$bo‘lsin.

U holda

$$cos^nx = cos^{2k+1}x = cos^{2k}x \cdot cosx = (cos^2x)^k \cdot cosx$$
$$= (1 - sin^2x)^k \cdot cosx$$

bo‘lib, integral ostidagi ifoda

$sin^mx \cdot cos^nx\, dx = sin^mx\,(1 - sin^2x)^k \cdot cosx\, dx$ ko‘rinishga keladi.

Agar bu yerda $\sin x = t$ deb olsak, u holda $cosx\, dx = dt$ bo‘lib, integral ostidagi ifoda

$$sin^mx\,(1 - sin^2x)^k \cdot cosx\, dx = t^m(1 - t^2)^k\, dt$$

ko‘rinishga keladi va integral darajali funksiyalar yig‘indisini integrallashdan iborat bo‘ladi.

5-misol. $\int cos^5x\, dx$ integralni hisoblang.

Yechish. $cos^5x = cos^4x \cdot cosx = (cos^2x)^2 \cdot cosx = (1 - sin^2x)^2 \cdot cosx$

bo‘lganligi uchun,

$\int cos^5x\, dx = \int (1 - sin^2x)^2 \cdot cosx\, dx = \{\sin x = t, cosx\, dx = dt\}$=

$$\int (1-t^2)^2\, dt = \int (1-t^2)^2\, dt = \int (1 - 2t^2 + t^4)dt =$$

$$= t - \frac{2t^3}{3} + \frac{t^5}{5} + C = -\sin x - \frac{2sin^3x}{3} + \frac{sin^5x}{5} + C.$$

3-hol. Sinus va kosinuslar daraja ko'rsatkichlari yig'indisi $m + n$ juft manfiy son, ya'ni $m + n = -2k$ $(k > 0, k \in Z)$.

Bu holda integral ostidagi ifoda ikki xil ko'rinishga ega bo'ladi:

1) Integral ostidagi funksiya kasr bo'lib, uning surati sinusning darajasidan, mahraji esa kosinusning darajasidan iborat bo'ladi yoki aksincha bo'lib, ularning daraja ko'rsatkichlari bir vaqtda juft yoki toq . $m + n$ manfiy bo'lganligidan mahrajning daraja ko'rsatkichi suratning daraja ko'rsatkichidan katta ekanligi kelib chiqadi.

2) Integral ostidagi funksiya surati o'zgarmas sondan, mahraji esa daraja ko'rsatkichlari bir xil (juft yoki toq) bo'lgan sinus va kosinusning ko'paytmasidan iborat.

Qaralayotgan holda $(m + n = -2k)$ $tg\,x = t$ yoki $ctg\,x = t$ almashtirish yordamida berilgan integral ko'phad yangi o'zgaruvchi t ning butun manfiy ko'rsatkichi daraja yig'indisining integralidan iborat bo'ladi. $tg\,x = t$ almashtirish qilinganda $sinx = \frac{t}{\sqrt{1+t^2}}$, $cosx = \frac{1}{\sqrt{1+t^2}}$, $dx = \frac{dt}{1+t^2}$ hosil bo'lishini, agar

$ctg\,x = t$ almashtirish qilinsa

$nx = \frac{1}{\sqrt{1+t^2}}$, $cosx = \frac{t}{\sqrt{1+t^2}}$, $dx = -\frac{dt}{1+t^2}$ hosil bo'lishini nazarda ımiz.

$\int \boldsymbol{sin^{2n}x\,dx}$; $\int \boldsymbol{cos^{2n}x\,dx}$; $(\boldsymbol{n > 0, n \in Z})$ ko'rinishdagi integrallar

$ın^2x = \frac{1}{2}(1 - \cos 2x)$ va $cos^2x = \frac{1}{2}(1 + \cos 2x)$

formulalar yordamida soddaroq integrallarga keltiriladi va hisoblanadi.

6-misol. $\int cos^4x\,dx = \int\ (cos^2x)^2dx = \int \left(\frac{1+cos2x}{2}\right)^2 dx = \frac{1}{4}\int(1 + 2cos2x + cos^2 2x)\,dx = \frac{1}{4}(x + sin2x + \int cos^2 2xdx) =$

$= \frac{1}{4}(x + sin2x + \int \frac{1+cos4x}{2} dx = \frac{1}{4}\left(x + sin2x + \frac{x}{2} + \frac{sin4x}{8}\right) + C.$

IV. $\int \boldsymbol{sin^{2m}x \cdot cos^{2n}x\, dx;\ n > 0, m > 0, n \in Z, m \in Z}$ ko'rinishdagi integrallarni hisoblashda ham

$sin^2x = \frac{1}{2}(1 - \cos 2x)$ va $cos^2x = \frac{1}{2}(1 + \cos 2x)$ formulalardan foydalaniladi.

Ba'zi hollarda bu formulalar bir necha marta qo'llanilishi mumkin.

7-misol. $\int sin^2x \cdot cos^2x\, dx$ integralni hisoblang.

Yechish: $\int sin^2x \cdot cos^2x\, dx = \frac{1}{2}\int(1 - \cos 2x)(1 + \cos 2x)dx = \frac{1}{2}\int(1 - cos^2 2x)\ dx = \frac{1}{2}(x - \int \frac{1}{2}(1 + \cos 4x)dx = \frac{1}{2}\left(x - \frac{1}{2}x - \frac{sin4x}{8}\right) + C = \frac{1}{4}x - \frac{sin4x}{16} + C.$

V. $\int \boldsymbol{R(sinx)cosxdx}$ integral $sinx = t,\ cosxdx = dt$ almashtirish yordamida $\int R(t)dt$ integralga keltiriladi.

$\int \boldsymbol{R(cosx)sinxdx}$ integral $cosx = t,\ sinxdx = -dt$ almashtirish yordamida $-\int R(t)dt$ integralga keltiriladi.

VI. $\int R(sinx, cosx)dx$ integral sin va cos ning ratsional funksiyasining integralidan iborat bo'ladi, $tg\frac{x}{2} = t\ \ (-\pi < x < \pi)$ almashtirish yordamida integrallanadi.

$$sinx = 2sin\frac{x}{2}cos\frac{x}{2} = \frac{2sin\frac{x}{2}cos\frac{x}{2}}{1} = \frac{2sin\frac{x}{2}cos\frac{x}{2}}{cos^2\frac{x}{2}+sin^2\frac{x}{2}} = \frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}};$$

$$cosx = cos^2\frac{x}{2} - sin^2\frac{x}{2} = \frac{cos^2\frac{x}{2}-sin^2\frac{x}{2}}{cos^2\frac{x}{2}+sin^2\frac{x}{2}} = \frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}}$$

$sinx = \frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{2t}{1+t^2};\qquad cosx = \frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{1-t^2}{1+t^2};\ dx = \frac{2dt}{1+t^2}$ bo'ladi.

8-misol. $\int \frac{dx}{3+5sinx+3cosx}$ aniqmas integralni toping.

Yechish: $tg\frac{x}{2} = t$ belgilashdan foydalanib, $sinx = \frac{2tg\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{2t}{1+t^2}$ va

$$cosx = \frac{1-tg^2\frac{x}{2}}{1+tg^2\frac{x}{2}} = \frac{1-t^2}{1+t^2}\ ; \quad dx = \frac{2dt}{1+t^2}$$ ekanligidan,

$$\int \frac{dx}{3+5sinx+3cosx} = = \int \frac{\frac{2dt}{1+t^2}}{3+5\frac{2t}{1+t^2}+3\frac{1-t^2}{1+t^2}} = \int \frac{\frac{2dt}{1+t^2}}{\frac{3+3t^2+10t+3-3t^2}{1+t^2}} =$$

$$= \int \frac{2dt}{10t+6} = \int \frac{dt}{5t+3} = \frac{1}{5}ln|5t+3| + C = \frac{1}{5}ln\left|5tg\frac{x}{2}+3\right| + C.$$

9-misol. $\int \frac{cos^3xdx}{sin^2x+sinx}$ integralni hisoblang.

Yechish. $\int \frac{cos^3xdx}{sin^2x+sinx} = \int \frac{cos^3xdx}{sinx(1+sinx)} = \int \frac{cosx(1-sin^2x)dx}{sinx(1+sinx)} =$

$$= \int \frac{cosx(1-sinx)dx}{sinx} = \int \frac{cosx}{sinx}dx - \int \frac{cosxsinx}{sinx}dx =$$

$$= \int \frac{d(sinx)}{sinx} - \int cosxdx = \ln(sinx) - sinx + C.$$

Ba'zi bir irratsional funksiyalarni integrallash.

Integral ostidagi funksiyada o'zgaruvchi $x, ax+b, ax^2+bx+c$ lar kasr darajalarida qatnashgan ayrim hollarda integrallarning hisoblanishini misollarda bayon etamiz.Shuni aytish kerakki, bunday hollarda integrallar o'zgaruvchilarni almashtirish yordamida ratsional funksiyalarga keltiriladi va hisoblanadi.

10-misol. Ushbu $\int \frac{\sqrt{x}}{1+\sqrt{x}}dx$ integral hisoblansin.

Yechish: Bu integralda $x = t^2$ almashtirish bajaramiz.Unda $dx = 2tdt\ bo'lib,$

$\frac{\sqrt{x}}{1+\sqrt{x}}dx = \int \frac{t}{1+t}2tdt = 2\int \frac{t^2}{1+t}dt$ bo'ladi, natijada ratsional funksiyani integrallashga keladi.

$$\int \frac{t^2}{1+t}dt = \int \left(t - 1 + \frac{1}{1+t}\right)dt = \frac{t^2}{2} - t + ln|t+1| + C$$

bo'lib,

$$2\int \frac{t^2}{1+t}dt = 2\left(\frac{t^2}{2} - t + ln|t+1|\right) + C = t^2 - 2t + 2ln|t+1| + C$$

bo'ladi.

11-misol. Ushbu $\int \frac{dx}{\sqrt{x}(1+\sqrt[3]{x})}$ integral hisoblansin.

Yechish: Bu integralda $x = t^6$ almashtirish bajaramiz.Unda $dx = 6t^5 dt\ bo'lib,$

$$\int \frac{dx}{\sqrt{x}(1+\sqrt[3]{x})} = 6\int \frac{t^5}{t^3(1+t^2)}dt = 6\int \frac{t^2+1-1}{(1+t^2)}dt =$$

$$= 6(\int dt - \int \frac{dt}{1+t^2}) = 6t - 6arctgt + C.$$

Mustaqil bajarish uchun misollar.

Quyidagi integrallarni hisoblang.

1. $\int sin3x \cdot cos4x\, dx$;
2. $\int cos5x \cdot cos2x\, dx$;
3. $\int sin2x \cdot sin\, 3x\, dx$;
4. $\int sin3x \cdot cos5x\, dx$;
5. $\int sin^4x \cdot cos^2x\, dx$;
6. $\int sin^5x \cdot cos^3x\, dx$;
7. $\int sin^6x\, dx$;
8. $\int sin^5x\, dx$;
9. $\int sin^2 5x\, dx$;
10. $\int \frac{dx}{sin^6x}$;
11. $\int \frac{dx}{5-3sinx}$;
12. $\int \frac{(2-sinx)dx}{2+cosx}$;
13. $\int cos7x \cdot cos3x\, dx$;
14. $\int cos6x \cdot cos4x\, dx$;
15. $\int sin5x \cdot sin\, x\, dx$;
16. $\int cosx \cdot cos3x\, dx$
17. $\int sin^3x \cdot cos^4x\, dx$;
18. $\int sin^2x \cdot cos^3x\, dx$;
19. $\int cos^6x\, dx$;
20. $\int cos^3x\, dx$;
21. $\int cos^2 3x\, dx$;
22. $\int \frac{dx}{cos^3x}$;
23. $\int \frac{dx}{5+4cosx}$;
24. $\int \frac{dx}{1+3cos^2x}$;
25. $\int \frac{\sqrt[6]{x}dx}{1+\sqrt[3]{x}}$;
26. $\int \frac{\sqrt{1+\sqrt[3]{x}}}{\sqrt[3]{x^2}}dx$
27. $\int \frac{\sqrt{x}}{x-\sqrt[3]{x^2}}dx$;
28. $\int \frac{\sqrt{x}}{1+\sqrt{x}}dx$;
29. $\int \frac{\sqrt{1+x}}{x}dx$;
30. $\int \frac{\sqrt{x}}{1+\sqrt[4]{x}}dx$;

22-§. Aniq integralga keltiriluvchi masalalar. Aniq integralning ta'rifi va uning asosiy xossalari. Nyuton-Leybnits formulasi. Aniq integralda o'zgaruvchini almashtirish. Bo'laklab integrallash.

1.Aniq integralga keltiriluvchi masalalar.

Aniq integralga olib keladigan ba'zi masalalarni ko'rib chiqaylik.

a) Egri chiziqli trapetsiyaning yuzini topish masalasi.

$[a, b]$ kesmada $y = f(x)$ funksiya berilgan bo'lsin.

$y = f(x)$ funksiya $x = a, x = b$ to'g'ri chiziqlar va Ox o'qi bilan chegaralangan shaklga egri chiziqli trapedsiya deyiladi.

$[a, b]$ kesmani n ta bo'lakka bo'lamiz,

$a = x_0 < x_1 < x_2 < \cdots < x_n = b$ va egri chiziqli trapetsiyani n ta kichik egri chiziqli trapetsiyalarga ajratamiz. Har bir $[x_{i-1}, x_i]$ kesmada ixtiyoriy ξ_i nuqtani olamiz va $f(\xi_i)$ ni o'tkazamiz.

$\Delta x_i = x_i - x_{i-1}$ egri chiziqli trapedsiyachalar asosi va balandligi $f(\xi_i)$ bo'lgan to'gri to'rtburchak chizamiz. Bu to'gri to'rtburchaklarni yuzalarining yig'indisi $\sum_{i=1}^{n} f(\xi_i)\,\Delta x_i$ dan iborat bo'ladi.Bu yigindi integral yig'indi deyiladi.

Egri chiziqli trapedsiyaning yuzi uchun

$S = \lim\limits_{\Delta x_i \to 0} \sum_{i=1}^{n} f(\xi_i)\,\Delta x_i$ ni qabul qilamiz.

2. Aniq integralning ta'rifi va uning asosiy xossalari.

Ta'rif: Integral yig'indining $max\Delta x_i \to 0$ dagi limitiga $y = f(x)$ funksiyaning $[a, b]$ kesmadagi aniq integrali deyiladi va quyidagicha belgilanadi,

$$\int_a^b f(x)dx = \lim_{max\Delta x_i \to 0} \sum_{i=1}^{n} f(\xi_i)\,\Delta x_i. \qquad (22.1)$$

Bu yerda $f(x)$ funksiya integrallanuvchi funksiya, a va b mos ravishda quyi va yuqori chegara, $f(x)dx$ ifoda integrallanuvchi ifoda.

Asosiy xossalari:

1. Quyi va yuqori chegaralarning o'rinlari almashsa integralning ishorasi teskarisiga o'zgaradi:

$$\int_a^b f(x)dx = -\int_b^a f(x)dx;$$

2. Integrallash chegaralari teng bo'lganda integralning qiymati 0 ga teng bo'ladi:

$$\int_a^a f(x)dx = 0$$

3. $[a, b]$ kesmani orasidan olingan d son uchun quyidagi tenglik o'rinli:

$$\int_a^b f(x)dx = \int_a^d f(x)dx + \int_d^b f(x)dx;$$

4. Chekli sondagi funksiyalar yig'indisining aniq integrali qo'shiluvchi funksiyalar aniq integrallarining yig'indisiga teng:

$\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx;$

5. O'zgarmas sonni aniq integral belgisi ostidan chiqarish mumkin:

$\int_a^b kf(x)dx = k\int_a^b f(x)dx;\ k = const;$

3. Nyuton-Leybnits formulasi.

Aniq integrallarni hisoblashda Nyuton-Leybnits formulasi

$\int_a^b f(x)dx = F(x)|_a^b = F(b) - F(a)$ dan foydalaniladi. (22.2)

1-misol: Aniq integrallarni hisoblang:

a) $\int_1^2 3x^2dx;$ b) $\int_0^2 \frac{dx}{(5x+1)^3};$ s) $\int_1^e \frac{lnx}{x}dx;$

Yechish. a) $\int_1^2 3x^2dx = \frac{3x^3}{3}|_1^2 = 2^3 - 1^3 = 7.$

b) $\int_0^2 \frac{dx}{(5x+1)^3} = \frac{1}{5}\int_0^2 (5x+1)^{-3}d(5x+1) = -\frac{1}{10}(5x+1)^{-2}|_0^2 = -\frac{11}{10}.$

s) $\int_1^e \frac{lnx}{x}dx = \int_1^e lnxd(lnx) = \frac{1}{2}ln^2x|_1^e = \frac{1}{2}ln^2e - \frac{1}{2}ln^21 = \frac{1}{2};$

4. **Aniq integralda o'zgaruvchini almashtirish. Bo'laklab integrallash.**

$\int_a^b f(x)dx$ integralni o'zgaruvchilarni almashtirish usuli bilan hisoblashda

$x = \varphi(t),\ \alpha \le t \le \beta$ almashtirishni qo'llaymiz. $[\alpha, \beta]$ kesmada

$x = \varphi(t)$, $\varphi'(t)$, $f(\varphi(t))$ funksiyalar uzluksiz va $\varphi(\alpha) = a$, $\varphi(\beta) = b$ bo'lsa

U holda quyidagi formula o'rinli:

$\int_a^b f(x)dx = \int_\alpha^\beta f(\varphi(t))\ \varphi'(t)dt.$

2-misol: $\int_1^4 \frac{1}{(1+\sqrt{x})^2} dx$ integralni hisoblang.

Yechish: $\int_1^4 \frac{1}{(1+\sqrt{x})^2} dx = \left\| \begin{matrix} 1+\sqrt{x} = t \\ x = (t-1)^2 \\ dx = 2(t-1)dt \\ x = 1\ da\ t = 2 \\ x = 4\ da\ t = 3 \end{matrix} \right\| = \int_1^3 \frac{2(t-1)}{t^2} dt = 2\int_1^3 (\frac{1}{t} - \frac{1}{t^2}) = 2\left(lnt + \frac{2}{t^3}\right)|_1^3 = 2\left(ln3 + \frac{2}{27} - ln1 - 2\right) = 2ln3 - \frac{104}{27};$

$[a, b]$ kesmada $u = u(x) va\ v = v(x)$ funksiyalar uzluksiz hosilalarga ega bo'lsin. Bo'laklab integrallash formulasi o'rinli bo'ladi.

Ya'ni,

$$\int_a^b udv = uv|_a^b - \int_a^b vdu.$$

3-misol. $\int_0^1 2xarctgxdx$ integralni hisoblang.

Yechish: $\int_0^1 2xarctgxdx = \left\| \begin{matrix} u = arctgx, & dv = 2xdx \\ du = \frac{1}{1+x^2}dx\,, & v = x^2 \end{matrix} \right\| =$

$$= arctgx \cdot x^2|_0^1 - \int_0^1 \frac{x^2}{1+x^2}dx = arctg1 - \int_0^1 \frac{x^2+1-1}{1+x^2}dx =$$

$$= \frac{\pi}{4} - \int_0^1 \left(1 - \frac{1}{1+x^2}\right)dx = \frac{\pi}{4} - (x - arctgx)\,|_0^1 = \frac{\pi}{4} - 1 + \frac{\pi}{4} = \frac{\pi}{2} - 1.$$

4-misol. $\int_0^1 \ln(1+x)dx$ integralni hisoblang.

Yechish: $\int_0^1 \ln(1+x)\,dx = \left\| \begin{matrix} u = \ln(1+x), & dv = dx \\ du = \frac{1}{1+x}dx, & v = x \end{matrix} \right\| = \ln(1+x) \cdot$

$x|_0^1 -$

$$-\int_0^1 \frac{x}{1+x}dx = ln2 - \int_0^1 \frac{x+1-1}{1+x}dx = ln2 - \int_0^1 \left(1 - \frac{1}{1+x}\right)dx =$$

$$= ln2 - (x - \ln(1+x))|_0^1 = ln2 - 1 + ln2 = 2ln2 - 1.$$

5. Aniq integral yordamida geometrik, fizik va texnik masalalar yechish.

Aniq integral faqatgina geometrik masalalarni yechishda keng qo`llanilib qolmasdan, balki aniq integral yordamida qator fizik, texnik va boshqa mazmundagi masalalarni yechishda ham foydalanish mumkin. Qanday mazmundagi masalalar yechilmasin ular bir umumiylikka ega. Barcha masalalarda cheksiz ko`p qo`shiluvchilarga ega bo`lgan yig`indining limiti hisoblanadi. Amaliy masalalarni yechishga aniq integralni qo`llashda quyidagi qoidaga rioya qilish lozim.

1. Erkli o`zgaruvchini tanlash, izlanayotgan miqdor yetarlicha kichkina qismlarga bo`linadiki, ularning soni ortib borgan sari har bir miqdor nolga intilsin.

2. Yuqori tartibli cheksiz kichik miqdorlar e'tiborga olinmasdan izlanayotgan miqdorning cheksiz kichik qismlari $f(x_i)\,\Delta x_i$ elementar qismga almashtiriladi.

3. Erkli o`zgaruvchini $a$ dan $b$ gacha o`zgarishda qarab, izlanayotgan miqdor integral yig`indining limiti sifatida aniqlanadi:

$$\lim_{\Delta x_i \to 0} \sum_{i=1}^{n} f(x_i)\,\Delta x_i = \int_a^b f(x)\,dx$$

a) Kuch ta'sirida bajariladigan ishni hisoblash masalasi.

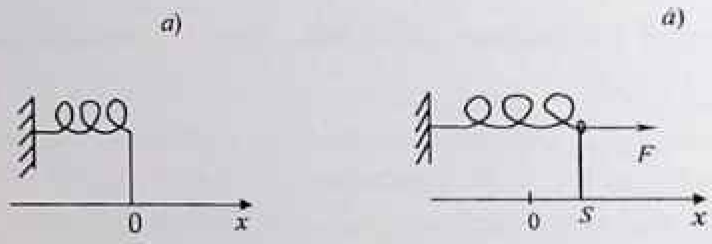
F o`zgaruvchan kuch ta'sirida to`g`ri chiziqli harakat qilayotgan moddiy nuqtaning bajargan ishi

$$A = \lim_{\Delta S \to 0} \sum_{i=1}^{n} F(S_i)\,\Delta S = \lim_{\Delta S \to 0} \sum_{i=a}^{b} F(S_i)\Delta S = \int_a^b F(S)\,ds \qquad (22.3)$$

formula bo`yicha hisoblanadi.

5-masala. $4H$ kuch prujinani 8 sm cho`zadi. Prujinani 8 sm cho`zish uchun qanday ish bajarish kerak?

Yechish. Guk qonuniga ko`ra prujinani $x$ kattalik qadar cho`zuvchi kuch $F=kx$ formula yordamida hisoblanadi, bunda k – o`zgarmas proportsionallik koeffitsienti



29-shakl

o nuqta prujinaning erkin holatiga to`g`ri keladi. k doimiyni aniqlaymiz. Masala shartidan $4=k\cdot 0{,}08$ tenglikni olamiz. Demak, $k=50$ va $F=50\,s$.(4) formula bo`yicha bajarilgan ishni hisoblaymiz:

$$A=\int_0^{0,08} 50\, s ds = 25\, s^2 \Big|_0^{0,08} = 25\cdot 0,08^2\, æ \; = 0,16\; æ$$

b) s **yuzaga suyuqlikning beradigan bosim kuchi**

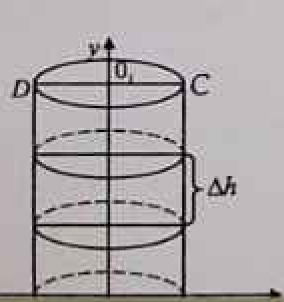
$$P=\gamma \int_a^b s\, dh \qquad (22.4)$$

formula bo`yicha topiladi.

6-masala. Silindrik bakning pastki asosi tekislikdagi teshikdan kirayotgan suv butun bakni to`ldiradi. Suvning silindr asosi yuziga beradigan bosim kuchini toping. Bakning balandligi h ga, asosi radiusi r ga teng.

Yechish. Suyuqlikning bak asosiga beradigan bosimi (30-shakl).

$$P=\rho g\int_{h_1}^{h_2} S(x)dx=\rho g\int_{h_1}^{h_2} h ds,\ gds,\ g=9,81\frac{i}{\tilde{n}\grave{a}\hat{e}^2}$$



formula bo`yicha hisoblanadi.

Bunda $ds \approx dv = \pi r^2 dh.$

Suvning bak asosiga beradigan bosim kuchini hisoblaymiz:

$$P=\rho g\int_0^h \pi r^2 h dh=\frac{\pi r^2 h^2}{2}\rho g.$$

(ρ - suyuqlikning zichligi)

v) **Bosib o'tilgan yo'lni tezlik bo'yicha topish masalasi**. Agar moddiy nuqta biror chiziq bo`ylab harakatlanib, uning tezligi moduli $t$ vaqtning funktsiyasi, ya'ni $\vartheta=f(t)$ bo`lsa, u holda nuqtaning $[t_1;t_2]$ vaqt oralig`ida bosib o`tgan yo'li

$$S=\int_{t_1}^{t_2}|f(t)|\,dt \qquad (22.5)$$

formula bo`yicha topiladi.

7-masala. Moddiy nuqta ixtiyoriy chiziq bo`ylab

$S=5-4t+t^2$ qonun bo`yicha harakatlanmoqda. $0\le t\le 5$ oraliqda nuqtaning holatini va bosib o`tgan yo`lini toping. (t - sekundlarda s va l metrlarda).

Yechish. 1)t = 5sek; $S=(5-4\cdot 5+25)=10;$

2) $V=S_t'=(5-4t+t^2)t'=-4+2t.$

Agar $t\in[0;2]$ bo`lsa, $\vartheta\le 0;$

Agar $t\in[2;5]$ bo`lsa, $\vartheta\ge 0.$

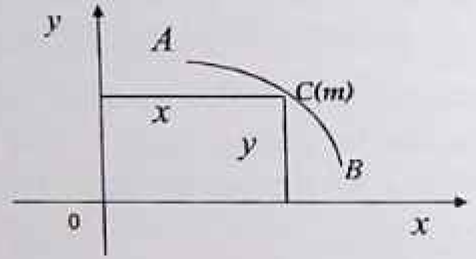
Moddiy nuqtaning e bosib o`tgan yo`li (23.5) formula bo`yicha topiladi:

$$e=\int_{t_1}^{t_2}|\vartheta|\,dt=\int_0^5|-4+2t|\,dt=-\int_0^2(-4+2t)\,dt+\int_2^5(-4+2t)\,dt=-$$

$$-\left(t^2-4t\right)\Big|_0^2+\left(t^2-4t\right)\Big|_2^5=-4+8+25-20-4+8=13\,i$$

4. Yassi egri chiziqning statik momentlari va og`irlik markazining koordinatalarini hisoblash.

Massasi m ga uzunligi Lga teng bo`lgan $AB$ yassi egri chiziq $y=f(x)$, $a\le x\le b$ funktsiya grafigidan iborat bo`lsin. Faraz qilaylik, AB egri chiziq bir jinsli bo`lsin, ya'ni massa taqsimotini ifodalovchi $\rho$ chiziqli zichlik egri chiziqning barcha nuqtalarida bir xil bo`lsin.



31-shakl

Soddalik uchun $\rho=1$ desak, egri chiziqning m massasi son jihatidan uning L uzunligiga teng bo`ladi.

$$m = L = \int_a^b \sqrt{1+y'^2}\ dx \qquad (22.6)$$

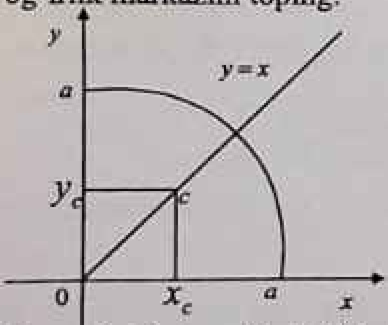
yassi egri chiziqning koordinatalar o`qlariga nisbatan statik momentlari.

$$M_x = \int_a^b yde = \int_a^b y\sqrt{1+y'^2}\ dx, \qquad (22.7)$$

$$M_y = \int_a^b xde = \int_a^b x\sqrt{1+y'^2}\ dx \qquad (22.8)$$

formulalar bo`yicha hisoblanadi.

8-masala. $x^2+y^2=a^2$ aylana yoyining 1-chorakda joylashgan qismining og`irlik markazini toping.



Yechish. Masala shartidan $x \geq 0, y \geq 0$ bo`lishligi kelib chiqadi.

Aylana tenglamasidan $y=\sqrt{a^2-x^2}$ bo`ladi

(4-rasm). Bu tenglikni differentsiallab, topamiz

32-shakl

$$y' = -\frac{2x}{2\sqrt{a^2 - x^2}} = -\frac{x}{\sqrt{a^2 - x^2}} = -\frac{x}{y}$$

Aylana yoyining 1-chorakdagi qismining uzunligini topamiz

$$\frac{1}{4} \cdot 2\pi a = \frac{\pi a}{2}$$

Bu yoyning massasi $m = \frac{\pi a}{2}$ bo`ladi.

(22.7) formula bo`yicha yoyning $ox$ o`qqa nisbatan statik momentini hisoblaymiz:

$$M_x = \int_a^b y\sqrt{1 + y'^2}\ dx = \int_0^a y\sqrt{1 + \left(-\frac{x}{y}\right)^2}\ dx = \int_0^a y \cdot \frac{1}{y}\sqrt{y^2 + x^2}\ dx = \int_0^a a dx = ax \Big|_0^a = a^2$$

(II) formula bo`yicha yoy og`irlik markazining ordinatasini topamiz.

$$y_c = \frac{Mx}{m} = \frac{a^2}{\frac{\pi a}{2}} = \frac{2a}{\pi}.$$

Aylana yoyi birinchi koordinatalar burchagining bissektrisasiga nisbatan simmetrik joylashgani uchun, bu yoyning og`irlik markazi $y = x$ to`g`ri chiziq ustida yotadi.

$$x_c = y_c = \frac{2a}{\pi}, \quad c\left(\frac{2a}{\pi}; \frac{2a}{\pi}\right)$$

5. Yassi figuraning statik momentlari va og`irlik markazining koordinatalarini hisoblash.

$y = f(x), a \le x \le b$ egri chiziq, ox o`q, $x = a$, va $x = b$ to`g`ri chiziqlar bilan chegaralangan moddiy yassi figura berilgan bo`lsin.

Yassi figuraning massasi uning yuzi bo`yicha tekis taqsimlangan bo`lsin, ya'ni massa zichligi o`zgarmas bo'lib, $\rho = 1$ bo`lsin. Bunday holda egri chiziqli trapetsiyaning massasi son jihatidan uning S yuziga teng bo`ladi:

$$m = S = \int_a^b y dx \qquad (22.9)$$

Koordinatalar o`qlariga nisbatan berilgan figuraning M_x va M_y statik momentlari

$$M_x = \frac{1}{2}\int_a^b y^2 dx, \qquad (22.10)$$

$$M_y = \int_a^b xy dx \qquad (22.11)$$

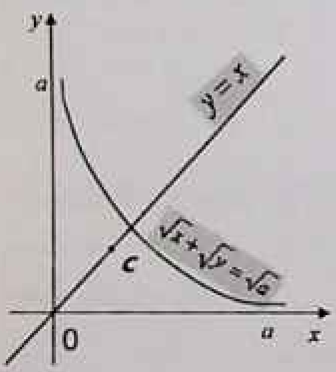
formulalar bo`yicha hisoblanadi.

Yassi figura og`irlik markazining koordinatalari ham yassi egri chiziq og`irlik markazining koordinatalaridek aniqlanadi va uning koordinatalri

$$x_c = \frac{M_y}{m} = \frac{\int_a^b xy dx}{\int_a^b y dx}, \quad y_c = \frac{M_x}{m} = \frac{1}{2}\cdot\frac{\int_a^b y^2 dx}{\int_a^b y dx} \qquad (22.12)$$

formulalar bo`yicha topiladi.

9-masala. $\sqrt{x}+\sqrt{y}=\sqrt{a}$ egri chiziq va koordinatalar o`qlari bilan chegaralangan figuraning og`irlik markazi koordinatalarini toping.



33-shakl

Yechish: Berilgan figura birinchi koordinatalar burchagining $y=x$ bissektrisasiga nisbatan simmet-rik joylashgan bo`ladi.

Shuning uchun jismning og`irlik markazi $y=x$ to`g`ri chiziq ustida yotadi (5-rasm).

$$\sqrt{x}+\sqrt{y}=\sqrt{a} \Rightarrow y=a+x-2\sqrt{ax},\ 0\le x\le a;$$
$$M_y=\int_0^a xydx=\int_0^a x\left(a+x-2\sqrt{ax}\right)dx=\frac{a^3}{30};$$
$$M_x=\frac{1}{2}\int_0^a y^2\ dx=\frac{1}{2}\ \int_0^a\left(a+x-2\sqrt{ax}\right)^2dx=\frac{a^3}{30};$$
$$m=S=\int_0^a ydx=\int_0^a\left(a+x-2\sqrt{ax}\right)dx=\frac{a^2}{6};$$
$$x_c=y_c=\frac{a}{5}\qquad c\left(\frac{a}{5};\frac{a}{5}\right)$$

Mustaqil bajarish uchun misol va masalalar.

1. $y=x^2+2$ parabolaning $0(0;0)$ va $A(2;6)$ nuqtalari orasidagi yoyining uzunligini toping. Javob: $L=\sqrt{17}+\frac{1}{L}\ln\left(4+\sqrt{17}\right)$

2. $y^2=(x-1)^3$ egri chiziqning $A(2;-1)$ va $B\,(5;-8)$ nuqtalari orasidagi yoyining uzunligini toping. Javob: $\frac{1}{27}\left(40\sqrt{40}-13\sqrt{13}\right)$

3. $y=x^2$ parabolaning $x=0$ va $x=2$ vertikal to`g`ri chiziqlar orasidagi yoyining uzunligini toping.

4. $y^2=x^3$ chiziqning $x=0$ va $x=1$ $(y\ge 0)$ vertikal chiziqlar orasidagi yoyining uzunligini toping. Javob: $\frac{8}{27}\left(\frac{13}{8}\sqrt{13}-1\right)$.

5. $y^2=x^3$ chiziqning $x=0$ va $x=\frac{4}{3}$ vertikal chiziqlar orasidagi yoyining uzunligini toping. Javob: $4\frac{64}{27}$.

6. $x^{2/3}+y^{2/3}=a^{2/3}$ astroida yoyining uzunligini toping. Javob: $6a$.

7. $9y^2=4x^3$ egri chiziqning (0; 0) va $\left(3;2\sqrt{3}\right)$ nuqtalari orasidagi yoyining uzunligini toping. Javob: $4\frac{2}{3}$.

8. $y=\frac{1}{2}x^2-1$ egri chiziqning ox o`q bilan kesishish nuqtalari orasidagi yoyining uzunligini hisoblang. Javob: $\sqrt{6}+\ln\left(\sqrt{2}+\sqrt{3}\right)$.

9. $y=x^3$ parabolaning 0(0;0) va $A\left(\frac{2}{3};\frac{8}{27}\right)$ nuqtalari orasidagi yoyining ox o`q atrofidagi aylanishidan hosil bo`lgan jism sirtining yuzini hisoblang.
Javob: $\frac{98\pi}{729}$.
10. $y^2=2x$ egri chiziq 0(0;0) va $A(1,5;\sqrt{3})$ nuqtalari orasidagi yoyining ox o`q atrofida aylanishidan hosil bo`lgan aylanish sirtning yuzini toping.
Javob: $\frac{7\pi}{3}$.
11. $y=\sin x$ sinusoidaning $0\le x\le\pi$ kesmadagi tarmog`ining $ox$ o`q atrofida aylanishidan hosil bo`lgan sirtning yuzini toping. Javob: $4\pi\left(\sqrt{2}+\ln\left(1+\sqrt{2}\right)\right)$.
12. x argument a dan b gacha o`zgarganda $x^2+y^2=R^2$ aylana yoyining $ox$ o`qi atrofida aylanishidan hosil bo`lgan Shar kamarining sirtini toping. $a$ va $b$ qanday bo`lganda. Bu sirt sfera sirtini ifoda qiladi?
Javob: $S=2\pi R(b-a)$. $a=-R, b=R$.
13. $4y-x^2=0$ parabolaning $x=2$ va $x=4$ vertikal to`g`ri chiziqlar orasidagi yoyining ox o`q atrofida aylanishidan hosil bo`lgan aylanma sirtning yuzini toping.
14. Prujinani 0,01 m qismi uchun $10H$ kuch kerak. Prujinani 0,04 m siqish uchun qancha ish bajariladi?
Javob: 0,8 j.
15. Prujinani 0,03 m siqish uchun 16 j ish bajariladi. 144 j ish bajarib, prujinani qancha siqish mumkin? Javob: 0,09 m.
16. Kanalning kesimi balandligi h asoslari a va b ga teng bo`lgan teng yonli rapetsiya shakliga ega. Kanalga quyiladigan suv tug`onni qanday kuch lan bosishini toping ($a>b$, $a-$ trapetsiyaning ustki asosi).
vob: $\frac{(a+2b)\,h^2}{6}\rho g$.
$\rho-$ suvning zichligi, $g-$erkin tushish tezlanishi).
17. Silindr shakldagi idish moy bilan to`ldirilgan. Agar idishning balandligi $H=3i$ va radiusi $R=1{,}5i$ bo`lsa, moyning idish yon sirtlariga beradigan bosimini hisoblang. Moyning zichligi $800\frac{\ddot{e}\ddot{a}}{i^3}$. Javob: $105920{,}1\acute{l}$.

18. $x^2+y^2=a^2$ aylananing ox o`q ustida yotgan qismining (yoyining) og`irlik markazini toping. Javob: $\left(0;\frac{10}{\pi}\right)$.

19. $x^{2/3}+y^{2/3}=a^{2/3}$ astrondaning birinchi chorakda yotgan yoyining og`irlik markazini toping. Javob: $\left(\frac{2}{5}a;\frac{2}{5}a\right)$.

20. $y=2x-x^2, y=0$ chiziqlar bilan chegaralangan figuraning og`irlik markazini toping. Javob: $\left(1;\frac{2}{5}\right)$.

21. $y=\cos x,\ y=0,\ x=0,\ x=\frac{\pi}{2}$ chiziqlar bilan chegaralangan figuraning og`irlik markazini toping. Javob: $\left(\frac{\pi-2}{2};\frac{\pi}{8}\right)$.

22. Bir jinsli yarim doiraning og`irlik markazini toping. Javob: $\left(0;\frac{4R}{3\pi}\right)$.

23. Uzunligi $a=25\ i$, radiusi $r=20i$ bo`lgan yarim silindr shaklda bo`lgan basseyndagi suvni chiqarish uchun bjariladigan ishni hisoblang.
Javob: $A=\frac{400000}{3}pg$.

24. Balandligi $h=3{,}5i$ va radiusi $r=1{,}5\ i$ bo`lgan silindr shaklida bo`lgan bakdagi benzin uning devorlariga beradigan bosimni hisoblang.
Javob: $18{,}345\ \pi y$

25. Diametri 20 m bo`lgan yarim sferik shakldagi idishdan suvni chiqarib tashlash uchun qancha ish bajariladi? Javob: $2{,}5\cdot10^{8}\pi k\bar{A}\cdot i$.

26. Kengligi 20 m va chuqurligi 5 m bo`lgan to`g`ri to`rtburchak shakldagi shlyuz-darvozaga suv qanday bosim beradi? Javob: 48 t.

Nazorat testlari.

1. $\int (3x^2+2x)dx$ ni toping.

A) $3x^3+2x^2+C$ B) $5x^3+2x^2+C$ C) x^3+2x^2+C **D)** x^3+x^2+C

2. $\int 2\cos 2x dx$ ni toping.

A) $\sin 2x+C$ B) $\frac{1}{2}\ln\frac{1}{|\cos x|}+C$ C) $-\frac{1}{2}\ln\frac{1}{|\cos x|}+C$ D) $\frac{3}{2}ctgx+C$

3. Integralni toping $\int_0^5 e^{\frac{x}{5}}dx$

A) $*5(e-1)$ B) $\frac{1}{5}(e-1)$ C) e^5+2 D) $5(e^5-1)$

4. $\int\frac{1}{x-2}dx$ integralni hisoblang.

A) $\ln|x-1|+2\sqrt{x}+C$ B) $2x^2+\ln|x-1|+C$ C) $\ln|x-2|+C$

5. $\int(2x^3+3)dx$ integralni hisoblang.

A) $\frac{x^4}{4}+3x+C$ **B)** $\frac{x^4}{2}+3x+C$

$3x^2+\frac{x^4}{4}+4x+C$ D) $\frac{x^4}{4}+e^x+\frac{6}{x}+C$

$\frac{2}{x^3}dx$ integralni hisoblang.

$-\frac{2}{x^2}+C$ B) $\frac{2}{x^2}+C$ C) $\sqrt{x}-\frac{1}{3x^2}+C$ **D)** $-\frac{1}{x^2}+C$

7. $\int 27x^2dx$ integralni hisoblang.

A) $81x^3+\frac{2}{x}+C$ B) $2x+\frac{27}{x^3}+C$ C) $9x^3+1-C$ **D)** $9x^3+C$

8. $\int\frac{2}{x^2+4}dx$ integralni hisoblang.

A) $\operatorname{arctg}\dfrac{x}{2}+C$ B) $2\operatorname{arctg}\dfrac{x}{2}+C$ C) $\operatorname{arctg}\dfrac{x}{4}+C$ D) $\operatorname{arctg}\dfrac{x}{4}+2x^3+C$

9. $\int\dfrac{2\ln x}{x}dx$ integralni hisoblang.

A) $\dfrac{(\ln x)^2}{2}+C$ B) $2(\ln x)^2+C$ C) $(\ln x)^2+C$ D) $\dfrac{\ln x}{2}+2^x+x^2+C$

10. $\int\dfrac{\cos x}{2\sin x}dx$ integralni hisoblang.

A) $\dfrac{1}{2}\ln|\sin x|+C$ B) $\frac{1}{2}\ln|\cos x|+\sin x-2x+C$

C) $\cos^2 x+\dfrac{1}{2}\ln|\cos x|+C$ D) $\dfrac{\ln x}{2}+2^x+x^2+C$

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MUNDARIJA

O‘QUV ADABIYOTINING
NASHR RUXSATNOMASI

O‘zbekiston Respublikasi Oliy ta’lim, fan va innovatsiyalar vazirligining 20 25 yil “ 21 ” Avgust dagi “316”-sonli buyrug‘iga asosan

M.N.SAMATOVA

(muallifning familiyasi, ismi-sharifi)

Yol harakatini tashkil etish, Transport vositalari muhandisligi

(ta’lim yo‘nalishi (mutaxassisligi))

ning talabalari (o‘quvchilari) uchun tavsiya etilgan

Oliy matematikadan misol va masalalar yechish

(o‘quv adabiyotining nomi va turi: darslik, o‘quv qo‘llanma)

ga O‘zbekiston Respublikasi Vazirlar Mahkamasi tomonidan litsenziya berilgan nashriyotlarda nashr etishga ruxsat berildi.

Vazir K.Sharipov

№ 900500